Investor Sophistication and Capital Income Inequality*

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Abstract

We propose an information-based general equilibrium model to explain levels and growth rates in capital income inequality among financial market participants. We show analytically that differences in information capacity of individuals lead to persistent differences in returns on their capital and resulting capital income. We show that the model can quantitatively account for the observed capital income polarization among U.S. households and the evolution of the disparity in investment returns among sophisticated and unsophisticated investors. We corroborate the validity of our mechanism using evidence on institutional holdings and asset turnover in the U.S. equity.

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The rise in wealth and income inequality in the United States and worldwide has been one of the most hotly discussed topics over the last few decades in both policy and academic circles.[1] One important component of total income which contributes to this inequality is capital income generated in financial markets. In the United States, capital income is by far the most polarized part of household income, and its polarization exhibits a strong upward trend.[2] A significant step towards understanding these patterns in the data is the vast literature in economics and finance[3] that extensively analyzes household behavior in financial markets and especially its impact on financial returns. Some of the robust general trends in household behavior are a growing non-participation in high-return investments and a decline in trading activity. Many hypothesize that an ever present and growing disparity in investor sophistication or access to superior investment technologies is partly responsible for these trends. An early articulation of this argument is in Arrow (1987); however, micro-founded general equilibrium treatments of such mechanisms are still missing.

In this paper, we provide a micro-founded mechanism for return differential and show that in a general equilibrium framework, it can go a long way in explaining the growth in capital income inequality, qualitatively and quantitatively. We propose heterogeneity in investor sophistication as the main friction in the model. Intuitively, if information about financial assets and its processing are costly, individuals with different access to financial resources may differ in terms of their capacity to acquire and process information. As a result, sophisticated investors earn higher returns on their wealth, and over time their capital income diverges from that of unsophisticated investors with relatively less information. In addition, unsophisticated investors perceive their informational disadvantage through asset prices and allocate their investments away from the allocations of informed investors, by

[1] For a summary of the literature, see Piketty and Saez (2003); Atkinson, Piketty, and Saez (2011). A comprehensive discussion of the topic is also provided in the 2013 Summer issue of the Journal Economic Perspectives.

[2] Using the data from the Survey of Consumer Finances we document that approximately 20% of households actively participate in financial markets. Capital income accounts for approximately 15% of this group's total income, ranging from 40% to less than 1%. Between 1989 and 2010, the ratio of the capital income of the group in the 90th percentile of the wealth distribution relative to that of the median group increased from 27 to 60.

[3] Most recently represented by Chien, Cole, and Lustig (2011) and Calvet, Campbell, and Sodini (2007).

either choosing to invest their capacity in different assets or not participating in the market for risky assets at all.

This basic intuition resonates well with robust empirical evidence that points to the growing presence in risky asset classes over the last 20-30 years of sophisticated, institutional investors (Gompers and Metrick (2001)). Specifically, the average institutional equity ownership has more than doubled over the last few decades and accounts for more than 60% of the total stock ownership. Our hypothesis also fits well with a puzzling phenomenon of the last two decades that points to a growing retrenchment of retail investors from trading and stock market ownership in general, even though direct transaction costs, if anything, have fallen significantly. We document that such avoidance of risky assets is present both for direct stock ownership and ownership of intermediated products, such as actively managed equity mutual funds. Specifically, we find that direct stock ownership has been falling steadily over the last 30 years, while flows into equity mutual funds coming from less sophisticated, retail investors began to decline starting from the early 2000s, having dropped by 2012 by a staggering 70% of their 2000 levels.

To formalize the economics of our arguments and to assess their qualitative and quantitative match to the data, we build a noisy rational expectations equilibrium model with endogenous information acquisition and capacity constraints in the spirit of Sims (2003). We generalize this theoretical framework by accounting for meaningful heterogeneity across both investors and assets. Specifically, we consider an economy with n risky assets and one riskless asset. The risky assets differ in terms of volatilities of their fundamental shocks. A fraction λ of investors are endowed with high capacity for processing information K_1 and the remaining fraction $1-\lambda$ have low, yet positive capacity K_2. Thus, everyone in the economy has the ability to learn about assets payoffs, but to different degrees. Investors have mean-variance preferences with equal risk aversion coefficients and (costly) learn about asset payoffs from optimal private signals. Based on her capacity and on the observed asset characteristics, an investor decides which assets to learn about, how much information to process about these assets, and how much wealth to invest in all assets available in the economy.

In equilibrium, learning exhibits specialization, preference for volatility, and strategic

substitutability. In a departure from existing work, not all assets are actively traded (namely learned about), and among the assets that are actively traded, not all are given the same attention by market participants. Specifically, although each investor chooses to learn about a single asset, the market as a whole learns about an endogenously determined number of assets, and the mass of investors choosing to learn about each asset varies with the volatility and liquidity of the asset.

We provide an analytical characterization of our model's three main predictions. First, we show that in the cross-section of investors, sophisticated investors generate higher returns and capital income relative to unsophisticated investors. This divergence in returns and incomes happens because of two forces. First, sophisticated investors have better information to identify profitable assets. Second, unsophisticated investors reduce their exposure to assets held by informed investors because, through the increase in prices, they find that these assets less compelling to hold. This latter effect is a direct consequence of general equilibrium forces and would not hold under partial equilibrium.

The second prediction investigates these effects by analyzing the response of our outcome variables to shocks to sophistication, which in our framework we model as shocks to information capacity. Specifically, we show that the return and income differentials increase with the *overall growth* in total market sophistication, which is meant as general progress in information processing technologies. This result holds even if we hold the relative sophistication of the two investor types constant. The intuition for this result is that the effects from the first prediction are additionally strengthened by the fact that the sophisticated investors find it easier to use the general increase in capacity because they are already starting from a high level of information processing capacity–in our world, the more you know, the easier it is to learn on the margin.

Finally, the third set of analytical results characterizes the growth of income polarization in response to a *relative* increase in sophistication between sophisticated and unsophisticated investors even if the total degree of market sophistication stays constant.

To test the limits of our theory and to provide additional identification of our mechanism, we develop additional analytical predictions of the model, which allow us to better identify

our sophistication mechanism. Specifically, we characterize the responses to total and relative sophistication shocks for market values, cross-asset exposure, and trading intensity. We show that with an increase in information capacity, sophisticated investors are more likely to invest in and learn about more volatile assets within a set of risky assets. Our mechanism also implies a robust, unique way in which investors expand their risky portfolio holdings as the total capacity in the economy expands. In particular, they keep moving down in the asset volatility dimension. At the same time, unsophisticated investors leave the risky assets and hold safer assets. Similar effects occur in terms of trading intensity. Sophisticated investors frequently trade their assets while unsophisticated investors turn over their risky assets much less. Finally, we show that the symmetric expansion in capacity leads to lower expected market returns. Our additional analytical results play an important role in that they cut against plausible alternative explanations, such as the model with heterogeneous risk aversion or differences in trading costs.

To evaluate the quantitative fit of our theoretical predictions relative to the data, we calibrate the model to the U.S. data spanning the period from 1989 to 2012. In our calibration, we fix the parameters based on the first half of our sample period, and treat the second subperiod data moments as a test for a dynamic exercise in the model of progress in information technology. Additionally, we construct the series of income inequality using the data from the Survey of Consumer Finances, and use institutional ownership data to measure equity ownership and returns of sophisticated and unsophisticated investors. Furthermore, we use data on mutual fund flows from Morningstar to evaluate investors' portfolio choices.

We find evidence that is qualitatively consistent with our analytical predictions and quantitatively with the calibrated model. First, sophisticated investors, on average, exhibit higher rates of returns that on an annual basis are approximately 2 percentage points higher in the model and 3 percentage points higher in the data. Second, we use the ratio of average financial wealth of the 10% wealthiest investors relative to 50% poorest investors in 1989 as a proxy for *initial* relative investor sophistication, and we posit that growth in financial wealth implies linear growth in investors' sophistication. We then show that introducing this feedback in our model generates *endogenous* evolution of capacity and capital income

that can match very accurately the observed capital income inequality growth in the data. While our model implies the average inequality growth between 1989 and 2010 of 98%, the same number in the data equals 90%. Moreover, we show that the model can closely match the evolution of the growth rate over the entire sample period. Given the good fit of the model, we hypothesize that our model can quantify the economic mechanism proposed first by Arrow (1987) in which financial wealth facilitates access to more sophisticated investment techniques, and hence begets even more wealth.

We employ a dynamic to further confirm our economic mechanism. We propose progress in information technology, which increases the average equity ownership rate of sophisticated investors from 23% (the data average for 1989-2000) to 43% (the data average for 2001-2012). We show that sophisticated investors increase their ownership of equities in a specific order. They first enter stocks that are most volatile and subsequently move into stocks with medium and lower volatility. At the same time, we show that sophisticated investors' entry into equity holdings drives higher asset turnover, in magnitudes consistent with the data, and both in the time series and in the cross-section of stocks.

As additional supporting evidence, we report a set of auxiliary predictions that qualitatively correspond to the analytical predictions of the model. In particular, we show that unsophisticated investors tend to hold increasingly larger fraction of their wealth in safer, liquid assets. They also tend to reduce their aggregate equity ownership. In the data, we observe a steady outflow of unsophisticated, retail money from risky assets, such as direct equity and equity mutual funds, while the flows from sophisticated investors into such assets are generally positive. Somewhat surprisingly, these outflows in the data continued until recently despite a large increase in the risky assets valuations.

Our paper spans three strands of literature: (1) the literature on household finance; (2) the literature on rational inattention; and (3) the literature on income inequality. While some of our contributions are specific to each of the individual streams our additional value added comes from the fact that we integrate the streams into one unified framework within our research context. Our results relate to a wide spectrum of research in household finance and portfolio choice. The main ideas we entertain build upon an empirical work on lim-

ited capital market participation (Mankiw and Zeldes (1991); Ameriks and Zeldes (2001)), growing institutional ownership (Gompers and Metrick (2001)), household trading decisions (Barber and Odean (2001), Campbell (2006), Calvet, Campbell, and Sodini (2009b, 2009a), Guiso and Sodini (2012)), and investor sophistication (Barber and Odean (2000, 2009), Calvet, Campbell, and Sodini (2007), Grinblatt, Keloharju, and Linnainma (2009)). While the majority of the studies attribute limited participation rates to either differences in stock market participation costs (Gomes and Michaelides (2005)) or preferences, we relate the decisions to differences in sophistication across investors.

Another building block of our paper is the literature on rational inattention and endogenous information capacity that originates with the papers of Sims (1998, 2003, 2006). More germane to our application are models of costly information of Van Nieuwerburgh and Veldkamp (2009, 2010), Mondria (2010), and Kacperczyk, Van Nieuwerburgh, and Veldkamp (2013). While the literature on endogenous information acquisition generally assumes that informed investors have homogenous information capacity, we study implications of the model with heterogeneous agents. We show that the implications of such models for portfolio decisions and asset prices are very different than those of the model with homogenous information. In addition, we study the implications of information frictions for income processes of investors and the equilibrium holdings of assets with different characteristics, such as volatility or turnover, all features which are absent in the present literature.

Our last building block constitutes the literature on income inequality that dates back to the seminal work by Kuznets and Jenks (1953) and has been subsequently propagated in the work of Piketty (2003), Piketty and Saez (2003), Alvaredo, Atkinson, Piketty, Saez, et al. (2013), Autor, Katz, and Kearney (2006), and Atkinson, Piketty, and Saez (2011). A vast majority of that literature explain total income inequality looking at the income earned in labor market (e.g., Acemoglu (1999, 2002); Katz and Autor (1999); Autor, Katz, and Kearney (2006, 2008); and Autor and Dorn (2013)); and they do not consider explanations that relate to informational sophistication of investors.

The closest paper in spirit to ours is the work by Arrow (1987) who also considers information differences as an explanation of income gap. However, his work does not consider

endogenous information acquisition and is not an equilibrium outcome of the economy with heterogeneously informed agents and multiplicity of assets. Both these elements are crucial for the results of our paper, and especially for showing the validity of our mechanism. Thanks to having a richer, equilibrium framework, we are able to calibrate the model to the data and show that it comes very close to the data moments. Finally, Arrow does not provide any evidence with respect to trading or volatility; to us, these elements are crucial to our identification. Another work related to ours is Peress (2004) who considers the role of wealth as a proxy for investor sophistication and studies its impact on income inequality. However, in contrast to our paper, his focus is largely on consequences of wealth differences for market participation without going into the micro foundation of how the different investors allocate their money, within disaggregated asset classes. Also, his model is fully based on differences in absolute risk aversion across agents while our model's predictions depend on differences in information capacity and impose no differences in risk aversion.

The rest of the paper proceeds as follows. In Section 1, we provide equilibrium framework to study behavior and income evolution of heterogeneously informed individuals. In Section 2 we derive analytical predictions, which we subsequently take to the data in Section 3. Section 4 concludes.

1 Theoretical Framework

We study a model of portfolio choice with endogenous information a la Grossman and Stiglitz (1980). We first present the investment environment. We then describe the investors' portfolio and information choice problems. Finally, we characterize the equilibrium and its properties.

1.1 Model Setup

The financial market consists of one riskless asset, with price normalized to 1 and payoff r, and n risky assets, indexed by i, with prices p_i, and independent payoffs $z_i \sim \mathcal{N}(\overline{z}_i, \sigma_i^2)$. The riskless asset has unlimited supply, and each risky asset has stochastic supply $x_i \sim \mathcal{N}(\overline{x}_i, \sigma_{xi}^2)$,

independent of payoffs and across assets.

These assets are traded by a continuum of atomless investors of mass one, indexed by j, with mean-variance utility over wealth W_j, and risk aversion coefficient denoted by $\rho > 0$. Each investor can choose to acquire information about some or all of the asset payoffs prior to making the portfolio allocation decision. Information is acquired in the form of endogenously designed signals which are then used to update the beliefs that inform the investor's portfolio allocation. The investor's signal choice is modeled following the rational inattention literature (Sims (2003)), using entropy reduction as a measure of the amount of acquired information. Each investor is modeled as though receiving information through a channel with fixed capacity.

In our modeling, we depart from existing work by introducing non-trivial heterogeneity in information capacity. Specifically, mass $\lambda \in (0, 1)$ of investors have high capacity for processing information, K_1, and are referred to as *sophisticated* investors, and mass $1 - \lambda$ of investors have low capacity for processing information, $0 < K_2 < K_1$, and are referred to as *unsophisticated* investors. Thus, everyone in the economy has the ability to learn about assets payoffs, but to different degrees.

