Restoring the Product Variety and Pro-competitive Gains from Trade with Heterogeneous Firms and Bounded Productivity*

by

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Abstract

The monopolistic competition model in international trade offers three sources of gains from trade that do not arise in competitive models: expansion in product variety; a pro-competitive reduction in the markups charged by firms; and the self-selection of more efficient firms into exporting. Recent literature on trade with heterogeneous firms has emphasized the third of these effects, and the first two effects are ruled out when using a Pareto distribution for productivity with a support that is unbounded above. The goal of this paper is to restore a role for product variety and pro-competitive gains from trade by using a bounded Pareto distribution for productivity.

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1. Introduction

The monopolistic competition model in international trade offers three sources of gains from trade that do not arise in competitive models. First, opening to trade can lead to expansion in product variety, as goods not available in autarky becomes imported. This first source is emphasized in the earliest writings by Krugman (1979) and throughout Helpman and Krugman (1985). A second source of gains emphasized by Krugman (1979) is that the pro-competitive effect of trade reduces the markups charged by firms, and therefore lowers consumer prices. In order for this fall in prices to translate in a *social* gain, and not just a redistribution from firms to consumers, we need the assumption of zero profits due to free entry. In that case, the reduced ratio of price to marginal cost implies a reduced ratio of average to marginal costs, so that firms are taking greater advantage of economies of scale (as discussed by Helpman and Krugman, 1985, p. 134). In this way, the consumer gains due to reduced markups become social gains because of the accompanying expansion of firm scale.

The third source of gains arises in the more recent models of monopolistic competition and trade with heterogeneous firms, due to Melitz (2003). In this case, trade will lead to the self-selection of more efficient firms into exporting, while less efficient firms exit the market, leading to rise in average productivity. This third source of gains has been the focus of recent literature. For example, if we add the assumption that firm productivity is unbounded above with a Pareto distribution, as in Chaney (2008), then it can be shown that the gains from trade in the Melitz (2003) model are *entirely* due to the selection of firms: the welfare gains from new imported varieties are just offset by the loss from fewer domestic varieties (Feenstra, 2010); and of course, there is no change in markups due to CES preferences. Even without the unbounded Pareto assumption, Melitz and Redding (2013) have recently argued that the rise in average productivity

due to firm selection and trade in the Melitz model is what distinguishes it most clearly from the homogeneous firm model of Krugman (1980). Even if we allow for non-CES preferences with heterogeneous firms, so that in principal a pro-competitive effect could operate, Arkolakis, Costinot, Donaldson and Rodriguez-Clare (ACDR, 2012) have recently shown that neither this effect nor product variety leads to any gains; so once again, the key source of gains from trade comes from the selection of firms.[1] That result in ACDR depends on the assumption of a Pareto distribution of productivity with a support that is unbounded above, which is the starting point for this paper.

The goal of this paper is to restore a role for product variety and pro-competitive gains from trade with heterogeneous firms, by using a bounded (or truncated) Pareto distribution for productivity. The empirical relevance of this approach is beyond question: Helpman, Melitz and Rubenstein (2008) have used the bounded Pareto to obtain a gravity equation in trade that is consistent with the many instances of zero trade flows between countries.[2] It is surprising, then, that the bounded Pareto has not received more theoretical attention (though it is consistent with Melitz, 2003, who did not constrain the distribution). One reason for the popularity of the unbounded Pareto is that, like CES preferences, it leads to highly tractable solutions for trade and welfare. A secondary goal of this paper is to show that the bounded Pareto distribution still yields tractable solutions, even with a class of preferences allowing for non-constant markups.

Specifically, we will work with a class of preferences introduced by Diewert (1976) known as the quadratic mean of order *r* (QMOR) expenditure function. This is perhaps the most

[1] ACDR further show that total gains are reduced by the pro-competitive effect when exporting firms only partially pass-through the effect of trade cost reductions to their prices. But that partial pass-through channel influences gains in their paper if and only if tastes are non-homothetic. In contrast, we assume homothetic tastes.

[2] Another motivation for using bounded productivity comes from the theory of globalization put forth by John Sutton and summarized in his Clarendon Lectures (Sutton, 2012). Sutton uses three assumptions to derive the interaction of firms as globalization proceeds, the third of which is "you can't make something out of nothing" (Sutton, 2012, p. 55). That assumption is intended to rule out unbounded productivity.

general parametric form for expenditure that is dual to homothetic preferences. It differs from the class of preferences used by ACDR because it is homothetic, and more important, because it gives an explicit functional form for the expenditure needed for one unit of utility – that is, for the cost of living.[3] In contrast, ACDR rely on an implicit solution for welfare by integrating from demand, which makes it challenging to deal with non-infinitesimal changes in product variety.[4] The QMOR expenditure function is introduced in section 2 where, because we are dealing with a monopolistic competition model, we assume that demand is symmetric across varieties and also that it has a finite reservation price. Given these properties, we establish the sign pattern of the parameters needed to ensure that the QMOR expenditure function is globally well-behaved: a feature that has not been assured in prior applications, mainly empirical.

Our use of the QMOR expenditure function sets this paper apart from other recent, theoretical literature dealing with variable markups in international trade. A more common choice is to use the additively separable utility function introduced by Krugman (1979), possibly with an explicit functional form for the sub-utility from each variety.[5] Zhelobodko *et al* (2010, 2011), Kichko *et al* (2013) and Dhingra and Morrow (2012) consider a broader class of additively separable functions than Krugman (1979) by allowing the elasticity of demand to be increasing or decreasing in quantity. These authors argue for a pro-competitive effect of trade in the latter case only (as assumed by Krugman and holding here). But when these authors consider heterogeneous firms, they do not appear to recognize that a Pareto distribution with unbounded support implies that the pro-competitive effect vanishes, as we shall explain here.

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[3] The class of preferences used by ACDR includes one homothetic case – the translog preferences – which are also included within the QMOR class. This is the only case that is common to both classes, as explained in section 2.

[4] In ongoing work, these authors propose a quantitative method to perform this integration, thereby obtaining welfare from estimated demand.

[5] Behrens and Murata (2007, 2012) use exponential functions and the latter paper includes pro-competitive effects, while Saure (2009) and Simonovska (2010) use a logarithmic function with displaced origin.

Another line of literature related to this paper assumes a finite number of firms, in which case markups are endogenous even with nested-CES preferences.[6] Initiated by Atkeson and Burstein (2008), this framework is used by Edmond, Midrigan and Xu (2012) to compute the pro-competitive gains from trade between the United States and Taiwan. Specializing to the case of Bertrand competition between firms, De Blas and Russ (2012) contrast the results obtained by Bernard, Eaton, Jensen, and Kortum (2003) using an infinite number of rivals to those obtained instead with a finite number of rivals; only in the latter case does a pro-competitive effect of trade operate. Our paper is most closely related to Holmes, Hsu and Lee (2013), who also use Bertrand competition and show that if and only if the distribution of productivities is unbounded Pareto, then trade leads to gains only through selection and not through markups. In these papers, Bertrand competition occurs between firms producing perfect substitutes, so there are no gains from product variety.

Before proceeding, we should give a brief intuition as to why the pro-competitive effect of trade vanishes with heterogeneous firms and the unbounded Pareto distribution. Suppose that we measure markups by the ratio (not the difference) of price and marginal cost. The most productive firm has zero cost, but a non-zero price, so its markup is infinite. The least productive surviving firm will have its marginal cost equal to the reservation price, so its markup is zero. This range of $[0,+\infty)$ for markups applies equally well to domestic and foreign firms, even if the latter face variable trade costs. Furthermore, the distribution of markups within this range is determined by the Pareto distribution of productivity. So changes in trade costs have no impact at all on the *distribution* of markups, from either domestic or foreign firms, but still affect the mass (or extensive margin) of exporters. The fixed distribution of markups no longer holds, however,

[6] Eaton, Kortum and Sotelo (2012) also consider a model with a finite but stochastic number of firms.

when productivity and markups are bounded above, since then the highest foreign markup depends on trade costs (so trade costs also affect the intensive margin).

Our paper proceeds as follows. We show in sections 2 and 3 that the QMOR expenditure function allows us to decompose the cost of living – and therefore welfare – into components that correspond to product variety, the pro-competitive effect, and the selection effect which is captured by average firm productivity. In the trade environments we shall consider, we are able to establish how these components change individually and jointly due to liberalization. This allows us to establish the gains comparing autarky to frictionless trade (section 4), and for small changes in trade costs around the frictionless equilibrium (section 5). Importantly, we contrast the *source* of gains with unbounded versus bounded Pareto, and show that it is only in the bounded case where the product variety and pro-competitive gains apply.

Finally, we are able to compare the magnitude of total gains from trade using unbounded versus bounded Pareto. Measured in relation to initial utility, we find that the proportionate rise in welfare due to trade liberalization is *largest* in the unbounded Pareto case, despite the fact that neither the product variety nor the pro-competitive channels operate in this case. Constraining the Pareto distribution to be bounded allows those extra sources of gains to operate, but reduces the gains due to firm selection, so that the total proportionate gains are lower. This result is related to Melitz and Redding (2013), who compare a heterogeneous firm model (with any productivity distribution) to a homogeneous firm model (i.e. with a degenerate distribution), both with CES preferences. They find higher proportionate gains when productivity is disperse across firms. We are using non-CES preferences, and find higher proportionate gains when productivity is the *most disperse* across firms, i.e. unbounded above. Conclusions are given in section 6, and the proofs of propositions are in the Appendix.

2. Consumer Preferences

Expenditure Function

We shall adopt the quadratic mean of order *r* (QMOR) expenditure function, which is defined by Diewert (1976, p. 130) over a discrete number of goods as:

$$\left[\sum_i \sum_j b_{ij} p_i^{r/2} p_j^{r/2}\right]^{1/r}, \quad r \neq 0,$$

where b_{ij} are parameters. We will consider the symmetric case where $b_{ii} = \alpha$, $b_{ij} = \beta$ for $i \neq j$, and the QMOR function is expressed over a continuum of goods indexed by ω:

$$e_r(\mathbf{p}) = \left[\alpha \int p_\omega^r d\omega + \beta \left(\int p_\omega^{r/2} d\omega\right)^2\right]^{1/r}, \quad r \neq 0. \tag{1}$$

This function is the expenditure needed to obtain one unit of utility, or the cost of living. For specific values of the parameters *r,* α and β, this expenditure function takes on familiar forms. For $\alpha > 0$, $\beta = 0$ and $r = (1-\sigma)$, the expenditure function is CES, so that $r < 0$ for $\sigma > 1$. For $r = 2$, we obtain a quadratic expenditure function, but without the additively separable outside good used by Melitz and Ottaviano (2008). For $r = 1$, we obtain what Diewert (1971) calls a Generalized Leontief function (since the dual to a Leontief production function is linear in prices like the first term of (1) for $r = 1$, while the second term adds generality). And as we show below, as $r \to 0$ then (1) approaches a translog function. So the quadratic mean of order *r* function nests the commonly used homothetic cases.

While the special cases of the quadratic mean of order *r* function have been applied empirically, it has not been applied in a monopolistic competition setting. To do so, we need to recognize that demand is positive if and only if prices are less than a reservation price p^*, equal across goods since the expenditure function is symmetric. In the CES case the reservation price is infinite, but we will focus here on finite reservation prices, while obtaining CES as a limiting

case. Goods that are not available should have their prices in (1) replaced by p^*, because that is the economically relevant price to evaluate expenditure, demand and welfare. To formalize this, without replacing any prices define $\Omega \equiv \{\omega \mid p_\omega \leq p^*\}$ as the set of available goods, with mass $N \equiv \int_\Omega d\omega > 0$. Denote the mass of all possible goods by $\tilde{N} \equiv \int d\omega > N$. Then replacing the prices in (1) by p^* for $\omega \neq \Omega$, we rewrite the expenditure function as:

$$e_r(\mathbf{p}) = \left[\alpha \int_\Omega p_\omega^r d\omega + \alpha(\tilde{N} - N)(p^*)^r + \beta \left(\int_\Omega p_\omega^{r/2} d\omega \right)^2 \right.$$
$$\left. + 2\beta(\tilde{N} - N)(p^*)^{r/2} \int_\Omega p_\omega^{r/2} d\omega + \beta(\tilde{N} - N)^2 (p^*)^r \right]^{1/r}.$$

Differentiating this expression with respect to p^*, dividing by $(\tilde{N} - N)$ and multiplying by utility u, we obtain the demand for a good with price p^*. Setting this demand equal to zero we solve for the reservation price:

$$p^* = \left(\frac{N}{N - [\tilde{N} + (\alpha / \beta)]} \right)^{2/r} \left(\int_\Omega \frac{1}{N} p_\omega^{r/2} d\omega \right)^{2/r}. \qquad (2)$$

The second term in (2) is a mean of order $r/2$ of the prices p_ω (also called a power mean), and for all values of r this mean lies between the minimum and maximum values of p_ω. The reservation price is above this mean price if and only if the first term in (2) is greater than unity. To ensure this and also rule out the CES case of an infinite reservation price, we assume:

Assumption 1

(a) If $r < 0$ then $\alpha > 0$, $\beta < 0$ and $[\tilde{N} + (\alpha / \beta)] < 0$;

(b) If $r > 0$ then $\alpha < 0$, $\beta > 0$ and $0 < [\tilde{N} + (\alpha / \beta)] < N$;

(c) As $r \to 0$ then $\alpha = \left(\frac{1}{\tilde{N}} - \frac{2\gamma}{r} \right)$ and $\beta = \frac{2\gamma}{r\tilde{N}}$ for any $\gamma > 0$.

It is readily confirmed that parts (a) and (b) of Assumption 1 ensure that the first term on the right of (2) exceeds unity, so the reservation price exceeds the mean price. Part (c) is consistent with (a) and (b) in the sense that either set of inequalities hold for small *r* so the first term of (2) is again greater than unity. Furthermore, in this limit it is shown by Diewert (1980, p. 451) and in the Appendix that the expenditure function in (1) approaches the translog form,

$$\ln e_0(\mathbf{p}) = \frac{1}{\tilde{N}}\int \ln p_\omega d\omega - \frac{\gamma}{2\tilde{N}}\int\int \ln p_\omega (\ln p_\omega - \ln p_{\omega'}) d\omega d\omega' .$$

While we have motivated Assumption 1 by the requirement that the reservation price in (2) exceeds the mean price, we should be more rigorous in checking that the QMOR expenditure function satisfies the necessary conditions for an expenditure function: that it is positive and non-decreasing, homogeneous of degree one and concave in prices. Concavity implies that demand curves slope downwards, which will ensure that the reservation price in (2) exceeds *all* prices p_ω for goods with positive demand, and not just the mean price. Confirming that these regularity conditions hold ensures that the expenditure function can be derived from a well-behaved homothetic utility function.

To check these conditions, substitute the reservation price (2) back into (1) to obtain expenditure defined over the available goods $\Omega \equiv \{\omega \mid p_\omega \le p^*\}$:

$$e_r(\mathbf{p}) = \left[\alpha\int_\Omega p_\omega^r d\omega - \left(\frac{\alpha}{N - [\tilde{N} + (\alpha/\beta)]}\right)\left(\int_\Omega p_\omega^{r/2} d\omega\right)^2\right]^{1/r} . \qquad (3)$$

We can compute demand by setting $p_{\omega'} = p$ for prices in a small interval $\omega' = [\omega, \omega + \varepsilon]$, differentiate (3) with respect to *p,* divide by ε, multiply by utility *u* and use (2) to obtain:

$$q_\omega(\mathbf{p}) = \alpha u\left[\frac{p_\omega}{e_r(\mathbf{p})}\right]^{r-1}\left[1 - \left(\frac{p^*}{p_\omega}\right)^{r/2}\right]. \qquad (4)$$

Note that this expression equals CES demand if we specify $\alpha > 0$, $\beta = 0$ and $r = (1-\sigma) < 0$ for $\sigma > 1$, in which case $p^* \to \infty$ and the final bracketed term above vanishes. For $p_\omega < p^*$ this final term has the same sign as α under Assumption 1, so demand is positive except when the price is greater than or equal to the reservation price p^*. This guarantees that the expenditure function is non-decreasing in prices.

