Understanding the Great Recession*
(Preliminary and Incomplete.)

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Abstract

We argue that the vast bulk of movements in aggregate real economic activity during the Great Recession were due to financial frictions interacting with the zero lower bound. We reach this conclusion looking through the lens of a New Keynesian model in which firms face moderate degrees of price rigidities and no nominal rigidities in the wage setting process. Our model does a very good job of accounting for the joint behavior of labor and goods markets, as well as inflation, during the Great Recession. According to the model the observed fall in TFP played a critical role in accounting for the small size of the drop in inflation that occurred during the Great Recession.

*The views expressed in this paper are those of the authors and do not necessarily reflect those of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

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1. Introduction

This paper seeks to understand the key forces driving the U.S. economy in the wake of the 2008 crisis. Plausible inference *requires* that the model provide a good description of key macro variables, including those describing labor market outcomes. To this end, we extend the medium-sized DSGE model in Christiano, Eichenbaum and Trabandt (2013) (CET) to endogenize labor force participation rates. This extension is important in light of the sharp drop in the labor force participation rate that occurred after 2008. To establish the empirical credibility of our model, we estimate its parameters using pre-2008 data. We argue that the model does a very good job of accounting for the dynamics of fourteen key macroeconomic variables over this period.

We use a simple method to quantify the business cycle movements of the macro variables in the post-2008 period. We then characterize the role of four different shocks in generating the behavior of the economy during the post-2008 period. In particular, we introduce two type of wedge shocks into our structural model. These wedges capture in a reduced form way frictions which are widely viewed as having been important during the post-2008 period. The first wedge is motivated by the literature stressing a reduction in consumption as a trigger for a zero lower bound episode (see Eggertsson and Woodford (2003), Eggertsson and Krugman (2011) and Guerrieri and Lorenzoni (2012)). For convenience we capture this idea as in Smets and Wouters (2007) by introducing a perturbation to agents' intertemporal Euler equations associated with saving. Second, motivated by the sharp movements in credit spreads observed in the post-2008 period, we introduce an additional wedge into households' first order condition for optimal capital accumulation. Simple financial friction models based on asymmetric information with costly monitoring imply that credit market frictions can be captured in a reduced form way as a tax on the gross return on capital (see Christiano and Davis (2006)). Also, motivated by models like Bigio (2012) and Kurlat (2012), we will allow this wedge to impact on the cost of working capital. We also incorporate into our analysis the observed decline in total factor productivity (TFP) as well as the initial rise and then decline in government consumption.

We argue that, contrary to a widespread view, NK models can account for the key features of the post-2008 US data with moderate degrees of price stickiness. The keys to this result are (i) a prolonged slowdown in TFP growth during the Great Recession, and (ii) the presence of a working capital channel on firms' purchases of intermediate factors of production that is subject to a wedge that rose in post-2008 period due to financial frictions. At the same time we argue that the vast bulk of the decline in real economic activity is due to the financial frictions as captured by the wedge shocks.

Significantly, our model accounts for the large decline in the labor force participation rate

and employment, as well as the persistent increase in unemployment. Our model is able to do so, even though we do not allow for any nominal rigidities in the wage setting process. All of the inertia in wages derives from the assumption by the alternating offer bargaining process proposed in Hall and Milgrom (2008) and extended in Christiano, Eichenbaum and Trabandt (2013).

Finally, our results indicate that the rise in government consumption associated with the American Recovery and Reinvestment Act of 2009 had a substantial expansionary effect with a peak multiplier in excess of 1.5. However, based on our model we cannot attribute the long duration of the Great Recession to the decline in government consumption that began around the start of 2011.

2. The Model

In this section, we describe a medium-sized DSGE model whose structure is, with one important exception, the same as the one CET. The exception is that we modify the framework to endogenize labor force participation rates.

2.1. Households and Labor Force Dynamics

The economy is populated by a large number of identical households. As in Andolfatto (1995) and Merz (1996) we assume that each household has a unit measure of workers. Members of the household can be engaged in three types of activities: (i) $(1 - L_t)$ members specialize on home production in which case we say they are not in the labor force, (ii) l_t members of the household are in the labor force and are employed in the production of a market good, and (iii) $(L_t - l_t)$ members of the household are unemployed, i.e. they are in the labor force but aren't employed.

We normalize the size of the population to 1. At the end of each time period, a fraction $1-\rho$ of randomly selected employed workers is separated from the firm they had been matched with. So a total of $(1 - \rho) l_{t-1}$ workers separate from firms and ρl_{t-1} workers remain attached to their firm. Let u_{t-1} denote the unemployment rate at time $t - 1$,so that the number of unemployed workers at time $t-1$ is $u_{t-1}L_{t-1}$. The sum of separated and unemployed workers is given by:

$$\begin{aligned}(1-\rho)l_{t-1} + u_{t-1}L_{t-1} &= (1-\rho)\, l_{t-1} + \frac{L_{t-1} - l_{t-1}}{L_{t-1}} L_{t-1} \\ &= L_{t-1} - \rho l_{t-1}. \end{aligned} \tag{2.1}$$

We assume that a separated worker and an unemployed worker have an equal probability, $1 - s$, of leaving or entering the non-participation state. It follows that $s\,(L_{t-1} - \rho l_{t-1})$

of separated and unemployed workers remain in the labor force and search for work. We refer to s as the 'staying rate'. The household chooses the total size of the labor force, L_t. This decision is equivalent to choosing r_t, the number of workers that it transfers from non-participation into the labor force. Given L_t, r_t solves:

$$L_t = s\left(L_{t-1} - \rho l_{t-1}\right) + r_t + \rho l_{t-1}. \tag{2.2}$$

The total number of workers searching for a job at the start of t is

$$s\left(L_{t-1} - \rho l_{t-1}\right) + r_t = L_t - \rho l_{t-1}. \tag{2.3}$$

Here we have used (2.2) to substitute out for r_t on the left hand side of the previous equation.

It is of interest to calculate the probability,r_t, that a non-participating worker is selected to be in the labor force. We denote this probability by e_t. Suppose that the $(1-s)\left(L_{t-1} - \rho l_{t-1}\right)$ workers who separated exogenously into the non-participation state do not return home in time to be included in the pool of workers relevant to the household's choice of r_t. Then the universe of workers from which the household can select r_t is $1 - L_{t-1}$. It follows that e_t, that a non-participating worker is selected to join the labor force is:[1]

$$e_t = \frac{r_t}{1 - L_{t-1}} = \frac{L_t - s\left(L_{t-1} - \rho l_{t-1}\right) - \rho l_{t-1}}{1 - L_{t-1}}. \tag{2.4}$$

The law of motion for employment is:

$$l_t = \left(\rho + x_t\right) l_{t-1} = \rho l_{t-1} + x_t l_{t-1}. \tag{2.5}$$

The job finding rate is the ratio of the number of new hires divided by the number of people searching for work, given by (2.3):

$$f_t = \frac{x_t l_{t-1}}{L_t - \rho l_{t-1}}. \tag{2.6}$$

[1] We include the staying rate, s, in our analysis for a substantive as well as a technical reason. The substantive reason is that, in the data, workers move in both directions between unemployment, non-participation and employment. The gross flows are much bigger than the net flows. Setting $s < 1$ helps the model account for these patterns. The technical reason for allowing $s < 1$ can be seen by setting $s = 1$ in (2.4). In that case, if the household wishes to make $L_t - L_{t-1} < 0$, it must set $e_t < 0$. That would require withdrawing from the labor force some workers who were unemployed in $t-1$ and stayed in the labor force as well as some workers who were separated from their firm and stayed in the labor force. But, if some of these workers are withdrawn from the labor force then their actual staying rate would be lower that the fixed number, s. So, the actual staying rate would be a non-linear function of $L_t - L_{t-1}$ with the staying rate being below s for $L_t - L_{t-1} < 0$ and equal to s for $L_t - L_{t-1} \geq 0$. This kink point is a non-linearity that would be hard to avoid because it occurs precisely at the model's steady state. Even with $s < 1$ there is a kink point, but it is far from steady state and so it can be ignored when we solve the model by perturbation methods.

2.2. Household Maximization

Members of the household derive utility from a market consumption good and a good produced at home. The home good is produced using labor of individuals who aren't in the labor force and unemployed individuals:

$$C_t^H = \eta_t^H \left(1 - L_t\right)^{1-\alpha_{c^H}} \left(L_t - l_t\right)^{\alpha_{c^H}} - \mathcal{F}(L_t, L_{t-1}; \eta_t^L) \tag{2.7}$$

The term $\mathcal{F}(L_t, L_{t-1}; \eta_t^L)$ captures the idea that is costly to change the number of people who specialize in home production,

$$\mathcal{F}(L_t, L_{t-1}; \eta_t^L) = 0.5\eta_t^L \phi_L \left(L_t/L_{t-1} - 1\right)^2 L_t. \tag{2.8}$$

We assume $\alpha_{c^H} < 1 - \alpha_{c^H}$, so that in steady state the unemployed contribute less to home production than do people who are out of the labor force. Finally, η_t^H and η_t^L are processes that ensure balanced growth. We discuss these processes in detail below.

Because workers experience no disutility from working, they supply their labor inelastically. An employed worker brings home the wages that it earns. Unemployed workers receives government-provided unemployment compensation which they give to the household. Unemployment benefits are financed by lump-sum taxes paid by the household. Workers maximize their expected income. By the law of large numbers, this strategy maximizes the total income of the household. Workers maximize expected income in exchange for perfect consumption insurance from the household. All workers have the same concave preferences over consumption. So the optimal insurance arrangement involves allocating the same level of the market good and the home good to all members of the household.

The representative household maximizes the objective function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{U}(C_t, C_t^H) \tag{2.9}$$

where

$$\mathcal{U}(C_t, C_t^H) = \frac{\left[\left(C_t - b\bar{C}_{t-1}\right) + \omega \left(C_t^H - b\bar{C}_{t-1}^H\right)\right]^{1-\Theta} - 1}{1 - \Theta} \tag{2.10}$$

Here, C_t and C_t^H denote market consumption and the consumption of a good produced at home. The parameter b controls the degree of habit formation in household preferences. We assume $0 \leq b < 1$. A bar over a variable indicates its economy-wide average value.

The flow budget constraint of the household is as follows:

$$\begin{aligned} & P_t C_t + P_{I,t} I_t + B_{t+1} && (2.11) \\ \leq\ & (R_{K,t} u_t^K - a(u_t^K) P_{I,t}) K_t + \left(L_t - l_t\right) P_t \eta_t^D D_t + l_t W_t + R_{t-1} B_t - T_t\ . && (2.12) \end{aligned}$$

The variable T_t denotes lump-sum taxes net of transfers and firm profits, B_{t+1} denotes beginning-of-period t purchases of a nominal bond which pays rate of return, R_t at the start of period $t+1$, and $R_{K,t}$ denotes the nominal rental rate of capital services. The variable u_t^K denotes the utilization rate of capital. As in Christiano, Eichenbaum and Evans (2005) (CEE), we assume that the household sells capital services in a perfectly competitive market, so that $R_{K,t}u_t^K K_t$ represents the household's earnings from supplying capital services. The increasing convex function $a(u_t^K)$ denotes the cost, in units of investment goods, of setting the utilization rate to u_t^K. The variable $P_{I,t}$ denotes the nominal price of an investment good and I_t denotes household purchases of investment goods. In addition, the nominal wage rate earned by an employed worker is W_t and $\eta_t^D D_t$ denotes exogenous unemployment benefits received by unemployed workers from the government. The term η_t^D is a process that ensures balanced growth and will be discussed below.

When the household chooses L_t it takes the aggregate job finding rate, f_t, and the law of motion linking L_t and l_t,

$$l_t = \rho l_{t-1} + f_t \left(L_t - \rho l_{t-1} \right). \tag{2.13}$$

as given. Relation (2.13) is consistent with the actual law of motion of employment because of the definition of f_t (see (2.6))

The household owns the stock of capital which evolves according to,

$$K_{t+1} = (1-\delta_K) K_t + \left[1 - S\left(I_t / I_{t-1} \right) \right] I_t. \tag{2.14}$$

The function $S(\cdot)$ is an increasing and convex function capturing adjustment costs in investment. We assume that $S(\cdot)$ and its first derivative are both zero along a steady state growth path.

At time 0, the household chooses state-contingent sequences, $\left\{ C_t^H, L_t, l_t, C_t,\ B_{t+1}, I_t, u_t^K, K_{t+1} \right\}_{t=0}^{\infty}$, to maximize utility, (2.9), subject to, (2.7), 2.8), (2.11), (2.13) and (2.14). The household takes $\{K_0, B_0, l_{-1}\}$ the state contingent sequence of prices and wages, $\{R_t, W_t, P_t, R_{K,t}, P_{I,t}\}_{t=0}^{\infty}$ as given.

2.3. Final Good Producers

A final homogeneous market good, Y_t, is produced by competitive and identical firms using the following technology:

$$Y_t = \left[\int_0^1 \left(Y_{j,t} \right)^{\frac{1}{\lambda}} dj \right]^{\lambda}, \tag{2.15}$$

where $\lambda > 1$. The representative firm chooses specialized inputs, $Y_{j,t}$, to maximize profits:

$$P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj,$$

subject to the production function (2.15). The firm's first order condition for the j^{th} input is:

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}}\right)^{\frac{\lambda}{\lambda-1}} Y_t. \tag{2.16}$$

2.4. Retailers

As in Ravenna and Walsh (2008), the j^{th} input good is produced by a monopolist *retailer*, with production function:

$$Y_{j,t} = k_{j,t}^{\alpha} \left(z_t h_{j,t}\right)^{1-\alpha} - \eta_t^{\phi}\phi. \tag{2.17}$$

The retailer is a monopolist in the product market and is competitive in the factor markets. Here $k_{j,t}$ denotes the total amount of capital services purchased by firm j. Also, $\eta_t^{\phi}\phi$ represents an exogenous fixed cost of production, where ϕ is a positive scalar and η_t^{ϕ} is a process, discussed below, that ensures balanced growth. We calibrate the fixed cost so that profits are zero along the balanced growth path. In (2.17), z_t is a technology shock whose properties are discussed below. Finally, $h_{j,t}$ is the quantity of an intermediate good purchased by the j^{th} retailer. This good is purchased in competitive markets at the price P_t^h from a *wholesaler*. Analogous to CEE, we assume that to produce in period t, the retailer must borrow $P_t^h h_{j,t}$ at the start of the period at the interest rate R_t. In this way, the marginal cost of a unit of $h_{j,t}$ is

$$P_t^h \left(\alpha R_t + (1-\alpha)\right), \tag{2.18}$$

where α is the fraction of the intermediate input that must be financed. We set $\alpha = 1/2$. The retailer repays the loan at the end of period t after receiving sales revenues. The j^{th} retailer sets its price, $P_{j,t}$, subject to the demand curve, (2.16), and the Calvo sticky price friction (2.19). In particular,

$$P_{j,t} = \begin{cases} P_{j,t-1} & \text{with probability } \xi \\ \tilde{P}_t & \text{with probability } 1-\xi \end{cases}. \tag{2.19}$$

Here, $\tilde{P}_t$ denotes the price set by the fraction $1-\xi$ of producers who can re-optimize. Note that we do not allow for price indexation. So, the model is consistent with the observation that many prices remain unchanged for extended periods of time (see Eichenbaum, Jaimovich and Rebelo, 2011, and Klenow and Malin, 2011).

2.5. Wholesalers and the Labor Market

A perfectly competitive representative wholesaler / firm produces the intermediate good using labor only. Let l_{t-1} denote employment of the wholesaler at the end of $t-1$. Consistent with our discussion above, a fraction $1-\rho$ of these workers separates exogenously from the

wholesaler at the end of period. A total of ρl_{t-1} workers are attached to the wholesaler at the start of period t. At the beginning of the period, the wholesaler pays a fixed cost, $\eta_t^\kappa \kappa$, to meet a worker with probability one. Here κ is a positive scalar and η_t^κ is a process, discussed below, that ensures balanced growth. To hire $x_t l_{t-1}$ workers, the wholesaler must post $x_t l_{t-1}/Q_t$ vacancies. Here Q_t denotes the aggregate vacancy filling rate which firms take as given. Posting vacancies is costless. The wholesaler meets $x_t l_{t-1}$ new workers. In equilibrium all of these workers will be employed by the firm so x_t is the wholesaler's hiring rate.

At the beginning of the period, the wholesaler is in contact with a total of l_t workers (see equation (2.5)). This pool of workers includes non-separated workers plus the new workers that the firm has just met. Each worker, engages in bilateral bargaining with a representative of the wholesaler, taking the outcome of all other negotiations as given. The equilibrium real wage rate, w_t, i.e. W_t/P_t, is the outcome of the alternating offer bargaining process described below. In equilibrium all bargaining sessions conclude successfully, so the representative wholesaler employs l_t workers. Production begins immediately after wage negotiations are concluded and the wholesaler sells the intermediate good at the real price, $\vartheta_t \equiv P_t^h/P_t$.

2.5.1. Wage Setting

Consistent with Hall and Milgrom (2008) and CET (2013), we assume that wages are determined according to the alternating offer bargaining protocol proposed in Rubinstein (1982) and Binmore, Rubinstein and Wolinsky (1986). Let w_t^p denote the expected present discounted value of the wage payments by a firm to a worker that it is matched with:

$$w_t^p = w_t + \rho E_t m_{t+1} w_{t+1}^p.$$

Here m_t is the time t discount factor which firms and workers view as an exogenous stochastic process beyond their control.

Let J_t denote the value, denominated in units of the final consumption good, to a firm of employing a worker in period t:

$$J_t = \vartheta_t^p - w_t^p.$$

Here ϑ_t^p denotes the expected present discounted value of the marginal revenue product associated with a worker to the firm:

$$\vartheta_t^p = \vartheta_t + \rho E_t m_{t+1} \vartheta_{t+1}^p. \tag{2.20}$$

Because there is free entry into the labor market, firm profits must be zero. It follows that

$$\eta_t^\kappa \kappa = J_t. \tag{2.21}$$

Let V_t denote the value to a worker of being matched with a firm that pays w_t in period t :

$$\begin{aligned} V_t &= w_t + E_t m_{t+1}[\rho V_{t+1} + (1-\rho)\, s\left(f_{t+1}\bar{V}_{t+1} + (1-f_{t+1})\, U_{t+1}\right) \\ &\quad + (1-\rho)\,(1-s)\, N_{t+1}]. \end{aligned} \tag{2.22}$$

Here, $\bar{V}_{t+1}$ denotes the value of working for another firm in period $t+1$. In equilibrium, $\bar{V}_{t+1} = V_{t+1}$. Also, U_{t+1} in (2.22) is the value of being an unemployed worker in period $t+1$ and N_{t+1} is the value of being out of the labor force in period $t+1$. The objects, s, ρ and f_{t+1} were discussed in the previous section. Relation (2.22) reflects our assumption that an employed worker remains in the same job with probability ρ, transits to another job without passing through unemployment with probability $(1-\rho)\, s f_{t+1}$, transits to unemployment with probability $(1-\rho)\, s\,(1-f_{t+1})$ and to non-participation with probability $(1-\rho)\,(1-s)$.

