Learning about the Neighborbood: A Model of Housing Cycles*

Michael Sockin† Wei Xiong‡

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Abstract

This paper develops a model to analyze information aggregation and learning in housing markets. In the presence of informational frictions, households face a realistic problem in learning about the quality of a neighborhood. As housing prices serve as a key signal in guiding households' expectations and coordinating their housing demand, higher housing prices may induce greater housing demand when the learning effect is sufficiently strong. This learning effect, together with extrapolative behavior, provides sharp predictions about housing cycles. In contrast to conventional wisdom, neighborhoods with intermediate supply elasticities are most prone to housing cycles induced by extrapolative behavior as the learning effect is particularly strong in these neighborhoods.

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†Princeton University. Email: msockin@princeton.edu.
‡Princeton University and NBER. Email: wxiong@princeton.edu.

People buy a house not just for shelter but also for the neighborhood to which the house belongs. There are many characteristics that affect the living conditions in a neighborhood, such as public safety, quality of local schools, and availability of local amenities like restaurants and public parks. In wealthier neighborhoods, the greater demand for public safety, high quality schools, and nice restaurants tends to attract a greater supply of these amenities, which in turn makes housing in these neighborhoods more desirable. The economic literature has recognized the importance of neighborhood effects in determining housing demand (e.g., Ioannides and Zabel, 2003) and a host of other urban issues (e.g., Durlauf, 2004; and Glaeser, Sacerdote, and Scheinkman, 2010).

It is important to recognize that the quality of a neighborhood, which is ultimately driven by the financial health of its residents and the strength of the local economy, is often not directly observable to potential home buyers. Home buyers also face other unobservable factors in housing markets, such as local housing supply conditions. These important informational frictions render local housing markets a useful platform to aggregate information. This fundamental aspect of housing markets, however, has received little attention in the academic literature, despite the great concerns from both public and policy circles across the world over housing boom and bust cycles and that information aggregation and learning have clearly important implications for housing market dynamics. In the paper, we aim to fill this gap by developing a model to analyze information aggregation and learning in housing markets and its implications for housing cycles.

Our current work integrates the standard framework for information aggregation in asset markets of Grossman and Stiglitz (1980) and Hellwig (1980) with a housing market in a local neighborhood. This setting allows us to extend the insights of market microstructure analysis to explore the real consequences of informational frictions in the market for a commodity that has a dual capacity as a durable consumption good and as a financial asset. This latter capacity of housing has been appreciated in the literature in explaining the recent U.S. housing boom and bust cycle (e.g., Glaeser, 2013; and Gao, 2013). Our paper introduces a conceptual framework for understanding how agents form expectations in housing markets and how these expectations interact with the local characteristics endemic to a neighborhood. This allows us to deliver sharp predictions about what cross-sectional variation across neighborhoods is important for agents when forming their expectations and how these expectations feed into housing prices.

We first present a static setting to highlight the basic information aggregation mechanism with perfectly rational households, and then extend the model to a dynamic setting with extrapolative households to characterize the implications of learning about the neighborhood for housing cycles.

The model presented herein features a continuum of households in a closed neighborhood. Each household specializes in producing a consumption good, which can be broadly interpreted as local services such as healthcare, restaurant services, and plumbing, and needs to trade his good for goods produced by other households in the neighborhood for consumption. The households have a Cobb-Douglas utility function over consumption of their own good, goods produced by other households, and housing. In choosing their housing demand, the households face uncertainty regarding the productivities of other households, a key aspect of the quality of the neighborhood, which ultimately determines demand for each household's good. In more desirable neighborhoods, households are more productive and there is a greater demand for each household's good, which in turn stimulates each household to demand more housing consumption. In this way, the complementarity in the households' goods consumption leads to complementarity in their housing demands.

In our model, each household possesses a private signal regarding the aggregate productivity of the households in the neighborhood. Through housing demand, the housing price aggregates the households' private signals and partially reflects the quality of the neighborhood. The presence of housing supply shock, however, prevents the housing price from perfectly revealing the quality of the neighborhood. In this way, the housing supply shock acts as a source of noise in the housing price. Despite the non-linearity in each household's housing demand, the Law of Large Numbers allows us to aggregate the households' housing demands in closed form and eventually to derive a unique log-linear equilibrium for the housing market. In this equilibrium, the housing price is a log-linear function of the unobservable quality of the neighborhood and the housing supply shock, and each household's housing demand is a log-linear function of its private signal and the housing price.

Learning about the neighborhood plays an important role in determining each household's housing demand. A higher housing price does not simply represent a larger cost of housing but also gives a positive signal about the neighborhood. The latter learning effect motivates each household to demand more housing, and is especially strong under the joint conditions that the households face substantial uncertainty in the quality of the neigh-

borhood and the housing price is reasonably informative. Under certain conditions on the model's parameters, the learning effect can even dominate the cost effect and cause each household to demand more housing despite a higher price. Interestingly, this positive price elasticity of housing demand, while reminiscent of extrapolative behavior of home buyers previously emphasized in the housing literature,[1] is driven by rational learning of households from the housing price. Our model particularly highlights the effects of housing demand complementarity and supply elasticity on the households' learning. A greater complementarity in households' consumption increases the neighborhood effect in housing demand and thus raises the households' incentives to learn.

The housing literature has highlighted that housing supply has an important effect on housing cycles (e.g., Malpezzi and Wachter (2002), Glaeser, Gyourko, and Saiz (2008), and Saiz (2010)). The conventional wisdom is that supply elasticity monotonically mitigates housing cycles. In our model, supply elasticity serves an additional role because households try to learn about a neighborhood from house prices. As the supply elasticity determines the weight of housing supply shocks in the housing price, it directly affects the informativeness of the housing price regarding the quality of the neighborhood.

As a result of informational frictions, the households' learning effect is more pronounced in neighborhoods with intermediate supply elasticities than with the most elastic or inelastic elasticities. In neighborhoods with very elastic supply, the price is uninformative, while in neighborhoods with very inelastic supply, the price is so revealing that the households face little uncertainty about the quality of the neighborhood. Netting out these two forces leads to the strongest reaction of demand to housing prices in neighborhoods with intermediate supply elasticities. This insight is new to the literature and can help explain the puzzling observation by Gao (2013) that areas with relatively elastic supply like Phoenix and Las Vegas had nevertheless experienced a dramatic boom and bust cycle during 2000s. In particular, Gao systematically documents that across different counties in the U.S., the magnitude of the housing price cycle around 2006 has an inverted U shape with respect to supply elasticity.

To fully characterize the implications of the learning effect for housing cycles, we extend our model into a dynamic setting in which households exhibit extrapolative behavior. These households mistakenly believe that fundamental shocks to the quality of the neighborhood

[1] Extrapolative behavior has been recognized as important for understanding a host of phenomena in stock markets, e.g. De Long, et al. (1990), Barberis, Shleifer and Vishny (1998), Greenwood and Shleifer (2013), Barberis, et al. (2013), as well as in housing markets, e.g. Case and Shiller (2005), Glaeser, Gyourko, and Saiz (2008), Piazzesi and Schneider (2009).

are more persistent than they actually are. Such extrapolative behavior allows the model to generate housing price momentum and dramatic boom and bust cycles. By anchoring this extrapolative behavior on the households' learning about the neighborhood, our model provides a sharp prediction that the housing cycle will have an inverted U shape relationship with supply elasticity.

The U.S. housing cycle of the 2000's has stimulated a lot of academic and policy interest in various aspects of housing market dynamics. While a large strand of the literature, including Mian and Sufi (2009), Keys et al (2010), and Landvoight, Piazzesi, and Schneider (2013), has emphasized credit expansion to subprime households as a key driver of this housing cycle, Glaeser, Gottlieb, and Gyourko (2010) point out that credit expansion, while clearly important, cannot explain the bulk of the housing price changes during this period. Earlier work on housing by Case and Shiller (2003), Glaeser, Gyourko, and Saiz (2008), and Piazzesi and Schneider (2009) among others, has highlighted the importance of extrapolative behavior by home buyers in explaining housing cycles. Our model builds upon these studies by focusing on how households' expectations drive housing prices. By anchoring their extrapolative behavior on their learning about the neighborhood, we derive sharp implications about where and when extrapolative learning is likely to be most severe.

The mechanism for driving housing cycles in our model is also different from the slow information diffusion model of Hong and Stein (1999), which generates momentum and reversals in asset prices through a slow diffusion of information among a population of investors. In their model, investors ignore the information revealed through the prices of traded assets, while in our model, learning from the housing price plays a central role in driving the housing cycle. By this nature of learning, our model also differs from Burnside, Eichenbaum, and Rebelo (2013), which offers a model of housing market boom and bust based on epidemic spreading of optimistic or pessimistic beliefs among home buyers through their social interactions.

The paper is organized as follows. Section 1 presents a baseline model with two periods and perfectly rational households to highlight the key mechanism of information aggregation and learning from prices in housing markets. Section 2 extends the model to have more periods and allow for extrapolative learning to characterize housing cycles. Finally, we conclude in Section 3. The technical proofs of propositions are provided in the Appendix.

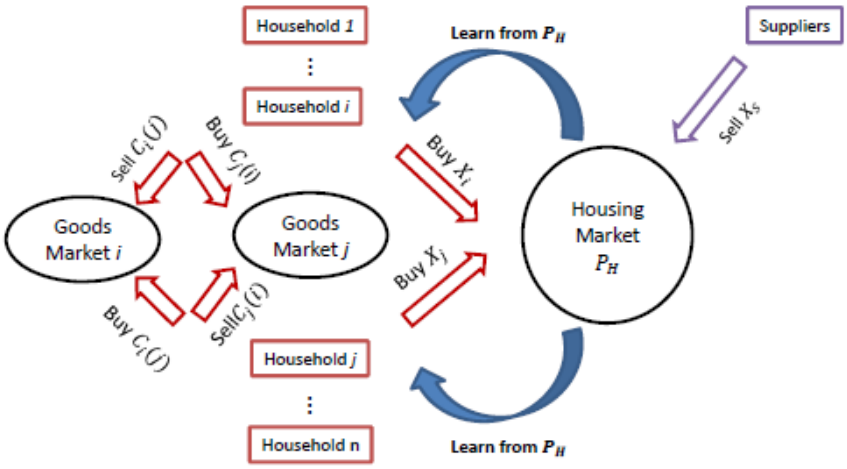


Figure 1: Structure of Baseline Model

1 The Baseline Model

In this section we develop a baseline model with two dates $t = 1, 2$ to analyze the effects of informational frictions on the housing market equilibrium in a closed neighborhood. A key feature of the baseline model is that the housing market is not only a place for households to trade housing but also a platform to aggregate private information about the unobservable quality of the neighborhood.

1.1 Model setting

Figure 1 illustrates the structure of the baseline model. There are two types of agents in the economy: households looking to buy housing in a neighborhood and home builders. Suppose that the neighborhood is new and all households need to purchase housing at the same time.[2] Each household cares about the quality of the neighborhood in which it lives, as it has to trade produced goods, such as healthcare, restaurants, luxury stores and schools, with other households in the same neighborhood. For simplicity, we assume that the neighborhood is closed and that each household is specialized in producing one good and can trade its good for other goods only with other households living in the same neighborhood. The quality of this closed neighborhood is thus reflected by the aggregate productivity of households in the neighborhood. A strong aggregate productivity implies a higher output for all households,

[2]For simplicity, we do not consider the endogenous decisions of households to choose the neighborhood and instead take the pool of households in the neighborhood as given. See Van Nieuwerburgh and Weill (2010) for a systematic treatment of moving decisions by households across neighborhoods.

which in turn leads to a stronger demand for each household's good.

In addition to consuming the produced goods, each household also wishes to consume housing. Through the trading of produced goods with each other, a strong aggregate productivity increases the demand for each household's good and its income, which thus motivates each household to demand more housing consumption. This gives rise to strategic complementarity in the households' housing demands.[3] Due to realistic informational frictions in gauging the quality of the neighborhood, the housing market provides an important platform for aggregating information. The resulting housing prices in turn serve as a key signal about the quality of the neighborhood.

At $t = 1$, households purchase houses from builders in a centralized market and decide how much of their goods to produce for trading with other households at $t = 2$. Each household will choose to purchase a bigger house if it expects to produce more goods in the second period.

1.1.1 Households

There is a continuum of households, indexed by $i \in [0, 1]$. Household i has a Cobb-Douglas utility function over housing H_i and a continuum of goods $\{C_j\}_{j\in[0,1]}$, given by

$$U\left(H_i, \{C_j(i)\}_{j\in[0,1]}\right) = \left(\frac{H_i}{1-\eta_H}\right)^{1-\eta_H} \left\{\frac{1}{\eta_H}\left(\frac{C_i(i)}{1-\eta_c}\right)^{1-\eta_c}\left(\frac{\int_{[0,1]/i} C_j(i)\,dj}{\eta_c}\right)^{\eta_c}\right\}^{\eta_H}, \quad (1)$$

where $C_j(i)$ is good j consumed by household i.[4] The parameters $\eta_H \in (0, 1)$ and $\eta_c \in (0, 1)$ measure the weights of different consumption components in the utility function. A higher η_H means a stronger complementarity between housing consumption and goods consumption, while a higher η_c means a stronger complementarity between consumption of the good produced by household i itself and consumption of the composite goods produced by the other households in the neighborhood.

The production function of household i is $e^{A_i} l_i$,where l_i is the labor supplied and A_i is the household's productivity. A_i is composed of a component A common to all households in the neighborhood and an idiosyncratic component ε_i:

$$A_i = A + \varepsilon_i,$$

[3]There are other types of social interactions between households living in a neighborhood, which are explored, for instance, in Durlauf (2004) and Glaeser, Sacerdote, and Scheinkman (2010).

[4]Our modeling choice of non-separable preferences for housing and consumption is similar to the CES specification of Piazzesi, Schneider, and Tuzel (2007).

where $A \sim \mathcal{N}\left(\bar{A}, \tau_A^{-1}\right)$ and $\varepsilon_i \sim \mathcal{N}\left(0, \tau_\varepsilon^{-1}\right)$ are both normally distributed. The aggregate productivity A represents the quality of the neighborhood. A higher A implies a more productive neighorhood.

Due to realistic informational frictions, neither A nor A_i is observable to the households. Instead, each household observes a noisy private signal about A at $t = 1$. Specifically, household i observes

$$s_i = A + \nu_i$$

where $\nu_i \sim \mathcal{N}\left(0, \tau_s^{-1}\right)$ is signal noise independent across households.

We assume that the household experiences a disutility for labor $\frac{l_i^{1+\psi}}{1+\psi}$ when producing its good, and that it maximizes its utility subject to its budget constraint

$$\begin{aligned} &\max_{\left\{H_i, \{C_i\}_{i\in[0,1]}, l_i\right\}} E\left[U\left(H_i, \{C_j(i)\}_{j\in[0,1]}\right) - \frac{l_i^{1+\psi}}{1+\psi}\middle| \mathcal{I}_i\right] \qquad (2) \\ s.t. \quad &: \quad P_H H_i + \int_0^1 P_j C_j(i)\, dj = P_i e^{A_i} l_i + \tau_i, \end{aligned}$$

where P_i is the price of the good it produces and τ_i is the income from building the house. We assume for simplicity that the housing supplier to household i is part of the household, so that $\tau_i = P_H H_i$ in equilibrium. The choice of goods consumption is made at $t = 2$ while the choice of labor and housing is made at $t = 1$ subject to each household's information set $\mathcal{I}_i = \{s_i, P_H\}$, which includes its private signal s_i and the housing price P_H.

