A Model of the Twin Ds: Optimal Default and Devaluation*

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Abstract

This paper characterizes jointly optimal default and exchange-rate policy. The theoretical environment is a small open economy with downward nominal wage rigidity as in Schmitt-Grohé and Uribe (2013) and limited enforcement of international debt contracts as in Eaton and Gersovitz (1981). It is shown that under optimal policy default is accompanied by large devaluations. At the same time, under fixed exchange rates, optimal default takes place in the context of large involuntary unemployment. Fixed-exchange-rate economies are found to be able to support less external debt than economies with optimally floating rates. In addition, the following three analytical results are presented: (1) Real economies with limited enforcement of international debt contracts in the tradition of Eaton and Gersovitz (1981) can be decentralized using capital controls. (2) Real economies in the tradition of Eaton and Gersovitz can be interpreted as the centralized version of models with downward nominal wage rigidity, optimal capital controls, and a full-employment exchange-rate policy. And (3) Full-employment is optimal in an economy with downward nominal wage rigidity, limited enforcement of debt contracts, and optimal capital controls. *(JEL E43, E52, F31, F34, F38, F41)*

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1 Introduction

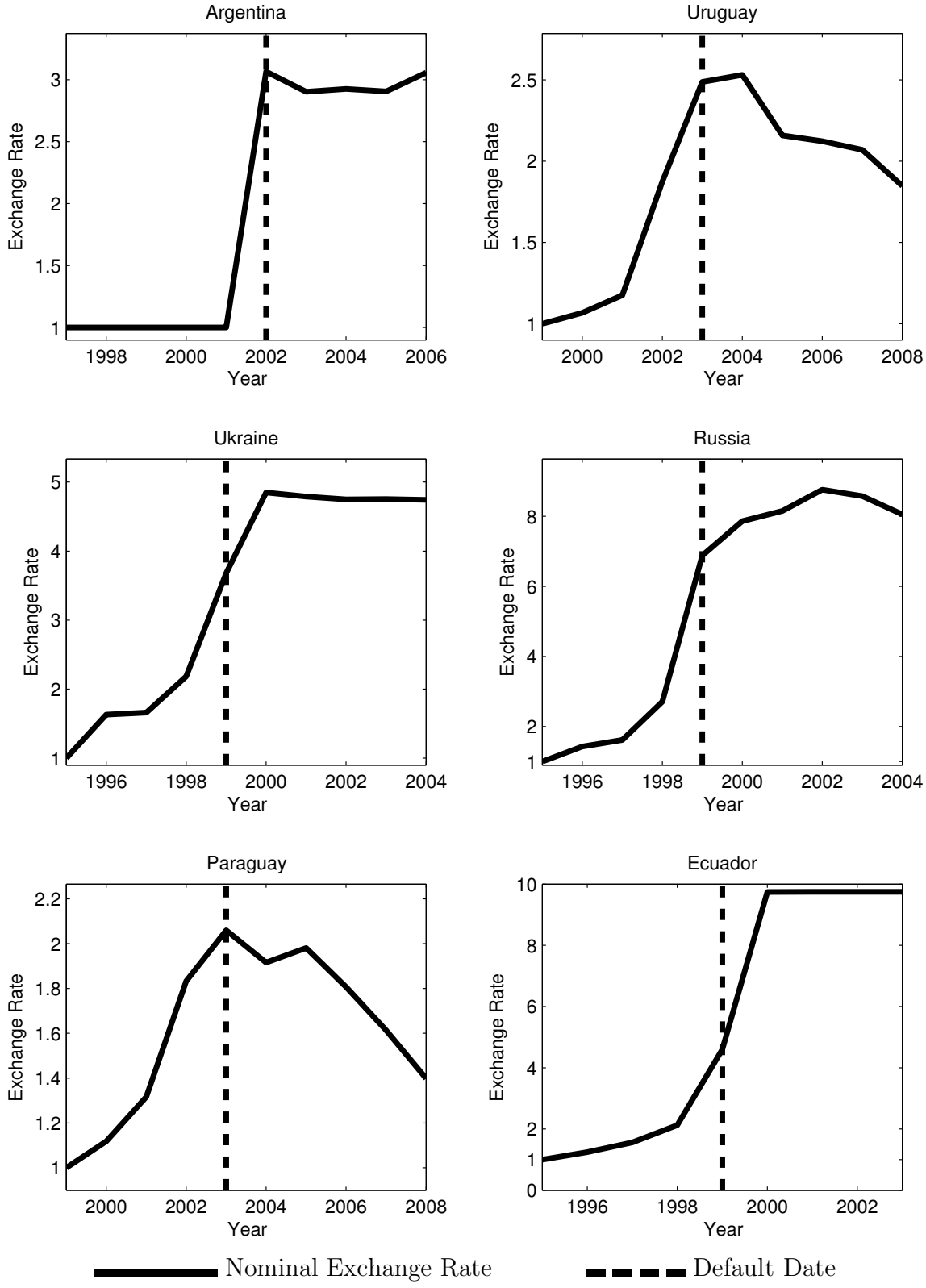
There exists a strong link between sovereign default and devaluation in emerging countries. For example, Reinhart (2002), using data for 58 countries over the period 1970 to 1999, estimates that the unconditional probability of a large devaluation in any 24-month period is 17 percent. At the same time, she estimates that conditional on the 24-month period containing a default event, the probability of a large devaluation increases to 84 percent. Reinhart refers to this phenomenon as the Twin Ds.

Figure 1 provides further evidence of the Twin Ds phenomenon. It displays the behavior of the nominal exchange rate around six well-known recent default episodes that were not included in Reinhart's study. In all cases, sovereign default is accompanied by large devaluations of the domestic currency.

The Twin Ds phenomenon suggests some connection between the decision to default and the decision to devalue. The existing theoretical literature has addressed the questions of optimal default and optimal devaluation extensively, but separately. Thus, existing studies, by design, are mute about the empirical regularity that defaults are typically accompanied by large devaluations. The present paper fills this void by presenting a model in which optimal default policy and optimal devaluation policy are determined jointly.

The theoretical environment embeds imperfect enforcement of debt contracts into a model with downward nominal wage rigidity. The imperfect-enforcement dimension of the model follows the seminal work of Eaton and Gersovitz (1981). The international financial market to which the small open economy has access is assumed to be incomplete and fully dollarized. Specifically, a non-state-contingent bond denominated in foreign currency is traded internationally each period. The country is assumed to lack a commitment mechanism (either moral or legal) to honor its international financial obligations, opening the possibility of sovereign default. Debt contracts are supported by the assumption that lenders have the ability to collectively punish delinquent countries by excluding them from financial markets both on the lending and borrowing sides. In addition, during periods of financial autarky

Figure 1: The Twin Ds: Six Examples



Note. For data sources see the appendix.

the country suffers an exogenous output loss.

Downward nominal wage rigidity is specified as in Schmitt-Grohé and Uribe (2013). The theoretical environment is that of a small-open economy with a traded and a nontraded sector. Nontradable goods are produced using labor and traded output is exogenous and stochastic. The labor market is perfectly competitive but may fail to clear because nominal wage growth is subject to a lower bound constraint. In this framework, negative external shocks cause contractions in aggregate demand, which may result in involuntary unemployment in the absence of appropriate policy intervention.

Unlike most of the related literature on sovereign default, our starting point is a decentralized economy. Individual households can borrow or lend in international financial markets and are subject to capital control taxes. In addition, households and firms interact in competitive factor and product markets in which prices are set in nominal terms. The government chooses optimally the paths of three policy instruments, the nominal exchange rate, the capital control tax, and the decision to default on the country's net foreign debt obligations.

The paper establishes two decentralization results that unfold twice the social planner real setup in which most models of default à la Eaton-Gersovitz are cast. The first unfolding allows households to make optimal consumption and savings decisions but maintains the assumption of a real economy. The second unfolding goes one step further and considers an environment in which all transactions are performed in nominal prices that adjust sluggishly. Specifically, the first unfolding demonstrates that real economies with limited enforcement of international debt contracts in the tradition of Eaton and Gersovitz (1981) can be decentralized using capital controls. This result is of interest because much of the existing literature on sovereign default is cast in terms of a social planner problem and does not discuss how to support the implied allocation as a competitive equilibrium. The issue of decentralization is not trivial because while individual households take credit market conditions (and in particular the interest rate) as given, the social planner internalizes that the cost of credit

depends on the country's external debt and other economic fundamentals. Capital controls, by altering the effective interest rate paid by domestic households, induce individuals to make borrowing decisions that are in line with the social planner's objectives.

The second decentralization result shows that real economies in the tradition of Eaton and Gersovitz can be interpreted as the centralized version of models with downward nominal wage rigidity, optimal capital controls, and optimal exchange-rate policy. This means that the real allocations typically characterized in the related literature on default can be viewed as stemming from more complex economies with nominal rigidities in which the government is continuously implementing the optimal exchange-rate policy.

An immediate payoff of this approach is that it allows for the characterization of the optimal devaluation policy. In particular, it allows one to address the question of whether the model captures the Twin Ds phenomenon as an optimal outcome. Further, the decentralized nominal economy features important macroeconomic indicators that do not appear in its centralized form. One such indicator is the rate of involuntary unemployment. In this regard, we show that the optimal exchange-rate policy brings about full employment at all times. This result suggests a policy-instrument specialization according to which exchange-rate policy is used to bring about full employment and capital control and default policies are used to induce efficient borrowing decisions.

An additional benefit of considering the decentralized version of the Eaton-Gersovitz economy is that it allows for the characterization of the equilibrium allocation when the devaluation policy is not set optimally. One such devaluation policy that is of particular empirical relevance is a currency peg. The reason is that a number of economies undergoing this type of exchange-rate arrangement, notably those in the periphery of the eurozone, have suffered sovereign debt crises with some ending in default.

