# A Model of Financialization of Commodities*

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**Abstract**

A sharp increase in the popularity of commodity investing in the past decade has triggered an unprecedented inflow of institutional funds into commodity futures markets. Such financialization of commodities coincided with significant booms and busts in commodity markets, raising concerns of policymakers. In this paper, we explore the effects of financialization in a model that features institutional investors alongside traditional futures markets participants. The institutional investors care about their performance relative to a commodity index. We find that if a commodity futures is included in the index, supply and demand shocks specific to that commodity spill over to all other commodity futures markets. In contrast, supply and demand shocks to a nonindex commodity affect just that commodity market alone. Moreover, prices and volatilities of all commodity futures go up, but more so for the index futures than for nonindex ones. Furthermore, financialization leads to an increase in correlations amongst commodity futures as well as in equity-commodity correlations. Consistent with empirical evidence, the increases in the correlations between index commodities exceed those for nonindex ones. We model explicitly demand shocks which allows us to disentangle the effects of financialization from the effects of rising demand for commodities, and find that in the presence of demand shocks the impact of institutions on futures prices becomes considerably stronger.

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# 1. Introduction

In the past decade the behavior of commodity prices has become highly abnormal. Unprecedented booms have been followed by busts, volatilities have spiked, and especially so during the 2007-2008 financial crisis. An emerging literature on financialization of commodities attributes this behavior to the emergence of commodities as an asset class, which has become widely held by institutional investors seeking diversification benefits (Singleton (2012), Tang and Xiong (2012)). Starting in 2004, institutional investors have been rapidly building their positions in commodity futures. CFTC staff report (2008) estimates institutional holdings to have increased from $15 billion in 2003 to over $200 billion in 2008. Many of the institutional investors hold commodities through a commodity futures index, such as the Goldman Sachs Commodity Index (GSCI), the Dow Jones UBS Commodity Index (DJ-UBS) or the S&P Commodity Index (SPCI). Tang and Xiong document that, interestingly, after 2004 the behavior of index commodities has become increasingly different from those of nonindex, with the former becoming more correlated with oil, an important index constituent, and more correlated with the equity market. Since institutional investors tend to trade in and out of equities and (index) commodities at the same time, their increased presence in the commodity futures markets could explain these effects. The financialization theory has far-reaching implication for regulation: the 2004-2008 boom in commodity prices has prompted many calls for curtailing positions of institutions whose trades may have generated the boom (see, e.g., Masters' (2008) testimony).

While the empirical literature on financialization of commodities has been influential and aided the discussion amongst policymakers, theoretical literature on the subject remains scarce. Our goal in this paper is to model the financialization of commodities and to disentangle the effects of institutional flows from the traditional demand and supply effects on commodity futures prices. We particularly focus on identifying the economic mechanisms through which institutions may influence commodity futures prices, volatilities, and their comovement.

We develop a multi-good, multi-asset dynamic model with institutional investors and standard futures markets participants. The institutional investors care about their perfor-

mance relative to a commodity index. They do so because their investment mandate specifies a benchmark index for performance evaluation or because their mandate includes hedging against commodity price inflation. We capture such benchmarking through the institutional objective function. Consistent with the extant literature on benchmarking (originating from Brennan (1993)), we postulate that the marginal utility of institutional investors increases with the index. In particular, institutional investors dislike to perform poorly when their benchmark index does well and so have an additional incentive to do well when their benchmark does well. Both classes of investors in our model invest in the commodity futures markets and the stock market. Prices in these markets fluctuate in response to three possible sources of shocks: (i) supply shocks, (ii) demand shocks, and (iii) fluctuations in assets under management of institutional investors. The latter source of risk captures the effects of financialization of commodity markets. To explore the differences between index and nonindex commodity futures, we include in the index only a subset of the traded futures contracts. We can then compare a pair of otherwise identical commodities, one of which belongs to the index and the other does not. By comparing our economy to an otherwise identical benchmark economy with no institutions, we are able to disentangle the effects of financialization from the effects of, say, reduced supply or rising demand for commodities. The model is solved in closed form, and all of our results below are derived analytically.

We first uncover that membership in the index creates a distinct spillover mechanism, arising due to the presence of institutions. Namely, supply and demand shocks that are specific to an index commodity get transmitted to all other index commodity futures, including nonindex ones. Since the marginal utility of institutions depends on the index value, so does the (common) discount factor in the economy. Through their effect on the index, shocks that are specific to index commodities affect the discount factor. Consequently, all assets in the economy are impacted by shocks to index commodities and the characteristics of index commodities. In contrast, the supply and demand shocks to a nonindex commodity affect just that commodity market alone. This spillover mechanism is key to our findings.

We find that the prices of all commodity futures go up with financialization. However, the price rise is higher for futures belonging to the index than for nonindex ones. This

happens because institutions care about the index. Since their marginal utility is increasing in the index level, they value assets that pay off more in the states when the index does well. Hence, relative to the benchmark economy without institutions, index futures are valued higher than nonindex. The larger the institutions, the more they distort pricing—or, more formally, the discount factor—making the above effects stronger.

The volatilities of both index and nonindex futures returns go up with financialization. The primary reason for this is that, absent institutions, there are only two sources of risk: supply and demand risks. With institutions present, some agents in the economy (institutional investors) face an additional risk of falling behind the index. This risk is reflected in the futures prices and it raises the volatilities of futures returns. While the volatilities of both index and nonindex futures rise, they do not, however, rise by the same magnitude. Institutions bid up prices and volatilities of index futures more than nonindex because index futures, by construction, pay off more when the index does well. The prices and volatilities of index futures become high enough to make them unattractive to the normal investors (standard market participants) so that they are willing to sell them to the institutions. Similarly, the institutional investors bid up the stock market value and volatility. This happens because the stock market payoff is positively correlated with that of the commodity price index, making the stock a good investment instrument for the institutions.

Furthermore, we find that financialization leads to an increase in the correlations amongst commodity futures as well as in the equity-commodity correlations. The frequently cited intuition for why the correlations should rise is that commodity futures markets have been largely segmented before the inflow of institutional investors in mid-2000s, and institutions who have entered these markets have linked them together, as well as with the stock market, through cross-holdings in their portfolios. We show that the argument does not need to reply on market segmentation. In our model the rise in the correlations occurs even under complete markets. Benchmarking institutional investors to a commodity index leads to the emergence of this index as a new (common) factor in commodity futures and stock returns, again due to the aforementioned spillover mechanism. In equilibrium, all assets load positively on this factor, which increases their covariances and their correlations. We

show that index commodity futures are more sensitive to this new factor, and so their covariances and correlations with each other rise more than those for otherwise identical nonindex commodities. A similar result also holds for equity-commodity correlations: the ones for index commodity futures rise by more than those for nonindex.

Finally, we seek to disentangle the effects of financialization from the effects of rising demand for commodities. For expositional simplicity, we consider demand shocks affecting one commodity only. We model the demand shocks so that the demand for that commodity is increasing in aggregate output (as in the model of oil prices of Dvir and Rogoff (2009)). In that setting, we uncover additionally that financialization increases sizeably both futures prices and volatilities, beyond the levels we find in the economy without demand shocks. In the presence of demand shocks, the index becomes more volatile and so the institutional investors' incentive to not fall behind the index strengthens further. In this setting, we find support for the view advocated in Kilian and Murphy (2012) that demand shocks are important in explaining commodity prices, but we also stress that financialization contributes sizably over and above the effects of rising demand. This contribution is significant not only for the commodity affected by demand shocks but also for commodities not affected by demand shocks, because demand shocks to an index commodity spill over to all other commodities in the presence of institutions.

Methodologically, this paper contributes to the asset pricing literature by providing a tractable multi-asset general equilibrium model with heterogeneous investors which is solved in closed form. While there is clearly a need for multi-asset models (e.g., to provide cross-sectional predictions for empirical asset pricing), such models have been notoriously difficult to solve analytically. Pavlova and Rigobon (2007) and Cochrane, Longstaff, and Santa-Clara (2008) discuss the complexities of such models and provide analytical solutions for the two-asset case. As Martin (2013) demonstrates, the general multi-asset case presents a formidable challenge. In contrast, our multi-asset model is surprisingly simple to solve. Our innovation is to replace Lucas trees considered in the above literature by zero-net-supply assets (futures) and model only the aggregate stock market as a Lucas tree. The model then becomes just as simple and tractable as a single-tree model.

## 1.1. Related Literature

This paper is related to several strands of literature. The two papers that have inspired this work are Singleton (2012) and Tang and Xiong (2012). Singleton examines the 2008 boom/bust in oil prices and argues that flows from institutional investors have contributed significantly to that boom/bust. Tang and Xiong document that the comovement between oil and other commodities has risen dramatically following the inflow of institutional investors starting from 2004, and that the commodities belonging to popular indices have been affected disproportionately more. There was no difference in comovement patterns of index and nonindex commodities pre-2004. Using a proprietary dataset from the CFTC, Buyuksahin and Robe (2010) investigate the recent increase in the correlation between equity indices and commodities and argue that this phenomenon is due to the presence of hedge funds that are active in both equity and commodity futures markets. Recently, Henderson, Pearson and Wang (2012) present new evidence on the financialization of commodity futures markets based on commodity-linked notes.

The impact of financialization on commodity futures and spot prices is the subject of much ongoing debate in the literature. Surveys of academic literature by Irwin and Sanders (2011) and Fattouh, Kilian, and Mahadeva (2012) reach a conclusion that while the participation of institutions in commodity futures markets has clearly increased significantly, it is not clear that this has affected commodity futures and spot prices. Kilian and Murphy (2012) attribute the 2003-2008 oil price surge to demand shocks and argue against speculative trading as an alternative explanation for this price surge. Hamilton and Wu (2012) examine whether purchases by index funds had an effect on commodity futures prices and reach a mostly negative conclusion.

While there is still lack of agreement on whether trades by institutional investors affect futures prices, it is reasonably well-established that such trades affect stock prices. Starting from Harris and Gurel (1986) and Shleifer (1986), a large body of work documented that prices of stocks that are added to the S&P 500 and other indices increase following the announcement and prices of stocks that are deleted drop—a phenomenon widely attributed to the price pressure from institutional investors. Relatedly, a variety of studies document

the so-called "asset class" effects: the "excessive" comovement of assets belonging to the same index or other visible category of stocks (e.g., Barberis, Shleifer, and Wurgler (2005) for the S&P500 vis-à-vis non-S&P500 stocks, Boyer (2011) for BARRA value and growth indices). These effects are attributed to the presence of institutional investors.

The closest theoretical work on the effects of institutions on asset prices and their dynamics is the one-good Lucas-tree economy of Basak and Pavlova (2012). Basak and Pavlova focus on index and asset class effects in the stock market. Our focus is on disentangling the effects of demand and supply shocks from those of financialization of commodity futures markets. Moreover, our key spillover mechanism is missing in their work; namely, the mechanism through which supply and demand shocks to cash flows of index assets get transmitted to all other assets, including the nonindex. Consequently, in contrast to our findings, their nonindex assets have zero correlation among themselves and with index assets, and the nonindex asset prices and volatilities are not affected by institutional investors. Finally, our work is able to offer an alternative way of formulating a multi-asset setting, which admits much tractability. Another related theoretical study of an asset-class effect is by Barberis and Shleifer (2003), whose explanation for this phenomenon is behavioral. By providing microfoundations for investors' demand schedules, we can establish a tighter link between the economic primitives and futures prices and their dynamics in our model.

Finally, there is a large and diverse literature going back to Keynes (1923) that studies the determination of commodity spot prices in production economies with storage and links the physical markets for commodities with the commodity futures markets.[1] We view our work as being complementary to this literature because in our work we simplify the physical markets for commodities and focus on the spillovers between the commodity futures markets for in a multi-commodity setting and the effects of index inclusion.

The remainder of the paper is organized as follows. Section 2 presents our model. Sec-

[1]In this strand of literature, a recent paper by Sockin and Xiong (2012) shows that price pressure from investors operating in futures markets (even if driven by nonfundamental factors) can be transmitted to spot prices of underlying commodities. Routledge, Seppi, and Spatt (2000) derive the term structure of forward prices for storable commodities, highlighting the importance of the non-negativity constraints on inventories. Acharya, Lochstoer, and Ramadorai (2012) stress the importance of capital constraints of futures' markets speculators and argue that frictions in financial (futures) markets can feedback into production decisions in the physical market.

tion 3 presents our main results on how institutional investors affect commodity futures prices, volatilities, and their comovement. Section 4 extends our framework to incorporate demand shocks. Section 5 concludes and the Appendix provides all proofs.

# 2. The Model

Our goal in this section is to develop a simple and tractable model of commodity futures markets in which prices fluctuate in response to three possible sources of shocks: (i) supply shocks, (ii) demand shocks, and (iii) fluctuations in assets under management of institutional investors. The former two sources of risk have been studied extensively in the literature. The third source of risk is new and it captures the effects of financialization of commodity markets. Having a theoretical model allows us to disentangle the effects of each of these three sources of risk on commodity prices and their dynamics.

We consider a pure-exchange multi-good, multi-asset economy with a finite horizon $T$. Uncertainty is resolved continuously, driven by a $K+1$-dimensional standard Brownian motion $\omega \equiv (\omega_0, \ldots, \omega_K)^\top$. All consumption in the model occurs at the terminal date $T$, while trading takes place at all times $t \in [0, T]$.

