Population Aging and Deflation: Japanese Experience

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1 Extended Abstract

Japan has been experiencing deflation for more than 15 years. Many view the persistent deflation as a result of insufficient aggregate demand and thus have criticized the Bank of Japan as being too passive and slow in combatting the deflationary pressure. This paper offers a different take on the underlying causes of deflation. That is, we explore a link between deflation and low-frequency structural changes facing the Japanese economy in the last two decades or so, namely population aging and its interaction with the labor market.

We develop a search/matching model embedded in a standard New Keynesian framework. The labor market in our model is designed to capture salient features of the Japanese labor market: In the model, workers are heterogeneous in their age and level of *firm-specific* human capital. It is well known that the Japanese labor market is characterized by a high degree of human capital specificity that is tied to a particular firm, unlike the U.S. labor market where human capital is considered occupation specific rather than firm specific.[1]

The key mechanism in our model in explaining persistent deflation lies in the declining unit labor cost of workers who have already accumulated firm specific skills (these workers are "old" workers in our model). Remember that, in the standard search/matching model with Nash bargaining, the worker's outside option value plays an important role in determining real wage (e.g., the lower value of being unemployed, say, due to a lower job finding rate, lowers the current wage). In our model with firm specific human capital, this channel takes a

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[1]We do not look into why the Japanese labor market has evolved into the current form. Rather, the emphasis on firm specific human capital in the Japanese labor market is taken as given in the paper. We discuss our modeling choice in the full draft.

more stringent form. For example, in an extreme case where human capital is completely firm specific, the outside option of the worker is to find a new job in which his prior experience is completely worthless. This fear factor in turn influences the wage bargaining at the worker's present job. We show that, in our model, this fear factor increases, as population aging progresses, thereby putting a gradual downward pressure on real wages of the old worker with firm specific skills. For the firm, this implies a lower unit labor cost, given that the firm pays lower wage even when the worker's productivity remains the same. Our model also features the mechanism through which young workers who have not accumulated firm specific skills are also adversely affected. This indirect effect results from the firm's reduced incentive to create new jobs. What happens in our model essentially is that the presence of a larger pool of older workers crowed out young workers' job opportunities. In the full draft, we present micro and macro evidence on the Japanese labor market that is broadly in line with these predictions.

We embed this labor market structure into a standard New Keynesian framework with nominal rigidities and show that population aging leads to persistent delation. It is important to note that a labor market with a high degree of firm specificity is particularly susceptible to "chronic" deflation in the face of population aging. In other words, aging and firm specificity of human capital are *both* essential in accounting for the persistent deflationary pressure. In this sense, the Japanese experience is quite peculiar and thus may not apply to other countries such as the U.S.

The analysis described so far assumes that the central bank follows the Taylor-type instrument rule in determining the policy interest rate. However, we also plan to study the optimal monetary policy, so that we can evaluate the Japan's actual monetary policy in light of the optimal policy. Below we present the current version of the model.

2 Model

Our model is designed to capture, what we believe, are salient features of the Japanese labor market, namely, aging and firm specificity of human capital. The economy consists of households, wholesale and retail firms, and a monetary authority. Wholesale firms produce an homogenous good which is sold in a competitive market to retail firms. Labor is the only input for production and there are three types of workers, as specified below. Hiring of workers is subject to search friction. Retail firms sell differentiated goods to households, and the retail sector is characterized by monopolistic competition and price stickiness as in standard new Keynesian (NK) models. A monetary authority determines the policy interest

rates following the Taylor type rule.

We first show the real model without nominal rigidities in order to investigate how aging can affect real wage or unit labor costs. Then, such a model is extended to incorporate nominal rigidities and monetary policy. With such a NK model with aging and firm specificity of human capital, we can understand the structural relationship between aging and deflation.

2.1 Labor Market: Real Model

In the economy, there are young and old workers who can work in the intermediate goods sector. A mass of ψ_t young workers are born every period and enter into the labor market as unemployed. Young workers become old with probability ϕ_t every period. Old workers die with probability γ_t every period. [2] When young workers become old, they also become "experienced," having higher labor productivity by a factor of $1 + \mu$. Young and old employed workers are subject to exogenous job separation probabilities, s_y and s_o, respectively. After the separations occur, workers enter into the matching market where workers look for a new job opportunity. We capture the specificity of human capital by assuming that experienced (i.e., old) workers lose their skills at the time of separation. When these workers lose their skills, the worker becomes inexperienced, losing their experience premium in their productivity. If human capital is fully firm specific, the separation results in a complete loss of skills, but we assume that only a fraction of human capital is firm specific. We capture this by assuming that the loss of the firm specific skill old occurs only with probability δ. With probability $1-\delta$, the worker remains experienced. In summary, there are three types of workers in the model: (i) young and inexperienced workers, (ii) old and experienced workers, and (iii) old and inexperienced workers. Note that all young workers are inexperienced and thus the terms "young" and "inexperienced" are used interchangeably in the model. Note also that all experienced workers are old workers. However, there are old workers that used to be experienced but are currently inexperienced due to job loss.