To study the equilibrium interaction of devaluation and default, the model's equilibrium dynamics are characterized numerically. As in much of the default literature, the model economy is assumed to be driven by exogenous and stochastic variations in the endowment

of tradable goods, which can also be interpreted as terms of trade shocks.

Under the optimal devaluation policy, the typical default episode occurs after a short string of increasingly negative endowment shocks. In the quarters prior to default, the consumption of tradables experiences a severe contraction. The generalized contraction in aggregate demand puts downward pressure on real wages. Absent any intervention by the central bank, downward nominal wage rigidity would prevent real wages from adjusting downwardly and involuntary unemployment would emerge. To avoid this scenario, the optimal policy calls for a large devaluation of the domestic currency of over 50 percent per year, which drastically reduces the real value of wages. Thus, the benevolent government's desire to preserve employment and contain the contraction in aggregate absorption of tradables during a severe external crisis gives rise endogenously to the Twin Ds, the joint occurrence of large devaluations and sovereign default.

The previous analysis makes it clear that in the present model the role of devaluations around default episodes is to avoid misalignments in real wages caused by downward nominal wage rigidity. This channel gives rise to large devaluations that are not followed by a string of subsequent large devaluations. That is, it produces paths for the level of the nominal exchange rate around defaults resembling step functions as in the six recent default episodes documented in figure 1.

Motivated by the recent debt crisis in the periphery of the eurozone, we numerically characterize the model's dynamics around a typical default under a currency peg. Under this exchange-rate regime, the central bank loses its ability to counteract the deleterious consequences of downward nominal wage rigidity during periods of depressed aggregate demand. As a consequence, the model predicts that the contraction around default episodes is accompanied by massive involuntary unemployment, which in the calibrated economy reaches 20 percent of the labor force.

The model predicts that economies whose currencies are pegged can support significantly less external debt than economies in which the exchange rate floats optimally. The reason is

that in fixed-exchange rate economies the incentives to default are stronger than in optimal-exchange rate economies. Under fixed exchange rates, default helps moderate not only the contraction in the domestic absorption of tradable goods (a contribution that is also present in the optimally floating regime), but also the magnitude of involuntary unemployment, a problem that is completely solved by appropriate devaluations under the optimal exchange-rate policy.

The present paper is related to at least three strands of the literature on macroeconomic policy in emerging economies. The real side of the model developed in this paper builds on recent contributions to the theory of sovereign default in the tradition of Eaton and Gersovitz, especially, Arellano (2008), Hatchondo, Martinez, and Sapriza (2010), Chatterjee and Eyigungor (2012), and Mendoza and Yue (2012). This literature has made significant progress in identifying features of the default model that help deliver realistic predictions for the average and cyclical behavior of key variables of the model, such as the level of external debt and the country interest rate premium. As mentioned earlier in this introduction, the nominal side of the present model draws heavily on Schmitt-Grohé and Uribe (2013). That paper analyzes the welfare costs of currency pegs vis-à-vis the optimal devaluation policy in the context of a small open economy with downward nominal wage rigidity. In that model, however, international debt contracts are assumed to be honored at all times. Other papers that study optimal exchange rate policy in the context of small open economies with nominal rigidities and perfect enforcement of debt contracts include Galí and Monacelli (2005) and Kollmann (2002). In contemporaneous work, Moussa (2013) builds a framework similar to the present one to study the role of debt denomination.

Devaluations around default can also be captured by models in which fiscal policy is unsustainable. Kriwoluzky, Müller, and Wolf (2014) study an environment in which default takes the form of a re-denomination of debt from foreign to domestic currency. In their model, debt redenomination cum devaluation lowers the real burden of debt, making fiscal policy sustainable. Finally, as mentioned earlier, the model presented in this paper is not

intended to explain default episodes that are followed by persistent increases in the rate of devaluation, like the ones observed in Latin America after the Mexican debt crisis of 1982. The persistent high levels of inflation that followed these early default episodes are typically attributed to a need to generate seignorage income to finance fiscal deficits. Yun (2014) argues that this phenomenon could also arise under the assumption that sovereign default causes the monetary authority to lose commitment.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 derives the competitive equilibrium. Section 4 characterizes analytically the equilibrium under optimal devaluation, default, and capital control policy. Section 5 analyzes numerically the typical default episode under the optimal policy in the context of a calibrated version of the model. Section 6 characterizes analytically and numerically the equilibrium dynamics under a currency peg. Section 7 concludes.

2 The Model

Here, we present a theoretical framework that embeds imperfect enforcement of international debt contracts à la Eaton and Gersovitz (1981) into the small open economy model with downward nominal wage rigidity of Schmitt-Grohé and Uribe (2013). We begin by describing the economic decision problem of households, firms, and the government interacting in a decentralized economic environment.

2.1 Households

The economy is populated by a large number of identical households with preferences described by the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{1}$$

where c_t denotes consumption. The period utility function U is assumed to be strictly increasing and strictly concave and the parameter β, denoting the subjective discount factor,

resides in the interval $(0, 1)$. The symbol $\mathbb{E}_t$ denotes the mathematical expectations operator conditional upon information available in period t. The consumption good is a composite of tradable consumption, c_t^T, and nontradable consumption, c_t^N. The aggregation technology is of the form

$$c_t = A(c_t^T, c_t^N), \tag{2}$$

where A is an increasing, concave, and linearly homogeneous function.

Motivated by the literature on the 'original sin,' which documents that virtually the totality of emerging-country external debt is denominated in foreign currency (see, for example, Eichengreen, Hausmann, and Panizza, 2005), we assume full liability dollarization. Specifically, households have access to a one-period, state non-contingent bond denominated in tradables. We let d_{t+1} denote the level of debt assumed in period t and due in period $t+1$ and q_t^d its price. The sequential budget constraint of the household is given by

$$P_t^T c_t^T + P_t^N c_t^N + \mathcal{E}_t d_t = P_t^T \tilde{y}_t^T + W_t h_t + (1 - \tau_t^d) q_t^d \mathcal{E}_t d_{t+1} + \mathcal{E}_t f_t + \Phi_t, \tag{3}$$

where P_t^T denotes the nominal price of tradable goods, P_t^N the nominal price of nontradable goods, $\mathcal{E}_t$ the nominal exchange rate defined as the domestic-currency price of one unit of foreign currency, $\tilde{y}_t^T$ the household's endowment of traded goods, W_t the nominal wage rate, h_t hours worked, τ_t^d a tax on debt, f_t a lump-sum transfer received from the government, and Φ_t nominal profits from the ownership of firms. Households are assumed to be subject to the natural debt limit, which prevents them from engaging in Ponzi schemes.

The variable $\tilde{y}_t^T$ is stochastic and is taken as given by the household. Households supply inelastically $\bar{h}$ hours to the labor market each period. Because of the presence of downward nominal wage rigidity, households may not be able to sell all of the hours they supply, which gives rise to the constraint

$$h_t \leq \bar{h}. \tag{4}$$

Households take employment, h_t, as exogenously given.

We assume that the law of one price holds for tradables. Specifically, letting P_t^{T*} denote the foreign currency price of tradables, the law of one price implies that

$$P_t^T = P_t^{T*}\mathcal{E}_t.$$

We further assume that the foreign-currency price of tradables is constant and normalized to unity, $P_t^{T*} = 1$. Thus, we have that the nominal price of tradables equals the nominal exchange rate,

$$P_t^T = \mathcal{E}_t.$$

Households choose contingent plans $\{c_t, c_t^T, c_t^N, d_{t+1}\}$ to maximize (1) subject to (2)-(4) and the natural debt limit, taking as given P_t^T, P_t^N, $\mathcal{E}_t$, W_t, h_t, Φ_t, q_t^d, τ_t^d, f_t, and $\tilde{y}_t^T$. Letting $p_t \equiv P_t^N/P_t^T$ denote the relative price of nontradables in terms of tradables and using the fact that $P_t^T = \mathcal{E}_t$, the optimality conditions associated with this problem are (2)-(4), the natural debt limit, and

$$\frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} = p_t, \tag{5}$$

$$\lambda_t = U'(c_t)A_1(c_t^T, c_t^N),$$

$$(1 - \tau_t^d)q_t^d\lambda_t = \beta\mathbb{E}_t\lambda_{t+1},$$

where λ_t/P_t^T denotes the Lagrange multiplier associated with (3).

2.2 Firms

Nontraded output, denoted y_t^N, is produced by perfectly competitive firms. Each firm operates a production technology of the form

$$y_t^N = F(h_t). \tag{6}$$

The function F is assumed to be strictly increasing and strictly concave. Firms choose the amount of labor input to maximize profits, given by

$$\Phi_t \equiv P_t^N F(h_t) - W_t h_t. \tag{7}$$

The optimality condition associated with this problem is $P_t^N F'(h_t) = W_t$. Dividing both sides by P_t^T yields

$$p_t F'(h_t) = w_t,$$

where $w_t = W_t/P_t^T$ denotes the real wage in terms of tradables.

2.3 Downward Nominal Wage Rigidity

In the present model, the financial friction stemming from the limited enforceability of debt contracts coexists with a nominal friction that takes the form of downward nominal wage rigidity. Specifically, following Schmitt-Grohé and Uribe (2013) we impose that

$$W_t \geq \gamma W_{t-1}, \quad \gamma > 0. \tag{8}$$

The parameter γ governs the degree of downward nominal wage rigidity. The higher is γ, the more downwardly rigid are nominal wages. Uribe and Schmitt-Grohé (2014) present evidence on downward nominal wage rigidity from developed, emerging, and poor countries, and estimate γ to be greater than 0.99 at a quarterly frequency.

