A Model of Financialization of Commodities*

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Abstract

A sharp increase in the popularity of commodity investing in the past decade has triggered an unprecedented inflow of institutional funds into commodity futures markets. Such financialization of commodities coincided with significant booms and busts in commodity markets, raising concerns of policymakers. In this paper, we explore the effects of financialization in a model that features institutional investors alongside traditional futures markets participants. The institutional investors care about their performance relative to a commodity index. We find that if a commodity futures is included in the index, supply and demand shocks specific to that commodity spill over to all other commodity futures markets. In contrast, supply and demand shocks to a nonindex commodity affect just that commodity market alone. Moreover, prices and volatilities of all commodity futures go up, but more so for the index futures than for nonindex ones. Furthermore, financialization—the presence of institutional investors—leads to an increase in correlations amongst commodity futures as well as in equity-commodity correlations. Consistent with empirical evidence, the increases in the correlations between index commodities exceed those for nonindex ones. We model explicitly demand shocks which allows us to disentangle the effects of financialization from the effects of demand and supply (fundamentals). We perform a simple calibration and find that financialization accounts for 11% to 17% of commodity futures prices and the rest is attributable to fundamentals.

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1. Introduction

In the past decade the behavior of commodity prices has become highly unusual. Commodity prices have reached all-time highs, and these booms have been followed by significant busts, with a major one occurring towards the end of the 2007-08 financial crisis. An emerging literature on financialization of commodities attributes this behavior to the emergence of commodities as an asset class, which has become widely held by institutional investors seeking diversification benefits (Buyuksahin and Robe (2012), Singleton (2012)). Starting in 2004, institutional investors have been rapidly building their positions in commodity futures. CFTC staff report (2008) estimates institutional holdings to have increased from $15 billion in 2003 to over $200 billion in 2008. Many of the institutional investors hold commodities through a commodity futures index, such as the Goldman Sachs Commodity Index (GSCI), the Dow Jones UBS Commodity Index (DJ-UBS) or the S&P Commodity Index (SPCI). Tang and Xiong (2012) document that, interestingly, after 2004 the behavior of index commodities has become increasingly different from those of nonindex, with the former becoming more correlated with oil, an important index constituent, and more correlated with the equity market. Since institutional investors tend to trade in and out of equities and (index) commodities at the same time, their increased presence in the commodity futures markets could explain these effects. The financialization theory has far-reaching implications for regulation: the 2004-2008 boom in commodity prices has prompted many calls for curtailing positions of institutions whose trades may have generated the boom (see, e.g., Masters' (2008) testimony).

While the empirical literature on financialization of commodities has been influential and has contributed to the policy debate, theoretical literature on the subject remains scarce. Our goal in this paper is to model the financialization of commodities and to disentangle the effects of institutional flows from the traditional demand and supply effects on commodity futures prices. We particularly focus on identifying the economic mechanisms through which institutions may influence commodity futures prices, volatilities, and their comovement.

We develop a multi-good, multi-asset dynamic model with institutional investors and standard futures markets participants. The institutional investors care about their perfor-

mance relative to a commodity index. They do so because their investment mandate specifies a benchmark index for performance evaluation or because their mandate includes hedging against commodity price inflation. We capture such benchmarking through the institutional objective function. Consistent with the extant literature on benchmarking (originating from Brennan (1993)), we postulate that the marginal utility of institutional investors increases with the index. In particular, institutional investors dislike to perform poorly when their benchmark index does well and so have an additional incentive to do well when their benchmark does well. Both classes of investors in our model invest in the commodity futures markets and the stock market. Prices in these markets fluctuate in response to three possible sources of shocks: (i) commodity supply shocks, (ii) commodity demand shocks, and (iii) (endogenous) fluctuations in assets under management of institutional investors. The latter source of risk captures the effects of financialization of commodity markets. To explore the differences between index and nonindex commodity futures, we include in the index only a subset of the traded futures contracts. We can then compare a pair of otherwise identical commodities, one of which belongs to the index and the other does not. The effects of financialization in our model are captured by comparing our economy with institutional investors to an otherwise identical benchmark economy with no institutions. The model is solved in closed form, and all of our results below are derived analytically.

We first uncover that membership in the index creates a novel spillover mechanism, arising due to the presence of institutions. Namely, supply and demand shocks that are specific to an index commodity get transmitted to all other commodity futures, including nonindex ones. Since the marginal utility of institutions depends on the index value, so does the (common) discount factor in the economy. Through their effect on the index, shocks that are specific to index commodities affect the discount factor. Consequently, all assets in the economy are impacted by shocks to index commodities and the characteristics of index commodities. In contrast, the supply and demand shocks to a nonindex commodity affect just that commodity market alone. This spillover mechanism is key to our findings.

We find that the prices of all commodity futures go up with financialization. Importantly, the price rise is higher for futures belonging to the index than for nonindex ones. This happens because institutions care about the index. Since their marginal utility is increasing

in the index level, they value assets that pay off more in the states when the index does well. Hence, relative to the benchmark economy without institutions, index futures are valued higher than nonindex. The larger the institutions, the more they distort pricing—or, more formally, the discount factor—making the above effects stronger.

The volatilities of both index and nonindex futures returns go up with financialization. The primary reason for this is that, absent institutions, there are only two sources of risk: supply and demand risks. With institutions present, some agents in the economy (institutional investors) face an additional risk of falling behind the index. This risk is reflected in the futures prices and it raises the volatilities of futures returns. While the volatilities of both index and nonindex futures rise, they do not, however, rise by the same magnitude. Institutions bid up prices and volatilities of index futures more than nonindex because index futures, by construction, pay off more when the index does well. The prices and volatilities of index futures become high enough to make them unattractive to the normal investors (standard market participants) so that they are willing to sell them to the institutions. Similarly, the institutional investors bid up the stock market value and volatility. This happens because the stock market payoff is positively correlated with that of the commodity price index, making the stock a good investment instrument for the institutions.

Furthermore, we find that financialization leads to an increase in the correlations amongst commodity futures as well as in the equity-commodity correlations. The frequently cited intuition for why the correlations should rise is that commodity futures markets have been largely segmented before the inflow of institutional investors in mid-2000s, and institutions who have entered these markets have linked them together, as well as with the stock market, through cross-holdings in their portfolios. We show that the argument does not need to reply on market segmentation. In our model the rise in the correlations occurs even under complete markets. Benchmarking institutional investors to a commodity index leads to the emergence of this index as a new (common) factor in commodity futures and stock returns, again due to the aforementioned spillover mechanism. In equilibrium, all assets load positively on this factor, which increases their covariances and their correlations. We show that index commodity futures are more sensitive to this new factor, and so their covariances and correlations with each other rise more than those for otherwise identical

nonindex commodities. A similar result also holds for equity-commodity correlations: the ones for index commodity futures rise by more than those for nonindex.

Finally, we seek to quantify the effects of financialization on commodity futures prices. We do this in a framework that features both supply and demand shocks. For expositional simplicity, we consider demand shocks affecting one commodity only. We model the demand shocks so that the demand for that commodity is increasing in aggregate output (as in the model of oil prices of Dvir and Rogoff (2009)). In that setting, we uncover additionally that financialization increases sizeably all futures prices, independent of whether there are demand shocks for the underlying commodity or not. Our simple calibration reveals that for the commodity affected by the demand shocks, 16.8% of its futures price is attributable to financialization and 83.2% to fundamentals (demand and supply). For index commodities unaffected by the demand shocks, financialization accounts for 11% of their futures prices. In the presence of demand shocks, the index becomes more volatile and so the institutional investors' incentive to not fall behind the index strengthens further. Our results support the view advocated in Kilian and Murphy (2013) that fundamentals, and especially demand shocks, are important in explaining commodity prices, but we also stress that financialization amplifies the effects of rising demand.[1] For example, a 33% increase in demand for a commodity raises the fraction of its futures price attributable to financialization from 16.8% to 24.9%.

Methodologically, this paper contributes to the asset pricing literature by providing a tractable multi-asset general equilibrium model with heterogeneous investors which is solved in closed form. While there is clearly a need for multi-asset models (e.g., to provide cross-sectional predictions for empirical asset pricing), such models have been notoriously difficult to solve analytically. Pavlova and Rigobon (2007) and Cochrane, Longstaff, and Santa-Clara (2008) discuss the complexities of such models and provide analytical solutions for the two-asset case. As Martin (2013) demonstrates, the general multi-asset case presents a formidable challenge. In contrast, our multi-asset model is surprisingly simple to solve. Our innovation is to replace Lucas trees considered in the above literature by zero-net-supply

[1]This amplification effect suggests that the specifications used in structural econometric models of commodity prices, such as in Kilian and Murphy, may not be time-invariant, and in particular the sensitivity of commodity prices to structural shocks may have changed since the inflow of institutional investors from 2004 onwards. This is a testable implication that we leave for future empirical work.

assets (futures) and model only the aggregate stock market as a Lucas tree. The model then becomes just as simple and tractable as a single-tree model.

1.1. Related Literature

This paper is related to several strands of literature. The two papers that have motivated this work are Singleton (2012) and Tang and Xiong (2012). Singleton examines the 2008 boom/bust in oil prices and argues that flows from institutional investors have contributed significantly to that boom/bust. Tang and Xiong document that the comovement between oil and other commodities has risen dramatically following the inflow of institutional investors starting from 2004, and that the commodities belonging to popular indices have been affected disproportionately more. There was no difference in comovement patterns of index and nonindex commodities pre-2004. Using a proprietary dataset from the CFTC, Buyuksahin and Robe (2012) investigate the recent increase in the correlation between equity indices and commodities and argue that this phenomenon is due to the presence of hedge funds that are active in both equity and commodity futures markets. Recently, Henderson, Pearson and Wang (2012) present new evidence on the financialization of commodity futures markets based on commodity-linked notes.

The impact of financialization on commodity futures and spot prices is the subject of much ongoing debate in the literature. Surveys of academic literature by Irwin and Sanders (2011) and Fattouh, Kilian, and Mahadeva (2013) challenge the view that increased speculation in oil futures markets in post-financialization period was an important determinant of oil prices. Kilian and Murphy (2013) attribute the 2003-2008 oil price surge to global demand shocks rather than speculative demand shifts. Hamilton and Wu (2012) examine whether commodity index-fund investing had a measurable effect on commodity futures prices and find little evidence to support this hypothesis.

While there is still lack of agreement on whether trades by institutional investors affect futures prices, it is reasonably well-established that such trades affect stock prices. Starting from Harris and Gurel (1986) and Shleifer (1986), a large body of work documented that prices of stocks that are added to the S&P 500 and other indices increase following the announcement and prices of stocks that are deleted drop—a phenomenon widely attributed

to the price pressure from institutional investors. Relatedly, a variety of studies document the so-called "asset class" effects: the "excessive" comovement of assets belonging to the same index or other visible category of stocks (e.g., Barberis, Shleifer, and Wurgler (2005) for the S&P500 vis-à-vis non-S&P500 stocks, Boyer (2011) for BARRA value and growth indices). These effects are attributed to the presence of institutional investors.

The closest theoretical work on the effects of institutions on asset prices is the Lucas-tree economy of Basak and Pavlova (2013). Basak and Pavlova focus on index and asset class effects in the stock market. Their model does not feature multiple commodities, nor is it designed to address some of the main issues in the debate on financialization; namely, how much of the rise in the commodity futures prices can be attributed to demand shocks and how much to financialization. Moreover, their model is missing our novel spillover mechanism whereby shocks to cash flows of index assets get transmitted to nonindex, and so "financialization" in their model would not affect prices of nonindex assets. Another related theoretical study of an asset-class effect is by Barberis and Shleifer (2003), whose explanation for this phenomenon is behavioral. However, they also do not explicitly model commodities and so cannot address some questions specific to the current debate on financialization of commodities.

Finally, there is a large and diverse literature going back to Keynes (1923) that studies the determination of commodity spot prices in production economies with storage and links the physical markets for commodities with the commodity futures markets.[2] We view our work as being complementary to this literature because in our work we simplify the physical markets for commodities and focus on the spillovers between the commodity futures markets for in a multi-commodity setting and the effects of index inclusion.

The remainder of the paper is organized as follows. Section 2 presents our model. Sec-

[2]In this strand of literature, a recent paper by Sockin and Xiong (2012) shows that price pressure from investors operating in futures markets (even if driven by nonfundamental factors) can be transmitted to spot prices of underlying commodities. Acharya, Lochstoer, and Ramadorai (2013) stress the importance of capital constraints of futures' markets speculators and argue that frictions in financial (futures) markets can feedback into production decisions in the physical market. In a similar framework, Gorton, Hayashi, and Rouwenhorst (2013) derive endogenously the futures basis and the risk premium and relate them to inventory levels. Routledge, Seppi, and Spatt (2000) derive the term structure of forward prices for storable commodities, highlighting the importance of the non-negativity constraints on inventories. Baker (2012) examines the effects of financialization in a model with storage. His interpretation of financialization is the reduction in transaction costs of households for trading futures, while we identify financialization with the presence of institutional investors.

tion 3 presents our main results on how institutional investors affect commodity futures prices, volatilities, and their comovement. Section 4 extends our framework to incorporate demand shocks. Section 5 concludes and the Appendix provides all proofs.

2. The Model

Our goal in this section is to develop a simple and tractable model of commodity futures markets in which prices fluctuate in response to three possible sources of shocks: (i) commodity supply shocks, (ii) commodity demand shocks, and (iii) endogenous fluctuations in assets under management of institutional investors. The former two sources of risk have been studied extensively in the literature. The third source of risk is new and it captures the effects of financialization of commodity markets. Having a theoretical model allows us to disentangle the effects of each of these three sources of risk on commodity prices and their dynamics.

We consider a pure-exchange multi-good, multi-asset economy with a finite horizon T. Uncertainty is resolved continuously, driven by a $K+1$-dimensional standard Brownian motion $\omega \equiv (\omega_0, \ldots, \omega_K)^\top$. All consumption in the model occurs at the terminal date T, while trading takes place at all times $t \in [0, T]$.