1.1.2 Builders

Home builders face a convex labor cost

$$\frac{k}{1+k} e^{-\xi} H_S^{\frac{1+k}{k}}$$

in supplying housing, where H_S is the quantity of housing supplied, $k \in (0,1)$ is a constant parameter, and ξ represents a supply shock. We assume that ξ is observed by builders but not households, and that from the the perspective of households $\xi \sim \mathcal{N}\left(\bar{\xi}, \tau_\xi^{-1}\right)$, where $\bar{\xi}$ is the predictable part of ξ and τ_ξ^{-1} is the variance of the component outside their expectations. The supply shock ξ acts as information noise for the households when they try to learn about the common productivity A from the housing price P_H.

Builders at $t = 1$ maximizes their profit subject to their noisy supply curve

$$\Pi(H_S) = \max_{H_S} P_H H_S - \frac{k}{1+k} e^{-\xi_i - \frac{1}{2}\tau_e^{-1}} H_S^{\frac{1+k}{k}}, \qquad (3)$$

It is easy to determine the builders' optimal supply curve:

$$H_S = P_H^k e^{k\xi}, \tag{4}$$

where k measures the price elasticity of housing supply.

1.1.3 Equilibrium

Our model features a noisy rational expectations equilibrium, which requires clearing of the goods and housing markets that are consistent with the optimal behavior of both households and builders:

- Household optimization: $\left\{H_i, \{C_i\}_{i\in[0,1]}, l_i\right\}$ solves each household's maximization problem in (2).
- Builder optimization: H_S solves the builders' maximization problem in (3).
- At $t = 2$, the market for each household's good clears:

$$\int_0^1 C_i(j)\, dj = e^{A_i} l_i, \quad \forall\ i \in [0,1].$$

- At $t = 1$, the housing market clears:

$$\int_{-\infty}^{\infty} X_i(s_i, P_H)\, d\Phi(\varepsilon_i) = P_H^k e^{k\xi},$$

  where each household's housing demand $H_i(s_i, P_H)$ depends on its private signal s_i and the housing price P_H. The demand from households is integrated over the idiosyncratic component of their private signals ε_i.

1.2 The equilibrium

1.2.1 Goods market equilibrium

We begin our analysis of the equilibrium with the goods markets at $t = 2$. Household i has $e^{A_i} l_i$ units of good i to consume and trade with the other households. It maximizes its utility function given in (2).[5] The following proposition describes the market equilibrium for each good.

[5] As mentioned previously, we agnostically treat a household's builder as being a part of its family.

Proposition 1 *Households i's optimal goods consumption is*

$$C_i(i) = (1-\eta_c)\, e^{A_i} l_i,\ C_j(i) = \eta_c e^{A_j} l_j.$$

The price of the good produced by household i is

$$P_i = \left(\frac{\eta_H}{1-\eta_H} H_i\right)^{1-\eta_H} \left(e^{A_i} l_i\right)^{(1-\eta_c)\eta_H - 1} \left(\int_{[0,1]/i} e^{A_j} l_j dj\right)^{\eta_c \eta_H}. \tag{5}$$

Proposition 1 shows that each household divides its consumption of goods between its own good and those produced by other households of the neighborhood with fractions $1-\eta_c$ and η_c, respectively. When $\eta = 1/2$, the household consumes its own good and the goods of its neighbors equally. The price of each good is determined by its choice of housing and its goods output relative to that of the rest of the neighborhood. One household's good is more valuable when the rest of the neighborhood produces more, and thus each household needs to take into account the labor decisions of the other households in its neighborhood when making its own decision.

1.2.2 Housing market equilibrium

We now turn our attention to the housing market equilibrium at $t = 1$. We first solve for the optimal labor and housing choices for a household given its utility function and budget constraint (2), as well as its optimal choices for consumption of goods at $t = 2$ characterized in Proposition 1.

Proposition 2 *Households i's optimal labor choice depends on its total housing expenditure*

$$l_i = \left(\frac{\eta_H}{1-\eta_H} P_H H_i\right)^{\frac{1}{1+\psi}},$$

and its demand for housing is

$$H_i = \left(\frac{\eta_H}{1-\eta_H}\right)^{-\frac{\psi}{\psi+\eta_c}} \left\{ e^{(\eta_H - 1 - \psi)\log P_H} E\left[e^{A_i(1-\eta_c)\eta_H} \left(\int_{[0,1]/i} e^{A_j} H_j^{\frac{1}{1+\psi}} dj\right)^{\eta_c \eta_H} \middle| \mathcal{I}_i \right]^{(1+\psi)} \right\}^{\frac{1}{(\psi+\eta_c)\eta_H}}. \tag{6}$$

Proposition 2 demonstrates that the labor chosen by a household toward producing its good is determined by its housing expenditure $P_H H_i$, and that its housing demand is determined by not only its own productivity A_i but also the aggregate productivity of the rest

of the neighborhood. This latter component arises from the social complementarity in the utility function of the household and captures the key idea that the household cares about the neighborhood due to its need to exchange goods with its neighbors.

By clearing the aggregate housing demand of the households with the supply from the builders, we derive the housing market equilibrium. Despite the nonlinearity in each household's demand and in the supply from the builders, we obtain a tractable unique log-linear equilibrium. The following proposition summarizes the housing price and each household's housing demand in this equilibrium.

Proposition 3 *At $t = 1$, the housing market has a unique log-linear equilibrium: 1) The housing price is a log-linear function of A and ξ:*

$$\log P_H = p_A A + p_\xi \xi + p_0, \tag{7}$$

with the coefficients p_A and p_ξ given by

$$p_A = \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1} - \frac{(\psi + \eta_c)\, kb}{\psi k + \frac{1+\psi}{\eta_H} - 1} \tau_s^{-1} \tau_A > 0, \tag{8}$$

$$p_\xi = -\frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1} - \frac{1 + \psi + \eta_c kb}{\psi k + \frac{1+\psi}{\eta_H} - 1} \frac{b\tau_\xi}{\tau_A + \tau_s + b^2 \tau_\xi} < 0, \tag{9}$$

where $b \in \left[0, \frac{1+\psi}{k} \frac{\tau_s}{(\psi+\eta_c)\tau_A + \psi\tau_s}\right]$ is the unique positive, real root of equation (29), and p_0 is given in equation (34).

2) The housing demand of household i is a log-linear function of its private signal s_i and $\log P_H$:

$$\log H_i = h_s s_i + h_P \log P_H + h_0, \tag{10}$$

with the coefficients h_s and h_P given by

$$h_s = kb > 0, \tag{11}$$

$$h_P = -\frac{1 - \eta_H + \psi}{\psi \eta_H} + \frac{b^2 \tau_\xi}{\psi p_A} \frac{(1 + \psi + \eta_c kb)}{(\tau_A + \tau_s + b^2 \tau_\xi)} \leq k, \tag{12}$$

and h_0 given by equation (23).

Proposition 3 establishes that the housing price P_H is a log-linear function of the common productivity of households A and the builder supply shock ξ, and that each household's housing demand is a log-linear function of its private signal s_i and the log housing price

$\log P_H$. Similar to Hellwig (1980), the housing price aggregates each household's dispersed private information to partially reveal A, and does not depend on the idiosyncratic noise in any individual household's signal because of the Law of Large Numbers. This last observation is the key to the tractibility of our model, and ensures that the demand from households retains a log-normal distribution after aggregation.

The supply shock ξ in the presence of informational frictions serves the same role as noise trading in standard models of asset market trading with dispersed information. This highlights an important channel for supply shocks to distort the expectations of potential home buyers that is not well-appreciated in the housing literature. Since households cannot perfectly disentangle changes in housing prices caused by supply shocks from those brought about by shocks to demand, they partially confuse a housing price change caused by a supply shock to be a signal about the quality of the community.

To facilitate our discussion of the impact of learning, it will be useful to introduce a perfect-information benchmark, in which all households perfectly observe the common productivity of the neighborhood A. The following proposition characterizes this benchmark equilibrium.

Proposition 4 *Consider a benchmark economy, in which households perfectly observe A (i.e., $s_i = A$ for all households). There is also a log-linear equilibrium, in which the housing price is*

$$\begin{aligned}\log P_H &= \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1} A - \frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1}\xi + \frac{\psi}{k\psi + \frac{1+\psi}{\eta_H} - 1}\log\left(\frac{1-\eta_H}{\eta_H}\right) \\ &\quad + \frac{\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1}\log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{(1+\psi)\,\eta_H}{2\left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)}\left((1-\eta_c)^2 + \eta_c^2\right)\tau_\varepsilon^{-1},\end{aligned}$$

and all households have the same housing demand

$$\log H = \frac{1+\psi}{\psi} A - \frac{1-\eta_H+\psi}{\psi\eta_H}\log P_H + \log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{(1+\psi)\,\eta_H}{2\psi}\left((1-\eta_c)^2 + \eta_c^2\right)\tau_\varepsilon^{-1}.$$

Furthermore, the economy with information frictions converges to this economy as $\tau_s \nearrow \infty$.

In this perfect-information economy, the housing price is once again a log-linear function of the common productivity A and the supply shock ξ, and each household's identical demand is a log-linear function of its private signal (the perfectly observed A) and the housing price $\log P_H$. In the absence of informational frictions, housing prices and demand conform to the

standard intuition concerning the impact of demand and supply shocks. While a shock to the common productivity A increases both the housing price and aggregate housing demand, a supply shock ξ reduces house prices but increases aggregate demand.

An important observation that is clear in the perfect-information setting is that the variance contribution of the supply shock ξ to the housing price $\log P_H$ is monotonically declining in the supply elasticity k. This suggests that variation in supply elasticities across communities has

1.3 The Impact of Learning

In the presence of informational frictions about the quality of the neighborhood A, each household needs to use its private signal s_i and the publicly observed housing price $\log P_H$ to learn about A. As the housing price $\log P_H$ is a linear combination of A and the housing supply shock ξ, the supply shock interferes with this learning process. Intuititively, a larger supply shock ξ, by depressing the housing price, will have an additional effect of reducing the households' expectations of A. This, in turn, reduces their housing demand and consequently further depresses the housing price. Thus the learning effect causes supply shocks to have a larger negative effect on the equilibrium housing price than they would in the perfect-information benchmark. Similarly, this learning effect also causes the quality of the neighborhood A to have a smaller positive effect than in the perfect-information benchmark because informational frictions cause households to partially discount the value of A. The following proposition formally establishes this learning effect on the housing price.

Proposition 5 *In the presence of informational frictions, coefficients* $p_A > 0$ *and* $p_\xi < 0$ *derived in Proposition 3 are both lower than their corresponding values in the perfect-information benchmark.*

Due to the households' learning of A, the housing price is not only the cost of acquiring a shelter but also a signal of the quality of the neighborhood. The housing demand of each household derived in (10) reflects both of these two effects. Specifically, we can decompose the price elasticity of each household's housing demand h_P in equation (12) into two components: the usual cost effect $-\frac{1-\eta_H+\psi}{\psi\eta_H} < 0$, which is negative and represents the standard downward sloping demand curve in the perfect-information benchmark in Proposition 4, and the second component $\frac{b^2\tau_\xi}{\psi p_A}\frac{(1+\psi+\eta_c kb)}{\left(\tau_A+\tau_s+b^2\tau_\xi\right)}$ is positive and represents the information effect induced by a higher housing price on the household's expectation regarding A.

The second component motivates the household to consume more housing through two related but distinct learning channels. The first is that a higher A implies a higher productivity for the household itself. A higher A also implies that other households are more productive, which in turn leads to more demand for the household's good. Both channels cause the household to expect a higher wealth at $t = 2$ and thus to demand more housing consumption at $t = 1$.

It is important to highlight that these learning effects induced by informational frictions exist because there is fundamental uncertainty in the quality of the neighborhood A. If $\tau_A \to \infty$ and the quality of the neighborhood were fixed at A_0, which would be akin to being a mature neighborhood, then the aforementioned distortions to housing prices and demand would vanish. Thus we would expect supply shocks to interfere with household learning after a structural change to the community, such as gentrification, or during a housing cycle, which motivates us to consider a dynamic extension of the model in the next section. This intuition about the necessity of uncertainty in A is formally etablished in the next proposition.

Proposition 6 *As $\tau_A \to \infty$, informational frictions in the economy vanish and p_ξ, h_s, and h_P converge to their perfect-information counterparts while $p_A \to 0$.*

To dissect the households' learning of A through the housing price, it is useful to examine the informativeness of the housing price $\log P_H$ regarding A. As $p_A A$ and $p_\xi \xi$ are the two linear components of $\log P_H$ in equation (7), we define the price informativeness π to be the ratio of the price variance induced by $p_A A$ relative to that by $p_\xi \xi$:

$$\pi = \frac{p_A^2 \tau_A^{-1}}{p_\xi^2 \tau_\xi^{-1}}.$$

This informativeness metric π has the interpretation of being similar to a measure of the signal-to-noise ratio in the housing price. The following proposition characterizes this price informativeness and shows that it varies with the housing supply elasticity and the degree of complementarity in households' goods consumption.

As the degree of complementarity in households' goods consumption η_c increases, there is a greater need for each household to trade goods with other households. This gives the publicly observed housing price a greater role in coordinating the households' housing demand. Each household increases the weight h_P it puts on the housing price and puts less weight h_s on its own private signal. As the informativeness of the housing price arises

from the aggregation of the households' signals through their individual housing demand, this lower weight on their private signals reduces the sensitivity of the housing price p_A to the common productivity A. This makes the equilibrium housing price less informative about A. Thus, as η_c increases, h_s and consequently p_A decrease, leading π to decrease. This mechanism underlies the comparative static that price informativeness is decreasing in η_c.

The housing supply shock ξ acts as information noise in the housing price and prevents it from fully revealing the quality of the neighborhood A. The supply elasticity k directly affects the amount of supply noise in housing price and thus the price informativeness. A larger supply elasticity implies a greater amount of supply noise in price and thus less price informativeness. In contrast to the the impact of an increase in η_c, an increase in k motivates households to put more weight h_s on their private signals since the housing price is less informative about A. Thus, as k increases, h_s increases while p_A and π both decrease.

These insights are summarized in the next proposition.

Proposition 7 *p_A and π are monotonically decreasing in the degree of complementarity in households' goods consumption η_c and the housing supply elasticity k. In contrast, h_s is monotonically increasing in k and decreasing in η_c.*

Proposition 7 suggests that housing prices in a neighborhood may be driven primarily by supply shocks not only because housing supply is elastic (large k), but also because of strong social effects (large η_c). Popular neighborhoods where there is a lot of social interaction, such as young adult neighborhoods, may have housing cycles that resemble those in elastic areas. A key difference between these two types of neighborhoods, however, is that, while there is great uncertainty about the quality of the community in both, households in more popular neighborhoods are less concerned about this uncertainty and consequently they are more likely to ignore private information that they may have about the community.