The presence of downwardly rigid nominal wages implies that the labor market will in general not clear. Instead, involuntary unemployment, given by $\bar{h} - h_t$, will be a regular feature of this economy. At any point in time, wages and employment must satisfy the slackness condition

$$(\bar{h} - h_t)(W_t - \gamma W_{t-1}) = 0. \tag{9}$$

This condition states that periods of unemployment ($h_t < \bar{h}$) must be accompanied by

a binding wage constraint. It also states that when the wage constraint is not binding ($W_t > \gamma W_{t-1}$), the economy must be in full employment ($h_t = \bar{h}$).

2.4 The Government

At the beginning of each period, the country can be either in good or bad financial standing in international financial markets. Let the variable I_t be an indicator function that takes the value 1 if the country is in good financial standing and 0 otherwise. If the economy starts period t in good financial standing ($I_{t-1} = 1$), the government can choose to default on the country's external debt obligations or to honor them. Default is defined as the full repudiation of external debt. If the government chooses to default, then I_t equals zero and the country enters immediately into bad standing. While in bad standing, the country is excluded from international credit markets, that is, it cannot borrow or lend from the rest of the world. Formally, letting d^e_{t+1} denote the amount of external debt assumed by the country in period t and due in period $t+1$, we have that

$$(1 - I_t)d^e_{t+1} = 0. \tag{10}$$

Following Arellano (2008), we assume that bad financial standing lasts for a random number of periods. Specifically, if the country is in bad standing in period t, it will remain in bad standing in period $t+1$ with probability $1-\theta$, and will regain good standing with probability θ. When the country regains access to financial markets, it starts with zero external obligations.

We assume that the government rebates the proceeds from the debt tax in a lump-sum fashion to households. In periods in which the country is in bad standing ($I_t = 0$), the government confiscates any payments of households to foreign lenders and returns the proceeds to households in a lump-sum fashion. The resulting sequential budget constraint

of the government is

$$f_t = \tau_t^d q_t^d d_{t+1} + (1 - I_t) d_t^e. \tag{11}$$

2.5 Foreign Lenders

Foreign lenders are assumed to be risk neutral. Let q_t denote the price of debt charged by foreign lenders to domestic borrowers during periods of good financial standing, and let r^* be a parameter denoting the foreign lenders' opportunity cost of funds. Then, q_t must satisfy the condition that the expected return of lending to the domestic country equal the opportunity cost of funds. Formally,

$$q_t = \frac{\text{1-Prob}\{I_{t+1} = 0 | I_t = 1\}}{1 + r^*}. \tag{12}$$

This expression can be equivalently written as

$$I_t \left[q_t - \frac{\mathbb{E}_t I_{t+1}}{1 + r^*} \right] = 0.$$

3 Competitive Equilibrium

In this section, we define the competitive equilibrium. Because all domestic households are identical, there is no borrowing or lending among them. This means that in equilibrium the household's asset position equals the country's net foreign asset position, that is,

$$d_t = d_t^e. \tag{13}$$

This expression implies that the debt tax, τ_t^d, can be interpreted as a capital control tax. Because when the country is in bad standing external debt is nil, the value of τ_t^d in bad standing periods is immaterial. Without loss of generality, we set $\tau_t^d = 0$ when $I_t = 0$, that

is,

$$(1 - I_t)\tau_t^d = 0. \tag{14}$$

In equilibrium, the market for nontraded goods must clear at all times. That is, the condition

$$c_t^N = y_t^N \tag{15}$$

must hold for all t.

We assume that each period the economy receives an exogenous and stochastic endowment equal to y_t^T per household. This is the sole source of aggregate fluctuations in the present model. Movements in y_t^T can be interpreted either as shocks to the physical availability of tradable goods or as shocks to the country's terms of trade. As in much of the literature on sovereign default, we assume that if the country is in bad financial standing, it suffers an output loss, which we denote by $L(y_t^T)$. The function $L(\cdot)$ is assumed to be nonnegative and nondecreasing. Thus the endowment received by the household, $\tilde{y}_t^T$, is given by

$$\tilde{y}_t^T = \begin{cases} y_t^T & \text{if } I_t = 1 \\ y_t^T - L(y_t^T) & \text{otherwise} \end{cases}. \tag{16}$$

As explained in much of the related literature, the introduction of an output loss during financial autarky improves the model's predictions along two dimensions. First, it allows the model to support more debt, as it raises the cost of default. Second, it discourages default in periods of relatively high output.

We assume that $\ln y_t^T$ obeys the law of motion

$$\ln y_t^T = \rho \ln y_{t-1}^T + \mu_t, \tag{17}$$

where μ_t is an i.i.d. innovation with mean 0 and variance σ_μ^2, and $|\rho| \in [0, 1)$ is a parameter.

In any period t in which the country is in good financial standing, the domestic price of

debt, q_t^d, must equal the price of debt offered by foreign lenders, q_t, that is

$$I_t(q_t^d - q_t) = 0. \tag{18}$$

Combining (3), (6), (7), (10), (11), (13), (15), (16), and (18) yields the following market-clearing condition for traded goods:

$$c_t^T = y_t^T - (1 - I_t)L(y_t^T) + I_t[q_t d_{t+1} - d_t].$$

Finally, let

$$\epsilon_t \equiv \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}$$

denote the gross devaluation rate of the domestic currency. We are now ready to define a competitive equilibrium.

Definition 1 (Competitive Equilibrium) *A competitive equilibrium is a set of stochastic processes* $\{c_t^T,\ h_t,\ w_t,\ d_{t+1},\ \lambda_t,\ q_t,\ q_t^d\}$ *satisfying*

$$c_t^T = y_t^T - (1 - I_t)L(y_t^T) + I_t[q_t d_{t+1} - d_t], \tag{19}$$

$$(1 - I_t)d_{t+1} = 0, \tag{20}$$

$$\lambda_t = U'(A(c_t^T, F(h_t)))A_1(c_t^T, F(h_t)), \tag{21}$$

$$(1 - \tau_t^d)q_t^d \lambda_t = \beta \mathbb{E}_t \lambda_{t+1}, \tag{22}$$

$$I_t(q_t^d - q_t) = 0, \tag{23}$$

$$\frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} = \frac{w_t}{F'(h_t)}, \tag{24}$$

$$w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t}, \tag{25}$$

$$h_t \leq \bar{h}, \tag{26}$$

$$(h_t - \bar{h}) \left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) = 0, \tag{27}$$

$$I_t \left[q_t - \frac{\mathbb{E}_t I_{t+1}}{1 + r^*} \right] = 0. \tag{28}$$

given processes $\{y_t^T, \epsilon_t, \tau_t^d, I_t\}$ *and initial conditions* w_{-1} *and* d_0.

4 Equilibrium Under Optimal Policy

This section characterizes the optimal default, devaluation, and capital-control policies. When the government can choose freely ϵ_t and τ_t^d, the competitive equilibrium can be written in a more compact form, as stated in the following proposition.

Proposition 1 (Competitive Equilibrium When ϵ_t and τ_t^d Are Unrestricted) *When the government can choose* ϵ_t *and* τ_t^d *freely, stochastic processes* $\{c_t^T, h_t, d_{t+1}, q_t\}$ *can be supported as a competitive equilibrium if and only if they satisfy (19), (20), (26), and (28), given processes* $\{y_t^T, I_t\}$ *and the initial condition* d_0.

The only nontrivial step in establishing this proposition is to show that if processes $\{c_t^T, h_t, d_{t+1}, q_t\}$ satisfy conditions (19), (20), (26), and (28), then they also satisfy the remaining conditions defining a competitive equilibrium, namely, conditions (21)-(25) and (27). To see this, pick λ_t to satisfy (21). When I_t equals 1, set q_t^d to satisfy (23) and set τ_t^d to satisfy (22). When I_t equals 0, set $\tau_t^d = 0$ (recall convention (14)) and set q_t^d to satisfy (22). Set w_t to satisfy (24). Set ϵ_t to satisfy (25) with equality. This implies that the slackness condition (27) is also satisfied. This establishes proposition 1.

The government is assumed to be benevolent. It chooses a default policy I_t to maximize the welfare of the representative household subject to the constraint that the resulting allocation can be supported as a competitive equilibrium. The Eaton-Gersovitz model imposes an additional restriction on the default policy. Namely, that the government has no commitment to honor past promises regarding debt payments or defaults. Further, the Eaton-Gersovitz default model assumes that the default decision in period t is an invariant function of the

aggregate state of the economy in period t. The states appearing in the conditions of the competitive equilibrium listed in proposition 1 are the endowment, y_t^T, and the stock of net external debt, d_t. Thus, we impose that the default decision is a time invariant function of these two variables. We can then define the optimal-policy problem as follows.

Definition 2 (Equilibrium under Optimal Policy) *An equilibrium under optimal policy is a set of processes* $\{c_t^T,\ h_t,\ d_{t+1},\ q_t,\ I_t\}$ *that maximizes*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, F(h_t))) \tag{29}$$

subject to

$$c_t^T = y_t^T - (1 - I_t)L(y_t^T) + I_t[q_t d_{t+1} - d_t], \tag{19}$$

$$(1 - I_t)d_{t+1} = 0, \tag{20}$$

$$h_t \leq \bar{h}, \tag{26}$$

$$I_t \left[q_t - \frac{\mathbb{E}_t I_{t+1}}{1 + r^*} \right] = 0 \tag{28}$$

and to the constraint that if $I_{t-1} = 1$*, then* I_t *is an invariant function of* y_t^T *and* d_t *and if* $I_{t-1} = 0$*, then* $I_t = 0$ *except when reentry to credit markets occurs exogenously, and the natural debt limit, given the initial conditions* d_0 *and* I_{-1}*.*

This problem is time consistent because none of the constraints contains a conditional expectation of a future nonpredetermined endogenous variable. To see that this is true for constraint (28), notice that by the restrictions imposed on the default decision, I_{t+1} depends only upon y_{t+1}^T and d_{t+1}, and that d_{t+1} is chosen in period t.

