Investor Sophistication and Capital Income Inequality*

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Abstract

What contributes to the growing income inequality across U.S. households? We develop an information-based general equilibrium model that links capital income derived from financial assets to a level of investor sophistication. Our model implies income inequality between sophisticated and unsophisticated investors that is growing in investors' aggregate and relative sophistication in the market. We show that our model is quantitatively consistent with the data from the U.S. market. In addition, we provide supporting evidence for our mechanism using a unique set of cross-sectional and time-series predictions on asset ownership and stock turnover.

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The rise in wealth and income inequality in the United States and worldwide has been one of the most hotly discussed topics over the last few decades in policy and academic circles.[1] An important component of total income is capital income generated in financial markets. In the United States, capital income is by far the most polarized part of household income, and its polarization exhibits a strong upward time trend.[2] A significant step towards understanding these patterns in the data is the vast literature in economics and finance[3] that extensively analyzes household behavior in financial markets and especially its impact on financial returns. Some of the robust general trends in the behavior are growing non-participation in high-return investments and a decline in trading activity. Anecdotal evidence suggests that an ever present and growing disparity in investor sophistication or access to superior investment technologies are partly responsible for these trends. An early articulation of this argument is Arrow (1987); however, micro-founded general equilibrium treatments of such mechanisms are still missing.

In this paper, we provide a micro-founded mechanism for the return differential and show that in a general equilibrium framework, it can go a long way in explaining the growth in capital income inequality, qualitatively and quantitatively. The main friction in the model is heterogeneity in investor sophistication. Intuitively, if information about financial assets and its processing are costly, individuals with different access to financial resources will differ in terms of their capacity to acquire and process information. Sophisticated investors have access to better information which allows them to earn higher income on the assets they hold. In addition, unsophisticated investors perceive their information disadvantage through asset prices and allocate their investments away from the allocations of informed investors. As a result, sophisticated investors earn higher returns on their wealth, and over time their capital

[1] For a summary of the literature, see Piketty and Saez (2003); Atkinson, Piketty, and Saez (2011). A comprehensive discussion of the topic is also provided in the 2013 Summer issue of the Journal Economic Perspectives.

[2] Using the data from the Survey of Consumer Finances we document that approximately 25% of households actively participate in financial markets. Capital income accounts for approximately 15% of this group's total income, ranging from 40% to less than 1%. Between 1989 and 2010, the ratio of the capital income of the group in the 90th percentile of the wealth distribution relative to that of the median group increased from 27 to 60.

[3] Most recently represented by Calvet, Campbell, and Sodini (2007) and Chien, Cole, and Lustig (2011).

income diverges from that of unsophisticated investors with relatively less information.

This basic intuition resonates well with robust empirical evidence that documents the growing presence of sophisticated, institutional investors in risky asset classes, over the last 20-30 years (Gompers and Metrick (2001)). Specifically, the average institutional equity ownership has more than doubled over the last few decades, and it accounts for more than 60% of the total stock ownership. Our hypothesis also fits well with a puzzling phenomenon of the last two decades that indicates a growing retrenchment of retail investors from trading and stock market ownership in general (Stambaugh (2014)),[4] even though direct transaction costs, if anything, have fallen significantly. We document such avoidance of risky assets both for direct stock ownership and ownership of intermediated products, such as actively managed equity mutual funds. Specifically, we find that direct stock ownership has been falling steadily over the last 30 years, while flows into equity mutual funds coming from less sophisticated, retail investors began to decline and turn negative starting from the early 2000s, implying a drop in cumulative flows by 2012 by an astounding 70% of their 2000 levels.

To formalize the economics of our arguments and to assess their qualitative and quantitative match to the data, we build a noisy rational expectations equilibrium model with endogenous information acquisition and capacity constraints in the spirit of Sims (2003). We generalize this theoretical framework by accounting for meaningful heterogeneity across both assets and investors. Specifically, we consider an economy with many risky assets and one riskless asset. The risky assets differ in terms of volatilities of their fundamental shocks. A fraction of investors are endowed with high capacity for processing information and the remaining fraction have lower, yet positive capacity. Thus, everyone in the economy has the ability to learn about assets payoffs, but to different degrees. Investors have mean-variance preferences with equal risk aversion coefficients and learn about assets payoffs from optimal private signals. Based on their capacity and the observed assets characteristics, investors

[4]We view the Stambaugh (2014) study as complementary to ours. It aims to explain the decreasing profit margins and activeness of active equity mutual funds using exogenously specified decline in individual investors' stock market participation. In contrast, our study endogenizes such decreasing participation as part of the mechanism which explains income inequality.

decide which assets to learn about, how much information to process about these assets, and how much wealth to invest.

In a departure from existing work, both the number of assets that are *actively* traded (i.e. learned about) in the market, and the mass of investors choosing to learn about each asset are determined endogenously. In equilibrium, learning exhibits specialization, preference for volatility and liquidity, and strategic substitutability. The mass of investors choosing to learn about each asset varies with the volatility and liquidity of the asset.

We provide an analytical characterization of the model's three main predictions. First, in the cross-section of investors, sophisticated investors generate higher returns and capital income relative to unsophisticated investors. This divergence in returns and incomes is driven by two forces: (i) sophisticated investors have better information to identify profitable assets, and (ii) unsophisticated investors reduce their exposure to assets held by sophisticated investors because, through the increase in prices, they find these assets less compelling to hold. The latter effect is a direct consequence of general equilibrium forces and would not hold under partial equilibrium.

The second set of analytical predictions investigates the response of our outcome variables to shocks to sophistication, which in our framework are modeled as shocks to information capacity. Specifically, the return and income differentials increase with the *overall growth* in aggregate market sophistication, which can be also understood as general progress in information-processing technologies. This result holds even if we keep the relative sophistication of the two investor types constant. The intuition for this result is that in our world, the more an investor knows, the easier it is for her to learn on the margin. As a result, the effects from our first prediction are additionally strengthened because sophisticated investors already start from a higher level of capacity to process information.

Finally, the third set of analytical results characterizes the growth of income inequality in response to a *relative* increase in sophistication between sophisticated and unsophisticated investors holding the total degree of market sophistication constant.

To test the limits of our theory and provide identification of the proposed mechanism, we develop a set of additional analytical predictions. These additional results play an important

role in that they cut against plausible alternative explanations, such as the model with heterogeneous risk aversion or differences in trading costs.

Specifically, we characterize responses to aggregate and relative sophistication shocks for market values, cross-asset exposure, and trading intensity. We show that sophisticated investors are more likely to invest in and learn about more volatile assets within a set of risky assets. Moreover, the mechanism implies a robust, unique way in which investors expand their risky portfolio holdings as the total capacity in the economy expands. In particular, they keep moving down in the asset volatility dimension. At the same time, unsophisticated investors abandon risky assets and hold safer assets. Similar effects occur in terms of trading intensity. Sophisticated investors frequently trade their assets while unsophisticated investors turn over their risky assets much less. Finally, we show that the symmetric expansion in capacity leads to lower expected market returns.

To evaluate the quantitative fit of our theoretical predictions to the data, we calibrate the model to the U.S. data spanning the period from 1989 to 2012. In our calibration, we fix the parameters based on the first half of our sample period, and treat the second subperiod data moments as a test for the dynamic effect coming from progress in information technology. On the data front, we construct a series of capital income inequality using data from the Survey of Consumer Finances and use institutional ownership data to measure equity ownership and returns of sophisticated and unsophisticated investors. Furthermore, we use data on mutual fund flows from Morningstar to evaluate investors' portfolio choices.

Both the analytical predictions from the model and the quantitative predictions from the parametrization are consistent with empirical evidence. Specifically, we conduct two quantitative exercises. First, in the *Aggregate Portfolios* exercise, we parameterize the model using stock-level micro data and aggregate household portfolios, which allows us to pin down details of the stochastic structure of assets payoffs. We show that sophisticated investors, on average, exhibit higher rates of returns that are approximately 2 percentage points per year higher in the model, compared to a 3 percentage point difference in the data.

Second, in the *Household Portfolios* exercise, we use the ratio of average financial wealth of the 10% wealthiest investors relative to 50% poorest investors in 1989 as a proxy for

initial relative investor sophistication, and posit that the growth in financial wealth implies linear growth in investors' sophistication. We then show that introducing this feedback in our model generates *endogenous* evolution of capacity and capital income that can match very accurately capital income inequality growth in the data: Our model implies the average inequality growth of 98% between 1989 and 2010, whereas the same number in the data equals 90%. Moreover, we can closely match the evolution of the growth rate over the entire sample period. Given the good fit of the model, we conclude that our model can quantify the economic mechanism proposed first by Arrow (1987) in which financial wealth facilitates access to more sophisticated investment techniques, and hence begets even more wealth.

In order to further confirm our economic mechanism, we compute dynamic predictions of our *Aggregate Portfolios* exercise. In particular, we introduce aggregate (not investor-specific) progress in information technology, which increases the average equity ownership rate of sophisticated investors from 23% (the data average for 1989-2000) to 43% (the data average for 2001-2012). In that exercise, we show that sophisticated investors increase their ownership of equities in a specific order, which we also confirm in the data. Specifically, they first enter stocks that are most volatile and subsequently move into stocks with medium and lower volatility. At the same time, we show that sophisticated investors' entry into equity induces higher asset turnover, in magnitudes consistent with the data, both in the time series and in the cross-section of stocks.

Finally, as additional supporting evidence, we report a set of auxiliary predictions that qualitatively correspond to the analytical predictions of the model. We show that unsophisticated investors tend to hold an increasingly larger fraction of their wealth in safer, liquid assets. They also tend to reduce their aggregate equity ownership: In the data, we observe a steady outflow of unsophisticated, retail money from risky assets, such as direct equity and equity mutual funds, while the flows from sophisticated investors into such assets are generally positive. Somewhat surprisingly, these outflows in the data continued until recently despite a large increase in the risky assets valuations.

Our paper spans three strands of literature: (1) the literature on household finance; (2) the literature on rational inattention; and (3) the literature on income inequality. While

some of our contributions are specific to each of the individual streams, our additional value added comes from the fact that we integrate the streams into one unified framework within our research context.

Our results relate to a wide spectrum of research in household finance and portfolio choice. The main ideas we entertain build upon an empirical work on limited capital market participation (Mankiw and Zeldes (1991); Ameriks and Zeldes (2001)), growing institutional ownership (Gompers and Metrick (2001)), household trading decisions (Barber and Odean (2001), Campbell (2006), Calvet, Campbell, and Sodini (2009b, 2009a), Guiso and Sodini (2012)), and investor sophistication (Barber and Odean (2000, 2009), Calvet, Campbell, and Sodini (2007), Grinblatt, Keloharju, and Linnainma (2009)). While the majority of the studies attribute limited participation rates to either differences in stock market participation costs (Gomes and Michaelides (2005)) or preferences, we relate the decisions to differences in sophistication across investors.

Another building block of our paper is the literature on rational inattention and endogenous information capacity that originates with the papers of Sims (1998, 2003, 2006). More germane to our application are models of costly information of Van Nieuwerburgh and Veldkamp (2009, 2010), Mondria (2010), and Kacperczyk, Van Nieuwerburgh, and Veldkamp (2013). While the literature on endogenous information acquisition generally assumes that informed investors have homogenous information capacity or face a homogeneous set of risky assets, we study implications of the model with heterogeneous agents in an environment with many heterogeneous assets. We solve for the endogenous allocation of investor types across assets types, and show that the implications of such a model for portfolio decisions and asset prices are very different than those of the model with homogeneity. In addition, we study the implications of information frictions for income processes of investors and the equilibrium holdings of assets with different characteristics, such as volatility or turnover, all features which are absent in the present literature.

Our last building block constitutes the literature on income inequality that dates back to the seminal work by Kuznets and Jenks (1953) and has been subsequently advanced by the work of Piketty (2003), Piketty and Saez (2003), Alvaredo, Atkinson, Piketty, Saez,

et al. (2013), Autor, Katz, and Kearney (2006), and Atkinson, Piketty, and Saez (2011). In contrast to our paper, a vast majority of that literature explain total income inequality looking at the income earned in labor market (e.g., Acemoglu (1999, 2002); Katz and Autor (1999); Autor, Katz, and Kearney (2006, 2008); and Autor and Dorn (2013)); and they do not consider explanations that relate to informational sophistication of investors.

The closest paper in spirit to ours is Arrow (1987) who also considers information differences as an explanation of income gap. However, his work does not consider heterogeneity across assets or investors and does not attempt a quantitative evaluation of the strength of the forces in general equilibrium. Both these elements are crucial for the results of our paper, and especially to establish the validity of our mechanism. Thanks to having a richer, equilibrium framework, we are able to parameterize the model and show that it comes very close to the data moments. Another work related to ours is Peress (2004) who examines the role that wealth and decreasing absolute risk aversion play in investors' acquisition of information and participation in risky assets. In contrast to that paper, we focus on micro foundations of how investors attain superior rates of return on equity. In addition, we model how different investors allocate their money across disaggregated risky asset classes. This allows us to test our information-based mechanism using micro-level data.

The rest of the paper proceeds as follows. In Section 1, we provide general equilibrium framework to study behavior and income evolution of heterogeneously informed individuals. In Section 2, we derive analytical predictions, which we subsequently take to the data. In Section 3, we establish our main results and provide additional evidence in favor of our proposed mechanism. Section 4 concludes. All the proofs and derivations are in the Appendix.

1 Theoretical Framework

We study portfolio decisions with endogenous information a la Grossman and Stiglitz (1980). We first describe the investment environment. Next, we present investors' portfolio and information choice problems. Finally, we characterize the equilibrium and its properties.

1.1 Model Setup

The financial market consists of one riskless asset, with price normalized to 1 and payoff r, and n risky assets, indexed by i, with prices p_i, and independent payoffs $z_i \sim \mathcal{N}(\overline{z}_i, \sigma_i^2)$. The riskless asset is assumed to be in unlimited supply, and each risky asset is available in (stochastic) supply $x_i \sim \mathcal{N}(\overline{x}_i, \sigma_{xi}^2)$, independent of payoffs and across assets.

Assets are traded by a continuum of atomless investors of mass one, indexed by j, with mean-variance utility over wealth W_j, and risk aversion coefficient $\rho > 0$. Prior to making their portfolio allocations, each investor can choose to obtain information about some or all of the risky assets payoffs. Information is obtained in the form of endogenously designed signals, which are then used to update the beliefs that inform the investor's portfolio allocation. The investor's signal choice is modeled following the rational inattention literature (Sims (2003)), using entropy reduction as a measure of the amount of acquired information. Each investor is modeled as though receiving information through a channel with fixed capacity.

In our modeling, we depart from existing work by assuming that all investors in the economy have the ability to learn about assets payoffs, but to different degrees. Specifically, mass $\lambda \in (0, 1)$ of investors have high capacity for processing information, K_1, and are referred to as *sophisticated* investors, and mass $1 - \lambda$ of investors have low capacity for processing information, K_2, and are referred to as *unsophisticated* investors, with $0 < K_2 < K_1 < \infty$.

Each decision period is split into two subperiods. In the first subperiod, investors solve the information acquisition problem. In the second subperiod, payoffs and assets supplies are realized, investors receive signals in accordance with their information acquisition strategy, and they choose their portfolio allocations.

1.2 Portfolio Decision

We begin by solving each investor's portfolio problem in subperiod 2, for a given information structure. Each investor chooses portfolio holdings to solve

$$\max_{\{q_{ji}\}_{i=1}^{n}} U_{2j} = E_{2j}\left(W_j\right) - \frac{\rho}{2} V_{2j}\left(W_j\right) \tag{1}$$

subject to the budget constraint

$$W_j = r\left(W_{0j} - \sum_{i=1}^{n} q_{ji} p_i\right) + \sum_{i=1}^{n} q_{ji} z_i, \tag{2}$$

where E_{2j} and V_{2j} denote the mean and variance conditional on investor j's information set in subperiod 2, W_{0j} is initial wealth (normalized to zero), and q_{ji} is the quantity invested by investor j in asset i.

The (standard) solution to the portfolio choice problem yields that the quantity invested in each asset i by investor j is given by Sharpe ratio, scaled by the inverse risk aversion coefficient:

$$q_{ji} = \frac{\widehat{\mu}_{ji} - r p_i}{\rho \widehat{\sigma}_{ji}^2}, \tag{3}$$

where $\widehat{\mu}_{ji}$ and $\widehat{\sigma}_{ji}^2$ are the mean and variance of investor j's posterior beliefs about the payoff z_i, conditional on the investor's information. If an investor chooses not to obtain information about a particular asset, then the investor's beliefs—and hence her portfolio holdings—for that asset's payoff are determined by her prior, which coincides with the unconditional distribution of payoffs.

