What Explains Japan's Persistent Deflation?*

(Preliminary and incomplete; please do not quote without permission)

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Abstract

While the asset price collapse of the late 80s in Japan might explain the disinflationary pressures that followed, it is hard to attribute the persistent deflation that the country has faced since the mid 90s to that initial shock. We argue that a failure to account for demographic trends when calibrating monetary policy might explain Japan's persistent deflation. The demographic transition, and in particular the increase in life expectancy, puts downward pressure on the real interest rate. Monetary policy rules that do not internalize this effect may end up being to restrictive, and lead to deflation in equilibrium. We calibrate a stylized model of a dynamic, monetary economy with a life-cycle structure to japanese data, and find that it accounts remarkably well for the low frequency developments in inflation and other macroeconomic variables since the early 90s in Japan.

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"*Seemingly, there would be no linkage between demography and deflation. But it may not be the case.*" Governor Masaaki Shirakawa, opening remarks at the 2012 BOJ-IMES Conference on "Demographic Changes and Macroeconomic Performance".

1 Introduction

While the asset price collapse of the late 80s in Japan might explain the disinflationary pressures that followed, it is hard to attribute the persistent deflation that the country has faced since the mid 90s to that initial shock. We argue that a failure to account for demographic trends when calibrating monetary policy might explain Japan's persistent deflation. The demographic transition – in particular the increase in life expectancy – puts downward pressure on the efficient real interest rate (i.e. the real interest rate that would prevail in an economy with flexible prices and no mark-up fluctuations). Monetary policy rules that do not internalize this effect may end up being to restrictive, and lead to deflation in equilibrium.

To make our point, we rely on a stylized model of a dynamic, monetary economy with a life-cycle structure, bulding on Gertler (1999). The point of departure for our analysis is that, for given retirement age, an increase in life expectancy lengthens the retirement period. Therefore, while the dependency ratio increases and consumption of retirees may actually increase, the longer retirement period increases the incentives to save throughout the life cycle. In practice, this effect may be stronger if the private sector believes that public pension systems may not be fully sustainable. The first result in the paper is that this aspect of the demographic transition puts downward pressure on the efficient real interest rate.[1]

In the baseline New Keynesian model (Clarida et al., 1999; Woodford, 2003), optimal policy (which coincides with a strict inflation-targeting regime) implies that the nominal interest rate "tracks" the efficient real interest rate. The demographic transition generates low-frequency variation in the level of the efficient real interest rate. While strict inflation targeting continues to appropriately internalize such exogenous pressures on equilibrium real interest rates, various interest rate policies may miss the consequences of the demographic transition. If the central bank sets the interest rate without taking into account the changes in the efficient real interest rate due to the increase in life expectancy, monetary policy becomes too restrictive, and may induce equilibrium deflation.

We calibrate the model to japanese data, and find that it accounts remarkably well for the low frequency developments in inflation and other macroeconomic variables since the early 90s in Japan.

[To be completed]

The rest of the paper proceeds as follows. Section 2 presents the model, with particular focus on the life-cycle dimension. Section 3 discusses the main quantitative experiments. Section 4 investigates if the decline in real interest rates and deflation could have been forecastes. Section 5 asks whether accounting for developments in productivity and fiscal policy changes our conclusions. Finally, Section 6 concludes.

[1] This finding may also bear important asset pricing implications. Favero et al. (2011) find that empirical models of the term structure that include demographic variables outperform traditional macro-finance frameworks that abstract from demographics.

