The relevance or otherwise of the central bank's balance sheet

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1 Introduction

This paper explores the impacts on an economy of a central bank changing the size and composition of its balance sheet. We compare "orthodox" monetary policy (that is the central bank changing the short term nominal interest rate and implemented through changing the rate paid on forms of money, eg reserves) with policies that involve the central bank trading in longer term bonds. Whether the central bank purchasing longer term bonds can affect the real economy is a key policy issue. Major central banks – The Fed, the Bank of England and the Bank of Japan – have massively expanded their balance sheets in recent years in an attempt to stimulate demand in the wake of the financial crisis of 2007-08. (The ECB also substantially increased its balance sheet – though less through outright purchases of securities). There is a good deal of empirical evidence that such purchases have had a positive impact – both on asset prices and on demand. (For a recent review of the empirical evidence on the impact of asset purchases – or quantitative easing – see the special issue of the Economic Journal, November 2012. For an overview of the empirical and theoretical issues see Joyce, Miles, Scott and Vayanos (2012)).

One of the ways in which such asset purchases could influence prices and demand is via portfolio balance effects – that is through the impact that changes in the relative amount of money and bonds in private sector portfolios has upon asset values and demand. It is the significance of this portfolio channel that we assess in this paper. As Durre and Pill (2012) note this is not only way in which expansion of the central balance sheet can affect the economy. Some central bank policies involve the substitution of flows of funds through the central bank balance sheet for flows between private financial firms. In times of stress this can keep credit flowing. But much recent empirical work (e.g. Gagnon et al (2010) and Joyce et al (2010)) focuses on the portfolio balance channel and finds it is significant.

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But the empirical evidence is not conclusive and its interpretation is clouded by the lack of a clear theoretical framework which allows one to understand how such central bank purchases might work. The clearest theoretical model of the impact of central bank asset purchases – that of Woodford and Eggertson (2003) – has the property that such asset purchases are completely ineffective. In their model quantitative easing (QE) is simply irrelevant – it achieves nothing. That is because in their model the private sector looks through the balance sheets of the central banks and the Treasury and sees that, with complete markets, variations in the balance sheet of the central bank can be perfectly offset and are therefore irrelevant. In the Woodford and Eggertson model an infinitely lived, representative household maximises utility in a world with compete markets and faces no limit on borrowing against future income. It is clear that with these assumptions central bank purchases – which are essentially swaps of assets with the representative agent – can do nothing because that single representative agent owns the balance sheet of the central bank and such swaps do not change its opportunity set. But such an idealised economy is not likely to be a reliable guide to the impact of central bank purchases even in "normal" times, let alone in the environment of recent years and in the aftermath of a near - total collapse of financial markets and where the supply of credit was seriously disrupted.

Our aim is to develop a (highly stylized) model that allows for heterogeneity amongst households and incomplete financial markets to assess what the impact of the central bank purchases might be. We use a model which uses the simplest way of introducing heterogeneity and less than complete markets – this is an overlapping generations model with young and old agents alive at the same time and in which agents with finite lives face constraints that stem from their inability to enter into contracts with yet-to-be-born people.

Using a simple calibrated model which does not have complete markets and where credit restrictions are binding we explore the effects of shifts in private sector financial portfolios brought about by central bank transactions. Specifically we adopt an overlapping generations set up where households cannot leave negative bequests, and the old cannot borrow from the young. In this model shocks to productivity generate risks. Short term and long term financial assets may allow households to respond to risks in different ways. So by introducing shocks to labour endowments in a model with agents of different ages (and where those shocks have a substantially different impact on the young and the old) we generate a demand for financial assets in an environment where long-term and short-term bonds are not perfect substitutes. The expectations theory of the term structure does not exactly hold. Debt management policies and monetary policy that involves the central bank shifting the composition of assets may then have real effects – but whether the effects are significant are not clear.

In fact we find that the ineffectiveness result of Woodford and Eggertson is surprisingly robust. But it is not universal and central bank purchases may have effects on the pattern of asset prices and on real decisions when the government conducts fiscal policy in specific ways. This link between changes in the balance sheet of the central bank and fiscal policy is an essential part of any model where the balance sheet of the central bank can generate gains and losses. This relation

between central bank purchases of assets and the fiscal position is important to spell out since changes in the central bank balance sheet, because they can generate profits or losses, must ultimately have fiscal implications.

Nonetheless our overall finding is that the portfolio balance effects of central bank asset purchases is weak in a wide range of environments. That is not to say that the big expansion of central bank balance sheets in recent years has been ineffective. Our finding is rather that the portfolio balance channel evaluated in an environment of normally functioning (though nonetheless incomplete) asset markets is fairly weak.

In the first part of this paper we describe the model and the solution technique we use. We then consider how variations in the central bank holding of government bonds (both nominal bonds and real bonds) affect the economy. Those impacts depend upon the way in which the fiscal implications of asset purchases are handled by the Treasury. We consider various assumptions about how the gains and losses of central bank operations affect taxes and show the effect this has.

2 Model Overview

Our framework is an OLG model without bequests and with two assets: money, and government bonds. Households live for two periods. When bonds are issued they also have a maturity of two periods. Each generation is born without an endowment of the single (non-durable) consumption good but the ability to transform their own labour into the consumption good. Production uses labour only; there is no capital accumulation. The production technology has stochastic productivity.

Each young household decides how much labour to supply to produce the consumption good, and how much of this to consume. They sell the remainder to old households, in exchange for money, and decide how many government bonds to buy. Because the consumption good perishes unless consumed, young households can only transfer wealth to when they are old by holding money or government bonds. Neither young nor old households can borrow.