Substituting in q_{ji} gives the following indirect utility function:

$$U_{2j} = \frac{1}{2\rho} \sum_{i=1}^{n} \left[\frac{\left(\widehat{\mu}_{ji} - r p_i\right)^2}{\widehat{\sigma}_{ji}^2} \right]. \tag{4}$$

1.3 Information Choice

In subperiod 1, each investor obtains information about assets payoffs in the form of signals, which are then used to update the beliefs that inform the investor's portfolio allocation. The investor chooses the allocation of information capacity across the different assets—the distribution of the signals—optimally, to maximize her ex-ante expected utility,

$$E_{1j}\left[U_{2j}\right] = \frac{1}{2\rho}\sum_{i=1}^{n}\left\{\left(\frac{1}{\widehat{\sigma}_{ji}^{2}}\right)E_{1j}\left[\left(\widehat{\mu}_{ji} - rp_i\right)^2\right]\right\}, \tag{5}$$

subject to a constraint on the total quantity of information conveyed by the signals,

$$I\left(z; s_j\right) \leq K_j, \tag{6}$$

where $I\left(z; s_j\right)$ denotes Shannon's (1948) mutual information, measuring the information about the vector of asset payoffs z conveyed by the vector of private signals s_j; and $K_j \in \{K_1, K_2\}$ denotes investor j's capacity for processing information.

Using $Var\left(x\right) = E\left[x^2\right] - \left[E\left(x\right)\right]^2$, the objective function in subperiod 1 becomes

$$E_{1j}\left[U_{2j}\right] = \frac{1}{2\rho}\sum_{i=1}^{n}\left[\left(\frac{1}{\widehat{\sigma}_{ji}^{2}}\right)\left(\widehat{V}_{ji} + \widehat{R}_{ji}^{2}\right)\right], \tag{7}$$

where $\widehat{R}_{ji}$ and $\widehat{V}_{ji}$ denote the ex-ante mean and variance of expected excess returns, $(\widehat{\mu}_{ji} - rp_i)$.

Each investor j receives a separate signal s_{ji} on each of the asset payoffs, z_i. For analytical tractability, we make the following assumption about the signal structure:

Assumption 1 *The signals s_{ji} are independent across assets.*[5]

It is important to note that we do not impose that all of these signals are informative. Assumption 1 implies that the total quantity of information obtained by an investor can be expressed as a sum of the quantities of information obtained for each asset:

[5]This assumption is standard in the literature. It is necessary for the analytical tractability of the model. Allowing for potentially correlated signals requires a strictly numerical approach, and is beyond the scope of this paper.

$$\sum_{i=1}^{n} I\left(z_i; s_{ji}\right) \leq K_j, \tag{8}$$

where $I\left(z_i; s_{ji}\right)$ measures the information about the asset payoff z_i conveyed by the private signal s_{ji}.

The information constraint (8) imposes a limit on the amount of entropy reduction that each investor can accomplish through the endogenously designed signal structure. Since perfect information requires infinite capacity, each investor compresses the payoff into a simpler representation, and hence necessarily faces some residual uncertainty about the realized payoffs. For each asset, investor j decomposes the payoff into a lower-entropy signal component, s_{ji}, and a residual component, δ_{ji}, that represents information lost due to the compression of the random variable z_i:

$$z_i = s_{ji} + \delta_{ji}.^{6} \tag{9}$$

For analytical tractability, we introduce the following additional assumption on the signal structure:

Assumption 2 *The signal s_{ji} is independent of the data loss δ_{ji}.*[7]

Since z_i is normally distributed, assumption 2 implies that s_{ji} and δ_{ji} are also normally distributed, by Cramer's Theorem:

$$s_{ji} \sim \mathcal{N}\left(\overline{z}_i, \sigma^2_{sji}\right) \quad \text{and} \quad \delta_{ji} \sim \mathcal{N}\left(0, \sigma^2_{\delta ji}\right),$$

[6]The literature on costly information often assumes an additive noise signal structure, where the signal is equal to the payoff plus noise. That specification has enabled a direct comparison to the literature on exogenous information. However, in the context of limited capacity, investors compress, or simplify the state of the world (in terms of entropy), rather than amplify it with noise. While conceptually closer to the information theoretic benchmark, this formulation of the state as a decomposition into the signal and data loss does not change the results in this particular application. For applications in which such compression is critical to obtaining the correct optimal signal structure, see the work by Matejka (2011), Matějka and Sims (2011), and Stevens (2012).

[7]The decomposition of the shock into independent components is optimal if the agent's signaling problem is to minimize the mean squared error of s_i for each i, (see, for example, Cover and Thomas (2006)). However, in general, the optimal signal structure may require correlation between the signal and the data loss (namely it may result in a higher posterior precision about asset payoffs). In our framework, we assume the independent decomposition to maintain analytical tractability.

with $\sigma_i^2 = \sigma_{sji}^2 + \sigma_{\delta ji}^2$.

Therefore, an investor's posterior beliefs about payoffs given signals are also normally distributed random variables, independent across assets, with mean and variance given by:

$$\widehat{\mu}_{ji} = s_{ji} \quad \text{and} \quad \widehat{\sigma}_{ji}^2 = \sigma_{\delta ji}^2. \tag{10}$$

A perfectly precise signal would be associated with no information loss, such that the variance of the data loss, and hence the investor's posterior uncertainty, would be zero. Conversely, a completely uninformative signal would result in the investor's posterior uncertainty being equal to her prior uncertainty, σ_i^2.

Using the signal structure and the implied ex-ante distribution of expected excess returns, the investor's objective becomes choosing the variance of the data loss, $\sigma_{\delta ji}^2$, for each asset i, to solve the following constrained optimization problem:

$$\max_{\{\sigma_{\delta ji}^2\}_{i=1}^n} \sum_{i=1}^{n} \left(\frac{\widehat{S}_i + \widehat{R}_i^2}{\sigma_{\delta ji}^2} \right), \tag{11}$$

subject to

$$\prod_{i=1}^{n} \left(\frac{\sigma_i^2}{\sigma_{\delta ji}^2} \right) \leq e^{2K_j}, \tag{12}$$

where

$$\widehat{R}_i \equiv \overline{z}_i - r\overline{p}_i \tag{13}$$

is the ex-ante mean of expected excess returns, common across investors, and where

$$\widehat{S}_i \equiv (1 - 2rb_i)\,\sigma_i^2 + r^2\sigma_{pi}^2 \tag{14}$$

is the component of the ex-ante variance of expected excess returns that is common across investors. The distribution of excess returns and the information constraint in equation (12) are derived in the Appendix.

The following proposition presents the solution to the information capacity allocation

problem of each investor.

Proposition 1 *In the solution to the maximization problem (11)-(12), each investor allocates her entire capacity to learning about a single asset. All assets that are actively traded (that is, learned about in equilibrium) belong to the set L of assets with maximal expected utility gains:*

$$L \equiv \left\{ i \,|\, i \in \arg\max_i G_i \right\}, \tag{15}$$

where the expected gain of asset i to the agent's utility is

$$G_i \equiv \frac{\widehat{S}_i + \widehat{R}_i^2}{\sigma_i^2}. \tag{16}$$

The linear objective function and the convex constraint imply a corner solution for the individual allocation of attention: each investor specializes, learning about one asset in the set of assets with maximal gains. Equilibrium conditions will then determine the set of assets L, as well as the mass of investors learning about each of the assets in the set L.

Using Proposition 1, and substituting the optimal capacity allocation in equation (12), we characterize the posterior beliefs of investor j learning about asset $l_j \in L$ by:

$$\widehat{\mu}_{ji} = \begin{cases} s_{ji} & \text{if } i = l_j, \\ \overline{z}_i & \text{if } i \neq l_j, \end{cases} \quad \text{and} \quad \widehat{\sigma}_{ji}^2 = \begin{cases} e^{-2K_j}\sigma_i^2 & \text{if } i = l_j, \\ \sigma_i^2 & \text{if } i \neq l_j. \end{cases} \tag{17}$$

Investors' posterior beliefs about payoffs are equal to their prior beliefs, for assets which they passively trade. On the other hand, for assets about which investors learn, the posterior variance is strictly lower, and it is decreasing in capacity K_j, whereas the posterior mean is equal to the received signal. Conditional on the realized payoff, the signal s_{ji} received by investor j about the actively traded asset i is a normally distributed random variable whose mean is a weighted average of the true realization, z_i, and the prior, $\overline{z}_i$,

$$E\left(s_{ji}|z_i\right) = \left(1 - e^{-2K_j}\right) z_i + e^{-2K_j}\overline{z}_i.$$

The higher is the capacity of an investor, the larger is the weight that the private signal puts on the realized payoff z_i relative to the investor's prior, $\overline{z}_i$.

1.4 Equilibrium

Given the solution to an individual investor's information allocation problem, the market clearing condition for each asset is given by

$$\int_{M_{1i}} \left(\frac{s_{ji} - rp_i}{e^{-2K_1} \rho \sigma_i^2} \right) dj + \int_{M_{2i}} \left(\frac{s_{ji} - rp_i}{e^{-2K_2} \rho \sigma_i^2} \right) dj + (1 - m_i) \left(\frac{\overline{z}_i - rp_i}{\rho \sigma_i^2} \right) = x_i, \tag{18}$$

where m_i denotes the mass of investors learning about asset i, M_{1i} denotes the set of sophisticated investors, of measure $\lambda m_i \geq 0$, who choose to learn about asset i, and M_{2i} denotes the set of unsophisticated investors, of measure $(1 - \lambda) m_i \geq 0$, who choose to learn about asset i.[8]

Following Admati (1985), we conjecture and verify that the equilibrium asset prices are a linear function of the underlying shocks. The following proposition presents the solution for equilibrium prices given the investors' individual allocation of capacity.

Proposition 2 *The price of asset i is given by*

$$p_i = a_i + b_i z_i - c_i x_i, \tag{19}$$

with

$$a_i = \frac{\overline{z}_i}{r\left(1 + \phi m_i\right)}, \quad b_i = \frac{\phi m_i}{r\left(1 + \phi m_i\right)}, \quad c_i = \frac{\rho \sigma_i^2}{r\left(1 + \phi m_i\right)}, \tag{20}$$

where $\phi \equiv \lambda \left(e^{2K_1} - 1\right) + (1 - \lambda) \left(e^{2K_2} - 1\right)$ is a measure of the total capacity for processing information available in the market, and m_i is the mass of investors learning about asset i.

The price of an asset reflects the asset's payoff only if at least some investors dedicate capacity to learning about this asset through private signals, such that $m_i > 0$. Conversely,

[8] Since the payoff factors are the same across all investors, regardless of investor type, the participation of sophisticated and unsophisticated investors in a particular asset will be proportional to their mass in the population.

for assets that are not actively learned about, the price only reflects the noisy supply, x_i.

Using the solution for equilibrium prices, the expected utility gain for each asset becomes

$$G_i = \frac{1 + \rho^2 \xi_i}{(1 + \phi m_i)^2}, \tag{21}$$

where $\xi_i \equiv \sigma_i^2 (\sigma_{xi}^2 + \overline{x}_i^2)$ is a term summarizing the properties of asset i. Equation (21) implies the following comparative statics:

$$\frac{\partial G_i}{\partial \xi_i} > 0, \quad \frac{\partial G_i}{\partial m_i} < 0, \quad \frac{\partial G_i}{\partial \phi} < 0, \quad \frac{\partial^2 G_i}{\partial \xi_i \partial \phi} < 0.$$

Hence, learning in the model exhibits preference for volatility (high ξ_i) and strategic substitutability (low m_i). The utility gain also falls with ϕ, the total amount of information in the market, but the decline is smaller for higher volatility assets.

We next determine which assets are learned about in equilibrium, and how the market allocates ϕ, the total information capacity available, across these assets. In a departure from existing work, we solve endogenously for m_i, the mass of investors learning about each asset. Then, for each asset, the quantity (ϕm_i) represents the amount of capacity that the market allocates to learning about asset i in equilibrium.

Without loss of generality, let assets in the economy be ordered in decreasing order of ξ_i. That is, for all $i \in \{1, ..., n-1\}$, $\xi_i > \xi_{i+1}$. The following proposition determines the set of assets that are learned about and the allocation of investor learning across assets.

Proposition 3 *The set L of assets that are learned about and the allocation of investor learning across assets, $\{m_i\}_{i=1}^n$, are determined by the following conditions:*

(i) There exists a threshold $\phi_1 > 0$ such that for aggregate information capacity below this threshold only the first asset is learned about in the market, ξ_1. That is, for $\phi < \phi_1$, $m_1 = 1$ and $m_i = 0$ for all $i > 1$, where

$$\phi_1 \equiv \sqrt{\frac{1 + \rho^2 \xi_1}{1 + \rho^2 \xi_2}} - 1. \tag{22}$$

(ii) For aggregate information capacity $\phi \geq \phi_1$, at least two assets are learned about in equilibrium. The market learns about assets in decreasing order of ξ_i. The equilibrium masses for these assets satisfy

$$\frac{1+\phi m_1}{1+\phi m_k} = \sqrt{\frac{1+\rho^2\xi_1}{1+\rho^2\xi_k}}, \quad \forall k \in \{2,...,L\}, \tag{23}$$

and

$$\sum_{i=1}^{L} m_i = 1. \tag{24}$$

(iii) Let h index an asset that is not learned about ($m_h = 0$). Then it must be the case that, for a given level of information capacity ϕ,

$$1+\rho^2\xi_h < \frac{1+\rho^2\xi_1}{(1+\phi m_1)^2}, \quad \forall h \in \{L+1,...,n\}. \tag{25}$$

The selection of investors into learning about different assets is pinned down by the $L-1$ indifference conditions (23), combined with the condition that each investor learns about some asset, (24). The equilibrium masses equate the utility gains across all assets that are learned about. As the overall capacity in the economy increases from zero, for instance through technological progress, investors first learn about the most volatile asset, and then start expanding their learning towards lower volatility assets. The aggregate amount of information capacity may not be high enough to enable the allocation of capacity to the lowest volatility assets, such that in equilibrium not all assets are necessarily learned about. Furthermore, for all $i \in \{1,...,L\}$, $m_i > m_{i+1}$. Since the expected utility gain is increasing in ξ_i and decreasing in m_i, the mass of investors learning about each assets is monotonic, in order to ensure that the gains are equated across all assets that are learned about.

The allocation of investors' masses is determined only by exogenous variables: $\{\xi_i\}_{i=1}^n$, ρ, and ϕ. In turn, the solution for $\{m_i\}_{i=1}^n$ pins down equilibrium prices, by Proposition 2, thereby completing the equilibrium solution.

In order to further characterize learning in the economy, we introduce the following notation:

Definition 1 *Let ϕ_k be a threshold for ϕ, such that for any $\phi < \phi_k$, at most k assets are actively traded (learned about) in equilibrium, while for $\phi \geq \phi_k$, at least $k+1$ assets are actively traded in equilibrium.*

Using the above definition, Proposition 3 implies that the threshold values of aggregate information capacity are monotonic: $0 < \phi_1 < \phi_2 < ... < \phi_n$. The following proposition further characterizes the solution to the aggregate allocation of investors to learning about different assets:

Proposition 4 *Suppose that $\phi_{k-1} \leq \phi < \phi_k$, such that k assets are actively traded in equilibrium, with $1 < k \leq n$. Then, the equilibrium allocation of active investors across assets, $\{m_i\}_{i=1}^n$, satisfies the following conditions:*

(i) There exists a threshold asset $\bar{\imath} < k$, such that for all assets $i \in \{1, ..., \bar{\imath}\}$, the mass m_i is decreasing in ϕ, and for all assets $i \in \{\bar{\imath}+1, ..., k\}$, the mass m_i is increasing in ϕ.

(ii) For all assets $i \in \{1, ..., k\}$, the quantity (ϕm_i), the total capacity that the market allocates to learning about asset i, is increasing in ϕ.

(iii) For all assets $i \in \{1, ..., k\}$, the quantity $m_i(e^{2K_j} - 1)$, the total capacity allocated to learning about asset i by investor group $j \in \{1, 2\}$, is increasing in K_j at an increasing rate.

Proposition 4 shows that as the amount of aggregate capacity in the market ϕ increases, the total amount of effective information (ϕm_i) allocated to each asset strictly increases for all assets that are learned about (part (ii)), even though the mass m_i of investors learning about the most volatile assets decreases, to make room for new assets to be learned about (part (i)). Furthermore, the amount of effective information allocated to each asset by each investor group $(m_i(e^{2K_j} - 1))$ also increases, but it increases by more for the group of sophisticated investors, who have higher capacity (part (iii)). In Section 2, we use this proposition to derive analytical predictions on the patterns of investment by investor type, in response to both cross-sectional dispersion and symmetric growth of capacity.

1.5 The Value of Prices

We have written the individual investor's information acquisition problem in terms of a constraint on the information conveyed about payoffs by private signals alone, disregarding the information content of prices. In the portfolio choice model with endogenous information choice, the investors are given some capacity for processing information, and they choose how to best allocate this capacity to learning about asset payoffs. We show that in such a model, prices are an inferior way to learn about payoffs when the investor has the opportunity to observe endogenously designed signals on the payoffs themselves.

Proposition 5 *The rationally inattentive investor chooses to devote all her capacity to learning about payoffs through private signals on asset payoffs, rather than devoting any capacity to learning about asset payoffs from prices.*

This result is an outcome of the information constraint alone, irrespective of the agent's objective function, as long as the objective function does not exhibit strategic complementarities. To see why the price is an inferiour source of information, note that the price is a signal on the asset's payoff whose precision is determined in equilibrium, and hence exogeously from the point of view of the individual investor. Moreover, it is a function not just of the payoff z_i, but also of the stochastic supply, x_i. Hence it is a "garbled" signal on z_i. Since the investors are not restricted in terms of the type of signals they can obtain (except for our two independence assumptions, which would not be violated if the investor were to choose to learn from prices), investors could, if it were optimal, choose to receive signals that would, in effect, mimic the properties of the equilibrium prices. But instead, they prefer to receive an ungarbled signal on the payoff directly, and they also prefer to receive an informative signal about a single asset (as per Proposition 1), rather than learning a little bit about all assets. Hence, prices are dominated, as a source of information, by the private signals.

