Globalization and Multiproduct Firms*

Volker Nocke†
University of Mannheim and CEPR

Stephen Yeaple‡
Pennsylvania State University and NBER

August 25, 2013

Abstract

We present an international trade model with multiproduct firms. Firms are heterogeneously endowed with two types of capabilities that jointly determine the trade-off within firms between managing a large portfolio of products and producing at low marginal cost. The model can explain many of the documented cross-sectional correlations in firm performance measures, including why larger firms are more productive and more diversified, and yet more diversified firms trade at a discount. Globalization is shown to induce heterogeneous responses across firms in terms of scope and productivity, some of which are consistent with existing empirical work, while others are potentially testable.

Keywords: multiproduct firms, trade liberalization, diversification discount, firm heterogeneity, productivity

JEL Classification: F12, F15

*We thank the Editor (Jan Eeckhout) and three anonymous referees for helpful comments.
†Email: nocke@uni-mannheim.de.
‡Email: sry3@psu.edu.

1 Introduction

Multiproduct firms dominate domestic and international commerce: they account for 91% of U.S. manufacturing sales (Bernard, Redding, and Schott, 2010) and 98% of the value of U.S. manufacturing exports (Bernard, Redding, and Schott, 2007). The empirical literature has documented many dimensions along which multiproduct firms differ in their performance from single-product firms. On average, multiproduct firms are larger than single-product firms (Bernard, Redding, and Schott, 2006) as well as more productive (Schoar, 2002). Yet financial markets tend to discount firm diversification: diversified firms exhibit, on average, lower market valuations relative to their book valuations than less diversified firms (Lang and Stulz, 1994).

Why is it that firms that manage a large number of products tend to have a low market-to-book ratio despite being more productive on average? More generally, what underlying mechanisms can generate the broad (and sometimes seemingly contradictory) range of cross-sectional correlations in firm performance measures? What is the likely impact of a major economic shock such as globalization on these correlations?

To address these questions, we extend the Melitz (2003) model in two directions. First, we allow each firm to choose the number of its products which involves an irrecoverable fixed cost per product. Second, we assume that firms are heterogeneously endowed with two types of capabilities, organizational capital and organizational efficiency. Organizational capital is a necessary (intangible) input into the production of each product managed by the firm. The more organizational capital is used in the production of a given product, the lower is that product's marginal cost. Because organizational capital is in fixed supply within the firm, the firm faces a trade-off between offering more products and producing each product at lower marginal cost (or, equivalently, higher total factor productivity). A firm's organizational efficiency determines the severity of that trade-off: the higher is a firm's organizational efficiency, the more effective is organizational capital in reducing the firm's marginal cost for a given product, and thus the higher the opportunity cost of adding an additional product.

We characterize firms' equilibrium choices of firm scope, scale, and export status as a function of their two-dimensional types. This allows us to derive a number of analytical predictions on the cross-sectional correlations of firm performance measures. We show that a firm's endowment of organizational efficiency uniquely determines the firm's optimal ratio of organizational capital to its (endogenous) number of products

and that this ratio is independent of the firm's endowment of organizational capital. As a firm's TFP depends only on that ratio as well as on its organizational efficiency, and positively so, this implies that a firm's equilibrium equilibrium level of TFP is increasing in its organizational efficiency but independent of its organizational capital. Among firms of a given size, there is co-existence of firms with few products but high TFP and firms with many products but low TFP. The model can therefore explain the "diversification discount puzzle:" holding firm size fixed, more diversified firms have a lower ratio of market to book value (Schoar, 2002). We also establish a condition on the distribution of organizational capital and organizational efficiency in the population of firms that implies a positive relationship between firm size and TFP, as found in the data (Bartelsman, Haltiwanger, and Scarpetta, 2013). Similar to Melitz (2003), a firm chooses to export if and only if its organizational efficiency is above a certain cutoff, which is independent of the firm's organizational capital.

A parameterized version of the model can simultaneously explain several more of the cross-sectional correlations in firm performance measures that have emerged as key stylized facts from the empirical literature. First, TFP and market-to-book ratio are positively correlated in the cross-section of firms (Schoar, 2002). Second, there is a positive cross-sectional correlation between the number of products a firm manages and the sales per product (Bernard, Redding, and Schott, 2006). Third, exporters are on average larger than non-exporters (Bernard and Jensen, 1999) but that correlation is far from perfect (Hallak and Sivadasan, 2011). Fourth, exporters sell on average more products than non-exporters (Bernard, Redding, and Schott, 2007).

In response to a bilateral trade liberalization, our model generates a heterogeneous response by firms. The induced change in the number of products managed is continuous in the size of the trade liberalization for both continuing exporters and continuing non-exporters but it is of opposite sign: continuing exporters increase their diversification while firms that continue to sell only domestically decrease their diversification. Firms that are induced to switch to exporting choose to drop the number of their products discontinuously so as to become leaner and meaner in the international market place. The implied TFP response by firms is consistent with Schoar (2002) who shows that an increase in the level of diversification of U.S. firms tends to be associated with a reduction in the TFP of incumbent plants. Interestingly, our model predicts that the implied response in the market-to-book ratio may be of the opposite sign to that of the implied TFP response.

In the parameterized version of the model, we show that a trade liberalization generates a substantial increase in the number of products managed by firms on average as well as an increase in industry-level TFP. Both of these findings are consistent with the empirical results in Bernard, Redding, and Schott (2011). We also show that the firms that switch to become exporters after the trade liberalization experience a larger increase in their TFP than those firms that continue to sell only domestically, which is what Lileeva and Trefler (2010) found in the case of the U.S.-Canadian Free Trade Agreement.

Our paper is most closely related to the nascent literature that is concerned with multiproduct firms in international trade (Eckel and Neary, 2010; Bernard, Redding, and Schott, 2011; Mayer, Melitz and Ottaviano, forthcoming; Dhingra, forthcoming). With the exception of Dhingra (forthcoming), in these papers firms draw a distribution of marginal costs (or, equivalently, product-specific preference parameters) for various products of different degrees of substitutability so that the marginal cost of any given product is exogenous. In doing so, these papers focus on the within-firm distribution of marginal costs. Only low marginal cost products are exported, and trade liberalization induces firms to shed weaker products to "focus on their core competencies." Instead, we abstract from within-firm heterogeneity in order to explore a rather different mechanism, namely one where a firm's marginal cost for any given product depends on how the firm solves the trade-off between product proliferation and specialization, and firms differ in the extent of this trade-off. This allows us to explain additional features of the data such as the diversification discount and the heterogeneous response of firms to a trade liberalization. Dhingra (forthcoming) differs from the above-mentioned papers in that, in her model, marginal costs are endogenous as firms can reduce their marginal costs by investing in process innovation. In contrast to our model, the scope of a firm is determined by the internalization of demand-side externalities ("cannibalization") at the firm level. This implies that, as in our model, a trade liberalization will generate different responses by different firms. However, as the mechanism relies on demand-side cannibalization, the model does not directly relate to the empirical literature on multiproduct firms as that literature is using data at a level of aggregation at which demand-side linkages are arguably negligible.[1] In contrast to Dhingra (forthcoming), we

[1]For instance, Bernard, Redding and Schott (2011) work with data disaggregated to the 5-digit SIC or about 1,500 products.

explain why larger firms are more productive in terms of TFP and also more diversified, and yet more diversified firms trade at a discount.

The plan of the paper is as follows. In the next section, we set out the closed economy model. In Section 3, we derive the equilibrium in the closed economy and show how different firms solve the trade-off between diversification and TFP differently. We analyze how equilibrium firm performance measures change with changes in a firm's organizational capital and organizational efficiency. We also demonstrate that the model gives rise to a diversification discount when controlling for firm size. In Section 4, we embed the model in an international trade setting with two identical countries. We characterize firms' exporting decisions as a function of their organizational efficiency and organizational capital. Further, we analyze the effects of globalization on firms' performance measures. The section closes with a numerical analysis of a parameterized version of the model. We conclude in Section 5.

2 The Closed Economy Model

We consider a discrete-time, infinite horizon model of a closed economy with a single (differentiated goods) sector and a single factor of production (labor). There is a mass L of identical consumers (workers) with a per-period CES utility function:

$$U_s = \left[\int_\Omega x_s(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}},$$

where $x_s(\omega)$ is consumption of product $\omega \in \Omega$ in period s, and $\sigma > 1$ is the elasticity of substitution between products. Products cannot be stored and there is no savings technology.

In each period, each worker supplies a single unit of labor. The economy-wide wage rate in period s is $w_s \equiv 1$ and serves as *numéraire*. Aggregate per-period income is thus equal to L. The resulting aggregate demand for product ω in period s is given by

$$X_s(\omega) = A_s p_s(\omega)^{-\sigma}, \tag{1}$$

where $p_s(\omega)$ is the price of product ω in period s and

$$A_s \equiv \frac{L}{\int_\Omega p_s(\omega)^{-(\sigma-1)} d\omega}$$

the residual demand level.

In each period, there is a sufficiently large mass of atomless and *ex ante* identical potential entrants. If a potential entrant decides not to enter, it obtains a profit of zero; if it does decide to enter, the firm has to incur an irrecoverable setup cost F^e. A fraction $F/F^e \in (0,1]$ of this entry cost is used to build firm-specific (but perfectly durable) capital equipment for which there is no resale market; the remaining fraction $(1-F/F^e) \in [0,1)$ is spent on intangibles (e.g., advice, know how). Upon entry, the firm receives a random draw of its time-invariant type $(\widetilde{\theta}, K)$ from a continuous distribution function $\widetilde{G}$ with associated density $\widetilde{g}$ and support $(0, 1/(\sigma-1)) \times [1,\infty)$. A firm's type $(\widetilde{\theta}, K)$ consists of two elements: its organizational efficiency $\widetilde{\theta}$ and its organizational capital K.

We think of 'organizational capital' and 'organizational efficiency' as being two types of firm capabilities that cannot be bought 'off the shelf.'[2] In our model, organizational capital is akin to a managerial input that is in fixed supply within the firm: the more of it is allocated to the production of one product, the less can be allocated to the production of another. Increasing the allocation of organizational capital to a given product allows the firm to produce that product at lower marginal cost. The rate at which an increase in organizational capital reduces marginal cost is what we call organizational efficiency. Firms are heterogeneous both in their endowment of organizational capital as well as in their organizational efficiency.

After learning its type, the entrant has to decide on the size of its product portfolio: for each of the N products it chooses to manage, the firm has to incur an irrecoverable one-time development cost f to build firm-product-specific (but perfectly durable) capital equipment. In each period, the firm has also to incur a constant labor cost per unit of output. The marginal cost of product ω, denoted $c(\omega; k_\omega, \widetilde{\theta})$, is decreasing in the amount of organizational capital, k_ω, that the firm chooses to spend on the product:

$$c(\omega; k_\omega; \widetilde{\theta}) = \begin{cases} z k_\omega^{-\widetilde{\theta}} & \text{if } k_\omega \geq 1, \\ \infty & \text{otherwise,} \end{cases}$$

where $z > 0$ is a cost parameter that is common to all firms and products.[3] The firm

[2] According to the literature on organizational capital (e.g., Prescott and Visscher, 1980; Atkeson and Kehoe, 2005), a key source of a firm's value lies in the 'architecture' of its organization, originating in "some framework of rules, routines, and tacit understandings that evolved over time" (Sutton, 2012, p. 12).

