International Liquidity and Exchange Rate Dynamics*

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Abstract

We provide a theory of the determination of exchange rates based on capital flows. Capital flows alter the balance sheet of financial intermediaries that vehicle these flows globally. Alterations to the balance sheets cause intermediaries to require different compensation for holding currency risk, thus impacting the level and volatility of exchange rates. Our mechanism of exchange rate determination in financial markets via the demand for assets differs starkly from the traditional modeling, which mostly relies on the demand for goods in order to determine exchange rates. The financial determination of exchange rates not only rationalizes the empirical disconnect between exchange rates and traditional macroeconomic fundamentals but also has real consequences for output and risk sharing. In a floating exchange rate regime, the exchange rate is sensitive to imbalances in financial markets and seldom performs the shock absorption role that is central to traditional theoretical macroeconomic analysis.

We reevaluate the Mundellian welfare analysis of fixed versus floating exchange rates and find novel trade-offs, which often suggest a case for heterodox policies, such as large-scale currency swaps, capital controls, and currency intervention. Our framework can accommodate a number of modeling features important to open economy analysis within an imperfect capital market model: non-tradables, production, money, sticky prices or wages, mark-ups and trade costs. Finally, our framework can rationalize a number of fundamental issues of international macroeconomics, such as: the exchange rate disconnect from macroeconomics fundamentals, Mussa's evidence on the relative volatility of exchange rates under fixed and floating regimes, external financial adjustment, the carry trade, the failure of purchasing power parity, and the failure of the Backus-Smith risk sharing condition.

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We provide a theory of exchange rate determination based on capital flows in imperfect financial markets. In our model, exchange rates are volatile and are largely disconnected from traditional macroeconomics fundamentals over the medium term; they are instead governed by financial forces. Global shifts in the demand and supply of assets result in large scale capital flows that are intermediated by the global financial system. The demand and supply of assets in different currencies and the willingness of the financial system to absorb the resulting imbalances are first order determinants of exchange rates. A framework to characterize such forces and their implications for welfare and policy, while desirable, has proven elusive.

In our model, intermediaries vehicle the flows globally and absorb part of the currency risk. Alterations in the size and composition of intermediaries' balance sheets induce them to differentially price currency risk, thus affecting both the level and the volatility of exchange rates.

Our theory of financial determination of the exchange rate contrasts with the traditional open macroeconomic model by introducing financial forces such as portfolio flows, intermediaries' balance sheets and their risk bearing capacity as first order determinants of exchange rates.

In Section 2 we present a minimal theory of exchange rate financial determination in a two-period two-country model where capital flows are intermediated by a global financial intermediary. Each country borrows or lends in its own currency and the intermediary absorbs all currency risk that is generated by the mismatch of global capital flows. Since the intermediary requires compensation for holding currency risk in the form of expected currency appreciation, exchange rates are jointly determined by global capital flows and by the intermediary's risk bearing capacity. Our theory, therefore, is an elementary one where supply and demand determine a price, the exchange rate, that clears markets.

The exchange rate appears to be disconnected from traditional macroeconomic fundamentals such as imports, exports, output or inflation in as much as the same fundamentals correspond to a different equilibrium exchange rate depending on the intermediary risk bearing capacity. A simple extension to a multi period model strengthens this intuition by solving the equilibrium exchange rate as a present value relationship. The exchange rate discounts future current account balances, but the rate of discounting is determined in financial markets and hinges on intermediaries' risk bearing capacity and balance sheet

risks. Changes in such capacity affect both the level and volatility of the exchange rate. Intermediaries act both as shock absorbers, by using their risk bearing capacity to accommodate flows that result from fundamental shocks, and are themselves the source of financial shocks that distort exchange rates.

Financial determination of exchange rates has real consequences for output and risk sharing. To more fully analyze these consequences, in Section 3 we introduce versions of the model with nominal exchange rates, monetary policy, and both flexible and sticky prices. In the presence of goods' prices that are sticky in the producer currency, a capital inflow or financial shock that produces an overly appreciated exchange rate causes a fall in demand for the country's exports and a corresponding fall in output.

Our theory yields novel predictions for the Mundellian welfare analysis of fixed versus floating exchange rates. In a financial world, exchange rate movements are dominated by financial factors and they seldom perform the benign expenditure switching role that is central to the traditional macroeconomic analysis. For example, the traditional rationale for prescribing floating exchange rates is that in the presence of asymmetric real shocks the exchange rate acts as a shock absorber by shifting global demand toward the country that has been most negatively affected by the shock via a depreciation of its currency.

By contrast, in Section 4 we show that in our framework the floating exchange rate is itself distorted by imbalances in financial markets and shocks to the financial system's risk bearing capacity; the exchange rate can often be the source of shocks to the real economy. Our welfare analysis suggests novel trade-offs that emerge when financial markets are disrupted or are overall less developed (such as those of emerging markets). Heterodox policies, such as large scale currency swaps, currency interventions, and capital controls, emerge as beneficial in these specific circumstances.

1 Related Literature

Two important papers were published in 1976, the now classic exchange rate overshooting model (Dornbusch (1976)) and the portfolio balance theory model (Kouri (1976)). While we incorporate important aspects of the Keynesian tradition that Dornbusch builds on, our model provides modern foundations to the spirit of the portfolio balance theory of exchange rates. Obstfeld and Rogoff (1995) brought the Keynesian approach into modern

international economics by providing micro-foundations to the dynamic version of the Mundell-Fleming-Dornbusch model. Their foundations have been essential not only to the analysis of the determination of exchange rates, but also to that of optimal policy and welfare. In this class of models, however, financial forces play little role. In the real version of these models exchange rates are mostly determined by the demand and supply of domestic and foreign goods. Even in the nominal versions of the models, where the nominal exchange rate is often expressed as the present discounted sum of future monetary policy and other macro fundamentals, the impact of finance is limited because most often the uncovered interest parity holds, the demand for money is tightly linked to consumption expenditures, and/or the model is linearized.

A vast parallel literature has explored the determinants of exchange rates in endowment or real business cycle models following Lucas (1982) and Backus, Kehoe, and Kydland (1992). In the context of asset pricing models Dumas (1992), Lewis (1998), Verdelhan (2010), Farhi and Gabaix (2011), and Colacito and Croce (2011) have explored the relationship between consumption and the real exchange rate following Backus and Smith (1993). Pavlova and Rigobon (2007) analyze a real model with complete markets where countries' representative agents have log preferences affected by taste shocks. This branch of the literature has mostly maintained the assumption of complete markets. We contribute to this literature not only by providing alternative drivers of exchange rates, intermediaries' risk bearing capacity and financial imbalances, which break the link between consumption and the real exchange rate, but also by focusing on incomplete and imperfect capital markets. These imperfections are crucial to our analysis of welfare and policy in open economy.

Finally, recent economic events have rekindled an interest in optimal policy and welfare analysis in open economy. Farhi, Gopinath, and Itskhoki (2012), Farhi and Werning (2012a,b) analyze policies such as capital controls, fiscal transfers and fiscal devaluations in the context of the neo-Keynesian model. Schmitt-Grohe and Uribe (2011b,a) also study the implications of nominal rigidities for pegged and floating exchange rates in the neo-Keynesian model. We contribute to this literature by analyzing policies in a world where financial flows are direct determinants of exchange rates and where the condition of financial markets is an important policy consideration. We find that public financial policies, which cannot be analyzed under complete markets or simple forms of

market incompleteness, provide new trade-offs and policy tools to the analysis of floating, managed or pegged exchange rates.

Relatively little research has been devoted to analyzing the relationship between portfolio flows and exchange rates. Notable exceptions are Evans and Lyons (2002), Hau and Rey (2006), Bacchetta and Van Wincoop (2010) who consider under different modeling assumptions (microstructure market making, equity portfolio flows, and overlapping generations of investors, respectively) the price impact of financial flows. Maggiori (2011), Shin and Bruno (2012) introduce financial intermediation as an important driver of global risk taking and capital flows.

We summarize our contribution as providing a tractable framework for the determination of exchange rates in financial markets via capital flows and the risk bearing capacity of financial intermediaries. Based on this framework, we analyze optimal policy and welfare in the presence of both financial and nominal frictions and characterize when and which financial policies can be used to re-equilibrate exchange rates. Our framework makes sense of a number of fundamental issues in open macroeconomics, such as: the exchange rate disconnect from macro fundamentals, Mussa's evidence on the relative volatility of exchange rates under fixed and floating regimes, external financial adjustment, the carry trade, the failure of purchasing power parity, and the failure of the Backus-Smith risk sharing condition.

Our framework is flexible in potentially accommodating within an imperfect capital market model a number of modeling features that are important in open economy analysis, such as non-tradable goods, sticky prices or wages, mark-ups, pricing to market, and trade costs. The framework, therefore, can easily be employed in future research to address a number of open questions in international macroeconomics.

2 Basic Gamma Model

2.1 Households

Consider a two-period world populated by two open economies: Japan and the USA. Each economy is populated by a continuum of households. The Japanese households'

flow utility at time t is:

$$U_t = \chi_t \ln C_{NT,t} + a_t \ln C_{H,t} + \iota_t \ln C_{F,t},$$

where C_{NT} is consumption of non-tradable goods (NT), C_H is consumption of domestic tradables, C_F is the consumption of foreign tradables, and $\{\chi_t, a_t, \iota_t\}$ are possibly random non-negative taste shifters. Intertemporal utility is defined as $\sum_{t=0}^{T} \beta^t U_t$. Hereafter we suppress time subscripts unless necessary to clarify the time varying nature of the variables.

The NT good is the numeraire in each economy and consequently its price equals one in domestic currency ($p_{NT} = 1$). NT are produced by an endowment process that for simplicity takes the form $C_{NT,t} = \chi_t$, unless otherwise stated. The static utility maximization problem takes the form:

$$\max_{C_{NT}, C_H, C_F} \quad \chi_t \ln C_{NT} + a_t \ln C_H + \iota_t \ln C_F + \lambda \left(PC - C_{NT} - p_H C_H - p_F C_F \right),$$

where PC is aggregate consumption expenditure and taken exogenous here. Standard optimality conditions imply $\lambda = \chi_t / C_{NT} = 1$, and

$$C_H = \frac{a_t}{p_H}, \qquad C_F = \frac{\iota_t}{p_F}.$$

Notice that the yen value of Japanese imports is $p_F C_F = \iota_t$.

The corresponding US utility is:

$$U_t^* = \chi_t^* \ln C_{NT}^* + \xi_t \ln C_H^* + b_t \ln C_F^*,$$

where C_H^* is US consumption of Japanese tradable goods. We assume that $C_{NT}^* = \chi_t^*$, similarly to Japan above, unless otherwise stated. The US static utility maximization problem leads to:

$$\lambda^* = \chi_t^* / C_{NT}^* = 1, \qquad C_H^* = \frac{\xi_t}{p_H^*}, \qquad C_F^* = \frac{b_t}{p_F^*}.$$

Hence, the dollar value of Japanese exports to the US is: $p_H^* c_H^* = \xi_t$.

The exchange rate e_t is defined as the quantity of Yen bought by 1 dollar, i.e. the

strength of US Dollar. For example, the yen value of Japanese exports is: $e_t \xi_t$.

Japanese net exports, expressed in Yen, are given by: $NX_t = e_t p^*_{H,t} C^*_{H,t} - p_{F,t} C_{F,t} = \xi_t e_t - \iota_t$.

We gather the results obtained so far in the Lemma below.

Lemma 1 *Expressed in Yen, Japanese exports to the US are $\xi_t e_t$, Japanese imports from the US are ι_t. Hence, Japanese net exports are:*

$$NX_t = \xi_t e_t - \iota_t.$$

Note that this result is independent of the pricing procedure (e.g. price stickiness under producer or local currency pricing). Under producer currency pricing (PCP) and in the absence of trade costs, the US dollar price of Japanese tradables is p_H/e while under local currency pricing (LCP) the price is simply p^*_H.

Financial markets are incomplete and each country trades a risk-free domestic currency bond.[1] The dynamic budget constraint for Japan is state by state:

$$\sum_t R^{-t} p_{H,t} Y_{H,t} = \sum_t R^{-t} \left(p_{H,t} C_{H,t} + p_{F,t} C_{F,t} \right). \tag{1}$$

Substituting the goods market clearing condition ($Y_{H,t} = C_{H,t} + C^*_{H,t}$) in the above equation yields:

$$\sum_t R^{-t} p_{H,t} C^*_{H,t} = \sum_t R^{-t} p_{F,t} C_{F,t},$$

i.e.:

$$\text{PV of Exports in Yen } = \text{PV of Imports in Yen.}$$

Substituting the previous expressions for the value in Yen of Japanese imports and exports one obtains:

$$\sum_t R^{-t} \left(\xi_t e_t - \iota_t \right) = 0.$$

If the country starts with a non-zero NFA position N_0, the modified balance equation

[1] The assumption, while stark, makes the analysis of the basic model most tractable. The market structure is enriched in the following sections. *Risk-free* here refers to paying one unit of NT goods in all states of the world and is therefore akin to "nominally risk free".

is:

$$N_0 + \sum_t R^{-t} \left(\xi_t e_t - \iota_t \right) = 0.$$

It is immediate that under financial autarky, i.e. if there is trade balance every period, that the equilibrium exchange rate is (when the initial NFA is 0):

$$e_t = \frac{\iota_t}{\xi_t}.$$

In financial autarky, the Yen depreciates ($\uparrow e$) whenever Japanese demand for US good increases or whenever US demand for Japanese goods falls. This has to occur because there is no mechanism, in this case, to absorb the excess demand/supply of Dollars versus Yen that a non-zero trade balance would generate.

2.2 Financiers

Suppose that global financial markets are imbalanced such that there is an excess supply of $Q_{H,0}$ yen versus dollar resulting for example from trade or portfolio flows. Who will be willing to give Japan those yens, and hold those dollars? We posit that the resulting imbalances are absorbed, at some premium, by world financiers.

We assume that there are a unit mass of world financial intermediaries, each managed by a world financier. Agents from the two countries are selected at random to run intermediaries for a single period.[2] Financiers start their job with no capital of their own and can trade bonds denominated in both currencies. Therefore, their balance sheet has the simple form of $q_{H,t} = -q_{F,t} e_t$, where $q_{H,t}$ is the yen value of Yen-denominated bonds the intermediary is long of and $q_{F,t}$ the corresponding value in dollars for Dollar-denominated bonds. At the end of the period financiers payout their profits and losses to the households.