Commodities. There are K commodities (goods), indexed by $k = 1, \ldots K$. The date-T supply of commodity k, D_{kT}, is the terminal value of the process D_{kt}, with dynamics

$$dD_{kt} = D_{kt}[\mu_k dt + \sigma_k d\omega_{kt}], \tag{1}$$

where μ_k and $\sigma_k > 0$ are constant. The process D_{kt} represents the arrival of news about D_{kT}. We refer to it as the commodity-k *supply news*. The price of good k at time t is denoted by p_{tk}. There is one further good in the economy, commodity 0, which we refer to as the generic good. This good subsumes all remaining goods consumed in the economy apart from the K commodities that we have explicitly specified above and it serves as the numeraire. The date-T supply of the generic good is D_T, which is the terminal value of the supply news process

$$dD_t = D_t[\mu dt + \sigma d\omega_{0t}], \tag{2}$$

where μ and $\sigma > 0$ are constant. Our specification implies that the supply news processes are uncorrelated across commodities ($dD_{kt}\, dD_{it} = 0$, $dD_{kt}\, dD_t = 0$, $\forall k$, $k \neq i$). This assumption is for expositional simplicity; it can be relaxed in future work.

Financial Markets. Available for trading are K standard futures contracts written on commodities $k = 1, \ldots, K$. A futures contract on commodity k matures at time T and delivers one unit of commodity k. The contract payoff at maturity is therefore p_{kT}. Each contract is continuously resettled at the futures price f_{kt} and is in zero net supply. The gains/losses on each contract are posited to follow

$$df_{kt} = f_{kt}[\mu_{f_{kt}} dt + \sigma_{f_{kt}} d\omega_t], \tag{3}$$

where $\mu_{f_{kt}}$ and the $K + 1$ vector of volatility components $\sigma_{f_{kt}}$ are determined endogenously in equilibrium (Section 3).

Our model makes a distinction between index and nonindex commodities because we seek to examine theoretically the asset class effect in commodity futures documented by Tang and Xiong (2012). A *commodity index* includes the first L commodities, $L \leq K$, and is defined as

$$I_t = \prod_{i=1}^{L} f_{it}^{1/L}. \tag{4}$$

This index represents a geometrically-weighted commodity index such as, for example, the S&P Commodity Index (SPCI). For expositional simplicity, our index weighs all commodities equally; this assumption is easy to relax.[3]

In addition to the futures markets, investors can trade in the stock market, S, and an instantaneously riskless bond. The stock market is a claim to the entire output of the economy at time T: $D_T + \sum_{k=1}^{K} p_{kT} D_{kT}$. It is in positive supply of one share and is posited to have price dynamics given by

$$dS_t = S_t[\mu_{St} dt + \sigma_{St} d\omega_t], \tag{5}$$

[3]To model other major commodity indices such as the Goldman Sachs Commodity Index and the Dow Jones UBS Commodity Index, it is more appropriate to define the index as $I_t = \frac{1}{L} \sum_{i=1}^{L} f_{it}$. Such specification is less tractable but one can show numerically that most of the implications are in line with those in our analysis below.

with μ_{St} and $\sigma_{St} > 0$ endogenously determined in equilibrium. The bond in zero net supply. It pays a riskless interest rate r, which we set to zero without loss of generality.[4]

We note that our formulation of asset cash flows is standard in the asset pricing literature. The main distinguishing characteristic of our model is that it avoids the complexities of multi-tree economies. This is because only the stock market is in positive net supply, while all other assets (futures) are in zero net supply. As we demonstrate in the ensuing analysis, this model is just as simple and tractable as a single-tree model.

Investors. The economy is populated by two types of market participants: normal investors, $\mathcal{N}$, and institutional investors, $\mathcal{I}$. The (representative) normal investor is a standard market participant, with logarithmic preferences over the terminal value of her portfolio:

$$u_{\mathcal{N}}(W_{\mathcal{N}T}) = \log(W_{\mathcal{N}T}), \tag{6}$$

where $W_{\mathcal{N}T}$ is (real) wealth or real consumption.

The institutional investor's objective function, defined over his terminal portfolio value (real consumption) $W_{\mathcal{I}T}$, is given by

$$u_{\mathcal{I}}(W_{\mathcal{I}T}) = (a + bI_T)\log(W_{\mathcal{I}T}), \tag{7}$$

where a, $b > 0$. The institutional investor is modeled along the lines of Basak and Pavlova (2013), who study institutional investors in the stock market and also provide microfoundations for such an objective function, as well as a status-based interpretation.[5] The objective function has two key properties: (i) it depends on the index level I_T and (ii) the marginal utility of wealth is increasing in the benchmark index level I_T. This captures the notion of benchmarking: the institutional investor is evaluated relative to his benchmark index and so he cares about the performance of the index. When the benchmark index is relatively high, the investor strives to catch up and so he values his marginal unit of performance

[4]This is a standard feature of models that do not have intermediate consumption. In other words, there is no intertemporal choice that would pin down the interest rate. Our normalization is commonly employed in models with no intermediate consumption (see e.g., Pastor and Veronesi (2012) for a recent reference).

[5]Direct empirical support for the status-based interpretation of our model is provided in Hong, Jiang, and Zhao (2011), who adopt the formulation in (7) in their analysis. Empirical work estimating objectives of institutional investors remains scarce, with a notable exception of Koijen (2013).

highly (his marginal utility of wealth is high). When the index is relatively low, the investor is less concerned about his performance (his marginal utility of wealth is low). We use the commodity market index as the benchmark index because in this work we attempt to capture institutional investors with the mandate to invest in commodities, most of whom are evaluated relative to a commodity index. An alternative interpretation of the objective function is that the institutional investor has a mandate to hedge commodity price inflation; i.e., deliver higher returns in states in which the commodity price index is high.[6]

In this multi-good world, (real) terminal wealth is defined as an aggregate over all goods, a consumption index (or real consumption). We take the index to be Cobb-Douglas, i.e.,

$$W_n = C_{n_0}^{\alpha_0} C_{n_1}^{\alpha_1} \cdot \ldots \cdot C_{n_K}^{\alpha_K}, \qquad n \in \{\mathcal{N}, \mathcal{I}\}, \tag{8}$$

where $\alpha_k > 0$ for all k. For the case of $\sum_{k=0}^{K} \alpha_k = 1$, the parameter α_k represents the expenditure share on good k, the fraction of wealth optimally demanded in good k. Here we are considering a general Cobb-Douglas aggregator in which the weights do not necessarily add up to one, and hence we label α_k as the "commodity demand parameter."[7] We take the commodity demand parameters to be the same for all investors in the economy. Heterogeneity in demand for specific commodities is not the dimension we would like to focus on in this paper.

A change in α_k represents a demand shift towards commodity k. A change in the demand parameter α_k is the simplest and most direct way of modeling a demand shift, i.e., an outward movement in the entire demand schedule, as typical in classical demand theory (Varian (1992)).[8] In Section 4, we allow the demand parameters α_k to be stochastic, in order to capture a more realistic environment with demand shocks. Until then, we keep them constant so as to isolate the effects of supply shocks and the effects of financialization (fluctuations in institutional wealth invested in the market) on commodity futures prices.

[6] Although the institutions are modeled similarly, our focus is different and our model generates a number of new insights, absent in Basak and Pavlova (Remark 1, Section 3).

[7] In what follows, we are interested in comparative statics with respect to α_k. The expenditure share on commodity k, $\alpha_k / \sum_{k=0}^{K} \alpha_k$, is monotonically increasing in α_k. Hence all our comparative statics for α_k are equally valid for expenditure shares $\alpha_k / \sum_{k=0}^{K} \alpha_k$.

[8] For example, an increase in demand for soya beans due to the invention of biofuels and concerns about the environment.

The institutional and normal investors are initially endowed with fractions $\lambda \in [0, 1]$ and $(1 - \lambda)$ of the stock market, providing them with initial assets worth $W_{\mathcal{I}0} = \lambda S_0$ and $W_{\mathcal{R}0} = (1-\lambda)S_0$, respectively.[9] The parameter λ thus represents the (initial) fraction of the institutional investors in the economy, and we will often refer to it as the size of institutions.

Starting with initial wealth W_{n0}, each type of investor $n = \mathcal{N}, \mathcal{I}$, dynamically chooses a portfolio process $\phi_n = (\phi_{n_1}, \ldots, \phi_{n_K})^\top$, where ϕ_n and ϕ_{n_S} denote the fractions of the portfolio invested in the futures contracts 1 through K and the stock market, respectively. The wealth process of investor n, W_n, then follows the dynamics

$$dW_{nt} = W_{nt} \sum_{k=1}^{K} \phi_{n_k t}[\mu_{f_k t} dt + \sigma_{f_k t} d\omega_t] + W_{nt}\phi_{n_S t}[\mu_{St} dt + \sigma_{St} d\omega_t]. \tag{9}$$

3. Equilibrium Effects of Financialization of Commodities

We are now ready to explore how the financialization of commodities affects equilibrium prices, volatilities, and correlations. In order to understand the effects of financialization, we will often make comparisons with equilibrium in a *benchmark economy*, in which there are no institutional investors. We can specify such an economy by setting $b = 0$ in (7), in which case the institution in our model no longer resembles a commodity index trader and behaves just like the normal investor. Another way to capture the benchmark economy within our model is to set the fraction of institutions, λ, to zero.

Equilibrium in our economy is defined in a standard way: equilibrium portfolios, asset and time-T commodity prices are such that (i) both the normal and institutional investors choose their optimal portfolios, and (ii) futures, stock, bond and time-T commodity markets clear. Letting $M_{t,T}$ to denote the (stochastic) discount factor or the pricing kernel in our model, by no-arbitrage, the futures prices are given by

$$f_{kt} = E_t[M_{t,T}\, p_{kT}]. \tag{10}$$

[9]The initial endowment of institutions comes from households (that are not explicitly modeled here), who delegate their assets to institutions to manage. Such households could be, for example, participants in defined benefit pension plans.

The discount factor $M_{t,T}$ is the marginal rate of substitution of any investor, e.g., the normal investor, in equilibrium.

To develop intuitions for our results, it is useful to examine the time-T prices prevailing in our equilibrium. These are reported in the following lemma.

Lemma 1 (Time-T equilibrium quantities). *In equilibrium with institutional investors, we obtain the following characterizations for the terminal date quantities.*

$$\textit{Commodity prices:}\quad p_{kT} = \frac{\alpha_k}{\alpha_0}\frac{D_T}{D_{kT}};\quad p_{kT} = \overline{p}_{kT}, \tag{11}$$

$$\textit{Commodity index:}\quad I_T = \frac{D_T}{\alpha_0}\prod_{i=1}^{L}\left(\frac{\alpha_i}{D_{iT}}\right)^{1/L};\quad I_T = \overline{I}_T, \tag{12}$$

$$\textit{Stock market value:}\quad S_T = D_T\sum_{k=0}^{K}\frac{\alpha_k}{\alpha_0};\quad S_T = \overline{S}_T, \tag{13}$$

$$\textit{Discount factor:}\quad M_{0,T} = \overline{M}_{0,T}\left(1+\frac{b\,\lambda(I_T - E[I_T])}{a + b\,E[I_T]}\right),\quad \overline{M}_{0,T} = \frac{e^{(\mu-\sigma^2)T}D_0}{D_T}, \tag{14}$$

where the expectation of the time-T index value, $E[I_T]$, is provided in the Appendix. The quantities with an upper bar denote the corresponding equilibrium quantities prevailing in the economy with no institutions.

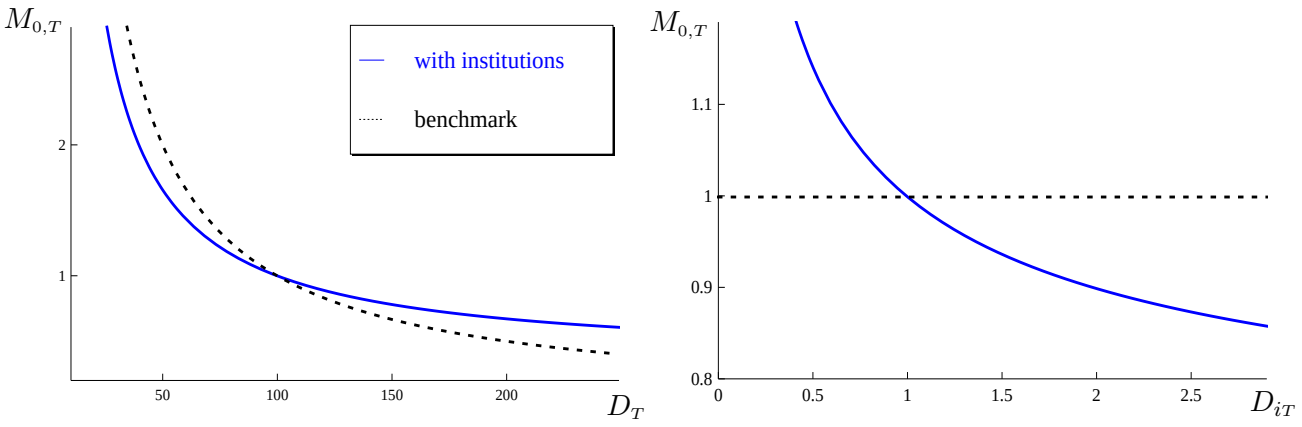
Lemma 1 reveals that the price of good k decreases with the supply of that good D_{kT}. As supply D_{kT} increases, good k becomes relatively more abundant. Hence, its price falls. A rise in the supply of the generic good D_T has the opposite effect. Now good k becomes more scarce relative to the generic good. Hence, its price rises. These are classical supply-side effects. These mechanisms are well explored in commodity markets and they are standard in multi-good models. A positive shift in α_k represents an increase in demand for good k. As a consequence, the price of good k goes up. This is a classical demand-side effect.

Since the index is given by $I_T = \prod_{i=1}^{L} p_{iT}^{1/L}$, the terminal index value inherits the properties of the individual commodity prices. In particular, it declines when the supply of any index commodity i D_{iT} goes up, and rises when the supply of the generic good D_T rises.

It is important to note that the time-T prices of commodities, and hence the commodity index coincide with their values in the benchmark economy with no institutions. We have

intentionally set up our model in this way. By effectively abstracting away from the effects of financialization on underlying cash flows in (10), we are able to elucidate the effects of institutions in the futures markets coming via the discount factor channel.

The stock market is a claim against the aggregate output of all goods in the economy, $D_T + \sum_{k=1}^{K} p_{kT} D_{kT}$, which in this model turns out to be proportional to the aggregate supply of the generic good D_T. So the aggregate wealth in the economy, the stock market value S_T, in equilibrium is simply a scaled supply of the generic good D_T. The quantity D is an important state variable in our model. In what follows, we will refer to it as (scaled) aggregate wealth, or, equivalently, (scaled) aggregate output.