We can eliminate utility u in (4) by using total expenditure $\int_\Omega p_\omega q_\omega(\mathbf{p}) d\omega = e_r(\mathbf{p}) u$. Evaluating the integral using (4), we solve for reduced-form expenditure:

$$e_r(\mathbf{p}) = p^* \left\{ \int_\Omega \alpha \left(\frac{p_\omega}{p^*} \right)^r \left[1 - \left(\frac{p^*}{p_\omega} \right)^{r/2} \right] d\omega \right\}^{1/r}. \qquad (5)$$

Again, for $p_\omega < p^*$ the term in square brackets in (5) has the same sign as α under Assumption 1, so expenditure is positive provided that a non-empty set of goods are purchased. Another condition that we need to confirm is that the expenditure function is concave in prices. Concavity implies that demand is downward sloping, and for the symmetric QMOR expenditure function, the reverse is also true as we show in the Appendix. We directly evaluate the elasticity of demand by differentiating (4) with respect to $p_\omega < p^*$, holding utility u and expenditure $e_r(\mathbf{p})$ constant, obtaining:

$$\eta_\omega \equiv -\frac{\partial \ln q_\omega}{\partial \ln p_\omega} = 1 - r + \underbrace{\frac{r}{2} \left\{ \left(\frac{p^*}{p_\omega} \right)^{r/2} \Big/ \left[\left(\frac{p^*}{p_\omega} \right)^{r/2} - 1 \right] \right\}}_{> 0}. \qquad (6)$$

The final term above approaches zero in the CES case when $\beta \to 0$ and $r = (1-\sigma) < 0$, so $p^* \to \infty$ and $\eta \to 1 - r = \sigma$. But with $\beta \neq 0$ under Assumption 1, this term is positive for $p_\omega < p^*$ and so $\eta_\omega > 1 - r$. With $r \leq 0$ we then obtain $\eta_\omega > 1 - r \geq 1 - \frac{r}{2} \geq 1$. For $0 < r \leq 2$, the term in curly brackets in (6) exceeds unity, so that $\eta_\omega > 1 - r + \frac{r}{2} = 1 - \frac{r}{2} \geq 0$. Summarizing, we have:

Proposition 1

Under Assumption 1, for $N > 0$ and $r \leq 2$, the QMOR expenditure function is globally positive, non-decreasing, homogeneous of degree one and concave in prices, with a finite reservation price.

For values of $r > 2$, the demand curves in (4) are still downward sloping in a neighborhood of the reservation price, but we cannot guarantee this property globally. Proposition 1 is that first time that the QMOR expenditure function has been shown to be globally well-behaved, and the fact that we can establish these properties is made possible by the assumed symmetry across goods. A final property is obtained by differentiating (6) and simplifying, to obtain:

$$\frac{\partial \eta_\omega}{\partial \ln(p_\omega / p^*)} = \left(\eta_\omega - 1 + r\right)\left(\eta_\omega - 1 + \tfrac{r}{2}\right) > 0\,,$$

so the elasticity is increasing in price, using the inequalities discussed above Proposition 1.

Welfare Gains

Having confirmed that the expenditure function is well-behaved for $r \leq 1$, we should explain how our demand system relates to that used by ACDR and also derive conditions to ensure welfare gains. Assume for convenience that labor is the only factor of production and each consumer has one unit, so that income equals the wage, w. Then $w = ue_r(\mathbf{p})$, which we can use in (4) and (5) to obtain the demand shares:

$$s_\omega(\mathbf{p}) \equiv \frac{p_\omega q_\omega(\mathbf{p})}{w} = \frac{d(p_\omega / p^*)}{D(\mathbf{p})}, \text{ with } d(p_\omega / p^*) \equiv \alpha\left(\frac{p_\omega}{p^*}\right)^r\left[1 - \left(\frac{p^*}{p_\omega}\right)^{r/2}\right], \qquad (7)$$

and $D(\mathbf{p}) \equiv \int_\Omega d(p_\omega / p^*)d\omega$. The term $D(\mathbf{p})$ in the denominator of the share expression ensures that the shares integrate to unity. In comparison, ACDR assume an expression for the demand shares that depend on the wage, the price p_ω, and also the reservation price p^*, but *does not*

involve any term like $D(\mathbf{p})$. Nevertheless, the shares integrate to unity because of the presence of the reservation price. It turns out that the ACDR demand shares and those in (7) have only one case in common: the translog case, as $r \to 0$. The translog demand shares are given by:

$$s_\omega(\mathbf{p}) = \frac{1}{N} - \gamma\left(\ln p_\omega - \int_\Omega \frac{1}{N} \ln p_\omega d\omega\right).$$

Setting the demand share equal to zero, we see that the reservation price is:

$$\ln p^* = \frac{1}{\gamma N} + \int_\Omega \frac{1}{N} \ln p_\omega d\omega, \tag{8}$$

so that the demand share is also expressed as $s_\omega(\mathbf{p}) = -\gamma\left(\ln p_\omega - \ln p^*\right)$. Integrating over the set of products Ω, we immediately see that the shares integrate to unity without the presence of a term like $D(\mathbf{p})$. Indeed, using Assumption 1(c) it can be shown that $\lim_{r\to 0} D(\mathbf{p}) = 1$ in the translog case.[7] ACDR consider a whole family of demand functions with the convenient property that the shares are defined using the price and the reservation price. All those demand functions except for the translog correspond to non-homothetic utility functions, while the translog is the only case in common with our homothetic expenditure function.

Combining (5) with the definition of $D(\mathbf{p})$, we see that:

$$e_r(\mathbf{p}) = p^* D(\mathbf{p})^{1/r}. \tag{9}$$

With labor as the only factor of production, and firm profits equal to zero under monopolistic competition, welfare is $u = w / e_r(\mathbf{p})$, so a drop in the expenditure function will indicate welfare gains. Proposition 1 guarantees that as the price of any good falls from the reservation price then welfare rises, so increased variety is beneficial for the consumer. Our goal here is to develop

[7] This is shown from (7), (8), Assumption 1(c), and $\lim_{r\to 0} \frac{2\gamma}{r}\left[(p^*/p)^{r/2} - 1\right] = \gamma \ln(p^*/p)$; see the Appendix.

more general sufficient conditions for welfare to rise from one equilibria to another. To achieve this, we need a characterization of the term *D*.

Such an characterization is easiest in the translog case, for which:

$$\lim_{r\to 0} \ln\left[D(\mathbf{p})^{1/r} \right] = -\frac{\gamma}{2}\int_{\Omega}\left[\ln\left(\frac{p_\omega}{p^*} \right) \right]^2 d\omega = -\frac{1}{2\gamma}\int_{\Omega} s_\omega(\mathbf{p})^2 .$$

The first equality can be shown by taking the limit of $\ln[D(\mathbf{p})^{1/r}]$ (see the Appendix), while the second follows from the translog share $s_\omega(\mathbf{p}) = -\gamma\left(\ln p_\omega - \ln p^*\right)$. Combining this result with the translog reservation price in (8), we find that log expenditure is:

$$\ln e_0(\mathbf{p}) = \underbrace{\int_{\Omega}\frac{1}{N}\ln p_\omega d\omega}_{\substack{\text{Pro-competitive} \\ \text{and Selection}}} + \underbrace{\frac{1}{\gamma N}}_{\substack{\text{Product} \\ \text{Variety}}} \underbrace{-\frac{1}{2\gamma}\int_{\Omega} s_\omega(\mathbf{p})^2}_{\text{Herfindahl}} .$$

Thus, expenditure is decomposed into three terms: the first reflects the pro-competitive and selection effects in lowering average prices; the second reflects the benefits of product variety; while the third is the Herfindahl index. Having more disperse expenditure shares will lower the Herfindahl index and *raise* expenditure, thereby lowering welfare. That counter-intuitive result is interpreted by Feenstra and Weinstein (2010) as reflecting "crowding" in product space.

For other values of *r*, we can still obtain a type of Herfindahl index by defining the "adjusted" demand shares:

$$z_\omega(\mathbf{p}) \equiv \frac{s_\omega(\mathbf{p})(p^*/p_\omega)^{r/2}}{\int_{\Omega} s_{\omega'}(\mathbf{p})(p^*/p_{\omega'})^{r/2} d\omega'} .$$

For the translog case, $r = 0$, these adjusted shares equal the conventional shares, while for the quadratic case, $r = 2$, these adjusted demand shares equal the *quantity* share of each product. Then defining the Herfindahl index, $H \equiv \int_{\Omega} z_\omega(\mathbf{p})^2 d\omega$, it is shown in the Appendix that,

$$D(\mathbf{p})^{1/r} = \underbrace{\left[-\alpha\left(\tilde{N}+\frac{\alpha}{\beta}\right)\right]^{1/r}}_{\text{constant}>0}\underbrace{\left[1-\left(\tilde{N}+\frac{\alpha}{\beta}\right)H\right]^{1/r}}_{\downarrow \text{ in } H}, \tag{10}$$

where the final term is decreasing in the Herfindahl index H (since $[\tilde{N}+(\alpha/\beta)]$ has the same sign as r, from Assumption 1).

Recall that expenditure is $e_r(\mathbf{p}) = p * D(\mathbf{p})^{1/r}$, from (9). As the reservation price falls, then so does expenditure and welfare rises. But that gain is offset if the Herfindahl index also falls. In the trade environments we shall consider, that result will be likely whenever variety increases: while there is not a one-to-one correspondence between changes in the mass of products N and the Herfindahl, in all cases that we examine an increase in N implies a lower Herfindahl, which in turn implies an increase in $D(\mathbf{p})^{1/r}$. It follows that if expenditure falls, then it falls by *less* than the reduction in the reservation price. The question is whether we achieve some bound to this offsetting effect on welfare due to crowding in product space. That question is answered in the affirmative, as shown by the following decomposition of expenditure:

Lemma 1

Under Assumption 1, the cost of living can be decomposed alternatively as:

$$\begin{aligned} e_r(\mathbf{p}) &= p * \underbrace{\left[-\alpha\left(\tilde{N}+\frac{\alpha}{\beta}\right)\right]^{1/r}}_{\text{constant}>0}\underbrace{\left[1-\left(\tilde{N}+\frac{\alpha}{\beta}\right)H\right]^{1/r}}_{\downarrow \text{ in } H} \\ &= \sqrt{p *}\underbrace{\left[-\alpha\left(\tilde{N}+\frac{\alpha}{\beta}\right)\right]^{1/r}}_{\text{constant}>0}\underbrace{\left[\int_{\Omega} s_\omega(\mathbf{p}) p_\omega^{-r/2} d\omega\right]^{-1/r}}_{\uparrow \text{ in prices } p_\omega} \end{aligned} \tag{11}$$

Sufficient conditions for a fall in the cost of living and rise in welfare are that: (i) the reservation price falls; and (ii) the Herfindahl index does not fall or the weighted-average price term on the second line does not rise.

The first line of (11), obtained by substituting (10) into (9), has already been discussed. It shows that if the reservation price is falling, but the Herfindahl index is also falling due to an increase in variety, then the decline in the cost of living (and increase in welfare) will be *less than* that in p^*. We have referred to this outcome as a crowding effect, though it also can be given another interpretation. Consider, for example, the CES case with $r = (1 - \sigma) < 0$ and $\beta \to 0$, for which there is no crowding in product space. The reservation price approaches ∞ in this case, but a limiting value of (11) still exists because $\lim_{\beta \to 0}[\tilde{N} + (\alpha / \beta)]^{1/r} \to 0$. Using the formula for p^* in (2), it can be shown that the first line of (11) approaches:

$$\lim_{\beta \to 0} e_r(\mathbf{p}) = \alpha^{1/r} \times \underbrace{\left(\int_\Omega p_\omega^{r/2} d\omega\right)^{2/r}}_{\lim_{\beta \to 0} p^* \left[-\left(\tilde{N} + \frac{\alpha}{\beta}\right)\right]^{2/r}} \times \frac{\left(\int_\Omega p_\omega^{r} d\omega\right)^{1/r}}{\underbrace{\left(\int_\Omega p_\omega^{r/2} d\omega\right)^{2/r}}_{\lim_{\beta \to 0} H}}, \quad \text{for } r < 0.$$

We see that $\lim_{\beta \to 0} H$ equals the ratio of two means: the mean of order r in the numerator, which is the exact index for the CES function; and the mean of order $r/2$ in the denominator, which uses an elasticity that is *too low* for the CES case, since $-r/2 = (\sigma - 1)/2 < (\sigma - 1)$. That numerator just cancels with the same term appearing in the reservation price in (2). A more general interpretation of the Herfindahl index, then, is that it is needed in (11) to correct the reservation price to obtain an exact measure of the cost of living.

Now suppose that the Herfindahl falls when comparing two equilibria, which will tend to increase the cost of living, but that the reservation price also falls. Can we easily determine whether welfare rises or falls? The second line of (11) gives an affirmative answer. Regardless of the change in the Herfindahl, the cost of living falls and welfare rises if – along with the fall in the reservation price – the share-weighted mean of order $-r/2$ shown on the second line does not rise. If the share-weighted mean of prices is falling, then it follows that the decline in the cost of

living and increase in welfare *exceeds* the fall in $\sqrt{p^*}$. So under these conditions, the changes in p^* and $\sqrt{p^*}$ effectively become bounds for the change in the cost of living.

In the next section we shall further decompose the reservation price into terms reflecting (i) product variety, (ii) the markups charged by firms, and (iii) an average of firm costs. The latter will reflect selection across firms. Using this decomposition in (11), we will obtain a decomposition of the cost of living into the three potential sources of gains from trade, together with an additional term (either the Herfindahl index or the share-weighted average of prices) that essentially combines all of these effects.[8]

3. Autarky Equilibrium

We have already assumed that labor is the only factor of production, and now we normalize the wage at unity. As in Melitz (2003), we assume that firms receive a random draw of productivity denoted by φ, so marginal costs are a/φ, where a is the labor need per unit of output for a firm with the lowest productivity of $\varphi = 1$. We will allow the Pareto distribution of productivity to have either an upper-bound in its support, as in Helpman, Melitz and Rubenstein (2008), or to be unbounded above:

Assumption 2

(a) The productivity distribution is Pareto, $G(\varphi) = (1-\varphi^{-\theta})/(1-b^{-\theta})$, $1 \le \varphi \le b$, where the upper bound is $b \in (1, +\infty]$ and $\theta > \max\{0, -r\}$;

(b) There is a sunk cost F of obtaining a productivity draw, but no fixed cost of production.

In part (a), we allow the Pareto distribution to be unbounded ($b = \infty$) or bounded ($b < \infty$). The

[8] Substituting the reservation price from (2) into (11), we obtain a further decomposition of expenditure into terms reflecting an exact price index similar to that in Diewert (1976), and changes in variety. See the Appendix.

restriction that $\theta > \{0, -r\}$ becomes $\theta > (\sigma - 1) > 0$ in the CES case, which is needed for certain first moments to converge in that case; this restriction is needed here for much the same reason. The assumption that there is no fixed cost of production in (b) is made for convenience, and follows Melitz and Ottaviano (2008).

The optimal price for a firm with productivity φ is $p = (a/\varphi)\eta/(\eta - 1)$. We follow ACDR and let $\mu \equiv p/(a/\varphi)$ denote the ratio of price to marginal cost, while $v \equiv p^*/(a/\varphi)$ denotes the ratio of the reservation price to marginal cost. From (6), the elasticity $\eta(p/p^*)$ is a function of the price relative to the reservation price, so $\eta(\mu/v)$ and using this notation the markup of the firm is written as:

$$\mu = \frac{\eta(\mu/v)}{\eta(\mu/v) - 1} \Leftrightarrow \left[\left(\frac{\mu}{v}\right)^{r/2} - \frac{1}{2}\right]\left[r(\mu - 1) + 1\right] = \frac{1}{2},$$

where the second expression follows from (6) and is used to solve for $\mu(v)$.[9] Differentiating this expression, it is shown in the Appendix that elasticity of the markup is $0 < v\mu'(v)/\mu < 1$, so that changes in marginal cost are only partially passed-through to prices.