*Commodities.* There are $K$ commodities (goods), indexed by $k = 1, \ldots K$. The date-$T$ supply of commodity $k$, $D_{kT}$, is the terminal value of the process $D_{kt}$, with dynamics

$$dD_{kt} = D_{kt}[\mu_k dt + \sigma_k d\omega_{kt}], \tag{1}$$

where $\mu_k$ and $\sigma_k > 0$ are constant. The process $D_{kt}$ represents the arrival of news about $D_{kT}$. We refer to it as the commodity-$k$ *supply news*. The price of good $k$ at time $t$ is denoted by $p_{tk}$. There is one further good in the economy, commodity 0, which we refer to as the generic good. This good subsumes all remaining goods consumed in the economy apart from the $K$ commodities that we have explicitly specified above and it serves as the numeraire. The date-$T$ supply of the generic good is $D_T$, which is the terminal value of the supply news process

$$dD_t = D_t[\mu dt + \sigma d\omega_{0t}], \tag{2}$$

where $\mu$ and $\sigma > 0$ are constant. Our specification implies that the supply news processes are uncorrelated across commodities ($dD_{kt}\, dD_{it} = 0$, $dD_{kt}\, dD_t = 0$, $\forall k$, $k \neq i$). This assumption is for expositional simplicity; it can be relaxed in future work.

*Financial Markets.* Available for trading are $K$ standard futures contracts written on commodities $k = 1, \ldots, K$. A futures contract on commodity $k$ matures at time $T$ and delivers one unit of commodity $k$. The contract payoff at maturity is therefore $p_{kT}$. Each contract is continuously resettled at the futures price $f_{kt}$ and is in zero net supply. The gains/losses on each contract are posited to follow

$$df_{kt} = f_{kt}[\mu_{f_{kt}} dt + \sigma_{f_{kt}} d\omega_t], \tag{3}$$

where $\mu_{f_{kt}}$ and the $K + 1$ vector of volatility components $\sigma_{f_{kt}}$ are determined endogenously in equilibrium (Section 3).

Our model makes a distinction between index and nonindex commodities because we seek to examine theoretically the asset class effect in commodity futures documented by Tang and Xiong (2012). A *commodity index* includes the first $L$ commodities, $L \leq K$, and is defined as

$$I_t = \prod_{i=1}^{L} f_{it}^{1/L}. \tag{4}$$

This index represents a geometrically-weighted commodity index such as, for example, the S&P Commodity Index (SPCI). For expositional simplicity, our index weighs all commodities equally; this assumption is easy to relax.[2]

In addition to the futures markets, investors can trade in the stock market, $S$, and an instantaneously riskless bond. The stock market is a claim to the entire output of the economy at time $T$: $D_T + \sum_{k=1}^{K} p_{kT} D_{kT}$. It is in positive supply of one share and is posited to have price dynamics given by

$$dS_t = S_t[\mu_{St} dt + \sigma_{St} d\omega_t], \tag{5}$$

[2]To model other major commodity indices such as the Goldman Sachs Commodity Index and the Dow Jones UBS Commodity Index, it is more appropriate to define the index as $I_t = \frac{1}{L} \sum_{i=1}^{L} f_{it}$. Such specification is less tractable but one can show numerically that most of the implications are in line with those in our analysis below.

with $\mu_{St}$ and $\sigma_{St} > 0$ endogenously determined in equilibrium. The bond in zero net supply. It pays a riskless interest rate $r$, which we set to zero without loss of generality.[3]

We note that our formulation of asset cash flows is standard in the asset pricing literature. The main distinguishing characteristic of our model is that it avoids the complexities of multi-tree economies. This is because only the stock market is in positive net supply, while all other assets (futures) are in zero net supply. As we demonstrate in the ensuing analysis, this model is just as simple and tractable as a single-tree model.

*Investors.* The economy is populated by two types of market participants: normal investors, $\mathcal{N}$, and institutional investors, $\mathcal{I}$. The (representative) normal investor is a standard market participant, with logarithmic preferences over the terminal value of her portfolio:

$$u_{\mathcal{N}}(W_{\mathcal{N}T}) = \log(W_{\mathcal{N}T}), \tag{6}$$

where $W_{\mathcal{N}T}$ is (real) wealth or real consumption.

The institutional investor's objective function, defined over his terminal portfolio value (real consumption) $W_{\mathcal{I}T}$, is given by

$$u_{\mathcal{I}}(W_{\mathcal{I}T}) = (a + bI_T)\log(W_{\mathcal{I}T}), \tag{7}$$

where $a$, $b > 0$. The institutional investor is modeled along the lines of Basak and Pavlova (2012), who study institutional investors in the stock market and also provide microfoundations for such an objective function, as well as a status-based interpretation.[4] The objective function has two key properties: (i) it depends on the index level $I_T$ and (ii) the marginal utility of wealth is increasing in the benchmark index level $I_T$. This captures the notion of benchmarking: the institutional investor is evaluated relative to his benchmark index and so he cares about the performance of the index. When the benchmark index is relatively high, the investor strives to catch up and so he values his marginal unit of performance

[3]This is a standard feature of models that do not have intermediate consumption. In other words, there is no intertemporal choice that would pin down the interest rate. Our normalization is commonly employed in models with no intermediate consumption (see e.g., Pastor and Veronesi (2012) for a recent reference).

[4]Direct empirical support for the status-based interpretation of our model is provided in Hong, Jiang, and Zhao (2011), who adopt the formulation in (7) in their analysis. Empirical work estimating objectives of institutional investors remains scarce, with a notable exception of Koijen (2012).

highly (his marginal utility of wealth is high). When the index is relatively low, the investor is less concerned about his performance (his marginal utility of wealth is low). We use the commodity market index as the benchmark index because in this work we attempt to capture institutional investors with the mandate to invest in commodities, most of whom are evaluated relative to a commodity index. An alternative interpretation of the objective function is that the institutional investor has a mandate to hedge commodity price inflation; i.e., deliver higher returns in states in which the commodity price index is high.[5]

In this multi-good world, (real) terminal wealth is defined as an aggregate over all goods, a consumption index (or real consumption). We take the index to be Cobb-Douglas, i.e.,

$$W_n = C_{n_0}^{\alpha_0} C_{n_1}^{\alpha_1} \cdot \ldots \cdot C_{n_K}^{\alpha_K}, \qquad n \in \{\mathcal{N}, \mathcal{I}\}, \tag{8}$$

where $\alpha_k > 0$ for all $k$. For the case of $\sum_{k=0}^{K} \alpha_k = 1$, the parameter $\alpha_k$ represents the expenditure share on good $k$, the fraction of wealth optimally demanded in good $k$. Here we are considering a general Cobb-Douglas aggregator in which the weights do not necessarily add up to one, and hence we label $\alpha_k$ as the "commodity demand parameter."[6] We take the commodity demand parameters to be the same for all investors in the economy. Heterogeneity in demand for specific commodities is not the dimension we would like to focus on in this paper.

A change in $\alpha_k$ represents a demand shift towards commodity $k$. A change in the demand parameter $\alpha_k$ is the simplest and most direct way of modeling a demand shift, i.e., an outward movement in the entire demand schedule, as typical in classical demand theory (Varian (1992)).[7] In Section 4, we allow the demand parameters $\alpha_k$ to be stochastic, in order to capture a more realistic environment with demand shocks. Until then, we keep them constant so as to isolate the effects of supply shocks and the effects of financialization (fluctuations in institutional wealth invested in the market) on commodity futures prices.

[5] Although the institutions are modeled similarly, our focus is different and our model generates a number of new insights, absent in Basak and Pavlova (Remark 1, Section 3).

[6] In what follows, we are interested in comparative statics with respect to $\alpha_k$. The expenditure share on commodity $k$, $\alpha_k / \sum_{k=0}^{K} \alpha_k$, is monotonically increasing in $\alpha_k$. Hence all our comparative statics for $\alpha_k$ are equally valid for expenditure shares $\alpha_k / \sum_{k=0}^{K} \alpha_k$.

[7] For example, an increase in demand for soya beans due to the invention of biofuels and concerns about the environment.

The institutional and normal investors are initially endowed with fractions $\lambda \in [0, 1]$ and $(1 - \lambda)$ of the stock market, providing them with initial assets worth $W_{\mathcal{I}0} = \lambda S_0$ and $W_{\mathcal{R}0} = (1-\lambda)S_0$, respectively.[8] The parameter $\lambda$ thus represents the (initial) fraction of the institutional investors in the economy, and we will often refer to it as the size of institutions.

Starting with initial wealth $W_{n0}$, each type of investor $n = \mathcal{N}, \mathcal{I}$, dynamically chooses a portfolio process $\phi_n = (\phi_{n_1}, \ldots, \phi_{n_k})^\top$, where $\phi_n$ and $\phi_{n_S}$ denote the fractions of the portfolio invested in the futures contracts 1 through $K$ and the stock market, respectively. The wealth process of investor $n$, $W_n$, then follows the dynamics

$$dW_{nt} = W_{nt} \sum_{k=1}^{K} \phi_{n_k t}[\mu_{f_k t} dt + \sigma_{f_k t} d\omega_t] + W_{nt}\phi_{n_S t}[\mu_{St} dt + \sigma_{St} d\omega_t]. \tag{9}$$

# 3. Equilibrium Effects of Financialization of Commodities

We are now ready to explore how the financialization of commodities affects equilibrium prices, volatilities, and correlations. In order to understand the effects of financialization, we will often make comparisons with equilibrium in a *benchmark economy*, in which there are no institutional investors. We can specify such an economy by setting $b = 0$ in (7), in which case the institution in our model no longer resembles a commodity index trader and behaves just like the normal investor. Another way to capture the benchmark economy within our model is to set the fraction of institutions, $\lambda$, to zero.

Equilibrium in our economy is defined in a standard way: equilibrium portfolios, asset and time-$T$ commodity prices are such that (i) both the normal and institutional investors choose their optimal portfolios, and (ii) futures, stock, bond and time-$T$ commodity markets clear. Letting $M_{t,T}$ to denote the (stochastic) discount factor or the pricing kernel in our model, by no-arbitrage, the futures prices are given by

$$f_{kt} = E_t[M_{t,T}\, p_{kT}]. \tag{10}$$

[8]The initial endowment of institutions comes from households (that are not explicitly modeled here), who delegate their assets to institutions to manage. Such households could be, for example, participants in defined benefit pension plans.

The discount factor $M_{t,T}$ is the marginal rate of substitution of any investor, e.g., the normal investor, in equilibrium.

To develop intuitions for our results, it is useful to examine the time-$T$ prices prevailing in our equilibrium. These are reported in the following lemma.

**Lemma 1 (Time-$T$ equilibrium quantities).** *In equilibrium with institutional investors, we obtain the following characterizations for the terminal date quantities.*

$$\textit{Commodity prices:}\quad p_{kT} = \frac{\alpha_k}{\alpha_0}\frac{D_T}{D_{kT}};\quad p_{kT} = \overline{p}_{kT}, \tag{11}$$

$$\textit{Commodity index:}\quad I_T = \frac{D_T}{\alpha_0}\prod_{i=1}^{L}\left(\frac{\alpha_i}{D_{iT}}\right)^{1/L};\quad I_T = \overline{I}_T, \tag{12}$$

$$\textit{Stock market value:}\quad S_T = D_T\sum_{k=0}^{K}\frac{\alpha_k}{\alpha_0};\quad S_T = \overline{S}_T, \tag{13}$$

$$\textit{Discount factor:}\quad M_{0,T} = \overline{M}_{0,T}\left(1+\frac{b\,\lambda(I_T - E[I_T])}{a + b\,E[I_T]}\right),\quad \overline{M}_{0,T} = \frac{e^{(\mu-\sigma^2)T}D_0}{D_T}, \tag{14}$$

*where the expectation of the time-$T$ index value, $E[I_T]$, is provided in the Appendix. The quantities with an upper bar denote the corresponding equilibrium quantities prevailing in the economy with no institutions.*

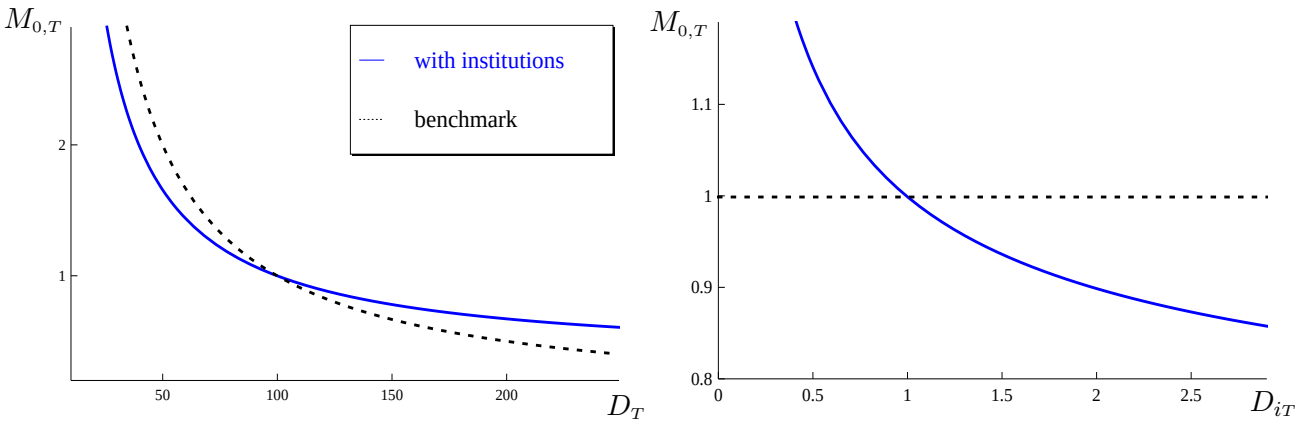
Lemma 1 reveals that the price of good $k$ decreases with the supply of that good $D_{kT}$. As supply $D_{kT}$ increases, good $k$ becomes relatively more abundant. Hence, its price falls. A rise in the supply of the generic good $D_T$ has the opposite effect. Now good $k$ becomes more scarce relative to the generic good. Hence, its price rises. These are classical supply-side effects. These mechanisms are well explored in commodity markets and they are standard in multi-good models. A positive shift in $\alpha_k$ represents an increase in demand for good $k$. As a consequence, the price of good $k$ goes up. This is a classical demand-side effect.