(a) Effect of aggregate output D_T **(b)** Effect of index commodity supply D_{iT}

Figure 1: **Discount factor.** This figure plots the discount factor in the presence of institutions against aggregate output D_T and against an index commodity supply D_{iT}. The dotted lines correspond to the discount factor in the benchmark economy with no institutions. The plots are typical. The parameter values, when fixed, are: $L = 2$, $K = 5$, $a = 1$, $b = 1$, $T = 5$, $\lambda = 0.4$, $\alpha_0 = 0.7$, $D_T = D_0 = 100$, $D_{kT} = D_{k0} = 1$, $\mu = \mu_k = 0.05$, $\sigma = 0.25$, $\sigma_k = 0.15$, $\alpha_k = 0.06$, $k = 1, \ldots K$ (see Section 4).

In the benchmark economy, the discount factor depends only on aggregate output D_T. It bears the familiar inverse relationship with aggregate output (dotted line in Figure 1a), implying that assets with high payoffs in low-D_T (bad) states get valued higher. In the presence of institutions, the discount factor is also decreasing in aggregate output D_T, albeit at a slower rate. That is, the presence of institutions makes the discount factor less sensitive

to news about aggregate output. Additionally, now the discount factor becomes dependent on the supply of each index commodity D_{iT} (Figure 1b). The channel through which institutions affect the discount factor is apparent from equation (14): the discount factor now becomes dependent on the performance of the index, pricing high-index states higher. This is the channel through which financialization affects asset prices in our model.

The new financialization channel works as follows. Institutional investors have an additional incentive to do well when the index does well. So relative to normal investors, they strive to align their performance with that of the index, performing better when the index does well in exchange for performing poorer when the index does poorly. This is optimal from their viewpoint because their marginal utility is increasing with the level of the index. As highlighted in our discussion of the equilibrium index value in (12), the index does well when the aggregate output D_T is high and supply of index commodity D_{iT} is low. Because of the additional demand from institutions, these states become more "expensive" relative to the benchmark economy (higher Arrow-Debreu state prices or higher discount factor $M_{0,T}$). The financialization channel thus counteracts the benchmark economy inverse relation between the discount factor $M_{0,T}$ and aggregate output, making the discount factor less sensitive to aggregate output D_T (as evident from Figure 1a). Additionally, it also makes the discount factor dependent and decreasing in each index commodity supply D_{iT}.

The graphs in Figure 1 are important because they underscore the mechanism for the valuation of assets in the presence of institutions. In particular, assets that pay off high in states in which the index does well (high D_T and low D_{iT}) are valued higher than in the benchmark economy with no institutions.

3.1. Equilibrium Commodity Futures Prices

Proposition 1 (Futures prices). *In the economy with institutions, the equilibrium futures price of commodity $k = 1, \ldots, K$ is given by*

$$f_{kt} = \overline{f}_{kt} \frac{a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{1_{\{k \le L\}} \sigma_k^2 (T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2 (T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}, \tag{15}$$

where the equilibrium futures price in the benchmark economy with no institutions $\overline{f}_{kt}$ *and the quantity* $g_i(t)$ *are given by*

$$\overline{f}_{kt} = \frac{\alpha_k}{\alpha_0} e^{(\mu - \mu_k - \sigma^2 + \sigma_k^2)(T-t)} \frac{D_t}{D_{kt}}, \quad g_i(t) = \frac{\alpha_i}{\alpha_0} e^{(\mu - \mu_i + (1/L+1)\sigma_i^2/2)(T-t)} \tag{16}$$

Consequently, in the presence of institutions,

(i) The futures prices are higher than in the benchmark economy, $f_{kt} > \overline{f}_{kt}$, $k = 1, \ldots, K$.

(ii) The index futures prices rise more than nonindex ones for otherwise identical commodities, i.e., for commodities i *and* k *with* $D_{it} = D_{kt}$, $\forall t$, $\alpha_i = \alpha_k$, $i \leq L$, $L < k \leq K$.

Proposition 1 reveals that the commodity futures prices in the benchmark economy with no institutions $\overline{f}_{kt}$ inherit the features of time-T futures prices highlighted in Lemma 1. The benchmark economy futures prices rise in response to positive news about aggregate output D_t and fall in response to positive news about the supply of commodity k, D_{kt}. In contrast, in the economy with institutions the commodity futures prices f_{kt} depend not only on own supply news D_{kt} but also those of all *index* commodities D_{it}. Other characteristics of index commodities such as expected growth in their supply μ_i, volatility σ_i and their demand parameters α_i now also affect the prices of all futures traded in the market. Note that, just like in the benchmark economy, supply news D_k and other characteristics of *nonindex* commodities have no spillover effects on other commodity futures.

To understand the price rises due to institutions, let us think about the valuation of assets in the context of two standard channels: the discount factor channel and the cashflow channel (see equation (10)). In this model, the futures cashflows are not affected by the presence of institutions (Lemma 1), and so the only channel through which futures prices are affected is the discount factor channel. From the discussion of Figure 1, futures that pay off high when the index does well, or equivalently when the aggregate output D_t is high and/or an index commodity supply D_{it} is low, command higher prices. The cashflows of all futures are increasing in aggregate output D_t (equation (11)). Therefore, all futures prices are higher in the presence of institutions (as reported in property (i) of Proposition 1). Additionally, index commodity futures (but not nonindex) pay off high when an index commodity is scarce. This compounds the aggregate output effect and implies that index commodity

futures prices are higher that those of otherwise identical nonindex commodity futures (as reported in property (ii) of Proposition 1). In other words, we can describe property (ii) as follows. In the economy with institutions, some agents care about the index. Since their marginal utility is increasing in the index level, they value assets that pay off more in the states when the index does well. Hence commodity futures that are members of the index have higher prices than the nonmembers.

Remark 1 (Difference from Basak and Pavlova (2013)). One major difference of this model from the the one-good stock market economy of Basak and Pavlova is that in their analysis nonindex security prices are unaffected by the presence of institutions, although the institutions are modeled similarly. Consequently, in contrast to our findings, their nonindex assets have zero correlation among themselves and with index assets, and the nonindex asset prices and volatilities are not affected by institutional investors. The key reason for these differences is that in Basak and Pavlova, cashflows of nonindex securities are exogenous and they are uncorrelated with the index. Here, nonindex cashflows, which are endogenously determined commodity prices, end up being correlated with the index. Tang and Xiong (2012) provide evidence that the financialization of commodities since 2004 has affected not only index commodities futures prices, volatilities and correlations, but also those of nonindex commodities. Unlike that of Basak and Pavlova, our model here is able to shed light on these important spillover effects from index commodities to nonindex ones. We quantify them in Section 4.

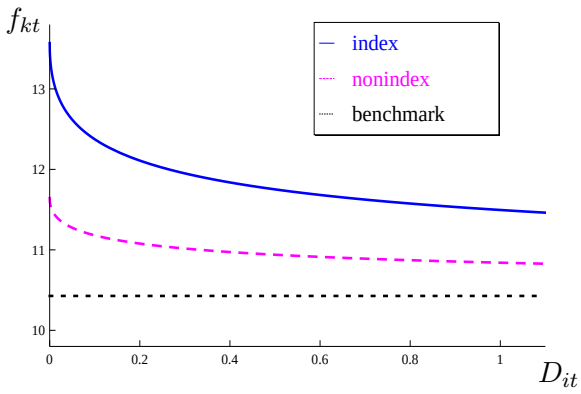
Corollary 1. *The equilibrium commodity futures prices have the following additional properties.*

(i) All commodity futures prices f_{kt} are increasing in the size of institutions λ, $k = 1, \ldots, K$.

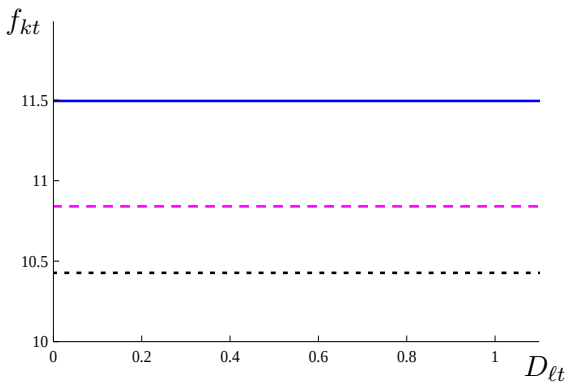
(ii) All commodity futures prices are more sensitive to aggregate output D_t than in the benchmark economy with no institutions; i.e., f_{kt} is increasing in D_t at a faster rate than does $\overline{f}_{kt}$, $k = 1, \ldots, K$. Moreover, index commodity futures are more sensitive to aggregate output that nonindex ones for otherwise identical commodities.

(iii) All commodity futures prices f_{kt}, $k = 1, \ldots, K$, react negatively to positive supply news of index commodities D_{it}, $i = 1, \ldots, L$, $k \neq i$, while in the benchmark economy such a price $\overline{f}_{kt}$ is independent of D_{it}. All prices f_{kt}, $k = 1, \ldots, K$, remain independent of nonindex commodities supply news $D_{\ell t}$, unless $k = \ell$.

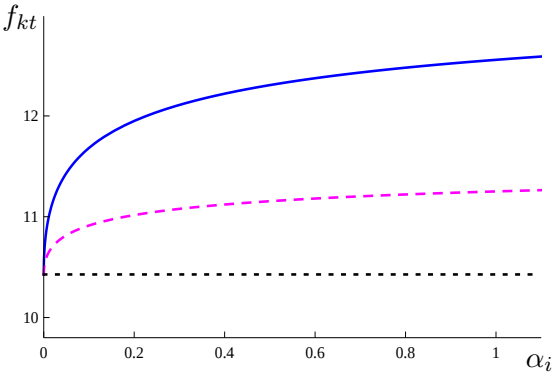
(iv) All commodity futures prices f_{kt}, $k = 1, \ldots, K$, react positively to a positive demand shift towards any index commodity α_i, $i = 1, \ldots, L$, $k \neq i$, while $\overline{f}_{kt}$ is independent of α_i. All prices f_{kt}, $k = 1, \ldots, K$, remain independent of nonindex commodities supply shifts α_ℓ, $\ell \neq k$.



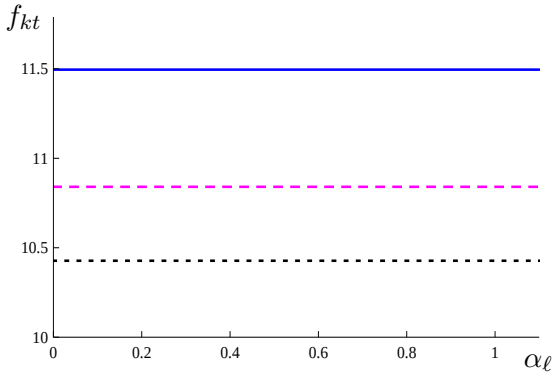
(a) Effect of index commodity supply news D_i



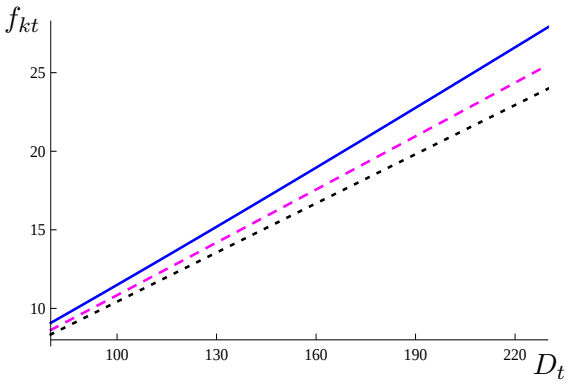
(b) Effect of nonindex commodity supply news D_ℓ



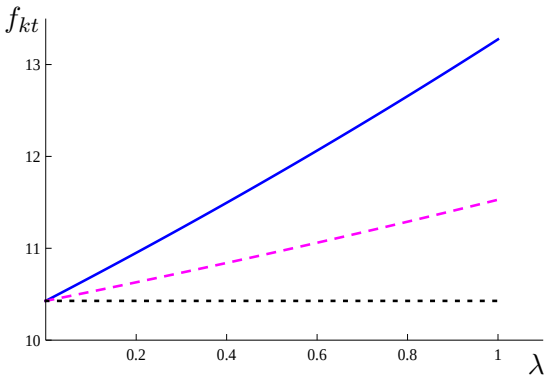
(c) Effect of index commodity demand parameter α_i



(d) Effect of index commodity demand parameter α_ℓ



(e) Effect of aggregate output D_t



(f) Effect of size of institution λ

Figure 2: **Futures prices.** This figure plots the equilibrium futures prices against several key quantities. The plots are typical. We set $t = 0.1$, $D_t = 100$, $D_{kt} = 1$, $k = 1, \ldots K$. The solid blue line is for index futures, the magenta dashed line is for nonindex futures, and the black dotted line is for the benchmark economy. The remaining parameter values (when fixed) are as in Figure 1.

Figure 2 illustrates the results of the corollary. To elucidate the intuitions, we start from properties (iii) and (iv) of the corollary. Panel (a) shows that, unlike in the benchmark economy, futures prices decrease in response to positive index commodities' supply news D_{it}. Institutional investors strive to align their performance with the index, and as a result distort prices the most when the index is high (relative to the benchmark economy). The index is high when D_{it} is low (supply of index commodity i is scarce) and low when D_{it} is high (supply is abundant). So the effects of the institutions on commodity futures prices f_{kt} are most pronounced for low D_{it} realizations and decline monotonically with D_{it}. These effects are absent in the benchmark economy in which agents are not directly concerned about the index. In contrast, futures prices f_{kt} do not react to news about supply of nonindex commodities (apart from that of own commodity k) because this news does not affect the performance of the index (panel (b) and Proposition 1).