We can now write the equilibrium conditions in autarky. A firm paying the sunk cost of F receives a draw of productivity φ with probability $g(\varphi) = G'(\varphi)$. We make a change of variables from φ to v. Since $v \equiv p^*/(a/\varphi)$ then $\varphi = av/p^*$, so using the Pareto distribution:

$$g(\varphi)d\varphi = \frac{\theta\varphi^{-\theta-1}}{1 - b^{-\theta}}d\varphi = \frac{\theta v^{-\theta-1}}{1 - b^{-\theta}}\left(\frac{p^*}{a}\right)^{\theta} dv = \left(\frac{p^*}{a}\right)^{\theta} g(v)dv \ . \qquad (12)$$

This change of variables – suggested by ACDR – will considerably simplify our expressions.

[9] The left side of the second expression can be evaluated at $\mu = 1$ and $\mu = v$ to show that it is above and below ½, so that a solution $\mu \in (1, v)$ where it equals ½ always exists.

Because there are no fixed costs of production, the lowest-productivity firm that will continue production will have marginal costs equal to the reservation price, so $v = 1$ is the lower bound. The upper bound for v, denoted by v^*, is obtained when productivity is $b = av / p^*$, so that:

$$v^* = bp^*/a.$$

Starting with the demand share $d / D(\mathbf{p})$ from (7), we multiply that by expenditure L to obtain total demand, and then by $(\mu - 1)/\mu$ to obtain profits. Using the bounds $v \in [1, v^*]$, the expected profit from entering the market must equal the sunk costs of F in equilibrium, so that:

$$\begin{aligned} F &= \int_1^{v^*} \left[\frac{\mu(v)-1}{\mu(v)}\right] d\left(\frac{\mu(v)}{v}\right)\left(\frac{L}{D(\mathbf{p})}\right)\left(\frac{p^*}{a}\right)^{\theta} g(v)dv \\ &= \frac{L\int_1^{v^*} \left[\frac{\mu(v)-1}{\mu(v)}\right] d\left(\frac{\mu(v)}{v}\right)\left(\frac{p^*}{a}\right)^{\theta} g(v)dv}{N_e \int_1^{v^*} d\left(\frac{\mu(v)}{v}\right)\left(\frac{p^*}{a}\right)^{\theta} g(v)dv}, \end{aligned} \tag{13}$$

where in the second line we substitute the expression for $D(\mathbf{p}) = \int_{\Omega} d(p_\omega / p^*) d\omega$. Rather than using the general notation Ω for the set of available products, with the change in variables in (12) we are now defining that set by the bounds for v and the mass of *entering* firms N_e. The mass of firms remaining after those with lowest productivity exit will be:

$$N = N_e \int_1^{v^*} \left(\frac{p^*}{a}\right)^{\theta} g(v)dv = N_e \left(\frac{p^*}{a}\right)^{\theta} G(v^*), \tag{14}$$

which also equals $N_e[1 - G(a/p^*)]$, where (a/p^*) is the productivity of the firm with marginal cost (a/φ) just equal to the reservation price. So as usual in the Melitz model, the mass of surviving firms N equals the mass of entering firms times the probability of survival, which is equivalently written as $[1 - G(a/p^*)] = (p^*/a)^{\theta} G(v^*)$.

We have already used the condition that expenditure equals the workforce L, so that full-employment holds. The remaining equilibrium condition is obtained from the reservation price in (2), re-written slightly by dividing by p^* and using the Pareto distribution:

$$N - [\tilde{N} + (\alpha / \beta)] = \left(N_e \int_1^{v^*} \left(\frac{\mu(v)}{v} \right)^{r/2} \left(\frac{p^*}{a} \right)^{\theta} g(v) dv \right). \qquad (15)$$

The solution to the equilibrium conditions (13)-(15) is summarized in the following result:

Proposition 2

Under Assumptions 1 and 2: (a) the autarky equilibrium conditions (13)-(15) have a positive solution for p^*, N_e and N; (b) if and only if $b = \infty$, the solution for N_e is proportional to the country size L, while the solution for N is independent of country size L.

The existence result in (a) relies on $\theta > \max\{0, -r\}$ in Assumption 2, so that the integrals in (13) and (15) remain bounded even for $v^* \to \infty$. The results in part (b), when productivity is unbounded, are obtained by inspection of the equilibrium conditions. In that case the upper-limit of integration in (13) is infinite, so the two integrals are constant and (13) becomes $F = \kappa_1 L / N_e$. It is immediate that the mass of entrants is proportional to country size in this case. This result is also obtained by ACDR, and follows from the "proportionality relation" between expected profits from entering the market and expected revenue. While these two variables are proportional for every firm in the CES case (i.e. regardless of productivity), they are proportional in *expected terms* for the general demand system that we or ACDR adopt, provided that the Pareto distribution is unbounded above and there are no fixed costs of production. Those two assumptions ensure that the upper ($v = \infty$) and lower ($v = 1$) limits of integration in (13) are exogenous.

There is a second implication of unbounded productivity that is less well known than the linear relationship between entry and country size, and that concerns the mass N of surviving firms. Substituting (14) into (15), the equilibrium condition for the reservation price becomes:

$$-\alpha\left(\tilde{N}+\frac{\alpha}{\beta}\right)=N\left\{\int_1^{v^*}\alpha\left[\left(\frac{\mu(v)}{v}\right)^{r/2}-1\right]g(v)dv\right\}, \qquad (16)$$

which equals a positive constant on the left, from Assumption 1. For $b = \infty$ and $v^* = bp^*/a = \infty$, it is immediate that (16) solves uniquely for N, independent of country size L. This surprising result is also found by Arkolakis, Costinot and Rodriguez-Clare (2010) for the translog case, and ACDR for their more general demand function. The finding that product variety is independent of country size holds only for $b = \infty$ and will have strong implications for the sources of gains from trade, examined in the following section.

Before turning to that discussion, we use the firm-level structure introduced in this section to further decompose the cost of living. The reservation price in Lemma 1 can be written as the product of terms that reflect the average of firm markups and costs, as follows:

Lemma 2

The reservation price in the closed economy is:

$$p^*=\underbrace{\left(\frac{N}{N-[\tilde{N}+(\alpha/\beta)]}\right)^{2/r}}_{\downarrow \text{ in variety } N}\underbrace{\left[\int_1^{v^*}\mu(v)^{r/2}\frac{\tilde{g}(v)}{\tilde{G}(v^*)}\right]^{2/r}}_{\text{Average markup}}\underbrace{\left[\int_1^{v^*}\left(\frac{p^*}{v}\right)^{r/2}\frac{g(v)}{G(v^*)}dv\right]^{2/r}}_{\text{Average of costs}}, \qquad (17)$$

where $\tilde{g}(v)\equiv g(v)/v^{r/2}$ is an "adjusted" density function with distribution $\tilde{G}(v^*)\equiv\int_1^{v^*}\tilde{g}(v)dv$.

The first term appearing on the right of (17) is the same variety term appearing in (2). The second term is a mean of order $r/2$ of the markups $\mu(v)$. To interpret the last term, recall that

$v \equiv p^*/(a/\varphi)$ is the ratio of the reservation price to marginal cost, so $p^*/v = \varphi/a$ is the marginal cost of a firm with productivity φ. The last term in (17) is therefore a mean of order $r/2$ of the marginal cost of firms. If all three terms in Lemma 2 fall and also the share-weighted price term in Lemma 1 does not rise, then we are assured of a welfare gain. We now examine trade environments allowing for such welfare gains.

4. Frictionless Trade

We initially consider frictionless trade, where in addition to the assumption of no fixed costs of production or export, we also ignore variable costs of trade (while introducing such trade costs in the next section). We suppose that the expenditure function in (1) along with Assumptions 1 and 2 holds across countries. In this environment, moving from autarky to frictionless trade is equivalent to growth in the labor force L. We have already shown in Proposition 2 that with an unbounded Pareto distribution, product variety N does not change but N_e rises in proportion to L. It follows that the probability of survival is falling, so there is a *positive selection effect:* only firms with productivity above a higher cutoff level produce in the larger market, while smaller firms are crowded out. Furthermore, this selection effect is the *only* source of welfare gain in the larger market: variety N is independent of L and it will follow that the Herfindahl index in (11) does not change; the average markup in (17) does not change because the upper-limit of integration is $v^* \equiv p^*b/a \to \infty$ as $b \to \infty$; and it follows from Lemmas 1 and 2 that only the fall in firms' costs changes the reservation price and welfare.

When productivity is bounded, however, then we shall find that all three sources of gains from country growth operate: variety increases, the average markup falls, and there is a positive selection effect. To show this, we perform the comparative statics on (13)-(15). Differentiating (16) and simplifying, we obtain:

$$d\ln N_e = -\theta(1+A)d\ln p^*, \qquad A \equiv \frac{N_e}{[\tilde{N}+(\alpha/\beta)]}\left(\frac{b^{-\theta}}{1-b^{-\theta}}\right)\left[1-\left(\frac{\mu(v^*)}{v^*}\right)^{r/2}\right],$$

where $v^* \equiv p^*b/a$. Re-express (13) by moving the denominator $D(\mathbf{p})$ to the left, obtaining:

$$0 = \int_1^{v^*}\left\{\left[\frac{\mu(v)-1}{\mu(v)}\right]L - FN_e\right\}d\left(\frac{\mu(v)}{v}\right)\left(\frac{p^*}{a}\right)^{\theta} g(v)dv.$$

Totally differentiating this condition, and substituting for $d\ln N_e$ from above, we obtain:

$$d\ln N_e = \left(\frac{1+A}{1+A+B}\right)d\ln L \quad \text{and} \quad d\ln p^* = \frac{-d\ln L}{\theta(1+A+B)}, \tag{18}$$

where $B \equiv \left[\frac{L}{F}\left(\frac{\mu(v^*)-1}{\mu(v^*)}\right) - N_e\right]\frac{d[\mu(v^*)/v^*]b^{-\theta}}{D(\mathbf{p})(1-b^{-\theta})}$.

To give the intuition for these results, consider the free entry condition (13). The markups appearing in the numerator of this expression are increasing as the reservation price rises or marginal cost falls, $\mu'(v) > 0$, and likewise for the Lerner index $[\mu(v)-1]/\mu(v)$. The rising markup follows from the fact that the demand elasticity in increasing in price, as noted earlier. So as the reservation price *falls*, the expected markup in the numerator of (13) falls relative to the integral in the denominator. It follows that N_e rises *less than proportionately* with L. That is shown in the first result in (18), where it is immediate that $A > 0$ for $b < \infty$, while $B > 0$ for $b < \infty$ because the Lerner index takes on its highest value at the upper bound v^*, so that:

$$\frac{L}{F}\left(\frac{\mu(v^*)-1}{\mu(v^*)}\right) > \frac{L}{F}\int_1^{v^*}\left(\frac{\mu(v)-1}{\mu(v)}\right)\left[\frac{d\left(\frac{\mu(v)}{v}\right)g(v)}{\int_1^{v^*} d\left(\frac{\mu(v')}{v'}\right)g(v')dv'}\right]dv = N_e\,.$$

The inequality states that the highest Lerner index exceeds its average, and then the equality follows directly from the free entry condition (13) and ensures that $B > 0$.

As productivity is unbounded and $b \to \infty$, then $A, B \to 0$, and so in that case we have $d \ln N_e = d \ln L$, as asserted in Proposition 2(b). For unbounded productivity we also have that the reservation price changes by $d \ln p^* = -d \ln L / \theta$ in (18) and, as discussed above, this change is purely due to the drop in the average of firm costs, i.e. the self-selection of more efficient firms in the larger market.

When productivity is bounded, the reservation price falls by less than $d \ln L / \theta$, as shown by (18) with $A, B > 0$. That means that the increased selection of firms is offset. We conclude that country growth leads to two opposing effects on product variety N: entry of firms N_e rises less than proportionately with L; but also the reservation price falls by less, so the increased selection is offset. It turns out that this second effect dominates so that product variety N *rises* with L. These various results are summarized as:

Proposition 3

Under Assumptions 1 and 2, an increase in country size L under frictionless trade leads to: (a) when $b = \infty$, then p^* falls only due to the drop in the average of firm costs, with the Herfindahl index H fixed; (b) when $b < \infty$, then variety N rises, the Herfindahl falls, and the average of firm costs, markups and the weighted-average price term in (11) all fall; (c) the proportional welfare gain when $b < \infty$ is *less than* that with $b = \infty$.

Part (a), with an unbounded Pareto distribution, has already been discussed above and the constant Herfindahl is shown in the Appendix. The rise in product variety with bounded Pareto, in part (b), has also been motivated by the comparative statics above and the falling Herfindahl is shown in the Appendix. The fact that the average of firm costs and markups both fall follows quite easily from those terms in (17): as the reservation price falls then so does $v^* \equiv p^* b/a$, and

so we are excluding the highest markup term $\mu(v^*)$ in (17); but because p^* appears explicitly within the integral of costs, we are also reducing the average of firm costs, as confirmed in the Appendix. The hardest piece to prove is that the share-weighted mean price term appearing in (11) also falls as country size increases, which together with the drop in the reservation price, ensures that the representative consumer gains in the larger country.

Part (c) shows that despite the fact that all three sources of gains from trade operate in the bounded Pareto case, the total proportional gains from trade are *smaller* with bounded than with unbounded Pareto. This result follows from our welfare decomposition in Lemma 1 and the comparative statics above. With $b = \infty$, we found that $d \ln p^* = -d \ln L / \theta$ and the Herfindahl index is fixed, so it follows immediately from the first line of (11) that the increase in welfare is $d \ln L / \theta$. But with $b < \infty$, we found above that $d \ln p^* < -d \ln L / \theta$, and we also confirm in the Appendix that the Herfindahl index is falling as variety increases. For both reasons, if follows that the frictionless trade leads to an increase in welfare that is *less than* $d \ln L / \theta$, obtained in the unbounded case.

The result in part (c) is related to that in Melitz and Redding (2013), who focus on the CES case only. They show that, provided there are fixed costs of exporting, then the gains from trade with heterogeneous firms exceed that with homogeneous firms as in the Krugman (1980) model. Homogeneous firms are an extreme case of bounded Pareto where there is a mass point at a single productivity. So Melitz and Redding (2013) are comparing any productivity distribution for heterogeneous firms (including bounded or unbounded Pareto) with a degenerate distribution with a single mass point. In comparison, we find that even without fixed costs of trade, the gains from frictionless trade with unbounded Pareto exceed those with bounded Pareto for the QMOR

class of preferences. So we are comparing the unbounded Pareto case to any bounded Pareto (but not including the degenerate case $b = 1$, ruled out in Assumption 2).

With this difference in our comparisons understood, the spirit of our results are similar: having a greater spread of productivities leads to higher proportional gains from trade. That is an especially surprising result in our context because by restricting the range of productivities we give scope for additional sources of gains from trade – due to product variety and reduced markups – that do not operate with the unbounded Pareto distribution. We have found with unbounded Pareto that these additional sources of gains necessarily reduces the self-selection of more efficient firms, so that the total proportional gains are lower.

This result can be usefully compared to the formula for welfare gains found by ACDR, which emphasizes the share of total consumption purchased from the domestic market. Denoting that fraction by λ, the domestic labor force by $\bar{L}$, and the world labor force by $L > \bar{L}$, then $\lambda = 1$ in autarky and $\lambda = \bar{L} / L$ with frictionless trade. With growth it follows that $d \ln \lambda = -d \ln L < 0$. Applying our result above that the welfare gain is $-d \ln p^* = d \ln L / \theta$ with unbounded Pareto, but *smaller* with bounded Pareto, we have therefore proved:

Corollary 1

The gain from frictionless trade equals $-d \ln \lambda / \theta > 0$ with an unbounded Pareto distribution, but is strictly less than this amount with a bounded Pareto distribution for productivity.

Our calculation of the gains from trade goes beyond ACDR by allowing for changes in product variety and markups under a bounded Pareto distribution, even though it is their own formula (obtained with unbounded Pareto) that becomes the upper bound.