Our financiers are meant to capture a broad array of financial institutions that intermediate global financial markets. These institutions range from global investment banks such as Goldman Sachs and JP Morgan, to macro and currency hedge funds such as Soros Fund Management, to active investment managers such as Pimco and Black Rock. While significant differences across these intermediaries certainly exist, we stress their common

[2]Being a financier in this set-up is an occupation for agents in the two countries rather than an entirely separate class of agents. The selection process is governed by a memoryless Poisson distribution.

characteristic of being active investors that profit from medium-term imbalances in international financial markets often by intermediating (*taking the other side*) of currency demand resulting from both trade flows and financial flows. They also share the characteristic of being subject to financial constraints that limit their ability to take positions based on their risk bearing capacity and existing balance sheet risks.[3]

We assume that each financier maximizes the expected value of the intermediary that she manages:

$$V_t = \left[q_{H,t} \; \beta \left(R_{t+1} - R^*_{t+1} \frac{e_{t+1}}{e_t} \right) \right].$$

In each period, after taking positions but before the shocks are realized, the financier can divert part of the funds she intermediates. If the financier diverts the funds, the intermediary is unwound and customers who had lent to the intermediary recover a portion $1 - \Gamma \left| \frac{q_{H,t}}{e_t} \right|$ of their credit position $\left| \frac{q_{H,t}}{e_t} \right|$.[4] Since creditors, when lending to the intermediary, correctly anticipate the incentives of the financier to divert funds, the financier is subject to a credit constraint of the form:

$$\underbrace{\frac{V_t}{e_t}}_{\substack{Intermediary\ Value \\ in\ Dollar}} \geq \underbrace{\left| \frac{q_{H,t}}{e_t} \right|}_{\substack{Total \\ Claims}} \quad \underbrace{\Gamma \left| \frac{q_{H,t}}{e_t} \right|}_{\substack{Diverted \\ Portion}} = \underbrace{\Gamma \left(\frac{q_{H,t}}{e_t} \right)^2}_{\substack{Total\ divertable \\ Funds}}.$$

Our functional assumption for the diverted funds is not only, as it will become clear below, one of convenience, but also stresses the idea that intermediaries are able to divert more and more funds as their balance sheets increase.[5] Foundations of financial constraints that share a similar spirit can be found in Gertler and Kiyotaki (2010) and Maggiori (2011).

[3]We do not provide here the contract-theory foundations underlying which assets and contracts the intermediaries trade in equilibrium. Rather we take as given the prevalence of short term debt in different currencies and analyze its equilibrium implications.

[4]Notice that the financier can divert funds from their creditors, but not from debtors. Given the balance sheet $q_{H,t} = -q_{F,t} e_t$, the Dollar-value of the financier's liabilities is always equal to $\left| \frac{q_{H,t}}{e_t} \right|$, irrespective of whether $q_{H,t}$ is positive or negative; hence the use of absolute value in the text above. More formally, we assume that the intermediary's creditors can recover a value in dollars equal to $min(1 - \Gamma(\frac{q_{H,t}}{e_t})^2, |\frac{q_{H,t}}{e_t}|)$.

[5]It is outside of the scope of this paper to provide deeper foundations for this constraint. However, such foundations could potentially be achieved in models of financial complexity where bigger balance sheets lead to more complex positions. These bigger positions, in turn, are more difficult and costly for creditors to unwind in order to recover their funds in case of default.

The optimization problem of the financier is:

$$\begin{aligned} \max_{q_{H,t}} \quad & V_t = \left[q_{H,t} \ \beta \left(R_{t+1} - R^*_{t+1} \frac{e_{t+1}}{e_t} \right) \right], \\ s.t. \quad & V_t \geq \Gamma \frac{q^2_{H,t}}{e_t}. \end{aligned}$$

Since the value of the intermediary is linear in the position $q_{H,t}$, while the right hand side of the constraint is convex in $q_{H,t}$, the constraint always binds. Intuitively, given any non-zero expected excess return in the currency market the financier will want to either borrow or lend as much as possible in Yen. The constraint limits the maximum position and therefore binds. Substituting the intermediary value in the constraint and re-arranging, we characterize the financier's optimal choice to intermediate global financial flows in different currencies as:

$$q_{H,t} = \frac{1}{\Gamma} E_t \left[e_t - e_{t+1} \frac{R^*_{t+1}}{R_{t+1}} \right].$$

Integrating the above demand function over the unit mass of financiers, we obtain the aggregate financiers' demand for assets:

$$Q_{H,t} = \frac{1}{\Gamma} E_t \left[e_t - e_{t+1} \frac{R^*_{t+1}}{R_{t+1}} \right].$$

Notice that the higher Γ, the lower the intermediary risk bearing capacity, the more segmented the asset market is. To understand the behavior of this demand, let us consider two polar opposite cases. When $\Gamma = 0$ financiers are able to absorb any imbalances, i.e. want to take infinite positions, whenever there is a positive excess return in currency markets. When $\Gamma \uparrow \infty$ financiers are unwilling to absorb any imbalances, i.e. want to take no positions, no matter what the expected returns from doing so are.

Since Γ, the financiers' risk bearing capacity, plays a crucial role in our theory we refer hereafter to the set-up described so far as the *basic Gamma model*.

We stress that the above demand function not only captures the spirit of international financial intermediation via the micro foundation of the constrained portfolio problem, but also behaves in aggregate similarly to the demand of a CARA agent with risk aversion Γ.[6] Similar demand functions have been central to the limits of arbitrage theory of

[6]Formally, it is equivalent to the first order approximation of a CARA demand around a constant

De Long, Shleifer, Summers, and Waldmann (1990), Shleifer and Vishny (1997), and Gromb and Vayanos (2002).

For simplicity, we assume (for now and for much of this paper) that financiers rebate their profits and losses to the US consumers, not Japanese consumers.[7] This asymmetry gives much tractability to the model, at fairly little cost in the economics. This assumption is relaxed in the Appendix where we derive the class of intermediaries' payoff functions to the two countries that still allow for a simple closed form solution.

2.3 Supply and Demand of Assets: Equilibrium Exchange Rate

Recall that for simplicity we are focusing on the case of imbalances resulting from trade flows. In this case, financiers trade dollar bonds with US household and Yen bonds with Japanese households. The financiers will be long $Q_H = -NX_0$ yen, and will be short $Q_F = -Q_H/e_0$ dollars.

Let us study the equilibrium when all interest rates are zero.[8] The key equations are the financiers' demand equation:

$$Q_{H,0} = \frac{1}{\Gamma} E\left[e_0 - e_1\right],$$

and the equilibrium "flow" demand for Yen at time $t = 0, 1$:

$$\xi_0 e_0 - \iota_0 + Q_{H,0} = 0, \tag{2}$$

$$\xi_1 e_1 - \iota_1 - Q_{H,0} = 0. \tag{3}$$

The first equation stresses that the time-zero total demand for Yen (against Dollar) is: $\xi_0 e_0 - \iota_0$ coming from the next exports of Japan, and Q_0 coming from the financiers. Recall that we assume that Japanese agents (exporters) do not wish to hold any currency exposure: they convert their US sales of ξ_0 dollars into Yen, for a demand $\xi_0 e_0$ of Yen. Likewise, US firms have ι_0 yen worth of exports to Japan, and sell them, as they want to keep only dollar balances.[9]

volatility of assets returns.

[7] For completeness, note that this assumption had already been implicitly made in deriving Japan inter temporal budget constraint in Equation (1).

[8] This is just for easiness of notation. In this framework, this requires $\beta = 1$.

[9] We could relax those assumptions: if exporters repatriated a fraction α of sales in their own currency, the first equation would become: $\alpha\left(\xi_0 e_0 - \iota_0\right) + Q_0 = 0$. The economics would remain similar.

At time 1, the same net export channel generates a demand for Yen of $\xi_1 e_1 - \iota_1$ while the financiers need to sell their position generating a supply of Yen of $Q_{H,0}$. Adding the last 2 equations gives the inter-temporal budget constraint:

$$\xi_1 e_1 + \xi_0 e_0 = \iota_0 + \iota_1. \tag{4}$$

Assume that $\xi_t = 1$ for $t = 1, 2$ for simplicity (for now at least), then:

$$E\left[e_1\right] = \iota_0 + \iota_1 - e_0 = \left[\left(1 + \Gamma\right) e_0 - \Gamma \iota_0\right],$$

hence the equilibrium exchange rate at time zero is:

$$e_0 = \frac{\left(1 + \Gamma\right) \iota_0 + E[\iota_1]}{2 + \Gamma}.$$

By simple substitution the equilibrium exchange rate at time one is:

$$\begin{aligned} e_1 &= \iota_0 + \iota_1 - e_0 = \{\iota_1\} + \iota_0 + E[\iota_1] - e_0 \\ &= \{\iota_1\} + \iota_0 + E[\iota_1] - \frac{\left(1 + \Gamma\right) \iota_0 + \iota_1}{2 + \Gamma} \\ &= \{\iota_1\} + \frac{\iota_0 + \left(1 + \Gamma\right) E[\iota_1]}{2 + \Gamma}, \end{aligned}$$

where we use the notation for random variable x, $\{x_t\} \equiv x_t - E_{t-1}[x_t]$. So that $\{x_t\}$ is the forecast error of the random variable x_t. We gather the result in the proposition below.

Proposition 1 *Assume that $\xi_t = 1$ for $t = 1, 2$, and that interest rates are zero in both countries. The exchange rate is:*

$$e_0 = \frac{\left(1 + \Gamma\right) \iota_0 + E[\iota_1]}{2 + \Gamma}, \tag{5}$$

$$e_1 = \{\iota_1\} + \frac{\iota_0 + \left(1 + \Gamma\right) E[\iota_1]}{2 + \Gamma}, \tag{6}$$

and the expected Yen appreciation is:

$$E\left[\frac{e_0 - e_1}{e_0}\right] = \frac{\Gamma\left(E\left[\iota_1\right] - \iota_0\right)}{\left(1 + \Gamma\right) \iota_0 + E[\iota_1]}.$$

The time zero exchange rate varies, depending on Γ, between two polar opposites: the UIP-based, and the financial autarky exchange rate, respectively. When $\Gamma \to \infty$, one has $e_0 = \frac{\iota_0}{\xi_0}$ which we have previously shown is the financial autarky value of the exchange rate. Intuitively, in this case financiers are so risk averse that no financial flows can occur between countries and, therefore, trade has to be balanced period by period. When $\Gamma = 0$, UIP holds and one obtains $e_0 = \frac{\iota_o + E[\iota_1]}{2}$. Intuitively, in this case financiers are so relaxed about risk (in fact, risk neutral) that they are willing to take infinite positions in currencies whenever there is a positive expected excess return by doing so. UIP only imposes a constant exchange rate in expectation $E[\Delta e] = 0$; the level of the exchange rate is then obtained from additionally using the inter temporal budget constraint in equation 4.

To further understand the effect of Γ, notice that at the end of period 0 (say, time 0^+), Japan's NFA are $N_{0^+} = e_0 - \iota_0 = \frac{E[\iota_1] - \iota_0}{2+\Gamma}$. Therefore, Japan has positive NFA at $t = 0^+$ *iff* $\iota_0 < E[\iota_1]$. If Japan has a positive NFA position, then the financiers are long Yen and short Dollar. For the financiers to bear this risk, they require a compensation: the Yen needs to appreciate on average. The required appreciation is generated by making the Yen weaker at time zero. The magnitude of the effect depends on the strength of the financiers risk bearing capacity (Γ) as formally shown here by taking partial derivatives: $\frac{\partial e_0}{\partial \Gamma} = \frac{\iota_0 - E[\iota_1]}{(2+\Gamma)^2} = \frac{-N_{0^+}}{2+\Gamma}$. We collect the result in the proposition below.

Proposition 2 *In the basic Gamma model,*

$$\frac{\partial e_0}{\partial \Gamma} = \frac{-N_{0^+}}{2+\Gamma}. \tag{7}$$

When there is a financial disruption ($\uparrow \Gamma$), countries that were positioned to be net external debtors ($N_{0^+} < 0$) experience a currency depreciation ($\uparrow e$), while the opposite is true for net creditor countries.

Intuitively, net external debtor countries have borrowed from the world financial system thus generating a long exposure for financiers to their currencies. Should the financial system risk bearing capacity be disrupted, these currencies depreciate to compensate financiers for the increased (perceived) risk. This situation formalizes a number of external crises where loosely defined global risk aversion shocks, embodied here in Γ, caused large

depreciations of the currencies of countries that had recently experienced large capital inflows.

To illustrate how the results readily extend to more general cases, we report below expressions for the case of different interest rates, stochastic shocks ξ_t, and an extension to multiple periods. Several more extensions can be found in the rest of the paper as well as the appendix.

Remark 1 *When ξ_t is stochastic, and under different interest rates, we have the slightly more complex expression:*

$$e_0 = \frac{\left(R^* E\left[\xi_1^{-1}\right] + \Gamma\right)\iota_0 + \frac{R^*}{R} E\left[\xi_1^{-1}\iota_1\right]}{\left(R^* E\left[\xi_1^{-1}\right] + \Gamma\right)\xi_0 + 1}.$$

Remark 2 *With arbitrary T periods, we have $\xi_t e_t - \iota_t + \frac{1}{\Gamma} E_t\left[e_t - e_{t+1}\right] = 0$, and for the budget constraint: $0 = \sum_{t=0}^{T} e_t \xi_t - \iota_t$. In the case $\xi_t = 1$ for all t, the equilibrium exchange rate is:*

$$e_0 = \frac{E_0\left[\sum_{t=0}^{T} \frac{\iota_t}{(1+\Gamma)^t}\right]}{\sum_{t=0}^{T} \frac{1}{(1+\Gamma)^t}}.$$

The risk aversion Γ is akin to some "myopia" about future flows.

Notice that the equation $e_t - \iota_t + \frac{1}{\Gamma} E_t\left[e_t - e_{t+1}\right] = 0$, that in each period has the simple but important interpretation of equating relative supply and demand of Yen vs Dollar, is also central to the tractability of our model. Formally it is a first order linear stochastic difference equation, the solution of which is easy to analyze by standard methods (for example by rolling the equation forward and imposing a no-bubble terminal condition as done above).