Money is remunerated at the policy rate set by the central bank. So money could be thought as Treasury-bills. But we could just as well think that there are 100% reserve backed commercial banks that are intermediaries between households and the central bank. Either way, we can think of money in this model as reserves of the central bank that are its liability and which pay a rate of interest equal to the central bank's policy rate. All money is interest bearing as long as the central bank sets a non-zero interest rate. We make this assumption because in developed economies non-interest bearing notes and coins are very much smaller than interest bearing accounts which can be easily used to finance transactions.

The Treasury issues bonds at a discount; bonds do not pay coupons. The amount issued is constant in each period. Bonds have a maturity of two periods at issuance. This fiscal rule is a very simple one which keeps the face value of

debt constant (the market value of government debt depends on real shocks (to productivity) and central bank policy. Taxes, which are lump sum, are varied to satisfy the fiscal rule. We abstract from credit risk of government bonds. There is no liquidity risk in the model.

Households pay state-dependent nominal lump sum taxes to the Treasury. Old households have simple decisions to make: they have no bequest motive so simply liquidate all their assets to finance the consumption good which they buy from young households. We assume old households do not supply labour.

What distinguishes money and bonds in our model? Money in this model is the only asset that can be used to buy goods. Bonds must be sold for money before they can be used for consumption. The central bank takes deposits, which we could think of as reserves that a commercial banking sector holds against deposits held by households. Reserves ("money") pay a 1 period interest rate set by the central bank. The central bank will use a policy rule to set the rate, which will be some function of the exogenous random variable in the model (productivity) and also a random shock. The two financial assets – money and bonds – differ because money has a known nominal value 1 period ahead (one plus the interest rate set by the central bank today) while newly issued bonds have a value one period ahead which depends on the interest rate that the central bank will set in the next period – which is not known today. Bonds with one period left to maturity are perfect substitutes for money because both assets have a know nominal value one period ahead. This means that the price of a one period bond is tied to the interest rate set by the central bank. The financial assets in the model are dramatically simple – in fact as simple as is possible while allowing longer dated government bonds to be imperfect substitutes for shorter dated financial assets.

The central bank balance sheet is straightforward: it holds 1 and 2 period bonds as assets which it acquires in exchange for issuing money (its liability). Any profits (or losses) made by the central bank from its portfolio of assets and liabilities is passed to the Treasury and taxes are raised or lowered accordingly so as to insure that the Treasury can continue to issue new bonds to replace those that mature.

3 Events

In each period:

1. The stochastic productivity of young households become know. Young households decide how much labour to supply to produce the consumption good.

2. The Treasury issues new 2-period bonds to replace those that mature and collects taxes from households to balance the budget. The central bank decides how many of these newly issued bonds to buy, and what interest rate to set for the remuneration of money.

3. Old households receive interest on their money holdings and sell their bonds (which now have a remaining maturity of one period) to the central bank in exchange for money. The central bank has to accept all 1-period bonds sold by old households at the price implied by its choice for the policy rate. One can think of these transactions as open market operations conducted to implement a particular decision over the policy rate.

4. Old households use their money to purchase some of the young households newly-produced consumption good.

4 Model: Detailed Specification

Our notation is as follows. We index individual households by j. Labour supply of a young household born in period t is $h_{j,t}$. Using this labour input, the household produces $y_{j,t} = \omega_t h_{j,t}^{\alpha}$ real units of the consumption good. ω_t is an aggregate productivity shock, distributed independently across periods according to a normal distribution with mean μ_ω and standard deviation σ_ω.The young household's real consumption is $c_{j,t}^Y$. Its nominal money holdings are $m_{j,t}^Y$, and its holdings of bonds issued in t and maturing in $t+2$ are denoted by $g_{j,t,t+2}^Y$. The market-clearing price of these bonds is $P_{t,t+2}^g$, and that of bonds with a remaining maturity of one period is $P_{t,t+1}^g$. The price of the consumption good is P_t^c. The lump-sum tax is denoted τ_t^Y if levied on the young, and τ_t^O if levied on the old. We do not indicate the dependence of the endogenous variables on the state variables of our model but it should be taken as read.

Each household's period utility is of the CRRA variety:

$$u(c,h) = \frac{\left(c^{1-\rho}(1-h)^{\rho}\right)^{1-\sigma_c} - 1}{1-\sigma_c} \tag{1}$$

and lifetime utility (recalling no-one can work when they are old) is:

$$U\left(c_{j,t}^Y, c_{j,t+1}^O, h_{j,t}\right) = u\left(c_{j,t}^Y, h_{j,t}\right) + \beta E_t\left[u\left(c_{j,t+1}^O, 0\right)\right] \tag{2}$$

where E_t is his expectation of the future given his information in period t (which include the realisation of period-t random variables and prices). A household's decision problem is to maximise lifetime utility over $\left\{h_{j,t}, m_{j,t}, g_{j,t,t+2}^Y\right\}$ subject to the budget constraints

$$P_t^c\left(y_{j,t} - c_{j,t}^Y\right) = m_{j,t}^Y + P_{t,t+2}^g g_{j,t,t+2}^Y + \tau_t^Y \tag{3}$$

$$m_{j,t}^Y(1+i_t) + P_{t+1,t+2}^g g_{j,t,t+2}^Y - \tau_t^O = P_{t+1}^c c_{j,t+1}^O \tag{4}$$

The left-hand side of (3) are the proceeds from selling the consumption good, and the right-hand side how the young household uses the proceeds: it holds some of it in money, uses some to purchase newly issued bonds, and pays some lump-sum taxes. Note that the young do not buy 1 period bonds which are perfect substitutes for money. We assume that the central bank stands ready to

swap one period bonds for money – these are open market operations required to establish a particular 1 period interest rate.