5. Variable Trade Costs

We now allow for variable costs of trade, but for simplicity, will suppose that the trading countries are symmetric. We shall let $\widetilde{C} \geq 2$ denote the number of (identical) countries in the world, but due to trade costs, each country does not necessarily trade with all others. We number countries by their proximity to an exporter, so $c = 1$ denotes the local market, $c = 2$ denotes the next closest market, etc. In equilibrium we allow for trade with whole countries or a fraction of a country, as explained below. We shall assume the following structure of trade costs:

Assumption 3

Numbering countries by their proximity to an exporter, delivering one unit to country c means that $\tau(c) = \tau_0 c^{\rho} \geq 1$ units must be sent, with $\tau_0 \geq 1$, $\rho \geq 0$ and $1 < c \leq \widetilde{C}$.

These costs apply onto to cross-border trade, while local sales ($c = 1$) have $\tau \equiv 1$. Notice that number of countries c that a nation is trading with plays the same role in Assumption 3 as distance does in an empirical specification of variable transport costs, while τ_0 plays the same role as a "border effect," i.e. the extra amount that must be sent regardless of distance. We can briefly provide a micro-structure that justifies the trade costs described in Assumption 3.

Suppose that countries are located evenly on a circle of circumference $\widetilde{C} \geq 2$, with the capital city at the center of each country. By construction, the capitals are distance ½ from each border, as shown in Figure 1 for a circle of circumference $\widetilde{C} = 4$. We assume that imported goods much reach the capital city (e.g. an airport) before being costlessly dispersed throughout the country. Then letting *dist* denote the distance a good travels to reach the border of an importing country, the good travels (½+*dist*) to reach the capital city. We assume that the

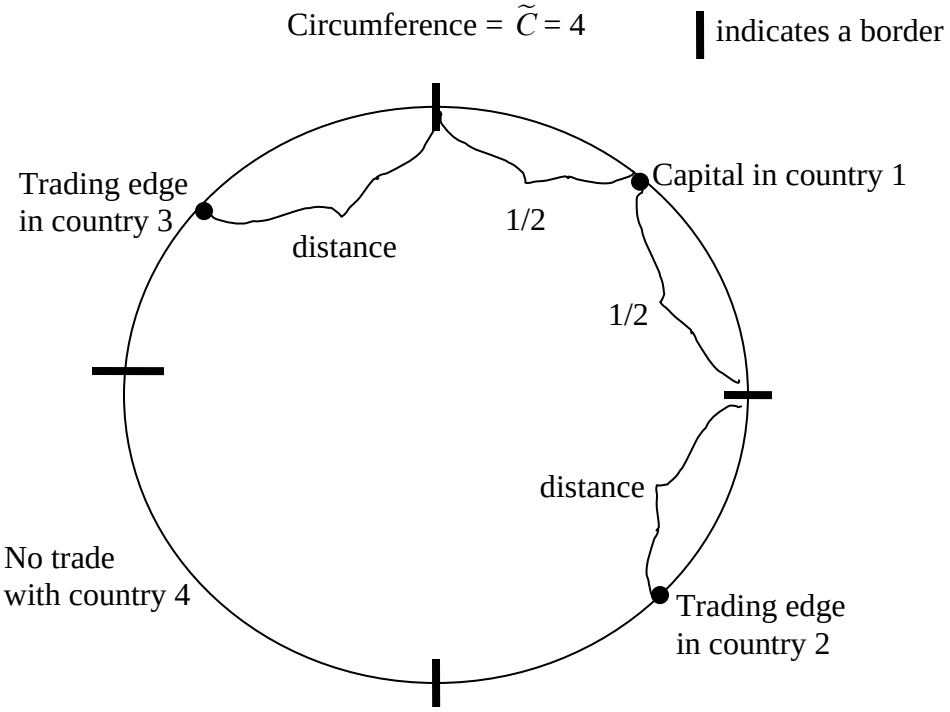


Figure 1: Geography of Trade with Four Countries

variable trade costs are $\tau \equiv \tau_0 \left(½ + dist\right)^{\rho}$. Since goods can enter from the border on either side, if *dist* = 1 then the importer is trading with $(c-1) = 2$ countries (not counting itself), while if *dist* = 2 then the importer is trading with $(c-1) = 4$ countries (not counting itself). Trade with a fraction of a country is also allowed, as illustrated Figure 1 where country 1 is trading with a fraction of the countries on either side up the "trading edge." In general we have $(c-1) = 2dist$ so that $dist = (c-1)/2$. Substituting this into the formula for trade costs we obtain:

$$\tau(c) \equiv \tau_0 \left(½ + dist\right)^{\rho} = \tau_0 [½ + (c-1)/2]^{\rho} = \tau_0 2^{-\rho} c^{\rho}.$$

Absorbing $2^{-\rho}$ into the border costs τ_0, we obtain Assumption 3.

Equilibrium Conditions

With Assumption 3, we can readily solve for the number C of countries that each nation trades with in the symmetric equilibrium. The most efficient firm in any country has marginal labor costs of a/b to produce one unit of output. Normalizing the wage at unity in every country, in equilibrium the marginal cost of producing enough to deliver one unit to the most distant country C will just equal the reservation price in that country:

$$\tau_0 C^{\rho}\left(\frac{a}{b}\right) = p^*, \text{ for } 1 < C < \widetilde{C}. \tag{19}$$

This equilibrium condition provides a very simple relation between the border effect τ_0 and the equilibrium number of trading partners. Of course, changes in the trade costs τ_0 will also affect the reservation price in (19), so we will need to specify all the equilibrium conditions to account for the endogenous response of both C and p^*. Note that if the trade cost τ_0 are sufficiently close to unity and ρ is close enough to zero to have $\tau_0 \widetilde{C}^{\rho}(a/b) < p^*$, then the most efficient firm from each country sells to every market, so that $C = \widetilde{C}$.

To write the other equilibrium conditions with trade, we revisit the change in variables introduced for the autarky economy. When a firm is selling to a foreign country, we let $v \equiv p^*/(\tau a/\varphi)$ denote the ratio of the reservation price to the marginal costs *inclusive* of the variable trade costs. It follows that $\varphi = \tau a v / p^*$, so that from (12):

$$g(\varphi)d\varphi = \frac{\theta\varphi^{-\theta-1}}{1-b^{-\theta}}d\varphi = \frac{\theta v^{-\theta-1}}{1-b^{-\theta}}\left(\frac{p^*}{a\tau}\right)^{\theta} dv = \left(\frac{p^*}{a\tau}\right)^{\theta} g(v)dv \ . \tag{20}$$

From the final expression in (20), we see that higher trade costs τ implies a lower density of firms in any interval dv, which shows how the trade costs affect the *extensive margin* of exporting firms. But in contrast to the unbounded Pareto case, trade costs now also affect the *intensive margin* of exporters, and of the highest-productivity exporter in particular.

The upper bound for v when selling to the domestic market is still denoted by $v^* = bp^*/a$, and the upper bound when selling to a foreign country c is:

$$v^*/\tau(c) = bp^*/a\tau(c). \tag{21}$$

With unbounded productivity, $b \to \infty$, the ratio of reservation price to marginal costs for foreign firms – inclusive of the variable trade costs – is in the range $[1,+\infty)$, the same as for home firms. So there is no difference in the distribution of marginal costs and prices charged by home and foreign firms: both countries have firms with essentially zero costs, charging an infinite markup, and firms with marginal costs equal to the reservation price, with zero markup. But with bounded productivity, we see from (21) that the ratio of the reservation price to marginal costs is in the range $v/\tau \in [1,\ bp^*/a\tau)$, which depends on the reservation price and trade costs. Now the price of the highest productivity firm is affected by trade costs, and we refer to this as an impact on the *intensive margin* of the highest productivity firm.

We continue to let N denote the total mass of products available to the representative consumer in each country, so this notation from section 2 stands. But in section 3, dealing with the autarky economy, we previously let N_e denote the mass of entering firms, while N was the mass of surviving firms. With trade we need to introduce a new notation for the mass of firms in a single country, so we now let M_e denote the mass of entering firms in a single country, and M denote the mass of surviving firms. These are related by the equilibrium condition (14), re-written using this new notation as,

$$M = M_e \int_1^{v^*} \left(\frac{p^*}{a}\right)^{\theta} g(v)dv = M_e \left(\frac{p^*}{a}\right)^{\theta} G(v^*). \tag{22}$$

Conditional on selling at home, the probability of firms in the interval dv selling to country c is then obtained by dividing (20) by the final terms in (22):

$$\frac{[p^*/a\tau(c)]^{\theta} g(v)}{(p^*/a)^{\theta} G(v^*)} dv = \frac{\tau(c)^{-\theta} g(v)}{G(v^*)} dv .$$

The total mass of products N available within a country is obtained by starting with the mass M available in each country, and then integrating over the conditional density above:

$$\begin{aligned} N &= M\left\{1 + \int_1^C \int_1^{v^*/\tau(c)} \frac{\tau(c)^{-\theta} g(v)}{G(v^*)} dv dc\right\} \\ &= M\left\{1 + \tau_0^{-\theta}\left[\frac{C^{(1-\rho\theta)}}{1-(v^*)^{-\theta}}\right] - \frac{(C-1)(v^*)^{-\theta}}{1-(v^*)^{-\theta}}\right\}, \end{aligned} \tag{23}$$

where $C^{(1-\rho\theta)} \equiv (C^{1-\rho\theta} - 1)/(1-\rho\theta)$ is the Box-Cox transformation of C.[10] We see from (23) that trade costs have a direct negative effect on the mass of products available in a country through $\tau_0^{-\theta}$, and also an indirect effect through the reservation price; both of these channels reflect the extensive margin of exporting firms, using the conditional density above. In addition, changes in trade costs have two further effects: through changing the range of countries C that are exporting to each destination; and through changing the mass of domestic products M.

While we will need to take into account all these effects, for the moment just concentrate on the direct impact of trade costs on the mass of exporting firms and countries, holding fixed the mass of domestic products M and also p^*. The effect of changing trade costs on the number of trading partners C can be obtained quite easily from the equilibrium condition (19). Using that condition to solve for C, substituting the result into (23), and differentiating with respect to trade costs while holding M and p^* fixed, we can obtain:

$$\left.\frac{d\ln N}{d\ln \tau_0}\right|_{M,p^*} = -\frac{\theta MC}{N}\left\{\frac{C^{(1-\rho\theta)}(v^*)^{-\theta}}{[1-(v^*)^{-\theta}]}\right\}.$$

[10] The result in (23) is obtained by first integrating over v, obtaining $\tau(c)^{-\theta} G[v^*/\tau(c)]/G(v^*)$; then using the Pareto distribution and trade costs in Assumptions 2 and 3; and then integrating over trading partners c.

This expression shows the partial effect of declining trade costs on expanding the range of available products, through the extensive margin of exporting firms and countries. The result is *increasing* in the number of trading partners *C,* because $C^{(1-\rho\theta)}$ in increasing in *C* regardless of the sign of $1-\rho\theta$. In other words, the greatest impact of reducing trade costs on product variety comes when a country is already trading with the most partners $(C = \widetilde{C})$, so the gain in variety comes exclusively from expanding the extensive margin of exporting firms rather than by expanding the range of exporting countries.

Returning to the full equilibrium conditions, they are (19) for the number of trading partners, (22) for the mass of domestically produced goods, and (23) for the mass of products available to consumers. We still need the free entry condition analogous to (13) in the closed economy, which is now written as:

$$F = \frac{LJ_\rho\left[\left(\frac{\mu(v)-1}{\mu(v)}\right)d\left(\frac{\mu(v)}{v}\right)\right]}{M_e J_\rho\left[d\left(\frac{\mu(v)}{v}\right)\right]}, \tag{24}$$

with,
$$J_\rho[f] \equiv \underbrace{\int_1^{v^*} f(v)\left(\frac{p^*}{a}\right)^\theta g(v)dv}_{\text{domestic market}} + \underbrace{\int_1^{C}\int_1^{v^*/\tau(c)} f(v)\left(\frac{p^*}{a\tau(c)}\right)^\theta g(v)dvdc}_{\text{export markets}}. \tag{25}$$

To move from the free entry condition in the closed economy to the open economy in (24), we have added the integrals over trading partners so that expected profits are computed over domestic sales plus exports. We have introduced the notation $J_\rho[f]$ as a functional depending on ρ, from the trade costs, that integrates any function $f(v)$ over the densities of firms selling in the domestic and all export markets. In each case, we use the density of these firms given by (12) and (20), including the terms $(p^*/a)^\theta$ and $(p^*/a\tau(c))^\theta$ appearing in these densities, reflecting the probability of selling domestically or exporting. Because these probabilities determine the mass

of firms, they influence the *extensive margin*. The similar terms $v^* \equiv bp^*/a$ and (v^*/τ_0) also appear in (25) in the upper limits of integration, where they influence the *intensive margin* of the highest-productivity firm selling at home and also exporting.

A final equilibrium condition is needed to solve for the reservation price, analogous to (15), and rewritten by adding the integral over trading partners:

$$N - \left(\tilde{N} + \frac{\alpha}{\beta}\right) = M_e J_\rho \left[\left(\frac{\mu(v)}{v}\right)^{r/2}\right]. \tag{26}$$

The complete set of equilibrium conditions are (19) and (22)–(26).

Before considering changes in trade costs, we also need to generalize Lemma 2 and define the average of firm costs and markups in the open economy, as follows:

Lemma 3

The reservation price in the open economy is:

$$p^* = \underbrace{\left(\frac{N}{N - [\tilde{N} + (\alpha/\beta)]}\right)^{2/r}}_{\downarrow \text{ in variety } N} \underbrace{\left\{\frac{\tilde{J}_\rho\left[\mu(v)^{r/2}\right]}{\tilde{J}_\rho[1]}\right\}^{2/r}}_{\text{Average markup}} \underbrace{\left\{\frac{J_\rho\left[\left(\frac{p^*}{v}\right)^{r/2}\right]}{J_\rho[1]}\right\}^{2/r}}_{\text{Average of costs}}, \tag{27}$$

where $\tilde{g}(v) \equiv g(v)/v^{r/2}$ is an "adjusted" density function with distribution $\tilde{G}(v^*) \equiv \int_1^{v^*} \tilde{g}(v)dv$, and $\tilde{J}_\rho[f]$ is defined as in (25) but using this density $\tilde{g}(v)$.

The first term on the right of (27) is identical to that in (17), where we continue to let $N \equiv \int_\Omega d\omega$ denote the mass of products available to consumers in any country. The set Ω now includes both domestic goods and imported products, and the integrals over these goods are taken using the functionals $J_\rho[f]$ and $\tilde{J}_\rho[f]$. The interpretation of the two other terms on the right is very similar to what we had in the closed economy. The second term is the average over domestic and

foreign firms of their price-cost ratios $\mu(v)$. Recalling that *v* denotes the ratio of the reservation price to a firm's marginal cost, then *p*/v* is the firm's marginal cost and the third term is the average over all domestic and foreign firms of their marginal costs.

Change in Trade Costs

It is useful to start with the share of expenditure coming from domestic production, or λ. Integrating over the demand shares in (17), we obtain in the symmetric equilibrium:

$$\lambda = \frac{M_e \int_1^{v^*} d\left(\frac{\mu(v)}{v}\right)\left(\frac{p^*}{a}\right)^{\theta} g(v)dv}{M_e J_\rho\left[d\left(\frac{\mu(v)}{v}\right)\right]}. \qquad (28)$$

Of course, the mass of entering firms cancels in the numerator and denominator. The change in this share due to a change in trade costs is greatly simplified if the functional in the denominator is constant. In fact, there are two cases where that results holds:

Lemma 4

The functional $J_\rho[f]$ is not affected by a change in trade costs if: (i) productivity is unbounded or product is bounded and the derivative is calculated at the frictionless equilibrium; and (ii) the change in the reservation price is $d\ln p^* = (1-\lambda)d\ln\tau_0$.