In contrast, in the literature on portfolio choice with exogenous imperfect information, investors are often assumed to learn about payoffs not only from their (exogenously given) private signals but also from equilibrium prices, which aggregate the information of all investors in the market.

The critical difference between endogenous and exogenous learning models is that in the endogenous information framework with capacity constraints on entropy reduction, learning from prices is no easier than learning from signals directly on the asset payoffs. There is no sense in which some random variables are easier to process than others: *all random variables are learnable*. In this framework, no variable is hidden, which means that prices lose their special role as a publicly available signal. On the other hand, processing any random variable is subject to the capacity constraint. Since equilibrium prices are simply linear combinations of the shocks in the model, from the point of view of the capacity constrained investor they are treated the same as any other random variable. Hence, processing prices consumes capacity just as processing the payoffs directly does. As a result, since prices consume capacity but are inferior signals per unit of capacity, the investors choose to not allocate any capacity to learning about payoffs from prices.[9]

2 Analytical Predictions

In this section, we present a set of analytical results implied by our model. We first present the predictions for capital income inequality followed by a set of theoretical predictions that are specific to our information-based mechanism. These results allow us to compare the model's implications with evidence from stock-level micro data.

2.1 Capital Income Inequality

Let π_{ji} denote the average profit per capita for investors of type $j \in \{1, 2\}$, from trading asset i:

$$\pi_{1i} \equiv \frac{Q_{1i}\left(z_i - rp_i\right)}{\lambda} \quad \text{and} \quad \pi_{2i} \equiv \frac{Q_{2i}\left(z_i - rp_i\right)}{1-\lambda}, \tag{26}$$

[9] In practice, it may be the case that individual investors are not as unconstrained in their choice of signals as modeled here. For instance, it may be the case that the relatively unsophisticated investor is restricted to processing information about payoffs only through publicly available signals, or that she cannot design signals on the payoffs themselves that are as precise as her capacity constraint would allow them to be. This would introduce a degree of *unlearnability* for asset payoffs, which may be an interesting avenue for future work.

where Q_{1i} and Q_{2i} are the aggregate holdings of asset i for sophisticated and unsophisticated investors, respectively, obtained by integrating holdings q_{ji} across investors of each type:

$$Q_{1i} = \lambda \left[\frac{(\overline{z}_i - rp_i) + m_i \left(e^{2K_1} - 1\right)(z_i - rp_i)}{\rho \sigma_i^2} \right], \tag{27}$$

and

$$Q_{2i} = (1-\lambda) \left[\frac{(\overline{z}_i - rp_i) + m_i \left(e^{2K_2} - 1\right)(z_i - rp_i)}{\rho \sigma_i^2} \right]. \tag{28}$$

Our first result is that heterogeneity in information capacity across investors drives capital income inequality as sophisticated investors generate higher income than unsophisticated ones. This is summarized in Proposition 6.

Proposition 6 *If $K_1 > K_2$ then $\sum_i \pi_{1i} - \sum_i \pi_{2i} > 0$.*

The informational advantage manifests itself in two ways. First, sophisticated investors achieve relatively higher profits by holding a different average portfolio (average effect). Second, they also achieve relatively higher profits by obtaining larger gains from shock realizations that are profitable relative to expectations, and incurring smaller losses on unprofitable shock realizations (dynamic effect). These two effects show up in the average level and the adjustment of holdings in response to shocks, and are summarized in Propositions 7 and 8.

Proposition 7 (Average Effect) *Let $K_1 > K_2$ and $\phi_{k-1} \leq \phi < \phi_k$, such that the first $k \in \{1, ..., n\}$ assets are learned about in equilibrium. The following statements hold:*

(i) For $i \in \{1, ..., k\}$, $E\left\{\frac{Q_{1it}}{\lambda} - \frac{Q_{2it}}{(1-\lambda)}\right\} > 0$, and for $i \in \{k+1, ..., n\}$, $\frac{Q_{1it}}{\lambda} - \frac{Q_{2it}}{(1-\lambda)} = 0$.

(ii) Suppose that $\overline{x}_i = \overline{x}$ and $\sigma_{xi} = \sigma_x$ for all i. For any two assets $i, l \in \{1, ..., k\}$, if $E(z_i - rp_i) > E(z_l - rp_l)$, then $E\left\{\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{(1-\lambda)}\right\} > E\left\{\frac{Q_{1l}}{\lambda} - \frac{Q_{2l}}{(1-\lambda)}\right\}$.

Proposition 7 demonstrates that sophisticated investors choose higher average holdings of risky assets (part (i)), and that on average, they also tilt their portfolios towards profitable assets more than unsophisticated investors do (part (ii)).

Proposition 8 (Dynamic Effect) *Let $K_1 > K_2$ and $\phi_{k-1} < \phi < \phi_k$, such that the first $k \in \{1, ..., n\}$ assets are learned about in equilibrium. For $i \in \{1, ..., k\}$, $\left[\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{(1-\lambda)}\right]$ is increasing in excess returns, $(z_i - rp_i)$.*

Proposition 8 illustrates the dynamic effect of investor sophistication. It shows that for every realized state x_i, z_i, sophisticated investors are able to adjust their portfolios (contemporaneously) upwards if the shock implies high returns and downwards if the shock implies low returns. Hence, also dynamically, they are able to outperform unsophisticated investors by responding to shock realizations in a way that increases their profits.

To see explicitly the impact on capital income inequality coming from the dynamic effect, we express the total capital income of an average sophisticated investor as[10]

$$\sum_{i=1}^{n} \pi_{1i} \equiv \sum_{i=1}^{n} \alpha_i \pi_{2i}, \tag{29}$$

where, by (27) and (28),

$$\alpha_i \equiv \frac{\pi_{1i}}{\pi_{2i}} = \frac{(\overline{z}_i - rp_i) + m_i(e^{2K_1} - 1)(z_i - rp_i)}{(\overline{z}_i - rp_i) + m_i(e^{2K_2} - 1)(z_i - rp_i)}, \quad \forall i. \tag{30}$$

That is, capital income of an average sophisticated investor can be expressed as a weighted sum of an average unsophisticated investor's capital income from each asset, but the weights depart from 1 whenever the asset is actively traded ($m_i > 0$).

To see the dynamic effect, consider how variation in the weights α_i drives income differences. For assets that are actively traded in equilibrium, they vary depending on the realization of the shocks z_i and x_i. There are two possible scenarios. First, $\pi_{2i} > 0$, which by (30) implies $\pi_{1i} > 0$ and $\alpha_i > 1$. Hence, sophisticated investors have a larger gain in their (positive) capital income from asset i. Second, $\pi_{2i} < 0$ and either (i) $\pi_{1i} < 0$ and $0 < \alpha_i < 1$, or (ii) $\pi_{1i} > 0$ and $\alpha_i < 0$. The first case implies that sophisticated investors put a smaller weight in their portfolio on the loss, while the second case means that the profit

[10]Here, we are implicitly assuming that profits are never exactly zero. For such case, the arguments extend trivially.

of sophisticated investors puts a negative weight on the loss. In both cases, sophisticated investors either incur a smaller loss or realize a bigger profit, state by state.

These arguments lead to the following comparative result: *increases* in sophistication heterogeneity lead to a *growing* capital income polarization. Intuitively, greater dispersion in information capacity means that, relative to unsophisticated investors, sophisticated investors receive higher-quality signals about the fundamental shocks x_i, z_i, and as a result, they respond more strongly to postive/negative realized excess profits $z_i - rp_i$. This is the essence of Proposition 9.

Proposition 9 *Consider an increase in capacity dispersion of the form* $K_1' = K_1 + \Delta_1 > K_1$, $K_2' = K_2 + \Delta_2 < K_2$, *with* Δ_1 *and* Δ_2 *chosen such that total information capacity* ϕ *remains unchanged. Then, the ratio* $\sum_i \pi_{1i} / \sum_i \pi_{2i}$ *increases, that is, capital income becomes more polarized.*

The results show that heterogeneity in capacity generates heterogeneity in portfolios, which in result decreases the relative participation of unsophisticated investors. Below, we explore the intuitive reasons behind unsophisticated investors' retrenchment from risky assets in the presence of informationally superior, sophisticated investors.

Intuition: Suppose that the realized state is $z_i > \bar{z}_i$, such that in equilibrium $z_i - rp_i > 0$, and consider a case of a *homogeneous* investor with capacity K_2 who receives a mean signal for her type, $S_2 = \bar{z}_i e^{-2K_2} + z_i(1 - e^{-2K_2})$. Her mean allocation choice is then

$$q_{2i} = e^{2K_2} \left(\frac{S_2 - rp_i}{\rho \sigma_i^2} \right),$$

where $e^{-2K_2}\sigma_i^2$ is the variance of the investor's posterior beliefs.

Let us also fix the allocation of investors to learning about different assets, $\{m_i\}_{i=1}^n$ at the equilibrium level, and perform an exogenous variation of increasing the capacity of mass $\gamma < m_i$ of investors to $K_1 > K_2$ so that they become more sophisticated. These new

sophisticated investors have average (across mass γ) demand given by

$$q_{1i} = e^{2K_1}\left(\frac{S_1 - rp_i}{\rho\sigma_i^2}\right),$$

where the mean signal they receive is $S_1 = \bar{z}_i e^{-2K_1} + z_i(1 - e^{-2K_1})$.

There are two effects that lead to an increased relative participation of sophisticated investors in risky assets in this example: a partial equilibrium one and a general equilibrium one.

First, *absent any price adjustment*, the partial equilibrium effect is that the remaining unsophisticated investors do not change their demand q_{1i} for asset i, but the new sophisticated investors now demand more, because $S_1 > S_2$ (we are considering a good state where $z_i > \bar{z}_i$), and also this signal is more precise ($e^{-2K_1}\sigma_i^2 < e^{-2K_2}\sigma_i^2$). Hence, in partial equilibrium, in which the price is exogenously given, we would observe growth in sophisticated investors' ownership: They would take bigger positions when they actively trade. However, we would see no change in the strategies of unsophisticated investors.

Second, there is the general equilibrium effect working through price adjustment, which makes unsophisticated investors perceive an *informational disadvantage* in trading asset i after sophisticated investors enter. In particular, in accordance with market clearing conditions (19) and (20), the price will adjust to the now greater demand from highly informed investors; in particular, it will be more informative about the fundamental shock z_i,[11] and since $z_i - rp_i > 0$, the equilibrium price will increase[12]. Through that price adjustment, both types of investors will see their returns go down, but only unsophisticated ones will choose to decrease their holdings–their signals are not of a high enough quality to sustain previous positions as the optimal choice. Through this general equilibrium effect, the entry of sophisticated investors spills over to an informational disadvantage for unsophisticated investors and causes their retrenchment from trading the asset.

[11] b_i will rise and a_i and c_i will drop, because we increased ϕ in the market for asset i by increasing total capacity of investors trading in that market.

[12] Both the price and its derivative with respect to ϕ in state x_i, z_i are proportional to $z_i - \bar{z}_i + \rho\sigma_i^2 x_i$.

2.2 Testing the Mechanism

In this section, we provide additional analytical characterization of our model's predictions. These analytical results, together with the quantitative predictions from our parameterized model, serve as a test of the main mechanism of the model when compared to the same features in the data.

We start with the characterization of properties of the market return in response to growth in the overall level of information in the economy. As aggregate information increases, prices contain a growing amount of information about the fundamental shocks, and excess market return drops. This is summarized in Proposition 10.

Proposition 10 (Market Value) *Growth in total information processing capacity leads to*

(i) higher average prices, $\frac{d\overline{p}_i}{d\phi} \geq 0$*, and hence a higher average value of the financial market.*

(ii) lower average market excess returns, $dE\left(z_{it} - rp_{it}\right)/d\phi \leq 0$.

Next, in Proposition 11, we consider the effects of a pure increase in dispersion of sophistication, without changing the aggregate level of sophistication in the economy. Such polarization in capacities implies polarization in holdings.

Proposition 11 *Consider an increase in capacity dispersion of the form* $K_1' = K_1 + \Delta_1 > K_1$, $K_2' = K_2 + \Delta_2 < K_2$*, with* Δ_1 *and* Δ_2 *chosen such that total information capacity* ϕ *remains unchanged. Then, the average ownership difference* $E\left\{\sum_i \frac{Q_{1i}}{\lambda} - \sum_i \frac{Q_{2i}}{1-\lambda}\right\}$ *increases, that is, sophisticated investors' ownership increases.*

Using Proposition 4, we can show that the aggregate symmetric growth in information technology, modeled as a common growth rate of both K_1 and K_2, leads to a growing retrenchment of unsophisticated investors and hence an increased ownership of risky assets by sophisticated (Proposition 12), as well as growing capital income polarization (Proposition 13).

Proposition 12 (Dynamic Ownership) *Consider symmetric information capacity, such that* $K_{1t} = K_t$ *and* $K_{2t} = K_t\gamma$, $\gamma \in (0,1)$, *and consider* $\phi_{k-1} < \phi < \phi_k$ *such that* $k > 1$ *assets are actively traded in equilibrium. In equilibrium, the average ownership share by sophisticated investors increases across all assets:*

$$dE\{\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{1-\lambda}\}/dK > 0.$$

Proposition 13 (Capital Income Polarization) *Consider symmetric information capacity, such that* $K_{1t} = K_t$ *and* $K_{2t} = K_t\gamma$, $\gamma \in (0,1)$, *and consider* $\phi_{k-1} < \phi < \phi_k$ *such that* $k > 1$ *assets are actively traded in equilibrium. In equilibrium, the average capital income becomes more polarized:*

$$dE\{\sum_i \pi_{1i} / \sum_i \pi_{2i}\}/dK > 0.$$

3 Results

In this section, we provide a discussion of the results corresponding to our analytical predictions. We first lay out empirical facts coming from household-level and institution-level data that motivate our investigation of capital income inequality. Further, we show the quantitative performance of our model that aims to explain these facts. To this end, we discuss the parametrization of the model and show the quantitative performance of the model for income inequality and the results that pin down our economic mechanism in the data. Next, we discuss alternative mechanisms that could potentially explain the data. Finally, we provide additional empirical evidence that supports our analytical predictions.

3.1 Motivating Facts: Capital Income Inequality

In this section, we discuss empirical evidence on capital income inequality. We first present results based on household-level data from the Survey of Consumer Finances (SCF). Next, we enhance these data with evidence based on institutional holdings data from Thom-

son Reuters.

3.1.1 Evidence from the Survey of Consumer Finances

We begin with summarizing the data on capital income and financial wealth inequality for U.S. households, based on data from the Survey of Consumer Finances. SCF provides information on a representative sample of U.S. households on a tri-annual basis. To map our sample to that from micro-level, financial market data, we use eight most recent surveys between 1989 and 2010.

In our framework, we assume that differences in investor sophistication can be mapped to differences in their wealth levels (Arrow (1987) and Calvet, Campbell, and Sodini (2009b)). Since capital income is generated by investments of disposable capital, we use financial wealth as our empirical proxy. To show the empirical distribution of financial wealth and income inequality in the data, we restrict our population to households with non-zero investment in either stocks or non-money market mutual funds (i.e. relatively active investors), and consider two subsets of households: a group of 10% of households with the highest level of total wealth at each point in time (sophisticated investors) and a group of 50% of households with the lowest level of total wealth (unsophisticated investors). For the two groups, we calculate average financial wealth and corresponding capital income and report the ratios of the two averages. Figure 1 shows the time series of the ratios for financial wealth (left panel) and capital income (right panel).

We find a significant dispersion in financial wealth across the two groups of households. The average financial wealth of sophisticated investors is at least 30 times larger than the average financial wealth of unsophisticated investors. Moreover, the dispersion in financial wealth exhibits a highly increasing trend over time: The financial wealth ratio almost doubles, from 30 in 1989 to more than 55 in 2010. This result conforms with anecdotal evidence of a growing polarization in wealth and earlier findings in Piketty and Saez (2003) and Atkinson, Piketty, and Saez (2011). It also suggests that financial sophistication became significantly more polarized in the last few decades.

