Very Long-Run Discount Rates

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January 2013

Abstract

We provide the first direct estimates of how agents trade off immediate costs and uncertain future benefits that occur in the very long run, 100 or more years away. We find that very long-run discount rates are low, much lower than implied by most economic theory. We estimate these discount rates by exploiting a unique feature of residential housing markets in England, Wales and Singapore, where residential property ownership takes the form of either leaseholds or freeholds. Leaseholds are temporary, tradable ownership contracts with maturities between 50 and 999 years, while freeholds are perpetual ownership contracts. The difference between leasehold and freehold prices represents the present value of perpetual rental income starting at leasehold expiry. We estimate the price discounts for varying leasehold maturities compared to freeholds via hedonic regressions using proprietary datasets of the universe of transactions in each country. Agents discount very long-run cash flows at low rates, assigning high present values to cash flows hundreds of years in the future. For example, 100-year leaseholds are valued up to 15% less than otherwise identical freeholds. This suggests that both long-term risk-free discount rates and long-term risk premia are low. Together with the relatively high average return to housing, this also implies a downward sloping term structure of discount rates. Our results provide a new testing ground for asset-pricing theories, and have direct implications for climate-change policy, long-run fiscal policy and the conduct of cost-benefit analyses.

JEL Codes: G11, G12, R30.

Keywords: Cost-Benefit Analysis, Asset Pricing, Climate Change, Real Estate, Bubbles.

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For latest version, visit http://ssrn.com/abstract_id=2346049. We thank Viral Acharya, John Campbell, John Cochrane, Xavier Gabaix, Bob Goldstein, Christian Gollier, Mervyn King, Ralph Koijen, Martin Lettau, Toby Moskowitz, William Nordhaus, Monika Piazzesi, Dick Thaler, Arthur Van Benthem, Jules Van Binsbergen, Stijn Van Nieuwerburgh, Martin Weitzman as well as seminar participants at NYU Stern and Notre Dame for helpful discussions. We gratefully acknowledge the generous research support from the NYU Stern Center for the Global Economy and Business as well as from the Fama-Miller Center and the Initiative on Global Markets at the University of Chicago Booth School of Business. We thank Hometrack, Trulia and iProperty for sharing part of their data.

Long-run discount rates play a central role in economics (Cochrane, 2011). For example, much of the debate around the optimal response to climate change centers on the trade-off between the immediate costs and the very long-term benefits of policies that aim to reduce global warming (Nordhaus, 2006; Weitzman, 2007; Barro, 2013; Pindyck, 2013). Unfortunately, there is little direct empirical evidence on how households discount payments over such long horizons because of the scarcity of finite, long-maturity assets necessary to estimate households' valuation of very long-run claims.

We provide the first direct estimates of households' discount rates for payments very far in the future, and find them to be low, much lower than implied by most economic theory. To estimate these long-run discount rates, we exploit a unique feature of residential housing markets in England, Wales and Singapore, where property ownership takes the form of either very long-term leaseholds or freeholds. Leaseholds are tradable ownership contracts with maturities ranging from 50 to 999 years, while freeholds are perpetual ownership contracts. The price difference between leaseholds and freeholds for otherwise identical properties captures the present value of perpetual rental income starting at leasehold expiry and, therefore, is informative about households' discount rates over that horizon.[1]

Our empirical analysis is based on proprietary information on the universe of residential property sales in England and Wales (2009-2013) and Singapore (1995-2013). These data contain information on transaction prices, leasehold terms and property characteristics such as location and structural attributes. We estimate long-run discount rates by comparing the prices of leaseholds with different maturities to the prices of freeholds across otherwise identical properties. We use hedonic regression techniques to control for possible heterogeneity between properties offered as leaseholds and properties offered as freeholds. This allows us to identify the discounts due to differences in lease length. We find that agents discount very long-run cash flows at very low rates; for example, 100-year leaseholds are valued up to 15% less than otherwise identical freeholds. Discounts are even greater at shorter maturities, growing to 30% for leaseholds with 50 to 70 years remaining. The discounts are zero for leaseholds with maturities of more than 700 years.

We show that these results suggest discount rates for very long-run cash flows that are substan-

[1]Focusing our analysis on real estate has several advantages. Real estate constitutes the most significant asset in most households' portfolios. Therefore, the term structure of discount rates applied to real estate cash flows contains important information about the time and risk preferences of households over long horizons. In addition, real estate is the only major asset class for which we have liquid markets in which agents trade finite-horizon contracts spanning hundreds of years. As such, it opens a new opportunities to study time and risk preferences over those horizons.

tially lower than those routinely implied by economic theory. This is because standard exponential discounting assigns little present value to distant payoffs even at moderately low discount rates. Together with a relatively high estimated return on real estate, these results also suggest a downward sloping term structure of discount rates.

The empirical results are consistent across England-and-Wales and Singapore, two housing markets with otherwise very different institutional settings. We minimize the concern that our results could be driven by unobservable quality differences across freehold and leasehold properties or by institutional differences between the two types of contracts by showing that there is no price difference between leaseholds with more than 700 years remaining and freeholds on observationally similar properties. Similarly, our results are not driven by potential frictions that might be important for short-maturity leasehold properties (50-70 years), such as financing frictions, since discounts to freeholds remain substantial even for leaseholds with 150 or even 250 years of maturity.[2]

To interpret the economic magnitude of the observed leasehold discounts, we first analyze the predictions from the classic Gordon-Growth valuation model (Gordon, 1982) with constant discount rates across maturities; then, we consider the impact of risk and frictions in more general models. In the Gordon-Growth model, rental income grows at rate g and is discounted at a constant rate r. To calibrate the model, we estimate unconditional expected housing returns r and rent growth g in the U.S., the U.K. and Singapore. Consistent with Shiller (2006), we find real rates of rent growth to be low, about 0.5% a year. Expected real returns to housing are relatively high, between 7% and 9% a year, and primarily driven by high rental yields. The Gordon-Growth model predicts that even with a conservative rate of return of 5.5% and optimistic rent growth of 2% the price discount of 100-year leaseholds relative to freeholds should be essentially zero. This simple model highlights that the challenge for economic theory is to *jointly* rationalize a high expected return to housing with the low discount rates necessary to match the observed discounts for long-dated leaseholds relative to freeholds. We call this the "long-run valuation puzzle."

[2]When we consider the possible role played by financing frictions for leaseholds, we identify two opposing forces. On the one hand, shorter leases could be attractive to buyers that are liquidity constrained. This effect makes leaseholds more desirable compared to freeholds and leads to smaller discounts. On the other hand, mortgage lenders typically require 30 years of unexpired lease term to remain at the end of the mortgage, suggesting that leaseholds have to be financed with shorter maturity mortgages once the lease length falls below 60 years. While this effect can contribute to lower valuations for short-term leases through the loss of collateral value, we show that it cannot quantitatively affect the discounts for longer-term leases. Intuitively, a lease that has 200 years remaining maturity will only incur potential losses to its collateral value 140 years from now. Any losses that occur that far into the future have little impact on present values at conventional discount rates.

We then consider whether the risk properties of housing can explain the long-run valuation puzzle. While the leading asset pricing models were not specifically designed to match the prices of very long-dated cash flows, we can study the term structure of discount rates that they imply for these cash flows (see Binsbergen, Brandt and Koijen, 2012). For cash flows with risk properties similar to those of rents, the external habit formation model of Campbell and Cochrane (1999) and the long-run risk model of Bansal and Yaron (2004) produce an upward sloping term structure of discount rates, while the rare disaster model of Barro (2006) and Gabaix (2012) generates a flat term structure of discount rates. These models thus produce a tension between rents that are sufficiently risky to generate high average expected returns to housing and the fact that, as rents become riskier, long-term cash flows are discounted at progressively higher rates thus generating smaller discounts for leaseholds with respect to freeholds. This exacerbates the long-run valuation puzzle.

A model that can rationalize the long-run valuation puzzle requires a downward sloping term structure of discount rates. Discount rates have to be sufficiently high in the short to medium run to contribute to high expected returns on housing, but also sufficiently low in the long run to match the observed value of long-run cash flows. Two existing classes of models can potentially generate this feature. A first class of models implies a downward sloping term structure of discount rates in (mostly) risk free environments. This class of models includes hyperbolic discounting, along the lines of Laibson (1997) and Luttmer and Mariotti (2003), and the gamma discounting of Weitzman (1998, 2001). A second class of models implies a downward sloping term structure of discount rates via declining risk premia for risky cash flows. While assets that provide a hedge against aggregate risks may naturally display downward sloping discount rates in the very long run (Weitzman, 2012; Martin, 2012), the challenge of the long-run valuation puzzle is to explain low long-run discounts rates on risky assets like housing. One model that achieves this is the reduced form model of Lettau and Wachter (2007, 2011), which generates a downward sloping term structure of discount rates for risky assets because claims to long-run cash flows have lower exposure to the unexpected innovation to rents (or dividends), which is the priced shock in the model.

In addition to analyzing the long-run time and risk preferences of households, our estimates are uniquely suited to directly test the classic theories of infinitely-lived rational bubbles of Blanchard and Watson (1982) and Froot and Obstfeld (1991). These theories study bubbles that in expectation grow faster than the discount rate and therefore imply a failure of the terminal condition that would

normally impose the present value of a payment occurring infinitely far into the future to be zero. We can directly test this condition by verifying whether leaseholds of very long maturity, 800 or more years, are valued identically to otherwise similar freeholds. Contrary to most of the empirical literature on bubbles, we do not need to assume a specific model of the "fundamental" value of the asset because *all* models that assume the absence of infinitely-lived rational bubbles, imply a zero value for claims to a payment at infinite maturity. We find no evidence of this type of bubbles, not even during periods of strong growth in house prices.

Implications Our paper contributes to three broad areas of economics and finance: environmental policy and intergenerational cost-benefit analysis, asset pricing, and real estate economics.

The literature on environmental policy has discussed extensively the importance of long-run discount rates in assessing the benefits of policies such as reducing carbon emissions (Gollier and Weitzman, 2010; Pindyck, 2013; Barro, 2013). For example, Stern (2007) calls for immediate action to reduce future environmental damage based on the assumption of very low discount rates. The authors argue that while agents discount the future over their lifetimes, they have an ethical impetus to care about future generations. This assumption has been criticized, amongst others, by Weitzman (2007) and Nordhaus (2006), who argued that "the Review's radical revision arises because of an extreme assumption about discounting [...] this magnifies enormously impacts in the distant future and rationalizes deep cuts in emissions, and indeed in all consumption, today." Much of the critique argued that asset markets reveal discount rates much higher than zero and often close to 6%, the private return to capital. However, such estimates are based on claims to infinite streams of cash flows and, as such, are not directly informative of long-run discount rates. We contribute to this literature by providing direct empirical evidence on long-run discount rates. Our long-run discount rates are higher than those in the Stern report but substantially smaller than those suggested by the unconditional return to the capital stock or housing.

Beyond the analysis of climate change, our estimates can provide an important input for cost-benefit analyses regularly conducted by government agencies across the world.[3] The U.S. Office of Management and Budget advises regulatory agencies to use both a 3% and a 7% annual discount rate in their analyses. If the regulatory action will have "important intergenerational benefits or costs,"

[3]For example, U.S. Executive Orders 13563 and 12866 require all government agencies to "propose or adopt a regulation only upon a reasoned determination that its benefits justify its costs."

they should also consider a sensitivity analysis using a lower but positive discount rate, ranging from 1 to 3 percent. The stated reasons for this wide range of applicable discount rates is that while "private markets provide a reliable reference for determining how society values time within a generation, [...] for extremely long time periods no comparable private rates exist." Our estimates provide such private market discounts rates for very long horizons and could help to guide government agencies in their choice of discount rates for benefit-cost analyses.

Our results are also informative for asset pricing theory. Our empirical evidence provides a new testing ground for the leading theoretical models of asset pricing as well as an input into the development of new theories. We view our paper as complementary to the recent and innovative contribution of Binsbergen, Brandt and Koijen (2012),[4] who show that the term structure of equity discount rates is downward sloping. First, we focus on real estate instead of equity; both are important components of households' portfolios. Second, our estimates are directly informative about (very) long-run discount rates, i.e. 80-250 years, while their estimates focus on (relatively) short-run discount rates (1-3 years in the original paper, and extended to 1-10 years in Binsbergen et al., 2013).

Finally, our results are of direct relevance for real estate economics and the ongoing effort to understand house prices. We add to the recent research effort to understand the return properties of real estate (Flavin and Yamashita, 2002; Lustig and Van Nieuwerburgh, 2005; Piazzesi, Schneider and Tuzel, 2007; Favilukis, Ludvigson and Van Nieuwerburgh, 2010) by focusing on a previously unexplored aspect of real estate: the term structure of house prices.

1 Housing Markets in Singapore and the United Kingdom

In this section we provide the relevant institutional details about the housing markets in the U.K. and in Singapore, focusing on the characteristics of freeholds and leaseholds. Appendix A.1 provides additional information, including details on the property taxation regimes.

1.1 Leaseholds and Freeholds in the U.K.

Property contracts in England and Wales come in two forms: permanent ownership, called a freehold, and long-duration, temporary ownership, called a leasehold. At least 1.43 million properties are

[4] A nascent literature motivated by this contribution includes Belo, Collin-Dufresne and Goldstein (2012) and Boguth et al. (2012).

owned as leaseholds (The Independent, 2013). Owning a leasehold provides ownership rights to the property for a period of time up to the maturity of the lease. Common initial leasehold maturities are 99, 125, 150, 250 or 999 years. During this period, ownership of the leaseholds entitles you to similar rights as the ownership of the freehold, including the right to rent out the property. Unlike in the case of commercial leases, the vast majority of the costs associated with a residential leasehold come up-front through the purchase price of the leasehold; annual payments are small to non-existent (see Appendix A.1.1). Leasehold properties are often sold in private secondary markets, in which case the buyer purchases the remaining term of the lease. Once the leasehold expires, the ownership reverts back to the freeholder, a process called "reversion". However, it is common for leaseholders to purchase leasehold extensions ahead of leasehold expiry. Over time, a number of laws described in Appendix A.1.1 have regulated the rights of leaseholders to extend their lease terms. For our sample period, leaseholds had the *right* to lease extensions at market prices. If leaseholder and freeholder cannot agree on the market price, it is determined by a government-run leasehold valuation tribunal.

1.2 Leaseholds and Freeholds in Singapore

Residential properties in Singapore are also either sold as freeholds or leaseholds, where the latter have initial terms of 99 years or 999 years.[5] By far the largest freeholder is the government of Singapore, represented by the Singapore Land Authority (SLA). As in the U.K., there is a vibrant private secondary market for leaseholds, where buyers purchase the remaining terms of the original leases.

At the expiration of the lease, the ownership interest reverts to the freeholder. Leaseholders may apply for a renewal of the lease with the SLA before the expiration of the lease. The granting of an extension is decided on a case-by-case basis; considerations include whether the development is in line with Government's planning intentions, is supported by the relevant agencies, and results in land use intensification, the mitigation of property decay and the preservation of community. If the extension is approved, the Chief Valuer determines the "land premium" that will be charged. The new lease will not exceed the original, and it will be the shorter of the original or the lease in line with the Urban Redevelopment Authority (URA) planning intention.

[5]There are also other types of less common lease structures. The first are private development 103-year leaseholds sold on freehold land. In addition, in November 2012 a plot of land at Jalan Jurong Kechil was the first to be sold for residential development under an initial 60-year lease agreement; though houses built there do not yet appear in our data.

2 Empirical Analysis

The estimation of the relative prices of leaseholds and freeholds is potentially challenging because the underlying properties are heterogeneous assets. Since leasehold and freehold properties could differ on important dimensions such as property size and location, comparing prices across properties requires us to control for these differences. We use hedonic regression techniques, which allow us to consider the variation in price over time and across lease terms for different properties while controlling for key characteristics of each property such as size, location and property age.

2.1 Analysis - United Kingdom

2.1.1 U.K. Residential Housing Data

We begin by analyzing data from England and Wales. We have obtained administrative transaction-level data on all residential housing sales from January 1st, 2009 to March 31st, 2013 from the U.K. Land Registry.[6] This initial dataset provides us with a total of 2.2 million housing transactions. The data include a leasehold indicator (whether or not the property is a leasehold or freehold), the price paid as well as some characteristics of the house: house type (detached, flat, semi-detached or terraced), full address with postcode, and a new construction indicator. In addition to these data, we have obtained a separate, proprietary dataset on details of each lease from the Land Registry - this provides information on the lease start date as well as the overall lease length. Figure 1 shows the distribution of remaining lease lengths for properties at their point of sale. There are many transactions with remaining lease lengths between 100 and 250 years, allowing us to trace out the term structure of leasehold discounts across different horizons. Finally, for a subset of the homes, we have been able to obtain information from "for sale" listings posted on a large U.K. property listings website. This provides us with property-level details on the number of bedrooms, bathrooms and the number of total rooms. Overall, we can match approximately 760,000 transactions to listings. Table 1 provides key summary statistics on our U.K. transaction sample.

[6] We are currently in the process of acquiring similar data for the period 1995-2009.

2.1.2 Price Variation by Lease Length Remaining

In this section we estimate the relative prices paid for leaseholds of varying remaining duration and freeholds for properties in England and Wales. Given the support of the "remaining lease length" distribution (see Figure 1) we construct J buckets for different remaining $MaturityGroups$. In particular, our $j = 1, ..., J$ buckets are: 70-84 years, 85-99 years, 100-124 years, 125-149 years, 150-300 years, and 700+ years. We then estimate regression (1). The unit of observation is a transaction i of a property of type g (e.g. detached, semi, terraced, flat/maisonette) in postal district h (of which we have 1,165 unique observations in the data) at time t (quarter or month). We assign each leasehold with remaining maturity T_i to one of the $MaturityGroup_j$ buckets depending on the number of years remaining on the lease at the point of sale. The excluded category are freeholds, so that the β_j coefficients capture the log-discount of leaseholds with that maturity relative to otherwise similar freeholds. Since we do not observe the size of the individual properties, our primary specification uses $log(Price)$ as the dependent variable. In a second set of results, we include $log(Price/Room)$ as the dependent variable.[7]

$$log(Price_{i,h,t,g}) = \alpha + \sum_{j=1}^{J} \beta_j \mathbf{1}_{\{T_i \in MaturityGroup_j\}} + \gamma Controls_i + \xi_h \times \psi_t \times \phi_g + \epsilon_{i,h,t,g} \quad (1)$$

We control for average prices in a property's geography by including postal district by time of sale by property type fixed effects. This means that we are identifying leasehold discounts by comparing leaseholds to freeholds for the same type of property that was sold in the same area and at the same time. We also include control (dummy) variables for whether the property is a new construction, as well as for the number of bedrooms, bathrooms, and the number of total rooms. Standard errors are clustered at the level of the fixed effects.[8]

Table 2 shows the results from regression (1). In column (1) we control for the time of sale in the interacted fixed effects by including the quarter of sale, in column (2) by including the month of

[7] We are in the process of obtaining additional hedonic property characteristics such as property size and age from a number of different sources for the next draft of this paper. As such, the current estimates for the U.K. should be considered as preliminary in that respect.

[8] Clustering standard errors addresses possible concerns about the correlation of regression residuals across different transactions within the unit of clustering. If this correlation was driven by unobserved characteristics or events that affected all properties within the level of fixed effects the same way, the fixed effects would already pick this up and robust OLS standard errors would be consistent. Therefore, given the large number of fixed effects, this is a very conservative strategy to estimate standard errors. See Petersen (2009) for details.

sale. In column (3), our preferred specification, we also interact our fixed effect with the number of bedrooms of the properties. This increases the number of fixed effects to 253,000. Here the identification of the β_j leasehold discount coefficients comes from comparing two properties of the same type with the same number of bedrooms sold in the same district and the same month. The results show that freeholds and leases with maturities of more than 700 years trade at approximately the same price: the coefficient on $\beta_{700+Years}$ is small and statistically indistinguishable from zero. However, leaseholds with shorter maturities trade at significant discounts to otherwise identical freeholds: leaseholds with 100 to 125 years remaining trade at a 15% discount to freeholds. Leaseholds between 150 and 300 remaining trade at a 7% discount.[9]

In columns (4) - (6) we include $log(Price/Room)$ as the dependent variable. The estimated log-discount of leasehold properties remains the same: while leases with 700+ years maturity remaining trade at the same price as freeholds, for shorter maturity leases there is a significant discount to the prices of freeholds. Figure 2 plots the coefficients β_i from regression (1). The top panel uses $log(Price)$ as the dependent variable, and corresponds to column (3) in Table 2, the bottom panel uses $log(Price/Room)$, and corresponds to column (6).

2.2 Market Segmentation

In our hedonic pricing regression 1 we are able to control for many characteristics of the property, such as property size and age. However, we observe no characteristics of the buyers or sellers in our transaction sample. This might cause concern that the clientele for leasehold and freehold properties could be very different, which could explain the price differences that we observe. To address this concern, we analyze data from the Survey of English Housing (SEH), which was a household-level conducted annually between 1994 and 2008. It covered a wide range of topics, including whether the property is owned as a freehold or leasehold, as well as detailed characteristics for the households. We focus on the sample of owner-occupieres (excluding renters). Overall, we have a sample of 201,933 responding households, which allows us to analyze whether households owning leasehold and freeholds do indeed differ on observable characteristis.