To further understand the impact of supply elasticity on the housing market equilibrium, we return to the intuition put forth in the introduction that, when supply is either very elastic or inelastic, there is very little for households to learn from the housing price. The next proposition formalizes this argument by characterizing the two limiting economies as the supply elasticity k goes to 0 and ∞.

Proposition 8 *As $k \to \infty$, the price and demand for housing converge to*

$$\log P_H = -\xi,$$

and

$$\log H_i = (1+\psi)\left(\psi + \frac{\eta_c \tau_A}{\tau_A + \tau_\varepsilon}\right)^{-1} \frac{\tau_\varepsilon}{\tau_A + \tau_\varepsilon} s_i - \frac{1}{\psi}\left(\frac{1+\psi}{\eta_H} - 1\right) \log P_H + h_0,$$

where h_0 is given in equation (35). As $k \to 0$, the price and demand for housing converge to

$$\log P_H = \frac{1+\psi}{\frac{1+\psi}{\eta_H} - 1} A + \left(\frac{1+\psi}{\eta_H} - 1\right)^{-1} \left(\psi \log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{1+\psi}{2}\left(\eta_c + (1-\eta_c)^2 \eta_H\right) \tau_\varepsilon^{-1}\right),$$

and $\log H_i = 0$.

As the supply elasticity becomes infinite, the housing price is completely driven by the supply shock ξ and contains no information for households about A. Thus they do not learn from the price, as is apparent from the loading of the households' housing demand on $\log P_H$, and use only their private signals to form expectations. As the supply elasticity goes to 0, the housing price is completely driven by A and fully reveals it. Interestingly, in this case, the households do not face any uncertainty about A and thus do not learn from the price either, as reflected by the lack of any loading of their housing demand on the price.

Between these two extreme cases, the households do learn from the housing price. The second component $\frac{b^2 \tau_\xi}{\psi p_A}\left(\frac{1+\psi+\eta_c k b}{\tau_A + \tau_s + b^2 \tau_\xi}\right) > 0$ in the price elasticity of the household's housing demand reflects this learning effect and acts to offset the usual negative cost effect. This learning effect tends to be significant when the households face substantial uncertainty about A and when the housing price is also reasonably informative about A. As a result of these two forces, the learning effect is strongest when the supply elasticity k takes an intermediate value. The following proposition provides a set of necessary and sufficient conditions under which the learning effect can even domoniate the cost effect, and lead to a price elasticity of housing demand that is positive. In particular, the conditions highlight that the learning effect dominates the cost effect when the housing supply elasticity falls within an intermediate range.

Proposition 9 *Two necessary and sufficient conditions ensure that $h_P > 0$: first*

$$\tau_\xi \geq 4\left(\frac{1+\psi-\eta_H}{(1+\psi)\eta_H}\right)^2 \left(\tau_A + \frac{\psi}{\psi+\eta_c}\tau_s\right);$$

and, second, the supply elasticity k within a range

$$k \in \left[\left(\frac{1+\psi}{2\left(\tau_A\left(\psi+\eta_c\right)+\psi\tau_s\right)}\sqrt{\frac{\left(\psi+\eta_c\right)\eta_H}{1+\psi-\eta_H}\tau_s\tau_\xi}-\rho\right)^2, \left(\frac{1+\psi}{2\left(\tau_A\left(\psi+\eta_c\right)+\psi\tau_s\right)}\sqrt{\frac{\left(\psi+\eta_c\right)\eta_H}{1+\psi-\eta_H}\tau_s\tau_\xi}+\rho\right)^2\right],$$

where $\rho=\frac{\tau_s^{1/2}}{\tau_A(\psi+\eta_c)+\psi\tau_s}\sqrt{\left(1+\psi\right)^2\frac{(\psi+\eta_c)\eta_H}{1+\psi-\eta_H}\tau_\xi-4\frac{1+\psi-\eta_H}{\eta_H}\left(\tau_A\left(\psi+\eta_c\right)+\psi\tau_s\right)}$.

That learning by households from the housing price is strongest at intermediate levels of supply elasticity, when informational frictions are most severe, is central to understanding the puzzling empirical findings documented in Gao (2013). Here severe informational frictions can lead to an upward-sloping demand curve for housing, which resembles extrapolative behavior by households.

2 An Extended Model of Housing Cycles

We now extend our model to multiple periods with overlapping generations of households and extrapolative learning to characterize housing cycles. We first introduce the model structure and then analyze the housing cycle under the setting.

2.1 Model Setting

There are five periods $t = 0, 1, 2, 3, 4$. In each of the periods, a generation of households arrives to the neighborhood. Each generation first chooses its housing decisions and labor decisions to produce their respective goods when they arrive, and then trade their goods with each other in the same generation and consume in the next period. To simplify the model, we avoid trading of either housing or goods across generations. Consider households that arrive at $t = \tau$. They acquire housing from builders of their generation at τ and then trade goods only with households of their own generation at $\tau+1$. This allows us to keep the same setting in our baseline model for each generation of households. The link through different generations goes through the persistence of the common productivity of each generation and the households' learning through the observation of the past housing prices.

At $t = 0$, generation 0 of households arrive at the neighborhood. They have the same utility function specified in equation (1) for consumption of their own good, goods produced

by others, and housing. We denote the common component of their productivities by A_0. They buy housing from builders of their own generation who have a supply shock ξ_0 and supply elasticity k. The housing price $P_H(0)$ serves as an anchor for the housing price fluctuations in the following periods. For simplicity, we suppose that A_0 is observable. Then, $P_H(0)$ comes directly from the perfect-information benchmark of the baseline model in Proposition 4:

$$\begin{aligned}\log P_H(0) &= \frac{1+\psi}{\psi k+\frac{1+\psi}{\eta_H}-1}A_0-\frac{\psi k}{\psi k+\frac{1+\psi}{\eta_H}-1}\xi_0+\frac{\psi}{k\psi+\frac{1+\psi}{\eta_H}-1}\log\left(\frac{1-\eta_H}{\eta_H}\right)\\ &+\frac{\psi}{\psi k+\frac{1+\psi}{\eta_H}-1}\log\left(\frac{1-\eta_H}{\eta_H}\right)+\frac{(1+\psi)\eta_H}{2\left(\psi k+\frac{1+\psi}{\eta_H}-1\right)}\left((1-\eta_c)^2+\eta_c^2\right)\tau_\varepsilon^{-1}.\end{aligned}$$

At $t=1$, another generation of households arrive to the neighborhood. They also have the same utility function specified in equation (1). We denote the common component of their productivities by A_1:

$$A_1 = A_0 + Z_1,$$

where $Z_1 \sim \mathcal{N}\left(0, \tau_A^{-1}\right)$ is a random shock to the common productivity. To highlight the households' learning in this period, we suppose that A_1 is not observable. Instead, each household of this generation observes a private signal

$$s_{i1} = A_1 + \nu_{i1},$$

where $\nu_{i1} \sim \mathcal{N}\left(0, \tau_s^{-1}\right)$ is independent and identically distributed signal noise. These households buy housing at $t=1$ from builders of their generation. The builders experience an unobservable supply shock $\xi_1 \sim \mathcal{N}\left(\bar{\xi}, \tau_\xi^{-1}\right)$, which is uncorrelated with all other shocks in the economy. The builders of this generation have the same supply elasticity k as the previous generation, as supply elasticity is often determined by zoning regulations of the neighborhood, which stays the same over time. The households of this generation also make their labor decisions at $t=1$ to produce goods, and then trade these goods with each other and consume at $t=2$.

The structure of generation 1 of households is exactly the same as our baseline model. As a result, the housing market equilibrium at $t=1$ follows directly from Proposition 3: The housing price is a log-linear function of A_1 and ξ_1:

$$\log P_H(1) = p_A A_1 + p_\xi \xi_1 + p_0.$$

The housing purchase by household i is a log-linear function of its private signal s_{i1} and $\log P_H(1)$:

$$\log H_{i1} = h_s s_{i1} + h_P \log P_H(1) + h_0.$$

We adopt the same notations for the coefficients derived in Proposition 3 except that $\bar{A}$ needs to be replaced by A_0.

At $t = 2$, another new generation of households come to the neighborhood. They again have the same utility function specified in equation (1) and the common component of their productivities by A_2:

$$A_2 = A_1 + Z_2,$$

where $Z_2 \sim \mathcal{N}\left(0, \tau_A^{-1}\right)$ is the fundamental shock to the common productivity. To examine the effect of extrapolative learning on the housing cycle, we further assume that households of generation 2 all mistakenly believe that the shock to A_2 contains a persistent component from the previous period shock Z_1. That is, they believe that

$$A_2 = A_1 + Z_2 + \phi Z_1,$$

where the parameter $\phi \geq 0$ measures the perceived persistence in the fundamental shock. This perceived persistent shock leads the households of generation 2 to extrapolate the past housing price change in forming their learning of A_2. This way of modeling extrapolative behavior is consistent with the model of Barberis, Shleifer and Vishny (1998) and conforms with the empirical observations of the expectations of households summarized in Glaeser (2013).

Each household of generation 2 observes a private signal about A_2:

$$s_{i2} = A_2 + \nu_{i2},$$

where $\nu_{i2} \sim \mathcal{N}(0, \tau_\varepsilon^{-1})$ is independent and identically distributed signal noise. Like before, the new generation of builders experience an unobservable supply shock $\xi_2 \sim \mathcal{N}\left(\bar{\xi}, \tau_\xi^{-1}\right)$, which is uncorrelated with ξ_1 and all other shocks in the economy. The households of generation 2 choose their housing demands and labor decisions to produce goods at $t = 2$, and then trade their goods with each other and consume at $t = 3$.

At $t = 2$, in learning about A_2 each household of generation 2 observes its private signal s_{i2}, the current housing price $P_H(2)$, and the past housing prices $P_H(0)$ and $P_H(1)$. The following proposition derives a unique log-linear equilibrium for the housing market.

Proposition 10 *At $t = 2$, the housing market has a unique log-linear equilibrium: 1) The housing price is a log-linear function of A_2, $\log P_H(1)$, and ξ_2:*

$$\log P_H(2) = p_{A2}A_2 + p_{P1}\log P_H(1) + p_{\xi 2}\xi_2 + p_{02},$$

with the coefficients p_{A2}, $p_{\xi 2}$, and p_{P1} given by

$$\begin{aligned}
p_{A2} &= \left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)^{-1}\left[1 + \psi - \frac{\tau_A + (1+\phi)^2\alpha}{1+(1+\phi)^2}(\psi + \eta_c)\tau_s^{-1}kx\right] > 0,\\
p_{\xi 2} &= -\left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)^{-1}\left[\psi k + \frac{x\tau_\xi(1+\psi+\eta_c kx)}{\tau_s + x^2\tau_\xi + \frac{1}{1+(1+\phi)^2}\tau_A + \frac{(1+\phi)^2}{1+(1+\phi)^2}\alpha}\right] < 0,\\
p_{P1} &= \left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)^{-1}\alpha\tau_s^{-1}(\psi + \eta_c)kx\left(\frac{1+\phi}{p_A}\right) > 0,
\end{aligned}$$

where $\alpha = \left[\tau_A^{-1} + \left(1+(1+\phi)^2\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}\right]^{-1}$ and $x \geq 0$ is the unique positive root of equation (46), and p_{02} given in equation (42).

2) The housing purchase by household i is a log-linear function of its private signal s_{i2}, $\log P_H(1)$, and $\log P_H(2)$:

$$\log H_{i2} = h_{s2}s_{i2} + h_{P1}\log P_H(1) + h_{P2}\log P_H(2) + h_{02},$$

with the coefficients h_{s2}, h_{P1}, and h_{P2} by

$$\begin{aligned}
h_{A2} &= kx > 0,\\
h_{P1} &= (k - h_{P2})p_{P1} > 0,\\
h_{P2} &= -\frac{1+\psi-\eta_H}{\psi\eta_H} + \frac{x^2\tau_\xi(1+\psi+\eta_c kx)}{\psi\left[\tau_s + x^2\tau_\xi + \frac{1}{1+(1+\phi)^2}\tau_A + \frac{(1+\phi)^2}{1+(1+\phi)^2}\alpha\right]}\frac{1}{p_{A2}},
\end{aligned}$$

and h_{02} by equation (38).

Finally, at $t = 3$, generation 3 of households arrive at the neighborhood. Just like the previous generations, they have the same utility function specified in equation (1) and have a common productivity A_3 given by

$$A_3 = A_2 + Z_3,$$

where $Z_3 \sim \mathcal{N}\left(0, \tau_A^{-1}\right)$ is the fundamental shock to this generation of households. They buy housing from builders of their own generation who have a supply shock ξ_3 and supply

elasticity k. To fascilitate our analysis of housing price fluctuations induced by extrapolative learning at $t=2$, we suppose that A_3 is observable so that the housing price $P_H(3)$ represents the long-run housing price when any bias induced by informational frictions and extrapolative behavior is corrected. We can again use Proposition 4 to derive the housing price:

$$\begin{aligned}\log P_H(3) &= \frac{1+\psi}{\psi k+\frac{1+\psi}{\eta_H}-1}A_3-\frac{\psi k}{\psi k+\frac{1+\psi}{\eta_H}-1}\xi_3+\frac{\psi}{k\psi+\frac{1+\psi}{\eta_H}-1}\log\left(\frac{1-\eta_H}{\eta_H}\right)\\ &\quad+\frac{\psi}{\psi k+\frac{1+\psi}{\eta_H}-1}\log\left(\frac{1-\eta_H}{\eta_H}\right)+\frac{(1+\psi)\eta_H}{2\left(\psi k+\frac{1+\psi}{\eta_H}-1\right)}\left((1-\eta_c)^2+\eta_c^2\right)\tau_\varepsilon^{-1}.\end{aligned}$$

The households of generation 3 produce their goods at $t=3$ and then trade the goods with each other and consume at $t=4$.

2.2 Housing Cycle

The extrapolative of generation-2 households can lead to a housing cycle in our model. To illustrate this cycle, we analyze the variance and autocovariances of the housing returns. We define

$$R_H(t_1,t_2)=\log P_H(t_2)-\log P_H(t_1)$$

as the housing return from t_1 to t_2. In our analysis, we focus on the following return variances: $Var\left[R_H(0,1)\right]$ and $Var\left[R_H(1,2)\right]$. The first reveals the price variance associated with the rational learning of generation-1 households, while the latter reveals the price variance induced by the extrapolative learning of generation-2 households. Furthermore, the short-run return covariance $Cov\left[R_H(0,1),R_H(1,2)\right]$ measures the short-run price momentum induced by generation-2 households, and the long-run return covariance $Cov\left[R_H(0,2),R_H(2,3)\right]$ measures the subsequent long-run price reversal.