Likewise, we find qualitatively similar patterns in capital income ratios: Income ratios

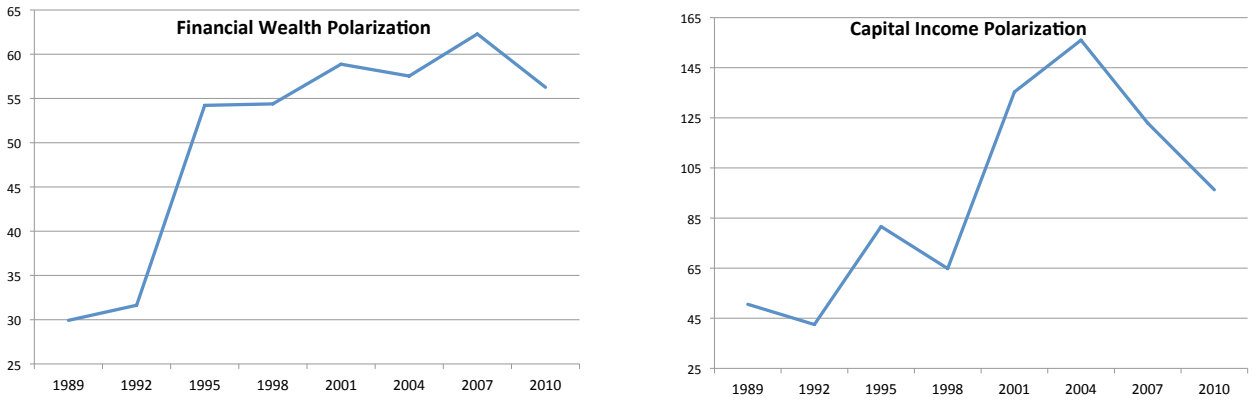


Figure 1: Financial Wealth and Capital Income Polarization: Survey of Consumer Finances.

are highly dispersed cross-sectionally, with sophisticated investors earning at the minimum 45 times more dollar income than unsophisticated ones. This dispersion also grows strongly over time up to 150 in 2004. Even though it subsequently diminishes slightly, it remains at a very high level of at least 100. Combining these two pieces together implies that sophisticated investors outperform unsophisticated investors in terms of their rates of returns on the invested capital. In the data, we find that the ratio in rates of returns for sophisticated vs. unsophisticated investors on average equals 1.7 and varies between 1.1 and 2.15 in the time series.

3.1.2 Evidence from Thomson Reuters

Our economic mechanism explaining income inequality has direct implications for investors' portfolio choices. Since the SCF data do not provide detailed information about investors' holdings and the returns they earn on these holdings, it is difficult to directly test some of the analytical predictions of our model. To accommodate such tests, we rely on portfolio holdings' data obtained from Thomson Reuters. These data contain a comprehensive sample of portfolios of publicly traded equity held by institutional investors. The data on holdings come from quarterly reports required by law and submitted by all institutional investors to the Securities and Exchange Commission (SEC). Relative to the SCF, the benefit of using the portfolio holdings data is its detailed micro-level structure, the drawback is

that we only observe a subset of investors who are not exactly the same investors as those in SCF; hence, any results on income and wealth inequality are difficult to obtain.

To provide a qualitative mapping between the two data sets, we define sophisticated investors as those classified as investment companies or independent advisors (types 3 and 4) in the Thomson data. These investors include wealthy individuals, mutual funds, and hedge funds. Among all types, these two groups are known to be particularly active in their information production efforts; in turn, other groups, such as banks, insurance companies, or endowments and pensions are more passive by nature. Our definition of unsophisticated investors is other shareholders who are not part of Thomson data. These are individual (retail) investors.

We provide evidence on growing capital income polarization, using data on aggregated financial performance of each group of investors over the time period 1989-2012. We proceed in three steps. First, we obtain the market value of each stock held by all investors of a given type. Market value of each stock is the product of the number of combined shares held by a given investor type and the price per share of that stock, obtained from CRSP. Since the number of shares held by unsophisticated investors is not directly observable we impute this value by taking the difference between the total number of shares available for trade and the number of shares held by *all* institutional investors. Second, we calculate the value shares of each stock in the aggregate portfolio by taking the ratio of market value of each stock relative to the total value of the portfolio of each type of investor. Third, we obtain the return on the aggregate portfolio by matching each asset share with their next month realized return and calculating the value-weighted aggregated return. We repeat this procedure separately for sophisticated and unsophisticated investors.

To compare financial performance of the two investor types we calculate cumulative values of \$1 invested by each group in January 1989 using time series of the aggregated monthly returns ending in December 2012. We present the two series in Figure 2.

Our results indicate that sophisticated investors systematically outperform unsophisticated investors. The value of \$1 invested in January 1989 grows to \$5.32 at the end 2012 for sophisticated investors and only to \$3.28 for unsophisticated investors. This result implies

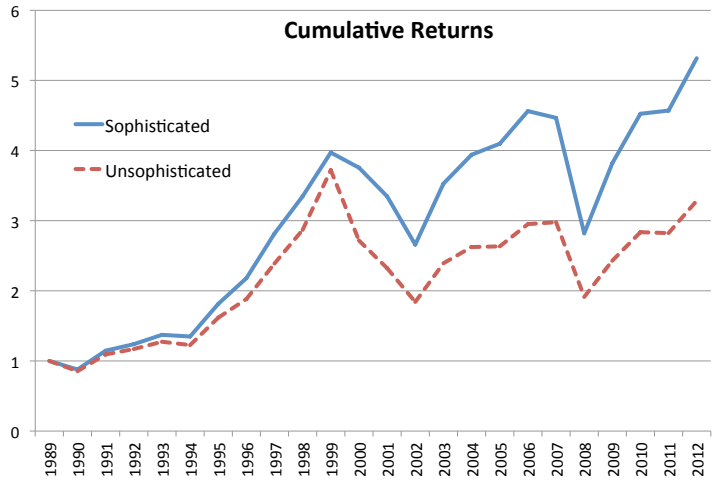


Figure 2: Cumulative Return in Equity Markets.

that sophisticated investors generate significantly more equity capital income per unit of financial wealth they invest, which in turn implies income inequality and growing polarization. These results are consistent with our earlier findings from SCF.

3.2 Parametrization

In this section, we describe the parametrization of the model that we subsequently use to assess the validity of our economic mechanism. We conduct two quantitative experiments: In the first experiment, which we label *Aggregate Portfolios*, we parameterize the model to stock-level micro data and aggregated investors' equity shares. Using these data allows us to match the general pattern of outperformance of sophisticated over unsophisticated investors. In addition, we are able to test the model's predictions regarding portfolio allocations and asset turnover across assets and over time. In the second experiment, labeled *Household Portfolios*, we use the parameterized environment from *Aggregate Portfolios* and endogenize information capacities by linking their relative values to relative financial wealth levels across investors types. Here, we examine whether the endogenous evolution of capacities, with an initial condition on relative capacities that is chosen based on the 1989 financial wealth levels, can account for the evolution of capital income inequality in the data. Below, we discuss the details of each parametrization exercise.

Aggregate Portfolios The starting point of this experiment is a parametrization of the model to match key moments of the data for the period 1989-2000. We think of this as the initial period in our model and treat it as a point of departure for our dynamic comparative statics exercises. To obtain the most comprehensive set of data targets and empirical statistics for testing the mechanism, we use the same classification of sophisticated and unsophisticated investors as in Section 3.1.2. This allows us to bring in data on the distribution of turnover and ownership by asset characteristics and over time, as well as time series data on differences in returns.

The key parameters of our model are the information capacity of each investor type (K_1 and K_2), the averages and volatilities of the fundamental shocks ($\bar{z}_i, \sigma_i$) and the supply shocks ($\bar{x}_i, \sigma_{xi}, , i = 1, ..., n$), the risk aversion parameter (ρ), and the fraction of sophisticated investors (λ).

For parsimony, we restrict some parameters and normalize the natural candidates. In particular, we normalize $\bar{x} = 5, \bar{z} = 10$ and restrict $\sigma_{xi} = \sigma_x$. To capture heterogeneity in assets returns, we set the lowest volatility $\sigma_n = 1$ and assume that volatility changes linearly across assets, which means that it can be parameterized by a single number, the slope of the line.[13] We pick the remaining parameters to match the following targets in the data (based on 1989-2000 averages): (i) aggregate equity ownership of sophisticated investors, equal to 23%; (ii) real risk-free interest rate, defined as the average nominal return on 3-month Treasury bills minus inflation rate, equal to 2.5%; (iii) average annualized stock market return in excess of the risk-free rate, equal to 11.9%; (iv) average monthly equity turnover, defined as the total monthly volume divided by the number of shares outstanding, equal to 9.7%; (v) the ratio of the 90th percentile to the median of the cross-sectional idiosyncratic volatility of stock returns, equal to 3.54. In addition, we arbitrarily set the fraction of assets about which agents learn to 50%.

To generate the dynamic predictions of our model, we assume a symmetric growth in information capacity of each investor type, in order to match the 2001-2012 average equity

[13] In particular, we set $\sigma_i = \sigma_n + \alpha(n - i)/n$ which, given our normalization of σ_n, leaves only α to be determined.

ownership rate of sophisticated investors, equal to 43%. We think of this approach as a way of modeling technological progress in investment technology which affects both types of investors in the same way—hence, the reported results are not driven by differential growth but come solely from the general equilibrium effects of our mechanism.

The above procedure leaves us with one key parameter left—the ratio of information capacity of sophisticated versus unsophisticated investors, K_1/K_2. We set this parameter to 10% in our parametrization. The parameters and model fit are presented in Tables 1 and 2.

Table 1: Parameter Values

Parameter	Value
K_1, K_2, λ, n	0.577, 0.0577, 0.2, 10
$\bar{z}_i$, $\bar{x}_i$	10, 5
σ_{xi}	0.41 for all assets i
$\{\sigma_i\}$, $i = 1, ..., 10$ assets	$\{1.5026, 1.4468, 1.3909, 1.3351, 1.2792, 1.2234, 1.1675, 1.1117, 1.0558, 1\}$

Table 2: Parametrization: Model Fit

Statistic	Data	Model
Market Return	11.9%	11.9%
Average Turnover	9.7%	9.7%
Sophisticated Investors' Ownership	23%	23%
Informed Trading	n.a.	50%

Household Portfolios In this experiment, we consider a version of our model in which information capacities evolve endogenously over time. In particular, to assess the ability of our mechanism to quantitatively account for the observed growth in income polarization at the household level (based on SCF), we link the growth in investors' information capacities to the growth in their financial wealth levels. The idea is that wealthier individuals have access to better information production or processing technologies, which in the language of our model means they have greater information capacity.

Specifically, we set the initial ratio of investors' information capacity, K_1/K_2 in the model, to the 1989 ratio of average financial wealth in the top 10% and the bottom 50% of the total wealth distribution of households with non-zero holdings of either stocks or mutual funds. In our data, this ratio is equal to 29.92. We pick the initial aggregate capacity level to match the excess return on the market portfolio, equal to 11.9% in the data. We then assume that the growth of each investor type's capacity is equal to their return on equity. All the remaining parameters, in particular the stochastic processes for payoffs and risk aversion, are the same as in the *Aggregate Portfolios* experiment.

3.3 Quantitative Results

In this section, we report quantitative results from our model for the two different parametrization exercises we perform. We first discuss our findings related to capital income inequality and its polarization. Then, we show results related to our specific economic mechanism and provide additional discussion of alternative hypotheses.

3.3.1 Income Inequality

Aggregate Portfolios We report the results in Table 3. The parameterized model implies a 2.1 percentage point advantage (12.7% versus 10.7%) in average portfolio returns between sophisticated and unsophisticated investors, which accounts for 70% of the difference in the data for the 1989-2000 period (13.4% versus 10.4%). We conclude that, quantitatively, the model can account for a significant fraction of the empirical difference in returns across these two investor types. Further, our mechanism has an economically large implication for the difference in performance across informational capacities, which suggests that a similarly large economic effect also exists within the household sector. If sophistication can be approximated by financial wealth (as implied by a setting in Arrow (1987)), then our mechanism would imply a growing disparity in capital incomes. We test this hypothesis in the *Household Portfolios* experiment below.

Table 3: Market Averages by Subperiod: Data and Model

	1989-2000		2001-2012	
Statistic	Data	Model	Data	Model
Market Return	11.9%	11.9%	2.4%	3.5%
Sophisticated Investors' Return	13.4%	12.7%	2.9%	3.7%
Unsophisticated Investors' Return	10.4%	10.7%	1.6%	3.4%
Average Equity Turnover	9.7%	9.7%	16%	14%
Sophisticated Investors' Ownership	23%	23%	43%	43%

Household Portfolios In this experiment, we investigate the endogenous propagation of heterogeneity in capacity across time by simulating the model for 21 years forward, which is the time span of our data set. As the outcome of the experiment, we obtain the endogenous capital income and financial wealth dispersion growth implied by our mechanism. Along the simulation period, capacity growth of each investor type is endogenously determined by the return on her wealth, as only the initial dispersion is exogenously determined. The results of this exercise are presented in Table 4.

Table 4: Capital Income and Wealth Dispersion: Data and Model

	Data 1989-2010	Model
Capital Income Dispersion Growth	90%	98%
Financial Wealth Dispersion Growth	88%	52%

We obtain a 98% growth in capital income inequality (90% in the data) and a 52% growth in financial wealth inequality (88% in the data), which is over 100% and 59% of the growth observed in the data, respectively. We conclude that our mechanism has a strong power to explain income and wealth polarization observed in the data. The time series for growth in capital income polarization in this model experiment and the SCF data is presented in Figure 3. The model matches well not only the overall growth but also the timing of the increase in capital income polarization.

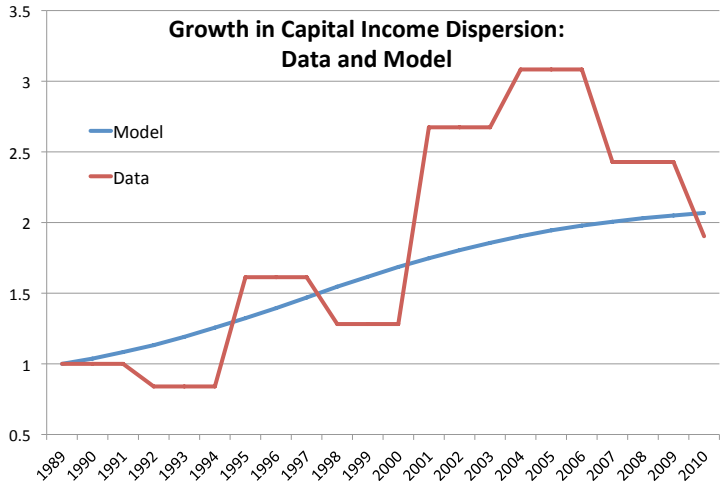


Figure 3: Cumulative Growth in Capital Income Dispersion

3.3.2 Testing the Mechanism

The results in the previous section demonstrate a significant potential of our information mechanism to account for the return differential and income inequality observed in the data. In this section, we provide a set of quantitative predictions from *Aggregate Portfolios*, which allow us to provide additional support for our mechanism by comparing it to the corresponding data moments. These are robust predictions of our mechanism and are proven analytically in Section 2. Below, we show the good fit of these results not only qualitatively but also quantitatively.

Market Averages Technological progress in information capacity in the model implies large changes in average market returns, cross-sectional return differential, and turnover. We report these statistics generated by the model and observed in the data in Table 3.

The changes implied by the model qualitatively match the changes in the data, but they also come close quantitatively. Both the model and the data imply a decrease in market return and a decrease in the return differential of portfolios held by sophisticated and unsophisticated investors. Intuitively, in the model, lower market return is a result of an increase in quantity of information: The price reflects that and tracks much more closely the actual return z than in the initial parametrization with lower overall capacity (for additional

intuition, see Proposition 10).

The model also predicts a sharp increase in average asset turnover, in magnitudes consistent with the data. As with the market return, this result is a direct implication of our mechanism and is not driven by changes in asset volatility. In fact, fundamental asset volatilities (σ_is) are held at the same level across the two sub-periods in the model. Intuitively, higher turnover in the model is driven by more informed trading by sophisticated investors, both due to their holding a larger share of the market as well as them receiving more precise signals about asset payoffs.

Ownership Investors in our model prefer to learn about assets with higher volatility. In particular, upon increasing their information capacity, they first invest it in the most volatile asset until the benefits from a unit of information become equalized with those of the second highest volatility asset, then third, and so forth (see Proposition 3). This process implies a particular way in which institutions expand their portfolio holdings as their capacity (through overall capacity) increases. Specifically, we should see that sophisticated investors exhibit the highest initial growth in ownership for the the highest volatility assets, then lower volatility assets, etc. Figure 4 shows the evolution of this growth in the model and in the data over the period 1989-2012.[14]

In Figure 5, we show *the change* in asset ownership by sophisticated investors over the periods 1989-2000 and 2001-2012, where assets are sorted by volatility of their returns. This cross-sectional change underlies the average ownership targets in the model of 23% in the initial period and 43% in the later period. Both the data and the model exhibit a hump-shaped profile of the increase and they are also very close quantitatively.

In conclusion, even though we parameterize the model to match the *aggregate* ownership levels of sophisticated investors in the pre- and post-2000 period, the model is also able to explain quantitatively *how* ownership increases *across* asset volatility classes, both in terms of timing of the growth levels and in terms of the absolute magnitudes of the ownership changes.

[14]To generate this graph in the model, we increase aggregate capacity from zero to the level that matches 48% sophisticated ownership, which is the last point in the data.