[3] The restriction $k_\omega \geq 1$ ensures that, holding the allocation of organizational capital to product ω

faces the following resource constraint on the allocation of its organizational capital:

$$\sum_{\omega \in \mathcal{I}} k_\omega \leq K,$$

where $\mathcal{I}$ is the set of products managed by the firm. That is, the firm's allocation of organizational capital over all of its products cannot exceed its endowment of organizational capital.

At the end of each period, an active firm (including its assets) dies with probability $1 - \beta \in (0, 1)$ and survives with the remaining probability β. For notational simplicity, there is no discounting (i.e., the discount factor is equal to one).

The sequence of moves in each period is as follows:

1. Potential entrants decide whether or not to enter the market.
2. Each new entrant decides how many products to manage and how much organizational capital to spend on each of them.
3. Each active firm (new entrant or surviving incumbent) sets the prices of its various products so as to maximize its profit. Profits are realized.
4. Each active firm dies with probability $1 - \beta$.

3 The Closed Economy: Analysis

In this section, we first derive the (stationary) equilibrium in the closed economy. We then analyze the implied cross-sectional correlations in firm performance measures. We provide conditions under which there is a positive cross-sectional correlation between total factor productivity (TFP) on the one hand and firm size and market-to-book ratio (Tobin's Q) on the other. We also show that the model predicts a *size premium* when controlling for firm scope, and a *diversification discount* when controlling for firm size.

3.1 Equilibrium in the Closed Economy

We solve for the equilibrium backwards. For notational convenience, it will prove useful to use the following monotonic transformation of managerial efficiency: $\theta \equiv \widetilde{\theta}(\sigma - 1)$. We

fixed, an increase in organizational efficiency θ (weakly) reduces the marginal cost of that product.

will henceforth refer to θ as the firm's organizational efficiency, and to (θ, K) as the firm's type, with associated distribution function G and density g on support $\Theta \equiv (0,1)\times[1,\infty)$.

Consider first firms' pricing decisions at stage 3. As each firm faces an iso-elastic demand function, given by (1), each firm optimally charges a constant markup over marginal cost for each one of its products. For a firm with organizational efficiency θ that has previously allocated k_ω units of organizational capital to product ω, the profit-maximizing price of that product is therefore given by

$$p(\omega; k_\omega; \theta) = \left(\frac{\sigma}{\sigma - 1}\right) c(\omega; k_\omega; \theta).$$

We now turn to firms' choice of scope at stage 2. Let $N(\theta, K)$ denote the number of products managed by a firm of type (θ, K).

The following lemma shows that a firm of type (θ, K) optimally allocates the same amount of organizational capital to each one of the $N(\theta, K)$ products it chooses to manage:

Lemma 1 *A firm of type (θ, K) chooses to manage no more than K products, i.e., $N(\theta, K) \leq K$. Moreover, it allocates $k_\omega = K/N(\theta, K)$ units of organizational capital to each one of its $N(\theta, K)$ products.*

Proof. See Appendix. ■

The lemma implies that the marginal cost of a firm of type (θ, K) is given by

$$c(\theta, K) = z\left(\frac{K}{N(\theta, K)}\right)^{-\frac{\theta}{\sigma-1}},$$

so that the firm optimally chooses to charge price

$$\begin{aligned} p(\theta, K) &= \left(\frac{\sigma}{\sigma - 1}\right) c(\theta, K) \\ &= \left(\frac{\sigma}{\sigma - 1}\right) z\left(\frac{K}{N(\theta, K)}\right)^{-\frac{\theta}{\sigma-1}}. \end{aligned} \tag{2}$$

The firm's per-period profit (gross of the sunk entry and product development costs) is given by

$$\begin{aligned} \pi(\theta, K) &= N(\theta, K)\left[p(\theta, K) - c(\theta, K)\right] A p(\theta, K)^{-\sigma} \\ &= N(\theta, K)(1-\beta) f \zeta \left(\frac{K}{N(\theta, K)}\right)^{\theta}, \end{aligned} \tag{3}$$

where

$$\zeta \equiv \frac{A}{\sigma(1-\beta)f}\left(\frac{\sigma-1}{\sigma z}\right)^{\sigma-1}, \tag{4}$$

$$A = \frac{(1-\beta)L}{M\left[\int_\Theta N(\theta,K)p(\theta,K)^{-(\sigma-1)}dG(\theta,K)\right]}, \tag{5}$$

and M is the mass of entrants in each period, and $M/(1-\beta)$ the mass of active firms. As ζ is proportional to A, we will henceforth (with a slight abuse of language) refer to ζ as to the markup-adjusted residual demand level.

Having sunk the entry and product development costs, an active firm's (market) value, $v(\theta,K)$, is the expected sum of future profits, i.e.,

$$\begin{aligned} v(\theta,K) &= \frac{\pi(\theta,K)}{1-\beta} \\ &= N(\theta,K)f\zeta\left(\frac{K}{N(\theta,K)}\right)^{\theta}, \end{aligned} \tag{6}$$

where the second equality follows from (3).

For simplicity, we will in the following focus on the case where $\zeta > 1$. It is straightforward to show that this assumption holds in equilibrium if the entry cost F^e is sufficiently large. Moreover, we will be abstracting from the integer constraints on the number of products so that N can take the value of any nonnegative real number.

By setting N arbitrarily small, and thus K/N arbitarily large, a firm can achieve arbitrarily low marginal cost, no matter what its type. This implies that the market value of a firm of type (θ,K) exceeds the (sunk) development costs of its $N(\theta,K)$ products: $v(\theta,K) > N(\theta,K)f$. As there are no other fixed costs, this implies that all entrants choose to be active. The firm's problem of choice of scope consists in choosing N so as to maximize its subsequent market value net of the product development costs:

$$\max_{N\geq 0} Nf\zeta\left(\frac{K}{N}\right)^{\theta} - Nf, \tag{7}$$

where the first term is the expected sum of future profits, conditional on having chosen to manage N products (see equation (6)), and the second term the cost of developing N products.

The following proposition characterizes the solution to this problem:

Proposition 1 *In equilibrium, a firm of type* (θ, K) *chooses to manage*

$$N(\theta, K) = \begin{cases} K & \text{if } \theta \in (0, \underline{\theta}] \\ K\left[(1-\theta)\zeta\right]^{1/\theta} & \text{if } \theta \in [\underline{\theta}, 1) \end{cases} \tag{8}$$

products, where $\underline{\theta} \equiv (\zeta - 1)/\zeta \in (0, 1)$.

Proof. See Appendix. ■

The proposition shows that the equilibrium number of products, $N(\theta, K)$, is proportional to the firm's organizational capital, K, independent of its organizational efficiency θ for $\theta < \underline{\theta}$, and strictly decreasing in θ for $\theta > \underline{\theta}$ (see the proof of the proposition). To understand why, consider the first-order condition to program (7):

$$\left[f\zeta\left(\frac{K}{N}\right)^{\theta} - f\right] - \theta f\zeta\left(\frac{K}{N}\right)^{\theta} = 0.$$

The first term on the LHS is the net profit of the marginal product, whereas the second term is the effect that the marginal product has on the total production costs of the N inframarginal products. Abstracting from the corner solution ($N = K$), the optimal number of products is achieved where these two effects balance each other. Note that organizational capital affects this trade-off only through the ratio K/N (the organizational capital per product). Hence, there is a uniquely optimal ratio K/N, which is increasing in organizational efficiency θ. As a result, the optimal number of products is proportional to organizational capital K and (weakly) decreasing in organizational efficiency θ.

Finally, we consider firms' entry decisions at stage 1. Since potential entrants are ex ante identical, free entry implies that they must be indifferent between entering and not:

$$\int_{\Theta} v^e(\theta, K) dG(\theta, K) - F^e = 0, \tag{9}$$

where

$$v^e(\theta, K) \equiv \frac{\pi(\theta, K)}{1-\beta} - N(\theta, K)f$$

is the value of an entrant of type (θ, K) after incurring the (type-independent) irrecoverable entry cost F^e but *before* incurring the irrecoverable product development costs.

Equilibrium in the labor market requires that

$$\begin{aligned} L &= \frac{M}{1-\beta}\int_{\Theta} N(\theta,K)\left[Ap(\theta,K)^{-\sigma}c(\theta,K)\right]dG(\theta,K) \\ &\quad +M\left[f\int_{\Theta} N(\theta,K)dG(\theta,K)+F^{e}\right]. \end{aligned}$$

The LHS of this equation is labor supply. The first term on the RHS is labor demand for production from the mass $M/(1-\beta)$ of active firms: a firm of type (θ, K) sells $Ap(\theta,K)^{-\sigma}$ units per product, resulting in labor demand of $Ap(\theta,K)^{-\sigma}c(\theta,K)$ for each of its $N(\theta,K)$ products. The second term on the RHS is labor demand from the mass M of entrants for product development and setup. Using equation (5), the labor market clearing condition simplifies to

$$L=\sigma M\left[f\int_{\Theta} N(\theta,K)dG(\theta,K)+F^{e}\right]. \tag{10}$$

Note that this equation implies that, in equilibrium, the mass M of entrants is proportional to the size of the economy, L.

An equilibrium in the closed economy is given by the collection $\{N(\cdot,\cdot), p(\cdot,\cdot), M, \zeta\}$ satisfying equations (2)–(5) and (8)–(10).

3.2 Cross-Sectional Correlation in Firm Performance Measures

We now investigate how various measures of firm performance – such as profit, sales, marginal cost, and Tobin's Q – vary with firm type in equilibrium.

Inserting (8) into (3), we can rewrite the per-period profit of a firm of type (θ, K) as

$$\pi(\theta,K)=\begin{cases} K(1-\beta)f\zeta & \text{if } \theta\in(0,\underline{\theta}], \\ K(1-\beta)f\zeta\left[(1-\theta)\zeta\right]^{\frac{1-\theta}{\theta}} & \text{if } \theta\in[\underline{\theta},1). \end{cases} \tag{11}$$

That is, per-period profit (and, by (6), market value) is proportional to the firm's endowment of organizational capital, K. However, $\pi(\theta,K)$ varies non-monotonically with the firm's organizational efficiency θ: $\partial\pi(\theta,K)/\partial\theta=0$ for $\theta\in(0,\underline{\theta})$, $\partial\pi(\theta,K)/\partial\theta<0$ for $\theta\in\left(\underline{\theta},\overline{\theta}\right)$ and $\partial\pi(\theta,K)/\partial\theta>0$ for $\theta\in\left(\overline{\theta},1\right)$.[4] At first, it may seem surprising

[4]Note that

$$\frac{d\left[(1-\theta)\zeta\right]^{\frac{1-\theta}{\theta}}}{d\theta}=\frac{1}{\theta}\left[(1-\theta)\zeta\right]^{(1-\theta)/\theta}\Gamma(\theta),$$

that per-period profit is decreasing in managerial efficiency on $\left(\underline{\theta}, \overline{\theta}\right)$. To understand this, recall that the per-period profit does not take into account any previously incurred sunk costs. Indeed, as we have seen above, holding organizational capital fixed, a firm endowed with greater managerial efficiency $\theta > \underline{\theta}$ optimally chooses to manage a smaller number of products, thus sinking a smaller amount of product development costs. It is straightforward to show that, from an ex ante point of view, being endowed with greater managerial efficiency is better for the firm: the value of an entrant, $v^e(\theta, K)$, is continuous and weakly increasing everywhere in θ (and strictly so for $\theta > \underline{\theta}$), holding K fixed.