It is again useful to first establish a perfect-information benchmark for our analysis of learning effects. Suppose that the common prodictivity A_t is observable in any period t. Then, it is clear that the housing price in each period follows directly from Proposition 4:

$$\begin{aligned}\log P_H(t) &= \frac{1+\psi}{\psi k+\frac{1+\psi}{\eta_H}-1}A_t-\frac{\psi k}{\psi k+\frac{1+\psi}{\eta_H}-1}\xi_t+\frac{\psi}{k\psi+\frac{1+\psi}{\eta_H}-1}\log\left(\frac{1-\eta_H}{\eta_H}\right)\\ &\quad+\frac{(1+\psi)\eta_H}{2\left(\psi k+\frac{1+\psi}{\eta_H}-1\right)}\left((1-\eta_c)^2+\eta_c^2\right)\tau_\varepsilon^{-1}.\end{aligned}$$

In this perfect-information benchmark, there is not any short-run price momentum because

$$\begin{aligned} & Cov\left[R_H\left(0,1\right),R_H\left(1,2\right)\right] \\ = \; & Cov\left[\frac{1+\psi}{\psi k+\frac{1+\psi}{\eta_H}-1}Z_1-\frac{\psi k}{\psi k+\frac{1+\psi}{\eta_H}-1}\left(\xi_1-\xi_0\right),\frac{1+\psi}{\psi k+\frac{1+\psi}{\eta_H}-1}Z_2+\frac{\psi k}{\psi k+\frac{1+\psi}{\eta_H}-1}\left(\xi_1-\xi_2\right)\right] \\ = \; & -\left(\frac{\psi k}{\psi k+\frac{1+\psi}{\eta_H}-1}\right)^2\tau_\xi^{-1}<0. \end{aligned}$$

As the common productivity in each period is observable, there is no learning by households. The negative autocovariance, derived above, in housing price comes from the transitory housing supply shocks.

Proposition 11 derives the price variance at $t=1$ and 2 and autocovariance in short-run and long-run housing returns in our main setting.

Proposition 11 *The housing price variance at $t=1$ is*

$$Var\left[\log P_H\left(1\right)\right]=p_\xi^2\tau_\xi^{-1}+p_A^2\tau_A^{-1},$$

and at $t=2$ is

$$Var\left[\log P_H\left(2\right)\right]=p_{\xi 2}^2\tau_\xi^{-1}+2p_{A2}^2\tau_A^{-1}+p_{P1}^2Var\left(\log P_H\left(1\right)\right).$$

The short-run housing return covariance is given by

$$Cov\left[R_H\left(0,1\right),R_H\left(1,2\right)\right]=-p_\xi^2\tau_\xi^{-1}+\left(p_{A2}-p_A\right)p_A\tau_A^{-1}+p_{P1}Var\left[\log P_H\left(1\right)\right]. \tag{13}$$

The long-run return covariance is given by

$$\begin{aligned} & Cov\left(R_H\left(2,3\right),R_H\left(0,2\right)\right) \\ = \; & 2\frac{1+\psi}{\psi k+\frac{1+\psi}{\eta_H}-1}p_{A2}\tau_A^{-1}+p_{P1}p_A\left(\frac{1+\psi}{\psi k+\frac{1+\psi}{\eta_H}-1}-2p_{A2}\right)\tau_A^{-1}-Var\left[\log P_H\left(2\right)\right]. \end{aligned}$$

The expression for the autocovariance in equation 13 illustrates two channels through which informational frictions and household learning affect the housing price dynamics. The first term term in equation 13 captures the negative autocovariance induced by transitory supply shocks, as in the perfect-information benchmark. The second term represents the autocovariance induced by the households' learning of persistent common productivity, which give rise to nonstationarity in the loadings on the fundamental shocks in the housing price

across periods. This term exists even in the absence of any extrapolative behavior. The third term captures the positive autocovariance generated by the households' overreaction to the past price change at $t = 1$. If the households' extrapolative behavior is sufficiently strong, it is possible to overcome the negative autocovariance induced by transotory supply shocks and possibly by the nonstationarity in the second term to lead to short-run housing price momentum at $t = 1$, which will naturally reverse at $t = 3$.

As we have learned from the literature, it is not surprising, given the presence of extrapolative behavior, to have short-run momentum and long-run reversal in the housing prices. What is more interesting is the potential interaction between extrapolative behavior and elasticity of housing supply through the households' learning through the equilibrium housing price. As we discussed in the introduction, Gao systematically documents a puzzling observation that during the recent U.S. housing cycle, the magnitude of the housing price boom and bust cycle experienced by different counties has an inverted U shape relationship with respect to elasticity of housing supply. This phenomenon is in sharp contrast to the conventional wisdom that supply elasticity monotonically mitigates housing cycles.

It is possible for our model to generate both short-run momentum and long-run revserval together with a non-monotonic relationship between the magnitude of the housing cycle with supply elasticity. We now demonstrate such as an outcome through a numerical illustration. As such an outcome requires a strong extrapolative learning force to overcome the transitory supply shocks imposed by our model, which may or may not be realistic, this outcome is unlikely to be general across a wide range of model parameter values. Thus, our illustration simply aims to demonstrate the possibility of this outcome rather than its prevalence.

Specifically, we choose the following values for the key model parameters:

$$\tau_A = 80, \tau_\xi = 20, \phi = 1, \tau_s = 20, \psi = \eta_H = \eta_c = 0.5.$$

In our illustration, we plot the housing price variances at $t = 1$ and 2 and the short-run housing return autocorrelation and the long-run return autocorrelation with respect to housing supply elasticity k. Figure 2 presents the perfect-information benchmark. Consistent with the conventional wisdom, the housing price variances at $t = 1$ and 2 are both monotonically decreasing with supply elasticity k. Furthermore, both the short-run and long-run housing return autocorrelations are negative and monotonically decreasing with k, which reflect the increasingly greater role played by transitory supply shocks in housing prices as k increases.

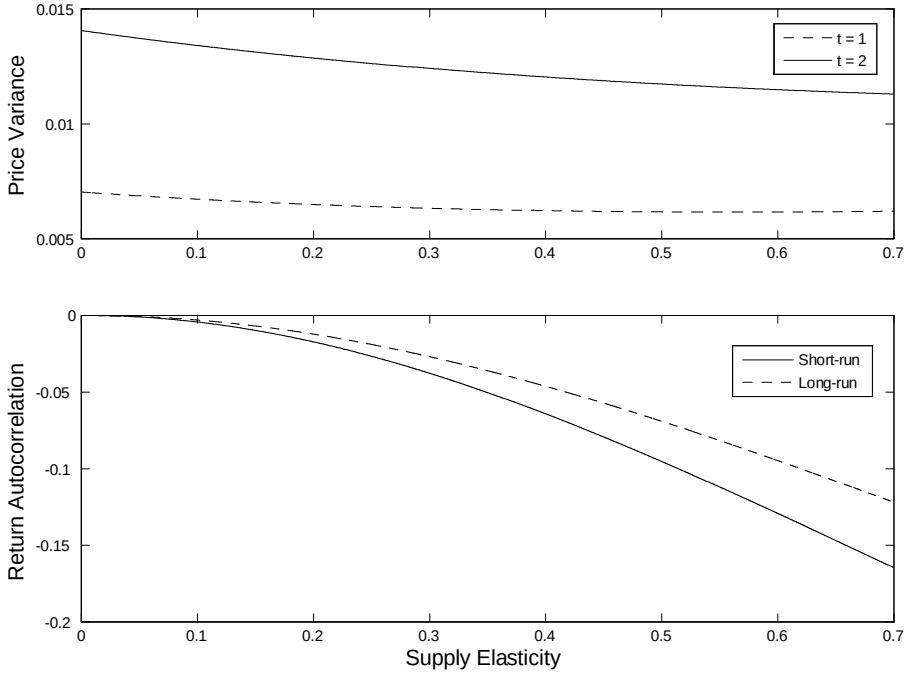


Figure 2: Price Variance and Return Autocorrelation in the Perfect-Information Benchmark

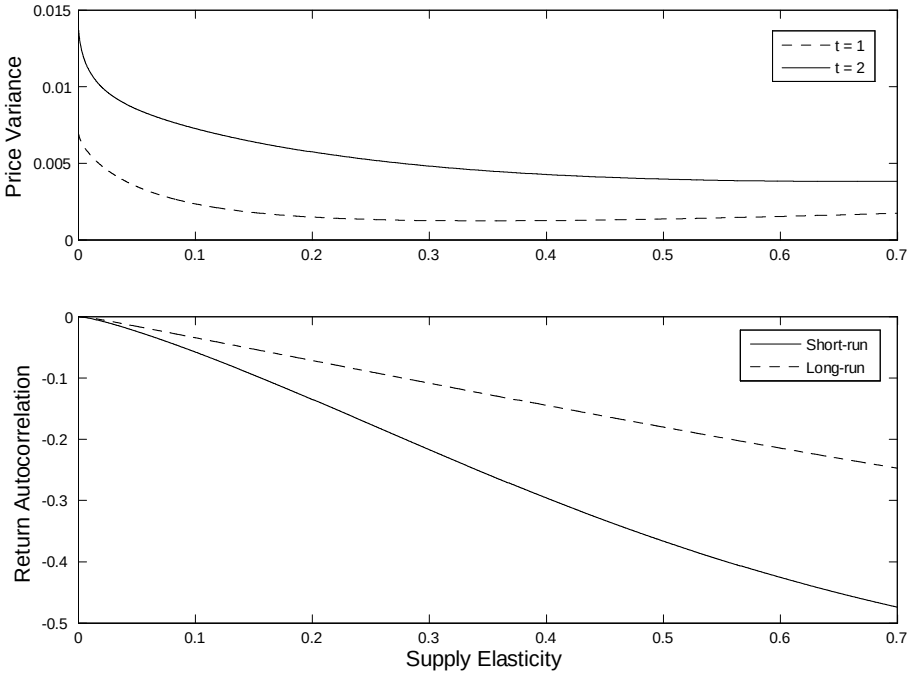


Figure 3: Price Variance and Return Autocorrelation with Rational Learning

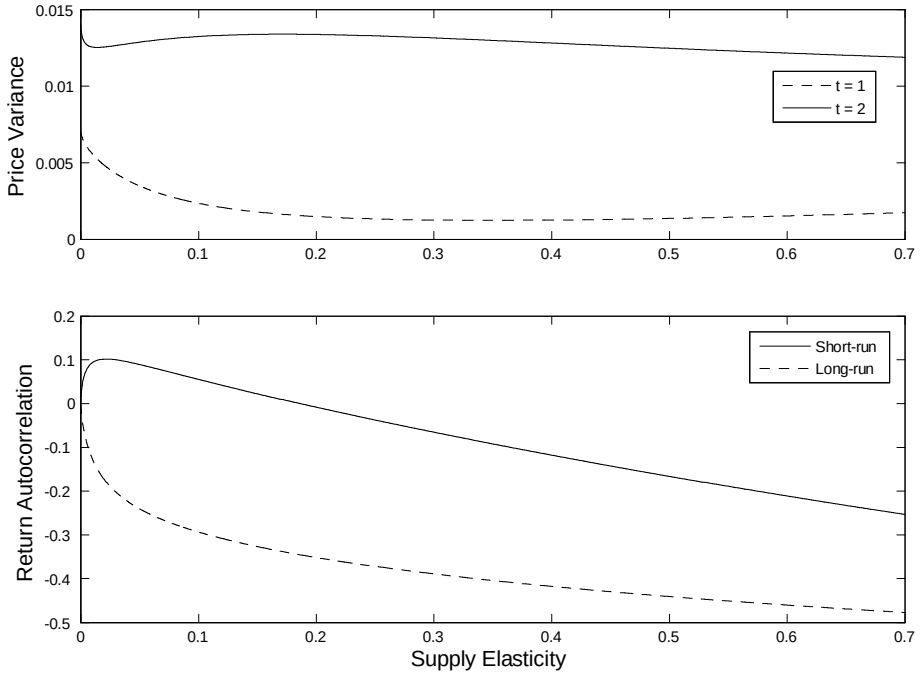


Figure 4: Price Variance and Return Autocorrelation with Extrapolative Learning

Figure 3 presents the case with informational frictions but without any extrapolative behavior (i.e., $\phi = 0$). This case serves as another benchmark to highlight the role of extrapolative behavior. Interestingly, despite the presence of informational frictions and household learning, the price variances at $t = 1$ and 2 remain monotonically decreasing with k, and the slopes of the decreasing curves become even steeper than that in the perfect-information benchmark. Both short-run and long-run return autocorrelations also remain negative.

Figure 4 presents the main case with extrapolative learning with $\phi = 1$. In the presence of extrapolative learning, the housing prices now exhibit positive short-run return autocorrelation (i.e., momentum) and, interestingly, the magnitude of the short-run autocorrelation has an inverted U shape with respect to supply elasticity k. This pattern reflects our earlier discussion that due to the households' endogeneous learning through the housing price, the effects of their extrapolative learning are most pronounced in areas with intermediate supply elasticities.

3 Conclusion

To be added.

Appendix Proofs of Propositions

A.1 Proof of Proposition 1

The first order conditions of household i's optimization problem in (2) respect to $C_i(i)$ and $C_j(i)$ at an interior point are

$$C_i(i) \quad : \quad \frac{\eta_H(1-\eta_c)}{C_i(i)} U\left(H_i, \{C_k(i)\}_{k\in[0,1]}\right) = \lambda_i P_i, \tag{14}$$

$$C_j(i) \quad : \quad \frac{\eta_H \eta_c}{C_j(i)} U\left(H_i, \{C_k(i)\}_{k\in[0,1]}\right) = \lambda_i P_j, \tag{15}$$

where λ_i is the Lagrange multiplier for the budget constraint. Dividing equations (14) and (15) leads to $\frac{\eta_c}{1-\eta_c}\frac{C_i(i)}{C_j(i)} = \frac{P_j}{P_i}$, which is equivalent to $P_j C_j(i) = \frac{\eta}{1-\eta} P_i C_i(i)$. By substituting this equation back to the household's budget constraint in (2), we obtain

$$C_i(i) = (1-\eta_c) e^{A_i} l_i.$$

The market clearing for the household's good requires that $\int_0^1 C_i(j)\,dj = e^{A_i} l_i$, which implies that $C_j(i) = \eta_c e^{A_i} l_i$. The symmetric problem of household j implies that $C_j(j) = (1-\eta_c) e^{A_j} l_j$, and the market clearing for its good implies $C_j(i) = \eta_c e^{A_j} l_j$.