A further implication of the restrictions imposed on the default decision I_t and of the assumption that traded output follows an autoregressive process of order one is that, by equation (28), the price of debt depends only upon y_t^T, and d_{t+1}, hence we can write equa-

tion (28) as

$$I_t \left[q_t - q(y_t^T, d_{t+1}) \right] = 0. \tag{30}$$

4.1 Optimality of Full Employment

We are ready to characterize the equilibrium process of employment under optimal policy. Consider the optimal policy problem stated in definition 2. Notice that h_t enters only in the in the objective function (29) and the constraint (26). Clearly, because U, A, and F are all strictly increasing, the solution to the optimal policy problem must feature full employment at all times, $h_t = \bar{h}$. We highlight this result in the following proposition:

Proposition 2 (Optimality of Full Employment) *The optimal-policy equilibrium features full employment at all times (i.e., $h_t = \bar{h}$ for all t).*

4.2 The Optimal-Policy Equilibrium As A Decentralization Of The Eaton-Gersovitz Model

We now show that the optimal-policy problem evaluated at $h_t = \bar{h}$ is identical to the Eaton-Gersovitz model as presented in Arellano (2008). To see this, we express the optimal policy problem in recursive form as follows. If the country is in good financial standing in period t, $I_{t-1} = 1$, the value of continuing to service the external debt, denoted $v^c(y_t^T, d_t)$, i.e., the value of setting $I_t = 1$, is given by

$$v^c(y_t^T, d_t) = \max_{\{c_t^T, d_{t+1}\}} \left\{ U\left(A\left(c_t^T, F(\bar{h}) \right) \right) + \beta \mathbb{E}_t v^g(y_{t+1}^T, d_{t+1}) \right\} \tag{31}$$

subject to

$$c_t^T + d_t = y_t^T + q(y_t^T, d_{t+1}) d_{t+1}, \tag{32}$$

where $v^g(y_t^T, d_t)$ denotes the value of being in good financial standing.

The value of being in bad financial standing in period t, denoted $v^b(y_t^T)$, is given by

$$v^b(y_t^T) = \left\{U\left(A\left(y_t^T - L(y_t^T), F(\bar{h})\right)\right) + \beta \mathbb{E}_t\left[\theta v^g(y_{t+1}^T, 0) + (1-\theta)v^b(y_{t+1}^T)\right]\right\}. \tag{33}$$

In any period t in which the economy is in good financial standing, it has the option to either continue to service the debt obligations or to default. It follows that the value of being in good standing in period t is given by

$$v^g(y_t^T, d_t) = \max\left\{v^c(y_t^T, d_t), v^b(y_t^T)\right\}. \tag{34}$$

The government chooses to default whenever the value of continuing to participate in financial markets is smaller than the value of being in bad financial standing, $v^c(y_t^T, d_t) < v^b(y_t^T)$. Let $D(d_t)$ be the default set defined as the set of tradable-output levels at which the government defaults on a level of debt d_t. Formally,[1]

$$D(d_t) = \left\{y_t^T : v^c(y_t^T, d_t) < v^b(y_t^T)\right\}. \tag{35}$$

We can then write the probability of default in period $t+1$, given good financial standing in period t, as

$$\text{Prob}\{I_{t+1} = 0 | I_t = 1\} = \text{Prob}\left\{y_{t+1}^T \in D(d_{t+1})\right\}.$$

Combining this expression with (12) and (30) yields

$$q(y_t^T, d_{t+1} = \frac{\text{1-Prob}\left\{y_{t+1}^T \in D(d_{t+1}) | y_t^T\right\}}{1+r^*}, \tag{36}$$

[1] A well-known property of the default set is that if $d < d'$, then $D(d) \subseteq D(d')$. To see this, note that the value of default, $v^b(y_t^T)$, is independent of the level of debt, d_t. At the same time, the continuation value, $v^c(y_t^T, d_t)$ is decreasing in d_t. To see this, consider two values of d_t, namely d and $d' > d$. Suppose that d^* and c^{T*} are the optimal choices of d_{t+1} and c_t^T when $d_t = d'$, given y_t^T. Notice that given d^*, y_t^T, and $d_t = d$, constraint (32) is satisfied for a value of c_t^T strictly greater than c^{T*}, implying that $v^c(y_t^T, d_t) > v^c(y_t^T, d')$ for $d < d'$. This means that, for a given value of y_t^T, if it is optimal to default when $d_t = d$, then it must also be optimal to default when $d_t = d' > d$.

Equations (31)-(46) are those of the Eaton-Gersovitz model as presented in Arellano (2008).

We have therefore demonstrated the equivalence between the optimal-policy problem stated in definition 2 and the Arellano (2008) model. We highlight this result in the following proposition:

Proposition 3 (Decentralization) *Real models of sovereign default in the tradition of Eaton and Gersovitz (1981) can be interpreted as the centralized version of economies with default risk, downward nominal wage rigidity, optimal capital controls, and optimal devaluation policy.*

Proposition 3 establishes that the allocation under optimal policy is isomorphic to the equilibrium of real models with limited enforcement in the tradition of Eaton and Gersovitz (1981) (such as Arellano, 2008). Unlike this family of models, however, the present model delivers precise predictions regarding the behavior of the nominal devaluation rate. In particular, the present formulation allows us to answer the question of why defaults are often accompanied by nominal devaluations, the Twin Ds phenomenon documented by Reinhart (2002). We will address this issue in more detail in the context of a quantitative version of the present model in section 5.

In the decentralization result of proposition 3, , capital controls play two roles. First, they allow for the internalization of the debt elasticity of the country premium. Second, together with the devaluation rate, capital controls play a key role in making full employment the optimal outcome. To see this, assume that capital controls are not part of the set of policy instruments available to the government. Suppose then that the process $\{\tau_t^d\}$ is exogenous and arbitrary. In this case, one must expand the set of constraints of the optimal-policy problem stated in definition 2 to include competitive-equilibrium conditions (21)-(23). This is because τ_t^d can no longer be set residually to ensure the satisfaction of these constraints. But clearly, there are no longer guarantees that the solution to the expanded optimal-policy problem will feature $h_t = \bar{h}$ for all t. It follows that when the government cannot set capital control taxes optimally, full employment is in general not optimal. No-

tice that even if the government cannot set capital controls optimally, it could achieve full employment at all times by appropriate use of the devaluation rate. But the resulting allocation would in general be suboptimal.[2] However, in the special case in which the function $U(A(c_t^T, c_t^N))$ is additively separable, which occurs when the intra- and intertemporal elasticities of consumption substitution equal each other, full employment reemerges as optimal. This is because when preferences are separable in tradable and nontradable consumption, competitive-equilibrium condition (21) is independent of h_t. This analysis establishes the following result.

Proposition 4 (Nonoptimality of Full Employment Without Capital Controls) *If capital controls are not available to the policy planner, full employment is in general not optimal. If $U(A(c_t^T, c_t^N))$ is separable in c_t^T and c_t^N, full employment is optimal even if capital controls are not available to the policy planner.*

4.3 Decentralization From Real To Real

In the previous section, we discussed the decentralization of the Eaton-Gersovitz model to a competitive economy with downward nominal rigidity. We established that capital controls and devaluation policy make the decentralization possible. Consider now the question of decentralizing the standard Eaton-Gersovitz model to a real competitive economy. To make the competitive economy real, suppose that nominal wages are fully flexible ($\gamma = 0$). In this case, the devaluation rate, ϵ_t, disappears from the set of competitive equilibrium conditions. Specifically, ϵ_t drops from conditions (25) and (27). The economy thus becomes purely real, and exchange-rate policy becomes irrelevant. However, clearly capital controls are still necessary to establish the equivalence between the optimal-policy problem and the standard default model, as they guarantee the satisfaction of the private-sector Euler equation (22). We therefore have the following result.

[2]This result is reminiscent of the one obtained by Ottonello (2014) in the context of a model with downward nominal wage rigidity à la Schmitt-Grohé and Uribe (2013) and collateral constraints à la Bianchi (2011).

Proposition 5 (Decentralization To A Real Economy) *Real models of sovereign default in the tradition of Eaton and Gersovitz (1981) can be decentralized to a real competitive economy via capital controls.*

This result is of interest because it highlights the fact that capital controls are present in all default models à la Eaton and Gersovitz even though they do not explicitly appear in the centralized analysis.

The need for capital controls in the decentralization of Eaton-Gersovitz-style models arises from the fact that the government internalizes the effect of aggregate external debt on the country premium, whereas individual agents take the country premium as exogenously given. Kim and Zhang (2012) also consider the case of decentralized borrowing and centralized default. However, we characterize the capital control scheme that results in an equilibrium allocation identical to that of a model with centralized borrowing and centralized default (the standard Eaton-Gersovitz-style setup). Specifically, both in the present setting and in Kim's and Zhang's borrowers do not internalize the fact that the interest rate depends on debt. However, in the present formulation households face capital control taxes that make them internalize the effect of borrowing on the country interest rate. By contrast, in the formulation of Kim and Zhang, capital control taxes are absent and hence the allocation under decentralized borrowing is different from the one under centralized borrowing.