The standard measure of firm size is firm sales (over all of the firm's products). As firms charge a fixed markup, per-period sales are proportional to per-period profit: $S(\theta, K) = \sigma\pi(\theta, K)$ for all (θ, K). From (11), firm sales can thus be written as

$$S(\theta, K) = \begin{cases} \sigma(1-\beta) f \zeta K & \text{if } \theta \in (0, \underline{\theta}], \\ \sigma(1-\beta) f \zeta K \left[(1-\theta)\zeta\right]^{\frac{1-\theta}{\theta}} & \text{if } \theta \in [\underline{\theta}, 1). \end{cases} \tag{12}$$

As firm sales are proportional to firm profit, the effects of changes in θ and K on $S(\theta, K)$ mirror those on $\pi(\theta, K)$.

Let us now turn to measures of firm productivity. One such measure is the firm's total factor productivity (TFP). As the entry cost F^e and the per-product development cost f are sunk, TFP is equal to the inverse of marginal cost, which in turn is given by

$$c(\theta, K) = \begin{cases} z & \text{if } \theta \in (0, \underline{\theta}], \\ z \left[(1-\theta)\zeta\right]^{\frac{1}{\sigma-1}} & \text{if } \theta \in [\underline{\theta}, 1). \end{cases} \tag{13}$$

TFP is thus independent of the firm's endowment of organizational capital K, and increasing in the firm's organizational efficiency θ for $\theta > \underline{\theta}$ (and independent of θ for $\theta < \underline{\theta}$).

Recall that the market value of a firm is the sum of expected future profits, $v(\theta, K) = \pi(\theta, K)/(1-\beta)$. Whereas the market value is thus forward looking, the book value is

---

where

$$\Gamma(\theta) \equiv -1 - \frac{\ln\left[(1-\theta)\zeta\right]}{\theta}.$$

We have $\Gamma(\underline{\theta}) = -1$, $\Gamma(\theta) > 0$ for θ sufficiently close to 1, and $\Gamma'(\theta) = \theta(1-\theta)^{-1} + \theta^{-2}\ln\left[(1-\theta)\zeta\right]$. Let $\overline{\theta} \in (\underline{\theta}, 1)$ be a solution to $\Gamma(\overline{\theta}) = 0$. (By continuity of Γ and the intermediate value theorem, $\overline{\theta}$ exists.) As $\Gamma'(\overline{\theta}) > 0$, $\overline{\theta}$ is unique.

backward looking and equal to the firm's historic expenditure on capital equipment (or, equivalently, its replacement cost):

$$\begin{aligned} b(\theta, K) &= F + N(\theta, K)f \\ &= \begin{cases} F + Kf & \text{if } \theta \in (0, \underline{\theta}], \\ F + Kf\left[(1-\theta)\zeta\right]^{1/\theta} & \text{if } \theta \in [\underline{\theta}, 1) \end{cases}. \end{aligned} \tag{14}$$

The market-to-book ratio of a firm of type (θ, K) – Tobin's Q – is thus given by

$$\begin{aligned} Q(\theta, K) &\equiv \frac{v(\theta, K)}{b(\theta, K)} \\ &= \begin{cases} \frac{f\zeta}{f + \frac{F}{K}} & \text{if } \theta \in (0, \underline{\theta}], \\ \frac{f}{(1-\theta)\left(f + \frac{F}{N(\theta, K)}\right)} & \text{if } \theta \in [\underline{\theta}, 1). \end{cases} \end{aligned} \tag{15}$$

It can easily be verified that $Q(\theta, K)$ is strictly increasing in the firm's endowment of organizational capital, K. Holding K fixed, $Q(\theta, K)$ is independent of θ for $\theta < \underline{\theta}$, and strictly increasing in organizational efficiency θ for θ sufficiently close to one. However, for intermediate values of θ, the market-to-book ratio $Q(\theta, K)$ may be increasing or decreasing in θ, depending on parameter values.

The empirical literature has shown that there is a tendency for larger firms to have higher TFP than smaller firms (Bartelsman, Haltiwanger, and Scarpetta, 2013) and that firms with higher TFP also exhibit a larger market-to-book ratio (Schoar, 2002). In our model, a firm's TFP is independent of organizational capital K and (weakly) increasing in organizational efficiency θ, firm size is increasing in K but non-monotonic in θ, whereas the market-to-book ratio is increasing in K but not increasing everywhere in θ. So, intuitively, one might expect that our model generates the cross-sectional correlations between TFP and firm size and between TFP and Tobin's Q found in the data if there is a sufficiently strong positive correlation between K and θ. Proposition 2 below formalizes this intuition for the case where the distribution of firm types can be characterized by an ordered pair $(\theta, K(\theta))$:

Proposition 2 *Suppose that the distribution of firm types could be characterized as an ordered pair $(\theta, K(\theta))$. Assume that the elasticity of K with respect to θ, $\kappa(\theta) \equiv \theta K'(\theta)/K(\theta)$, satisfies $\kappa(\theta) > 0$ for $\theta \in (0, \underline{\theta})$, and*

$$\kappa(\theta) > 1 + \ln[(1-\theta)\zeta]^{1/\theta} \equiv \Upsilon(\theta) \text{ for } \theta \in (\underline{\theta}, 1), \tag{16}$$

where $\Upsilon(\theta) < 1$. Then:

1. *Marginal cost $c(\theta, K(\theta))$ is (weakly) decreasing and firm sales $S(\theta, K(\theta))$ increasing in θ for all $\theta \in (0, 1)$. That is, there is a positive cross-sectional correlation between TFP and firm size.*

2. *Marginal cost $c(\theta, K(\theta))$ is (weakly) decreasing and Tobin's $Q(\theta, K(\theta))$ increasing in θ for all $\theta \in (0, 1)$. That is, there is a positive cross-sectional correlation between TFP and the market-to-book ratio.*

Proof. See Appendix. ■

The empirical literature has established that larger firms tend to be more productive in terms of TFP (Bartelsman, Haltiwanger, and Scarpetta, 2013) and more diversified in terms of the number of products they manage (Bernard, Redding, and Schott, 2006). At first glance, these results may seem to be at odds with a very well known finding in the corporate finance literature – the *diversification discount puzzle* (Lang and Stulz, 1994), according to which more diversified firms tend to be less productive in terms of Tobin's Q. The diversification discount holds, in particular, when controlling for firm size (Schoar, 2002).[5]

The following proposition shows that these seemingly contradictory empirical findings are consistent with each other. Our model can not only account for the observed correlations between TFP and size and between TFP and Tobin's Q but it predicts a *size premium* when controlling for diversification and a *diversification discount* when controlling for firm size:

Proposition 3 *Holding diversification $N(\theta, K)$ fixed, there is a positive cross-sectional relationship between firm size $S(\theta, K)$ and the market-to-book ratio $Q(\theta, K)$. Holding firm size $S(\theta, K)$ fixed, there is a negative cross-sectional relationship between firm scope $N(\theta, K)$ and the market-to-book ratio $Q(\theta, K)$.*

Proof. See Appendix. ■

[5]Several explanations of the diversification discount puzzle have been proposed in the corporate finance literature. For instance, Rajan, Servaes and Zingales (2000) provide an explanation based on agency costs that result in the misallocation of resources across divisions. Maksimovic and Phillips (2002) argue that the diversification discount puzzle can better be explained by comparative advantage across sectors. There are also some who argue that the diversification discount may in fact be a statistical artifact of selection (Villalonga, 2004).

To understand why our model predicts a size premium, holding diversification fixed, suppose two firms optimally choose the same number of products but one firm is larger than the other in terms of sales. Then, it has to be the case that both firms have the same book value but that the larger firm has higher TFP and therefore a higher market value. To understand why our model predicts a diversification discount, holding firm size fixed, suppose two firms are of the same size but one firm chooses to manage a larger number of products than the other. Then, it has to be the case that both firms have the same market value (which is proportional to firm size) but that the more diversified firm had to have made a larger investment in product-specific capital in the past and therefore has a larger book value.

4 The Open Economy

In this section, we extend the model to incorporate a simple trading environment between two identical countries, home and foreign. We first derive the equilibrium and various firm performance measures (and their cross-sectional correlations). We then analyze the effects of globalization (a symmetric fall in trade costs) on equilibrium, showing that our model generates a number of predictions that are consistent with the empirical findings in the literature. In the last part of this section, we numerically analyze a parameterized version of the model, which gives rise to several additional cross-sectional correlations that have been documented in the empirical literature.

When choosing at stage 2 how many products to manage, a new entrant also decides for each of its products whether to sell it only domestically or both domestically and abroad. If it chooses to export any particular product, the entrant must incur at that stage a one-time irrecoverable cost of f^x to set up a firm-product-specific distribution system. At stage 3, firms have to pay an iceberg-type trading cost $\tau > 1$ for each unit shipped to the foreign market.

4.1 Equilibrium in the Open Economy

As in the closed economy, the profit-maximizing price of a firm of type (θ, K) that has allocated k_ω units of organizational capital to product ω involves a constant markup over marginal cost: the firm's domestic price is $p(\omega; k_\omega; \theta) = (\sigma/(\sigma-1))\, c(\omega; k_\omega; \theta)$ whereas the price charged abroad (in case the firm chooses to serve that market) is $p^*(\omega; k_\omega; \theta) =$

$\tau p(\omega; k_\omega; \theta)$, reflecting the higher cost of serving the foreign market.

Turning to the allocation of organizational capital and export decisions, the following lemma provides a preliminary result:

Lemma 2 *A firm of type (θ, K) chooses to manage no more than K products, i.e., $N(\theta, K) \leq K$. Generically, it exports all of its products, denoted $\delta^x(\theta, K) = 1$, or none, $\delta^x(\theta, K) = 0$. In either case, the firm allocates the same amount $k_\omega = K/N(\theta, K)$ of its organizational capital to each one of its $N(\theta, K)$ products.*

Proof. See Appendix. ■

It follows from this lemma that all of the $N(\theta, K)$ products of firm (θ, K) have the same marginal cost,

$$c(\theta, K) = z \left(\frac{K}{N(\theta, K)} \right)^{-\frac{\theta}{\sigma-1}}, \tag{17}$$

so that the firm optimally charges price

$$p(\theta, K) = \left(\frac{\sigma}{\sigma - 1} \right) c(\theta, K), \tag{18}$$

for all of its products in the domestic market, and – provided the firm chooses to export ($\delta^x(\theta, K) = 1$) – price

$$p^*(\theta, K) = \tau \left(\frac{\sigma}{\sigma - 1} \right) c(\theta, K). \tag{19}$$

for all of its products in the foreign market. The firm's stage-2 problem of choice of scope, allocation of organizational capital, and export status therefore simplifies to

$$\max_{N \in [0,K], \delta^x \in \{0,1\}} N \left[f\zeta \left(1 + \delta^x \rho\right) \left(\frac{K}{N} \right)^{\theta} - \left(f + \delta^x f^x\right) \right], \tag{20}$$

where $\rho \equiv \tau^{-(\sigma-1)}$ is a measure of trade freeness.