The first order condition in equation (14) also gives the price of the good produced by household i. Since the household's budget constraint in (2) is entirely in nominal terms, the price system is only identified up to λ_i, the Lagrange multiplier. We therefore normalize λ_i to 1. Given that $C_i(i) = (1-\eta_c) e^{A_i} l_i$ and $C_j(i) = \eta_c e^{A_j} l_j$, it follows that

$$\begin{aligned} P_i &= \eta_H \frac{1-\eta_c}{C_i(i)} U\left(H_i, \{C_j(i)\}_{j\in[0,1]}\right) \\ &= \left(\frac{\eta_H}{1-\eta_H} H_i\right)^{1-\eta_H} \left(e^{A_i} l_i\right)^{(1-\eta_c)\eta_H - 1} \left(\int_{[0,1]/i} e^{A_j} l_j dj\right)^{\eta_c \eta_H}. \end{aligned}$$

A.2 Proof of Proposition 2

At $t=0$, the first order conditions for household i's choices of H_i and l_i at an interior point are

$$H_i \quad : \quad \frac{1-\eta_H}{H_i} E\left[U\left(H_i, \{C_j(i)\}_{j\in[0,1]}\right) \middle| \mathcal{I}_i\right] = \lambda_i P_H, \tag{16}$$

$$l_i \quad : \quad l_i^{\psi} = \lambda_i E\left[P_i e^{A_i} \middle| \mathcal{I}_i\right]. \tag{17}$$

Taking expectations of equations (14) and (15) from Proposition 1, and imposing $\lambda_i = 1$ to equation (16), one arrives at

$$\eta_H (1-\eta_c) P_H H_i = (1-\eta_H) E\left[P_i C_i(i) \middle| \mathcal{I}_i\right] = (1-\eta_H)(1-\eta_c) E\left[P_i e^{A_i} l_i \middle| \mathcal{I}_i\right],$$

and therefore

$$P_H H_i = \frac{1-\eta_H}{\eta_H} l_i E\left[e^{A_i} P_i \middle| \mathcal{I}_i\right].$$

From equation (17), it follows that

$$l_i^{\psi} = E\left[e^{A_i} P_i \middle| \mathcal{I}_i\right],$$

from which we see that

$$l_i = \left(\frac{\eta_H}{1-\eta_H} P_H H_i\right)^{\frac{1}{1+\psi}}. \tag{18}$$

It follows then, from equations (5) and (18), that

$$\frac{\eta_H}{1-\eta_H} P_H H_i = \left(\frac{\eta_H}{1-\eta_H}\right)^{(1-\eta_H)\frac{1+\psi}{1+\psi-\eta_H}} E\left[e^{A_i(1-\eta_c)\eta_H} H_i^{1-\eta_H} \left(H_i^{-\frac{1}{1+\psi}} \int_{[0,1]/i} e^{A_j} H_j^{\frac{1}{1+\psi}} dj\right)^{\eta_c \eta_H} \middle| \mathcal{I}_i\right]^{\frac{1+\psi}{1+\psi-\eta_H}}.$$

Note that integrating over the continuum of other households' housing decisions is equivalent to taking an expectation with respect to another household's housing decision. We then obtain equation (6).

A.3 Proof of Proposition 3

We first conjecture that each household's housing purchasing and the housing price take the following log-linear forms:

$$\log H_i = h_P \log P_H + h_s s_i + h_0, \tag{19}$$

$$\log P_H = p_A A + p_\xi \xi + p_0, \tag{20}$$

where the coefficients h_0, h_P, h_s, p_0, p_A, and p_ξ will be determined by equilibrium conditions.

Given the conjectured functional form for H_i, we can expand equation (6). It follows that

$$\begin{aligned}
& E\left[e^{A_i(1-\eta_c)\eta_H} \left(\int_{[0,1]/i} e^{A_j} H_j^{\frac{1}{1+\psi}} dj\right)^{\eta_c\eta_H} \middle| \mathcal{I}_i\right] \\
= \; & E\left[e^{A_i(1-\eta_c)\eta_H} \left(\int_{[0,1]/i} e^{\left(1+\frac{1}{1+\psi}h_s\right)A_j} e^{\frac{1}{1+\psi}(h_0+h_P\log P_H)} dj\right)^{\eta_c\eta_H} \middle| \mathcal{I}_i\right] \\
= \; & e^{\frac{\eta_c\eta_H}{1+\psi}(h_0+h_P\log P_H)} E\left[e^{A_i(1-\eta_c)\eta_H} \left(\int_{[0,1]/i} e^{\left(1+\frac{1}{1+\psi}h_s\right)A_j} dj\right)^{\eta_c\eta_H} \middle| \mathcal{I}_i\right] \\
= \; & e^{\frac{\eta_c\eta_H}{1+\psi}(h_0+h_P\log P_H)} e^{\frac{\eta_c\eta_H}{2}\left(1+\frac{h_s}{1+\psi}\right)^2\tau_\varepsilon^{-1}} e^{\frac{1}{2}(1-\eta_c)^2\eta_H^2\tau_\varepsilon^{-1}} E\left[e^{A\eta_H\left(1+\eta_c\frac{h_s}{1+\psi}\right)} \middle| \mathcal{I}_i\right]
\end{aligned}$$

where the last step uses the fact that A is independent of ε_j and the second step exploits the Law of Large Number for the continuum when integrating over households, which still holds if we subtract sets of measure 0 from the integral.

Define

$$z \equiv \frac{\log P_H - p_0 - p_\xi \bar{\xi}}{p_A} = A + \frac{p_\xi}{p_A}\left(\xi - \bar{\xi}\right),$$

which is a sufficient statistic of information contained in P_H. Then, conditional on observing its own signal s_i and the housing price P_H, household i's expectation of A is

$$E\left[A \mid s_i, \log P_H\right] = E\left[A \mid s_i, z\right] = \frac{1}{\tau_A + \tau_s + \frac{p_A^2}{p_\xi^2}\tau_\xi}\left(\tau_A \bar{A} + \tau_s s_i + \frac{p_A^2}{p_\xi^2}\tau_\xi z\right),$$

and its conditional variance of A is

$$Var\left[A \mid s_i, \log P_H\right] = \left(\tau_A + \tau_s + \frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}.$$

Therefore,

$$\begin{aligned}\log E\left[e^{A\eta_H\left(1+\eta_c\frac{h_A}{1+\psi}\right)}\middle|\mathcal{I}_i\right] &= \eta_H\left(1+\eta_c\frac{h_s}{1+\psi}\right)\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}\left(\tau_A\bar{A}+\tau_s s_i+\frac{p_A^2}{p_\xi^2}\tau_\xi z\right)\\ &\quad+\frac{1}{2}\eta_H^2\left(1+\eta_c\frac{h_s}{1+\psi}\right)^2\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}.\end{aligned}$$

Then,

$$\begin{aligned}&E\left[e^{A_i(1-\eta_c)\eta_H}\left(\int_{[0,1]/i}e^{A_j}H_j^{\frac{1}{1+\psi}}dj\right)^{\eta_c\eta_H}\middle|\mathcal{I}_i\right]\\ =\;&\eta_H\left(1+\eta_c\frac{h_s}{1+\psi}\right)\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}\left(\tau_A\bar{A}+\tau_s s_i+\frac{p_A}{p_\xi^2}\tau_\xi\left(\log P_H-p_0-p_\xi\bar{\xi}\right)\right)\\ &+\frac{\eta_c\eta_H}{1+\psi}h_P\log P_H+\frac{\eta_c\eta_H}{1+\psi}h_0+\frac{1}{2}\left(\eta_c\eta_H\left(1+\frac{h_s}{1+\psi}\right)^2+(1-\eta_c)^2\eta_H^2\right)\tau_\varepsilon^{-1}\\ &+\frac{1}{2}\eta_H^2\left(1+\eta_c\frac{h_s}{1+\psi}\right)^2\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}.\end{aligned}$$

Therefore, equation (6) gives $\log H_i$ as

$$\begin{aligned}
\log H_i &= \frac{1}{\psi+\eta_c}\left(1+\psi+\eta_c h_s\right)\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}\tau_s s_i \\
&+\frac{1}{\psi+\eta_c}\left(\eta_c h_P-\frac{1+\psi-\eta_H}{\eta_H}+\left(1+\psi+\eta_c h_s\right)\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}\frac{p_A}{p_\xi^2}\tau_\xi\right)\log P_H \\
&+\frac{1}{\psi+\eta_c}\left(1+\psi+\eta_c h_s\right)\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}\left(\tau_A\bar{A}-\frac{p_A}{p_\xi^2}\tau_\xi\left(p_0+p_\xi\bar{\xi}\right)\right) \\
&+\frac{\psi}{\psi+\eta_c}\log\left(\frac{1-\eta_H}{\eta_H}\right)+\frac{\eta_c}{\psi+\eta_c}h_0+\frac{1}{2}\frac{1+\psi}{\psi+\eta_c}\left(\eta_c\left(1+\frac{h_s}{1+\psi}\right)^2+\left(1-\eta_c\right)^2\eta_H\right)\tau_\varepsilon^{-1} \\
&+\frac{1}{2}\frac{1+\psi}{\psi+\eta_c}\eta_H\left(1+\eta_c\frac{h_s}{1+\psi}\right)^2\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}
\end{aligned}$$

Matching coefficients with the conjectured log-linear form in (19), it follows that

$$h_s = \frac{1}{\psi+\eta_c}\left(1+\psi+\eta_c h_s\right)\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}\tau_s, \tag{21}$$

$$h_P = \frac{1}{\psi}\left(-\frac{1+\psi-\eta_H}{\eta_H}+\left(1+\psi+\eta_c h_s\right)\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}\frac{p_A}{p_\xi^2}\tau_\xi\right), \tag{22}$$

$$\begin{aligned}
h_0 &= \frac{1+\psi+\eta_c h_s}{\psi}\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}\left(\tau_A\bar{A}-\frac{p_A}{p_\xi^2}\tau_\xi\left(p_0+p_\xi\bar{\xi}\right)\right) \\
&+\log\left(\frac{1-\eta_H}{\eta_H}\right)+\frac{1}{2}\frac{1+\psi}{\psi}\left(\eta_c\left(1+\frac{h_s}{1+\psi}\right)^2+\left(1-\eta_c\right)^2\eta_H\right)\tau_\varepsilon^{-1} \\
&+\frac{1}{2}\frac{1+\psi}{\psi}\eta_H\left(1+\eta_c\frac{h_s}{1+\psi}\right)^2\left(\tau_A+\tau_s+\frac{p_A^2}{p_\xi^2}\tau_\xi\right)^{-1}.
\end{aligned} \tag{23}$$

By aggegating households' housing demand and the builders' supply and imposing market clearing in the housing market, we have

$$h_0+h_P\left(p_0+p_A A+p_\xi\xi\right)+h_s A+\frac{1}{2}h_s^2\tau_s^{-1}=k\left(\xi+p_0+p_A A+p_\xi\xi\right).$$

Matching coefficients of the two sides of the equation leads to the following three conditions

$$h_0+h_P p_0+\frac{1}{2}h_s^2\tau_s^{-1} = kp_0, \tag{24}$$

$$h_P p_A+h_s = kp_A, \tag{25}$$

$$h_P p_\xi = k+kp_\xi. \tag{26}$$

It follows from equation (26) that

$$p_\xi = -\frac{k}{k - h_P}, \tag{27}$$

and further from equation (25) that

$$p_A = \frac{h_s}{k - h_P}. \tag{28}$$

Thus, by taking the ratio of equations (28) and (27), we arrive at

$$\frac{p_A}{p_\xi} = -\frac{h_A}{k}.$$

Substituting $\frac{p_A}{p_\xi} = -\frac{h_s}{k}$ into equation (21), and defining $b = -\frac{p_A}{p_\xi}$, we arrive at

$$kb^3\tau_\xi + \left(\tau_A + \frac{\psi}{\psi + \eta_c}\tau_s\right) kb - \frac{1+\psi}{\psi + \eta_c}\tau_s = 0. \tag{29}$$

We see from equation (29) that b has at most one positive root since the above 3rd order polynomial has only one sign change, by Descartes' Rule of Signs. By setting $b \to -b$, we see that there is no sign change, and therefore b has no negative root. Furthermore, by the Fundamental Theorem of Algebra, the roots of the polynomial (29) exist. Thus, it follows that equation (29) has only one real, nonnegative root $b \geq 0$ and 2 complex roots.[6]

Furthemore, by dropping the cubic term from equation (29), one arrives at an upper bound for b:

$$b \leq \frac{1+\psi}{k} \frac{\tau_s}{(\psi + \eta_c)\,\tau_A + \psi\tau_s}.$$

Since $\frac{h_s}{k} = -\frac{p_A}{p_\xi} = b$, we can recover $h_s = kb > 0$ and $p_\xi = -\frac{1}{b}p_A < 0$. From equation (22) and $b = -\frac{p_A}{p_\xi}$, it follows that

$$h_P = \frac{1}{\psi}\left(1 - \frac{1+\psi}{\eta_H} + \frac{b^2\tau_\xi}{\tau_A + \tau_s + b^2\tau_\xi}\left(1 + \psi + \eta_c kb\right)\frac{1}{p_A}\right). \tag{30}$$

From equation (26), one also has that $h_P = k\left(1 + p_\xi^{-1}\right)$. Since $p_\xi \leq 0$, it follows that $h_P < k$ whenever $k > 0$.

From $h_s = kb$ and equations (28) and (30), we arrive at

$$p_A = \left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)^{-1}\left(\psi kb + \frac{b^2\tau_\xi}{\tau_A + \tau_s + b^2\tau_\xi}\left(1 + \psi + \eta_c kb\right)\right) > 0. \tag{31}$$

[6]The uniqueness of the positive, real root also follows from the fact that the LHS of the polynomial equation is monotonically increasing in b.

One arrives at p_ξ from recognizing that $p_\xi = -\frac{1}{b}p_A$. Manipulating equation (29), we first we recognize that

$$1 + \psi + \eta_c kb = \left(\tau_A + \tau_s + b^2\tau_\xi\right)(\psi + \eta_c)\, kb\tau_s^{-1}. \tag{32}$$

Substituting equation (32) into equation (31), and invoking equation (29) to replace $k\tau_\xi b^3$, one arrives at

$$p_A = \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1} - \frac{(\psi + \eta_c)\, kb}{\psi k + \frac{1+\psi}{\eta_H} - 1}\tau_s^{-1}\tau_A. \tag{33}$$

From $h_A = kb$, $b = -\frac{p_A}{p_\xi}$, and equations (30), (24) and (23), one also finds that

$$\begin{aligned} p_0 \;=\; & \left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)^{-1}\left(\psi \log\left(\frac{1-\eta_H}{\eta_H}\right) + (1 + \psi + \eta_c kb)\,\frac{\tau_A\bar{A} + b\tau_\xi\bar{\xi}}{\tau_A + \tau_s + b^2\tau_\xi} + \frac{\psi}{2}h_s^2\tau_s^{-1}\right. \\ & \left. + \frac{1+\psi}{2}\left(\eta_c\left(1 + \frac{kb}{1+\psi}\right)^2 + (1-\eta_c)^2\,\eta_H\right)\tau_\varepsilon^{-1} + \frac{1+\psi}{2}\eta_H\left(1 + \frac{\eta_c kb}{1+\psi}\right)^2\left(\tau_A + \tau_s + b^2\tau_\xi\right)^{-1}\right). \end{aligned} \tag{34}$$

Given p_0, p_A, and $b = -\frac{p_A}{p_\xi}$, we can recover h_0 from equation (23).

Since we have explicit expressions for all other equilibrium objects as functions of b, and b exists and is unique, it follows that an equilibrium in the economy exists and is unique.

