Understanding Uncertainty Shocks and the Role of Black Swans

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Abstract

A recent literature explores many ways in which uncertainty shocks can have important economic effects. But how large are uncertainty shocks and where do they come from? Researchers typically estimate a model with stochastic volatility, using all available data, then condition on the estimated model to infer volatility. This volatility is the uncertainty of an agent who knows the true probability of outcomes and whose only uncertainty is about what the draw from that distribution will be. We model a Bayesian forecaster who uses new data released each quarter to re-estimate the parameters that govern the shape of the probability distribution of GDP growth. Although the forecaster's parameter revisions are small, the probability of black swans (extreme events) is very sensitive to these revisions. Our real-time measure of GDP forecast uncertainty reveals that changes in the risk of a black swan explain most of the shocks to uncertainty.

Some times are uncertain times for the aggregate economy. At other times, events appear to be predictable and confidence is high. An active emerging literature argues that changes in uncertainty can explain asset pricing, banking crises, business cycle fluctuations, and the 2007 financial crisis. But these theories do not explain why people suddenly become uncertain. We examine whether a simple, Bayesian belief formation process can generate large enough fluctuations in uncertainty to explain recessions and financial crises.

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One source of uncertainty shocks is volatility shocks. The variance of unpredictable innovations to output could rise. If an agent knows the true law of motion for the economy, then we show that all uncertainty shocks come from changes in volatility. But if an agent does not know the true law of motion of the economy because he does not know the true parameter values that govern the model, then new sources of uncertainty shocks emerge. We estimate two Bayesian forecasting models and compute uncertainty (conditional standard deviation) of the forecast of next quarter's GDP growth, recognizing that since the estimated parameters have standard errors, they are themselves uncertain. In each quarter, we update the forecast model parameter estimates, using all U.S. GDP growth data that was available at that time.

Acknowledging parameter uncertainty uncovers two new sources of uncertainty shocks. One source of shocks is surprises: events that are improbable under the previous period's estimated model. If an event occurs that a model says is improbable, a Bayesian agent concentrates less probability weight on that model's parameters; standard errors on the parameter estimate often rises and uncertainty increases. Of course, an outlier event also leads a stochastic volatility model to predict higher volatility. That makes these mechanisms hard to distinguish. But more importantly, uncertainty shocks generated from this first mechanism are tiny.

The second source of uncertainty is something we call "black swan risk," which is the conditional probability of a growth realization more negative than any observed so far. When the forecasting model implies a normal distribution of outcomes, the probability of an n-standard-deviation event is constant. But when we allow our forecaster to estimate a distribution with skewness, the probability of negative outliers can fluctuate. For example, a mildly negative observation can lead the forecaster to estimate stronger negative skewness, which makes extreme negative outcomes more likely and thus raises uncertainty. Because the probability of tail events is very sensitive to the parameters that regulate the curvature of the probability distribution, black swan risk is very volatile and generates large changes in uncertainty. When we apply this model to GDP data, we find that most of the shocks to uncertainty are explained by changes in the estimated probabilities of black swans.

We define macroeconomic uncertainty as the standard deviation of next-period GDP growth y_{t+1}, conditional on all information observed through time t: $Std[y_{t+1}|\mathcal{I}_t]$. We use

this definition because in most models, this is the theoretically-relevant moment. When there is an option value of waiting, beliefs with a higher conditional variance (imprecise beliefs) raise the value of waiting to observe additional information. Thus, it is uncertainty in the form of a higher conditional variance that typically delays consumption or investment and thus depresses economic activity.

To build up intuition for how estimation uncertainty behaves, we start with one of the simplest forecasting models: a linear model of GDP growth with a hidden Markov state and homoskedastic innovations. We hold the volatility of the innovations fixed so that we can isolate the effect of estimation uncertainty shocks. Each period t, our forecaster observes time-t GDP growth and uses the complete history of GDP data to estimate her model and forecast GDP growth in $t+1$. In such a setting, an agent with known model parameters faces no uncertainty shocks because the estimated model is a homoskedastic one. But the forecaster with estimation uncertainty experiences changes in the conditional variance of her forecasts. These are uncertainty shocks.

However, a linear model with normally distributed shocks generates only small uncertainty shocks, those shocks are not counter-cyclical, and its forecasts badly miss a key feature of forecast data: The average forecast is significantly lower than the average GDP growth realization. For an unbiased forecaster with a linear model, this should not be the case. But if the true distribution of GDP growth is not known bus is estimated to be negatively skewed, we show that expected values can be systematically lower than the realizations. Adding extra uncertainty, such as parameter uncertainty, amplifies this effect. Therefore, we explore a non-linear forecasting model and find that combining non-linear forecasting with parameter uncertainty generates medium-sized shocks to uncertainty, particularly in recessions. Finally, we allow agents to learn about the non-linearity in their forecasting model. This generates the largest uncertainty shocks.

We compare our model-based uncertainty series to commonly-used uncertainty proxies and find that it is less variable, but more persistent than the proxy variables. The most highly correlated proxies are the price of a volatility option (VIX) or forecast dispersion. But neither achieves more than a 40% correlation with our uncertainty measure.

A key message of our paper is that understanding and measuring economic uncertainty requires relaxing the full-information assumptions of rational expectations econometrics. In

econometrics, rational expectations typically describes the assumption that agents know the true law of motion of the economy. If agents have rational expectations, they have no uncertainty about their model. Their only uncertainty is about realizations of model innovations. To measure the uncertainty of such a forecaster, it makes sense to estimate a model on as much data as possible, take the parameters as given, and estimate the conditional standard deviation of model innovations. This is what stochastic volatility estimates typically are. But in reality, the macroeconomy is not governed by a simple, known model and we surely don't know its parameters. Instead, our forecast data (from the Survey of Professional Forecasters or SPF) suggests that forecasters estimate simple models to approximate complex processes and constantly use new data to update our beliefs. Forecasters are not irrational. They simply do not know the economy's true data-generating process. In such a setting, uncertainty and volatility can behave quite differently.

Related literature A new and growing literature uses uncertainty shocks as a driving process to explain business cycles (e.g., Bloom, Floetotto, Jaimovich, Sapora-Eksten, and Terry (2012), Basu and Bundick (2012) Christiano, Motto, and Rostagno (2012), Bianchi, Ilut, and Schneider (2012)), to explain investment dynamics Bachmann and Bayer (2012a), to explain asset prices (e.g., Bansal and Shaliastovich (2010), Pastor and Veronesi (2012)), and to explain banking panics (Bruno and Shin, 2012). These papers are complementary to ours. We explain where uncertainty shocks come from, while these papers trace out the economic and financial consequences of the shocks.[1]

A small set of related theories appeal to nonlinearities in a production economy to explain why uncertainty fluctuates (Van Nieuwerburgh and Veldkamp (2006), Fajgelbaum, Schaal, and Taschereau-Dumouchel (2013), Bachmann and Moscarini (2012)). Our model differs because it does not depend on an economic environment, only on a statistical law of motion. Our uncertainty shocks arise from agents applying Bayes law when parameters are unknown.

Our exercise also speaks to a set of papers that measure uncertainty shocks in various ways. Bloom (2009), Baker, Bloom, and Davis (2012), Stock and Watson (2012), Jurado,

[1] In contrast, Bachmann and Bayer (2012b) argue that there is little impact of uncertainty on economic activity.

Ludvigson, and Ng (2013) Fernandez-Villaverde, Kuester, Guerron, and Rubio-Ramirez (2012), Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe (2011), Nakamura, Sergeyev, and Steinsson (2012), Justiniano and Primiceri (2008) and Born, Peter, and Pfeifer (2011) document the properties of uncertainty shocks in the U.S. and in emerging economies, while Bachmann, Elstner, and Sims (2012) use forecaster data to measure ex-ante and ex-post uncertainty in Germany. While our paper also engages in a measurement exercise, what we add to this literature is a quantitative model of how and why such shocks arise.

The idea that the probability of rare events is time-varying also appears in models of discrete, rare events (e.g., Rietz (1998), Barro (2005), Gabaix (2012)). However, these papers assume that there is a time-varying probability of a rare event and derive the consequences of that for asset prices. Our paper is a complement to this literature; it describes a learning process that can generate volatile, counter-cyclical beliefs about the probabilities of extreme, negative outcomes. Using only the assumptions of Bayesian updating and the idea of parameter uncertainty in non-normal probability distributions, we show as a result that estimated probabilities of tail outcomes are volatile and that economic uncertainty is sensitive to these probabilities. This finding helps to justify the assumptions of the previous literature.