Supply and demand matter in the Gamma model We now further illustrate how *supply and demand of assets* do matter for the *financial determination of the exchange rate*. Consider the case where US consumers have, at time 0, an inelastic demand (e.g. some noise traders) f of Yen bonds funded by an offsetting position $-f/e_0$ in Dollar bonds. The flow equations are now:

$$e_0\xi_0 - \iota_0 + Q + f = 0, \tag{8}$$

$$e_1\xi_1 - \iota_1 - Q - f = 0, \tag{9}$$

as at time 1, financiers and US consumers repatriate their holdings. The financiers' demand is still $\Gamma Q_{H,0} = E[e_0 - e_1]$. The equilibrium exchange rate is derived in the proposition below.

Proposition 3 (Demand pressure in the FX market) *Assume $\xi_t = 1$ for $t = 1, 2$. With an inelastic time-0 extra demand f for Yen bonds by US consumers, the exchange rates at time 0 and 1 are:*

$$e_0 = \frac{(1+\Gamma)\,\iota_0 + E[\iota_1] - f\Gamma}{2+\Gamma}, \tag{10}$$

$$e_1 = \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\,E[\iota_1] + f\Gamma}{2+\Gamma}. \tag{11}$$

Hence, an extra demand f for Yen at time 0 makes the Yen appreciate at time 0, makes the Yen depreciate at time 1. However, it does not change the time-average value of the Yen: $e_0 + e_1 = \iota_0 + \iota_1$ independently of f.

Proof: Adding equations 8 and 9 gives:

$$e_0 + e_1 = \iota_0 + \iota_1.$$

Substituting this in the time zero flow equation and using the financiers' demand gives:

$$\begin{aligned} 0 &= e_0\xi_0 - \iota_0 + Q + f \\ &= e_0 - \iota_0 + \frac{e_0 - E[e_1]}{\Gamma} + f \\ &= e_0 - \iota_0 + \frac{e_0 - (\iota_0 + E[\iota_1] - e_0)}{\Gamma} + f, \end{aligned}$$

rearranging:

$$e_0 = \frac{(1+\Gamma)\,\iota_0 + E[\iota_1] - f\Gamma}{2+\Gamma}.$$

Finally:

$$\begin{aligned} e_1 = \iota_0 + \iota_1 - e_0 &= \{\iota_1\} + \iota_0 + E[\iota_1] - e_0 \\ &= \{\iota_1\} + \iota_0 + E[\iota_1] - \frac{(1+\Gamma)\,\iota_0 + E[\iota_1] - f\Gamma}{2+\Gamma} \\ &= \{\iota_1\} + \frac{\iota_0 + (1+\Gamma)\,E[\iota_1] + f\Gamma}{2+\Gamma}. \end{aligned}$$

□

An increase in US demand for Yen bonds needs to be absorbed by the financiers who need to sell Yen bonds and buy Dollar bonds. To induce the financiers to provide the desired bonds the Yen needs to appreciate on impact, *as a result of the capital flow*, to then be expected to depreciate thus generating an expected gain for the financier's short Yen position. This example emphasizes that our model is an elementary one where a relative price, the exchange rate, has to move to equate the supply and demand of two assets, Yen and Dollar bonds.

This framework can analyze concrete situations such as the recent large scale capital flows from developed countries into emerging market bond markets, say from US investors into Brazilian Real bonds, that put upward pressure on the receiving countries' currencies. While such flows and their impact on currencies have been paramount in the logic of market participants and policy makers, they had thus far proven elusive in a formal theoretical analysis.

Hau, Massa, and Peress (2010) provide direct evidence that plausibly exogenous capital flows impact the exchange rate in a manner consistent with the Gamma model. They show that following a restating of the weights of the MSCI World Equity Index countries that as a result experienced capital inflows (because their weight in the index increased) had their currency appreciate.

To stress the difference between our basic Gamma model of the financial determination of exchange rates and the traditional macroeconomic framework we illustrate next two polar cases that have been popular in the previous literature: the UIP-based exchange rate, and the complete market exchange rate.

Much of the now classic international macroeconomic analysis spurring from Dornbusch (1976) and Obstfeld and Rogoff (1995) either directly assumes that UIP holds or effectively imposes it by solving a first order linearization of the model.[10] The closest analog to this literature in the basic Gamma model is the case when $\Gamma = 0$ so that UIP holds by assumption. In this world, financiers are so relaxed about supplying liquidity to satisfy shifts in the world demand for assets that such shifts have no price impact. Consider the case of US investors suddenly wanting to buy Brazilian real bonds, the financiers would simply take the other side of the investor portfolio demand with no effect on the

[10]Intuitively, a first order linearization imposes certainty equivalence on the model and therefore kills any risk premia such as those that could generate a deviation from UIP.

exchange rate between Dollar and Real. In fact, equation 10 confirms that if $\Gamma = 0$, then portfolio flow f has no impact on the equilibrium exchange rate.

Another strand of the literature emanating from Lucas (1982) and Backus, Kehoe, and Kydland (1992) has analyzed risk premia predominately under complete markets. We now show that the exchange rate in a set-up with complete markets but otherwise identical to ours is *constant*. We only sketch here the logic and the main equations while a full treatment is relegated to the appendix. Under complete markets, the marginal utility of US and Japanese agents, expressed in a common currency, must be equal. Intuitively, the full risk sharing that occurs under complete markets calls for Japan and the US to have the same marginal benefit from consuming an extra unit. In our set-up this risk sharing condition takes the simple form:

$$\frac{P_t C_t}{P_t^* C_t^*} = \nu^{-1} e_t \frac{\theta_t}{\theta_t^*},$$

where the total consumption expenditure is $PC = C_{NT} + p_H C_H + p_F C_F$ and $\theta \equiv \chi + a + \iota$ and $\theta^* \equiv \chi^* + b + \xi$ are preference shifters. Simple substitution of the optimality conditions of the consumers' static maximization problems yield $PC = \theta$ and $P^*C^* = \theta^*$. Thus one obtains the conclusion that $e_t = \nu$, i.e. the exchange rate is constant and equal to the Negishi weight ν. We formalize this result in the Lemma below.

Lemma 2 *Consider an economy identical to the set-up of the basic Gamma model, other than the fact that financial markets are complete, then the equilibrium exchange rate is constant:*

$$e_t = \nu,$$

where ν is the relative Negishi weight of the US.

The exchange rate disconnect The Meese and Rogoff (1983) result on the inability of economic fundamentals such as output, inflation, exports and imports to predict, or even contemporaneously co-move with, exchange rates has had a chilling and long-lasting effect on theoretical research in the field. The disconnect between exchange rates and macro fundamentals is a central feature of the Gamma model. To stress this feature, we show here in the context of the basic model that two economies with identical macroeconomic fundamentals feature entirely unconnected equilibrium exchange rates depending

on the imbalances in financial markets: the financiers risk bearing capacity Γ and their balance sheet exposure Q_H. In later sections and the appendix, we confirm that this property is also at the core of generalized versions of the basic Gamma model.

Consider a world, which we will call *Tranquil Times*, in which intermediaries risk bearing capacity is $\Gamma_D > 0$ and the starting intermediary balance sheet is $Q^T_{H,0^-} = -f < 0$.

Consider an alternate world, which we will call *Distressed Times*, in which financial imbalances are higher and risk bearing capacity is lower. Higher financial imbalances are given by $Q^D_{H,0^-} = -(f + \Delta f) < 0$, where $\Delta f > 0$. The lower risk bearing capacity is given by $\Gamma^D > \Gamma^T$.

We stress that the two economies have otherwise identical fundamentals. Simple algebra, entirely similar to the derivations in the previous sections, reveals that:

$$e_0^T - e_0^D \propto (\Gamma_D - \Gamma_T)\left[E[\iota_1] - \iota_0 + 2f\right] + \Gamma_D\left(2 + \Gamma_T\right)\Delta_f, \tag{12}$$

i.e. the equilibrium exchange rates in the two economies are potentially very different from each other. To understand the disconnect between traditional fundamentals and exchange rates in the Basic Gamma model, let us consider each of the two terms on the right hand side of Equation (12).

The first term has two components: $[E[\iota_1] - \iota_0 + 2f]$ summarizes the common fundamentals of the two economies, while $(\Gamma_D - \Gamma_T)$ highlights that the same fundamentals imply different exchange rates based on the different risk bearing capacity of the financiers. Recall that whenever $E[\iota_1] - \iota_0 > 0$, trade flows induce the financiers to lend in Dollar and borrow in Yen. Similarly a positive f represents a short Yen and long Dollar starting imbalance for the financiers. Therefore, the term $[E[\iota_1] - \iota_0 + 2f]$ summarizes, based on the common fundamentals of the two economies, how many dollars the financiers are long of. Consider the case when $[E[\iota_1] - \iota_0 + 2f] > 0$ and the financiers are, therefore, long dollars. In this case, the lower the risk bearing capacity (the higher $(\Gamma_D - \Gamma_T) > 0$), the more the dollar has to depreciate at time zero to induce the financiers to hold the position via expected capital gains ($e_0^T - e_0^D \uparrow$).

The second term on the right hand side of Equation (12) shows that different starting balance sheets for the intermediaries also lead to different equilibrium exchange rates. Since $\Delta f > 0$ the financiers are shorter Yen and longer Dollar in the Distressed than in

the Tranquil economy. This further financial imbalance requires a deprecated, at time zero, Dollar in the Distressed economy compared to the Tranquil one in order for the financiers to intermediate the flows. How strong this effect is depends on the level of the risk bearing capacity in the two economies (the term $\Gamma_D(2+\Gamma_T)$): the lower the risk bearing capacity, the stronger the effect.

2.4 Production and Price Rigidities

Clearing the goods market The reader will have noticed that very little so far has been said about output. In many instances it is simplest to consider the endowment economy version of the basic Gamma model, where $\{Y_{NT}, Y_H, Y^*_{NT}, Y_F\}$ are all exogenous processes. We collect here this basic result assuming that all prices are flexible and that the low of one price (LOP) holds, so that:

$$p_H = p^*_H e, \quad p_F = p^*_F e.$$

Summing US and Japanese demand for Japanese tradable goods, we obtain the world demand:

$$C_H = \frac{a_t}{p_H}, \qquad C^*_H = \frac{\xi_t e_t}{p_H}, \qquad D_{H,t} \equiv C_H + C^*_H = \frac{a_t + \xi_t e_t}{p_{Ht}}.$$

Clearing the good market, $Y_H = D_H$, yields the equilibrium price of Japanese tradables:

$$p_H = \frac{a_t + \xi_t e_t}{Y_H}.$$

An entirely similar argument yields: $p^*_F = \frac{b_t + \frac{\iota_t}{e_t}}{Y_F}$.

Equilibrium Production without price rigidities Let us introduce a minimal model of production that will allow to formalize the effects of the exchange rate on output and unemployment. While we maintain the assumption that non-tradable goods in each country are an endowment, we now assume that tradable goods in each country are produced with a technology linear in labor with productivities $\{A, A^*\}$. For much of the analysis and unless otherwise stated, we assume $\{A = 1, A^* = 1\}$. L is the available labor supply in each country and labor is supplied inelastically . Simple profit maximization at the firm level yields a Yen wage of $w_H = p_H$. Under flexible price, good market clearing

then implies full employment $Y_H = L$ and a Japanese tradable price of:

$$p_H^\circ = \frac{a_t + e_t \xi_t}{L},$$

where circle in p° denotes an ideal, frictionless quantity. The resulting consumption shares of the Japanese tradables are :

$$C_H = \frac{a_t}{a_t + e_t \xi_t} L, \qquad C_H^* = \frac{e_t \xi_t}{a_t + e_t \xi_t} L.$$

Likewise for US tradables the equilibrium features both full employment $Y_F = L$ and a Dollar price for the goods of:

$$p_F^{*\circ} = \frac{b_t + \iota_t / e_t}{L}.$$

Equilibrium Production with price rigidities Let us now assume that wages are "downward rigid" in domestic currency and preset at $\{\bar{p}_H, \bar{p}_F^*\}$, where these "prices" are exogenous. Let us further assume that firms do not engage in pricing to market, so that prices are sticky in producer currency (PCP). Firm profit maximization then implies that: $p_H = \max(\overline{p}_H, p_H^\circ)$, or more explicitly:

$$p_H = \max\left(\overline{p}_H, \frac{a_t + e_t \xi_t}{L}\right),$$

Hence:

$$Y_H = \min\left(\frac{a_t + e_t \xi_t}{\overline{p}_H}, L\right).$$

When demand is sufficiently low ($a_t + e_t \xi_t$ low), then output is demand-determined (it depends on e_t, ξ_t, and a_t) and there is unemployment $L^u \equiv L - Y_H$.

Notice that in this case the exchange rate e_t has an *expenditure-switching* effect: if the Yen depreciates ($e_t \uparrow$), unemployment falls and output expands. Formally, if $\frac{a_t + e_t \xi_t}{\overline{p}_H} < L$, as e_t increases, Y_H increases and consequently L^u falls. Intuitively, since Japanese tradables prices are sticky in Yen, when the Yen depreciates these goods become cheap for US consumers to buy, thus increasing US demand for Japanese tradables. In a world that is demand constrained, this expansion in demand for Japanese tradable is met by expanding production, thus raising Japanese output and employment.

Clearly, a similar expression and mechanism apply to US tradables:

$$Y_F = \min\left(\frac{b_t + \iota_t/e_t}{\bar{p}_F^*}, L\right).$$

The expenditure switching role of exchange rates has been central to the Keynesian analysis of open macroeconomics of Keynes (1924), Mundell (1963), Dornbusch (1976), Obstfeld and Rogoff (1995). In the Gamma model it is enriched by being the central channel for the transmission of financial forces that affect the exchange rate, such as the risk bearing capacity and balance sheet of the financiers, into output and employment.

The financial determination of exchange rates has real consequences. Let us reconsider our earlier example of a sudden inflow of capital from US investors into Brazilian Real bonds. The exchange rate in this economy with production and sticky prices is still characterized by equation 10. As previously discussed the capital inflow in Brazil causes the Real to appreciate ($\frac{\partial e_0}{\partial f} = -\frac{\Gamma}{2+\Gamma} < 0$) and if the flow is sufficiently strong (f sufficiently high) or the financiers risk bearing capacity sufficiently low (Γ sufficiently high) the appreciation (the increase in e_0) can be so strong as to make Brazilian goods uncompetitive on international markets; the corresponding fall in world demand for Brazilian output ($\downarrow C_H^* = \frac{\iota_0}{e_0 \bar{p}_F^*}$) causes an economic slump in Brazil with following output and increasing unemployment.