To avoid a taxonomy of cases, we impose in the following (as in the closed economy case) an implicit restriction on parameters such that the markup-adjusted residual demand level ζ, defined as before in (4), satisfies[6]

$$\frac{\ln(1 + f^x/f)}{\ln(1 + \rho)} > \zeta > 1. \tag{21}$$

Given this assumption, the following proposition states the solution to program (20):

[6] This restriction ensures that, in equilibrium, any firm (θ, K) that chooses to export sets $N(\theta, K) < K$, i.e., $\theta^x > \underline{\theta}$, where the export cutoff θ^x will be defined in Proposition 4.

Proposition 4 *In the equilibrium of the open economy, the export decision of a firm of type* (θ, K) *is given by*

$$\delta^x(\theta, K) = \begin{cases} 0 & \text{if } \theta \in (0, \theta^x), \\ 1 & \text{if } \theta \in (\theta^x, 1), \end{cases} \tag{22}$$

where

$$\theta^x \equiv 1 - \frac{\ln(1+\rho)}{\ln(1+f^x/f)} \in (\underline{\theta}, 1). \tag{23}$$

The firm's equilibrium number of products is

$$N(\theta, K) = \begin{cases} K & \text{if } \theta \in (0, \underline{\theta}], \\ K\left((1-\theta)\zeta\right)^{\frac{1}{\theta}} & \text{if } \theta \in [\underline{\theta}, \theta^x), \\ K\left[\left(\frac{1+\rho}{1+f^x/f}\right)(1-\theta)\zeta\right]^{\frac{1}{\theta}} & \text{if } \theta \in (\theta^x, 1). \end{cases} \tag{24}$$

Proof. See Appendix. ■

Proposition 4 demonstrates that a firm's export decision is independent of its organizational capital K, depending only on its organizational efficiency θ: a firm chooses to export, $\delta^x(\theta, K) = 1$, if and only if $\theta > \theta^x$. The reason why a firm's endowment of organizational capital does not affect its export decision is that the only fixed costs of exporting are at the product level but not at the firm level. The reason why a firm chooses to export if and only if its organizational efficiency is sufficiently large is that a firm with greater organizational efficiency optimally chooses to have lower marginal cost (as we have already seen in the closed economy case), thus allowing the firm to make a sufficiently high gross profit per product abroad to cover the one-time product-level fixed cost of exporting. As we discuss in the next subsection, $N(\theta, K)$ is discontinuous at $\theta = \theta^x$: the number of products managed by a non-exporter with organizational efficiency just below θ^x is discretely larger than that managed by an exporter with organizational efficiency just above θ^x. By doing so, an exporter saves on the product-level export cost and sells more units per product at lower marginal cost.

Given that a firm either exports all of its products or none, its per-period profit (having sunk the entry, product development and export costs) can be written as

$$\pi(\theta, K) = N(\theta, K)(1-\beta)f\zeta\left[1 + \delta^x(\theta, K)\rho\right]\left(\frac{K}{N(\theta, K)}\right)^{\theta}. \tag{25}$$

The free entry condition is again given by

$$\int_{\Theta} v^e(\theta, K) dG(\theta, K) - F^e = 0, \tag{26}$$

where the value $v^e(\theta, K)$ of a new entrant of type (θ, K) now accounts for the fact that a new entrant may choose to become an exporter:

$$v^e(\theta, K) = \frac{\pi(\theta, K)}{1-\beta} - N(\theta, K)(f + \delta^x(\theta, K) f^x). \tag{27}$$

The labor market clearing condition in the open economy is given by

$$\begin{aligned} L &= \frac{AM}{1-\beta}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \int_\Theta (1+\delta^x(\theta,K)\rho)\, N(\theta,K) c(\theta,K)^{-(\sigma-1)} dG(\theta,K) \\ &\quad + M\left[\int_\Theta (f + \delta^x(\theta,K) f^x) N(\theta,K) dG(\theta,K) + F^e\right] \\ &= \sigma M\left[\int_\Theta (f + \delta^x(\theta,K) f^x) N(\theta,K) dG(\theta,K) + F^e\right], \end{aligned} \tag{28}$$

where the second equality follows from the fact that

$$A = \frac{(1-\beta)L}{M\left[\int_\Theta (1+\delta^x(\theta,K)\rho)\, N(\theta,K) p(\theta,K)^{-(\sigma-1)} dG(\theta,K)\right]}. \tag{29}$$

As in the closed economy case, the equilibrium mass M of entrants is thus proportional to the size of the economy, L.

An equilibrium in the open economy is given by the collection $\{N(\cdot,\cdot), p(\cdot,\cdot), p^*(\cdot,\cdot), \delta^x(\cdot,\cdot), M, \zeta\}$ satisfying equations (17)–(19) and (22)–(29).

4.2 Cross-Sectional Correlations in Firm Performance Measures

We now turn to the key firm performance measures and their cross-sectional correlations.

Inserting (24) into (17), we obtain marginal cost (the inverse of TFP) as a function of firm type:

$$c(\theta, K) = \begin{cases} z & \text{if } \theta \in (0, \underline{\theta}) \\ z\left((1-\theta)\zeta\right)^{\frac{1}{\sigma-1}} & \text{if } \theta \in (\underline{\theta}, \theta^x) \\ z\left[(1-\theta)\zeta\left(\frac{1+\rho}{1+f^x/f}\right)\right]^{\frac{1}{\sigma-1}} & \text{if } \theta \in (\theta^x, 1) \end{cases}.$$

As in the closed economy case, the marginal cost of the firm is independent of K, and strictly decreasing in θ for all $\theta > \underline{\theta}$. However, mirroring our earlier observation that

$N(\theta, K)$ drops discretely with an increase in θ at the export threshold $\theta = \theta^x$, marginal cost is discontinuous at $\theta = \theta^x$. A firm with organizational efficiency $\theta = \theta^x$ is indifferent between exporting and selling only domestically. If it chooses to export, the firm optimally increases the TFP of its production processes by focusing its organizational capital on fewer products. The opportunity cost of becoming productive enough to export is the reduction in domestic profits due to the reduced product range.

Inserting (24) into (25), we obtain the per-period profit of a firm of type (θ, K):

$$\pi(\theta, K) = \begin{cases} K(1-\beta)f\zeta & \text{if } \theta \in (0, \underline{\theta}], \\ K(1-\beta)f\zeta\left[(1-\theta)\zeta\right]^{(1-\theta)/\theta} & \text{if } \theta \in [\underline{\theta}, \theta^x], \\ K\left(\frac{1-\beta}{1-\theta}\right)(f+f^x)\left[\left(\frac{1+\rho}{1+f^x/f}\right)(1-\theta)\,\zeta\right]^{\frac{1}{\theta}} & \text{if } \theta \in [\theta^x, 1). \end{cases} \tag{30}$$

As in the closed economy case, the firm's sales are proportional to its per-period profit:

$$S(\theta, K) = \sigma\pi(\theta, K) \tag{31}$$

From (30) and (31), it follows that per-period profit and sales are discontinuous in θ at the export cutoff θ^x, jumping up as the number of products managed drops.[7]

As before, the firm's market value is $v(\theta, K) = \pi(\theta, K)/(1-\beta)$ whereas its book value is the replacement cost of its tangible assets that it previously accumulated for entry, product development and export distribution: $b(\theta, K) = F + N(\theta, K)f$ if $\theta < \theta^x$ and $b(\theta, K) = F + N(\theta, K)(f + f^x)$ if $\theta > \theta^x$. The firm's value of Tobin's Q, which is the ratio of the two, is thus given by

$$Q(\theta, K) = \begin{cases} \frac{f\zeta}{f+\frac{F}{K}} & \text{if } \theta \in (0, \underline{\theta}), \\ \frac{f}{(1-\theta)\left(f+\frac{F}{N(\theta,K)}\right)} & \text{if } \theta \in (\underline{\theta}, \theta^x), \\ \frac{f+f^x}{(1-\theta)\left(f+f^x+\frac{F}{N(\theta,K)}\right)} & \text{if } \theta \in (\theta^x, 1), \end{cases} \tag{32}$$

where $N(\theta, K)$ is given by (24).

[7] To see this, note that

$$\begin{aligned} \ln\left(\frac{\lim_{\theta\downarrow\theta^x} S(\theta, K)}{\lim_{\theta\uparrow\theta^x} S(\theta, K)}\right) &= \ln(1+\rho) + \left(\frac{1}{\theta^x} - 1\right)\ln\left[\frac{1+\rho}{1+f^x/f}\right] \\ &= \frac{1-\theta^x}{\theta^x} + \ln\left(1+\frac{f^x}{f}\right) > 0, \end{aligned}$$

where we have used the definition of θ^x to establish the last equality.

As the following proposition shows, our result on the diversification discount (holding firm size fixed) carries over the open economy setting:

Proposition 5 *Consider two firms of different types, (θ, K) and (θ', K'), with the same level of sales, $S(\theta, K) = S(\theta', K')$. Then, the firm that produces the larger number of products will have a lower market-to-book ratio: $N(\theta', K') > N(\theta, K)$ implies $Q(\theta', K') < Q(\theta, K)$. That is, when controlling for firm size, there is a diversification discount in the equilibrium of the open economy.*

Proof. See Appendix. ■

4.3 The Effects of Globalization on Firm Performance

We now explore the effects of a reduction in the iceberg-type trade cost τ (and thus of an increase in the trade freeness parameter ρ) on firms' decisions and the resulting impact on firm performance measures. We confine attention to changes that are small enough to preserve the parameter restriction (21). (In the following, we will index post-liberalization variables by a prime.)

The following lemma shows how trade liberalization affects the effective market size for exporters and non-exporters.

Lemma 3 *Consider an increase in trade freeness from ρ to $\rho' > \rho$. This lowers the effective market size facing non-exporters, i.e., $\zeta' < \zeta$, and raises the effective market size facing exporters, i.e., $\zeta'(1+\rho') > \zeta(1+\rho)$.*

Proof. See Appendix. ■

An immediate implication of the lemma is that trade liberalization results in an increase in welfare by inducing a lower price index (or, equivalently, an increase in the markup-adjusted residual demand level). The lemma also makes clear that a fall in trade costs reduces the effective market size facing firms that do not export while raising the effective market size of exporting firms. As exporting becomes more attractive and the domestic market less attractive, the cutoffs for maximal diversification and exporting change, as the next proposition shows.

Proposition 6 *Consider an increase in trade freeness from ρ to $\rho' > \rho$. This induces the thresholds for exporting and for maximal diversification to fall: $\theta^{x\prime} < \theta^x$ and $\underline{\theta}' < \underline{\theta}$.*

Proof. Equation (23) immediately implies that $\theta^{x\prime} < \theta^x$. Lemma 1, which establishes that $\zeta' < \zeta$, and the fact that $\underline{\theta} \equiv (\zeta - 1)/\zeta$ is increasing in ζ, imply that $\underline{\theta}' < \underline{\theta}$. ■

As in Melitz (2003), an increase in the freeness of trade lowers the (organizational) efficiency threshold above which a firm selects into exporting: Following a trade liberalization, any firm (θ, K) with $\theta \in (\theta^{x\prime}, \theta^x)$ will switch from non-exporting to exporting. In our setting, there is also another form of selection. As the effective size of the domestic market becomes smaller, due to a reduction in trade costs for foreign firms, the threshold above which firms opt to be less than maximally diversified falls as well.