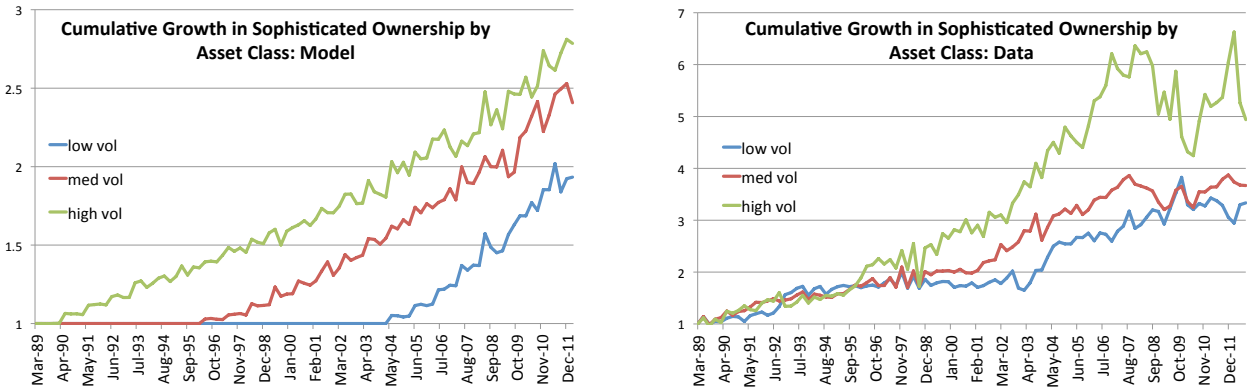


Figure 4: Cumulative Growth in Sophisticated Investors' Ownership: Data and Model

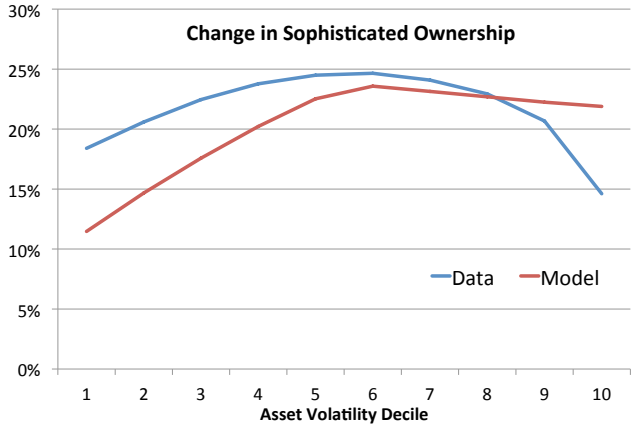


Figure 5: Absolute Change in Sophisticated Investors' Ownership

Turnover Our model implies cross-sectional variation in asset turnover, driven by differential investment of investors' information capacity. Intuitively, if an asset is more attractive and investors invest more in it, then there are more investors with precise signals about assets returns, and these investors want to act on the better information by taking larger and more volatile positions. Since the sophisticated investors receive more precise signals, and they have preference towards high-volatility assets, we should see a positive relationship between volatility and turnover. We report turnover in relation to return volatility in the model and in the data in Table 5.

The first two rows compare data and the model prediction for the initial parametrization to 1989-2000 data. Both data and model show increasing patterns in turnover as volatility

Table 5: Turnover by Asset Volatility

Volatility quintile	1	2	3	4	5	Mean
1989-2000						
Data	5%	8.5%	10.5%	12.5%	11.5%	9.7%
Model	9%	9%	9.3%	9.9%	10.8%	9.7%
2001-2012						
Data	11%	14.6%	17%	18.4%	19.3%	16%
Model	12.5%	13.6%	14.2%	15%	15.4%	14%

goes up, which are quantitatively close to each other. In the next two rows, we compare data for the 2001-2012 period to results generated from the dynamic exercise in the model in which we increase overall capacity. The model implies an increase in average turnover compared to an earlier period and additionally matches the cross-sectional pattern of the increase. This effect is purely driven by our information friction, since the fundamental volatilities remain constant over time in this exercise.[15]

3.3.3 Additional Supporting Evidence

So far, we presented quantitative results supporting our analytical predictions that are based on our parameterized model. Specifically, our theoretical predictions imply that differences in capital income generally can stem from two sources: heterogeneity in prices of investable assets and the differential exposure of investors to holding such assets. In this section, we provide additional evidence on each of these channels that offers support for our predictions qualitatively but cannot be assessed quantitatively.

Unsophisticated Investors' Retrenchment In this section, we show that cross-sectional differences in assets holdings of investors with different levels of sophistication are consistent with predictions of our model and thus contribute to capital income inequality and its growing polarization. Our main prediction is that unsophisticated investors should

[15]Our model also implies a positive turnover-ownership relationship, which we further confirm in the data. This result is also consistent with the empirical findings in Chordia, Roll, and Subrahmanyam (2011).

be more likely to invest in assets with lower expected values. In the quantitative tests of the model in Figure 4, we show that sophisticated investors allocate their wealth first into assets with highest level of volatility and subsequently into assets with lower levels of volatility. Now, we provide additional evidence which suggests similar investors' preferences.

Our first piece of evidence is based on SCF data regarding households' holdings in liquid wealth. The idea of this test is that unsophisticated investors should be more likely to invest in safe (liquid) assets. SCF provides detailed classification of wealth invested in such assets that include checking accounts, call accounts, money market accounts, coverdell accounts, and 529 educational state-sponsored plans. As before, in each period, we divide households into two groups: top 10% and bottom 50% of the wealth distribution. For each of the groups, we calculate the average ratio of liquid wealth to total financial wealth. Higher ratios would imply greater exposure to low-profit assets. We present the two time series in Figure 6.

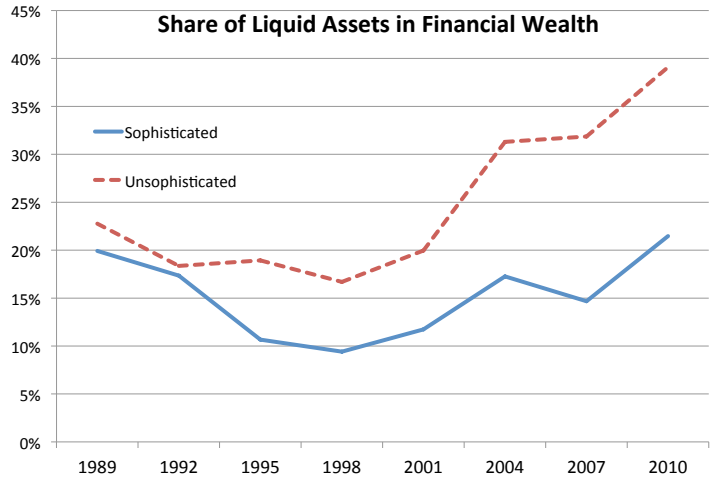


Figure 6: Share of Liquid Wealth in Financial Wealth: Survey of Consumer Finances.

We find evidence that strongly supports predictions from our model. First, the average ratio of liquid wealth for sophisticated investors, equal to 15.3%, is significantly lower than that for unsophisticated investors, which in our sample equals 25%. In addition, while the exposure to liquid assets by sophisticated investors is generally non-monotonic (u-shaped), similar investment for unsophisticated investors exhibits a strong positive time trend, especially in the last 20 years: the average investment goes up from 16.7% in 1998 to 39% in 2010. This evidence strongly supports our economic mechanism in that differences in information

capacity lead to retrenchment by unsophisticated investors from risky assets and relocation to safer assets.

We further confirm this claim using evidence on institutional holdings from Thomson Reuters. To this end, we calculate average (equal-weighted) equity ownership of sophisticated investors (mutual funds and hedge funds) and unsophisticated (retail) investors. We report the respective time series quarterly averages of the ownership over the period 1989-2012 in Figure 7.

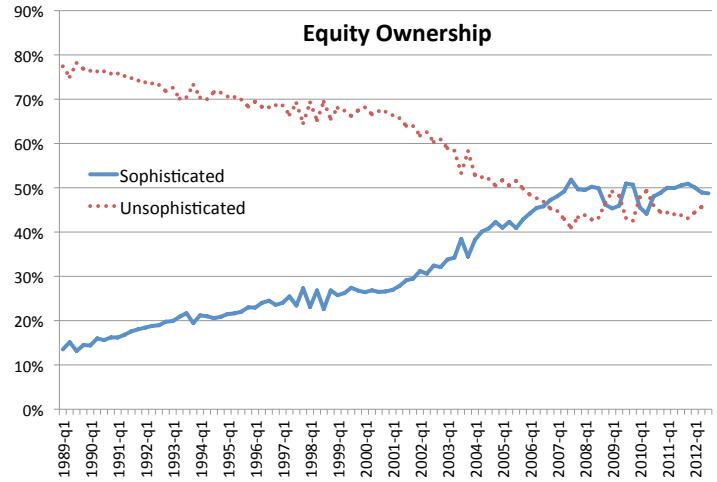


Figure 7: Equity Ownership by Sophistication Type.

The results paint a picture that is generally consistent with our model's predictions. Although the average ownership level of unsophisticated investors is higher in an unconditional sample and equals 61%, the time series evidence clearly indicates a very strong pattern: the average equity ownership for unsophisticated investors goes down while that for sophisticated investors significantly goes up.[16] We argue that this evidence is consistent with the view that the observed expansion of relative financial wealth presented in Figure 1 drives the expansion of information capacities. Realizing a positive shock to information capacity sophisticated investors enter the profitable equity market at the expense of unsophisticated investors who perceive the informational disadvantage in the market and as a result move

[16]The visible positive trend in active ownership has been documented before by Gompers and Metrick (2001) and is even stronger if one accounts for differences in market values across assets and the preference of sophisticated investors for large-cap stocks.

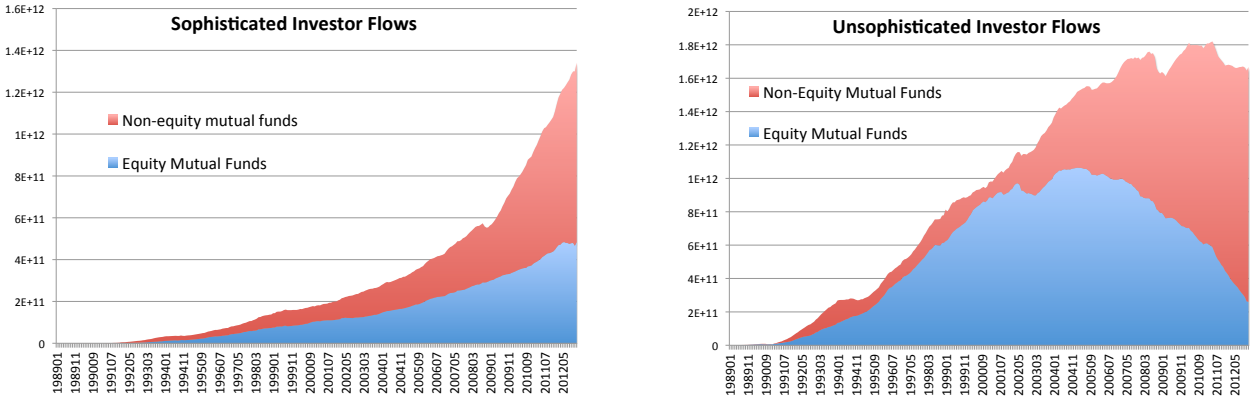


Figure 8: Cumulative Flows to Mutual Funds by Sophistication Type: Equity vs. Non-Equity

away from equity. Notably, the retrenchment of unsophisticated investors from directly holding equity happened despite the overall strong performance of equity markets over the same time period. This suggests that investors do not simply respond to past trends in equity returns.

As a final auxiliary prediction we consider money flows into mutual funds. The idea is that equity mutual funds are more risky than non-equity funds. As such, unsophisticated investors should be less likely to invest in the former, especially if information capacity gets more polarized.

To test this prediction in the data we use mutual fund data from Morningstar. Morningstar classifies different funds into those serving institutional investors and individuals whose investment is at least $100,000 (institutional funds) and those serving individual investors with investment value less than $100,000 (retail funds). For the purpose of testing our predictions, we define sophisticated investors as those investing in institutional funds and unsophisticated investors as those investing in retail funds. Subsequently, we calculate cumulative aggregated dollar flows into equity and non-equity funds, separately for each investment type. Our data span the period 1989-2012. We present the results in Figure 8.

We find that the cumulative flows from sophisticated investors into equity and non-equity funds increase steadily over the whole sample period. In contrast, the flows from

unsophisticated investors display a visibly different pattern. The flows into equity funds keep increasing until 2000 but subsequently decrease at a significant rate of more than 3 times by 2012. Moreover, the decrease in cumulative flows to equity mutual funds coincides with a significant increase in cumulative flows to non-equity funds. Overall, these findings support predictions of our model: Sophisticated investors have a large exposure to risky assets and subsequently add extra exposure to less risky assets, whereas unsophisticated investors leave riskiest assets and move into safer assets as they perceive higher information disadvantage.

One could note that the increase in equity flows by unsophisticated investors in the early period of our sample is inconsistent with our model. We argue that this result could still be rationalized by contrasting it with the steady decrease in holdings of individual equity documented earlier. To the extent that individual equity holdings are more risky than diversified equity portfolios, such as mutual funds, this only means that in the earlier period unsophisticated investors reallocate their wealth from riskier to safer asset class.

Stock Selection Ability The second building block of our economic mechanism is the ability of sophisticated investors to better choose assets. Our quantitative evaluation maps the model prediction to the observed differences in performance between sophisticated and unsophisticated investors. Here, we provide an additional qualitative result in which we show that equity holdings of sophisticated investors are higher for stocks which realize higher returns.

To conduct this test, we obtain data on stock returns come from Center for Research on Security Prices (CRSP), and for each stock we calculate the market shares of sophisticated investors. Next, we estimate the regression model over the period January 1989-December 2012 with stock/month as a unit of observation. Our dependent variable is the share of sophisticated investors in month t and the independent variable is return corresponding to the stock in month t. Our regression model includes year-month fixed effects and standard errors are clustered at the stock level to account for the cross-sectional correlation in the data. We report the results of this estimation in Table 6.

Table 6: Future Returns Explain Sophisticated Investors' Ownership

Variable	Value	Standard Error
Future Return	0.048	0.00845
Constant	0.300	0.00007
Year-Month-Fixed Effects		Yes
Number of Observations		1,525,787

We find strong evidence that sophisticated investors in our sample tend to invest more in stocks that generate higher returns (which is consistent with our model's prediction summarized in Proposition 7). Hence, we conclude that sophisticated investors in our sample exhibit superior stock-selection ability. This finding is consistent with a number of other studies that show the strong existence of stock-picking ability among sophisticated investors, such as actively managed equity mutual funds (e.g. Daniel, Grinblatt, Titman, and Wermers (1997), Cohen, Coval, and Pástor (2005), Kacperczyk, Sialm, and Zheng (2005), Kacperczyk and Seru (2007)). At the individual level, there is ample anecdotal evidence that shows superior investment ability of wealthy investors such as Warren Buffett or Carl Icahn.

Overall, our evidence is consistent with the premise of our economic mechanism that sophisticated investors are good at choosing assets and relocating their resources to the most profitable ones.

3.3.4 Discussion of Alternative Mechanisms

Our evidence so far strongly suggests that heterogeneity in information quality has a strong ability to explain cross-sectional and time-series patterns in capital income inequality, while simultaneously producing results that are consistent with other micro-level financial data. While the information friction constitutes a plausible economic mechanism, there might be other mechanisms which could also be used as explanations of the data. In this section, we consider two such explanations: heterogeneity in risk aversion and differences in transaction costs. We discuss their respective merits in terms of the ability to explain the empirical facts, both within the context of our model and also in a more general setting.

The first potential explanation is that income inequality in the data is driven by differences in risk aversion across economic agents. In particular, if one group of investors is less risk averse they would hold a greater share of risky assets with higher expected returns and hence would have higher expected capital income.

We argue that both within our CARA model and an alternative CRRA specification, such result is unlikely. In particular, heterogeneity in risk aversion per se would produce a growing (in difference of risk aversions) ownership of sophisticated investors in risky assets, but it would not generate any difference in investor-specific rates of return on equity. Additionally, a growing risk aversion disparity would generate a uniform proportional retrenchment of the high risk aversion agents from equity, which is inconsistent with evidence in Figure 4. That evidence, supported additionally by regression results in Section 3.3.3, suggests that the excess market performance is driven by sophisticated investors explicitly picking different portfolio shares (as opposed to pure timing). Finally, differences in risk aversion across agents cannot explain other micro-level facts in the data, such as the asymmetric ownership across asset classes (in Figure 4) or turnover profile of assets (in Table 5).

We are not the first ones to point out that preference-based approaches to explaining household portfolio choice suffer from serious drawbacks. Dumas (1989) and more recently Chien, Cole, and Lustig (2011) have argued that differences in risk preferences cannot account for observed differences in rates of returns across agents with different degrees of sophistication.

The second alternative mechanism aims to explain the data using differences in transaction costs across agents. To the extent that less sophisticated investors face higher transaction costs in risky asset markets they would be willing to participate less, as argued in Gomes and Michaelides (2005) and others.

While this explanation might have some merit to explain cross-sectional patterns in the data, we believe it is less likely to explain the time-series results. In particular, we observe that more sophisticated households generate significantly greater gap in their incomes over time, which is hard to reconcile with the fact that there was not much change in the overall quantity of transaction costs, as reported in French (2008). In fact, if anything, growth

in internet access and services made an access to more direct investing extremely easy and relatively less costly for the average citizen as opposed to just the few privileged ones.

Overall, while we believe that alternative mechanisms can be certainly at play it is hard to use them to explain the full set of results we document in this paper.

4 Concluding Remarks

What contributes to the growing income inequality across households? This question has been of great economic and policy relevance for at least several decades starting with a seminal work by Kuznets. We approach this question from the perspective of capital income that is known to be highly unequally distributed across individuals. We propose a theoretical information-based framework that links capital income derived from financial assets to a level of investor sophistication. Our model implies the presence of income inequality between sophisticated and unsophisticated investors that is growing in the extent of total sophistication in the market and in relative sophistication across investors. Additional predictions on asset ownership, market returns, and turnover help us pin down the economic mechanism and rule out alternative explanations. The quantitative predictions of the model match qualitatively and quantitatively the observed data.