There are two subperiods in each period. In the first subperiod, investors solve the information acquisition problem. In the second subperiod, payoffs and asset supplies are realized, investors receive signals on payoffs in accordance with their information acquisition strategy, and solve the portfolio optimization problem, and equilibrium prices clear markets.

1.2 Portfolio Decision

We begin by solving each investor's portfolio problem in subperiod 2, for a given information structure. Each investor chooses portfolio holdings to solve

$$\max_{\{q_{ji}\}_{i=1}^{n}} U_{2j} = E_{2j}\left(W_j\right) - \frac{1}{2}\rho V_{2j}\left(W_j\right) \tag{1}$$

subject to the budget constraint

$$W_j = r\left(W_{0j} - \sum_{i=1}^{n} q_{ji}p_i\right) + \sum_{i=1}^{n} q_{ji}z_i, \tag{2}$$

where E_{2j} and V_{2j} denote the mean and variance conditional on the investor j's information set in subperiod 2, W_{0j} is initial wealth (which will be normalized to zero), and q_{ji} is the quantity invested by investor j in asset i.

The (standard) solution to the portfolio choice problem yields that the quantity invested in each asset by investor j is given by the Sharpe ratio scaled by the inverse risk-aversion coefficient:

$$q_{ji} = \frac{\widehat{\mu}_{ji} - rp_i}{\rho\widehat{\sigma}^2_{ji}}, \tag{3}$$

where $\widehat{\mu}_{ji}$ and $\widehat{\sigma}^2_{ji}$ are the mean and variance of investor j's posterior beliefs about the payoff z_i, conditional on that investor's information choice. If an investor chooses not to acquire a signal about a particular asset, then the investor's beliefs–and hence portfolio holdings–for that asset are determined by her prior, which coincides with the unconditional distribution of payoffs.

Substituting q_{ji} yields the following indirect utility function:

$$U_{2j} = \frac{1}{2\rho}\sum_{i=1}^{n}\left[\frac{\left(\widehat{\mu}_{ji} - rp_i\right)^2}{\widehat{\sigma}^2_{ji}}\right]. \tag{4}$$

1.3 Information Choice

In subperiod 1, each investor acquires information about asset payoffs in the form of signals which are then used to update the beliefs that inform the investor's portfolio allocation. For analytical tractability, we make the following assumption about the signal structure:[4]

Assumption 1 *Each investor j receives a separate signal s_{ji} on each of the asset payoffs z_i, and these signals are independent across assets.*

[4]This assumption is standard in the literature. It is necessary for the analytical tractability of the model. Allowing for potentially correlated signals requires a strictly numerical approach, and is beyond the scope of this paper.

The investor chooses the optimal allocation of information capacity across the different assets–the distribution of each signal–to maximize ex-ante expected utility, $E_{1j}\left[U_{2j}\right]$,

$$\max \frac{1}{2\rho} \sum_{i=1}^{n} \left\{ \left(\frac{1}{\widehat{\sigma}_{ji}^2} \right) E_{1j} \left[\left(\widehat{\mu}_{ji} - r p_i \right)^2 \right] \right\}, \tag{5}$$

subject to a constraint on the total quantity of information conveyed by the signals,

$$\sum_{i=1}^{n} I\left(z_i; s_{ji}\right) \leq K_j, \tag{6}$$

where $I\left(z_i; s_{ji}\right)$ denotes Shannon's (1948) mutual information, measuring the information about the asset payoff z_i conveyed by the private signal s_{ji}; and $K_j \in \{K_1, K_2\}$ denotes investor j's capacity for processing information.[5]

The information constraint (6) imposes a limit on the amount of entropy reduction that each investor can accomplish through the endogenously designed signal structure. Since perfect information requires infinite capacity, there will necessarily be some residual uncertainty about the realized payoffs that each investor will face. For each asset, investor j decomposes[6] the payoff into a lower-entropy signal component, s_{ji}, and a residual component, δ_{ji}, that represents data lost due to the compression of z_i:

$$z_i = s_{ji} + \delta_{ji}. \tag{7}$$

We introduce the following assumption on the signal structure.

Assumption 2 *The signal s_{ji} is independent of the data loss δ_{ji}.*

[5] Assumption 1 guarantees that the total quantity of information acquired by the investor can be expressed as the sum of the quantities of information obtained for each asset, as in equation 6.

[6] The existing literature on costly information typically assumes an additive noise signal structure, where the signal is equal to the payoff plus noise. That specification has enabled a direct comparison to the literature on exogenous information. However, in the context of limited capacity, our agents simplify the state of the world (in terms of entropy), rather than amplifying it with noise. While conceptually closer to the information theoretic benchmark, this formulation nonetheless does not change the results in this particular application. (For applications where the compression is critical, see the work on discreteness in rational inattention by Matejka (2011), Matejka and Sims (2010), and Stevens (2012)).

Since z_i is normally distributed, this assumption[7] implies that s_{ji} and δ_{ji} are also normally distributed, by Cramer's Theorem:

$$s_{ji} \sim \mathcal{N}\left(\overline{z}_i, \sigma^2_{sji}\right) \quad \text{and} \quad \delta_{ji} \sim \mathcal{N}\left(0, \sigma^2_{\delta ji}\right),$$

with $\sigma^2_i = \sigma^2_{sji} + \sigma^2_{\delta ji}$.

Therefore, the investor's posterior beliefs about payoffs given signals are also normally distributed random variables, independent across assets, with mean and variance given by:

$$\widehat{\mu}_{ji} = s_{ji} \quad \text{and} \quad \widehat{\sigma}^2_{ji} = \sigma^2_{\delta ji}. \tag{8}$$

The investor's objective becomes then choosing the variance of the data lost, $\sigma^2_{\delta ji}$, for each asset i, to solve the following constrained optimization problem:[8]

$$\max_{\{\sigma^2_{\delta ji}\}_{i=1}^n} \sum_{i=1}^{n} \left(\frac{\widehat{S}_i + \widehat{R}_i^2}{\sigma^2_{\delta ji}} \right), \tag{9}$$

subject to

$$\prod_{i=1}^{n} \left(\frac{\sigma^2_i}{\sigma^2_{\delta ji}} \right) \leq e^{2K_j}, \tag{10}$$

where

$$\widehat{R}_i \equiv \overline{z}_i - r\overline{p}_i \tag{11}$$

is the mean of expected excess returns, which is common across investors, and where

$$\widehat{S}_i \equiv (1 - 2rb_i)\,\sigma^2_i + r^2\sigma^2_{pi} \tag{12}$$

[7]The decomposition of the shock into independent components can be shown to be optimal if the agent's signaling problem is to minimize the mean squared error of s_i for each i. (See, for example, Cover and Thomas, 2005). However, in general, the optimal signal structure may require correlation between the signal and the data loss (namely it may result in higher posterior precision about asset payoffs). In our framework, we assume the independent decomposition to maintain analytical tractability. This puts a lower bound on the severity of the information friction.

[8]The distribution of excess returns, used in equations (11)-(12), the objective function in equation (9) and the information constraint in equation (10) are derived in the Appendix.

is the component of the variance of expected excess returns that is also common across investors.

The following Proposition characterizes the equilibrium policy of investors for information capacity allocation.

Proposition 1 *In the solution to the maximization problem (9)-(10), each investor allocates all capacity to learning about a single asset. All assets that are actively traded (i.e. learned about in equilibrium) belong to the set of assets with maximal expected gain factors, L:*

$$L \equiv \left\{ i \,|\, i \in \arg\max_i G_i \right\}, \tag{13}$$

where the gain factor of asset i is defined as

$$G_i \equiv \frac{\widehat{S}_i + \widehat{R}_i^2}{\sigma_i^2}. \tag{14}$$

Proof. In Appendix. ■

Using Proposition 1, and substituting the optimal capacity allocation in equation (10), we characterize the posterior beliefs of investor j learning about asset $l_j \in L$ by:

$$\widehat{\mu}_{ji} = \begin{cases} s_{ji} & \text{if } i = l_j, \\ \overline{z}_i & \text{if } i \neq l_j, \end{cases} \quad \text{and} \quad \sigma^2_{\delta ji} = \begin{cases} e^{-2K_j}\sigma_i^2 & \text{if } i = l_j, \\ \sigma_i^2 & \text{if } i \neq l_j. \end{cases} \tag{15}$$

The investors' posterior beliefs about payoffs are equal to their prior beliefs, for assets which they passively trade. On the other hand, for the asset about which each investor learns, the posterior variance is strictly lower and decreasing in capacity K_j, whereas the posterior mean is equal to the received signal. Each signal s_{ji} received by an investor of type j is a weighted average of the true realization, z_i, and the prior, $\overline{z}_i$, with mean

$$E\left(s_{ji}|z_i\right) = \left(1 - e^{-2K_j}\right) z_i + e^{-2K_j}\overline{z}_i. \tag{16}$$

The higher is the capacity of the investor, the larger is the weight that the investor's signal puts on the realization z_i relative to the prior, $\overline{z}_i$.

1.4 Equilibrium

Given the solution to an individual investor's information allocation problem, the market clearing condition for each asset is given by

$$\int_{M_{1i}} \left(\frac{s_{ji} - rp_i}{e^{-2K_1} \rho \sigma_i^2} \right) dj + \int_{M_{2i}} \left(\frac{s_{ji} - rp_i}{e^{-2K_2} \rho \sigma_i^2} \right) dj + (1 - m_i) \left(\frac{\overline{z}_i - rp_i}{\rho \sigma_i^2} \right) = x_i, \tag{17}$$

where m_i denotes the mass of investors learning about asset i, M_{1i} denotes the set of sophisticated investors, of measure $\lambda m_i \geq 0$, who choose to learn about asset i, and M_{2i} denotes the set of unsophisticated investors, of measure $(1 - \lambda) m_i \geq 0$, who choose to learn about asset i.[9]

Following Admati (1985), we conjecture and verify that the equilibrium asset prices are a linear function of the underlying shocks, which we derive in the Lemma below.

Lemma 1 *The price of asset i is given by*

$$p_i = a_i + b_i z_i - c_i x_i, \tag{18}$$

with

$$a_i = \frac{\overline{z}_i}{r\left(1 + \phi m_i\right)}, \quad b_i = \frac{\phi m_i}{r\left(1 + \phi m_i\right)}, \quad c_i = \frac{\rho \sigma_i^2}{r\left(1 + \phi m_i\right)}, \tag{19}$$

where $\phi \equiv \lambda \left(e^{2K_1} - 1\right) + (1 - \lambda) \left(e^{2K_2} - 1\right)$ is a measure of the average capacity for processing information available in the market, and where m_i is the mass of investors learning about asset i.

Proof. In Appendix. ■

[9] Since the payoff factors are the same across all investors, regardless of investor type, the participation of sophisticated and unsophisticated investors in a particular asset will be proportional to their mass in the population.

We next determine what assets are learned about in equilibrium, and how the market overall chooses to allocate information capacity across these assets. In a departure from existing work, we solve for m_i, the endogenous mass of investors learning about each asset.

Using equilibrium prices in the investor's information allocation solution, we obtain the following expression for the expected gain factor:

$$G_i = \frac{1 + \rho^2 \xi_i}{(1 + \phi m_i)^2}, \tag{20}$$

where

$$\xi_i \equiv \sigma_i^2 \left(\sigma_{xi}^2 + \overline{x}_i^2 \right) \tag{21}$$

is a summary statistic of the properties of asset i, and depends only on exogenous parameters. Equation (20) implies that learning in the model exhibits preference for volatility and strategic substitutability (preference for low aggregate learning rate m_i).

The following Lemma shows that as overall capacity in the economy increases from zero, investors will learn about the most volatile asset first, and then start expanding their learning towards the next highest volatility asset.

Lemma 2 *Without loss of generality, let assets in the economy be ordered such that, for all* $i = 1, ...n-1$, $\xi_i > \xi_{i+1}$. *Then, the following conditions hold:*

(i) For aggregate information capacity such that $0 < \phi < \phi_1$, *where*

$$\phi_1 \equiv \sqrt{\frac{1 + \rho^2 \xi_1}{1 + \rho^2 \xi_2}} - 1, \tag{22}$$

only one asset is learned about in the market, and this asset is the asset with the largest idiosyncratic factor, ξ_1.

(ii) For aggregate information capacity such that $\phi \geq \phi_1$, *at least two assets are learned about in equilibrium. As* ϕ *increases, the market learns about new assets in a decreasing order of* ξ_i. *Let* m_k *and* m_l *denote the masses of investors learning about two assets* k, l, *and*

let h index an asset that is not learned about in equilibrium ($m_h = 0$). These masses satisfy

$$\left(\frac{1+\phi m_k}{1+\phi m_l}\right)^2 = \frac{1+\rho^2\xi_k}{1+\rho^2\xi_l}, \quad \forall k,l \in L, \tag{23}$$

and

$$\frac{1+\rho^2\xi_k}{(1+\phi m_k)^2} > 1+\rho^2\xi_h, \quad \forall k \in L, h \notin L. \tag{24}$$

Proof. In Appendix. ■

The average capacity measure ϕ_1 determines the threshold quantity of information in the market above which investors expand to more than one asset. As the market's capacity for processing information grows further above the threshold, for instance through technological progress, investors expand to learning about lower and lower volatility assets.

The selection of investors into learning about different assets is pinned down by the indifference conditions (36), combined with the condition that every investor learns about some asset, $\sum_{i=1}^{L} m_i = 1$. In order to present a complete characterization of learning in the economy, we introduce the following notation:

Definition 1 *Let ϕ_k be a threshold for ϕ such that for any $\phi < \phi_k$, at most k assets are actively traded (learned about) in equilibrium, while for $\phi \geq \phi_k$, at least k assets are actively traded in equilibrium.*

Then, by Lemma 2, $0 < \phi_1 \leq \phi_2 \leq ... \leq \phi_n$.

Lemma 3 *Suppose that $\phi_{k-1} \leq \phi < \phi_k$ such that $k > 1$ assets are actively traded in equilibrium. Then, the equilibrium allocation of active investors across assets, $\{m_i\}_{i=1}^n$, satisfies the following:*

(i)

$$\begin{aligned} m_1 &= \frac{1+\frac{1}{\phi}\sum_{j=2}^{k}(1-c_{j1})}{1+\sum_{j=2}^{k} c_{j1}}, \\ m_i &= c_{i1}m_1 - \frac{1}{\phi}(1-c_{i1}) \text{ for } 1 < i \leq k, \\ m_i &= 0 \text{ for } i > k, \end{aligned}$$

where $c_{i1} = \sqrt{\frac{1+\rho^2\xi_i}{1+\rho^2\xi_1}} < 1$.