4.4 The Optimal Devaluation Policy

We now wish to characterize the behavior of the devaluation rate in the optimal-policy equilibrium. In the context of a version of the present model with perfect enforcement of international debt contracts, Schmitt-Grohé and Uribe (2013) show that there exists a whole family of optimal devaluation policies given by

$$\epsilon_t \geq \gamma \frac{w_{t-1}}{w^f(c_t^T)}, \tag{37}$$

where $w^f(c_t^T)$ denotes the full-employment real wage, defined as

$$w^f(c_t^T) \equiv \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h}).$$

Given the assumed properties of the aggregator function A, the full-employment real wage, $w^f(c_t^T)$, is strictly increasing in the absorption of tradable goods. Here, we show that this family of devaluation policies is also optimal in the present environment with imperfect enforcement of debt contracts. To see this, notice that because in the optimal-policy equilibrium $h_t = \bar{h}$ for all t, competitive-equilibrium condition (24) implies that $w_t = w^f(c_t^T)$, for all $t \geq 0$. Combining this expression with (25) yields (37). We summarize this result in the following proposition.

Proposition 6 (The Optimal Devaluation Policy) *The optimal devaluation policy satisfies*

$$\epsilon_t \geq \gamma \frac{w^f(c_{t-1}^T)}{w^f(c_t^T)},$$

for all $t > 0$.

According to this proposition, the government must devalue in periods in which consumption of tradables experiences a sufficiently large contraction. To the extent that contractions of this type coincide with default episodes, the current model will predict that devaluations and default happen together. The next section explores this possibility quantitatively.

5 The Twin Ds

Proposition 6 establishes the existence of an entire family of devaluation policies that are consistent with the optimal allocation. From this family, we select the one that stabilizes nominal wages, which are the source of nominal rigidity in the present model. Specifically,

we assume a devaluation rule of the form

$$\epsilon_t = \frac{w_{t-1}}{w^f(c_t^T)}.$$

For $\gamma < 1$, this policy rule clearly belongs to the family of optimal devaluation policies given in (37). An additional property of this devaluation rule is that it guarantees price stability in the long run.

5.1 Functional Forms, Calibration, And Computation

We calibrate the model to the Argentine economy. The time unit is assumed to be one quarter. Table 1 summarizes the parameterization. We adopt a period utility function of the CRRA type

$$U(c) = \frac{c^{1-\sigma} - 1}{1-\sigma},$$

and set $\sigma = 2$ as in much of the related literature. We assume that the aggregator function takes the CES form

$$A(c^T, c^N) = \left[a(c^T)^{1-\frac{1}{\xi}} + (1-a)(c^N)^{1-\frac{1}{\xi}}\right]^{\frac{1}{1-\frac{1}{\xi}}}.$$

Following Uribe and Schmitt-Grohé (2014), we set $a = 0.26$, and $\xi = 0.5$. We assume that the production technology is of the form

$$y_t^N = h_t^\alpha,$$

and set $\alpha = 0.75$ as in Uribe and Schmitt-Grohé (2014). We normalize the time endowment $\bar{h}$ at unity. Based on the evidence on downward nominal wage rigidity reported in Uribe and Schmitt-Grohé (2014), we set the parameter γ equal to 0.99, which implies that nominal wages can fall up to 4 percent per year. We also follow these authors in measuring tradable

Table 1: Calibration

Parameter	Value	Description
γ	0.99	Degree of downward nominal wage rigidity
σ	2	Inverse of intertemporal elasticity of consumption
y^T	1	Steady-state tradable output
$\bar{h}$	1	Labor endowment
a	0.26	Share of tradables
ξ	0.5	Elasticity of substitution between tradables and nontradables
α	0.75	Labor share in nontraded sector
β	0.85	Quarterly subjective discount factor
r^*	0.01	world interest rate
θ	0.0385	Probability of reentry
δ_1	-0.35	parameter of output loss function
δ_2	0.4403	parameter of output loss function
ρ	0.9317	serial correlation of $\ln y_t^T$
σ_μ	0.037	std. dev. of innovation μ_t
		Discretization of State Space
n_y	200	Number of output grid points (equally spaced in logs)
n_d	200	Number of debt grid points (equally spaced)
n_w	125	Number of wage grid points (equally spaced in logs)
$[\underline{y}^T, \overline{y}^T]$	[0.6523,1.5330]	traded output range
$[\underline{d}, \overline{d}]^{float}$	[0,1.5]	debt range under optimal float
$[\underline{d}, \overline{d}]^{peg}$	[-1,1.25]	debt range under peg
$[\underline{w}, \overline{w}]^{peg}$	[1.25,4.25]	wage range under peg

Note. The time unit is one quarter.

output as the sum of GDP in agriculture, forestry, fishing, mining, and manufacturing in Argentina over the period 1983:Q1 to 2001:Q4. We obtain the cyclical component of this time series by removing a quadratic trend. The OLS estimate of the AR(1) process (17) yields $\rho = 0.9317$ and $\sigma_\mu = 0.037$. Following Chatterjee and Eyigungor (2012), we set $r^* = 0.01$ per quarter and $\theta = 0.0385$. The latter value implies an average exclusion period of about 6.5 years. Following these authors, we assume that the output loss function takes the form

$$L(y_t^T) = \max\left\{0, \delta_1 y_t^T + \delta_2 (y_t^T)^2\right\}.$$

We set $\delta_1 = -0.35$ and $\delta_2 = 0.4403$. We calibrate β, the subjective discount factor, at 0.85. The latter three parameter values imply that under the optimal policy the average debt to traded GDP ratio in periods of good financial standing is 60 percent per quarter, that the frequency of default is 2.6 times per century, and that the average output loss is 7 percent per year conditional on being in financial autarky. The predicted average frequency of default is in line with the Argentine experience since the late 19th century (see Reinhart et al., 2003). The implied average output loss concurs with the estimate reported by Zarazaga (2012) for the Argentine default of 2001. The implied debt-to-traded-output ratio is in line with existing default models in the Eaton-Gersovitz tradition, but below the debt levels observed in Argentina since the 1970s. The assumed value of β is low compared the values used in models without default, but not uncommon in models à la Eaton-Gersovitz (see, for example, Mendoza and Yue, 2012).

We approximate the equilibrium by value function iteration over a discretized state space. We assume 200 grid points for tradable output and 200 points for debt. The transition probability matrix of tradable output is computed using the simulation approach proposed by Schmitt-Grohé and Uribe (2013).

5.2 Equilibrium Dynamics Around A Typical Default Episode

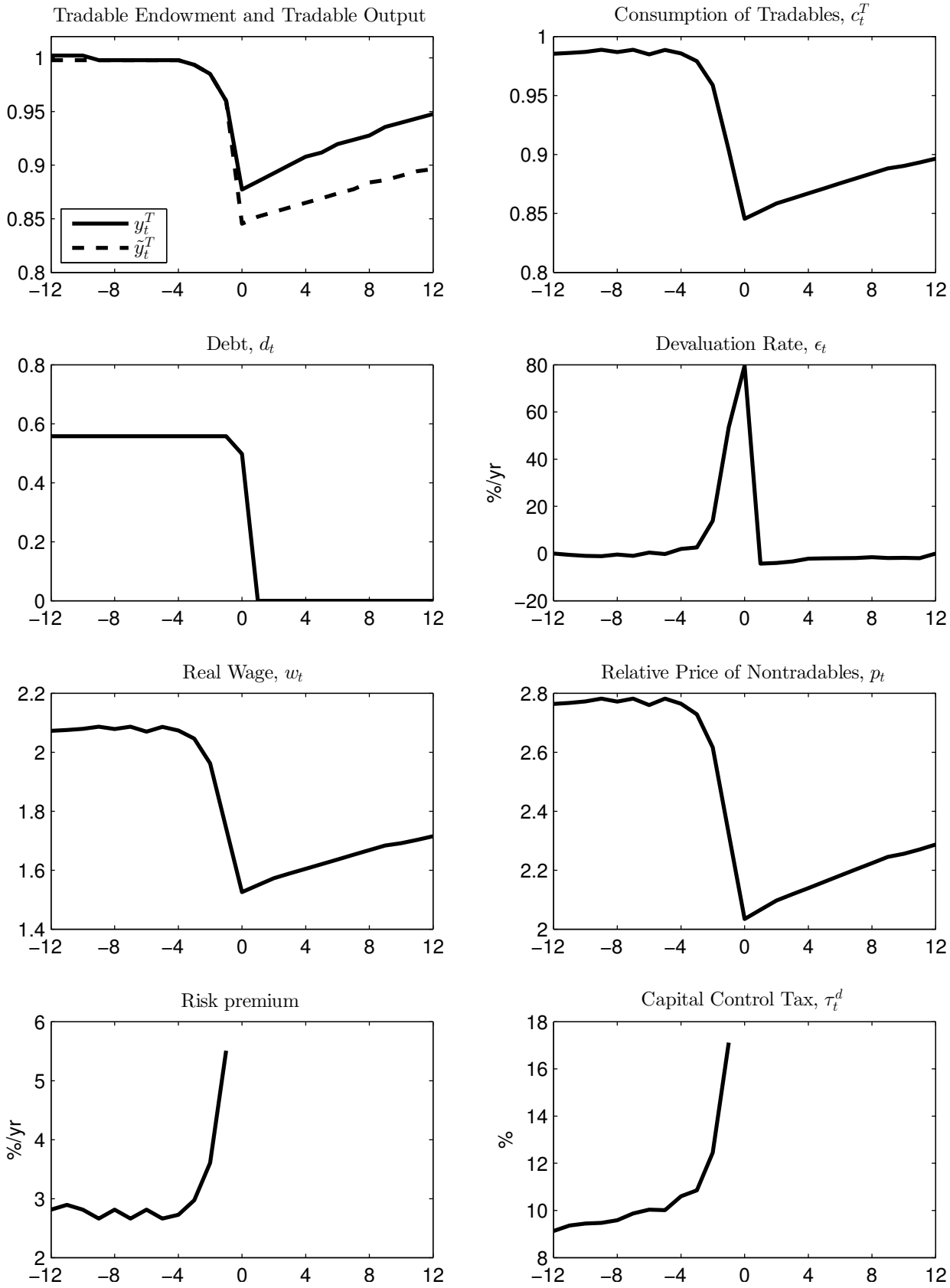
We wish to numerically characterize the behavior of key macroeconomic indicators around a typical default event. To this end, we simulate the calibrated model for 1.1 million quarters and discard the first 0.1 million quarters. We then identify all default episodes. For each default episode we consider a window that begins 12 quarters before the default date and ends 12 quarters after the default date. For each macroeconomic indicator of interest, we compute the median period-by-period across all windows. The date of the default is normalized to 0.