The left-hand side of (4) denotes the nominal wealth after taxes when old, $w^O_{j,t+1}$: this is the sum of remunerated money holdings, the receipts from selling bonds (now 1-period) to the central bank, minus tax payments. (Notice that one could equally write this problem as one of choosing any other three of the period-t decision variables $\left\{c^Y_{j,t}, h_{j,t}, m_{j,t}, g^Y_{j,t,t+2}\right\}$; it is clearly optimal for the old households to spend their entire nominal wealth on the consumption good in the absence of a bequest motive.)

4.1 First-order conditions

Substituting the budget constraints for a young household's consumption into the lifetime utility yields

$$\begin{aligned} & U \\ = \; & \frac{1}{1-\sigma_c}\left(\left(\left(\omega_t h^\alpha_{j,t} - \left(\tau^Y_t + m^Y_{j,t} + P^g_{t,t+2} g^Y_{j,t,t+2}\right)/P^c_t\right)^{1-\rho}\left(1-h_{j,t}\right)^\rho\right)^{1-\sigma_c} - 1\right) \\ & + \beta E_t\left[\frac{1}{1-\sigma_c}\left(\left(\left(m^Y_{j,t}\left(1+i_t\right) + P^g_{t+1,t+2} g^Y_{j,t,t+2} - \tau^O_t\right)/P^c_{t+1}\right)^{(1-\rho)(1-\sigma_c)} - 1\right)\right] \end{aligned} \tag{5}$$

The optimal solution has first order conditions

- with respect to money holdings, $m_{j,t}$:

$$\frac{\partial U}{\partial m^Y_{j,t}} = u'_c\left(c^Y_{j,t}, h_{j,t}\right)\left(-\frac{1}{P^c_t}\right) + \beta E\left[u'_c\left(c^O_{j,t+1}, 0\right)\frac{1+i_t}{P^c_{t+1}}\right] = 0 \tag{6}$$

  where we denote the marginal utility with respect to consumption as

$$u'_c\left(c,h\right) = \left(1-\rho\right)c^{(1-\sigma_c)(1-\rho)-1}\left(1-h\right)^{\rho(1-\sigma_c)} \tag{7}$$

- with respect to holdings of bonds with a remaining maturity of two periods, $g^Y_{j,t,t+2}$:

$$\frac{\partial U}{\partial g^Y_{j,t}} = u'_c\left(c^Y_{j,t}, h_{j,t}\right)\left(-\frac{P^g_{t,t+2}}{P^c_t}\right) + \beta E\left[u'_c\left(c^O_{j,t+1}, 0\right)\frac{P^g_{t+1,t+2}}{P^c_{t+1}}\right] = 0 \tag{8}$$

- with respect to labour supply, $h_{j,t}$

$$\left(1-h_{j,t}\right)\frac{1-\rho}{\rho}y'_{j,t} = c^Y_{j,t} \tag{9}$$

The first-order conditions can be expressed as

$$u_c'\left(c_{j,t}^Y, h_{j,t}\right) = \beta E\left[(1+i_t)\frac{P_t^c}{P_{t+1}^c}u_c'\left(c_{j,t+1}^O, 0\right)\right] \quad (10)$$

$$u_c'\left(c_{j,t}^Y, h_{j,t}\right) = \beta E\left[\frac{P_{t,t+1}^g}{P_{t,t+2}^g}\frac{P_t^c}{P_{t+1}^c}u_c'\left(c_{j,t+1}^O, 0\right)\right] \quad (11)$$

$$(1-h_{j,t})\frac{1-\rho}{\rho}y_{j,t}' = c_{j,t}^Y \quad (12)$$

$$y_{j,t} = \omega_t h_{j,t}^\alpha \quad (13)$$

We introduce the minimum amount of heterogeneity needed by assuming all agents of a given generation are the same. We use capital letters to denote their decisions. The market clearing conditions for the consumption good and newly issued bond are:

$$C_t^O = Y_t - C_t^Y \quad (14)$$

$$G_{t,t+2}^Y = \gamma - G_t^{CB} \quad (15)$$

(14) states that in equilibrium, the period t per-person consumption of the old and the young must equal per-person production in t, Y_t. (15) states that per-person purchases of newly issued bonds must equal the net per-person supply of bonds: the difference between issuance, γ, and the amount of newly issued bonds that the central bank buys (per person), G_t^{CB}.

Because the old spend all their nominal wealth on the consumption good, we have

$$P_t^c C_t^O = W_t^O \quad (16)$$

The post-tax per-person nominal wealth of the old is the sum of their remunerated reserve holdings, plus the value of their bonds, minus lump-sum taxes:

$$W_t^O = M_{t-1}^Y(1+i_{t-1}) + P_{t,t+1}^g G_{t-1,t+1}^Y - \tau_t^O \quad (17)$$

The per-person money holdings of the young are given by the difference between their revenue from selling their consumption good to the old, $P_t^c C_t^O = W_t^O$, and the value of their purchases of bonds:

$$M_t^Y = W_t^O - G_{t,t+2}^Y P_{t,t+2}^g \quad (18)$$

$P_{t,t+1}^g$, the price at which the central bank buys government bonds with a remaining maturity of one period from the old, depends on the type of government bond that the Treasury issues. This is explained in the next two sections.