The unemployment pool consists of these three types of workers. We assume that the matching market is divided by the skill level, meaning that experienced workers and inexperienced workers participate in the different matching markets. In other words, the firms hire workers separately in the labor market for different types of jobs: (i) jobs that require previous experience thus suitable only for experienced (and hence old) workers and (ii) entry-level jobs that any workers can be employed at. We call the matching market for the first type of job "E-matching market" and for the latter type of job "I-matching market." The compo-

[2] See Cheron et al. and Esteban-Pretel and Fujimoto for search/matching models with a more explicit demographic structure.

sition of this market will be affected by the demographic structure as well as the extent of the specificity of human capital (i.e., δ). The two matching markets are represented by the following matching functions:

$$m^i = \bar{m}^i (u_t^i)^\alpha (v_t^i)^{1-\alpha}, \tag{1}$$

$$m^e = \bar{m}^e (u_t^e)^\alpha (v_t^e)^{1-\alpha}, \tag{2}$$

where the subscripts i and e indicate the I-matching market and E-matching market, respectively; m, u, v, $\bar{m}$, and α are, respectively, the number of matches, the number of job seekers, the number of job openings, the scale parameter, and the elasticity with respect to unemployment. The job finding rate $\frac{m}{u}$, and job filling rate $\frac{m}{v}$ in each market are denoted, respectively by $f(\theta)$ and $q(\theta)$, with subscript either i or e. θ is market tightness is the ratio of vacancies to unemployment in each market.

2.1.1 Labor Market Equilibrium Conditions

Given the transition probabilities specified above, we write the Bellman equations for each type of workers. Subscripts, y, o, and $\tilde{o}$ in the Bellman equations below, respectively, indicate young workers, experienced old workers, and inexperienced old workers.

Old workers. Let us start with the experienced worker. If he is employed, the value H^o evolves according to:

$$H_t^o = w_t^o + \mathbb{E}_t \tilde{\beta}_{t,t+1}(1-\gamma_t)\Big[(1-s^e)H_{t+1}^o + s^e\big\{\delta U_{t+1}^{\tilde{o}} + (1-\delta)U_{t+1}^o\big\}\Big], \tag{3}$$

where $\tilde{\beta}$ is the stochastic discount factor, which will be specified below, w_t^o is current-period real wage paid to the worker, s^e is the job destruction probability of the experienced match, U^o is the value of the old worker who is currently in the unemployment pool as an experienced worker, and $U_t^{\tilde{o}}$ is the value of the unemployed old worker who has become inexperienced at the time of job loss. When the experienced worker is in the unemployment pool, the value follows:

$$U_t^o = b^o u_t^o + \mathbb{E}_t \tilde{\beta}_{t,t+1}(1-\gamma_t)\Big[f^e(\theta_t^e)H_{t+1}^o + (1-f^e(\theta_t^e))U_{t+1}^o\Big], \tag{4}$$

where b_t^o represents the value of leisure of the old worker. Next, the value of the job filled with the experienced worker J^o is written as:

$$J_t^o = (1+\mu)n_t^o - w_t^o n_t^o + \mathbb{E}_t \tilde{\beta}_{t,t+1}(1-\gamma_t)(1-s^e)J_{t+1}^o. \tag{5}$$

Note that$(1+\mu)y_t$ is the flow output produced by the worker-firm relationship. As mentioned above, μ is the experience premium over productivity of the inexperienced worker, y_t. Also note (5) imposes that the firm gets 0 when separation occurs as a consequence of free entry.