Since the index is given by $I_T = \prod_{i=1}^{L} p_{iT}^{1/L}$, the terminal index value inherits the properties of the individual commodity prices. In particular, it declines when the supply of any index commodity $i$ $D_{iT}$ goes up, and rises when the supply of the generic good $D_T$ rises.

It is important to note that the time-$T$ prices of commodities, and hence the commodity index coincide with their values in the benchmark economy with no institutions. We have

intentionally set up our model in this way. By effectively abstracting away from the effects of financialization on underlying cash flows in (10), we are able to elucidate the effects of institutions in the futures markets coming via the discount factor channel.

The stock market is a claim against the aggregate output of all goods in the economy, $D_T + \sum_{k=1}^{K} p_{kT} D_{kT}$, which in this model turns out to be proportional to the aggregate supply of the generic good $D_T$. So the aggregate wealth in the economy, the stock market value $S_T$, in equilibrium is simply a scaled supply of the generic good $D_T$. The quantity $D$ is an important state variable in our model. In what follows, we will refer to it as (scaled) aggregate wealth, or, equivalently, (scaled) aggregate output.




**(a)** Effect of aggregate output $D_T$ **(b)** Effect of index commodity supply $D_{iT}$

Figure 1: **Discount factor.** This figure plots the discount factor in the presence of institutions against aggregate output $D_T$ and against an index commodity supply $D_{iT}$. The dotted lines correspond to the discount factor in the benchmark economy with no institutions. The plots are typical. The parameter values (when fixed) are: $L = 2$, $K = 4$, $a = 1$, $b = 1$, $T = 5$, $\lambda = 0.4$, $\alpha_0 = 0.7$, $D_T = D_0 = 100$, $D_{kT} = D_{k0} = 1$, $\mu = \mu_k = 0.01$, $\sigma = \sigma_k = 0.15$, $\alpha_k = 0.075$, $k = 1, \ldots K$.

In the benchmark economy, the discount factor depends only on aggregate output $D_T$. It bears the familiar inverse relationship with aggregate output (dotted line in Figure 1a), implying that assets with high payoffs in low-$D_T$ (bad) states get valued higher. In the presence of institutions, the discount factor is also decreasing in aggregate output $D_T$, albeit at a slower rate. That is, the presence of institutions makes the discount factor less sensitive

to news about aggregate output. Additionally, now the discount factor becomes dependent on the supply of each index commodity $D_{iT}$ (Figure 1b). The channel through which institutions affect the discount factor is apparent from equation (14): the discount factor now becomes dependent on the performance of the index, pricing high-index states higher. This is the channel through which financialization affects asset prices in our model.

The new financialization channel works as follows. Institutional investors have an additional incentive to do well when the index does well. So relative to normal investors, they strive to align their performance with that of the index, performing better when the index does well in exchange for performing poorer when the index does poorly. This is optimal from their viewpoint because their marginal utility is increasing with the level of the index. As highlighted in our discussion of the equilibrium index value in (12), the index does well when the aggregate output $D_T$ is high and supply of index commodity $D_{iT}$ is low. Because of the additional demand from institutions, these states become more "expensive" relative to the benchmark economy (higher Arrow-Debreu state prices or higher discount factor $M_{0,T}$). The financialization channel thus counteracts the benchmark economy inverse relation between the discount factor $M_{0,T}$ and aggregate output, making the discount factor less sensitive to aggregate output $D_T$ (as evident from Figure 1a). Additionally, it also makes the discount factor dependent and decreasing in each index commodity supply $D_{iT}$.

The graphs in Figure 1 are important because they underscore the mechanism for the valuation of assets in the presence of institutions. In particular, assets that pay off high in states in which the index does well (high $D_T$ and low $D_{iT}$) are valued higher than in the benchmark economy with no institutions.

## 3.1. Equilibrium Commodity Futures Prices

**Proposition 1 (Futures prices).** *In the economy with institutions, the equilibrium futures price of commodity $k = 1, \ldots, K$ is given by*

$$f_{kt} = \overline{f}_{kt} \frac{a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{1_{\{k \le L\}} \sigma_k^2 (T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2 (T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}, \tag{15}$$

*where the equilibrium futures price in the benchmark economy with no institutions* $\overline{f}_{kt}$ *and the quantity* $g_i(t)$ *are given by*

$$\overline{f}_{kt} = \frac{\alpha_k}{\alpha_0} e^{(\mu - \mu_k - \sigma^2 + \sigma_k^2)(T-t)} \frac{D_t}{D_{kt}}, \quad g_i(t) = \frac{\alpha_i}{\alpha_0} e^{(\mu - \mu_i + (1/L+1)\sigma_i^2/2)(T-t)} \tag{16}$$

*Consequently, in the presence of institutions,*

*(i) The futures prices are higher than in the benchmark economy,* $f_{kt} > \overline{f}_{kt}$, $k = 1, \ldots, K$.

*(ii) The index futures prices rise more than nonindex ones for otherwise identical commodities, i.e., for commodities* $i$ *and* $k$ *with* $D_{it} = D_{kt}$, $\forall t$, $\alpha_i = \alpha_k$, $i \leq L$, $L < k \leq K$.

Proposition 1 reveals that the commodity futures prices in the benchmark economy with no institutions $\overline{f}_{kt}$ inherit the features of time-$T$ futures prices highlighted in Lemma 1. The benchmark economy futures prices rise in response to positive news about aggregate output $D_t$ and fall in response to positive news about the supply of commodity $k$, $D_{kt}$. In contrast, in the economy with institutions the commodity futures prices $f_{kt}$ depend not only on own supply news $D_{kt}$ but also those of all *index* commodities $D_{it}$. Other characteristics of index commodities such as expected growth in their supply $\mu_i$, volatility $\sigma_i$ and their demand parameters $\alpha_i$ now also affect the prices of all futures traded in the market. Note that, just like in the benchmark economy, supply news $D_k$ and other characteristics of *nonindex* commodities have no spillover effects on other commodity futures.

To understand the price rises due to institutions, let us think about the valuation of assets in the context of two standard channels: the discount factor channel and the cashflow channel (see equation (10)). In this model, the futures cashflows are not affected by the presence of institutions (Lemma 1), and so the only channel through which futures prices are affected is the discount factor channel. From the discussion of Figure 1, futures that pay off high when the index does well, or equivalently when the aggregate output $D_t$ is high and/or an index commodity supply $D_{it}$ is low, command higher prices. The cashflows of all futures are increasing in aggregate output $D_t$ (equation (11)). Therefore, all futures prices are higher in the presence of institutions (as reported in property (i) of Proposition 1). Additionally, index commodity futures (but not nonindex) pay off high when an index commodity is scarce. This compounds the aggregate output effect and implies that index commodity

futures prices are higher that those of otherwise identical nonindex commodity futures (as reported in property (ii) of Proposition 1). In other words, we can describe property (ii) as follows. In the economy with institutions, some agents care about the index. Since their marginal utility is increasing in the index level, they value assets that pay off more in the states when the index does well. Hence commodity futures that are members of the index have higher prices than the nonmembers.

**Remark 1 (Difference from Basak and Pavlova (2012)).** One major difference of this work from that of the one-good stock market economy of Basak and Pavlova is that in that analysis nonindex security prices are unaffected by the presence of institutions, although the institutions are modeled similarly. In that model, cashflows of nonindex securities are exogenous and they are uncorrelated with the index. Here, nonindex cashflows, which are endogenously determined commodity prices, end up being correlated with the index. Tang and Xiong (2012) provide evidence that the financialization of commodities since 2004 has affected not only index commodities futures prices, volatilities and correlations, but also those of nonindex commodities. Unlike that of Basak and Pavlova, our model here is able to shed light on these important spillover effects from index commodities to nonindex ones.

**Corollary 1.** *The equilibrium commodity futures prices have the following additional properties.*

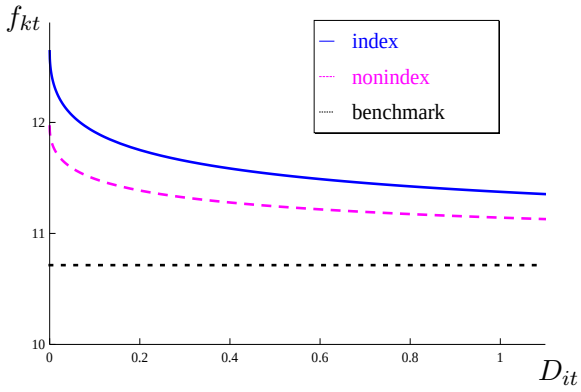
*(i) All commodity futures prices $f_{kt}$ are increasing in the size of institutions $\lambda$, $k = 1, \ldots, K$.*

*(ii) All commodity futures prices are more sensitive to aggregate output $D_t$ than in the benchmark economy with no institutions; i.e., $f_{kt}$ is increasing in $D_t$ at a faster rate than does $\overline{f}_{kt}$, $k = 1, \ldots, K$. Moreover, index commodity futures are more sensitive to aggregate output that nonindex ones for otherwise identical commodities.*

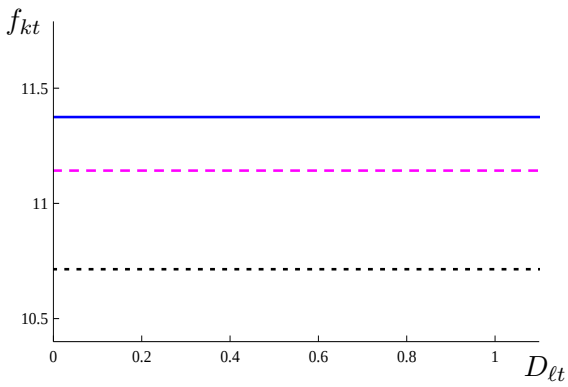
*(iii) All commodity futures prices $f_{kt}$, $k = 1, \ldots, K$, react negatively to positive supply news of index commodities $D_{it}$, $i = 1, \ldots, L$, $k \neq i$, while in the benchmark economy such a price $\overline{f}_{kt}$ is independent of $D_{it}$. All prices $f_{kt}$, $k = 1, \ldots, K$, remain independent of nonindex commodities supply news $D_{\ell t}$, unless $k = \ell$.*

*(iv) All commodity futures prices $f_{kt}$, $k = 1, \ldots, K$, react positively to a positive demand shift towards any index commodity $\alpha_i$, $i = 1, \ldots, L$, $k \neq i$, while $\overline{f}_{kt}$ is independent of $\alpha_i$. All prices $f_{kt}$, $k = 1, \ldots, K$, remain independent of nonindex commodities supply shifts $\alpha_\ell$, $\ell \neq k$.*

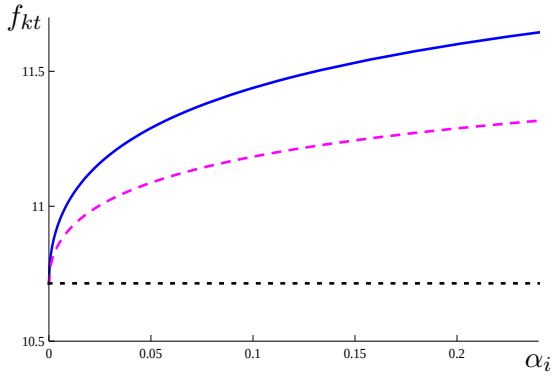
Figure 2 illustrates the results of the corollary. To elucidate the intuitions, we start from properties (iii) and (iv) of the corollary. Panel (a) shows that, unlike in the benchmark