4.2 Monetary policy

The central bank's policy instruments are the nominal interest rate at which it remunerates money ('Bank Rate'), and the amount of newly issued government bonds that it buys. We will refer to the purchase of newly issued government

bonds as 'quantitative easing', or QE. We also make the inconsequential assumption that it buys all bonds with a remaining maturity of one period from old households. The central bank's assets therefore comprise all government bonds with a remaining maturity of one period, and those newly issued government bonds which it chooses to buy as part of its QE operation. Its liabilities consist of money which we can think of as issued directly to households or else as held as reserves by commercial banks who issue deposits to households of exactly equivalent value. We assume that the central bank has no equity; instead, it is indemnified for any losses that it may make by the Treasury, and transfers any profits to the Treasury.

We assume that the central bank follows a policy rule that only depends on the exogenous state variables: the productivity shock, ω_t, and the random innovations to the level of Bank Rate and its purchases of newly issued bonds. The innovations are independently normally distributed with zero mean and standard deviation $\sigma_{\varepsilon i}$ and $\sigma_{\varepsilon g}$, respectively. In addition, we assume that the policy rules are linear subject to a zero lower bound for Bank Rate ($i_t \geq 0$), and that the central bank's purchases are capped by the Treasury's (constant) issuance of bonds, and that it cannot issue its own liabilities ($G_t^{CB} \in [0, \gamma]$). Within these bounds, the policy rules take the form

$$i_t = a_1 + a_2 (\omega_t - \mu_\omega) + \varepsilon_{i,t} \tag{19a}$$

$$G_t^{CB} = b_1 + b_2 (\omega_t - \mu_\omega) + \varepsilon_{g,t} \tag{19b}$$

where a, b are scalars.

4.3 Fiscal policy

Fiscal policy only involves setting the size of the nominal lump-sum taxes to levy on households. There are no government expenditures other than transfers to the central bank on maturity of the bonds, and those required to indemnify the central bank for any losses it may make. Government revenues consist only of taxes levied on households and of profits that the central bank may make.

We assume that the government aims to balance its budget in each period by levying an appropriate amount of lump-sum taxes. The amount of bonds issued is assumed to be constant in each period. (We set this quantity at $\gamma = 0.5$ so that at any time there are bonds with aggregate face value of 1 outstanding). The costs of servicing the outstanding zero-coupon debt are booked on an accrual basis. In each period, the tax is then equal to the nominal return that that period's old earned on their portfolio:

$$\tau_{t+1}^Y + \tau_{t+1}^O = i_t M_t^Y + \left(P_{t+1,t+2}^g - P_{t,t+2}^g\right)\left(\gamma - G_t^{CB}\right)$$

We will consider different scenarios, which differ by the generation that is subject to taxation (either the young or the old), and by the type of government bond that the Treasury issues: either nominal bonds, or real (that is price-level linked) bonds.

Neither type of government bond pays a coupon. Nominal bonds are redeemed at a price of 1 at maturity. The central bank purchases nominal bonds with a remaining maturity of one period at the price at which young households would be indifferent between holding these bonds and money. In contrast, price-level linked bonds with a remaining maturity of one period are purchased by the central bank at the price of the consumption good prevailing at that time. Price-level linked bonds therefore provide a real savings opportunity for households, in contrast to nominal bonds. Below we will concentrate on results where the government only issues nominal bonds (since it turns out that results are very similar if the government issues real bonds).

5 Calibration

We should think of a period as about half an adult life – so of the order of 25 years. We set the discount factor to 0.66, implying a discount rate of 0.5 (or 50%). With a 25 year period that corresponds to a discount rate of about 2% a year. That 2% a year number is a natural way of thinking about the average level of the policy rate set by the central bank, so in calibrations we will often use a policy rule which implies that on average Bank Rate is 50% a period, or about 2% a year (though that rate will vary depending on the outcome for productivity). There is inevitably a tension between wanting the model to be simple (so using two period lives) and realism. Two period lives means periods must be long. That stretches the nature of the monetary policy decision uncomfortably, because we want the policy rate to be set for one period. But for our purposes what really matters is that we have one asset (a bond) with a life which is significantly longer than the period for which the interest rate set by the central bank can be known with some certainty.

We assume a coefficient of relative risk aversion of 3 and an exponent on labour supply in the utility function of 0.5. The average level of productivity is 1; its standard deviation is 0.1. The exponent on labour in the production function is 2/3. The random component of the central bank's interest rate rule is 0.1 – so that 1 standard deviation either side of the mean level of bank rate is 10 percentage points. That would cover a range from 40% to 60% (roughly 1.6% a year to 2.4%). The random component of the central bank's purchases of newly issued bonds is 0.1 (which is 20% of all the new bonds issued).

Table 1 summarises the parameters of the model.