Next, consider the situation in which the old worker is employed as an inexperienced worker. The value $H^{\tilde{o}}$ evolves according to:

$$H_t^{\tilde{o}} = w_t^{\tilde{o}} + \mathbb{E}_t \tilde{\beta}_{t,t+1}(1-\gamma_t)\Big[(1-s^i)H_{t+1}^{\tilde{o}} + s^i U_{t+1}^{\tilde{o}}\Big], \tag{6}$$

where $w_t^{\tilde{o}}$ is current-period real wage paid to the worker, s^i is the job destruction probability of the inexperienced match, and $U_t^{\tilde{o}}$ is the value of the worker who is looking for a job as an inexperienced worker after the job loss. When the old worker is in the matching market as an inexperienced worker, the value is written as:

$$U_t^{\tilde{o}} = b^o + \mathbb{E}_t \tilde{\beta}_{t,t+1}(1-\gamma_t)\Big[f^i(\theta_t^i)H_{t+1}^{\tilde{o}} + (1-f^i(\theta_t^i))U_{t+1}^{\tilde{o}}\Big]. \tag{7}$$

The interpretation is straightforward. Note that once the worker has become inexperienced, there is no chance that he regains the skills and ever becomes an experienced worker. The value of the job that employs the inexperienced old worker is written as follows.

$$J_t^{\tilde{o}} = y_y - w_t^{\tilde{o}} + \mathbb{E}_t \tilde{\beta}_{t,t+1}(1-\gamma_t)(1-s^i)J_{t+1}^{\tilde{o}}, \tag{8}$$

where, as in (5), we impose the free entry condition.

Young workers. The continuation values of the young worker with and without job are written as follows.

$$\begin{aligned} H_t^y = w_t^y &+ \mathbb{E}_t \tilde{\beta}_{t,t+1}(1-\phi_t)\Big[(1-s^i)H_{t+1}^y + s^i U_{t+1}^y\Big] \\ &+ \tilde{\beta}\phi_t\Big[(1-s^e)H_{t+1}^o + s^e\big\{\delta U_{t+1}^{\tilde{o}} + (1-\delta)U_{t+1}^o\big\}\Big], \end{aligned} \tag{9}$$

$$\begin{aligned} U_t^y = b^y &+ \mathbb{E}_t \tilde{\beta}_{t,t+1}(1-\phi_t)\Big[f^i(\theta_t^i)H_{t+1}^y + (1-f^i(\theta_t^i))U_{t+1}^y\Big] \\ &+ \tilde{\beta}_{t,t+1}\phi_t\Big[f^i(\theta_t^i)H_{t+1}^o + (1-f^i(\theta_t^i))\big\{\delta U_{t+1}^{\tilde{o}} + (1-\delta)U_{t+1}^o\big\}\Big], \end{aligned} \tag{10}$$

where ϕ_t in both equations capture the possibility that each worker becomes old in each period. Note that we assume that the possibility of job finding realizes before the probability of becoming old. If the worker is young today, then job finding occurs based on that status. The worker then may or may not become old.[3] The interpretation of (9) and (10) are straightforward. There is one issue, though, in (10) that needs some clarification. That is,

[3]In the discrete time setup, we have to take a stand on which event happens first and this ordering appears more reasonable than the opposite one.

when the worker fails to find a job and becomes old, these workers are subject to the δ risk, as represented in the expression in the curly brackets in (10).

Lastly, the value of the job is:

$$J_t^y = y_t - w_t^y + \mathbb{E}_t \tilde{\beta}_{t,t+1}\big[(1-\phi_t)(1-s^i)J_{t+1}^y + \phi_t(1-s^e)J_{t+1}^o\big].$$

Surplus. We assume that the worker and the firm divides the surplus over their respective outside option values, according to a fixed share η being the worker bargaining power and $1-\eta$ being the firm bargaining power. The total surplus for the experienced match is written as: $S_t^o = H_t^o + J_t^o - \big[\delta U_{t+1}^{\tilde{o}} + (1-\delta)U^o\big]$. By using the expressions for the continuation values previously specified in this expression results in:

$$S_t^o = (1+\mu)y_t - b^o + \mathbb{E}_t\tilde{\beta}_{t,t+1}(1-\gamma_t)\Big[(1-s^e-\eta f^e(\theta_t^e))S_{t+1}^o + (1-f^e(\theta_t^e))\delta(U_{t+1}^{\tilde{o}} - U_{t+1}^o)\Big]. \quad (11)$$

Note that the surplus depends on the difference in the values of unemployment $U^{\tilde{o}} - U^o$ whose evolution can be characterized by:

$$U_t^{\tilde{o}} - U_t^o = \mathbb{E}_t\tilde{\beta}_{t,t+1}(1-\gamma_t)\Big[\eta(f^i(\theta_t^i)S_{t+1}^{\tilde{o}} - f^e(\theta_t^e)S_{t+1}^o) + (1-f^e(\theta_t^e)\delta)(U_{t+1}^{\tilde{o}} - U_{t+1}^o)\Big]. \quad (12)$$