(ii)

$$\begin{aligned} \frac{dm_1}{d\phi} &= -\frac{1}{\phi^2}\frac{\sum_{j=2}^{k}(1-c_{j1})}{1+\sum_{j=2}^{k}c_{j1}} < 0, \\ \frac{dm_i}{d\phi} &= \frac{1}{\phi^2}\left[1 - c_{i1}\frac{k}{1+\sum_{j=2}^{k}c_{j1}}\right]. \end{aligned}$$

(iii) There exists $\bar{\imath} < k$*, such that for all assets* i *with* $\bar{\imath} < i \leq k$*,* $\frac{dm_i}{d\phi} > 0$ *and for all* $i \leq \bar{\imath}$*,* $\frac{dm_i}{d\phi} < 0$.

(iv) $\frac{d(\phi m_i)}{d\phi} \geq 0$ *for all* i*, with equality for* $i > k$.

Proof. In Appendix. ■

The allocation of masses is determined only by exogenous variables: ξ_i, ρ, and ϕ. In turn, the solution for $\{m_i\}$ pins down equilibrium prices, by Lemma 1, thereby completing the characterization of equilibrium.

2 Analytical Predictions

In this section, we present a set of analytical qualitative results for our environment. First, we present the model's predictions for capital income inequality. To provide additional verification of our information-based mechanism, we then derive a set of additional theoretical predictions that are specific to our mechanism. These results allow us to compare the model's implications with evidence from stock-level micro-data, for evaluation against alternative mechanisms for capital income polarization.

2.1 Capital Income Inequality

Let π_{ji} denote the average profit per capita for investors of type $j \in \{1, 2\}$, from trading asset i:

$$\pi_{1it} \equiv \frac{Q_{1it}\left(z_{it} - rp_{it}\right)}{\lambda} \quad \text{and} \quad \pi_{2it} \equiv \frac{Q_{2it}\left(z_{it} - rp_{it}\right)}{1-\lambda}, \tag{25}$$

where Q_{1i} and Q_{2i} are the aggregate holdings levels of asset i for sophisticated and unsophisticated investors respectively, obtained by integrating holdings q_{ij} across investors of each type:

$$Q_{1it} = \lambda \left[\frac{(\overline{z}_i - rp_{it}) + m_i \left(e^{2K_1} - 1\right)(z_{it} - rp_{it})}{\rho \sigma_i^2} \right], \tag{26}$$

and

$$Q_{2it} = (1-\lambda) \left[\frac{(\overline{z}_i - rp_{it}) + m_i \left(e^{2K_2} - 1\right)(z_{it} - rp_{it})}{\rho \sigma_i^2} \right]. \tag{27}$$

The first result from the model is that heterogeneity in information capacity across investors is the sole driver of capital income inequality – the sophisticated agents will generate higher income than the unsophisticated agents. This is summarized in Proposition 2.

Proposition 2 $\sum_i \pi_{1it} - \sum_i \pi_{2it} > 0$ *if and only if* $K_1 > K_2$.

Proof. In Appendix. ■

The informational advantage manifests in the model in two ways. First, sophisticated investors achieve relatively higher profits by holding a different average portfolio (average effect). Second, they also achieve relatively higher profits by obtaining larger gains from shock realizations that are profitable relative to expectations, and incurring smaller losses on unprofitable shock realizations (dynamic effect). These two effects show up in the average level and the adjustment of holdings in response to shocks, and are summarized in Propositions 3 and 4 below. Proposition 3 shows the average effect, and demonstrates that sophisticated investors choose higher average holdings of risky assets (part (i)), and that they also on average tilt their portfolios towards more profitable assets more than the unsophisticated investors (part (ii)).

Proposition 3 (Average Effect) *Let* $K_1 > K_2$ *and* $\phi_{k-1} \leq \phi < \phi_k$, *such that the first* $k \in \{1, ..., n\}$ *assets are learned about in equilibrium. Let* i *denote the index of an asset,* $i \in \{1, ..., n\}$. *The following statements hold:*

(i) If $i > k$, *then* $\frac{Q_{1it}}{\lambda} - \frac{Q_{2it}}{(1-\lambda)} = 0$, *and if* $i \leq k$, *then* $E\left\{\frac{Q_{1it}}{\lambda} - \frac{Q_{2it}}{(1-\lambda)}\right\} > 0$.

(ii) Suppose that $\overline{x}_i = \overline{x}$ *and* $\sigma_{xi} = \sigma_x$ *for all* i. *For any two assets* $i, l \leq k$, *if* $E(z_i - rp_i) > E(z_l - rp_l)$, *then* $E\left\{\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{(1-\lambda)}\right\} > E\left\{\frac{Q_{1l}}{\lambda} - \frac{Q_{2l}}{(1-\lambda)}\right\}$.

Proof. In Appendix. ■

Next, Proposition 4 below illustrates the dynamic effect of investor sophistication. It shows that for every realized state z_i, x_i, the sophisticated investors are able to adjust their portfolios (contemporaneously) upwards if the shock implies high returns and downwards if the shock implies low returns. Hence, also dynamically, they are able to outperform unsophisticated investors by responding to shock realizations in a way that increases their profits.

Proposition 4 (Dynamic Effect) *Let $K_1 > K_2$ and $0 < \phi < \phi_k$, such that the first $k \in \{1, ..., n\}$ assets are learned about in equilibrium. For $i \leq k$, $\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{(1-\lambda)}$ is increasing in excess returns, $z_i - rp_i$.*

Proof. In Appendix. ■

To see explicitly the dynamic effect's impact on capital income inequality, we can express the total capital income of the average sophisticated investor as[10]

$$\sum_{i=1}^{n} \pi_{1i} \equiv \sum_{i=1}^{n} \alpha_i \pi_{2i}, \tag{28}$$

where, by (26) and (27),

$$\alpha_i \equiv \frac{\pi_{1i}}{\pi_{2i}} = \frac{(\overline{z}_i - rp_i) + m_i(e^{2K_1} - 1)(z_i - rp_i)}{(\overline{z}_i - rp_i) + m_i(e^{2K_2} - 1)(z_i - rp_i)}, \quad \forall i. \tag{29}$$

That is, the average sophisticated investor's capital income can be expressed as a weighted sum of the average unsophisticated investor's capital income from each asset, but the weights depart from 1 whenever the asset is actively traded ($m_i > 0$).

To see the dynamic effect, consider how variation in the weights α_i drives income differences. For assets that are actively traded in equilibrium, they vary depending on the realization of the shocks z_i and x_i. There are two possible scenarios. First, $\pi_{2i} > 0$, which by (29) implies $\pi_{1i} > 0$ and $\alpha_i > 1$. Hence, the sophisticated agents have a larger gain in

[10] We are implicitly assuming that profits are never exactly zero. For such case, the arguments extend trivially.

their (positive) capital income from asset i. Second, $\pi_{2i} < 0$ and either (i) $\pi_{1i} < 0$ and $0 < \alpha_i < 1$, or (ii) $\pi_{1i} > 0$ and $\alpha_i < 0$. The first case implies that the sophisticated investors put a lower weight in their portfolio on the loss, while the second case means that the profit of the sophisticated investor puts negative weight on the loss. In both cases, sophisticated investors either incur a smaller loss or realize a bigger profit, state by state.

The arguments above lead us to establish a comparative result: *increases* in heterogeneity in sophistication lead to a *growing* capital income polarization. Intuitively, higher information capacity dispersion means that relative to the unsophisticated investors, the sophisticated investors receive higher quality signals about the fundamental shocks z_i, x_i, and as a result they will respond more strongly to postive/negative realized excess profits $z_i - rp_i$. This is the essence of Proposition 5.

Proposition 5 *Consider a increase in capacity dispassion of the form $K_1' = K_1 + \Delta_1 > K_1$, $K_2' = K_2 + \Delta_2 < K_2$, with Δ_1 and Δ_2 chosen such that total information capacity $\phi = const$. Then, the ratio $\sum_i \pi_{1i} / \sum_i \pi_{2i}$ increases, i.e. capital incomes become more polarized.*

Proof. In Appendix. ■

The results above show that capacity heterogeneity generates heterogeneity in portfolios, and in particular it decreases the relative participation of the unsophisticated investors. Below, we explore the intuitive reasons behind unsophisticated investor retrenchment from risky assets in the presence of informationally superior, sophisticated investors.

Intuition: Example Suppose that the realized state is $z_i > \bar{z}_i$ such that in equilibrium $z_i - rp_i > 0$ and consider a case of an *homogeneous* investors with capacity K_2 who received at mean signal for his type, $S_2 = \bar{z}_i e^{-2K_2} + z_i(1 - e^{-2K_2})$. Their mean allocation choice is then

$$q_{2i} = e^{2K_2} \left(\frac{S_2 - rp_i}{\rho \sigma_i^2} \right),$$

where $e^{-2K_2}\sigma_i^2$ is the variance of the investor posterior beliefs.

Suppose we fix the allocation of agents to learning about different assets, $\{m_i\}_{i=1}^n$ at the equilibrium level and, we perform an exogenous variation of increasing the capacity of

mass $\gamma < m_i$ of investors to $K_1 > K_2$–they become sophisticated investors. These new sophisticated investors have mean (across mass γ) demand given by

$$q_{1i} = e^{2K_2}\left(\frac{S_1 - rp_i}{\rho\sigma_i^2}\right),$$

where the mean signal they receive is $S_1 = \bar{z}_i e^{-2K_1} + z_i(1 - e^{-2K_1})$.

There are two effect that lead to increased relative participation of sophisticated investors in risky assets in this example: a partial equilibrium one and a general equilibrium one.

First, *absent price adjustment*, the partial equilibrium effect is that the remaining unsophisticated investors don't change their demand q_{1i} for asset i, but the new sophisticated investors now demand more, because $S_1 > S_2$ (we are considering a good state where $z_i > \bar{z}_i$), and also this signal is more precise ($e^{-2K_1}\sigma_i^2 < e^{-2K_2}\sigma_i^2$). Hence, in partial equilibrium where the price is exogenously given, we observe growth in sophisticated ownership due to this effect: Sophisticated investors take bigger positions when they actively trade. However, we would see no change in the strategies of unsophisticated investors.

Second, there is the general equilibrium effect working through price adjustment, which makes unsophisticated investors perceive an *informational disadvantage* in trading asset i after sophisticated investors enter. In particular, in accordance with market clearing conditions (18) and (19), the price process will adjust to the now greater demand from highly informed investors – in particular it will be more informative about the fundamental z_i,[11] and since $z_i - rp_i > 0$, the equilibrium price will increase[12]. Through that price adjustment, both types of agents will see their returns go down, but only the unsophisticated agents will choose to decrease their holdings–their signals are not high enough quality to sustain previous positions as optimal choice. Through this general equilibrium effect, the entry of sophisticated investors spills over to an informational disadvantage of the unsophisticated investors and causes their retrenchment from trading the asset.

[11] b_i will rise and a_i and c_i will drop, because we just increased ϕ in the market for asset i by increasing total capacity of agents trading in that market.

[12] Both the price and its derivative with respect to ϕ in state z_i, x_i are proportional to $z_i - \bar{z}_i + \rho\sigma_i^2 x_i$.

2.2 Testing the Mechanism

Below, we provide additional analytical characterization of the predictions of our model. These analytical results, together with the quantitative predictions from our parameterized model, serve as tests of the main mechanism of the model when compared to the same features of the data.

We start with the characterization of the properties of the market return in response to growth in the overall level of information in the economy. As aggregate information increases, prices contain a growing amount of information about the fundamental shocks, and market excess return drops. This is summarized in Proposition 6.

Proposition 6 (Market Value) *A symmetric growth in information processing capacity leads to*

(i) higher average prices, $\frac{d\overline{p}_i}{d\phi} \geq 0$*, and hence a higher average value of the financial market.*

(ii) lower average market excess returns, $dE\left(z_{it} - rp_{it}\right)/d\phi \geq 0$.

Proof. In Appendix. ■

Next, in Proposition 7, we consider the effects of pure increase in dispersion of sophistication, without changing the aggregate level of sophistication in the economy. Such polarization in capacities implies polarization in holdings.

Proposition 7 *Consider a increase in capacity dispersion of the form* $K_1' = K_1 + \Delta_1 > K_1$*,* $K_2' = K_2 + \Delta_2 < K_2$*, with* Δ_1 *and* Δ_2 *chosen such that total information capacity* $\phi = const$*. Then, the average ownership difference* $E\left\{\sum_i \frac{Q_{1i}}{\lambda} - \sum_i \frac{Q_{2i}}{1-\lambda}\right\}$ *increases, i.e. sophisticated ownership increases.*

Proof. In Appendix. ■

To consider the effects of aggregate symmetric growth in information technology in the economy, we first need to establish an auxiliary result:

Lemma 4 *Consider symmetric information capacity, i.e. such that* $K_{1t} = K_t$ *and* $K_{2t} = K_t\gamma$, $\gamma \in (0,1)$, *and consider* $0 < \phi < \phi_k$ *is such that* $k > 1$ *assets are actively traded in equilibrium. Then the following is true:*

$$\frac{d[m_i(e^{2K_1} - 1)]}{dK} > 0 \text{ and } \frac{d[m_i(e^{2K_1} - 1)]}{dK} > \frac{d[m_i(e^{2K_2} - 1)]}{dK}$$

Proof. In Appendix. ■

With the result from Lemma 4 at hand, we show that aggregate symmetric growth in information technology, modeled as a common growth rate of both K_1 and K_2, leads to a growing retrenchment of unsophisticated investors and hence increased sophisticated ownership of equity (Proposition 8), as well as growing capital income polarization (Proposition 9)

Proposition 8 (Dynamic Ownership) *Consider symmetric information capacity, i.e. such that* $K_{1t} = K_t$ *and* $K_{2t} = K_t\gamma$, $\gamma \in (0,1)$, *and consider* $0 < \phi < \phi_k$ *is such that* $k > 1$ *assets are actively traded in equilibrium. In equilibrium, the average sophisticated ownership share increases across all assets:*

$$dE\{\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{1-\lambda}\}/dK > 0.$$

Proof. In Appendix. ■

Proposition 9 (Capital Income Polarization) *Consider symmetric information capacity, i.e. such that* $K_{1t} = K_t$ *and* $K_{2t} = K_t\gamma$, $\gamma \in (0,1)$, *and consider* $0 < \phi < \phi_k$ *is such that* $k > 1$ *assets are actively traded in equilibrium. In equilibrium, the average capital incomes become more polarized:*

$$dE\{\sum_i \pi_{1i} / \sum_i \pi_{2i}\}/dK > 0.$$

3 Results

In this section, we provide a discussion of the results corresponding to our theoretical predictions. We first lay out empirical facts coming from household-level and institution-level data that motivate our investigation of capital income inequality. Next, we show the quantitative performance of our model that aims to explains these facts. To this end, we discuss the parameterization of the model and then show the quantitative performance of the model for income inequality and the results that pin down our economic mechanism in the data. Next, we discuss alternative mechanisms that could potentially explain the data. Finally, we provide additional empirical evidence that supports our analytical predictions.