6 Results for when taxes fall on the old

When the Treasury taxes the old, its balanced budget rule means that the nominal wealth of the old in each period is constant in equilibrium. Because taxes are lump sum, young households retain their marginal incentive to adjust their portfolio to promised returns. This leads to a particularly simple (but we think still interesting) formulation of the problem. In an equilibrium in which all

households of a single generation make the same decisions, the post-tax nominal wealth of the old is

$$\begin{aligned} W_t^O &= M_{t-1}^Y (1+i_{t-1}) + P_{t,t+1}^g G_{t-1,t+1}^Y - \tau_t^O \\ &= M_{t-1}^Y (1+i_{t-1}) + P_{t,t+1}^g \left(\gamma - G_{t-1}^{CB}\right) \\ &\quad -i_{t-1} M_{t-1}^Y - \left(P_{t,t+1}^g - P_{t-1,t+1}^g\right)\left(\gamma - G_{t-1}^{CB}\right) \\ &= M_{t-1}^Y + \left(\gamma - G_{t-1}^{CB}\right) P_{t-1,t+1}^g \\ &= W_{t-1}^O \end{aligned} \tag{20}$$

The first line here substitutes the young's government bond purchases, $G_{t-1,t+1}^Y$, using the market clearing condition of newly issued bonds, $\gamma = G_{t-1}^{CB} + G_{t-1,t+1}^Y$. The last equality at (20) simply recognises that the post-tax nominal wealth of the old is spent on the consumption good that the young produce, who retain some of the money received (M_{t-1}^Y) and use the remainder to purchase newly issued government bonds at price $P_{t-1,t+1}^g$. The tax rule cuts the link between periods that the autoregressive process for nominal wealth would otherwise provide: taxing the old means that $W_t^O = W_{t-1}^O$, and all exogenous state variables are by assumption independently distributed. Shocks therefore only affect the endogenous variables in the period in which they occur. One period later, absent further shocks, the model is back in stochastic steady state.

We first assume that all bonds are nominal debt. This means that money and nominal government bonds with a remaining maturity of one period are perfect substitutes: at given market prices, their nominal return is known with certainty. In equilibrium, we must therefore have $P_{t,t+1}^g = 1/(1+i_t)$.

Table 2 shows the (stochastic) steady state and the impulse response function for shocks to bank rate and to central bank purchases of 2 period bonds. In Table 2A we show results for a specific rule for the policy rate. Here the central bank sets its interest rate to 50% per period when all shocks are at their mean, and reduces the policy rate if productivity is above average. It does so in a way so that when the productivity shock is 10% above average (1 standard deviation) the policy rate is 10pp lower. This rule is symmetric for below average productivity. The central bank buys on average 50% of all newly issued government bonds. These purchases do not vary with the productivity shock.

Column 1 shows the stochastic steady state – this is the equilibrium of the model when all the shocks are at zero (so productivity is 1) but all households base decisions on the true processes for random variables. Column 2 shows the outcomes when productivity is 1 standard deviation above the mean; column 3 shows the outcomes a period after such a productivity shock.

Column 4 shows the impact of a 1 standard deviation positive shock to the policy rate in the first period; column 5 shows outcomes a period later (when we are back to the stochastic steady state).

Column 6 shows the impact of a 1 standard deviation expansion of asset purchases – a swap of central bank money for 2 period bonds. The size of this expansion in purchases is 20% of all bond issues (=0.1). Column 7 shows what then happens 1 period later, where we return to the stochastic steady state.

In steady state, and following the particular rule for the policy rate, inflation is expected to be almost zero (1.8% over a period that we can think of as about 25 years). This means that expected real returns are close to the nominal policy rate. Notice however that the expected equilibrium return on bonds is slightly higher than that on money (just over 51% on bonds against about 49% on money). This failure of the pure expectations theory reflects the nature of the monetary policy rule: high productivity means that the policy rate falls, so the price at which the central bank purchases bonds with a remaining maturity of one period increases in high productivity states of the world. Correspondingly, low productivity states are ones in which the price at which the central bank purchases these bonds is low. High productivity also means high production, so the price of the consumption good is low. So high productivity is good not only for the young, but also for the old. To be encouraged to hold an asset whose returns are positively correlated with a state that is good anyway, households need to expect to earn a higher return on bonds than on money.

In response to higher productivity (column 2) households increase their labour supply slightly. Together with higher productivity, this translates into substantially higher production. This extra output, even with a policy rate that boosts the wealth of the old, means the price level is lower. So the central bank monetary policy rule does not preserve price stability in the face of real shocks. The price level is about 10% lower when productivity is higher. This also means that starting from a position of high productivity, inflation over the next period is expected to be higher. Higher expected inflation adds to the impact of the lower policy rate. So the expected real returns of both money and bonds both fall when productivity today is high. Nevertheless, the positive income effect of higher productivity means that the savings rate slightly increases.

Columns 4-5 show the results for the case in which the central bank's interest rate rule is shocked. Column 4 shows the shock period; column 5 the following period (which is back to the stochastic steady state). The central bank's policy rate increases from 50% to 60% as a result of the policy shock. This causes the price level to be lower by about 3%. Expected inflation to the next period becomes positive. This means that the increase in the nominal interest rate translates into a (slightly smaller) increase in the real interest rate for both money and bonds. With higher returns on savings, young households increase their labour supply and their savings rate rises.

Columns 6-7 show the results when the central bank's bond purchase rule is shocked. These outcomes are the relevant ones for considering the impact of central bank asset purchases. Column 6 shows what happens in the shock period; column 7 is the following period (which with taxes levied on the old is back to the steady state). The central bank's purchases of government bonds increase from 0.25 (50% of newly issued bonds) to 0.35 as a result of the shock. The impacts of this are very small. As bonds become scarcer in the portfolio of young households, and money more plentiful, the expected return on newly issued bonds falls slightly, and that on money rises. These effects are too small to lead to significant variations in real variables.

In Table 2B, we consider a different central bank policy rule for setting

interest rates. Here the central bank sets its interest rate to 50% per period when all shocks are at their mean, and increases it by 10pp when the productivity shock is 10% above average (1 standard deviation). This rule is symmetric with respect to productivity. This somewhat perverse rule means that a positive supply shock leads to a tightening in policy. As before, the central bank buys on average 50% of all newly issued government bonds. These purchases do not vary with the productivity shock.