Table 4 shows the results for important household characteristics. In columns (1) and (2) we show the mean and standard deviation of the outcome variable in the sample. For example, average

[9]The percentage discount is calculated as $1 - e^{\beta}$.

weekly income of the household head was about £350. Column (3) shows the unconditional difference between households owning leaseholds and freeholds: households owning leaseholds have, on average, £48 less weekly income. However, we saw above that leaseholds and freeholds pertain to different property types in general, something we control for in our hedonic analysis. Since we would expect buyers of apartments (which are predominantly leaseholds) to be different to buyers of houses (which are predominantly freeholds). To analyze the conditional differences in buyer characteristics, we run regression 2, where we control for property type by region fixed effects, just as in the hedonic pricing regression 1. Geographic controls are more coarse, since the (SEH) only reports 354 unique local authority codes.

$$Outcome_i = \alpha + \beta Leasehold_i + \xi X_i + \phi_{PropertyType \times Region} + \varepsilon_i. \tag{2}$$

Columns (4) and (5) show the point estimates and clustered standard errors of the estimate of β, with column (5) controlling for other property characteristics that are observed in both the SEH and the transaction dataset, such as the number of rooms, the property age, and the floor on which the property is located. The evidence shows that the sample of households owning freeholds and leaseholds looks very similar. Depending on the specification, the weekly income of households owning leasehold properties is between £3 less and £8 more, relative to a sample mean and standard deviation of £350 and £450 respectively. Similarly, they are between 1.3 and 1.5 years younger, relative to a sample mean and standard deviation of 52 and 16. The number of household members and number of dependent children is essentially identical across groups owning freeholds and leaseholds. This is reassuring, since it makes it less likely that our results are driven by clientele effects related to differential bequest motives. Leasehold owners are no more likely to be first-time buyers or be married, and only 2% more likely to have a mortgage. Finally, there is not differential satisfaction with the state of the neighborhood across owner types.

2.3 Data - Singapore

We have obtained transaction-level price data for all private residential transactions in Singapore from the Urban Redevelopment Authority. We do not use transaction prices for property sales by the HDB, which usually happen at below-market value (see Appendix A.1.2). We observe approximately

380,000 arms-length transactions between January 1995 and September 2013. For each transaction there is information on the transaction price and date, the lease terms, property characteristics such as size and age, as well as the precise location of the property. Table **??** provides an overview of the transaction sample used in the regressions. There are between 10,000 and 40,000 transactions per year. Many of transactions are for newly built apartments, with the average transacted home being less than 5 years of age. Between 30% and 60% of all private transactions each year are recorded for freehold properties. For leasehold property transactions we observe substantial dispersion in the lease length remaining at the time of sale, as shown in Figure 3. In the top panel we show the remaining lease length of leases initially written for 99 years. In the bottom panel we show the equivalent distribution for leases of initially 999 years. There are essentially no transactions of leasehold properties with 100 to 800 years remaining on the lease.

2.4 Analysis - Singapore

To analyze the relative price paid for leaseholds and freeholds we run regression (3). The unit of observation is a property i of type h (e.g., apartment, condominium, detached house, executive condominium, semi-detached house and terrace house), of title type s (either "strata" or "land", see appendix A.1.2), in geography g, sold at time t. As before, for leaseholds the variable T_i captures the number of years remaining on the lease at the time of sale. The key dependent variable is the price per square foot paid in the transaction.[10] As for the previous analysis for England and Wales, we split the 99-year leases into J buckets with different groups of lease length remaining ($MaturityGroup_j$).[11] We form buckets of leases with 50-69 years, 70-84 years, 85-89 years, 90-94 years and 95-99 years remaining. We also include a dummy variable for all 999-year leases, all of which have at least 800 years remaining when we observe the transaction. The excluded category are the freeholds.

$$\ln\left(\frac{Price}{Sqft}\right)_{i,h,s,g,t} = \alpha + \sum_{j=1}^{J} \beta_j \mathbf{1}_{\{T_i \in MaturityGroup_j\}} + \gamma Controls_{i,t} + \\ + \xi_h \times \rho_s \times \phi_g \times \psi_t + \epsilon_{i,h,s,g,t} \tag{3}$$

[10] In order to avoid our results being primarily driven by extreme outliers such as luxury condominiums, we winsorize the price per square foot at the 1% level. This adjustments has little effect on the estimated coefficients.

[11] See Figure 3 for a distribution of the lease length remaining at the time of sale in our dataset.

The results from this regression are shown in Table 3. In column (1) we control for 5-digit postcode by property type by title type by transaction quarter fixed effects. Beyond these 94,700 fixed effects, our other control variables include property age, size and type, as well as the total number of units in a development. Standard errors are clustered at the level of the fixed effect. The results are very consistent with our findings for the U.K. The price per square foot paid for freeholds and otherwise similar 999-year leaseholds is economically and statistically identical. On the other hand, leases with durations of 100 years or less sell at a significant discount to otherwise identical freeholds. For example, a lease with 95-99 years remaining maturity trades at a 12.7% discount, a lease with 70-84 years remaining maturity trades at a 23% discount. The regression has an extremely high adjusted R^2 of above 95%. This suggests that there remains no significant variation in prices that is not yet explained by our control variables, and that our discounts are thus unlikely to be driven by unobserved heterogeneity between freehold and leasehold properties.[12]

In column (2) we also interact the fixed effects with property type to further ensure that our results are not driven by observed differences between leasehold and freehold properties. In column (3) we further control for the transaction month rather than the transaction quarter. This is to address possible concerns that leaseholds and freeholds might transact at different times in the quarter, which, combined with aggregate market price movements over time could potentially explain our findings. While these additions increase the total number of fixed effects to approximately 98,000 and 140,000 respectively, the estimated discounts across all maturities remain the same in both specifications.

In column (4), rather than controlling for the of the property age directly, we only focus on the sale of newly-built properties. The estimates for 95-99 year leases are unaffected. For leases with shorter maturities the estimates move somewhat. However, since most leases get topped up to 99-years when the property gets rebuilt, there are essentially no transactions to estimate the discount of new properties with 80 years lease length remaining. In column (5) we restrict the transactions to those where the buyer is not the HDB. The results are very similar to those in columns (1) - (3), suggesting that sales to the HDB generally happen at market value.

Finally, Figure 4 plots the coefficients β_j from regression (3) as reported in column (3) of Table 3. This provides a graphical display of the term structure of leasehold discounts.

[12]The adjusted R^2 remains at 95.3% if we exclude those instances where we only observe one transaction for a particular fixed effect, in which the fixed effects perfectly explains the transaction price.

2.4.1 Time Series of Discounts

We also investigate the returns of different constant-maturity time series of leasehold and freehold properties. We do this analysis for Singapore only, since our time series extends back to 1995 (as opposed to 2009 for the U.K.). Analyzing time series movements of house prices is challenging, because the characteristics of houses sold may vary over time. This means that comparing average transaction prices across different time periods is inadequate. Many time series of house prices such as the Case-Shiller indices for the U.S. are thus constructed using a repeat-sales methodology. This approach assumes that the characteristics of individual houses do not change over time, and elicits market prices movements by analyzing the appreciation of individual properties. However, when analyzing the time series movements of leaseholds, a repeat sales approach is in adequate. This is because in between two sales of the same leaseholds the lease length has declined, so that the change in the transaction price would underestimate market-wide increases of prices holding all else fixed.

In order to analyze the time series properties of the return series we therefore need to keep the lease length of the properties fixed over time. To do this we estimate regression (4) separately for houses within each maturity group $j \in J$. We include 4-digit postcode by property type by title type fixed effects. As before, we also control for the age of the property (by including a dummy variable for every possible age in years), the size of the property (by including a dummy for each of 40 equally sized groups capturing property size) and the total number of units in the property.

$$\forall j \in J: \qquad \ln\left(\frac{Price}{Sqft}\right)_{i,h,s,g,t} = \alpha + \sum_{t=1996}^{2013} \beta_t^j I_{(Year=t)} + \gamma ControlVars_{i,t} + \phi_g \times \xi_h \times \chi_s + \epsilon_{i,h,s,g,t} \qquad (4)$$

The time series of $e^{\beta_t^j}$ is the price index for lease type j. The top panel of Figure 5 shows these price indices for the same J buckets as in Figure 4.[13] While definitely correlated, the time series of the constant-maturity price series are different across lease lengths. In particular, the short-end of the maturity structure (50-70 years) seems to appreciate faster than leases of longer maturity. To get a clearer picture of the average returns across maturities, the bottom panel of Figure 5 plots average yearly returns by maturity bin with standard errors. While these graphs are obtained using only the capital gains series, rents conditional on observable characteristics are likely to be the same across maturities. This suggests that the pattern for average returns will follow that of capital gains. The

[13] Average lease length remaining within each bin remains approximately even over time.

figure suggests a pattern of decreasing discount rates by maturity (with the exception of the very-long term leaseholds and freeholds). We interpret these results as suggestive that expected returns are decreasing across maturities, consistent with the results of Binsbergen, Brandt and Koijen (2012) who look at short-end US equity dividend strips of maturity of up to 10 years. Due the short time series for our returns the standard errors around the estimates are high and these results should only be interpreted as suggestive.

3 Discussion and Interpretation

The previous section presented new facts about the pricing of freeholds and leaseholds of different maturities. Leaseholds with over 700 years of maturity trade at the same price as freeholds for otherwise identical properties. Discounts on leaseholds with maturities of 70-250 years range from 25% for maturities of 70 years, to $12-15\%$ at 100 years, to $6-8\%$ at 200 years. In this section we discuss the implications of these discounts for households time and risk preferences over long horizons. We first study a simple model with constant discount rates. While this model imposes a high degree of abstraction, it illustrates the main challenge that our empirical results present for economic theory: to *jointly* match the leasehold discounts and the average return to housing. We then verify that not even the leading asset pricing models offer a resolution to this empirical challenge. We finally provide a reduced form analysis of what models would have to match, namely a decreasing term structure of discount rates, in order to rationalize our empirical findings.

3.1 Constant Discount Rates and Leasehold Discounts

We start by considering a simple extension of the classic valuation model of Gordon (1982). We assume that rents (cash flows) arising in each future period are discounted at a constant rate r, so that the t-period discount function is e^{-rt}. We also assume that rents are expected to grow at a constant rate g, so that expected rents evolve according to $E_t[D_{t+s}] = D_t e^{gs}$.[14]

[14]Technically, g is the sum of the expected growth rate of rents and a Jensen inequality term. Given the low variance of rent growth and in the interest of intuitive results, we ignore the latter term and refer to g as the expected growth rate of rents.

In this model a claim to the rents for T periods, the T-maturity leasehold, is valued at:

$$P_t^T = \int_t^{t+T} e^{-r(s-t)} D_t e^{g(s-t)} ds = \frac{D_t}{r-g}(1 - e^{-(r-g)T}). \tag{5}$$

Correspondingly, the infinite maturity claim, the freehold, is valued at:

$$P_t = \lim_{T\to\infty} P_t^T = \frac{D_t}{r-g}.$$

The above valuation formula for infinite maturity claims is the classic formula by Gordon (1982). The discount for a T-maturity leasehold with respect to the freehold ($Disc_t^T$) is:

$$Disc_t^T \equiv \frac{P_t^T}{P_t} - 1 = -e^{-(r-g)T}. \tag{6}$$

Therefore, the discount depends directly on the difference $r - g$. For any given maturity, the discount decreases (in absolute value) the higher the discount rate r and the lower the growth rate of rents g. The first effect occurs because a higher discount rate reduces the present value of rents occurring far into the future. The second effect occurs because a higher growth rate of rents increases the actual rents occurring in the future.

3.2 Housing Returns and Rent Growth Rates

To further evaluate our simple model we need to calibrate r and g to the real expected return on housing and rent growth rate, respectively. We estimate the expected return to housing and the growth rate of rents for both the U.K. and Singapore using several methodologies and sample periods. We summarize our findings in Table 5 and leave the details of the methodologies to Appendix A.2.

The top panel of Table 5 presents the estimated average housing returns for the U.K. and Singapore, as well as the U.S..[15] These are real net returns to housing because they account for maintenance, depreciation, taxes and inflation. Average real net returns are in the range $8 - 10\%$ for all countries considered. To be as conservative as possible, we choose a baseline estimate of: $r = 6.5\%$, almost two percentage points below the lowest return observed in any country in our sample. This benchmark

[15] U.S. housing returns, while not the focus of this paper, provide a useful benchmark because they have been the subject of an extensive literature (Gyourko and Keim, 1992; Flavin and Yamashita, 2002; Lustig and Van Nieuwerburgh, 2005; Piazzesi, Schneider and Tuzel, 2007).

is consistent with estimates for the U.S. in Flavin and Yamashita (2002), who find a real return to housing of 6.6%, and Favilukis, Ludvigson and Van Nieuwerburgh (2010), who find a real return of 9-10% before depreciation and property taxes.

The bottom panel of Table 5 shows that average real rental growth rates are approximately 0.5% in all three countries. In an effort to be conservative, we choose the maximum observed value and set our baseline g to 0.7%.

Overall, our estimates are consistent with the notion that average house price growth over long periods of time is relatively low and the key driver of real housing returns is the high rental yield (see Shiller, 2006). Our estimated average capital gains are positive but relatively small despite focusing on samples and countries that are often regarded as having experienced major growth in house prices.

3.3 The Long Run Valuation Puzzle

At the estimated benchmark values of $r = 6.5\%$ and $g = 0.7\%$, the constant-discount-rates model from Section 3.1 implies a leasehold discount at 100 years of $Disc^{100} = -e^{-0.06*100} = -0.25\%$. In other words, the 100-year leasehold would be valued only 0.25% less than the freehold. The discount we find in the data is 12%, orders of magnitudes higher. More generally, the white bars in Figure 6 compare the logarithmic discounts obtained under our baseline calibration for different leasehold maturities with those observed in the data for the U.K. and Singapore (data is in black bars). The 700+ year leaseholds are valued at a 0% discount to freeholds both in the data and in the model. However, the model cannot match the discounts observed for leaseholds with maturities of 250 years or less. For example, for leaseholds with 50-70 years remaining, we observe a log discount of 38% in the data. The log discount in the model is a mere 2.8%. Intuitively, a model of exponential discounting assigns essentially zero present value to cash flows occurring 100 or more years into the future when discounting at an effective rate $r - g$ of 6% or more.

This intuition is robust to even more conservative calibrations of r and g. We evaluate a "high rent growth rate" scenario by setting $g = 2\%$,[16] and a "low expected returns" scenario with $r = 5.5\%$

[16] One might conjecture that "super-star" cities like Singapore or London might experience high rent growth in the future (Gyourko, Mayer and Sinai, 2006). However, since rents and consumption are cointegrated we would not expect rents to grow at a faster rate than consumption in the long run. Rent growth rates higher than consumption would have the implication that over time a larger and larger fraction of consumption expenditures would be devoted to housing. We also note that the past low growth rate of rents occurred in a period when London and Singapore were already rising capitals

per year, significantly less than our lowest estimate. Figure 6 also shows the discounts obtained in the high-rent-growth and low-expected-return scenarios. Both robustness exercises only slightly increase the model implied discounts. Even the calibration that allows for both low returns and high rent growth cannot match the data, especially at longer horizons.

While the long-run discounts could be matched by an *unrealistic* calibration with a constant net discount rate of $r - g = 2\%$, this calibration would not be consistent with the high average return to housing. Recall that r is the expected return to owning a freehold property. The simple constant-discount-rates model thus highlights the challenge for economic theory posed by our results: to *jointly* rationalize both a high expected return to housing and the low long-run discount rates necessary to match the observed discounts for long-dated leaseholds relative to freeholds. We call this joint problem the "long-run valuation puzzle".

3.4 General Formulas for: Discount Rates, Leasehold Discounts, and Expected Returns

We now analyze the long-run valuation puzzle through the lense of asset pricing theory. We first derive a general formula that links the price discounts between freeholds and leaseholds to stochastic discount factors and the behavior of rents. Consider a claim to the risky rent at time T, denoted D_T. The present value at time t is the expected dividend $E_t[D_T]$ discounted with some discount factor $R_{t,t+T}$:

$$P_t^{D_T} = \frac{E_t[D_T]}{R_{t,t+T}} \tag{7}$$

The price of a safe security with maturity T (which pays 1 for sure at time T) is:

$$P_t^{1_T} = \frac{1}{R^f_{t,t+T}}$$

$R^f_{t,t+T}$ is the total return on the investment in the safe security when held to maturity (up to T). Since the rent D_T is risky, we would expect that $R_{t,t+T} > R^F_{t,t+T}$: risky cash flows are discounted at a higher rate than they would be if they were safe. Therefore, we can always decompose $R_{t,t+T}$ into a discount factor that would be applied even if D_T were certain, and an additional discount that compensates

of the world.

the agents for risk (the risk premium $RP_{t,t+T}$):

$$R_{t,t+T} = R^f_{t,t+T} + RP_{t,t+T}$$

Asset pricing theory relates the discount factors $R_{t,t+T}$ and $R^F_{t,t+T}$ to a "stochastic discount factor" $\xi_{t,t+T}$ that represents marginal utility in different states of the world.[17]

$$P^{D_T}_t = E_t[\xi_{t,t+T} D_T], \tag{8}$$

The values of $R_{t,t+T}$, $RP_{t,t+T}$, and $\xi_{t,t+T}$ are directly related by no-arbitrage conditions. In particular (see Appendix A.3.1):

$$RP_{t,t+T} \equiv -\frac{Cov_t[\xi_{t,t+T}, R_{t,t+T}]}{Var[\xi_{t,t+T}]} \frac{Var[\xi_{t,t+T}]}{E_t[\xi_{t,t+T}]} \equiv \beta_{t,t+T}\lambda_{t,t+T}.$$

The risk premium has the opposite sign to the covariance between the stochastic discount factor and the rent ($Cov_t[\xi_{t,t+T}, D_T]$). A claim that pays a higher rent in states of the world when extra resources are less valuable, i.e. when $\xi_{t,t+T}$ is low, is less desirable and thus discounted at higher rate. Such an asset is risky, and its risk premium is positive. The risk premium can be decomposed into an asset-specific "quantity of risk" term $(\beta_{t,t+T})$, which summarizes how strongly the payoff co-varies with the stochastic discount factor, and a "price of risk" term $(\lambda_{t,t+T})$, that only depends on the discount factor $\xi_{t,t+T}$ and summarizes the compensation required for each unit of risk at that horizon.

We now derive a general representation for the leasehold discount. Intuitively, the difference in price between the freehold and the T-maturity leasehold is the present value of perpetual rents starting at lease expiry, T periods from now. This, in turn, is equal to the present value at time t of a freehold starting at time T. We can compute this present value by applying the valuation formula in equation (7):

$$P_t - P^T_t = \frac{E_t[P_T]}{R^f_{t,t+T} + RP_{t,t+T}}.$$

[17] It is a fundamental theorem of finance that such (strictly positive) discount factor exists under the assumption of no-arbitrage. We stress that the formulas above does not require assumptions about households preferences or market completeness.

We obtain percentage discounts by dividing both sides by the value of the T-term leasehold (P_t^T):

$$Disc^T = -\frac{E_t[P_T]/P_t}{R^f_{t,t+T} + RP_{t,t+T}} \tag{9}$$

Equation (9) shows that the leasehold discounts estimated in Section 2 are related to two basic forces: the expected capital appreciation on the freehold (the numerator), and the discount factor (the denominator). The discounts are bigger the more households expect the price of the freehold to increase over the length of the leasehold. This is because the leaseholder does not benefit from these capital gains while the freeholder does. The discounts are also bigger the lower the discount factor, since this attaches higher present value to rents occurring far into the future.[18]

3.5 Risk, Return and Leasehold Discounts in Asset Pricing Models

We now turn to fully specified general equilibrium asset pricing models that pin down both the expected return to housing $E[r_t]$ and the discounts of leaseholds at different maturities. These models also allow us to decompose the total discount factor $R_{t,t+T}$ at each maturity into the risk-free component, $R^f_{t,t+T}$, and the risk-premium, $RP_{t,t+T}$. We consider here the leading models of asset pricing: the external habit formation model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), and the variable rare disaster model of Barro (2006) and Gabaix (2012).[19] These models were not specifically designed to understand the term structure of discount rates in the housing market – and especially the very far end of the term structure – and therefore our empirical findings are a new testing ground for these theories.

Our evaluation of these models complements Binsbergen, Brandt and Koijen (2012) who focus on the models' ability to reconcile the expected returns of short-dated dividend strips (up to 3 years) with the equity premium. We therefore only discuss the models briefly and point out which elements are most important for the valuation of long-dated claims to housing. We deviate as little as possible from the original papers' calibrations of the stochastic discount factor and cash-flows. In each model, we calibrate housing to be a risky asset with average growth rate of dividends of 0.7% and an exposure to risk that ensures an expected return of 6.5%.

[18] Notice that we can recover the Gordon-Growth implied discounts in equation (6) by substituting the Gordon-Growth assumptions in equation (9): $R^f_{t,t+T} = e^{-rT}$; $RP_{t,t+T} = 0$; $E_t[P_T]/P_t = e^{gT}$.

[19] For the rare disaster model see also: Rietz (1988); Gourio (2012); Martin (2013).

Figure 7 shows the discounts for long-dated leaseholds relative to freeholds implied by these models together with those observed in the data. In all cases the models cannot match the discounts and tend to produce even smaller discounts than those of the constant-discount-rates model.

In the long-run risk model of Bansal and Yaron (2004) agents have a preference for early resolution of uncertainty and are concerned about shocks that persistently affect the growth rate of consumption.[20] Therefore, agents dislike claims to very long-term cash flows that are exposed to these long-run risks.[21] The model matches the expected return to housing only if housing is exposed to long-run risks. However, the model also implies that leaseholds with higher maturity are more exposed to long-run risks, and command higher risk premia. This upward sloping term structure of risk premia produces even smaller discounts for leaseholds relative to freeholds compared to the constant discount rate model.[22]

In the external habit model of Campbell and Cochrane (1999) agents care about their surplus consumption relative to a habit level, which itself depends on the history of aggregate consumption.[23] Negative shocks to consumption, with which rents are correlated, induce increases in risk premia because they bring current consumption closer to the habit level. Long-term claims, due to their high duration, are particularly exposed to these shocks and are therefore particularly risky. The model implies an upward sloping term structure of risk premia that contributes to generate low discounts for leaseholds compared to freeholds.