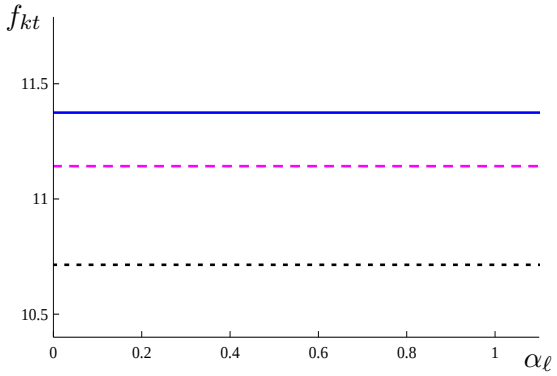

**(a)** Effect of index commodity supply news $D_i$



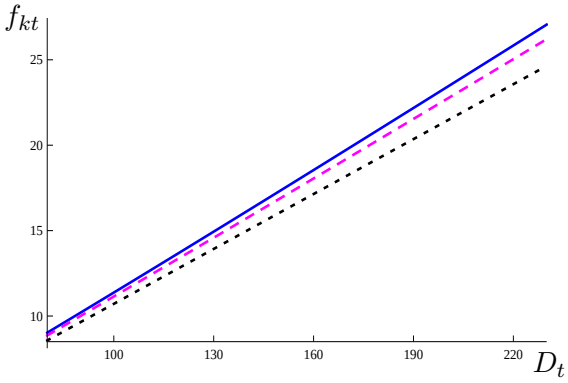

**(b)** Effect of nonindex commodity supply news $D_\ell$



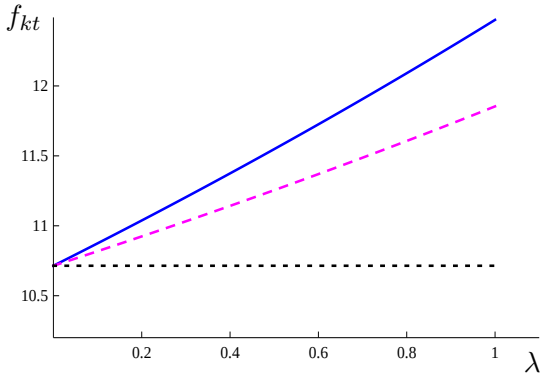

**(c)** Effect of index commodity demand parameter $\alpha_i$




**(d)** Effect of index commodity demand parameter $\alpha_\ell$




**(e)** Effect of aggregate output $D_t$




**(f)** Effect of size of institution $\lambda$

Figure 2: **Futures prices.** This figure plots the equilibrium futures prices against several key quantities. The plots are typical. We set $t = 0.1$, $D_t = 100$, $D_{kt} = 1$, $k = 1, \ldots K$. The solid blue line is for index futures, the magenta dashed line is for nonindex futures, and the black dotted line is for the benchmark economy. The remaining parameter values (when fixed) are as in Figure 1.

economy, futures prices decrease in response to positive index commodities' supply news $D_{it}$. Institutional investors strive to align their performance with the index, and as a result distort prices the most when the index is high (relative to the benchmark economy). The index is high when $D_{it}$ is low (supply of index commodity $i$ is scarce) and low when $D_{it}$ is high (supply is abundant). So the effects of the institutions on commodity futures prices $f_{kt}$ are most pronounced for low $D_{it}$ realizations and decline monotonically with $D_{it}$. These effects are absent in the benchmark economy in which agents are not directly concerned about the index. In contrast, futures prices $f_{kt}$ do not react to news about supply of nonindex commodities (apart from that of own commodity $k$) because this news does not affect the performance of the index (panel (b) and Proposition 1).

The demand-side effects on commodity futures prices are presented in panels (c)–(d). In contrast to the benchmark economy in which futures prices depend only on own commodity demand parameter $\alpha_k$, in panel (c) it emerges that futures prices increase in demand parameters $\alpha_i$ for all commodities that are members of the index. An upward shift in demand for any index commodity leads to an increase in that commodity's price (a classical demand argument, see Lemma 1) and therefore leads to an increase in the value of the index. Since the marginal utility of the institutions is increasing in the index, the effects on prices become increasingly more pronounced as $\alpha_k$ increases. In contrast, these effects are not present for nonindex commodities (panel (d)). A shift in demand for those commodities leave the index unaffected and hence makes futures prices independent of demand shifts towards nonindex commodities (changes in $\alpha_\ell$), apart from own demand shift. A caveat to this discussion is that we are not formally modeling demand shifts in this section, but merely presenting comparative statics with respect to demand parameters $\alpha_k$. In an economy with demand uncertainty, investors take into account of this uncertainty in their optimization (Section 4).

Panel (e) demonstrates that aggregate output news $D_t$ have stronger effects on futures prices $f_{kt}$ than in the benchmark economy with no institutions. This is because good news about aggregate output not only increases the cashflows of all futures contracts (increases $p_{kT}$) but also increases the value of the index. This latter effect is responsible for the amplification of the effect of aggregate output news depicted in panel (e). The higher the

aggregate output, the higher the index and hence the stronger the amplification effect. Finally, panel (f) shows that commodity futures prices rise when there are more institutions in the market. The more institutions there are, the stronger their effect on the discount factor and hence on all commodity futures prices. Finally, all panels in Figure 2 illustrate that in the presence of institutions, index futures rise more than nonindex, as already highlighted in Proposition 1.

## 3.2. Futures Volatilities and Correlations

The past decade in commodity futures markets has been characterized by an increase in volatility, with booms and busts in commodity markets attracting unprecedented attention of policymakers and commentators. We explore commodity futures volatilities in this section in order to highlight the sources of this increased volatility. Our objective is to demonstrate how standard demand and supply risks can be amplified in the presence of institutions.

Propositions 2 reports the futures return volatilities in closed form.[9]

**Proposition 2 (Volatilities of commodity futures).** *In the economy with institutions, the volatility vector of loadings of index commodity futures k returns on the Brownian motions are given by*

$$\sigma_{f_k t} = \overline{\sigma}_{f_k} + h_{kt}\,\sigma_{It}, \quad h_{kt} > 0, \quad k = 1, \ldots, L, \tag{17}$$

*and nonindex by*

$$\sigma_{f_k t} = \overline{\sigma}_{f_k} + h_t\,\sigma_{It}, \quad h_t > 0, \quad k = L+1, \ldots, K, \tag{18}$$

*where $\overline{\sigma}_{f_k}$ is the corresponding volatility vector in the benchmark economy with no institutions and $\sigma_{I_t}$ is the volatility vector for the conditional expectation of the index $E_t[I_T]$, given by*

$$\overline{\sigma}_{f_k} = (\sigma,\, 0, \ldots, -\sigma_k, 0, \ldots, 0),\ \sigma_{It} = (\sigma,\, -\tfrac{1}{L}\sigma_1, \ldots, -\tfrac{1}{L}\sigma_L,\, 0, \ldots, 0), \tag{19}$$

*and where $h_t$ and $h_{kt}$ are strictly positive stochastic processes provided in the Appendix with the property $h_{kt} > h_t$.*

*Consequently, in the presence of institutions,*

*(i) The volatilities of all futures prices, $\|\sigma_{f_k t}\|$, are higher than in the benchmark economy, $k = 1, \ldots K$.*

[9]The notation $||z||$ denotes the square root of the dot product $z \cdot z$.

*(ii) The volatilities of index futures rise more than those of nonindex for otherwise identical commodities, i.e., for commodities $i$ and $k$ with $D_{it} = D_{kt}$, $\forall t$, $\alpha_i = \alpha_k$, $i \leq L$, $L < k \leq K$.*

The general formulae presented in Proposition 2 can be decomposed into individual loadings of futures returns on the primitive sources of risk in our model, the Brownian motions $\omega_0$, $\omega_1, \ldots, \omega_K$.

| | *Sources of risk associated with* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Generic | Index commodities | | | | | Nonindex commodities | | |
| | $\omega_0$ | $\omega_1$ | $\ldots$ | $\omega_k$ | $\ldots$ | $\omega_L$ | $\omega_{L+1}$ | $\ldots$ | $\omega_K$ |
| *Loadings* | | | | | | | | | |
| Benchmark $\overline{\sigma}_{f_k}$ | $\sigma$ | 0 | $\ldots$ | $-\sigma_k$ | $\ldots$ | 0 | 0 | $\ldots$ | 0 |
| Index $\sigma_{f_k}$ | $\sigma(1+h_{kt})$ | $-\sigma_1 \frac{1}{L} h_{kt}$ | $\ldots$ | $-\sigma_k(1+\frac{1}{L}h_{kt})$ | $\ldots$ | $-\frac{1}{L}\sigma_L h_{kt}$ | 0 | $\ldots$ | 0 |

(a) Index commodity futures $k = 1, \ldots, L$

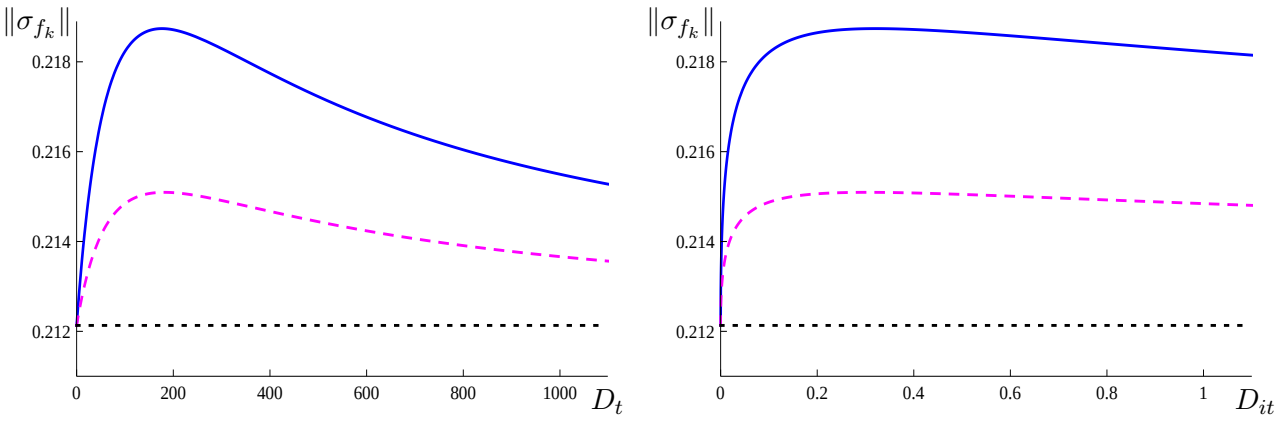
| | *Sources of risk associated with* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Generic | Index commodities | | | Nonindex commodities | | | | |
| | $\omega_0$ | $\omega_1$ | $\ldots$ | $\omega_L$ | $\omega_{L+1}$ | $\ldots$ | $\omega_k$ | $\ldots$ | $\omega_K$ |
| *Loadings* | | | | | | | | | |
| Benchmark $\overline{\sigma}_{f_k}$ | $\sigma$ | 0 | $\ldots$ | 0 | 0 | $\ldots$ | $-\sigma_k$ | $\ldots$ | 0 |
| Nonindex $\sigma_{f_k}$ | $\sigma(1+h_t)$ | $-\sigma_1 \frac{1}{L} h_t$ | $\ldots$ | $-\frac{1}{L}\sigma_L h_t$ | 0 | $\ldots$ | $-\sigma_k$ | $\ldots$ | 0 |

(b) Nonindex commodity futures $k = L+1, \ldots, K$

Table 1: **Individual volatility components of futures prices.**

Recall that in our model the supply news of individual commodities $D_{kt}$ are independent of each other and of the generic good supply news $D_t$. Each of these processes is driven by own Brownian motion. Since in the benchmark economy the futures price depends only on own $D_{kt}$ and aggregate output $D_t$, it is exposed to only two primitive sources of risk: Brownian motions $\omega_k$ and $\omega_0$. In the presence of institutions, futures prices become additionally dependent on supply news of all index commodities and therefore exposed to sources

of uncertainty $\omega_1, \ldots \omega_L$. (The dependence is negative, as illustrated in Corollary 1 and Figure 2a.) Additionally, as argued in Corollary 1 and Figure 2e, shocks to $D_t$ are amplified in the presence of institutions. Proposition 2 formalizes these intuitions by explicitly reporting the loadings on $\omega_0$, $\omega_1, \ldots, \omega_K$, the driving forces behind $D$, $D_1, \ldots, D_K$, respectively. Hence, commodity futures become more volatile for two reasons: (i) their volatilities are amplified because prices react stronger to news about aggregate output $D_t$ and (ii) there is now dependence on additional sources of risk driving index commodity supply news $D_1, \ldots, D_L$. As discussed earlier, the fundamental reason behind this result is that institutions have an additional incentive to do well when the index does well, and any shock that affects the index becomes an additional source of risk for the institutions.



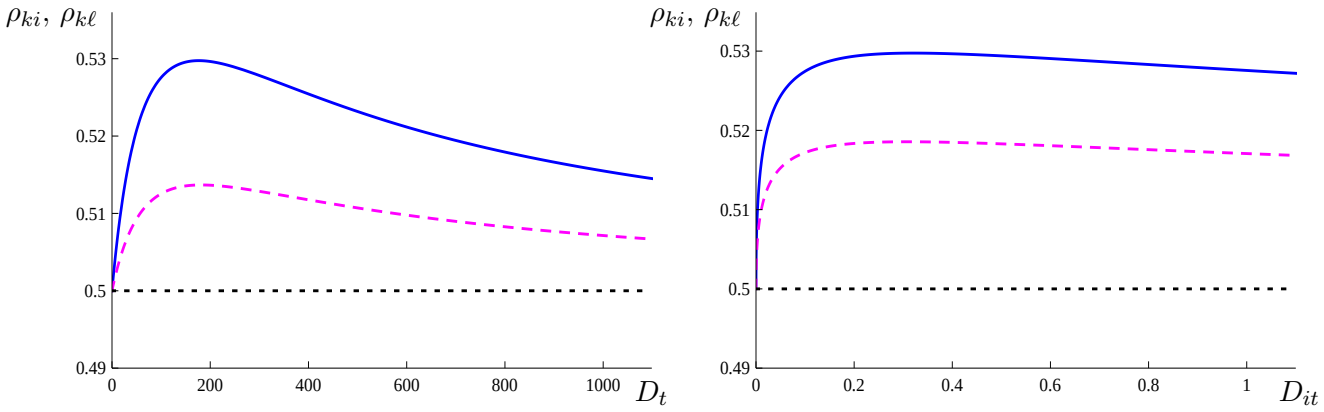

**(a)** Effect of aggregate output news $D$

**(b)** Effect of index commodity supply news $D_i$

Figure 3: **Commodity futures volatilities.** This figure plots the commodity futures volatility $||\sigma_{f_k t}||$ in the presence of institutions against aggregate output news $D_t$ and against index commodity supply news $D_{it}$, $i \neq k$. As in Figure 2, the solid blue line is for index futures, the magenta dashed line is for nonindex futures, and the black dotted line is for the benchmark economy. The parameter values are as in Figure 2.