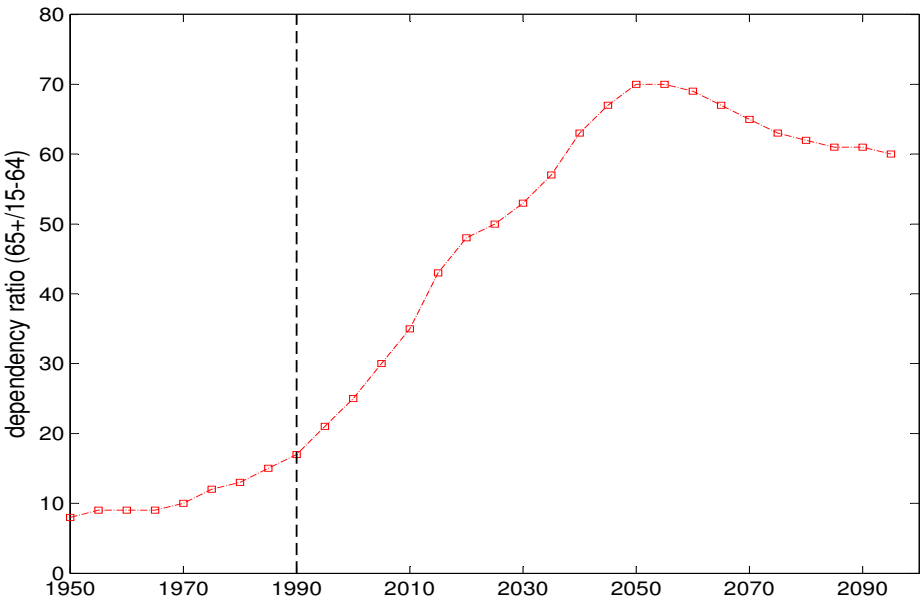


Figure 1: Dependency ratio for Japan. Source: United Nations World Population Database 2010.

2 The Model

The economic actors in the model are households, firms, and the government. Individuals are born workers and supply inelastically one unit of labor while in the labor force. Retirees consume out of their savings. Income is either consumed or saved in the three available assets: physical capital, a nominal government bond, and shares of goods producing firms. Intermediate goods producing firms are monopolistic competitive and produce differentiated goods. Final goods producing firms are perfectly competitive and produce the homogeneous consumption good. The government consists of a fiscal authority and a central bank. The fiscal authority takes spending as given and decides the mix of lump-sum taxes and one-period nominal debt to satisfy its budget constraint. The central bank sets monetary policy.

We abstract from aggregate uncertainty and consider the effects of unexpected one-time changes in demographic parameters in an otherwise perfect-foresight environment. The only source of uncertainty that may potentially affect agents' behavior stems from idiosyncratic retirement and death risk.

2.1 Households and Life-Cycle Structure

At any given point in time, individuals belong to one of two groups: workers (w) or retirees (r). At time $t-1$, workers have mass N^w_{t-1} and retirees have mass N^r_{t-1}. Between periods $t-1$ and t, a worker remains in the labor force with probability ω_t, and retires otherwise. If retired, an individual survives

from period $t-1$ to period t with probability γ_t.[2] In period t, $(1-\omega_t+n_t)\,N^w_{t-1}$ new workers are born. Consequently, the law of motion for the aggregate labor force is

$$N^w_t = (1-\omega_t+n_t)\,N^w_{t-1} + \omega_t N^w_{t-1} = (1+n_t)\,N^w_{t-1}, \tag{1}$$

so that n_t represents the growth rate of the labor force between periods $t-1$ and t. The number of retirees evolves over time according to

$$N^r_t = (1-\omega_t)\,N^w_{t-1} + \gamma_t N^r_{t-1}. \tag{2}$$

From (1) and (2), we define the dependency ratio ($\psi_t \equiv N^r_t/N^w_t$), which summarizes the relevant heterogeneity in the population and evolves according to

$$(1+n_t)\,\psi_t = (1-\omega_t) + \gamma_t \psi_{t-1}. \tag{3}$$

Workers inelastically supply one unit of labor, while retirees do not work.[3] Preferences for an individual of group $z = \{w, r\}$ are a restricted version of the recursive non-expected utility family (Kreps and Porteus, 1978; Epstein and Zin, 1989) that assumes risk neutrality

$$V^z_t = \left\{(C^z_t)^\rho + \beta^z_{t+1}\left[E_t\left(V_{t+1} \mid z\right)\right]^\rho\right\}^{\frac{1}{\rho}}, \tag{4}$$

where C^z_t denotes consumption and V^z_t stands for the value of utility in period t. Retirees and workers have different discount factors to account for the probability of death

$$\beta^z_{t+1} = \begin{cases} \beta\gamma_{t+1} & \text{if } z = r \\ \beta & \text{if } z = w \end{cases}$$

The expected continuation value in (4) differs across workers and retirees because of the different possibilities to transition between groups

$$E_t\left\{V_{t+1} \mid z\right\} = \begin{cases} V^r_{t+1} & \text{if } z = r \\ \omega_{t+1} V^w_{t+1} + (1-\omega_{t+1})\,V^r_{t+1} & \text{if } z = w \end{cases}$$

This life-cycle model is analytically tractable because the transition probabilities ω and γ are independent of age and of the time since retirement. With standard risk-averse preferences, however, this assumption would imply a strong precautionary saving motive for young agents, which is hard to reconcile with actual consumption/savings choices. Risk-neutral preferences with respect to income

[2] Because retirement is an absorbing state in this model, the probability of retiring is perhaps best interpreted as the risk of becoming permanently unable to supply labor.