The demand-side effects on commodity futures prices are presented in panels (c)–(d). In contrast to the benchmark economy in which futures prices depend only on own commodity demand parameter α_k, in panel (c) it emerges that futures prices increase in demand parameters α_i for all commodities that are members of the index. An upward shift in demand for any index commodity leads to an increase in that commodity's price (a classical demand argument, see Lemma 1) and therefore leads to an increase in the value of the index. Since the marginal utility of the institutions is increasing in the index, the effects on prices become increasingly more pronounced as α_k increases. In contrast, these effects are not present for nonindex commodities (panel (d)). A shift in demand for those commodities leave the index unaffected and hence makes futures prices independent of demand shifts towards nonindex commodities (changes in α_ℓ), apart from own demand shift. A caveat to this discussion is that we are not formally modeling demand shifts in this section, but merely presenting comparative statics with respect to demand parameters α_k. In an economy with demand uncertainty, investors take into account of this uncertainty in their optimization (Section 4).

Panel (e) demonstrates that aggregate output news D_t have stronger effects on futures prices f_{kt} than in the benchmark economy with no institutions. This is because good news about aggregate output not only increases the cashflows of all futures contracts (increases

p_{kT}) but also increases the value of the index. This latter effect is responsible for the amplification of the effect of aggregate output news depicted in panel (e). The higher the aggregate output, the higher the index and hence the stronger the amplification effect. Finally, panel (f) shows that commodity futures prices rise when there are more institutions in the market. The more institutions there are, the stronger their effect on the discount factor and hence on all commodity futures prices. Finally, all panels in Figure 2 illustrate that in the presence of institutions, index futures rise more than nonindex, as already highlighted in Proposition 1.

3.2. Futures Volatilities and Correlations

The past decade in commodity futures markets has been characterized by an increase in volatility, with booms and busts in commodity markets attracting unprecedented attention of policymakers and commentators. We explore commodity futures volatilities in this section in order to highlight the sources of this increased volatility. Our objective is to demonstrate how standard demand and supply risks can be amplified in the presence of institutions.

Propositions 2 reports the futures return volatilities in closed form.[10]

Proposition 2 (Volatilities of commodity futures). *In the economy with institutions, the volatility vector of loadings of index commodity futures k returns on the Brownian motions are given by*

$$\sigma_{f_k t} = \overline{\sigma}_{f_k} + h_{kt}\,\sigma_{It}, \quad h_{kt} > 0, \quad k = 1, \ldots, L, \tag{17}$$

and nonindex by

$$\sigma_{f_k t} = \overline{\sigma}_{f_k} + h_t\,\sigma_{It}, \quad h_t > 0, \quad k = L+1, \ldots, K, \tag{18}$$

where $\overline{\sigma}_{f_k}$ is the corresponding volatility vector in the benchmark economy with no institutions and σ_{I_t} is the volatility vector for the conditional expectation of the index $E_t[I_T]$, given by

$$\overline{\sigma}_{f_k} = (\sigma,\, 0, \ldots, -\sigma_k, 0, \ldots, 0),\ \sigma_{It} = (\sigma,\, -\tfrac{1}{L}\sigma_1, \ldots, -\tfrac{1}{L}\sigma_L,\, 0, \ldots, 0), \tag{19}$$

and where h_t and h_{kt} are strictly positive stochastic processes provided in the Appendix with the property $h_{kt} > h_t$.

Consequently, in the presence of institutions,

[10]The notation $||z||$ denotes the square root of the dot product $z \cdot z$.

Sources of risk associated with

	Generic	Index commodities					Nonindex commodities		
	ω_0	ω_1	...	ω_k	...	ω_L	ω_{L+1}	...	ω_K
Loadings									
Benchmark $\overline{\sigma}_{f_k}$	σ	0	...	$-\sigma_k$	...	0	0	...	0
Index σ_{f_k}	$\sigma(1+h_{kt})$	$-\sigma_1 \frac{1}{L} h_{kt}$	...	$-\sigma_k(1+\frac{1}{L}h_{kt})$	...	$-\frac{1}{L}\sigma_L h_{kt}$	0	...	0

(a) Index commodity futures $k = 1, \ldots, L$

Sources of risk associated with

	Generic	Index commodities			Nonindex commodities				
	ω_0	ω_1	...	ω_L	ω_{L+1}	...	ω_k	...	ω_K
Loadings									
Benchmark $\overline{\sigma}_{f_k}$	σ	0	...	0	0	...	$-\sigma_k$	...	0
Nonindex σ_{f_k}	$\sigma(1+h_t)$	$-\sigma_1 \frac{1}{L} h_t$	...	$-\frac{1}{L}\sigma_L h_t$	0	...	$-\sigma_k$	...	0

(b) Nonindex commodity futures $k = L+1, \ldots, K$

Table 1: **Individual volatility components of futures prices.**

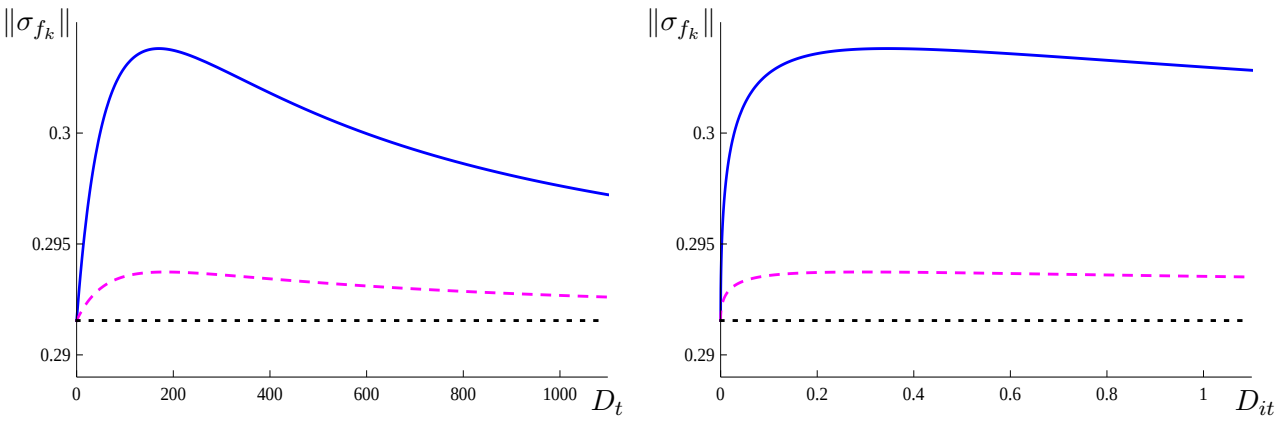
(i) The volatilities of all futures prices, $\|\sigma_{f_{kt}}\|$, are higher than in the benchmark economy, $k = 1, \ldots K$.

(ii) The volatilities of index futures rise more than those of nonindex for otherwise identical commodities, i.e., for commodities i and k with $D_{it} = D_{kt}$, $\forall t$, $\alpha_i = \alpha_k$, $i \leq L$, $L < k \leq K$.

The general formulae presented in Proposition 2 can be decomposed into individual loadings of futures returns on the primitive sources of risk in our model, the Brownian motions ω_0, $\omega_1, \ldots, \omega_K$.

Recall that in our model the supply news of individual commodities D_{kt} are independent of each other and of the generic good supply news D_t. Each of these processes is driven by own Brownian motion. Since in the benchmark economy the futures price depends only on own D_{kt} and aggregate output D_t, it is exposed to only two primitive sources of risk: Brownian motions ω_k and ω_0. In the presence of institutions, futures prices become addi-

tionally dependent on supply news of all index commodities and therefore exposed to sources of uncertainty $\omega_1, \ldots \omega_L$. (The dependence is negative, as illustrated in Corollary 1 and Figure 2a.) Additionally, as argued in Corollary 1 and Figure 2e, shocks to D_t are amplified in the presence of institutions. Proposition 2 formalizes these intuitions by explicitly reporting the loadings on ω_0, $\omega_1, \ldots, \omega_K$, the driving forces behind D, $D_1, \ldots, D_K$, respectively. Hence, commodity futures become more volatile for two reasons: (i) their volatilities are amplified because prices react stronger to news about aggregate output D_t and (ii) there is now dependence on additional sources of risk driving index commodity supply news $D_1, \ldots, D_L$. As discussed earlier, the fundamental reason behind this result is that institutions have an additional incentive to do well when the index does well, and any shock that affects the index becomes an additional source of risk for the institutions.



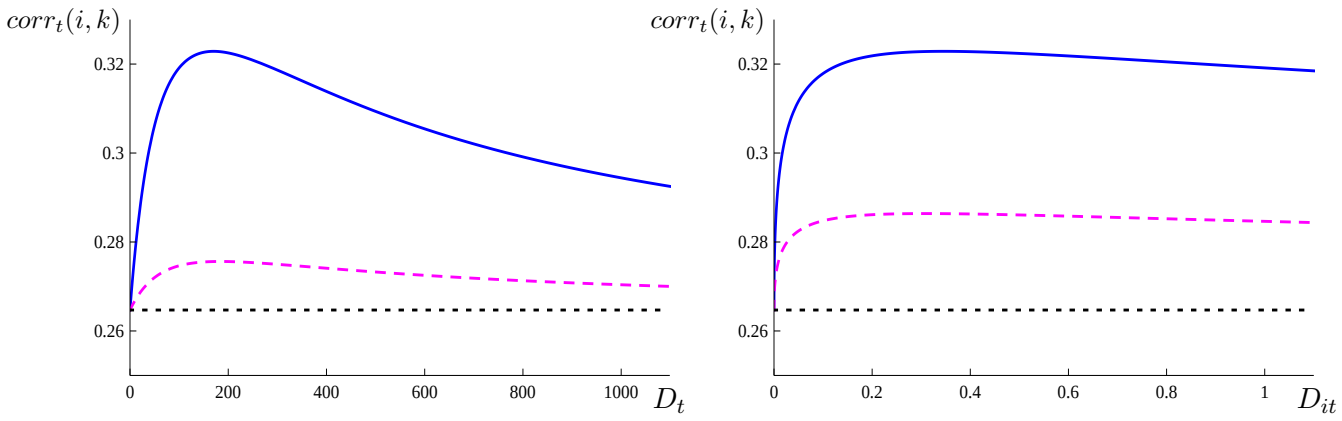
(a) Effect of aggregate output news D

(b) Effect of index commodity supply news D_i

Figure 3: **Commodity futures volatilities.** This figure plots the commodity futures volatility $||\sigma_{f_k t}||$ in the presence of institutions against aggregate output news D_t and against index commodity supply news D_{it}, $i \neq k$. As in Figure 2, the solid blue line is for index futures, the magenta dashed line is for nonindex futures, and the black dotted line is for the benchmark economy. The parameter values are as in Figure 2.

Figure 3 illustrates the above discussion. It also reveals that the volatilities of index and nonindex futures are differentially affected by the presence of institutions. Tang and Xiong (2012) document that since 2004, and especially during 2008, index commodities have exhibited higher volatility increases than nonindex ones. Our results are consistent with these

findings. Institutions bid up volatilities of index futures more than nonindex because index futures, by construction, pay off more when the index does well. The volatilities of index futures become high enough to make them unattractive to the normal investors (standard market participants) so that they are willing to sell the index futures to the institutions.



(a) Effect of aggregate output news D

(b) Effect of index commodity supply news D_i

Figure 4: **Futures returns correlations.** This figure plots return correlations of two index futures $corr_t(i,k)$ and two nonindex futures $corr_t(\ell,k)$ in the presence of institutions against aggregate output news D_t and against index commodity supply news D_{it}, $i \neq k$. As in Figure 2, the solid blue line is for index futures, the magenta dashed line is for nonindex futures, and the black dotted line is for the benchmark economy. The parameter values are as in Figure 2.

We next turn to examining the (instantaneous) correlations of futures returns, defined as $corr_t(i,k) = \sigma_{f_it} \cdot \sigma_{f_kt}/(\|\sigma_{f_it}\| \, \|\sigma_{f_kt}\|)$. Recent evidence indicates that financialization of commodities markets has coincided with a sharp increase in the correlations across a wide range of commodity futures returns. Tang and Xiong (2012) document that the average correlation of non-energy commodity futures with oil has increased from 0.1 in 1990s and early 2000s to about 0.5 in 2009. The increase in the correlations is especially pronounced for the index futures returns. Tang and Xiong find that the average correlation of nonindex futures returns with oil rose to 0.2 while that of index commodities exceeded 0.5. Tang and Xiong hypothesize that the commodity markets have been largely segmented before 2000, and the inflow of institutional investors who hold multiple commodities in the same portfolio has linked together the commodity futures markets and increased the correlations among com-

modities, and especially the index ones. Our model shows that one does not need to rely on the market segmentation assumption to produce these effects. Arguably, commodity market speculators investing across commodity markets have been present before 2004. Our model produces both the increase in the correlations amongst commodities and the higher increase in the correlations of index commodities under the complete markets assumption.[11] The key mechanism that we stress is that in the presence of institutional investors benchmarked to a commodity index. This index (more precisely, $E_t[I_T] = D_t \prod_{i=1}^{L} (g_i(t)/D_{it})^{1/L}$) emerges as a common factor in returns of all commodities, raising their correlations. However, the sensitivity to this new factor is higher for index commodity futures (Proposition 2), which is the primary reason why their returns become more correlated than those of nonindex futures. We note that the above intuition is precise for covariances. However, it carries through also to the correlations because the effect of rising volatilities is smaller than the effect of rising covariances. Figure 4 illustrates this discussion and presents the correlations occurring in our model.

3.3. Transmission to Stock Market

Since investors in our model invest both in the futures and stock markets, one may expect that the effects we find in the futures market may get transmitted to the stock market. This is indeed the case in our model. Proposition 3 demonstrates that the discount factor, affected by financialization, makes the stock market price and volatility dependent on the characteristics of the index commodities.

Proposition 3 (Stock market level and volatility). *In the economy with institutions, the equilibrium stock market level and volatility vector are given by*

$$S_t = \overline{S}_t \frac{a + b(1-\lambda) D_0 \prod_{i=1}^{L} (g_i(0)/D_{i0})^{1/L} + b\,\lambda\, D_t \prod_{i=1}^{L} (g_i(t)/D_{it})^{1/L}}{a + b(1-\lambda) D_0 \prod_{i=1}^{L} (g_i(0)/D_{i0})^{1/L} + b\,\lambda\, e^{-\sigma^2 (T-t)} D_t \prod_{i=1}^{L} (g_i(t)/D_{it})^{1/L}}, \tag{20}$$

$$\sigma_{St} = \overline{\sigma}_S + h_{St}\sigma_{It}, \quad h_{St} > 0, \tag{21}$$

[11] This result can be shown analytically when the volatilities of commodity supply news are the same, i.e., $\sigma_k = \sigma_j$, $\forall k, j = 1, \ldots, K$. For different volatility supply news parameters, all cross correlations (including the stock) can be analytically shown to increase for $L = 1$.