Figure 3 illustrates the above discussion. It also reveals that the volatilities of index and nonindex futures are differentially affected by the presence of institutions. Tang and Xiong (2012) document that since 2004, and especially during 2008, index commodities have exhibited higher volatility increases than nonindex ones. Our results are consistent with these findings. Institutions bid up volatilities of index futures more than nonindex because index

futures, by construction, pay off more when the index does well. The volatilities of index futures become high enough to make them unattractive to the normal investors (standard market participants) so that they are willing to sell the index futures to the institutions.




**(a)** Effect of aggregate output news $D$

**(b)** Effect of index commodity supply news $D_i$

Figure 4: **Futures returns correlations.** This figure plots return correlations of two index futures $\rho_{kit}$ and two nonindex futures $\rho_{k\ell t}$ in the presence of institutions against aggregate output news $D_t$ and against index commodity supply news $D_{it}$, $i \neq k$. As in Figure 2, the solid blue line is for index futures, the magenta dashed line is for nonindex futures, and the black dotted line is for the benchmark economy. The parameter values are as in Figure 2.

We next turn to examining the (instantaneous) correlations of futures returns, defined as $\rho_{kit} = \sigma_{f_k t} \cdot \sigma_{f_i t} / (\|\sigma_{f_k t}\| \, \|\sigma_{f_i t}\|)$. Recent evidence indicates that financialization of commodities markets has coincided with a sharp increase in the correlations across a wide range of commodity futures returns. Tang and Xiong (2012) document that the average correlation of non-energy commodity futures with oil has increased from 0.1 in 1990s and early 2000s to about 0.5 in 2009. The increase in the correlations is especially pronounced for the index futures returns. Tang and Xiong find that the average correlation of nonindex futures returns with oil rose to 0.2 while that of index commodities exceeded 0.5. Tang and Xiong hypothesize that the commodity markets have been largely segmented before 2000, and the inflow of institutional investors who hold multiple commodities in the same portfolio has linked together the commodity futures markets and increased the correlations among commodities, and especially the index ones. Our model shows that one does not need to rely on

the market segmentation assumption to produce these effects. Arguably, commodity market speculators investing across commodity markets have been present before 2004. Our model produces both the increase in the correlations amongst commodities and the higher increase in the correlations of index commodities under the complete markets assumption.[10] The key mechanism that we stress is that in the presence of institutional investors benchmarked to a commodity index. This index (more precisely, $E_t[I_T] = D_t \prod_{i=1}^{L} (g_i(t)/D_{it})^{1/L}$) emerges as a common factor in returns of all commodities, raising their correlations. However, the sensitivity to this new factor is higher for index commodity futures (Proposition 2), which is the primary reason why their returns become more correlated than those of nonindex futures. We note that the above intuition is precise for covariances. However, it carries through also to the correlations because the effect of rising volatilities is smaller than the effect of rising covariances. Figure 4 illustrates this discussion and presents the correlations occurring in our model.

## 3.3. Transmission to Stock Market

Since investors in our model invest both in the futures and stock markets, one may expect that the effects we find in the futures market may get transmitted to the stock market. This is indeed the case in our model. Proposition 3 demonstrates that the discount factor, affected by financialization, makes the stock market price and volatility dependent on the characteristics of the index commodities.

**Proposition 3 (Stock market level and volatility).** *In the economy with institutions, the equilibrium stock market level and volatility vector are given by*

$$S_t = \overline{S}_t \frac{a + b(1-\lambda) D_0 \prod_{i=1}^{L} (g_i(0)/D_{i0})^{1/L} + b\,\lambda\, D_t \prod_{i=1}^{L} (g_i(t)/D_{it})^{1/L}}{a + b(1-\lambda) D_0 \prod_{i=1}^{L} (g_i(0)/D_{i0})^{1/L} + b\,\lambda\, e^{-\sigma^2 (T-t)} D_t \prod_{i=1}^{L} (g_i(t)/D_{it})^{1/L}}, \tag{20}$$

$$\sigma_{St} = \overline{\sigma}_S + h_{St}\sigma_{It}, \quad h_{St} > 0, \tag{21}$$

*where $\overline{S}_t$ and $\overline{\sigma}_S$ are the corresponding quantities in the benchmark economy with no insti-*

[10] This result can be shown analytically when the volatilities of commodity supply news are the same, i.e., $\sigma_k = \sigma_j$, $\forall k, j = 1, \ldots, K$. For different volatility supply news parameters, all cross correlations (including the stock) can be analytically shown to increase for $L = 1$.

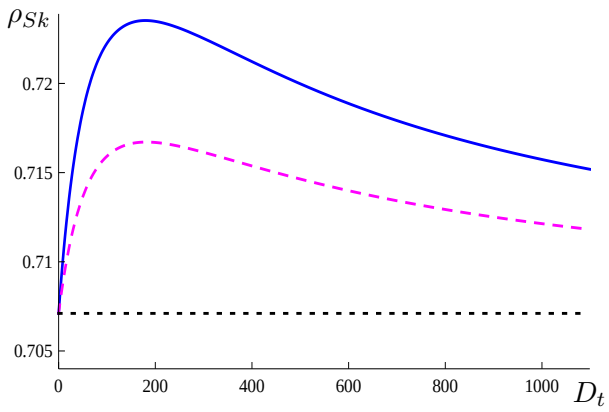
*tutions, given by*

$$\overline{S}_t = \frac{\sum_{k=0}^{K} \alpha_k}{\alpha_0} e^{(\mu-\sigma^2)(T-t)} D_t, \quad \overline{\sigma}_S = \sigma, \tag{22}$$

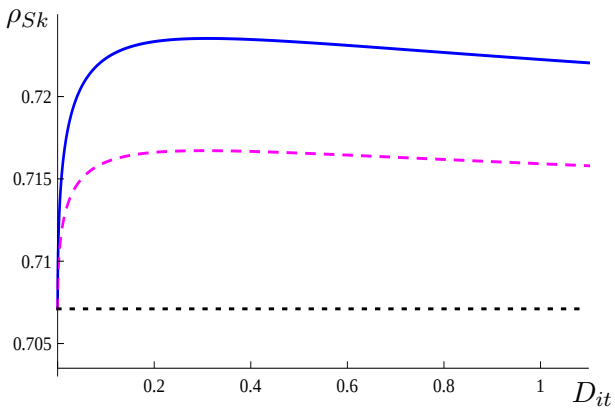
*and $h_{St}$ is a strictly positive stochastic process provided in the Appendix, and $\sigma_{It}$ is as in Proposition 2.*

*Consequently, in equilibrium, the stock market level and its volatility $\|\sigma_{St}\|$ are increased in the presence of institutions.*

Proposition 3 reveals that the stock market is higher in the presence of institutional investors. This is because the stock market pays off in high aggregate output (high-$D_T$) states, which are also the states in which the commodity index does well. The institutional investors who desire payoffs in those states bid up the stock price. For the same reason, they also bid up the stock return volatility, making the stock a less attractive investment for the normal investors.




**(a)** Effect of aggregate output news $D$




**(b)** Effect of index commodity supply news $D_i$

Figure 5: **Equity-futures correlations.** This figure plots return correlations of the stock market with index futures and the stock market with nonindex futures in the presence of institutions against aggregate output news $D_t$ and against index commodity supply news $D_{it}$, $i \neq k$. As in Figure 2, the solid blue line is for index futures, the magenta dashed line is for nonindex futures, and the black dotted line is for the benchmark economy. The parameter values are as in Figure 2.

The quantities $\rho_{Skt} = \sigma_{f_S t} \cdot \sigma_{f_k t} / (\|\sigma_{St}\| \, \|\sigma_{f_k t}\|)$, for all $k$, are the (instantaneous) equity-futures correlations in our model. These correlations always rise in the presence of insti-

tutions. In other words, we do get a theoretical confirmation within our model to support the assertion that the recent rise in the equity-commodity correlations can be attributed to financialization.[11] Figure 5 depicts the equity-commodity correlations in our model. The correlations of the stock market and the commodity futures returns goes up because both the stock market and the commodities returns depend positively on the new common factor: the commodity index. The correlations of the stock market and the index commodities is higher than that with the nonindex because the index commodity futures have a higher loading on the new factor.

## 3.4. Commodity Spot Prices

Commodity spot prices are important determinants of the cost of living worldwide. Spiralling food and energy prices observed in recent years have sparked an intense debate whether the inflow of institutional investors into the futures markets may be pushing millions of households below the poverty line. In his congressional testimony, Masters (2008) argues that the price spiral is unequivocally due to the inflow of institutional commodity investors. In a rigorous study, Singleton (2012) establishes this link for oil prices.

The framework we have developed so far does not carry direct implications for time-$t$ commodity spot prices $p_t$. To formally determine prices $p_t$, one would need to add spot market clearing at all interim periods $t < T$ and intertemporal consumption. However, we may attempt to extrapolate from our model and conjecture the types of implications that one would expect from a fully-fledged model with intertemporal consumption. Let us make several additional assumptions. First, assume that the commodities are storable (until the maturity of the futures contract). Second, assume that a trader can freely buy or sell (short) a commodity at any time $t \leq T$. Shorting a commodity is understood as a reduction of inventories of the commodity that the trader is holding. Under these assumptions, it is possible to construct an arbitrage strategy of replicating a futures contract in the physical commodity market. Finally, let us set each commodity's convenience yield/storage costs to

[11] Tan and Xiong document that the correlation between GSCI commodity index and the S&P500 rose after 2004, and have been especially high in 2008. Relatedly, Buyuksahin and Robe (2009) find that the GSCI-S&P500 correlation rose since the 2008 financial crisis, but not before.

be a constant fraction $\delta_k$ of its price $k = 1, \dots K$. The relationship between futures and spot commodity prices is then provided by the familiar cost-of-carry formula[12]

$$f_{kt} = p_{kt} e^{\delta_k (T-t)}. \tag{23}$$

Consequently, the time-$t$ commodity prices for commodities $k = 1, \dots, K$ are as in Proposition 1, replacing $f_{kt}$ by $p_{kt} e^{\delta_k (T-t)}$. Furthermore, all comparative statics reported in Proposition 1 and Corollary 1 go through for commodity spot prices. Admittedly, this result has been obtained under very strong assumptions. The assumption of a constant cost of carry is simplistic. We recognize that convenience yields are stochastic in practice, driven by a number of factors (Cassasus and Collin-Dufresne (2005)). In future work, it would be useful to consider stochastic convenience yields and investigate potentially interesting effects financialization may have had on convenience yields. Moreover, an ideal model would need to introduce storage explicitly. We leave this for future research.

# 4. Economy with Demand Shocks

In this section we introduce demand shocks to our baseline model. While our setup with supply-side-only uncertainty is capable of delivering most of our insights, we need a richer model to explore the quantitative effects of financialization. As we demonstrate in this section, the presence of demand shocks alone can generate an increase in futures prices and volatilities. But, importantly, demand shocks also sizeably magnify the effects of financialization. Furthermore, it has been argued extensively in the literature that demand shocks are very important in explaining the behavior of prices of oil and other commodities (see Fattouh, Kilian, and Mahadeva (2012) for a survey). For example, Kilian and Murphy (2012) reach a conclusion that the 2004-2008 surge in oil prices can be attributed to demand shocks. Within our model we can disentangle how much of a rise in futures prices and their comovement is due to positive demand shocks and how much of it is due to financialization.

To model demand shocks, we make the following modification to our model. In the

[12]The formula does not feature the interest rate $r$ because we have normalized $r$ to zero.

consumption index (8) of the investors, repeated here for expositional clarity,

$$W_n = C_{n_0}^{\alpha_0} C_{n_1}^{\alpha_1} \cdot \ldots \cdot C_{n_K}^{\alpha_K}, \qquad n \in \{\mathcal{N}, \mathcal{I}\}, \tag{24}$$

we allow one of the demand parameters, $\alpha_1$, to be stochastic. Shocks to $\alpha_1$ then represent shifts in demand for good 1 in the commodity index; we hereafter refer to them as demand shocks. We do not consider shocks to demand for other goods, but our model can be extended to incorporate such shocks. We assume that $\alpha_1$ is a strictly positive process with dynamics

$$d\alpha_{1t} = \alpha_{1t}\sigma_\alpha dw_{0t}, \tag{25}$$

where $\sigma_\alpha > 0$ is constant. Implicit in this assumption is that $\alpha_1$ is driven by the same source of risk, Brownian motion $w_0$, as the (scaled) aggregate output $D$–i.e., $\alpha_1$ has a one-to-one mapping with aggregate output. Now an investor's time-$T$ demand for good 1 is not simply a (decreasing) function of its price $p_{1T}$, but also an (increasing) function of the aggregate output $D_T$ (through $\alpha_{1T}$). The latter assumption has recently been advocated by Dvir and Rogoff (2009) in their model of oil prices.