Despite extensive debates on the possibility of this type of adverse effect on the real economy of fluctuations in exchange rates due to capital flows and imbalances in financial markets, there are few formal models able to analyze the necessary theoretical foundations. We focused here on providing such foundations in a positive model, Section **??** provides the corresponding normative analyses.

LCP World Let us know consider prices that are sticky in the export destination currency (LCP). Assume that prices for Japanese tradable goods are exogenously set at $\{\bar{p}_H, \bar{p}_H^*\}$ in Yen in Japan and in Dollar in the US, respectively.

Japanese output is now given by:

$$Y_H = min\left(\frac{a_t}{\bar{p}_H} + \frac{\xi_t}{\bar{p}_H^*}, L\right).$$

Notice that while the exchange rate no longer affects output, Japanese next exports

in Yen still have the same value: $NX_t = e_t\xi_t - i_t$. Consequently the exchange rate is unchanged from the previous formulae.

Devereux and Engel (2003) stressed the absence of exchange rate effects on output under LCP. The empirical evidence shows that in practice a combination of PCP, LCP and limited pass-trough are present in the data.For much of this paper, we focus on PCP as the most illustrative case. Nothing is lost in our qualitative analysis by allowing for somewhat more limited pass-trough of exchange rate changes to local prices of internationally traded goods.

3 Nominal exchange rate

We have thus far considered a real model, we now investigate a nominal version of the model where the nominal exchange rate is determined, similarly to our baseline model, in financial markets.[11] The monetary model not only shows that the insights of the real model hold more generally, but also provides new insights on monetary policy in the presence of incomplete markets and financial intermediation.

We assume that money is only used domestically and that his demand is captured, in reduced form, in the utility function of consumers in each country. A vast literature has focused on foundations of the demand for money; such foundations are beyond the scope of this paper and consequently we focus on the simplest approach that delivers a plausible demand for money and much tractability. In the appendix we show versions of the model with Taylor rules or targeting of nominal expenditures by the central bank as common in the new Keynesian analysis.

Japan's flow utility is:

$$U_t = \omega_t \ln \frac{M_t}{P_t} + \chi_t \ln C_{NT} + a_t \ln C_{H,t} + \iota_t \ln C_{F,t},$$

where M is the amount of money outstanding and P is the nominal price level so that

[11]Notice that we have indeed set-up the "real" model in the previous section in such a way that non-tredables in each country play a role very similar to money and where, therefore, the exchange rate is rather similar to a nominal exchange rate (see Obstfeld and Rogoff (1996)[Ch. 8.3]). In this section we make such analogy clearer. The appendix provides a full discussion of the CPI-based real exchange rate in the models of the previous and current sections.

$\frac{M}{P}$ are real money balances. Corresponding US utility is:

$$U^* = \omega_t^* \ln \frac{M_t^*}{P_t^*} + \chi_t^* \ln C_{NT} + \xi_t \ln C_{H,t}^* + b_t \ln C_{F,t}^*.$$

Money is the numeraire in each economy, with local currency price equal to one. The static utility maximization problem is entirely similar to the one in the basic Gamma model:

$$\max_{\frac{M}{P}, C_{NT}, C_H, C_F} \omega_t \ln \frac{M}{P} + \chi_t \ln C_{NT} + a_t \ln C_H + \iota_t \ln C_F + \lambda_t \left(PC - M - p_{NT} C_{NT} - p_H C_H - p_F C_F \right),$$

where C is a basket of goods $C \equiv (C_{NT})^{\chi_t'} (C_H)^{a_t'} (C_F)^{\iota_t'} (\frac{M}{P})^{\omega_t'}$. We use the generic definition of $x' \equiv \frac{x}{\theta}$ and correspondingly $x^{*\prime} \equiv \frac{x^*}{\theta^*}$, where $\theta \equiv \omega + \chi + a + \iota$ and $\theta^* \equiv \omega^* + \chi^* + b + \xi$. PC is total nominal expenditure which is taken as exogenous here. Standard optimization arguments lead to demand functions:

$$M = \omega_t' \, PC, \qquad C_{NT} = \chi_t' \, \frac{PC}{p_{NT}},$$
$$C_H = a_t' \, \frac{PC}{p_H}, \qquad C_F = \iota_t' \, \frac{PC}{p_F}.$$

The demand equations simply state that total consumption is allocated to the various goods as a function of their relative price and their relative taste shock. Money demand, in the top equation, is proportional to total nominal consumption expenditures; the coefficient of proportionality is potentially stochastic.

Let us define $m \equiv \frac{M}{\omega'}$ and $m^* \equiv \frac{M^*}{\omega^{*\prime}}$. Notice that since money is non-tradable across countries or with the financiers (but bonds that pay in units of money are tradable with the financiers as in the previous section), the money market clearing implies that the central bank can pin down the level of nominal expenditure ($m = PC, m^* = P^* C^*$).[12] For most of the paper it is convenient to consider the *cashless* limit of our economies by taking the limit case when $\{M, M^*, \omega, \omega^*\} \downarrow 0$ and $\{m, m^*\}$ are finite (see Woodford (1998, 2011)).

Remark 3 *Money Vs NT.*

[12]The central bank in each period choses money supply after the preference shocks are realized so that m and m^* are policy variables. We abstract here from issues connected with the zero lower bound (ZLB) on nominal interest rates. Extensions that include the ZLB are explicitly modeled in the Appendix.

Notice the strong duality between money in the present set-up and non-tradable goods in the basic Gamma model of Section 2. If $M_t = \omega_t$ and $C_{NT,t} = \chi_t$, one recovers the previous Section equations, because the demand for money implies $P_t C_t = \theta_t$, in which case the demand for non-tradables implies that $p_{NT,t} = 1$.

The nominal exchange rate e_t is defined as the strength of the US dollar, so that an increase in e is a Dollar appreciation. US demand for Japanese goods is:

$$C_H^* = \xi_t^{'} \frac{P^* C^*}{p_H^*}$$

Therefore Japanese nominal exports in Yen are:

$$p_H^* C_H^* \; e = \xi_t^{'} \; m^* e$$

Japanese nominal net exports in Yen are: $NX = p_H^* C_H^* e - p_F C_F$, or more explicitly:

$$NX = \xi_t^{'} m^* e - \iota_t^{'} \; m.$$

Solving for the Nominal Exchange Rate Let us start by focusing on the case when both nominal risk-less rates are zero. The financiers demand equation takes the by now familiar form: $Q_{H,0} = \frac{1}{\Gamma} E\left[e_0 - e_1\right]$.

Starting from period 1, the dynamic budget constraint implies: $N_0 + NX_1 = 0$. Substituting the expression for next exports gives the period 1 nominal exchange rate:

$$e_1 = \frac{\iota_1^{'} \; m_1 - N_0}{\xi_1^{'} \; m_1^*}. \tag{13}$$

Autarky Economy Suppose countries were in financial autarky and therefore $N_0 = 0$, then the exchange rate is given by:

$$e = \underbrace{\frac{\iota^{'}}{\xi^{'}}}_{\substack{Trade \\ Effect}} \underbrace{\frac{m}{m^*}}_{\substack{Monetary \\ Effect}}. \tag{14}$$

The first terms is generated by the demand for goods as in the autarky case of the basic Gamma model. The second term highlight a basic force of the nominal model: the Yen depreciates whenever there is a relative expansion of Japanese monetary policy compared

to Japanese money demand ($\uparrow m$) or whenever there is a corresponding tightening of monetary policy in the US ($\downarrow m^*$). Intuitively a Japanese relative monetary expansion increases Japanese nominal expenditure in Yen, including the expenditure on imports. Since the US nominal expenditure in Dollar on Japanese goods in unchanged, a Dollar appreciation is necessary to restore nominal trade balance.

The flow equations of Yen in period one and two are given by:

$$\xi_0^{'} \; m_0^* \; e_0 - \iota_0^{'} \; m_0 + Q_{H,0} = 0, \tag{15}$$

$$\xi_1^{'} \; m_1^* \; e_1 - \iota_1^{'} \; m_1 - Q_{H,0} = 0. \tag{16}$$

Performing substitutions entirely similar to those in the basic Gamma model (using also the financiers' demand equation) yields the equilibrium nominal exchange rate in period zero which we report in the proposition below.

Proposition 4 *In the nominal version of the basic Gamma model, the equilibrium nominal exchange rate is given by:*

$$e_0 = \frac{\iota_0^{'} \; m_0 \left\{ \Gamma + E_0 \left[\left(\xi_1^{'} m_1^* \right)^{-1} \right] \right\} + E_0 \left[\frac{\iota_1^{'} m_1}{\xi_1^{'} m_1^*} \right]}{1 + \xi_0^{'} \; m_0^* \left\{ \Gamma + E_0 \left[\left(\xi_1^{'} m_1^* \right)^{-1} \right] \right\}}.$$

The appendix extends the result in Proposition 4 to allow for different interest rates across countries.

3.1 Production and Price Rigidities

The simplest case to consider is, once again, the endowment economy. Assume that $\{Y_{H,t}, Y_{F,t}\}$ are produced by endowments and that the LOP holds, then we have:

$$\begin{aligned} Y_H &= C_H + C_H^* \\ &= a_t^{'} \frac{PC}{p_H} + \xi_t^{'} \frac{P^* C^*}{p_H^*} \\ &= \frac{1}{p_H} \left(a_t^{'} \; m + \xi_t^{'} \; m^* e \right) \end{aligned}$$

so that:

$$p_H = \frac{a_t^{'} \; m + \xi_t^{'} \; m^* \; e}{Y_H}.$$

Finally the consumption share can easily be derived by substituting this price in the Japanese demand function for domestic goods:

Let us now consider a production set-up identical to the basic Gamma model in Section 2.4 other than having wages possibly being downward rigid in nominal terms here. It follows by a simple extension of the calculations for the basic Gamma model that when wages are flexible one has $Y_H = L$ and

$$p_H^\circ = \frac{a_t^{'} m + \xi_t^{'} m^* e_t}{Y_H}.$$

When wages are rigid in domestic currency we are in a PCP world, where $p_H = \max(p_H^\circ, \bar{p}_H)$ and:

$$Y_H = min\left(\frac{a_t^{'} m + e_t \xi_t^{'} m^*}{\bar{p}_H}, L\right)$$

Monetary Non-neutrality In this economy both unexpected and expected monetary shocks have real effects whenever prices are rigid at a sufficiently high level for output to be demand determined.

Recall from equation 13 that an unexpected (so that it does not affect N_0) Japanese monetary expansion weakens the Yen ($\uparrow e_1$). The expansion will have two effects on output, both positive. The first effect is domestic; by increasing nominal expenditure the central bank reduces the distortion due to price rigidity by raising all prices relative to the price of domestic tradables, the resulting boost in internal demand increases Japanese output. To see this notice that Japan demand for its domestic goods depends positively on its own monetary supply: $\frac{a_t^{'}}{\bar{p}_H}\, m_t$. The second effect is international; by weakening the Yen, Japanese goods in the US become more attractive, the resulting boost in foreign demand increases Japanese output. Tho see this notice that US demand for Japanese goods depends positively on Japanese monetary supply: $e_1 \frac{\xi_1^{'}}{\bar{p}_H} m_1^* = \frac{\iota_1^{'}}{\bar{p}_H} m_1 - \frac{N_0}{\bar{p}_H}$. We collect this result i the proposition below under the further simplifying assumption that $N_0 = 0$.

Proposition 5 *When wages are downward rigid in domestic currency, a Japanese unexpected monetary expansion in period 1 increases that period's output whenever output is demand driven. Furthermore, under the assumption that $N_0 = 0$, one has:*

$$Y_{H,1} = min\left(\frac{(a_1^{'} + i_1^{'}) m_1}{\bar{p}_H}, L\right)$$

So that $\frac{\partial Y_{H,1}}{\partial m^s} \geq = 0$*, with strict inequality whenever* $Y_{H,1} < L$*.*

Anticipated monetary policy can also have real effects.

LCP World Let us know consider prices that are sticky in the export destination currency (LCP).

Assume that prices for Japanese tradable goods are exogenously set at $\{\bar{p}_H, \bar{p}_H^*\}$ in Yen in Japan and in Dollar in the US, respectively.

Japanese output is now given by:

$$Y = min\left(\frac{a_t' m_t}{\bar{p}_H} + \frac{\xi_t' m_t^*}{\bar{p}_H^*}, L\right)$$

Notice that while the exchange rate no longer affects output, Japanese next exports in Yen still have the same value: $NX_t = e_t \xi_t' m_t^* - i_t' m_t$. Consequently the exchange rate is unchanged from the previous formulae.

4 Welfare and Heterodox Policies

The Gamma model of exchange rates considered so far in the positive analysis has made clear that exchange rates are affected by financial forces and can behave quite differently, as they do in the data, from those implied by the traditional macroeconomic analysis. We have also shown how the financial determination of exchange rates has real consequences for output and risk sharing.

We now turn to the normative analysis of the Gamma model and show that in a world of imperfect capital markets not only the exchange rate seldom performs the expenditure-switching role central to the Mundellian prescription of floating exchange rates, but actually a number of heterodox policies such as capital controls, large scale currency swaps, and currency intervention become beneficial in specific circumstances.

The Mundellian prescription of floating exchange rates rests on the idea that a country hit by a negative real shock would be helped by a depreciating currency that would in turn, under some form of sticky prices, boost its exports and therefore alleviate the adverse impact of the shock on output and employment. We have shown that not only this mechanism is not prevalent in the Gamma model (or in the data), but actually that

the exchange rate can in itself be the source of transmissions of financial shocks and financial imbalances to the real economy.

The possibility of such perverse effects of floating exchange rates has been the subject of extensive debates, culminating in the threat of *currency wars*, but its theoretical analysis and the development of policies to improve welfare has been elusive. Even the nascent research effort on analyzing the welfare consequences of capital controls, fiscal transfers, and fiscal devaluations has been carried out within the context of the new Keynesian model either under complete markets or by imposing the UIP condition.

4.1 Policy Instruments a Gamma world

We describe in this section the effects of policy instruments on the equilibrium exchange rate. We consider two classes of instruments: outright interventions (purchases and sales) in the currency market, and taxation of capital flows. The analysis of these instruments is not only of interest per se, but also the foundation of optimal policy in the next sections.