It is convenient to rewrite (2.22) as follows:

$$V_t = w_t^p + A_t, \tag{2.23}$$

where

$$\begin{aligned} A_t &= (1-\rho)\, E_t m_{t+1}\left[s f_{t+1}\bar{V}_{t+1} + s\,(1-f_{t+1})\, U_{t+1} + (1-s)\, N_{t+1}\right] \\ &\quad + \rho E_t m_{t+1} A_{t+1}. \end{aligned} \tag{2.24}$$

Note that V_t consists of two components. The first is the expected present value of the wages received by a worker from a firm that he is matched with at time t. The second corresponds to the expected present value of the payments that a worker receives in all dates and states when he is separated from that firm.

The value of unemployment, U_t, is given by,

$$U_t = \eta_t^D D_t + \tilde{U}_t. \tag{2.25}$$

Recall that $\eta_t^D D_t$ represents unemployment compensation at time t. The variable $\tilde{U}_t$ denotes the continuation value of unemployment:

$$\tilde{U}_t \equiv E_t m_{t+1}\left[s f_{t+1} V_{t+1} + s\,(1-f_{t+1})\, U_{t+1} + (1-s)\, N_{t+1}\right]. \tag{2.26}$$

Expression (2.26) reflects our assumption that an unemployed worker finds a job in the next period with probability $s f_{t+1}$, remains unemployed with probability $s\,(1-f_{t+1})$ and exits the labor force with probability $1-s$.

The value of non-participation is:

$$\begin{aligned} N_t &= 0 + \tilde{N}_t \\ \tilde{N}_t &= E_t m_{t+1}\left[e_{t+1}\left(f_{t+1} V_{t+1} + (1-f_{t+1}) U_{t+1}\right) + (1-e_{t+1})\, N_{t+1}\right]. \end{aligned} \tag{2.27}$$

Expression (**??**) reflects our assumption that a non-participating worker is selected to join the labor force with probability e_t, defined in (2.4).

The structure of alternating offer bargaining is the same as it is in CET.[2] Each matched worker-firm pair (both those who just matched for the first time and those who were matched in the past) bargain over the current wage rate, w_t. Each time period (a quarter) is subdivided into M periods of equal length, where M is even. The firm make a wage offer at the start of the first subperiod. It also make an offer at the start of every subsequent odd subperiod in the event that all previous offers have been rejected. Similarly, workers make a wage offer at the start of all even subperiods in case all previous offers have been rejected. Because M is even, the last offer is made, on a take-it-or-leave-it basis, by the worker. When the firm rejects an offer it pays a cost, $\eta_t^\gamma \gamma$, of making a counteroffer. Here γ is a positive scalar and $\eta_t^\gamma$is a process that ensures balanced growth.

In subperiod $j = 1, ..., M-1,$ the recipient of an offer can either accept or reject it. If the offer is rejected the recipient may declare an end to the negotiations or he may plan to make a counteroffer at the start of the next subperiod. In the latter case there is a probability, δ, that bargaining breaks down and the wholesaler and worker revert to their outside option. For the firm, the value of the outside option is zero and for the worker the outside option is unemployment.[3] Given our assumptions, workers and firms never choose to terminate bargaining and go to their outside option.

It is always optimal for the firm to offer the lowest wage rate subject to the condition that the worker does not reject it. To know what that wage rate is, the wholesaler must know what the worker would counteroffer in the event that the firm's offer was rejected. But, the worker's counteroffer depends on the firm's counter offer in case the worker's counteroffer is rejected. We solve the firm's initial offer beginning from worker's final offer and working backwards. Since workers and firms know everything about each other, the firm's opening wage offer is always accepted. The firm must know what all the counteroffers would be in each of the $M-1$ future subperiods in order to determine what its opening offer is. Our environment is sufficiently simple that the solution to the bargaining problem has the following straightforward characterization:

$$\alpha_1 J_t = \alpha_2 \left(V_t - U_t\right) - \alpha_3 \eta_t^\gamma \gamma + \alpha_4 \left(\vartheta_t - \eta_t^D D_t\right) \tag{2.28}$$

[2] We assume that the outside option for a worker is always unemployment and not out of the labor force. That is, when bargaining breaks down, workers are send to unemployment.

[3] We could allow for the possibility that when negotiations break down the worker has a chance of leaving the labor force. To keep our anlysis realtively simple, we do not allow for that possibility here.

where $\beta_i = \alpha_{i+1}/\alpha_1$, for $i = 1, 2, 3$ and,

$$\begin{aligned}
\alpha_1 &= 1 - \delta + (1-\delta)^M \\
\alpha_2 &= 1 - (1-\delta)^M \\
\alpha_3 &= \alpha_2 \frac{1-\delta}{\delta} - \alpha_1 \\
\alpha_4 &= \frac{1-\delta}{2-\delta}\frac{\alpha_2}{M} + 1 - \alpha_2.
\end{aligned}$$

See the technical appendix for a detailed derivation of (2.28) and the procedure that we use for solving the bargaining problem

2.6. Market Clearing, Monetary Policy and Functional Forms

The total supply of the intermediate good is given by l_t which equals the total quantity of labor used by the wholesalers. So, clearing in the market for intermediate goods requires

$$h_t = l_t, \tag{2.29}$$

where

$$h_t \equiv \int_0^1 h_{j,t} dj.$$

The capital services market clearing condition is:

$$u_t^K K_t = \int_0^1 k_{j,t} dj.$$

Market clearing for final goods requires:

$$C_t + (I_t + a(u_t^K)K_t)/\Psi_t + \eta_t^{\kappa} \kappa x_t l_{t-1} + G_t = Y_t. \tag{2.30}$$

The right hand side of the previous expression denotes the quantity of final goods. The left hand side represents the various ways that final goods are used. Homogeneous output, Y_t, can be converted one-for-one into either consumption goods, goods used to hire workers, or government purchases, G_t. In addition, some of Y_t is absorbed by capital utilization costs. Finally, Y_t can be used to produce investment goods using a linear technology in which one unit of the final good is transformed into Ψ_t units of I_t. Perfect competition in the production of investment goods implies,

$$P_{I,t} = \frac{P_t}{\Psi_t}.$$

We adopt the following specification of monetary policy:

$$\ln(R_t/R) = \rho_R \ln(R_{t-1}/R) + (1-\rho_R)\left[r_\pi \ln(\pi_t/\pi) + r_y \ln(\mathcal{Y}_t/\mathcal{Y})\right] + \sigma_R \varepsilon_{R,t}.$$

Here, π denotes the monetary authority's target inflation rate, which is also the steady state inflation rate in the model. The shock, $\varepsilon_{R,t}$, is a unit variance, zero mean disturbance to monetary policy. Also, R and $\mathcal{Y}$ denote the steady values of R_t and $\mathcal{Y}_t$. The variable, $\mathcal{Y}_t$, denotes Gross Domestic Product (GDP):

$$\mathcal{Y}_t = C_t + I_t/\Psi_t + G_t.$$

Here, G_t denotes government consumption and is assumed to have the following representation:

$$G_t = \eta_t^g g_t. \tag{2.31}$$

Here, η_t^g is a process that guarantees balanced growth.

Working with data from Fernald (2012) we find that the growth rate of total factor productivity is well described by an $i.i.d.$ process. Accordingly, we assume that $\ln \mu_{z,t} \equiv \ln (z_t/z_{t-1})$ is $i.i.d.$ We also assume that $\ln \mu_{\Psi,t} \equiv \ln (\Psi_t/\Psi_{t-1})$ follows an AR(1) process. The parameters that control the standard deviations of both processes are denoted by (σ_z, σ_Ψ). The autocorrelation of $\ln \mu_{\Psi,t}$ is denoted by ρ_Ψ.

Our model exhibits growth stemming from neutral and investment-specific technological progress. The variables $Y_t/\Phi_t, C_t/\Phi_t$, w_t/Φ_t and $I_t/(\Psi_t\Phi_t)$ converge to constants in nonstochastic steady state, where

$$\Phi_t = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t$$

is a weighted average of the sources of technological progress. If objects like the fixed cost of production, the cost of hiring, the cost to a firm of preparing a counteroffer, government purchases, and unemployment transfer payments were constant, they would become irrelevant over time. To avoid this implication, it is standard in the literature to suppose that such objects grow at the same rate as output, which in our case is given by Φ_t. An unfortunate implication of this assumption is that technology shocks of both types immediately affect the vector of objects

$$\Omega_t = \left[\eta_t^g, \eta_t^D, \eta_t^\gamma, \eta_t^\kappa, \eta_t^\phi, \eta_t^L, \eta_t^H\right]'. \tag{2.32}$$

It seems hard to justify such an assumption. To avoid this problem, we proceed as in Christiano, Trabandt and Walentin (2012) and Schmitt-Grohé and Uribe (2012) who assume that government purchases, G_t, are a distributed lag of unit root technology shocks, i.e. G_t is cointegrated with Y_t but has a smoother stochastic trend. In particular, we assume that $\Omega_{i,t}$ evolves according to:

$$\Omega_{i,t} = \Phi_{t-1}^\theta \left(\Omega_{i,t-1}\right)^{1-\theta}. \tag{2.33}$$

Here $\Omega_{i,t}$ refers to the i^{th} element of Ω_t and $0 < \theta \leq 1$ is a parameter to be estimated. Note that $\Omega_{i,t}$ grows at the same rate as Φ_t in the long-run. When θ is very close to zero, $\Omega_{i,t}$

is virtually unresponsive in the short-run to an innovation in either of the two technology shocks, a feature that we find very attractive on a priori grounds.

We assume that the cost of adjusting investment takes the form:

$$\begin{aligned} S\left(I_t/I_{t-1}\right) &= 0.5\exp\left[\sqrt{S''}\left(I_t/I_{t-1}-\mu\cdot\mu_\Psi\right)\right] \\ &\quad +0.5\exp\left[-\sqrt{S''}\left(I_t/I_{t-1}-\mu\cdot\mu_\Psi\right)\right]-1. \end{aligned}$$

Here, μ and μ_Ψ denote the unconditional growth rates of Φ_t and Ψ_t. The value of I_t/I_{t-1} in nonstochastic steady state is $(\mu\cdot\mu_\Psi)$. In addition, S'' represents a model parameter that coincides with the second derivative of $S(\cdot)$, evaluated in steady state. It is straightforward to verify that $S(\mu\cdot\mu_\Psi)=S'(\mu\cdot\mu_\Psi)=0$.

We assume that the cost associated with setting capacity utilization is given by,

$$a(u_t^K)=0.5\sigma_a\sigma_b(u_t^K)^2+\sigma_b\left(1-\sigma_a\right)u_t^K+\sigma_b\left(\sigma_a/2-1\right)$$

where σ_a and σ_b are positive scalars. We normalize the steady state value of u_t^K to one. This pins down the value of σ_b given an estimate of σ_a.

Finally, we discuss how vacancies are determined. We posit a standard matching function:

$$x_t l_{t-1}=\sigma_m\left(1-\rho l_{t-1}\right)^\sigma\left(l_{t-1}v_t\right)^{1-\sigma}, \tag{2.34}$$

where $l_{t-1}v_t$ denotes the total number of vacancies and v_t denotes the vacancy rate. Given x_t and l_{t-1}, we use (2.34) to solve for v_t. Recall that we defined the total number of vacancies by $x_t l_{t-1}/Q_t$. We can solve for the aggregate vacancy filling rate Q_t using

$$Q_t=\frac{x_t}{v_t}. \tag{2.35}$$

The equilibrium of our model has a particular recursive structure. We can first solve all model variables, apart from v_t and Q_t. These two variables can then be solved for using (2.34) and (2.35).

3. Econometric Methodology

We estimate our model using a Bayesian variant of the strategy in CEE that minimizes the distance between the dynamic response to three shocks in the model and the analog objects in the data. The latter are obtained using an identified VAR for post-war quarterly U.S. times series that include key labor market variables. The particular Bayesian strategy that we use is the one developed in Christiano, Trabandt and Walentin (2011), henceforth CTW.

CTW estimate a 14 variable VAR using 14 variables quarterly data that are seasonally adjusted and cover the period 1951Q1 to 2008Q4.[4]To facilitate comparisons, our analysis is based on the same VAR that CTW use. As in CTW, we identify the dynamic responses to a monetary policy shock by assuming that the monetary authority sees the current and lagged values of all the variables in the VAR and a monetary policy shock affects only the Federal Funds Rate contemporaneously. As in Altig, Christiano, Eichenbaum and Linde (2011), Fisher (2006) and CTW, we make two assumptions to identify the dynamic responses to the technology shocks: (i) the only shocks that affect labor productivity in the long-run are the innovations to the neutral technology shock, z_t, and the innovation to the investment-specific technology shock, Ψ_t and (ii) the only shock that affects the price of investment relative to consumption in the long-run is the innovation to Ψ_t. These assumptions are satisfied in our model. Standard lag-length selection criteria lead CTW to work with a VAR with 2 lags.[5]

We include the following variables in the VAR:[6]

$$\begin{pmatrix} \Delta \ln(\text{relative price of investment}_t) \\ \Delta \ln(\text{real } GDP_t/\text{hours}_t) \\ \Delta \ln(GDP \text{ deflator}_t) \\ \text{unemployment rate}_t \\ \ln(\text{capacity utilization}_t) \\ \ln(\text{hours}_t) \\ \ln(\text{real } GDP_t/\text{hours}_t) - \ln(\text{real wage}_t) \\ \ln(\text{nominal } C_t/\text{nominal } GDP_t) \\ \ln(\text{nominal } I_t/\text{nominal } GDP_t) \\ \ln(\text{vacancies}_t) \\ \text{job separation rate}_t \\ \text{job finding rate}_t \\ \ln(\text{hours}_t/\text{labor force}_t) \\ \text{Federal Funds rate}_t \end{pmatrix}. \tag{3.1}$$

Given an estimate of the VAR we compute the implied impulse response functions to the three structural shocks. We stack the contemporaneous and 14 lagged values of each of these impulse response functions for 13 of the variables listed above in a vector, $\hat{\psi}$. We do not include the job separation rate because that variable is constant in our model. We include the job separation rate in the VAR to ensure the VAR results are not driven by an omitted variable bias.

The logic underlying our model estimation procedure is as follows. Suppose that our

[4]There is an ongoing debate over whether or not there is a break in the sample period that we use. Implicitly, our analysis sides with those authors who argue that the evidence of parameter breaks in the middle of our sample period is not strong. See for example Sims and Zha (2006) and Christiano, Eichenbaum and Evans (1999).

[5]See CTW for a sensitivity analysis with respect to the lag length of the VAR.

[6]See section A of the technical appendix in CTW for details about the data.

structural model is true. Denote the true values of the model parameters by θ_0. Let $\psi(\theta)$ denote the model-implied mapping from a set of values for the model parameters to the analog impulse responses in $\hat{\psi}$. Thus, $\psi(\theta_0)$ denotes the true value of the impulse responses whose estimates appear in $\hat{\psi}$. According to standard classical asymptotic sampling theory, when the number of observations, T, is large, we have

$$\sqrt{T}\left(\hat{\psi}-\psi(\theta_0)\right) \overset{a}{\sim} N(0, W(\theta_0,\zeta_0)).$$

Here, ζ_0 denotes the true values of the parameters of the shocks in the model that we do not formally include in the analysis. Because we solve the model using a log-linearization procedure, $\psi(\theta_0)$ is not a function of ζ_0. However, the sampling distribution of $\hat{\psi}$ is a function of ζ_0. We find it convenient to express the asymptotic distribution of $\hat{\psi}$ in the following form:

$$\hat{\psi} \overset{a}{\sim} N(\psi(\theta_0), V), \tag{3.2}$$

where

$$V \equiv \frac{W(\theta_0,\zeta_0)}{T}.$$

For simplicity our notation does not make the dependence of V on θ_0, ζ_0 and T explicit. We use a consistent estimator of V. Motivated by small sample considerations, that estimator has only diagonal elements (see CTW). The elements in $\hat{\psi}$ are graphed in Figures $3-5$ (see the solid lines). The gray areas are centered, 95 percent probability intervals computed using our estimate of V.

In our analysis, we treat $\hat{\psi}$ as the observed data. We specify priors for θ and then compute the posterior distribution for θ given $\hat{\psi}$ using Bayes' rule. This computation requires the likelihood of $\hat{\psi}$ given θ. Our asymptotically valid approximation of this likelihood is motivated by (3.2):

$$f\left(\hat{\psi}|\theta,V\right) = \left(\frac{1}{2\pi}\right)^{\frac{N}{2}} |V|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\hat{\psi}-\psi(\theta)\right)' V^{-1}\left(\hat{\psi}-\psi(\theta)\right)\right]. \tag{3.3}$$

The value of θ that maximizes the above function represents an approximate maximum likelihood estimator of θ. It is approximate for three reasons: (i) the central limit theorem underlying (3.2) only holds exactly as $T \to \infty$, (ii) our proxy for V is guaranteed to be correct only for $T \to \infty$, and (iii) $\psi(\theta)$ is calculated using a linear approximation.

Treating the function, f, as the likelihood of $\hat{\psi}$, it follows that the Bayesian posterior of θ conditional on $\hat{\psi}$ and V is:

$$f\left(\theta|\hat{\psi},V\right) = \frac{f\left(\hat{\psi}|\theta,V\right)p(\theta)}{f\left(\hat{\psi}|V\right)}. \tag{3.4}$$

Here, $p(\theta)$ denotes the priors on θ and $f\left(\hat{\psi}|V\right)$ denotes the marginal density of $\hat{\psi}$:

$$f\left(\hat{\psi}|V\right) = \int f\left(\hat{\psi}|\theta, V\right) p(\theta)\, d\theta.$$

The mode of the posterior distribution of θ can be computed by maximizing the value of the numerator in (3.4), since the denominator is not a function of θ. The marginal density of $\hat{\psi}$ is required for an overall measure of the fit of our model. To compute the marginal likelihood, we use the standard Laplace approximation. In our analysis, we also find it convenient to compute the marginal likelihood based on a subset of the elements in $\hat{\psi}$ (see Appendix B for details).

4. Results for the Estimated Model

This section presents results for the estimated model. First, we discuss the posterior modes of the estimated structural parameters. Second, we discuss the ability of the model to account for the dynamic response of the economy to a monetary policy shock, a neutral technology shock and an investment-specific technology shock. Finally, we discuss the macroeconomic effects of an increase in unemployment benefits and government purchases of goods in normal times, i.e. when the ZLB on the nominal interest is not binding.

We set the values for a subset of the model parameters a priori. These values are reported in Panel A of Table 3. We also set the steady state values of five model variables, listed in Panel B of Table 3. We specify β so that the steady state annual real rate of interest is three percent. In addition we set the coefficient Θ, that governs the curvature of the household's instantaneous utility function, to three. The depreciation rate on capital, δ_K, is set to imply an annual depreciation rate of 10 percent. The values of μ and μ_Ψ are equal to the sample average of real per capita GDP and real investment growth in our sample. We assume the monetary authority's inflation target is 2.5 percent and that the profits of intermediate good producers are zero in steady state. We set the rate at which vacancies create job-worker meetings, Q, to 0.7, as in den Haan, Ramey and Watson (2000) and Ravenna and Walsh (2008). We set the steady state unemployment rate to the average unemployment rate in our sample , implying a steady state value of u equal to 0.055. We set the parameter M to 60 which roughly corresponds to the number of business days in a quarter. We assume $\rho = 0.9$, which implies a match survival rate that is consistent with both HM and Shimer (2012a).[7]

[7] Denote the probability that a worker separates from a job at a monthly rate by $1-\tilde{\rho}$. The probability that a person employed at the end of a quarter separates in the next three months is $(1-\tilde{\rho})+\tilde{\rho}(1-\tilde{\rho})+\tilde{\rho}^2(1-\tilde{\rho}) = (1-\tilde{\rho})\left(1+\tilde{\rho}+\tilde{\rho}^2\right)$. Shimer (2012a) reports that $\tilde{\rho} = 1-0.034$, implying a quarterly separation rate of 0.0986. HM assume a similar value of 0.03 for the monthly separation rate. This value is also consistent with Walsh's (2003) summary of the empirical literature.