In the variable disasters model of Barro (2006) and Gabaix (2012) consumption growth is subject to rare but large negative shocks, the disasters.[24] Agents dislike assets that are exposed to these disas-

[20] We calibrate the model following the parametrization of Bansal and Yaron (2004). The baseline calibration of the risky asset in that paper implies an expected return of 6.5% a year and, consequently, we maintain the same calibration here. We only modify the average growth rate of cash flows (rents) to match the observed 0.7% annual growth rate of rents as in our baseline calibration.

[21] Dew-Becker and Giglio (2013) show that half the total price of risk in the long-run risk model comes from fluctuations in consumption with cycles longer than 230 years and three quarters of the risk prices come from fluctuations longer than 75 years. These horizons correspond closely to the maturities of the leaseholds we consider in this paper.

[22] The result is entirely driven by risk premia that increase with maturity. As pointed out by Beeler and Campbell (2012), risk-free bond yields in this model actually decrease with maturity. The same result obtains when agents have power utility but are ambiguity averse, as in Hansen and Sargent (2001). When the agent has a preference for robustness, he can be viewed as having a reference distribution for the relevant shocks (the true distribution) and a worst-case distribution, which is what he uses to actually price assets. Under the worst-case distribution he places relatively more weight on some bad states of the world, which correspond to states with persistently low consumption growth. Therefore, the model has the same asset pricing implications as the long-run risk model.

[23] We calibrate the model following the parametrization of Campbell and Cochrane (1999). We impose an average growth rate of rents of 0.7% per year, and a correlation of rent growth and consumption growth of 0.27 to ensure that expected returns on housing are 6.5%.

[24] We calibrate the model in Gabaix (2012) to match the expected return on housing from our baseline calibration (6.5%). This requires modeling housing as a slightly safer claim than equity with respect to disasters with an average resilience of

ters. While the presence of rare disasters increases risk premia, it does so uniformly across maturities because claims to cash flows at all horizons are equally exposed to the disaster risk. Therefore, discount rates will be the same at all horizons and equal to the average return (6.5%). Therefore, this model's performance is similar to that of the constant discount rate model in Section 3.1: since cash flows far into the future are discounted at the relatively high average rate of return, the rare disaster model is not able to match the observed discounts between leaseholds and freeholds.

In general these models are subject to a tension between rents that are sufficiently risky to generate a high average expected return to housing and the fact that, as rents become riskier, long-term cash flows are discounted at progressively higher rates, generating small leasehold discounts. Therefore, taking into account that rents are risky and housing commands a nontrivial risk premium exacerbates the long-run valuation puzzle within these models. While it is beyond the scope of this paper to suggest modifications to allow these models to better match the data, the next section illustrates which characteristics a model would require in order to rationalize the long-run valuation puzzle.

3.6 Reduced Form Models of Discount Factors

We find our estimates to be consistent with a downward sloping term structure of discount rates. Discount rates have to be sufficiently high in the short to medium run to contribute to high average expected returns on housing, but also sufficiently low in the long run to match the observed discounts applied to long-run cash flows. Two existing classes of models can potentially generate this feature: models with hyperbolic discount and the reduced form model of Lettau and Wachter (2007).

Lettau and Wachter (2007) propose a reduced-form model in which the only priced shock is the unexpected innovation in rents (dividends). Unexpected rent growth today is negatively correlated with future rent growth. Therefore, long-term claims to future rents are safer than short-term claims because short-term claims do not benefit as much from the future increase in rent growth that follows a negative shock. Figure 8 shows that this model is able to match the magnitudes of the discounts at different horizons. The intuition is that long-term claims in the model are relatively safe and therefore cash flows arising many years into the future are discounted at low rates of around 2.9% a year. Combined with our baseline calibration of rent growth at 0.7%, the model is able to generate

0.1 instead of 0.09. We also modify the growth rate of dividends (rents) to match the 0.7% annual growth rate of rents, as in our baseline calibration.

leasehold discounts as large as those in the data. At the same time, the model is able to match the 6.5% expected return to housing because it implies high short term discount rates, as high as 20% per year for the first few years. While this model is not a micro-founded general equilibrium model, its functional form is very informative about the characteristics that any general equilibrium model would require in order to match the observed discounts.

We also consider models that feature a variation in the (subjective) discount rate across horizons. We follow Laibson (1997) and Luttmer and Mariotti (2003) in considering the possibility that agents attach higher discounts to short term cash flows than they do to long term cash flows. Rather than considering the full general equilibrium environment in the original references, we focus on a simple reduced form approach by directly postulating the discount function.[25]

We consider a mix of hyperbolic and exponential discounting by assuming that the discount function follows: $f(t) = \frac{e^{-\rho t}}{1+\kappa t}$, where $\rho > 0$ is the subjective discount rate associated with exponential discounting and $\kappa > 0$ is the hyperbolic parameter. Intuitively, if $\kappa = 0$ we recover exponential discounting at $e^{-\rho t}$, while if $\rho = 0$ we recover hyperbolic discounting at $\frac{1}{1+\kappa\, t}$. This mixed form of discounting tends to behave like hyperbolic discounting in the short run and like exponential discounting in the long run. The relative importance of short-run and long run is the subject of the calibration below.[26] We resume our earlier assumption from Section 3.1 that rents grow at constant rate g. In this case, the T-maturity leasehold is valued at:

$$P_0^T = \int_0^T \frac{e^{-(\rho-g)s}}{1+\kappa s} D_0 ds$$

Appendix A.3.2 derives analytic expressions for the resulting value, as well as for the value of the freehold. We only report here the model-implied discount between a T-maturity leasehold and the freehold:

$$Disc_0^T = \frac{Ei\left(\frac{(T\kappa+1)(g-\rho)}{\kappa}\right) - Ei\left(\frac{g-\rho}{\kappa}\right)}{\Gamma\left(0, \frac{\rho-g}{\kappa}\right)} - 1.$$

[25] Our approach is limited but extremely tractable. We do not analyze the rich problem of time inconsistency that could arise in the presence of hyperbolic discounting nor do we consider the potentially countervailing equilibrium consequences highlighted by Barro (1999). By directly assuming the reduced form discount function we are implicitly postulating that one could build fully fledged general equilibrium models that imply such discount function.

[26] While we adopt the continuous time formulation of Luttmer and Mariotti (2003), this is also in the spirit of Laibson (1997) where agents aggressively discount the immediate future (the hyperbolic discounting occurs in the short run), but have a constant discount rate between any two periods in the future (in the long run, the exponential discounting prevails).

where $Ei(x)$ is the Exponential Integral and $\Gamma(x)$ is the Upper Incomplete Gamma function, both discussed in appendix A.3.2. Figure 8 shows the discounts implied by a calibration of the hyperbolic-exponential model that at the same time matches the observed discounts of leaseholds of different maturities and the average return to housing. The calibration is obtained by setting κ to 10% and ρ to 1.4%. This calibration implies comparatively higher discount rates for short term than for long term cash flows. To illustrate this property, we analyze below the evolution of the marginal discount rate:

$$r(t) \equiv -\frac{\dot{f(t)}}{f(t)} = \rho + \frac{\kappa}{1+\kappa t},$$

where $r(t)$ is the marginal discount rate and $\dot{f(t)}$ is the time derivative of the discount function f. Indeed, the analysis shows that the very short run discount rate is $\rho + \kappa = 11.4\%$, while the long run marginal discount rate approaches $\rho = 1.4\%$.[27]

A similar declining term structure of discount rates could have been derived in reduced form in an environment where agents have constant discount rates in as long as there is uncertainty about the appropriate level of that discount rate. Weitzman (1998, 2001), in fact, points out not only that disagreement about the discount rate implies that long-term cash flows should be discounted at the lowest discount rate that is assumed to occur with positive probability,[28] but also that if the uncertainty takes the form of a Gamma distribution the effective discount function behaves similarly to the hyperbolic one.

Overall, we find that both the Lettau and Wachter (2007) model and the hyperbolic-exponential discount function show that a downward sloping term structure of discount rates is necessary in order to jointly rationalize the expected returns to housing and the long-term discount. These positive results, however, have to be interpreted conservatively. We view both models as convenient functional forms to understand the patterns in the data rather than fully specified general equilibrium models, and do not judge their ability to fit other stylized facts of asset pricing. It remains an open and promising question for future work to explore models that could reconcile the long-run valuation puzzle as well as match other stylized facts of asset pricing.[29]

[27] The results are obtained as the limits when $t \downarrow 0$ and $t \to \infty$.

[28] See also Dybvig, Ingersoll and Ross (1996).

[29] Intuitively, it could be interesting to analyze models that mix risk premia that vary at different horizons with declining rates of time preferences.

3.7 Liquidity and Financing Frictions

There are two frictions that could affect the relative pricing of leasehold and freehold properties: a liquidity and a financing friction.[30]

Leaseholds, in particular short dated ones, require lower upfront payment to take ownership of a property (even if only for a limited number of years). If households have high future income that cannot be immediately monetized, these shorter leaseholds are a more attractive investment than longer leaseholds or freeholds, in particular given the ability to top up leaseholds at the leaseholder's request. This liquidity effect makes shorter leaseholds relatively more expensive, thus reducing the discounts compared to a frictionless benchmark. Since this effect worsens the long-run valuation puzzle, we do not assess its quantitative implications.

A second potential source of concern is that properties with short maturity leases are harder to finance (i.e. their collateral value is low) than long maturity ones. In particular, both in Singapore and the U.K. it becomes difficult to obtain a mortgage using short maturity leaseholds as collateral. Mortgage lenders in the U.K. typically require 30 years unexpired lease term to remain at the end of the mortgage (Council of Mortgage Lenders, 2013). Mortgages generally have maturities between 10 and 30 years with the most common term length being 25 years in the U.K. market. This means that leasehold purchases have to be financed with shorter duration mortgages once the lease length falls below 55 or 60 years. The loss in "collateral value" for these leaseholds could contribute to the large discounts we observe in the data, particularly for leases in the $50-70$ years of maturity basket.

It is beyond the scope of this paper to provide a full general equilibrium model of housing in the presence of collateral and borrowing constraints. Instead we consider a simple deviation from the constant-discount-rates model in Section 3.1 and check whether the quantitative implication of a reduced form collateral constraint can help explain the observed discounts. Assume that for the last $\bar{T}$ years of lease maturity the house has lower collateral value. We model this has an effective rent for the last $\bar{T}$ years that is a fraction $(1-\alpha)$ of the original rent. This loss corresponds to the per-period shadow value of liquidity (i.e., the per-period cost to the buyer of having to use own resources to finance the house instead of an external mortgage). Alternatively, we can interpret α as the total loss in value once the leasehold reaches 60 years of remaining maturity due to the fact that

[30] In Appendix A.1.1 we discuss that ground rents and service charges cannot explain the observed discounts between leaseholds and freeholds. Many other frictions, such as long-run uncertainty about the enforcement property rights in countries such as Singapore should reduce the relative valuation of freeholds and leaseholds.

new potential buyers will no longer have access to long-maturity mortgages and might need to make a larger downpayment. The value of the lease and the implied leasehold discounts with respect to freeholds are given by:

$$\begin{aligned} P_t^T &= \int_t^{t+T} e^{-\rho(s-t)} D_t e^{g(s-t)} (1 - \alpha \mathbf{1}_{\{s>t+T-\bar{T}\}}) ds; \\ Disc_t^T &= e^{-(\rho-g)T} + \alpha \left(e^{-(\rho-g)(T-\bar{T})} - e^{-(\rho-g)T} \right) - \mathbf{1}_{\{T<\bar{T}\}} \alpha \left(e^{-(\rho-g)(T-\bar{T})} - 1 \right). \end{aligned} \tag{10}$$

The formulas are derived and discussed in more detail in Appendix (A.3.3). In Figure 9 we set $\bar{T} = 60$ and solve the model for different values of of α. We parametrize r and g as in our baseline estimate at 6.5% and 0.7%, respectively. For the proportional value of the collateralizability of the house (α) we explore a range between 5% and 20%, which we believe to be a conservative estimate. The figure shows that even for α as high as 20%, the model cannot match the empirical discounts at essentially any horizon.

Most importantly even *unrealistically* high assumptions on the loss of collateral value for short duration leaseholds cannot help to explain the discounts for leases of long maturities (for example 150 or 250 years). Intuitively, a lease that has 200 years left today will only incur potential losses of its collateral value 140 years from now, when the lease will have 60 years left. Any losses that occur so far into the future have little impact on present values at conventional discount rates.

4 Implications of the Findings

4.1 Asset Pricing

Our estimates of very long-run discount rates provide a novel testing ground for theoretical asset pricing models, and are complementary to those in Binsbergen, Brandt and Koijen (2012). We provide evidence for very long-run discount rates (80-250 years) for residential housing while they focus on the short-run (1-3 years) discount rates in equity markets. Despite the different asset classes, data, and methodologies we find similar qualitative patterns: short-run discount rates are higher than long-run discount rates. The leading general equilibrium asset pricing models such as the habit model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), and the rare disaster model of Barro (2006) and Gabaix (2012) cannot match this pattern. While these models

where not specifically set-up to match the horizon-variation in discount rates, the evidence provides a new stringent testing ground for future theoretical advances.

It is important to emphasize that without a structural model we cannot disentangle the risk-free component, $R^f_{t,t+T}$, from the risk premium, $RP_{t,t+T}$, in our estimates of total discount rates, $R_{t,t+T}$. Our estimates imply that the total discount rates are declining over the horizon T and are as low as 2% for horizons of 100 or more years. Since rents are risky (see Appendix A.2.3), the risk premium ($RP_{t,t+T}$) is likely to be positive.[31] Long-run rents in cities such as London or Singapore could carry substantial systematic risk, since they load heavily on the performance of the global economy. If this were true then our estimates would imply that agents are not afraid of these long-run risks (i.e. have low aversion to these risks): the price of risk ($\lambda_{t,t+T}$ in Section 3.4) is low for large T. If the risk premium is non-negative, then the risk free component also has to be low to match the total discount rate. This implies that agents attach high present values to payments that occur for sure 100 or more years from now.

To sum up, we conclude that our estimates imply two novel facts. First, agents have low discount rates (in the range of 1%) for risk-free payments occurring far into the future (100 years or more). Second, given plausible estimates of the riskiness of long-run rents, agents have low aversion over risks that materialize in the very long-run.

4.2 Environmental Policy

> *Any consideration of the costs of meeting climate objectives requires confronting one of the thorniest issues in all climate-change economics: how should we compare present and future costs and benefits? [...] A full appreciation of the economics of climate change cannot proceed without dealing with discounting.* **(Nordhaus, 2013)**

The above quote from Nordhaus's recent book on the economics of climate change summarizes a long debate on the appropriate discount factors to use in evaluating environmental policies. The economics literature on climate change, starting with the seminal paper of Nordhaus (1973), has pointed out that discounting is of central relevance to the tradeoff between immediate costs, through loss

[31] In fact, one could argue that since rents and consumption are likely to be cointegrated in the long-run, claims to long-run rents should be as risky as claims to long-run consumption. For example, Jeske, Krueger and Mitman (2011) show that the share of consumption expenditures on housing over total consumption in the U.S has been remarkably constant at 14.1% over the past 40 years.

of output, and uncertain benefits that occur very far into the future.[32] Estimates of the appropriate discount rate range from the zero discounting of Stern (2007) to as high as 10% per year based on the returns to risky private investments.

The debate has tried to infer discount rates from the realized returns of traded assets such as private capital, equity, bonds, and real estate. These estimates of average returns, however, reveal only the average returns on these assets. Our estimates in Table 5 find that the average real returns to residential housing are above 8%. However, the crucial estimates to evaluate climate-policy are the discount rates for cash-flows very far into the future.[33] We found such discount rates to be much lower than those implied by average returns and of the order of 2%.

Our direct empirical estimates of discount rates are consistent both with the decreasing term structure of discount rates suggested by Weitzman (1998, 2001) on the basis of survey evidence as well as on theoretical grounds, and with the survey data from Layton and Brown (2000), who surveyed 376 subjects to elicit their preferences for mitigating impacts of climate change that will occur in the distant future.[34]

We summarize our contribution to this important debate as providing the first direct estimates of discount rates at horizons that are relevant for climate-change policies (e.g. 70-100 years and beyond), using market prices for traded, quantitatively important assets. Our findings that long-run discount rates are low, much lower than average returns, can potentially suggest a radical reassessment of the value of climate policies.

It is important to recall from Section 4.1 that our estimates combine risk-free discounts and risk premia. Both are estimated to be low. As noted in Barro (2013), the contribution of the time value of money and risk to long-term discount rates might have opposite prediction for climate policies. Our results imply that agents have low long-run risk-free discount rates and are relatively less concerned towards long-run risks. This implies that households are willing to invest in policies that reduce with certainty the adverse effects of climate change, even if the benefits will only arise far in the future.

[32] See also: Lind (1982); Cline (1992); Nordhaus (1992); Arrow et al. (1996); Weitzman (1998); Nordhaus (2001); Groom et al. (2005); Gollier (2006); Dasgupta (2007); Nordhaus (2007); Weitzman (2007); Gollier and Weitzman (2010); Gerlagh and Liski (2012); Goulder and Williams (2012); Pindyck (2013); Weitzman (2013).

[33] Of course in models with constant discount rates the average return provides all the necessary information. We have, however, shown in Section 3 that such models cannot be reconciled with the data.

[34] The "stated preference" approach was used in these papers since, as Layton and Brown (2000) remark, "we are aware of no markets that reveal the preferences of those alive today to help others 150 years in the future. Short of a national plebiscite, survey or other experimental methods will be the only way to determine whether those alive today are willing to spend hundreds of billions of dollars annually to mitigate uncertain damages in the future."

However, agents appear relatively unwilling to invest in policies that only reduce the risk of even potentially large, distant environmental disasters.

Our estimates of very long-run discount rates can also inform other inter-generational cost-benefit analyses regularly undertaken by governments (Feldstein, 1964; Layard and Glaister, 1994; Stiglitz, 1994; Arrow et al., 2013; Damon, Mohlin and Sterner, 2013) such as the evaluation of long-term infrastructure projects (e.g. the Hoover dam). Similarly, Auerbach, Gokhale and Kotlikoff (1994) have pointed out that an analysis of optimal fiscal policy requires taking a stance on long-run discount rates to evaluate the present value of leaving large debts to future generations.

4.3 Real Estate

Our results are also of direct relevance for real estate economics and the ongoing effort to understand house prices. We add to the recent research effort to understand the return properties of real estate (Flavin and Yamashita, 2002; Piazzesi, Schneider and Tuzel, 2007; Favilukis, Ludvigson and Van Nieuwerburgh, 2010) by focusing on a previously unexplored aspect of real estate: the term structure of house prices. Our findings that the term structure for housing discount rates is downward sloping poses yet unexplored questions for modeling house prices.

4.4 Rational Bubbles

Our estimates of long-run discount rates can also be used to *directly* test for the presence of infinitely-lived rational bubbles. The existence of bubbles is one of the most fundamental, oldest, and most difficult questions in economics. In their recent survey of the literature on bubbles, Brunnermeier and Oehmke (2013) emphasize that "identifying bubbles in the data is a challenging task. The reason is that in order to identify a bubble, one needs to know an asset's fundamental value, which is usually difficult to measure." We show that this is not the case for our tests, which are *model independent*.

The classic infinitely-lived rational bubble models of Blanchard and Watson (1982) and Froot and Obstfeld (1991) feature a failure of the no-bubble condition, which is routinely imposed in most economic models. The no-bubble condition requires that the present value of a payment occurring in the limit as the horizon goes to infinity is zero:

$$\lim_{T\to\infty} E_t[\xi_{t,T} P_T] = 0,$$

where $\xi_{t,T}$ is a model-implied discount factor between date t and T and P_T the price of the asset at time T. Our data is uniquely suited to test this condition because we can estimate the present value of a claim to rents occurring at very long horizons, for example $T = 999$ years. More formally:

$$P_t - P_t^T \approx \lim_{T\to\infty} E_t[\xi_{t,T} P_T], \qquad \text{for large T.}$$

Intuitively, the difference in value between a freehold (P_t) and a 999-maturity leasehold (P_t^{999}) is the present value of the claim to rents starting 999 years from today and extending to the infinite future (i.e. the present value of a freehold 999 years from now, $E_t[\xi_{t,999}\ P_{999}]$). Therefore we can test whether the no-bubble condition holds, on average, by testing whether the discount of very long leases to freeholds is zero. We correspondingly formulate our null hypothesis of no-bubbles as: $Disc^T = 0$ for $T > 800$ years.

The estimates of extreme long-run discounts for Singapore and the UK are reported in Figures 4 and 2. In all cases the point estimates of the discounts are negligible and not statistically significant for T sufficiently large, 800 or more years. We conclude that there is no evidence in our data supporting the presence of infinitely-lived rational bubbles.[35] As an even more stringent test, Figure 10 shows that there is no evidence of a bubble at any point in time between 1995 and 2013 in Singapore. It does so by showing that the price of freeholds P_t and those of 999-year leasehold P_t^{999} are essentially identical, and certainly within the 1% confidence interval of each other, at all points in time. This test is of particular interest because it shows the absence of an infinitely-lived rational bubble even at the peak of the housing market in 2013 after years of strong house price growth, when many commentators where hinting at the presence of a large bubble.