4.1.1 Bond purchases or FX swaps

We show that currency interventions in the forms of foreign currency bond purchases or FX swaps can stabilize the exchange rate.

A one-period economy The world ends at the end of the period. Before knowing the realization of ι_0, the Japanese government enters into a currency swap: it's long, on a notional q, the Dollar, and short the Yen. Hence, its NFA is $N = e\xi - \iota + q\left(e - E\left[e\right]\right)$ and the balance condition is $N = 0$:

$$e\xi - \iota + q\left(e - E\left[e\right]\right) = 0 \tag{17}$$

In this simplified one period model, we do not have to consider the financiers. Without the swap, we would have $e = \xi/\iota$. However, with the swap, the government gets $q\left(e - E\left[e\right]\right)$. Suppose the ξ is known, and ι is random. Then, taking the expectation, $E\left[e\right]\xi - E\left[i\right] = 0$ and

$$E\left[e\right] = E\left[\iota\right]/\xi$$

But the realized FX is

$$e = \frac{\iota + qE[e]}{\xi + q} \tag{18}$$

Hence, as $q \to \infty$, $var(e) \to 0$. We conclude

Proposition 6 *The FX swap can stabilize the exchange rate. In the limit of an infinite notational swap, the exchange rate is constant.*

A two-period economy Here, the government enters into the swap with the financiers, at the end of time 0. Hence, equilibrium condition are:

$$\sum_{t=0}^{1} e_t \xi_t - \iota_t + q(e_1 - e_0) = 0$$

the last part being the proceed from the swap. The financiers' demand equation implies:

$$Ee_1 = e_0 + \Gamma(e_0 \xi_0 - \iota_0 - q e_0)$$

As Japan via the swap is long q dollars, financiers are short the corresponding amount.

The period one equilibrium exchange rate satisfies:

$$e_1 = -e_0 \frac{\xi_0 - q}{\xi_1 + q} + \frac{\iota_0 + \iota_1}{\xi_1 + q} \tag{19}$$

While by substituting the above expression in the financiers' demand equation, one has:

$$-e_0 \frac{\xi_0 - q}{\xi_1 + q} + \frac{\iota_0 + E\iota_1}{\xi_1 + q} = e_0(1 + \Gamma(\xi_0 - q)) - \Gamma \iota_0$$

i.e.

$$e_0 = \frac{\iota_0 + E\iota_1 + \Gamma \iota_0 (\xi_1 + q)}{\xi_0 + \xi_1 + \Gamma(\xi_0 - q)(\xi_1 + q)}$$

We see that, at $q = 0$, $de_0/dq > 0$ iff $\Gamma > 0$: as Japan is buying a long dollar position, the dollar appreciates.[13]

Given (19), we have:

$$\{e_1\} = \frac{\{\iota_1\}}{\xi_1 + q}$$

Hence indeed, a higher q increases the volatility of the exchange rate.

[13]There's a limit q^M to the feasible q: at $q = q^M$, the dollar appreciates infinitely.

4.1.2 Taxing capital flows

Another instrument is to tax financial returns. Specifically we assume that each financier return is taxed via a proportional tax, while overall proceeds of the tax are rebated in lump sum to financiers. Hence, the financiers' demand, becomes:

$$Q = \frac{E\left[e_0 - e_1\right](1-\tau)}{\Gamma}.$$

where τ is the tax rate. Notice that this is equivalent to changing Γ to an effective Γ^{eff}

$$\Gamma^{eff} = \frac{\Gamma}{1-\tau}.$$

so the demand is $Q = \frac{E[e_0 - e_1]}{\Gamma^{eff}}$.

Lemma 3 *A tax τ on finance is equivalent to changing the Γ to $\Gamma^{eff} = \frac{\Gamma}{1-\tau}$.*

A high tax makes the effective Γ higher, i.e. finance less effective.

4.2 Welfare Preliminaries: First-best Exchange Rate

We first establish the first best (FB) equilibrium from a welfare perspective.

First best exchange rate In the decentralized allocation (for a given date):

$$\begin{aligned} c_{NT} &= \frac{\lambda\chi}{p_{NT}} = \lambda\chi, \qquad c^*_{NT} = \lambda^*\chi^* \\ c_H &= \frac{a\lambda}{p_H}, \qquad c^*_H = \frac{\xi\lambda^*}{p_H/e} \end{aligned} \tag{20}$$

so that

$$e_t = \frac{\frac{c^*_H}{\xi\lambda^*}}{\frac{c_H}{a\lambda}}$$

Suppose that the Negishi weight is ν. The planner maximizes $U + \nu U^*$ subject to the resource constraint; hence, in particular $\max_{c_H + c^*_H \leq Y_H} a \ln c_H + \nu\xi \ln c^*_H$, which gives

$$\frac{a}{c_H} = \frac{\nu\xi}{c^*_H}$$

Hence, in the FB equilibrium the exchange rate satisfies:

$$e^{FB} = \nu \frac{\lambda}{\lambda^*} = \nu \frac{c_{NT}/\chi}{c^*_{NT}/\chi^*}$$

We state the result more generally in the Lemma below.

Lemma 4 *The first best exchange rate, when the relative Negishi weight on the US is ν, satisfies:*

$$e^{FB}_t = \nu \frac{\lambda_t}{\lambda^*_t} = \nu \frac{c_{NT,t}/\chi_t}{c^*_{NT,t}/\chi^*_t}. \tag{21}$$

Since producing output is not costly in our simple production structure, the planner always chooses $Y_H = Y_F = L$.

4.3 Optimal currency intervention in the FX market

We consider the case where the first best exchange rate e^{FB}_1 is stochastic: this increases somewhat the realism of the analysis, and forces the financiers to bear some risk, but is not otherwise essential. Specifically, in light of (21), we assume $c_{NT,1}$ is variable: its value is $1+\pi_1$, where π_1 can be also interpreted as a stochastic time-1 monetary expansion. For notational simplicity, we set many parameters at 1: ν, $c_{NT,0}, \iota_t, \chi_t, \chi^*_t, c^*_{NT,t}$ for $t = 0, 1$ are all 1. Hence, by Lemma 4, the first best exchange rate is:

$$e^{FB}_1 = 1 + \pi_1.$$

Positive analysis The Japanese next exports in Yen at time 1 are: $e_1 - 1 - \pi_1$.

Let us study a small intervention by Japan: at time 0, Japan buys q dollar bonds (to depreciate the yen), and sells them at time 1. The market clearing conditions for Yen bonds are:

$$e_0 - 1 - qe_0 + Q_{H,0} = 0,$$
$$e_1 - 1 + qe_1 - Q_{H,0} - \pi_1 = 0,$$

these imply:

$$e_0 + e_1 - 2 + q\,(e_1 - e_0) - \pi_1 = 0.$$

Let us solve the impact of a small swap q on the exchange rate: it will create a change $\widehat{e}_t = e_t - e_*$. First, when $q = 0$, we have $\widehat{e}_0 = 0, \widehat{e}_1 = \pi_1$ (recall that $E_0[\pi_1] = 0$). So, for general q, $\widehat{e}_0 = O(q)$ and $\widehat{e}_1 = \pi_1 + O(q)$, so $q(e_1 - e_0) = q\pi_1 + O(q^2)$, the last equation above then gives:

$$\widehat{e}_0 + \widehat{e}_1 + q\pi_1 - \pi_1 + O\left(q^2\right) = 0 \tag{22}$$

hence

$$\widehat{e}_0 + E[\widehat{e}_1] = 0 + O\left(q^2\right)$$

That is, q keeps unchanged the average expected value of the exchange rate over the two periods.

Combining the financiers' demand and the flow equation for the Yen market in period-0 gives $e_0 - 1 - qe_0 + E[e_0 - e_1]/\Gamma = 0$, i.e.

$$\widehat{e}_0 - q + E[\widehat{e}_0 - \widehat{e}_1]/\Gamma = 0 + O\left(q^2\right)$$

and as $E[\widehat{e}_1] = -\widehat{e}_0$, we get

$$\widehat{e}_0 - q + \frac{2\widehat{e}_0}{\Gamma} = 0$$

i.e.

$$\widehat{e}_0 = \frac{q}{1 + 2/\Gamma}$$

finally $\widehat{e}_1$ is given by (22).

We conclude:

Proposition 7 *If the Japanese government enters into a (long) Dollar for Yen swap with notional q at time 0, the exchange rates is (for small q):*

$$e_0 = 1 + \frac{q}{1 + 2/\Gamma} + O\left(q^2\right) \tag{23}$$

$$e_1 = 1 - \frac{q}{1 + 2/\Gamma} + (1 - q)\pi_1 + O\left(q^2\right) \tag{24}$$

The intervention's impact on the *average* exchange rate is only second order: it creates a depreciation at time 0, and an appreciate at time 1.[14]

[14]This is general. Consider the case $\xi_t = \xi$ for all t. Then, a small q will induce a small change $\delta(e_1 - e_0) = O(q)$, so we'll have $\sum_t \delta e_t = 0 + O\left(q^2\right)$. The exchange rate will go down by de_0, and will compensate next period: $de_1 = -de_0$ $(+O\left(q^2\right))$.

Normative analysis Consider the symmetric case

$$a_t = \xi_t, b_t = \iota_t$$

and the values are all 1, so that in the frictionless case, countries are symmetric. Call s_t the real share of Japan's consumption. Given (20),

$$s_t = \frac{c_H}{c_H + c_H^*} = \frac{\lambda_t}{\lambda_t + \lambda_t^* e_t}$$

In the scenario analyzed here where with many unit values, $\lambda_t^* = c_{NT,t}/\chi_t = 1$, $\lambda_t = c_{NT,t}/\chi_t = 1 + \pi_1$, (with $\pi_0 = 0$), we have

$$s_t = \frac{1}{1 + e_t/(1 + \pi_t)}$$

The interpretation is that s_t is higher when the Japanese exchange rate $e_t/(1 + \pi_t)$ appreciates.

So, Japanese consumption of good g is $c_{gt} = s_t Y_g$, where Y_g is the world production of good g. Japan's welfare at t is (dropping the non-tradables endowment term):

$$U_t = a_t \ln C_H + \iota_t \ln C_F = a_t \ln s_t Y_H^w + \iota_t \ln s_t Y_F^w$$

$$U_t = (a_t + \iota_t) \ln s_t + a_t \ln Y_H + \iota_t \ln Y_F^*$$

Hence, Japanese welfare goes up when its share s_t is higher and when world production is higher (Y_H, Y_F^* higher).

We revisit the situation with unemployment seen earlier. We shall see that the intervention on a scale q, by inducing a time-0 Yen devaluation (i) improves by a first order effect the production at time 0 (ii) leads only to a second order loss in the total share of consumption $\ln s_0 + \ln s_1 = O(q^2)$. Hence, the intervention is desirable. In attention, it is desirable also for the US (the US benefits from the increase in world production, and experiences only a second order change from the intertemporal distortion).

Let us see that analytically. Call $\widehat{\ln s_t} = \ln s_t - \ln \frac{1}{2}$ the change induced by the policy

q. As $s_t = 1/\left(1 + e_t/\left(1 + \pi_t\right)\right)$, a purchase q induces a change (by Proposition 7)

$$\widehat{\ln s_0} = \frac{\widehat{e}_0}{1 + e_*} = \frac{q}{2\left(1 + 2/\Gamma\right)} + O\left(q^2\right)$$

$$\widehat{\ln s_1} = \frac{\widehat{e}_1/(1 + \pi_1)}{1 + e_*} = \frac{-\left(\frac{1}{1+2/\Gamma} + \pi_1\right) q}{2} + O\left(q^2\right)$$

Note that

$$E_0 \sum_{t=0}^{1} d \ln s_t = 0 + O\left(q^2\right),$$

so indeed the sum of the distortions is second order.

In terms of GDP, however, there is an improvement that we shall now calculate. We are at time 0 and assume PCP in the goods market. p_H° the frictionless market clearing price for the home good

$$p_H^\circ = \frac{\xi_0\left(\lambda_0 + \lambda_0^* e_*\right)}{L} = \frac{2}{L}$$

and say that the actual price is $\kappa > 0$ percent higher:

$$\overline{p}_H = p_H^\circ\left(1 + \kappa\right)$$

Then:

$$\begin{aligned} Y_H &= \min\left(\frac{\xi_0\left(\lambda_0 + \lambda_0^* e_0\right)}{\overline{p}_H}, L\right) = \min\left(\frac{1 + e_0}{p_H^\circ\left(1 + \kappa\right)}, L\right) = L \min\left(\frac{1+e_0}{\frac{2}{L}\left(1 + \kappa\right)}, L\right) \\ &= L \min\left(\frac{1+e}{2\left(1 + \kappa\right)}, 1\right) \end{aligned}$$

so Y_H (home production of tradables) increases in e for $e \in [0, 1 + 2\kappa]$. The exchange rate that would resolve unemployment is:

$$e_0 = 1 + 2\kappa.$$

Utility of the country, as a function of e is

$$\begin{aligned} U &= E_0 \left[U_0 + U_1 \right] = E_0 \sum_{t=0}^{1} \left(\xi_t + \iota_t \right) \ln s_t + \xi_t \ln Y_H + \iota_t \ln Y_F^* \\ &= \xi_0 \ln \min \left(\frac{1+e_0}{2+2\kappa}, 1 \right) + U^{FB} + O\left(q^2\right) \end{aligned}$$

where $U^{FB} = E_0 \sum_{t=0}^{1} \left(\xi_t + \iota_t \right) \ln \frac{1}{2} + \xi_t \ln L + \iota_t \ln L^*$ is the utility at the first best of production.

Hence, utility is strictly increasing in $e \in [0, 1+2\kappa]$, and the optimal intervention would yield $e = 1 + 2\kappa$. Given Proposition 7, $e_0 = 1 + \frac{1}{1+2/\Gamma} q + O\left(q^2\right)$, the optimal q satisfies $\frac{1}{1+2/\Gamma} q = 2\kappa$, i.e.

$$q^{Opt} = \left(1 + \frac{2}{\Gamma} \right) 2\kappa + O\left(\kappa^2\right)$$

Hence, for a price too high by κ percent, the intervention q should be $q = \left(1 + \frac{2}{\Gamma}\right) 2\kappa$. The intervention needs to be bigger when financial markets are more relaxed (low Γ). This occurs because arbitrage will tend to "undo" the intervention.