Although our empirical findings are strictly based on the U.S. market, our model should have similar implications for other financial markets. For example, qualitatively, we know that income inequality in emerging markets tends to be even larger than the one documented for the U.S. To the extent that financial sophistication in such markets is much more skewed one could rationalize within our framework the differences in capital incomes. Similarly, the U.S. market is considered to be the most advanced in terms of its total sophistication, which is possibly why we find a greater dispersion in capital income compared to other developed markets, such as those in Europe or Asia.

More generally, one could argue that although the overall growth of investment resources and competition across investors with different skill levels are generally considered as a positive aspect of a well-functioning financial market, our work suggests that one should

assess any policy targeting overall information environment in financial markets as potentially exerting an offsetting and negative effect on socially relevant issues, such as distribution of income. We leave detailed evaluation of such policies for future research.

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Appendix

Theoretical Framework

The distribution of excess returns. Let $\widehat{R}_{ji}$ and $\widehat{V}_{ji}$ denote the mean and variance of the ex-ante distribution (in subperiod 1) of the posterior beliefs about excess returns, $\widehat{\mu}_{ji} - rp_i$. We have that $E_{1j}\left(\widehat{\mu}_{ji}\right) = \overline{z}_i$, and hence

$$\widehat{R}_i = \overline{z}_i - r\overline{p}_i,$$

the same across all investors j. The variance of posterior beliefs about excess returns is

$$\widehat{V}_{ji} = Var_{1j}\left(\widehat{\mu}_{ji}\right) + r^2\sigma_{pi}^2 - 2rCov\left(\widehat{\mu}_{ji}, p_i\right).$$

From the distribution of posterior beliefs,

$$Var_{1j}\left(\widehat{\mu}_{ji}\right) = \frac{Cov^2\left(z_i, s_{ji}\right)}{\sigma_{sji}^2}.$$

The signal structure implies that $Cov\left(z_i, s_{ji}\right) = \sigma_{sji}^2$ such that $Var_{1j}\left(\widehat{\mu}_{ji}\right) = \sigma_{sji}^2$. We compute $Cov\left(\widehat{\mu}_{ji}, p_i\right)$ exploiting the fact that posterior beliefs and prices are conditionally independent given payoffs, and hence

$$Cov\left(\widehat{\mu}_{ji}, p_i\right) = \frac{Cov\left(\widehat{\mu}_{ji}, z_i\right) Cov\left(z_i, p_i\right)}{\sigma_i^2},$$

with $Cov\left(z_i, p_i\right) = b_i\sigma_i^2$ and $Cov\left(\widehat{\mu}_{ji}, z_i\right) = \sigma_{sji}^2$. Then, $\widehat{V}_{ji}$ becomes

$$\widehat{V}_{ji} = \left(1 - 2rb_i\right)\sigma_{sji}^2 + r^2\sigma_{pi}^2.$$

Equivalently,

$$\widehat{V}_{ji} = \widehat{S}_i - \left(1 - 2rb_i\right)\sigma_{\delta ji}^2$$

where

$$\widehat{S}_i \equiv \left(1 - 2rb_i\right)\sigma_i^2 + r^2\sigma_{pi}^2$$

is the same across investors. ■

The objective function in (11). The investor's objective is to maximize ex-ante expected utility,

$$U_{1j} = \frac{1}{2\rho}\sum_{i=1}^{n}\left[\left(\frac{1}{\widehat{\sigma}_{ji}^2}\right)\left(\widehat{V}_{ji} + \widehat{R}_{ji}^2\right)\right],$$

where $\widehat{R}_{ji}$ and $\widehat{V}_{ji}$ denote the ex-ante mean and variance of excess returns, $(\widehat{\mu}_{ji} - rp_i)$. Using

$\widehat{\sigma}_{ji}^2 = \sigma_{\delta ji}^2$ and the distribution of excess returns derived above, the objective function becomes

$$U_{1j} = \frac{1}{2\rho}\sum_{i=1}^{n}\left(\frac{\widehat{S}_i + \widehat{R}_i^2}{\sigma_{\delta ji}^2}\right) - \frac{1}{2\rho}\sum_{i=1}^{n}\left(1 - 2rb_i\right),$$

where the second term is independent of the investor's choices. Hence the investor's objective becomes choosing the variance $\sigma_{\delta ji}^2$ for each asset i to solve

$$\max_{\{\sigma_{\delta ji}^2\}_{i=1}^{n}} \sum_{i=1}^{n}\left(\frac{\widehat{S}_i + \widehat{R}_i^2}{\sigma_{\delta ji}^2}\right),$$

where, from the derivation of excess returns, $\widehat{R}_i \equiv \overline{z}_i - r\overline{p}_i$, and $\widehat{S}_i \equiv \left(1 - 2rb_i\right)\sigma_i^2 + r^2\sigma_{pi}^2$. ■

The information constraint in (12). For each asset i, the entropy of $z_i \sim f\left(z_i\right) = \mathcal{N}\left(\overline{z}_i, \sigma_i^2\right)$ is

$$\begin{aligned}
H\left(z_i\right) &= \int f\left(z_i\right)\ln\frac{1}{f\left(z_i\right)}dz_i \\
&= \int f\left(z_i\right)\ln\left\{\sqrt{2\pi\sigma_i^2}\exp\left[\frac{\left(z_i - \overline{z}_i\right)^2}{2\sigma_i^2}\right]\right\}dz_i \\
&= \int f\left(z_i\right)\left[\frac{1}{2}\ln\left(2\pi\sigma_i^2\right) + \frac{\left(z_i - \overline{z}_i\right)^2}{2\sigma_i^2}\right]dz_i \\
&= \frac{1}{2}\ln\left(2\pi\sigma_i^2\right) + \frac{1}{2\sigma_i^2}\int f\left(z_i\right)\left(z_i - \overline{z}_i\right)^2 dz_i = \\
&= \frac{1}{2}\ln\left(2\pi e\sigma_i^2\right).
\end{aligned}$$

The signal structure, $z_i = s_{ji} + \delta_{ji}$, implies that

$$\begin{aligned}
I\left(z_i; s_{ji}\right) &= H\left(z_i\right) + H\left(s_{ji}\right) - H\left(z_i, s_{ji}\right) \\
&= \frac{1}{2}\log\left(2\pi e\sigma_i^2\right) + \frac{1}{2}\log\left(2\pi e\sigma_{sji}^2\right) - \frac{1}{2}\log\left[\left(2\pi e\right)^2\left|\Sigma_{z_i s_{ji}}\right|\right] \\
&= \frac{1}{2}\log\left(\frac{\sigma_i^2\sigma_{sji}^2}{\left|\Sigma_{z_i s_{ji}}\right|}\right) = \frac{1}{2}\log\left(\frac{\sigma_i^2}{\sigma_{\delta ji}^2}\right),
\end{aligned}$$

where $\left|\Sigma_{z_i s_{ji}}\right|$ is the determinant of the variance-covariance matrix of z_i and s_{ji},

$$\left|\Sigma_{z_i s_{ji}}\right| = \sigma_{sji}^2\sigma_{\delta ji}^2.$$

Across assets,

$$I\left(z;s_j\right)=\sum_{i=1}^{n}I\left(z_i;s_{ji}\right)=\frac{1}{2}\sum_{i=1}^{n}\log\left(\frac{\sigma_i^2}{\sigma_{\delta ji}^2}\right)=\frac{1}{2}\log\left(\prod_{i=1}^{n}\frac{\sigma_i^2}{\sigma_{\delta ji}^2}\right).$$

Hence, the information constraint becomes

$$\prod_{i=1}^{n}\left(\frac{\sigma_i^2}{\sigma_{\delta ji}^2}\right)\leq e^{2K_j},$$

which completes the derivation. ■

Proof of Proposition 1. The linear objective function and the convex constraint imply a corner solution for the optimal allocation of attention for each investor. Let l_j index the asset to which investor j dedicates her entire capacity. Then, from the information constraint, the investor's posterior variance is given by

$$\sigma_{\delta ji}^2=\begin{cases}e^{-2K_j}\sigma_i^2 & \text{if } i=l_j,\\ \sigma_i^2 & \text{if } i\neq l_j.\end{cases}$$

The investor's objective becomes

$$\sum_{i=1}^{n}\left(\frac{\widehat{S}_i+\widehat{R}_i^2}{\sigma_{\delta ji}^2}\right)=\left(e^{2K_j}-1\right)\left(\frac{\widehat{S}_{l_j}+\widehat{R}_{l_j}^2}{\sigma_{l_j}^2}\right)+\sum_{i=1}^{n}\left(\frac{\widehat{S}_i+\widehat{R}_i^2}{\sigma_i^2}\right)$$

Let G_i denote the utility gain of asset i, defined as

$$G_i\equiv\frac{\widehat{S}_i+\widehat{R}_i^2}{\sigma_i^2}.$$

Then, the investor's objective becomes

$$\sum_{i=1}^{n}\left(\frac{\widehat{S}_i+\widehat{R}_i^2}{\sigma_{\delta ji}^2}\right)=\left(e^{2K_j}-1\right)G_{l_j}+\sum_{i=1}^{n}G_i.$$

Since $e^{2K_j}>1$, the objective is maximized by allocating all capacity to the asset with the largest utility gain: $l_j\in\arg\max_i G_i$. ■

Proof of Proposition 2. Market clearing for each asset $i\notin L$ that is not learned about in equilibrium is given by

$$\frac{\overline{z}_i-rp_i}{\rho\sigma_i^2}=x_i.$$

Market clearing for each asset $i \in L$ that is learned about in equilibrium is given by

$$\int_0^1 \left(\frac{s_{ji} - rp_i}{\rho e^{-2K_j} \sigma_i^2} \right) dj = x_i.$$

Let $m_i \in (0, 1]$ denote the mass of investors learning about asset $i \in L$. Since the gain factor G_i for each i is the same across all investors, regardless of investor type, the participation of sophisticated and unsophisticated investors in learning about a particular asset will be proportional to their mass in the population. Hence, let M_{1i} denote the set of sophisticated investors who choose to learn about asset i, of measure $\lambda m_i \geq 0$, and let M_{2i} denote the set of unsophisticated investors who choose to learn about asset i, of measure $(1 - \lambda) m_i \geq 0$. Then, market clearing becomes

$$\int_{M_{1i}} \left(\frac{s_{ji} - rp_i}{e^{-2K_1} \rho \sigma_i^2} \right) dj + \int_{M_{2i}} \left(\frac{s_{ji} - rp_i}{e^{-2K_2} \rho \sigma_i^2} \right) dj + (1 - m_i) \left(\frac{\overline{z}_i - rp_i}{\rho \sigma_i^2} \right) = x_i.$$

Each signal s_{ji} received by an investor of type j is a normally distributed random variable whose mean is weighted average of the true realization, z_i, and the prior, $\overline{z}_i$:

$$E\left(s_{ji} | z_i\right) = \left(1 - e^{-2K_j}\right) z_i + e^{-2K_j} \overline{z}_i.$$

Hence

$$\int_{M_{1i}} s_{ji} dj = \lambda m_i \left[\left(1 - e^{-2K_1}\right) z_i + e^{-2K_1} \overline{z}_i \right]$$

and

$$\int_{M_{2i}} s_{ji} dj = (1 - \lambda) m_i \left[\left(1 - e^{-2K_2}\right) z_i + e^{-2K_2} \overline{z}_i \right].$$

The market clearing equation above can be written as

$$\alpha_1 \overline{z}_i + \alpha_2 z_i - x_i = \alpha_3 r p_i,$$

where

$$\begin{aligned}
\alpha_1 &\equiv \frac{\lambda m_i}{\rho \sigma_i^2} + \frac{(1 - \lambda) m_i}{\rho \sigma_i^2} + \frac{1 - m_i}{\rho \sigma_i^2} \\
\alpha_2 &\equiv \frac{\lambda m_i}{\rho \sigma_i^2} \left(e^{2K_1} - 1\right) + \frac{(1 - \lambda) m_i}{\rho \sigma_i^2} \left(e^{2K_2} - 1\right) \\
\alpha_3 &\equiv \frac{\lambda m_i}{\rho \sigma_i^2} e^{2K_1} + \frac{(1 - \lambda) m_i}{\rho \sigma_i^2} e^{2K_2} + \frac{1 - m_i}{\rho \sigma_i^2}.
\end{aligned}$$

Defining

$$\phi \equiv \lambda \left(e^{2K_1} - 1\right) + (1 - \lambda) \left(e^{2K_2} - 1\right),$$

we obtain the identification of the coefficients in

$$p_i = a_i + b_i z_i - c_i x_i$$

as

$$a_i = \frac{\overline{z}_i}{r\left(1+\phi m_i\right)}, \quad b_i = \frac{\phi m_i}{r\left(1+\phi m_i\right)}, \quad c_i = \frac{\rho \sigma_i^2}{r\left(1+\phi m_i\right)},$$

which completes the proof. ■

Proof of Proposition 3. For any K_1, $K_2 > 0$, at least one asset will be learned about in the economy.

(i) First, suppose that only one asset is learned about, and let this asset be denoted by l: $m_l = 1$ and $m_i = 0$ for all $i \neq l$. This implies that $G_l > G_i$ for any $i \neq l$, i.e.

$$\frac{1+\rho^2 \xi_l}{\left(1+\phi\right)^2} > 1 + \rho^2 \xi_i,$$

or, equivalently,

$$\frac{1+\rho^2 \xi_l}{1+\rho^2 \xi_i} > \left(1+\phi\right)^2.$$

Since $1+\phi > 1$, the inequality holds only if $\xi_l > \xi_i$ for any $i \neq l$. We have assumed, without loss of generality, that assets in the economy are ordered such that, for all $i \in \{1, ..., n-1\}$, $\xi_i > \xi_{i+1}$. Hence, $l = 1$: the asset learned about is the asset with the highest value of ξ_1. Moreover, since the left-hand side of the inequality is decreasing in ξ_i, the threshold for starting to learn about the second asset, namely the point at which the inequality above no longer holds, taking shocks and risk aversion as given, is

$$\phi_1 \equiv \sqrt{\frac{1+\rho^2 \xi_1}{1+\rho^2 \xi_2}} - 1.$$

At this threshold market capacity, investors begin learning about the second asset, ξ_2.

Second, let $0 < \phi < \phi_1$. Suppose that more than one asset is learned about. To induce investors to dedicate capacity to more than one asset, it must be the case that equilibrium masses the utility gains from learning across these assets. In particular, suppose that the first L assets are learned about (since the gains are increasing in ξ_i). Then it must be the case that $G_1 = G_2$, i.e.

$$\frac{1+\rho^2 \xi_1}{\left(1+\phi m_1\right)^2} = \frac{1+\rho^2 \xi_2}{\left(1+\phi m_2\right)^2},$$

with $m_1 + m_2 + \Delta = 1$, where Δ is the residual mass allocated to the remaining $L-2$ assets. Equivalently,

$$\frac{1+\phi m_2}{1+\phi m_1} = \sqrt{\frac{1+\rho^2 \xi_1}{1+\rho^2 \xi_2}}.$$

Using the definition of ϕ_1, this becomes

$$\frac{1+\phi m_2}{1+\phi m_1} = \phi_1 + 1,$$

which, rearranging, yields

$$\phi_1 + \phi_1\phi m_1 + \phi m_1 = \phi m_2$$
$$\Leftrightarrow (\phi_1 - \phi) + \phi_1\phi m_1 + 2\phi m_1 + \phi\Delta = 0.$$

But this is a contradiction, since we have assumed that $\phi < \phi_1$ and all the other terms are positive. Hence, for $0 < \phi < \phi_1$, only one asset is learned about in equilibrium.

(ii) For $\phi \geq \phi_1$, at least two assets are learned about in equilibrium. The gain factors must be equated for all assets learned about in equilibrium, $G_1 = G_k$, for any $k \in \{2, ..., L\}$, which yields

$$\frac{1+\phi m_1}{1+\phi m_k} = \sqrt{\frac{1+\rho^2\xi_1}{1+\rho^2\xi_k}}, \quad \forall k \in \{2, ..., L\}.$$

(iii) Any asset not learned about in equilibrium must have a strictly lower gain factor, $G_h < G_1$, for any $h \in \{L+1, ..., n\}$, or equivalently,

$$1 + \rho^2\xi_h < \frac{1+\rho^2\xi_1}{(1+\phi m_1)^2}.$$

■

Proof of Proposition 4. We begin by deriving expressions for the masses m_i. The necessary and sufficient set of conditions for determining $\{m_i\}_{i=1}^n$ in equilibrium are

$$\frac{1+\phi m_i}{1+\phi m_1} = c_{i1} \qquad \forall i \in \{2, ..., k\}, \tag{31}$$

$$\sum_{i=1}^{k} m_i = 1. \tag{32}$$

and $m_i = 0$ for any $i \in \{k+1, ..., n\}$, where

$$c_{i1} \equiv \sqrt{\frac{1+\rho^2\xi_i}{1+\rho^2\xi_1}} < 1.$$

The first set of equalities in (31) yields

$$m_i = c_{i1}m_1 - \frac{1}{\phi}(1 - c_{i1}) \qquad \forall i \in \{2, ..., k\}. \tag{33}$$

Plugging m_i into the feasibility constraint (32), we obtain

$$1 = \sum_{i=1}^{k} m_i = m_1 + m_1 \sum_{i=2}^{k} c_{i1} - \frac{1}{\phi} \sum_{i=2}^{k} (1 - c_{i1})$$

which results in a solution for m_1 given by

$$m_1 = \frac{1 + \frac{1}{\phi} \sum_{i=2}^{k} (1 - c_{i1})}{1 + \sum_{i=2}^{k} c_{i1}}. \tag{34}$$

Note that equation (33) implies that the masses m_i are strictly decreasing across the assets that are learned about:

$$m_1 > m_2 > ... > m_k,$$

since $c_{11} = 1$ and $c_{i1} < 1$ is strictly decreasing in i.