The theoretical part of our paper grows out of an existing literature that estimates Bayesian forecasting models with model uncertainty. Cogley and Sargent (2005) use such a model to understand the behavior of monetary policy, while Johannes, Lochstoer, and Mou (2011) estimate a similar type of model on consumption data to capture properties of asset prices. While the mechanics of model estimation are similar, the focus on non-linear filtering, uncertainty shocks and uncertainty proxies distinguish our paper. Nimark (2012) also generates increases in uncertainty by assuming that only outlier events are reported. Thus, the publication of a signal conveys both the signal content and information that the true event is far away from the mean. Such signals can increase agents' uncertainty. But that paper does not attempt to quantitatively explain the fluctuations in uncertainty measures. Our paper is in the spirit of Hansen's Ely Lecture (Hansen, 2007) and Chen, Dou, and Kogan (2013) which advocate putting agents in the model on equal footing with an econometrician who is learning about his environment over time and struggling with

model selection.

1 A Linear-Normal Forecasting Model

The purpose of the model is to explain why relaxing rational expectations and assuming that agents do not know the true distribution of outcomes with certainty opens up an additional source of uncertainty shocks. To isolate this new source of uncertainty shocks, we consider first a homoskedastic model. If an agent knew the true model (they had rational expectations), their uncertainty would be only about the random realizations of future shocks. Since the variance of these shocks is assumed to be constant over time, there would be no uncertainty shocks.

We consider a forecaster who observes real-time GDP growth data, in every quarter and forecasts the next period's growth. The agent knows that GDP growth comes from a linear model with a hidden Markov state, but does not know the parameters of this model. Each period, he starts with prior beliefs about these parameters and the current state, observes the new GDP data, and updates his beliefs using Bayes' law.

1.1 Definitions

A model, denoted $\mathcal{M}$, has a vector a parameters θ. Together, $\mathcal{M}$ and θ determine a probability distribution over a sequence of outcomes y_t. Let $y^t \equiv \{y_\tau\}_{\tau=0}^t$ denote a series of data available to the forecaster at time t. In every model, agent i's information set $\mathcal{I}_{it}$ will include the model $\mathcal{M}$ and the history y^t of y observations up to and including time t. The state S_t, innovations, and the parameters θ are never observed.

The agent, who we call a forecaster and index by i, is not faced with any economic choices. He simply uses Bayes' law to forecast future y outcomes. Specifically, at each date t, the agent conditions on his information set $\mathcal{I}_{it}$ and forms beliefs about y_{t+1}. We call the expected value $E\left(y_{t+1}|\mathcal{I}_{it}\right)$ an agent i's *forecast* and the square root of the conditional variance $Var\left(y_{t+1}|\mathcal{I}_{it}\right)$ is what we call *uncertainty*. Forecasters' forecasts will differ from the realized growth rate. This difference is what we call a forecast error.

Definition 1. *An agent i's forecast error is the distance, in absolute value, between the forecast and the realized growth rate: $FE_{i,t+1} = |y_{t+1} - E[y_{t+1}|\mathcal{I}_{it}]|$.*

We date the forecast error $t+1$ because it depends on a variable y_{t+1} that is not observed at time t. Similarly, an average forecast error is

$$\bar{FE}_{t+1} = \frac{1}{N_t} \sum_{i=1}^{I_t} FE_{i,t+1}. \tag{1}$$

We define forecast errors and uncertainty over 1-period-ahead forecasts because that is the horizon we focus on in this paper. But future work could use these same tools to measure uncertainty at any horizon.

Definition 2. *Uncertainty is the standard deviation of the time-$(t+1)$ GDP growth, conditional on an agent's time-t information:* $U_{it} = \sqrt{E\left[\left(y_{t+1} - E[y_{t+1}|\mathcal{I}_{it}]\right)^2 \middle| \mathcal{I}_{it}\right]}$.

Volatility is the same standard deviation as before, but now conditional on the history y^t, the model $\mathcal{M}$ and the parameters θ:

Definition 3. *Volatility is the standard deviation of the unexpected innovations in y_{t+1}, taking the model and its parameters as given:*
$VOL_t = \sqrt{E\left[\left(y_{t+1} - E[y_{t+1}|y^t, \theta, \mathcal{M}]\right)^2 \middle| \mathcal{I}_{it}, \theta\right]}$.

If an agent knew the parameters (i.e., if $\mathcal{I}_{it} = \{y^t, \mathcal{M}, \theta\}$), then uncertainty and volatility would be identical. Thus, the only source of uncertainty shocks would be volatility shocks.

Many papers equate volatility, uncertainty and squared forecast errors. These definitions allow us to understand the conditions under which these are equivalent. Volatility and uncertainty are both ex-ante measures because they are time-t expectations of $t+1$ outcomes, which are time-t measurable. However, forecast errors are an ex-post measure because it is not measurable at the time when the forecast is made. Substituting definition 1 into definition 3 reveals that $U_{it} = \sqrt{E[FE^2_{i,t+1}|\mathcal{I}_{it}]}$. So, uncertainty squared is the same as the expected squared forecast error. Of course, what people measure with forecast errors is typically not the *expected* squared forecast error. It is an average of *realized* squared forecast errors: $\sqrt{1/N_t \sum_i FE^2_{i,t+1}}$.

1.2 The linear forecasting model

We begin by examining the following continuous-state hidden Markov process for y_t[2]

$$y_t = \alpha + s_t + \sigma\varepsilon_{y,t} \quad (2)$$

$$s_t = \rho s_{t-1} + \sigma_s\varepsilon_{s,t} \quad (3)$$

where $\varepsilon_{y,t}$ and $\varepsilon_{s,t}$ are standard normal random variables independent of each other. In this model, the parameters are $\theta \equiv [\alpha, \rho, \sigma, \sigma_s]'$.

Information assumptions Each forecaster has an identical information set: $\mathcal{I}_{it} = \{y^t, \mathcal{M}\}$, $\forall i$. The model $\mathcal{M}$ is described by (2) and (3). The state s_t and the parameters θ are never observed.

To compute forecasts and the process for uncertainty, we use Bayesian updating A forecast is a conditional expectation of next-period growth, where the expectation is taken over unknown parameters, states, and growth realizations. Using the law of iterated expectations, we can write this forecast as:

$$E\left(y_{t+1}|y^t\right) = \int\int\int y_{t+1} p\left(y_{t+1}|\theta, s_{t+1}, y^t\right) p\left(s_{t+1}|\theta, y^t\right) p\left(\theta|y^t\right) d\theta ds_{t+1} dy_{t+1} \quad (4)$$

The first probability density function, $p\left(y_{t+1}|\theta, s_{t+1}, y^t\right)$, is the probability of $t+1$ gdp growth, given the state and the parameters. From 2, we know that, conditional on the state, gdp growth is normally distributed with mean $\alpha + s_{t+1}$ and variance σ^2.

The second probability density function, $p\left(s_{t+1}|\theta, y^t\right)$, is the probability of a hidden state in a Kalman filtering system. It is also conditionally normally distributed. When the parameters are known, (2) and (3) form the observation and state equations of a Kalman filtering system. The following recursive equations describe the conditional mean and variance of the first two probability terms, jointly:

$$E[y_{t+1}|y^t, \theta, \mathcal{M}] = \rho E[y_t|y^{t-1}, \theta, \mathcal{M}] + \rho K_t y_t \quad (5)$$

where the term $K_t = K_t = (I + \sigma^2 Var[y_t|y^{t-1}, \theta, \mathcal{M}])^{-1}$ is the Kalman gain and the

[2]We have also explored a version of the model with a discrete hidden state. The results are very similar and are reported in Appendix B.

conditional variance of the estimate is

$$Var[y_{t+1}|y^t, \theta, \mathcal{M}] = \left((1-\rho^2)\sigma^2 + \sigma_S^2\right) I \tag{6}$$

Volatility is $\sqrt{Var[y_{t+1}|y^t, \theta, \mathcal{M}]}$, which is a constant. Constant volatility may or may not be a realistic feature of the data. But it is a helpful starting point because it will allow us to isolate the fluctuations in uncertainty that come from parameter uncertainty.