Proposition 8 *We normalize the exchange rate to 1. Suppose that the home sector is priced too high, by a fraction κ. Then, the following intervention improves welfare (in Japan and the US): the Japanese government buys a value $q \in \left[0, q^{Opt}\right]$ worth of US bonds, where the optimal scale of this intervention is*

$$q^{Opt} = \left(1 + \frac{2}{\Gamma} \right) 2\kappa + O\left(\kappa^2\right) \tag{25}$$

This result is in normalized units. More generally, some simple calculations show that

$$q^{Opt} = \left(\xi_* + \frac{2}{\Gamma} \right) \frac{1 + e_*}{e_*} \kappa + O\left(\kappa^2\right).$$

4.4 Taxing capital flows

Positive analysis Suppose that ι_0 is high, but $E\left[\iota_1\right]$ is low:

$$E\left[\iota_1\right] < \iota_0 \tag{26}$$

Then, at time zero the Yen is appreciated - so that the economy can potentially feature

unemployment. We study here how a tax on capital flows is then beneficial.

As seen in section 4.1.2, this is equivalent to changing Γ into $\Gamma/(1-\tau)$. This leads to a value of the exchange rate (see equations 5 and A.7-A.8):

Proposition 9 *If finance is taxed at a rate τ, the exchange rate is:*

$$\begin{aligned} e_0(\tau) &= \frac{\left(1+\frac{\Gamma}{1-\tau}\right)\iota_0 + E[\iota_1]}{2+\frac{\Gamma}{1-\tau}} \\ e_1(\tau) &= \frac{\iota_0 + \left(1+\frac{\Gamma}{1-\tau}\right)E[\iota_1]}{2+\frac{\Gamma}{1-\tau}} + \{\iota_1\} \end{aligned} \tag{27}$$

Note that

$$e_0 + e_1 = \iota_0 + \iota_1 \tag{28}$$

As Γ goes from 0 to ∞, e_0 goes from $(\iota_0 + E[\iota_1])/2$ to ι_0. Hence, given (26), a tax on capital flows devalues the exchange rate, and so can help unemployment.

Let us parametrize the scale δe_0 of the policy, which will change e_0 to $e_0' = e_0 + \delta e_0$. For each $\delta e_0 \in [0, \lim_{\tau\to\infty} e_0(\tau) - e_0(0)] = [0, \frac{\iota_0 - E[\iota_1]}{2}]$, there is a corresponding tax τ that gives:

$$\delta e_0(\tau) = e_0(\tau) - e_0(0)$$

with $\delta e_0(\tau)$ an increasing function. But it's easier to express everything in term of δe_0 directly.

Normative analysis To keep the analytics to a minimum, we make the countries very symmetric:

$$\xi_t = 1,\ \iota_0 + E\iota_1 = 2,\ a_t = \xi_t, b_t = \iota_t, \lambda_t = \lambda_t^* = 1$$

so that in the frictionless case, they consume the same proportion of all traded goods, $s_t = \frac{1}{2}$.[15] Call s_t the real share of Japan's consumption. Japanese consumption of good g is $c_{gt} = s_t Y_g$, where Y_g is the world production of good g. Japan's welfare at t is (dropping the non-tradables endowment term):

[15]This is true in the limit of the small financial sector, which rebates infinitesimal financial profits. Otherwise, the country getting a higher fraction of the financial profits will be a bit richer, and consume more. That can be compensated by a small transfer that ensures $\lambda_t = \lambda_t^* = 1$.

$$U_t = a_t \ln C_H + \iota_t \ln C_F = a_t \ln s_t Y_H^w + \iota_t \ln s_t Y_F^w$$

$$U_t = (a_t + \iota_t) \ln s_t + \xi_t \ln Y_H + \iota_t \ln Y_F^*$$

Hence, Japanese welfare goes up when its share s_t is higher and when world production is higher (Y_H, Y_F^* higher).

We revisit the situation with unemployment seen earlier. We shall see that time-0 Yen devaluation $\delta e_0 > 0$ (i) improves by a first order effect the production at time 0 (as it increases the demand for Japanese goods, hence lowers Japanese unemployment) (ii) leads only to a second order loss in the total share of consumption $\ln s_0 + \ln s_1 = O\left(\delta e_0^2\right)$. Hence, the intervention is desirable. In attention, it is good also for the US (the US benefits from the increase in world production, and experiences only a second order change from the intertemporal distortion).

Let us see that analytically. Call $\widehat{\ln s_t} = \ln s_t\left(\delta e_0\right) - \ln s_t\left(0\right)$ the change induced by the policy δe_0. Given (20), $s_t = \frac{c_H}{c_H + c_H^*} = \frac{\lambda_t}{\lambda_t + \lambda_t^* e_t}$, i.e.

$$s_t = \frac{1}{1 + e_t}$$

Hence, the policy change δe_t induces a change in share:

$$\widehat{\ln s_t} = \frac{-\delta e_t}{\left(1 + e_t\left(0\right)\right)^2} + O\left(\delta e_t^2\right) = -\frac{\delta e_t}{4} + O\left(\delta e_t^2\right)$$

As (28) implies $\delta e_0 + \delta e_1 = 0$, we have

$$E_0 \sum_{t=0}^{1} \widehat{\ln s_t} = 0 + O\left(\delta e_0^2\right)$$

so indeed the sum of the distortions is second order.

In term of GDP, however, there's an improvement that we shall now calculate. We are at time 0. Given the price $\overline{p}_H$, home production is:

$$Y_H\left(e_0\right) = \min\left(\frac{\lambda_0 a_0 + \lambda_0^* \xi_0 e_0}{\overline{p}_H}, L\right)$$

so that it is increasing in $e_0 \in [0, \overline{e}_0]$, where

$$\overline{e}_0 \equiv \frac{\overline{p}_H L - \lambda_0 a_0}{\lambda_0^*}$$

(recall that we normalized $\xi_0 = 1$).

Let us assume that:

$$\overline{e}_0 > e_0(0),$$

i.e., there is unemployment at time 0 without intervention. Utility of Japan is

$$\begin{aligned} U = E_0[U_0 + U_1] &= E_0 \sum_{t=0}^{1} (a_t + \iota_t) \ln s_t + a_t \ln Y_H + \iota_t \ln Y_F^* \\ &= \ln \frac{Y_H(e_0)}{L} + U^{FB} + O\left(\delta e_0^2\right) \end{aligned}$$

where $U^{FB} = E_0 \sum_{t=0}^{1} (a_t + \iota_t) \ln \frac{1}{2} + a_t \ln L + \iota_t \ln L^*$ is the utility at the first best of production.

The next Proposition holds for small δe_0^2, i.e. small $|\overline{e}_0 - e_0(0)|$.

Proposition 10 *Suppose that the there is unemployment in the Japan, and that $\iota_0 > E[\iota_1]$, i.e. Japan is running a trade deficit. We assume that $\overline{e}_0 \equiv \frac{\overline{p}_H L - \lambda_0 a_0}{\lambda_0^*} < \iota_0$. Define τ^{opt} by $e_0(\tau^{opt}) = \overline{e}_0$, where $e_0(\tau)$ is given in (27). The following intervention improves welfare (in Japan and the US): impose a financial tax $\tau \in [0, \tau^{opt}]$ (in particular, on financial flows to Japan). As the tax is bigger, the Yen appreciates and welfare improves by a first order amount.*

4.5 Discussion: Policy instruments

In our basic context with a floor on the nominal wage, monetary policy would work well too (we do not analyze this formally because we do not wish to replicate the massive body of existing work on monetary policy): inflation would make the minimal wage not binding.

Hence, there are at least three policy tools, (i) monetary policy (ii) FX interventions, (iii) taxation of finance. In general, all might be used, and FX interventions are particularly potent when Γ is high – when financial market are disrupted. While (i) is well-understood, our analysis allows to analyze (ii) and (iii) in a way that was diffi-

cult in previous treatments; that's because it takes seriously financial imperfections in international markets.

Which tools are better? Let us give a few pointers to the relevant forces, without a systematic treatment.

First, in some cases monetary policy ceases to work: namely, when there is a real rigidity (e.g., when the minimal wage in the export sector is real, i.e. in terms of the nontraded good). Then only FX interventions (ii) and (iii), creating "real devaluations" work. In addition, monetary policy has other goals (e.g. price stability), that prevent starting surprise inflation to cure unemployment.

Which are more desirable, between (ii) and (iii)? From a purely theoretical point of view, one might prefer (iii), as it uses a "comparative advantage" of the government – who can tax, while in (ii) the government is allowed to do a "tactical" currency intervention; but consumers are assumed not to be able to do that, though financiers can.

From a practical point of view, (ii) seems easier: any government can go ahead and buy bonds, while a "taxation of finance" poses difficult problems of implementability and tax evasion (e.g. via off-shore accounts of disguised trades in the form of over the counter derivatives). Still, those are not insurmountable, at least in the short run.

5 Revisiting canonical issues with the Gamma model

5.1 Nominal and Real Exchange Rates: the Mussa Evidence

Mussa (1986) highlighted that before 1973, when nominal exchange rates were broadly constant under the fixed-exchange-rate Bretton Woods system, real exchange rates were relatively stable, while after 1973, when nominal exchange rates were allowed to float, real exchange rates become volatile and in particular as volatile as nominal ones.

The Gamma model rationalizes such evidence as a combination of sticky prices, the importance of non-tradable goods, and (mostly) nominal exchange rates that are driven by financial imbalances and financial shocks.

To understand Mussa's evidence recall that the real exchange rate can be defined as the ratio of two broad price levels, one in each country, expressed in the same numeraire. It is most common to use consumer price indices (CPI) adjusted by the nominal exchange rate, in which case one has:

$$\mathcal{E} = \frac{P^* e}{P}.$$

Mussa's evidence is then simply the statement that, independently from variations in e, CPI indices and their ratio are relatively stable ($Var_t[P^*/P]$ is low), while under the modern float nominal exchange rates are volatile ($Var[e]$ is high).

Consider the nominal version of the basic Gamma model in Section 3. We show in the Appendix that standard calculations imply that the real CPI-based exchange rate is:

$$\mathcal{E} = \tilde{\theta} \frac{(p_H^*)^{\xi'} (p_F^*)^{b'} (p_{NT}^*)^{\chi^{*'}}}{(p_H)^{a'} (p_F)^{\iota'} (p_{NT})^{\chi'}} \; e_t,$$

where $\tilde{\theta}$ is a function of exogenous demand shocks that does not play a major role in the economics of our results. Intuitively, one has $Var[\mathcal{E}] \approx Var[e]$ if the term containing relative prices above is relatively stable. Various theoretical combinations could in principle achieve such result, but focus here only on the case that is supported by the empirical evidence. Consider a world where non-tradables are a large component of the CPI basket ($\chi' \approx \chi^{*'}$ close to 0.7), the demand shares of tradables are relatively similar across countries ($\xi' \approx a'$, $b' \approx \iota'$), and prices of non-tradables are relatively stable in domestic currency ($C_{NT} \approx \chi' m$, $C_{NT}^* \approx \chi^{*'} m^*$), then the first term in the above equation is relatively stable.

Under floating exchange rates, we showed in Section 3 that the nominal exchange rate is driven by financial forces such as capital flows, and financiers' risk bearing capacity and balance sheet exposures. Such forces make the nominal exchange rate potentially volatile and disconnected by traditional macroeconomic fundamentals. Thus the nominal version of the basic Gamma model can reconcile Mussa's evidence on nominal and real exchange rate behavior during the modern float period.

At the simplest level, Mussa's evidence for the behavior of exchange rates under the fixed-exchange-rate Bretton Woods system is reconciled in our model by making the nominal exchange rate constant $e = \bar{e}$ while maintaining the above assumptions that induce a stable P^*/P. It is, however, also interesting to understand how a fixed exchange rate regime works in the basic Gamma model.

In the most basic case, under a fixed exchange rate, liquidity in Yen and Dollar

are perfect substitutes for world financiers. Hence $\Gamma = 0$.[16] The financiers' demand equation immediately imposes $R = R^*$. Intuitively, interest rates have to be equalized across countries to induce the financiers to treat liquidity in Yen and Dollar as perfect substitutes and not put pressure on the nominal exchange rate. Notice that in this case monetary authorities lose control over domestic monetary policy. The set of feasible monetary policy is given by two equations (for simplicity derived under $R \approx 1$):

$$\iota_1 m_1 + \iota_0 m_o = \bar{e}(\xi_1 m_1^* + \xi_0 m_0^*),$$
$$E\left[\frac{m_0^*}{m_1^*}\frac{\theta_1}{\theta_0}\right] = E\left[\frac{m_0}{m_1}\frac{\theta_1^*}{\theta_0^*}\right].$$

The first condition is Japan's dynamic budget constraint, while the second restricts (via the agents' Euler equations in each country) monetary policies to engineer the same interest rates.

The logic of the basic *Trilemma* of open macroeconomics is at work here (Obstfeld, Shambaugh, and Taylor (2005)). The world can only contemporaneously achieve two out of the following three objectives: (1) monetary policy independence across countries, (2) open capital accounts, and (3) fixed exchange rates. In the above examples, authorities chose (2) and (3) and sacrificed (1). Alternative it is simple to understand how a degree of monetary policy independence could be regained by imposing capital controls, thus somewhat sacrificing (2). Recall from our policy analysis in Section 4.4 that a form of capital controls in our model is to tax financiers activities. Consider a proportional tax on financiers' holdings of US bonds by τ, so that they perceive the interest rate to be $R^*(1-\tau)$. Then the financier's demand equation implies that $R = R^*(1-\tau)$ thus restoring some relative monetary independence based on the choice of τ.

5.2 Carry trade and Financial shocks

Our setup gives rise to a profitable carry trade. Let us give the intuition in terms of the most basic model first and then extend it to a set-up with shocks to the financiers' risk bearing capacity (Γ shocks).

[16]While the property is intuitive, it is taken as exogenous here. When full micro foundations are added to the financier's demand, the property arises because under a fixed exchange rate returns to currency speculation are deterministic and financiers are willing to take infinite positions whenever such returns are different from zero.

First, imagine a world in which countries are in financial autarky because the financiers have zero risk bearing capacity ($\Gamma = \infty$), suppose that Japan has a 1% interest rate while the US has a 5% interest rate, and that all periods ($t = 0, ..., T$) are ex-ante identical with $\xi_t = 1$ and ι_t a martingale. Thus, we have $e_0 = E[e_1] = ... = E[e_T]$. A small financier with some available risk bearing capacity, say a small hedge fund, could take advantage of the difference, and pocket the 4% interest rate differential. In this case, there is a very profitable carry trade. As the financial sector risk bearing capacity expands (Γ becomes smaller, but still positive), this carry trade becomes less profitable, but does not disappear entirely unless $\Gamma = 0$, in which case, the UIP condition holds. Intuitively, the carry trade in the basic Gamma model reflects the risk compensation necessary to induce the financiers to intermediate global financial flows.