A.4 Proof of Proposition 4

When all households observe A directly, there are no longer information frictions in the economy. Since the households' idiosyncratic productivity components are unobservable, they are now symmetric. Then, it follows that $H_j = H_i = H$. Imposing this symmetry in equation (6), we see that each household's housing demand is then given by

$$\log H = \frac{1+\psi}{\psi}A - \frac{(1+\psi-\eta_H)}{\psi\eta_H}\log P_H + \log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{(1+\psi)\,\eta_H}{2\psi}\left((1-\eta_c)^2 + \eta_c^2\right)\tau_\varepsilon^{-1}.$$

By market clearing, $\log H = k\left(\xi + \log P_H\right)$, it follows that

$$\begin{aligned} \log P_H \;=\; & \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1}A - \frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1}\xi + \frac{\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1}\log\left(\frac{1-\eta_H}{\eta_H}\right) \\ & + \frac{(1+\psi)\,\eta_H}{2\left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)}\left((1-\eta_c)^2 + \eta_c^2\right)\tau_\varepsilon^{-1}. \end{aligned}$$

This characterizes the economy in the limit as information frictions dissipate.

To see that the economy with information frictions (finite τ_s) converges to this perfect-information limit, we consider a sequence of τ_s that converges to ∞. From equation (29), it follows that, as $\tau_s \nearrow \infty$, $b \to \frac{1+\psi}{\psi k}$. Since $h_s = kb$, it follows that

$$h_s \to \frac{1+\psi}{\psi}.$$

Taking the limit $\tau_s \nearrow \infty$ in equation (31), recognizing that $h_s = kb$ remains finite in the limit, we see that

$$p_A \to \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1}.$$

Since $p_\xi = -\frac{1}{b} p_A$, it follows that

$$p_\xi \to -\frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1}$$

In addition, from equation (30), we find that as $\tau_s \nearrow \infty$,

$$h_P \to \frac{1}{\psi}\left(1 - \frac{1+\psi}{\eta_H}\right)$$

Finally, from equations (34) and (23), it follows that

$$\begin{aligned} p_0 &\to \frac{\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1} \log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{(1+\psi)\,\eta_H}{2\left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)} \left((1-\eta_c)^2 + \eta_c^2\right) \tau_\varepsilon^{-1}, \\ h_0 &\to \log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{(1+\psi)\,\eta_H}{2\psi} \left((1-\eta_c)^2 + \eta_c^2\right) \tau_\varepsilon^{-1}. \end{aligned}$$

Thus, we see that the economy with information frictions converges to the perfect-information benchmark as $\tau_s \nearrow \infty$.

A.5 Proof of Proposition 6

Rewriting equation (29) as

$$kb^3 \tau_A^{-1} \tau_\xi + \left(1 + \frac{\psi}{\psi + \eta_c} \tau_A^{-1} \tau_s\right) kb - \frac{1+\psi}{\psi + \eta_c} \tau_A^{-1} \tau_s = 0,$$

it follows that, as $\tau_A \to \infty$, that $b \to 0$. Thus $p_A \to 0$ and

$$p_\xi = -\left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)^{-1} \left(\psi k + \frac{b\tau_\xi}{\tau_A + \tau_s + b^2 \tau_\xi} \left(1 + \psi + \eta_c k b\right)\right) \to -\frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1},$$

which is its value in the perfect-information benchmark characterized in Proposition 4. Since $b \to 0$, it follows that $h_s = kb \to 0$ and $h_P \to -\frac{1+\psi-\eta_H}{\psi \eta_H}$ as $\tau_A \to \infty$, which are their perfect-information counterparts.

A.6 Proof of Proposition 5

From equation (33), it is clear that

$$p_A = \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1} - \frac{(\psi + \eta_c)\, kb}{\psi k + \frac{1+\psi}{\eta_H} - 1} \tau_s^{-1} \tau_A < \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1}.$$

Thus, it follows that p_A is always lower than its corresponding values in the perfect-information benchmark.

Similarly, since $p_\xi = -\frac{1}{b}p_A$, it follows with equation (31) that we can express p_ξ as

$$p_\xi = -\frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1} - \frac{1}{\psi k + \frac{1+\psi}{\eta_H} - 1}\left(\psi k + \frac{b\tau_\xi}{\tau_A + \tau_s + b^2\tau_\xi}\left(1 + \psi + \eta_c k b\right)\right) < -\frac{\psi k}{\psi k + \frac{1+\psi}{\eta_H} - 1},$$

which is the corresponding value of p_ξ in the perfect-information benchmark.

A.7 Proof of Proposition 7

We first recognize that the price informativeness π can be written as

$$\pi = \frac{p_A^2 \tau_A^{-1}}{p_\xi^2 \tau_\xi^{-1}} = \frac{\tau_\xi}{\tau_A} b^2.$$

Note that b is determined by the polynomial equation (29). We define the LHS of the equation as $G(b)$:

$$G(b) = kb^3\tau_\xi + \left(\tau_A + \frac{\psi}{\psi + \eta_c}\tau_s\right) kb - \frac{1+\psi}{\psi+\eta_c}\tau_s = 0.$$

Comparative statics of b with respect to k and η_c reveal, by the Implicit Function Theorem, that

$$\frac{\partial b}{\partial k} = -\frac{\partial G/\partial k}{\partial G/\partial b} = -\frac{b^3\tau_\xi + \left(\tau_A + \frac{\psi}{\psi+\eta_c}\tau_s\right) b}{3kb^2\tau_\xi + \left(\tau_A + \frac{\psi}{\psi+\eta_c}\tau_s\right) k} = -\frac{b^2\tau_\xi + \tau_A + \frac{\psi}{\psi+\eta_c}\tau_s}{2kb^3\tau_\xi + \frac{1+\psi}{\psi+\eta_c}\tau_s} b^2 < 0,$$

by invoking equation (29) and that $b \geq 0$, and

$$\frac{\partial b}{\partial \eta_c} = -\frac{\partial G/\partial \eta_c}{\partial G/\partial b} = -\frac{1+\psi-\psi kb}{3kb^2\tau_\xi + \left(\tau_A + \frac{\psi}{\psi+\eta_c}\tau_s\right) k} \frac{\tau_s}{(\psi+\eta_c)^2} = -\frac{1+\psi-\psi kb}{2kb^3\tau_\xi + \frac{1+\psi}{\psi+\eta_c}\tau_s} \frac{\tau_s b}{(\psi+\eta_c)^2}.$$

Since, from Proposition 3, $b \leq \frac{1+\psi}{k}\frac{\tau_s}{(\psi+\eta_c)\tau_A + \psi\tau_s}$, it follows that

$$1 + \psi - \psi kb \geq (1+\psi)\frac{(\psi+\eta_c)\tau_A}{(\psi+\eta_c)\tau_A + \psi\tau_s} > 0,$$

Thus, $\frac{\partial b}{\partial \eta_c} < 0$. Since price informativeness $\pi = \frac{\tau_\xi}{\tau_A} b^2$, it follows that π is monotonically decreasing in both k and η_c.

We recall the expressions for $\frac{\partial b}{\partial k} < 0$ and $\frac{\partial b}{\partial \eta_c} < 0$ from the proof of Proposition 7.

Now consider $h_s = kb$. It is straightforward to see that

$$\frac{\partial h_s}{\partial k} = b + \frac{\partial b}{\partial k} = \left(\frac{kb^3\tau_\xi + \frac{1+\psi}{\psi+\eta_c}\tau_s - \left(\tau_A + \frac{\psi}{\psi+\eta_c}\tau_s\right) b}{2kb^3\tau_\xi + \frac{1+\psi}{\psi+\eta_c}\tau_s}\right) b = \frac{2kb^4\tau_\xi}{2kb^3\tau_\xi + \frac{1+\psi}{\psi+\eta_c}\tau_s} > 0,$$

by invoking equation (29). It follows that $\frac{\partial h_s}{\partial k} > 0$.

From the proof of Proposition 5, it is clear that we can express p_A as

$$p_A = \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1}\left(1 - \frac{1+\psi+\eta_c kb}{\tau_A + \tau_s + b^2\tau_\xi}\right).$$

Since $\left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)^{-1}$ is decreasing in k, $-\left(\tau_A + \tau_s + b^2\tau_\xi\right)^{-1}$ is decreasing in k because $\frac{\partial b}{\partial k} < 0$, and kb is increasing k, it follows that $\frac{\partial p_A}{\partial k} < 0$.

Since $\frac{\partial b}{\partial \eta_c}$, it immediately follows that, since $h_s = kb$, $\frac{\partial h_s}{\partial b} < 0$. In addition,

$$\frac{\partial}{\partial \eta_c}(\eta_c b) = b + \eta_c \frac{\partial b}{\partial \eta_c} = \frac{b}{2kb^3\tau_\xi + \frac{1+\psi}{\psi+\eta_c}\tau_s}\left(2kb^3\tau_\xi + \frac{1+\psi}{(\psi+\eta_c)^2}\psi\tau_s + \frac{\psi kb\eta_c\tau_s}{(\psi+\eta_c)^2}\right) > 0.$$

Thus it also follows, by similar logic to that above, that $\frac{\partial p_A}{\partial \eta_c} < 0$.

A.8 Proof of Proposition 8

We first consider the limiting case for the economy as $k \to \infty$. From equation (29), it is apparent that as $k \to \infty$, the first and third terms dominate and either $b = 0$ or $b = \pm i\sqrt{\tau_\xi^{-1}\left(\tau_A + \frac{\psi}{\psi+\eta_c}\tau_\varepsilon\right)}$. Thus, as $k \to \infty$, one has that $b \to 0$. Therefore, $p_A \to 0$ and the housing price is completely driven by the supply shock ξ. Given that $b = -\frac{p_A}{p_\xi}$, we can rewrite equation (30) as

$$h_P = \frac{1}{\psi}\left(1 - \frac{1+\psi}{\eta_H} - (1+\psi+\eta_c kb)\frac{b\tau_\xi}{\tau_A + \tau_\varepsilon + b^2\tau_\xi}\frac{1}{p_\xi}\right).$$

Let us assume that p_ξ is bounded in the limit. Then, as $k \to \infty$ and $b \to 0$, one has that

$$h_P \to -\frac{1}{\psi}\left(\frac{1+\psi}{\eta_H} - 1\right).$$

Then, it follows from equation (27) that $p_\xi \to -1$, since h_P remains bounded in the limit. This confirms the initial conjecture that p_ξ is bounded as $k \to \infty$.

From equation (21), it is straightforward to see that, as $k \to \infty$,

$$h_s \to (1+\psi)\left(\psi + \frac{\eta_c\tau_A}{\tau_A + \tau_\varepsilon}\right)^{-1}\frac{\tau_\varepsilon}{\tau_A + \tau_\varepsilon}.$$

Since h_A remains bounded in the limit, it is easy to see from equation (34) that $p_0 \to 0$ as $k \to \infty$. It further follows from equation (23) that in the limit

$$\begin{aligned} h_0 \;\to\; & \log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{1+\psi}{2\psi}\left(\eta_c\left(1 + \frac{h_A}{1+\psi}\right)^2 + (1-\eta_c)^2\eta_H\right)\tau_\varepsilon^{-1} \\ & + \frac{1+\psi}{2\psi}\eta_H\left(1 + \eta_c\frac{h_A}{1+\psi}\right)^2(\tau_A + \tau_s)^{-1} + \frac{1+\psi+\eta_c h_A}{\psi}\frac{\tau_A}{\tau_A + \tau_s}\bar{A}. \end{aligned} \tag{35}$$

To consider the case $k \to 0$, we first rewrite equation (29) as

$$k\tau_\xi + \left(\tau_A + \frac{\psi}{\psi + \eta_c}\tau_s\right) kb^{-2} - \frac{1+\psi}{\psi+\eta_c}\tau_s b^{-3} = 0.$$

We see that, as $k \to 0$, the last term of the equation dominates and $b \to \infty$. Thus as $k \to 0$, $b \to \infty$, and therefore $p_\xi \to 0$ and the demand shock A completely drives the housing price. From equation (21), it follows that as $k \to 0$ one has that $h_s \to 0$. Since h_A remains bounded in the limit, and $h_A = kb$, it follows from equation (31) that

$$p_A \to \frac{1+\psi}{\frac{1+\psi}{\eta_H} - 1}.$$

Since $h_s \to 0$, and h_A and p_A remain bounded as $k \to 0$, we also see from equation (25) that $h_P \to 0$.Since h_A remains bounded in the limit, it is easy to see from equation (34) that as $k \to 0$.

$$p_0 \to \left(\frac{1+\psi}{\eta_H} - 1\right)^{-1} \left(\psi \log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{1+\psi}{2}\left(\eta_c + (1-\eta_c)^2 \eta_H\right)\tau_\varepsilon^{-1}\right). \tag{36}$$

It further follows from equation (23) that in the limit $h_0 \to 0$.

A.9 Proof of Proposition 9

First, we rewrite the condition $h_P \geq 0$ with equations (30) and (31) as

$$b^2\tau_\xi\left(1 + \psi + \eta_c kb\right)\frac{k\eta_H}{1+\psi-\eta_H} \geq k\left(\tau_A + \tau_s\right)b + kb^3\tau_\xi.$$

Then, substituting for $k\tau_\xi b^3$ with equation (29), the above condition reduces to

$$b \geq b^* = \sqrt{\frac{1+\psi-\eta_H}{k\left(\psi+\eta_c\right)\eta_H}\tau_\xi^{-1}\tau_s}.$$

It follows that, since $b^* \leq b$ and the LHS of equation (29) is monotonically increasing in b, that the LHS of equation (29) evaluated at b^* must be negative. Let $y = \sqrt{k}$. Then,

$$y^2 - \left(\frac{1+\psi}{2\left(\tau_A\left(\psi+\eta_c\right)+\psi\tau_s\right)}\sqrt{\frac{\left(\psi+\eta_c\right)\eta_H}{1+\psi-\eta_H}\tau_s\tau_\xi}\right)y + \frac{1+\psi-\eta_H}{\eta_H}\frac{\tau_s}{\tau_A\left(\psi+\eta_c\right)+\psi\tau_s} \leq 0.$$

Thus, it follows that it is necessary and sufficient for $h_P \geq 0$ that

$$y \in \left[\frac{1+\psi}{2\left(\tau_A\left(\psi+\eta_c\right)+\psi\tau_s\right)}\sqrt{\frac{\left(\psi+\eta_c\right)\eta_H}{1+\psi-\eta_H}\tau_s\tau_\xi} - \rho, \frac{1+\psi}{2\left(\tau_A\left(\psi+\eta_c\right)+\psi\tau_s\right)}\sqrt{\frac{\left(\psi+\eta_c\right)\eta_H}{1+\psi-\eta_H}\tau_s\tau_\xi} + \rho\right],$$

where

$$\rho = \frac{\tau_s^{1/2}}{\tau_A(\psi+\eta_c)+\psi\tau_s}\sqrt{(1+\psi)^2\frac{(\psi+\eta_c)\eta_H}{1+\psi-\eta_H}\tau_\xi - 4\frac{1+\psi-\eta_H}{\eta_H}(\tau_A(\psi+\eta_c)+\psi\tau_s)},$$

provided that

$$\tau_\xi \geq 4\left(\frac{1+\psi-\eta_H}{(1+\psi)\eta_H}\right)^2\left(\tau_A+\frac{\psi}{\psi+\eta_c}\tau_s\right).$$

Recognizing that $y=\sqrt{k}$, the second condition in the proposition follows.