(i) Note that for any $\phi > 0$, m_1 changes continuously with ϕ. Differentiating equation (34) with respect to ϕ, we obtain

$$\frac{dm_1}{d\phi} = -\frac{1}{\phi^2} \left[\frac{\sum_{i=2}^{k} (1 - c_{i1})}{1 + \sum_{i=2}^{k} c_{i1}} \right] < 0,$$

since for all $i > 1$, $0 < c_{i1} < 1$. Hence, m_1 is decreasing in ϕ.

Likewise, for each asset $i \in \{2, ..., k\}$, m_i changes continuously with ϕ. Differentiating (33) with respect to ϕ, we obtain

$$\frac{dm_i}{d\phi} = c_{i1} \left(\frac{dm_1}{d\phi} \right) + \frac{1}{\phi^2} (1 - c_{i1}).$$

Substituting in the derivative of m_1 and rearranging we obtain

$$\frac{dm_i}{d\phi} = \frac{1}{\phi^2} \left[1 - c_{i1} \left(\frac{k}{\sum_{j=1}^{k} c_{j1}} \right) \right],$$

where we have used the fact that $c_{11} = 1$. Since $c_{i1} < 1$ and c_{i1} is strictly decreasing in i, then $\frac{dm_i}{d\phi}$ is increasing in i.

Next, consider the case of a local increase in ϕ to some $\phi' < \phi_k$, such that no new assets are learned about in equilibrium. Since $\Sigma_i m_i = 1$, there must be at least one asset $i > 2$ for which $\frac{dm_i}{d\phi} > 0$, and this asset defines the cutoff $\bar{i}$.

Finally, consider the case of an increase in ϕ to some ϕ' with $\phi_k \leq \phi' < \phi_{k+x}$, such that $x \geq 1$ new assets are learned about in equilibrium[17]. Let the equilibrium masses associated with aggregate capacity ϕ' be denoted by m_i' for $i \in \{1, ..., k + x\}$. For the new assets, $m_i' > m_i = 0$ for all $i \in \{k + 1, ..., k + x\}$, hence the mass is increasing in ϕ. From above,

[17] With $x \leq n - k$, since there are only n assets in the economy.

$m_1' < m_1$ and $m_i' < m_i$ for at least one asset $i \in \{1, ..., k\}$. Since $\Sigma_i m_i = 1$, the new cutoff will be some $\bar{i}' > \bar{i}$.

(ii) First, consider the case of a local increase in ϕ to some $\phi' < \phi_k$, such that no new assets are learned about in equilibrium. For assets that are not learned about, $i > k$, $m_i = 0$ both before and after the capacity increase, hence $\frac{d(\phi m_i)}{d\phi} = 0$. By Proposition 3,

$$\frac{1 + \phi m_i}{1 + \phi m_l} = c_{il}, \ \forall i, l \leq k.$$

where $c_{il} \equiv \sqrt{\frac{1+\rho^2 \xi_i}{1+\rho^2 \xi_l}} > 0$. Equivalently,

$$1 + \phi m_i = (1 + \phi m_l)\, c_{il}.$$

Totally differentiating, $\frac{d(\phi m_i)}{d\phi}$ is given by

$$m_i + \phi \frac{dm_i}{d\phi} = \left(m_l + \phi \frac{dm_l}{d\phi} \right) c_{il}. \tag{35}$$

Suppose that there exists an asset i such that $\frac{d(\phi m_i)}{d\phi} \leq 0$. Then for all other assets $l \leq k$, $l \neq i$, the right-hand side of equation (35) must be negative. Since $c_{il} > 0$, $m_l > 0$, and $\phi > 0$, then we must have that $\frac{dm_l}{d\phi} < 0$ as well. Hence for all assets learned about, the mass decreases with the increase in ϕ. But $\Sigma_i m_i = 1$, hence there must be at least one asset for which $\frac{dm_i}{d\phi} \geq 0$, which is a contradiction. Hence for all $i \leq k$, $\frac{d(\phi m_i)}{d\phi} > 0$.

Second, consider the case of an increase in ϕ to some ϕ' with $\phi_k \leq \phi' < \phi_{k+x}$, such that $x \geq 1$ new assets are learned about in equilibrium[18]. For assets which remain passively traded, $i > k + x$, $m_i = m_i' = 0$; hence, there is no change in the aggregate capacity allocated to these assets. For the new assets that are actively traded, $i \in \{k + 1, ..., k + x\}$, $m_i' > m_i = 0$; hence, $\phi' m_i' > \phi m_i$. Finally, from Proposition 3, an asset i that was actively traded both before and after the increase, $i \leq k$, had, prior to the increase, a utility gain strictly larger than that of an asset l that was previously not learned about:

$$c_{il} - 1 - \phi m_i > 0$$

and has, after the increase, a utility gain equal to that of asset l,

$$1 + \phi' m_i' = (1 + \phi' m_l')\, c_{il}.$$

Substituting the inequality for c_{il} into the equality above, $\phi' m_i' - \phi m_i > \phi' m_l' (1 + \phi m_i)$. Since the right-hand side is positive, it follows that $\phi' m_i' > \phi m_i$, which completes the proof.

(iii) Let $K_1 = K$ and $K_2 = \gamma K$, for some $\gamma \in (0, 1)$, and consider the case of a local

[18] With $x \leq n - k$, since there are only n assets in the economy.

increase in capacity K such that no new assets are learned about in equilibrium. Let $m_{i\phi} \equiv \frac{dm_i}{d\phi}$. The derivatives we are interested in are

$$\begin{aligned}\frac{d[m_i(e^{2K}-1)]}{dK} &= 2e^{2K}m_i + m_{i\phi}(e^{2K}-1)\frac{d\phi}{dK} \\ \frac{d[m_i(e^{2K\gamma}-1)]}{dK} &= 2\gamma e^{2K\gamma}m_i + m_{i\phi}(e^{2K\gamma}-1)\frac{d\phi}{dK}\end{aligned}$$

where

$$\frac{d\phi}{dK} = 2\lambda e^{2K} + 2\gamma(1-\lambda)e^{2K\gamma} > 0.$$

First, consider the case in which $m_{i\phi} > 0$. Then, since $e^{2K} > e^{2K\gamma} > \gamma e^{2K\gamma}$, we have that

$$\frac{d[m_i(e^{2K}-1)]}{dK} > \frac{d[m_i(e^{2K\gamma}-1)]}{dK} > 0.$$

Next, consider the case in which $m_{i\phi} < 0$. Factoring out $2e^{2K}$ yields

$$\begin{aligned}\frac{d\left[m_i\left(e^{2K}-1\right)\right]}{dK} &= 2e^{2K}\left\{m_i + m_{i\phi}\left(e^{2K}-1\right)\left[\lambda + (1-\lambda)\gamma e^{2K(\gamma-1)}\right]\right\} \\ &= 2e^{2K}\left\{m_i + m_{i\phi}\left[\lambda\left(e^{2K}-1\right) + (1-\lambda)\gamma\left(e^{2K}-1\right)e^{2K(\gamma-1)}\right]\right\} \\ &= 2e^{2K}\left\{m_i + m_{i\phi}\left[\lambda\left(e^{2K}-1\right) + (1-\lambda)\gamma\left(e^{2K\gamma}-e^{2K(\gamma-1)}\right)\right]\right\} \\ &> 2e^{2K}\left\{m_i + m_{i\phi}\left[\lambda\left(e^{2K}-1\right) + (1-\lambda)\left(e^{2K\gamma}-1\right)\right]\right\},\end{aligned}$$

where the inequality follows from $m_{i\phi} < 0$, $\gamma < 1$, $e^{2K} > 1$, and $e^{2K(\gamma-1)} < 1$. Using the definition of ϕ, we obtain

$$\frac{d\left[m_i\left(e^{2K}-1\right)\right]}{dK} > 2e^{2K}\left(m_i + \phi m_{i\phi}\right) = 2e^{2K}\left[\frac{d\left(\phi m_i\right)}{d\phi}\right] > 0,$$

where the last inequality follows from part (ii) above.

Note that

$$\lambda\left\{\frac{d[m_i(e^{2K}-1)]}{dK}\right\} + (1-\lambda)\left\{\frac{d[m_i(e^{2K\gamma}-1)]}{dK}\right\} = \left[\frac{d\left(\phi m_i\right)}{d\phi}\right]\left(\frac{d\phi}{dK}\right).$$

Plugging in $d\phi/dK$,

$$\lambda\left\{\frac{d[m_i(e^{2K}-1)]}{dK}\right\} + (1-\lambda)\left\{\frac{d[m_i(e^{2K\gamma}-1)]}{dK}\right\} = \lambda\left\{2e^{2K}\left[\frac{d\left(\phi m_i\right)}{d\phi}\right]\right\} + (1-\lambda)\left\{2\gamma e^{2K\gamma}\left[\frac{d\left(\phi m_i\right)}{d\phi}\right]\right\}.$$

Since the first term on the left-hand side is greater than the first term on the right-hand side, and since $2e^{2K} > 2\gamma e^{2K\gamma}$, it must be the case that the second element of this weighted

average is smaller, which implies that

$$\frac{d[m_i(e^{2K}-1)]}{dK} > \frac{d[m_i(e^{2K\gamma}-1)]}{dK},$$

which concludes the proof. ■

Proof of Proposition 5. We consider the choice of an individual investor, taking the choices of all other investors as given, characterized by the solution in the main text.

First, suppose that it is possible for an individual investor to observe the price p_i with no idiosyncratic error. The information contained in the price about the payoff is

$$I(z_i; p_i) = \frac{1}{2}\log\left(\frac{\sigma_i^2}{\sigma_{yi}^2}\right)$$

where σ_{yi}^2 is the variance of the investor's posterior beliefs about the payoff of asset i conditional on the price of asset i, and it is given by

$$\sigma_{yi}^2 = \frac{c_i^2\sigma_{xi}^2\sigma_i^2}{c_i^2\sigma_{xi}^2 + b_i^2\sigma_i^2}.$$

Hence

$$I(z_i; p_i) = \frac{1}{2}\log\left(\frac{c_i^2\sigma_{xi}^2 + b_i^2\sigma_i^2}{c_i^2\sigma_{xi}^2}\right).$$

We need to consider three cases: (1) No other investor is learning through their private signals about this asset, in which case $b_i = 0$ and $I(z_i; p_i) = 0$, making prices irrelevant for the inference regarding payoffs.

(2) Other investors are learning through their private signals, making the price of this asset informative, and the individual investor's capacity is sufficiently high to process the price, namely $I(z_i; p_i) \leq K_j$. Equivalently,

$$\frac{c_i^2\sigma_{xi}^2 + b_i^2\sigma_i^2}{c_i^2\sigma_{xi}^2} \leq e^{2K_j}.$$

Using the solution for equilibrium prices,

$$\left(\frac{\phi m_i}{\rho\sigma_i\sigma_{xi}}\right)^2 \leq e^{2K_j} - 1. \tag{36}$$

Hence the quantity of information conveyed by prices is determined by the amount of capacity (ϕm_i) that the market overall allocates to learning about asset i. If the market's allocation happens to coincide with the individual investor's allocation, then the investor is indifferent between learning from prices and learning from her own private signals. But in general, the exogenous price signal can do no better than the optimally chosen endogenous signal.

(3) Other investors are learning through their private signals, making the price of this

asset informative, but the individual investor's capacity is too low to process the price, namely

$$\frac{c_i^2\sigma_{xi}^2 + b_i^2\sigma_i^2}{c_i^2\sigma_{xi}^2} > e^{2K_j}.$$

In this case, we need to relax the assumption that the investor can observe the price perfectly, and consider the case in which the investor treats the price as any other random variable that cannot be processed perfectly for free. Suppose that the investor allocates capacity to learning the price of asset i. This investor will observe a compressed representation of the price, s_{ji}^p, that is the result of decomposing the price into the part that is processed by the investor and the part that is lost in the compression:

$$p_i = s_{ji}^p + \varepsilon_{ji}.$$

Using the same two independence assumptions that were used in setting up the private payoff signal,

$$\begin{aligned} s_{ji}^p &\sim \mathcal{N}\left(\overline{p}_i, \sigma_{spji}^2\right), \\ \varepsilon_{ji} &\sim \mathcal{N}\left(0, \sigma_{\varepsilon ji}^2\right), \\ \sigma_{pi}^2 &= \sigma_{spji}^2 + \sigma_{\varepsilon ji}^2. \end{aligned}$$

These signals must satisfy the capacity constraint,

$$\prod_{i=1}^{n}\left(\frac{\sigma_{pi}^2}{\sigma_{\varepsilon ji}^2}\right) \leq e^{2K_j},$$

where

$$I\left(p_i; s_{ji}^p\right) = \frac{1}{2}\log\left(\frac{\sigma_{pi}^2}{\sigma_{\varepsilon ji}^2}\right).$$

The investor acquires the same quantity of information as before. However, the information that this signal vector conveys about the payoff vector z will be shown to be in fact smaller:

$$I\left(z; s_j^p\right) < I\left(z; s_j\right).$$

And as a result the investors' posterior beliefs will be more uncertain.

By our independence assumptions,

$$I\left(z; s_j^p\right) = \sum_{i=1}^{n} I\left(z_i; s_{ji}^p\right)$$

and

$$\begin{aligned} I\left(z_i; s_{ji}^p\right) &= H\left(z_i\right) + H\left(s_{ji}^p\right) - H\left(z_i, s_{ji}^p\right) \\ &= \frac{1}{2}\log\left(2\pi e\sigma_i^2\right) + \frac{1}{2}\log\left(2\pi e\sigma_{spji}^2\right) - \frac{1}{2}\log\left[\left(2\pi e\right)^2\left|\Sigma_{z_i s_{pji}}\right|\right] \\ &= \frac{1}{2}\log\left(\frac{\sigma_i^2\sigma_{spji}^2}{\left|\Sigma_{z_i sp_{ji}}\right|}\right), \end{aligned}$$

where $\left|\Sigma_{z_i s_{pji}}\right|$ is the determinant of the variance-covariance matrix of z_i and s_{ji}^p,

$$\left|\Sigma_{z_i s_{pji}}\right| = \sigma_i^2\sigma_{spji}^2 - Cov^2\left(z_i, s_{pji}\right).$$

Hence

$$\begin{aligned} \left|\Sigma_{z_i s_{pji}}\right| &= \sigma_i^2\sigma_{spji}^2 - \frac{b_i^2\sigma_i^2\sigma_i^2\sigma_{spji}^2\sigma_{spji}^2}{\sigma_{pi}^2\sigma_{pi}^2} \\ &= \sigma_i^2\sigma_{spji}^2\left[1 - \frac{b_i^2\sigma_i^2\sigma_{spji}^2}{\sigma_{pi}^2\sigma_{pi}^2}\right] \\ &= \sigma_i^2\sigma_{spji}^2\left[\frac{\sigma_{pi}^2\sigma_{pi}^2 - b_i^2\sigma_i^2\sigma_{spji}^2}{\sigma_{pi}^2\sigma_{pi}^2}\right] \end{aligned}$$

so that

$$I\left(z_i; s_{ji}^p\right) = \frac{1}{2}\log\left(\frac{\sigma_{pi}^2\sigma_{pi}^2}{\sigma_{pi}^2\sigma_{pi}^2 - b_i^2\sigma_i^2\sigma_{spji}^2}\right).$$

Using $p_i = s_{ji}^p + \varepsilon_{ji}$

$$\begin{aligned} I\left(z_i; s_{ji}^p\right) &= \frac{1}{2}\log\left(\frac{\sigma_{pi}^2\sigma_{pi}^2}{\sigma_{pi}^2\sigma_{pi}^2 - b_i^2\sigma_i^2\sigma_{pi}^2 + b_i^2\sigma_i^2\sigma_{\varepsilon ji}^2}\right) \\ &= \frac{1}{2}\log\left(\frac{\sigma_{pi}^2}{\sigma_{pi}^2 - b_i^2\sigma_i^2 + \frac{b_i^2\sigma_i^2}{\sigma_{pi}^2}\sigma_{\varepsilon ji}^2}\right). \end{aligned}$$

Using $\sigma_{pi}^2 = b_i^2\sigma_i^2 + c_i^2\sigma_{xi}^2$,

$$I\left(z_i; s_{ji}^p\right) = \frac{1}{2}\log\left(\frac{\sigma_{pi}^2}{c_i^2\sigma_{xi}^2 + \frac{b_i^2\sigma_i^2}{\sigma_{pi}^2}\sigma_{\varepsilon ji}^2}\right).$$