[3] Gertler (1999) shows how to introduce variable labor supply in this framework without sacrificing its analytical tractability. The demographic trends documented in Section 1 should induce individuals to supply more hours and increase participation rates. The data for all advanced economies, instead, display the opposite tendency, that is, a more or less pronounced downward trend for both variables (see the appendix of Ferrero, 2010). We thus view the assumption of inelastic labor supply as a natural benchmark for the purposes of our paper. At the same time, government policies around the world are attempting to fight this course, delaying the retirement age. We return to this issue at the end of the paper.

fluctuations prevent a counterfactual excess of savings by young workers (Farmer, 1990; Gertler, 1999). The separation of the coefficient of intertemporal substitution ($\sigma \equiv (1-\rho)^{-1}$) from risk aversion implied by (4) allows for a reasonable response of consumption and savings to changes in interest rates.

Households consume a homogeneous final good C_t and allocate their wealth among investment in new physical capital K_t, nominal bonds issued by the government B_t, and shares of intermediate goods producers x_{Ft}. Households rent the capital stock to intermediate goods producers at a real rate R_t^K and bear the cost of depreciation $\delta \in (0,1)$. Government bonds B_t pay a gross nominal return R_t. Shares of intermediate goods producing firms trade at the price P_{Ft} (relative to the price of one unit of the final good P_t) and pay a real dividend D_{Ft}.

2.1.1 Retirees

An individual born in period j and retired in period τ chooses consumption $C_t^r(j,\tau)$ and assets $K_t^r(j,\tau)$, $B_t^r(j,\tau)/P_t$ and $x_{Ft}^r(j,\tau)$, for $t \geq \tau$ to solve

$$V_t^r(j,\tau) = \max\left\{[C_t^r(j,\tau)]^\rho + \beta\gamma_{t+1}\left[V_{t+1}^r(j,\tau)\right]^\rho\right\}^{\frac{1}{\rho}}, \tag{5}$$

subject to

$$\begin{aligned} C_t^r(j,\tau) + K_t^r(j,\tau) + \frac{B_t^r(j,\tau)}{P_t} + P_{Ft}x_{Ft}^r(j,\tau) \\ = \frac{1}{\gamma_t}\left\{[R_t^K + (1-\delta)]K_{t-1}^r(j,\tau) + \frac{R_{t-1}}{\pi_t}\frac{B_{t-1}^r(j,\tau)}{P_{t-1}} + (P_{Ft}+D_{Ft})x_{Ft-1}^r(j,\tau)\right\} \end{aligned} \tag{6}$$

where $\pi_t \equiv P_t/P_{t-1}$ is gross inflation in period t. Additionally, the optimization problem is also subject to the consistency requirement that the retiree's initial asset holdings upon retirement correspond to the assets held in the last period as a worker, that is,

$$\begin{aligned} K_{\tau-1}^r(j,\tau) &= K_{\tau-1}^w(j) \\ B_{\tau-1}^r(j,\tau) &= B_{\tau-1}^w(j) \\ x_{F\tau-1}^r(j,\tau) &= x_{F\tau-1}^w(j) \end{aligned} \tag{7}$$

At the beginning of each period, retirees turn their wealth over to a perfectly competitive mutual fund industry which invests the proceeds and pays back a premium over the market return equal to $1/\gamma_t$ to compensate for the probability of death (Blanchard, 1985; Yaari, 1965). A retiree who survives between periods $t-1$ and t then makes investment decisions right at the end of the period.

Appendix B.1 derives the Euler equations for government bonds, capital and equity that characterize the problem of a retiree. In the absence of aggregate uncertainty, the real returns on the three assets are equalized

$$\frac{R_t}{\pi_{t+1}} = R_{t+1}^K + (1-\delta) = \frac{P_{Ft+1}+D_{Ft+1}}{P_{Ft}}. \tag{8}$$

We define total real assets for a retiree as

$$A_t^r(j,\tau) \equiv K_t^r(j,\tau) + \frac{B_t^r(j,\tau)}{P_t} + P_{Ft}x_{Ft}^r(j,\tau). \tag{9}$$

Due to the equality of real returns, the retiree's budget constraint (6) can be rewritten compactly as

$$C_t^r(j,\tau) + A_t^r(j,\tau) = \frac{R_{t-1}A_{t-1}^r(j,\tau)}{\gamma_t\pi_t}. \tag{10}$$