3.1 Motivating Facts: Capital Income Inequality

In this section, we provide empirical evidence on capital income inequality. We first discuss results based on household-level data from Survey of Consumer Finances (SCF). Next, we enhance these data with evidence based on institutional holdings data from Thomson Reuters.

3.1.1 Evidence from Survey of Consumer Finances

We begin with summarizing the data on capital income and financial wealth polarization for U.S. households. These data come from the Survey of Consumer Finances and have been used before in studies on income and wealth distribution. SCF provides evidence on a representative sample of U.S. households on a tri-annual basis. To map our results to micro-level from financial markets we use 8 most recent surveys between 1989 and 2010.

In our framework, we assume that differences in investor sophistication can be mapped to differences in their wealth levels (Arrow (1987) and Calvet, Campbell, and Sodini (2009b)). Since capital income is generated by investments of disposable capital, we use financial wealth as our empirical proxy. To show the empirical distribution of wealth and income polarization in the data, we restrict our population to households with non-zero financial wealth and consider two subsets of households: a group of 10% of households with the highest

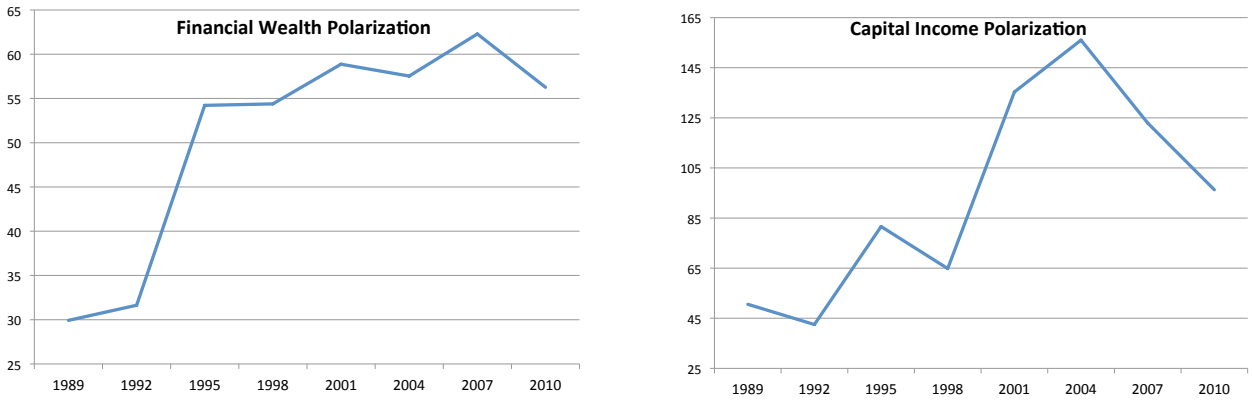


Figure 1: Financial Wealth and Capital Income Polarization: Survey of Consumer Finances.

level of financial wealth in each point in time (sophisticated investors) and a group of 50% of households with the lowest level of financial wealth (unsophisticated investors). For the two groups, we calculate average financial wealth and corresponding capital income and report the ratios of the two averages. Figure 1 shows the time series of the ratios for financial wealth (left panel) and capital income (right panel).

We find a significant dispersion in financial wealth across the two groups of households. Sophisticated investors have an average levels of financial wealth that is at least 30 times larger than the average wealth of unsophisticated investors. Moreover, the difference in wealth exhibits a highly increasing trend over time: the financial wealth ratio almost doubles, from 30 in 1989 to more than 55 in 2010. This result conforms well to anecdotal evidence of a growing polarization in wealth and earlier findings in Piketty and Saez (2003) and Atkinson, Piketty, and Saez (2011). It also suggests that financial sophistication became significantly more polarized in the last few decades.

Likewise, we find qualitatively similar patterns in capital income ratios: Income ratios are highly dispersed cross-sectionally, with sophisticated investors earning at the minimum 45 times more dollar income than unsophisticated ones. This dispersion also grows strongly over time up to 150 in 2004. Even though it subsequently diminishes slightly, still it remains at a very high level of at least 100. Combining these two pieces together implies that sophisticated investors outperformed unsophisticated investors in terms of their rates of

returns on the invested capital. In the data, we find that the ratio in rates of returns for sophisticated vs. unsophisticated on average equals 1.7 and varies between 1.1 and 2.15 in the time series.

3.1.2 Evidence from Thomson Reuters

Our economic mechanism for income inequality involves a portfolio choice problem as a building block. Since the SCF data do not provide detailed information about investors' holdings and the returns they earn on these holdings it is difficult to directly test some of the analytical predictions of our model. To accommodate such tests, we rely on portfolio holdings data obtained from Thomson Reuters. These data contain a comprehensive sample of portfolios of publicly traded equity held by institutional investors. These holdings come from quarterly reports required by law and submitted by all institutional investors to Securities and Exchange Commission (SEC). Relative to SCF, the benefit of using the data is its detailed micro-level structure, the drawback is that we only observe a subset of investors and these are not exactly the same investors as those in SCF.

To provide a qualitative mapping between the two data sets, we define sophisticated investors as investment companies and independent advisors (type 3 and type 4 in the data). These investors include wealthy individuals, mutual funds and hedge funds. Among all institutional types these two groups are known to be particularly active in their information production efforts; in turn, other groups such as banks, insurance companies, or endowments and pensions are more passive by nature. Our group of unsophisticated investors are other shareholders who are not classified as institutional types. By definition, these remaining investors must be individual (retail) investors. We provide evidence on growing capital income polarization using data on aggregated financial performance of each group of investors over the time period 1989-2012.

We proceed in three steps. First, we obtain the market value of each stock held by all investors of a given type. Market value of each stock is the product of the number of combined shares held by a given investor type and the price per share of that stock obtained from CRSP. Since the number of shares held by unsophisticated investors is not

directly observable we impute this value by taking the difference between the total number of shares available for trade and the number of shares held by *all* institutional investors. Second, we calculate the value shares of each stock in the aggregate portfolios by taking the ratio of market value of each stock relative to the total value of the portfolio. Third, we obtain the return on the aggregated portfolio by matching each asset share with their next month realized return and calculating the value-weighted aggregated return. We repeat this procedure separately for sophisticated and unsophisticated investors.

To compare financial performance of the two investor types we calculate cumulative values of $1 invested by each group in 1989 using time series of the aggregated monthly returns ending in December 2012. We present the two series in Figure 2.

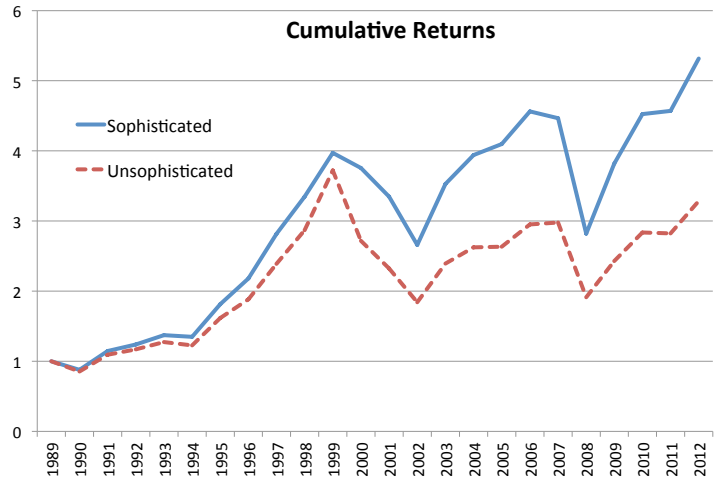


Figure 2: Cumulative Return in Equity Markets.

Our results indicate that sophisticated (institutional) investors systematically outperform unsophisticated (retail) investors. The value of $1 invested in January 1989 grows to $5.32 at the end 2012 for sophisticated investors and only to $3.28 for unsophisticated investors. This result implies that sophisticated investors generate significantly more equity capital income per unit of financial wealth they invest, which in turn implies income inequality and growing polarization. These results are consistent with our earlier findings from SCF.

3.2 Parametrization

In this section, we describe the parametrization of the model that we subsequently use to assess the validity of our economic mechanism. We conduct two quantitative experiments: In the first experiment, which we label *Aggregate Portfolios*, we parameterize the model to stock-level micro data and aggregated investors equity shares. Using these data allows us to match the general pattern of outperformance of sophisticated (institutional) over unsophisticated (retail) investors. In addition, we are able to test the model's predictions regarding portfolio allocations and asset turnover across assets and over time. In the second experiment, labeled *Household Portfolios*, we use the parameterized environment from *Aggregate Portfolios* and endogenize information capacities by linking their relative values to relative wealth levels across investors types. Here, we examine whether the endogenous evolution of capacities, with an initial condition on relative capacities that is chosen based on the 1989 wealth levels, can account for the evolution of capital income inequality in the data.

Aggregate Portfolios The starting point of this experiment is to parameterize the model so that we can match key moments of the data for the period 1989-2000. We think of this as the initial period in our model and treat is as a point of departure for our dynamic comparative statics exercises. To obtain the most comprehensive set of data targets and empirical statistics for testing the mechanism, we use the same classification of sophisticated and unsophisticated investors. This allows us to bring in data on the distribution of turnover and ownership by asset characteristics and over time, as well as time series data on differences in returns.

The key parameters of our model are the information capacity of each investor type (K_1 and K_2), the averages and volatilities of the fundamental shocks ($\bar{z}_i, \sigma_i$) and the supply shocks ($\bar{x}_i, \sigma_{xi}, , i = 1, ..., n$), the risk aversion parameter (ρ), and the fraction of sophisticated investors (λ).

For parsimony, we restrict some parameters and normalize the natural candidates. In particular, we normalize $\bar{x} = 5, \bar{z} = 10$ and restrict $\sigma_{xi} = \sigma_x$. To capture heterogeneity in assets returns, we set the lowest volatility $\sigma_n = 1$ and assume that volatility changes linearly

across assets, which means that it can be parameterized by a single number, the slope of the line.[13] We pick the remaining parameters to match the following targets in the data (based on 1989-2000 averages): (i) aggregate equity ownership of sophisticated investors, equal to 23%; (ii) real risk-free interest rate, defined as the average nominal return on 3-month Treasury bills minus inflation rate, equal to 2.5%; (iii) average annualized stock market return in excess of the risk-free rate, equal to 11.9%; (iv) average monthly equity turnover, defined as the total monthly volume divided by the number of shares outstanding, equal to 9.7%; (v) the ratio of the 90th percentile to the median of the cross-sectional idiosyncratic volatility of stock returns, equal to 3.54. In addition, we arbitrarily set the fraction of assets about which agents learn to 50%.

To generate the dynamic predictions of our model, we assume a symmetric growth in information capacity of each investor type, in order to match the 2001-2012 average equity ownership rate of sophisticated investors, equal to 43%. We think of this approach as a way of modeling technological progress in investment technology which affects both types of investors in the same way–hence, the reported results are not driven by differential growth but come solely from the general equilibrium effects of our mechanism.

The above procedure leaves us with one key parameter left—the ratio of information capacity of sophisticated versus unsophisticated investors, K_1/K_2. We set this parameter to 10% in our parametrization. The parameters and model fit are presented in Tables 1 and 2.

Table 1: Parameter Values

Parameter	Value
K_1, K_2, λ, n	1.571, 0.1571, 0.2, 10
$\bar{z}_i$, $\bar{x}_i$	10, 5
σ_{xi}	0.41 for all assets i
$\{\sigma_i\}$, $i = 1, ..., 10$ assets	$\{1.5026, 1.4468, 1.3909, 1.3351, 1.2792, 1.2234, 1.1675, 1.1117, 1.0558, 1\}$

[13] In particular, we set $\sigma_i = \sigma_n + \alpha_(n - i)$ which, given our normalization of σ_n, leaves only α to be determined.

Table 2: Parametrization: Model Fit

Statistic	Data	Model
Market Return	11.9%	11.9%
Average Turnover	9.7%	9.7%
Sophisticated Investors' Ownership	23%	23%
Informed Trading	n.a.	50%

Household Portfolios In this experiment, we consider a version of our model in which information capacities evolve endogenously over time. In particular, to assess the ability of our mechanism to quantitatively account for the observed growth in income polarization at the household level (based on SCF), we link the growth in investors' information capacities to the growth in their financial wealth levels. The idea is that wealthier individuals have access to better information production or processing technologies, which in the language of our model means they have greater information capacity.

Specifically, we assume that information capacity depends linearly on financial wealth of each investor type and type 1 (2) denotes a sophisticated (unsophisticated) investor. We set the initial ratio of investors' 1 and 2 information capacity, K_1/K_2 in the model, to the 1989 ratio of average financial wealth in the top 10% and the bottom 50% of the financial wealth distribution of households with non-zero holdings of either stocks or mutual funds. In our data, this ratio is equal to 29.92. We pick the initial aggregate capacity level to match the excess return on the market portfolio, equal to 11.9% in the data. We initialize the investors at the same financial wealth level, which is chosen to match the average capital income relative to financial wealth using the same population of households.[14] This value equals 9.5% in 1989. All the remaining parameters, in particular the stochastic processes for payoffs and risk aversion, are the same as in the *Aggregate Portfolios* experiment.

[14]In the model, investment decisions are independent of wealth and only depend on the level of information capacity; hence, the levels do not matter for rates of return on equity—we normalize them for each investor type and look only at the effects of polarization in capacities.