The strength of directly testing the no-bubble condition is that all models that assume the absence of infinitely-lived rational bubbles have the same implication: that the fundamental present value of a payment occurring in infinite time ($\lim_{T\to\infty} E_t[\xi_{t,T} P_T]$) is equal to zero. We do not need to specify a model (a choice of $\xi_{t,T}$ and of a stochastic process for rents) in order to obtain a fundamental value to compare to the valuation in the data. All no-bubble models imply that such fundamental value is zero. Our direct testing methodology is made possible by the uniqueness of our data that allows us to identify the terminal no-bubble condition. Such tests have been elusive because we do not

[35] While the literature has already put forward theoretical arguments for the fragility of the existence of infinitely-lived rational bubbles (for example Tirole (1985)), our tests provide direct empirical evidence for the absence of such bubbles.

normally observe traded claims to payments that only occur extremely far into the future. Our direct tests contrasts sharply with a large previous literature (for example: Flood and Garber (1980); Evans (1991); Diba and Grossman (1988*b*,*a*); West (1987)) that had to either deal with the thorny problem of establishing fundamental values or find indirect ways to test for bubbles.[36]

We note, however, that our bubble tests should not be interpreted as providing evidence for the absence of *all types* of bubbles. We provide evidence against a specific, in the theoretical literature very common, type of bubble: the infinitely-lived rational bubble. Our tests are uninformative with respect to the presence of finitely-lived bubbles of the kind described for example in Abreu and Brunnermeier (2003) and DeMarzo, Kaniel and Kremer (2008).

5 Conclusions

We provide novel estimates of very long-run discount rates by exploring unique features of the U.K. and Singapore housing markets where properties trade as either freeholds (infinite maturity ownership) or leaseholds of various maturities. We find that low long-run discount rates, much lower than routinely assumed by economic theory, are necessary in order to explain both the relatively high expected return to housing and the observed discounts between long-run leaseholds and freeholds. Our results provide new insight on the term structure of house prices, a new testing ground for theoretical asset pricing models, and a direct estimate of the long-run discount rates that are crucial to evaluate environmental policies and other immediate actions that only have payoffs very far into the future.

[36] See Flood and Hodrick (1990) for a review of the deep econometric problems that had a chilling effect on the empirical literature attempting to test for the presence of bubbles.

References

Abreu, Dilip, and Markus K Brunnermeier. 2003. "Bubbles and crashes." *Econometrica*, 71(1): 173–204.

Arrow, Kenneth, Maureen Cropper, Christian Gollier, Ben Groom, Geoffrey Heal, Richard Newell, William Nordhaus, Robert Pindyck, William Pizer, Paul Portney, et al. 2013. "How Should Benefits and Costs Be Discounted in an Intergenerational Context? The Views of an Expert Panel." *The Views of an Expert Panel (December 19, 2013). Resources for the Future Discussion Paper*, , (12-53).

Arrow, K. J., W. Cline, K. G. Maler, M. Munasinghe, R. Squitieri, and J. Stiglitz. 1996. *Intertemporal Equity, Discounting, and Economic Efficiency, in "Climate Change 1995: Economic and Social Dimensions of Climate Change"*. Cambridge University Press.

Auerbach, Alan J, Jagadeesh Gokhale, and Laurence J Kotlikoff. 1994. "Generational accounting: a meaningful way to evaluate fiscal policy." *The Journal of Economic Perspectives*, 8(1): 73–94.

Bansal, Ravi, and Amir Yaron. 2004. "Risks for the long run: A potential resolution of asset pricing puzzles." *The Journal of Finance*, 59(4): 1481–1509.

Barro, Robert J. 1999. "Ramsey meets Laibson in the neoclassical growth model." *The Quarterly Journal of Economics*, 114(4): 1125–1152.

Barro, Robert J. 2006. "Rare disasters and asset markets in the twentieth century." *The Quarterly Journal of Economics*, 121(3): 823–866.

Barro, Robert J. 2013. "Environmental protection, rare disasters, and discount rates." *NBER Working Paper No. 19258*.

Beeler, Jason, and John Y. Campbell. 2012. "The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment." *Critical Finance Review*, 1: 141–182.

Belo, Frederico, Pierre Collin-Dufresne, and Robert S Goldstein. 2012. "Endogenous dividend dynamics and the term structure of dividend strips." National Bureau of Economic Research.

Binsbergen, Jules van, Michael Brandt, and Ralph Koijen. 2012. "On the Timing and Pricing of Dividends." *American Economic Review*, 102(4): 1596–1618.

Binsbergen, Jules van, Wouter Hueskes, Ralph Koijen, and Evert Vrugt. 2013. "Equity yields." *Journal of Financial Economics*.

Blanchard, Olivier J, and Mark W Watson. 1982. "Bubbles, rational expectations and financial markets." *Crises in the Economic and Financial Structure: Bubbles, Bursts, and Shocks*. P. Wachtel. D.C. Heath & Company.

Boguth, Oliver, Murray Carlson, AJ Fisher, and Mikhail Simutin. 2012. "Leverage and the limits of arbitrage pricing: Implications for dividend strips and the term structure of equity risk premia." *Working Paper*.

Brunnermeier, Markus K., and Martin Oehmke. 2013. "Bubbles, Financial Crises, and Systemic Risk." *Handbook of the Economics of Finance*. Amsterdam:Elsevier.

Campbell, John Y, and John H Cochrane. 1999. "By Force of Habit: A Consumption Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy*, 107(2): 205–251.

Campbell, Sean D, Morris A Davis, Joshua Gallin, and Robert F Martin. 2009. "What moves housing markets: A variance decomposition of the rent–price ratio." *Journal of Urban Economics*, 66(2): 90–102.

Cline, William R. 1992. *The Economics of Global Warming*. Institute for International Economics.

Cochrane, John H. 2011. "Presidential address: Discount rates." *The Journal of Finance*, 66(4): 1047–1108.

Council of Mortgage Lenders. 2013. "What minimum unexpired lease term does the lender accept?" *CLM Lender's Handbook*.

Damon, Maria, Kristina Mohlin, and Thomas Sterner. 2013. "Putting a Price on the Future of Our Children and Grandchildren." *The Globalization of Cost-Benefit Analysis in Environmental Policy*, 47.

Dasgupta, Partha. 2007. "Commentary: The Stern Reviewï£¡s Economics of Climate Change." *National Institute Economic Review*, 199: 4–70.

DeMarzo, Peter M, Ron Kaniel, and Ilan Kremer. 2008. "Relative wealth concerns and financial bubbles." *Review of Financial Studies*, 21(1): 19–50.

Dew-Becker, Ian, and Stefano Giglio. 2013. "Asset pricing in the frequency domain: theory and empirics." *Unpublished manuscript, University of Chicago*.

Diba, Behzad T, and Herschel I Grossman. 1988*a*. "Explosive rational bubbles in stock prices?" *The American Economic Review*, 78(3): 520–530.

Diba, Behzad T, and Herschel I Grossman. 1988*b*. "The theory of rational bubbles in stock prices." *The Economic Journal*, 98(392): 746–754.

Dybvig, Philip H, Jonathan E Ingersoll, and Stephen A Ross. 1996. "Long forward and zero-coupon rates can never fall." *Journal of Business*, 1–25.

Evans, George W. 1991. "Pitfalls in testing for explosive bubbles in asset prices." *The American Economic Review*, 81(4): 922–930.

Favilukis, Jack, Sydney C Ludvigson, and Stijn Van Nieuwerburgh. 2010. "The macroeconomic effects of housing wealth, housing finance, and limited risk-sharing in general equilibrium." National Bureau of Economic Research.

Feldstein, Martin S. 1964. "The social time preference discount rate in cost benefit analysis." *The Economic Journal*, 74(294): 360–379.

Flavin, Marjorie, and Takashi Yamashita. 2002. "Owner-occupied housing and the composition of the household portfolio." *The American Economic Review*, 92(1): 345–362.

Flood, Robert P, and Peter M Garber. 1980. "Market fundamentals versus price-level bubbles: the first tests." *The Journal of Political Economy*, 745–770.

Flood, Robert P, and Robert J Hodrick. 1990. "On testing for speculative bubbles." *The Journal of Economic Perspectives*, 4(2): 85–101.

Froot, Kenneth A, and Maurice Obstfeld. 1991. "Intrinsic Bubbles: The Case of Stock Prices." *The American Economic Review*.

Gabaix, Xavier. 2012. "Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance." *The Quarterly Journal of Economics*, 127(2): 645–700.

Gerlagh, Reyer, and Matti Liski. 2012. "Carbon Prices for the Next Thousand Years." CESifo Group Munich.

Gollier, C. 2006. "An evaluation of the Stern Report on the economics of climate change." *IDEI working papers no. 464*.

Gollier, Christian, and Martin L Weitzman. 2010. "How should the distant future be discounted when discount rates are uncertain?" *Economics Letters*, 107(3): 350–353.

Gordon, Myron J. 1982. *The investment, financing, and valuation of the corporation.* Greenwood Press.

Goulder, Lawrence H, and Roberton C Williams. 2012. "The choice of discount rate for climate change policy evaluation." *Climate Change Economics*, 3(04).

Gourio, Francois. 2012. "Disaster Risk and Business Cycles." *The American Economic Review*, 102(6): 2734–2766.

Groom, Ben, Cameron Hepburn, Phoebe Koundouri, and David Pearce. 2005. "Declining discount rates: the long and the short of it." *Environmental and Resource Economics*, 32(4): 445–493.

Gyourko, J., C.J. Mayer, and T.M. Sinai. 2006. "Superstar cities." *NBER Working Paper*, 12355.

Gyourko, Joseph, and Donald B Keim. 1992. "What does the stock market tell us about real estate returns?" *Real Estate Economics*, 20(3): 457–485.

Hansen, Lars P., and Thomas J. Sargent. 2001. "Robust control and model uncertainty." *American Economic Review*, 91: 60–66.

Jeske, Karsten, Dirk Krueger, and Kurt Mitman. 2011. "Housing and the macroeconomy: The role of bailout guarantees for government sponsored enterprises." National Bureau of Economic Research.

Laibson, David. 1997. "Golden eggs and hyperbolic discounting." *The Quarterly Journal of Economics*, 112(2): 443–478.

Layard, Richard, and Stephen Glaister. 1994. *Cost-benefit analysis.* Cambridge University Press.

Layton, David F, and Gardner Brown. 2000. "Heterogeneous preferences regarding global climate change." *Review of Economics and Statistics*, 82(4): 616–624.

Leigh, Wilhelmina A. 1980. "Economic depreciation of the residential housing stock of the United States, 1950-1970." *The Review of Economics and Statistics*, 62(2): 200–206.

Lettau, Martin, and Jessica A Wachter. 2007. "Why Is Long-Horizon Equity Less Risky? A Duration-Based Explanation of the Value Premium." *The Journal of Finance*, 62(1): 55–92.

Lettau, Martin, and Jessica A Wachter. 2011. "The term structures of equity and interest rates." *Journal of Financial Economics*, 101(1): 90–113.

Lind, Robert, ed. 1982. *Discounting for Time and Risk in Energy Policy.* Resources for the Future.

Lustig, Hanno N, and Stijn G Van Nieuwerburgh. 2005. "Housing collateral, consumption insurance, and risk premia: An empirical perspective." *The Journal of Finance*, 60(3): 1167–1219.

Luttmer, Erzo GJ, and Thomas Mariotti. 2003. "Subjective discounting in an exchange economy." *Journal of Political Economy*, 111(5): 959–989.

Martin, Ian. 2012. "On the Valuation of Long-Dated Assets." *Journal of Political Economy*, 120(2): 346–358.

Martin, Ian. 2013. "Consumption-Based Asset Pricing with Higher Cumulants." *Review of Economic Studies*, 80.

Nordhaus, W. D. 1973. "The Allocation of Energy Resources." *Brookings Papers on Economic Activity*, 3: 529–576.

Nordhaus, W. D. 1992. "An Optimal Transition Path for Controlling Greenhouse Gases." *Science*, 258: 1315–19.

Nordhaus, W. D. 2001. "Climate Change: Global Warming Economics." *Science*, 294: 1283–84.

Nordhaus, W. D. 2007. "A Review of the Stern Review on the Economics of Climate Change." *Journal of Economic Literature*, 45: 686–702.

Nordhaus, William D. 2006. "The" Stern review" on the economics of climate change." National Bureau of Economic Research.

Nordhaus, William D. 2013. *The Climate Casino: Risk, Uncertainty, and Economics for a Warming World.* Yale University Press.

Petersen, M.A. 2009. "Estimating standard errors in finance panel data sets: Comparing approaches." *Review of Financial Studies*, 22(1): 435.

Piazzesi, Monika, Martin Schneider, and Selale Tuzel. 2007. "Housing, consumption and asset pricing." *Journal of Financial Economics*, 83(3): 531–569.

Pindyck, Robert. 2013. "Climate Change Policy: What Do the Models Tell Us?" *Journal of Economic Literature*, 51(3): 860–872.

Rietz, Thomas A. 1988. "The equity risk premium a solution." *Journal of monetary Economics*, 22(1): 117–131.

Shiller, Robert J. 2006. "Long-term perspectives on the current boom in home prices." *The Economists' Voice*, 3(4).

Stern, Nicholas. 2007. "The economics of climate change: the Stern report." *Cambridge, UK*.

Stiglitz, Joseph E. 1994. *Discount Rates, in "Cost-benefit analysis".* Cambridge University Press.

The Independent. 2013. "Shortening leaseholds are exposing more than a million homeowners to major risk." *Published on April 8, 2013*.

Tirole, Jean. 1985. "Asset bubbles and overlapping generations." *Econometrica*, 1499–1528.

Weitzman, Martin L. 1998. "Why the Far-Distant Future Should Be Discounted at Its Lowest Possible Rate." *Journal of Environmental Economics and Management*, 36(3): 201 – 208.

Weitzman, Martin L. 2001. "Gamma discounting." *American Economic Review*, 260–271.

Weitzman, Martin L. 2007. "A Review of the Stern Review on the Economics of Climate Change." *Journal of Economic Literature*, 45(3): 703–724.

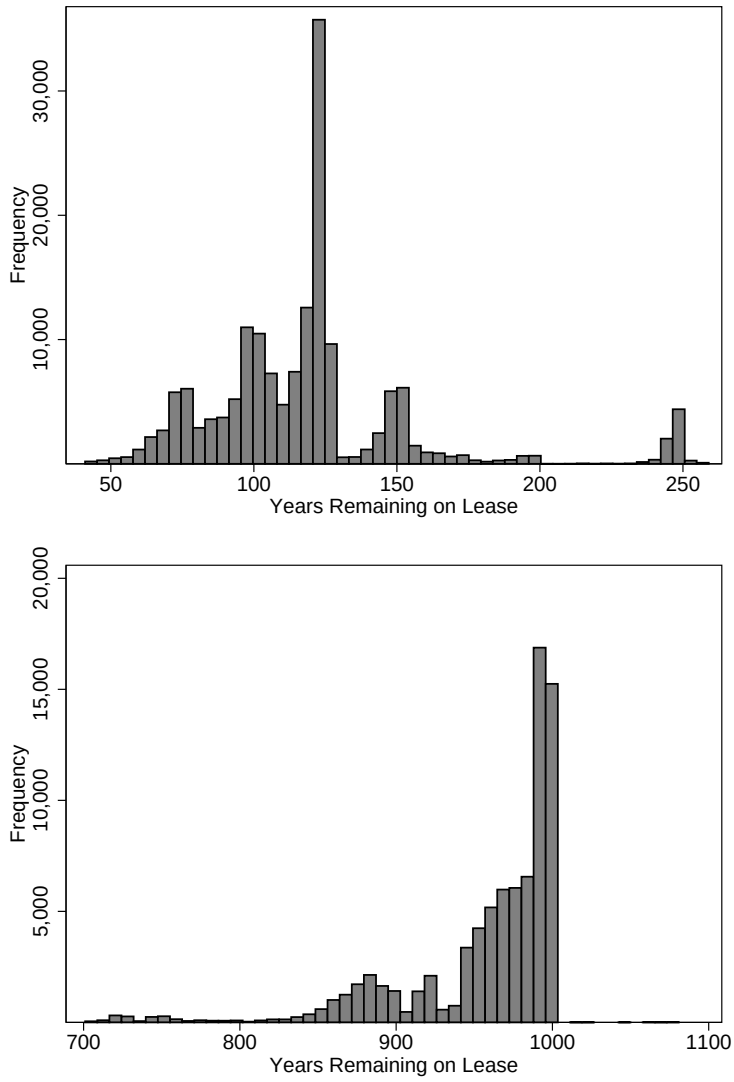
Weitzman, Martin L. 2012. "Rare Disasters, Tail-Hedged Investments, and Risk-Adjusted Discount Rates." National Bureau of Economic Research.

Weitzman, Martin L. 2013. "Tail-Hedge Discounting and the Social Cost of Carbon." *Journal of Economic Literature*, 51(3): 873–882.

West, Kenneth D. 1987. "A specification test for speculative bubbles." *The Quarterly Journal of Economics*, 102(3): 553–580.

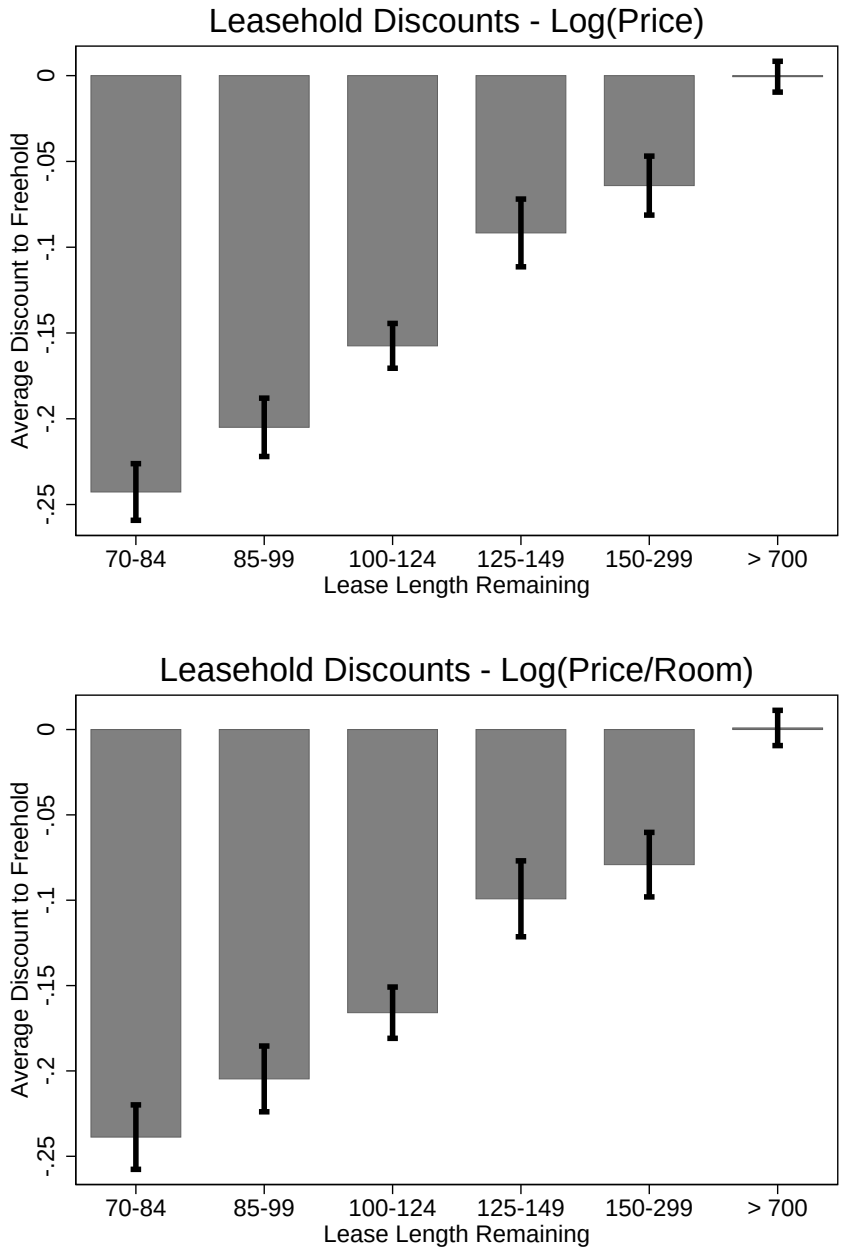
Figures

Figure 1: Distribution of Remaining Lease Lengths at Sale (U.K.)