In Appendix B.1 we show that consumption is a fraction of total wealth

$$C_t^r(j,\tau) = \xi_t^r\left[\frac{R_{t-1}A_{t-1}^r(j,\tau)}{\gamma_t\pi_t}\right], \tag{11}$$

where the marginal propensity to consume for a retiree ξ_t^r satisfies the first-order non-linear difference equation

$$\frac{1}{\xi_t^r} = 1 + \gamma_{t+1}\beta^\sigma\left(\frac{R_t}{\pi_{t+1}}\right)^{\sigma-1}\frac{1}{\xi_{t+1}^r}. \tag{12}$$

From (10) and (11), asset holdings evolve according to

$$A_t^r(j,\tau) = (1-\xi_t^r)\left[\frac{R_{t-1}A_{t-1}^r(j,\tau)}{\gamma_t\pi_t}\right].$$

Finally, the Appendix also shows that the value function for a retiree is linear in consumption

$$V_t^r(j,\tau) = (\xi_t^r)^{\frac{\sigma}{1-\sigma}}C_t^r(j,\tau). \tag{13}$$

2.1.2 Workers

Workers start their life with zero assets. We write the optimization problem for a worker born in period j in terms of total assets $A_t^w(j) \equiv K_t^w(j) + B_t^w(j)/P_t + P_{Ft}x_{Ft}^w(j)$. Specifically, a worker chooses consumption $C_t^w(j)$ and assets $A_t^w(j)$ for $t \geq j$ to solve

$$V_t^w(j) = \max\left\{[C_t^w(j)]^\rho + \beta\left[\omega_{t+1}V_{t+1}^w(j) + (1-\omega_{t+1})V_{t+1}^r(j,t+1)\right]^\rho\right\}^{\frac{1}{\rho}}, \tag{14}$$

subject to

$$C_t^w(j) + A_t^w(j) = \frac{R_{t-1}A_{t-1}^w(j)}{\pi_t} + \frac{W_t}{P_t} - T_t^w \tag{15}$$

and $A_j^w(j) = 0$, where W_t represents the nominal wage and T_t^w is the total amount of lump-sum taxes in real terms paid by each worker. Workers do not turn their wealth over to the mutual fund industry, and hence do not receive the additional return that compensates for the probability of death.[4] The value function $V_{t+1}^r(j,t+1)$ is the solution of the utility maximization problem for a retiree described above and enters the continuation value of workers, who have to take into account the possibility that

[4] Allowing workers access to the mutual fund industry would provide complete insurance against the probability of retirement, hence shutting down most of the interesting life-cycle dimensions of the model.

retirement occurs between periods t and $t+1$.

In Appendix B.2 we present the complete solution of a worker's optimization problem and show that workers' consumption is a fraction of total wealth, defined as the sum of financial and non-financial ("human") wealth

$$C_t^w(j) = \xi_t^w \left[\frac{R_{t-1} A_{t-1}^w(j)}{\pi_t} + H_t^w \right], \tag{16}$$

where H_t^w represents the present discounted value of current and future real wages net of taxation and is independent of individual-specific characteristics

$$H_t^w \equiv \sum_{v=0}^{\infty} \frac{(W_{t+v}/P_{t+v} - T_{t+v}^w)}{\prod_{s=1}^{v} \left(\frac{\Omega_{t+s} R_{t+s-1}}{\pi_{t+s}} / \omega_{t+s-1,t+s} \right)} = \frac{W_t}{P_t} - T_t^w + \frac{\omega_{t+1} H_{t+1}^w}{\Omega_{t+1} R_t / \pi_{t+1}}. \tag{17}$$

As for retirees, workers' marginal propensity to consume ξ_t^w also evolves according to a first-order non-linear difference equation

$$\frac{1}{\xi_t^w} = 1 + \beta^\sigma \left(\frac{\Omega_{t+1} R_t}{\pi_{t+1}} \right)^{\sigma-1} \frac{1}{\xi_{t+1}^w}. \tag{18}$$

The adjustment term Ω_t that appears in (17) and (18) corresponds to

$$\Omega_t \equiv \omega_t + (1-\omega_t) \left(\frac{\xi_t^r}{\xi_t^w} \right)^{\frac{1}{1-\sigma}}.$$

In the definition of non-financial wealth (17), the term $\frac{\Omega_{t+1} R_t}{\pi_{t+1}} / \omega_{t+1}$ constitutes the real effective discount rate for a worker. The first component of the (higher) discounting captures the effect of the finite lifetime horizon (less value attached to the future). The term ω_{t+1} augments the actual discount factor because workers need to finance consumption during the retirement period (positive probability of retiring).