3.3 Quantitative Results

In this section, we report quantitative results from our model for the two different parametrization exercises. We first discuss our findings related to capital income inequality and its polarization. Next, we show results related to our specific economic mechanism and provide additional discussion of alternative hypotheses.

3.3.1 Income Inequality

Aggregate Portfolios We report the results in Table 3. The parameterized model implies a 2.1 percentage point advantage (12.7% versus 10.7%) in average portfolio returns between sophisticated and unsophisticated investors, which accounts for 70% of the difference in the data for the 1989-2000 period (13.4% versus 10.4%). We conclude that, quantitatively, the model can account for a significant fraction of the empirical difference in returns across these two investor types. Further, our mechanism has an economically large implication for the difference in performance across informational capacities, which suggests that a similarly large economic effect also exists within the household sector. If sophistication can be approximated by financial wealth (as implied by a setting in Arrow (1987)), then our mechanism would imply a growing disparity in capital incomes. We test this hypothesis in the *Household Portfolios* experiment below.

Table 3: Market Averages by Subperiod: Data and Model

	1989-2000		2001-2012	
Statistic	Data	Model	Data	Model
Market Return	11.9%	11.9%	3.5%	2.4%
Sophisticated Investors' Return	12.7%	16%	3.7%	2.9%
Unsophisticated Investors' Return	10.7%	13%	3.4%	1.6%
Average Equity Turnover	9.7%	9.7%	14%	16%
Sophisticated Investors' Ownership	23%	23%	43%	43%

Household Portfolios In this experiment, we investigate the endogenous propagation of heterogeneity in capacity across time by simulating the model for 21 years forward, which is

the time span of our data set. As the outcome of the experiment, we obtain the endogenous capital income and financial wealth dispersion growth implied by our mechanism. Along the simulation period, capacity growth of each investor type is endogenously determined by the return on her wealth, as only the initial dispersion is exogenously determined. The results of this exercise are presented in Table 4.

Table 4: Capital Income and Wealth Dispersion: Data and Model

	Data 1989-2010	Model
Capital Income Dispersion Growth	90%	98%
Financial Wealth Dispersion Growth	88%	52%

We obtain a 98% growth in capital income inequality (90% in the data) and a 52% growth in financial wealth inequality (88% in the data), which is over 100% and 59% of the growth observed in the data, respectively. We conclude that our mechanism has a strong power to explain income and wealth polarization observed in the data. The model matches both the level and slope of capital income dispersion, as shown in Figure 3.

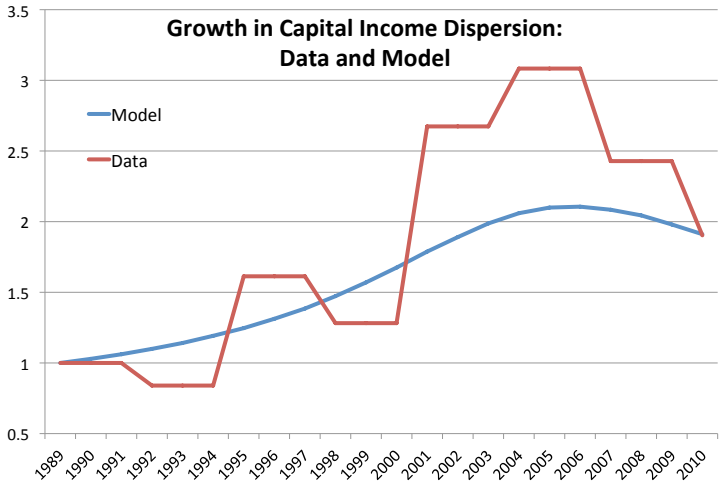


Figure 3: Cumulative Growth in Capital Income Dispersion

3.3.2 Testing the Mechanism

The results in the previous section demonstrate a significant potential of our information mechanism to account for the return differentials and income inequality observed in the data. In this section, we provide a set of quantitative predictions from *Aggregate Portfolios*, which allow us to provide additional support for our mechanism. These are robust predictions of our mechanism and are proven analytically in Section 2. Below, we show the robustness of these results not only qualitatively but also quantitatively.

Market Averages Technological progress in information capacity in the model implies large changes in average market returns, cross-sectional return differential, and turnover. We report these statistics generated by the model and observed in the data in Table 3.

The changes implied by the model all qualitatively match the changes in the data, but they also come close quantitatively. Both the model and the data imply a decrease in market return and a decrease in the return differential of portfolios held by sophisticated and unsophisticated investors. Intuitively, in the model, lower market return is a result of an increase in quantity of information: The price reflects that and tracks much more closely the actual return z than in the initial parametrization with lower overall capacity (for additional intuition, see proposition 6).

The model also predicts a sharp increase in average asset turnover, in magnitudes consistent with the data. As with the market return, this result is a direct implication of our mechanism and is not driven by changes in asset volatility. Intuitively, higher turnover in the model is driven by more informed trading by sophisticated investors, both due to their holding a larger share of the market as well as them receiving more precise signals about asset payoffs.

Ownership Investors in our model prefer to learn about assets with higher volatility. In particular, upon increasing their information capacity, they first invest it in the most volatile asset until the benefits from a unit of information become equalized with those of the second highest volatility asset, then third, and so forth (see lemma 2). This process

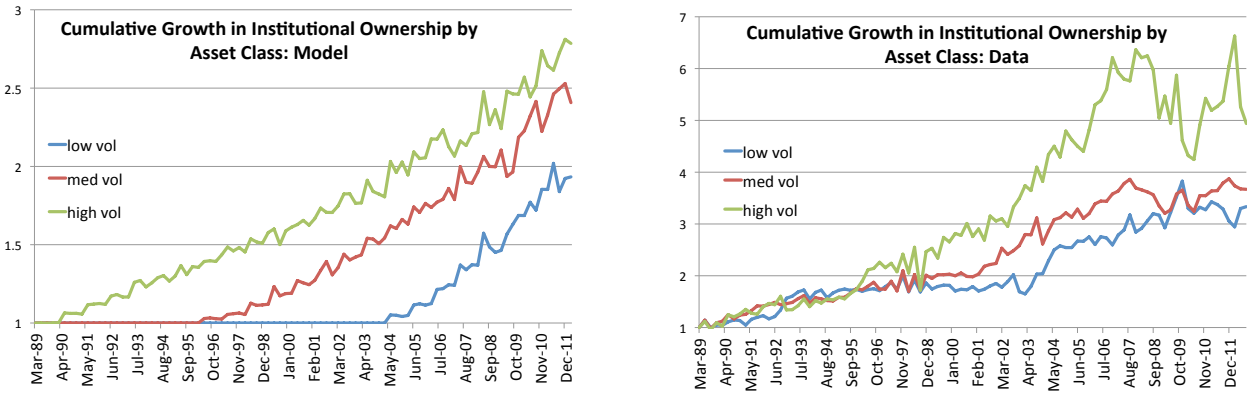


Figure 4: Cumulative Growth in Sophisticated Investors' Ownership: Data and Model

implies a particular way in which institutions expand their portfolio holdings as their capacity (through overall capacity) increases. Specifically, we should see that sophisticated investors exhibit the highest initial growth in ownership for the the highest volatility assets, then lower volatility assets, etc. Figure 4 shows the evolution of this growth in the model and in the data over the period 1989-2012.[15]

In Figure 5, we show *the change* in asset ownership by sophisticated investors over the periods 1989-2000 and 2001-2012, where assets are sorted by volatility of their returns. This cross-sectional change underlies the average ownership targets in the model of 23% in the initial period and 43% in the later period. Both the data and the model exhibit a hump-shaped profile of the increase and they are also very close quantitatively.

In conclusion, even though we parameterize the model to match the *aggregate* ownership levels of sophisticated investors in the pre- and post-2000 period, the model is also able to explain quantitatively *how* ownership increases *across* asset volatility classes, both in terms of timing of the growth levels and in terms of the absolute magnitudes of the ownership changes.

Turnover Our model implies cross-sectional variation in asset turnover, driven by differential investment of investors' information capacity. Intuitively, if an asset is more attractive

[15]To generate this graph in the model, we increase aggregate capacity from zero to the level that matches 48% institutional ownership, which is the last point in the data.

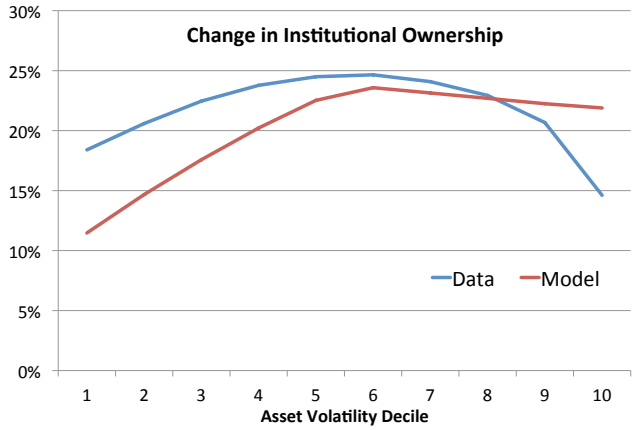


Figure 5: Absolute Change in Sophisticated Investors' Ownership

and investors invest more in it, then there are more investors with precise signals about assets returns, and these investors want to act on the better information by taking larger and more volatile positions. Since the sophisticated investors receive more precise signals, and they have preference towards high-volatility assets, we should see a positive relationship between volatility and turnover. We report turnover in relation to return volatility in the model and in the data in Table 5.

Table 5: Turnover by Asset Volatility

Volatility quintile	1	2	3	4	5	Mean
1989-2000						
Data	5%	8.5%	10.5%	12.5%	11.5%	9.7%
Model	9%	9%	9.3%	9.9%	10.8%	9.7%
2001-2012						
Data	11%	14.6%	17%	18.4%	19.3%	16%
Model	12.5%	13.6%	14.2%	15%	15.4%	14%

The first two rows compare data and the model prediction for the initial parametrization to 1989-2000 data. Both data and model show increasing patterns in turnover as volatility goes up, which are quantitatively close to each other. In the next two rows, we compare data for the 2001-2012 period to results generated from the dynamic exercise in the model in which we increase overall capacity. The model implies an increase in average turnover

compared to an earlier period and additionally matches the cross-sectional pattern of the increase. This effect is purely driven by our information friction, since the fundamental volatilities remain constant over time in this exercise.[16]

3.3.3 Additional Supporting Evidence

So far, we presented quantitative results supporting our analytical predictions that are based on quantitative calibration of the model. Specifically, our theoretical predictions imply that differences in capital income generally can stem from two sources: heterogeneity in prices of investable assets and the differential exposure of investors to holding such assets. In this section, we provide additional evidence on each of these channels that offers support to our predictions qualitatively but cannot be assessed quantitatively.

Unsophisticated Investors' Retrenchment In this section, we show that cross-sectional differences in assets holdings of investors with different levels of sophistication are consistent with predictions of our model and thus contribute to capital income inequality and its growing polarization. Our main prediction is that unsophisticated investors should be more likely to invest in assets with lower expected values. In the quantitative tests of the model in Figure 4, we show that sophisticated investors allocate their wealth first into assets with highest level of volatility and subsequently into assets with lower levels of volatility. Now, we provide additional evidence which suggests similar investors' preferences.

Our first piece of evidence is based on SCF data regarding households' holdings in liquid wealth. The idea of this test is that unsophisticated investors should be more likely to invest in safe (liquid) assets. SCF provides detailed classification of wealth invested in such assets that include checking accounts, call accounts, money market accounts, coverdell accounts, and 529 educational state-sponsored plans. As before, in each period, we divide households into two groups: top 10% and bottom 50% of the financial wealth distribution. For each of the groups, we calculate the average ratio of liquid wealth to total financial wealth. Higher

[16] Our model also implies a positive turnover-ownership relationship, which we further confirm in the data. This result is also consistent with the empirical findings in Chordia, Roll, and Subrahmanyam (2011).

ratios would imply greater exposure to low-profit assets. We present the two time series in Figure 6.

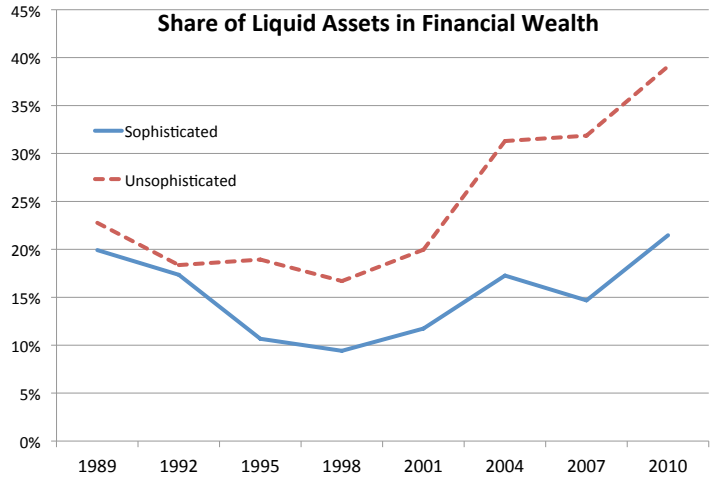


Figure 6: Share of Liquid Wealth in Financial Wealth: Survey of Consumer Finances.

We find evidence that strongly supports predictions from our model. First, the average ratio of liquid wealth for sophisticated investors, equal to 15.3%, is significantly lower than that for unsophisticated investors, which in our sample equals 25%. In addition, while the exposure to liquid assets by sophisticated investors is generally non-monotonic (u-shaped), similar investment for unsophisticated investors exhibits a strong positive time trend, especially in the last 20 years: the average investment goes up from 16.7% in 1998 to 39% in 2010. This evidence strongly supports our economic mechanism in that differences in information capacity lead to retrenchment by unsophisticated investors from risky assets and relocation to safer assets.

We further confirm this claim using evidence on institutional holdings from Thomson Reuters. To this end, we calculate average (equal-weighted) equity ownership of sophisticated investors (mutual funds and hedge funds) and unsophisticated (retail) investors. We report the respective time series quarterly averages of the ownership over the period 1989-2012 in Figure 7.