Finally, the third probability density function is the probability of the parameter vector θ, conditional on the t-history of observed GDP data. To compute posterior beliefs about parameters, we employ a Markov Chain Monte Carlo (MCMC) technique.[3] At each date t, the MCMC algorithm produces a sample of possible parameter vectors, $\left\{\theta^d\right\}_{d=1}^{D}$, such that the probability of any parameter vector θ^d being in the sample is equal to the posterior probability of those parameters, $p\left(\theta^d|y^t\right)$. Therefore, we can compute an approximation to any integral by averaging over sample draws: $\int f(\theta)p(\theta|y^t)d\theta \approx 1/D \sum_d f(\theta^d)$.

To estimate uncertainty, we compute these probability density terms and integrate numerically to get a forecast. In similar fashion, we also calculate $E\left(y_{t+1}^2|y^t\right)$. Applying the variance formula $Var\left(y_{t+1}|y^t\right) = E\left(y_{t+1}^2|y^t\right) - E\left(y_{t+1}|y^t\right)^2$, and taking the square root yields uncertainty: $U_t = \sqrt{Var\left(y_{t+1}|y^t\right)}$.

1.3 Data Description

There are two pieces of data that we use to evaluate and estimate our forecasting model. The first is real-time GDP data from the Philadelphia Federal Reserve. The variable we denote y_t is the growth rate of GDP. Specifically, it is the log-difference of the seasonally-adjusted real GDP series, times 400, so that it can be interpreted as an annualized percentage change. We use real-time data because we want to accurately assess what agents know at each date. Allowing them to observe final GDP estimates, that are not known until 2 years later, is not consistent with the goal. Therefore, y_t represents the estimate of GDP growth between then end of quarter $t-1$ and quarter t, based on the GDP estimates available at time t. Similarly, y^t is the history of GDP growth up to and including period t, based on the data available at time t.

[3] More details are presented in the Appendix. Also, see Johannes, Lochstoer, and Mou (2011) for a recursive implementation of a similar discrete-state problem of sampling from the sequence of distributions.

We use the second set of data, professional GDP forecasts, to evaluate our forecasting models. We describe below the four key moments that we use to make that assessment. The data come from the Survey of Professional Forecasters, released by the Philadelphia Federal Reserve. The data are a panel of individual forecaster predictions of real US output for both the current quarter and for one quarter ahead from quarterly surveys from 1968 Q4 to 2011 Q4. In each quarter, the number of forecasters varies from quarter-to-quarter, with an average of 40.5 forecasts per quarter.

Formally, $t \in \{1, 2, \ldots, T\}$ is the quarter in which the survey of professional forecasters is given. Let $i \in \{1, 2, \ldots, I\}$ index a forecaster and $I_t \subset \{1, 2, \ldots, I\}$ be the subset of forecasters who participate in a given quarter. Thus, the number of forecasts made at time t is $N_t = \sum_{i=1}^{I} \mathbb{I}(i \in I_t)$. Finally, let y_{t+1} denote the GDP growth rate over the course of period t. Thus, if GDP_t is the GDP at the end of period t, observed at the start of quarter $t+1$, then $y_{t+1} \equiv ln(GDP_t) - ln(GDP_{t-1})$. This timing convention may appear odd. But we date the growth $t+1$ because it is not known until the start of date $t+1$. The growth forecast is constructed as $E_{it}[y_{t+1}] = ln(E_{it}[GDP_t]) - ln(GDP_{t-1})$.

1.4 Estimation and results: linear-normal model

Our forecaster starts with a prior distribution of parameters, $p(\theta)$, described in table 1. The prior means are chosen to match the mean, variance, and persistence of the GDP

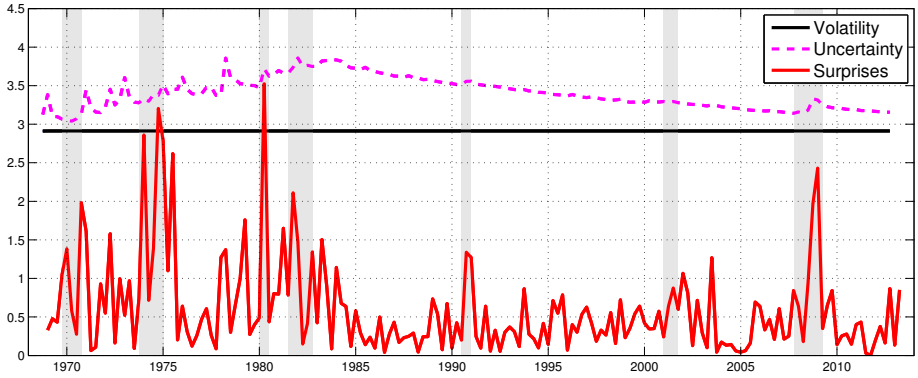
Table 1: Prior assumptions

parameter	mean	stdev
ρ	0.5	0.08
σ^2	3	0.5
σ_s^2	5	1

growth data between 1968-2012.[4] The standard deviations for prior beliefs are chosen so that uncertainty is not increasing or decreasing on average at the start of the simulation. If prior variances were too high or low, then uncertainty would spike or fall as agents started learning. Since we don't want priors to be a source of uncertainty shocks, we used this as

[4]The appendix explores priors estimated on data from 1947-68.

a criterion when choosing prior variance. We endow the agent with the prior belief that $y_t = 2.68$, its long-run mean. Starting in quarter 4 of 1968, each period, the agent observes y_t, and updates his beliefs about future GDP growth using (4).



To understand what properties of our forecasts come from parameter uncertainty, Figure 1.4 compares our uncertainty to that of an agent who takes the model parameters as given (labeled θ known). To keep the results comparable, the parameters that are used to compute volatility are the same as the mean prior beliefs in our model.[5]

The main takeaway is that estimation uncertainty generates uncertainty shocks, albeit small ones. With known parameters, stdev$(U_t) = 0$. When parameters are updated every period, stdev$(U_t) = 0.20$. What triggers these parameter uncertainty shocks is primarily surprises. We define a surprise to be an event that was unlikely, given the previous period's estimated model:

$$Surprise_t = \frac{|y_t - E(y_t|y^{t-1})|}{U_{t-1}}.$$

Figure 1.4 plots this surprise measure against the uncertainty measure. What it reveals is that periods with large surprises are periods when uncertainty increases. But also that the uncertainty generated by the surprise persists, long after the surprise has passed.

Our results also expose three aspects of our forecasts that do not look realistic. 1) Our forecasters' uncertainty is not counter-cyclical (Correl(U_t,GDP) = 13%). Every common

[5]The appendix reports results where parameters come from matching moments of the data from 1947-1968.

proxy for uncertainty is counter-cyclical and most theories use uncertainty to explain the onset of a recession. So, a forecasting model that fails to deliver this feature is suspect. 2) The model does not explain the low average forecasts of GDP observed in the professional forecaster data. The true average of GDP growth over 1968:Q4-2012:Q4 is 2.68%. The average professional forecast of GDP growth is 2.24%, almost half a percentage point lower. This model fails to explain that gap.[6] 3) There is obviously no dispersion in forecasts. There is only one forecaster. Yet, forecast dispersion is a prominent and interesting feature of the data. Later, we will introduce heterogeneous signals to examine its role.

In the next section, we examine a nonlinear forecasting model that remedies the first two problems: It generates counter-cyclical uncertainty, forecasts that are low on average, and larger uncertainty shocks.

2 A Skewed Forecasting Model

The linear-normal model of the previous section was a starting point. It is a commonly-used framework where we shut down all sources of uncertainty shocks besides our estimation uncertainty and could see that mechanism at work. But that model misses important features of the forecast data and its uncertainty shocks are modest. One reason that uncertainty varies so little is that all the random variables are normally distributed. The normal distribution has the unusual properties that the conditional variance is the same, no matter what the conditional mean is and that an n-standard deviation event is always equally unlikely. Since uncertainty is a conditional variance, the normal distribution shuts down much scope for changes in uncertainty. Therefore, we explore a model that has non-normal shocks.

Moving away from normality, there is an infinite set of possibilities. But one natural choice is to focus on a distribution with skewness. Negative skewness is a salient feature of GDP data. In the real GDP (1968:Q4-2012:Q4) data we use for our forecasting model simulations, the skewness of GDP growth is -0.30. Skewness is also a feature of many models. Models where workers lose jobs quickly and find jobs gradually, or models where

[6] This gap only arises in final GDP estimates. The average initial GDP announcement has 2.3% growth on average, in line with the forecasts. But if these initial announcements are themselves BEA forecasts of what the final GDP estimate will be, there is still a puzzle about why early estimates are systematically lower than final estimates.

borrowing constraints amplify downturns are just a couple of examples of models that generate booms that are more gradual than crashes. Finally, we find that a model with skewness both does a better job of matching features of forecast data and generates much larger uncertainty shocks.