Suppose that the investor chooses to allocate all her capacity to learning about the price of

a particular asset l. Then, for this asset, the information obtained about payoffs is

$$I\left(z_i; s_{ji}^p\right) = \frac{1}{2}\log\left(\frac{\sigma_{pi}^2}{c_i^2\sigma_{xi}^2 + b_i^2\sigma_i^2 e^{-2K_j}}\right)$$

or

$$I\left(z_i; s_{ji}^p\right) = \frac{1}{2}\log\left(\frac{e^{2K_j}\sigma_{pi}^2}{e^{2K_j}c_i^2\sigma_{xi}^2 + b_i^2\sigma_i^2}\right)$$

$$I\left(z_i; s_{ji}^p\right) = \frac{1}{2}\log\left(e^{2K_j}\right) + \frac{1}{2}\log\left(\frac{\sigma_{pi}^2}{e^{2K_j}c_i^2\sigma_{xi}^2 + b_i^2\sigma_i^2}\right)$$

versus, when learning directly about the payoff,

$$I\left(z_i; s_{ji}\right) = \frac{1}{2}\log\left(e^{2K_j}\right).$$

In equilibrium,

$$e^{2K_j}c_i^2\sigma_{xi}^2 + b_i^2\sigma_i^2 > \sigma_{pi}^2$$

since, using the formula for the variance of prices, and the fact that $e^{2K_j} > 1$ for any $K_j > 0$,

$$\left(e^{2K_j} - 1\right)c_i^2\sigma_{xi}^2 > 0.$$

Hence

$$I\left(z_i; s_{ji}^p\right) < I\left(z_i; s_{ji}\right),$$

which completes the proof. ■

Analytical Predictions

Proof of Proposition 6. Using equations (27)-(28), the difference in profits for asset i is given by

$$\pi_{1i} - \pi_{2i} = \frac{m_i\left(e^{2K_1} - e^{2K_2}\right)\left(z_i - rp_i\right)^2}{\rho\sigma_i^2} \geq 0.$$

This difference is zero if $m_i = 0$ or $K_1 = K_2$. For $K_1 > K_2$, it is strictly positive for assets that are learned about in equilibrium (i.e., if $m_i > 0$). Also, $K_1 > K_2 > 0$ implies $\phi > 0$. It follows that $m_i > 0$ for at least one i. ■

Proof of Proposition 7. Using equations (27)-(28), the ownership difference for asset i becomes

$$\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{(1-\lambda)} = m_i\left(e^{2K_1} - e^{2K_2}\right)\left(\frac{z_i - rp_i}{\rho\sigma_i^2}\right).$$

(i) For $i > k$, $m_i = 0$, and hence the ownership difference is equal to zero. For $i \leq k$,

$m_i > 0$, and the expected ownership differential is given by

$$E\left\{\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{(1-\lambda)}\right\} = \frac{m_i \overline{x}_i \left(e^{2K_1} - e^{2K_2}\right)}{1 + \phi m_i}, \tag{37}$$

where we have used the fact that expected excess returns are, by equations (19) and (20),

$$E\left(z_i - rp_i\right) = \frac{\rho \sigma_i^2 \overline{x}_i}{1 + \phi m_i}. \tag{38}$$

Since $K_1 > K_2$ and $\overline{x}_i > 0$, the result follows.

(ii) First, we show that if $E(z_i - rp_i) > E(z_l - rp_l)$, then $m_i > m_l$. Since $i, l < k$, their gain factors are equated, $G_i = G_l$. Using (38), and the fact that $\overline{x}_i = \overline{x}$ and $\sigma_{xi} = \sigma_x$ for all i, the gain factor of asset i can be written as

$$G_i = \frac{1 + \rho^2\left(\sigma_x^2 + \overline{x}^2\right)\sigma_i^2}{\rho^2 \overline{x}^2 \sigma_i^4}\left[E(z_i - rp_i)\right]^2,$$

and a corresponding expression holds for G_l. The inequality in excess returns implies that

$$\frac{1 + \rho^2\left(\sigma_x^2 + \overline{x}^2\right)\sigma_i^2}{\sigma_i^4} < \frac{1 + \rho^2\left(\sigma_x^2 + \overline{x}^2\right)\sigma_l^2}{\sigma_l^4},$$

which reduces to $\sigma_i^2 > \sigma_l^2$. Proposition 3 implies that m_i is increasing in ξ_i, which, under the maintained assumptions that $\overline{x}_i$ and σ_{xi}^2 are equal across i, implies that m_i is increasing in σ_i^2. Hence, $m_i > m_l$.

Next, from the expression for the expected ownership differential in (37), the difference in expected relative ownership across the two assets is

$$E\left\{\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{(1-\lambda)}\right\} - E\left\{\frac{Q_{1l}}{\lambda} - \frac{Q_{2l}}{(1-\lambda)}\right\} = \frac{\overline{x}\left(e^{2K_1} - e^{2K_2}\right)(m_i - m_l)}{(1 + \phi m_i)(1 + \phi m_l)} > 0,$$

which completes the proof. ■

Proof of Proposition 8. Using equations (27)-(28), the state-by-state ownership difference for asset i becomes

$$\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{(1-\lambda)} = m_i\left(e^{2K_1} - e^{2K_2}\right)\left(\frac{z_i - rp_i}{\rho \sigma_i^2}\right).$$

If $i \leq k$, the equilibrium level of $m_i > 0$ is an ex-ante decision, and hence it is constant across realizations. The result follows. ■

Proof of Proposition 9. Our derivation keeps the aggregate information quantity ϕ constant, and hence the masses m_i unchanged, by equation (23), which in turn implies that prices also remain unchanged, by equations (19) and (20). By equations (26), (27), and (28),

relative capital income is

$$\frac{\sum_i \pi_{1i}}{\sum_i \pi_{2i}} = \frac{\sum_i \{(\overline{z}_i - rp_i)(z_i - rp_i) + m_i(e^{2K_1} - 1)(z_i - rp_i)^2\}}{\sum_i \{(\overline{z}_i - rp_i)(z_i - rp_i) + m_i(e^{2K_2} - 1)(z_i - rp_i)^2\}}.$$

Since $K_1' > K_1$ and $K_2' < K_2$, each element of $\sum_i \pi_{1i}$ increases and each element of $\sum_i \pi_{2i}$ decreases. ■

Proof of Proposition 10. (i) From equations (19) and (20), the average equilibrium price of asset i can be expressed as

$$\overline{p}_i = \frac{1}{r}\left(\overline{z}_i - \frac{\rho\sigma_i^2 \overline{x}_i}{1 + \phi m_i}\right).$$

For $i > k$, $m_i = 0$, and $\overline{p}_i$ remains unchanged. For $i \leq k$, $m_i > 0$, and $\overline{p}_i$ is increasing in ϕm_i, which in turn is increasing in ϕ, per Proposition 4.

(ii) Equilibrium expected excess returns are

$$E\left(z_{it} - rp_{it}\right) = \frac{\rho\sigma_i^2 \overline{x}_i}{1 + \phi m_i}.$$

For $i > k$, $m_i = 0$, and expected excess returns remain unchanged. For $i \leq k$, $m_i > 0$, and the expected excess return of asset i is decreasing in ϕm_i, which in turn is increasing in ϕ, per Proposition 4. ■

Proof of Proposition 11. The average ownership difference is given by

$$E\left\{\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{(1-\lambda)}\right\} = \frac{m_i \overline{x}_i \left(e^{2K_1} - e^{2K_2}\right)}{1 + \phi m_i}.$$

For our designed deviation of information capacities, the aggregate information quantity ϕ constant, and hence the masses m_i are unchanged by equation (23). Polarization in e^{2K_1} versus e^{2K_2} gives the result. ■

Proof of Proposition 12. Using equations (27) and (28), the expected difference in asset ownership is given by

$$E\left\{\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{1-\lambda}\right\} = \frac{1 + m_i\left(e^{2K_1} - 1\right)}{1 + \phi m_i}\overline{x}_i - \frac{1 + m_i\left(e^{2K_2} - 1\right)}{1 + \phi m_i}\overline{x}_i.$$

Since average quantities have to be equal to average supply $\bar{x}_i$, it is enough to show that the first element of the sum is increasing. It is given by

$$\frac{dE\{\frac{Q_{1i}}{\lambda}\}}{dK} = \frac{\frac{d[m_i(e^{2K}-1)]}{dK}(1 + \phi m_i) - \frac{d\phi m_i}{d\phi}\frac{d\phi}{dK} m_i(e^{2K} - 1)}{(1 + \phi m_i)^2}\overline{x}_i$$

The sign of the expression is determined by the sign of

$$\begin{aligned} sign\left(\frac{dE\{\frac{Q_{1i}}{\lambda}\}}{dK}\right) &= sign\left(\frac{d[m_i(e^{2K}-1)]}{dK} - \frac{d\phi m_i}{d\phi}\frac{d\phi}{dK}m_i(e^{2K}-1)\frac{1}{1+\phi m_i}\right) \\ &= sign\left(\frac{d[m_i(e^{2K}-1)]}{dK} - \frac{d\phi m_i}{d\phi}(e^{2K}-1)\frac{2(\lambda e^{2K}+(1-\lambda)\gamma e^{2K\gamma})}{\lambda e^{2K}+(1-\lambda)e^{2K\gamma}}\right) \end{aligned}$$

In the proof of Proposition 4, we show that

$$\frac{d[m_i(e^{2K}-1)]}{dK} > 2e^{2K}\frac{d(\phi m_i)}{d\phi} > 0.$$

Using that expression, we obtain

$$sign\left(\frac{dE\{\frac{Q_{1i}}{\lambda}\}}{dK}\right) = sign\left(2e^{2K} - (2e^{2K}-2)\frac{\lambda e^{2K}+(1-\lambda)\gamma e^{2K\gamma}}{\lambda e^{2K}+(1-\lambda)e^{2K\gamma}}\right) > 0,$$

where the last inequality is guaranteed by $\frac{\lambda e^{2K}+(1-\lambda)\gamma e^{2K\gamma}}{\lambda e^{2K}+(1-\lambda)e^{2K\gamma}} < 1$. ■

Proof of Proposition 13. Using equations (26) and (27), the expected income from holding asset i for the sophisticated investors is given by:

$$E(\pi_{1i}) = \frac{m_i(e^{2K}-1)(\sigma_i^2+\rho^2\xi_i) - \phi m_i\sigma_i^2 + \rho^2\xi_i}{\rho(1+\phi m_i)^2}$$

and hence, the ratio of expected profits is

$$\frac{E\pi_{1i}}{E\pi_{2i}} = \frac{m_i(e^{2K}-1)(\sigma_i^2+\rho^2\xi_i) - \phi m_i\sigma_i^2 + \rho^2\xi_i}{m_i(e^{2K\gamma}-1)(\sigma_i^2+\rho^2\xi_i) - \phi m_i\sigma_i^2 + \rho^2\xi_i}$$

which can be written as

$$\frac{E\pi_{1i}}{E\pi_{2i}} = \frac{m_i(e^{2K}-1)\alpha - \phi m_i + \omega}{m_i(e^{2K\gamma}-1)\alpha - \phi m_i + \omega}$$

where

$$\alpha = 1 + \frac{\rho^2\xi_i}{\sigma_i^2} \text{ and } \omega = \alpha - 1.$$

Then consider the difference between old and new expected profit between two levels of overall capacity $K^* > K$, with K^* associated with the endogenous mass of investors m_i^* and K with m_i:

$$\Delta \equiv \frac{m_i^*(e^{2K^*}-1)\alpha - \phi^* m_i^* + \omega}{m_i^*(e^{2K^*\gamma}-1)\alpha - \phi^* m_i^* + \omega} - \frac{m_i(e^{2K}-1)\alpha - \phi m_i + \omega}{m_i(e^{2K\gamma}-1)\alpha - \phi m_i + \omega}.$$

We will show that $\Delta > 0$, i.e.

$$\frac{m_i^*(e^{2K^*}-1)\alpha - \phi^* m_i^* + \omega}{m_i^*(e^{2K^*\gamma}-1)\alpha - \phi^* m_i^* + \omega} > \frac{m_i(e^{2K}-1)\alpha - \phi m_i + \omega}{m_i(e^{2K\gamma}-1)\alpha - \phi m_i + \omega}.$$

Suppose that expected profits for each investor are positive (which must be true for them to hold the asset), then the above is equivalent to

$$[m_i^*(e^{2K^*}-1)\alpha-\phi^* m_i^*+\omega][m_i(e^{2K\gamma}-1)\alpha-\phi m_i+\omega] > [m_i(e^{2K}-1)\alpha-\phi m_i+\omega][m_i^*(e^{2K^*\gamma}-1)\alpha-\phi^* m_i^*+\omega].$$

Multiplying through and rearranging,

$$\begin{aligned}
& \alpha\omega[m_i^*(e^{2K^*}-1) - m_i(e^{2K}-1) - (m_i^*(e^{2K^*\gamma}-1) - m_i(e^{2K\gamma}-1))] \\
& +m_i^*(e^{2K^*}-1)\alpha m_i(e^{2K\gamma}-1)\alpha - m_i^*(e^{2K^*}-1)\alpha\phi m_i \\
& -\phi^* m_i^* m_i(e^{2K\gamma}-1)\alpha \\
> & \\
& +m_i(e^{2K}-1)\alpha m_i^*(e^{2K^*\gamma}-1)\alpha - m_i(e^{2K}-1)\alpha\phi^* m_i^* \\
& -\phi m_i m_i^*(e^{2K^*\gamma}-1)\alpha
\end{aligned}$$

Since the first term in square brackets is positive by Proposition 4, for our result to hold it is enough to show that (factoring out $\alpha m_i^* m_i > 0$)

$$\begin{aligned}
& \alpha[(e^{2K^*}-1)(e^{2K\gamma}-1) - (e^{2K}-1)(e^{2K^*\gamma}-1)] - (e^{2K^*}-1)\phi - \phi^*(e^{2K\gamma}-1) \\
> \quad & -(e^{2K}-1)\phi^* - \phi(e^{2K^*\gamma}-1)
\end{aligned}$$

which can be written as

$$\alpha[(e^{2K^*}-1)(e^{2K\gamma}-1) - (e^{2K}-1)(e^{2K^*\gamma}-1)] - [(e^{2K\gamma}-e^{2K})\phi^* + \phi(e^{2K^*}-e^{2K^*\gamma})] > 0$$

To obtain a closed-form expression for the second bracketed term, plug in the definition of ϕ, to obtain

$$\begin{aligned}
& (e^{2K\gamma}-e^{2K})[\lambda(e^{2K^*}-1) + (1-\lambda)(e^{2K^*\gamma}-1)] + (e^{2K^*}-e^{2K^*\gamma})[\lambda(e^{2K}-1) + (1-\lambda)(e^{2K\gamma}-1)] \\
= \quad & (e^{2K\gamma}-1)\lambda(e^{2K^*}-1) + (e^{2K\gamma}-1)(1-\lambda)(e^{2K^*\gamma}-1) \\
& -(e^{2K}-1)\lambda(e^{2K^*}-1) - (e^{2K}-1)(1-\lambda)(e^{2K^*\gamma}-1) \\
& +(e^{2K^*}-1)\lambda(e^{2K}-1) + (e^{2K^*}-1)(1-\lambda)(e^{2K\gamma}-1) \\
& -(e^{2K^*\gamma}-1)\lambda(e^{2K}-1) - (e^{2K^*\gamma}-1)(1-\lambda)(e^{2K\gamma}-1) \\
= \quad & (e^{2K^*}-1)(e^{2K\gamma}-1) - (e^{2K}-1)(e^{2K^*\gamma}-1)
\end{aligned}$$

Hence, a sufficient condition for $\Delta > 0$ is

$$(\alpha-1)[(e^{2K^*}-1)(e^{2K\gamma}-1)-(e^{2K}-1)(e^{2K^*\gamma}-1)]>0 \tag{39}$$

Since $\alpha>1$, it is enough to show that the term in square brackets is positive. To see that, define $f(K^*)=\left(e^{2K^*}-1\right)\left(e^{2K\gamma}-1\right)-\left(e^{2K}-1\right)\left(e^{2K^*\gamma}-1\right)$ and notice that $f(K)=0$. Furthermore, also notice that $f'(K^*=K)=0$ and $f'(K^*)=0$ for all K^* if $\gamma\in\{0,1\}$, and that $f'(K^*)$ has a single maximum with respect to γ for each K^*, and that maximum is attained at $\gamma\in(0,1)$. To see that, calculate

$$\begin{aligned} f'_\gamma &\equiv \frac{df'(K^*)}{d\gamma}=2\left(2Ke^{2K^*}e^{2K\gamma}-e^{2K^*\gamma}\left(e^{2K}-1\right)(1+2\gamma K)\right) \\ &> 2e^{2K^*\gamma}e^{2K}\left(2K+\left(\frac{1}{e^{2K}}-1\right)(1+2\gamma K)\right). \end{aligned}$$

Clearly, $f'_\gamma=0$ for a single value of γ. Additionally, by the arguments in the proof of Proposition 4, we know that at $\gamma=0$, $f'_\gamma=0$. Hence, for any K^*, K, $f'=0$ for $\gamma\in\{0,1\}$, f' is increasing in γ at $\gamma=0$ and f' has a single maximum with respect to γ. It follows that for all γ between zero and one, $f'(K^*)>0$, and hence equation (39) is satisfied. ■