Finally, we assume that the steady state value of the ratio of government consumption to gross output is 0.20.

All remaining model parameters are estimated subject to the restrictions summarized in Table 1. Table 2 presents prior and posterior distributions for all of the estimated objects in the model.

4.1. The Estimated Model

A number of features of the posterior mode of the estimated parameters of our model are worth noting. First, the posterior mode of ξ implies a moderate degree of price stickiness, with prices changing on average once every 3 quarters. This value lies within the range reported in the literature. For example, according to Nakamura and Steinsson (2012), the recent micro-data based literature finds that the price of the median product changes roughly every 1.5 quarters when sales are included, and every 3 quarters when sales are excluded. Second, the posterior mode of δ implies that there is a roughly 0.1% chance of an exogenous break-up in negotiations when a wage offer is rejected. Third, the posterior modes of our model parameters, along with the assumption that the steady state unemployment rate equals 5.5%, implies that it costs firms 0.78 days of marginal revenue to prepare a counteroffer during wage negotiations (see Table 3). Fourth, the posterior mode of steady-state hiring costs as a percent of gross output is equal to 0.5%. This result implies that steady-state hiring costs as a percent of total wages of newly-hired workers is equal to 6.8%.[8] Silva and Toledo (2009) report that, depending on the exact costs included, the value of this statistic is between 4 and 14 percent, a range that encompasses the corresponding statistic in our model

Fifth, the posterior mode of the replacement ratio is 0.33. To put this number in perspective, consider the following narrow measure of the fraction of unemployment benefits to wages. The numerator of the fraction is total payments of the government for unemployment insurance divided by the total number of unemployed people. The denominator of the fraction is total compensation of labor divided by the number or employees, i.e. wage per worker. The average of the numerator divided by denominator in our sample period is 13.7%. This fraction represents the lower bound on the average replacement rate. Shimer (2005) argues in favor of 0.4 allowing for a broader interpretation of unemployment insurance, while Hall (2008) suggests 0.7, based on a broader interpretation that permits utility from leisure. It is well know that Diamond (1982), Mortensen (1985) and Pissarides (1985) (DMP) style models require a replacement rate in excess of 0.90 to account for fluctuations

[8] Table 4 reports the hiring cost to gross output ratio in steady state which is defined as: $s_l = 100\kappa n_\kappa x l/y$. Here n_κ is equal to Ω_t^κ/Φ_t evaluated at steady state. Given s_l and the real wage, w, it is straightforward to compute hiring costs as a share of the wage of newly hired workers: $100 n_\kappa \kappa/w$.

in labor markets (see for example the extended discussion in CET). For the reasons stressed in CET, alternating offer bargaining between workers and firms mutes the sensitivity of real wages to aggregate shocks. This property underlies our model's ability to account for estimated response of the economy to monetary policy shocks and shocks to neutral and capital embodied technology with a low replacement ratio.

Sixth, the posterior mode of s implies that a separated or unemployed worker leaves the labor force with probability 0.24. Seventh, the posterior mode for α_{C^h} is 0.01, implying that people outside of the labor force account for virtually all of home production. Eight, the posterior mode of θ which governs the responsiveness of the elements of Ω_t to technology shocks, is close to zero (0.072). So, variables like government purchases and unemployment benefits which are affected by those processes, are very unresponsive in the short-run to technology shocks. Finally, the posterior modes of the parameters governing monetary policy are similar to those reported in the literature (see for example Justiniano, Primiceri, and Tambalotti, 2010).

The solid black lines in Figures 1-4 present the impulse response functions to a monetary policy shock, a neutral-technology shock and an investment-specific technology shock, implied by the estimated VAR. The grey areas represent 95 percent probability intervals. The solid lines with the circles correspond to the impulse response functions of our model evaluated at the posterior mode of the structural parameters. Figure 1 shows that the model does very well at reproducing the estimated effects of an expansionary monetary policy shock, including the hump-shaped rises real GDP and hours worked, the rise in the labor force participation rate and the muted response of inflation. Notice that real wages respond by much less than hours worked to a monetary policy shock. Even though the maximal rise in hours worked is roughly 0.14%, the maximal rise in real wages is only 0.06%. Significantly, the model accounts for the hump-shaped fall in the unemployment rate as well as the rise in the job finding rate and vacancies that occur after an expansionary monetary policy shock. The model does understate the rise in the capacity utilization rate. The sharp rise of capacity utilization in the estimated VAR may reflect that our data on the capacity utilization rate pertains to the manufacturing sector, which may overstate the average response across all sectors in the economy.

From Figure 2 we see that the model does a good job of accounting for the estimated effects of a neutral technology shock. Note that the model is able to account for the initial rise and subsequent persistent decline in the unemployment rate. The model also accounts for the initial declines and subsequent rises in vacancies and the job finding rate after a positive neutral technology shock. The model overstates somewhat the longer-term effect on the labor force participation rate of a technology shock. Finally, it is important to note that our VAR estimates imply that inflation falls sharply after a positive neutral technology

shock. This finding is consistent with results in Christiano, Eichenbaum and Vigfusson (2005, 2006), Altig, Christiano, Eichenbaum and Linde (2011) and Paciello (2011) and Sims (2011). Our model is able to account for the important pattern, a result that we return too when discussing the behavior of inflation during the Great Recession.

Viewed as a whole, the results of this section provide evidence that our model does well at accounting for the cyclical properties of key labor market and other macro variables, as measured by their response, in the non-ZLB period, to a monetary policy shock, a neutral technology shock and an investment specific technology shock.

4.1.1. A shock to government consumption

We conclude by discussing the model's implications for government spending in 'normal' times, i.e. a period during which the zero lower bound on the nominal interest rate is not binding. Figure 5 displays the response of the economy to a 1 percentage point increase in g_t.The solid and dotted lines display the impulse response functions of variables when g_t follows an AR coefficient, ρ_g, with 0.6 and 0.9,respectively. In both cases, the rise in g_t induces a rise hours worked, the labor force, hours worked, real GDP and the job finding rate. Real wages and inflation rise but by relatively small amounts. At the same time the rise in g_t leads to a persistent fall in the unemployment rate. In both cases, consumption initially rises. When ρ_g is equal to 0.9, consumption eventually falls due to the stronger negative wealth effect from the larger rise in taxes associated with larger rise in the present value of government spending. In both cases, investment falls in response to an increase in government consumption, with the fall larger when ρ_g is equal to .9.

The basic intuition for how a government spending shock affects the economy is similar to the traditional NK model, except for the mechanism in the labor market. As in standard NK sticky price models, an expansionary shock to government purchases leads to an increase in the demand for final goods. This rise induces an increase in the demand for the output of sticky price retailers. Since they must satisfy demand, the retailers purchase more of the wholesale good. Therefore, the relative price of the wholesale good increases and the marginal revenue product associated with a worker rises. Other things equal, this motivates wholesalers to hire more workers and increases the probability that an unemployed worker finds a job. The latter effect induces a rise in workers' disagreement payoffs. The resulting increase in workers' bargaining power generates a rise in the real wage and provides an incentive for the household to increase the size of the labor force.. Given our assumptions about parameter values, alternating offer bargaining mutes the increase in real wages, thus allowing for a large rise in employment, a substantial decline in unemployment, and a small rise in inflation.

5. The Great Recession

In this section we analyze the behavior of the economy in the last 4 years. The section is organized as follows. First, we describe our characterization of the events over that period. Second, we describe the shocks that we subject the model to in order to key characteristics of the post-2008 data. Third, discuss our strategy for solving the model with in the presence of a binding ZLB on the nominal interest rate. Finally, we discuss our empirical results

5.1. Characterizing the Recession

The solid line in Figure 6 displays the behavior of key macroeconomic variables since 2001. In the case of variables that grow over time, we fit a linear trend from 2001 to 2008Q2, represented by the dark-dashed line. To characterize what the data would have looked like absent the shocks that caused the financial crisis, we extrapolate the trend line (see the thin dashed line). With one exception we assume that the other variables would have followed a no-change trajectory after 2008Q2, had the financial crisis not occurred. The exception is the interest rate spread, which we measure using the estimates provided in Gilchrist and Zakrajšek (2012) (GZ). As Figure 6 shows, the spread was already unusually high in 2008Q2. Our projection as of 2008Q2 is that the GZ spread falls to 1 percent, its value during the relatively tranquil period, 1990-1997. The specific path taken by our projection extends a linear trend computed over the period 2001-2008Q2.[9]

The projections for the labor force and employment after 2008Q2 are perhaps controversial because of ongoing demographic changes in the U.S. population. That said, our no-change projection for the labor force to population ratio is very similar to a forecast made by the Bureau of Labor Statistics in November, 2007.[10] In addition, our no-change forecast of the employment to population ratio is consistent with a forecast made by the Bureau of Labor Statistics in 2008.

The distance between the solid lines and the thin-dashed lines represents our statistical estimate of the economic effects of the shocks that hit the economy in 2008Q3 and later (see Figure 7). Of course, the dynamics of the data over this period also reflect the effects of shocks that occurred before 2008Q3. Our procedure assumes that those effects are relatively small. An alternative strategy for computing the post-2008Q2 projections would use more sophisticated time series methods that integrate the effects of these shocks. For example, an alternative projection for unemployment would take into account that unemployment tends to rise in the latter stages of a recession, and that a recession had started in late

[9] The latter is the capacity utilization-adjusted measure of TFP computed by John Fernald and available at http://www.frbsf.org/economic-research/economists/jfernald/quarterly_tfp.xls

[10] See Erceg and Levin (2013), Figure 1.

2007. The alternative projection would have reduced somewhat our estimate of the impact of the financial crisis shocks on unemployment. We plan to investigate alternative ways of computing our projections.

5.2. The Shocks Driving the Great Recession

We suppose that the Great Recession was triggered by four shocks that occurred beginning in part by significant shocks to financial markets that had not occurred to any important way before 2008Q3. Two of our shocks are wedges which capture in a reduced form way frictions which are widely viewed as having been important during the Great Recession. The other sources of shocks in the Great Recession were government consumption and technology.

We begin by discussing the two financial shocks. The first is motivated by the literature stressing a reduction in consumption as a trigger for a ZLB episode (see, e.g., Christiano, Eichenbaum and Rebelo (2011), Eggertsson and Krugman (2012), Eggertsson and Woodford (2003), Guerrieri and Lorenzoni (2012)). For convenience we capture this idea as in Smets and Wouters (2007) by introducing a perturbation, Δ_t^b, to agents' intertemporal Euler equations associated with saving. With this wedge, the Euler equation associated with the nominally risk free bond is given by:

$$1 = (1 + \Delta_t^b) E_t m_{t+1} R_t / \pi_{t+1}.$$

The Euler equation implied by optimality of the household's capital accumulation decision is given by:

$$1 = (1 + \Delta_t^b)(1 - \Delta_t^k) E_t m_{t+1} R_{t+1}^k / \pi_{t+1}.$$

We refer to Δ_t^b and $1 - \Delta_t^k$ as the *consumption wedge* and the *financial wedge*, respectively. A simple financial friction model based on asymmetric information with costly monitoring implies that credit market frictions can be captured as a tax, Δ_t^k, on the gross return on capital (see Christiano and Davis (2006)). In our model, the quarterly return on capital invested in period t, net of the financial wedge, is $\left(1 - \Delta_t^k\right) R_{t+1}^k$.

Recall that firms finance a fraction, α, of the intermediate input in advance (see (2.18)). We suppose that the financial wedge also applies to working capital loans. As a result, we replace (2.18) with

$$P_t^h \left(\alpha R_t \left(1 + \Delta_t^k \right) + (1 - \alpha) \right), \tag{5.1}$$

where $\alpha = 1/2$, as before.

We measure the financial wedge using the GZ interest rate spread. The latter is based on the average credit spread on senior unsecured bonds issued by nonfinancial firms covered in Compustat and by the Center for Research in Security Prices. The average and median duration of the bonds in GZ's data set is 6.47 and 6.00 years, respectively. We interpret the

GZ spread as an indicator of Δ_t^k. We suppose that the Δ_t^k's are related to the GZ spread as follows:

$$\Gamma_t = E\left[\frac{\Delta_t^k + \Delta_{t+1}^k + ... + \Delta_{t+23}^k}{6}|\Omega_t\right], \tag{5.2}$$

where Γ_t denotes the GZ spread minus the projection of that spread as of 2008Q2 (see the dashed line in Figure 6). Also, Ω_t denotes the information available to agents at time t. In (5.2) we sum over Δ_{t+j}^k for $j = 0, ..., 23$ because Δ_t^k is a tax on the one quarter return to capital while Γ_t applies to $t + j$, $j = 0, 1, ..., 23$ (i.e., 6 years). Also, we divide the sum in (5.2) by 6 to take into account that Δ_t^k is measured in quarterly decimal terms while our empirical measure of Γ_t is measured in annual decimal terms.

We feed the sequence of Γ_t's displayed in Figure 7 for $t \geq 2008Q3$ to our model. However, we do not assume that agents have perfect foresight over the Γ_t's. Instead, we assume that at time t agents observe Γ_{t-s}, $s \geq 0$ and that they forecast future values of Γ_t using a mean-zero, first order autoregressive representation (AR(1)), with autoregressive coefficient, $\rho_\Gamma = 0.8$.

We now turn to a discussion of total factor productivity (TFP), which is graphed in Figure 6.[11] Figure 7 displays the difference between the level of TFP and its linear trend computed between 2008Q3 and 2013Q2. The difference between the data and its trend is graphed in Figure 7. The detrended data are clearly very volatile. In our view some of this volatility reflects measurement error because of the well known difficulties of correctly computing TFP at a quarterly frequency. This consideration leads us to assume that actual detrended TFP is a smoothed version of measured detrended TFP (see Figure 7). We feed the growth rate, $\log(z_t/z_{t-1})$, of this measure to the model for $t \geq 2008Q3$. At time t agents are assumed to observe TFP growth in $t - s$, $s \geq 0$. They forecast future values of TFP using a particular unobserved components model.

The assumption we make about the time series representation of TFP growth is motivated by the persistence in the observed slowdown in TFP after a long period of relatively rapid growth (see Fernald (2012) for an extended discussion). Our model of TFP is designed to capture the idea that in real time, agents would have been slow to realize the persistence with which TFP had dropped.

The components model used by agents to forecast TFP growth is

$$\log(z_t/z_{t-1}) = \mu_z + p_t + \varepsilon_t - \varepsilon_{t-1},$$

where $p_t = \rho_p p_{t-1} + \varepsilon_t^p$. Here, ε_t^p and ε_t are mutually uncorrelated at all leads and lags and each of the two variables is uncorrelated over time. Here, $\rho_p = 0.8$ and the standard

[11] The latter is the capacity utilization-adjusted measure of TFP computed by John Fernald and available at http://www.frbsf.org/economic-research/economists/jfernald/quarterly_tfp.xls

deviations of ε_t is λ times the standard deviation of ε_t^p, where $\lambda = 3$. The solid trend line for TFP drawn in Figure 7 simulates the p_t process under the assumption that $\varepsilon_t^p = -0.0035$, $p_{t-1} = 0$, $\varepsilon_{t+s} = 0$ for $s = -1, 0, 1, ...$ for $t = 2008Q3$. We assigned the value of 0.8 to ρ_p and -0.35 percent to ε_t^p in order to produce the smooth detrended TFP curve in Figure 7. So, we are taking the position that the level of TFP dropped by 0.35 percent in 2008Q3 and continued to fall until it reached its asymptote 1.75 percent below its unshocked path.

We assume that in period t agents only use current and past observations on TFP growth itself (i.e., the solid line in Figure 7 up to period t) to forecast future TFP growth. They do not directly observe ε_t^p and ε_t. Because ε_t plays an important role in the dynamics of $\log(z_t)$, they persistently believe that the drop in TFP is temporary. Over time they come to understand that TFP has in fact dropped permanently.

Next we consider the government consumption shock, (2.31). The variable η_t^g defined in (2.31) is computed using the smoothed TFP numbers in Figure 7 using (2.32) and (2.33).[12] Then, g_t is actual government consumption in Figure 6, divided by η_t^g. Agents forecast the period t value of η_t^g using current and past realizations of the technology shocks, as discussed above. In forecasting g_t agents assume $\log(g_t/g)$ is the realization of a zero-mean AR(1) process with coefficient, $\rho_g = 0.75$.

Finally, we do not have data on Δ_t^b. So, we set it to a constant value 1.25 percent per year for 16 quarters. After that, it refers to its steady state of zero.

There are 6 free parameters in the above calculations. They are the autoregressive parameters, ρ_g, ρ_Γ, ρ_p, the relative volatility parameter, λ, the initial drop in TFP, ε_t^p in 2008Q3, and the jump in Δ_t^b. . We chose these parameters to maximize the model's ability to account for the post 2008Q2 behavior of our 9 endogenous variables (see Figure 7).

5.3. Solving the Model

Our solution method requires agents' forecasts of the future values of shocks. The other shocks are assumed to be stochastic, so we have to specify their joint distribution. To this end, we describe the state space - observer form for the stochastic shocks. Let the state associated with the exogenous shocks be denoted by the $M \times 1$ vector ξ_t. The law of motion of ξ_t has the following form,

$$\xi_t = F\xi_{t-1} + \varepsilon_t,$$

[12] In our calculations we assume that the investment specific technology shock simply remains on its steady state growth path after 2008. We plan to relax this assumption in subsequent drafts. .

where ε_t is an iid sequence of shocks, uncorrelated with ξ_{t-1}. Let y_t denote the 3×1 vector of exogenous variables that are observed by agents:

$$y_t = \begin{pmatrix} \log(g_t/g) \\ \Gamma_t \\ \mu_{z,t} - \mu_z \end{pmatrix},$$

where g and μ_z represent the steady state values of g_t and $\mu_{z,t}$ ($\equiv \log(z_t/z_{t-1})$) respectively. We assume that the steady state value of Γ_t is zero. This vector is related to the state by the $3 \times M$ matrix H :

$$y_t = H\xi_t.$$

We posit that each element of y_t is the sum of two stochastic processes, one of which is much more persistent than the other. We assume that agents only observe y_t and its past history. Our solution strategy requires that we compute $P[y_{t+j}|y_{t-s}, s \geq 0]$ for $j > 0$. We do this using standard Kalman filtering formulas (see the appendix for details). We assume that the only shocks operating in the post-2008 period are the ones in y_t. An additional source of variation comes from the deterministic sequence of consumption wedges, Δ_t^b.