Next, the surplus value of the inexperienced match that employs an old worker is $S_t^{\tilde{o}} = H_t^{\tilde{o}} + J_t^{\tilde{o}} - U_t^{\tilde{o}}$. The evolution of this surplus takes the following simple form:

$$S_t^{\tilde{o}} = y_t - b^o + \mathbb{E}_t\tilde{\beta}_{t,t+1}(1-\gamma_t)(1-s^i - f^i(\theta_t^i)\eta)S_{t+1}^{\tilde{o}}. \quad (13)$$

Lastly, one can obtain the surplus value function for the young worker as follows:

$$S_t^y = y_t - b_t^y + \mathbb{E}_t\tilde{\beta}_{t,t+1}(1-\phi_t)\Big[(1-s^i - f^i(\theta_t^i)\eta)S_{t+1}^y\Big] + \phi_t\Big[(1-s^e - f^i(\theta_t^i)\eta)S_{t+1}^o\Big]. \quad (14)$$

Job creation conditions. The representative wholesale firm maximizes the value of the firm by posting vacancies (for the two types of jobs) up to the point where the marginal cost of positing a vacancy equals the marginal return from the vacancy. These conditions are written as:

$$c^i = \mathbb{E}_t\tilde{\beta}_{t,t+1}q^i(\theta_t^i)(1-\eta)\Big[\lambda S_{t+1}^y + (1-\lambda)S_{t+1}^{\tilde{o}}\Big], \quad (15)$$

$$c^e = \mathbb{E}_t\tilde{\beta}_{t,t+1}q^e(\theta_t^e)(1-\eta)S_{t+1}^o. \quad (16)$$

Equation (15) represents the job creation condition for jobs for inexperienced workers, while Equation (16) is the job creation condition for jobs for experienced workers. Note that λ in (15) is the share of young workers in the matching market for inexperienced workers. The equilibrium values of the model are characterized by the value functions (11), (12), (13), (14), and the above two job creation conditions.

Output. We assume that output of wholesale goods, denoted by y_t, is obtained by linearly aggregating over the output produced by the three types of workers and thus can be written as:

$$Q_t = y_t \left[n_t^y + n_t^{\tilde{o}} + (1+\mu) n_t^o \right], \tag{17}$$

where n_t is the employment stock in period t with each superscript indicating the worker type.

2.1.2 Stock-Flow Relationships

New born workers h_t enter the labor market as unemployed. We also suppose that production and matching activities are mutually exclusive activities and both take place during the period. We have to make some timing assumptions regarding the events that occur at the end of each period after production or matching activities. These timing assumptions are quantitatively innocuous The timing is as follows.

1. During each period, production and matching take place.
2. At the end of each period, some old workers die.
3. Some young workers become old.
4. Separations for each group of workers occur.

Under these timing assumptions, the transition law of motion for each stock is expressed as follows.

$$n_{t+1}^y = n_t^y - \phi_t n_t^y - (1-\phi_t) s^y n_t^y + f^i(\theta_t^i)(1-\phi_t) u_t^y, \tag{18}$$
$$u_{t+1}^y = u_t^y + h_t + (1-\phi_t) s^i n_t^y - f^i(\theta_t^i) u_t^y - (1 - f^i(\theta_t^i)) \phi_t u_t^y, \tag{19}$$
$$n_{t+1}^o = n_t^o - \gamma_t n_t^o - (1-\gamma_t) s^e n_t^o + (1-s^e)\phi_t n_t^y + \phi_t f^i(\theta_t^i) u_t^y + (1-\gamma_t) f^e(\theta_t^e) u_t^o, \tag{20}$$
$$u_{t+1}^o = u_t^o - f^e(\theta_t^e) u_t^o - (1 - f^e(\theta_t^e))\gamma_t u_t^o + (1-\delta)(1-\gamma_t) s^e n_t^o + (1 - f^i(\theta_t^i))\phi_t u_t^y, \tag{21}$$
$$n_{t+1}^{\tilde{o}} = n_t^{\tilde{o}} - \gamma_t n_t^{\tilde{o}} - (1-\gamma_t) s^i n_t^{\tilde{o}} + (1-\gamma_t) f^i(\theta_t^i) u_t^{\tilde{o}}, \tag{22}$$
$$u_{t+1}^{\tilde{o}} = u_t^{\tilde{o}} - f^i(\theta_t^i) u_t^{\tilde{o}} - (1 - f^i(\theta_t^i))\gamma_t u_t^{\tilde{o}} + (1-\gamma_t) s^i n_t^{\tilde{o}} + (1-\gamma_t)\delta s^e n_t^o \tag{23}$$