To establish this result, consider first the case where productivity is unbounded. The upper limits of integration in (25) for *v* are ∞, and the mass of trading partners becomes $C = \widetilde{C}$, because some firm is efficient enough to export to every country. So (25) simplifies to:

$$J_\rho[f] \equiv \int_1^{\infty} f(v)\left(\frac{p^*}{a}\right)^{\theta} g(v)dv\left[1+\int_1^{\widetilde{C}} \tau(c)^{-\theta}dc\right] = \int_1^{\infty} f(v)\left(\frac{p^*}{a}\right)^{\theta} g(v)dv \underbrace{\left[1+\tau_0^{-\theta}\widetilde{C}^{(1-\rho\theta)}\right]}_{\text{Inverse of } \lambda}, \qquad (29)$$

where the final term is obtained by completing the integral over trade costs. That final term also measures the ratio of total product variety available in each country to the domestic varieties available, since from (23):

$$N = M\underbrace{\left[1+\tau_0^{-\theta}\widetilde{C}^{(1-\rho\theta)}\right]}_{\text{Inverse of }\lambda}, \qquad \text{as } v^* \to \infty . \tag{30}$$

As we have already noted, with unbounded productivity there is no difference between the home and foreign firms on the intensive margin (their range of prices and markups is the same), but only on the extensive margin (reflecting the mass of home and foreign firms). It follows that the share of consumption purchased from home firms is simply $\lambda = M/N$, which equals the inverse of the bracketed term in (29) and (30), as indicated.

It is now a straightforward matter to differentiate (29) to obtain:

$$d\ln J_\rho[f] = \theta d\ln p^* - \underbrace{\theta\left[\frac{\tau_0^{-\theta}\widetilde{C}^{(1-\rho\theta)}}{1+\tau_0^{-\theta}\widetilde{C}^{(1-\rho\theta)}}\right]}_{(1-\lambda)} d\ln\tau_0 = 0 \Leftrightarrow d\ln p^* = (1-\lambda)d\ln\tau_0.$$

Thus, this functional is constant if $d\ln p^* = (1-\lambda)d\ln\tau_0$. To interpret this result, a drop in trade costs lowers the reservation price, which reduce the value of the functional (over sales, markups, or any other function f) in the domestic market. But since $d\ln p^* < d\ln\tau_0$, then exporters face enhanced opportunities abroad since the drop in their trade costs exceeds the fall in the foreign reservation prices. These two effects both operate on the extensive margin (reflecting the probability of being a domestic firm or exporter in a range dv), and just cancel.

Surprisingly, we obtain the same result when we allow for bounded productivity and introduce an intensive margin, provided that we evaluate the change in the functional around the frictionless equilibrium with $\tau_0 = 1$ and $\rho = 0$. Because we are evaluating derivatives with respect

to trade costs τ_0, it is mathematically correct to evaluate the equilibrium conditions at $\rho = 0$, totally differentiate with respect to τ_0, and then evaluate at $\tau_0 = 1$. This technique simplifies the analysis considerably. In particular, at the frictionless equilibrium, any exporting firm will sell to all countries so the equilibrium condition (19) will hold instead as $\tau_0 \widetilde{C}^{\rho}(a/b) \leq p^*$.[11] Except in the knife-edge case where $\widetilde{C}^{\rho}(a/b) = p^*$, the strict inequality $\tau_0 \widetilde{C}^{\rho}(a/b) < p^*$ will continue to hold for τ_0 in a neighborhood of unity. Therefore, for $\rho = 0$ we can again hold the number of trading partner fixed at $C = \widetilde{C}$ even as we change the trade costs slightly.

Evaluated with $\rho = 0$ and $C = \widetilde{C}$, the functional becomes:

$$J_0[f] = \int_1^{v^*} f(v)\left(\frac{p^*}{a}\right)^{\theta} g(v)dv + (\widetilde{C}-1)\int_1^{v^*/\tau_0} f(v)\left(\frac{p^*}{a\tau_0}\right)^{\theta} g(v)dv. \tag{33}$$

We can readily compute the impact of a change around the frictionless equilibrium, as follows:

$$\left.\frac{\partial J_0[f]}{\partial \ln p^*}\right|_{\tau_0=1} = \widetilde{C}\left[\theta\int_1^{v^*} f(v)\left(\frac{p^*}{a}\right)^{\theta} g(v)dv + f(v^*)\left(\frac{p^*}{a}\right)^{\theta} g(v^*)v^*\right],$$

$$\left.\frac{\partial J_0[f]}{\partial \ln \tau_0}\right|_{\tau_0=1} = -(\widetilde{C}-1)\left[\theta\int_1^{v^*} f(v)\left(\frac{p^*}{a}\right)^{\theta} g(v)dv + f(v^*)\left(\frac{p^*}{a}\right)^{\theta} g(v^*)v^*\right].$$

The first term within the brackets reflects the change in the extensive margin, while the second term reflects the change in the intensive margin. The total change in the functional is constant if:

$$\left.\frac{\partial J_0[f]}{\partial \ln p^*}\right|_{\tau_0=1} d\ln p^* + \left.\frac{\partial J_0[f]}{\partial \ln \tau_0}\right|_{\tau_0=1} d\ln\tau_0 = 0 \Leftrightarrow d\ln p^* = \underbrace{\left(\frac{\widetilde{C}-1}{\widetilde{C}}\right)}_{(1-\lambda)} d\ln\tau_0 . \tag{34}$$

[11] As discussed just after (19), if $\tau_0 \widetilde{C}^{\rho}(a/b) < p^*$, then the most efficient firm from each country sells to every market, so that $C = \widetilde{C}$.

This result shows that the import competition created by the fall in trade costs and the reservation price is just offset by the export opportunities created, so that the functional is constant, when (34) holds. This is the same formula for the price change result that we found above, for unbounded productivity, when evaluating the effect of trade costs on the extensive margin only. Now we see that this result also holds when incorporating the intensive margin and when evaluated at the frictionless equilibrium. In that case, the share of expenditure λ is simply $1/\widetilde{C},$ so that the change in the reservation price in (34) is once again $d\ln p^* = (1-\lambda)d\ln\tau_0$, as we found with the unbounded Pareto.

We now argue that the change in the reservation price of $d\ln p^* = (1-\lambda)d\ln\tau_0$ is in fact the *equilibrium* change. This result follows because the various functionals appearing in the equilibrium conditions are all (locally) constant. In the free entry condition (24), for example, the functionals evaluate expected profits, and since these are constant it follows that the mass of entrants in each country, M_e, is constant in the denominator.[12] Then from the equilibrium condition (26), it is immediate that product variety N is constant. Furthermore, since the denominator of (24) is the term $D(\mathbf{p})$ introduced earlier, it is constant and then so is the Herfindahl index. We confirm in the Appendix that these various results are enough to ensure that the change in prices $d\ln p^* = (1-\lambda)d\ln\tau_0$ is the equilibrium change. Finally, because the Herfindahl index does not change, it follows from Lemma 1 the change in welfare equals $-d\ln p^* = -(1-\lambda)d\ln\tau_0$. We summarize these results with:

[12] That result may appear surprising in view of Proposition 2, which stated that country growth leads to more entrants. But the worldwide mass of entrants that are potentially exporting to each country is still growing. We could define that worldwide mass as N_e, computed as in (23) but with M_e on the right. with a fall in trade costs, it would follow that N_e is rising.

Proposition 4

Under Assumptions 1–3, a small reduction in trade costs has the following results whether productivity is unbounded, or bounded with the change evaluated at the frictionless equilibrium:
(a) no change in the mass of entrants M_e, the mass of varieties N, or the Herfindahl index H;
(b) the same proportionate fall in the reservation price and rise in welfare of $-(1-\lambda)d\ln\tau_0$,
which is due only to the selection of firms.

The final result in Proposition 4, that only the selection effect operates around the frictionless trade equilibrium, follows because the functionals defining the average markup in Lemma 3 are locally constant so there is no change in the average markup. To interpret this result, a drop in trade costs lowers the reservation price and therefore lowers the markups charged by home firms, while the lowest-productivity firms exit. But since $d\ln p^* < d\ln\tau_0$, then foreign exporters have a greater drop in their trade costs than in the home reservation price, and they will raise their markups. That allows some low-productivity foreign firms to enter and begin exporting to the home market. In the end, with unbounded productivity the home and foreign markups are both in the range [1, +∞), before and after the drop in trade costs. The changes in the extensive margin of domestic and foreign firms due to falling trade costs have no effect on this distribution of markups, or on the average markup.

What we now find is that this result holds even with bounded productivity. In that case, the markup charged by the highest productivity home firm is reduced, and the markup charged by the highest productivity foreign exporter is increased. These changes in the intensive margin also cancel when evaluated for a small change in trade costs at the frictionless equilibrium, so

there is no pro-competitive effect. In addition, we have found that the mass of varieties N is locally constant, so that there is no variety effect either.

Because we already know the impact of a large change in trade costs from Proposition 3, moving from autarky to frictionless trade, the results in Proposition 4 cannot hold for all parameter values away from the frictionless equilibrium. Rather, for some parameter values we must observe the same qualitative results as reported in Proposition 3:

Corollary 2

With bounded productivity, a reduction in trade costs τ_0 evaluated at some values of $\tau_0 > 1$ or $\rho > 0$ leads to: (a) a fall in the reservation price and rise in welfare less than $-(1-\lambda)d\ln\tau_0$; (b) reduced gains due to the selection effect, but positive gains due to increased product variety and a reduction in the average markup.

We have not established that the results in Corollary 2 hold *for all* values of $\tau_0 > 1$ and $\rho > 0$, though we would conjecture that such a result holds.

There is one final result that *differs* for bounded and unbounded productivities, even at the frictionless equilibrium. While we have found that $d\ln p^* = (1-\lambda)d\ln\tau_0$ holds in either case, that is not the formula used by ACR and ACDR to characterize the gains from reduced trade costs. Rather, these authors focus on the change in the share of consumption coming from domestic producers, or λ in (28). We have already argued that the functional in the denominator of this expression is constant with unbounded productivity, and locally constant with bounded productivity. So given the change in prices $d\ln p^* = (1-\lambda)d\ln\tau_0$, the change in λ comes from the numerator only, and is readily calculated as:

$$d\ln\lambda = \left\{\theta + \left[\frac{d\left(\frac{\mu(v^*)}{v^*}\right)g(v^*)v^*}{\int_1^{v^*} d\left(\frac{\mu(v)}{v}\right)g(v)dv}\right]\right\} d\ln p^* \Rightarrow \begin{cases} -d\ln p^* = -\dfrac{d\ln\lambda}{\theta} & \text{if } b = \infty \\ -d\ln p^* < -\dfrac{d\ln\lambda}{\theta} & \text{if } d\ln p^* < 0 \text{ and } b < \infty. \end{cases}$$

With unbounded productivity, since $\lim_{v^*\to\infty} g(v^*)v^* = 0$ we obtain the formula for welfare gain found by ACR and ACDR, $-d\ln p^* = -d\ln\lambda/\theta$. With bounded productivity, however, the fall in the share of consumption coming from home production *exceeds* that obtain with unbounded productivity. It follows that the formula $-d\ln\lambda/\theta$ now *overstates* the welfare gains from reducing trade costs:

Corollary 3

The gain from a small reduction in trade costs equals $-d\ln\lambda/\theta > 0$ with an unbounded Pareto distribution for productivity, but is strictly less than this amount with a bounded Pareto distribution when evaluated at the frictionless equilibrium.

This result is very similar in spirit to Corollary 1, and shows that the formula for gains from trade from ACDR holds as an upper bound in the two cases we have considered: a large reduction in trade costs from autarky to frictionless trade; and a small reduction in trade costs when evaluated at the frictionless equilibrium.

To explain why the share of share of consumption coming from home production falls more when productivity is bounded, we note that the fall in the mass of surviving domestic firms M is also larger with bounded productivity, since from (22) we obtain (with M_e fixed):

$$d\ln M = \left[\theta + \frac{g(v^*)v^*}{G(v^*)}\right] d\ln p^*.$$

The fall in the mass of domestic firms due to a reduction in trade costs is therefore $d\ln M =$

$\theta d \ln p^* = \theta(1-\lambda) d \ln \tau_0 < 0$ when productivity is unbounded. From the above equation, domestic varieties fall even more when productivity is bounded (and evaluated at the frictionless equilibrium). We interpret than extra drop in the mass of firms as resulting from the extra import competition in the intensive margin, as applies when productivity is bounded.

6. Conclusions

Our goal in this paper has been to evaluate the gains from trade when firm markups are endogenous. To achieve that, we have introduced a quite general class of preferences represented by the quadratic mean of order *r* expenditure function, due to Diewert (1976). Prior applications of this expenditure/cost function have been mainly empirical, i.e. estimating the function for specific values of the parameter *r.* In that case, the concavity and other properties of the function are checked at the estimated parameters. For theoretical purposes, we want to ensure that the function is globally well behaved. We have shown that this is the case for a symmetric function and the parameter values in Assumption 1.

Despite the general class of preferences we use, however, the crucial feature of the model allowing for multiple sources of gains from trade comes from the supply side of the model. With heterogeneous firms, the very simple result of Krugman (1979) linking a drop in the markup to (frictionless) trade no longer applies necessarily. We have shown that this result does not apply when the distribution of firm productivity is Pareto and the support is unbounded above, allowing for infinite productivity and zero costs. That assumption was not part of the Melitz (2003) model, but was introduced by Chaney (2008), enabling him to derive a "distorted" gravity equation even with heterogeneous firms. The finding that this structure rules out a pro-competitive gain from trade, and also rules out any variety gain, should be enough to suggest that alternative forms for the distribution of productivity should be investigated. Here we have

focused on the Pareto distribution with a support that is bounded above. It is know from the work of Helpman, Melitz and Rubenstein (2008) that this distribution allows for a tractable gravity equation, at least in its empirical specification, and we have shown that it can also be used to obtain theoretical results.

We have investigated two case of trade liberalization. The first, following Krugman (1979), was growth in country size. That exercise is meant to capture the movement from autarky to frictionless trade with another country. Equivalently, we can think of trade costs falling from some high level leading to autarky to zero, so the results for this case correspond to a *large* change in trade costs. We found in Proposition 3 that the product variety and pro-competitive gains from trade operate if and only if productivity is bounded above. This result shows that the main analytical results of ACDR, who assume that productivity is unbounded, are quite special and only allow for gains due to firm selection. Despite that limitation, their formula applies as an *upper bound* to the proportional gains obtained from frictionless trade when productivity is bounded and all three types of gains operate. When variety and pro-competitive gains operate (due to bounded productivity), then the selection effect is sufficiently offset that the total gains are reduced.

The second case we have investigated is a small change in trade costs in a symmetric equilibrium. When productivity is unbounded then this small change can be evaluated in the presence of trade frictions, but when productivity is bounded then we focus on a small change around the frictionless equilibrium. In both cases, we find the import competition due to the a reduction in trade costs is just offset by the export opportunities created. For example, the fall in domestic markups is just offset by a rise in foreign markups, so that the average markup does not change. More generally, the fall in trade costs has no impact on the entry of domestic firms,

product variety available to consumers, or on the average markup, so that the gains from trade are due entirely to the exit of less efficient firms. If productivity is bounded and we move away from the frictionless equilibrium, then the product variety and pro-competitive gains must come back into play.

There are a number of directions for further research. First, neither of the two cases of trade liberalization we have examined took advantage of the geography of trade and trade costs as illustrated in Figure 1 and Assumption 3: with finite reservation prices, bounded productivity and positive trade costs, there are countries not trading with others. In contrast, the two cases we considered implied that each country trades with all others (since either there is a firm productive enough to do so, or with bounded productivity we evaluate the change in trade costs at the frictionless equilibrium). We suggested in the previous section that even when countries trade will all others, there is still a potentially large impact of trade liberalization on variety through the extensive margin of exporting firms. Still, it would be of interest to examine how product variety changes when *new countries* begin exporting, especially since that is the focus of the empirical work in Broda and Weinstein (2006), for example, and is very easily measured in the trade data.

Second, we have not allowed for fixed costs of domestic production or exporting. The presence of such fixed costs would make the lowest profitable marginal cost endogenous (instead of unity as used here because we divide marginal cost by the reservation price). That change alone would re-introduce the product variety and pro-competitive channels as potential channels of gain. While it is challenging to allow for fixed costs and endogeneity of the lowest marginal cost in general, its formulation is simplified in either the translog or the CES cases, which

deserve more attention. Of course, with fixed costs then we should also allow these to fall as an alternative force leading to increased trade, and evaluate the gains.