We now turn to the equilibrium conditions of the model. Three of the equilibrium conditions are different from the ones used in the analysis of pre-2008 data. First, the monetary policy rule must be adjusted to respect the zero lower bound on the nominal rate of interest. Let Z_t denote a gross 'shadow' rate of interest, which satisfies the following Taylor-style monetary policy rule:[13]

$$\begin{aligned} \ln(Z_t/R) &= \rho_R \ln(Z_{t-1}/R) + (1-\rho_R)\left[r_\pi \ln(\pi_t/\pi) + r_y \ln(\mathcal{Y}_t/\mathcal{Y})\right], \\ R_t &= \max\{1, Z_t\}. \end{aligned}$$

The other two changes reflect the introduction of the consumption and financial wedges. Finally, we add (5.2) to the system of equilibrium conditions.

Let ϱ_t denote the $N \times 1$ vector of period t endogenous variables. We express the equilibrium conditions of the model as follows:

$$E\left[f\left(\varrho_{t+1}, \varrho_t, \varrho_{t-1}, y_t, y_{t+1}, \Delta_t^b, \Delta_{t+1}^b\right)|\Omega_t\right] = 0, \tag{5.3}$$

where the information set is given by

$$\Omega_t = \left\{\varrho_{t-1-j}, y_{t-j},\ j \geq 0\right\}.$$

Our solution strategy proceeds as follows. Consistent with our discussion above we fix a set of values for y_t for the period after 2008Q2. We suppose that at date t agents observe

[13] Although this is a formulation of policy is often used in the analysis of the zero lower bound, it does not allow us to study 'forward guidance'. By forward guidance we mean that the interest rate is kept at its lower bound until certain endogenous variables cross specified thresholds.

y_{t-s}, $s \geq 0$ for each t after 2008Q2. At each such date t, they compute forecasts, y_{t+1}^t, y_{t+2}^t, $y_{t+2}^t, ...$, of the future values of y_t using the techniques that are appropriate given the state space - observer setup described above. It is convenient to use the notation $y_t^t \equiv y_t$.

We adopt an analogous notation for ϱ_t. In particular, denote the expected value of ϱ_{t+j} formed at time t by ϱ_{t+j}^t, where $\varrho_{t+j}^t \equiv \varrho_t$ for $j = 0$. The equilibrium value of ϱ_t is the first element in the sequence, ϱ_{t+j}^t, $j \geq 0$. To compute this sequence we require y_{t+j}^t, $j \geq 0$, and ϱ_{t-1}. For t greater than 2008Q3 $\varrho_{t-1} = \varrho_{t-1}^{t-1}$. For t corresponding to 2008Q3, we simply set ϱ_{t-1} to its non-stochastic steady state value. We now discuss how we computed ϱ_{t+j}^t, $j \geq 0$. We do so by solving the equilibrium conditions and imposing certainty equivalence. In particular, ϱ_t^t must satisfy:

$$\begin{aligned} & E\left[f\left(\varrho_{t+1}, \varrho_t, \varrho_{t-1}, y_t, y_{t+1}, \Delta_t^b, \Delta_{t+1}^b\right)|\Omega_t\right] \\ \simeq \quad & f\left(\varrho_{t+1}^t, \varrho_t^t, \varrho_{t-1}^{t-1}, y_t^t, y_{t+1}^t, \Delta_t^b, \Delta_{t+1}^b\right) = 0. \end{aligned}$$

Evidently, to solve for ϱ_t^t requires ϱ_{t+1}^t. Relation (5.3) implies:

$$\begin{aligned} & E\left[f\left(\varrho_{t+2}, \varrho_{t+1}, \varrho_t, y_{t+1}, y_{t+2}, \Delta_{t+1}^b, \Delta_{t+2}^b\right)|\Omega_t\right] \\ \simeq \quad & f\left(\varrho_{t+2}^t, \varrho_{t+1}^t, \varrho_t^t, y_{t+1}^t, y_{t+2}^t, \Delta_{t+1}^b, \Delta_{t+2}^b\right) = 0. \end{aligned}$$

Proceeding in this way, we obtain a sequence of equilibrium conditions involving ϱ_{t+j}^t, $j \geq 0$. Solving for this sequence requires a terminal condition. We obtain this condition by imposing that ϱ_{t+j}^t converges to the non-stochastic steady state value of ϱ_t.

5.4. Results

Our results are reported in Figures 7-13. Figure 7 displays the simulation of our model when the shocks are chosen in the manner discussed above. The key result is that the model does a very good job at accounting for the behavior of 9 endogenous variables in the post 2008 period. Notice in particular that the model is able to account for the modest decline in real wages despite the absence of nominal rigidities in wage setting. Also, notice that the model accounts very well for the average level of inflation despite the fact that we assume only a moderate degree of price stickiness. Firms change prices on average once a year. Finally, model also accounts well for the key labor market variables: labor force participation, employment and unemployment.

The subsequent figures decompose the impact of different shocks in the Figure 7 results. We determine the role of a shock by setting that shock to its steady state value and redoing the simulations underlying Figure 7. Figure 8 displays the effect of TFP. For convenience, the solid line reproduces the corresponding solid line in Figure 7. The dashed line displays the behavior of the economy when TFP shocks are shut down (i.e., $\varepsilon_t^p = 0$ in 2008Q3).

Comparing the solid and dashed lines, we see that the TFP shocks certainly contributed to the decline in aggregate economic activity in the Great Recession. But, those contributions were relatively small. By contrast, the TFP slowdown had a major impact on inflation. Had it not been for the TFP decline, there would have been substantial deflation, as predicted by very simple New Keynesian models that do not allow for a drop in TFP. This result is anticipated by the VARs estimated in the pre-2008 period. This is because a positive neutral technology shock (e.g., TFP) has a very substantial and sharply estimated negative impact on inflation. The pre-2008 model accounts for that fact.

Medium sized DSGE models often abstract from the working capital channel. A natural question is how important is that channel in allowing our model to account for the moderate degree of inflation during the Great Recession. To answer that question, we redo the simulation underlying Figure 7, replacing (5.1) with (2.18). The results are displayed in Figure 9. The key result is that the increased interest rate in the working capital channel plays a critical role in the model to account the moderate amount of inflation that occurred during the Great Recession.

Figures 10 and 11 report the effects of the financial and consumption wedges, respectively. These shocks play a vital role in driving the economy into the ZLB. They also account for the vast bulk of movements in real quantities. The financial wedge is overwhelmingly the most important shock for investment. At the same time, the consumption wedge plays a very important in accounting for the large drop in consumption. Notice that the model attributes the substantial drop in the labor force participation rate is almost entirely to the consumption and financial wedges. In this way, the model is consistent with the fact that labor force participation rates are not very cyclical during normal recessions, while being very cyclical during the Great Recession.

We now turn to Figures 12, which displays the role of government consumption in the Great Recession. Government consumption passes through two phases. The first occurs at the beginning and involves the expansion associated with the American Recovery and Reinvestment Act of 2009. The second phase involves a contraction that began around the beginning of 2011. The first phase involves a maximum rise of 3 percent in government consumption (i.e., 0.6 percent of GDP) and a maximum rise of 1 percent in GDP. This implies a maximum government consumption multiplier of 1/.6 or 1.67. In the second phase it is clear that the government spending, while contractionary, has a very small multiplier effect on output. It is difficult to attribute the long duration of the Great Recession to the recent decline in government consumption. These findings are consistent with results reported in Christiano, Eichenbaum and Rebelo (2011). They show that a rise in government consumption that is expected to not extend beyond the ZLB has a large multiplier effect. They also show that a rise in government consumption that is expected to extend beyond the

ZLB has a relatively small multiplier effect. A feature of our simulations is that the increase in government consumption in the first phase is never expected by agents to persist beyond the ZLB. But, the second phase in which there is a decrease in government consumption does cause agents to expect that decrease to persist beyond the end of the ZLB.

6. Conclusion

This paper argues that the vast bulk of movements in aggregate real economic activity during the Great Recession were due to financial frictions interacting with the zero lower bound. We reach this conclusion looking through the lens of a New Keynesian model in which firms face moderate degrees of price rigidities and no nominal rigidities in the wage setting process. Our model does a very good job of accounting for the joint behavior of labor and goods markets, as well as inflation, during the Great Recession. According to the model the observed fall in TFP played a critical role in accounting for the small size of the drop in inflation that occured during the Great Recession.

References

[1] Aguiar, Mark, Erik Hurst and Loukas Karabarbounis, 2012, "Time Use During the Great Recession," *American Economic Review*, forthcoming.

[2] Andolfatto, D., 1996, "Business Cycles and Labor Market Search," *American Economic Review*, 86(1), March, pp. 112-132.

[3] Bartsch, Kristina J., 2009, The employment projections for 2008–18, *Monthly Labor Review*, November.

[4] Binmore, Ken, 1985, "Bargaining and Coalitions," in A. Roth, ed., Game-Theoretic Models of Bargaining, Cambridge: Cambridge University Press, 1985.

[5] Binmore, Ken, Ariel Rubinstein, and Asher Wolinsky, 1986, "The Nash Bargaining Solution in Economic Modelling," *RAND Journal of Economics*, 17(2), pp. 176-88..

[6] Christiano, Lawrence J., Cosmin Ilut, Roberto Motto and Massimo Rostagno, 2008, "Monetary Policy and Stock Market Boom-Bust Cycles," European Central Bank Working Paper no. 955, October.

[7] Christiano, Lawrence J., Cosmin Ilut, Roberto Motto and Massimo Rostagno, 2011, "Monetary Policy and Stock Market Booms," in Macroeconomic Challenges: the Decade Ahead, Federal Reserve Bank of Kansas City (Policy Symposium, Jackson Hole Wyoming).

[8] Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans, 1999, "Monetary Policy Shocks: What Have We Learned and to What End?," *Handbook of Macroeconomics*, edited by John B. Taylor and Michael Woodford, pages 65-148, Amsterdam, New York and Oxford: Elsevier Science, North-Holland.

[9] Christiano, Lawrence J., Martin Eichenbaum and Charles L. Evans, 2005, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), pp. 1-45.

[10] Christiano, Lawrence J., Mathias Trabandt and Karl Walentin, 2011, "DSGE Models for Monetary Policy Analysis," in Benjamin M. Friedman, and Michael Woodford, editors: *Handbook of Monetary Economics*, Vol. 3A, The Netherlands: North-Holland.

[11] Christiano, Lawrence J., Mathias Trabandt and Karl Walentin, 2011b, "Introducing Financial Frictions and Unemployment into a Small Open Economy Model," *Journal of Economic Dynamics and Control*, 35(12), pp. 1999-2041.

[12] Christiano, Lawrence J., Mathias Trabandt and Karl Walentin, 2012, "Involuntary Unemployment and the Business Cycle," manuscript, Northwestern University, 2012.

[13] den Haan, Wouter, Garey Ramey, and Joel Watson, 2000, "Job Destruction and Propagation of Shocks," *American Economic Review*, 90(3), pp. 482-98.

[14] Diamond, Peter A., 1982, "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy*, 90(5), pp. 881-894.

[15] Eichenbaum, Martin, Nir Jaimovich and Sergio Rebelo, 2011, "Reference Prices and Nominal Rigidities,", *American Economic Review*, 101(1), pp. 242-272.

[16] Eggertsson, Gauti and Michael Woodford "The zero interest-rate bound and optimal monetary policy." Brookings Panel on Economic Activity, 2003.

[17] Erceg, Christopher J., Henderson, Dale, W. and Andrew T. Levin, 2000, "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 46, pp. 281-313.

[18] Erceg, Christopher J., and Andrew T. Levin,' Labor Force Participation and Monetary Policy in the Wake of the Great Recession', manuscript, 2013.

[19] Fernald, John, 2012, "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity," Manuscript, Federal Reserve Bank of San Francisco.

[20] Fisher, Jonas, 2006, "The Dynamic Effects of Neutral and Investment-Specific Technology Shocks," *Journal of Political Economy*, 114(3), pp. 413-451.

[21] Fujita, Shigeru and Gary Ramey, 2009, "The Cyclicality of Separation and Job Finding Rates", *International Economic Review*, 50(2), pp. 415-430.

[22] Gilchrist, Simon and Egon Zakrajšek, 2012, Credit Spreads and Business Cycle Fluctuations, *American Economic Review*, 102(4): 1692–1720.

[23] Hall, Robert E., 2005, "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, 95(1), pp. 50-65.

[24] Hall, Robert E. and Paul R. Milgrom, 2008, "The Limited Influence of Unemployment on the Wage Bargain," *The American Economic Review*, 98(4), pp. 1653-1674.

[25] Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti, 2010, "Investment Shocks and Business Cycles," *Journal of Monetary Economics* 57(2), pp. 132-145.

[26] Klenow, Pete and Benjamin Malin, 2011, "Microeconomic Evidence on Price-Setting," in the Handbook of Monetary Economics 3A, editors: B. Friedman and M. Woodford, Elsevier, pp. 231-284.

[27] Lorenzoni, Guido and Veronica Guerrieri, Credit Crises, Precautionary Savings and the Liquidity Trap," (2012).

[28] Merz, M., 1995, "Search in the Labor Market and the Real Business Cycle," *Journal of Monetary Economics*, 36(2), November, pp. 269-300.

[29] Mortensen, Dale T., 1982, "Property Rights and Efficiency in Mating, Racing, and Related Games," *American Economic Review*, 72(5), pp. 968–79.

[30] Nakamura, Emi and Jon Steinsson, 2012, "Price Rigidity: Microeconomic Evidence and Macroeconomic Implications," manuscript, Columbia University.

[31] Paciello, Luigi, (2011), "Does Inflation Adjust Faster to Aggregate Technology Shocks than to Monetary Policy Shocks," *Journal of Money, Credit and Banking*, 43(8).

[32] Pissarides, Christopher A., 1985, "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages," *American Economic Review*, 75(4), pp. 676-90.

[33] Ravenna, Federico and Carl Walsh, 2008, "Vacancies, Unemployment, and the Phillips Curve," *European Economic Review,* 52, pp. 1494-1521.

[34] Rogerson, Richard, and Johanna Wallenius, 2009, "Micro and Macro Elasticities in a Life Cycle Model with Taxes," *Journal of Economic Theory*, 144(6), pp. 2277-2292.

[35] Rubinstein, Ariel, 1982, "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50(1), pp. 97-109.

[36] Schmitt-Grohé, Stephanie, and Martin Uribe, 2012, "What's News in Business Cycles?," *Econometrica*, 80, pp. 2733-2764.

[37] Shimer, Robert, 2005a, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *The American Economic Review*, 95(1), pp. 25-49.

[38] Shimer, Robert, 2005b, "The Cyclicality of Hires, Separations, and Job-to-Job Transitions," *Federal Reserve Bank of St. Louis Review.*

[39] Shimer, Robert, 2012a, "Reassessing the Ins and Outs of Unemployment," *Review of Economic Dynamics*, 15(2), pp. 127-48.

[40] Shimer, Robert, 2012b, "Job Search, Labor Force Participation, and Wage Rigidities," in Daron Acemoglu, Manuel Arellano, and Eddie Dekel, eds., Advances in Economics and Econometrics: Theory and Applications, Tenth World Conference, forthcoming.

[41] Sims, Christopher A. and Tao Zha, 2006, "Were There Regime Switches in U.S. Monetary Policy?," *American Economic Review*, 96(1), pp. 54-81.

[42] Smets, Frank and Raf Wouters, 2003, "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, MIT Press, 1(5), pp. 1123-1175.

[43] Smets, Frank and Rafael Wouters, 2007, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), pp. 586-606.

[44] Walsh, Carl, 2003, "Labour Market Search and Monetary Shocks," Chapter 9 in *Dynamic Macroeconmic Analyis, Theory and Policy in General Equilibrium*, edited by Sumru Altug, Jagjit S. Chadha and Charles Nolan, Cambridge University Press..

[45] Yun, Tack, 1996, "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics*, 37(2), pp. 345-370.

[46]

Table 1: Non-Estimated Model Parameters and Calibrated Variables

Parameter	Value	Description
	Panel A: Parameters	
δ_K	0.025	Depreciation rate of physical capital
β	0.9968	Discount factor
ρ	0.9	Job survival probability
M	60	Max. bargaining rounds per quarter (alternating offers model)
Θ	3	Inverse elasticity of substitution
$400\log(\mu)$	1.7	Annual output per capita growth rate
$400\log(\mu \cdot \mu_\Psi)$	2.9	Annual investment per capita growth rate
	Panel B: Steady State Values	
$400(\pi - 1)$	2	Annual net inflation rate
$profits$	0	Intermediate goods producers profits
Q	0.7	Vacancy filling rate
u	0.05	Unemployment rate
L	0.67	Labor force to population ratio
G/Y	0.2	Government consumption to gross output ratio