2.1.3 Wages

From the continuation values and the surplus sharing rules we considered above, we can obtain the following expressions for the real wages of each type of workers. For example,

wage for the experienced worker can be derived by using the relation $\eta J^o = (1-\eta)\big[H^o - (\delta U^{\tilde{o}} + (1-\delta)U^o)\big]$. One can obtain:

$$w_t^o = \eta(1+\mu)y_t + (1-\eta)b^o + \mathbb{E}_t\tilde{\beta}_{t,t+1}(1-\gamma_t)(1-\eta)\Big[f^e(\theta_t^e)\eta S_{t+1}^o - (1-f^e(\theta_t^e))\delta(U_{t+1}^{\tilde{o}} - U_{t+1}^o)\Big]. \tag{24}$$

A similar algebra can be used to obtain wages for the other two types of workers:

$$w_t^y = \eta y_t + (1-\eta)b^y + \mathbb{E}_t\tilde{\beta}_{t,t+1}\Big[f^i(\theta_t^i)(1-\eta)\eta\big\{(1-\phi_t)S_{t+1}^y + \phi_t S_{t+1}^o\big\}\Big], \tag{25}$$

$$w_t^{\tilde{o}} = \eta y_t + (1-\eta)b^o + \mathbb{E}_t\tilde{\beta}_{t,t+1}(1-\lambda)f^i(\theta_t^i)\eta(1-\eta)S_{t+1}^{\tilde{o}}. \tag{26}$$

Under plausible calibrations, we obtain $w_t^o > w_t^y > w_t^{\tilde{o}}$. The latter inequality will prevail because the old (inexperienced) worker is no more productive than the young worker, yet faces the death probability and has no upside potential. Having an upside potential for the future drives up the present wage because of the bargaining. In the aggregate, we match the upward sloping wage profile because experienced workers are more productive, but we can show that the slope of the profile becomes flatter as the aging progresses in the economy.

Aging implies the decline in the average wage. Aging means that there are more flows of workers who are hit by the δ shock, which crowds the I-matching. market. Note that the firm gains more by meeting with the young worker in the matching market when $S_t^y > S_t^{\tilde{o}}$ holds, implying that when there are more older workers in the I-matching market, the firm's job creation incentive is reduced.[4] Moreover, this will in turn put the downward pressure on the wages of the experienced (old) workers as their outside option gets worse and worse. Importantly, these effects are quantitatively larger in the economy in which the degree of human capital specificity (δ) is large.

2.1.4 System of Equations: Real Model

Given $\tilde{\beta}_{t,t+1} = \beta$, the real model consists of 15 equations (11) to (26) together with two definitions

$$\theta_t^i := \frac{v_t^i}{u_t^i}, \tag{27}$$

and

$$\theta_t^e := \frac{v_t^e}{u_t^e}, \tag{28}$$

[4]To see $S_t^y > S_t^{\tilde{o}}$, note that the productivity levels of these two types are the same. However, young workers embodies higher capital value due to their upside potential while older workers don't have this upside potential anymore.

for 17 endogenous variables S_t^o, $U_t^{\tilde{o}} - U_t^o$, $S_t^{\tilde{o}}$, S_t^y, y_t, n_t^y, u_t^y, n_t^o, u_t^o, $n_t^{\tilde{o}}$, $u_t^{\tilde{o}}$, w_t^o, w_t^y, $w_t^{\tilde{o}}$, θ_t^i, θ_t^e, v_t^i and v_t^e.

A mass of total population L_t evolves according to

$$L_{t+1} = (1 - \gamma_t + \psi_t)\, L_t. \tag{29}$$

Also, the assumption that new born workers h enter the labor market as unemployed implies

$$h_t = \psi_t L_t \tag{30}$$

In this model, aging can be replicated by changing ϕ_t γ_t and ψ_t.

We detrend 9 equations (17) to (23), and (27) and (28) by L_t using equation (29) and (30). The detrened real model consists of the following 17 equations.