Updating non-normal variables is typically cumbersome. Combining this with parameter uncertainty typically requires particle filtering, which is possible, but slow. We make this problem tractable by doing a change of measure. The Radon-Nikodym theorem tells us that, for any measure g that is absolutely continuous with respect to a normal distribution, we can find a change-of-measure function f such that $g(x) = \int f(x)d\Phi(x)$, where Φ is a normal cdf. If we estimate such an f function, then we use f^{-1} to take data from a skewed distribution and transform it into normal data that we can then use the tools from the last section to learn about.

Of course, allowing a forecaster to explore the whole function space of possible f's is not viable. We focus the problem by considering a family of functions and allowing the forecaster to consider parameter estimates that govern the properties of the distribution. The change of measure function should have three desirable properties: 1) Its range is the real line; 2) it has a small number of parameters to estimate; and 3) it can be either globally concave or globally convex, depending on the estimated parameters. A class of transformations that satisfy this criteria is[7]

$$f(\widetilde{X}_t) = c + b\ exp(-\widetilde{X}_t) \tag{7}$$

If we have estimates for b and c, we can do a change of variable: Use $f^{-1}(y_t)$ to transform GDP growth into a variable $\widetilde{X}$, which is a normally-distributed continuous variable with a hidden persistent state.

$$\begin{aligned} \widetilde{X}_t &= S_t + \sigma\epsilon_t \\ S_t &= \rho S_{t-1} + \sigma_S\varepsilon_t \end{aligned}$$

[7]Of course, the choice of class of transformations can affect our uncertainty estimates. However, the key property of this function is its degree of curvature. We allow our agent to estimate a parameter that governs this curvature. Any function with similar curvature, such as a polynomial or sine function, would have a similar mechanism at work.

where $\epsilon_t \sim N(0,1)$ and $\varepsilon_t \sim N(0,1)$. This procedure allows our forecaster to consider a family of non-normal distributions of GDP growth and convert each one into a linear-normal filtering problem with unknown parameters that can be solved using the same tools as in the previous section. The only additional complication is that the parameters b and c also need to be estimated.

To isolate the effect of learning about the non-normality of the conditional distribution, we compute two sets of results. In the first set, we assume that the forecaster knows the non-linear mapping. He is uncertain about the $\widetilde{X}$ process and what parameters govern it. But he knows the relationship between the variable $\widetilde{X}$ and GDP (y). The parameter ratio c/b in equation (7) affects the skewness of the resulting distribution of GDP growth. Therefore, we choose $c/b = 24$ so that the skewness of y matches the skewness of GDP growth in the data from 1952-1968:Q3.[8] The remaining b parameter can then be estimated just like the mean of y_t was before. Then, we can use linear updating MCMC techniques to form beliefs about the $\widetilde{X}_t$ process. For each parameter draw θ_i from the MCMC algorithm, we compute $E[y_t|\mathcal{I}_t, \theta_i]$ and $E[y_t^2|\mathcal{I}_t, \theta_i]$. We average these expectations over all parameter draws and compute uncertainty as $U_t = E[y_t^2|\mathcal{I}_t] - E[y_t|\mathcal{I}_t]^2$.

In the second set of results, we allow the forecaster to update beliefs about c/b each period and to consider his uncertainty about these parameters when computing conditional standard deviations. This slight change in the model is the piece that generates the largest uncertainty shocks.

2.1 Results: Uncertainty Shocks

Figure 1 compares the time series of uncertainty in each of our models.In the non-linear model where the change of measure function f is known, the presence of unknown parameters raises uncertainty above what the model with state uncertainty alone predicts. It does so directly, but also indirectly, as the unknown parameters make the state harder to infer. But by the end of the sample, beliefs about parameters have largely converged and the uncertainty levels are more similar to the other models.

Column (3) of Table 2 shows that updating beliefs about the non-normality of the GDP

[8] We choose this time period because the agents would know this data before the first forecast is made in 1968. If we instead use the 1968-2012 sample, the results are nearly identical.

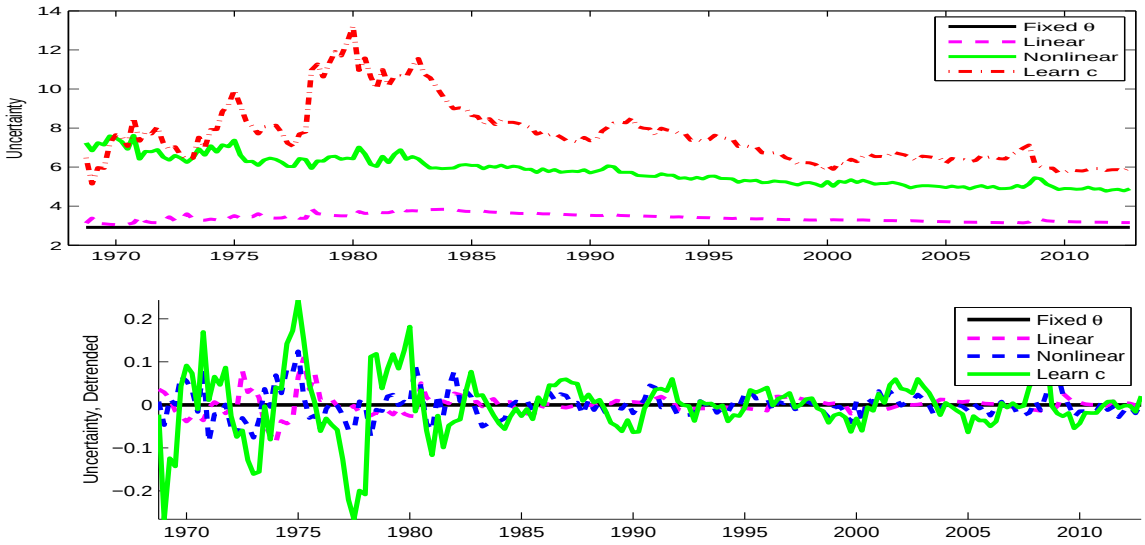


Figure 1: Uncertainty (U_t) implied by each of the models. The top panel plots the raw series, while the second shows the same uncertainty series in percentage deviations from trend.

growth distribution has a large effect on uncertainty. Such learning increases the average level of uncertainty by 32%. More strikingly, it more than doubles the size of uncertainty shocks. The standard deviation of the uncertainty series was 0.71% with a fixed, non-linear transformation and rises to 1.60% when our forecaster updates beliefs about the shape of the distribution. One can interpret the magnitude of this standard deviation relative to the mean. A 1-standard deviation shock to uncertainty raises uncertainty 21% above its mean. That is quite a volatile process and offers quite a contrast to the relatively modest changes in volatility typically measured.

Uncertainty is very persistent. It is highest at the start of the sample, when data is most scarce, and then slowly decays over the rest of the sample. As noted by Collin-Dufresne, Johannes, and Lochstoer (2013), the persistent uncertainty process comes from the nature of learning: A single large shock to GDP growth results in a quick reevaluation of the parameter and model probabilities. These revisions in beliefs act as permanent, non-stationary shocks even when the underlying shock is transitory.

Since using growth rates of GDP is a form of trend-removal, it makes sense to correlate a stationary series with another stationary series. Therefore, we detrend volatility and

model:		linear (1)	skewed (2)	learn c (3)
Mean	U_t	3.38%	5.79%	7.65%
	V_t	2.91%	6.82%	6.82%
Std deviation	U_t	0.21%	0.71%	1.60%
	V_t	0%	0.37%	0.37%
Autocorrelation	U_t	0.95	0.95	0.95
	V_t	0	0.47	0.47
	Detrended uncertainty/volatility			
Corr($\tilde{U}_t$, y_t)		0.28	-0.90	-0.34
Corr($\tilde{V}_t$, y_t)		0	-0.92	-0.92
Corr($\tilde{U}_t$, y_{t+1})		0.24	-0.22	-0.10
Corr($\tilde{V}_t$, y_{t+1})		0	-0.23	-0.23
		Forecast properties		
	data	linear	nonlinear	learn c
Mean forecast	2.24%	3.06%	2.24%	2.21%
Mean $\lvert F\,Err\rvert$	2.20%	2.31%	2.35%	2.40%

Table 2: **Properties of model uncertainty series.** Forecasts are computed using equations (4), (7) and (15). Volatilities are computing assuming that the true parameters θ are the maximum likelihood estimates, using all gdp data available at time t.

uncertainty in order to discern the nature of their cyclical components (Table 2, middle panel). We remove the low-frequency changes in uncertainty, using a bandpass filter to filter out frequencies lower than once every 32 quarters. Then, we compute a log deviation from this long-run trend.

$$\tilde{U}_t \equiv ln(U_t) - ln(U_t^{trend}) \tag{8}$$

The resulting series, plotted in figure 1, reveals large, highly counter-cyclical uncertainty shocks. In each of the recessions since 1968, uncertainty has risen sharply, 10-20% above trend.