The next proposition formally considers how the choice of firm scope is affected by a trade liberalization.

Proposition 7 *Consider an increase in trade freeness from ρ to $\rho' > \rho$. This causes firms that initially sold only domestically to drop products, i.e., $N(\theta, K)' \leq N(\theta, K)$ for all $\theta \in (0, \theta^x)$, with a strict inequality if $\theta \in (\underline{\theta}', \theta^x)$, and all continuing exporters to increase the number of products they manage, i.e., $N(\theta, K)' > N(\theta, K)$ for all $\theta \in (\theta^x, 1)$.*

Proof. See Appendix. ■

Trade liberalization causes firms that do not export prior to the trade shock to drop product lines. This effect is especially strong for firms that are induced by the trade liberalization to switch to exporting because an exporter optimally wants to be "leaner and meaner," as discussed before. On the other hand, for continuing exporters the trade shock results in a larger effective market size to which they respond by adding more products. These results suggest systematic and asymmetric changes in firms' marginal costs, which the following corollary substantiates.

Corollary 1 *Consider an increase in trade freeness from ρ to $\rho' > \rho$. For firms that initially sold only domestically, this results in higher TFP, i.e., $c(\theta, K)' \leq c(\theta, K)$ for all $\theta \in (0, \theta^x)$, with a strict inequality if $\theta \in (\underline{\theta}', \theta^x)$. For continuing exporters, this results in lower TFP, i.e., $c(\theta, K)' > c(\theta, K)$ for all $\theta \in (\theta^x, 1)$.*

Proof. This follows directly from the definition of marginal cost (the inverse of TFP) and Proposition 7. ■

The changes in the number of product lines managed by firms of different types implies a particular productivity effect that varies across firms. Those non-exporters that choose to drop products experience an increase in their TFP as these firms become

"leaner and meaner." Those firms that switch to become exporters after the reduction in trade costs also see their TFP rise: as they face the first-order effect associated with paying the additional fixed cost f^x per product, they choose to become "leaner and meaner," too. Finally, continuing exporters see their TFP fall as they adjust to an effectively larger market by expanding their product scope.

To complete our analysis, the following proposition considers how a trade liberalization affects Tobin's Q across firms.

Proposition 8 *Consider an increase in trade freeness from ρ to $\rho' > \rho$. There exists a threshold value of organizational efficiency, $\widehat{\theta} \in (\theta^{x\prime}, \theta^x)$, such that any firm (θ, K) with organizational efficiency below that threshold experiences a reduction in their market-to-book ratio, i.e., $Q(\theta, K)' < Q(\theta, K)$ if $\theta < \widehat{\theta}$, while the opposite holds for any other firm, i.e., $Q(\theta, K)' > Q(\theta, K)$ if $\theta > \widehat{\theta}$.*

Proof. See Appendix. ■

Tobin's Q falls for non-exporters because the trade liberalization induces a decrease in the home market's markup-adjusted residual demand level ζ, which directly reduces the profitability of such firms. For exporting firms the increase in access to the foreign market more than compensates for this fall in ζ as $(1+\rho')\zeta' > (1+\rho)\zeta$. Hence, we see that there is a negative relationship between the effect of trade liberalization on a firm's TFP and the effect on its Tobin's Q! The model thus cautions analysts to carefully consider what drives firms' performance measures. Falling TFP can be associated with rising profitability if greater access to foreign markets induces firms to diversify their product mix. On the other hand, non-exporting firms are hurt by the trade liberalization and see their profitability fall but optimally respond by becoming leaner, which increases their TFP.

4.4 Numerical Example

To further elaborate on the model's implications, we explore a numerical example. We discipline the choice of parameters in our example by matching three well known facts about the distribution of firm performance measures. Holding fixed this parameterization, the model implies a rich set of cross-sectional correlations in other firm performance measures that are consistent with the sign of the correlations found in the empirical lit-

erature.[8] We also consider the comparative static of a 10 percent multilateral decrease in iceberg trade costs and show that the model implies heterogeneous responses across firms that are also consistent with recent empirical work.

To implement our numerical example, we approximate the continuous distribution function G by drawing 10,000 times a pair (θ, K) from a Gumbel copula with Pareto marginals for K and Power marginals for θ. Given an initial guess of the mass of entrants, M, we allow firms to choose the number of their products and their export status and then calculate their profits as a function of their type and M. Adjusting M if average realized profits are non-zero, we iterate until the free entry condition holds. We choose the distributional parameters and the export costs so that the following empirical observations hold in the numerical example: (i) the size distribution of firms is consistent with Zipf's law, (ii) 20% of firms export, and (iii) conditional on exporting, a firm's export sales are roughly 15% of its total revenue.[9] Finally, we choose the sunk entry cost relative to the sunk cost of opening a plant so that most firms are not maximally diversified.[10]

We first discuss the parameterized model's implications for the relationship between size, TFP, and export status. The results are shown in Figure 1, where the logarithm of firm size for non-exporters (shown as dark triangles) and exporters (shown as light Xs) is plotted against the logarithm of firm TFP.

The model is consistent with several well-documented features of the data. First, as found by Bartelsman, Haltiwanger, and Scarpetta (2013), there is a strong positive correlation between the logarithm of aggregate sales and the logarithm of TFP. Second, as documented by Bernard and Jensen (1999), exporters have on average larger aggregate sales than non-exporters. Third, despite the fact that exporters are on average larger

[8] We view our numerical exercise as illustrative of the kinds of correlations the model can generate rather than as a calibration exercise. The latter would require a more formal modeling of country asymmetries that is outside the scope of the present paper.

[9] Bernard, Redding and Schott (2007) report in the appendix to their paper that 20% of manufacturing firms export and that the share of their exports in total revenue is 15%.

[10] The parameter values chosen in the simulation are as follows. Without loss of generality, we set $\beta = 0$. To be consistent with a 25% mark-up we set $\sigma = 5$. We set the fixed cost of opening a plant to $f = 4$ and the trade costs to $f^x = 1.5$ and $\tau = 1.5$. We assume that all of the entry cost is spent on tangible assets, i.e., $F^e = F$, and choose this cost to be $F = 30$, and the population size to be $L = 300$. Turning to distributional parameters, the Gumbel dependence parameter is set to 4, the Pareto marginals for K have a shape parameter of 1.05, and the Power distribution for θ has a shape parameter of 0.25. Note that this choice of parameters satisfies condition (21).

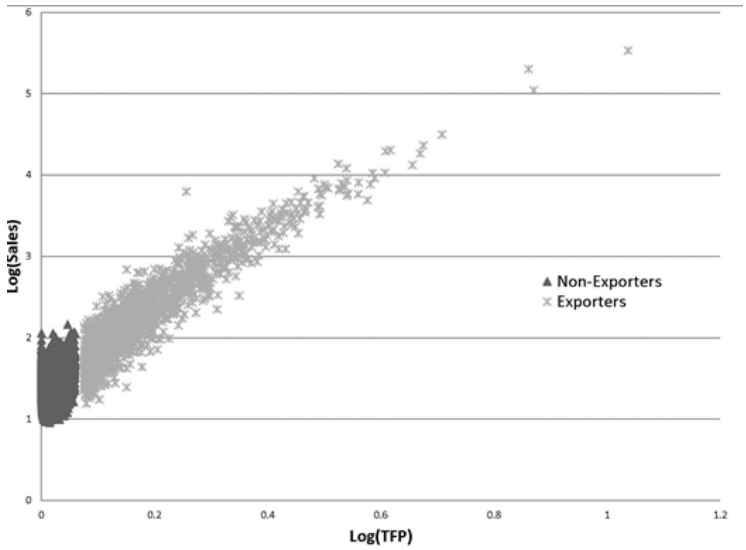


Figure 1: The figure plots the log of firm sales (on the vertical axis) against the log of TFP (on the horizontal axis), both for non-exporters (triangles) and exporters (Xs).

than non-exporters, there is no strict sorting of firms into export mode based solely on firm size as can be seen from the substantial overlap in firm sizes in Figure 1. This is consistent with the empirical observation made in Hallak and Sivadasan (2011) that there is no strict sorting on the basis of firm size in the data.

Turning to financial performance measures, Figure 2 shows the simulated relationship between the logarithm of Tobin's Q for non-exporters (depicted as dark triangles) and exporters (depicted as light Xs) and the logarithm of TFP. Consistent with the results of Schoar (2002), firms that display high levels of TFP tend to have higher levels of Tobin's Q. Note, however, that for a given level of TFP there is substantial heterogeneity in the realized Tobin's Q, which reflects variation in the extent of diversification across firms. It is this variability that is associated with the diversification discount that arises generically in our model (see Propositions 3 and 5 for the closed economy and open economy, respectively).

In the data, there is a positive link between the number of products a firm manages and its aggregate sales and the likelihood that the firm exports (Bernard, Redding, and Schott, 2006 and 2007). The relationship between these firm performance measures in our numerical example is shown in Figure 3. There is a striking correlation between the number of products a firm manages and its aggregate sales. Less obvious from

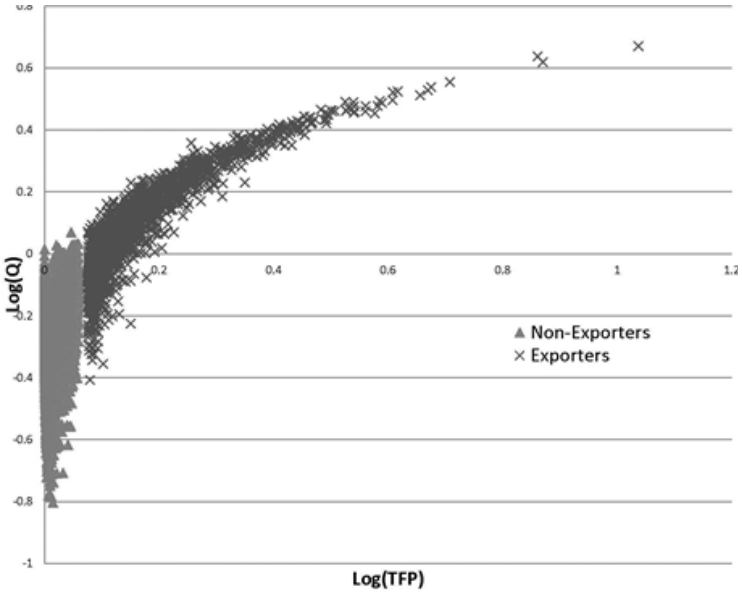


Figure 2: The figure plots the log of Tobin's Q (on the vertical axis) against the log of TFP (on the horizontal axis), both for non-exporters (triangles) and exporters (Xs).

the figure is that the correlation between the logarithm of the number of products a firm manages and the logarithm of sales per product is also positive (0.36). Further, the simulation results indicate that although there is substantial overlap between the number of products managed by exporters and non-exporters, on average exporters manage 45% more products than firms that do not export. Both of these implications are consistent with the data.