The results paint a picture that is generally consistent with our model's predictions. Although the average ownership level of unsophisticated investors is higher in an unconditional sample and equals 61%, the time series evidence clearly indicates a very strong pattern: the

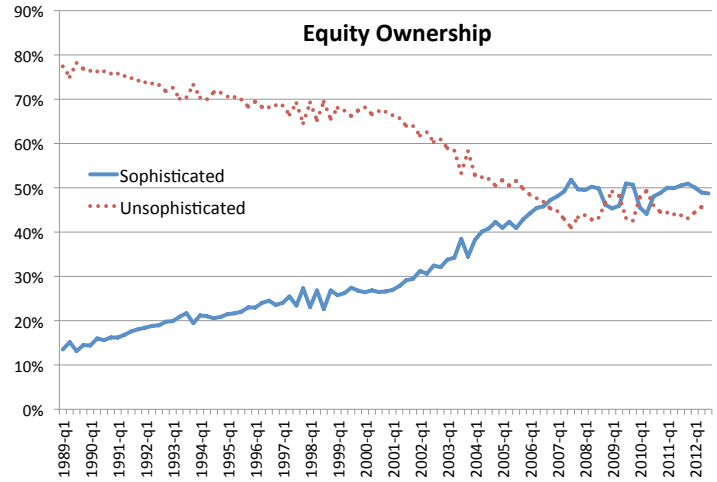


Figure 7: Equity Ownership by Sophistication Type.

average equity ownership for unsophisticated investors goes down while that for sophisticated investors significantly goes up.[17] We argue that this evidence is consistent with the observed expansion of relative information capacities presented in Figure 1. Realizing a positive shock to information capacity sophisticated investors enter profitable equity market at the expense of unsophisticated investors who perceive the informational disadvantage in the market and as a result move away from equity. Notably, the retrenchment of unsophisticated investors from directly holding equity happened despite the overall strong performance of equity markets over the same time period. This suggests that investors do not simply respond to past trends in equity returns.

As a final auxiliary prediction we consider money flows into mutual funds. The idea is that equity mutual funds are more risky than non-equity funds. As such, unsophisticated investors should be less likely to invest in the former, especially if information capacity gets more polarized.

To test this prediction in the data we use mutual fund data from Morningstar. Morningstar classifies different funds into those serving institutional investors and individuals whose investment is at least $100,000 (institutional funds) and those serving individual in-

[17]The visible positive trend in active ownership has been documented before by Gompers and Metrick (2001) and is even stronger if one accounts for differences in market values across assets and the preference of sophisticated investors for large-cap stocks.

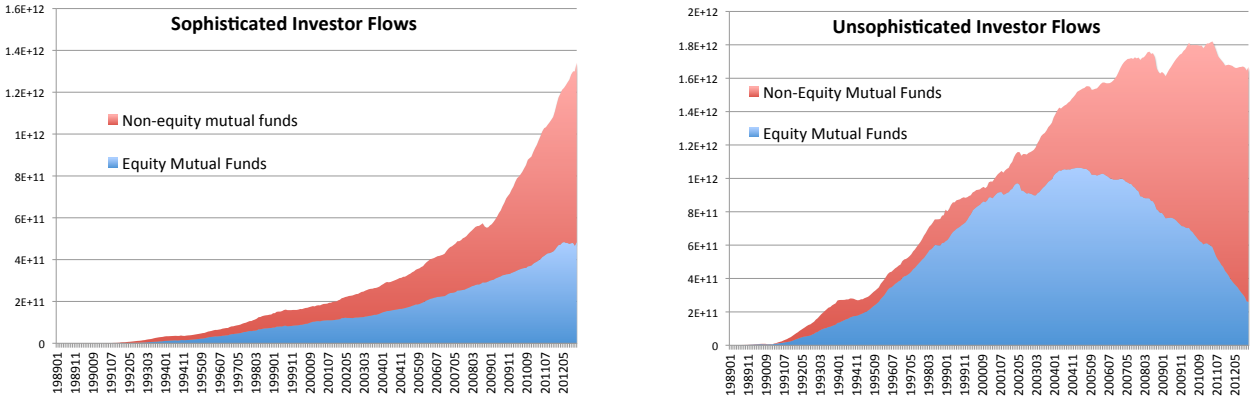


Figure 8: Cumulative Flows to Mutual Funds by Sophistication Type: Equity vs. Non-Equity

vestors with investment value less than $100,000 (retail funds). For the purpose of our predictions, we define sophisticated investors as institutional funds and unsophisticated investors as retail funds. Subsequently, we calculate cumulative aggregated dollar flows into equity and non-equity funds, separately for each investment type. Our data span the period 1989-2012. We present the results in Figure 8.

We find that the cumulative flows from sophisticated investors into equity and non-equity funds increase steadily over the whole sample period. In contrast, the flows from unsophisticated investors display a visibly different pattern. The flows into equity funds keep increasing until 2000 but subsequently decrease at a significant rate of more than 3 times by 2012. Moreover, the decrease in cumulative flows to equity mutual funds coincides with a significant increase in cumulative flows to non-equity funds. Overall, these findings support predictions of our model: Sophisticated investors have a large exposure to risky assets and subsequently add extra exposure to less risky assets, whereas unsophisticated investors leave riskiest assets and move into safer assets as they perceive higher information disadvantage.

One could note that the increase in equity flows by unsophisticated investors in the early period of our sample is inconsistent with our model. We argue that this result could still be rationalized by contrasting it with the steady decrease in holdings of individual equity

documented earlier. To the extent that individual equity holdings are more risky than diversified equity portfolios, such as mutual funds, this only means that in the earlier period unsophisticated investors reallocate their wealth from riskier to safer asset class.

Stock Selection Ability The second building block of our economic mechanism is the ability of sophisticated investors to better choose assets. Our quantitative evaluation maps the model prediction to the observed differences in performance between sophisticated and unsophisticated investors. Here, we provide an additional qualitative result in which we show that equity holdings of sophisticated investors are higher for stocks which subsequently realize higher returns.

To conduct this test, we obtain data on stock returns come from Center for Research on Security Prices (CRSP), and for each stock we calculate the market shares of sophisticated investors. Next, we estimate the regression model over the period January 1989-December 2012 with stock/month as a unit of observation. Our dependent variable is the share of sophisticated investors at month t and the independent variable is return corresponding to the stock. Our regression model includes year-month-fixed effects and standard errors are clustered at the stock level to account for the cross-sectional correlation in the data. We report the results of this estimation in Table 6.

Table 6: Contemporaneous Returns Explain Sophisticated Investors' Ownership

Variable	Value	Standard Error
Contemporaneous Return	0.048	0.00845
Constant	0.300	0.00007
Year-Month-Fixed Effects		Yes
Number of Observations		1,525,787

We find strong evidence that sophisticated investors in our sample tend to invest more in stocks that subsequently generate higher returns. Hence, we conclude that sophisticated investors in our sample exhibit superior stock-selection ability. This finding is consistent with a number of other studies that show the strong existence of stock-picking ability among sophisticated investors, such as actively managed equity mutual funds (e.g. Daniel, Grinblatt,

Titman, and Wermers (1997), Cohen, Coval, and Pástor (2005), Kacperczyk, Sialm, and Zheng (2005), Kacperczyk and Seru (2007)). At the individual level, there is ample anecdotal evidence that shows superior investment ability of wealthy investors such as Warren Buffett or Carl Icahn.

Overall, our evidence is consistent with the premise of our economic mechanism that sophisticated investors are good at choosing assets and relocating their resources to the most profitable ones.

3.3.4 Discussion of Alternative Mechanisms

Our evidence so far strongly suggests that heterogeneity in information quality has strong ability to explain cross-sectional and time-series patterns in capital income inequality, while simultaneously producing results that are consistent with other micro-level financial data. While the information friction constitutes a plausible economic mechanism, there might be other mechanisms which could also be used as explanations of the data. In this section, we consider two such explanations: heterogeneity in risk aversion and differences in transaction costs. We discuss their respective merits in terms of the ability to explain the empirical facts, both within the context of our model and also in a more general setting.

The first potential explanation is that income inequality in the data is driven by differences in risk aversion across economic agents. In particular, if one group of investors is less risk averse they would hold a greater share of risky assets with higher expected returns and hence would have higher expected capital income.

We argue that both within our CARA model and an alternative CRRA specification, such result is unlikely. In particular, heterogeneity in risk aversion per se would produce a growing (in difference of risk aversions) ownership of sophisticated investors in risky assets, but it would not generate any difference in investor-specific rates of return on equity. Additionally, a growing risk aversion disparity would generate a uniform proportional retrenchment of the high risk aversion agents from equity, which is inconsistent with evidence in Figure 4. That evidence, supported additionally by regression results in Section 3.3.3, suggests that the excess market performance is driven by sophisticated investors explicitly picking different

portfolio shares (as opposed to pure timing). Finally, differences in risk aversion across agents cannot explain other micro-level facts in the data, such as the asymmetric ownership across asset classes (in Figure 4) or turnover profile of assets (in Table 5).

We are not the first ones to point out that preference-based approaches to explaining household portfolio choice suffer from serious drawbacks. Dumas (1989) and more recently Chien, Cole, and Lustig (2011) have argued that differences in risk preferences cannot account for observed differences in rates of returns across agents with different degrees of sophistication.

The second alternative mechanism aims to explain the data using differences in transaction costs across agents. To the extent that less sophisticated investors face higher transaction costs in risky asset markets they would be willing to participate less, as argued in Gomes and Michaelides (2005) and others.

While this explanation might have some merit to explain cross-sectional patterns in the data, we believe it is less likely to explain the time-series results. In particular, we observe that more sophisticated households generate significantly greater gap in their incomes over time, which is hard to reconcile with the fact that there was not much change in the overall quantity of transaction costs, as reported in French (2008). In fact, if anything, growth in internet access and services made an access to more direct investing extremely easy and relatively less costly for the average citizen as opposed to just the few privileged ones.

Overall, while we believe that alternative mechanisms can be certainly at play it is hard to use them to explain the full set of results we document in the paper.

4 Concluding Remarks

What contributes to the growing income inequality across households? This question has been of great economic and policy relevance for at least several decades starting with a seminal work by Kuznets. We approach this question from the perspective of capital income that is known to be highly unequally distributed across individuals. We propose a theoretical information-based framework that links capital income derived from financial assets

to a level of investor sophistication. Our model implies the presence of income inequality between sophisticated and unsophisticated investors that is growing in the extent of total sophistication in the market and in relative sophistication across investors. Additional predictions on asset ownership, market returns, and turnover help us pin down the economic mechanism and rule out alternative explanations. The quantitative predictions of the model match qualitatively and quantitatively the observed data.

Although our empirical findings are strictly based on the U.S. market, our model should have similar implications for other financial markets. For example, qualitatively, we know that income inequality in emerging markets tends to be even larger than the one documented for the U.S. To the extent that financial sophistication in such markets is much more skewed one could rationalize within our framework the differences in capital incomes. Similarly, the U.S. market is considered to be the most advanced in terms of its total sophistication, which is possibly why we find a greater dispersion in capital income compared to other developed markets, such as those in Europe or Asia.

More generally, one could argue that although the overall growth of investment resources and competition across investors with different skill levels are generally considered as a positive aspect of a well-functioning financial market, our work suggests that one should assess any policy targeting overall information environment in financial markets as potentially exerting an offsetting and negative effect on socially relevant issues, such as distribution of income. We leave detailed evaluation of such policies for future research.

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Appendix

Theoretical Framework

Proof: The distribution of excess returns. Let $\widehat{R}_{ji}$ and $\widehat{V}_{ji}$ denote the mean and variance of the ex-ante distribution (in subperiod 1) of the posterior beliefs about excess returns, $\widehat{\mu}_{ji} - rp_i$. We have that $E_{1j}\left(\widehat{\mu}_{ji}\right) = \overline{z}_i$,and hence

$$\widehat{R}_i = \overline{z}_i - r\overline{p}_i, \tag{30}$$

the same across all investors j.The variance of posterior beliefs about excess returns, $\widehat{V}_{ji}$, is given by

$$\widehat{V}_{ji} = Var_{1j}\left(\widehat{\mu}_{ji}\right) + r^2\sigma_{pi}^2 - 2rCov\left(\widehat{\mu}_{ji}, p_i\right).$$

From the distribution of posterior beliefs,

$$Var_{1j}\left(\widehat{\mu}_{ji}\right) = \frac{Cov^2\left(z_i, s_{ji}\right)}{Var\left(s_{ji}\right)}.$$

The signal structure implies that $Cov\left(z_i, s_{ji}\right) = Var\left(s_{ji}\right)$ such that $Var_{1j}\left(\widehat{\mu}_{ji}\right) = Var\left(s_{ji}\right)$. We compute $Cov\left(\widehat{\mu}_{ji}, p_i\right)$ exploiting the fact that posterior beliefs and prices are conditionally independent given payoffs, and hence

$$Cov\left(\widehat{\mu}_{ji}, p_i\right) = \frac{Cov\left(\widehat{\mu}_{ji}, z_i\right) Cov\left(z_i, p_i\right)}{\sigma_i^2},$$

with $Cov\left(z_i, p_i\right) = b_i\sigma_i^2$ and $Cov\left(\widehat{\mu}_{ji}, z_i\right) = Var\left(s_{ji}\right)$. Then, $\widehat{V}_{ji}$ becomes

$$\widehat{V}_{ji} = \left(1 - 2rb_i\right) Var\left(s_{ji}\right) + r^2\sigma_{pi}^2.$$

Equivalently,

$$\widehat{V}_{ji} = \widehat{S}_i - \left(1 - 2rb_i\right)\sigma_{\delta ji}^2$$

where

$$\widehat{S}_i \equiv \left(1 - 2rb_i\right)\sigma_i^2 + r^2\sigma_{pi}^2 \tag{31}$$

is the same across investors. ■

Proof: Objective function in (9). The agent's objective is to maximize ex-ante expected utility,

$$\max \frac{1}{2\rho}\sum_{i=1}^{n}\left\{\left(\frac{1}{\widehat{\sigma}_{ji}^2}\right) E_{1j}\left[\left(\widehat{\mu}_{ji} - rp_i\right)^2\right]\right\}.$$