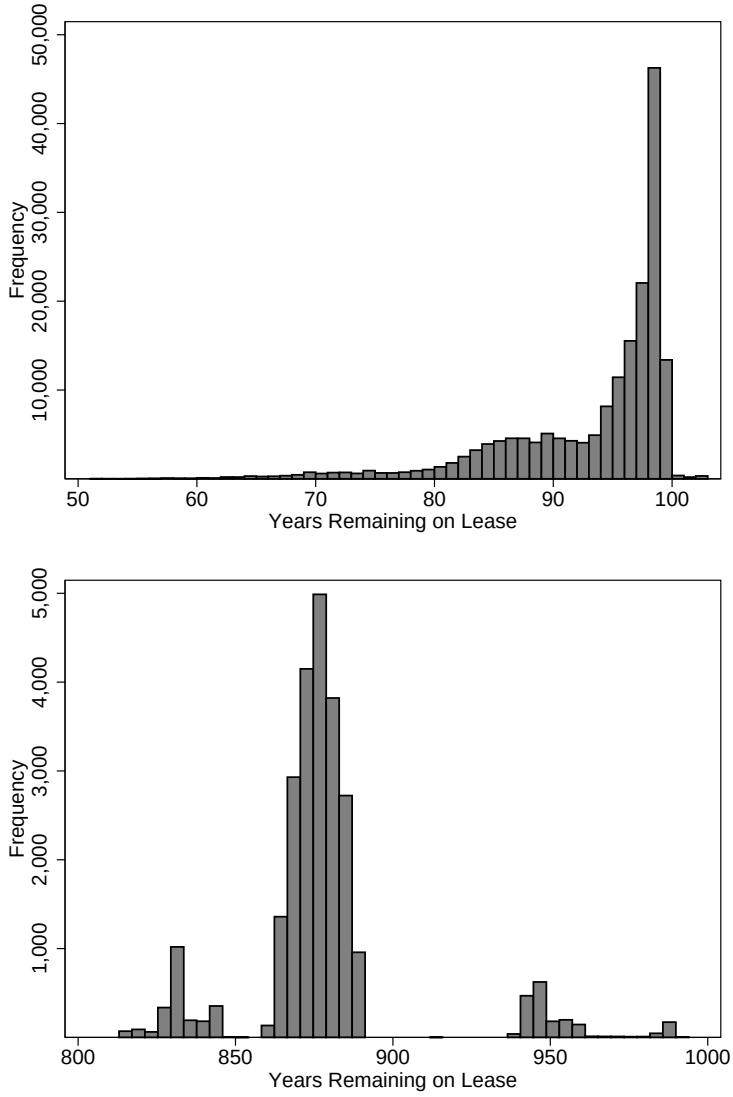
Note: This figure shows the distribution of years remaining on the lease for the leasehold transactions in our U.K. transaction sample.

Figure 2: Price Discount by Remaining Lease Length (U.K.) – Houses with hedonics



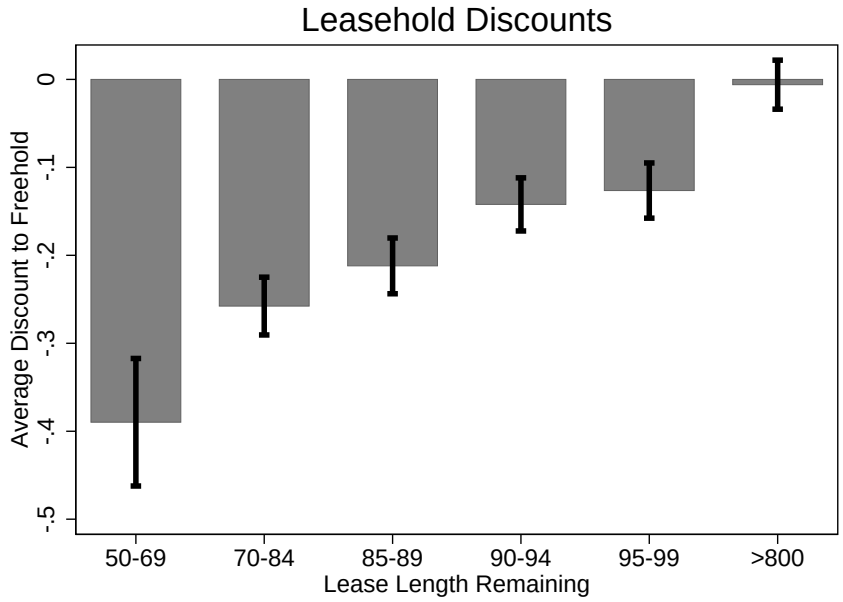
Note: This figure shows β_j coefficients from regression (1). To convert into percentage discounts for leasehold properties of a certain maturity, construct $e^{\beta} - 1$. In the top panel the dependent variable is the log price paid for properties in the U.K. between 2009 and 2013, corresponding to column (3) in Table 2, in the bottom panel it is the log price per room, corresponding to column (6) in Table 2. We only include properties which we could match to property listings with information on the number of bedrooms and bathrooms. We include postal district by property type by transaction month by number of bedrooms fixed effects. We also control for the number of bathrooms and the total number of rooms, as well as wehther the property is a new construction. The bars indicate the 95% confidence interval of the estimate using standard errors clustered at the level of the fixed effects.

Figure 3: Distribution of Remaining Lease Lengths at Sale (Singapore)



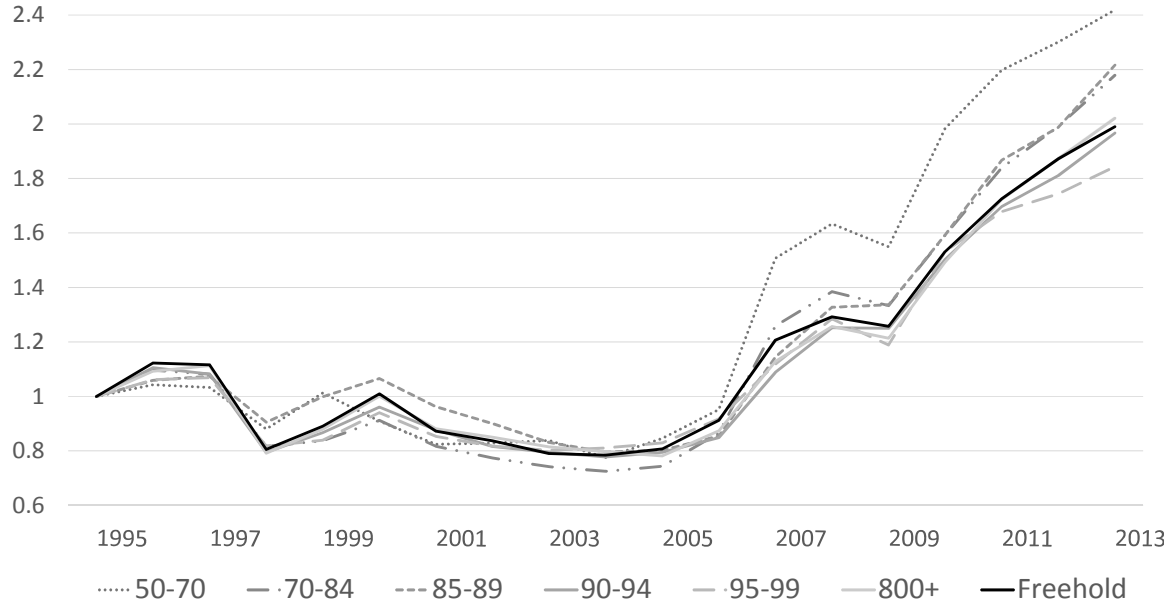
Note: This figure shows the distribution of years remaining on the lease for the leasehold transactions in our Singapore transaction sample.

Figure 4: Price Discount by Remaining Lease Length (Singapore)

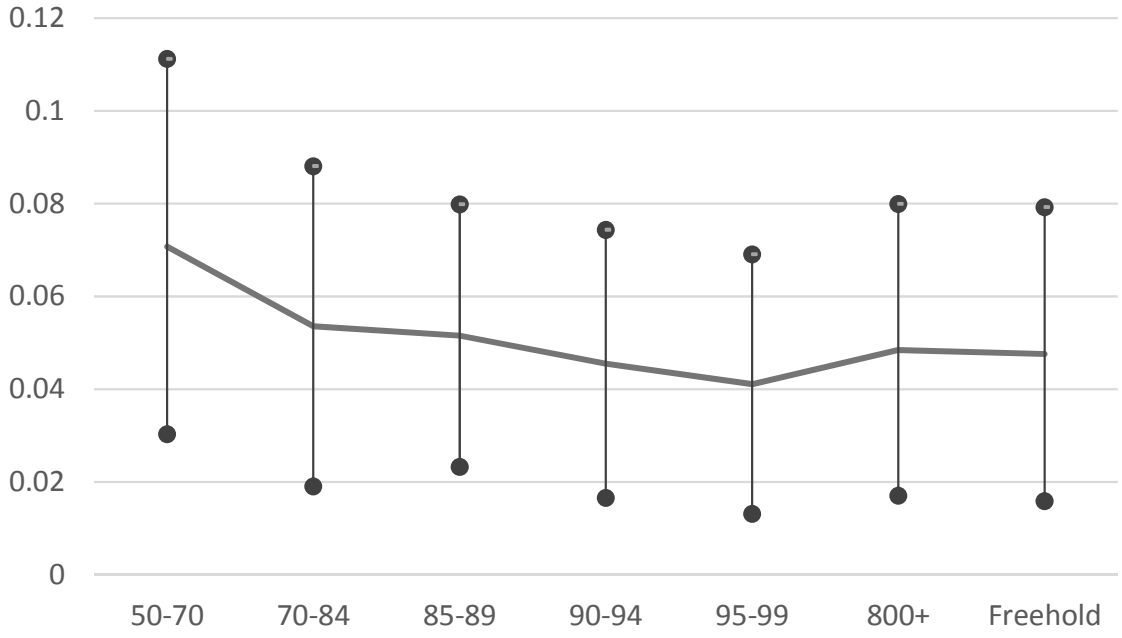


Note: This figure shows β_j coefficients from regression (3). To convert into percentage discounts for leasehold properties of a certain maturity, construct $e^{\beta} - 1$. The dependent variable is the log price per square foot paid for properties sold by private parties in Singapore between 1995 and 2013. We include fixed effect at the 5-digit postcode by property type (apartment, condominium, detached house, executive condominium, semi-detached house and terrace house) by title type (Strata or Land) by transaction month. We control for the age of the property (by including a dummy variable for every possible age in years), the size of the property (by including a dummy for each of 40 equally sized groups capturing property size) and the total number of units in the property. The bars indicate the 95% confidence interval of the estimate using standard errors clustered at the level of the fixed effect.

Figure 5: Price Index and Annualized Capital Gain by Lease Type - Singapore



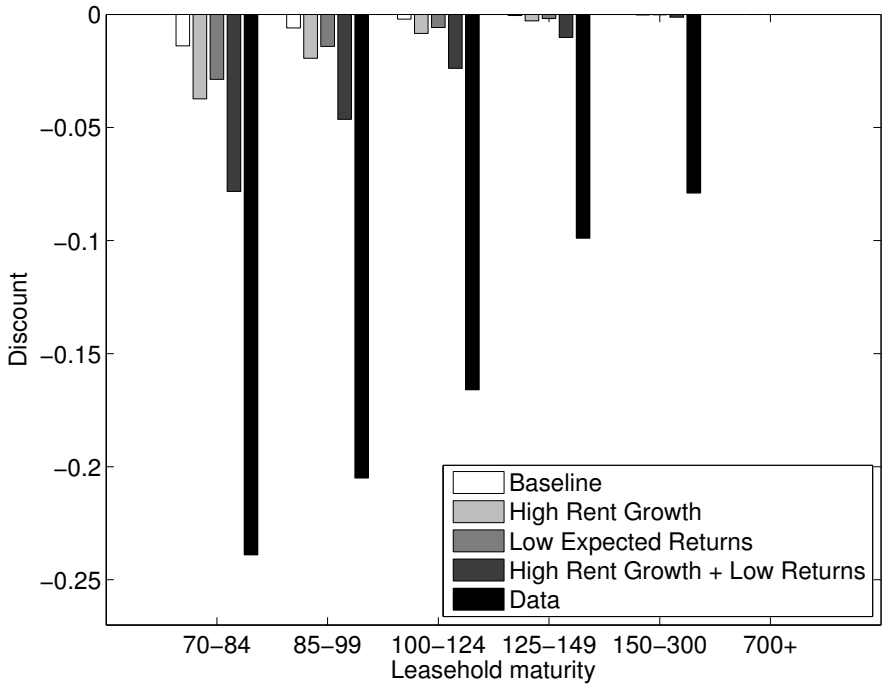
(a) Price Index



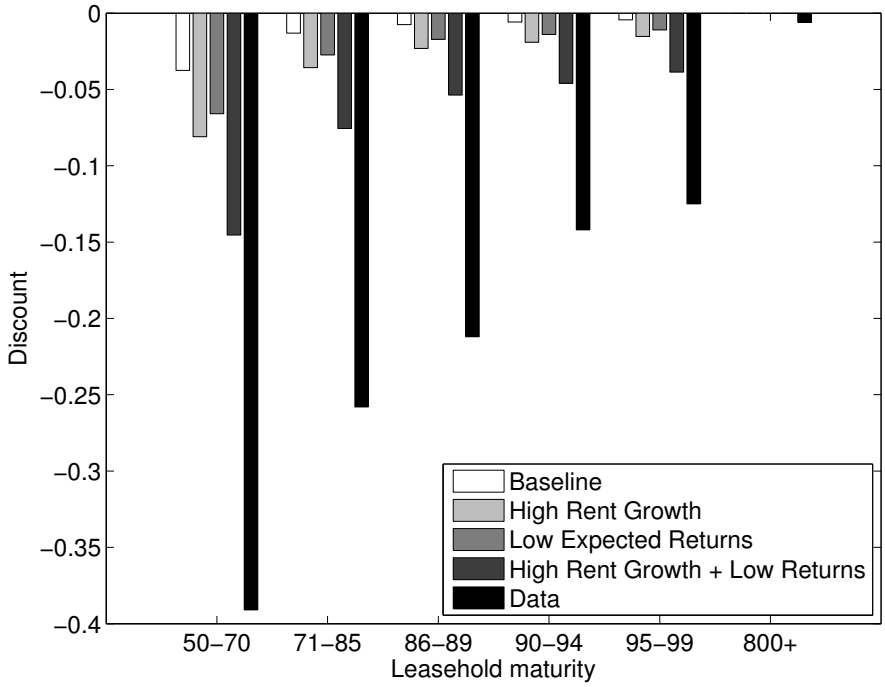
(b) Annualized Capital Gain

Note: The top panel of this figure shows hedonic price indices for different lease length properties in Singapore, using the regression estimates from regression (4). We include 4-digit postcode by property type by title type fixed effects. We also control for the age of the property (by including a dummy variable for every possible age in years), the size of the property (by including a dummy for each of 40 equally sized groups capturing property size) and the total number of units in the property. The bottom panel shows the average annualized capital gains for leaseholds of different maturities. We plot point estimates and 1 standard deviation error bounds.

Figure 6: Constant Discount Model: Discounts vs. Data



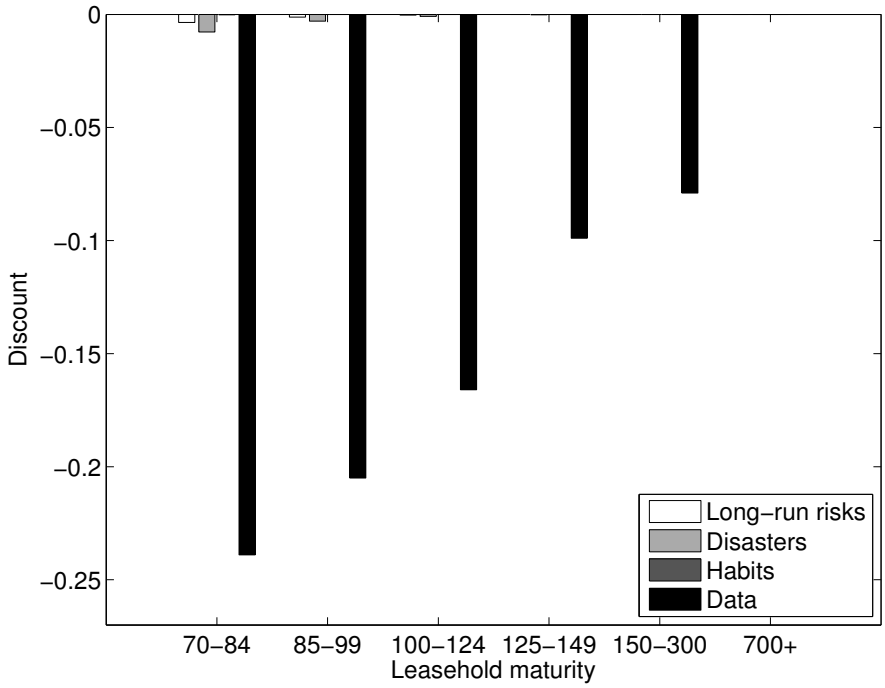
(a) U.K.



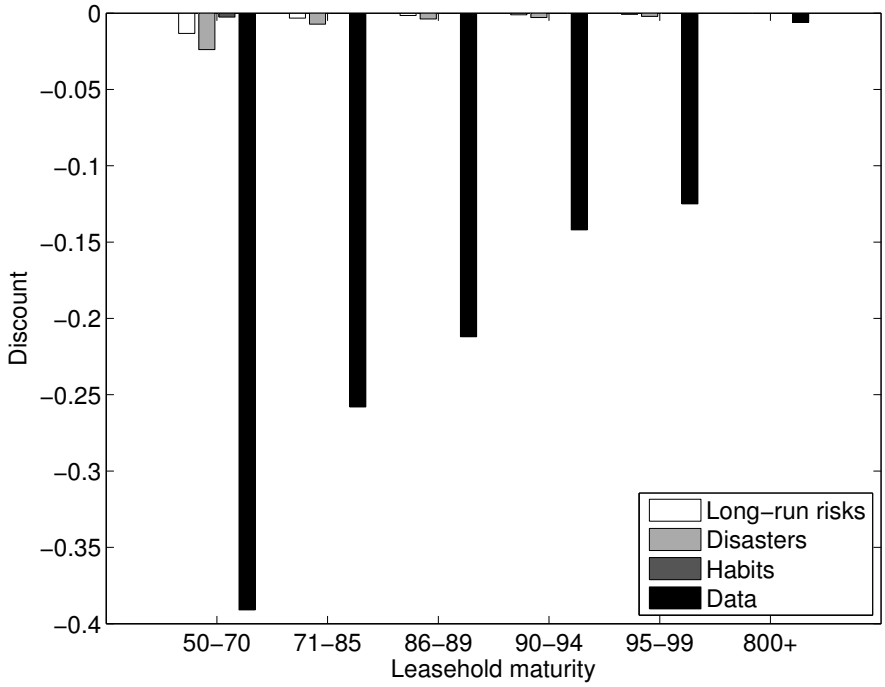
(b) Singapore

Note: The figure shows the discounts for leaseholds observed in the U.K. (top panel) and Singapore (bottom panel) together with discounts predicted by a number of parameterizations of the constant discount model. The baseline calibration has $r = 6.5\%$ and $g = 0.7\%$. A "low expected return" calibration takes $r = 5.5\%$, while a "high rent growth" calibration takes $g = 2\%$.

Figure 7: Asset Pricing Model Discounts vs. Data



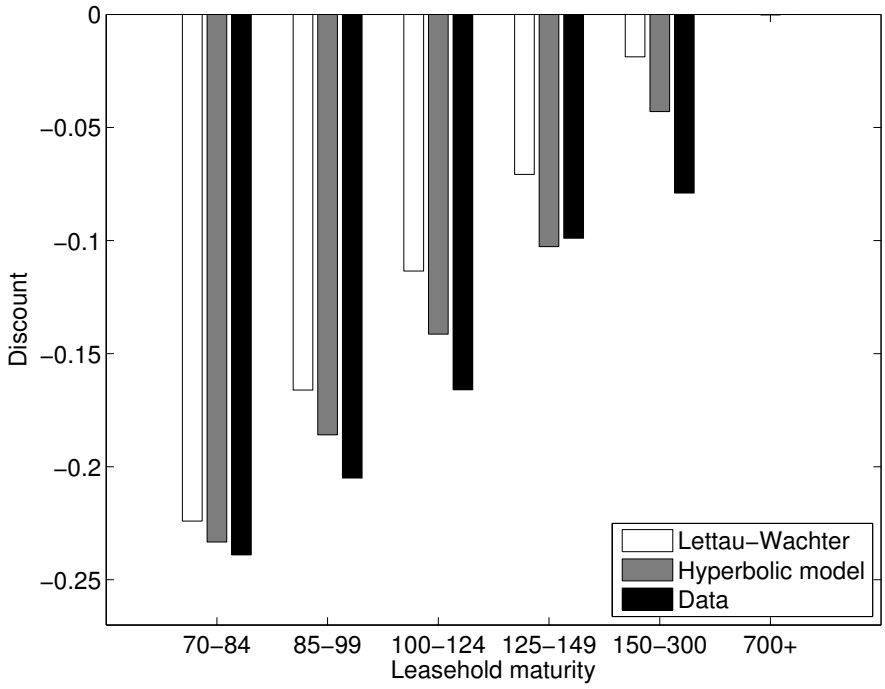
(a) U.K.



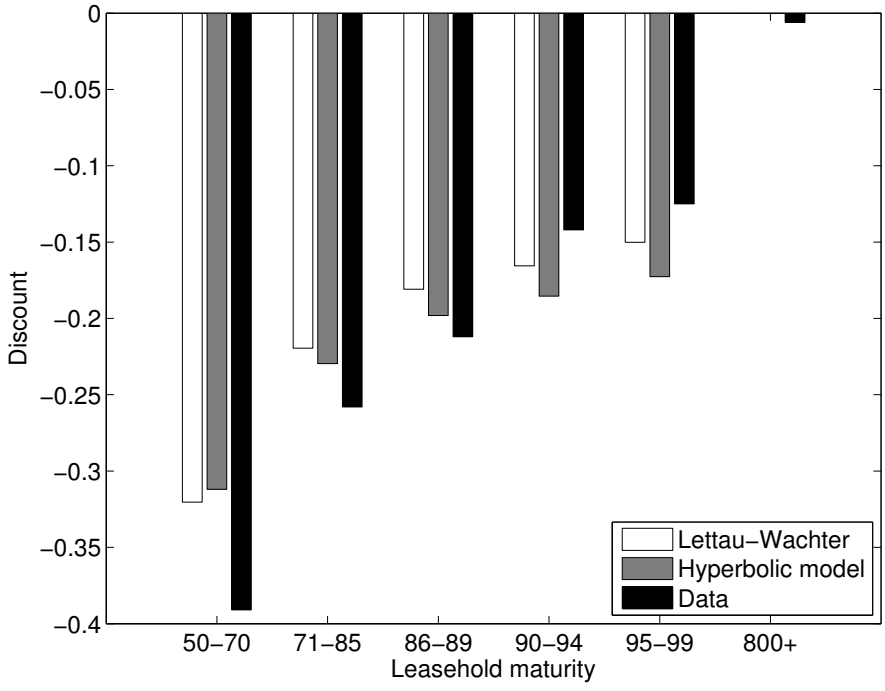
(b) Singapore

Note: The figure shows the discounts for leaseholds observed in the U.K. (top panel) and Singapore (bottom panel) together with discounts predicted by the long-run risk model, the variable rare-disaster model, and the habit-formation model. The calibrations impose that housing has expected return of 6.5% and growth rate of rents of 0.7%.