A substantial empirical literature has developed in the last ten years that documents the impact both at the industry level and the firm level of a reduction in global trading costs. We conclude this section by comparing two equilibria that differ only in the size of trade costs. In particular, we consider how the number of products managed by firms and the TFP of those firms change when trade costs are reduced by 10%.

Table 1 reports the average fractional changes in the number of products managed (first row) and TFP (second row) that are induced by the trade liberalization. These induced changes are computed for three groups of firms. The first column shows the average fractional changes across all firms. The second row shows the average fractional changes for firms that do not export prior to the trade liberalization but choose to export thereafter, whereas the third column reports the average fractional changes for firms that

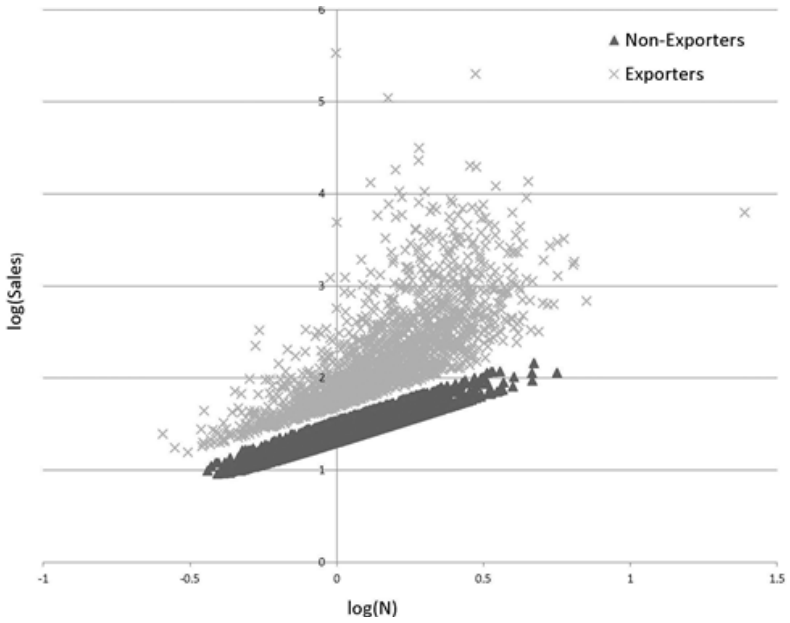


Figure 3: The figure plots the log of firm sales (on the vertical axis) against the log of a firm's number of products (on the horizontal axis), both for non-exporters (triangles) and exporters (Xs).

Change in	**All Firms**	**Switchers**	**Never Export**
Number of Products	-18%	-34%	-21%
TFP	+0.5%	+4.3%	0.4%

Table 1: The table shows the effect of the trade liberalization on the number of products and TFP by firms' export status.

never export.

The results of our comparative static are broadly consistent with those found in the empirical literature. As documented by Bernard, Redding, and Schott (2011), a trade liberalization is associated with a substantial reduction in the number of products managed by firms on average (column 1, row 1) and an increase in industry-level TFP (column 1, row 2).

Recently, Lileeva and Trefler (2010) have shown that Canadian firms exposed to the U.S.-Canadian Free Trade Agreement tariff reductions responded very differently depending on whether they switched to exporting after the trade liberalization or not. In particular, firms that remained non-exporters even after the trade liberalization experienced modest or no productivity changes while firms that switched to exporting experienced very substantial improvements in TFP. As illustrated in Table 1, our model generates heterogeneous responses across firms similar to those found in the data. Comparing columns 2 and 3 of Table 1, we see that those firms that are induced to export (column 2) choose to reduce the number of their products and therefore experience large increases in their TFP, while the responses of non-exporting firms (column 3) are substantially weaker.

Note that the positive link between the *change* in the number of products managed and the *change* in measured TFP illustrated in Table 1 is also consistent with existing empirics. Schoar (2002) shows that an increase in the level of diversification of U.S. firms tends to be associated with a reduction in the TFP of incumbent plants. Our model produces exactly such a relationship when an external shock, such as a change in trade costs, induces firms to alter their scope.

5 Conclusion

We developed an international trade model with multiproduct firms. In the model, firms are heterogeneously endowed with a stock of organizational capital and the efficiency with which organizational capital can be used to reduce marginal cost. This gives rise to a trade-off between focusing on managing few products at low marginal cost and many products at high marginal cost. Depending on their endowment, different firms solve this tradeoff differently.

Our model generates cross-sectional correlations in firm performance measures that are qualitatively consistent with the data. In particular, our model can simultaneously give rise to two seemingly contradictory empirical correlations: a positive correlation between firm size and firm TFP and a negative correlation between Tobin's Q and the number of products managed by a firm.

Our model generates heterogeneous responses across firms to a trade liberalization that are also consistent with the recent empirical literature. Trade can cause a decrease in the average number of products managed by firms and an increase in average productivity that is particularly pronounced among firms that are induced to export. The model also generates interesting new predictions. For instance, while a trade liberalization ought to raise the market-to-book ratio of exporting firms, the increase in this financial indicator should be negatively correlated with measured TFP.

To make our analysis as transparent as possible, we simplified along several dimensions. For instance, we treated all products as being perfectly symmetric. Allowing for heterogeneity across products would allow for richer resource allocation issues to arise within the firm. Moreover, we have kept the analysis static, assuming that endowments of organizational capital are randomly assigned to firms, thereby avoiding the analysis of the accumulation of organizational capital within the firm. We leave this for future research.

6 Appendix: Proofs

Proof of Lemma 1. Let $\mathcal{I}$ denote the firm's set of products. Conditional on having incurred the irrecoverable development cost for its $N = \#\mathcal{I}$ products, the firm optimally

allocates its organizational capital so as to maximize the sum of its future profits:

$$\max_{\{k_\omega\}_{\omega\in\mathcal{I}}} \frac{A}{(1-\beta)\sigma}\left(\frac{\sigma-1}{\sigma z}\right)^{\sigma-1} \sum_{\omega\in\mathcal{I}^+\equiv\{\omega\in\mathcal{I}|k_\omega\geq 1\}} (k_\omega)^\theta$$
$$\text{subject to } \sum_{\omega\in\mathcal{I}} k_\omega \leq K.$$

Note that the objective function is increasing and concave in the k_ω's. For a given set $\mathcal{I}$ of products, it is thus optimal for the firm to exhaust all of its endowment of organizational capital (i.e., set $\sum_{\omega\in\mathcal{I}} k_\omega = K$) and, for each $\omega \in \mathcal{I}$, to choose either $k_\omega = k \geq 1$ or $k_\omega = 0$. However, it cannot be optimal to chose a set $\mathcal{I}$ of products and then allocate $k_\omega = 0$ to some product $\omega \in \mathcal{I}$ (resulting in infinite marginal cost for that product); in that case, the firm would have increased its profit by choosing not to develop that good and saving development cost f. Hence, a firm of type (θ, K) chooses to manage no more than K products, $N(\theta, K) \leq K$, and sets $k_\omega = K/N(\theta, K)$ for each one of its $N(\theta, K)$ products. ■

Proof of Proposition 1. Let $\widetilde{N}(\theta, K)$ denote the solution to the first-order condition of program (7), i.e.,

$$(1-\theta) f \zeta K^\theta \widetilde{N}(\theta, K)^{-\theta} - f = 0,$$

or

$$\widetilde{N}(\theta, K) = K\left[(1-\theta)\zeta\right]^{\frac{1}{\theta}}.$$

Note that $\widetilde{N}(\theta, K) > 0$ for all $(\theta, K) \in \Theta$; that is, each entrant chooses to be active. By Lemma 1, the solution to the first-order condition, $\widetilde{N}(\theta, K)$, is the solution to the problem of profit maximization only if $\widetilde{N}(\theta, K) \leq K$. The value-maximizing number of products is thus given by

$$N(\theta, K) = \min\left\{K, \widetilde{N}(\theta, K)\right\}.$$

Next, we show that $\widetilde{N}(\theta, K)$ is strictly decreasing in θ. Taking the partial derivative of $\widetilde{N}(\theta, K)$ with respect to θ, and dividing by K, we obtain

$$\begin{aligned}\frac{\partial \widetilde{N}(\theta, K)}{\partial \theta}\frac{1}{K} &= -\frac{\zeta}{\theta}\left[(1-\theta)\zeta\right]^{\frac{1-\theta}{\theta}} - \frac{1}{\theta^2}\left[(1-\theta)\zeta\right]^{\frac{1}{\theta}} \ln\left((1-\theta)\zeta\right) \\ &= -\frac{\zeta}{\theta^2}\left[(1-\theta)\zeta\right]^{\frac{1-\theta}{\theta}} \Psi(\theta),\end{aligned}$$

where

$$\Psi(\theta) \equiv \theta + (1-\theta)\ln\left((1-\theta)\zeta\right). \tag{33}$$

Hence, $\partial\widetilde{N}(\theta,K)/\partial\theta < 0$ if and only if $\Psi(\theta) > 0$. Now, we have $\Psi(0) = \ln(\zeta) > 0$ as $\zeta > 1$ by assumption. Further,

$$\Psi'(\theta) = -\ln\left((1-\theta)\zeta\right),$$

$$\Psi''(\theta) = \frac{1}{1-\theta} > 0,$$

so that $\Psi(\theta)$ achieves its unique minimum at $\theta^m \equiv (\zeta-1)/\zeta$, which is the unique solution on $(0,1)$ to $\Psi'(\theta^m) = 0$. But note that $\Psi(\theta^m) = \theta^m > 0$, implying that $\Psi(\theta) > 0$ for all θ. Hence, $\partial\widetilde{N}(\theta,K)/\partial\theta < 0$, and thus

$$N(\theta,K) = \begin{cases} K & \text{if } \theta \in (0,\underline{\theta}], \\ \widetilde{N}(\theta,K) & \text{if } \theta \in [\underline{\theta},1), \end{cases}$$

where $\underline{\theta} \equiv (\zeta-1)/\zeta$ is such that $\widetilde{N}(\underline{\theta},K) = K$. ■

Proof of Proposition 2.