Finally, while the bounded Pareto given us a particularly sharp comparison with the unbounded case, we should also allow for other distributions of firm productivity. Head and Mayer (reference needed) argue that the log-normal distribution offers a better approximation to actual firm sizes than the Pareto. Edmond, Midrigan and Xu (2012) allow for a "double" Pareto as a way to better approximate the actual distribution, and Holmes, Hsu and Lee (2013) also investigate distributions other than the Pareto. These authors show that the pro-competitive gains from trade tend to be greater outside the simple Pareto case, while Feenstra and Weinstein (2010) estimate these gains for the United States without making any assumption on the distribution. For all these reasons, we expect that future literature will be less likely to adopt the unbounded Pareto case as representative when investigating the gains from trade.

Appendix:

Translog expenditure function:

We start with the translog expenditure function, re-written slightly from the main text as:

$$\ln e_0(\mathbf{p}) = \frac{1}{\tilde{N}}\int \ln p_\omega d\omega - \frac{\gamma}{2}\int \ln p_\omega^2 d\omega + \frac{\gamma}{2\tilde{N}}\int\int \ln p_\omega \ln p_{\omega'} d\omega d\omega',$$

using $\tilde{N} = \int d\omega$. Our goal is to show that this is the limit of the QMOR function as $r\to 0$, under Assumption 3(c). We show this in reverse. By using $\ln x = \lim_{r\to 0}\left(\frac{x^r - 1}{r}\right) = \lim_{r/2\to 0}\left(\frac{x^{r/2}-1}{r/2}\right)$ we re-write the translog function as:

$$\lim_{r\to 0}\left(\frac{e_r^r - 1}{r}\right) = \frac{1}{\tilde{N}}\int \lim_{r\to 0}\left(\frac{p_\omega^r - 1}{r}\right)d\omega - \frac{\gamma}{2}\int \lim_{r/2\to 0}\left(\frac{p_\omega^{r/2}-1}{r/2}\right)^2 d\omega + \frac{\gamma}{2\tilde{N}}\int\int \lim_{r/2\to 0}\left(\frac{p_\omega^{r/2}-1}{r/2}\right)\lim_{r/2\to 0}\left(\frac{p_{\omega'}^{r/2}-1}{r/2}\right)d\omega d\omega'.$$

Dropping the limit operator, multiplying through by r and completing the squares, we obtain:

$$e_r(\mathbf{p})^r - 1 = \frac{1}{\tilde{N}}\int (p_\omega^r - 1)d\omega - \frac{2\gamma}{r}\int (p_\omega^r - 2p_\omega^{r/2} + 1)d\omega + \frac{2\gamma}{r\tilde{N}}\int\int (p_\omega^r p_{\omega'}^r - p_\omega^r - p_{\omega'}^r + 1)d\omega d\omega'.$$

Because $\tilde{N} = \int d\omega$ and using $\alpha = \left(\frac{1}{\tilde{N}} - \frac{2\gamma}{r}\right)$ and $\beta = \frac{2\gamma}{r\tilde{N}}$ from Assumption 1(c), this becomes:

$$e_r(\mathbf{p})^r = \alpha\int p_\omega^r d\omega + \beta\int\int p_\omega^r p_{\omega'}^r\, d\omega d\omega' + \frac{2\gamma}{r}\int (2p_\omega - 1)d\omega - \frac{2\gamma}{r\tilde{N}}\int\int (p_\omega^r + p_{\omega'}^r - 1)d\omega d\omega'.$$

The last terms cancel, and so raising both side to the power $1/r$ we obtain (1). Doing the same steps in reverse, it follows that the translog function is the limiting value of (1) as $r\to 0$.

The other property we wish to establish is the limiting value of $D(\mathbf{p})$ as $r\to 0$. Using the definition in (7) with $\alpha = \left(\frac{1}{\tilde{N}} - \frac{2\gamma}{r}\right)$:

$$\lim_{r\to 0} D(\mathbf{p}) = \int_\Omega \lim_{r\to 0}\left(\frac{1}{\tilde{N}} - \frac{2\gamma}{r}\right)\left(\frac{p_\omega}{p^*}\right)^r\left[1 - \left(\frac{p^*}{p_\omega}\right)^{r/2}\right]d\omega = \gamma\int_\Omega \lim_{r\to 0}\left[\frac{(p^*/p_\omega)^{r/2} - 1}{r/2}\right]\left(\frac{p_\omega}{p^*}\right)^r d\omega,$$

which equals $\int_{\Omega}\gamma\left(\ln p^{*}-\ln p_{\omega}\right)d\omega=\int_{\Omega}s_{\omega}(\mathbf{p})d\omega=1$, as desired. We also need to check the limiting value of $D(\mathbf{p})^{1/r}$. It is convenient to start with the formula for $D(\mathbf{p})^{1/r}$ established below in (A1). Again using the parameter values in Assumption 1(c), we obtain:

$$\lim_{r\to 0}\ln\left[D(\mathbf{p})^{1/r}\right]=\lim_{r\to 0}\frac{1}{r}\ln\left\{1-\frac{r}{2\gamma\tilde{N}}+\left(\frac{1}{\tilde{N}}-\frac{2\gamma}{r}\right)\int_{\Omega}\left[\left(\frac{p_{\omega}}{p^{*}}\right)^{r/2}-1\right]^{2}d\omega\right\}.$$

If a finite limit exists, then the expression in curly brackets must approach unity, so its natural log approaches zero. In that case, we can use the approximation $\ln(1+x)\approx x$, which will be arbitrarily close for $x\to 0$, to rewrite the above expression as:

$$\lim_{r\to 0}\ln\left[D(\mathbf{p})^{1/r}\right]=\lim_{r\to 0}\frac{1}{r}\left\{-\frac{r}{2\gamma\tilde{N}}+\left(\frac{1}{\tilde{N}}-\frac{2\gamma}{r}\right)\int_{\Omega}\left[\left(\frac{p_{\omega}}{p^{*}}\right)^{r/2}-1\right]^{2}d\omega\right\}.$$

Eliminating terms in the curly brackets that approach zero, we obtain:

$$\lim_{r\to 0}\ln\left[D(\mathbf{p})^{1/r}\right]=\lim_{r\to 0}\frac{1}{r}\left\{-\frac{2\gamma}{r}\int_{\Omega}\left[\left(\frac{p_{\omega}}{p^{*}}\right)^{r/2}-1\right]^{2}d\omega\right\}=-\frac{\gamma}{2}\int_{\Omega}\left[\lim_{r\to 0}\frac{(p_{\omega}/p^{*})^{r/2}-1}{r/2}\right]^{2}d\omega,$$

which gives the result shown in the main text since the above limit is $\ln(p_{\omega}/p^{*})$. Having established these translog properties referred to in the main text, we prove all remaining results for the QMOR function with $r\neq 0$, appealing to continuity to establish them for $r=0$.

Proof of Proposition 1:

(a) Consider the expenditure function (3) for $r\leq 1$, $r\neq 0$, with the set of available goods defined by $\Omega\equiv\{\omega\mid p_{\omega}\leq p^{*}\}$ using the reservation price in (2). Multiplying *all* prices (including prices for goods not available and also p^{*}) by $\lambda>0$ has no effect on Ω, so that $e_{r}(\lambda\mathbf{p})=\lambda e_{r}(\mathbf{p})$ in (3)

and the expenditure function is homogeneous of degree one. We have already confirmed in the text that demand in non-negative, so that the expenditure function is non-decreasing in prices, and strictly positive for expenditure on a non-empty set of goods. The remaining condition we need to confirm is concavity in prices, which is shown in several steps.

` Letting $\eta_\omega \equiv -\partial \ln q_\omega / \partial \ln p_\omega$ denote the elasticity of demand from (6), we argued in the main text that $\eta_\omega > 1-r \geq 0$ for $r \leq 1$ and also $\eta_\omega \geq 0$ for $1 < r \leq 2$ We use these conditions to establish the concavity of the expenditure function. It is convenient to use the following characterization of concavity for a function defined over R^n, but which we apply to the expenditure function defined over a continuum of prices:

Lemma A1 (Marshall and Olkin, 1979, p. 446): Let $S \in R^n$ be an open convex set, φ: $S \rightarrow R$. For $x, y \in S$, define the one variable function $f(\lambda) = \varphi(\lambda x + (1-\lambda)y)$ on the interval (0, 1). Then φ is concave on S if and only if f is concave on (0, 1) for all $x, y \in S$.

Following this Lemma, we consider prices $p_\omega(\lambda) \equiv \lambda p_\omega + (1-\lambda)p'_\omega$ and reservation prices $p^*(\lambda)$ obtained by using $p_\omega(\lambda)$ in (2), along with $\Omega_\lambda \equiv \{\omega \mid p_\omega \leq p^*(\lambda)\}$. Then from (3),

$$f(\lambda) \equiv e_r(\lambda \mathbf{p} + (1-\lambda)\mathbf{p}') = \left[\alpha \int_{\Omega_\lambda} p_\omega(\lambda)^r d\omega - A\left(\int_{\Omega_\lambda} p_\omega(\lambda)^r d\omega \right)^2 \right]^{1/r},$$

where $A \equiv \alpha / [N - (\tilde{N} + (\alpha / \beta))]$. Small changes in the set Ω_λ have no effect on expenditure because these borderline goods have zero demand. So differentiating, we obtain:

$$f'(\lambda) = \alpha e_r^{1-r} \int_{\Omega_\lambda} p_\omega(\lambda)^{r-1} \left[1 - \left(\frac{p^*(\lambda)}{p_\omega(\lambda)} \right)^{r/2} \right] (p_\omega - p'_\omega) d\omega = \frac{1}{u} \int_{\Omega_\lambda} q_\omega(\lambda)(p_\omega - p'_\omega) d\omega,$$

where the first equality uses $p^*(\lambda)$ from (2) and the second uses demand $q_\omega(\lambda)$ defined as in (4) using prices $p_\omega(\lambda)$ and utility *u.* With fixed income, utility depends on prices and λ. For example, with income of unity, utility is $u = 1/e_r(\lambda\mathbf{p} + (1-\lambda)\mathbf{p}')$. Using the first expression above to again differentiate e_r^{1-r}, and the second expression to differentiate $q_\omega(\lambda)$ holding utility fixed, we obtain :

$$f''(\lambda) = \frac{(1-r)f'(\lambda)^2}{e_r} + \frac{1}{u}\int_{\Omega_\lambda} \frac{\partial q_\omega(\lambda)}{\partial p_\omega(\lambda)}(p_\omega - p'_\omega)^2 d\omega$$

$$= \frac{(1-r)f'(\lambda)^2}{e_r} - \frac{1}{u}\int_{\Omega_\lambda} \eta_\omega(\lambda) q_\omega(\lambda)\frac{(p_\omega - p'_\omega)^2}{p_\omega(\lambda)} d\omega,$$

where the second line uses $\eta_\omega(\lambda)$ defined as the elasticity of demand at prices $p_\omega(\lambda)$. For $1 < r \le 2$ we have shown that $\eta_\omega \ge 0$, so we immediately obtain $f''(\lambda) \le 0$, as desired.

For $r \le 1$ we use $\eta_\omega(\lambda) \ge 1 - r$ to obtain:

$$f''(\lambda) \le \frac{(1-r)}{u}\left[\int_{\Omega_\lambda} q_\omega(\lambda)h(\lambda)(p_\omega - p'_\omega)d\omega - \int_{\Omega_\lambda} q_\omega(\lambda)\frac{(p_\omega - p'_\omega)^2}{p_\omega(\lambda)} d\omega\right],$$

where $h(\lambda) \equiv \dfrac{f'(\lambda)}{e_r} = \dfrac{\int_{\Omega_\lambda} q_\omega(\lambda)(p_\omega - p'_\omega)d\omega}{\int_{\Omega_\lambda} q_\omega(\lambda)p_\omega(\lambda)d\omega}$. It follows that $f''(\lambda) \le 0$ if the expression in brackets above is less than or equal to zero:

$$h(\lambda)\int_{\Omega_\lambda} q_{\omega'}(\lambda)(p_{\omega'} - p'_{\omega'})d\omega' \le \int_{\Omega_\lambda} q_{\omega'}(\lambda)(p_{\omega'} - p'_{\omega'})^2 / p_{\omega'}(\lambda)d\omega',$$

where we introduce the notation ω' for convenience. This inequality is established by expanding the square $\left\{\left[\frac{p_{\omega'}(\lambda)}{p_\omega(\lambda)}\right]^{1/2}(p_\omega - p'_\omega) - \left[\frac{p_\omega(\lambda)}{p_{\omega'}(\lambda)}\right]^{1/2}(p_{\omega'} - p'_{\omega'})\right\}^2 \ge 0,$ and using the definition of $h(\lambda)$. QED

Proof of Lemma 1:

To establish the first line of (11), we need to show that (10) holds. From (2), dividing by p^* and rearranging terms, we obtain:

$$\alpha\left[N-\left(\tilde{N}+\frac{\alpha}{\beta}\right)\right]=\int_{\Omega}\alpha\left(\frac{p_\omega}{p^*}\right)^{r/2}.$$

Noting that $N=\int_\Omega d\omega$, we have $-\alpha[\tilde{N}+(\alpha/\beta)]=\int_\Omega \alpha\left[(p_\omega/p^*)^{r/2}-1\right]d\omega,$ which is a positive constant from Assumption 1. Subtracting this term from the definition of $D(\mathbf{p})$, and completing the square, we obtain,

$$D(\mathbf{p})^{1/r}=\left\{-\alpha\left(\tilde{N}+\frac{\alpha}{\beta}\right)+\alpha\int_{\Omega}\left[\left(\frac{p_\omega}{p^*}\right)^{r/2}-1\right]^2 d\omega\right\}^{1/r}. \tag{A1}$$

Using (7), the "adjusted" demand shares are:

$$z_\omega(\mathbf{p})\equiv\frac{s_\omega(\mathbf{p})(p^*/p_\omega)^{r/2}}{\int_\Omega s_\omega(\mathbf{p})(p^*/p_\omega)^{r/2}d\omega}=\frac{\alpha\left[(p_\omega/p^*)^{r/2}-1\right]}{\int_\Omega\alpha\left[(p_\omega/p^*)^{r/2}-1\right]d\omega},$$

The denominator on the right is a positive constant $-\alpha[\tilde{N}+(\alpha/\beta)]$, as noted above. Then defining the Herfindahl index, $H\equiv\int_\Omega z_\omega(\mathbf{p})^2 d\omega$, it follows from (A1) that (10) holds. Using this in (9), we obtain the first line of (11).

To obtain the second line of (11), we develop a second expression for the term $D(\mathbf{p})$. From (7), we can write:

$$\int_\Omega s_\omega(\mathbf{p})\left(\frac{p^*}{p_\omega}\right)^{r/2}d\omega=\frac{1}{D(\mathbf{p})}\int_\Omega\alpha\left[\left(\frac{p^*}{p_\omega}\right)^{r/2}-1\right]d\omega,$$

where the integral on the right is once again $-\alpha[\tilde{N}+(\alpha/\beta)]>0.$ It follows that,

$$D(\mathbf{p})^{1/r} = \left[-\alpha\left(\tilde{N} + \frac{\alpha}{\beta}\right)\right]^{1/r}\left[\int_{\Omega} s_{\omega}(\mathbf{p}) p_{\omega}^{-r/2}\right]^{-1/r} \frac{1}{\sqrt{p^*}}, \tag{A2}$$

Using this in (9), we obtain the second line of (11).