Using $\widehat{\sigma}_{ji}^2 = \sigma_{\delta ji}^2$ and the distribution of excess returns derived above, the objective function becomes

$$U_{1j} = \frac{1}{2\rho}\sum_{i=1}^{n}\left(\frac{\widehat{S}_i + \widehat{R}_i^2}{\sigma_{\delta ji}^2}\right) - \frac{1}{2\rho}\sum_{i=1}^{n}\left(1 - 2rb_i\right),$$

where the second term is independent of the investor's choices. Hence the investor's objective becomes choosing the variance $\sigma^2_{\delta ji}$ for each asset i to solve

$$\max_{\{\sigma^2_{\delta ji}\}_{i=1}^n} \sum_{i=1}^n \left(\frac{\widehat{S}_i + \widehat{R}_i^2}{\sigma^2_{\delta ji}} \right), \tag{32}$$

where, from the derivation of excess returns, $\widehat{R}_i \equiv \overline{z}_i - r\overline{p}_i$, and $\widehat{S}_i \equiv (1 - 2rb_i)\,\sigma_i^2 + r^2\sigma^2_{pi}$. ■

Proof: Information constraint in (10). For each asset i, the entropy of $z_i \sim f(z_i) = N(\overline{z}_i, \sigma_i^2)$ is

$$\begin{aligned}
H(z_i) &= \int f(z_i) \ln \frac{1}{f(z_i)} dz_i \\
&= \int f(z_i) \ln \left\{ \sqrt{2\pi\sigma_i^2} \exp \left[\frac{(z_i - \overline{z}_i)^2}{2\sigma_i^2} \right] \right\} dz_i \\
&= \int f(z_i) \left[\frac{1}{2} \ln \left(2\pi\sigma_i^2 \right) + \frac{(z_i - \overline{z}_i)^2}{2\sigma_i^2} \right] dz_i \\
&= \frac{1}{2} \ln \left(2\pi\sigma_i^2 \right) + \frac{1}{2\sigma_i^2} \int f(z_i) (z_i - \overline{z}_i)^2 dz_i = \\
&= \frac{1}{2} \ln \left(2\pi e \sigma_i^2 \right).
\end{aligned}$$

Hence, the signal structure, $z_i = s_{ji} + \delta_{ji}$, implies that

$$\begin{aligned}
I(z_i; s_{ji}) &= H(z_i) + H(s_{ji}) - H(z_i, s_{ji}) \\
&= \frac{1}{2} \log \left(2\pi e \sigma_i^2 \right) + \frac{1}{2} \log \left(2\pi e \sigma^2_{sji} \right) - \frac{1}{2} \log \left[(2\pi e)^2 \left| \Sigma_{z_i s_{ji}} \right| \right] \\
&= \frac{1}{2} \log \left(\frac{\sigma_i^2 \sigma^2_{sji}}{\left| \Sigma_{z_i s_{ji}} \right|} \right) = \frac{1}{2} \log \left(\frac{\sigma_i^2}{\sigma^2_{\delta i}} \right),
\end{aligned}$$

where $\left| \Sigma_{z_i s_{ji}} \right|$ is the determinant of the variance-covariance matrix of z_i and s_{ji}, $\left| \Sigma_{z_i s_{ji}} \right| = \sigma^2_{sji}\sigma^2_{\delta ji}$. Hence, across assets,

$$I(z; s_j) = \frac{1}{2} \sum_{i=1}^n \log \left(\frac{\sigma_i^2}{\sigma^2_{\delta ji}} \right) = \frac{1}{2} \log \left(\prod_{i=1}^n \frac{\sigma_i^2}{\sigma^2_{\delta ji}} \right).$$

Hence, the information constraint becomes

$$\prod_{i=1}^n \left(\frac{\sigma_i^2}{\sigma^2_{\delta ji}} \right) \leq e^{2K_j}. \tag{33}$$

■

Proof of Proposition 1. The linear of the objective function and the convex constraint imply a corner solution for the optimal allocation of attention for each individual investor, and an interior allocation of attention across a subset of assets in the overall economy. ■

Proof of Lemma 1. Market clearing for each asset $i \notin L$ that is not learned about in equilibrium is given by

$$\frac{\overline{z}_i - rp_i}{\rho\sigma_i^2} = x_i.$$

Market clearing for each asset $i \in L$ that is learned about in equilibrium is given by

$$\int_0^1 \left(\frac{s_{ji} - rp_i}{\rho e^{-2K_j}\sigma_i^2}\right) dj = x_i.$$

Not all investors will choose to learn about a particular asset that is learned about in equilibrium. Let m_i denote the mass of investors learning about asset $i \in L$. Since the gain factor G_i for each i is the same across all investors, regardless of investor type, the participation of sophisticated and unsophisticated investors in learning about a particular asset will be proportional to their mass in the population. Hence, let M_{1i} denote the set of sophisticated investors who choose to learn about asset i, of measure $\lambda m_i \geq 0$, and M_{2i} the set of unsophisticated investors who choose to learn about asset i, of measure $(1-\lambda)\, m_i \geq 0$. Then market clearing becomes

$$\int_{M_{1i}} \left(\frac{s_{ji} - rp_i}{e^{-2K_1}\rho\sigma_i^2}\right) dj + \int_{M_{2i}} \left(\frac{s_{ji} - rp_i}{e^{-2K_2}\rho\sigma_i^2}\right) dj + (1 - m_i)\left(\frac{\overline{z}_i - rp_i}{\rho\sigma_i^2}\right) = x_i.$$

Each signal s_{ji} received by an investor of type j is a weighted average of the true realization, z_i, and the prior, $\overline{z}_i$, with mean

$$E\left(s_{ji}|z_i\right) = \left(1 - e^{-2K_j}\right) z_i + e^{-2K_j}\overline{z}_i.$$

Hence

$$\int_{M_{1i}} s_{ji} dj = \lambda m_i \left[\left(1 - e^{-2K_1}\right) z_i + e^{-2K_1}\overline{z}_i\right]$$

and

$$\int_{M_{2i}} s_{ji} dj = (1-\lambda)\, m_i \left[\left(1 - e^{-2K_2}\right) z_i + e^{-2K_2}\overline{z}_i\right].$$

The market clearing equation above can be written as

$$\alpha_1 \overline{z}_i + \alpha_2 z_i - x_i = \alpha_3 rp_i,$$

where

$$\alpha_1 \equiv \frac{\lambda m_i}{\rho\sigma_i^2} + \frac{(1-\lambda)\, m_i}{\rho\sigma_i^2} + \frac{1-m_i}{\rho\sigma_i^2}$$
$$\alpha_2 \equiv \frac{\lambda m_i}{\rho\sigma_i^2}\left(e^{2K_1}-1\right) + \frac{(1-\lambda)\, m_i}{\rho\sigma_i^2}\left(e^{2K_2}-1\right)$$
$$\alpha_3 \equiv \frac{\lambda m_i}{\rho\sigma_i^2}e^{2K_1} + \frac{(1-\lambda)\, m_i}{\rho\sigma_i^2}e^{2K_2} + \frac{1-m_i}{\rho\sigma_i^2}.$$

Defining

$$\phi \equiv \lambda\left(e^{2K_1}-1\right) + (1-\lambda)\left(e^{2K_2}-1\right),$$

we obtain the identification of the coefficients in

$$p_i = a_i + b_i z_i - c_i x_i$$

as

$$a_i = \frac{\overline{z}_i}{r\,(1+\phi m_i)}, \quad b_i = \frac{\phi m_i}{r\,(1+\phi m_i)}, \quad c_i = \frac{\rho\sigma_i^2}{r\,(1+\phi m_i)}. \tag{34}$$

■

Proof of Lemma 2. For any K_1, $K_2 > 0$, at least one asset will be learned about in the economy. Suppose only one asset is learned about, and let this asset be denoted by l. This implies that the masses are $m_l = 1$ and $m_i = 0$ for all any $i \neq l$, and that $G_l > G_i$ for any $i \neq l$, i.e.

$$\frac{1+\rho^2\xi_l}{(1+\phi)^2} > 1+\rho^2\xi_i.$$

Since $\phi > 0$, the inequality holds if and only if $\xi_l > \xi_i$ for any $i \neq l$. We have assumed, without loss of generality, that assets in the economy are ordered such that, for all $i = 1, ...n-1$, $\xi_i > \xi_{i+1}$.Hence, $l = 1$: the asset learned about is the asset with the highest idiosyncratic term ξ_1.

Finally, the threshold for starting to learn about the second asset, namely the point at which the inequality above no longer holds, taking shocks and risk aversion as given, is

$$\phi_1 \equiv \sqrt{\frac{1+\rho^2\xi_1}{1+\rho^2\xi_2}} - 1. \tag{35}$$

At this threshold market capacity, the investors in the economy begin also learning about at least one other asset, and that asset is ξ_2. Hence, for aggregate information capacity such that $0 < \phi < \phi_1$, only one asset is learned about, and that asset is ξ_1. For $\phi \geq \phi_1$, at least two assets are learned about in equilibrium. For any assets learned about in equilibrium, the gain factors must be equated, which yields

$$\left(\frac{1+\phi m_k}{1+\phi m_l}\right)^2 = \frac{1+\rho^2\xi_k}{1+\rho^2\xi_l}, \quad \forall k, l \in L. \tag{36}$$

Moreover, any asset not learned about in equilibrium must have a strictly lower gain factor, and

hence

$$\frac{1+\rho^2\xi_k}{(1+\phi m_k)^2} > 1+\rho^2\xi_h, \quad \forall k \in L, h \notin L. \tag{37}$$

■

Proof of Lemma 3. *(i)* The necessary and sufficient set of conditions for characterization of m_i's in equilibrium is

$$\frac{1+\phi m_i}{1+\phi m_1} = c_{i1} \qquad \forall i \leq k, \tag{38}$$

$$\sum_{i=1}^{k} m_i = 1. \tag{39}$$

The first set of equalities in (38) yields

$$m_i = c_{i1} m_1 - \frac{1}{\phi}(1-c_{i1}) \qquad \forall i \in \{2,...,k\}. \tag{40}$$

Plugging into the feasibility constraint (39), we obtain

$$1 = \sum_{i=1}^{k} m_i = m_1 + m_1 \sum_{i=2}^{k} c_{i1} - \frac{1}{\phi}\sum_{i=2}^{k}(1-c_{i1})$$

which results in a solution for m_1 given by

$$m_1 = \frac{1+\frac{1}{\phi}\sum_{i=2}^{k}(1-c_{i1})}{(1+\sum_{i=2}^{k} c_{i1})}. \tag{41}$$

Since for $i > k$, $m_i = 0$ holds trivially, this completes the proof of (i).

(ii) Differentiating (41) with respect to ϕ, we obtain

$$\frac{dm_1}{d\phi} = -\frac{1}{\phi^2}\frac{\sum_{i=2}^{k}(1-c_{i1})}{(1+\sum_{i=2}^{k} c_{i1})}.$$

For all $i > 1$, $c_{i1} < 1$ and hence $\frac{dm_1}{dt\phi} < 0$. Differentiating (40) with respect to ϕ gives

$$\frac{dm_i}{d\phi} = c_{i1}\frac{dm_1}{d\phi} + \frac{1}{\phi^2}(1-c_{i1})$$

which gives (ii):

$$\begin{aligned}\frac{dm_i}{d\phi} &= \frac{1}{\phi^2}\left[1 - c_{i1} - c_{i1}\frac{\sum_{j=2}^{k}(1-c_{j1})}{1+\sum_{j=2}^{k}c_{j1}}\right] = \\ \frac{1}{\phi^2}\left[1 - c_{i1}\frac{1+\sum_{j=2}^{k}c_{i1}+\sum_{j=2}^{k}(1-c_{j1})}{1+\sum_{j=2}^{k}c_{j1}}\right] &= \frac{1}{\phi^2}\left[1 - c_{i1}\frac{k}{1+\sum_{j=2}^{k}c_{j1}}\right]. \qquad (42)\end{aligned}$$

(iii) First notice that $\frac{dm_1}{d\phi} < 0$ together with the feasibility constraint implies that $\frac{dm_i}{d\phi} > 0$ for at least one $i \leq k$. The cutoff condition is then a direct consequence of the fact that $c_{11} = 1$ and that c_{i1} is strictly decreasing in i.

Finally, consider the derivative to total information devoted to asset i with respect to aggregate capacity ϕ:

$$\begin{aligned}\frac{d\phi m_i}{d\phi} &= m_i + \frac{1}{\phi}\left[1 - c_{i1} - c_{i1}\frac{\sum_{j=2}^{k}(1-c_{j1})}{1+\sum_{j=2}^{k}c_{j1}}\right] \\ &= c_{i1}\frac{1+\frac{1}{\phi}\sum_{i=2}^{k}(1-c_{i1})}{(1+\sum_{i=2}^{k}c_{i1})} - \frac{1}{\phi}(1-c_{i1}) + \frac{1}{\phi}\left[1 - c_{i1} - c_{i1}\frac{\sum_{j=2}^{k}(1-c_{j1})}{1+\sum_{j=2}^{k}c_{j1}}\right] \\ &= c_{i1}\frac{1}{(1+\sum_{i=2}^{k}c_{i1})} > 0.\end{aligned}$$

(vi) First, consider the case of a local increase in ϕ to some $\phi' < \phi_k$, such that no new assets are learned about in equilibrium. Suppose that $\phi_{k-1} \leq \phi < \phi_k$, such that only k assets are learned about in equilibrium. Consider the case of a local increase in ϕ to some $\phi' < \phi_k$, such that no new assets are learned about in equilibrium. For $i > k$, $m_i = 0$ both before and after the capacity increase, hence $\frac{d(\phi m_i)}{d\phi} = 0$. For $i, l \leq k$, by lemma 4,

$$\frac{1+\phi m_i}{1+\phi m_l} = c_{il},\ \forall i, l \leq k. \qquad (43)$$

where $c_{il} \equiv \sqrt{\frac{1+\rho^2\xi_i}{1+\rho^2\xi_l}} > 0$, a constant. Then

$$1 + \phi m_i - (1+\phi m_l)\, c_{il} = 0. \qquad (44)$$

Totally differentiating, we have that

$$m_i + \phi\frac{dm_i}{d\phi} = \left(m_l + \phi\frac{dm_l}{d\phi}\right) c_{il}. \qquad (45)$$

Suppose that there exists an asset i such that $\frac{d(\phi m_i)}{d\phi} \leq 0$. Then for all assets $l \leq k$, $\left(m_l + \phi\frac{dm_l}{d\phi}\right) c_{il} \leq 0$. Since $c_{il} > 0$, $m_l > 0, \phi > 0$, then we must have that $\frac{dm_l}{d\phi} < 0$. Hence

for all assets learned about, $\frac{dm_i}{d\phi} < 0$. But since $\Sigma_i m_i = 1$, there must be at least one asset for which $\frac{dm_i}{d\phi} \geq 0$, which is a contradiction. Hence for all $i \leq k$, $\frac{d(\phi m_i)}{d\phi} > 0$.