Figure 2 displays the dynamics around a typical default episode. The model predicts that defaults occur after a sudden contraction in tradable output. As shown in the upper left panel, y_t^T is at its mean level of unity until three quarters prior to the default. Then a string of three negative shocks drives y_t^T 1.3 standard deviations below average.[3] At this point (period 0), the government defaults, triggering a loss of output $L(y_t^T)$, as shown by the difference between the solid and the broken lines in the upper left panel. After the default, tradable output begins to recover. Thus, the period of default coincides with the trough of the contraction in tradable endowment, y_t^T. The same is true for GDP measured in terms of tradables. Therefore, the model captures the empirical regularity regarding the cyclical behavior of output around default episodes identified by Levy-Yeyati and Panizza (2011), according to which default marks the end of a contraction and the beginning of a recovery.

As can be seen from the left panel of the second row of the figure, the model predicts that the country does not smooth out the temporary decline in the tradable endowment. Instead, the country sharply adjusts the consumption of tradables downward, by 14 percent. The contraction in consumption is actually larger than the contraction in traded output so that the trade balance improves. In fact, the trade balance surplus is large enough to generate a slight decline in the level of external debt. These dynamics seem at odds with the quintessen-

[3] One may wonder whether a fall in traded output of this magnitude squares with a default frequency of only 2.6 per century. The reason why it does is that it is the sequence of output shocks that matters. The probability of traded output falling from its mean value to 1.3 standard deviations below mean in only three quarters is much lower than the unconditional probability of traded output being 1.3 standard deviations below mean.

Figure 2: A Typical Default Episode Under Optimal Policy



tial dictum of the intertemporal approach to the balance of payments according to which countries should finance temporary declines in income by external borrowing. The country deviates from this prescription because foreign lenders raise the interest rate premium prior to default. This increase in the cost of credit discourages borrowing and induces agents to postpone consumption.

Both the increase in the country premium and the contraction in tradable output in the quarters prior to default cause a negative wealth effect that depresses the desired consumption of nontradables. In turn the contraction in the demand for nontradables puts downward pressure on the price of nontradables. However, firms in the nontraded sector are reluctant to cut prices given the level of wages, for doing so would generate losses. Thus, given the real wage, the decline in the demand for nontradables, translates into a decline in the supply of nontradables and hence unemployment since nontradables are produced using only labor. The excess supply of labor creates downward pressure on nominal wages. However, due to downward nominal wage rigidity, nominal wages fail to decline to a point consistent with clearing of the labor market. To avoid unemployment, the government devalues the currency sharply by about 50 percent (see the right panel on the second row of figure 2). The devaluation lowers real wages in terms of tradables (left panel of row 3 of the figure) which fosters employment. In this way, the government prevents the crisis, which originates in the external sector, from spreading into the nontraded sector.

The prediction of a large devaluation around the default date is in line with the empirical evidence reported by Reinhart (2002) indicating that defaults are typically accompanied by large devaluations. It is in this regard that the model captures what Reinhart refers to as the Twin Ds.

The default crisis is characterized by a sharp real exchange-rate depreciation, as shown by the collapse in the relative price of nontradables (see the right panel on the third row of figure 2). This relative price change conveys a signal to consumers to switch expenditures away from tradables and toward nontradables.

Finally, the bottom right panel of figure 2 shows that the government increases capital controls sharply in the three quarters prior to the default from 9 to 16 percent. It does so as a way to make private agents internalize an increased sensitivity of the interest rate premium with respect to debt. The debt elasticity of the country premium is larger during the crisis because foreign lenders understand that the lower is output the higher the incentive to default, as the output loss, that occurs upon default, $L(y_t^T)$, decreases in absolute and relative terms as y_t^T falls. This capital control tax is implicitly present in every default model à la Eaton-Gersovitz. By analyzing the decentralized version of the model economy, the present analysis makes it explicit.

6 Equilibrium Under Fixed Exchange Rates

In this section, we study how suboptimal exchange-rate policy affects external debt, optimal default decisions, and equilibrium unemployment in the context of the present model. To this end, we consider the polar case of a fixed exchange-rate regime. This monetary arrangement is of particular empirical relevance because fixed exchange rates are often observed in reality, and because sovereign defaults have been observed in the context of fixed exchange rates pre and post default. Prominent examples of this type of phenomenon are countries in the periphery of Europe, such as Greece and Cyprus, in the aftermath of the global contraction of 2008. We are particularly interested in the predictions of the model regarding unemployment around default episodes and in the ability of peggers to support debt in equilibrium vis-a-vis economies with optimally floating rates.

Formally, we now assume that

$$\epsilon_t = 1. \tag{38}$$

Given this policy, we assume that the government sets the default and capital control polices in an optimal fashion.

Definition 3 (Peg-Constrained Optimal Equilibrium) *An optimal-policy equilibrium under a currency peg is a set of processes* $\{c_t^T,\, h_t,\, w_t,\, d_{t+1},\, \lambda_t,\, q_t^d,\, \tau_t^d,\, q_t,\, I_t\}_{t=0}^{\infty}$ *that maximizes*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, F(h_t))) \tag{29}$$

subject to (19)-(24), (26), (28),

$$w_t \geq \gamma w_{t-1}, \tag{39}$$

$$(h_t - \bar{h})\,(w_t - \gamma w_{t-1}) = 0, \tag{40}$$

and to the constraint that if $I_{t-1} = 1$*, then* I_t *is an invariant function of* y_t^T*,* d_t*, and* w_{t-1}*, and if* $I_{t-1} = 0$*, then* $I_t = 0$ *except when reentry to credit markets occurs exogenously, and the natural debt limit, given the initial conditions* d_0*,* w_{-1}*, and* I_{-1}*.*

Note that now the default decision depends not only on y_t^T and d_t, as in the case in which the devaluation rate was a policy instrument available to the government, but also on the past real wage w_{t-1}. This is because, under a currency peg, the competitive equilibrium conditions (i.e., the constraints faced by the policy planner) always include the past wage. Consequently, by equation (28) the price of debt, q_t, depends on the triplet (y_t^T, d_{t+1}, w_t).

Our strategy to characterize the peg-constrained optimal-policy equilibrium is again to consider a less constrained maximization problem and then show that the solution to this problem also satisfies the constraints of the peg-constrained optimal-policy problem listed in definition 3. The less constrained problem consists in dropping conditions (21)-(23) and (40) from the set of constraints in definition 3 and choosing processes $\{c_t^T, h_t, w_t, d_{t+1}, q_t, I_t\}$ to maximize the utility function (29). To see that the solution to this less restrictive problem satisfies the constraints dropped from the definition of the optimal-policy equilibrium, set λ_t to satisfy (21). If $I_t = 1$, the set q_t^d to satisfy (23) and set τ_t^d to satisfy (22). If $I_t = 0$, then, by the convention (14) $\tau_t^d = 0$, and set q_t^d to satisfy (22).

It remains to show that (40) is also satisfied. The proof is by contradiction and adapted

from Schmitt-Grohé and Uribe (2013). Suppose, contrary to what we wish to show, that the solution to the less constrained problem implies $h_t < \bar{h}$ and $w_t > \gamma w_{t-1}$ at some date $t' \geq 0$. Consider now a perturbation to the allocation that solves the less constrained problem consisting in a small increase in hours at time t' from $h_{t'}$ to $\tilde{h}_{t'} \leq \bar{h}$. Clearly, this perturbation does not violate the resource constraint (19), since hours do not enter in this equation. From (24) we have that the real wage falls to $\tilde{w}_{t'} \equiv \frac{A_2(c^T_{t'}, F(\tilde{h}_{t'}))}{A_1(c^T_{t'}, F(\tilde{h}_{t'}))} F'(\tilde{h}_{t'}) < w_{t'}$. Because A_1, A_2, and F' are continuous functions, expression (39) is satisfied provided the increase in hours is sufficiently small. In period $t' + 1$, restriction (39) is satisfied because $\tilde{w}_{t'} < w_{t'}$. We have therefore established that the perturbed allocation satisfies the restrictions of the less constrained problem. Finally, the perturbation is clearly welfare increasing because it raises the consumption of nontradables in period t' without affecting the consumption of tradables in any period or the consumption of nontradables in any period other than t'. It follows that an allocation that does not satisfy the slackness condition (40) cannot be a solution to the less constrained problem. This completes the proof that the allocation that solves the less constrained problem is also feasible in the optimal-policy problem. It follows that the allocation that solves the less constrained problem is indeed the optimal allocation.