Figure 8: Lettau-Wachter and Hyperbolic Discounting Model Discounts vs. Data



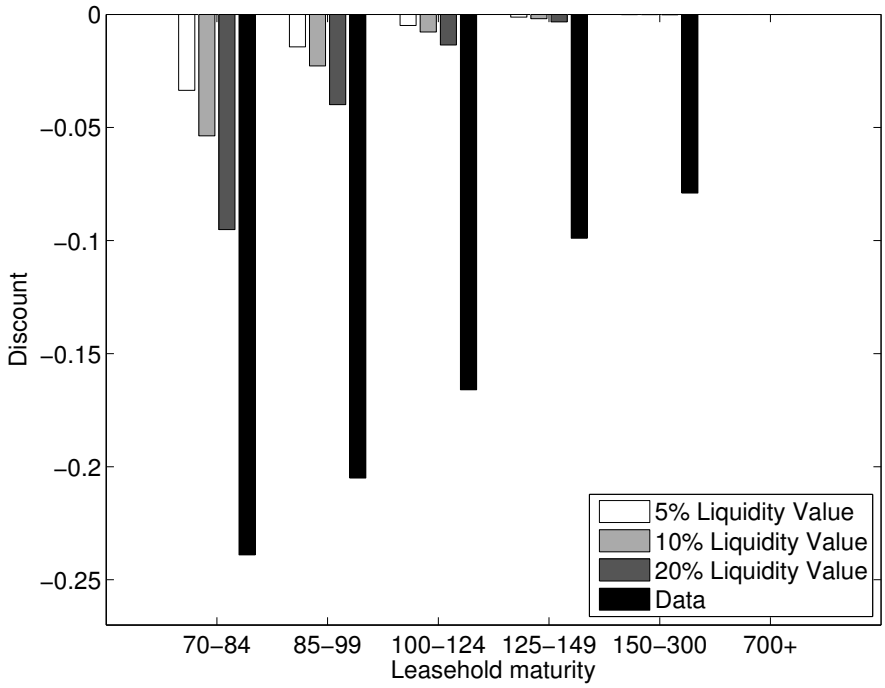
(a) U.K.



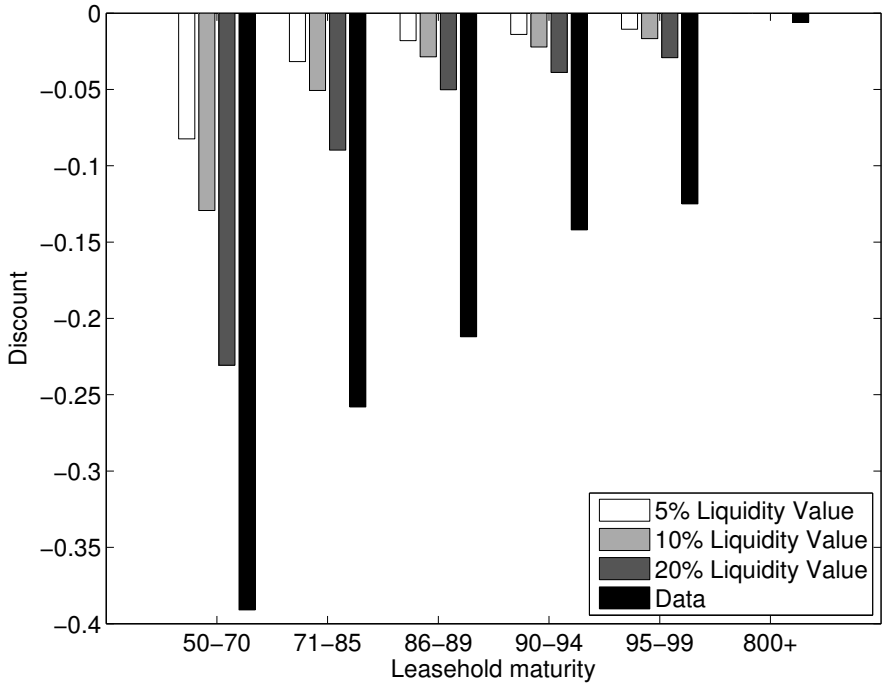
(b) Singapore

Note: The figure shows the discounts for leaseholds observed in U.K. (top panel) and Singapore (bottom panel) together with the discounts predicted by a parameterizations of the Lettau-Wachter and hyperbolic discounting models in section 3.6 using $r = 6.5\%$ and $g = 0.7\%$, $\kappa = 10\%$ and $\rho = 1.4\%$.

Figure 9: Liquidity Discounts vs. Data



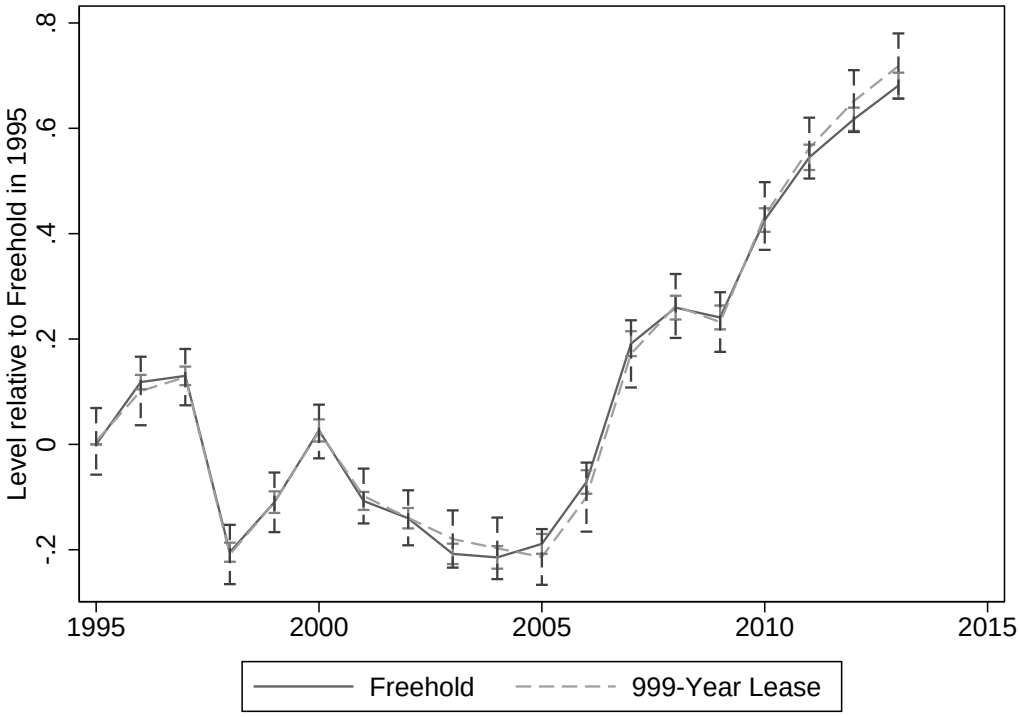
(a) U.K.



(b) Singapore

Note: The figure shows the discounts for leaseholds observed in the U.K. (top panel) and Singapore (bottom panel) together with discounts discounts predicted by a parameterizations of the liquidity discounting model in section 3.7 using $r = 6.5\%$ and $g = 0.7\%$.

Figure 10: Time Series of 999-Year Leases and Freeholds



Note: The figure shows the a time series of the price level of 999-Year leaseholds and freeholds in Singapore between 1995 and 2013. Estimates are obtained from a regression of log(price/sqft) on 5-digit postcode by property type by title type fixed effects, the same control variables as Table 3 and a separate dummy for each year by lease type (Freehold, 99-Year Lease, 999-Year Lease). All price levels are relative to freeholds in 1995. The bars indicate the 95% confidence interval of the estimate using standard errors clustered at the level of the fixed effect.

Tables

Table 1: Data Sample - UK

Year	N	Price ('000)	Beds	Baths	Total Rooms	Share Leaseholds
2009	192,949	212.2	2.97	1.48	6.09	11.2%
2010	200,644	233.4	2.98	1.49	6.14	11.6%
2011	189,958	229.0	2.98	1.58	6.10	11.2%
2012	185,847	238.0	2.99	1.56	6.10	11.3%
2013	85,204	237.1	2.97	1.50	6.05	10.7%

Note: This table shows the sample and key summary statistics by year for our U.K. transaction sample. Price is reported in thousands of Pound Sterling.

Table 2: Impact of Lease Type on Price - United Kingdom

	log(Price)			log(Price / Room)		
	(1)	(2)	(3)	(4)	(5)	(6)
Lease Length Remaining						
70-84 Years	-0.255***	-0.258***	-0.243***	-0.249***	-0.249***	-0.239***
	(0.006)	(0.007)	(0.008)	(0.007)	(0.008)	(0.010)
85-99 Years	-0.216***	-0.214***	-0.205***	-0.214***	-0.212***	-0.205***
	(0.008)	(0.008)	(0.009)	(0.008)	(0.008)	(0.010)
100-124 Years	-0.151***	-0.159***	-0.158***	-0.157***	-0.164***	-0.166***
	(0.007)	(0.006)	(0.007)	(0.008)	(0.007)	(0.008)
125-149 Years	-0.093***	-0.098***	-0.092***	-0.101***	-0.106***	-0.099***
	(0.009)	(0.008)	(0.010)	(0.010)	(0.010)	(0.011)
150-299 Years	-0.059***	-0.061***	-0.064***	-0.076***	-0.078***	-0.079***
	(0.009)	(0.008)	(0.009)	(0.010)	(0.009)	(0.010)
> 700 Years	-0.011**	-0.009**	-0.001	-0.009*	-0.007	0.001
	(0.005)	(0.004)	(0.005)	(0.005)	(0.005)	(0.005)
Fixed Effects	District × Prop Type × Quarter	District × Prop Type × Month	District × Prop Type × Month × Beds	District × Prop Type × Quarter	District × Prop Type × Month	District × Prop Type × Month × Beds
Controls	✓	✓	✓	✓	✓	✓
R-squared	0.746	0.743	0.751	0.651	0.646	0.657
N	851,483	851,483	851,483	764,524	764,524	764,524

Note: This table shows results from regression (1). To convert into percentage discounts for leasehold properties, construct $e^{\beta_j} - 1$. The dependent variables are the log price (columns 1-3) and the log price per room (columns 4-6) for properties sold in England and Wales between 2009 and 2013. We include postal district by property type by transaction time fixed effects. In columns (1) and (4) the transaction time is the transaction quarter, in the other columns the transaction month. In columns (3) and (6) we also interact the fixed effects with the number of beds in the property. We also control for the number of bedrooms, bathrooms and the total number of rooms, as well as whether the property is a new construction. Standard errors are clustered at the level of the fixed effect. Significance Levels: * (p<0.10), ** (p<0.05), *** (p<0.01).

Year	N	Price ('000)	Size (sqft)	Age	99-Year Lease	999-Year Lease
1995	12,412	1,149	1,758	3.42	34%	9%
1996	18,434	1,269	1,676	2.44	37%	14%
1997	12,534	1,179	1,709	2.49	53%	7%
1998	13,095	806	1,689	2.14	64%	5%
1999	23,500	991	1,827	2.92	42%	8%
2000	12,615	1,188	1,925	4.05	43%	8%
2001	11,577	883	1,732	3.35	57%	4%
2002	17,618	853	1,631	2.83	52%	6%
2003	9,807	826	1,656	4.23	50%	6%
2004	11,231	894	1,701	4.64	42%	6%
2005	16,771	1,037	1,848	5.09	37%	6%
2006	24,261	1,276	1,845	5.44	35%	6%
2007	39,203	1,625	1,719	5.14	40%	8%
2008	13,919	1,357	1,598	5.67	45%	7%
2009	32,967	1,362	1,550	4.87	43%	8%
2010	34,481	1,586	1,490	5.58	48%	6%
2011	25,236	1,475	1,341	4.54	58%	4%
2012	36,652	1,453	1,268	4.27	63%	4%
2013	15,215	1,539	1,248	3.57	69%	4%

Note: This table shows the sample and key summary statistics by year for our Singapore transaction sample. Price is reported in thousands of Singapore Dollars.

Table 3: Impact of Lease Type on Price per Square Foot - Singapore

	(1)	(2)	(3)	(4)	(5)
Lease Length Remaining					
50-70 Years	-0.392***	-0.384***	-0.391***	-0.446***	-0.441***
	(0.024)	(0.028)	(0.037)	(0.041)	(0.042)
70-85 Years	-0.259***	-0.266***	-0.258***	-0.468***	-0.265***
	(0.011)	(0.014)	(0.017)	(0.034)	(0.022)
85-89 Years	-0.196***	-0.213***	-0.212***	-0.100***	-0.219***
	(0.011)	(0.014)	(0.016)	(0.032)	(0.021)
90-94 Years	-0.143***	-0.146***	-0.142***	-0.172***	-0.146***
	(0.012)	(0.014)	(0.015)	(0.028)	(0.021)
95-99 Years	-0.136***	-0.122***	-0.125***	-0.129***	-0.128***
	(0.013)	(0.014)	(0.016)	(0.028)	(0.020)
> 800 Years	0.002	-0.008	-0.006	0.023	-0.000
	(0.011)	(0.013)	(0.014)	(0.032)	(0.019)
Fixed Effects	5-digit PC × Title Type × Quarter	5-digit PC × Prop Type × Title Type × Quarter	5-digit PC × Prop Type × Title Type × Month	5-digit PC × Prop Type × Title Type × Month	5-digit PC × Prop Type × Title Type × Month
Controls	✓	✓	✓	✓	✓
Restrictions	.	.	.	New Only	Private Buyer
R-squared	0.967	0.968	0.972	0.976	0.969
N	378,768	378,768	378,768	223,810	220,044

Note: This table shows results from regression (3). To convert into percentage discounts for leasehold properties, construct $e^{\beta_j} - 1$. The dependent variable is the price per square foot paid for properties sold by private parties in Singapore between 1995 and 2013. We include fixed effect at the 5-digit postcode by property type (apartment, condominium, detached house, executive condominium, semi-detached house and terrace house) by title type (Strata or Land) by transaction date. In columns (1) and (2), the transaction date interaction is for the transaction quarter, in column (3) - (5) the transaction month. We control for the age of the property (by including a dummy variable for every possible age in years), the size of the property (by including a dummy for each of 40 equally sized groups caputring property size), and the total number of units in the property. In column (4) we only focus on properties that were built within the last 3 years of our transaction date; in column (5) we only focus on properties that were bought by a private individual (and not the HDB). Standard errors are clustered at the level of the fixed effect. Significance Levels: * (p<0.10), ** (p<0.05), *** (p<0.01).

Table 4: Characteristics of Buyers of Leaseholds and Freeholds

	Sample		Leasehold Δ		
	Mean	St. Dev.	Unconditional	Conditional I	Conditional II
	(1)	(2)	(3)	(4)	(5)
Age Head of Household (years)	52.30	16.01	-2.68	-1.54	-1.30
				(0.21)	(0.20)
Weekly Income (£)	350.2	450.6	-48.07	-3.01	5.60
				(4.56)	(4.45)
Number of people in household	2.53	1.27	-0.48	-0.03	0.02
				(0.01)	(0.01)
Number of dependent children	0.55	0.94	-0.19	-0.01	0.02
				(0.01)	(0.01)
Head of Household Married	0.64	0.48	-0.21	-0.01	0.01
				(0.01)	(0.01)
First Time Buyer	0.40	0.48	0.11	-0.00	-0.01
				(0.01)	(0.01)
Currently Has Mortgage	0.59	0.49	0.03	0.02	0.02
				(0.01)	(0.01)
Very Satisfied with Neighborhood	0.47	0.50	-0.06	0.00	0.00
				(0.00)	0.00

Note: This table provides summary statistics on characteristics of owners of freeholds and leaseholds in the Survey of English Housing. The underlying contains information on 201,933 households in England surveyed between 1994 and 2008. The first two columns provide the sample means of the outcome variables. Column (3) provides the unconditional average difference between leasehold owners and freehold owners (e.g., on average, heads of household owning leasehold properties are 2.7 years younger than heads of households owning freehold properties). Columns (4) and (5) show the β coefficient of the following regression: $Outcome_i = \alpha + \beta Leasehold_i + \xi X_i + \phi_{PropertyType \times Region} + \varepsilon_i$. Column (4) does not include any additional controls in X_i, column (5) includes dummy variables for property age, the number of rooms and the floor on which the property is on (the control variables that overlap with the transaction dataset). In other words, these columns show the difference between freehold and leasehold owning households, conditional on living on the same property type (flat, semi-detached house, etc.) and living in the same local authority. Standard errors are clustered at the level of the fixed effect.

Table 5: Expected Returns and Rental Growth

	United States		Singapore		United Kingdom	
	Balance Sheet	Price/Rent	Balance Sheet	Price/Rent	Balance Sheet	Price/Rent
Gross Return	10.3%	10.7%	10.4%	10.3%	12.5%	10.9%
Rental Yield	8.3%	9.8%	6.1%	6.0%	9.7%	6.9%
Capital Gain	2.0%	0.8%	4.3%	4.3%	2.8%	4%
Depreciation	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%
Taxes	0.67%	0.67%	0.5%	0.5%	0%	0%
Net Return	**8.1%**	**8.5%**	**8.4%**	**8.3%**	**11%**	**9.4%**
Sample	1953-2012	1988-2012	1985-2012	1990-2012	1989-2012	1996-2012
Real Rent Growth		0.1%		0.2%		0.7%
Sample		1988-2012		1990-2012		1996-2012

Note: This table shows our estimates for net return to housing and real rent growth in the U.S., the U.K. and in Singapore. See appendix A.2 for details.

A.1 Institutional Appendix

A.1.1 United Kingdom

McMichael (1921) argues that the history of leasehold property ownership in England has its roots in feudalism, a system of land use and ownership that was common in Europe between the tenth and thirteenth centuries. Land was owned and controlled by a military or political sovereign ruler, who gave portions of land he or she owned to a number of lords as "tenants-in-chief." The lord, in turn, could allow another person, a vassal, to use smaller portions of the land in return for pledging allegiance and military or other service to the lord. See Cheshire and Burn (2006) for a detailed review of real property law.[37]

Over time, a number of laws have regulated the rights of leaseholders to extend their lease terms. There are three key Acts of Parliament that regulate this process. The 1967 Leasehold Reform Act en ables tenants of houses (not flats) held on long leases to acquire either the freehold (a process called "enfranchisement") or an extended lease term. The 1993 Leasehold Reform, Housing and Urban Development Act conferred rights to collective enfranchisement and lease extension on groups of flat owners in the same building who have been in occupation for a number of years. The Commonhold and Leasehold Reform Act of 2002 extended the right to lease extensions to individuals who have owned (but not necessarily occupied) flats for at least two years.

The above laws codify the bargaining process for a lease extension in the following way. First, the leaseholder files a proposal for extension, with an offered premium. The freeholder reverts with a counteroffer, and the two parties can then bargain on the final price of the extension. If the two parties cannot agree on a price, the leaseholder can ask a special tribunal, the Leasehold Valuation Tribunal, to assess the "fair value" of the extension. The "fair value" that the law refers to is intended to be the market value: the amount that the property "might be expected to realize if sold on the open market by a willing seller to a willing buyer". Essentially, the law guarantees that the leaseholder is able to remain in the property for a longer term if she is willing to pay the market value for the extension. This removes some of the bargaining frictions that can be associated with the cost of moving and that result in a potential hold-up problem by the freeholder. Using an analogy to the bond market, this is equivalent to saying that the short-term investor can roll over her investment at what will be the prevailing interest rate without paying major transaction costs when doing so. Therefore, the reduction of this friction suggests that the price paid for a leasehold will more closely reflect the value of the rental income that accrues over the term of the leasehold.

[37] McMichael (1921) describes the historical debate regarding the origins of common lease length terms of 99, 125 and 999 years: *Matthew Bacon, author of "A Treatise on Leases and Terms for Years" published in London, England, in 1798, explains in various parts of his book that the ninety-nine year period represents three lives, but Bacon does not indicate why such a term was selected as the length of time a lease was to prevail. It is supposed by some that there was an English common law which prevented a lessor from granting a lease for 100 years and that it was therefore made for a somewhat briefer period, but no real evidence has ever been found to substantiate this theory.* 1000-year leases were also common, with Jack Cade in Shakespeare's *Henry IV, Part II* exclaiming that "Now I am so hungry, that if I might have a lease of my life for a thousand years, I could stay no longer." McMichael (1921) also discusses theories of moving from 1000 year to 999 year leases: *Lord Coke, who lived in the reign of Queen Elizabeth, in his writings on the subject of leases suggested that a lease for 1,000 years might on its face suggest fraud and it is thought that to avoid such a contingency the lessors of those early days set upon 999 years as the extreme limit for the life of a lease. Such leases, in any event, were made at that time.*

Her Majesty Revenue and Customs (HMRC), the tax authority for England and Wales, gives equal treatment to the price paid for any term of leasehold or for a freehold when levying Stamp Duty Land Tax (SDLT) on residential property transactions.[38] The HMRC does not levy property taxes on actual ownership, it only taxes transactions (changes in ownership).