The dynamics of asset holdings can then be obtained from the budget constraint of a worker and the consumption function (16)

$$A_t^w(j) + \frac{\omega_{t+1} H_{t+1}^w}{\Omega_{t+1} R_t / \pi_{t+1}} = (1 - \xi_t^w) \left[\frac{R_{t-1} A_{t-1}^w(j)}{\pi_t} + H_t^w \right].$$

Finally, as for retirees, workers' value function is also linear in their consumption

$$V_t^w(j) = (\xi_t^w)^{\frac{\sigma}{1-\sigma}} C_t^w(j), \tag{19}$$

2.1.3 Aggregation of Households' Decisions

The marginal propensities to consume of workers and retirees are independent of individual characteristics. Hence, given the linearity of the consumption functions, aggregate consumption of workers

(C_t^w) and retirees (C_t^r) have the same form as (11) and (16)[5]

$$C_t^w = \xi_t^w \left(\frac{R_{t-1} A_{t-1}^w}{\pi_t} + H_t \right), \tag{20}$$

$$C_t^r = \xi_t^r \left(\frac{R_{t-1} A_{t-1}^r}{\pi_t} \right), \tag{21}$$

where A_{t-1}^z is total financial wealth that members of group $z = \{w, r\}$ carry from period $t-1$ into period t, and the aggregate value of human wealth H_t evolves according to

$$H_t = \frac{W_t N_t^w}{P_t} - T_t + \frac{\omega_{t+1} H_{t+1}}{(1 + n_{t+1}) \Omega_{t+1} R_t / \pi_{t+1}}. \tag{22}$$

The aggregate consumption function C_t is the weighted sum of (21) and (20). If $\lambda_t \equiv A_t^r / A_t$ denotes the share of total financial wealth A_t held by retirees, the aggregate consumption function is

$$C_t = \xi_t^w \left[(1 - \lambda_{t-1}) \frac{R_{t-1} A_{t-1}}{\pi_t} + H_t \right] + \xi_t^r \left(\lambda_{t-1} \frac{R_{t-1} A_{t-1}}{\pi_t} \right). \tag{23}$$

Relative to the standard neoclassical growth model, the distribution of assets across cohorts is an additional state variable which keeps track of the heterogeneity in wealth accumulation due to the life-cycle structure. Aggregate assets for retirees depend on the total savings of those who are retired in period t as well as on the total savings of the fraction of workers who retire between periods t and $t+1$

$$A_t^r = \frac{R_{t-1} A_{t-1}^r}{\pi_t} - C_t^r + (1 - \omega_{t+1}) \left(\frac{R_{t-1} A_{t-1}^w}{\pi_t} + \frac{W_t N_t^w}{P_t} - T_t - C_t^w \right). \tag{24}$$

Aggregate assets for workers depend only on the savings of the fraction of workers who remain in the labor force

$$A_t^w = \omega_{t+1} \left(\frac{R_{t-1} A_{t-1}^w}{\pi_t} + \frac{W_t N_t^w}{P_t} - T_t - C_t^w \right). \tag{25}$$

The law of motion for the distribution of financial wealth across groups obtains from substituting expressions (21) and (25) into (24)

$$[\lambda_t - (1 - \omega_{t+1})] A_t = \omega_{t+1} (1 - \xi_t^r) \lambda_{t-1} \frac{R_{t-1} A_{t-1}}{\pi_t}. \tag{26}$$

Expression (26) relates the evolution of the distribution of wealth λ_t to the aggregate asset position A_t. From expression (9) and its counterpart for workers, total assets equal the sum of the aggregate capital stock, government bonds and the real market value of intermediate goods producing firms[6]

$$A_t = K_t + \frac{B_t}{P_t} + P_{Ft}. \tag{27}$$

[5] An aggregate variable Q_t^z for group $z = \{w, r\}$ takes the form $Q_t^z \equiv \int_0^{N_t^z} Q_t^z(i)\, di$.

[6] Without loss of generality, we set the net supply of shares of those firms equal to one (i.e. $x_{Ft} = 1$).

2.2 Firms and Production

Two types of firms operate in the economy. A continuum of monopolistically competitive firms hire labor from workers and rent capital from both workers and retirees to produce differentiated intermediate goods. Competitive retailers combine these intermediate goods to produce a homogeneous final good, which is then used for both consumption and investment.