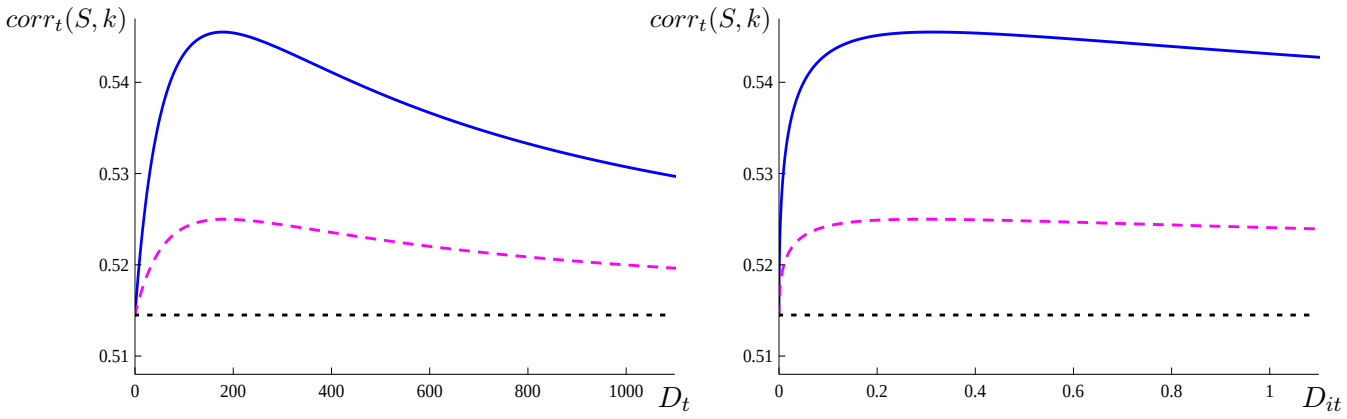
where $\overline{S}_t$ *and* $\overline{\sigma}_S$ *are the corresponding quantities in the benchmark economy with no institutions, given by*

$$\overline{S}_t = \frac{\sum_{k=0}^{K} \alpha_k}{\alpha_0} e^{(\mu-\sigma^2)(T-t)} D_t, \quad \overline{\sigma}_S = \sigma, \tag{22}$$

and h_{St} *is a strictly positive stochastic process provided in the Appendix, and* σ_{It} *is as in Proposition 2.*

Consequently, in equilibrium, the stock market level and its volatility $\|\sigma_{St}\|$ *are increased in the presence of institutions.*

Proposition 3 reveals that the stock market is higher in the presence of institutional investors. This is because the stock market pays off in high aggregate output (high-D_T) states, which are also the states in which the commodity index does well. The institutional investors who desire payoffs in those states bid up the stock price. For the same reason, they also bid up the stock return volatility, making the stock a less attractive investment for the normal investors.



(a) Effect of aggregate output news D

(b) Effect of index commodity supply news D_i

Figure 5: **Equity-futures correlations.** This figure plots return correlations of the stock market with index futures and the stock market with nonindex futures in the presence of institutions against aggregate output news D_t and against index commodity supply news D_{it}, $i \neq k$. As in Figure 2, the solid blue line is for index futures, the magenta dashed line is for nonindex futures, and the black dotted line is for the benchmark economy. The parameter values are as in Figure 2.

The quantities $corr_t(S,k) = \sigma_{fSt} \cdot \sigma_{f_kt}/(\|\sigma_{St}\|\,\|\sigma_{f_kt}\|)$, for all k, are the (instantaneous) equity-futures correlations in our model. These correlations always rise in the presence of

institutions. In other words, we do get a theoretical confirmation within our model to support the assertion that the recent rise in the equity-commodity correlations can be attributed to financialization.[12] Figure 5 depicts the equity-commodity correlations in our model. The correlations of the stock market and the commodity futures returns goes up because both the stock market and the commodities returns depend positively on the new common factor: the commodity index. The correlations of the stock market and the index commodities is higher than that with the nonindex because the index commodity futures have a higher loading on the new factor.

3.4. Commodity Spot Prices

Commodity spot prices are important determinants of the cost of living worldwide. Spiralling food and energy prices observed in recent years have sparked an intense debate whether the inflow of institutional investors into the futures markets may be pushing millions of households below the poverty line. In his congressional testimony, Masters (2008) argues that the price spiral is unequivocally due to the inflow of institutional commodity investors. In a formal study, Singleton (2012) presents evidence in favor of this view.

The framework we have developed so far does not carry direct implications for time-t commodity spot prices p_t. To formally determine prices p_t, one would need to add spot market clearing at all interim periods $t < T$ and intertemporal consumption. However, we may attempt to extrapolate from our model and conjecture the types of implications that one would expect from a fully-fledged model with intertemporal consumption. Let us make several additional assumptions. First, assume that the commodities are storable (until the maturity of the futures contract). Second, assume that a trader can freely buy or sell (short) a commodity at any time $t \leq T$. Shorting a commodity is understood as a reduction of inventories of the commodity that the trader is holding. Under these assumptions, it is possible to construct an arbitrage strategy of replicating a futures contract in the physical commodity market. Finally, let us set each commodity's convenience yield/storage costs to

[12]Tan and Xiong document that the correlation between GSCI commodity index and the S&P500 rose after 2004, and have been especially high in 2008. Relatedly, Buyuksahin and Robe (2012) find that the GSCI-S&P500 correlation rose since the 2008 financial crisis, but not before.

be a constant fraction δ_k of its price $k = 1, \dots K$. The relationship between futures and spot commodity prices is then provided by the familiar cost-of-carry formula[13]

$$f_{kt} = p_{kt} e^{\delta_k (T-t)}. \tag{23}$$

Consequently, the time-t commodity prices for commodities $k = 1, \dots, K$ are as in Proposition 1, replacing f_{kt} by $p_{kt} e^{\delta_k (T-t)}$. Furthermore, all comparative statics reported in Proposition 1 and Corollary 1 go through for commodity spot prices. Admittedly, this result has been obtained under very strong assumptions. The assumption of a constant cost of carry is simplistic. We recognize that convenience yields are stochastic in practice, driven by a number of factors (Gibson and Schwartz (1990), Cassasus and Collin-Dufresne (2005)). In future work, it would be useful to consider stochastic convenience yields and investigate potentially interesting effects financialization may have had on convenience yields. Moreover, an ideal model would need to introduce storage explicitly. We leave this for future research.

4. Economy with Demand Shocks

In this section we introduce commodity demand shocks to our baseline model. While our setup with supply-side-only uncertainty is capable of delivering most of our insights, we need a richer model to explore the quantitative effects of financialization. As we demonstrate in this section, the presence of demand shocks alone can generate an increase in futures prices and volatilities. But, importantly, demand shocks also sizeably magnify the effects of financialization. Furthermore, it has been argued extensively in the literature that demand shocks are very important in explaining the behavior of prices of oil and other commodities (see Fattouh, Kilian, and Mahadeva (2013) for a survey). For example, Kilian and Murphy (2013) reach a conclusion that the 2004-2008 surge in oil prices can be attributed to demand shocks. Within our model we can disentangle how much of a rise in futures prices and their comovement can be attributed to positive demand shocks alone and how much to financialization.

To model demand shocks, we make the following modification to our model. In the

[13]The formula does not feature the interest rate r because we have normalized r to zero.

consumption index (8) of the investors, repeated here for expositional clarity,

$$W_n = C_{n_0}^{\alpha_0} C_{n_1}^{\alpha_1} \cdot \ldots \cdot C_{n_K}^{\alpha_K}, \qquad n \in \{\mathcal{N}, \mathcal{I}\}, \tag{24}$$

we allow one of the demand parameters, α_1, to be stochastic. Shocks to α_1 then represent shifts in demand for good 1 in the commodity index; we hereafter refer to them as demand shocks. We do not consider shocks to demand for other goods, but our model can be extended to incorporate such shocks. We assume that α_1 is a strictly positive process with dynamics

$$d\alpha_{1t} = \alpha_{1t}\sigma_\alpha dw_{0t}, \tag{25}$$

where $\sigma_\alpha > 0$ is constant. Implicit in this assumption is that α_1 is driven by the same source of risk, Brownian motion w_0, as the (scaled) aggregate output D—i.e., α_1 has a one-to-one mapping with aggregate output. Now an investor's time-T demand for good 1 is not simply a (decreasing) function of its price p_{1T}, but also an (increasing) function of the aggregate output D_T (through α_{1T}). The latter assumption has recently been advocated by Dvir and Rogoff (2009) in their model of oil prices. In a calibration that follows, we associate commodity 1 with energy. We therefore frequently refer to commodity 1 as energy and the remaining commodities as non-energy. By construction, futures on commodity 1 are included in the index. This is consistent with the data: energy futures are included in all popular commodity indices.

Proposition 4 reports the equilibrium futures prices and their return volatilities in the economy with demand shocks in closed form. The equilibrium stock market level and volatility, not presented here for brevity, are provided explicitly in the Appendix.

Proposition 4 (Futures prices and volatilities with demand shocks). *In the economy with institutions and demand shocks, the equilibrium futures price of commodity $k = 1, \ldots, K$ and its associated volatility vector of loadings are given by*

$$f_{kt} = \overline{f}_{kt} \frac{A + b\,\lambda e^{(1_{\{k\le L\}}\sigma_k^2/L + 1_{\{k=1\}}(\sigma_\alpha^2/L + \sigma\sigma_\alpha))(T-t)} \alpha_{1t}^{1/L} D_t \prod_{i=1}^{L} (\hat{g}_i(t)/D_{it})^{1/L}}{A + b\,\lambda e^{-(\sigma^2 + \sigma\sigma_\alpha/L)(T-t)} \alpha_{1t}^{1/L} D_t \prod_{i=1}^{L} (\hat{g}_i(t)/D_{it})^{1/L}}, \quad A, \hat{g}_i(t) > 0, \tag{26}$$

$$\sigma_{f_k t} = \overline{\sigma}_{f_k} + \hat{h}_{kt}\sigma_{It}, \quad \hat{h}_{kt} > 0, \tag{27}$$

where $\overline{f}_{kt}$ is the equilibrium futures price in the benchmark economy with no institutions, $\overline{\sigma}_{kt}$ its corresponding volatility vector, and σ_{I_t} is the volatility vector of the conditional expected index $E_t[I_T]$, given by

$$\overline{f}_{kt} = \frac{\left(\alpha_k 1_{\{k>1\}} + \alpha_{1t} 1_{\{k=1\}}\right) e^{(\mu-\mu_k-\sigma^2+\sigma_k^2)(T-t)}}{\alpha_0} \frac{D_t}{D_{kt}}, \tag{28}$$

$$\overline{\sigma}_f = (\sigma + \sigma_\alpha 1_{\{k=1\}}, 0, \ldots, -\sigma_k, 0, \ldots, 0), \tag{29}$$

$$\sigma_I = (\sigma + \tfrac{1}{L}\sigma_\alpha, -\tfrac{1}{L}\sigma_1, \ldots, -\tfrac{1}{L}\sigma_L, 0, \ldots, 0), \tag{30}$$

and the constant A, the deterministic quantity $\hat{g}_i(t)$ and the stochastic process $\hat{h}_{kt}$ are explicitly provided in the Appendix.

Consequently, in equilibrium, all futures prices and their volatilities $\|\sigma_{f_k}\|$ are higher than in the benchmark economy.

Proposition 4 confirms our earlier result that all futures prices are higher in the presence of institutions, with prices of index futures exceeding those of nonindex ones. The distinguishing feature of our economy with demand shocks is that these effects become stronger than in the economy without demand shocks. Below we undertake a simple calibration in order to assess the quantitative importance of our results.

Since we have taken commodity 1 to represent energy, we calibrate the demand parameter α_{1t} from the energy expenditure share in total consumption. The expenditure share in our model is given by $\alpha_{1t}/(\sum_{k=0,k\neq 1}^{K} \alpha_k + \alpha_{1t})$. For convenience, we set our baseline parameters such that $\sum_{k=0,k\neq 1}^{K} \alpha_k + \alpha_{1t} = 1$ at time t. We obtain data on the energy expenditure share in the US from BEA Table 2.3.5U from 1959:M1 through 2012:M12.[14] The average expenditure share in the sample is about 6%, and so we set $\alpha_{1t} = 0.06$. As also noted by Hamilton (2013), the energy expenditure share series is very volatile. Our estimate of σ_α obtained from the series is 9.8%. Finally, the series does not have a deterministic trend, and we cannot reject the null that it has a unit root, which supports our specification in (25). The expenditure share on the generic good is taken to be 70%, and the remaining expenditure is spread equally across the remaining commodities (other than energy). We calibrate the volatility of the process for the generic good supply news D_t from the equations for stock market volatility (A42)–(A43) in the Appendix using the value of 16% for the aggregate US stock

[14] Hamilton (2013) uses the same data source in his detailed analysis of the energy expenditure share.