Second, suppose capacity increases from ϕ to ϕ', with $\phi_{k-1} \leq \phi < \phi_k \leq \phi' < \phi_{k+x}$, such that $x \geq 1$ new assets are learned about in equilibrium, with $x \leq n - k$, since there are only n assets in the economy. For $i > k + x$, $m_i = m_i' = 0$, hence there is no change in the aggregate capacity allocated to these assets. For $i \in \{k+1, ..., k+x\}$, $m_i' > m_i = 0$, hence $\phi' m_i' > \phi m_i$. For two assets i, l with $i \leq k$ and $k + 1 < l \leq k + x$,

$$1 + \phi m_i - c_{il} > 0 \quad \text{and} \quad 1 + \phi' m_i' - (1 + \phi' m_l')\, c_{il} = 0. \tag{46}$$

Hence, $\phi' m_i' - \phi m_i > \phi' m_l' (1 + \phi m_i)$. Since the right hand side is positive, $\phi' m_i' > \phi m_i$. ■

Analytical Predictions

Proof of Proposition 2. Using equations (26)-(27), the difference in profits for asset i is given by

$$\pi_{1it} - \pi_{2it} = \frac{m_i \left(e^{2K_1} - e^{2K_2}\right) (z_{it} - rp_{it})^2}{\rho \sigma_i^2} \geq 0. \tag{47}$$

The difference in equation (47) is zero if $m_i = 0$ or $K_1 = K_2$. For $K_1 > K_2$, it is strictly positive for assets that are learned about in equilibrium (i.e. if $m_i > 0$). Also $K_1 > K_2$ implies $\phi > 0$. It follows that $m_i > 0$ for at least one i. ■

Proof of Proposition 3. Using equations (26)-(27), the ownership difference for asset i becomes

$$\frac{Q_{1it}}{\lambda} - \frac{Q_{2it}}{(1-\lambda)} = m_i \left(e^{2K_1} - e^{2K_2}\right) \left(\frac{z_{it} - rp_{it}}{\rho \sigma_i^2}\right). \tag{48}$$

(i) For $i > k$, $m_i = 0$, and hence the ownership difference is equal to zero. For $i \leq k$, $m_i > 0$, and the expected ownership differential is given by

$$E\left\{\frac{Q_{1it}}{\lambda} - \frac{Q_{2it}}{(1-\lambda)}\right\} = \frac{m_i \overline{x}_i \left(e^{2K_1} - e^{2K_2}\right)}{1 + \phi m_i}, \tag{49}$$

where we have used the fact that expected excess returns are, by equations (18) and (19),

$$E\left(z_{it} - rp_{it}\right) = \frac{\rho \sigma_i^2 \overline{x}_i}{1 + \phi m_i}. \tag{50}$$

Since $K_1 > K_2$ and $\overline{x}_i > 0$, the result follows.

(ii) First, we show that if $E(z_i - rp_i) > E(z_l - rp_l)$, then $m_i > m_l$. Since $i, l < k$, their gain factors are equated, $G_i = G_l$. Using (50), and the fact that $\overline{x}_i = \overline{x}$ and $\sigma_{xi} = \sigma_x$ for all i, the gain factor of asset i can be written as

$$G_i = \frac{1 + \rho^2 \left(\sigma_x^2 + \overline{x}^2\right) \sigma_i^2}{\rho^2 \overline{x}^2 \sigma_i^2} \left[E(z_{it} - rp_{it})\right]^2, \tag{51}$$

and a corresponding expression holds for G_l. The inequality in excess returns implies that

$$\frac{1+\rho^2\left(\sigma_x^2+\overline{x}^2\right)\sigma_i^2}{\sigma_i^2} < \frac{1+\rho^2\left(\sigma_x^2+\overline{x}^2\right)\sigma_l^2}{\sigma_l^2}, \tag{52}$$

which reduces to $\sigma_i^2 > \sigma_l^2$. Equation (36) implies $\frac{dm_i}{d\xi_i} > 0$, which under the maintained assumptions on $\overline{x}_i$ and σ_{xi}^2 implies that $\frac{dm_i}{d\sigma_i^2} > 0$. Hence $m_i > m_l$.

Next, from the expression for the expected ownership differential in (49), the difference in expected relative ownership across the two assets is

$$E\left\{\frac{Q_{1i}}{\lambda}-\frac{Q_{2i}}{(1-\lambda)}\right\}-E\left\{\frac{Q_{1l}}{\lambda}-\frac{Q_{2l}}{(1-\lambda)}\right\}=\frac{\overline{x}\left(e^{2K_1}-e^{2K_2}\right)(m_i-m_l)}{(1+\phi m_i)(1+\phi m_l)}>0.$$

■

Proof of Proposition 4. Using equations (26)-(27), the state-by-state ownership difference for asset i becomes

$$\frac{Q_{1it}}{\lambda}-\frac{Q_{2it}}{(1-\lambda)}=m_i\left(e^{2K_1}-e^{2K_2}\right)\left(\frac{z_{it}-rp_{it}}{\rho\sigma_i^2}\right). \tag{53}$$

If $i < k$, the equilibrium level of $m_i > 0$ is an ex-ante decision, and hence is constant across realizations. The result follows. ■

Proof of Proposition 5. Our deviation keeps the aggregate information quantity ϕ constant, and hence the masses m_is unchanged by equation (36), which in turn implies that prices also remain unchanged, by equations (18) and (19). By equations (25), (26) and (27), relative capital income is

$$\frac{\sum_i \pi_{1i}}{\sum_i \pi_{2i}}=\frac{\sum_i\{(\overline{z}_i-rp_i)(z_i-rp_i)+m_i(e^{2K_1}-1)(z_i-rp_i)^2\}}{\sum_i\{(\overline{z}_i-rp_i)(z_i-rp_i)+m_i(e^{2K_2}-1)(z_i-rp_i)^2\}}.$$

Since $K_1' > K_1$ and $K_2' < K_2$, each element of $\sum_i \pi_{1i}$ increases and each element of $\sum_i \pi_{2i}$ decreases. ■

Proof of Proposition 6. (i) From equations (18) and (19), the average equilibrium price of asset i can be expressed as

$$\overline{p}_i=\frac{1}{r}\left(\overline{z}_i-\frac{\rho\sigma_i^2\overline{x}_i}{1+\phi m_i}\right).$$

For $i > k$, $m_i = 0$, and $\overline{p}_i$ remains unchanged. For $i \leq 0$, $m_i > 0$, and $\overline{p}_i$ is increasing in ϕm_i, which in turn is increasing in ϕ, per lemma 3.

(ii) Equilibrium expected excess returns are

$$E\left(z_{it}-rp_{it}\right)=\frac{\rho\sigma_i^2\overline{x}_i}{1+\phi m_i}. \tag{54}$$

For $i > k$, $m_i = 0$, and expected excess returns remain unchanged. For $i \leq 0$, $m_i > 0$, and the expected excess return of asset i is decreasing in ϕm_i, which in turn is increasing in ϕ, per lemma

3. ■

Proof of Proposition 7. The average ownership difference is given by

$$E\left\{\frac{Q_{1it}}{\lambda}-\frac{Q_{2it}}{(1-\lambda)}\right\}=\frac{m_i\overline{x}_i\left(e^{2K_1}-e^{2K_2}\right)}{1+\phi m_i}. \tag{55}$$

For our designed deviation of information capacities, the aggregate information quantity ϕ constant, and hence the masses m_is unchanged by equation (36). Polarization in e^{2K_1} versus e^{2K_2} gives the result. ■

Proof of Lemma 4. Denote $m_{i\phi}\equiv\frac{dm_i}{d\phi}$. Then the derivatives we are interested in are

$$\begin{aligned}\frac{d[m_i(e^{2K}-1)]}{dK} &= 2e^{2K}m_i+m_{i\phi}(e^{2K}-1)\frac{d\phi}{dK}\\ \frac{d[m_i(e^{2K\gamma}-1)]}{dK} &= 2\gamma e^{2K}m_i+m_{i\phi}(e^{2K\gamma}-1)\frac{d\phi}{dK}\end{aligned}$$

where

$$\frac{d\phi}{dK}=2\lambda e^{2K}+2\gamma(1-\lambda)e^{2K\gamma}.$$

Consider first the case where $m_{i\phi}>0$. Then, because for $m_i>0$, $d\phi/dK>0$, and $e^{2K}>\gamma e^{2K\gamma}$, we have

$$\frac{d[m_i(e^{2K}-1)]}{dK}>\frac{d[m_i(e^{2K\gamma}-1)]}{dK}>0.$$

Now, consider a case where $m_{i\phi}<0$. Pluggining in and factoring out $2e^{2K}$ gives

$$\begin{aligned}\frac{d[m_i(e^{2K}-1)]}{dK} &= 2e^{2K}\left(m_i+m_{i\phi}\frac{2\lambda e^{2K}+2\gamma(1-\lambda)e^{2K\gamma}}{2}\frac{e^{2K}-1}{e^{2K}}\right)\\ &= 2e^{2K}\left(m_i+m_{i\phi}[\lambda e^{2K}+\gamma(1-\lambda)e^{2K\gamma}](1-\frac{1}{e^{2K}})\right)\\ &= 2e^{2K}\left(m_i+m_{i\phi}[\lambda(e^{2K}-1)+\gamma(1-\lambda)(e^{2K\gamma}-\frac{e^{2K\gamma}}{e^{2K}})]\right)\\ &> 2e^{2K}\left(m_i+m_{i\phi}[\lambda(e^{2K}-1)+(1-\lambda)(e^{2K\gamma}-1)]\right)\\ &= 2e^{2K}\left(m_i+m_{i\phi}\phi\right).\end{aligned} \tag{56}$$

where the inequality comes from the fact that $m_{i\phi}<0$ and,

$$e^{2K\gamma}-1>\gamma(e^{2K\gamma}-\frac{e^{2K\gamma}}{e^{2K}})>0.$$

This last sequence of inequalities follows from the evaluation of $F(\gamma)=e^{2K\gamma}-1-\gamma(e^{2K\gamma}-\frac{e^{2K\gamma}}{e^{2K}})$.

In particular, its derivative is

$$F'^{2K\gamma}[2K + (\frac{1}{e^{2K}} - 1)(1 + 2\gamma K)]$$

Clearly, $F'(\gamma) = 0$ has a unique solution, and $sign(F'(0)) = sign(2K + (\frac{1}{e^{2K}} - 1))$, which is positive for all $K > 0$ – hence, $F'(0) > 0$. On the other extreme, $sign(F'(0)) = sign(1 + 2K - e^{2K})$, which is negative for all $K > 0$. This implies that F has a maximum at $\gamma \in (0, 1)$, which together with the fact that $F(1) = F(0) = 0$ implies that for all $K > 0$, $\gamma \in (0, 1)$, $F(\gamma) > 0$. With this result in hand, we can use (56) to get

$$\frac{d[m_i(e^{2K} - 1)]}{dK} > 2e^{2K}\left(m_i + m_{i\phi}\phi\right) > 0,$$

where the last inequality follows from part (iv) of Lemma 3. Note that

$$\lambda\frac{d[m_i(e^{2K} - 1)]}{dK} + (1 - \lambda)\frac{d[m_i(e^{2K\gamma} - 1)]}{dK} = \frac{d\phi m_i}{d\phi}\frac{d\phi}{dK} = \frac{d\phi m_i}{d\phi}(\lambda 2e^{2K} + 2\gamma(1 - \lambda)e^{2K\gamma})$$

Since by previous paragraph, $\frac{d[m_i(e^{2K}-1)]}{dK} > 2e^{2K}\frac{d\phi m_i}{d\phi}$, and $e^{2K} > e^{2K\gamma}$, it must be that the second element of the weighted average is smaller, which implies

$$\frac{d[m_i(e^{2K} - 1)]}{dK} > \frac{d[m_i(e^{2K\gamma} - 1)]}{dK}.$$

■

Proof of Proposition 8. Expected difference in asset ownership, by equations (26) and (27) are given by

$$E\{\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{1 - \lambda}\} = \frac{1 + m_i\left(e^{2K_1} - 1\right)}{1 + \phi m_i}\overline{x}_i - \frac{1 + m_i\left(e^{2K_2} - 1\right)}{1 + \phi m_i}\overline{x}_i.$$

Since average quantitates have to be equal to average supply $\bar{x}_i$, it is enough to show that the first element of the sum is increasing. It is given by

$$\frac{dE\{\frac{Q_{1i}}{\lambda}\}}{dK} = \frac{\frac{d[m_i(e^{2K}-1)]}{dK}(1 + \phi m_i) - \frac{d\phi m_i}{d\phi}\frac{d\phi}{dK}m_i(e^{2K} - 1)}{(1 + \phi m_i)^2}\overline{x}_i$$

The sign of the expression is determined by the sign of

$$\begin{aligned} sign\left(\frac{dE\{\frac{Q_{1i}}{\lambda}\}}{dK}\right) &= sign\left(\frac{d[m_i(e^{2K} - 1)]}{dK} - \frac{d\phi m_i}{d\phi}\frac{d\phi}{dK}m_i(e^{2K} - 1)\frac{1}{1 + \phi m_i}\right) \\ &= sign\left(\frac{d[m_i(e^{2K} - 1)]}{dK} - \frac{d\phi m_i}{d\phi}(e^{2K} - 1)\frac{2(\lambda e^{2K} + (1 - \lambda)\gamma e^{2K\gamma})}{\lambda e^{2K} + (1 - \lambda)e^{2K\gamma}}\right) \end{aligned}$$

In the proof of Lemma 4, we showed that

$$\frac{d[m_i(e^{2K}-1)]}{dK} > 2e^{2K}\frac{d\phi m}{d\phi} > 0.$$

Using that expression above gives

$$sign\left(\frac{dE\{\frac{Q_{1i}}{\lambda}\}}{dK}\right) > 2e^{2K} - (2e^{2K}-2)\frac{\lambda e^{2K} + (1-\lambda)\gamma e^{2K\gamma}}{\lambda e^{2K} + (1-\lambda)e^{2K\gamma}} > 0,$$

where the last inequality is guaranteed by $\frac{\lambda e^{2K}+(1-\lambda)\gamma e^{2K\gamma}}{\lambda e^{2K}+(1-\lambda)e^{2K\gamma}} < 1$. ■