Let us consolidate this simple intuition in the basic Gamma model. In the most basic model, the different interest rates arise from different rates of time preferences, such that $R = \beta^{-1}$ and $R^* = \beta^{*-1}$. Without loss of generality, assume $R < R^*$ so that Yen is the "funding" currency, and Dollar the investment currency. The expected return on the "carry trade" is:

$$\overline{R}^c \equiv E_0 \left[\frac{R^*}{R} \frac{e_1}{e_0} - 1 \right]$$

The calculations in Lemma 6 reveal that:

$$\begin{aligned} \overline{R}^c &= \frac{E\left[\frac{R^*}{R} e_1 - e_0\right]}{e_0} = \frac{-\Gamma Q_{H,0}}{e_0} \\ &= -\Gamma \left(\frac{\iota_0 - e_0}{e_0} \right) = \Gamma \left(1 - \frac{\iota_0}{e_0} \right). \end{aligned} \tag{29}$$

Recall that:

$$e_0 = \frac{(R^* + \Gamma)\, \iota_0 + \frac{R^*}{R} E[\iota_1]}{R^* + \Gamma + 1},$$

so that we conclude:

$$\overline{R}^c = \Gamma R \left(1 - \iota_0 \frac{R^* + \Gamma + 1}{(R^* + \Gamma)\, \iota_0 + \frac{R^*}{R} E[\iota_1]} \right).$$

We collect this result in the proposition below.

Proposition 11 *The expected return to the carry trade in the basic Gamma model is:*

$$\overline{R}^c = \Gamma \frac{\frac{R^*}{R} E[\iota_1] - \iota_0}{(R^* + \Gamma)\iota_0 + \frac{R^*}{R} E[\iota_1]} \tag{30}$$

Hence the carry trade return is bigger (i) when R^*/R is higher (ii) when the funding country is a net foreign creditor.

To gain further intuition on the result above, consider first the case where $\iota_0 = E[\iota_1]$. The first order approximation to R^c in the case of a small interest rate differential $R^* - R$ is:

$$R^c = \frac{\Gamma}{2 + \Gamma}(R^* - R).$$

Notice that we have both $\frac{\partial R^c}{\partial \Gamma} > 0$ and $\frac{\partial R^c}{\partial (R^* - R)} > 0$, so that the profitability of the carry trade increases the more limited the risk bearing capacity of the financiers and the largest the interest rate differential. The first effect occurs because, given an interest rate differential, expected returns to the carry trade have to increase whenever the risk bearing capacity of the financiers goes down to induce them to intermediate financial flows. The second effect occurs because, given a level of risk bearing capacity for the financiers, an increase in the interest rate differential will not be offset one to one by the expected exchange rate change due to the risk premium.

The effects of broadly defined "global risk aversion" on the profitability of the carry trade have been central to the empirical analysis of for example Dumas and Solnik (1995), Lustig et al. (2011). Here we have shown that the carry trade is more profitable the lowest the risk bearing capacity of the financiers, in the next subsection we will formally account for shocks to such capacity in the form of a stochastic Γ.

In addition to a pure carry force due to the interest rate differential, our model gives a theoretical foundation to the fact that debtor countries' currencies are riskier. We expand here on the result formalized in Proposition 2 that showed how net external debtor countries' currencies depreciate whenever risk bearing capacity decreases ($\uparrow \Gamma$). Consider the general case, with potentially $\iota_0 \neq E[\iota_1]$, and recall that Japanese net exports are:

$$N_{0^+} = e_0 - \iota_0 = \frac{\frac{R^*}{R} E[\iota_1] - \iota_0}{R^* + \Gamma + 1}$$

Even if both countries have the same interest rate, a currency trade long the surplus country's currency and short the deficit country's currency will have a positive expected return. In fact, the formula above and the expression for currency returns in equation (30) show that $N_0^+ > 0 \iff \overline{R}^c < 0$. Intuitively, a debtor country has borrowed from world financial markets and therefore its currency has to depreciate on impact and be expected to appreciate, thus generating a positive currency excess return, to compensate the financiers for holding the currency risk. In our basic Gamma model, net exports in period zero are almost always the same as net foreign assets. We stress, however, that our results hold more generally in set-ups with either multiple periods or non-zero starting net foreign asset positions that decouple net exports from net foreign assets. The basic driving force is that countries that borrow from world financiers tend to have a risky currency that is both weaker on average and more exposed to shocks to the financiers' risk bearing capacity. The key mechanism is to ask oneself, are the world financiers long or short such a currency? How averse to bearing risk are the financiers? Those are the key determinants of currency excess returns.

Della Corte, Riddiough, and Sarno (2013) offer empirical evidence consistent with our theoretical predictions. They show that net debtor countries' currencies have higher returns that net creditors' currencies and that net debtor countries' currencies tend to be on the receiving end of carry trade related speculative flows.

5.2.1 The risks and returns of the carry trade

We now expand on the risks of the carry trade by studying a three period ($t = 0, 1, 2$) model with stochastic shocks to the financiers' risk bearing capacity in the middle period. To keep the analysis streamlined, we take period 2 to be the "long run". Intuitively period-2 will be a long run steady state where countries have zero net foreign assets and run a zero trade balance. This allows to quickly focus on a short-medium run exchange rate dominated by financial forces and a long run exchange rate completely anchored by fundamentals. We jump into the analysis, and provide many of the background details of this model in the appendix. The flow demand equations in the Yen / Dollar market

are:

$$e_t - \iota_t + Q_{H,t} = 0 \text{ for } t = 0, 1,$$
$$e_2 - \iota_2 = 0,$$

with the financiers' demand for Yen:

$$Q_{H,t} = \frac{e_0 - E\left[e_1\right] \frac{R^*}{R}}{\Gamma_t}.$$

We take time-1 financial conditions, Γ_1 to be stochastic. We define the "certainty equivalent" $\overline{\Gamma}_1$ by $\frac{\overline{\Gamma}_1}{1+\overline{\Gamma}_1} \equiv E_0\left[\frac{\Gamma_1}{1+\Gamma_1}\right]$.

Let us define $\mathcal{R}^* \equiv R^*/R$ the ratio of gross interest rates. A carry trade is originated whenever $\mathcal{R}^* > 1$. After a bit of algebra (solving backwards: first for e_1, then for e_0), the equilibrium exchange rate dynamics are:

$$e_0 = \frac{\Gamma_0\left(1+\overline{\Gamma}_1\right)\iota_0 + E_0\left[\overline{\Gamma}_1 \mathcal{R}^* \iota_1 + \left(\mathcal{R}^*\right)^2 \iota_2\right]}{\left(\Gamma_0+1\right)\left(\overline{\Gamma}_1+1\right)}, \tag{31}$$

$$e_1 = \frac{\Gamma_1 \iota_1 + \mathcal{R}^* E_1\left[\iota_2\right]}{\Gamma_1 + 1}, \tag{32}$$

$$e_2 = \iota_2. \tag{33}$$

Recall that the carry trade return between period 0 and 1 is:

$$R_1^c \equiv \frac{\mathcal{R}^* e_1 - e_0}{e_0}. \tag{34}$$

Interestingly here the carry trade has also "exposure to financial conditions". Notice that $\frac{\partial e_1}{\partial \Gamma_1} < 0$ in equation (32), so that the Yen (the funding currency) appreciates whenever there is a negative shock to the financiers' risk bearing capacity ($\uparrow \Gamma_1$, $\downarrow e_1$). Since the carry trade here is short Yen and long Dollar, we correspondingly have: $\frac{\partial R_1^c}{\partial \Gamma_1} < 0$, the carry trade does badly whenever there is a negative shock to the financiers' risk bearing capacity ($\uparrow \Gamma_1$). This is consistent with the intuition and the empirical findings in Brunnermeier, Nagel, and Pedersen (2008); we obtain this effect here in the context of an equilibrium model. The model potentially allows for different shapes in the reactions of the level and dynamics of the exchange rate to interest rate changes (see Engel (2012)).We

formalize and prove the results obtained so far in the proposition below.

Equation (32) shows that the carry trade is also exposed to fundamentals shocks, i.e. it does badly if ι_1 is lower than expected: notice that $\frac{\partial e_1}{\partial \iota_1} > 0$, and consequently $\frac{\partial E_0\left[R_1^c\right]}{\partial \iota_1} > 0$.

Proposition 12 *Assume that $R^* > R$ and $\iota_0 = E_0[\iota_1] = E_1[\iota_2]$. Consider the returns to the carry trade, R_1^c, and the corresponding expected return $\overline{R}_1^c \equiv E_0[R_1^c]$. We have:*

1. *An adverse shock to financiers affects the returns to carry trade negatively : $\frac{\partial R_1^c}{\partial \Gamma_1} < 0$.*
2. *The carry trade has positive expected returns: $\overline{R}_1^c > 0$.*
3. *The expected return to the carry trade is higher the worse the financial conditions are at time 0 $\left(\frac{\partial \overline{R}_1^c}{\partial \Gamma_0} > 0\right)$, or the better the financial conditions are expected to be at time 1 $\left(\frac{\partial \overline{R}_1^c}{\partial \overline{\Gamma}_1} < 0\right)$, and the higher the interest rate differential $\left(\frac{\partial \overline{R}_1^c}{\partial R^*} > 0\right)$.*

Proof. We have already derived Claim 1. For Claim 2, we can calculate, from equations 31, 32 and 34:

$$\begin{aligned} E_0[R_1^c] &= \mathcal{R}^* \frac{\Gamma_0\left(1+\overline{\Gamma}_1+\mathcal{R}^*\right)}{\Gamma_0\left(1+\overline{\Gamma}_1\right)+\overline{\Gamma}_1\mathcal{R}^*+\left(\mathcal{R}^*\right)^2} \\ &= \mathcal{R}^*\Gamma_0 \frac{\overline{\Gamma}_1+1+\mathcal{R}^*}{\overline{\Gamma}_1(\Gamma_0+\mathcal{R}^*)+\Gamma_0+\left(\mathcal{R}^*\right)^2} > 0. \end{aligned}$$

Hence, the expected carry trade return is positive.

For Claim 3, recall that a function $\frac{ax+b}{cx+d}$ is increasing in x iff $\Delta^x \equiv ad - bc > 0$. For Γ_0,

$$\Delta^{\Gamma_0} = \left(1+\overline{\Gamma}_1+\Gamma_*\right)\left(\overline{\Gamma}_1\mathcal{R}^*+\left(\mathcal{R}^*\right)^2\right) > 0$$

which proves $\frac{\partial \overline{R}_1^c}{\partial \Gamma_0} > 0$.

For $\overline{\Gamma}_1$, the discriminant is

$$\frac{\Delta^{\overline{\Gamma}_1}}{\mathcal{R}^*\Gamma_0} = \Gamma_0 + \left(\mathcal{R}^*\right)^2 - \left(1+\mathcal{R}^*\right)\left(\Gamma_0+\mathcal{R}^*\right) = -\mathcal{R}^*\left(1+\Gamma_0\right) < 0$$

so that $\frac{\partial \overline{R}_1^c}{\partial \overline{\Gamma}_1} < 0$.

Finally, for $\mathcal{R}^*$, we simplify compute:

$$\frac{\partial \overline{R}_1^c}{\partial \mathcal{R}^*} = \frac{\Gamma_0\left(1+\Gamma_0\right)\left(1+\overline{\Gamma}_1\right)\left(2\mathcal{R}^*+\overline{\Gamma}_1\right)}{\left(\Gamma_0\left(1+\overline{\Gamma}_1\right)+\overline{\Gamma}_1\mathcal{R}^*+\left(\mathcal{R}^*\right)^2\right)^2} > 0.$$

□

The Fama regression The now classic UIP regression of Fama (1984) is in levels:[17]

$$\frac{e_1 - e_0}{e_0} = \alpha + \beta \left(R - R^* \right) . + \varepsilon_1 \tag{35}$$

Under UIP, we would find $\beta = 1$. However, most studies find $\beta < 1$, and even sometimes $\beta < 0$.

Proposition 13 *In the model considered in this section, one has* $\beta = \frac{1+\overline{\Gamma}_1-\Gamma_0}{(1+\Gamma_0)\left(1+\overline{\Gamma}_1\right)}$. *Therefore,* $\beta < 1$. *In addition, one has* $\beta < 0$ *iff* $\overline{\Gamma}_1 + 1 < \Gamma_0$, *i.e. if risk bearing capacity is low in period 0 compared to period 1.*

Proof. The regression corresponds to:

$$\beta = \frac{-\partial E\left[\frac{e_1}{e_0} - 1\right]}{\partial R^*}.$$

Calculations show (evaluating at $R^* = R$ for simplicity):

$$\beta = \frac{1 + \overline{\Gamma}_1 - \Gamma_0}{\left(1 + \Gamma_0\right)\left(1 + \overline{\Gamma}_1\right)} \tag{36}$$

Hence, $\beta \leq \frac{1+\overline{\Gamma}_1}{(1+\Gamma_0)\left(1+\overline{\Gamma}_1\right)} = \frac{1}{1+\Gamma_0} < 1$. □

Intuitively financial market imperfections always lead to $\beta < 1$ and very bad current market imperfections compared to future ones lead to $\beta < 0$. This occurs because any positive Γ leads to a positive risk premium on currencies that the financiers are long of and hence to a deviation from UIP ($\beta < 1$), and in addition if financial conditions are particularly worse today compared to tomorrow the risk premium is so big as to induce currencies that have high interest rates to appreciate on average ($\beta < 0$).

[17]The regression is most commonly performed in its logarithmic approximation version, but the levels prove more convenient for our theoretical treatment without loss of economic content. See Engel (2013) for a recent survey of the literature based on this classic regression.