The assertion on the relationship between marginal cost and θ follows directly from (13). To see that $S(\theta,K(\theta))$ is increasing in θ for all $\theta \in (0,1)$, consider equation (12). Note first that $\partial S(\theta,K)/\partial\theta = 0$ and $\partial S(\theta,K)/\partial K > 0$ for $\theta \in (0,\underline{\theta})$. So, $\kappa(\theta) > 0$ implies that $S(\theta,K(\theta))$ is increasing in θ for $\theta \in (0,\underline{\theta})$. For $\theta \in (\underline{\theta},1)$ we have:

$$\frac{d\ln S(\theta,K(\theta))}{d\theta} = \frac{K'(\theta)}{K(\theta)} - \frac{1}{\theta}\left(1 + \ln[(1-\theta)\zeta]^{1/\theta}\right),$$

which is strictly positive if and only if (16) holds. To see that $Q(\theta,K(\theta))$ is increasing in θ for all $\theta \in (0,1)$, consider equation (15). Note first that $\partial Q(\theta,K)/\partial\theta = 0$ and $\partial Q(\theta,K)/\partial K > 0$ for $\theta \in (0,\underline{\theta})$. So, $\kappa(\theta) > 0$ implies that $Q(\theta,K(\theta))$ is increasing in θ for $\theta \in (0,\underline{\theta})$. For $\theta \in (\underline{\theta},1)$ we have:

$$\begin{aligned}\frac{d\ln Q(\theta,K(\theta))}{d\theta} &= \frac{K'(\theta)}{K(\theta)} - \frac{1}{\theta}\left(\frac{1}{1-\theta} + \ln[(1-\theta)\zeta]^{1/\theta}\right) + \frac{1}{1-\theta} \\ &\quad + \frac{f[(1-\theta)\zeta]^{1/\theta}\left(\frac{K(\theta)}{\theta(1-\theta)} + \frac{K(\theta)}{\theta}\ln[(1-\theta)\zeta]^{1/\theta} - K'(\theta)\right)}{F + K(\theta)f[(1-\theta)\zeta]^{1/\theta}},\end{aligned}$$

which is strictly positive if and only if

$$\begin{aligned} & \frac{K'(\theta)}{K(\theta)}\left[1-\frac{f[(1-\theta)\zeta]^{1/\theta}}{\frac{F}{K(\theta)}+f[(1-\theta)\zeta]^{1/\theta}}\right] \\ > \; & \frac{1}{\theta}\left(\frac{1}{1-\theta}+\ln[(1-\theta)\zeta]^{1/\theta}\right)\left[1-\frac{f[(1-\theta)\zeta]^{1/\theta}}{\frac{F}{K(\theta)}+f[(1-\theta)\zeta]^{1/\theta}}\right]-\frac{1}{1-\theta}, \end{aligned}$$

or

$$\begin{aligned} \kappa(\theta) \; > \; & \frac{1}{1-\theta}+\ln[(1-\theta)\zeta]^{1/\theta}-\frac{\theta}{1-\theta}\left[\frac{\frac{F}{K(\theta)}+f[(1-\theta)\zeta]^{1/\theta}}{\frac{F}{K(\theta)}}\right] \\ = \; & \Upsilon(\theta)-\frac{\theta}{(1-\theta)}\frac{fK(\theta)}{F}[(1-\theta)\zeta]^{1/\theta}, \end{aligned}$$

where $\Upsilon(\theta)$ is as defined in (16). Hence, $\kappa(\theta) > \Upsilon(\theta)$ implies that $Q(\theta, K(\theta))$ is increasing in θ for $\theta \in (\underline{\theta}, 1)$. ■

Proof of Proposition 3. First, consider two firms, (θ', K') and (θ'', K''), that share the same degree of diversification, $N(\theta', K') = N(\theta'', K'')$. As $b(\theta, K) = F + N(\theta, K)f$, this implies that the two firms have the same book value: $b(\theta', K') = b(\theta'', K'')$. But then one firm has a larger market-to-book ratio than the other, $Q(\theta', K') > Q(\theta'', K'')$, if and only if the former has a larger market value, $v(\theta', K') > v(\theta'', K'')$, which holds if and only if that firm is larger, $S(\theta', K') > S(\theta'', K'')$ as $v(\theta, K) = S(\theta, K)/[\sigma(1-\beta)]$. Hence, there is a positive relationship between firm size and Tobin's Q, holding firm scope fixed.

Consider now two firms, (θ', K') and (θ'', K''), with the same level of sales, $S(\theta', K') = S(\theta'', K'')$. As $v(\theta, K) = S(\theta, K)/[\sigma(1-\beta)]$, this implies that the two firms have the same market value: $v(\theta', K') = v(\theta'', K'')$. Thus, one firm has a larger market-to-book ratio than the other, $Q(\theta', K') > Q(\theta'', K'')$, if and only if the former has a smaller book value, $b(\theta', K') < b(\theta'', K'')$. But as $b(\theta, K) = F + N(\theta, K)f$, this holds if and only if the former is less diversified, $N(\theta', K') < N(\theta'', K'')$. Hence, there is a negative relationship between diversification and Tobin's Q, holding firm size fixed. ■

Proof of Lemma 2. Let $\mathcal{I}$ denote the firm's set of products. Conditional on having incurred the irrecoverable development cost for its $N = \#\mathcal{I}$ products, and given its choice of which product(s) to export (if any), the firm optimally allocates its organizational

capital so as to maximize the sum of its future profits:

$$\max_{\{k_\omega\}_{\omega\in\mathcal{I}}} \zeta f \sum_{\omega\in\mathcal{I}^+\equiv\{\omega\in\mathcal{I}|k_\omega\geq 1\}} \left[1+\chi(\omega)\rho\right](k_\omega)^\theta$$
$$\text{subject to } \sum_{\omega\in\mathcal{I}} k_\omega \leq K$$

where $\rho \equiv \tau^{-(\sigma-1)}$ is a measure of trade freeness, $\chi(\omega) = 1$ if the firm chooses to export product ω, and $\chi(\omega) = 0$ otherwise. Note that the objective function is increasing and concave in k_ω for $k_\omega \geq 1$ but independent of k_ω for $k_\omega < 1$. For a given set $\mathcal{I}$ of products, it is thus optimal for the firm to exhaust all of its endowment of organizational capital (i.e., set $\sum_{\omega\in\mathcal{I}} k_\omega = K$) and, for each $\omega \in \mathcal{I}$, to choose $k_\omega \in \{0, k^x\}$ with $k^x \geq 1$ if $\chi(\omega) = 1$, and $k_\omega \in \{0, k^d\}$ with $k^d \geq 1$ if $\chi(\omega) = 0$. However, it cannot be optimal to chose a set $\mathcal{I}$ of products and then allocate $k_\omega = 0$ to some product $\omega \in \mathcal{I}$ (resulting in infinite marginal cost for that product); in that case, the firm would have increased its profit by choosing not to develop that good and saving development cost f (as well as f^x if $\chi(\omega) = 1$). Hence, a firm of type (θ, K) chooses to manage no more than K products, $N(\theta, K) \leq K$.

The Lagrangian associated with the firm's stage-2 decisions can thus be written as

$$L = f\zeta N\left[(1-\delta)\left(k^d\right)^\theta + \delta(1+\rho)\left(k^x\right)^\theta\right] - \lambda N\left[(1-\delta)k^d + \delta k^x - \frac{K}{N}\right],$$

where δ is the share of exported products, and λ the Lagrange multiplier on the firm's organizational capital constraint. As the Lagrangian is linear in δ, it is optimal for the firm to set $\delta \in \{0, 1\}$, i.e., to export either all of its products or none. ■

Proof of Proposition 4. Suppose first that the firm chooses not to export, $\delta^x = 0$, so that program (20) simplifies to (7). As we have seen in Section 3.1, the solution to this program is $N^d(\theta, K) = \min\{K, K\left[(1-\theta)\zeta\right]^{1/\theta}\}$ and the resulting expected sum of future profits (net of product development costs)

$$v^d(\theta, K) = \max\left\{Kf\left[\zeta - 1\right], Kf\left[(1-\theta)\zeta\right]^{1/\theta}\left(\frac{\theta}{1-\theta}\right)\right\}, \tag{34}$$

where the first argument on the RHS is positive by (21).Next, suppose that the firm chooses to export, $\delta^x = 1$, so that program (20) becomes

$$\max_{N\in[0,K]} N\left[f\zeta(1+\rho)\left(\frac{K}{N}\right)^\theta - (f+f^x)\right].$$

From the first-order condition,

$$\left[f\zeta\left(1+\rho\right)\left(\frac{K}{N}\right)^{\theta}-\left(f+f^{x}\right)\right]-\theta f\zeta\left(1+\rho\right)\left(\frac{K}{N}\right)^{\theta}=0,$$

and the constraint $N \leq K$, we obtain the solution to this program:

$$N^{x}(\theta,K)=\min\left\{K,K\left[\left(\frac{1+\rho}{1+f^{x}/f}\right)\left(1-\theta\right)\zeta\right]^{\frac{1}{\theta}}\right\}.$$

The resulting expected sum of future profits (net of the product development and export costs) is

$$v^{x}(\theta,K)=\max\left\{Kf\left[\zeta\left(1+\rho\right)-\left(1+f^{x}/f\right)\right],Kf\left[\left(\frac{1+\rho}{1+f^{x}/f}\right)\left(1-\theta\right)\zeta\right]^{\frac{1}{\theta}}\left(1+f^{x}/f\right)\left(\frac{\theta}{1-\theta}\right)\right\}.$$

As $x/\ln x$ is increasing in x for $x>1$, condition (21) implies that the first argument in the max-function is negative. Hence, $v^{x}(\theta,K)$ simplifies to

$$v^{x}(\theta,K)=Kf\left[\left(\frac{1+\rho}{1+f^{x}/f}\right)\left(1-\theta\right)\zeta\right]^{\frac{1}{\theta}}\left(1+\frac{f^{x}}{f}\right)\left(\frac{\theta}{1-\theta}\right)$$

The firm optimally chooses not to export, $\delta^{x}(\theta,K)=0$, if $v^{d}(\theta,K)>v^{x}(\theta,K)$, and to export, $\delta^{x}(\theta,K)=1$, if the inequality is reversed. Now, $v^{x}(\theta,K)$ is larger than the second argument in the max-function on the RHS of (34) if and only if

$$\left(\frac{1+\rho}{1+f^{x}/f}\right)^{\frac{1}{\theta}}\left(1+f^{x}/f\right)>1,$$

or

$$\theta>\theta^{x}\equiv1-\frac{\ln\left(1+\rho\right)}{\ln\left(1+f^{x}/f\right)}.$$

Note that $\theta^{x}<1$. Condition (21) implies that $\theta^{x}>\underline{\theta}$. As the second argument in the max-function on the RHS of (34) is larger than the first argument in that same max-function if and only if $\theta>\underline{\theta}$, we have $v^{x}(\theta,K)>v^{d}(\theta,K)$ if and only if $\theta>\theta^{x}$. Equations (22) and (24) follow. ■

Proof of Proposition 5. For two firms that are either both exporters ($\min\{\theta,\theta'\}>\theta^{x}$) or both non-exporters ($\max\{\theta,\theta'\}<\theta^{x}$), the argument in the proof of Proposition 3 carries over to the open economy case. What still needs to be shown is that the result

obtains for an exporting firm with $\theta > \theta^x$ when compared to a non-exporting firm for which $\theta' < \theta^x$. From the expression for Tobin's Q in (32), we have

$$\frac{Q(\theta,K)}{Q(\theta',K')} = \frac{(1-\theta')\left(f+\frac{F}{N(\theta',K')}\right)}{(1-\theta)\left(f+f^x+\frac{F}{N(\theta,K)}\right)}\left(\frac{f+f^x}{f}\right). \tag{35}$$

As the two firms have the same sales level, $S(\theta,K) = S(\theta',K')$, by assumption, equation (31) implies

$$N(\theta,K) = N(\theta',K')\left(\frac{1-\theta}{1-\theta'}\right)\left(\frac{f}{f+f^x}\right).$$