Table 2B shows that the impact of a positive productivity shock – given this monetary policy rule of tightening policy after a positive supply shock – is to drive the level of prices lower than when the central bank responds by lowering interest rates. So this interest rate rule creates more volatility in inflation in the face of productivity shocks. Such variability is unhelpful so it is not surprising that this policy rule creates slightly lower expected welfare than the rule used for table 2A. It is worth noting too that under the different policy rule for the nominal interest rate on money, the payoff on government bonds is now negatively correlated with the good state (high productivity): high productivity means a high nominal interest rate, so a low price of bonds that mature in the following period. So households are willing to accept a lower expected return on bonds than on money.

Impulse responses are also similar to those of Table 2A. Column 2 shows that in response to a positive productivity shock today, the increase in expected inflation more than offsets the increase in the policy rate. So the expected real returns of both money and bonds fall, albeit by less in Table 2A. The savings ratio is therefore slightly higher than in Table 2A. Crucially, we find once again that variations in central bank purchases of bonds have very little affect on the wider economy – neither returns on financial assets nor real decisions are much affected.

Table 2C varies the central bank's bond purchase rule. The central bank now buys on average no newly issued government bonds. As before, these purchases do not vary with the productivity shock. And as in Table 2A, the central bank sets its interest rate to 50% per period when all shocks are at their mean, and reduces it by 10pp when the productivity shock is 10% above average.

The steady state is very similar to that of Table 2A. The impact of a surprise purchase of government bonds (column 6) on the expected real returns of money and bonds is again small, indeed it is somewhat smaller than in Table 2A. (This is probably because the shock only reduces the supply of newly issued bonds to households from 0.5 to 0.4, a relatively smaller decline than in Table 2A, where it fell from 0.25 to 0.15 in response to the shock.)

The results on the impact of central bank purchases of real (indexed link) bonds are very similar (and are not reported): once again central bank asset purchases did little to asset prices or to real outcomes.

We also experimented with much larger moves in the central bank rate in response to shocks to productivity, and somewhat greater shocks to productivity. In all cases we found weak responses to central bank asset purchases.

7 Impact of taxing the young

In this case, we have

$$\tau_t^Y = i_{t-1} M_{t-1}^Y + \left(P_{t,t+1}^g - P_{t-1,t+1}^g\right)\left(\gamma - G_{t-1}^{CB}\right) \tag{21}$$

This adds four lagged dependent variables to our model: i_{t-1}, M_{t-1}^Y, $P_{t-1,t+1}^g$, and G_{t-1}^{CB} and therefore considerably complicates the solution of our model. In the next iteration of this paper we will report results when it is the young who pay taxes.

8 Conclusions

Models in which there is a single representative agent that lives infinitely long in a world of complete markets are models where a set of assumptions is made that imply that changes in the balance sheet of the central bank have no impact. We have developed a simple and highly stylized model of the economy where we assess whether shifts in the balance sheet of the central bank have an impact where we no longer have complete markets and where there is heterogeneity across agents each of which has a finite life. Analytical solutions to that model are not available. So we turn to simulations of a calibrated version of this OLG model. We find that across a fairly wide set of environments – with different rules for the setting of interest rates by the central banks and different ways in which the central bank sets asset purchases – the impact of asset purchases working through a portfolio re-balancing channel is weak.

That result does not show that central bank asset purchases (quantitative easing) does not work. And it certainly does not show that the major expansion of the balance sheets of the central banks in the US, UK, Japan and in the euro zone in recent years have been ineffective. We reach a different conclusion, which is that the main way in which such balance sheet changes have worked is probably not through the operation of a portfolio rebalancing of private sector portfolios undertaken in an environment of normally functioning financial markets. Durre and Pill (2012) argue that the main way in which the central bank using its balance sheet has affected the wider economy is through providing alternatives to intermediation flowing through markets that have been disrupted. As they put it:

"Our characterisation of the transmission mechanism of non-standard policy measures is based on the central bank's offering its own balance sheet as a vehicle for intermediating those intra-financial flows that are disrupted as the financial market seizes up".

We think our results provides some indirect evidence in support of that characterisation.

9 Annex: Numerical solution method

9.1 Overview

Independently of the scenario, we proceed by setting up a grid of conjectured equilibrium prices (and taxes) for each state, and solve households' optimisation problem for this set of prices. We then adjust prices according to excess demand or supply for each good.

9.2 Details

We describe here the details for the case in which the government issues nominal bonds and taxes are levied on the old.

1. We set up a grid of conjectured equilibrium prices and taxes $\left(P_t^c, P_{t,t+2}^g, \tau_{t+1}^O\right)_0$ and central bank decisions (i_t, G_t^{CB}). Period-t variables are unknown functions of the contemporaneous state variables $(\omega_t, \varepsilon_{i,t}, \varepsilon_{g,t})$, while τ_{t+1}^O (which in equilibrium taxes the return from t to $t+1$ of old households' portfolio in equilibrium) depends additionally on $(\omega_{t+1}, \varepsilon_{i,t+1}, \varepsilon_{g,t+1})$. We do not assume that these functions have a specific form but just choose a value (a number) for each price at each state.