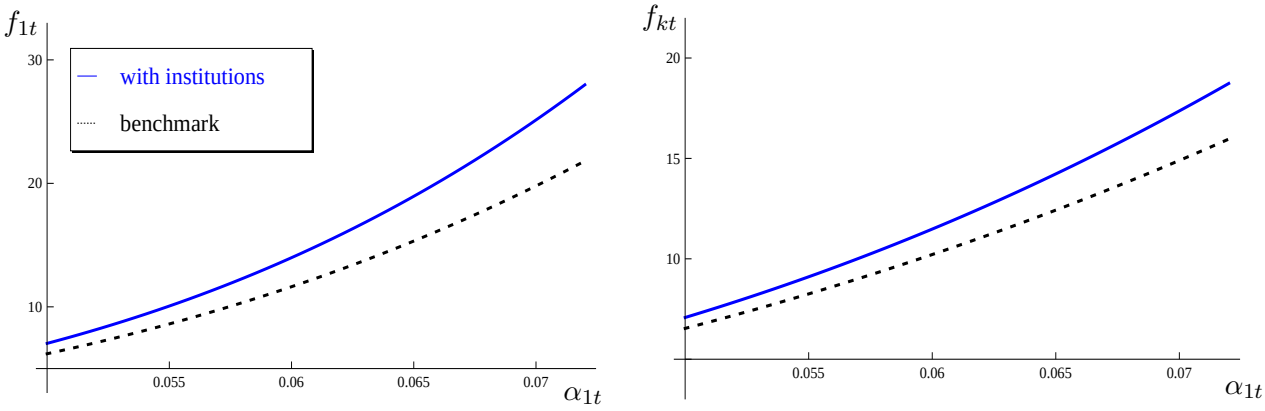
Parameter or State Variable	Symbol	Value
Mean growth rate of generic good's supply news	μ	0.05
Volatility of generic good's supply news	σ	0.15
Mean growth rate of commodity k supply news, $k = 1, \ldots, K$	μ_k	0.05
Volatility of commodity 1 (energy) supply news	σ_1	0.33
Volatility of commodity $k \neq 1$ (non-energy) supply news	σ_k	0.25
Volatility of commodity 1 demand shocks	σ_α	0.098
Demand parameter, generic good	α_0	0.7
Demand parameter, commodity $k = 2, \ldots K$	α_k	0.06
Number of commodities	K	5
Number of commodities in the index	L	2
Terminal date	T	5 years
Current date	t	0.1 years
(Initial) fraction of institutions in the economy	λ	0.4
Objective function parameters	a, b	1
Time-0 and time-t supply of generic good	D_0, D_t	100
Time-0 and time-t supply of commodity k, $k = 1, \ldots, K$	D_{k0}, D_{kt}	1
Time-0 and time-t demand parameter for energy	α_{10}, α_{1t}	0.06

Table 2: **Calibrated values of model parameters and state variables.**

market volatility in the data. The calibrated value for the volatility of generic good's supply news, σ, is around 15%, which is consistent with the aggregate dividend volatility in the data. The model-implied volatility parameters of the processes for D_1 and D_k, $k = 2, \ldots, K$ are calibrated from equation (29) using data on the average volatilities of the energy-sector and non-energy sector futures from Gorton, Hayashi, and Rouwenhorst (2013). We set the mean growth rates $\mu = \mu_k = 0.05$ for all k. We have tried a wide range of alternative values for these parameters, and our results do not vary much. The horizon T is set in line with typical performance evaluation horizons of fund managers, usually 3-5 years (BIS (2003)). These and the remaining parameter values for our baseline case are summarized in Table 2.

Figure 6 illustrates the results of the proposition and disentangles the contribution of financialization over and above that of fundamentals (demand and supply). We vary the energy demand parameter α_{1t} to highlight the contribution of a rising/decreasing demand. As one can see from the figure, increasing demand for energy pushes up its futures price even in the benchmark economy with no institutions (the dashed lines).[15] But in the presence

[15]The benchmark price of energy futures directly depends on the energy demand parameter α_{1t} (see (28)). However, there is also an indirect dependence of benchmark futures prices of all commodities on α_{1t}. This is because α_{1t} and the aggregate output D_t are driven by the same source of risk, the Browninan motion ω_0, and a rise in α_{1t} always coincides with a rise in D_t. Since all commodity futures prices depend positively on



(a) Effect of energy demand parameter α_{1t} **(b)** Spillover to non-energy futures

Figure 6: **Futures prices.** Panel (a) plots index futures 1 price in the economy with demand shocks (solid blue line). Panel (b) plots futures k, $1 < k \leq K$, price in the economy with demand shocks (solid blue line). Both plots are against the energy demand parameter α_{1t}. The dotted black lines are for the corresponding prices in the benchmark economy with no institutions. The parameter values are as in Table 2.

of institutions, the futures price increases even more, and especially so in the presence of demand shocks (solid blue lines). This is because there is now an additional risk in the economy—shifts in demand for energy—that affects the value of the index. Therefore an asset whose payoff is positively correlated with these demand shocks—the energy futures—becomes even more valuable than in the economy without demand shocks. There is also an important spillover of demand shocks to energy on the other futures prices. These spillovers are illustrated in Figure 6(b). The spillovers occur because the rise in demand for energy is positively related to increases in aggregate output, and all prices are increasing in aggregate output. Institutions bid up prices of all futures because they all pay off higher in high-energy-demand states—the states when the value of the index is high.

Above effects are quantitatively important. As revealed by Table 3, for our baseline parameterization, we find that 16.8% of the energy futures price is attributable to financialization—the presence of institutions—and 83.2% to fundamentals (demand and supply). The effects of financialization are somewhat smaller for commodities unaffected by demand shocks, but

D_t (see (28)), the futures prices then rise with α_{1t} even in the economy without institutions.

		σ_k		
		0.19	0.24	0.29
	0.24	14.39%	14.43%	14.46%
σ_1	0.29	16.79%	**16.83%**	16.86%
	0.34	19.68%	19.72%	19.76%

(a) Commodity futures $k = 1$ (energy)

		σ_k		
		0.19	0.24	0.29
	0.24	9.09%	11.00%	13.35%
σ_1	0.29	9.16%	**11.04%**	13.40%
	0.34	9.19%	11.08%	13.44%

(b) Commodity futures k, $1 < k \leq L$ (non-energy)

Table 3: **Fraction of futures prices explained by financialization,** $(f_{kt} - \overline{f}_{kt})/\overline{f}_{kt}$**. Scenario 1: Baseline calibration** (bold cells). The parameter values are as in Table 2 with energy demand parameter $\alpha_{1t} = 0.06$.

they are still sizeable. For example, financialization accounts for 11% of the price of other index commodity futures. We perform a sensitivity analysis around our calibrated values for the supply news volatilities and report the resulting values in Table 3. Our results are not out of line with the findings of Kilian and Murphy (2013) that fluctuations in fundamentals are important in explaining the fluctuations in commodity prices, but we also stress a significant contribution of financialization.

Table 3 also highlights that the magnitudes of the impact of financialization on futures prices are quite sensitive to the volatility of the supply news: the more volatile the individual commodity supply news are, the bigger the fraction of the commodity futures prices that is explained by financialization. In unreported analysis, we find that the effects of financialization are also stronger the bigger the aggregate output news volatility σ and the bigger the demand uncertainly σ_α. These comparative statics may explain why the debate whether institutional investors influence commodity futures prices has been especially intense during the 2007-2008 financial crisis. The high uncertainty during the crisis has amplified the effects of financialization, pushing prices much higher than what could have been justified by fundamentals (supply and demand) alone.

Kilian and Murthy argue further that most of the 2003-2008 increase in energy prices (specifically, oil prices) was due to global demand shocks. Our model delivers this result, but also uncovers an important interaction: the effects of financialization become stronger with higher global demand. To illustrate this implication, we explore within our model the effects of an upward demand shift for energy from the calibrated value of $\alpha_{1t} = 0.06$ to

		σ_k		
		0.19	0.24	0.29
σ_1	0.24	20.48%	21.70%	22.86%
	0.29	23.60%	**24.93%**	26.21%
	0.34	27.24%	28.70%	30.09%

(a) Commodity futures $k = 1$ (energy)

		σ_k		
		0.19	0.24	0.29
σ_1	0.24	13.34%	16.90%	21.28%
	0.29	13.37%	**16.92%**	21.31%
	0.34	13.40%	16.94%	21.34%

(b) Commodity futures k, $1 < k \leq L$ (non-energy)

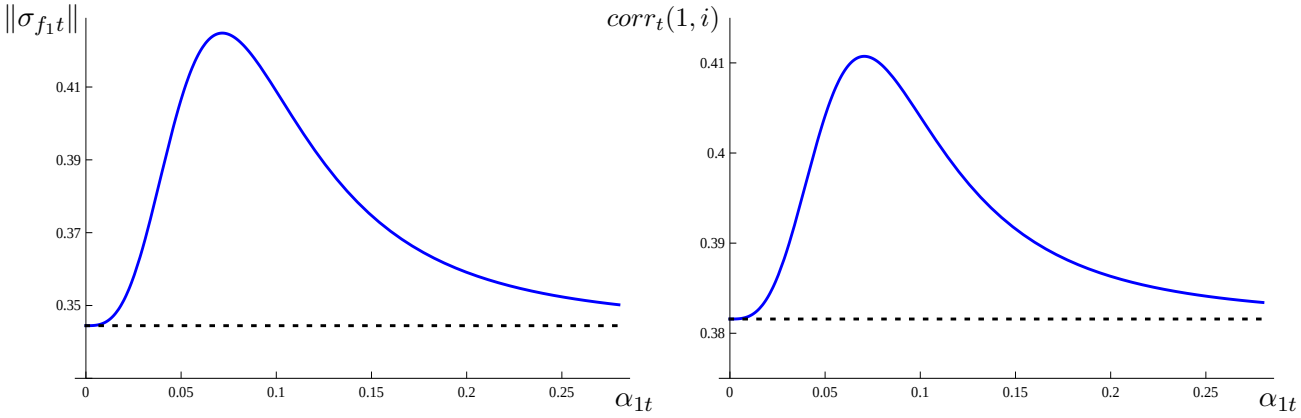
Table 4: **Fraction of futures prices explained by financialization,** $(f_{kt} - \overline{f}_{kt})/\overline{f}_{kt}$**. Scenario 2: Increased demand for energy** (bold cells). The effect of increasing the energy demand parameter α_{1t} from $\alpha_{1t} = 0.06$ to 0.08. The remaining parameter values are as in Table 2.

0.08—a 33% increase. Table 4 presents the (recomputed) fraction of futures prices that is attributable to financialization. As one can see clearly, financialization becomes significantly more important. For our baseline parameter values for the volatilities of supply news, the fraction attributable to financialization rises from 16.8% to 24.9% for the energy futures and from 11% to 16.9% for non-energy.[16]

Proposition 4 also confirms that our remaining results of Section 3 continue to hold in the presence of demand shocks. In particular, futures return volatilities are higher in the presence of institutions. Moreover, we find via numerical analysis that they become even higher in the presence of demand shocks. To understand the intuition behind this new result, it is useful to note that the energy (commodity 1) futures is exposed to an additional source of risk, demand shocks, and so it is more volatile even in the benchmark with no institutions. Additionally, the membership of energy futures in the index makes the index riskier than in the economy without demand shocks. Since falling behind the index is a source of risk for institutional investors, all futures prices depend on the expected index, as we have highlighted before. The (expected) index appears as a new risk factor in the futures prices, and this factor is now more volatile (higher $||\sigma_I||$). Consequently, all futures prices

[16]Tables 3 and 4 demonstrate that our results are quite sensitive to the calibration of the supply news volatilities σ_k, $k = 1, \ldots, K$. For robustness, we re-examine our results using a calibration of these parameters based on a study of Vassilev (2010). Vassilev's study includes fewer commodities, and a calibration based on his data yields $\sigma_1 = 0.33$ and $\sigma_k = 0.25$. (The remaining parameter values remain as in our baseline calibration in Table 2). For this new calibration, we find that the fraction attributable to financialization rises from 16.8% to 19.1% for the energy futures and from 11% to 11.5% for non-energy. For an upward demand shift for energy from $\alpha_{1t} = 0.06$ to 0.08, these fractions become 28.2% and 17.8%, respectively.

are more volatile as well. In terms of magnitudes, for our baseline parameter values, the volatility of energy futures prices in our model rises from 38.2% to 41% with financialization. Figure 7(a) presents the sensitivity of these magnitudes to the energy demand parameter α_{1t}. In the data, the volatility of energy futures has been highly time-varying, e.g., rising from 30% to 60% in 2008, and then falling back down. It is also time-varying in our analysis, and the size of the increase in the volatility due to financialization depends positively on the aggregate output news volatility.



(a) Volatility of energy futures

(b) Energy/non-energy futures return correlation

Figure 7: **Volatilities and correlations.** Panel (a) plots return volatility of energy futures. Panel (b) plots return correlations of energy futures with non-energy, $corr_t(1,i)$, $i \neq 1$. Both plots are against the energy demand parameter α_{1t}. The dotted lines are for the corresponding correlations in the benchmark economy with no institutions. The parameters are from Table 2.

The commodity futures return correlations are also higher with financialization. As before, this is because the expected index emerges as a common factor affecting all assets in the economy, and hence the covariances of all assets with each other increase more than in the economy without demand shocks. The same ends up being true for the corresponding correlations. To illustrate the effects of financialization on the correlations quantitatively, in Figure 7(b) we plot the commodity futures return correlations in the economies with and without institutions. Again, we focus on the energy futures contract. We find that the correlation increases are sizeable. For example, for our baseline parameter values, the

correlation of energy futures with non-energy rises from 34.4% to 42% with financialization. While sizeable, the increase is smaller than that documented by Tang and Xiong (2012): in their sample, the correlations of oil futures returns with non-energy commodity futures rises from 10% pre-2004 to about 50% in 2009. We conjecture that our results on the correlations may be sensitive to the assumptions about the nature of the demand shocks in the model. We leave it for future research to explore a more general specification of the demand shocks to commodity 1 as well as the more general case of demand shocks to more than one commodity. To fully address the question of the effects of financialization on cross-commodity correlations and to generate the increases in the correlations of the magnitude documented by Tan and Xiong, we believe that one needs a model with a common demand shock to a group of commodities (e.g., demand for metals by China). It would be interesting to disentangle the effects of correlated demand shocks from the effects of financialization, which as we have shown, also increases cross-commodity correlations.

5. Conclusion

In this paper we have explored theoretically how the presence of institutional investors may affect commodity futures prices and their dynamics. We have found that in the presence of institutions futures prices of all commodities rise, with futures prices of index commodities increasing by more. We have also found that in the presence of institutional investors shocks to fundamentals (demand and supply) of index commodities get transmitted to prices of all other commodities. Furthermore, the volatilities of all commodity futures rise in the presence of institutions, with those of index commodities increasing by more. These effects are more pronounced in the presence of demand shocks. Finally, the presence of institutions leads to an increase in the cross-commodity and equity-commodity correlations, with those for index commodity futures increasing by more.

This paper focuses on commodity futures markets, and only very briefly touches upon the linkages between commodity futures and spot markets. It would be interesting to improve our theoretical understanding of whether price pressure from institutional investors operating in futures markets may be transmitted to spot prices of underlying commodities. Sockin and

Xiong (2012) is one nice recent attempt to model this, but there is room for more. Also left for future research is the analysis of the open interest in the futures markets. In a recent paper, Hong and Yogo (2012) document that open interest predicts asset prices and macroeconomic variables. It would be interesting to examine whether our model delivers this intriguing finding. Finally, our analysis of financialization is based on comparing the economies with and without institutional investors, and we do not address the issue of why the institutions entered the commodity futures markets in the first place. The question of what prompted their increased participation post-2004 is left for future work.