We now pose the peg-constrained optimal-policy equilibrium in recursive form. This representation is of great convenience for the quantitative analysis that follows. For a government in good financial standing at the beginning of period t, the value of continuing to service its debt, denoted $v^c(y^T_t, d_t, w_{t-1})$, is given by

$$v^c(y^T_t, d_t, w_{t-1}) = \max_{\{c^T_t, d_{t+1}, h_t, w_t\}} \left\{ U\left(A\left(c^T_t, F(h_t)\right)\right) + \beta \mathbb{E}_t v^g(y^T_{t+1}, d_{t+1}, w_t) \right\} \tag{41}$$

subject to

$$c^T_t + d_t = y^T_t + q(y^T_t, d_{t+1}, w_t) d_{t+1}, \tag{42}$$

$$\frac{A_2(c^T_t, F(h_t))}{A_1(c^T_t, F(h_t))} = \frac{w_t}{F'(h_t)}, \tag{24}$$

$$w_t \geq \gamma w_{t-1}, \tag{39}$$

$$h_t \leq \bar{h}, \tag{26}$$

where $v^g(y_t^T, d_t, w_{t-1})$ denotes the value function associated with entering period t in good financial standing, for an economy with tradable output y_t^T, external debt d_t, and past real wage w_{t-1}.

The value of being in bad financial standing in period t, denoted $v^b(y_t^T, w_{t-1})$, is given by

$$v^b(y_t^T, w_{t-1}) = \max_{\{h_t, w_t\}} \left\{ U\left(A\left(y_t^T - L(y_t^T), F(h_t)\right)\right) + \beta \mathbb{E}_t \left[\theta v^g(y_{t+1}^T, 0, w_t) + (1-\theta) v^b(y_{t+1}^T, w_t)\right] \right\}, \tag{43}$$

subject to

$$\frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} = \frac{w_t}{F'(h_t)}, \tag{24}$$

$$w_t \geq \gamma w_{t-1}, \tag{39}$$

$$h_t \leq \bar{h}. \tag{26}$$

The value of being in good standing in period t is given by

$$v^g(y_t^T, d_t, w_{t-1}) = \max\left\{ v^c(y_t^T, d_t, w_{t-1}), v^b(y_t^T, w_{t-1}) \right\}. \tag{44}$$

Note that now the values of default, continuation, and good standing, $v^b(y_t^T, w_{t-1})$, $v^c(y_t^T, d_t, w_{t-1})$, and $v^g(y_t^T, d_t, w_{t-1})$, respectively, depend on the past real wage, w_{t-1}. This is because under downward nominal wage rigidity and a suboptimal exchange-rate policy, the past real wage, by placing a lower bound on the current real wage, can prevent the labor market from clearing, thereby causing involuntary unemployment and suboptimal consumption of nontradable goods.

Under a currency peg, the default set is defined as

$$D(d_t, w_{t-1}) = \left\{ y_t^T : v^b(y_t^T, w_{t-1}) > v^c(y_t^T, d_t, w_{t-1}) \right\}. \tag{45}$$

The price of debt must satisfy the condition that the expected return of lending to the domestic country equals the opportunity cost of funds. Formally,

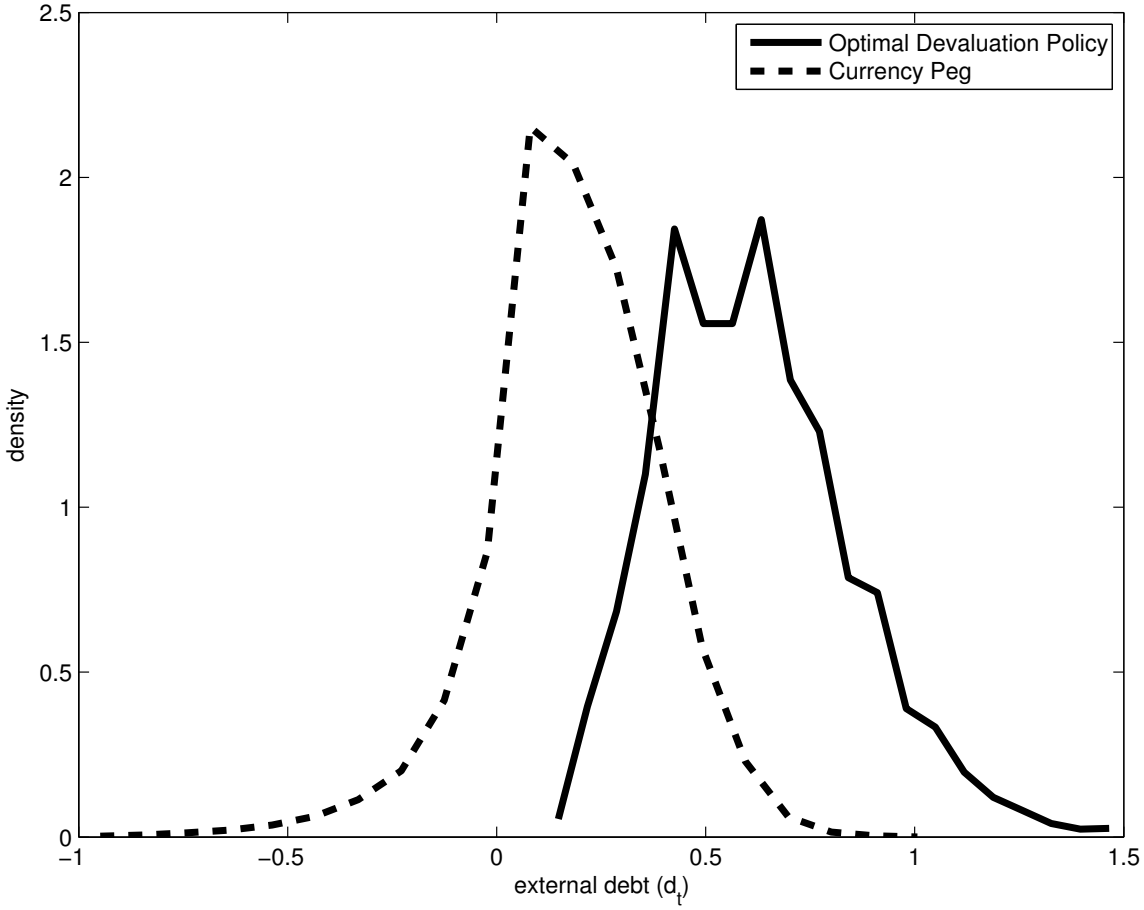
$$q_t = \frac{\text{1-Prob}\left\{y_{t+1}^T \in D(d_{t+1}, w_t)\right\}}{1 + r^*}. \tag{46}$$

Next, we characterize numerically the dynamics implied by the model under a currency peg. The calibration of the model is as shown in table 1. Relative to the case of optimal devaluations, the equilibrium under a currency peg features an additional state variable, namely the past real wage, w_{t-1}. We discretize this state variable with a grid of 125 points, equally spaced in logs, taking values between 1.25 and 4.25. This additional endogenous state variable introduces two computational difficulties. First, it significantly expands the number of points in the discretized state space, from 40 thousand to 5 million. Second, it introduces a simultaneity problem that can be a source of non convergence of the numerical algorithm. The reason is that the price of debt, $q(y_t^T, d_{t+1}, w_t)$, depends on the current wage, w_t. At the same time, the price of debt determines consumption of tradables, which, in turn, affects employment and the wage rate itself. To overcome this source of non-convergence, we develop a procedure to find the exact policy rule for the current wage given the pricing function $q(\cdot, \cdot, \cdot)$ for each possible debt choice d_{t+1}. With this wage policy rule in hand, the debt policy rule is found by value function iteration. This step delivers a new debt pricing function, which is then used in the next iteration.

6.1 Debt Sustainability Under A Currency Peg

Under a currency peg the economy can support significantly less debt than under the optimal devaluation policy. Figure 3 displays with a solid line the distribution of external debt under a currency peg, conditional on the country being in good financial standing. For comparison, the figure also displays the distribution of debt under the optimal devaluation policy. The median debt falls from 0.6 (or also 60 percent of tradable output) under the

Figure 3: Distribution of External Debt



Note. Debt distributions are conditional on being in good financial standing.

optimal devaluation policy to 0.2 (or 20 percent of tradable output) under a currency peg. This reduced debt capacity is a consequence of the fact that, all other things equal, the benefits from defaulting are larger under a currency peg than under optimal devaluation policy. The reason is that under a currency peg, default has two benefits. One is to spur the recovery in the consumption of tradables, since the repudiation of external debt frees up resources otherwise devoted to servicing the external debt. The second, related to the first, is to lessen the unemployment consequences of the external crisis. Recall that in equilibrium c_t^T is a shifter of the demand for labor (see equation 24). The first benefit is also present under optimal devaluation policy. But the second is not, for the optimal devaluation policy, by itself, can bring about the first-best employment outcome.

6.2 Typical Default Episodes With Fixed Exchange Rates

Figure 4 displays with solid lines the model dynamics around typical default episodes. The typical default episode is constructed in the same way as in the case of optimal devaluations. To facilitate comparison, figure 4 reproduces with broken lines the typical default dynamics under the optimal devaluation policy.

As shown in the upper left panel of the figure, under a currency peg, the contraction in the tradable endowment, y_t^T, that precedes default is significantly more protracted than under the optimal devaluation policy. Under the peg, the tradable endowment starts falling 12 quarters prior to default, compared to only 3 quarters under optimal devaluation. In addition, the contraction is deeper under a currency peg (16 percent) than under the optimal devaluation policy (13 percent). The prediction that default is preceded by a persistent slump is consistent with observed defaults that occurred in countries undergoing long currency pegs. For example, the Greek default of 2012 occurred in the context of a contraction that had started in 2008.