Proposition 4 reports the equilibrium futures prices and their return volatilities in the economy with demand shocks in closed form. The equilibrium stock market level and volatility, not presented here for brevity, are provided explicitly in the Appendix.

**Proposition 4 (Futures prices and volatilities with demand shocks).** *In the economy with institutions and demand shocks, the equilibrium futures price of commodity $k = 1, \ldots, K$ and its associated volatility vector of loadings are given by*

$$f_{kt} = \overline{f}_{kt} \frac{A + b\,\lambda e^{(1_{\{k\le L\}}\sigma_k^2/L + 1_{\{k=1\}}(\sigma_\alpha^2/L + \sigma\sigma_\alpha))(T-t)} \alpha_{1t}^{1/L} D_t \prod_{i=1}^{L} (\hat{g}_i(t)/D_{it})^{1/L}}{A + b\,\lambda e^{-(\sigma^2 + \sigma\sigma_\alpha/L)(T-t)} \alpha_{1t}^{1/L} D_t \prod_{i=1}^{L} (\hat{g}_i(t)/D_{it})^{1/L}}, \quad A, \hat{g}_i(t) > 0, \tag{26}$$

$$\sigma_{f_k t} = \overline{\sigma}_{f_k} + \hat{h}_{kt}\sigma_{It}, \quad \hat{h}_{kt} > 0, \tag{27}$$

*where $\overline{f}_{kt}$ is the equilibrium futures price in the benchmark economy with no institutions, $\overline{\sigma}_{kt}$ its corresponding volatility vector, and $\sigma_{I_t}$ is the volatility vector of the conditional expected index $E_t[I_T]$, given by*

$$\overline{f}_{kt} = \frac{\left(\alpha_k 1_{\{k>1\}} + \alpha_{1t} 1_{\{k=1\}}\right) e^{(\mu - \mu_k - \sigma^2 + \sigma_k^2)(T-t)}}{\alpha_0} \frac{D_t}{D_{kt}}, \tag{28}$$

$$\overline{\sigma}_f = (\sigma + \sigma_\alpha 1_{\{k=1\}}, 0, \ldots, -\sigma_k, 0, \ldots, 0), \tag{29}$$

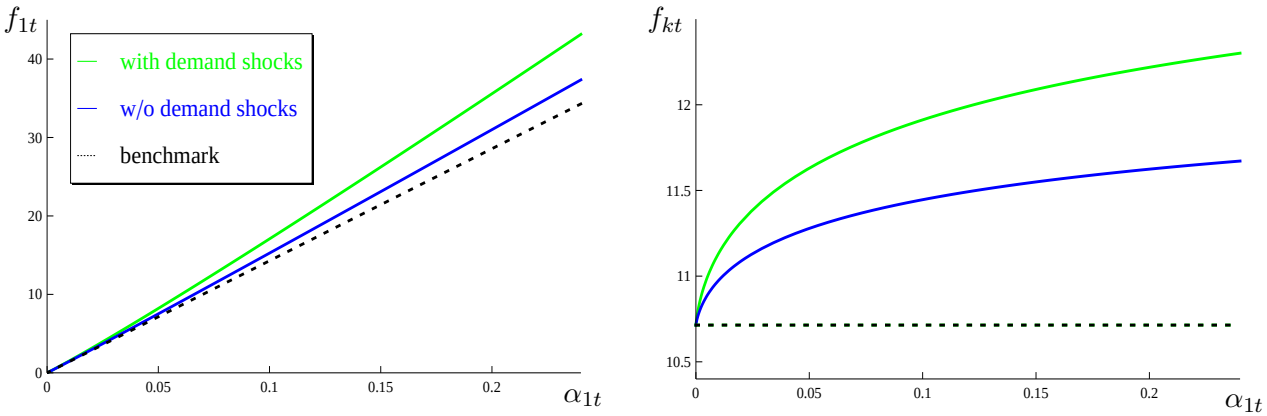
$$\sigma_I = (\sigma + \tfrac{1}{L}\sigma_\alpha, -\tfrac{1}{L}\sigma_1, \ldots, -\tfrac{1}{L}\sigma_L, 0, \ldots, 0), \tag{30}$$

*and the constant $A$, the deterministic quantity $\hat{g}_i(t)$ and the stochastic process $\hat{h}_{kt}$ are explicitly provided in the Appendix.*

*Consequently, in equilibrium, all futures prices and their volatilities $\|\sigma_{f_k}\|$ are higher than in the benchmark economy.*

Proposition 4 confirms our earlier result that all futures prices are higher in the presence of institutions, with prices of index futures exceeding those of nonindex ones. The distinguishing feature of our economy with demand shocks is that these effects become stronger. Figure 6(a) illustrates the results of the proposition and disentangles the effects of financialization from a rising demand for commodity 1. As one can see from the figure, increasing demand for commodity 1 pushes up its futures price even in the benchmark economy with no institutions (the dashed line). But in the presence of institutions, the futures price increases even more, and especially so in the presence of demand shocks (solid soft green line). This is because there is now an additional risk in the economy—demand shocks to commodity 1—that affects the value of the index. Therefore an asset whose payoff is positively correlated with these demand shocks—the futures on commodity 1—becomes even more valuable than in the economy without demand shocks. There is also an important spillover of demand shocks to commodity 1 on the other futures prices. These spillovers are illustrated in Figure 6(b). The spillovers in the presence of demand shocks are stronger than in the economy without demand shocks because the rise in demand for commodity 1 is positively related to increases in aggregate output (and all prices are increasing in aggregate output). Institutions bid up prices of all futures because they all pay off higher in high-demand states—the states when the value of the index is high. These effects are quantitatively important. Our results not out of line with the findings of Kilian and Murphy (2012) that fluctuations in demand are important in explaining the fluctuations in commodity prices (panel (a)), but we also stress a significant contribution of financialization (panel (a) and especially panel (b)).

Proposition 4 also confirms that our remaining results of Section 3 continue to hold in the presence of demand shocks. In particular, futures return volatilities are higher in the presence of institutions. Moreover, we find via numerical analysis that they become even higher in the presence of demand shocks. To understand the intuition behind this new result, it is useful to note that the commodity 1 futures is exposed to an additional source of risk, demand




**(a)** Effect of (own) demand parameter $\alpha_{1t}$

**(b)** Spillover

Figure 6: **Effects of demand shocks on futures prices.** Panel (a) plots index futures 1 price in the economy with demand shocks (solid soft green line) and without demand shocks (solid blue line) against the demand parameter $\alpha_{1t}$. Panel (b) plots futures $k$ price in the economy with demand shocks (solid soft green line) and without demand shocks (solid blue line) against the demand parameter $\alpha_{1t}$. The dotted black lines are for the corresponding prices in the benchmark economy with no institutions. We set $k = 2 \leq L$, $\sigma_\alpha = 0.15$. The remaining parameter values are as in Figure 2.

shocks, and so it is more volatile even in the benchmark with no institutions. Additionally, the membership of commodity 1 in the index makes the index riskier than in the economy without demand shocks. Since falling behind the index is a source of risk for institutional investors, all futures prices depend on the expected index, as we have highlighted before. The (expected) index appears as a new risk factor in the futures prices, and this factor is now more volatile (higher $||\sigma_I||$). Consequently, all futures prices are more volatile as well.

The commodity futures return correlations are also higher in the presence of institutions. As before, this is because the expected index emerges as a common factor affecting all assets in the economy, and hence the covariances of all assets with each other increase more than in the economy without demand shocks. The same ends up being true for the corresponding correlations. To illustrate the relative importance of demand shocks on the correlations quantitatively, in Figure 7 we plot the commodity futures return correlations in the economies with and without demand shocks. We focus on the futures contract on (the index) commodity 1. Apparent from Figure 7 is the fact that the correlation of commodity 1

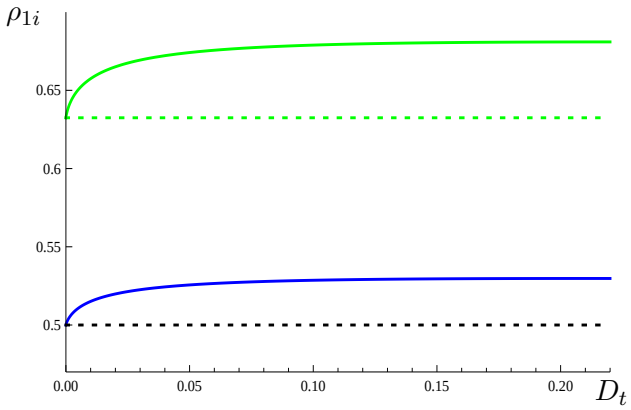



Figure 7: **Effects of demand shocks on correlations.** This figure plots return correlations of future 1 and another index future, $\rho_{1it}$, in the economy with demand shocks (solid soft green line) and without demand shocks (solid blue line) against aggregate output news $D_t$. The dotted soft green and black lines are for the corresponding correlations in the benchmark economy with no institutions. The remaining parameter values are as in Figure 6.

future with other futures increases significantly. Interestingly, a considerable part of the increase in the correlation is due to the introduction of demand uncertainty per se, even in the absence of financialization. However, there is also an interaction effect: in the presence of demand shocks, the effects of financialization on futures return correlations become stronger (although quantitatively not by much). Similar results hold for equity-commodity return correlations. We conjecture that our results on the correlations may be sensitive to the assumptions about the nature of demand shocks in the model. We leave it for future research to explore a more general specification of the demand shocks to commodity 1 as well as the more general case of demand shocks to more than one commodity. To fully address the question of the effects of financialization on cross-commodity correlations, we believe that one needs a model with a common demand shock to a group of commodities (e.g., demand for metals by China). It would be interesting to disentangle the effects of correlated demand shocks from the effects of financialization, which as we have shown, also increases cross-commodity correlations.

# 5. Conclusion

In this paper we have explored theoretically how the presence of institutional investors may affect commodity futures prices and their dynamics. We have found that in the presence of institutions futures prices of all commodities rise, with futures prices of index commodities increasing by more. We have also found that in the presence of institutional investors shocks to fundamentals (demand and supply) of index commodities get transmitted to prices of all other commodities. Furthermore, the volatilities of all commodity futures rise in the presence of institutions, with those of index commodities increasing by more. These effects are more pronounced in the presence of demand shocks. Finally, the presence of institutions leads to an increase in the cross-commodity and equity-commodity correlations, with those for index commodity futures increasing by more.

This paper focuses on commodity futures markets, and only very briefly touches upon the linkages between commodity futures and spot markets. It would be interesting to improve our theoretical understanding of whether price pressure from institutional investors operating in futures markets may be transmitted to spot prices of underlying commodities. Sockin and Xiong (2012) is one nice recent attempt to model this, but there is room for more. Also left for future research is the analysis of the open interest in the futures markets. In a recent paper, Hong and Yogo (2012) document that open interest predicts asset prices and macroeconomic variables. It would be interesting to examine whether our model delivers this intriguing finding.

# Appendix A

**Proof of Lemma 1.** We first determine the institutional and normal investors' optimal demands in each commodity. Since the securities market is dynamically complete in our setup with $K+1$ risky securities and $K+1$ sources of risk $\omega$, there exists a state price density process, $\xi$, such that the time-$t$ value of a payoff $Q_T$ at time $T$ is given by $E_t\big[\xi_T Q_T\big]/\xi_t$. In our setting, the state price density is a martingale. Accordingly, investor $n$'s, $n = \mathcal{N}, \mathcal{I}$, dynamic budget constraint (9) can be restated as

$$E_t\big[\xi_T \sum_{k=0}^{K} p_{kT} C_{n_k T}\big] = \xi_t W_{nt}. \tag{A1}$$

Maximizing the institutional investor's expected objective function (7), with the Cobb-Douglas aggregator (8) substituted in, subject to (A1) evaluated at time $t = 0$ leads to the institution's optimal demand in commodity $k = 1, \ldots, K$ and generic good, respectively, as

$$C_{\mathcal{I}_k T} = \frac{\alpha_k \left(a + bI_T\right)}{y_I p_{kT} \xi_T}, \quad C_{\mathcal{I}_0 T} = \frac{\alpha_0 \left(a + bI_T\right)}{y_I \xi_T}, \tag{A2}$$

where $1/y_I$ solves (A1) evaluated at $t = 0$. Substituting (A2) into (A1) at $t = 0$, we obtain

$$\frac{1}{y_I} = \frac{\lambda \xi_0 S_0}{\sum_{j=0}^{K} \alpha_j \left(a + bE\left[I_T\right]\right)}.$$

Consequently, the institution's optimal commodity demands are given by

$$C_{\mathcal{I}_k T} = \frac{\alpha_k}{\sum_{j=0}^{K} \alpha_j} \frac{\lambda \xi_0 S_0}{p_{kT} \xi_T} \frac{a + bI_T}{a + bE\left[I_T\right]}, \qquad k = 1, \ldots, K, \tag{A3}$$

$$C_{\mathcal{I}_0 T} = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{\lambda \xi_0 S_0}{\xi_T} \frac{a + bI_T}{a + bE\left[I_T\right]}. \tag{A4}$$

Similarly, we obtain the normal investor's optimal commodity demands at time $T$ as

$$C_{\mathcal{N}_k T} = \frac{\alpha_k}{\sum_{j=0}^{K} \alpha_j} \frac{(1-\lambda)\, \xi_0 S_0}{p_{kT} \xi_T}, \qquad k = 1, \ldots, K, \tag{A5}$$

$$C_{\mathcal{N}_0 T} = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{(1-\lambda)\, \xi_0 S_0}{\xi_T}. \tag{A6}$$

We now proceed to determine the equilibrium prices at time $T$. To obtain the equilibrium state price density, we impose the market clearing condition for the generic good, $C_{\mathcal{N}_0 T} + C_{\mathcal{I}_0 T} = D_T$, and substitute (A4) and (A6) to obtain

$$\frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{\xi_0 S_0}{\xi_T} \left(1 - \lambda + \lambda \frac{a + bI_T}{a + bE\left[I_T\right]}\right) = D_T,$$

which after rearranging leads to the equilibrium terminal state price density:

$$\xi_T = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{\xi_0 S_0}{D_T} \left( 1 + \frac{\lambda b \left( I_T - E\left[ I_T \right] \right)}{a + bE\left[ I_T \right]} \right). \tag{A7}$$

The equilibrium state price density in the benchmark economy with no institutions is obtained by considering the special case of $b = 0$ in (A7). The time-$T$ discount factor is defined as $M_{0,T} = \xi_T / \xi_0$, which after substituting (A7) leads to the expression (14) reported in Lemma 1.