Appendix A

Proof of Lemma 1. We first determine the institutional and normal investors' optimal demands in each commodity. Since the securities market is dynamically complete in our setup with $K+1$ risky securities and $K+1$ sources of risk ω, there exists a state price density process, ξ, such that the time-t value of a payoff Q_T at time T is given by $E_t\big[\xi_T Q_T\big]/\xi_t$. In our setting, the state price density is a martingale. Accordingly, investor n's, $n = \mathcal{N}, \mathcal{I}$, dynamic budget constraint (9) can be restated as

$$E_t\big[\xi_T \sum_{k=0}^{K} p_{kT} C_{n_k T}\big] = \xi_t W_{nt}. \tag{A1}$$

Maximizing the institutional investor's expected objective function (7), with the Cobb-Douglas aggregator (8) substituted in, subject to (A1) evaluated at time $t = 0$ leads to the institution's optimal demand in commodity $k = 1, \ldots, K$ and generic good, respectively, as

$$C_{\mathcal{I}_k T} = \frac{\alpha_k \left(a + bI_T\right)}{y_I p_{kT} \xi_T}, \quad C_{\mathcal{I}_0 T} = \frac{\alpha_0 \left(a + bI_T\right)}{y_I \xi_T}, \tag{A2}$$

where $1/y_I$ solves (A1) evaluated at $t = 0$. Substituting (A2) into (A1) at $t = 0$, we obtain

$$\frac{1}{y_I} = \frac{\lambda \xi_0 S_0}{\sum_{j=0}^{K} \alpha_j \left(a + bE\left[I_T\right]\right)}.$$

Consequently, the institution's optimal commodity demands are given by

$$C_{\mathcal{I}_k T} = \frac{\alpha_k}{\sum_{j=0}^{K} \alpha_j} \frac{\lambda \xi_0 S_0}{p_{kT} \xi_T} \frac{a + bI_T}{a + bE\left[I_T\right]}, \qquad k = 1, \ldots, K, \tag{A3}$$

$$C_{\mathcal{I}_0 T} = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{\lambda \xi_0 S_0}{\xi_T} \frac{a + bI_T}{a + bE\left[I_T\right]}. \tag{A4}$$

Similarly, we obtain the normal investor's optimal commodity demands at time T as

$$C_{\mathcal{N}_k T} = \frac{\alpha_k}{\sum_{j=0}^{K} \alpha_j} \frac{(1-\lambda)\, \xi_0 S_0}{p_{kT} \xi_T}, \qquad k = 1, \ldots, K, \tag{A5}$$

$$C_{\mathcal{N}_0 T} = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{(1-\lambda)\, \xi_0 S_0}{\xi_T}. \tag{A6}$$

We now proceed to determine the equilibrium prices at time T. To obtain the equilibrium state price density, we impose the market clearing condition for the generic good, $C_{\mathcal{N}_0 T} + C_{\mathcal{I}_0 T} = D_T$, and substitute (A4) and (A6) to obtain

$$\frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{\xi_0 S_0}{\xi_T} \left(1 - \lambda + \lambda \frac{a + bI_T}{a + bE\left[I_T\right]}\right) = D_T,$$

which after rearranging leads to the equilibrium terminal state price density:

$$\xi_T = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{\xi_0 S_0}{D_T} \left(1 + \frac{\lambda b \left(I_T - E\left[I_T\right]\right)}{a + bE\left[I_T\right]}\right). \tag{A7}$$

The equilibrium state price density in the benchmark economy with no institutions is obtained by considering the special case of $b = 0$ in (A7). The time-T discount factor is defined as $M_{0,T} = \xi_T/\xi_0$, which after substituting (A7) leads to the expression (14) reported in Lemma 1.

To determine the equilibrium commodity prices at T, we impose the market clearing condition $C_{\mathcal{N}_k T} + C_{\mathcal{I}_k T} = D_{kT}$ for each commodity $k = 1, \ldots, K$, and substitute (A3) and (A5) to obtain

$$\frac{\alpha_k}{\sum_{j=0}^{K} \alpha_j} \frac{\xi_0 S_0}{p_{kT} \xi_T} \left(1 - \lambda + \lambda \frac{a + bI_T}{a + bE\left[I_T\right]}\right) = D_{kT},$$

which after substituting the equilibrium state price density (A7) and rearranging leads to the equilibrium commodity price expressions (11) in Lemma 1. Substituting the equilibrium commodity prices (11) that are in the commodity into the definition of the index (4) leads to the equilibrium commodity index value (12). Moreover, substituting the equilibrium commodity prices (11) into the stock market terminal value $S_T = D_T + \sum_{k=1}^{K} p_{kT} D_{kT}$ leads to the expression (13) in Lemma 1. To determine the unconditional expectation of the index, we make use of the fact that D_T, D_{iT}, $i = 1, \ldots, L$, are lognormally distributed and hence obtain

$$E\left[I_T\right] = E\left[\frac{D_T}{\alpha_0} \prod_{i=1}^{L} \left(\frac{\alpha_i}{D_{iT}}\right)^{1/L}\right] = e^{\left(\mu - \frac{1}{L}\sum_{i=1}^{L}\left(\mu_i - \frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)\right)T} \frac{D_0}{\alpha_0} \prod_{i=1}^{L} \left(\frac{\alpha_i}{D_{i0}}\right)^{1/L}. \tag{A8}$$

Finally, we note that the equilibrium commodity and stock prices at time T are as in the benchmark economy with no institutions (the special case of $b = 0$, $a = 1$).

Q.E.D.

Proof of Proposition 1. By no arbitrage, the futures price of commodity $k = 1, \ldots, K$ in our setup is given by

$$f_{kt} = \frac{E_t\left[\xi_T p_{kT}\right]}{\xi_t}. \tag{A9}$$

We proceed by first determining the equilibrium state price density process ξ. Since the state price density process is a martingale, its time-t value is given by

$$\begin{aligned} \xi_t &= E_t\left[\xi_T\right] \\ &= \bar{\xi} E_t\left[1/D_T\right] \left(a + b\left(1 - \lambda\right) E\left[I_T\right] + \lambda b \frac{E_t\left[I_T/D_T\right]}{E_t\left[1/D_T\right]}\right), \end{aligned} \tag{A10}$$

where the second equality follows by substituting ξ_T from (A7) and rearranging, and

$$\bar{\xi} = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{\xi_0 S_0}{a + bE\left[I_T\right]}. \tag{A11}$$

Substituting (12) and using the fact that D_T, D_{iT}, $i = 1, \ldots, L$, are lognormally distributed, we obtain

$$\begin{aligned} E_t\left[I_T/D_T\right] &= \frac{1}{\alpha_0} E_t\left[\prod_{i=1}^{L} \left(\alpha_i/D_{iT}\right)^{1/L}\right] \\ &= \frac{1}{\alpha_0} e^{-\frac{1}{L}\sum_{i=1}^{L}\left(\mu_i - \frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)(T-t)} \prod_{i=1}^{L} \left(\alpha_i/D_{it}\right)^{1/L}. \end{aligned} \tag{A12}$$

Substituting (A8), (A12) and $E_t\left[1/D_T\right] = e^{-\left(\sigma^2-\mu\right)(T-t)}/D_t$ into (A10), we obtain

$$\xi_t = \bar{\xi}\frac{e^{-\left(\sigma^2-\mu\right)(T-t)}}{D_t}\left(a + b\left(1-\lambda\right)D_0 \prod_{i=1}^{L}\left(g_i\left(0\right)/D_{i0}\right)^{1/L} + b\lambda e^{-\sigma^2(T-t)} D_t \prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}\right), \tag{A13}$$

where $g_i\left(t\right)$ is as given in (16).

To compute the expected deflated futures payoff of commodity $k = 1, \ldots, K$, we substitute (A7) and (11), and rearrange to obtain

$$E_t\left[\xi_T p_{kT}\right] = \bar{\xi}\frac{\alpha_k}{\alpha_0} E_t\left[1/D_{kT}\right]\left(a + b\left(1-\lambda\right)E\left[I_T\right] + b\,\lambda\,\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]}\right), \tag{A14}$$

where $\bar{\xi}$ is as in (A11).

For nonindex futures contracts $k = L+1, \ldots, K$, we proceed by considering

$$\begin{aligned} E_t\left[I_T/D_{kT}\right] &= \frac{1}{\alpha_0} E_t\left[D_T/D_{kT}\prod_{i=1}^{L}\left(\alpha_i/D_{iT}\right)^{1/L}\right] \\ &= \frac{1}{\alpha_0} E_t\left[D_T\prod_{i=1}^{L}\left(\alpha_i/D_{iT}\right)^{1/L}\right] E_t\left[1/D_{kT}\right], \end{aligned}$$

where in the first equality we have substituted (12) and in the second we have made use of the fact that D_{kT} is independent of D_T, D_{iT}, $i = 1, \ldots, L$. Consequently, using the fact that D_T, D_{iT}, $i = 1, \ldots, L$, are lognormally distributed, we obtain

$$\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]} = D_t \prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}, \tag{A15}$$

where $g_i\left(t\right)$ is as in (16). Substituting (A13)–(A15), (A8) and $E_t\left[1/D_{kT}\right] = e^{\left(\sigma_k^2-\mu_k\right)(T-t)}/D_{kt}$ into (A9), and rearranging, we arrive at the equilibrium nonindex futures price expression

reported in (15) for $k = L+1, \ldots, K$. The equilibrium futures price $\bar{f}_k$ in the benchmark economy with no institutions (16) follows by considering the special case of $a = 1$, $b = 0$ in (15).

For index futures contracts $k = 1, \ldots, L$, we substitute (12) and again compute

$$\begin{aligned} E_t\left[I_T/D_{kT}\right] &= \frac{1}{\alpha_0} E_t\left[D_T/D_{kT} \prod_{i=1}^{L} \left(\alpha_i/D_{iT}\right)^{1/L}\right] \\ &= \frac{1}{\alpha_0} e^{\left(-\mu+\mu_k+\left(\frac{1}{L}+1\right)\sigma_k^2-\frac{1}{L}\sum_{i=1}^{L}\left(\mu_i-\frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)\right)(T-t)} \frac{D_t}{D_{kt}} \prod_{i=1}^{L} \left(\alpha_i/D_{it}\right)^{1/L}. \end{aligned}$$

So, using $E_t\left[1/D_{kT}\right] = e^{\left(\sigma_k^2-\mu_k\right)(T-t)}/D_{kt}$ we obtain

$$\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]} = e^{\frac{1}{L}\sigma_k^2(T-t)} D_t \prod_{i=1}^{L} \left(g_i\left(t\right)/D_{it}\right)^{1/L}, \tag{A16}$$

where $g_i\left(t\right)$ is as in (16). Substituting (A13), (A14), (A16), and (A8) into (A9) and rearranging leads to the equilibrium index futures price expression reported in (15) for $k = 1, \ldots, L$. The property (i) that the futures prices are higher than in the benchmark economy follows by observing that the factor multiplying $\bar{f}_{kt}$ in expression (15) is strictly greater than one. Similarly, the property (ii) that the index futures price rise is higher than that of nonindex futures follows by observing that the factor multiplying $\bar{f}_{kt}$ in expression (15) is higher for an otherwise identical index futures.

Q.E.D.

Proof of Corollary 1. The stated properties follow by taking the appropriate partial derivatives of the expressions (15)–(16), and comparing the relevant magnitudes of the partial derivatives of interest.

Q.E.D.

Proof of Proposition 2. We write the equilibrium index futures price in (15) for $k = 1, \ldots, L$ as

$$f_{kt} = \bar{f}_{kt} \frac{Z_t}{Y_t}, \tag{A17}$$

where

$$\overline{f}_{kt} = \frac{\alpha_k}{\alpha_0} e^{(\mu-\mu_k-\sigma^2+\sigma_k^2)(T-t)} \frac{D_t}{D_{kt}},$$

$$Z_t = a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{\sigma_k^2(T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L},$$

$$Y_t = a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2(T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}.$$

where $g_i(t)$ is as in (16).

Applying It's Lemma to both sides of (A13), we obtain

$$\sigma_{f_k t} = \overline{\sigma}_{f_k} + \sigma_{Zt} - \sigma_{Yt}, \tag{A18}$$

where

$$\begin{aligned}
\overline{\sigma}_{f_k} &= (\sigma,\, 0, \ldots, -\sigma_k, 0, \ldots, 0) \\
\sigma_{Zt} &= \frac{b\,\lambda\, e^{\sigma_k^2(T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{\sigma_k^2(T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} \sigma_{It} \\
\sigma_{Yt} &= \frac{b\,\lambda\, e^{-\sigma^2(T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2(T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} \sigma_{It},
\end{aligned}$$

and σ_{It} is the volatility vector of $D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L} = E_t\left[I_T\right]$ given by

$$\sigma_{It} = \left(\sigma,\, -\tfrac{1}{L}\sigma_1, \ldots, -\tfrac{1}{L}\sigma_L,\, 0, \ldots, 0\right).$$

We note that $Y_t \sigma_{Yt} = Z_t \sigma_{Zt} e^{-\left(\sigma^2 + \sigma_k^2/L\right)(T-t)}$. Hence, we have

$$\begin{aligned}
Z_t \sigma_{Zt} Y_t - Y_t \sigma_{Yt} Z_t &= Z_t \sigma_{Zt} \left(Y_t - e^{-\left(\sigma^2+\sigma_k^2/L\right)(T-t)} Z_t\right) \\
&= Z_t \sigma_{Zt} \left(1 - e^{-\left(\sigma^2+\sigma_k^2/L\right)(T-t)} Z_t\right) \left(a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L}\right),
\end{aligned} \tag{A19}$$

where the second equality follows by substituting Z_t and Y_t and manipulating terms. Substituting (A19) into the expression $\sigma_{Zt} - \sigma_{Yt} = \left(Z_t \sigma_{Zt} Y_t - Y_t \sigma_{Yt} Z_t\right)/Y_t Z_t$, and then into (A18) leads to the equilibrium volatility vector of loadings of index commodity futures in (17) where

$$\begin{aligned}
h_{kt} = &\frac{b\,\lambda\, e^{\sigma_k^2(T-t)/L} \left(1 - e^{-\left(\sigma^2+\sigma_k^2/L\right)(T-t)}\right) \left(a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L}\right)}{a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{\sigma_k^2(T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} \\
&\times \frac{D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2(T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} > 0,
\end{aligned} \tag{A20}$$

where $g_i(t)$ is as in (16).