As in the case of optimal exchange rate policy, the decline in tradable output is accompanied by a significant contraction in tradable consumption (see the top right panel of the

Figure 4: A Typical Default Episode Under A Currency Peg

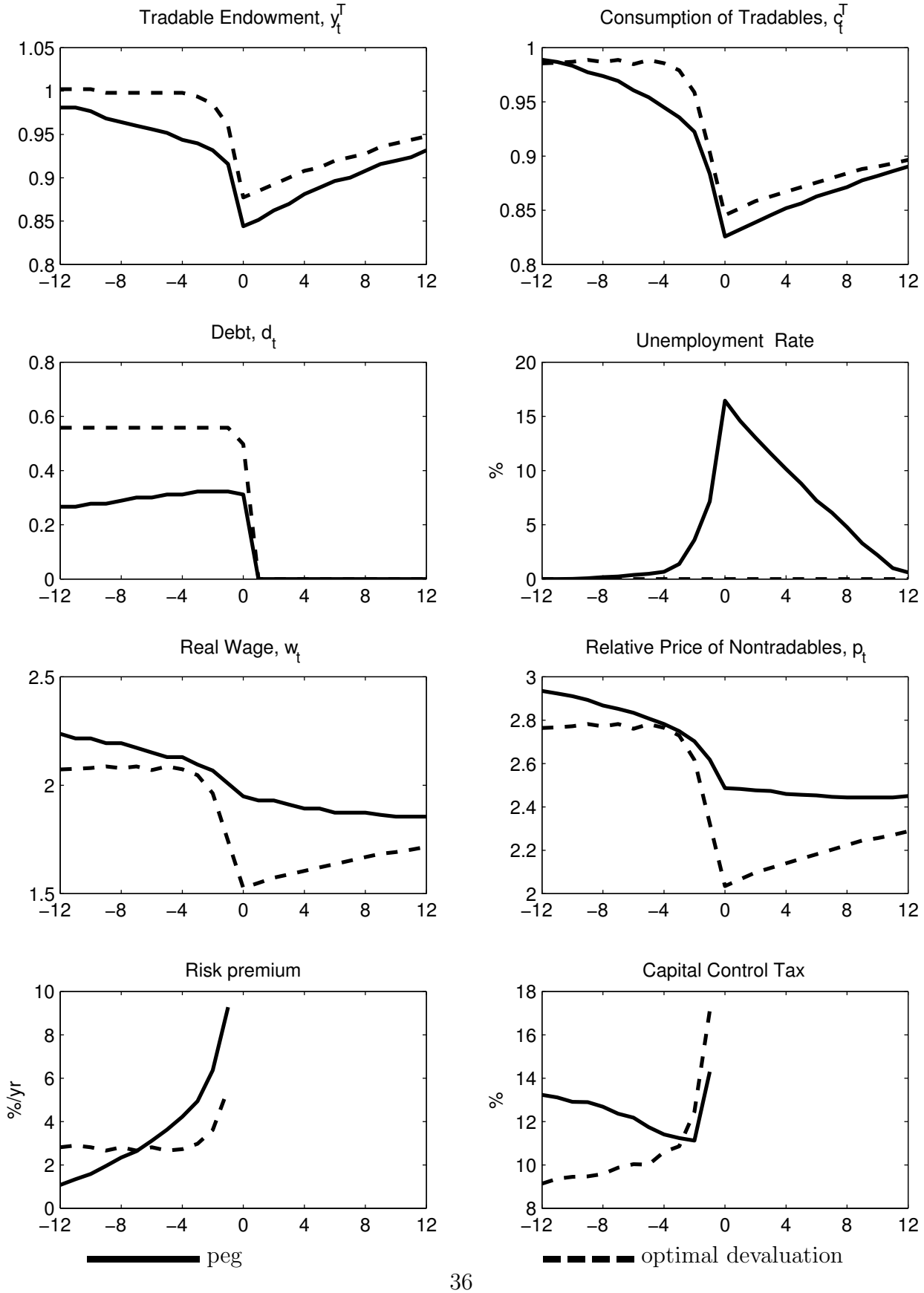


figure). However, unlike the case of optimal devaluation policy, the contraction in aggregate demand leads to massive involuntary unemployment. Starting 8 quarters prior to default, the unemployment rate increases steadily from 0 to 10 percent in the quarter prior to default. This situation reaches great-depression proportions in the quarter of default (period 0), as the rate of involuntary unemployment jumps from 10 to 20 percent. After the default, labor-market conditions gradually improve as domestic absorption recovers. Involuntary unemployment is caused by a failure of real wages to decline in a context of highly depressed aggregate demand (see the left panel of row 3 of figure 4). In turn, the downward rigidity of the real wage is due to the fact that nominal wages are downwardly rigid and that the nominal exchange rate is fixed.

As shown in the left panel of row two of figure 4, the pre-default contraction is characterized by a steady increase in external debt. This prediction stands in contrast to what happens under optimal devaluation policy. The difference is explained by the fact that under a currency peg, the government has a greater incentive to smooth consumption of tradables to contain the consequences of the external crisis on unemployment. It does so by lowering capital control taxes (see the bottom right panel of the figure), which amounts to a reduction in the effective interest rate at which domestic households can borrow. Notice that contrary to what happens under the optimal devaluation policy, under a currency peg capital controls fall in the pre-default period.

Under a currency peg, capital controls are driven by two opposing forces. On the one hand, they are used to make households internalize the fact that the country interest-rate is increasing in the level of debt. This channel is also present under the optimal devaluation policy and induces the government to increase capital control taxes as the external crisis deepens. On the other hand, as mentioned above, the peg-constrained government has an incentive to lower capital control taxes to ameliorate the effects of the contraction in tradable absorption on unemployment. This second channel, first identified in Schmitt-Grohé and Uribe (2013), dominates during the pre-default recession.

Two variables highlight the elevated vulnerability of the peg economy relative to the economy with an optimal float around default episodes: the real exchange rate and the country interest-rate premium. The right panel on the third row of figure 4 displays the behavior of the relative price of nontradables. A fall in this variable means that the real exchange rate depreciates as tradables become more expensive relative to nontradables. Under the optimal policy, the real exchange rate depreciates sharply around the default date, inducing agents to switch expenditure away from tradables and toward nontradables. This redirection of aggregate spending stimulates the demand for labor (since the nontraded sector is labor intensive) and prevents the emergence of involuntary unemployment. Under the currency peg, by contrast, the real exchange rate depreciates insufficiently, inducing a much milder expenditure switch toward nontradables, and thus failing to avoid unemployment. The reason why the relative price of nontradables is reluctant to decline under the peg is that real wages, and hence the labor cost faced by firms, stay too high due to the combination of downward nominal wage rigidity and a currency peg.

The second indicator of macroeconomic fragility is the country premium, shown in the bottom left panel of the figure. Under the peg, the cost of credit increases monotonically over the 12 quarters preceding the default, with the country premium reaching 10 percent in the quarter prior to default. The peak of the country premium is twice as high under the peg as under the optimal devaluation policy. This difference is explained by two factors: first, in the peg economy the typical default occurs for more severe contractions in the traded sector than is the case under the optimal devaluation policy. Second, in the peg economy the steady and significant increase in unemployment makes default more attractive.

7 Conclusion

Much of the existing literature on sovereign default in the Eaton-Gersovitz (1981) tradition is cast in the form of a social planner problem, in which a centralized authority makes default

decisions and determines the consumption of private households and the path of external debt. In this environment private households are modeled as hand-to-mouth consumers who cannot participate in credit markets. The main analytical contributions of this paper are two decentralization results. The first decentralization result is that real models of sovereign default in the spirit of Eaton-Gersovitz (1981) can be viewed as the centralized version of real economies with default risk in which private households do participate in financial markets and are subject to capital control taxes. Capital controls are set to induce households to mimic the social planner's asset and consumption plans. This result makes explicit the presence of a policy instrument that makes the decisions of atomistic households compatible with those of the social planner. This instrument is implicit in all existing Eaton-Gersovitz default models but is not seen because the economy is folded into a social planner's problem. The second decentralization result unfolds the social planner's problem one step further. It shows that real models of sovereign default in the Eaton-Gersovitz (1981) tradition can be viewed as the centralized version of economies with default risk and downward nominal wage rigidity in which the government chooses optimally the default policy, the devaluation policy, and the capital controls policy.

These decentralization results make it possible to characterize the behavior of devaluations and capital controls associated with the optimal default policy. Calibrated versions of the model show that the typical default episode is accompanied by large devaluations. For plausible calibrations, the devaluation rate is as high as 50 percent during default episodes. Hence the Twin Ds phenomenon identified in Reinhart (2002) emerges endogenously as the optimal outcome.

The central role of devaluations around default episodes is to fend off involuntary unemployment. In the presence of downward nominal wage rigidity, devaluations reduce real wages and hence marginal costs of production. In this way, it becomes possible for firms that are faced with weaker demand to lower prices. Because default takes place when aggregate demand is highly depressed, the optimal policy calls for large devaluations.

By contrast, under a currency peg the government is unable to reduce the real value of wages by devaluing the domestic currency. Hence, involuntary unemployment emerged in periods of low aggregate demand. As default episodes typically occur is periods of exceptionally depressed aggregate demand, they are accompanied by massive unemployment.

The presence of unemployment in the fixed-exchange-rate economy strengthens the incentives to default, because the repudiation of debt frees up resources that contribute to economic recovery. As a result, the peg economy faces larger risk premia and can support less external debt than the optimal exchange-rate economy.

Data Appendix

This appendix describes the data sources used to construct figure 1. The average annual nominal exchange rate is taken from World Development Indicators. The exchange rate is expressed in units of local currency per US$, [code: `PA.NUS.FCRF`], and normalized so that the first observation is unity. The default dates are taken from Moody's (2011) and Standard & Poor's (2006).

Table 2: Dates of Default

Country	Default Episode
Argentina	2002
Uruguay	2003
Ukraine	1999
Russia	1999
Paraguay	2003
Ecuador	1999

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