Table 2: Priors and Posteriors of Model Parameters

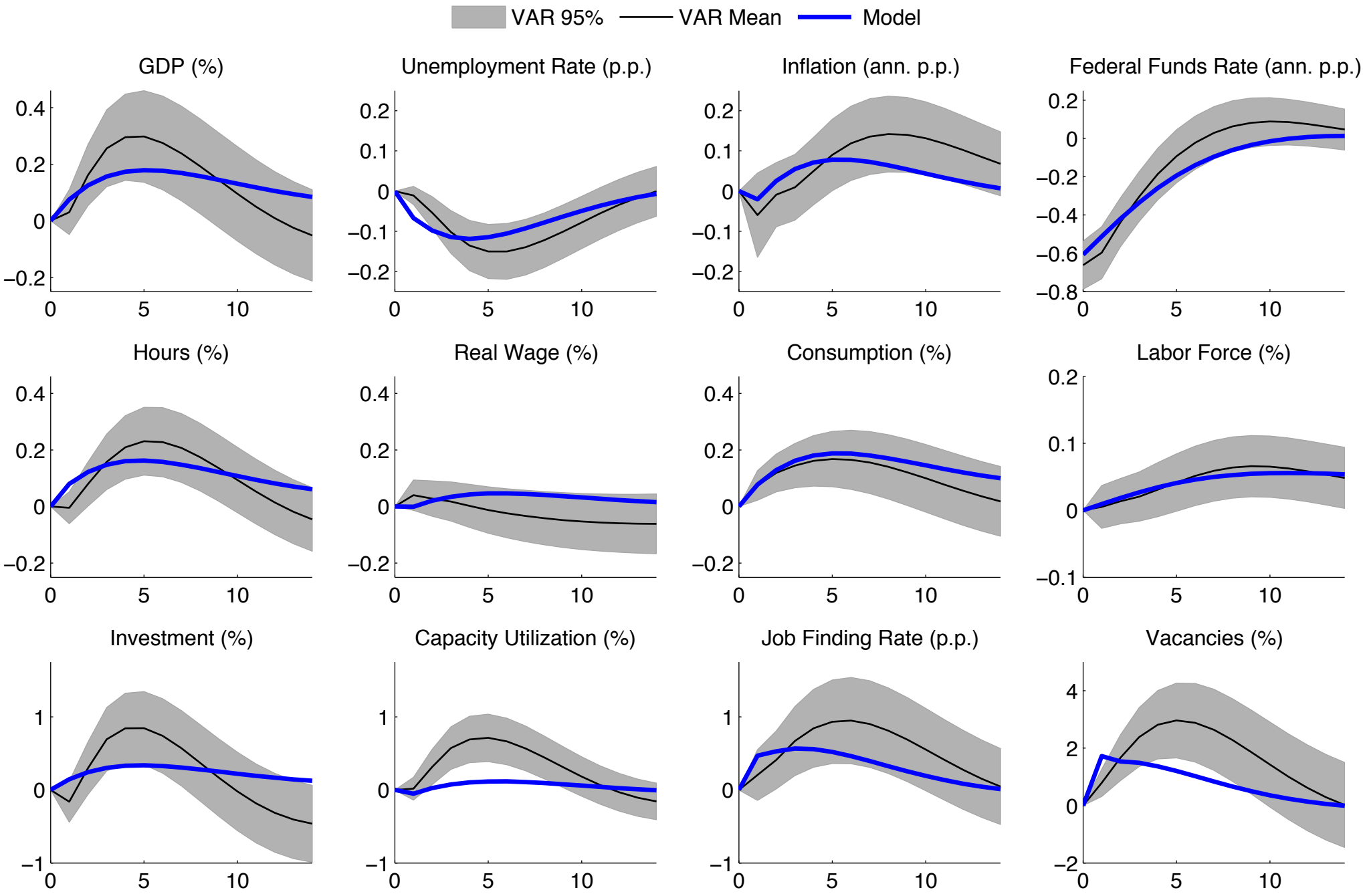
		Prior		Posterior	
		Distribution	Mean,Std.	Mode	Std.
Price Setting Parameters					
Price Stickiness	ξ	Beta	0.66,0.15	**0.702**	0.019
Price Markup Parameter	λ	Gamma	1.20,0.05	**1.294**	0.042
Monetary Authority Parameters					
Taylor Rule: Interest Rate Smoothing	ρ_R	Beta	0.70,0.15	**0.838**	0.016
Taylor Rule: Inflation Coefficient	r_π	Gamma	1.70,0.15	**1.514**	0.117
Taylor Rule: GDP Coefficient	r_y	Gamma	0.10,0.05	**0.036**	0.013
Preferences and Technology					
Market and Home Consumption Habit	b	Beta	0.50,0.15	**0.851**	0.019
Capacity Utilization Adjustment Cost	σ_a	Gamma	0.50,0.30	**0.077**	0.054
Investment Adjustment Cost	$S^{''}$	Gamma	8.00,2.00	**11.70**	1.596
Capital Share	α	Beta	0.33,0.03	**0.214**	0.016
Technology Diffusion	θ	Beta	0.50,0.20	**0.072**	0.015
Labor Market Parameters					
Probability of Bargaining Breakup	100δ	Gamma	0.50,0.40	**0.100**	0.027
Replacement Ratio	D/w	Beta	0.25,0.05	**0.328**	0.058
Hiring Cost to Output Ratio	s_l	Gamma	1.00,0.30	**0.479**	0.147
Labor Force Adjustment Cost	ϕ_L	Gamma	125,20.0	**190.8**	19.60
Unempl'd Share in Home Production	α_{c^H}	Beta	0.03,0.01	**0.013**	0.004
Probability of Staying in Labor Force	s	Beta	0.90,0.05	**0.735**	0.088
Matching Function Parameter	σ	Beta	0.50,0.10	**0.554**	0.038
Shocks					
Std. Monetary Policy	σ_R	Gamma	0.65,0.05	**0.607**	0.035
Std. Neutral Technology	σ_{μ_z}	Gamma	0.10,0.05	**0.131**	0.016
Std. Invest. Technology	σ_Ψ	Gamma	0.10,0.05	**0.127**	0.019
AR(1) Invest. Technology	ρ_Ψ	Beta	0.75,0.10	**0.658**	0.062

Notes: s_l denotes the steady state hiring or search cost to gross output ratio (in percent).

Table 3: Model Steady States and Implied Parameters

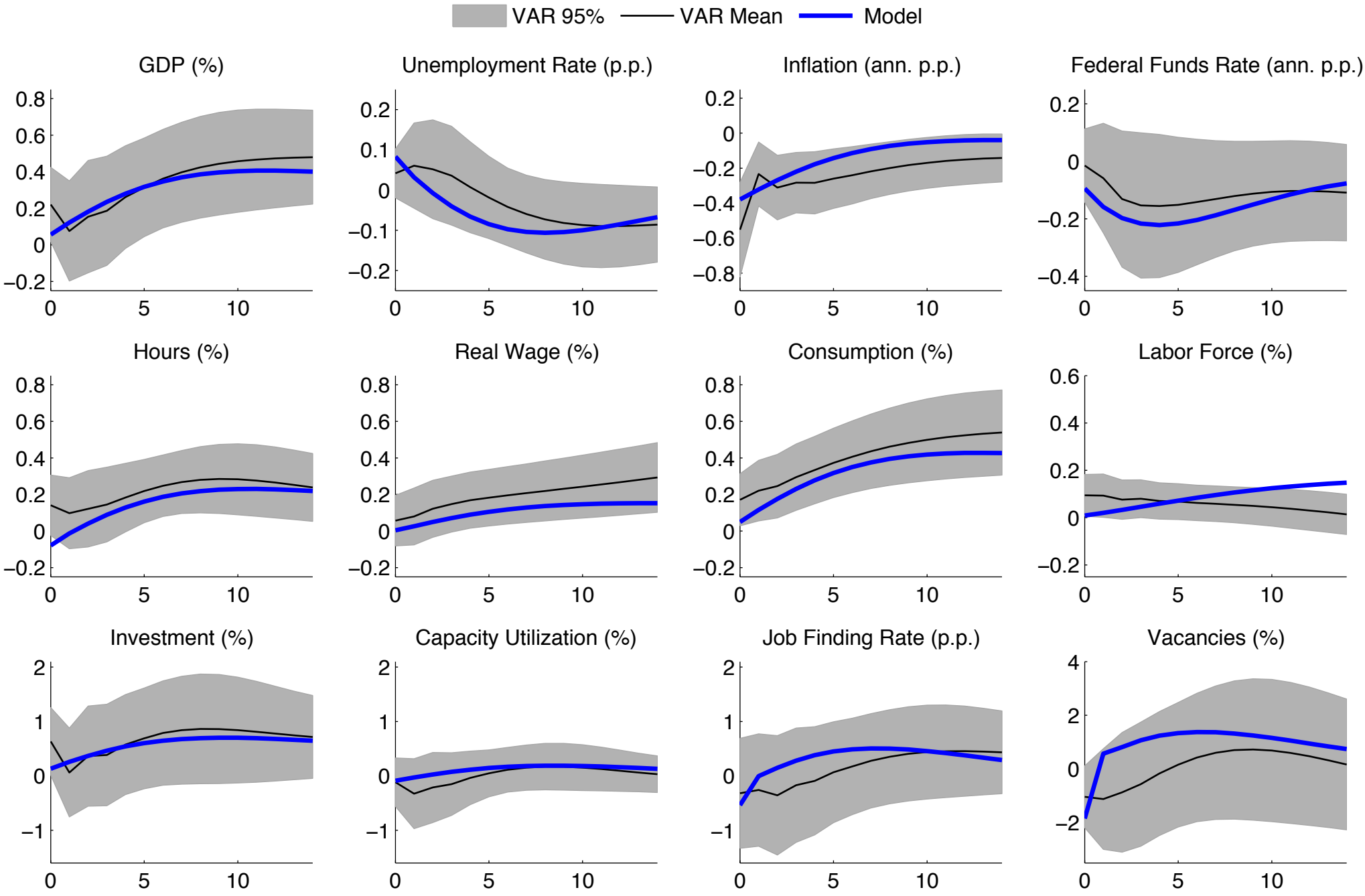
Variable	At Estimated Posterior Mode	Description
K/Y	6.07	Capital to gross output ratio (quarterly)
C/Y	0.60	Market consumption to gross output ratio
I/Y	0.20	Investment to gross output ratio
l	0.63	Employment to population ratio
R	1.0125	Gross nominal interest rate (quarterly)
R^{real}	1.0075	Gross real interest rate (quarterly)
mc	0.77	Marginal cost (inverse markup)
σ_b	0.036	Capacity utilization cost parameter
Y	0.74	Gross output
ϕ/Y	0.29	Fixed cost to gross output ratio
σ_m	0.66	Level parameter in matching function
f	0.63	Job finding rate
ϑ	0.91	Marginal revenue of wholesaler
x	0.1	Hiring rate
J	0.06	Value of firm
V	184.6	Value of work
U	181.2	Value of unemployment
N	176.3	Value of not being in the labor force
v	0.14	Vacancy rate
e	0.08	Probability of leaving non-participation
ω	0.97	Home consumption weight in utility
C^H	0.30	Home consumption
w	0.91	Real wage
$\gamma/(\vartheta/M)$	0.78	Counteroffer costs as share of daily revenue

Figure 1: Impulse Responses to a Monetary Policy Shock



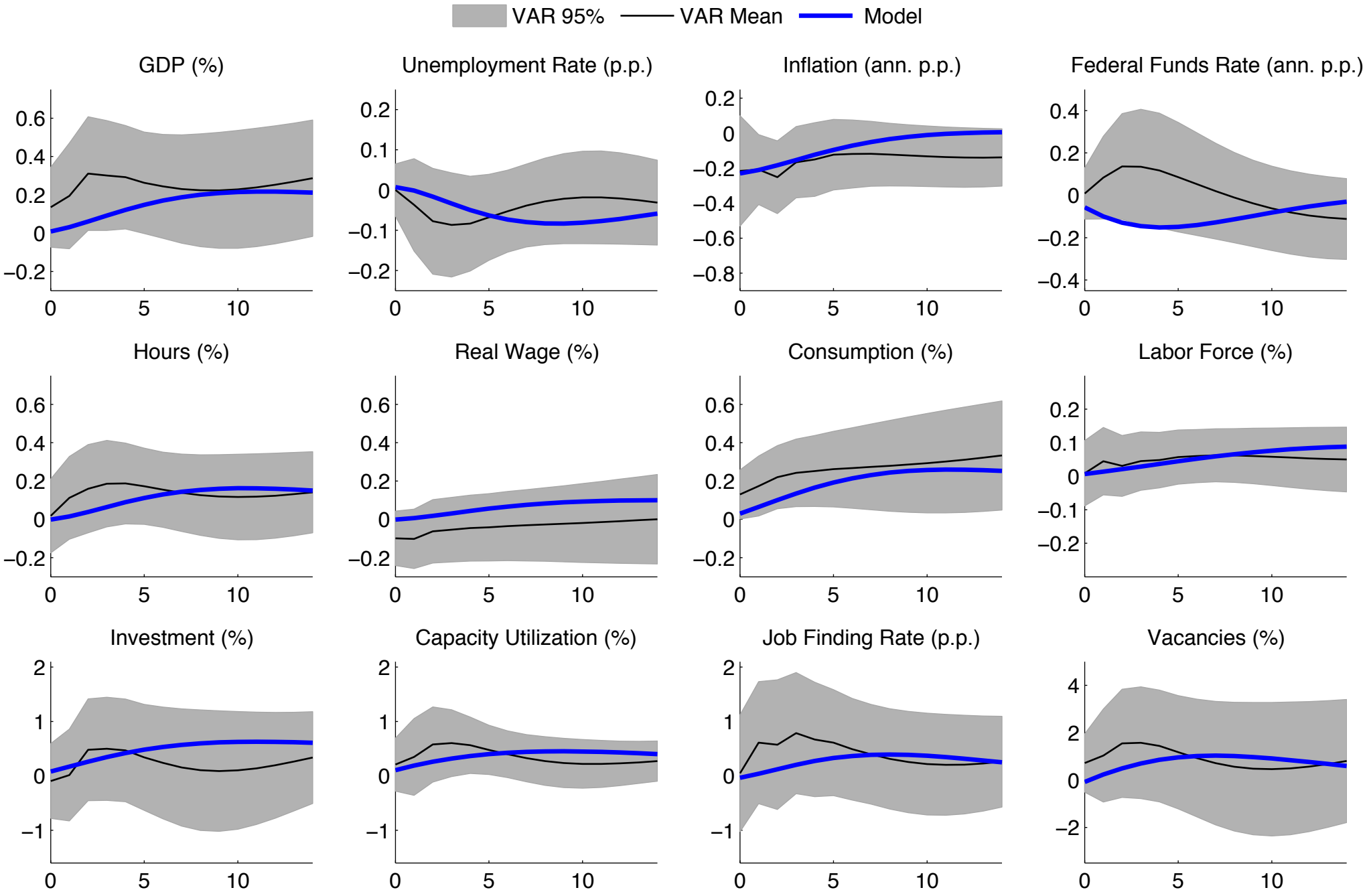
Notes: x–axis in quarters.

Figure 2: Impulse Responses to a Neutral Technology Shock



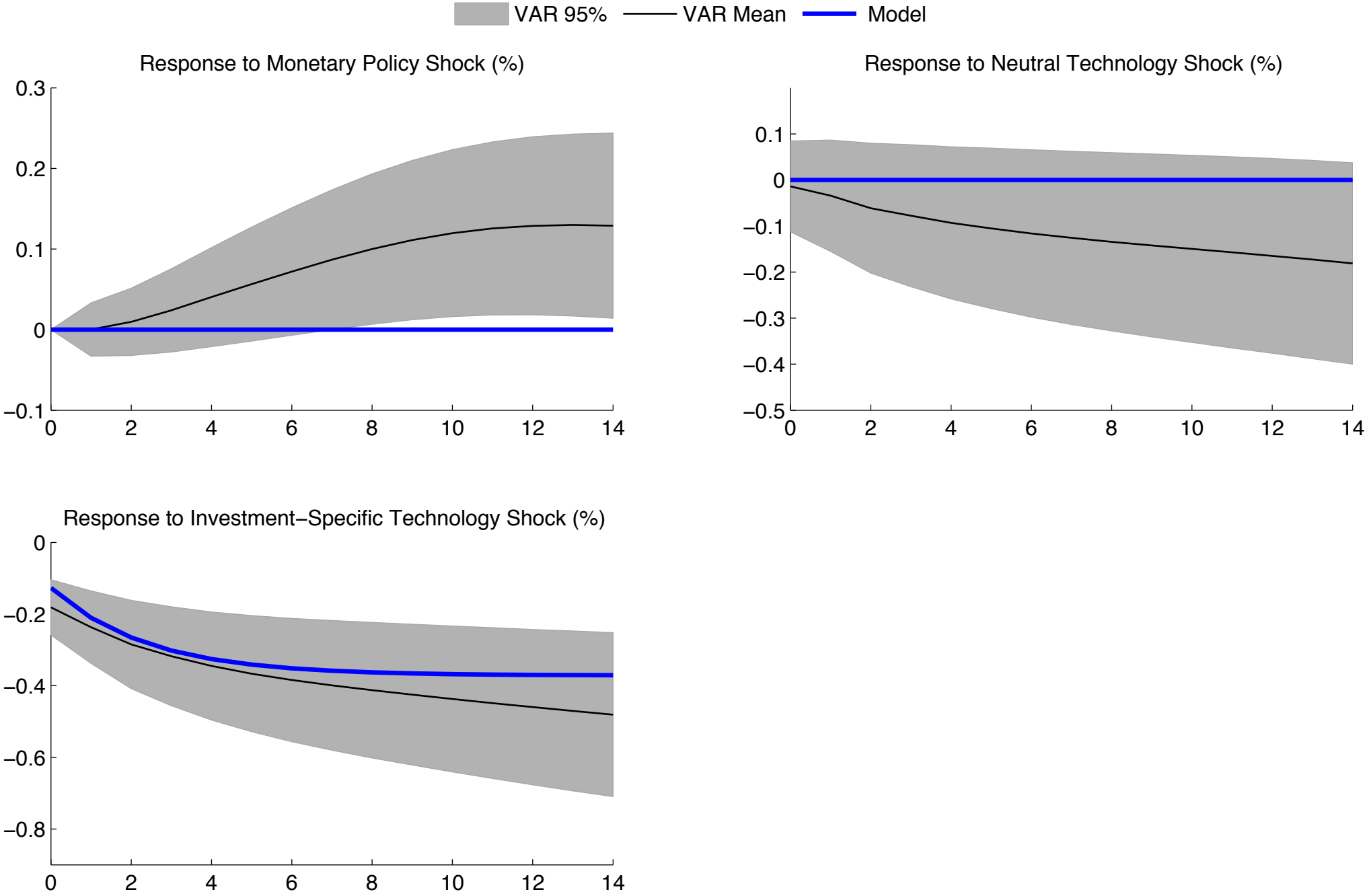
Notes: x−axis in quarters.

Figure 3: Impulse Responses to an Investment–Specific Technology Shock



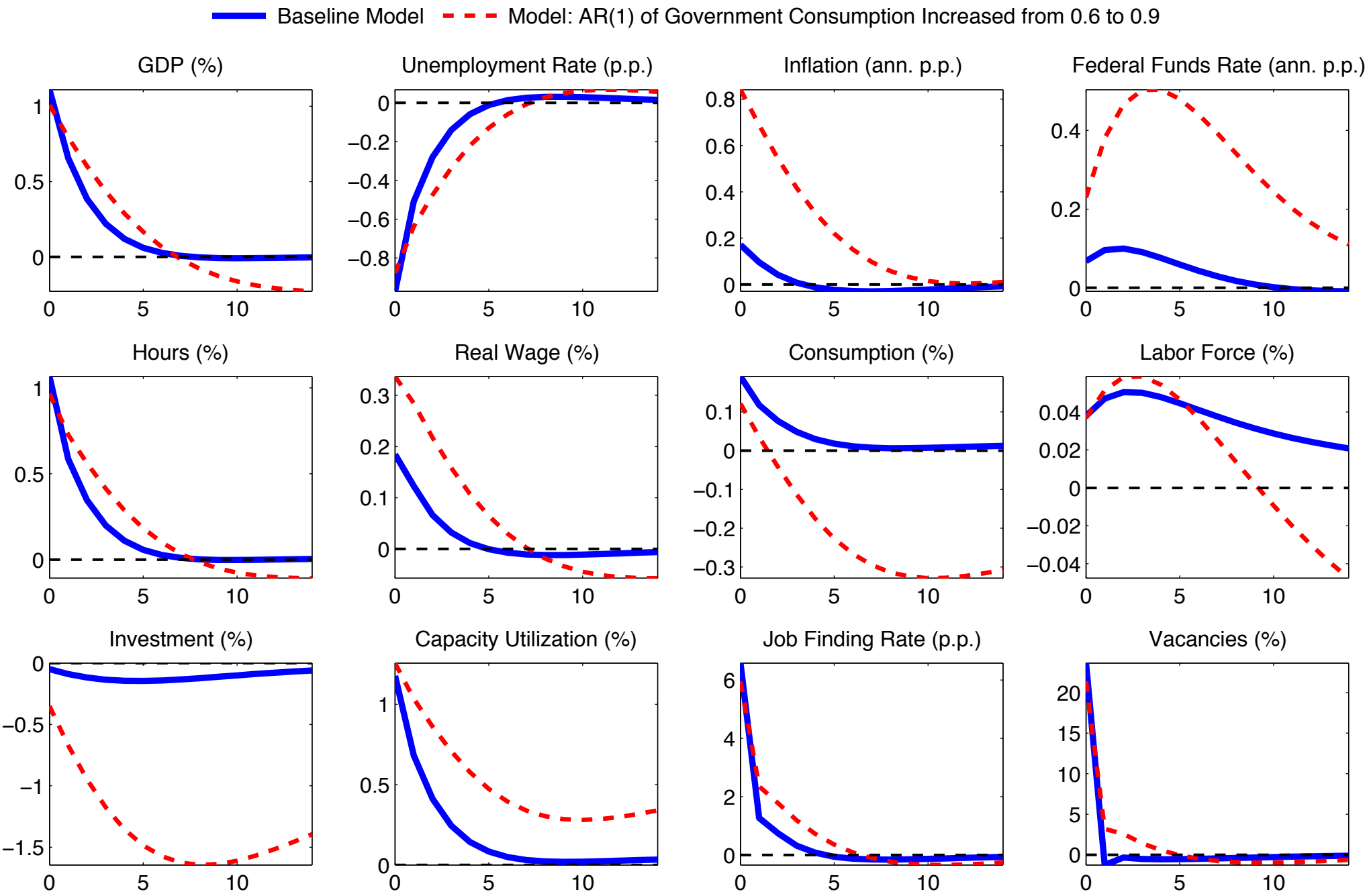
Notes: x–axis in quarters.