In the U.K. there are two other possible institutional features that might reduce the value of leaseholds relative to freeholds: ground rents and service charges. However, both of those are far too small in magnitude to explain the estimated difference. In fact, since ground rents and management fees are present for leaseholds of all maturities, the fact that 800+ year leaseholds trade at the same price as otherwise identical freeholds shows that they cannot contribute significantly to leaseholds.

A lessee generally has to pay annual ground rent to the freeholder. The original rationale for the ground rent was that the purchase price of the lease only covered the temporary ownership of the structure, but not the land the property sits on. The land still belongs to the freeholder who has the right to request that the lessee makes regular payments for the use of the land, the ground rent.

Ground rent payments are generally very small (50-100 pounds per year) for a typical property and in many cases are either zero or a symbolic amount ("a peppercorn"). In fact, all leases extended under the Leasehold Reform Act of 1993 are set as such peppercorn levels. Even in cases where the ground rent is in principle positive, it is often zero in practice, because for the rent to be collected the freeholder has to make a specific written request to the lessee. Oftentimes such requests are not made because the amount collected would be too small to cover the administrative costs. Ground rents are customized on a property by property basis and no centralized database exists. This makes it hard to control for them in the regression analysis. We stress, however, that the amounts involved for almost all properties are so small as not to constitute a problem for our analysis.

Similarly, for leasehold apartments the lessee sometimes has to pay a service charge to a Management Agency appointed by the freeholder. In apartment buildings sold as leaseholds, the freeholder still manages the common areas of the building and appoints a Management Agency to do so. The service charge is the amount that the lessees pay every year (or as a one-off if major works are carried out) to the freehold's Management Agency to cover the cost of the maintenance of common areas. The quota that each lessee pays depends on his share of the building.

While maintenance costs can be a non-trivial amount, as long as the maintenance is carried out at fair value (the private market cost of the works) service charges are not a problem for our analysis. While of course we cannot rule out that some freeholders attempt to extract monopoly rents via the service charge, there are strong mitigating factors that alleviate this concern. First, in many cases it is actually efficient to have the freeholder manage the property because she will in general own the freehold of many properties (e.g. a landed estate) and can enjoy the resulting economies of scale in the management of the properties. Second, the lessees can ask for the right to manage (RTM) the property and appoint their own management agency.

[38] The first £125,000 are exempt from stamp duty, with rates rising progressively thereafter. For more details see: http://www.hmrc.gov.uk/sdlt/calculate/leasehold.htm.

A.1.2 Singapore

Residential properties in Singapore can be classified into land titles or strata titles. Land title properties occupy land that is exclusive to the owner (like a detached house), whereas a strata title comprises units in cluster housing (flats or apartments) or in condominium developments. Owners of strata properties enjoy exclusive title only to the airspace of their individual unit. The land that the development is built on is shared by all the owners of the project, based on the share of the strata title unit owned by each owner. Owners are free to sell their individual unit. In order to sell the land, they will have to go via an "en bloc" sale, which requires a minimum of 80% of the owners' consent.[39]

A large fraction of the Singaporean housing stock consists of Housing and Development Board (HDB) properties, mostly in the form of flats. The HDB flats are part of a state-subsidized home-ownership program and leases are often granted at below market values. We exclude these properties from our analysis and focus instead on the private market.

Finally, property taxes are independent of the form and duration of ownership. Property taxes are levied on the *Annual Value* (AV), the tax-authority assessed 1-year rental income of the property. For rental properties, the tax rate is set at 10% of AV; for owner-occupied properties, it rises from 0% on the first $6,000 to a marginal rate of 6% for AVs exceeding $65,000.[40] The rental income, and therefore the Annual Value, of a property is unaffected by the length of the lease under which the property is owned. Property transactions are also subject to stamp duty irrespective of the form and duration of ownership.[41]

[39] 80% consent is necessary if the development is at least 10 years old and 90% consent is necessary if the development is less than 10 years old.

[40] Starting from January 1, 2014, property taxes were made more progressive. For details see: http://www.iras.gov.sg/irasHome/page04.aspx?id=2094. This is after the end of our sample, and should thus

[41] Stamp duties are transaction taxes, and are assessed on the purchase value of the property. The first $180,000 are assessed at 1%, the next $180,000 at 2% and each additional increase in the sales prices at 3%. See http://www.iras.gov.sg/irasHome/page04.aspx?id=8748fordetails.

A.2 Data and Empirical Appendix

A.2.1 Real Housing Returns

In this section we estimate r and g for the US, the UK and Singapore. We briefly describe our methodology and findings, and provide the details of the data and estimation procedure in section A.2.4 below . We employ two complementary approaches to estimating average returns to housing. The first approach, which we call the balance-sheet approach, is based on the total value of the residential housing stock and the total value of housing services consumed (the dividend from that stock). We obtain this information from countries' national accounts.[42] We control for the growth of the housing stock over time in order to back out the return series for a representative house. The second approach, which we label the price-rent approach, starts from the price-rent ratio estimated in a baseline year and constructs a time series of returns by combining a house price index and a rental income index. This approach focuses on a representative portfolio of houses and, therefore, does not need to correct for changes in the housing stock. After adjusting for inflation, both methods provide estimates of the gross real returns to housing ($E[R^G]$). To compute net returns, we subtract maintenance costs and depreciation (δ) and any tax-related decreases in return (τ). We estimate net returns as $r = E[R] = E[R^G] - \delta - \tau$.

The top panel of Table 5 presents the estimated average housing returns for the US, England-and-Wales, and Singapore. Our estimates for housing returns in the US follow Favilukis, Ludvigson and Van Nieuwerburgh (2010).[43] While U.S. housing returns are not the focus of this paper, they provide a useful benchmark because they have been the subject of an extensive literature Gyourko and Keim (1992); Flavin and Yamashita (2002); Lustig and Van Nieuwerburgh (2005); Piazzesi, Schneider and Tuzel (2007). The balance-sheet and the price-rent approaches provide similar estimates for the average annual real gross return ($E[R^G]$): 10.3% and 10.7% respectively. We calibrate the impact of maintenance and depreciation (δ) at 1.5% and the property tax impact τ at 0.67%.[44] We conclude that average real net returns in the U.S. housing market are between 8% and 8.5%. This is similar to the estimates in Flavin and Yamashita (2002), who find a real return to housing of 6.6%, and Favilukis, Ludvigson and Van Nieuwerburgh (2010), who find a real return of 9-10% before netting out depreciation and property taxes.

Column three and four in Table 5 report our estimates for the Singaporean housing market. The balance-sheet and price-rent approaches provide similar estimates for the average annual real gross

[42] To determine the total consumption of housing services, these measures impute the value of the owner-occupied equivalent rents, the housing services consumed by individuals from living in their own house. See Mayerhauser and Reinsdorf (2006) and McCarthy and Peach (2010) for a description of the construction of these measures.

[43] We thank Stijn van Nieuwerburgh for sharing the data and for insightful discussions on estimating housing returns.

[44] Malpezzi, Ozanne and Thibodeau (1987) provide an overview of the literature on depreciation. For example, (Leigh, 1980) estimates the annual depreciation rate of housing units in the U.S. to be between 0.36% and 1.36%. Depreciation is also a key calibration parameter for much of a recent literature in macroeconomics that considers households' portfolio and consumption decisions with housing as an additional asset. Cocco (2005) chooses a depreciation rate equal to 1% on an annual basis; Díaz and Luengo-Prado (2008) include an annual depreciation rate of 1.5%. Property taxes in the U.S. are levied at the state level and, while there is variation across states, are generally around 1% of house prices. Property taxes, however, are deductible from federal income tax. We assume that the deductibility reflects a marginal U.S. federal income tax rate of 33%. The net impact is therefore $(1 - 0.33) * 0.01 = 0.67\%$.

return ($E[R^G]$): 10.3% and 10.4%, respectively. We assume the cost of maintenance and depreciation to be 1.5%, in line with the estimates for the U.S., and the property tax impact to be 0.5%.[45] A conservatively low estimate of the real net returns in the Singapore housing market is therefore between 8.3% and 8.4%.

The two rightmost columns of Table 5 report the estimates for the housing market in England and Wales. The balance-sheet and the price-rent approaches provide similar estimates for the average annual real gross return ($E[R^G]$): 12.5% and 10.9%, respectively. We maintain the calibration for the cost of maintenance and depreciation at 1.5%. There are no property taxes to be considered in England and Wales. Average real net returns in the U.K. housing market are approximately $9-11\%$.

Overall, the estimates show that real expected returns for housing are between 8% and 10% for all countries in our international panel. These estimates are in line with the existing literature, and robust to the different methodologies.[46] Our estimates for the U.S. and England-and-Wales are consistent with the notion (see Shiller (2006)) that average house price growth over extended periods of time is relatively low and the key driver of real housing returns is the high rental yield. Our estimated average capital gains are positive but relatively small (even for Singapore where they are the highest) despite focusing on samples and countries that are often regarded as having experienced major growth in house prices.

A.2.2 Real Rental Growth

In order to calibrate the parameter governing rent growth (g), we estimate the average growth rate of rental income, obtained directly from rental indexes. The national accounts and the rental index provide similar growth rate estimates on the sample where both are available.

The estimated real growth rate of rents is close to zero. For the U.S., our estimate (0.2%) is in line with the estimates of Campbell et al. (2009) that obtain a median growth rate of 0.4% per year. We obtain an identically low estimate (0.2%) of average annual rental growth for Singapore, while the U.K. estimate is somewhat higher at 0.7%. As for the case of real average house price growth, our estimates of small-to-negligible real rent growth are in line with Shiller (2006). In our baseline estimates, we calibrate g to be 0.2%.

A.2.3 The riskiness of housing returns

While for the purpose of our calibration we are directly interested in the expected return r and the growth rate of rents g, it is interesting to study how risky housing assets are. How are housing

[45] Singapore levies a 10% annual tax on the estimated rental income of the property. A lower tax rate applies to owner-occupied properties (6%), but we use the more conservative (higher) rate for rental properties. See section 1.2 for details. The tax impact on returns is the tax rate times the average rent-price ratio, estimated at 5%. Hence, $\tau = 0.1 * 0.05 = 0.5\%$.

[46] We also note that since most movements in rent-price ratios are driven by movements in house prices and not by movements in rents Shiller (2007), our estimates of returns are relatively unaffected by the time period chosen. For example, since 2013 rent-price ratios in the U.S. have declined to approximately their 2000 levels (see Figure A.2), ending the sample in 2005 would have produced a slightly lower average rent-price ratio. However, focusing on that period would also exclude the house price crash from our estimates of capital gains, thus leading to higher estimated average capital gains. In the overall estimates of expected returns, the higher estimated capital gains would be offset by a lower estimated rent-price ratio.

returns correlated with the marginal utility of consumption? Figure A.3 plots the growth rates of rents and personal consumption expenditures (PCE) in the U.S. since 1929. In periods of falling PCE, in particular the Great Depression, rents also fell noticeably. The bottom panel shows that there is a (weak) positive relationship between the growth rates of rents and personal consumption expenditures. This suggests that housing rents tend to increase when consumption increases and the marginal utility of consumption is low. Table A.2 uses data from Mack and Martínez-García (2011) to report the correlation between annual real house price growth and personal disposable income in a panel of 21 developed and emerging countries. The average correlation is 37%, with a minimum of 5.4% for Luxembourg and a maximum of 63.1% for Spain. Overall, this evidence suggests that housing returns are risky.

A.2.4 Details on Estimation Procedures

This section describes the methodology and data used to compute average real returns and rent growth for residential properties. We report the details of the calculations in an online appendix.

The balance-sheet approach Following Favilukis, Ludvigson and Van Nieuwerburgh (2010), this approach uses information about the value of the stock of residential real estate to estimate the value (price) of housing and total household expenditure on housing as a measure of the value of rents in each period. Since we are only interested in the return to a representative property, we need to control for changes in the total housing stock. We proxy for the change in the stock by population growth, assuming that at least over long periods the per capita stock of housing is constant. We derive the gross return to housing in each period as:

$$R^G_{t+1} = \frac{V^H_{t+1} + CE^H_t}{V^H_t} \frac{\pi_t}{\pi_{t+1}} \frac{L_t}{L_{t+1}},$$

where V^H is the value of the housing stock, CE^H is the household expenditure on housing, π is the CPI price level index, and L is population.

- For the U.S. we follow Favilukis, Ludvigson and Van Nieuwerburgh (2010) and use data from the Flow of Funds (obtained from the Federal Reserve Board and the Federal Reserve Bank of St. Louis). For the value of the housing stock we sum the value of two series: owner-occupied real estate and tenant-occupied real estate (FL155035005, FL115035023) from the Flow of Funds. From the Federal Reserve Bank of St. Louis we obtain: (i) household expenditure on housing in each period, series number DHUTRC1A027NBEA of the National Income and Product Accounts (personal consumption expenditures - services: housing and utilities); (ii) Population estimates (POP); and (iii) the Consumer Price Index (USACPIBLS).

- For the U.K., using the same procedure, we combine the value of the total stock of housing (series CGRI) and the total expenditures on housing (series ADIZ) from the National Accounts (available from the Office of National Statistics). From the same source, we obtain the CPI

(series D7BT). We adjust for the change in the stock of housing using the population growth series from ONS for England and Wales.

- We use a similar procedure for Singapore. From the National Accounts (from singstat.gov.sg), we obtain the value of the private residential stock of housing (series M013181.1.1.1 P017199) and the private consumption expenditure on housing and utilities (series M013131.1.4 P017135). We obtain the series for the population growth (that proxies for the change in the stock of housing wealth) from the World Bank (series SP.POP.GROW). Finally, we obtain the CPI series from the National Statistical Office (singstat.gov.sg).

The price-rent approach This approach constructs a time series of returns by combining information from a house price index, a rent index, and an estimate of the price-to-rent ratio in a baseline year. Without loss of generality suppose we have the rent-to-price ration at time $t = 0$. We can derive the time series of the rent-to-price ratio as:

$$\frac{P_t}{D_{t+1}} = \frac{P_t}{P_{t-1}} \frac{D_t}{D_{t+1}} \frac{P_{t-1}}{D_t}; \qquad \frac{P_0}{D_1} \quad given.$$

where P is the price index and D the rental index. Notice that, given a baseline year $\frac{P_0}{D_1}$, only information about the growth rates in prices and rents are necessary for the calculations.

We then compute real returns using the formula:

$$R^G_{t+1} = \left(\frac{D_{t+1}}{P_t} + \frac{P_{t+1}}{P_t} \right) \frac{\pi_t}{\pi_{t+1}}.$$

- For the U.S. we follow Favilukis, Ludvigson and Van Nieuwerburgh (2010) and use the Case-Shiller 10-city house price index (series SPCS10RSA from the Federal Reserve Bank of St. Louis), and compute rent growth using the BLS shelter index (the component of CPI related to shelter, item CUSR0000SAH1 from the Federal Reserve Bank of St. Louis). However, differently from Favilukis, Ludvigson and Van Nieuwerburgh (2010), we choose 2012 as a baseline year for the rent-price ratio, which is estimated at 0.1, because of the availability of high quality data for that year. We obtained two independent estimates for the rent-price ratio in the base year of 2012. The first estimate is the price-rent ration implied by the balance-sheet approach. The second estimate is a direct estimate obtained using data by the real estate portal Trulia. Figure A.1 shows the distribution of rent-price ratios across the 100 largest MSAs provided by Trulia.[47] Both independent estimates imply a rent-price ratio of 10% in 2012. Figure A.2 suggests that these rent-price ratios are close to their long-run average.

- For Singapore we obtain a time series of price and rental indices for the whole island from the Urban Redevelopment Authority (the official housing arm of the government: ura.gov.sg).

[47] We thank Jed Kolko and Trulia for providing these data. Trulia observes a large set of both for-sale and for-rent listings. The rent-price ratio is constructed using a MSA-level hedonic regression of log(price) on property attributes, zip code fixed effects, and a dummy for whether the unit is for sale or for rent. The rent-to-price ratio is constructed by inverting the exponent of the coefficient on this dummy variable.

To estimate the baseline rent-price ratio, we use data from for-sale and for-rent listings provided by iProperty.com, Asia's largest online property listing portal. We observe approximately 105,000 unique listings from the year 2012, about 46% of which are for-rent listings. To estimate the rent-price ratio we run the following regression which pools both types of listing, which follows a similar methodology as Figure A.1 in the construction of rent-price ratios for the U.S.:

$$\ln\left(ListingPrice\right)_{i,t} = \alpha + \beta_i ForRent_i + \gamma Controls_{i,t} + \epsilon_{i,t} \tag{A.1}$$

The dependent variable, *ListingPrice* is equal to the list-price in "for-sale" listings, and equal to the annual rent in "for-rent" listings. $ForRent_i$ is an indicator variable that is equal to one if the listing is a for-rent listing. The results are reported in Table A.1. In column (1) we control for postal code by quarter fixed effects. The estimate coefficient on β_i suggests a rent-price ratio of $e^{\beta_i} = 4.5\%$. In columns (2) - (4) we also control for other characteristics of the property, such as the property type, the number of bedrooms, bathrooms as well as the property type, size, age and the floor. In columns (3) and (4) we tighten fixed effects to the month by postal code level and the month by postal code by number of bedrooms level respectively. In all specifications the estimated rent-price ratio fro 2012 is 4.5%. Finally, note that if we instead used the rent-price ratio obtained from the Balance Sheet approach as a baseline estimate in 2012, we would obtain a higher total return (as the baseline in 2012 would be 6% rather than 4.5%). We choose 4.5% to be as conservative as possible.

- For England and Wales we use the house price index from the UK Land Registry to compute price appreciation and we use the CPI component "Actual rents for housing" (series D7CE) from the Office of National Statistics as a rental index. As a baseline we used the 6% rent-price ratio in 2012 obtained from the balance-sheet approach.

A.3 Theoretical Appendix

A.3.1 The Stochastic Discount Factor

Starting with the fundamental valuation equation $P_t^{D_T} = E_t[\xi_{t,t+T} D_T]$ and the definition of return $R_{t,t+T} = \frac{D_T}{P_t^{D_T}}$, we have:

$$1 = E_t[\xi_{t,t+T} R_{t,t+T}] = E_t[\xi_{t,t+T}] E_t[R_{t,t+T}] + Cov_t[\xi_{t,t+T}, R_{t,t+T}].$$

Re-arranging we obtain:

$$E_t[R_{t,t+T}] = E_t[\xi_{t,t+T}]^{-1} \left(1 - Cov_t[\xi_{t,t+T}, R_{t,t+T}]\right) = R^f_{t+t+T} - Cov_t[\xi_{t,t+T}, R_{t,t+T}] E_t[\xi_{t,t+T}]^{-1},$$

where the last equality follows from the definition $R^f_{t+t+T} \equiv E_t[\xi_{t,t+T}]^{-1}$. Finally, we re-arrange the definition of returns, take conditional expectations, and substitute in the above derivation for expected returns to write:

$$P_t^{D_T} = \frac{D_T}{R_{t,t+T}} = \frac{D_T}{R^f_{t+t+T} - Cov_t[\xi_{t,t+T}, R_{t,t+T}] E_t[\xi_{t,t+T}]^{-1}} = \frac{D_T}{R^f_{t+t+T} - \frac{Cov_t[\xi_{t,t+T}, R_{t,t+T}]}{Var_t[\xi_{t,t+T}]} \frac{Var_t[\xi_{t,t+T}]}{E_t[\xi_{t,t+T}]}},$$

which provides the expressions in the main body of the paper by defining:

$$\begin{aligned} RP_{t,t+T} &\equiv \beta_{t,t+T} \lambda_{t,t+T}; \\ \beta_{t,t+T} &\equiv -\frac{Cov_t[\xi_{t,t+T}, R_{t,t+T}]}{Var_t[\xi_{t,t+T}]}; \\ \lambda_{t,t+T} &\equiv \frac{Var_t[\xi_{t,t+T}]}{E_t[\xi_{t,t+T}]}. \end{aligned}$$

A.3.2 Details on Hyperbolic-Exponential Discounting

We include here details for the derivations in Section 3.6 of the paper. First, let us focus on a model of pure hyperbolic discounting. In continuous time, the hyperbolic discount function is simply $\frac{1}{1+\kappa s}$ where $\kappa > 0$ is the subjective hyperbolic parameter. To gather intuition, assume that rents were constant at D. Let us value the T lease contract. For simplicity consider the $t = 0$ starting condition.

$$P_t^T = \int_0^T \frac{1}{1+\kappa\, s} D ds = D \frac{\ln(1+\kappa\, T)}{\kappa}.$$

The obvious problem with this type of discounting when applied to longer term assets is that the valuation of claims diverges (even without dividend growth) as the horizon T increases ($T \to \infty$). A similar problem occurs for the gamma discounting of Weitzman (2001) that derives a similar functional form for its effective discount rate.