6 Generalization of the model

6.1 Infinite horizon equilibrium

We now provide an extension of the model to an infinite horizon. We proceed in continuous time. Equations are:

$$E\left[de_t\right] = -\Gamma Q_{H,t} dt \text{ (financiers' demand equation)}$$

$$dQ_{H,t} = -\left(\xi_t e_t - \iota_t\right) dt - \phi_Q Q_{H,t} \text{ (financiers' flows)}$$

$$dN_t = rN_t + \xi_t e_t - \iota_t \text{ (Japan net foreign asset position)}$$

the last equation can be rewritten as:

$$N_t + \int_{s\geq 0} e^{-rs}\left(\xi_{t+s} e_{t+s} - \iota_{t+s}\right) ds = 0 \text{ (Japan dynamic external budget constraint)}$$

We assume $\iota_t = \iota_* + \widehat{\iota}_t$, and $d\widehat{\iota}_t = -\phi_\iota \widehat{\iota}_t$+noise. We assume $\xi_t = 1$ to simplify the algebra. The following Proposition derives the equilibrium exchange rate.

Proposition 14 *The equilibrium exchange rate is:*

$$e_t = \iota_* + \frac{r}{r+\phi_\iota}\widehat{\iota}_t - \left(r - \frac{\Gamma}{r+\phi_Q}\right) N_t + \frac{\Gamma}{r+\phi_Q} Q_{H,t} \tag{37}$$

The interpretation: $e_t = \iota_* + \frac{r}{r+\phi_\iota}\widehat{\iota}_t - rN_t$ is just the traditional macroeconomic solution. The new term is the Γ term. The first part, $\frac{\Gamma}{r+\phi_Q} Q_{H,t}$, means: if $Q_{H,t}$ is high, then the financiers are long the Yen, so they are more exposed to a fall in the Yen. To compensate them for such risk the Yen is weaker. The "$\frac{\Gamma}{r+\phi_Q} N_t$" term means: suppose that Japan has negative NFA, dynamic solvency implies that it will need to run positive trade balances in the future, so the financiers will need to borrow Yen and hold Dollar in the future, hence the Dollar will have a positive risk premium in the future: that depreciates the Dollar today.

7 Conclusion

We presented a model of exchange rate determination in imperfect capital markets where global financiers' intermediate capital flows. Exchange rates are determined by the balance sheet risks and risk bearing capacity of these financiers. The exchange rate in our model is relatively disconnected from traditional macroeconomic fundamentals, such as output, inflation and the trade balance and instead more connected to financial forces such as the demand for assets denominated in different currencies. We have shown how seemingly heterodox policies, such as foreign currency swaps and capital controls, can be welfare improving in this context.

Our model is tractable, with simple to derive closed form solutions, and can be generalized to address a number of both classic and new issues in international macroeconomic analysis. In the last sections of the paper we showed how our model makes sense of some fundamental regularities of international macroeconomics, such as Mussa's evidence on the behavior of nominal and real exchange rates and the carry trade; extensions of our model are likely to shed light on a number of other puzzles, making further work an exciting research endeavor.

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Online Appendix: International Liquidity and Exchange Rate Dynamics

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Not for Publication

A.1 Value of the exchange rate in more general setups

We record here the value of the FX in more general setups: when R, R^*, etc.

With a short and a long period, and demand shocks from financiers Let's now work with the demand for yen by the Financiers:

$$Q = \frac{e_0 - Ee_1 + f}{\Gamma} \tag{A.1}$$

where f is a demand shock for the Yen. We also have periods of unequal length: period "1" lasts for T periods. Assume $\xi_t = 1$

$$e_0 - \iota_0 + Q = 0 \tag{A.2}$$
$$T(e_1 - \iota_1) - Q = 0 \tag{A.3}$$

Let's solve this. Adding the two budget constraints:

$$e_0 + Te_1 = \iota_0 + T\iota_1 \tag{A.4}$$
$$e_1 = \iota_1 + \frac{\iota_0 - e_0}{T} \tag{A.5}$$

so $e_0 - \iota_o + q^{Fin} = 0$ gives:

$$\Gamma(e_0 - \iota_0) + e_0 - \left(\iota_1 + \frac{\iota_0 - e_0}{T}\right) + f = 0$$

$$e_0 = \frac{(1 + \Gamma T)\iota_0 + T(\iota_1 - f)}{1 + \Gamma T + T} \tag{A.6}$$

$$e_1 = \iota_1 + \frac{\iota_0 - e_0}{T} = \{\iota_1\} + E[e_1]$$

In the deterministic case

$$e_1 = \iota_1 + \frac{\iota_0 - e_0}{T}$$
$$e_1 = \frac{\iota_0 + (1 + \Gamma T)\iota_1 + f}{1 + \Gamma T + T}$$

We record this in the next Lemma.

Lemma 5 *In the case where "period 1" lasts for T periods, and financiers' time 0 demand is $q^{Fin} = \frac{e_0 - Ee_1 + f}{\Gamma}$, the FX values are:*

$$e_0 = \frac{(1+\Gamma T)\,\iota_0 + T\,(E[\iota_1] - f)}{1+\Gamma T+T} \tag{A.7}$$

$$e_1 = \frac{\iota_0 + (\Gamma T + T)\,E[\iota_1] + f}{1+\Gamma T+T} + \{\iota_1\} \tag{A.8}$$

normalizing $\xi_t = 1$. In the limit $T = \infty$,

$$e_0 = \frac{\Gamma\iota_0 + E[\iota_1] - f}{\Gamma+1} \tag{A.9}$$

$$e_1 = \iota_1 \tag{A.10}$$

We see that an "f" expectational shock can be very powerful: even for small Γ, it does affect e_0. Whereas, for small Γ, and ι_0 shock is quite diluted (multiplied by $\Gamma/(1+\Gamma)$).

Short run / long run analysis: quicker derivation The $T = \infty$ limit is more quickly derived as follows: In the long run, $e_1 = \iota_1$ is pinned downed by the fundamentals: $e_1 - \iota_1 - \frac{Q_0}{T} = 0$ becomes $e_1 - \iota_1 = 0$. So, the equations become:

$$\begin{aligned} e_0 - \iota_0 + Q_0 &= 0 \\ e_1 - \iota_1 &= 0 \end{aligned}$$

A.1.1 When countries have different interest rates

When the gross interest rates are R and R^*, the financiers' demand for Yen becomes:

$$q^F = \frac{e_0 - E[e_1]\frac{R^*}{R}}{\Gamma} \tag{A.11}$$

The equations are:

$$\begin{aligned} \xi_0 e_0 - \iota_0 + Q_0 &= 0 \\ \xi_1 e_1 - \iota_1 - RQ_0 &= 0 \end{aligned}$$

so

$$\xi_0 e_0 - \iota_0 + \frac{\xi_1 e_1 - \iota_1}{R} = 0$$

i.e. the present value of next exports is 0.

Solving this, in the case $\xi_t = 1$, gives:

$$e_1 = \iota_1 + R\,(\iota_0 - e_0)$$

$$\begin{aligned} 0 &= \Gamma\,(e_0 - \iota_0) + \Gamma q^F \\ &= \Gamma\,(e_0 - \iota_0) + e_0 - E[e_1]\frac{R^*}{R} \\ &= \Gamma\,(e_0 - \iota_0) + e_0 - (E\iota_1 + R\,(\iota_0 - e_0))\frac{R^*}{R} \end{aligned}$$

i.e.

$$e_0 = \frac{(R^* + \Gamma)\,\iota_0 + \frac{R^*}{R} E[\iota_1]}{R^* + \Gamma + 1} \tag{A.12}$$

Then, $e_1 = \iota_1 + R(\iota_0 - e_0)$ gives:

$$\begin{aligned} e_1 &= \{\iota_1\} + E[\iota_1] + R(\iota_0 - e_0) \\ &= \{i_1\} + (R\iota_0 + E[\iota_1]) - R\frac{(R^* + \Gamma)\,\iota_0 + \frac{R^*}{R} E[\iota_1]}{R^* + \Gamma + 1} \\ e_1 &= \{i_1\} + \frac{R\iota_0 + (\Gamma + 1) E[\iota_1]}{R^* + \Gamma + 1} \end{aligned}$$

Lemma 6 *When countries have different interest rates,*

$$e_0 = \frac{(R^* + \Gamma)\,\iota_0 + \frac{R^*}{R} E[\iota_1]}{R^* + \Gamma + 1} \tag{A.13}$$

$$e_1 = \{i_1\} + \frac{R\iota_0 + (\Gamma + 1) E[\iota_1]}{R^* + \Gamma + 1} \tag{A.14}$$

For the record, this implies

$$E\left[\frac{e_1}{e_0} - 1\right] = \frac{(R - R^* - \Gamma)\,\iota_0 + \left(\Gamma + 1 - \frac{R^*}{R}\right) E[\iota_1]}{(R^* + \Gamma)\,\iota_0 + \frac{R^*}{R} E[\iota_1]} \tag{A.15}$$

Nominal Interest Rates In the nominal extension to the basic Gamma model in Section 3, the domestic saving decision in each country is characterized by the Euler equations:

$$\begin{aligned} \theta_t \frac{1}{C_t P_t} &= \beta R_{t+1} E_t\left[\theta_{t+1} \frac{1}{C_{t+1} P_{t+1}}\right] \\ \theta_t^* \frac{1}{C_t^* P_t^*} &= \beta R_{t+1}^* E_t\left[\theta_{t+1}^* \frac{1}{C_{t+1}^* P_{t+1}^*}\right] \end{aligned}$$

Notice that the above equations fully determine the nominal interest rate because monetary policy controls the level of nominal expenditure. For example the Yen nominal interest rate is pinned down by:

$$\begin{aligned} R_{t+1} &= \frac{1}{\beta} E_t\left[\frac{m_t}{m_{t+1}} \frac{\theta_{t+1}}{\theta_t}\right]^{-1} \\ R_{t+1}^* &= \frac{1}{\beta} E_t\left[\frac{m_t^*}{m_{t+1}^*} \frac{\theta_{t+1}^*}{\theta_t^*}\right]^{-1} \end{aligned}$$

The central bank, therefore, fully controls the nominal risk-free rate by altering (intertemporally) the money supply. An increase in the expected growth of the money supply relative to money demand increase the nominal interest rate.

Note that we have assumed that money is never used as a store of wealth. Implicitly we are assuming that money has value, hence the utility term, because within each period it facilitates some transactions. To fix ideas it has value in at the beginning of the period say t^-, but by the end of the period (t^+) after transactions have occurred it has no value.

This allows the decision to hold money to be static rather than inter-temporal, thus simplifying the equations without much loss of economics.

A.2 Some Proofs

Proof of Proposition 1 In the general case, the budget constraint gives:

$$e_1 = \xi_1^{-1}\left(\iota_0 + \iota_1 - \xi_0 e_0\right)$$

while the financiers' condition is

$$\begin{aligned} E\left[e_1\right] &= e_0 + \Gamma\left(\xi_0 e_0 - \iota_0\right) \\ &= \left(1 + \Gamma\xi_0\right) e_0 - \Gamma\iota_0 \end{aligned}$$

Hence

$$E\left[\xi_1^{-1}\left(\iota_0 + \iota_1\right)\right] - E\left[\xi_1^{-1}\xi_0\right] e_0 = \left(1 + \Gamma\xi_0\right) e_0 - \Gamma\iota_0$$

i.e.

$$\begin{aligned} e_0 &= \frac{E\left[\xi_1^{-1}\left(\iota_0 + \iota_1\right)\right] + \Gamma\iota_0}{\left(1 + \Gamma\xi_0\right) + E\left[\xi_1^{-1}\xi_0\right]} \\ &= \frac{E\left[\xi_1^{-1} + \Gamma\right]\iota_0 + E\left[\xi_1^{-1}\iota_1\right]}{E\left[\xi_1^{-1} + \Gamma\right]\xi_0 + 1} \\ &= \frac{\iota_0 + \frac{E\left[\xi_1^{-1}\iota_1\right]}{E\left[\xi_1^{-1}+\Gamma\right]}}{\xi_0 + \frac{1}{E\left[\xi_1^{-1}+\Gamma\right]}} \end{aligned} \tag{A.16}$$

Proof of Proposition 14 **Solution**: with $\xi_t = 1$ for all dates, must be:

$$e_t = \iota_* + A\widehat{\iota_t} + BN_t + DQ_t$$

It's easy to solve by method of undetermined coefficients. Then:

$$\begin{aligned} E_t\left[\frac{dN}{dt}\right] &= rN_t + e_t - i_t = \left(r + B\right) N_t + DQ_t + \left(A - 1\right)\widehat{\iota_t} \\ E_t\left[\frac{dQ}{dt}\right] &= -\left(e_t - \iota_t\right) - \phi_Q = -BN_t + \left(-D - \phi_Q\right) Q_t + \left(-A + 1\right)\widehat{\iota_t} \end{aligned}$$

$$0 = E\left[\frac{de_t}{dt}\right] + \Gamma Q_t = BE_t\left[\frac{dN}{dt}\right] + DE_t\left[\frac{dQ_t}{dt}\right] + \Gamma Q_t + AE_t\left[\frac{d\widehat{\iota_t}}{dt}\right]$$

$$\begin{aligned} 0 &= B\left[\left(r + B\right) N_t + DQ_t + \left(A - 1\right)\widehat{\iota_t}\right] + D\left[-BN_t + \left(-D - \phi_Q\right) Q_t + \left(-A + 1\right)\widehat{\iota_t}\right] + \Gamma Q_t - A\phi_\iota\widehat{\iota_t} \\ &= B\left(r + B - D\right) N_t + \left[D\left(B - D - \phi_Q\right) + \Gamma\right] Q_t + \left[\left(B - D\right)\left(A - 1\right) - A\phi_\iota\right]\widehat{\iota_t} \end{aligned}$$

So we have $B\left(r + B - D\right) = 0$. The root $B = 0$ is non-economic: it's rather like a bubble solution, where the FX is disconnected from fundamentals. Indeed, the value of

the FX should depend on N_t. So, we have $r + B - D = 0$:

$$B - D = -r$$

and setting to 0 the Q_t coefficient gives:

$$D = \frac{\Gamma}{r + \phi_Q}$$

and setting to 0 the $\widehat{\iota}_t$ coefficient gives: $-r\left(A - 1\right) - A\phi = 0$, i.e.

$$A = \frac{r}{r + \phi_\iota}$$

so finally

$$e_t = \iota_* + \left(-r + D\right) N_t + DQ_t + A\widehat{\iota}_t$$

$$e_t = \iota_* + \frac{r}{r + \phi_\iota}\widehat{\iota}_t - \left(r - \frac{\Gamma}{r + \phi_Q}\right) N_t + \frac{\Gamma}{r + \phi_Q} Q_t \tag{A.17}$$