To determine the volatility vector of loadings of nonindex futures $k = L+1, \ldots, K$, as reported in (18), we follow the same steps as above for index futures, and obtain the stochastic process h_t as

$$h_t = \frac{b\,\lambda\left(1-e^{-\sigma^2(T-t)}\right)\left(a+b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L}\right)}{a+b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L}+b\,\lambda\,D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}} \times \frac{D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}}{a+b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L}+b\,\lambda\,e^{-\sigma^2(T-t)}D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}} > 0, \tag{A21}$$

where $g_i(t)$ is as in (16).

The property that volatilities of all futures prices are higher than in the benchmark economy follow immediately from (17)–(18). To prove property (ii), we note that for commodities i and k with $D_{it} = D_{kt}$, $\alpha_i = \alpha_k$, we have $h_{kt} > h_t$ from (A20)–(A21), and hence the volatility increase for an index futures is higher than that for an otherwise identical nonindex futures.

Q.E.D.

Proof of Proposition 3. By no arbitrage, the stock market level is given by

$$S_t = \frac{E_t\left[\xi_T D_T\right]}{\xi_t}. \tag{A22}$$

To compute the expected deflated stock market payoff, we substitute (A7) and (12) to obtain

$$E_t\left[\xi_T D_T\right] = \bar{\xi}\sum_{k=0}^{K}\frac{\alpha_k}{\alpha_0}\left(a+b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L}+b\,\lambda\,D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}\right), \tag{A23}$$

where we have used the fact that D_T, D_{iT}, $i = 1, \ldots, L$ are lognormally distributed, and $\bar{\xi}$ is as in (A11) and $g_i(t)$ is as in (16). Substituting (A23) and (A13) into (A22), and manipulating, leads to the reported equilibrium stock market level in (20). The equilibrium stock market level $\bar{S}_t$ in the benchmark economy (22) follows by considering the special case of $a = 1$, $b = 0$ in (13).

To derive the stock market volatility vector (21), we follow the same steps for the index futures in the Proof of Proposition 2, and obtain the stochastic process h_{St} to be as in (A21). The property that the stock market level and its volatility are higher than those in the benchmark follow straightforwardly from the expressions (20)–(22).

Q.E.D.

Proof of Proposition 4. We first consider the investors' optimal demands in each com-

modity. Maximizing the institutional investor's expected objective function (7), subject to (A1) evaluated at $t = 0$ leads to the institution's optimal demand in commodity $k = 2, \ldots, K$ and generic good as in (A2) of Section 3, and demand in commodity 1 as

$$C_{\mathcal{I}_1 T} = \frac{\alpha_{1T}\,(a + bI_T)}{y_I p_{1T} \xi_T}. \tag{A24}$$

Here, $1/y_I$ solves (A1) evaluated at $t = 0$, and using the lognormal distribution property of D_T, D_{kT}, α_{1T}, is given by

$$\frac{1}{y_I} = \frac{\lambda\,\xi_0 S_0}{a(\sum_j \alpha_j + \alpha_1) + b(\sum_j \alpha_j + \alpha_1 e^{(\sigma\sigma_\alpha + \sigma_\alpha^2/L)T}) E\,[I_T]}, \tag{A25}$$

where henceforth the summation $\sum_j$ denotes the summation over all commodities but the first, i.e., $j = 0, 2, \ldots, K$, and $\alpha_1 \equiv \alpha_{10}$ denotes the initial value of the process α_{1t}. Similarly, we obtain the normal investor's optimal commodity demands at time T for $k = 0, 2, \ldots, K$ to be as previously in (A5)–(A6), and for commodity 1 as

$$C_{\mathcal{N}_1 T} = \frac{\alpha_{1T}\,(1 - \lambda)\,\xi_0 S_0}{(\sum_j \alpha_j + \alpha_1) p_{1T} \xi_T}. \tag{A26}$$

To determine the equilibrium state price density, we impose market clearing for the generic good, $C_{\mathcal{N}_0 T} + C_{\mathcal{I}_0 T}$, substitute (A4) and (A6), and rearrange to obtain at T

$$\xi_T = \bar{\xi} \frac{1}{D_T} \left(A + b\lambda I_T\right), \tag{A27}$$

where

$$\bar{\xi} = \frac{\alpha_0 \xi_0 S_0}{a(\sum_j \alpha_j + \alpha_1) + b(\sum_j \alpha_j + \alpha_1 e^{(\sigma\sigma_\alpha + \sigma_\alpha^2/L)T}) E\,[I_T]}, \tag{A28}$$

$$A = a + b\,(1 - \lambda)\,\frac{\sum_j \alpha_j + \alpha_1 e^{\left(\sigma\sigma_\alpha + \sigma_\alpha^2/L\right)T}}{\sum_j \alpha_j + \alpha_1} E\,[I_T]\,. \tag{A29}$$

To determine the equilibrium commodity prices at T, we impose the market clearing condition $C_{\mathcal{N}_k T} + C_{\mathcal{I}_k T} = D_{kT}$ for each commodity $k = 1, \ldots, K$, and substitute (A3), (A5), (A24) and (A26) and the equilibrium state price density (A27) to obtain the same commodity prices (11) as in Lemma 1 for $k = 2, \ldots, K$, and for commodity 1 we obtain

$$p_{1T} = \frac{\alpha_{1T}}{\alpha_0} \frac{D_T}{D_{1T}}. \tag{A30}$$

Substituting the equilibrium commodity prices (11), (A30), into the definition of the index (4) leads to the time-T equilibrium commodity index value

$$I_T = \frac{\alpha_{1T}^{1/L}}{\alpha_0 \alpha_1^{1/L}} D_T \prod_{i=1}^{L} \left(\alpha_i / D_{iT}\right)^{1/L}. \tag{A31}$$

Hence, making use of the lognormal property of D_T, D_{iT}, α_{1T} we deduce the unconditional expected index value to be

$$E\left[I_T\right] = e^{\left(\mu - \frac{1}{L}\left(\sigma\sigma_\alpha + \frac{1}{2}\left(\frac{1}{L}-1\right)\sigma_\alpha^2\right) - \frac{1}{L}\sum_{i=1}^{L}\left(\mu_i - \frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)\right)T} \frac{D_0}{\alpha_0}\prod_{i=1}^{L}\left(\frac{\alpha_i}{D_{i0}}\right)^{1/L}. \tag{A32}$$

We now determine the equilibrium futures prices. First, the equilibrium time-t state price density follows by taking the conditional expectation of (A27), substituting (A31), using the lognormality of D_T, D_{iT}, α_{1T}, and after some algebra we get

$$\xi_t = \bar{\xi} e^{\left(\sigma^2 - \mu\right)(T-t)} \frac{1}{D_t}\left(A + b\,\lambda e^{-\left(\sigma^2 + \sigma\sigma_\alpha/L\right)(T-t)} \alpha_{1t}^{1/L} D_t \prod_{i=1}^{L}\left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}\right), \tag{A33}$$

where $\bar{\xi}$ and A are as in (A28)–(A29), and

$$\hat{g}_i\left(t\right) = \frac{\alpha_i}{\alpha_0 \alpha_1^{1/L}} e^{\left(\mu + \frac{1}{L}\left(\sigma\sigma_\alpha + \frac{1}{2}\left(\frac{1}{L}-1\right)\sigma_\alpha^2\right) - \mu_i + \frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)(T-t)}. \tag{A34}$$

To compute the expected deflated futures payoff of commodity $k = 1, \ldots, K$, we substitute (A27), (A11) and (A30), and rearrange to obtain

$$E_t\left[\xi_T p_{kT}\right] = \frac{\bar{\xi}}{\alpha_0}\left(\alpha_k 1_{\{k>1\}} + \alpha_{1t} 1_{\{k=1\}}\right) \frac{e^{\left(\sigma_k^2 - \mu_k\right)(T-t)}}{D_{kt}}\left(A + b\lambda \frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]}\right). \tag{A35}$$

For nonindex futures contracts $k = L+1, \ldots, K$, using the lognormality of D_T, D_{kT}, α_{1T}, and substituting (A31) we obtain

$$\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]} = \alpha_{1t}^{1/L} D_t \prod_{i=1}^{L}\left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}, \tag{A36}$$

where $\hat{g}_i\left(t\right)$ is as in (A34). For index futures contracts except for the first commodity futures, $k = 2, \ldots, L$, we get

$$\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]} = e^{\frac{1}{L}\sigma_k^2(T-t)} \alpha_{1t}^{1/L} D_t \prod_{i=1}^{L}\left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}. \tag{A37}$$

Finally, for the first index futures contract $k = 1$, we deduce

$$\frac{E_t\left[I_T/D_{1T}\right]}{E_t\left[1/D_{1T}\right]} = e^{\left(\frac{1}{L}\left(\sigma_1^2 + \sigma_\alpha^2\right) + \sigma\sigma_\alpha\right)(T-t)} \alpha_{1t}^{1/L} D_t \prod_{i=1}^{L}\left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}. \tag{A38}$$

Substituting (A35)–(A38) and (A33) into (A9) and rearranging leads to the equilibrium index futures price expression reported in (26). The equilibrium futures price $\bar{f}_k$ in the benchmark economy with no institutions (28) follows by considering the special case of $a = 1$, $b = 0$ in (26). The property that the futures prices are higher than in the benchmark economy

follows by observing that the factor multiplying $\bar{f}_{kt}$ in expression (26) is strictly greater than one.

To derive the equilibrium volatility vector of loadings, we apply It's Lemma to the futures price expression (26), and follow similar steps to those in the proof of Proposition 2 to deduce (27) in Proposition 4, where

$$\hat{h}_{kt} = \frac{b\,\lambda\,A\left(1-e^{-\left(\sigma^2+\sigma\sigma_\alpha/L+1_{\{k\leq L\}}\sigma_k^2/L+1_{\{k=1\}}\left(\sigma\sigma_\alpha+\sigma_\alpha^2/L\right)\right)(T-t)}\right)e^{\left(1_{\{k\leq L\}}\sigma_k^2/L+1_{\{k=1\}}\left(\sigma\sigma_\alpha+\sigma_\alpha^2/L\right)\right)(T-t)}}{A+b\,\lambda e^{\left(1_{\{k\leq L\}}\sigma_k^2/L+1_{\{k=1\}}(\sigma\sigma_\alpha+\sigma_\alpha^2/L)\right)(T-t)}\alpha_{1t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}}$$

$$\times\frac{\alpha_{1t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}}{A+b\,\lambda e^{-(\sigma^2+\sigma\sigma_\alpha/L)(T-t)}\alpha_{1t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}}>0, \tag{A39}$$

where A and $\hat{g}_i(t)$ are as in (A29) and (A34), respectively. The property that volatilities of all futures price returns are higher than in the benchmark economy follows immediately from (27) since $h_{kt}>0$.

Finally, to determine the stock market level and volatility in equilibrium, we note that in equilibrium the stock market terminal value is given by

$$S_T=\frac{\sum_j\alpha_j+\alpha_{1T}}{\alpha_0}D_T,$$

where we have substituted (11) and (A24). Following similar steps in the determination of equilibrium futures prices above, we arrive at the following equilibrium stock market level and its associated vector of loadings in the presence of institutions and demand shocks:

$$S_t=\bar{S}_t\frac{A+b\,\lambda\frac{\sum_j\alpha_j+\alpha_{1t}e^{\left(\sigma\sigma_\alpha+\sigma_\alpha^2/L\right)(T-t)}}{\sum_j\alpha_j+\alpha_{1t}}\alpha_{1t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}}{A+b\,\lambda e^{-(\sigma^2+\sigma\sigma_\alpha/L)(T-t)}\alpha_{1t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}}, \tag{A40}$$

$$\sigma_{St}=\bar{\sigma}_{St}+\sigma_{Qt}-\sigma_{Yt}, \tag{A41}$$

where $\bar{S}_t$ and $\bar{\sigma}_S$ are the corresponding quantities in the benchmark economy with no institutions, given by

$$\bar{S}_t=\frac{\sum_j\alpha_j+\alpha_{1t}}{\alpha_0}e^{\left(\mu-\sigma^2\right)(T-t)}D_t, \tag{A42}$$

$$\bar{\sigma}_{St}=\left(\sigma+\frac{\alpha_{1t}}{\sum_j\alpha_j+\alpha_{1t}}\sigma_\alpha,0,\ldots,0\right), \tag{A43}$$

A and $\hat{g}_i(t)$ are as in (A29) and (A34), respectively, and

$$\sigma_{Qt} = \frac{b\,\lambda\,Q_t}{A + b\,\lambda\,Q_t}\left(\sigma + \left(\frac{1}{L} + h_{\alpha t}\right)\sigma_\alpha, -\frac{1}{L}\sigma_1, \ldots, -\frac{1}{L}\sigma_L, 0, \ldots, 0\right),$$

$$\sigma_{Yt} = \frac{b\,\lambda e^{-\left(\sigma^2+\sigma\sigma_\alpha/L\right)(T-t)}\alpha_{1t}^{1/L} D_t \prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}}{A + b\,\lambda e^{-(\sigma^2+\sigma\sigma_\alpha/L)(T-t)}\alpha_{1t}^{1/L} D_t \prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}}\sigma_{It},$$

$$Q_t = \frac{\sum_j \alpha_j + \alpha_{1t} e^{\left(\sigma\sigma_\alpha+\sigma_\alpha^2/L\right)(T-t)}}{\sum_j \alpha_j + \alpha_{1t}}\alpha_{1t}^{1/L} D_t \prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L},$$

$$h_{\alpha t} = \frac{\alpha_{1t}\sum_j \alpha_j \left(e^{\left(\sigma\sigma_\alpha+\sigma_\alpha^2/L\right)(T-t)} - 1\right)}{\left(\sum_j \alpha_j + \alpha_{1t}\right)\left(\sum_j \alpha_j + \alpha_{1t} e^{(\sigma\sigma_\alpha+\sigma_\alpha^2/L)(T-t)}\right)} > 0$$

with σ_{It} being as in Proposition 4.

Q.E.D.

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