To determine the equilibrium commodity prices at $T$, we impose the market clearing condition $C_{\mathcal{N}_k T} + C_{\mathcal{I}_k T} = D_{kT}$ for each commodity $k = 1, \ldots, K$, and substitute (A3) and (A5) to obtain

$$\frac{\alpha_k}{\sum_{j=0}^{K} \alpha_j} \frac{\xi_0 S_0}{p_{kT} \xi_T} \left( 1 - \lambda + \lambda \frac{a + bI_T}{a + bE\left[ I_T \right]} \right) = D_{kT},$$

which after substituting the equilibrium state price density (A7) and rearranging leads to the equilibrium commodity price expressions (11) in Lemma 1. Substituting the equilibrium commodity prices (11) that are in the commodity into the definition of the index (4) leads to the equilibrium commodity index value (12). Moreover, substituting the equilibrium commodity prices (11) into the stock market terminal value $S_T = D_T + \sum_{k=1}^{K} p_{kT} D_{kT}$ leads to the expression (13) in Lemma 1. To determine the unconditional expectation of the index, we make use of the fact that $D_T$, $D_{iT}$, $i = 1, \ldots, L$, are lognormally distributed and hence obtain

$$E\left[ I_T \right] = E\left[ \frac{D_T}{\alpha_0} \prod_{i=1}^{L} \left( \frac{\alpha_i}{D_{iT}} \right)^{1/L} \right] = e^{\left( \mu - \frac{1}{L} \sum_{i=1}^{L} \left( \mu_i - \frac{1}{2} \left( \frac{1}{L} + 1 \right) \sigma_i^2 \right) \right) T} \frac{D_0}{\alpha_0} \prod_{i=1}^{L} \left( \frac{\alpha_i}{D_{i0}} \right)^{1/L}. \tag{A8}$$

Finally, we note that the equilibrium commodity and stock prices at time $T$ are as in the benchmark economy with no institutions (the special case of $b = 0$, $a = 1$).

*Q.E.D.*

**Proof of Proposition 1.** By no arbitrage, the futures price of commodity $k = 1, \ldots, K$ in our setup is given by

$$f_{kt} = \frac{E_t\left[ \xi_T p_{kT} \right]}{\xi_t}. \tag{A9}$$

We proceed by first determining the equilibrium state price density process $\xi$. Since the state price density process is a martingale, its time-$t$ value is given by

$$\begin{aligned} \xi_t &= E_t\left[ \xi_T \right] \\ &= \bar{\xi} E_t\left[ 1/D_T \right] \left( a + b\left( 1 - \lambda \right) E\left[ I_T \right] + \lambda b \frac{E_t\left[ I_T / D_T \right]}{E_t\left[ 1/D_T \right]} \right), \end{aligned} \tag{A10}$$

where the second equality follows by substituting $\xi_T$ from (A7) and rearranging, and

$$\bar{\xi} = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{\xi_0 S_0}{a + bE\left[I_T\right]}. \tag{A11}$$

Substituting (12) and using the fact that $D_T$, $D_{iT}$, $i = 1, \ldots, L$, are lognormally distributed, we obtain

$$\begin{aligned} E_t\left[I_T/D_T\right] &= \frac{1}{\alpha_0} E_t\left[\prod_{i=1}^{L} \left(\alpha_i/D_{iT}\right)^{1/L}\right] \\ &= \frac{1}{\alpha_0} e^{-\frac{1}{L}\sum_{i=1}^{L}\left(\mu_i - \frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)(T-t)} \prod_{i=1}^{L} \left(\alpha_i/D_{it}\right)^{1/L}. \end{aligned} \tag{A12}$$

Substituting (A8), (A12) and $E_t\left[1/D_T\right] = e^{-\left(\sigma^2-\mu\right)(T-t)}/D_t$ into (A10), we obtain

$$\xi_t = \bar{\xi}\frac{e^{-\left(\sigma^2-\mu\right)(T-t)}}{D_t}\left(a + b\left(1-\lambda\right)D_0 \prod_{i=1}^{L}\left(g_i\left(0\right)/D_{i0}\right)^{1/L} + b\lambda e^{-\sigma^2(T-t)} D_t \prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}\right), \tag{A13}$$

where $g_i\left(t\right)$ is as given in (16).

To compute the expected deflated futures payoff of commodity $k = 1, \ldots, K$, we substitute (A7) and (11), and rearrange to obtain

$$E_t\left[\xi_T p_{kT}\right] = \bar{\xi}\frac{\alpha_k}{\alpha_0} E_t\left[1/D_{kT}\right]\left(a + b\left(1-\lambda\right)E\left[I_T\right] + b\,\lambda\,\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]}\right), \tag{A14}$$

where $\bar{\xi}$ is as in (A11).

For nonindex futures contracts $k = L+1, \ldots, K$, we proceed by considering

$$\begin{aligned} E_t\left[I_T/D_{kT}\right] &= \frac{1}{\alpha_0} E_t\left[D_T/D_{kT}\prod_{i=1}^{L}\left(\alpha_i/D_{iT}\right)^{1/L}\right] \\ &= \frac{1}{\alpha_0} E_t\left[D_T\prod_{i=1}^{L}\left(\alpha_i/D_{iT}\right)^{1/L}\right] E_t\left[1/D_{kT}\right], \end{aligned}$$

where in the first equality we have substituted (12) and in the second we have made use of the fact that $D_{kT}$ is independent of $D_T$, $D_{iT}$, $i = 1, \ldots, L$. Consequently, using the fact that $D_T$, $D_{iT}$, $i = 1, \ldots, L$, are lognormally distributed, we obtain

$$\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]} = D_t \prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}, \tag{A15}$$

where $g_i\left(t\right)$ is as in (16). Substituting (A13)–(A15), (A8) and $E_t\left[1/D_{kT}\right] = e^{\left(\sigma_k^2-\mu_k\right)(T-t)}/D_{kt}$ into (A9), and rearranging, we arrive at the equilibrium nonindex futures price expression

reported in (15) for $k = L+1, \ldots, K$. The equilibrium futures price $\bar{f}_k$ in the benchmark economy with no institutions (16) follows by considering the special case of $a = 1$, $b = 0$ in (15).

For index futures contracts $k = 1, \ldots, L$, we substitute (12) and again compute

$$\begin{aligned} E_t\left[I_T/D_{kT}\right] &= \frac{1}{\alpha_0} E_t\left[D_T/D_{kT} \prod_{i=1}^{L} \left(\alpha_i/D_{iT}\right)^{1/L}\right] \\ &= \frac{1}{\alpha_0} e^{\left(-\mu+\mu_k+\left(\frac{1}{L}+1\right)\sigma_k^2-\frac{1}{L}\sum_{i=1}^{L}\left(\mu_i-\frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)\right)(T-t)} \frac{D_t}{D_{kt}} \prod_{i=1}^{L} \left(\alpha_i/D_{it}\right)^{1/L}. \end{aligned}$$

So, using $E_t\left[1/D_{kT}\right] = e^{\left(\sigma_k^2-\mu_k\right)(T-t)}/D_{kt}$ we obtain

$$\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]} = e^{\frac{1}{L}\sigma_k^2(T-t)} D_t \prod_{i=1}^{L} \left(g_i\left(t\right)/D_{it}\right)^{1/L}, \tag{A16}$$

where $g_i\left(t\right)$ is as in (16). Substituting (A13), (A14), (A16), and (A8) into (A9) and rearranging leads to the equilibrium index futures price expression reported in (15) for $k = 1, \ldots, L$. The property (i) that the futures prices are higher than in the benchmark economy follows by observing that the factor multiplying $\bar{f}_{kt}$ in expression (15) is strictly greater than one. Similarly, the property (ii) that the index futures price rise is higher than that of nonindex futures follows by observing that the factor multiplying $\bar{f}_{kt}$ in expression (15) is higher for an otherwise identical index futures.

*Q.E.D.*

**Proof of Corollary 1.** The stated properties follow by taking the appropriate partial derivatives of the expressions (15)–(16), and comparing the relevant magnitudes of the partial derivatives of interest.

*Q.E.D.*

**Proof of Proposition 2.** We write the equilibrium index futures price in (15) for $k = 1, \ldots, L$ as

$$f_{kt} = \bar{f}_{kt} \frac{Z_t}{Y_t}, \tag{A17}$$

where

$$\overline{f}_{kt} = \frac{\alpha_k}{\alpha_0} e^{(\mu-\mu_k-\sigma^2+\sigma_k^2)(T-t)} \frac{D_t}{D_{kt}},$$

$$Z_t = a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{\sigma_k^2(T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L},$$

$$Y_t = a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2(T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}.$$

where $g_i(t)$ is as in (16).

Applying It's Lemma to both sides of (A13), we obtain

$$\sigma_{f_k t} = \overline{\sigma}_{f_k} + \sigma_{Zt} - \sigma_{Yt}, \tag{A18}$$

where

$$\begin{aligned}
\overline{\sigma}_{f_k} &= (\sigma,\, 0, \ldots, -\sigma_k, 0, \ldots, 0) \\
\sigma_{Zt} &= \frac{b\,\lambda\, e^{\sigma_k^2(T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{\sigma_k^2(T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} \sigma_{It} \\
\sigma_{Yt} &= \frac{b\,\lambda\, e^{-\sigma^2(T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2(T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} \sigma_{It},
\end{aligned}$$

and $\sigma_{It}$ is the volatility vector of $D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L} = E_t\left[I_T\right]$ given by

$$\sigma_{It} = \left(\sigma,\, -\tfrac{1}{L}\sigma_1, \ldots, -\tfrac{1}{L}\sigma_L,\, 0, \ldots, 0\right).$$

We note that $Y_t \sigma_{Yt} = Z_t \sigma_{Zt} e^{-\left(\sigma^2 + \sigma_k^2/L\right)(T-t)}$. Hence, we have

$$\begin{aligned}
Z_t \sigma_{Zt} Y_t - Y_t \sigma_{Yt} Z_t &= Z_t \sigma_{Zt} \left(Y_t - e^{-\left(\sigma^2+\sigma_k^2/L\right)(T-t)} Z_t\right) \\
&= Z_t \sigma_{Zt} \left(1 - e^{-\left(\sigma^2+\sigma_k^2/L\right)(T-t)} Z_t\right) \left(a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L}\right),
\end{aligned} \tag{A19}$$

where the second equality follows by substituting $Z_t$ and $Y_t$ and manipulating terms. Substituting (A19) into the expression $\sigma_{Zt} - \sigma_{Yt} = \left(Z_t \sigma_{Zt} Y_t - Y_t \sigma_{Yt} Z_t\right)/Y_t Z_t$, and then into (A18) leads to the equilibrium volatility vector of loadings of index commodity futures in (17) where

$$\begin{aligned}
h_{kt} = &\frac{b\,\lambda\, e^{\sigma_k^2(T-t)/L} \left(1 - e^{-\left(\sigma^2+\sigma_k^2/L\right)(T-t)}\right) \left(a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L}\right)}{a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{\sigma_k^2(T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} \\
&\times \frac{D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2(T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} > 0,
\end{aligned} \tag{A20}$$

where $g_i(t)$ is as in (16).

To determine the volatility vector of loadings of nonindex futures $k = L+1, \ldots, K$, as reported in (18), we follow the same steps as above for index futures, and obtain the stochastic process $h_t$ as

$$h_t = \frac{b\,\lambda\left(1-e^{-\sigma^2(T-t)}\right)\left(a+b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L}\right)}{a+b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L}+b\,\lambda\,D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}} \times \frac{D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}}{a+b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L}+b\,\lambda\,e^{-\sigma^2(T-t)}D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}} > 0, \tag{A21}$$

where $g_i(t)$ is as in (16).

The property that volatilities of all futures prices are higher than in the benchmark economy follow immediately from (17)–(18). To prove property (ii), we note that for commodities $i$ and $k$ with $D_{it} = D_{kt}$, $\alpha_i = \alpha_k$, we have $h_{kt} > h_t$ from (A20)–(A21), and hence the volatility increase for an index futures is higher than that for an otherwise identical nonindex futures.