2. Starting from initial guesses for the three household choice variables $\left(h_{j,t}, m_{j,t}^Y, g_{j,t,t+2}^Y\right)$, we minimise the joint error of the first-order conditions at each state. This is done using Matlab's $fmincon$ function, where the constraints are

   (a) the young cannot issue money nor bonds: $m_{j,t}^Y, g_{j,t,t+2}^Y \geq 0$

   (b) the nominal wealth of the old after taxes must be positive: $w_{j,t}^O \geq 0$

   (c) the young cannot supply a negative amount of labour: $h_{j,t} \geq 0$

   The expectations are solved using Gauss-Hermite integration over our three normally distributed exogenous variables. (We used five nodes for the results presented in the main part of this paper.) This yields an array of values for each of the young HH's choice variables for each realisation of the state variables.

3. Only now do we impose that in equilibrium, all households of each generation make the same decision X_t, and set $x_{j,t} = X_t$ for each endogenous variable $x_{j,t} \in \left\{c_{j,t}^Y, h_{j,t}, m_{j,t}^Y, g_{j,t,t+2}^Y\right\}$. ($X_t$ denotes cross-sectional averages over $x_{j,t}$.)

(a) We compute aggregate nominal wealth of the old in $t+1$ after taxes:

$$\begin{aligned} & W_{t+1}^{O}\left(\omega_t, \varepsilon_{i,t}, \varepsilon_{g,t}, \omega_{t+1}, \varepsilon_{i,t+1}, \varepsilon_{g,t+1}\right) \\ = \; & i_t\left(W^{O}-P_{t,t+2}^{g}\left(\gamma-G_t^{CB}\right)-P_{t,t+3}^{g}\gamma\right) \\ & +\left(P_{t+1,t+3}^{g}-P_{t,t+3}^{g}\right)\gamma+\left(1/\left(1+i_{t+1}\right)-P_{t,t+2}^{g}\right) \\ & \cdot\left(\gamma-G_t^{CB}\right) \\ & -\tau_{t+1}^{O} \end{aligned} \tag{22}$$

where W^O is the 'target' for the nominal wealth of the old (which we set to 1).

(b) We revise our best guesses for the prices by comparing relative excess demands and supply for the consumption good and the government bonds.

(c) We recalculate the tax rate as

$$\tau_{t+1}^{O}\left(\omega_t, \varepsilon_{i,t}, \varepsilon_{g,t}, \omega_{t+1}, \varepsilon_{i,t+1}, \varepsilon_{g,t+1}\right) = W_{t+1}^{O} - W^{O} \tag{23}$$

4. We iterate steps two and three until the relative difference between demand and supply in the markets for the consumption good and the bonds is smaller than 10^{-5}. This is usually reached after fifteen rounds of price changes.

10 References

References:

Durre, A. and Pill, H (2012): "Central Bank Balance Sheets as Policy Tools", Bank for International Settlement papers, no 66.

Eggertson, G. and Woodford, M. (2003): "The Zero Bound on Interest Rate and Optimal Monetary Policy", Brookings Papers on Economic Activity, 2003, 1. Pages 139-211

Gagnon, J; Raskin, M; Remache, J; and Sack, B. (2010): "Large Scale Asset Purchases by the Federal Reserve: Did they Work?" FRB New York staff report no 441.

Joyce, M; Miles, D; Scott, A; and Vayanos, D ""Quantitative Easing and Unconventional Monetary Policy", The Economic Journal, November 2012, vol 122, pp 271-289.

Joyce, M; Lasaosa, A.; Stevens, I and Tong, M. (2010): "The Financial Market Impact of Quantitative Easing", Bank of England Working paper no. 393.

Table 1: Calibration

Parameters		**Base calibration**
β	Discount factor	0.67
ρ	Exponent of leisure in utility function	0.5
σ_c	CRRA coefficient	3
α	Exponent of labour in production function	0.67
γ	Amount of bonds issued in each period	0.5
W^o	Nominal wealth of old HHs net of taxes in the scena	1
Shocks		
μ_ω	mean productivity shock	1
σ_ω	SD of productivity shock	0.1
$\sigma_{\varepsilon,i}$	SD of CB interest rate innovation	0.1
$\sigma_{\varepsilon,g}$	SD of CB bond purchase innovation	0.1

Table 2A: Government bonds, not price level linked: Policy rate falls in response to positive supply shock. CB buys half of newly issued bonds when all shocks are at their means

Shock (+1SD) to...		Productivity		Policy rate		CB bond purchases	
Variable	**1: sSS**	**2: Shock**	**3: Shock +1**	**4: Shock**	**5: Shock +1**	**6: Shock**	**7: Shock +1**
Productivity shock	1	1.1	1	1	1	1	1
CB innovation on Bank	0	0	0	0.1	0	0	0
CB innovation on bond purchases	0	0	0	0	0	0.1	0
Bank Rate	0.5000	0.4000	0.5000	0.6000	0.5000	0.5000	0.5000
CB bond purchases	0.2500	0.2500	0.2500	0.2500	0.2500	0.3500	0.2500
Labour supply	0.5300	0.5316	0.5300	0.5344	0.5300	0.52998	0.53001
Production	0.6549	0.7219	0.6549	0.6585	0.6549	0.65490	0.65492
Consumption of the young	0.3872	0.4240	0.3872	0.3825	0.3872	0.38720	0.38717
Consumption of the old	0.2678	0.2979	0.2678	0.2760	0.2678	0.2677	0.2678
Savings ratio	0.4088	0.4127	0.4088	0.4191	0.4088	0.4088	0.4088
Bond purchases of young HHs	0.2500	0.2500	0.2500	0.2500	0.2500	0.1500	0.2500
Money holdings of the young	0.8884	0.8801	0.8884	0.8953	0.8884	0.9330	0.8884
Price of consumption good	3.7347	3.3697	3.7347	3.6242	3.7347	3.7355	3.7347
Price of bonds	0.4463	0.4797	0.4463	0.4190	0.4463	0.4467	0.4463
Utility of young	-2.2478	-2.0246	-2.2478	-2.3078	-2.2478	-2.2474	-2.2478
Expected utility of old HH next period	-1.4010	-1.4010	-1.4010	-1.4010	-1.4010	-1.4010	-1.4010
Expected inflation	0.0180	0.1327	0.0180	0.0494	0.0180	0.0178	0.0180
Expected real return on bonds	0.5115	0.2745	0.5115	0.5635	0.5115	0.5104	0.5115
Expected real return on money	0.4957	0.2630	0.4957	0.5471	0.4957	0.4960	0.4957