In the paper, therefore, we augmented the hyperbolic discount function to include an exponential term: $\frac{e^{-\rho s}}{1+\kappa s}$, where $\rho > 0$ is the subjective discount rate associated with exponential discounting. This

form of discounting tends to behave like hyperbolic discounting in the short run and like exponential discounting in the long run. The T-maturity leasehold is valued at:

$$P_0^T = \int_0^T \frac{e^{-(\rho-g)s}}{1+\kappa s} D_0 ds = D_0 \frac{e^{\frac{\rho-g}{\kappa}} \left(Ei\left(\frac{(T\kappa+1)(g-\rho)}{\kappa}\right) - Ei\left(\frac{g-\rho}{\kappa}\right)\right)}{\kappa},$$

where $Ei(x)$ is the Exponential Integral function defined as:

$$Ei(x) \equiv -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt.$$

The freehold is correspondingly valued at:

$$P_0 = D_0 \frac{e^{\frac{\rho-g}{\kappa}} \Gamma\left(0, \frac{\rho-g}{\kappa}\right)}{\kappa},$$

where $\Gamma(x)$ is the Upper Incomplete Gamma Function defined as:

$$\Gamma(0,x) \equiv \int_x^{\infty} \frac{e^{-t}}{t} dt.$$

The discount is now:

$$Disc_0^T = \frac{Ei\left(\frac{(T\kappa+1)(g-\rho)}{\kappa}\right) - Ei\left(\frac{g-\rho}{\kappa}\right)}{\Gamma\left(0, \frac{\rho-g}{\kappa}\right)} - 1.$$

The marginal discount rate discussed in the main body of the paper can be derived by defining the discount function as $f(t) = exp\left(-\int_o^t r(s)ds\right)$. Then an application of Leibniz's rule for differentiation under the integral sign yields: $\dot{f(t)} = -r(t)f(t)$, where $\dot{f(t)}$ is the time derivative of function f. Hence, the result in the paper that $r(t) = -\frac{\dot{f(t)}}{f(t)}$. Finally, applying this formula to the exponential-hyperbolic discount function, $f(t) = \frac{e^{-\rho t}}{1+\kappa t}$, one obtains the result in the paper that:

$$r(t) = -\frac{\dot{f(t)}}{f(t)} = \rho + \frac{\kappa}{1+\kappa t}.$$

A.3.3 Details on Financing Frictions

We assume that for the last $\bar{T}$ years of lease maturity the house has lower collateral value. We model this has an effective rent for the last $\bar{T}$ years that is a fraction $(1-\alpha)$ of the original rent. The value of

the lease now follows:

$$\begin{aligned} P_t^T &= \int_t^{t+T} e^{-\rho(s-t)} D_t e^{g(s-t)} (1 - \alpha \mathbf{1}_{\{s>t+T-\bar{T}\}}) ds = \\ &= \int_t^{t+T} e^{-\rho(s-t)} D_t e^{g(s-t)} ds - \alpha \int_{t+T-\bar{T}}^{t+T} e^{-\rho(s-t)} D_t e^{g(s-t)} ds + \\ &\quad + \mathbf{1}_{\{T<\bar{T}\}} \alpha \int_{t+T-\bar{T}}^{t} e^{-\rho(s-t)} D_t e^{g(s-t)} ds \\ &= \frac{D_t}{\rho - g} \left[1 - e^{-(\rho-g)T} - \alpha \left(e^{-(\rho-g)(T-\bar{T})} - e^{-(\rho-g)T} \right) + \mathbf{1}_{\{T<\bar{T}\}} \alpha \left(e^{-(\rho-g)(T-\bar{T})} - 1 \right) \right] \quad \text{(A.2)} \end{aligned}$$

Notice that the first multiplicative term in equation (A.2) is simply the valuation of the freehold under the Gordon-Growth formula $\left(\frac{D_t}{\rho-g}\right)$. The first term inside the squared bracket $\left(1 - e^{-(\rho-g)T}\right)$ is the Gordon-Growth price adjustment for the value of a T-maturity leasehold as shown in equation (5). The second term inside the squared bracket $\left(\alpha \left(e^{-(\rho-g)(T-\bar{T})} - e^{-(\rho-g)T} \right)\right)$ is the loss in value for the T-maturity leasehold due to the frictions. Notice that this term is zero whenever there are no frictions ($\alpha = 0$ and or $\bar{T} = 0$). The last term inside the squared bracket $\left(\mathbf{1}_{\{T<\bar{T}\}} \alpha \left(e^{-(\rho-g)(T-\bar{T})} - 1 \right)\right)$ captures the notion that if a leasehold has already less than $\bar{T}$ years left than it would be valued at:

$$P_t^T = \frac{D_t(1-\alpha)}{\rho - g}(1 - e^{-(\rho-g)T}),$$

so that the leasehold is then valued as if the rents where only a fraction$(1-\alpha)$ of the original ones. Notice that the value of the freehold is unaffected by the frictions because by definition it never loses its collateral value:

$$P_t = \lim_{T\to\infty} P_t^T = \frac{D_t}{\rho - g}.$$

The model implied discounts are now:

$$Disc_t^T = e^{-(\rho-g)T} + \alpha \left(e^{-(\rho-g)(T-\bar{T})} - e^{-(\rho-g)T} \right) - \mathbf{1}_{\{T<\bar{T}\}} \alpha \left(e^{-(\rho-g)(T-\bar{T})} - 1 \right).$$

Let us focus on the case in which $T > \bar{T}$, i.e. if we are valuing a leasehold with maturity beyond the problematic threshold. Notice the following effects:

1. $\frac{\partial Disc_t^T}{\partial \alpha} > 0$, the discount increases the greater the per-period collateral benefit.
2. $\frac{\partial Disc_t^T}{\partial \bar{T}} > 0$, the discount increases whenever the threshold for financing increases.
3. $\frac{\partial Disc_t^T}{\partial \alpha \partial T} < 0$ and $\lim_{T\to\infty} \frac{\partial Disc_t^T}{\partial \alpha} = 0$, the marginal effect of the loss in collateral value on the discount decreases with maturity of the lease and is zero in the limit of very long leases.

The last property is the most relevant for our robustness exercise. It states that no matter how high the frictions are ($\uparrow \alpha$), their effect decreases with the length of the lease. As we have shown in the main body of the paper, this effect makes the frictions quantitatively incapable of explaining the observed discounts, especially for long term leases (100 or 200 years for example).

Appendix References

Cheshire, Geoffrey Chevalier, and Edward Hector Burn. 2006. *Cheshire's Modern Law of Real Property.* Butterworth.

Cocco, Joao F. 2005. "Portfolio choice in the presence of housing." *Review of Financial studies*, 18(2): 535–567.

Díaz, Antonia, and María José Luengo-Prado. 2008. "On the user cost and homeownership." *Review of Economic Dynamics*, 11(3): 584–613.

Favilukis, Jack, Sydney C Ludvigson, and Stijn Van Nieuwerburgh. 2010. "The macroeconomic effects of housing wealth, housing finance, and limited risk-sharing in general equilibrium." National Bureau of Economic Research.

Flavin, Marjorie, and Takashi Yamashita. 2002. "Owner-occupied housing and the composition of the household portfolio." *The American Economic Review*, 92(1): 345–362.

Gyourko, Joseph, and Donald B Keim. 1992. "What does the stock market tell us about real estate returns?" *Real Estate Economics*, 20(3): 457–485.

Lustig, Hanno N, and Stijn G Van Nieuwerburgh. 2005. "Housing collateral, consumption insurance, and risk premia: An empirical perspective." *The Journal of Finance*, 60(3): 1167–1219.

Mack, Adrienne, and Enrique Martínez-García. 2011. "A cross-country quarterly database of real house prices: a methodological note." *Federal Reserve Bank of Dallas Globalization and Monetary Policy Institute Working Paper*, , (99).

Malpezzi, Stephen, Larry Ozanne, and Thomas G Thibodeau. 1987. "Microeconomic estimates of housing depreciation." *Land Economics*, 63(4): 372–385.

Mayerhauser, Nicole, and Marshall Reinsdorf. 2006. "Housing Services in the National Economic Accounts." *US Bureau of Economic Analysis*.

McCarthy, Jonathan, and Richard Peach. 2010. "The measurement of rent inflation." *FRB of New York Staff Report*, , (425).

McMichael, Stanley L. 1921. *Long Term Land Leaseholds: Including Ninety-nine Year Leases.* SL McMichael.

Piazzesi, Monika, Martin Schneider, and Selale Tuzel. 2007. "Housing, consumption and asset pricing." *Journal of Financial Economics*, 83(3): 531–569.

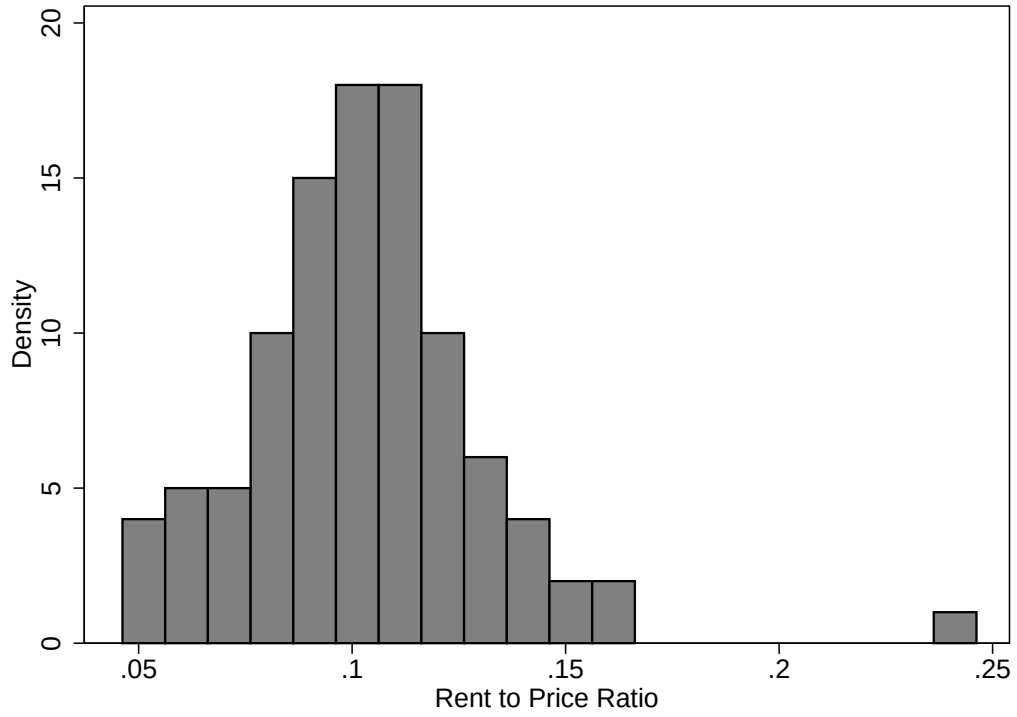
Shiller, Robert J. 2006. "Long-term perspectives on the current boom in home prices." *The Economists' Voice*, 3(4).

Shiller, Robert J. 2007. "Understanding recent trends in house prices and home ownership." National Bureau of Economic Research.

Weitzman, Martin L. 2001. "Gamma discounting." *American Economic Review*, 260–271.

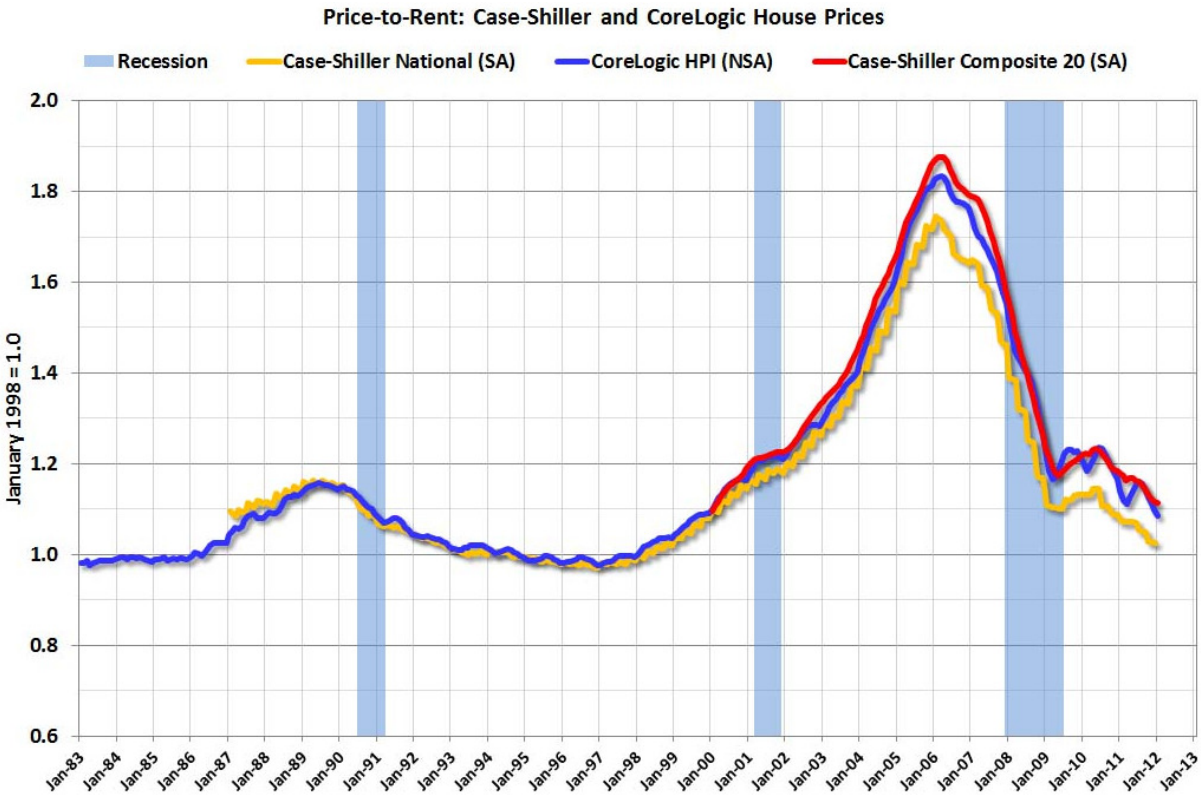
Appendix Figures

Figure A.1: Cross-Sectional Distribution of Price-Rent Ratio in the U.S.



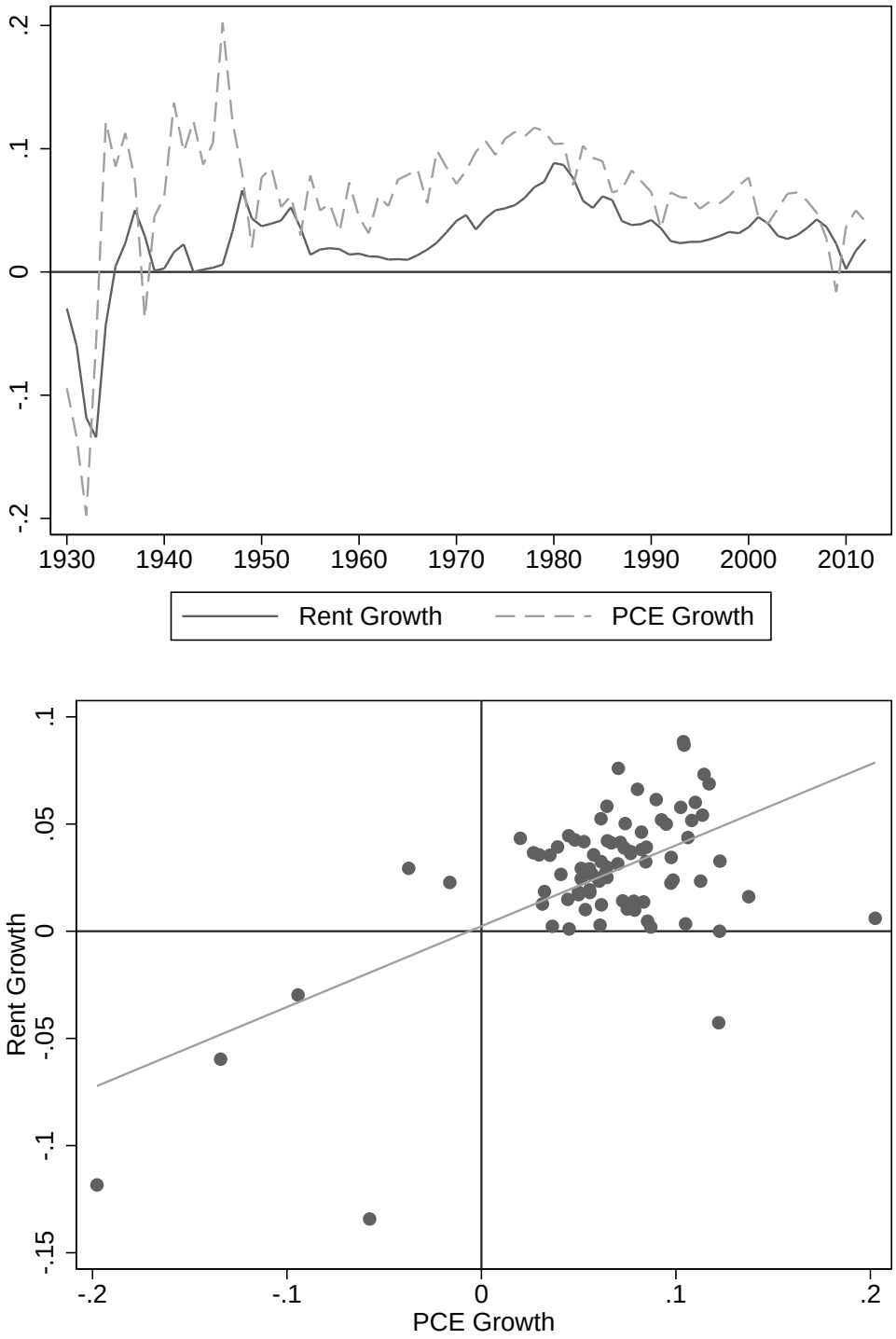
Note: The figure shows the distribution of the rent-to-price ratio for the 100 largest MSAs in the U.S. in September 2013 as constructed by Trulia, which observes a large set of both for-sale and for-rent listings. It is constructed using a metro-level hedonic regression of ln(price) on property attributes, zipcode fixed effects, and a dummy for whether the unit is for rent. The rent-to-price ratio is constructed by taking the exponent of the coefficient on this dummy variable.

Figure A.2: Price-Rent Ratio Timeseries in the U.S.



Note: The figure shows the time series of the price-rent ratio in the U.S. as constructed by http://www.calculatedriskblog.com/.

Figure A.3: Rent Growth vs. PCE Growth in U.S.



Note: The figure shows the annual growth rates of the "Consumer Price Index for All Urban Consumers: Rent of primary residence" (FRED ID: CUUR0000SEHA) and "Personal Consumption Expenditure" (FRED ID: PCDGA) since 1929.

Appendix Tables

Table A.1: Rent-Price Ratio Singapore - 2012

	(1)	(2)	(3)	(4)
For-Rent Dummy	-3.095***	-3.131***	-3.123***	-3.107***
	(0.044)	(0.019)	(0.014)	(0.025)
Fixed Effects	Quarter × Postal Code	Quarter × Postal Code	Month × Postal Code	Month × Postal Code × Bedrooms
Controls	·	✓	✓	✓
Implied Rent-Price Ratio	4.5%	4.4%	4.4%	4.5%
R-squared	0.804	0.873	0.872	0.872
N	106,145	105,189	105,189	105,189

Note: This table shows results from regression (A.1). To convert into rent-price ratios, we take e^{β}. The dependent variables is the price (for-sale price or annualized for-rent price) for properties listed on iProperty.com in Singapore in 2012. Fixed effects are included as indicated. In columns (2) and (4) we also control for characteristics of the property: we include dummy variables for the type of property (condo, house, etc.), indicators for the number of bedrooms and bathrooms, property age, property size (by adding dummy variables for 50 equal-sized buckets), information on the kitchen (cermamic, granite, etc.), which floor the property is on and the tenure type for leaseholds. Standard errors are clustered at the level of the fixed effect. Significance Levels: * (p<0.10), ** (p<0.05), *** (p<0.01).

Table A.2: Time Series Properties of Real House Price Growth

	Real HP Growth		Real PDI Growth		
	Mean	Std. Dev.	Mean	Std. Dev.	Correlation
Australia	3.20%	6.89%	1.43%	2.77%	0.093
Belgium	2.80%	5.87%	1.17%	2.27%	0.436
Canada	2.51%	7.63%	1.37%	2.10%	0.489
Switzerland	0.94%	4.73%	1.12%	1.63%	0.445
Germany	-0.29%	2.31%	1.27%	1.70%	0.288
Denmark	1.57%	8.99%	1.09%	2.29%	0.211
Spain	2.05%	8.26%	0.83%	2.46%	0.631
Finland	2.04%	8.19%	2.07%	3.21%	0.482
France	2.52%	5.23%	1.22%	1.58%	0.358
U.K.	3.53%	8.54%	2.20%	2.74%	0.355
Ireland	3.70%	9.73%	1.83%	3.59%	0.529
Italy	0.60%	8.28%	0.82%	2.44%	0.325
Japan	-0.24%	4.28%	1.55%	1.40%	0.587
S. Korea	0.59%	7.70%	3.95%	4.58%	0.235
Luxembourg	3.94%	6.68%	2.84%	3.75%	0.054
Netherlands	2.32%	9.43%	0.48%	3.25%	0.472
Norway	2.76%	7.23%	2.22%	2.52%	0.064
New Zealand	2.20%	7.73%	0.98%	3.45%	0.530
Sweden	1.50%	7.27%	1.34%	2.28%	0.431
U.S.	1.13%	3.89%	1.60%	1.56%	0.371
S. Africa	0.88%	9.65%	0.53%	3.05%	0.373

Note: This table shows time series properties of quarterly frequency annual growth rats of real house prices and personal disposable income, as collected by Mack and Martínez-García (2011). Columns (1) and (2) show the mean and standard deviation of real house price growth. Columns (3) and (4) the mean and standard deviation of real personal disposable income growth. Column (5) shows the correlation of real house price growth with real personal disposble income growth.