Finally, keep in mind that there is still no stochastic volatlity in this model. To the extent that we believe that there are volatility shocks to GDP, this would create additional shocks to uncertainty, above and boynd those we have already measured. These series are not yet a complete picture of macroeconomic uncertainty. Instead, they are a look at what part of uncertainty is missed when we just measure volatility.

2.2 Black Swan Risk

To understand why uncertainty varies so much, it is helpful to look at the probability of tail events. Since our estimated probability distribution is negatively skewed, negative outliers are more likely than positive ones. For a concrete example, let's consider the probability of a particular negative growth event. The mean of GDP growth is 2.68%, while its standard deviation is 3.38%. If GDP growth were normally distributed, then $y_{t+1} \leq -6.8\%$ would be a 1-in-100-year event (Pr= 0.0025 quarterly). Let's call this rare event, which is more negative than any observed in our data, a black swan.

$$Black\,Swan\,Risk_t = Prob[y_{t+1} \leq -6.8\%|\mathcal{I}_t]. \tag{9}$$

The correlation between black swan risk and uncertainty is 91%. This illustrates that when distributions of outcomes are being re-estimated each period, uncertainty shocks arise in times when the estimated probabilities of extreme events changes.

When the distribution of GDP growth is non-normal and states and parameters change over time, the probability of this black swan event fluctuates. Figure 2 plots the estimated black swan probability each period. The black swan probability varies considerably, from

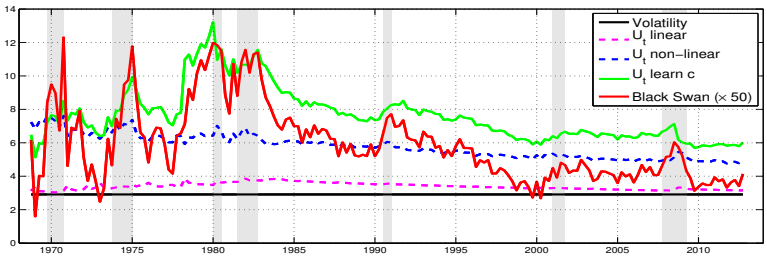


Figure 2: When the probability of a black swan event is high, uncertainty is high. *Black Swan Risk* is defined in (9). It is evaluated in the model where the skewness parameter c is updated every period.

under 4% in the 1960's to over 20% in the 1970's. Leading up to the 2008 financial crisis, the black swan probability rose from 7% in 2006 Q1 to over 12% in Q3 of 2008. This does highlight the fact that the volatility of black swan risk falls over time in this model as our forecaster gathers more information. As the parameter estimates become more precise, they become less sensitive to new information. One way to prevent this kind of convergence

would be to use a model with time-varying parameters.

What this comparison teaches us is that when we compute uncertainty to include the uncertainty that comes from estimating parameters, and if we consider a model that has a skewed distribution of outcomes, then most of the changes in uncertainty come from changes in the estimated probability of rare events. Most of these uncertainty shocks were not present when we did not allow the forecaster to update his skewness beliefs by changing the c parameter. When we allow for learning about skewness, new pieces of data cause changes in the skewness estimates. Tail event probabilities are very sensitive to this skewness parameter. When the probability of extreme events is high, uncertainty is high as well.

2.3 Why Does the Non-normal Forecast Model Perform So Differently?

The key to understanding the counter-cyclical nature of uncertainty and the low average forecast is the curvature of the change of measure function (7). When we estimate the change of measure function f, we consistently find that the coefficient b is negative, meaning that the transformation is concave. A concave transformation of a normal variable puts more weight on very low realizations and makes very high realizations extremely unlikely. (See (Albagli, Hellwig, and Tsyvinski, 2013).) In other words, the concave transformation creates a negatively-skewed variable. This fits the data because the unconditional distribution of GDP growth rates is negatively skewed.

One economic interpretation of this concave change of measure is to think of $\widetilde{X}_t$ as an economic fundamental condition. When the economy is functioning very well (high $\widetilde{X}_t$), then improving its efficiency results in a small increase in GDP. But if there is a high degree of dysfunction or inefficiency (low $\widetilde{X}$), then the economy can easily fall into a deep depression. Most modern macroeconomic models are not linear. Many generate exactly this type of effect through borrowing or collateral constraints, other financial accelerator mechanisms, matching frictions, or information frictions. Even a simple diminishing returns story could explain such a concave mapping.

The concavity of the change of measure explains why uncertainty becomes counter-cyclical. The concave line in Figure 3 is a mapping from x into GDP growth, y. The slope of this curve is a Radon-Nikodym derivative. A given amount of uncertainty is like a band

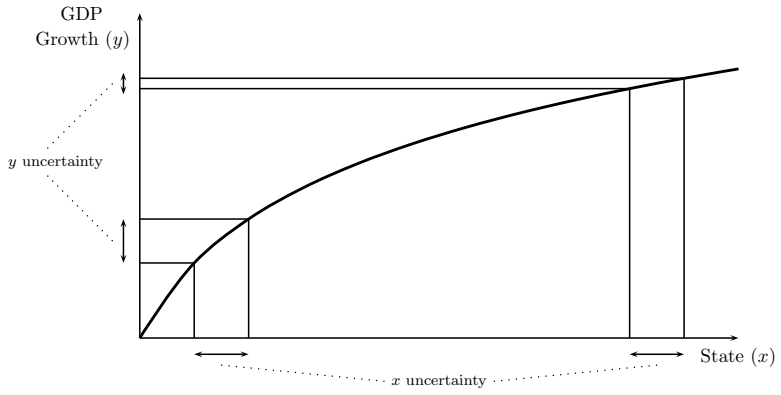


Figure 3: Nonlinear change of measure and counter-cyclical uncertainty. A given amount of uncertainty about x creates more uncertainty about y when x is low than it does when x is high.

of possible x's. (If x was uniform, the band would represent the positive-probability set and the width of the band would measure uncertainty about x.) If that band is projected on to the y-space, the implied amount of uncertainty about y depends on the state x. When x is high, the mapping is flat, and the resulting width of the band projected on the y-axis (y uncertainty) is small. When x is low, the band projected on the y axis is larger and uncertainty is high. (See (Straub and Ulbricht, 2013) for theoretical results relating concavity to uncertainty.)

Learning about b and c causes this concave curve to shift over time. As the slope of the curve shifts, the amount of y uncertainty that a given amount of x uncertainty generates changes. This is the single largest source of uncertainty shocks in our model. Notice in Table 2 that one difference between uncertainty with known c/b and with distribution learning is that the uncertainty is significantly less counter-cyclical when forecasters are learning about the non-normality of the distribution. The reason for this is that when GDP is high, it increases the estimated mean of the GDP process. This makes the negative outliers in the sample lie further away from the mean and increases the sample skewness. Since the beliefs about c are tied to model skewness, this causes a decline in c, which increases the slope of the change of measure function and as a result, increases uncertainty.