By substituting the reservation prices from (2) into (11), we can further obtain:

$$e_r(\mathbf{p}) = \underbrace{\left(\frac{-\alpha N[\tilde{N} + (\alpha/\beta)]}{N - [\tilde{N} + (\alpha/\beta)]}\right)^{1/r}}_{\downarrow \text{ in N}} \times \underbrace{\left(\frac{\int_{\Omega} \frac{1}{N} p_{\omega}^{r/2} d\omega}{\int_{\Omega} s_{\omega}(\mathbf{p}) p_{\omega}^{-r/2} d\omega}\right)^{1/r}}_{\text{Exact price index without variety}}$$

This final expression for the cost of living consists of a term that declines as product variety rises, and an exact price index *without* accounting for variety. This formula for the exact price index is very similar to that found in Diewert (1976, p. 131), and can be shown to be identical to that formula for the symmetric case we are considering here. QED

Proof of Lemma 2:

Working from (2), the reservation price equals:

$$\begin{aligned} p^* &= \left(\frac{N}{N - [\tilde{N} + (\alpha/\beta)]}\right)^{2/r} p^* \left[\int_{\Omega} \frac{1}{N}\left(\frac{p_{\omega}}{p^*}\right)^{r/2} d\omega\right]^{2/r} \\ &= \left(\frac{N}{N - [\tilde{N} + (\alpha/\beta)]}\right)^{2/r} p^* \left[\int_{1}^{v^*}\left(\frac{\mu(v)}{v}\right)^{r/2} \frac{g(v)}{G(v^*)}\right]^{2/r} \\ &= \left(\frac{N}{N - [\tilde{N} + (\alpha/\beta)]}\right)^{2/r} \left[\int_{1}^{v^*} \mu(v)^{r/2} \frac{\tilde{g}(v)}{\widetilde{G}(v^*)}\right]^{2/r} p^* \left[\frac{\widetilde{G}(v^*)}{G(v^*)}\right]^{2/r} \end{aligned}$$

The first line follows from (2) using simple algebra, and the second line follows by expressing the integral using the change in variables in (12), where the term $(p^*/a)^{\theta}$ from (12) cancels in the numerator and denominator of the conditional density $g(v)/G(v^*)$. Then the third line follows by defining the "adjusted" density function $\tilde{g}(v) \equiv g(v)/v^{r/2}$, with $\widetilde{G}(v^*) \equiv \int_1^{v^*} \tilde{g}(v)dv$.

Using the definition of the "adjusted" distribution function, the final term on the third line above is easily re-written as it appears in Lemma 2. QED

Proof of Proposition 2:

(a) Using (16) and $N = N_e(p*/a)^{\theta}$ to substitute for N_e in (13) we obtain:

$$\frac{F}{L} = \left(\frac{p*}{a}\right)^{\theta} \left\{ \frac{\int_1^{v*} \left[\frac{\mu(v)-1}{\mu(v)}\right] d\left(\frac{\mu(v)}{v}\right) g(v)dv \int_1^{v*} \alpha \left[\left(\frac{\mu(v)}{v}\right)^{r/2} - 1\right] g(v)dv}{-\alpha\left(N + \frac{\alpha}{\beta}\right)\int_1^{v*} d\left(\frac{\mu(v)}{v}\right) g(v)dv} \right\}. \qquad \text{(A4)}$$

We need to show that the three integrals in (A4) are bounded as $v* = bp*/a \to \infty$. The integral in the denominator is:

$$\int_1^{v*} d\left(\frac{\mu(v)}{v}\right) g(v)dv \le \int_1^{v*} d\left(\frac{1}{v}\right) g(v)dv \quad \text{for } \mu(v) \ge 1, \text{ since } d' \le 0$$

$$\le \begin{cases} \int_1^{v*} \alpha v^{-r} g(v)dv & \text{for } \alpha > 0,\ r < 0 \text{ in (7)} \\ \int_1^{v*} |\alpha| v^{-r/2} g(v)dv & \text{for } \alpha < 0,\ r > 0 \text{ in (7)} \end{cases}$$

where the second line follows by inspection of $d(1/v)$ in (7). Using $g(v) = \theta v^{-\theta-1}/[1-b^{-\theta}]$, it is readily confirmed that $\theta > \max\{0,-r\}$ in Assumption 2 is sufficient for this integral in the denominator to be finite even as $v* = bp*/a \to \infty$. The first integral in the numerator is smaller than that in the denominator, so it is also bounded. The second integral in the numerator of (A4) can be shown to be bounded by a similar set of inequalities to that used above. It follows that the right-hand side of (A4) approaches infinity as $p* \to \infty$.

We next consider $p* \to 0$. Since the first integral in the numerator of (A4) is less than the integral in the denominator, their ratio is less than unity. The second integral in the numerator of (A4) approaches zero as $v* = bp*/a \to 0$. It follows that the right-hand side approaches zero as

$p^* \to 0$. Therefore, there will exist a value for $p^* \in (0,+\infty)$ such that (A4) holds, and this is the equilibrium value. Then N is computed from (16) and N_e from (14). Part (b) is proved in the main text. QED

To establish Proposition 3 we use the following result on the elasticity of the markup $\mu(v)$:

Lemma A2: $\dfrac{\mu' v}{\mu} = \left\{ \dfrac{(\eta - 1 + r)^2 / \eta(\eta - 1)}{1 + \left[(\eta - 1 + r)^2 / \eta(\eta - 1) \right]} \right\}$, so $0 < \dfrac{\mu' v}{\mu} < 1$ and $\lim\limits_{v \to 1} \dfrac{\mu' v}{\mu} = \dfrac{1}{2}$.

Proof: In the main text we report that $\left[\left(\dfrac{\mu(v)}{v} \right)^{r/2} - \dfrac{1}{2} \right] \left[r(\mu(v) - 1) + 1 \right] = \dfrac{1}{2}$, which follows from (6). Rearranging this expression we obtain,

$$-r\left(\mu(v) - 1\right) = \left[1 - \left(\frac{v}{\mu(v)} \right)^{r/2} \right] \Bigg/ \left[1 - \frac{1}{2} \left(\frac{v}{\mu(v)} \right)^{r/2} \right], \tag{A5}$$

and differentiating we can obtain:

$$\frac{\mu' v}{\mu} = \frac{\left(\dfrac{v}{\mu} \right)^{r/2} \left[\dfrac{1}{2\mu} + \dfrac{r(\mu - 1)}{2\mu} \right] \Bigg/ \left[1 - \dfrac{1}{2} \left(\dfrac{v}{\mu(v)} \right)^{r/2} \right]}{1 + \left\{ \left(\dfrac{v}{\mu} \right)^{r/2} \left[\dfrac{1}{2\mu} + \dfrac{r(\mu - 1)}{2\mu} \right] \Bigg/ \left[1 - \dfrac{1}{2} \left(\dfrac{v}{\mu(v)} \right)^{r/2} \right] \right\}}.$$

Substituting from Lemma A2 and (A5), and using $1/\mu = (\eta - 1)/\eta$ and $(1 - \mu)/\mu = 1/\eta$, which implies $\left[\frac{1}{2\mu} + \frac{r(\mu - 1)}{2\mu} \right] = (\eta - 1 + r)/2\eta$, we obtain the desired result. The limiting value for $\mu' v/\mu$ as $v \to 1$ and $\eta \to \infty$ is obtained using L'Hôpitals Rule. QED

Proof of Proposition 3:

(a) To establish part (a), we need to show that the Herfindahl index is fixed in the unbounded Pareto case; the remaining statement have been proved in the text. Using (10) and (A2), we have:

$$\left[1-\left(\tilde{N}+\frac{\alpha}{\beta}\right)H\right]^{1/r}=\left[\int_{\Omega}\frac{d(p_\omega/p^*)}{D(\mathbf{p})}\left(\frac{p_\omega}{p^*}\right)^{-r/2}d\omega\right]^{-1/r}$$

$$=\left[\frac{N_e\int_1^{v^*}d\left(\frac{\mu(v)}{v}\right)\left(\frac{\mu(v)}{v}\right)^{-r/2}\left(\frac{p^*}{a}\right)^{\theta}g(v)dv}{N_e\int_1^{v^*}d\left(\frac{\mu(v)}{v}\right)\left(\frac{p^*}{a}\right)^{\theta}g(v)dv}\right]^{-1/r} \qquad \text{(A6)}$$

where in the second line we have converted from integrals over ω to integrals over v, using the change in variables in (12). Clearly, various terms in the numerator and denominator of the second line cancel. When $b = \infty$, then we are left with two integrals with an upper limit of integration of $v^* = \infty$, and the ratio of these integrals are independent of p^* and N_e . Therefore, the Herfindahl is fixed.

(b) (i) To show that N rises with L, we differentiate (14) and use the Pareto distribution:

$$d\ln N = d\ln N_e+\theta\left[\frac{(a/p^*)^{-\theta}}{(a/p^*)^{-\theta}-b^{-\theta}}\right]d\ln p^*$$

$$=\left\{\frac{N_e}{[\tilde{N}+(\alpha/\beta)]}\left(\frac{b^{-\theta}}{1-b^{-\theta}}\right)\left[1-\left(\frac{\mu(v^*)}{v^*}\right)^{r/2}\right]-\left[\frac{b^{-\theta}}{(a/p^*)^{-\theta}-b^{-\theta}}\right]\right\}\frac{d\ln L}{(1+A+B)}$$

$$=\left\{\frac{\left[\left(\frac{\mu(v^*)}{v^*}\right)^{r/2}-1\right]}{\int_1^{v^*}\left[\left(\frac{\mu(v)}{v}\right)^{r/2}-1\right]\left(\frac{p^*}{a}\right)^{\theta}g(v)dv}-\left[\frac{1-b^{-\theta}}{(a/p^*)^{-\theta}-b^{-\theta}}\right]\right\}\frac{b^{-\theta}d\ln L}{(1-b^{-\theta})(1+A+B)}$$

where the second line follow by substituting from (18) and simplifying; and the third line follows from (14) and (16). The term in curly brackets is positive if and only if,

$$\int_1^{v^*}\left[\frac{\left(\frac{\mu(v)}{v}\right)^{r/2}-1}{\left(\frac{\mu(v^*)}{v^*}\right)^{r/2}-1}\right]\left(\frac{p^*}{a}\right)^{\theta}g(v)dv<\left[\frac{(a/p^*)^{-\theta}-b^{-\theta}}{1-b^{-\theta}}\right]=\int_1^{v^*}\left(\frac{p^*}{a}\right)^{\theta}g(v)dv\,.$$

This inequality holds because $\mu(1) = 1$, while $\mu(v)/v$ declines to $\mu(v^*)/v^* < 1$ for $v^* > 1$, and

Lemma A2 ensures that this decline in monotonic. It follows that the term in square brackets in the first integral above is less than unity for $v < v^*$.

(ii) Next, we consider the average of costs in (17). Differentiating using $d\ln v^* = d\ln p^*$,

$$d\ln\left\{\sqrt{p^*}\left[\frac{\tilde{G}(v^*)}{G(v^*)}\right]^{1/r}\right\} = \left\{\frac{1}{2}+\frac{v^*}{r}\left[\frac{\tilde{g}(v^*)}{\tilde{G}(v^*)}-\frac{g(v^*)}{G(v^*)}\right]\right\}d\ln p^*$$

$$= \left\{\frac{1}{2}+\frac{g(v^*)v^*}{rG(v^*)}\left[\frac{(v^*)^{-r/2}-\int_1^{v^*}v^{-r/2}\frac{g(v)}{G(v^*)}dv}{\int_1^{v^*}v^{-r/2}\frac{g(v)}{G(v^*)}dv}\right]\right\}d\ln p^*$$

Integrating by parts we have $\int_1^{v^*}v^{-r/2}\frac{g(v)}{G(v^*)}dv = (v^*)^{-r/2}+\frac{r}{2}\int_1^{v^*}v^{-\frac{r}{2}-1}\frac{G(v)}{G(v^*)}dv$, so that,

$$d\ln\left\{\sqrt{p^*}\left[\frac{\tilde{G}(v^*)}{G(v^*)}\right]^{1/r}\right\} = \frac{(1-C_1)}{2}d\ln p^*, \text{ with } C_1 \equiv \frac{g(v^*)v^*}{G(v^*)}\left[\frac{\int_1^{v^*}v^{-\frac{r}{2}-1}\frac{G(v)}{G(v^*)}dv}{\int_1^{v^*}v^{-r/2}\frac{g(v)}{G(v^*)}dv}\right].$$

Note that C_1 can be re-written as $C_1 = \frac{g(v^*)v^*}{G(v^*)}\Big/\int_1^{v^*}\frac{g(v)v}{G(v)}\left[v^{-\frac{r}{2}-1}\frac{G(v)}{G(v^*)}\Big/\int_1^{v^*}v^{-\frac{r}{2}-1}\frac{G(v)}{G(v^*)}dv\right]dv$,

where the final expression in square brackets acts as weights, i.e. it integrates to unity. It follows that $0 < C_1 < 1$ if $g(v^*)v^*/G(v^*)$ is declining in v^*, as is confirmed for the Pareto distribution.

For the average of markups in (17), differentiation gives us:

$$d\ln\left[\int_1^{v^*}\mu(v)^{r/2}\frac{\tilde{g}(v)}{\tilde{G}(v^*)}\right]^{1/r} = \frac{v^*\tilde{g}(v^*)}{r\tilde{G}(v^*)}\left[\frac{\mu(v^*)^{r/2}-\int_1^{v^*}\mu(v)^{r/2}\frac{\tilde{g}(v)}{\tilde{G}(v^*)}dv}{\int_1^{v^*}\mu(v)^{r/2}\frac{\tilde{g}(v)}{\tilde{G}(v^*)}dv}\right]d\ln p^* \ .$$

Integrating by parts we obtain, $\int_1^{v^*}\mu(v)^{r/2}\frac{\tilde{g}(v)}{\tilde{G}(v^*)}dv = \mu(v^*)^{r/2}-\frac{r}{2}\int_1^{v^*}\mu(v)^{r/2}\left(\frac{\mu' v}{\mu}\right)\frac{\tilde{G}(v)/v}{\tilde{G}(v^*)}dv$, so,

$$d\ln\left[\int_1^{v^*}\mu(v)^{r/2}\frac{\tilde{g}(v)}{\tilde{G}(v^*)}\right]^{1/r} = \frac{C_2}{2}d\ln p^*, \text{ with } C_2 \equiv \frac{v^*\tilde{g}(v^*)}{\tilde{G}(v^*)}\left[\frac{\int_1^{v^*}\mu(v)^{r/2}\left(\frac{\mu' v}{\mu}\right)\frac{\tilde{G}(v)/v}{\tilde{G}(v^*)}dv}{\int_1^{v^*}\mu(v)^{r/2}\frac{\tilde{g}(v)}{\tilde{G}(v^*)}dv}\right].$$

Because $0 < \mu'v/\mu < 1$ from Lemma A2, we see that C_2 is less than,

$$C_2 < \frac{\tilde{g}(v^*)v^*}{\widetilde{G}(v^*)} \Big/ \int_1^{v^*} \frac{\tilde{g}(v)v}{\widetilde{G}(v)} \left[\mu(v)^{r/2} \frac{\widetilde{G}(v)/v}{\widetilde{G}(v^*)} \Big/ \int_1^{v^*} \mu(v)^{r/2} \frac{\widetilde{G}(v)/v}{\widetilde{G}(v^*)} dv \right] dv ,$$

where the final expression in square brackets acts as weights, i.e. it integrates to unity. It follows that $0 < C_2 < 1$ if $\tilde{g}(v^*)v^*/\widetilde{G}(v^*)$ is declining in v^*, as is readily confirmed for the "adjusted" Pareto distribution defined in the main text.