$$\begin{aligned}
S_t^o &= (1+\mu)y_t - b^o \\
&\quad + \mathbb{E}_t\tilde{\beta}_{t,t+1}(1-\gamma_t)\Big[(1 - s^e - \eta f^e(\theta_t^e))S_{t+1}^o + (1 - f^e(\theta_t^e))\delta(U_{t+1}^{\tilde{o}} - U_{t+1}^o)\Big], \\
U_t^{\tilde{o}} - U_t^o &= \mathbb{E}_t\tilde{\beta}_{t,t+1}(1-\gamma_t)\Big[\eta(f^i(\theta_t^i)S_{t+1}^{\tilde{o}} - f^e(\theta_t^e)S_{t+1}^o) + (1 - f^e(\theta_t^e)\delta)(U_{t+1}^{\tilde{o}} - U_{t+1}^o)\Big], \\
S_t^{\tilde{o}} &= y_t - b^o + \mathbb{E}_t\tilde{\beta}_{t,t+1}(1-\gamma_t)(1 - s^i - f^i(\theta_t^i)\eta)S_{t+1}^{\tilde{o}}, \\
S_t^y &= y_t - b^y \\
&\quad + \mathbb{E}_t\tilde{\beta}_{t,t+1}(1-\phi_t)\Big[(1 - s^i - f^i(\theta_t^i)\eta)S_{t+1}^y\Big] + \phi_t\Big[(1 - s^e - f^i(\theta_t^i)\eta)S_{t+1}^o\Big], \\
c^i &= \mathbb{E}_t\tilde{\beta}_{t,t+1}q^i(\theta_t^i)(1-\eta)\Big[\lambda S_{t+1}^y + (1-\lambda)S_{t+1}^{\tilde{o}}\Big], \\
c^e &= \mathbb{E}_t\tilde{\beta}_{t,t+1}q^e(\theta_t^e)(1-\eta)S_{t+1}^o, \\
\bar{q}_t &= y_t\left[\bar{n}_t^y + \bar{n}_t^{\tilde{o}} + (1+\mu)\bar{n}_t^o\right] \\
(1 - \gamma_t + \psi_t)\,\bar{n}_{t+1}^y &= \bar{n}_t^y - \phi_t\bar{n}_t^y - (1-\phi_t)s^y\bar{n}_t^y + f^i(\theta_t^i)(1-\phi_t)\bar{u}_t^y, \\
(1 - \gamma_t + \psi_t)\,\bar{u}_{t+1}^y &= \bar{u}_t^y + \psi_t + (1-\phi_t)s^i\bar{n}_t^y - f^i(\theta_t^i)\bar{u}_t^y - (1 - f^i(\theta_t^i))\phi_t\bar{u}_t^y, \\
(1 - \gamma_t + \psi_t)\,\bar{n}_{t+1}^o &= \bar{n}_t^o - \gamma_t\bar{n}_t^o - (1-\gamma_t)s^e\bar{n}_t^o + (1-s^e)\phi_t\bar{n}_t^y + \phi_t f^i(\theta_t^i)\bar{u}_t^y + (1-\gamma_t)f^e(\theta_t^e)\bar{u}_t^o, \\
(1 - \gamma_t + \psi_t)\,\bar{u}_{t+1}^o &= \bar{u}_t^o - f^e(\theta_t^e)\bar{u}_t^o - (1 - f^e(\theta_t^e))\gamma_t\bar{u}_t^o + (1-\delta)(1-\gamma_t)s^e\bar{n}_t^o + (1 - f^i(\theta_t^i))\phi_t\bar{u}_t^y, \\
(1 - \gamma_t + \psi_t)\,\bar{n}_{t+1}^{\tilde{o}} &= \bar{n}_t^{\tilde{o}} - \gamma_t\bar{n}_t^{\tilde{o}} - (1-\gamma_t)s^i\bar{n}_t^{\tilde{o}} + (1-\gamma_t)f^i(\theta_t^i)\bar{u}_t^{\tilde{o}}, \\
(1 - \gamma_t + \psi_t)\,\bar{u}_{t+1}^{\tilde{o}} &= \bar{u}_t^{\tilde{o}} - f^i(\theta_t^i)\bar{u}_t^{\tilde{o}} - (1 - f^i(\theta_t^i))\gamma_t\bar{u}_t^{\tilde{o}} + (1-\gamma_t)s^i\bar{n}_t^{\tilde{o}} + (1-\gamma_t)\delta s^e\bar{n}_t^o, \\
w_t^o &= \eta(1+\mu)y_t + (1-\eta)b^o + \mathbb{E}_t\tilde{\beta}_{t,t+1}(1-\gamma_t)(1-\eta)\Big[f^e(\theta_t^e)\eta S_{t+1}^o \\
&\quad - (1 - f^e(\theta_t^e))\delta(U_{t+1}^{\tilde{o}} - U_{t+1}^o)\Big], \\
w_t^y &= \eta y_t + (1-\eta)b^y + \mathbb{E}_t\tilde{\beta}_{t,t+1}\Big[f^i(\theta_t^i)(1-\eta)\eta\big\{(1-\phi_t)S_{t+1}^y + \phi_t S_{t+1}^o\big\}\Big],
\end{aligned}$$

$$w_t^{\tilde{o}} = \eta y_t + (1-\eta) b^o + \mathbb{E}_t \tilde{\beta}_{t,t+1} (1-\lambda) f^i(\theta_t^i) \eta (1-\eta) S_{t+1}^{\tilde{o}},$$
$$\theta_t^i = \frac{\bar{v}_t^i}{\bar{u}_t^i},$$
$$\theta_t^e = \frac{\bar{v}_t^e}{\bar{u}_t^e}.$$

Variables with upper bar denote per capita variables.