Aside from generating larger uncertainty shocks, the nonlinear model also explains the low GDP growth forecasts in the professional forecaster data. The average forecast

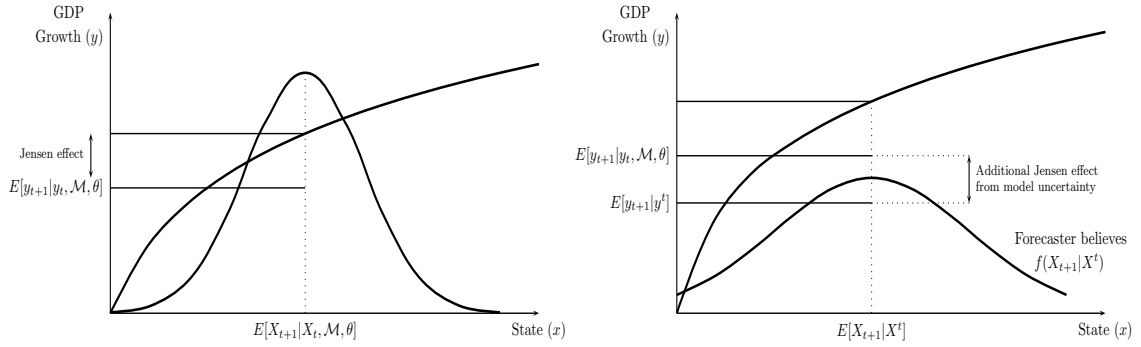


Figure 4: Explaining why average forecasts are lower than mean GDP growth. The result has two key ingredients: The forecaster faces more uncertainty than he would if he knew the true distribution of outcomes, and a Jensen inequality effect from the concave change of measure.

is 2.2% in the model and 2.2% in the forecaster (SPF) data.[9] Thus, the first puzzle is why the nonlinear model produces these average forecasts. If GDP growth is a concave transformation of a linear-normal underlying variable, Jensen's inequality tells us that expected values will be systematically lower than the median realization. But by itself, Jensen's inequality does not explain the forecast bias because the expected GDP growth and the mean GDP growth should both be lowered by the concave transformation (see figure 4, left panel). It must be that there is some additional uncertainty in expectations, making the Jensen inequality effect larger for forecasts than it is for the unconditional mean of the true distribution (see figure 4, right panel). This would explain why our results tell us that most of the time the sample mean is greater than the average forecast ($E[y_{t+1}|\theta] > E[y_{t+1}|y^t]$). If the agent knew the true parameters, he would have less uncertainty about y_{t+1}. Less uncertainty would make the Jensen effect smaller and raise his estimate of y_{t+1}, on average. Thus, it is the combination of parameter uncertainty and a skewed distribution that can explain the forecast bias.[10]

[9] Elliott and Timmermann (2008) confirm our finding of forecast bias. They argue that stock analysts over-estimate earnings growth and the Federal Reserve under-estimates GDP growth.

[10] When we use first-release GDP data, this forecast bias disappears. But that is consistent with an interpretation of this first release as itself a forecast. The first release may be lower, on average because it is more uncertain and the added uncertainty, combined with the non-linear model, lowers the average initial announcement.

3 Data Used to Proxy for Uncertainty

Our model generates a measure of economic uncertainty. Next, we compare our measure to others. Commonly used measures for uncertainty include the VIX, forecast dispersion, squared forecast errors, and GARCH volatility estimates. First, we describe each measure and examine the theoretical difference with ours. Then, we compare the statical properties.

3.1 Time-varying volatility models

One common procedure for estimating the size of volatility shocks is to estimate an ARMA process that allows for stochastic volatility. In order to compare such a volatility measure to our uncertainty series, we estimate a GARCH model of GDP growth, that allows for time-variation in the variance of the innovations. All data is quarterly and all of these series are non-stationary. To obtain stationary series, we use annualized growth rates.

The GARCH process that generates the best fit is one with an AR(1) process for GDP growth (y) and a GARCH(1) process for volatility, σ^2_{t+1}, which includes 1 lagged variance:

$$y_{t+1} = 3.38 + 0.41 y_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim \mathcal{N}(0, \sigma^2_{t+1}) \tag{10}$$

$$\sigma^2_{t+1} = 0.52 + 0.76\sigma^2_t + 0.24\epsilon^2_t \tag{11}$$

We assume that the errors in our models, ϵ_t, are Gaussian and estimate the process using our full sample of data (1947-2012). The complete set of estimates, stationarity tests, as well as more detail about the model selection process are reported in appendix B.

Our forecasting model assumed that innovations to the state and observation equations had constant variance. We estimate homoskedastic models to test the hypothesis of homoskedasticity with an ARCH-LM test. The evidence for time-varying volatility is inconclusive.[11]

[11] The log likelihood of the highest-likelihood heteroskedastic model is only 3% higher than the best-fitting homoskedastic model. With an ARCH-LM test, one cannot reject the null hypothesis of homoskedasticity (pvalue is 0.22). We come to a similar conclusion if we estimate the model, starting in 1947, use a stochastic volatility model, or relax the distributional assumptions. Our baseline analysis assumes that errors ϵ_t have a Gaussian distribution. Using distributions with fatter tails (student-t) yields no difference in estimations or significance. Furthermore, we explore further lags of all variables; either coefficients were not significantly different from zero or the log-likelihood was reduced. Finally, we included different lags of linear terms for ϵ_t and variances σ^2_t in the GARCH specification. Again, the estimated parameters were not significant or

But we could incorporate volatility shocks in our forecasting model by allowing the shock variances σ^2 and/or σ_s^2 to have their own stochastic processes. Increases in volatility would increase uncertainty directly and indirectly, by making other parameters harder to learn. Future work will explore this interaction.

3.2 Forecast dispersion

Some authors use forecast dispersion (D_t in equation 13) as a measure of uncertainty.[12] One advantage of this measure is that it is typically regarded as "model-free." It turns out that dispersion is not model-free. It is only equivalent to uncertainty in models with uncorrelated signal noise and no parameter uncertainty.

Any unbiased forecast can be written as the difference between the true variable being forecast and some forecast noise that is orthogonal to the forecast. In other words,

$$y_{t+1} = E[y_{t+1}|\mathcal{I}_t] + \eta_t + e_{it} \tag{12}$$

where the forecast error ($\eta_t + e_{it}$) is mean-zero and orthogonal to the forecast. We can further decompose any forecast error into a component that is common to all forecasters η_t and a component that is the idiosyncratic error e_{it} of forecaster i.

Dispersion is the average squared difference of each forecast from the average forecast. We can write each forecast as $y_{t+1} - \eta_t - e_{it}$. Then, with a large number of forecasters, we can apply the law of large numbers, set the average e_{it} to 0 and write the average forecast as $\bar{E}[y_{t+1}|\mathcal{I}_t] = y_{t+1} - \eta_t$. Thus,

$$D_t \equiv \frac{1}{N}\sum_i (E[y_{t+1}|\mathcal{I}_t] - \bar{E}[y_{t+1}|\mathcal{I}_t])^2 = \frac{1}{N}\sum_i e_{it}^2 \tag{13}$$

Whether dispersion accurately reflects uncertainty depends on private or public nature of information. Imprecise private information (high e_{it}^2) generates forecast dispersion, while imprecise public information or estimation error typically does not. To understand the importance of this distinction, we explore a forecasting model with forecast dispersion.

the log-likelihood was reduced.

[12] See e.g. Baker, Bloom, and Davis (2012) or Diether, Malloy, and Scherbina (2002), or Johnson (2004).

A Model with Heterogeneous Signals Exploring a model with heterogenous signals allows to to better understand the realtionship between forecast dispersion and uncertainty. It also allows us to address the concern that professional economic forecasters have access to much more information that just past GDP realizations. In this model, the signals will be a stand-in for all this other macroeconomic data that forecasters can use. Since the linear-normal model is the simplest setting to illustrate these effects, we augment that model with signals.

To model this additional information and the heterogeneity of forecasts, we consider a setting where forecasters update beliefs as in section 1. But each period, each forecaster i observes an additional signal z_{it} that is the next period's GDP growth, with common signal noise and idiosyncratic signal noise:

$$z_{it} = y_{t+1} + \eta_t + \epsilon_{it} \tag{14}$$

where $\eta_t \sim N(0, \sigma_\eta^2)$ is common to all forecasters and $\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$ is i.i.d. across forecasters.

We calibrate the two signal noise variances σ_η^2 and σ_ϵ^2 to match two moments. The first is the average dispersion of forecasts. Time-t forecast dispersion is defined in (13), where $\bar{E}_t = 1/N_t \sum_i E_{it}[y_{t+1}]$ is the average time-t growth forecast and N_t is the number of forecasters making forecasts at time t. The time-series average of D_t in the survey of professional forecasters is 1.6%.

The second calibration moment is the average forecast error, $1/T \sum_t FE_t$. In the data, this average forecast error is 1.91%. Note that this average error is larger than the dispersion. It tells us that signal noise is not exclusively private signal noise. We choose σ_η and σ_ϵ such that average dispersion and forecast errors are identical in the model and in the data. The resulting estimated signal variances are $\sigma_\eta = 1.42\%$ and $\sigma_\epsilon = 2.31\%$.