2.2.1 Final Goods Producers

Competitive retailers produce a final good Y_t combining intermediate goods $Y_t(i)$, with $i \in (0,1)$, according to a constant elasticity of substitution technology with elasticity $\theta > 1$

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}.$$

Profit maximization yields the demand for the i^{th} intermediate good as a function of its relative price and total demand

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} Y_t, \tag{28}$$

where $P_t(i)$ is the price of good i. The zero-profit condition for intermediate goods producers yields an expression for the price of one unit of the final good as a function of the intermediate goods prices

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}.$$

2.2.2 Intermediate Goods Producers

Intermediate goods producers combine labor hired from workers and capital rented from both workers and retirees to produce differentiated inputs according to a standard Cobb-Douglas labor-augmenting technology

$$Y_t(i) = [X_t N_t^w(i)]^{\alpha} K_{t-1}(i)^{1-\alpha}, \tag{29}$$

where $\alpha \in (0,1)$ is the labor share and the technology factor X_t grows exogenously at rate x_t

$$X_t = (1 + x_t) X_{t-1}.$$

Intermediate goods producers operate in monopolistic competition and set prices subject to (quadratic) adjustment costs (Rotemberg, 1982). As usual, we can split the problem of these firms in two steps. The first step is to obtain the firm's marginal cost from its cost minimization problem. To this end, firm i chooses how much labor to hire and how much capital to rent in order to solve

$$\min_{N_t^w(i), K_{t-1}(i)} \frac{W_t}{P_t} N_t^w(i) + R_t^K K_{t-1}(i), \tag{30}$$

subject to its technological constraint (29). The first-order conditions for labor and capital are

$$\frac{W_t}{P_t} = mc_t(i)\frac{\alpha Y_t(i)}{N_t^w(i)} \tag{31}$$

$$R_t^K = mc_t(i)\frac{(1-\alpha)Y_t(i)}{K_{t-1}(i)}, \tag{32}$$

where $mc_t(i)$ is the Lagrange multiplier on the technological constraint. Taking the ratio between the two first-order conditions shows that the capital-labor ratio in this economy is independent of firm characteristics. Because, via the production function, the ratios of both employment and capital to output can be expressed as a function of the capital-labor ratio, the first-order conditions of the minimization problem above also imply that the marginal cost is independent of firm characteristics. Furthermore, we can use the production function to derive an expression for the marginal cost as a function of factor prices only

$$mc_t = \frac{\left(\frac{W_t/P_t}{X_t}\right)^{\alpha}(R_t^K)^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}. \tag{33}$$

The second step of the intermediate goods producers' problem is the dynamic price-setting decision. For simplicity, we assume that firms use the real interest rate to discount future profits. A generic firm i solves

$$\max_{P_t(i)} \sum_{t=0}^{\infty} \frac{1}{\prod_{s=1}^{t} R_{t+s-1}/\pi_{t+s}} \underbrace{\left[\left(\frac{P_t(i)}{P_t} - mc_t\right) Y_t(i) - \frac{\phi_P}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 Y_t\right]}_{\equiv D_{Ft}(i)}, \tag{34}$$

subject to (28). In a symmetric equilibrium (i.e. $P_t(i) = P_t, \forall i$), the first-order condition for this problem yields a dynamic Phillips curve that links current inflation to current marginal cost and discounted future inflation

$$(\pi_t - 1)\,\pi_t = \frac{\theta - 1}{\phi_P}\left(\frac{\theta}{\theta - 1} mc_t - 1\right) + \frac{1}{R_t/\pi_{t+1}}\frac{Y_{t+1}}{Y_t}(\pi_{t+1} - 1)\,\pi_{t+1}. \tag{35}$$

2.3 Fiscal and Monetary Policies

The government issues nominal debt B_t and levies lump-sum taxes to finance a given stream of spending G_t. The flow government budget constraint in nominal terms is

$$B_t = R_{t-1}B_{t-1} + P_t(G_t - T_t). \tag{36}$$

For simplicity, we assume that the ratio between government spending and GDP is constant ($G_t = gY_t$). We also impose a fiscal rule that requires the government to keep real debt constant as a fraction of GDP

$$\frac{B_t}{P_t} = bY_t. \tag{37}$$

In the baseline analysis, we assume the central bank follows a strict inflation-targeting rule

$$\pi_t = 1. \tag{38}$$

Because of the absence of shocks that create a tradeoff between inflation and marginal costs, this economy features the "divine coincidence" (Blanchard and Galí, 2007). Therefore, a policy of strict inflation targeting replicates the flexible price equilibrium allocation, that is, the same allocation that would arise if ϕ_P were to be equal to zero.