Figure 4: Impulse Responses of Relative Price of Investment



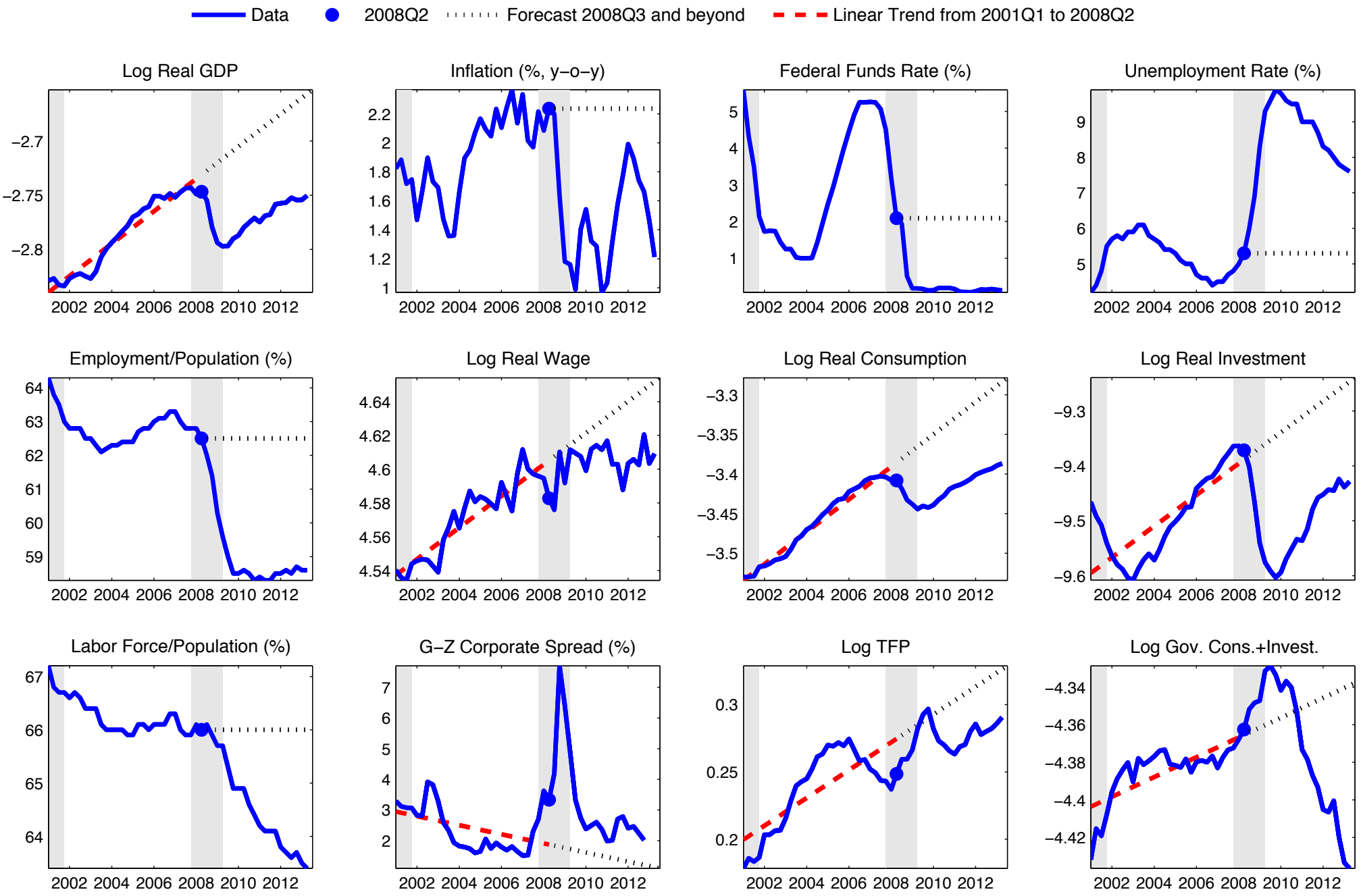
Notes: x−axis in quarters.

Figure 5: Impulse Responses to a Government Consumption Shock



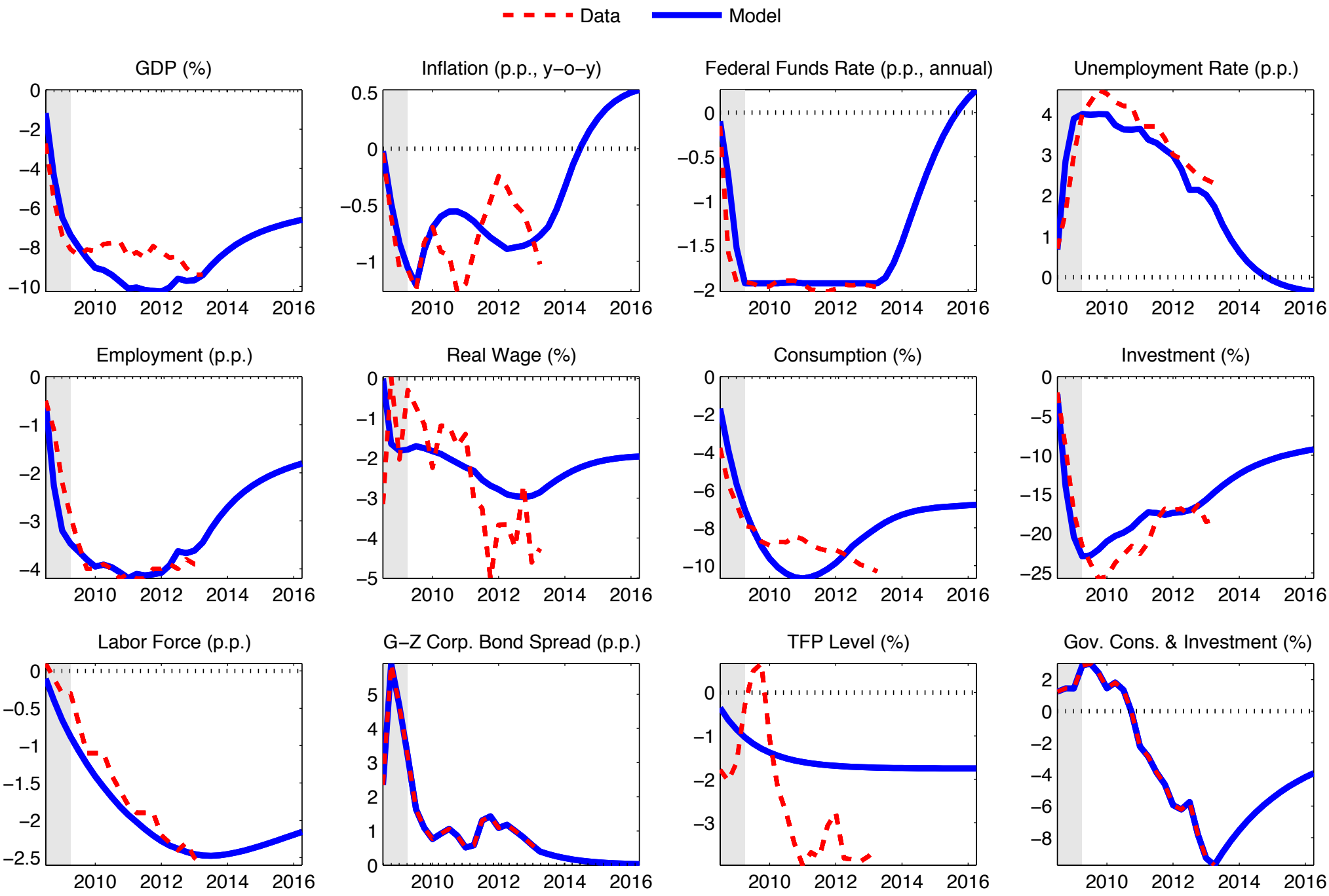
Notes: x–axis in quarters. Government consumption rises by 1% of steady state output in t=0; AR(1) is 0.6 or 0.9.

Figure 6: The Great Recession in the U.S.



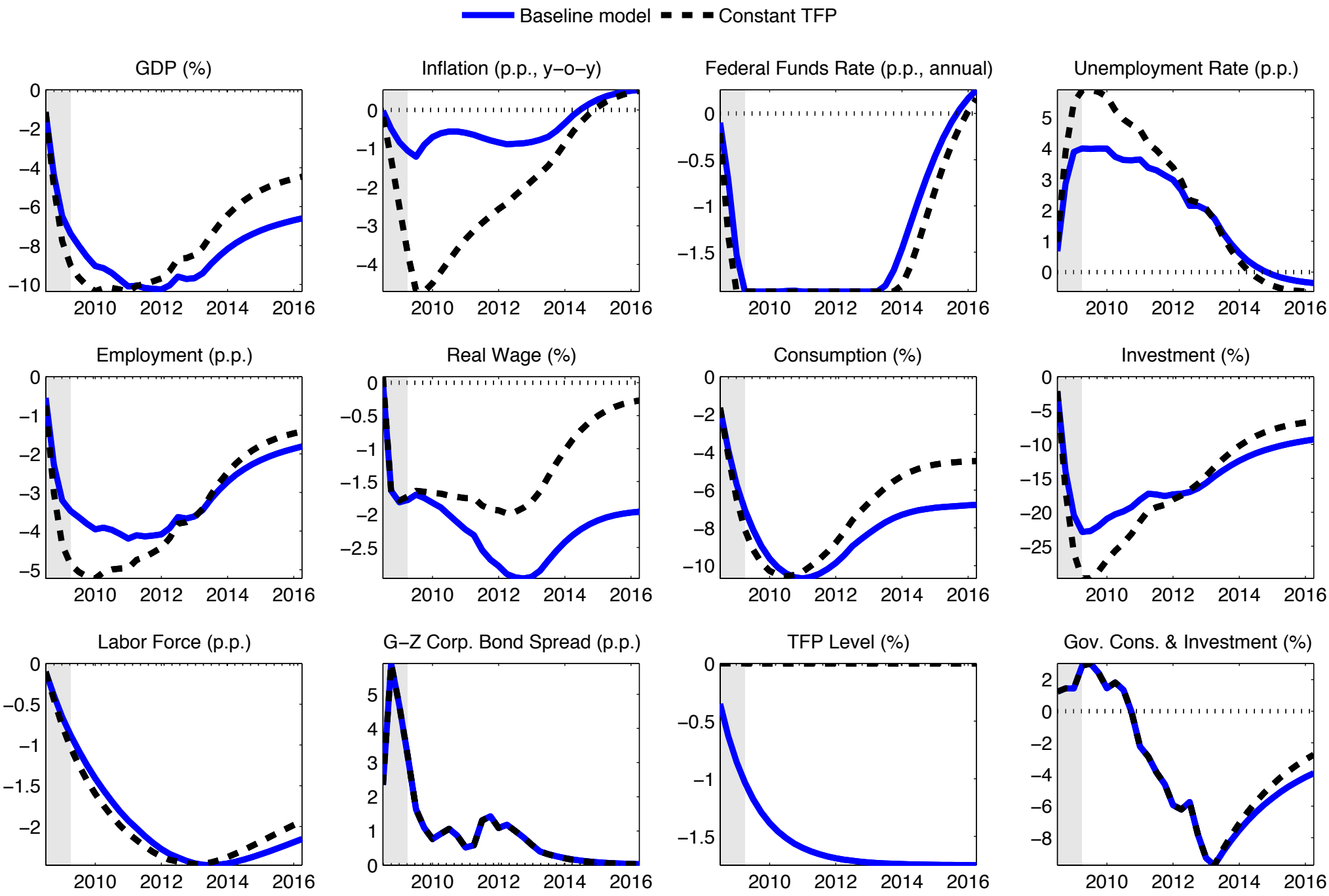
Notes: Gray areas indicate NBER recession dates.

Figure 7: The U.S. Great Recession: Data vs. Medium–sized Model



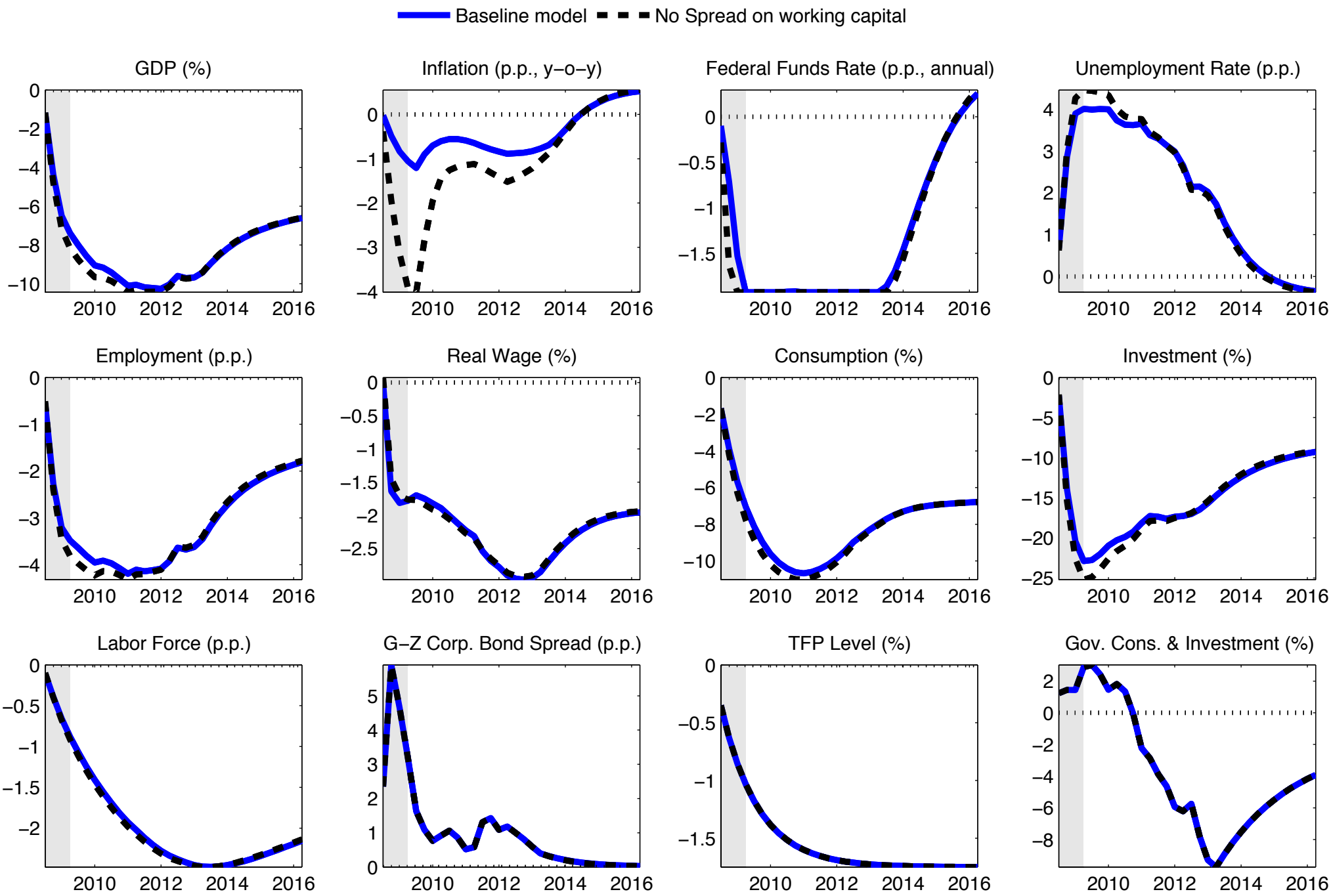
Notes: Data are the differences between raw data and forecasts, see Figure 6. Gray areas indicate NBER recession dates.

Figure 8: The U.S. Great Recession: Effects of TFP



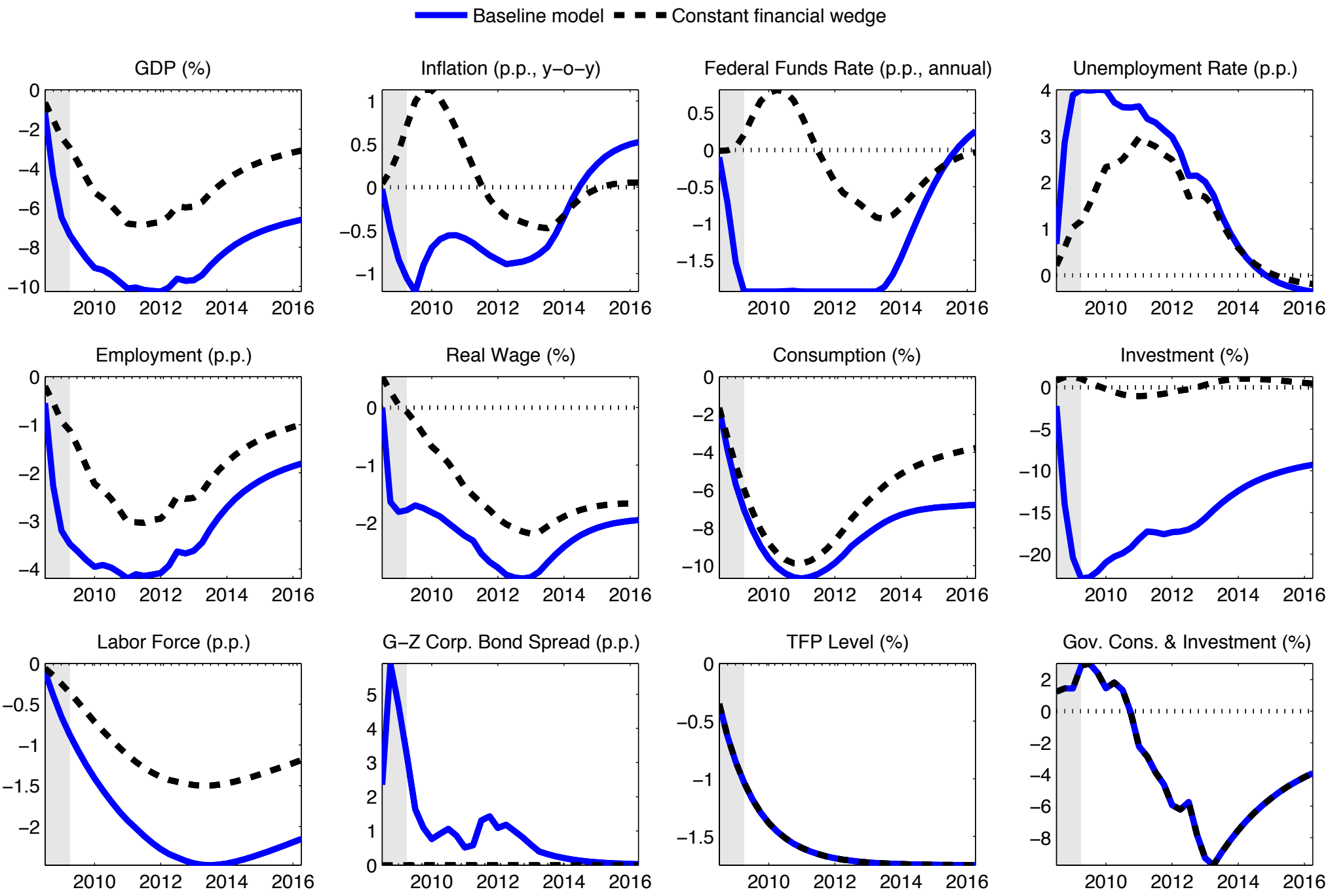
Notes: Baseline as in Figure 7. Gray areas are NBER recession dates.