Our forecasters use prior GDP growth realizations to estimate the parameters of the model, just as in (4). Then, they use their additional, heterogeneous signal to update these prior beliefs using Bayes' law:

$$E\left(y_{t+1}|y^t, z_{it}\right) = \int y_{t+1} \frac{f(z_{it}|y^{t+1}) f(y_{t+1}|y^t)}{\int f(z_{it}|y', y^t) f(y'|y^t) dy'} dy_{t+1}. \tag{15}$$

Comparing signal model to forecast data The main result is that forecast dispersion reduces uncertainty and average forecast errors, in line with the data. But it does not create uncertainty shocks. Table 3 reveals that the uncertainty shocks in this model and the linear model without signals are similar and small.

	volatility V_t	uncertainty U_t
Mean	2.73%	2.11%
Std deviation	0.21%	0.05%
Autocorrelation	0	0.95
Detrended:	volatility V_t	uncertainty U_t
Corr with y_t	0	0.27
Corr with y_{t+1}	0	0.24
Forecast properties	data	signals
Mean forecast	2.24%	2.63%
Mean $\lvert F\,Err\rvert$	2.20%	2.20%

Table 3: **Volatility, uncertainty and forecasts in the heterogeneous signal model.** Forecasts are computed using (15). Volatilities are computing assuming that the true parameters θ are the maximum likelihood estimates, using all gdp data available at time t and all signals observed through time t.

An insight we get from this model is that parameter uncertainty does not show up as forecast dispersion. Parameters are estimated on a whole set of historical data. That data is common to all forecasters. The differences in signals about next-quarter GDP growth create differences in forecasts. But for parameter estimation, there are many pieces of data that forecasters have in common and one signal that differs. So the parameter estimates mostly reflect the common historical data that forecasters observe. Since they have little heterogeneity, they do not contribute to changes in forecast dispersion. Estimation error looks more like the public component of the forecast error η_t than it does the private component ϵ_t.

This model matches the mean forecast error better than the previous models because it allows forecasts to be more highly correlated with future GDP growth. With a signal about future GDP, forecasters in this model are more likely to revise their forecasts slightly up when growth will be high and down when it will be low, achieving a higher correlation between forecast and GDP growth.

The bottom line is that forecast dispersion is not synonymous with uncertainty. Build-

ing forecast heterogeneity in the model, similar to what we see in the data, is not a mechanism for generating large uncertainty shocks. Uncertainty shocks that come from estimation uncertainty are unlikely to be captured by a dispersion measure because most of the relevant data used for estimation is public, not private. With a nonlinear forecasting model, more subtle relationships between dispersion and uncertainty can arise. These effects are explored in Orlik, Veldkamp, and Kozeniauskas (2013).

3.3 Mean-squared forecast errors

A measure related to forecast dispersion that captures both private and common forecast errors is the forecast mean-squared error.

We define a forecast mean-squared error (MSE_{t+1}) of a forecast of y_{t+1} made in quarter t as the square root of the average squared distance between the forecast and the realized value

$$MSE_{t+1} = \sqrt{\frac{\sum_{i\in I_t}(E[y_{t+1}|\mathcal{I}_{it}] - y_{t+1})^2}{N_t}}. \tag{16}$$

If forecast errors were completely idiosyncratic, with no common component, then dispersion in forecasts and mean-squared forecasting errors would be equal. To see this, note that $FE^2_{jt} = (E[y_{t+1}|\mathcal{I}_{jt}] - y_{t+1})^2$. We can split up FE^2_{jt} into the sum $((E[y_{t+1}|\mathcal{I}_{jt}] - \bar{E}_t[y_{t+1}]) + (\bar{E}[y_{t+1}] - y_{t+1}))^2$, where $\bar{E}_t[y_{t+1}] = \int_j E[y_{t+1}|\mathcal{I}_{jt}]$ is the average forecast. If the first term in parentheses is orthogonal to the second, $1/N\sum_j FE^2_{jt} = MSE^2_t$ is simply the sum of forecast dispersion and the squared error in the average forecast: $E[y_{t+1}|\mathcal{I}_{jt}] - \bar{E}_t[y_{t+1}])^2 + (\bar{E}_t[y_{t+1}] - y_{t+1})^2$.

We can then use this insight along with our forecast data to evaluate the extent to which variation in mean-squared errors (MSE) comes from changes in the accuracy of average forecasts and how much comes from changes in dispersion. We estimate a regression of the GDP forecast mean-squared error, MSE^2, defined in (16) on $(\bar{E}_t[y_{t+1}] - y_{t+1})^2$. We find that the R^2 of this regression is 80%. The remaining variation is due to changes in forecast dispersion. Since most of the fluctuation in MSE comes from changes in average forecast errors, it tells us that using forecast dispersion as a proxy for uncertainty will miss an important source of variation.

3.4 VIX and confidence measures

We discuss the volatility and forecast-based measures in most detail because there are measures that we can relate explicitly to our theory. Other proxy variables for uncertainty are interesting but have a less clear connection to our model. The market volatility index (VIX) is a traded blend of options that measures expected percentage changes of the S&P500 in the next 30 days. It captures expected volatility of equity prices. But it would take a rich and complicated model to link macroeconomic uncertainty to precise movements in the VIX. Nevertheless, we can compare its statistical properties to those of the uncertainty measure in our model. Figure 5 does just this.

Another commonly cited measure of uncertainty is business or consumer confidence. The consumer confidence survey asks respondents whether their outlook on future business or employment conditions is "positive, negative or neutral." Likewise, the index of consumer sentiment asks respondents whether future business conditions and personal finances will be "better, worse or about the same." While these indices are indeed negatively correlated with the GARCH-implied volatility of GDP, they are not explicitly questions about uncertainty. Furthermore, we would like to use a measure that we can compare to the forecasts in our model. Since it is not clear what macro variable "business conditions" or "personal finances" corresponds to, it is not obvious what macro variable respondents are predicting.

3.5 Comparing uncertainty proxies to model-generated uncertainty

Figure 5 plots each of the uncertainty proxies. There is considerable comovement, but also substantial variation in the dynamics of each process. These are clearly not measures of the same stochastic process, each with independent observation noise. Furthermore, they have properties that are quite different from our model-implied uncertainty metric. Table 4 shows that our uncertainty metric is less volatile, moderately counter-cyclical, but the raw uncertainty series is more persistent than the proxy variables.

3.6 Inferring uncertainty from probability forecasts

One way to infer the uncertainty of an economic forecaster is to ask them about the probabilities of various events. The survey of professional forecasters does just that. They

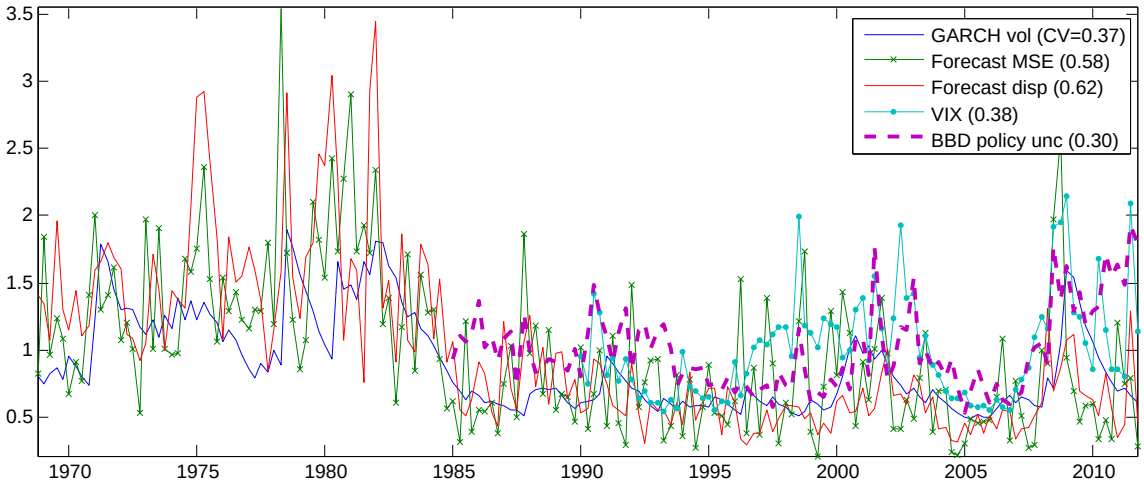


Figure 5: Comparing variables used to measure uncertainty in the literature.

ask about the probability that GDP growth exceeds 6%, is between 5-5.9%, between 4-4.9%, ..., between -1 and -2%, and below -2%. However, the survey only reports a single probability weight that is averaged across all forecasters.