Table 2B: Government bonds, not price level linked: Policy rate rises in response to positive supply shock. CB buys half of newly issued bonds when all shocks are at their means

Shock (+1SD) to...		Productivity		Policy rate		CB bond purchases	
Variable	**1: sSS**	**2: Shock**	**3: Shock +1**	**4: Shock**	**5: Shock +1**	**6: Shock**	**7: Shock +1**
Productivity shock	1	1.1	1	1	1	1	1
CB innovation on Bank Rate	0	0	0	0.1	0	0	0
CB innovation on bond purchases	0	0	0	0	0	0.1	0
Bank Rate	0.5000	0.6000	0.5000	0.6000	0.5000	0.5000	0.5000
CB bond purchases	0.2500	0.2500	0.2500	0.2500	0.2500	0.3500	0.2500
Labour supply	0.5306	0.5415	0.5306	0.5350	0.5306	0.53058	0.53059
Production	0.6554	0.7311	0.6554	0.6590	0.6554	0.65539	0.65540
Consumption of the young	0.3866	0.4121	0.3866	0.3819	0.3866	0.38656	0.38655
Consumption of the old	0.2688	0.3190	0.2688	0.2771	0.2688	0.2688	0.2688
Savings ratio	0.4102	0.4364	0.4102	0.4205	0.4102	0.4102	0.4102
Bond purchases of young HHs	0.2500	0.2500	0.2500	0.2500	0.2500	0.1500	0.2500
Money holdings of the young	0.8861	0.8930	0.8861	0.8931	0.8861	0.9316	0.8861
Price of consumption good	3.7196	3.1653	3.7196	3.6099	3.7196	3.7199	3.7196
Price of bonds	0.4556	0.4279	0.4556	0.4278	0.4556	0.4558	0.4556
Utility of young	-2.2556	-2.1494	-2.2556	-2.3158	-2.2556	-2.2554	-2.2556
Expected utility of old HH next period	-1.4169	-1.4169	-1.4169	-1.4169	-1.4169	-1.4169	-1.4169
Expected inflation	0.0307	0.2232	0.0307	0.0624	0.0307	0.0306	0.0307
Expected real return on bonds	0.4621	0.3215	0.4621	0.5123	0.4621	0.4617	0.4621
Expected real return on money	0.5066	0.3623	0.5066	0.5586	0.5066	0.5067	0.5066

Table 2C: Government bonds, not price level linked: Policy rate falls in response to positive supply shock. CB buys no newly issued bonds when all shocks are at their means

Shock (+1SD) to...		Productivity		Policy rate		CB bond purchases	
Variable	**1: sSS**	**2: Shock**	**3: Shock +1**	**4: Shock**	**5: Shock +1**	**6: Shock**	**7: Shock +1**
Productivity shock	1	1.1	1	1	1	1	1
CB innovation on Bank Rate	0	0	0	0.1	0	0	0
CB innovation on bond purchases	0	0	0	0	0	0.1	0
Bank Rate	0.5000	0.4000	0.5000	0.6000	0.5000	0.5000	0.5000
CB bond purchases	0.0000	0.0000	0.0000	0.0000	0.0000	0.1000	0.0000
Labour supply	0.5300	0.5316	0.5300	0.5344	0.5300	0.53000	0.53001
Production	0.6549	0.7219	0.6549	0.6585	0.6549	0.65492	0.65492
Consumption of the young	0.3872	0.4239	0.3872	0.3825	0.3872	0.38718	0.38718
Consumption of the old	0.2677	0.2980	0.2677	0.2760	0.2677	0.2677	0.2677
Savings ratio	0.4088	0.4128	0.4088	0.4191	0.4088	0.4088	0.4088
Bond purchases of young HHs	0.5000	0.5000	0.5000	0.5000	0.5000	0.4000	0.5000
Money holdings of the young	0.7768	0.7604	0.7768	0.7905	0.7768	0.8215	0.7768
Price of consumption good	3.7349	3.3690	3.7349	3.6243	3.7349	3.7350	3.7349
Price of bonds	0.4463	0.4792	0.4463	0.4190	0.4463	0.4463	0.4463
Utility of young	-2.2477	-2.0250	-2.2477	-2.3077	-2.2477	-2.2477	-2.2477
Expected utility of old HH next period	-1.4009	-1.4009	-1.4009	-1.4009	-1.4009	-1.4009	-1.4009
Expected inflation	0.0179	0.1329	0.0179	0.0493	0.0179	0.0179	0.0179
Expected real return on bonds	0.5115	0.2755	0.5115	0.5636	0.5115	0.51147	0.51150
Expected real return on money	0.4958	0.2628	0.4958	0.5472	0.4958	0.49585	0.49584