Inserting this expression into (35), and rewriting, we obtain

$$\frac{Q(\theta,K)}{Q(\theta',K')} = \frac{f+\frac{F}{N(\theta',K')}}{\left(\frac{1-\theta}{1-\theta'}\right)f+\frac{F}{N(\theta',K')}} > 1,$$

where the inequality follows as $\theta > \theta'$ by hypothesis. ■

Proof of Lemma 3. Inserting the expressions for $\pi(\theta,K)$ and $N(\theta,K)$, the free entry condition (26) can be rewritten as

$$\begin{aligned} & f\int_1^\infty K\left\{(\zeta-1)\int_0^{\underline{\theta}} g(\theta,K)d\theta + \int_{\underline{\theta}}^{\theta^x}\left(\frac{\theta}{1-\theta}\right)\left[(1-\theta)\zeta\right]^{\frac{1}{\theta}} g(\theta,K)d\theta \right. \\ & \left. + \int_{\theta^x}^1 \left(\frac{\theta\left(1+f^x/f\right)}{1-\theta}\right)\left[\left(\frac{1+\rho}{1+f^x/f}\right)(1-\theta)\zeta\right]^{\frac{1}{\theta}} g(\theta,K)d\theta\right\} dK - F^e \\ = \; & 0. \end{aligned} \tag{36}$$

Totally differentiating this expression yields

$$\begin{aligned} & \frac{d\zeta}{\zeta} f\int_1^\infty K\left\{\int_0^{\underline{\theta}} \zeta g(K,\theta)d\theta + \int_{\underline{\theta}}^{\theta^x}\left(\frac{1}{1-\theta}\right)\left[(1-\theta)\zeta\right]^{\frac{1}{\theta}} g(\theta,K)d\theta \right. \\ & + \left. \int_{\theta^x}^1 \left(\frac{1+f^x/f}{1-\theta}\right)\left[\left(\frac{1+\rho}{1+f^x/f}\right)(1-\theta)\zeta\right]^{\frac{1}{\theta}} g(\theta,K)d\theta\right\} dK \\ & + \frac{d\rho}{(1+\rho)} f\int_1^\infty K\int_{\theta^x}^1 \left(\frac{1+f^x/f}{1-\theta}\right)\left[\left(\frac{1+\rho}{1+f^x/f}\right)(1-\theta)\zeta\right]^{\frac{1}{\theta}} g(\theta,K)d\theta dK \\ = \; & 0, \end{aligned}$$

which establishes that $d\zeta d\rho < 0$, and thus $\zeta' < \zeta$. Now suppose that $\zeta(1+\rho)$ were to fall as well so that $\zeta'(1+\rho') \leq \zeta(1+\rho)$. Then, the LHS of (36) would be negative after the trade liberalization, a contradiction. Hence, $\zeta'(1+\rho') > \zeta(1+\rho)$. ■

Proof of Proposition 7. Consider first a firm (θ, K) with $\theta \in (0, \underline{\theta}']$. By Proposition 6 and the definition of the threshold $\underline{\theta}$, we have $N(\theta, K)' = N(\theta, K) = K$ if $\theta \in (0, \underline{\theta}']$ and $N(\theta, K)' < N(\theta, K) = K$ if $\theta \in (\underline{\theta}', \underline{\theta}]$. Consider now a firm (θ, K) with $\theta \in (\underline{\theta}, \theta^{x\prime}) \cup (\theta^x, 1)$. Differentiating equation (24) in conjunction with Lemma 3 implies that $N(\theta, K)' < N(\theta, K)$ if $\theta \in (\underline{\theta}, \theta^{x\prime})$, and $N(\theta, K)' > N(\theta, K)$ if $\theta \in (\theta^x, 1)$. Finally, consider a firm (θ, K) with $\theta \in (\theta^{x\prime}, \theta^x)$. From (24), we obtain the ratio between the number of products post-liberalization and pre-liberalization:

$$\frac{N(\theta, K)'}{N(\theta, K)} = \left(\frac{\zeta'}{\zeta}\right)^{\frac{1}{\theta}} \left(\frac{1+\rho'}{1+f^x/f}\right)^{\frac{1}{\theta}} < 1,$$

where the inequality follows from Lemma 3 and the parameter restriction (21). ■

Proof of Proposition 8. Consider first a firm (θ, K) with $\theta \in (0, \underline{\theta}')$. From equation (32) and the fact that $\zeta' < \zeta$ by Lemma 3, it follows immediately that $Q(\theta, K)' < Q(\theta, K)$ for such a firm.

Consider now a firm (θ, K) with $\theta \in (\underline{\theta}', \underline{\theta})$. From (32) and (24), the ratio between Tobin's Q post-liberalization and pre-liberalization equals

$$\frac{Q(\theta, K)'}{Q(\theta, K)} = \frac{f + \frac{F}{K}}{(1-\theta)\,\zeta\left(f + \frac{F}{K[(1-\theta)\zeta']^{1/\theta}}\right)}.$$

By definition of the thresholds $\underline{\theta}$ and $\underline{\theta}'$, we have for any $\theta \in (\underline{\theta}', \underline{\theta})$ that $(1-\theta)\,\zeta > 1 > (1-\theta)\,\zeta'$. Hence, $Q(\theta, K)' < Q(\theta, K)$.

Next, consider a firm (θ, K) with $\theta \in (\underline{\theta}, \theta^{x\prime}) \cup (\theta^x, 1)$. Proposition 7 combined with (32) immediately implies that $Q(\theta, K)' < Q(\theta, K)$ if $\theta \in (\underline{\theta}, \theta^{x\prime})$ and $Q(\theta, K)' > Q(\theta, K)$ if $\theta \in (\theta^x, 1)$. Finally, consider a firm (θ, K) with $\theta \in (\theta^{x\prime}, \theta^x)$. Note that, from (24) and (30), that for those firms that are not maximally diversified we have the following relationship between the number of products and per-period profit: $\pi(\theta, K) = (1-\beta)(1-\theta)^{-1}\left[f + \delta^x(\theta, K) f^x\right] N(\theta, K)$. For such firms, Tobin's Q can be written as

$$Q(\theta, K) = \left[1 - \theta + \frac{(1-\beta)F}{\pi(\theta, K)}\right]^{-1}.$$

Hence, the ratio between Tobin's Q post-liberalization and pre-liberalization equals

$$\frac{Q(\theta, K)'}{Q(\theta, K)} = \frac{1 - \theta + \frac{(1-\beta)F}{\pi(\theta, K)}}{1 - \theta + \frac{(1-\beta)F}{\pi(\theta, K)'}}.$$

To sign the effect of an increase in trade freeness on Tobin's Q it thus suffices to establish whether a firm's profits have risen or fallen. Given Lemma 1, it follows immediately that $\lim_{\theta \downarrow \theta^{x\prime}} [\pi(\theta, K)' - \pi(\theta, K)] < 0$ and $\lim_{\theta \uparrow \theta^{x}} [\pi(\theta, K)' - \pi(\theta, K)] > 0$. To complete the proof, it suffices to show that $\pi(\theta, K)'/\pi(\theta, K)$ is monotonically increasing in θ. From (30), and noting that any firm with $\theta \in (\theta^{x\prime}, \theta^{x})$ is switching from non-exporting to exporting, we have

$$\begin{aligned} \ln\left(\frac{\pi(\theta, K)'}{\pi(\theta, K)}\right) &= \ln\left(1 + \frac{f^x}{f}\right) + \frac{1}{\theta}\ln\left[\left(\frac{1+\rho'}{1+f^x/f}\right)\frac{\zeta'}{\zeta}\right] \\ &= \ln\left(1 + \frac{f^x}{f}\right)\left(1 - \frac{\theta^{x'}}{\theta}\right) - \frac{1}{\theta}\ln\left(\frac{\zeta}{\zeta'}\right), \end{aligned}$$

where the second line follows from (23). As $\theta > \theta^{x'}$ and $\zeta > \zeta'$, this expression is strictly increasing in θ and is equal to zero at

$$\theta = \widehat{\theta} \equiv \theta^{x\prime} + \frac{\ln(\zeta/\zeta')}{\ln(1 + f^x/f)}.$$

■

References

[1] Atkeson, A., and P. Kehoe, "Modeling and Measuring Organization Capital," *Journal of Political Economy* 113 (2005), 1026-1053.

[2] Bartelsman, E., J. Haltiwanger, and S. Scarpetta, "Cross-Country Differences in Productivity: The Role of Allocation and Selection," American Economic Review 103 (2013), 305-334.

[3] Bernard, A. B., and J. B. Jensen, "Exceptional Exporter Performance: Cause, Effect, or Both?" *Journal of International Economics* 47 (1999), 1-25.

[4] Bernard, A. B., S. J. Redding, and P. K. Schott, "Multi-Product Firms and Trade Liberalization," NBER Working Paper 12782, 2006.

[5] Bernard, A. B., S. J. Redding, and P. K. Schott, "Firms in International Trade," Journal of Economic Perspectives 21 (2007), 105-130.

[6] Bernard, A. B., S. J. Redding, and P. K. Schott, "Multiple Product Firms and Product Switching," *American Economic Review* 100 (2010), 70-91.

[7] Bernard, A. B., S. J. Redding, and P. K. Schott, "Multiproduct Firms and Trade Liberalization," *Quarterly Journal of Economics* 126 (2011), 1271-1318.

[8] Dhingra, S., "Trading Away Wide Brands for Cheap Brands," *American Economic Review* (forthcoming).

[9] Eckel, C., and J. P. Neary, "Multi-Product Firms and Flexible Manufacturing in the Global Economy," *Review of Economic Studies* 77 (2010), 188-217.

[10] Hallak, J. C. and J. Sivadasan, "Firms Exporting Behavior under Quality Constraints," Mimeo, University of Michigan, 2011.

[11] Lang, L., and R. Stulz, "Tobin's Q, Diversification, and Firm Performance," *Journal of Political Economy* 102 (1994), 1248-1280.

[12] Lileeva, A., and D. Trefler, "Improved Access to Foreign Markets Raises Plant-Level Productivity...for Some Products," *Quarterly Journal of Economics* 125 (2010), 1051-1099.

[13] Maksimovic, V., and G. Phillips, "Do Conglomerate Firms Allocate Resources Inefficiently Across Industries? Theory and Evidence," *Journal of Finance* 57 (2002), 721-767.

[14] Mayer, T., M. J. Melitz, and G. I. P. Ottaviani, "Market Size, Competition, and the Product Mix of Exporters," *American Economic Review* (forthcoming).

[15] Melitz, M. J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica* 71 (2003), 1695-1725.

[16] Prescott, E., and M. Visscher, "Organization Capital," *Journal of Political Economy* 88 (1980), 446-461.

[17] Rajan, R. G., H. Servaes, and L. Zingales, "The Cost of Diversity: The Diversification Discount and Inefficient Investment," *Journal of Finance* 55 (2000), 35-80.

[18] Schoar, A., "Effects of Corporate Diversification on Productivity," *Journal of Finance* 57 (2002), 2379-2403.

[19] Sutton, J., *Competing in Capabilities: The Globalization Process* (Oxford: Oxford University Press, 2012).

[20] Villalonga, B., "Diversification Discount or Premium? New Evidence from the Business Information Tracking Sites," *Journal of Finance* 59 (2004), 479-506.