A.10 Proof of Proposition 10

We first conjecture that $\log P_H(2)$ and $\log H_{i2}$ take the following log-linear functional forms

$$\begin{aligned}\log P_H(2) &= p_{A2}A_2 + p_{P1}\log P_H(1) + p_{\xi 2}\xi_2 + p_{02}, \\ \log H_{i2} &= h_{s2}s_{i2} + h_{P1}\log P_H(1) + h_{P2}\log P_H(2) + h_{02}.\end{aligned} \tag{37}$$

Under the conjectured functional form for H_{i2}, equation (6) allows us to expand each generation-2 household's housing demand. It follows that

$$\begin{aligned}
& E\left[e^{A_{i2}(1-\eta_c)\eta_H}\left(\int_{[0,1]/i} e^{A_{j2}}H_{j2}^{\frac{1}{1+\psi}}dj\right)^{\eta_c\eta_H}\middle|\mathcal{I}_{i2}\right] \\
= & E\left[e^{A_{i2}(1-\eta_c)\eta_H}\left(\int_{[0,1]/i} e^{\left(1+\frac{1}{1+\psi}h_{s2}\right)A_{j2}}e^{\frac{1}{1+\psi}(h_{02}+h_{P1}\log P_H(1)+h_{P2}\log P_H(2))}dj\right)^{\eta_c\eta_H}\middle|\mathcal{I}_{i2}\right] \\
= & e^{\frac{\eta_c\eta_H}{1+\psi}(h_{02}+h_{P1}\log P_H(1)+h_{P2}\log P_H(2))}E\left[e^{\eta_H\left(1+\frac{\eta_c}{1+\psi}h_{s2}\right)A_2}e^{\varepsilon_i(1-\eta_c)\eta_H}\left(\int_{[0,1]/i} e^{\left(1+\frac{1}{1+\psi}h_{s2}\right)\varepsilon_{j2}}dj\right)^{\eta_c\eta_H}\middle|\mathcal{I}_{i2}\right] \\
= & e^{\frac{\eta_c\eta_H}{1+\psi}(h_{02}+h_{P1}\log P_{H1}+h_{P2}\log P_{H2})}e^{\frac{\eta_H}{2}\left((1-\eta_c)^2\eta_H+\eta_c\left(1+\frac{1}{1+\psi}h_{s2}\right)^2\right)\tau_\varepsilon^{-1}}E\left[e^{\eta_H\left(1+\frac{\eta_c}{1+\psi}h_{s2}\right)A_2}\middle|\mathcal{I}_{i2}\right]
\end{aligned}$$

Define

$$\begin{aligned}
z_1 &= (1+\phi)\frac{\log P_{H1}-p_0-p_\xi\bar{\xi}}{p_A} = (1+\phi)A_1 + (1+\phi)\frac{p_\xi}{p_A}\left(\xi_1-\bar{\xi}\right), \\
z_2 &= \frac{\log P_{H2}-p_{P1}\log P_{H1}-p_{02}-p_{\xi 2}\bar{\xi}}{p_{A2}} = A_2 + \frac{p_{\xi 2}\left(\xi_2-\bar{\xi}\right)}{p_{A2}},
\end{aligned}$$

which are sufficient statistics for A_1 and A_2 in the observed prices. Let us define $Q_i = \left[\begin{array}{ccc} s_{i2} & z_2 & z_1 \end{array}\right]'$ as the vector of signals observed by household i. Then, we can express the vector Q_i as

$$Q_i = \left[\begin{array}{c} A_2+\sqrt{b}u_1 \\ A_2+\sqrt{c}u_2 \\ A_1+\sqrt{d}u_2 \end{array}\right].$$

where $b = \tau_s^{-1}$, $c = \frac{p_{\xi 2}^2}{p_{A2}^2}\tau_\xi^{-1}$, $d = (1+\phi)^2 \frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}$, and $[u_1, u_2, u_3]' \sim \mathcal{N}(0, I_3)$. We recall that $A_2 = A_1 + \phi Z_1 + Z_2$ under household i's belief, and further define $\beta = (1+\phi)^2$. Then, household i's belief about A_2, $E[A_2 \mid \mathcal{I}_{i2}]$, after observing the vector of signals Q_i, is summarized in the following lemma.

Lemma 12 *Conditional on observing the vector of signals Q_i, the conditional expectation of A_2 of household i $E[A_2 \mid \mathcal{I}_{i2}]$ is given by*

$$E[A_2 \mid \mathcal{I}_{i2}] = \frac{\frac{d/a}{\beta(a+d)+d}}{\frac{1}{b}+\frac{1}{c}+\frac{1}{(1+\beta)a}+\frac{\beta}{1+\beta}\frac{\beta}{\beta(a+d)+d}} A_0 + \frac{\left[\begin{array}{ccc} \frac{1}{b} & \frac{1}{c} & \frac{\beta}{\beta(a+d)+d} \end{array}\right] Q_i}{\frac{1}{b}+\frac{1}{c}+\frac{1}{(1+\beta)a}+\frac{\beta}{1+\beta}\frac{\beta}{\beta(a+d)+d}},$$

with conditional variance

$$Var[A_2 \mid \mathcal{I}_{i2}] = \left(\frac{1}{b}+\frac{1}{c}+\frac{1}{(1+\beta)a}+\frac{\beta}{1+\beta}\frac{\beta}{\beta(a+d)+d}\right)^{-1}.$$

This is the analogue of the traditional formula for Bayesian updating of normally distributed random variables with normal signals, except that one of the signals only contains a piece of the variable being updated. Specifically, the above formula takes into account that z_1 contains information about A_1 which is related to A_2 by $A_2 = A_1 + \phi Z_1 + Z_2$ under household $i's$ beliefs. Thus

$$\begin{aligned} E[A_2 \mid \mathcal{I}_{i2}] &= \left(\tau_s + \frac{p_{A2}^2}{p_{\xi 2}^2}\tau_\xi + \frac{1}{1+(1+\phi)^2}\tau_A + \frac{(1+\phi)^2}{1+(1+\phi)^2}\alpha\right)^{-1} \cdot \\ &\quad \left(\alpha\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}\tau_A A_0 + \tau_s s_{i2} + \frac{p_{A2}^2}{p_{\xi 2}^2}\tau_\xi z_2 + \alpha z_1\right), \end{aligned}$$

where $\alpha = \left(\tau_A^{-1} + \left(1+(1+\phi)^2\right)\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}\right)^{-1}$, and

$$Var[A_2 \mid \mathcal{I}_{i2}] = \left(\tau_s + \frac{p_{A2}^2}{p_{\xi 2}^2}\tau_\xi + \frac{1}{1+(1+\phi)^2}\tau_A + \frac{(1+\phi)^2}{1+(1+\phi)^2}\alpha\right)^{-1}.$$

Substituting this into our expression for $E\left[e^{A_i(1-\eta_c)\eta_H}\left(\int_{[0,1]/i}e^{A_{j2}}H_{j2}^{\frac{1}{1+\psi}}dj\right)^{\eta_c\eta_H}\middle|\mathcal{I}_{i2}\right]$ and then into our $t=2$ analogue of equation (6), we get $\log H_{i2}$

$$
\begin{aligned}
&\log H_{i2}\\
=\;&\frac{\psi}{\psi+\eta_c}\log\left(\frac{1-\eta_H}{\eta_H}\right)+\frac{\eta_H-1-\psi}{(\psi+\eta_c)\eta_H}\log P_H(2)\\
&+\frac{1}{\psi+\eta_c}(1+\psi+\eta_c h_{s2})\left(\tau_s+\frac{p_{A2}^2}{p_{\xi2}^2}\tau_\xi+\frac{1}{1+(1+\phi)^2}\tau_A+\frac{(1+\phi)^2}{1+(1+\phi)^2}\alpha\right)^{-1}\tau_s s_{i2}\\
&+\frac{1}{\psi+\eta_c}\eta_c h_{P2}\log P_H(2)+\frac{1}{\psi+\eta_c}\eta_c h_{P1}\log P_H(1)\\
&+\frac{1+\psi+\eta_c h_{s2}}{\psi+\eta_c}\left(\tau_s+\frac{p_{A2}^2}{p_{\xi2}^2}\tau_\xi+\frac{1}{1+(1+\phi)^2}\tau_A+\frac{(1+\phi)^2}{1+(1+\phi)^2}\alpha\right)^{-1}\frac{p_{A2}}{p_{\xi2}^2}\tau_\xi\log P_H(2)\\
&+\frac{1+\psi+\eta_c h_{s2}}{\psi+\eta_c}\left(\tau_s+\frac{p_{A2}^2}{p_{\xi2}^2}\tau_\xi+\frac{1}{1+(1+\phi)^2}\tau_A+\frac{(1+\phi)^2}{1+(1+\phi)^2}\alpha\right)^{-1}\alpha\left(\frac{1+\phi}{p_A}\right)\log P_H(1)\\
&-\frac{1}{\psi+\eta_c}(1+\psi+\eta_c h_{s2})\left(\tau_s+\frac{p_{A2}^2}{p_{\xi2}^2}\tau_\xi+\frac{1}{1+(1+\phi)^2}\tau_A+\frac{(1+\phi)^2}{1+(1+\phi)^2}\alpha\right)^{-1}\frac{p_{A2}}{p_{\xi2}^2}\tau_\xi p_{P1}\log P_H(1)\\
&+\frac{\eta_c}{\psi+\eta_c}h_{02}+\frac{1}{2}\frac{1+\psi}{\psi+\eta_c}\left((1-\eta_c)^2\eta_H+\eta_c\left(1+\frac{1}{1+\psi}h_{s2}\right)^2\right)\tau_\varepsilon^{-1}\\
&+\frac{1}{\psi+\eta_c}(1+\psi+\eta_c h_{s2})\left(\tau_s+\frac{p_{A2}^2}{p_{\xi2}^2}\tau_\xi+\frac{1}{1+(1+\phi)^2}\tau_A+\frac{(1+\phi)^2}{1+(1+\phi)^2}\alpha\right)^{-1}\alpha\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}\tau_A A_0\\
&-\frac{1}{\psi+\eta_c}(1+\psi+\eta_c h_{s2})\left(\tau_s+\frac{p_{A2}^2}{p_{\xi2}^2}\tau_\xi+\frac{1}{1+(1+\phi)^2}\tau_A+\frac{(1+\phi)^2}{1+(1+\phi)^2}\alpha\right)^{-1}\frac{p_{A2}}{p_{\xi2}^2}\tau_\xi\left(p_{02}+p_{\xi2}\bar{\xi}\right)\\
&-\frac{1}{\psi+\eta_c}(1+\psi+\eta_c h_{s2})\left(\tau_s+\frac{p_{A2}^2}{p_{\xi2}^2}\tau_\xi+\frac{1}{1+(1+\phi)^2}\tau_A+\frac{(1+\phi)^2}{1+(1+\phi)^2}\alpha\right)^{-1}\alpha(1+\phi)\frac{p_0+p_\xi\bar{\xi}}{p_A}\\
&+\frac{1}{2}\frac{1+\psi}{\psi+\eta_c}\eta_H\left(1+\frac{\eta_c}{1+\psi}h_{s2}\right)^2\left(\tau_s+\frac{p_{A2}^2}{p_{\xi2}^2}\tau_\xi+\frac{1}{1+(1+\phi)^2}\tau_A+\frac{(1+\phi)^2}{1+(1+\phi)^2}\alpha\right)^{-1}.
\end{aligned}
$$

Matching coefficients with the conjectured solution, it follows that

$$h_{02} = \log\left(\frac{1-\eta_H}{\eta_H}\right) + \frac{1}{2}\frac{1+\psi}{\psi}\left(\left(1-\eta_c\right)^2\eta_H + \eta_c\left(1+\frac{1}{1+\psi}h_{s2}\right)^2\right)\tau_\varepsilon^{-1}$$
$$+\frac{1+\psi+\eta_c h_{s2}}{\psi}\left(\tau_s + \frac{p_{A2}^2}{p_{\xi 2}^2}\tau_\xi + \frac{1}{1+\left(1+\phi\right)^2}\tau_A + \frac{\left(1+\phi\right)^2}{1+\left(1+\phi\right)^2}\alpha\right)^{-1}\alpha\frac{p_\xi^2}{p_A^2}\tau_\xi^{-1}\tau_A A_0$$
$$-\frac{1+\psi+\eta_c h_{s2}}{\psi}\left(\tau_s + \frac{p_{A2}^2}{p_{\xi 2}^2}\tau_\xi + \frac{1}{1+\left(1+\phi\right)^2}\tau_A + \frac{\left(1+\phi\right)^2}{1+\left(1+\phi\right)^2}\alpha\right)^{-1}\frac{p_{A2}}{p_{\xi 2}^2}\tau_\xi\left(p_{02}+p_{\xi 2}\bar{\xi}\right)$$
$$-\frac{1+\psi+\eta_c h_{s2}}{\psi}\left(\tau_s + \frac{p_{A2}^2}{p_{\xi 2}^2}\tau_\xi + \frac{1}{1+\left(1+\phi\right)^2}\tau_A + \frac{\left(1+\phi\right)^2}{1+\left(1+\phi\right)^2}\alpha\right)^{-1}\alpha\left(1+\phi\right)\frac{p_0+p_\xi\bar{\xi}}{p_A}$$
$$+\frac{\eta_H}{2}\frac{1+\psi}{\psi}\left(1+\frac{\eta_c}{1+\psi}h_{s2}\right)^2\left(\tau_s + \frac{p_{A2}^2}{p_{\xi 2}^2}\tau_\xi + \frac{1}{1+\left(1+\phi\right)^2}\tau_A + \frac{\left(1+\phi\right)^2}{1+\left(1+\phi\right)^2}\alpha\right)^{-1}. \quad (38)$$

and

$$h_{s2} = \frac{1}{\psi+\eta_c}\left(1+\psi+\eta_c h_{s2}\right)\left(\tau_s + \frac{p_{A2}^2}{p_{\xi 2}^2}\tau_\xi + \frac{1}{1+\left(1+\phi\right)^2}\tau_A + \frac{\left(1+\phi\right)^2}{1+\left(1+\phi\right)^2}\alpha\right)^{-1}\tau_s, \quad (39)$$

$$h_{P1} = \frac{1+\psi+\eta_c h_{s2}}{\psi}\left(\tau_s + \frac{p_{A2}^2}{p_{\xi 2}^2}\tau_\xi + \frac{1}{1+\left(1+\phi\right)^2}\tau_A + \frac{\left(1+\phi\right)^2}{1+\left(1+\phi\right)^2}\alpha\right)^{-1}\left(\alpha\frac{1+\phi}{p_A} - \frac{p_{A2}}{p_{\xi 2}^2}\tau_\xi p_{P1}\right), \quad (40)$$

$$h_{P2} = \frac{\eta_H - 1 - \psi}{\psi\eta_H} + \frac{1+\psi+\eta_c h_{s2}}{\psi}\left(\tau_s + \frac{p_{A2}^2}{p_{\xi 2}^2}\tau_\xi + \frac{1}{1+\left(1+\phi\right)^2}\tau_A + \frac{\left(1+\phi\right)^2}{1+\left(1+\phi\right)^2}\alpha\right)^{-1}\frac{p_{A2}}{p_{\xi 2}^2}\tau_\xi \quad (41)$$

Aggegating household demand and builder supply directly and imposing market clearing in the housing market, one also has that

$$h_{02} + h_{P1}\log P_{H1} + h_{P2}\left(p_{02} + p_{A2}A_2 + p_{P1}\log P_{H1} + p_{\xi 2}\xi_2\right) + h_{s2}A_2 + \frac{1}{2}h_{s2}^2\tau_\varepsilon^{-1}$$
$$= k\left(\xi_2 + p_{02} + p_{A2}A_2 + p_{P1}\log P_{H1} + p_{\xi 2}\xi_2\right).$$

Matching coefficients with the conjecture solution equation (37) gives rise to the following five conditions

$$h_{02} + h_{P2}p_{02} + \frac{1}{2}h_{s2}^2\tau_\varepsilon^{-1} = kp_{02}, \quad (42)$$
$$h_{P1} + h_{P2}p_{P1} = kp_{P1}, \quad (43)$$
$$h_{P2}p_{A2} + h_{s2} = kp_{A2}, \quad (44)$$
$$h_{P2}p_{\xi 2} = k\left(1+p_{\xi 2}\right). \quad (45)$$

This confirms the log-linear form of $P_H\left(2\right)$ and therefore the conjectured form of the equilibrium.