2.4 Equilbrium

Given the dynamics for the demographic processes n_t, ω_t, and γ_t and the growth rate of productivity x_t, an (imperfectly competitive) equilibrium for this economy is a sequence of quantities $\{C_t^r, C_t^w, C_t, A_t^r, A_t^w, A_t, \lambda_t, H_t, Y_t, K_t, mc_t, I_t, D_{Ft}, V_t^r, V_t^w, V_t, B_t/P_t, T_t\}$, marginal propensities to consume $\{\xi_t^r, \xi_t^w, \epsilon_t, \Omega_t\}$, prices $\{R_t, R_t^K, P_{Ft}, W_t/P_t, \pi_t\}$, and dependency ratios ψ_t such that:

1. Retirees and workers maximize utility subject to their budget constraints, taking prices and wages as given, as outlined in sections 2.1.1 and 2.1.2.

2. Final goods producers maximize profits subject to their technology constraint (section 2.2.1). Intermediate good producers maximize profits subject to their technology constraint and taking the demand for their differentiated product as given (section 2.2.2).

3. The fiscal authority chooses the mix of debt and taxes to satisfy its budget constraint and the central bank decides upon monetary policy (section 2.3).

4. The markets for labor, capital and goods clear. In particular, the economy-wide resource constraint is

$$\left[1 - \frac{\phi_P}{2}\left(\pi_t - 1\right)^2\right] Y_t = C_t + I_t + G_t, \tag{39}$$

   where investment I_t is defined by the law of motion of capital

$$K_t = (1 - \delta)K_{t-1} + I_t. \tag{40}$$

Appendix A.3 lists all the equilibrium conditions. We focus on an equilibrium with constant productivity growth (i.e. $x_t = x, \forall t$) and constant probability of retirement (i.e. $\omega_t = \omega, \forall t$). We calculate the steady state and characterize the dynamics (see Appendix A.4 for details) of variables expressed in efficiency units (i.e., $q_t \equiv Q_t/(X_t N_t^w)$ for any variable Q_t). We discuss the demographic variables that drive the transition (the growth rate of the number of workers n_t and the probability of surviving γ_t) in the next section.

Table 1: Parameter values and steady state exogenous variables.

Parameter		Value	Description
ω	=	0.9948	Probability of remaining in the labor force
x	=	0.27%	Productivity growth rate
α	=	0.623	Labor share
δ	=	0.025	Depreciation rate
θ	=	12	Elasticity of substitution among varieties
ϕ_P	=	133	Price adjustment cost
σ	=	0.5	Elasticity of intertemporal substitution
g	=	0.17	Government spending (% of GDP)
b	=	4	Government debt (% of GDP)
β	=	0.9982	Individual discount factor

3 Quantitative Analysis

In this section, we present the results of the main experiment. We feed the model with data on demographic variables. The initial shock is unanticipated but agents are perfectly aware of the entire path of the exogenous variables thereafter. Two factors drive the demographic transition in the model: (i) A decrease in the growth rate of the labor force n_t and (ii) An increase in the probability of surviving γ_t. The next section describes the calibration of the parameters and of the exogenous processes for the demographic variables.

3.1 Calibration

Each period is one quarter. We initialize the model in a steady state which corresponds to 1990. Individuals are born at age 20. The average duration of employment equals $1/(1-\omega)$. We calibrate the probability of remaining in the labor force ω to 0.9948, which implies an average retirement age of 68, corresponding to the average effective retirement age in Japan between 1990 and 2005 (OECD Pensions Indicators).

To calibrate the probability of surviving, we use data and projections from the United Nation World Population Prospects (the 2010 revision) on life expectancy at birth. In steady state, the expected duration of retirement is equal to $1/(1-\gamma)$. The initial value of γ (equal to 0.9821) is calibrated to match an average 14 years retirement period, which corresponds to 79 years of life expectancy as for Japan in 1990. The final value of γ (equal to 0.9907) is calibrated to match an average 27 years retirement period, which corresponds to 92 years of life expectancy as projected for Japan in 2090. As in the data, this transition occurs smoothly in the model over 100 years.[7]

We calibrate the growth rate of the labor force n using data and projections from the National Institute of Population and Social Security Research. The annualized growth rate of the Japanese labor force was 0.8% 1990 and is projected to reach a minimum of -1.82% in 2040 before recovering somewhat (the projection for 2060 is -1.26%). We smooth the actual projections with a process that

[7] Data on life expectancy at 20 between 1965 and 2010 (thanks to Naohiro Ogawa and Rikiya Matsukura for providing us with these data) display no difference with life expectancy at birth plus twenty.