Figure 9: The U.S. Great Recession: Effects of Spread on Working Capital



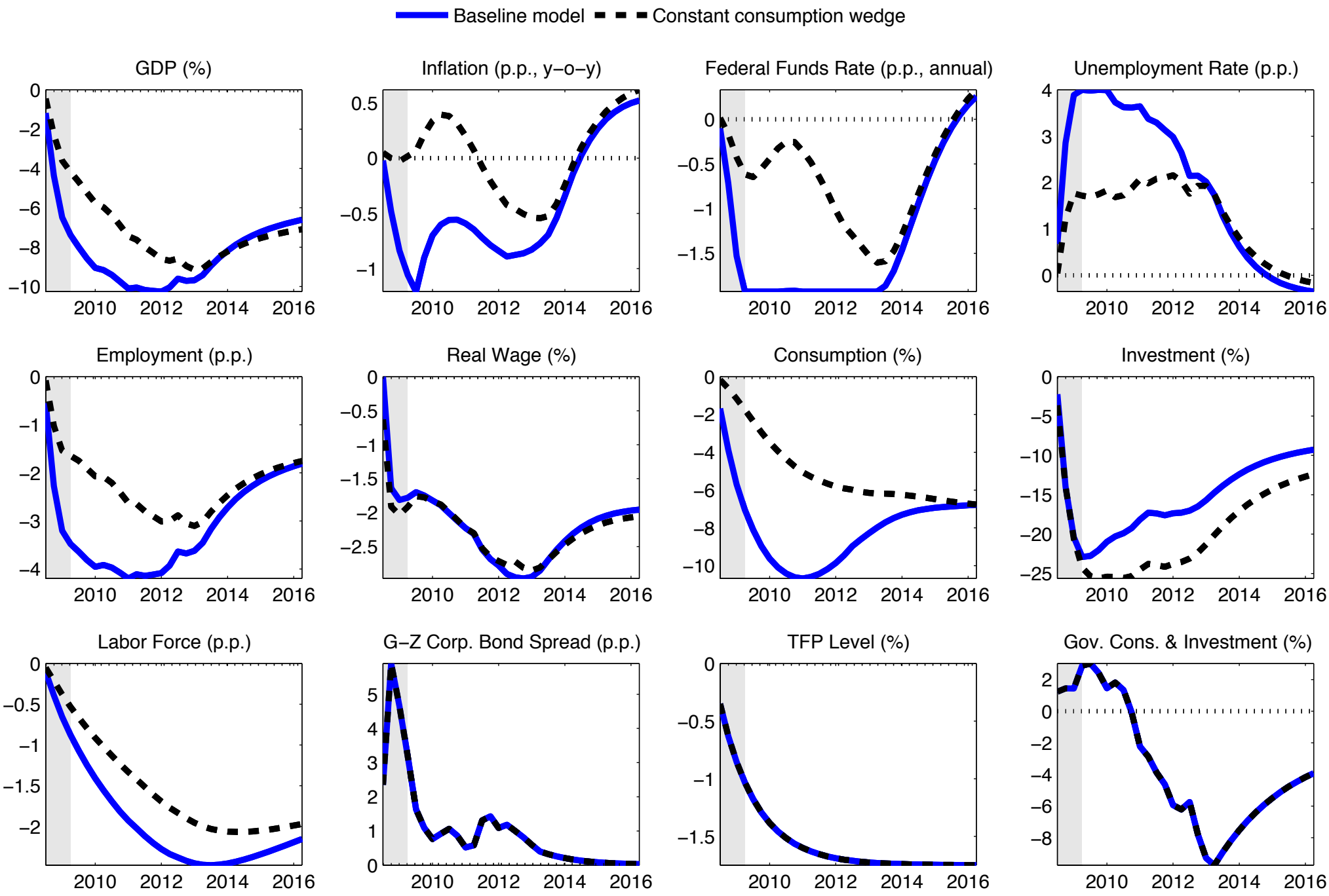
Notes: Baseline as in Figure 7. Gray areas are NBER recession dates.

Figure 10: The U.S. Great Recession: Effects of Financial Wedge



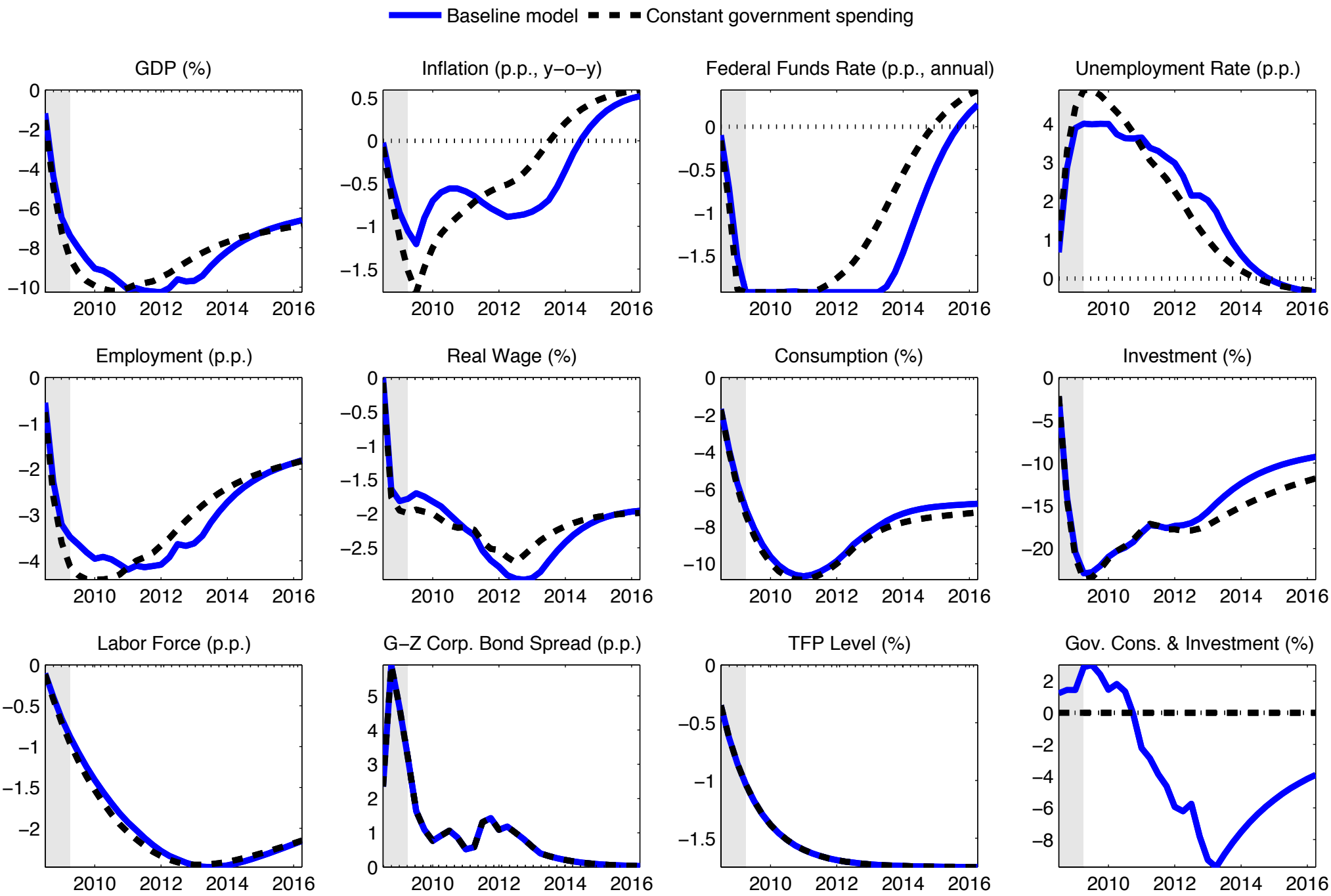
Notes: Baseline results as in Figure 7. Gray areas are NBER recession dates.

Figure 11: The U.S. Great Recession: Effects of Consumption Wedge



Notes: Baseline results as in Figure 7. Gray areas are NBER recession dates.

Figure 12: The U.S. Great Recession: Effects of Government Consumption & Investment



Notes: Baseline results as in Figure 7. Gray areas are NBER recession dates.

Technical Appendix
"Unemployment and the Great Recession"

Lawrence J. Christiano, Martin S. Eichenbaum, Mathias Trabandt

Abstract

In this technical appendix, we provide the derivations for the alternating offer bargaining sharing rule. In addition, we provide the equilibrium equations for the estimated medium-sized DSGE model developed in the main text.

A. The Equilibrium Wage Rate

TO BE EDITED. Note that D and γ need to be replaced by $\eta_t^D D$ and $\eta_t^\gamma \gamma$ everywhere below.

We develop an analytic expression relating the equilibrium wage rate to economy-wide variables taken as given by firms and workers when bargaining.

It is useful to re-state the indifference conditions for the worker and the firm given in the main text:

$$w_{j,t}+\tilde{w}_t^p+A_t=\delta\left[\frac{M-j+1}{M}D+\tilde{U}_t\right]+(1-\delta)\left[\frac{D}{M}+w_{j+1,t}+\tilde{w}_t^p+A_t\right] \qquad \text{for } j=1,3,..,M-1$$

$$\frac{M-j+1}{M}\vartheta_t+\tilde{\vartheta}_t^p-(w_{j,t}+\tilde{w}_t^p)=(1-\delta)\left[-\gamma+\frac{M-j}{M}\vartheta_t+\tilde{\vartheta}_t^p-(w_{j+1,t}+\tilde{w}_t^p)\right] \qquad \text{for} j=2,4,..,M-2$$

$$\frac{\vartheta_t}{M}+\tilde{\vartheta}_t^p-(w_{j,t}+\tilde{w}_t^p)=0 \qquad \text{for } j=M$$

Rewrite the previous expressions and abbreviate variables taken as given during the wage bargaining:

$$w_{j,t}+\tilde{w}_t^p=\underbrace{\frac{D}{M}+\delta\left(U_t-A_t\right)}_{a}-\underbrace{\frac{\delta D}{M}}_{c_j}j+(1-\delta)\left(w_{j+1,t}+\tilde{w}_t^p\right) \qquad \text{for } j=1,3,5,...,M-1$$

$$w_{j,t}+\tilde{w}_t^p=\underbrace{\frac{\vartheta_t}{M}+\delta\vartheta_t^p+(1-\delta)\gamma}_{b}-\underbrace{\frac{\delta\vartheta_t}{M}}_{d_j}j+(1-\delta)\left(w_{j+1,t}+\tilde{w}_t^p\right) \qquad \text{for } j=2,4,5,...,M-2$$

$$w_{j,t}+\tilde{w}_t^p=\left(\frac{1-M}{M}\right)\vartheta_t+\vartheta_t^p \qquad \text{for } j=M$$

Or, in short:

$$\begin{aligned} w_{j,t} + \tilde{w}_t^p &= a - c_j + (1-\delta)\left(w_{j+1,t} + \tilde{w}_t^p\right) \text{ for } j = 1,3,5,...,M-1 \\ w_{j,t} + \tilde{w}_t^p &= b - d_j + (1-\delta)\left(w_{j+1,t} + \tilde{w}_t^p\right) \text{ for } j = 2,4,5,...,M-2 \end{aligned}$$

Write out:

$$\begin{aligned} w_t^p = w_{1,t} + \tilde{w}_t^p &= a - c_1 + (1-\delta)\left(w_{2,t} + \tilde{w}_t^p\right) \\ w_{2,t} + \tilde{w}_t^p &= b - d_2 + (1-\delta)\left(w_{3,t} + \tilde{w}_t^p\right) \\ w_{3,t} + \tilde{w}_t^p &= a - c_3 + (1-\delta)\left(w_{4,t} + \tilde{w}_t^p\right) \\ w_{4,t} + \tilde{w}_t^p &= b - d_4 + (1-\delta)\left(w_{5,t} + \tilde{w}_t^p\right) \\ &... \\ w_{M-1,t} + \tilde{w}_t^p &= a - c_{M-1} + (1-\delta)\left(w_{M,t} + \tilde{w}_t^p\right) \end{aligned}$$

Substituting several times results in the following pattern:

$$\begin{aligned} w_t^p &= a + (1-\delta)^2 a + (1-\delta)^4 a + (1-\delta)^6 a \\ &\quad + (1-\delta) b + (1-\delta)^3 b + (1-\delta)^5 b \\ &\quad - c_1 - (1-\delta)^2 c_3 - (1-\delta)^4 c_5 - (1-\delta)^6 c_7 \\ &\quad - (1-\delta) d_2 - (1-\delta)^3 d_4 - (1-\delta)^5 d_6 \\ &\quad + (1-\delta)^7 \left(w_{8,t} + \tilde{w}_t^p\right) \end{aligned}$$

Rearrange:

$$\begin{aligned} w_t^p &= a + (1-\delta)^2 a + (1-\delta)^4 a + (1-\delta)^6 a + ... + (1-\delta)^{M-2} a \\ &\quad + (1-\delta) b + (1-\delta)^3 b + (1-\delta)^5 b + ... + (1-\delta)^{M-3} b \\ &\quad - c_1 - (1-\delta)^2 c_3 - (1-\delta)^4 c_5 - (1-\delta)^6 c_7 - ... - (1-\delta)^{M-2} c_{M-1} \\ &\quad - (1-\delta) d_2 - (1-\delta)^3 d_4 - (1-\delta)^5 d_6 - ... - (1-\delta)^{M-3} d_{M-2} \\ &\quad + (1-\delta)^{M-1} \left(w_{M,t} + \tilde{w}_t^p\right) \end{aligned}$$

Or, equivalently:

$$\begin{aligned} w_t^p &= a\left[1 + (1-\delta)^2 + (1-\delta)^4 + (1-\delta)^6 + ... + (1-\delta)^{M-2}\right] \\ &\quad b(1-\delta)\left[1 + (1-\delta)^2 + (1-\delta)^4 + (1-\delta)^6 + ... + (1-\delta)^{M-4}\right] \\ &\quad - c_1 - (1-\delta)^2 c_3 - (1-\delta)^4 c_5 - (1-\delta)^6 c_7 - ... - (1-\delta)^{M-2} c_{M-1} \\ &\quad - (1-\delta) d_2 - (1-\delta)^3 d_4 - (1-\delta)^5 d_6 - ... - (1-\delta)^{M-3} d_{M-2} \\ &\quad + (1-\delta)^{M-1}\left[\left(\frac{1-M}{M}\right)\vartheta_t + \vartheta_t^p\right] \end{aligned} \tag{A.1}$$

Note that:

$$\begin{aligned} S &= 1 + x + x^2 + x^3 + .. + x^n \\ xS &= x + x^2 + x^3 + .. + x^n + x^{n+1} \end{aligned}$$

Substract and rearrange:

$$S = \frac{1 - x^{n+1}}{1 - x}$$

So that:

$$1 + x + x^2 + x^3 + .. + x^n = \frac{1 - x^{n+1}}{1 - x} \tag{A.2}$$

Using (A.2), we can write the square brackets multiplying a and b in (A.1) as:

$$\left[1 + \underbrace{(1-\delta)^2}_{x} + \underbrace{(1-\delta)^4}_{x^2} + \underbrace{(1-\delta)^6}_{x^3} + ... + \underbrace{(1-\delta)^{M-2}}_{x^{(M-2)/2}}\right] = \frac{1 - (1-\delta)^M}{1 - (1-\delta)^2}$$

and

$$\left[1 + \underbrace{(1-\delta)^2}_{x} + \underbrace{(1-\delta)^4}_{x^2} + \underbrace{(1-\delta)^6}_{x^3} + ... + \underbrace{(1-\delta)^{M-4}}_{x^{(M-4)/2}}\right] = \frac{1 - (1-\delta)^{M-2}}{1 - (1-\delta)^2}$$

Hence,

$$\begin{aligned} w_t^p &= \frac{1 - (1-\delta)^M}{1 - (1-\delta)^2} a + b\,(1-\delta)\,\frac{1 - (1-\delta)^{M-2}}{1 - (1-\delta)^2} + (1-\delta)^{M-1}\left[\left(\frac{1-M}{M}\right)\vartheta_t + \vartheta_t^p\right] \\ &- \left[c_1 + (1-\delta)^2 c_3 + (1-\delta)^4 c_5 + ... + (1-\delta)^{M-2} c_{M-1}\right] \\ &- (1-\delta)\left[d_2 + (1-\delta)^2 d_4 + (1-\delta)^4 d_6 + ... + (1-\delta)^{M-4} d_{M-2}\right] \end{aligned} \tag{A.3}$$

The square bracket in the last line in (A.3) can be written as,

$$\begin{aligned} &\left[d_2 + (1-\delta)^2 d_4 + (1-\delta)^4 d_6 + ... + (1-\delta)^{M-4} d_{M-2}\right] \\ = \;& 2\frac{\delta\vartheta_t}{M}\left[1 + (1-\delta)^2 2 + (1-\delta)^4 3 + ... + (1-\delta)^{M-4}\,\frac{(M-2)}{2}\right] \end{aligned}$$

Note that differentiating both sides of:

$$1 + x + x^2 + x^3 + .. + x^n = \frac{1 - x^{n+1}}{1 - x}$$

yields

$$1 + 2x + 3x^2 + .. + nx^{n-1} = \frac{-(n+1)x^n\,(1-x) + (1 - x^{n+1})}{(1-x)^2}$$

Hence, the square bracket of the last line in (A.3) can be expressed more compactly as:

$$\begin{aligned} &\left[1 + \underbrace{(1-\delta)^2}_{x} 2 + \underbrace{(1-\delta)^4}_{x^2} 3 + \underbrace{(1-\delta)^6}_{x^3} 4 + ... + \underbrace{(1-\delta)^{M-4}}_{x^{(M-4)/2}} \underbrace{\frac{(M-2)}{2}}_{n}\right] \\ = \;& \frac{\left(1 - (1-\delta)^M\right) - \frac{M}{2}\,(1-\delta)^{(M-2)}\left(1 - (1-\delta)^2\right)}{\left(1 - (1-\delta)^2\right)^2} \end{aligned}$$