It follows from equations (44) and (45):

$$\frac{h_{s2}}{k} = -\frac{p_{A2}}{p_{\xi 2}},$$

Define $x = -\frac{p_{A2}}{p_{\xi 2}}$. Then substituting $x = -\frac{p_{A2}}{p_{\xi 2}}$ and $h_{s2} = kx$ into equation (39), one arrives at a third-order polynomial that identifies x:

$$k\tau_\xi x^3 + \left(\frac{\psi}{\psi + \eta_c}\tau_s + \frac{1}{1 + (1+\phi)^2}\tau_A + \frac{(1+\phi)^2}{1 + (1+\phi)^2}\alpha \right) kx - \frac{1+\psi}{\psi + \eta_c}\tau_s = 0. \tag{46}$$

We see from equation (46) that x has at most one positive root and zero negative roots by Descartes' Rule of Signs. Thus, it follows that equation (46) has only one real, nonnegative root and 2 complex roots that exist. Thus x exists and is unique.

Given x, we can recover h_{s2} as $h_{s2} = kx$ and $p_{\xi 2} = -\frac{1}{x}p_{A2}$, as well as h_{P2} from equation (41)

$$h_{P2} = -\frac{1+\psi-\eta_H}{\psi\eta_H} + \frac{1+\psi+\eta_c kx}{\psi} \frac{x^2\tau_\xi}{\tau_s + x^2\tau_\xi + \frac{1}{1+(1+\phi)^2}\tau_A + \frac{(1+\phi)^2}{1+(1+\phi)^2}\alpha} \frac{1}{p_{A2}}. \tag{47}$$

Furthermore, given $h_{s2} = kx$ and h_{P2} from equation (47), we can recover p_{A2} from equation (44) as

$$p_{A2} = \left(\psi k + \frac{1+\psi}{\eta_H} - 1 \right)^{-1} \left(\psi kx + (1+\psi+\eta_c kx) \frac{x^2\tau_\xi}{\tau_s + x^2\tau_\xi + \frac{1}{1+(1+\phi)^2}\tau_A + \frac{(1+\phi)^2}{1+(1+\phi)^2}\alpha} \right) > 0. \tag{48}$$

Manipulating equation (46), we first we recognize that

$$1 + \psi + \eta_c kx = \left(\tau_s + \tau_\xi x^2 + \frac{1}{1 + (1+\phi)^2}\tau_A + \frac{(1+\phi)^2}{1 + (1+\phi)^2}\alpha \right) (\psi + \eta_c)\, \tau_s^{-1} kx. \tag{49}$$

Substituting equation (49) into equation (48), and invoking equation (46) to replace $k\tau_\xi b^3$, one arrives at

$$p_{A2} = \frac{1}{\psi k + \frac{1+\psi}{\eta_H} - 1} \left(1 + \psi - \frac{\tau_A + (1+\phi)^2 \alpha}{1 + (1+\phi)^2} (\psi + \eta_c)\, \tau_s^{-1} kx \right).$$

From equation (43), it follows that $h_{P1} = (k - h_{P2})\, p_{P1}$, and substituting this, equation (41), and $h_{A2} = kx$ into equation (40), one arrives at

$$p_{P1} = \left(\psi k + \frac{1+\psi}{\eta_H} - 1 \right)^{-1} \frac{\alpha \left(\frac{1+\phi}{p_A} \right)}{\tau_s + x^2\tau_\xi + \frac{1}{1+(1+\phi)^2}\tau_A + \frac{(1+\phi)^2}{1+(1+\phi)^2}\alpha} (1 + \psi + \eta_c kx) > 0,$$

from which we can recover h_{P1}. Substituting equation (49) reduces the above to

$$p_{P1} = \left(\psi k + \frac{1+\psi}{\eta_H} - 1\right)^{-1} \alpha \tau_s^{-1} (\psi + \eta_c) kx \left(\frac{1+\phi}{p_A}\right).$$

Futhermore, since $h_{P2} < k$, it follows that $h_{P1} > 0$.

In addition, it is easy to see that p_{02} and h_{02} can be recovered from (42) and (38).

Since we have explicit expressions for all other equilibrium objects as functions of x, and x exists and is unique, it follows that an equilibrium in the economy exists and is unique.

A.11 Proof of Proposition 11

We first derive the short-run return covariance:

$$\begin{aligned}
& Cov\left[R_H(0,1), R_H(1,2)\right] \\
= \; & Cov\left[p_A A_1 + p_\xi \xi_1 - \frac{(1+\psi) A_0 - \psi k \xi_0}{\psi k + \frac{1+\psi}{\eta_H} - 1},\ p_{A2} A_2 + p_{\xi 2} \xi_2 + (p_{P1} - 1) p_A A_1 + (p_{P1} - 1) p_\xi \xi_1\right] \\
= \; & (p_{P1} - 1) p_\xi^2 \tau_\xi^{-1} + Cov\left[p_A A_1 - \frac{1+\psi}{\psi k + \frac{1+\psi}{\eta_H} - 1} A_0,\ p_{A2} A_2 + (p_{P1} - 1) p_A A_1\right],
\end{aligned}$$

since all supply shocks (ξ_0, ξ_1, ξ_2) are mutually independent and independent of all other shocks. Furthermore, since $A_1 = A_0 + Z_1$ and $A_2 = A_0 + Z_1 + Z_2$, it follows that

$$\begin{aligned}
& Cov\left[R_H(0,1), R_H(1,2)\right] \\
= \; & (p_{P1} - 1) p_\xi^2 \tau_\xi^{-1} + Cov\left[p_A Z_1, p_{A2} Z_2 + (p_{A2} + (p_{P1} - 1) p_A) Z_1\right] \\
= \; & (p_{P1} - 1) p_\xi^2 \tau_\xi^{-1} + (p_{A2} + (p_{P1} - 1) p_A) p_A \tau_A^{-1},
\end{aligned}$$

since A_0 is observable and Z_1, and Z_2 are mutually independent. We can decompose the short-run return covariance as

$$Cov\left[R_H(0,1), R_H(1,2)\right] = -p_\xi^2 \tau_\xi^{-1} + (p_{A2} - p_A) p_A \tau_A^{-1} + p_{P1}\left(p_\xi^2 \tau_\xi^{-1} + p_A^2 \tau_A^{-1}\right)$$

where the first term captures the negative auto-covariance induced by transitory supply shocks, the second and third terms captures auto-covariance induced by extrapolative learning.

We now derive the long-term return covariance:

$$\begin{aligned}
& Cov\left[R_H\left(0,2\right),R_H\left(2,3\right)\right] \\
= \; & Cov\left[p_{A2}A_2+p_{\xi 2}\xi_2+p_{P1}\log P_H\left(1\right)-\frac{1+\psi}{\psi k+\frac{1+\psi}{\eta_H}-1}A_0-\frac{\psi k}{\psi k+\frac{1+\psi}{\eta_H}-1}\xi_0,\right. \\
& \left.\frac{1+\psi}{\psi k+\frac{1+\psi}{\eta_H}-1}A_3-\frac{\psi k}{\psi k+\frac{1+\psi}{\eta_H}-1}\xi_3-p_{A2}A_2-p_{\xi 2}\xi_2-p_{P1}\log P_H\left(1\right)\right] \\
= \; & Cov\left[p_{A2}A_2+p_{\xi 2}\xi_2+p_{P1}\log P_H\left(1\right),-p_{A2}A_2-p_{\xi 2}\xi_2-p_{P1}\log P_H\left(1\right)\right] \\
= \; & 2\frac{1+\psi}{\psi k+\frac{1+\psi}{\eta_H}-1}p_{A2}\tau_A^{-1}+p_{P1}p_A\left(\frac{1+\psi}{\psi k+\frac{1+\psi}{\eta_H}-1}-2p_{A2}\right)\tau_A^{-1} \\
& -\left(2p_{A2}^2\tau_A^{-1}+p_{\xi 2}^2\tau_\xi^{-1}+p_{P1}^2Var\left(\log P_H\left(1\right)\right)\right),
\end{aligned}$$

since $(Z_1, Z_2, Z_3, \xi_1, \xi_2, \xi_3)$ are mutually independent shocks and A_0 is known at $t=0$. Since $Var\left(\log P_H\left(2\right)\right)=p_{\xi 2}^2\tau_\xi^{-1}+2p_{A2}^2\tau_A^{-1}+p_{P1}^2Var\left(\log P_H\left(1\right)\right)$, we obtain $Cov\left[R_H\left(0,2\right),R_H\left(2,3\right)\right]$ given in the proposition.

A.12 Proof of Lemma 1

Define Q_i to be the 3×1 vector of signals observed by household i that take the form

$$Q_i=\left[\begin{array}{c} A_2+\sqrt{b}u_1 \\ A_2+\sqrt{c}u_2 \\ A_1+\sqrt{d}u_2 \end{array}\right].$$

where $[u_1,u_2,u_3]'\sim\mathcal{N}\left(0,I_3\right)$ and we remind that $A_2=A_2+Z_2+\phi Z_1$. The households have a prior $A_2\sim\mathcal{N}\left(A_0,\left(1+\beta\right)a\right)$. Then, conditional on Q_i, household $i's$ belief about A_2 is given by

$$E\left[A_2\mid\mathcal{I}_{i2}\right]=A_0+Cov\left[A_2,Q_i\right]'Var\left[Q_i\right]^{-1}\left(Q_i-A_0\mathbf{1}_{3\times 1}\right),$$

where

$$Var\left[Q_i\right]=\left[\begin{array}{ccc} \left(1+\beta\right)a+b & \left(1+\beta\right)a & \beta a \\ \left(1+\beta\right)a & \left(1+\beta\right)a+c & \beta a \\ \beta a & \beta a & \beta a+d \end{array}\right],$$

and

$$Cov\left[A_2,Q_i\right]'=\left[\begin{array}{ccc} \left(1+\beta\right)a & \left(1+\beta\right)a & \beta a \end{array}\right]$$

It follows that the inverse of this matrix, which can be found by using cofactors, is

$$Var\left[Q_i\right]^{-1}=\frac{1}{\left|Var\left[Q_i\right]\right|}\left[\begin{array}{ccc} \left(a+c\right)\left(\beta a+d\right)+\beta ad & -\beta a\left(a+d\right)-ad & -\beta ac \\ -\beta a\left(a+d\right)-ad & \left(a+b\right)d+\left(a+b+d\right)\beta a & -\beta ab \\ -\beta ac & -\beta ab & \left(1+\beta\right)a\left(b+c\right)+bc \end{array}\right],$$

and, through a series of row manipulations,

$$|Var\left[Q_i\right]| = \left| \begin{array}{ccc} b & -c & 0 \\ a+b & a & -d \\ -\beta b & 0 & \beta\left(a+d\right)+d \end{array} \right| = a\left(b+c\right)\left(\beta\left(a+d\right)+d\right)+bc\left(\beta a+d\right).$$

Thus it follows that

$$\begin{aligned} & Cov\left[A_2, Q_i\right]' Var\left[Q_i\right]^{-1} \\ = & \frac{1}{a\left(b+c\right)\left(\beta\left(a+d\right)+d\right)+bc\left(\beta a+d\right)} \left[\begin{array}{ccc} ac\left(\beta\left(a+d\right)+d\right) & ab\left(\beta\left(a+d\right)+d\right) & \beta abc \end{array} \right] \\ = & \frac{1}{\frac{1}{b}+\frac{1}{c}+\frac{1}{(1+\beta)a}+\frac{\beta}{1+\beta}\frac{\beta}{\beta(a+d)+d}} \left[\begin{array}{ccc} \frac{1}{b} & \frac{1}{c} & \frac{\beta}{\beta(a+d)+d} \end{array} \right] \end{aligned}$$

Substituting this expression for $Cov\left[A_2, Q_i\right]' Var\left[Q_i\right]^{-1}$ into our general expression for household $i's$ beliefs $E\left[A_2 \mid \mathcal{I}_{i2}\right]$, we see that

$$E\left[A_2 \mid \mathcal{I}_{i2}\right] = \frac{\frac{d/a}{\beta(a+d)+d}}{\frac{1}{b}+\frac{1}{c}+\frac{1}{(1+\beta)a}+\frac{\beta}{1+\beta}\frac{\beta}{\beta(a+d)+d}} A_0 + \frac{1}{\frac{1}{b}+\frac{1}{c}+\frac{1}{(1+\beta)a}+\frac{\beta}{1+\beta}\frac{\beta}{\beta(a+d)+d}} \left[\begin{array}{ccc} \frac{1}{b} & \frac{1}{c} & \frac{\beta}{\beta(a+d)+d} \end{array} \right] Q_i.$$

Furthermore, he conditional variance of household $i's$ beliefs $Var\left[A_2 \mid \mathcal{I}_{i2}\right]$ is given by

$$Var\left[A_2 \mid \mathcal{I}_{i2}\right] = \left(1+\beta\right)a - Cov\left[A_2, Q_i\right]' Var\left[Q_i\right]^{-1} Cov\left[A_2, Q_i\right],$$

from which it is straightforward to see that

$$Var\left[A_2 \mid \mathcal{I}_{i2}\right] = \left(\frac{1}{b}+\frac{1}{c}+\frac{1}{\left(1+\beta\right)a}+\frac{\beta}{1+\beta}\frac{\beta}{\beta\left(a+d\right)+d}\right)^{-1}.$$

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