2.2 Household

We adapt a large family assumption, so that the risk of job loss is pooled among all members of the household. Under this assumption, the existing representative household choose the path of per capita consumption by maximizing the discounted sum of the flow utility:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \left[\beta (1-\gamma_t)\right]^j u\left(c_{t+j}\right),$$

where $u(.)$ is the utility function and c_t is per capita consumption of the retail good.[5] The budget constraint is written as:

$$P_t c_t + \frac{a_{t+1}}{1-\gamma_t} \leq (1+i_{t-1}) a_t + W_t^o n_t^o + W_t^{\tilde{o}} n_t^{\tilde{o}} + W_t^y n_t^y + \Pi_t + t_t,$$

where p_t and a_t denote the consumer price index (CPI) and holdings of the financial asset that is zero net supply, respectively. There exists annuity market so that all bequests are equally distributed to existing household members. W_t^o, $W_t^{\tilde{o}}$ and W_t^y denote nominal wages for experienced workers, old but inexperienced workers and inexperienced workers, respectively. Note also that:

$$w_t^o := \frac{W_t^o}{P_t}, w_t^{\tilde{o}} := \frac{W_t^{\tilde{o}}}{P_t}, \text{and } w_t^y := \frac{W_t^y}{P_t}.$$

Π_t is the profits from the ownership of retail and wholesale goods sector and t_t denotes transfer and tax. Transfer equates per capita consumption between new born and surviving households. Each household maximizes the welfare with respect to consumption and the asset holding. From the first-order condition, we obtain the standard consumption Euler equation:

$$u'\left(c_t\right) = \beta \mathbb{E}_t \frac{1+i_t}{1+\pi_{t+1}^C} u'\left(c_{t+1}\right), \tag{31}$$

where π_t^C denotes the CPI inflation rate defined as:

$$\pi_t^C := \frac{P_t}{P_{t-1}} - 1.$$

[5]Note that labor supply margin is irrelevant for this problem given that workers supply labor inelastically.

The stochastic discount factor between t and $t+j$ is written as:

$$\tilde{\beta}_{t,t+j} := \beta^j \mathbb{E}_t \frac{u'(c_{t+j})}{u'(c_t)}.$$

Consumption aggregated over all goods is:

$$c_t = \left[\int_0^1 c_t(i)^{1-\frac{1}{\lambda}} \, \mathrm{d}i \right]^{\frac{\lambda}{\lambda-1}},$$

where i is the index for each differentiated retail goods. By solving the firm's cost minimization problem, we obtain the following Hicksian demand function:

$$c_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\lambda} c_t. \tag{32}$$

The CPI is given by:

$$P_t = \left[\int_0^1 P_t(i)^{1-\lambda} \, \mathrm{d}i \right]^{\frac{1}{1-\lambda}} \tag{33}$$

2.3 Retail Goods Producer

The retail goods producer i maximizes the present discounted value of profits:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \xi^{t+j} \tilde{\beta}_{t,t+j} \Pi_{t+j}(i),$$

where

$$\Pi_t(i) = (1+\tau) P_t(i) c_t(i) - P_t^W c_t(i),$$

subject to equation (32). τ is the subsidy to eliminate monopolistic rents in the steady state. Each retail producer resets its price with probability $1-\xi$ following Calvo (1983) - Yun (1996) framework.

The optimal price-setting condition implies:

$$\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\xi)^j \frac{u'(c_{t+j}) c_{t+j}}{u'(c_t) c_t} \left(\frac{P_t}{P_{t+j}} \right)^{-\lambda-1} \left[\frac{P_t^*}{P_{t+j}} \frac{P_t}{P_{t+j}} - \frac{\lambda}{(1+\tau)(\lambda-1)} \frac{P_{t+j}^W}{P_{t+j}} \right] = 0, \tag{34}$$

where P_t^* is the optimal price set at time t. From equation (33), under Calvo pricing,

$$P_t^{1-\lambda} = \xi \int_0^1 P_{t-1}(i)^{1-\lambda} \, \mathrm{d}i + (1-\xi)(P_t^*)^{1-\lambda} = \xi P_{t-1}^{1-\lambda} + (1-\xi)(P_t^*)^{1-\lambda},$$

or

$$\frac{P_t^*}{P_t} = \left[\frac{1-\xi\left(\frac{1}{1+\pi_t^C}\right)^{1-\lambda}}{1-\xi}\right]^{\frac{1}{1-\lambda}}.$$