*Q.E.D.*

**Proof of Proposition 3.** By no arbitrage, the stock market level is given by

$$S_t = \frac{E_t\left[\xi_T D_T\right]}{\xi_t}. \tag{A22}$$

To compute the expected deflated stock market payoff, we substitute (A7) and (12) to obtain

$$E_t\left[\xi_T D_T\right] = \bar{\xi}\sum_{k=0}^{K}\frac{\alpha_k}{\alpha_0}\left(a+b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L}+b\,\lambda\,D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}\right), \tag{A23}$$

where we have used the fact that $D_T$, $D_{iT}$, $i = 1, \ldots, L$ are lognormally distributed, and $\bar{\xi}$ is as in (A11) and $g_i(t)$ is as in (16). Substituting (A23) and (A13) into (A22), and manipulating, leads to the reported equilibrium stock market level in (20). The equilibrium stock market level $\bar{S}_t$ in the benchmark economy (22) follows by considering the special case of $a = 1$, $b = 0$ in (13).

To derive the stock market volatility vector (21), we follow the same steps for the index futures in the Proof of Proposition 2, and obtain the stochastic process $h_{St}$ to be as in (A21). The property that the stock market level and its volatility are higher than those in the benchmark follow straightforwardly from the expressions (20)–(22).

*Q.E.D.*

**Proof of Proposition 4.** We first consider the investors' optimal demands in each com-

modity. Maximizing the institutional investor's expected objective function (7), subject to (A1) evaluated at $t = 0$ leads to the institution's optimal demand in commodity $k = 2, \ldots, K$ and generic good as in (A2) of Section 3, and demand in commodity 1 as

$$C_{\mathcal{I}_1 T} = \frac{\alpha_{1T}\,(a + bI_T)}{y_I p_{1T} \xi_T}. \tag{A24}$$

Here, $1/y_I$ solves (A1) evaluated at $t = 0$, and using the lognormal distribution property of $D_T, D_{kT}, \alpha_{1T}$, is given by

$$\frac{1}{y_I} = \frac{\lambda\,\xi_0 S_0}{a(\sum_j \alpha_j + \alpha_1) + b(\sum_j \alpha_j + \alpha_1 e^{(\sigma\sigma_\alpha + \sigma_\alpha^2/L)T}) E\,[I_T]}, \tag{A25}$$

where henceforth the summation $\sum_j$ denotes the summation over all commodities but the first, i.e., $j = 0, 2, \ldots, K$, and $\alpha_1 \equiv \alpha_{10}$ denotes the initial value of the process $\alpha_{1t}$. Similarly, we obtain the normal investor's optimal commodity demands at time $T$ for $k = 0, 2, \ldots, K$ to be as previously in (A5)–(A6), and for commodity 1 as

$$C_{\mathcal{N}_1 T} = \frac{\alpha_{1T}\,(1 - \lambda)\,\xi_0 S_0}{(\sum_j \alpha_j + \alpha_1) p_{1T} \xi_T}. \tag{A26}$$

To determine the equilibrium state price density, we impose market clearing for the generic good, $C_{\mathcal{N}_0 T} + C_{\mathcal{I}_0 T}$, substitute (A4) and (A6), and rearrange to obtain at $T$

$$\xi_T = \bar{\xi} \frac{1}{D_T} \left(A + b\lambda I_T\right), \tag{A27}$$

where

$$\bar{\xi} = \frac{\alpha_0 \xi_0 S_0}{a(\sum_j \alpha_j + \alpha_1) + b(\sum_j \alpha_j + \alpha_1 e^{(\sigma\sigma_\alpha + \sigma_\alpha^2/L)T}) E\,[I_T]}, \tag{A28}$$

$$A = a + b\,(1 - \lambda)\,\frac{\sum_j \alpha_j + \alpha_1 e^{\left(\sigma\sigma_\alpha + \sigma_\alpha^2/L\right)T}}{\sum_j \alpha_j + \alpha_1} E\,[I_T]\,. \tag{A29}$$

To determine the equilibrium commodity prices at $T$, we impose the market clearing condition $C_{\mathcal{N}_k T} + C_{\mathcal{I}_k T} = D_{kT}$ for each commodity $k = 1, \ldots, K$, and substitute (A3), (A5), (A24) and (A26) and the equilibrium state price density (A27) to obtain the same commodity prices (11) as in Lemma 1 for $k = 2, \ldots, K$, and for commodity 1 we obtain

$$p_{1T} = \frac{\alpha_{1T}}{\alpha_0} \frac{D_T}{D_{1T}}. \tag{A30}$$

Substituting the equilibrium commodity prices (11), (A30), into the definition of the index (4) leads to the time-$T$ equilibrium commodity index value

$$I_T = \frac{\alpha_{1T}^{1/L}}{\alpha_0 \alpha_1^{1/L}} D_T \prod_{i=1}^{L} \left(\alpha_i / D_{iT}\right)^{1/L}. \tag{A31}$$

Hence, making use of the lognormal property of $D_T$, $D_{iT}$, $\alpha_{1T}$ we deduce the unconditional expected index value to be

$$E\left[I_T\right] = e^{\left(\mu - \frac{1}{L}\left(\sigma\sigma_\alpha + \frac{1}{2}\left(\frac{1}{L}-1\right)\sigma_\alpha^2\right) - \frac{1}{L}\sum_{i=1}^{L}\left(\mu_i - \frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)\right)T} \frac{D_0}{\alpha_0}\prod_{i=1}^{L}\left(\frac{\alpha_i}{D_{i0}}\right)^{1/L}. \tag{A32}$$

We now determine the equilibrium futures prices. First, the equilibrium time-$t$ state price density follows by taking the conditional expectation of (A27), substituting (A31), using the lognormality of $D_T$, $D_{iT}$, $\alpha_{1T}$, and after some algebra we get

$$\xi_t = \bar{\xi} e^{\left(\sigma^2 - \mu\right)(T-t)} \frac{1}{D_t}\left(A + b\,\lambda e^{-\left(\sigma^2 + \sigma\sigma_\alpha/L\right)(T-t)} \alpha_{1t}^{1/L} D_t \prod_{i=1}^{L}\left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}\right), \tag{A33}$$

where $\bar{\xi}$ and $A$ are as in (A28)–(A29), and

$$\hat{g}_i\left(t\right) = \frac{\alpha_i}{\alpha_0 \alpha_1^{1/L}} e^{\left(\mu + \frac{1}{L}\left(\sigma\sigma_\alpha + \frac{1}{2}\left(\frac{1}{L}-1\right)\sigma_\alpha^2\right) - \mu_i + \frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)(T-t)}. \tag{A34}$$

To compute the expected deflated futures payoff of commodity $k = 1, \ldots, K$, we substitute (A27), (A11) and (A30), and rearrange to obtain

$$E_t\left[\xi_T p_{kT}\right] = \frac{\bar{\xi}}{\alpha_0}\left(\alpha_k 1_{\{k>1\}} + \alpha_{1t} 1_{\{k=1\}}\right) \frac{e^{\left(\sigma_k^2 - \mu_k\right)(T-t)}}{D_{kt}}\left(A + b\lambda \frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]}\right). \tag{A35}$$

For nonindex futures contracts $k = L+1, \ldots, K$, using the lognormality of $D_T$, $D_{kT}$, $\alpha_{1T}$, and substituting (A31) we obtain

$$\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]} = \alpha_{1t}^{1/L} D_t \prod_{i=1}^{L}\left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}, \tag{A36}$$

where $\hat{g}_i\left(t\right)$ is as in (A34). For index futures contracts except for the first commodity futures, $k = 2, \ldots, L$, we get

$$\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]} = e^{\frac{1}{L}\sigma_k^2(T-t)} \alpha_{1t}^{1/L} D_t \prod_{i=1}^{L}\left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}. \tag{A37}$$

Finally, for the first index futures contract $k = 1$, we deduce

$$\frac{E_t\left[I_T/D_{1T}\right]}{E_t\left[1/D_{1T}\right]} = e^{\left(\frac{1}{L}\left(\sigma_1^2 + \sigma_\alpha^2\right) + \sigma\sigma_\alpha\right)(T-t)} \alpha_{1t}^{1/L} D_t \prod_{i=1}^{L}\left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}. \tag{A38}$$

Substituting (A35)–(A38) and (A33) into (A9) and rearranging leads to the equilibrium index futures price expression reported in (26). The equilibrium futures price $\bar{f}_k$ in the benchmark economy with no institutions (28) follows by considering the special case of $a = 1$, $b = 0$ in (26). The property that the futures prices are higher than in the benchmark economy

follows by observing that the factor multiplying $\bar{f}_{kt}$ in expression (26) is strictly greater than one.

To derive the equilibrium volatility vector of loadings, we apply It's Lemma to the futures price expression (26), and follow similar steps to those in the proof of Proposition 2 to deduce (27) in Proposition 4, where

$$\hat{h}_{kt} = \frac{b\,\lambda\,A\left(1-e^{-\left(\sigma^2+\sigma\sigma_\alpha/L+1_{\{k\leq L\}}\sigma_k^2/L+1_{\{k=1\}}\left(\sigma\sigma_\alpha+\sigma_\alpha^2/L\right)\right)(T-t)}\right)e^{\left(1_{\{k\leq L\}}\sigma_k^2/L+1_{\{k=1\}}\left(\sigma\sigma_\alpha+\sigma_\alpha^2/L\right)\right)(T-t)}}{A+b\,\lambda e^{\left(1_{\{k\leq L\}}\sigma_k^2/L+1_{\{k=1\}}(\sigma\sigma_\alpha+\sigma_\alpha^2/L)\right)(T-t)}\alpha_{1t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}}$$

$$\times\frac{\alpha_{1t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}}{A+b\,\lambda e^{-(\sigma^2+\sigma\sigma_\alpha/L)(T-t)}\alpha_{1t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}}>0, \tag{A39}$$

where $A$ and $\hat{g}_i(t)$ are as in (A29) and (A34), respectively. The property that volatilities of all futures price returns are higher than in the benchmark economy follows immediately from (27) since $h_{kt}>0$.

Finally, to determine the stock market level and volatility in equilibrium, we note that in equilibrium the stock market terminal value is given by

$$S_T=\frac{\sum_j\alpha_j+\alpha_{1T}}{\alpha_0}D_T,$$

where we have substituted (11) and (A24). Following similar steps in the determination of equilibrium futures prices above, we arrive at the following equilibrium stock market level and its associated vector of loadings in the presence of institutions and demand shocks:

$$S_t=\bar{S}_t\frac{A+b\,\lambda\frac{\sum_j\alpha_j+\alpha_{1t}e^{\left(\sigma\sigma_\alpha+\sigma_\alpha^2/L\right)(T-t)}}{\sum_j\alpha_j+\alpha_{1t}}\alpha_{1t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}}{A+b\,\lambda e^{-(\sigma^2+\sigma\sigma_\alpha/L)(T-t)}\alpha_{1t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}}, \tag{A40}$$

$$\sigma_{St}=\bar{\sigma}_{St}+\sigma_{Qt}-\sigma_{Yt}, \tag{A41}$$

where $\bar{S}_t$ and $\bar{\sigma}_S$ are the corresponding quantities in the benchmark economy with no institutions, given by

$$\bar{S}_t=\frac{\sum_j\alpha_j+\alpha_{1t}}{\alpha_0}e^{\left(\mu-\sigma^2\right)(T-t)}D_t, \tag{A42}$$

$$\bar{\sigma}_{St}=\left(\sigma+\frac{\alpha_{1t}}{\sum_j\alpha_j+\alpha_{1t}}\sigma_\alpha,0,\ldots,0\right), \tag{A43}$$

$A$ and $\hat{g}_i(t)$ are as in (A29) and (A34), respectively, and

$$\sigma_{Qt} = \frac{b\,\lambda\,Q_t}{A + b\,\lambda\,Q_t}\left(\sigma + \left(\frac{1}{L} + h_{\alpha t}\right)\sigma_\alpha, -\frac{1}{L}\sigma_1, \ldots, -\frac{1}{L}\sigma_L, 0, \ldots, 0\right),$$

$$\sigma_{Yt} = \frac{b\,\lambda e^{-\left(\sigma^2+\sigma\sigma_\alpha/L\right)(T-t)}\alpha_{1t}^{1/L} D_t \prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}}{A + b\,\lambda e^{-(\sigma^2+\sigma\sigma_\alpha/L)(T-t)}\alpha_{1t}^{1/L} D_t \prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L}}\sigma_{It},$$

$$Q_t = \frac{\sum_j \alpha_j + \alpha_{1t} e^{\left(\sigma\sigma_\alpha+\sigma_\alpha^2/L\right)(T-t)}}{\sum_j \alpha_j + \alpha_{1t}}\alpha_{1t}^{1/L} D_t \prod_{i=1}^{L}\left(\hat{g}_i(t)/D_{it}\right)^{1/L},$$

$$h_{\alpha t} = \frac{\alpha_{1t}\sum_j \alpha_j \left(e^{\left(\sigma\sigma_\alpha+\sigma_\alpha^2/L\right)(T-t)} - 1\right)}{\left(\sum_j \alpha_j + \alpha_{1t}\right)\left(\sum_j \alpha_j + \alpha_{1t} e^{(\sigma\sigma_\alpha+\sigma_\alpha^2/L)(T-t)}\right)} > 0$$

with $\sigma_{It}$ being as in Proposition 4.

*Q.E.D.*

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