Finally, the terms involving c in (A.3) can be rewritten as:

$$\begin{aligned}
& \left[c_1 + (1-\delta)^2 c_3 + (1-\delta)^4 c_5 + (1-\delta)^6 c_7 + ... + (1-\delta)^{M-2} c_{M-1}\right] \\
= & \frac{\delta D}{M}\left[1 + (1-\delta)^2 3 + (1-\delta)^4 5 + (1-\delta)^6 7 + ... + (1-\delta)^{M-2}(M-1)\right] \\
= & \frac{\delta D}{M} 2\left[1/2 + (1-\delta)^2 2 + (1-\delta)^4 3 + (1-\delta)^6 4 + ... + (1-\delta)^{M-2} M/2\right] \\
& -\frac{\delta D}{M}\left[1 + (1-\delta)^2 + (1-\delta)^4 + (1-\delta)^6 + ... + (1-\delta)^{M-2}\right] + \frac{\delta D}{M} \\
= & 2\frac{\delta D}{M}\frac{\left(1-(1-\delta)^{M+2}\right) - (1+\frac{M}{2})(1-\delta)^M\left(1-(1-\delta)^2\right)}{\left(1-(1-\delta)^2\right)^2} \\
& -\frac{\delta D}{M}\frac{1-(1-\delta)^M}{1-(1-\delta)^2}
\end{aligned}$$

Pulling everything together, we can write (A.3) as:

$$\begin{aligned}
w_t^p = & \frac{1-(1-\delta)^M}{1-(1-\delta)^2}\left[\frac{D}{M} + \delta\left(U_t - A_t\right)\right] \\
& + (1-\delta)\frac{1-(1-\delta)^{M-2}}{1-(1-\delta)^2}\left[\frac{\vartheta_t}{M} + \delta\vartheta_t^p + (1-\delta)\gamma\right] \\
& -2\frac{\delta D}{M}\frac{\left(1-(1-\delta)^{M+2}\right) - (1+\frac{M}{2})(1-\delta)^M\left(1-(1-\delta)^2\right)}{\left(1-(1-\delta)^2\right)^2} \\
& +\frac{\delta D}{M}\frac{1-(1-\delta)^M}{1-(1-\delta)^2} \\
& -(1-\delta)2\frac{\delta\vartheta_t}{M}\frac{\left(1-(1-\delta)^M\right) - \frac{M}{2}(1-\delta)^{(M-2)}\left(1-(1-\delta)^2\right)}{\left(1-(1-\delta)^2\right)^2} \\
& +(1-\delta)^{M-1}\left[\left(\frac{1-M}{M}\right)\vartheta_t + \vartheta_t^p\right]
\end{aligned}$$

Collecting terms gives:

$$\begin{aligned} w_t^p &= \frac{1-(1-\delta)^M}{1-(1-\delta)^2}\delta\left(U_t - A_t\right) + \left[(1-\delta)\,\delta\frac{1-(1-\delta)^{M-2}}{1-(1-\delta)^2} + (1-\delta)^{M-1}\right]\vartheta_t^p \\ &+ (1-\delta)\,\frac{1-(1-\delta)^{M-2}}{1-(1-\delta)^2}\,(1-\delta)\,\gamma \\ &+ \left[\begin{array}{c} (1+\delta)\,\frac{1-(1-\delta)^M}{1-(1-\delta)^2} \\ -2\delta\frac{\left(1-(1-\delta)^{M+2}\right)-(1+\frac{M}{2})(1-\delta)^M\left(1-(1-\delta)^2\right)}{\left(1-(1-\delta)^2\right)^2} \end{array}\right]\frac{D}{M} \\ &+ \left[\begin{array}{c} (1-\delta)\,\frac{1-(1-\delta)^{M-2}}{1-(1-\delta)^2} + (1-\delta)^{M-1}\,(1-M) \\ -(1-\delta)\,2\delta\frac{\left(1-(1-\delta)^M\right)-\frac{M}{2}(1-\delta)^{(M-2)}\left(1-(1-\delta)^2\right)}{\left(1-(1-\delta)^2\right)^2} \end{array}\right]\frac{\vartheta_t}{M} \end{aligned}$$

Simplifying, using straightforward algebra yields:

$$\begin{aligned} (2-\delta)\,w_t^p &= \left(1-(1-\delta)^M\right)\left(U_t - A_t\right) + \left(1-\delta+(1-\delta)^M\right)\vartheta_t^p \\ &+ \frac{1}{\delta}\left((1-\delta)^2 - (1-\delta)^M\right)\gamma \\ &+ \frac{(1-\delta)^M\,(1-\delta-(2-\delta)\,M) - (1-\delta)}{2-\delta}\left[\frac{\vartheta_t}{M} - \frac{D}{M}\right] \end{aligned}$$

After some further rewriting, we can express the previous expression as the following alternating offer bargaining sharing rule:

$$(\alpha_1+\alpha_2)\,w_t^p = \alpha_1\vartheta_t^p + \alpha_2\left(U_t - A_t\right) + \alpha_3\gamma - \alpha_4\left(\vartheta_t - D\right)$$

where

$$\begin{aligned} \alpha_1 &= 1-\delta+(1-\delta)^M \\ \alpha_2 &= 1-(1-\delta)^M \\ \alpha_3 &= \alpha_2\frac{1-\delta}{\delta} - \alpha_1 \\ \alpha_4 &= \frac{1-\delta}{2-\delta}\frac{\alpha_2}{M} + 1 - \alpha_2. \end{aligned}$$

Note that $\alpha_1, .., \alpha_4 > 0$. Alternatively, we can write the alternating offer bargaining sharing rule in terms of the following variables:

$$\alpha_1 J_t = \alpha_2\left(V_t - U_t\right) - \alpha_3\gamma + \alpha_4\left(\vartheta_t - D\right).$$

Finally, notice that for $M \to \infty$, the sharing rule becomes:

$$J_t = \frac{1}{1-\delta}\left[V_t - U_t - \frac{(1-\delta)^2}{\delta}\gamma\right].$$

B. Medium-sized DSGE Model

Here, we list the dynamic equilibrium equations for the medium-sized DSGE model with alternating offer bargaining and hiring costs (see section 2 in the main text). We also provide steady state calculations.

B.1. Medium-Sized Model: Scaled Dynamic Equations

$$
\begin{aligned}
\text{Cons. FOC (1)} \quad &: \quad \psi_t = \left[(c_t - bc_{t-1}/\mu_t) + \omega\left(c_t^H - bc_{t-1}^H/\mu_t\right)\right]^{-\Theta} \\
\text{Bond FOC (2)} \quad &: \quad 1 = (1+\Delta_t^b) E_t m_{t+1} R_t/\pi_{t+1} \\
\text{Invest. FOC (3)} \quad &: \quad 1 = p_{k',t}\Upsilon_t\left[1 - \tilde{S}_t - \tilde{S}_t'\mu_t\mu_{\Psi,t} i_t/i_{t-1}\right] \\
&\quad + E_t m_{t+1}\mu_{t+1} p_{k',t+1}\Upsilon_{t+1}\tilde{S}_{t+1}'\left(i_{t+1}/i_t\right)^2\mu_{\Psi,t+1}\mu_{t+1} \\
\text{Capital FOC (4)} \quad &: \quad 1 = (1+\Delta_t^b)(1-\Delta_t^k) E_t m_{t+1} R_{t+1}^k/\pi_{t+1} \\
\text{LOM capital (5)} \quad &: \quad \bar{k}_t = \left(1-\delta^k\right)/(\mu_t\mu_{\Psi,t})\bar{k}_{t-1} + \Upsilon_t\left(1-\tilde{S}_t\right) i_t \\
\text{Cost. minim. (6)} \quad &: \quad 0 = a'\left(u_t^k\right) u_t^k \bar{k}_{t-1}/(\mu_{\Psi,t}\mu_t) - \alpha/(1-\alpha)\vartheta_t\left[\nu^f R_t + 1 - \nu^f\right] l_t \\
\text{Production (7)} \quad &: \quad y_t = \mathring{p}_t^{\lambda/(\lambda-1)}\left[\epsilon_t\left(u_t^k\bar{k}_{t-1}/(\mu_t\mu_{\Psi,t})\right)^{\alpha} l_t^{1-\alpha} - n_t\phi\right] \\
\text{Resources (8)} \quad &: \quad y_t = n_t g_t + c_t + i_t + a\left(u_t^k\right)\bar{k}_{t-1}/(\mu_{\psi,t}\mu_t) + n_t\kappa_t x_t l_{t-1} \\
\text{Taylor rule (9)} \quad &: \quad \ln\left(R_t/R\right) = \rho_R \ln\left(R_{t-1}/R\right) \\
&\quad + (1-\rho_R)\left[r_\pi \ln\left(\pi_t/\pi\right) + r_y \ln\left(\mathcal{Y}_t/\mathcal{Y}\right)\right] + \sigma_R\varepsilon_{R,t}/400 \\
\text{Pricing 1 (10)} \quad &: \quad F_t = \psi_t y_t + \beta\xi E_t\left(\tilde{\pi}_{t+1}/\pi_{t+1}\right)^{1/(1-\lambda)} F_{t+1}\mu_{t+1}^{1-\Theta} \\
\text{Pricing 2 (11)} \quad &: \quad K_t = \lambda\psi_t y_t mc_t + \beta\xi E_t\left(\tilde{\pi}_{t+1}/\pi_{t+1}\right)^{\lambda/(1-\lambda)} K_{t+1}\mu_{t+1}^{1-\Theta} \\
\text{Pricing 3 (12)} \quad &: \quad (1-\xi)\left(K_t/F_t\right)^{1/(1-\lambda)} = 1 - \xi\left(\tilde{\pi}_t/\pi_t\right)^{1/(1-\lambda)} \\
\text{Price disp. (13)} \quad &: \quad \mathring{p}_t^{\lambda/(1-\lambda)} = (1-\xi)^{1-\lambda}\left[1 - \xi\left(\tilde{\pi}_t/\pi_t\right)^{1/(1-\lambda)}\right]^{\lambda} + \xi\left[\tilde{\pi}_t/\pi_t\mathring{p}_{t-1}\right]^{\lambda/(1-\lambda)} \\
\text{PV wages (14)} \quad &: \quad w_t^p = w_t + \rho E_t m_{t+1}\mu_{t+1} w_{t+1}^p \\
\text{PV revenue (15)} \quad &: \quad \vartheta_t^p = \vartheta_t + \rho E_t m_{t+1}\mu_{t+1}\vartheta_{t+1}^p \\
\text{Free entry (16)} \quad &: \quad n_t\kappa = J_t
\end{aligned}
$$

$$
\begin{aligned}
\text{Firm value (17)} &: J_t = \vartheta_t^p - w_t^p \\
\text{Work value (18)} &: V_t = w_t^p + A_t \\
\text{Cont. value (19)} &: A_t = (1-\rho) E_t m_{t+1}\mu_{t+1} \left[s f_{t+1} V_{t+1} + s\left(1-f_{t+1}\right) U_{t+1} + (1-s) N_{t+1} \right] \\
&\quad +\rho E_t m_{t+1}\mu_{t+1} A_{t+1} \\
\text{Unemp. value (20)} &: U_t = n_t D_t + E_t m_{t+1}\mu_{t+1}[s f_{t+1} V_{t+1} + s(1-f_{t+1}) U_{t+1} + (1-s) N_{t+1}] \\
\text{Sharing rule (21)} &: \alpha_1 J_t = \alpha_2 \left(V_t - U_t\right) - \alpha_3 n_t \gamma + \alpha_4 \left(\vartheta_t - n_t D_t\right) \\
\text{Real GDP (22)} &: \mathcal{Y}_t = n_t g_t + c_t + i_t \\
\text{Unemp. rate (23)} &: u_t^r = \left(L_t - l_t\right)/L_t \\
\text{Finding rate (24)} &: f_t = x_t l_{t-1}/(L_t - \rho l_{t-1}) \\
\text{Matching fun. (25)} &: x_t l_{t-1} = \sigma_m \left(L_t - \rho l_{t-1}\right)^{\sigma} \left(v_t^{tot}\right)^{1-\sigma} \\
\text{Vacancies (26)} &: \tilde{v}_t = v_t l_{t-1} \\
\text{LOM empl. (27)} &: l_t = \left(\rho + x_t\right) l_{t-1} \\
\text{Comp. Tech. (28)} &: \ln \mu_t = \alpha/(1-\alpha) \ln \mu_{\Psi,t} + \ln \mu_{z,t} \\
\text{Neutr. Tech. (29)} &: \ln \mu_{z,t} = (1-\rho_{\mu_z}) \ln \mu_z + \rho_{\mu_z} \ln \mu_{z,t-1} + \sigma_{\mu_z} \varepsilon_{\mu_z,t}/100 \\
\text{Invest. Tech. (30)} &: \ln \mu_{\Psi,t} = (1-\rho_{\mu_\Psi}) \ln \mu_\Psi + \rho_{\mu_\Psi} \ln \mu_{\Psi,t-1} + \sigma_{\mu_\Psi} \varepsilon_{\mu_\Psi,t}/100 \\
\text{Tech. diffus. (31)} &: n_t = n_{t-1}^{1-\theta} \mu_t^{-1} \text{ where } n_t = \Omega_t/\Phi_t
\end{aligned}
$$

$$
\begin{aligned}
\text{Non-participation (32)} &: N_t = 0 + E_t m_{t+1}\mu_{t+1} \left[e_{t+1} \left(f_{t+1} V_{t+1} + \left(1-f_{t+1}\right) U_{t+1} \right) + \left(1-e_{t+1}\right) N_{t+1} \right] \\
\text{Exit prob. non-part. (33)} &: e_t = \left[L_t - s\left(L_{t-1} - \rho l_{t-1}\right) - \rho l_{t-1} \right] / \left[1 - L_{t-1} \right] \\
\text{FOC employment (34)} &: p_{l,t} = -\alpha_u \zeta_t/\psi_t \left[c_t^H + \mathcal{F}_t \right] / \left[L_t - l_t \right] + w_t - n_t D_t \\
&\quad +E_t m_{t+1}\mu_{t+1} p_{l,t+1} \rho \left[1 - f_{t+1} \right] \\
\text{FOC LFP (35)} &: \zeta_t/\psi_t \left(\left[1-\alpha_u \right] / \left[1 - L_t \right] - \alpha_u / \left[L_t - l_t \right] \right) \left(c_t^H + \mathcal{F}_t \right) = n_t D_t \\
&\quad p_{l,t} f_t - \zeta_t/\psi_t \mathcal{F}_{L,t} - E_t m_{t+1}\mu_{t+1} \zeta_{t+1}/\psi_{t+1} \mathcal{F}_{L,t}^{'} \\
\text{Home consumption (36)} &: c_t^H = n_t \left(1 - L_t\right)^{1-\alpha_u} \left(L_t - l_t\right)^{\alpha_u} - \mathcal{F}_t \\
\text{Marg. utility home cons.(37)} &: \zeta_t = \omega \psi_t \\
\text{Asset pricing kernel (38)} &: m_{t+1} = \beta \psi_{t+1}/\psi_t \left(\mu_{t+1}\right)^{-\Theta} \\
\text{Return on capital wedge (39)} &: \Delta_t^k = \rho_k \Delta_{t-1}^k + \sigma_k \varepsilon_{k,t}/100 \\
\text{Fisher equation wedge (40)} &: \Delta_t^b = \rho_b \Delta_{t-1}^b + \sigma_b \varepsilon_{b,t}/100
\end{aligned}
$$

Check: 40 equations in the following 40 endogenous unkowns:

$$
\begin{aligned}
&\psi_t \; c_t \; R_t \; \pi_t \; p_{k',t} \; i_t \; u_t^k \; \bar{k}_t \; \vartheta_t \; l_t \; y_t \; \mathring{p}_t \; x_t \; F_t \; K_t \; S_t \; U_t \; J_t \\
&w_t \; v_t \; f_t \; u_t \; \mathcal{Y}_t \; \vartheta_t^p \; w_t^p \; A_t \; \tilde{v}_t \; n_t \; \mu_t \; \mu_{\Psi,t} \; \mu_{z,t} \\
&N_t \; m_{t+1} \; e_t \; p_{l,t} \; \zeta_t \; c_t^H \; L_t \; \Delta_t^b \; \Delta_t^k
\end{aligned}
$$

In the above 40 equations, it is useful to define several abbreviated variables that are

functions of the 40 endogenous variables. In particular,

$$
\begin{aligned}
\text{Cap. util. cost (42)} &: \quad a(u_t^k) = 0.5\sigma_b\sigma_a\left(u_t^k\right)^2 + \sigma_b\left(1-\sigma_a\right)u_t^k + \sigma_b\left(\sigma_a/2-1\right)\\
\text{Cap. util. deriv. (43)} &: \quad a'(u_t^k) = \sigma_b\sigma_a u_t^k + \sigma_b\left(1-\sigma_a\right)\\
\text{Invest. adj. cost (44)} &: \quad \tilde{S}_t = 0.5\exp\left[\sqrt{\tilde{S}''}\left(\mu_t\mu_{\Psi,t}i_t/i_{t-1} - \mu\cdot\mu_\Psi\right)\right]\\
&\quad +0.5\exp\left[-\sqrt{\tilde{S}''}\left(\mu_t\mu_{\Psi,t}i_t/i_{t-1} - \mu\cdot\mu_\Psi\right)\right] - 1\\
\text{Invest. adj. deriv. (45)} &: \quad \tilde{S}_t' = 0.5\sqrt{\tilde{S}''}\exp\left[\sqrt{\tilde{S}''}\left(\mu_t\mu_{\Psi,t}i_t/i_{t-1} - \mu\cdot\mu_\Psi\right)\right]\\
&\quad -0.5\sqrt{\tilde{S}''}\exp\left[-\sqrt{\tilde{S}''}\left(\mu_t\mu_{\Psi,t}i_t/i_{t-1} - \mu\cdot\mu_\Psi\right)\right]\\
\text{Capital return (46)} &: \quad R_t^k = \pi_t/(\mu_{\Psi,t}p_{k',t-1})\left(u_t^k a'\left(u_t^k\right) - a(u_t^k) + (1-\delta^k)p_{k',t}\right)\\
\text{Marginal cost (47)} &: \quad mc_t = \tau_t\left(\mu_{\Psi,t}\mu_t\right)^\alpha\vartheta_t\left[\nu^f R_t + 1 - \nu^f\right]\left(u_t^k\bar{k}_{t-1}/l_t\right)^{-\alpha}/(\epsilon_t\left(1-\alpha\right))\\
\text{Price indexation (48)} &: \quad \tilde{\pi}_t = \pi_{t-1}^{\kappa^f}\pi^{1-\kappa^f-\varkappa^f}\breve{\pi}^{\varkappa^f}\\
\text{LFP adj. cost (49)} &: \quad \mathcal{F}_t = 0.5n_t^L\phi_L\left(L_t/L_{t-1}-1\right)^2 L_t\\
\text{LFP adj. deriv. (50)} &: \quad \mathcal{F}_{L,t} = 0.5n_t^L\phi_L\left(L_t/L_{t-1}-1\right)^2 + n_t^L\phi_L\left(L_t/L_{t-1}-1\right)L_t/L_{t-1}\\
\text{LFP adj. deriv (51)} &: \quad \mathcal{F}_{L,t}^{'} = -E_t n_{t+1}^L\phi_L\left(L_{t+1}/L_t-1\right)L_{t+1}^2/L_t^2
\end{aligned}
$$

We adopt $\kappa^f = 0, \varkappa^f = \breve{\pi} = 1$ which corresponds to the case of no indexation of prices. We set $\nu^f = 1$ which corresponds to the working capital specification in the main text. The variables Υ_t, ϵ_t and τ_t are exogenous and set equal to 1 for all t. In addition, g_t and D_t are also exogenous processes.

TO BE EDITED. Add steady state equations.