By following Yun (2005), equation (34) can be transformed into recursive representations:

$$\left[\frac{1-\xi\left(\frac{1}{1+\pi_t^C}\right)^{1-\lambda}}{1-\xi}\right]^{\frac{1}{1-\lambda}} F_t = K_t, \tag{35}$$

where

$$F_t = 1 + \beta\xi\mathbb{E}_t \frac{u'(c_{t+1})\, c_{t+1}}{u'(c_t)\, c_t}\left(\frac{1}{1+\pi_{t+1}^C}\right)^{-\lambda} F_{t+1}, \tag{36}$$

and

$$K_t = \frac{\lambda}{(1+\tau)(\lambda-1)}\frac{P_t^w}{P_t} + \beta\xi\mathbb{E}_t \frac{u'(c_{t+1})\, c_{t+1}}{u'(c_t)\, c_t}\left(\frac{1}{1+\pi_{t+1}^C}\right)^{-\lambda-1} K_{t+1}. \tag{37}$$

Linear approximation of these three equations around deterministic steady states leads to the so-called new Keynesian Phillips curve:

$$\hat{\pi}_t^C = \beta\hat{\pi}_{t+1}^C + \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}\frac{P_t^w}{P_t},$$

when the subsidy eliminates monopolistic rents:

$$\frac{\lambda}{(1+\tau)(\lambda-1)} = 1.$$

2.4 Monetary Authority and Market Clearing Condition

The monetary authority sets the nominal interest rate using the following Taylor-type rule:

$$1 + i_t = \frac{1}{\beta}\left(1+\pi_t^C\right)^{\alpha_\pi}\left(\frac{\bar{q}_t}{\bar{q}_{t-1}}\right)^{\alpha_y}. \tag{38}$$

The goods market clearing condition is written as:

$$P_t c_t L_t = P_t^w\left(q_t - c^i v_t^i - c^e v_t^e\right),$$

or

$$c_t + p_t^w\left(c^i\bar{v}_t^i + c^e\bar{v}_t^e\right) = p_t^w\bar{q}_t, \tag{39}$$

where we define

$$p_t^w := \frac{P_t^w}{P_t}.$$

2.5 System of Equations: NK Model

In addition to 17 equations above, 6 equations (31), (35), (36), (37), (38), and (39) determine the rational expectations equilibrium of i_t, π_t^C, F_t, K_t, C_t, and P_t^W:

$$\begin{aligned}
u'(c_t) &= \beta \mathbb{E}_t \frac{1+i_t}{1+\pi_{t+1}^C} u'(c_{t+1}), \\
K_t &= \left[\frac{1-\xi\left(\frac{1}{1+\pi_t^C}\right)^{1-\lambda}}{1-\xi} \right]^{\frac{1}{1-\lambda}} F_t, \\
F_t &= 1 + \beta\xi \mathbb{E}_t \frac{u'(c_{t+1})\, c_{t+1}}{u'(c_t)\, c_t} \left(\frac{1}{1+\pi_{t+1}^C}\right)^{-\lambda} F_{t+1}, \\
K_t &= \frac{\lambda}{(1+\tau)(\lambda-1)} p_t^w + \beta\xi \mathbb{E}_t \frac{u'(c_{t+1})\, c_{t+1}}{u'(c_t)\, c_t} \left(\frac{1}{1+\pi_{t+1}^C}\right)^{-\lambda-1} K_{t+1}, \\
1+i_t &= \frac{1}{\beta}\left(1+\pi_t^C\right)^{\alpha_\pi} \left(\frac{\bar{q}_t}{\bar{q}_{t-1}}\right)^{\alpha_y}, \\
\bar{q}_t &= \frac{1}{p_t^w} c_t + c^i \bar{v}_t^i + c^e \bar{v}_t^e.
\end{aligned}$$

The relative price of intermediate goods p_t^w is determined from the goods market clearing condition. Since the marginal costs for intermediate goods producers are characterized by real wages determined through Nash bargaining in equations (24) to (26), these wages drive the dynamics of the relative price of intermediate goods and therefore CPI inflation rates. As explained above, aging gradually decreases real wages through less bargaining power of the old workers. Thus, if monetary policy does not follow complete price stability, namely α^π is not very high, chronic deflation remains as long as aging continues. Later, we will check whether the optimal monetary policy in this labor search/match model with some additional real distortions recommends the central bank to deviate from the complete price stability.