Since this data does not completely describe the distribution of y_{t+1} beliefs, computing a variance requires some approximation. The most obvious approximation is to assume a discrete distribution. For example, when agents assign a probability to $1-2\%$ GDP growth, we treat this as if that is the probability placed on the outcome of 1.5% GDP growth. When the agent says that there is probability $p_{6.5}$ of growth above 6%, we treat this as probability $p_{6.5}$ placed on the outcome $y_{t+1} = 6.5\%$. And if the agent reports probability $p_{-2.5}$ of growth below -2%, we place probability of $p_{-2.5}$ on $y_{t+1} = -2.5\%$. Then the expected rate of GDP growth is $\bar{y} = \sum_{m \epsilon M} p_m m$ for $M = \{-2.5, -1.5, \ldots, 6.5\}$. Finally, the conditional variance of beliefs about GDP growth are $var[y|\mathcal{I}] = \sum_{m \epsilon M} p_m (m - \bar{y})^2$.

The resulting conditional variance series is not very informative. It hardly varies (range is $[0.0072, 0.0099]$). It does not rise substantially during the financial crisis. In fact, it suggests that uncertainty in 2008 was roughly the same as it was in 2003. The reason this measure does not detect high uncertainty in exteme events is that the growth rates are top- and bottom-coded. All extremely bad GDP events are grouped in the bin "growth less than 2%." All the uncertainty was about how bad this recession might be. Instead,

	Mean	Standard deviation	autocorr	correlation with g_{t+1}	correlation with $\tilde{U}_t$
forecast MSE	2.64%	1.53	0.48	0.04	-3.02%
forecast dispersion	1.54%	0.95	0.74	-0.19	19.98%
GARCH volatility	3.65%	1.35	0.90	0.06	7.26%
GARCH real-time	3.45%	1.04	0.90	-0.07	8.32%
VIX	20.55	7.81	0.58	-0.41	35.80%
BBD policy uncertainty	105.95	31.79	0.65	-0.41	20.95%
NL model uncertainty ($\tilde{U}_t$)	0	0.03	0.41	-0.23	100%

Table 4: **Properties of forecast errors and volatility series for macro variables.** Forecast MSE and dispersion are defined in (16) and (13) and use data from 1968q4-2011q4. Growth forecast is constructed as $ln(E_t(GDP_t)) - ln(E_t(GDP_{t-1}))$. GARCH volatility is the σ_{t+1} that comes from estimating (11), using data from 1947q2-2012q2. At each date t, GARCH real-time is the σ_{t+1} that comes from estimating (11) using only data from 1947:Q2 through date t. VIX_t is the Chicago Board Options Exchange Volatility Index closing price on the last day of quarter t, from 1990q1-2011q4. BBD policy uncertainty is the Baker, Bloom, and Davis (2012) economic policy uncertainty index for the last month of quarter t, from 1985q1-2011q4. $\tilde{U}_t$ is nonlinear model uncertainty, measured as the log deviation from trend (eq. 8).

what the probability bins reveal is a high probability weight on growth below 2%. Since most of the probability is concentrated in one bin, it makes the uncertainty look low. The bottom line is that while using surveys to ask about ex-ante probabilities of GDP events is a promising approach to measuring uncertainty, the available data does not seem useful for our purposes.

4 Conclusions

Most approaches to measuring economic uncertainty ignore parameter estimation uncertainty. Sometimes referred to as "rational expectations econometrics," this approach entails estimating a model and then treating the estimated parameters as truth to infer what the volatility of innovations was in each period in the past. In equating the volatility with uncertainty, the econometrician is assuming that the uncertain agent knows the true distribution of outcomes at every moment in time and is only uncertain about which outcome will be chosen from this distribution. Assuming such precise knowledge of the economic model rules out most uncertainty and ignores many sources of uncertainty shocks.

We measure uncertainty for an agent who is not endowed with knowledge of the true economic model. Instead, this agent needs to estimate the model of the economy, just like an econometrician. The conditional variance of this agent's forecast, his uncertainty, is much higher and varies more than volatility does. When the agent considers skewed distributions of outcomes, new data or real-time revisions to existing data can change his beliefs about the skewness and kurtosis of the distribution, and thus the probability of extreme events. Small changes in the estimated shape of a probability distribution can increase or decrease the probability of these tail events many-fold. Because these events are so far from the mean outcome, changes in their probability have a large effect on conditional variance, which translates into large shocks to uncertainty. Thus, our message is that it is beliefs about these black swans, extreme events that are never observed, but whose probability is inferred from a forecasting model, that are responsible for much of the shocks to economic uncertainty.

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A Estimating the model

We estimate the parameters of the model using random-walk Metropolis Hastings algorithm. The likelihood function of a given vector of parameters, θ, (which we need for the accept/reject step of the algorithm) is given by

$$p\left(y^T|\theta\right) = \prod_{t=0}^{T-1} p\left(y_{t+1}|y^t,\theta\right)$$

In turn, the predictive distribution of the data, $p\left(y_{t+1}|y^t,\theta\right)$ can be obtained as an integral against the filtering distribution

$$p\left(y_{t+1}|y^t,\theta\right) = \int\int p\left(y_{t+1}|s_{t+1},\theta\right) p\left(s_{t+1}|s_t,\theta\right) p\left(s_t|y^t,\theta\right) ds_t ds_{t+1}$$

We apply Kalman filtering techniques to obtain the distribution of the filtered values of the hidden state, $p\left(s_t|y^t,\theta\right)$, which will be characterized by the following moments $\widehat{s}_t \equiv E\left[s_t|y^{t-1},\theta\right]$ and $Var_{s,t} \equiv E\left[\left(s_t-\widehat{s}_t\right)\left(s_t-\widehat{s}_t\right)'\right]$. The predictive distribution of the data is then given by the following moments $E\left[y_t|y^{t-1},\theta\right] = \widehat{s}_t$ and $Var\left[y_t|y^{t-1},\theta\right] = Var_{s,t} + \sigma^2$.

B Estimated ARCH/GARCH process for GDP

In this section we present the GARCH models that we estimate to infer volatility proxies for GDP. We estimate volatility using an ARMA model with ARCH or GARCH errors. The estimation procedure is maximum likelihood. We considered several models and chose the AR and MA orders based on the significance of additional variables and their effect on the log-likelihood. Similarly, we considered different lags of linear terms for ϵ_t and variances σ_t^2 in the GARCH specification and used the significance and effect of additional variables on the log-likelihood to inform the specification choice.

We use quarterly GDP growth rate data for 1947:Q2–2012:Q2.[13] The subsample of the GDP data which matches the uncertainty data is 1968:Q4 to 2011:Q4. The results of ADF tests indicate that the series for the full sample and matching subsample are stationary.

Best-fitting homoskedastic model Recall that we defined $y_{t+1} \equiv \ln(gdp_t) - \ln(gdp_{t-1})$. The best-fitting processes is an ARMA(1,0):

$$\Delta gdp_{t+1} = 3.25 + 0.37\Delta gdp_t + \epsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, 14.43)$$

The log-likelihood is -713.27.

Best-fitting heteroskedastic model For heteroskedastic processes, the best specification is ARMA(1,0) for growth and GARCH(1) for variance:

$$\Delta gdp_{t+1} = 3.38 + 0.41\Delta gdp_t + \epsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, \sigma^2_{t+1})$$

$$\sigma^2_{t+1} = 0.52 + 0.76\sigma^2_t + 0.24\epsilon^2_t$$

The log-likelihood is -693.90.

For each sample period of each variable we use Augmented Dickey-Fuller (ADF) tests to check that the series is stationary. This test is based on estimating an AR model for the data. For each sample we use the best-fitting AR model as the basis for the test. Each of the series we use passed the stationarity test.

[13]Data source: Bureau of Economic Analysis (http://www.bea.gov/national/index.htm#gdp). We are using the seasonally adjusted annual rate for the quarterly percentage change in real GDP. Note that this data is for the actual percentage change, not an approximation. The version of the data is 27 July 2012, downloaded 2 August 2012.