(iii) Finally, we need to show that the share-weighted mean of prices in (11) falls as the labor force L grows. The share-weighted mean of prices after cancelling common terms in (A6) is,

$$\left[\int_\Omega s_\omega(\mathbf{p}) p_\omega^{-r/2} d\omega \right]^{-1/r} = \frac{\sqrt{p^*}}{\widetilde{D}(v^*)^{-1/r}} \left[\int_1^{v^*} d\left(\frac{\mu(v)}{v} \right) \left(\frac{\mu(v)}{v} \right)^{-r/2} g(v) dv \right]^{-1/r} ,$$

where we define $\widetilde{D}(v^*) \equiv \int_1^{v^*} d\left(\frac{\mu(v)}{v} \right) g(v) dv$. The above expression changes by:

$$d \ln \left[\int_\Omega s_\omega(\mathbf{p}) p_\omega^{-r/2} d\omega \right]^{-1/r} = \left\{ \frac{1}{2} - \frac{1}{r} \left[\frac{d\left(\frac{\mu(v^*)}{v^*} \right) \left(\frac{\mu(v^*)}{v^*} \right)^{-r/2} g(v^*)v^*}{\int_1^{v^*} d\left(\frac{\mu(v)}{v} \right) \left(\frac{\mu(v)}{v} \right)^{-r/2} g(v) dv} - \frac{d\left(\frac{\mu(v^*)}{v^*} \right) g(v^*)v^*}{\widetilde{D}(v^*)} \right] \right\} d \ln p^* .$$

Integrating by parts using $d\widetilde{D} \equiv d[\mu(v)/v] g(v) dv$, we obtain:

$$\int_1^{v^*} \left(\frac{\mu(v)}{v} \right)^{-r/2} d\widetilde{D} = \widetilde{D}(v^*) \left(\frac{\mu(v^*)}{v^*} \right)^{-r/2} + \frac{r}{2} \int_1^{v^*} \left(\frac{\mu(v)}{v} \right)^{-r/2} \left(\frac{\mu'(v)}{\mu} - \frac{1}{v} \right) \widetilde{D}(v) dv .$$

Expressing the long term in square brackets above with a common denominator, we can substitute the above integral and rearrange terms to obtain:

$$d \ln \left[\int_\Omega s_\omega(\mathbf{p}) p_\omega^{-r/2} d\omega \right]^{-1/r} = \frac{C_3}{2} d \ln p^*, \; C_3 \equiv \left\{ 1 + \frac{d\left(\frac{\mu(v^*)}{v^*} \right) g(v^*)}{\widetilde{D}(v^*)/v^*} \left[\frac{\int_1^{v^*} \left(\frac{\mu(v)}{v} \right)^{-r/2} \left(\frac{\mu'(v)v}{\mu} - 1 \right) \frac{\widetilde{D}(v)}{v} dv}{\int_1^{v^*} d\left(\frac{\mu(v)}{v} \right) \left(\frac{\mu(v)}{v} \right)^{-r/2} g(v) dv} \right] \right\}$$

From Lemma A2 we have that $0 < \mu'(v)v/\mu < 1$ for $v > 1$, and so $-1 < [\mu'(v)v/\mu] - 1 < 0$. It

follows that $C_3 < 1$ and a sufficient condition for $C_3 > 0$ is:

$$\frac{d\left(\frac{\mu(v^*)}{v^*}\right)g(v^*)}{\widetilde{D}(v^*)/v^*} < \frac{\int_1^{v^*} d\left(\frac{\mu(v)}{v}\right)\left(\frac{\mu(v)}{v}\right)^{-r/2} g(v)dv}{\int_1^{v^*}\left(\frac{\mu(v)}{v}\right)^{-r/2}\frac{\widetilde{D}(v)}{v}dv} = \int_1^{v^*} d\left(\frac{\mu(v)}{v}\right)\frac{g(v)v}{\widetilde{D}(v)}\left[\frac{\left(\frac{\mu(v)}{v}\right)^{-r/2}\frac{\widetilde{D}(v)}{v}}{\int_1^{v^*}\left(\frac{\mu(v)}{v}\right)^{-r/2}\frac{\widetilde{D}(v)}{v}dv}\right]dv,$$

where the equality follows by rearranging terms. Notice that the final term in square brackets acts as a weight, i.e. it integrates to unity. It is immediate that the inequality will hold if $d\left(\frac{\mu(v^*)}{v^*}\right)g(v^*)v^*/\widetilde{D}(v^*)$ is declining in v^*. To verify this, we take logs and differentiate,

$$\frac{d\ln}{d\ln v^*}\left[d\left(\frac{\mu(v^*)}{v^*}\right)g(v^*)v^*/\widetilde{D}(v^*)\right] = -(\eta(v^*)-1)\left(\frac{\mu'(v^*)v^*}{\mu(v^*)}-1\right)-\theta-d\left(\frac{\mu(v^*)}{v^*}\right)\frac{g(v^*)v^*}{\widetilde{D}(v^*)} < 0$$

$$\Leftrightarrow -\theta-(\eta(v^*)-1)\left(\frac{\mu'(v^*)v^*}{\mu(v^*)}-1\right) < d\left(\frac{\mu(v^*)}{v^*}\right)\frac{g(v^*)v^*}{\widetilde{D}(v^*)}.$$

From the definition $\widetilde{D}(v^*) \equiv \int_1^{v^*} d\left(\frac{\mu(v)}{v}\right)g(v)dv$, we can integrate by parts to obtain:

$$\widetilde{D}(v^*) \equiv d\left(\frac{\mu(v^*)}{v^*}\right)g(v^*)v^* - \int_1^{v^*}\left[g'(v)v-(\eta(v)-1)\left(\frac{\mu'(v)v}{\mu(v)}-1\right)g(v)\right]d\left(\frac{\mu(v)}{v}\right)dv,$$

so that, $d\left(\frac{\mu(v^*)}{v^*}\right)\frac{g(v^*)v^*}{\widetilde{D}(v^*)} = 1-\int_1^{v^*}\left[\theta+1+(\eta(v)-1)\left(\frac{\mu'(v)v}{\mu(v)}-1\right)\right]d\left(\frac{\mu(v)}{v}\right)\frac{g(v)}{\widetilde{D}(v^*)}dv.$

Substituting this expression above, the inequality we wish to establish is:

$$-\left[\theta+1+(\eta(v^*)-1)\left(\frac{\mu'(v^*)v^*}{\mu(v^*)}-1\right)\right] < -\int_1^{v^*}\left[\theta+1+(\eta(v)-1)\left(\frac{\mu'(v)v}{\mu(v)}-1\right)\right]d\left(\frac{\mu(v)}{v}\right)\frac{g(v)}{\widetilde{D}(v^*)}dv$$

$$= -\left[\theta+1+(\eta(v)-1)\left(\frac{\mu'(v)v}{\mu(v)}-1\right)\right]\frac{\widetilde{D}(v)}{\widetilde{D}(v^*)}\Bigg|_1^{v^*} + \int_1^{v^*}\frac{\widetilde{D}(v)}{\widetilde{D}(v^*)}\frac{d}{dv}\left[(\eta(v)-1)\left(\frac{\mu'(v)v}{\mu(v)}-1\right)\right]dv$$

$$= -\left[\theta+1+(\eta(v^*)-1)\left(\frac{\mu'(v^*)v^*}{\mu(v^*)}-1\right)\right] + \int_1^{v^*}\frac{\widetilde{D}(v)}{\widetilde{D}(v^*)}\frac{d}{dv}\left[\eta(v)-1\left(\frac{\mu'(v)v}{\mu(v)}-1\right)\right]dv,$$

where in the second line we integrate by parts, with $\lim_{v\to 1}(\eta(v)-1)\widetilde{D}(v) = 0$ as can be confirmed using L'Hôpitals Rule and Lemma A2. Therefore, the inequality we wish to establish is that the final integral above is positive, and a sufficient condition for this to hold is that:

$$\frac{d}{dv}\left[(\eta(v)-1)\left(\frac{\mu'(v)v}{\mu(v)}-1\right)\right]>0. \qquad \text{(A6)}$$

Using Lemma A2, we see that the expression in square brackets approaches $-\infty$ as $v \to 1$ while it equals a finite negative number for $v = v^*$, so the positive relationship with v shown in (A6) holds globally. To confirm (A6) locally, we substitute for $\mu'v/\mu$ using Lemma A2 and differentiate using $\eta'(v) < 0$, obtaining a long polynomial involving r and η. It can be confirmed that the polynomial is negative, so that with $\eta'(v) < 0$, then the positive sign in (A6) is obtained.

(c) We have shown that $d\ln\left[\int_\Omega s_\omega(\mathbf{p})p_\omega^{-r/2}d\omega\right]^{-1/r} = \frac{C_3}{2}d\ln p^*$, with $0 < C_3 < 1$. It follows from the first two lines of (11) that,

$$d\ln\left[1-\left(\tilde{N}+\frac{\alpha}{\beta}\right)H\right]^{1/r} = d\ln\left\{\left[\int_\Omega s_\omega(\mathbf{p})p_\omega^{-r/2}d\omega\right]^{-1/r} / \sqrt{p^*}\right\} = \tfrac{1}{2}(C_3-1)d\ln p^*.$$

Since $\left[1-\left(\tilde{N}+\frac{\alpha}{\beta}\right)H\right]^{1/r}$ is decreasing in H (from Assumption 1), then if the reservation price falls so $\frac{1}{2}(C_3-1)d\ln p^* > 0,$ it follows that H must also be falling. QED

Proof of Lemma 3:

Working from (2), the reservation price equals:

$$p^* = \left(\frac{N}{N-\left(\tilde{N}+\frac{\alpha}{\beta}\right)}\right)^{2/r} p^*\left[\int_\Omega \frac{1}{N}\left(\frac{p_\omega}{p^*}\right)^{r/2}d\omega\right]^{2/r} = \left(\frac{N}{N-\left(\tilde{N}+\frac{\alpha}{\beta}\right)}\right)^{2/r} p^*\left\{\frac{J_\rho\left[\left(\frac{\mu(v)}{v}\right)^{r/2}\right]}{J_\rho[1]}\right\}^{2/r}.$$

The first equality follows from (2) using simple algebra, and the second equality follows by integrating over all domestic and foreign products in the set Ω, and expressing the integral using the change in variables in (12) and (20) and then using the functional in (25). Then by definition of the "adjusted" density function $\tilde{g}(v) \equiv g(v)/v^{r/2}$, used in the functional $\tilde{J}[f]$, we have:

$$p^* \frac{J_\rho\left[\left(\frac{\mu(v)}{v}\right)^{r/2}\right]}{J_\rho[1]} = \frac{\tilde{J}_\rho\left[\mu(v)^{r/2}\right]}{\tilde{J}_\rho[1]}\left\{p^* \frac{\tilde{J}_\rho[1]}{J_\rho[1]}\right\} = \frac{\tilde{J}_\rho\left[\mu(v)^{r/2}\right]}{\tilde{J}_\rho[1]}\left\{\frac{J_\rho\left[\left(\frac{p^*}{v}\right)^{r/2}\right]}{J_\rho[1]}\right\},$$

so that Lemma 3 is established. QED

Proof of Proposition 4:

We need to prove that the change in price $d\ln p^* = (1-\lambda)d\ln\tau_0$ is the equilibrium change, and will do so by also showing that M_e and N are constant. It will follow that the term $D(\mathbf{p})$ in the denominator of (24) is constant, and it follows that the Herfindahl index is also constant. From Lemma 1 the change in welfare equals therefore equals $-d\ln p^* = -(1-\lambda)d\ln\tau_0$.

(i) Consider first the case of unbounded productivity. By integrating over trade costs, the free entry condition (24) becomes:

$$F = \frac{L\left[1+\tau_0^{-\theta}\tilde{C}^{(1-\rho\theta)}\right]\int_1^\infty\left[\frac{\mu(v)-1}{\mu(v)}\right]d\left(\frac{\mu(v)}{v}\right)\left(\frac{p^*}{a}\right)^\theta g(v)dv}{M_e\left[1+\tau_0^{-\theta}\tilde{C}^{(1-\rho\theta)}\right]\int_1^\infty d\left(\frac{\mu(v)}{v}\right)\left(\frac{p^*}{a}\right)^\theta g(v)dv}.$$

The term involving trade costs in brackets cancels in the numerator and denominator as does the term $(p^*/a)^\theta$. The remaining integrals are constant because their upper-limits do not depend on p^* or trade costs. It follows immediately that the mass of entering firms M_e is constant.

Likewise using $v^* \to \infty$ and $v^*/\tau(c) \to \infty$, condition (26) becomes:

$$\begin{aligned} N - \left(\tilde{N} + \frac{\alpha}{\beta}\right) &= M_e\left(\frac{p^*}{a}\right)^\theta\left[1+\tau_0^{-\theta}\tilde{C}^{(1-\rho\theta)}\right]\int_1^\infty\left(\frac{\mu(v)}{v}\right)^{r/2} g(v)dv \\ &= M\left[1+\tau_0^{-\theta}\tilde{C}^{(1-\rho\theta)}\right]\int_1^\infty\left(\frac{\mu(v)}{v}\right)^{r/2} g(v)dv = N\int_1^\infty\left(\frac{\mu(v)}{v}\right)^{r/2} g(v)dv, \end{aligned} \tag{A7}$$

where the first equality follows by evaluating the integrals in (26), the second follows from using

$M = M_e(p^*/a)^{\theta}$, and the third line follows from using (30). Again, the final integral is constant because its upper-limit does not depend on p^* or trade costs. From this condition we therefore uniquely solve for product variety N available to consumers, independent of trade costs. Then differentiating the first equality in (A7) for fixed N and M_e, we obtain:

$$0 = \theta d\ln p^* - \underbrace{\theta\left(\frac{\tau_0^{-\theta}\widetilde{C}^{(1-\rho\theta)}}{1+\tau_0^{-\theta}\widetilde{C}^{(1-\rho\theta)}}\right)}_{(1-\lambda)} d\ln\tau_0 \Leftrightarrow d\ln p^* = (1-\lambda)d\ln\tau_0 .$$

It follows that $d\ln p^* = (1-\lambda)d\ln\tau_0$ is the equilibrium change in the reservation price.

(ii) Next, consider the case of bounded productivity. We will argue that $d\ln p^* = (1-\lambda)d\ln\tau_0$ satisfies all the equilibrium conditions and implies that M_e and N are locally constant around the frictionless equilibrium. We have already argued in the text that $d\ln p^* = (1-\lambda)d\ln\tau_0$ implies that the functional on the right of (26) is locally constant, so that:

$$d\ln N\left(\frac{N}{N-[\tilde{N}+(\alpha/\beta)]}\right) = d\ln M_e . \tag{A8}$$

We can also use the equilibrium conditions (22) and (23), with $\rho = 0$ and $C = \widetilde{C}$, to obtain:

$$N = \underbrace{M_e\left(\frac{p^*}{a}\right)^{\theta} G(v^*)}_{=M}\underbrace{\left\{1+\tau_0^{-\theta}\left[\frac{\widetilde{C}-1}{1-(v^*)^{-\theta}}\right]-\frac{(\widetilde{C}-1)(v^*)^{-\theta}}{1-(v^*)^{-\theta}}\right\}}_{\equiv h(\tau_0,v^*)} .$$

We can then calculate the total change in N around the frictionless equilibrium as:

$$\begin{aligned} d\ln N &= d\ln M_e + \left[\theta + \frac{g(v^*)v^*}{G(v^*)}\right]d\ln p^* + \left.\frac{\partial\ln h}{\partial\ln\tau_0}\right|_{\tau_0=1} d\ln\tau_0 + \left.\frac{\partial\ln h}{\partial\ln v^*}\right|_{\tau_0=1} d\ln p^* \\ &= d\ln M_e + \left[\theta + \frac{\theta(v^*)^{-\theta}}{1-(v^*)^{-\theta}}\right]\left(\frac{\widetilde{C}-1}{\widetilde{C}}\right)d\ln\tau_0 - \frac{\theta}{\widetilde{C}}\left[\frac{\widetilde{C}-1}{1-(v^*)^{-\theta}}\right]d\ln\tau_0 + (0)d\ln p^* \\ &= d\ln M_e \quad . \end{aligned}$$

To move from the first line to the second, it is readily confirmed from the definition of $h(\tau_0, v^*)$ that the final derivative is zero when evaluated at $\tau_0 = 1$, while the derivative with respect to τ_0 is as shown. The second term on the second line is obtained by substituting $d\ln p^* = (1-\lambda)d\ln\tau_0$ with $\lambda = 1/\widetilde{C}$ and using the Pareto distribution to evaluate $g(v^*)v^*/G(v^*)$. It is readily seen that the second and third terms sum to zero, so that the final line is obtained. From Assumption 1 we have $[\tilde{N} + (\alpha/\beta)] \neq 0$, so the only way that $d\ln N = d\ln M_e$ is consistent with (A8) is if $d\ln N = d\ln M_e = 0$. It follows that these are the equilibrium changes, in conjunction with $d\ln p^* = (1-\lambda)d\ln\tau_0$. QED

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