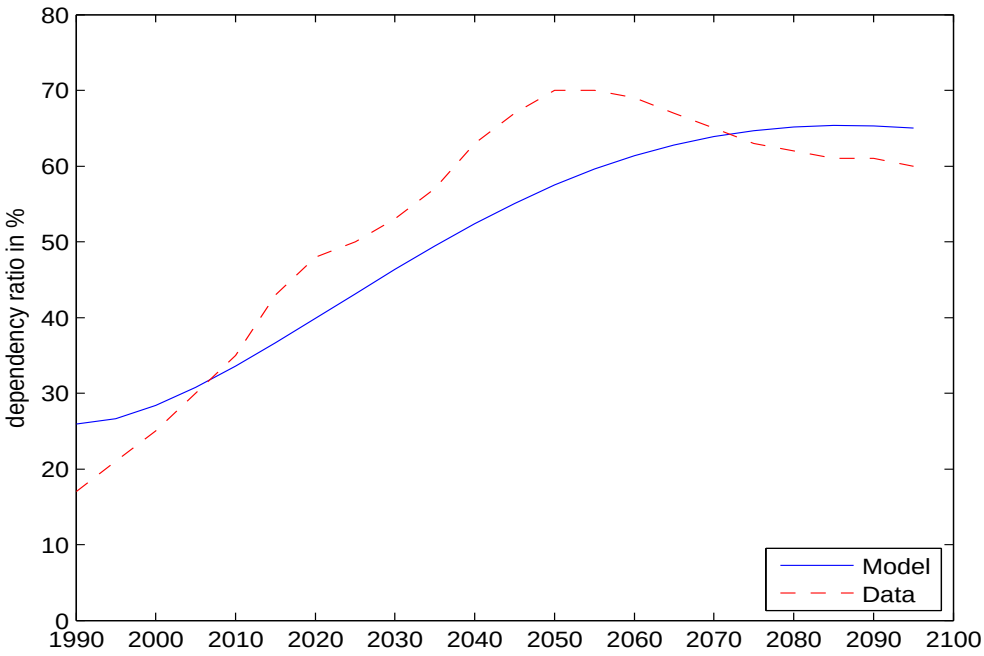


Figure 2: The continuous blue line is the dependency ratio in the model (number of retirees divided by number of workers). The dashed red line is the dependency ratio in the data (number of people 65 and over divided by number of people between the age of 15 and 64).

matches the initial value, reaches a minimum of 1.3% in 50 years and slowly recovers toward zero in the next 75 years.[8]

In spite of the simple demographic structure embedded in our model, we obtain a profile for the dependency ratio which captures the essence of the aging process in Japan fairly well (figure 2).

We calibrate x to match the average quarterly growth rate of labor-augmenting productivity calculated consistently with the methods in Hayashi and Prescott (2002) for the period 1990-2011, equal to 0.14% (0.56% annualized).[9] We adopt the same labor share α (equal to 0.623) and depreciation rate δ (equal to 0.025, or 10% in annualized terms) used to construct the capital stock.

We set θ equal to 12 to generate a 9% steady state markup, in line with the estimates for Japan in Høj et al. (2007). The price adjustment cost parameter ϕ_P is chosen so that the linearized Phillips curve has the same slope of a model with staggered price setting (Calvo, 1983) with four quarters average frequency of price changes, in line with the evidence for Japan in Higo and Saita (2007). The elasticity of intertemporal substitution σ equals 0.5 (Hall, 2009).

In the baseline exercise, we fix the fiscal variables (in percent of GDP) at their average value between 1990 and 2011 to highlight the key mechanism related to demographics. Government consumption as a fraction of GDP equals 17%. Government debt substantially differs whether we consider the gross or net series. In principle, the net series would be more appropriate. However, we worry

[8] The actual projections minimum of -1.82 in 2040 is a temporary blip. The forecast equals -1.3% in 2035 and -1.26% in 2060.

[9] Thanks to Nao Sudo for providing us with the data to compute the productivity series.

that some assets on the government balance sheet may be overvalued, possibly because of book-value accounting. Hence, we take the simple average of the series of gross and net government debt as a fraction of GDP from the IMF World Economic Outlook database, which yields a value for debt as a fraction of annualized GDP equal to 100%.

Conditional on the other parameters, we pick the individual discount factor β to match an annualized 4.6% real interest rate in the initial steady state, consistent with the data for Japan in 1990.[10]

Given our calibration, the model almost exactly matches the initial steady state consumption and investment ratios to GDP (53.2% and 29.8%, respectively) with their empirical averages over the period 1980-1990 (54.8% and 29.8%, respectively). The differences reflect the fact that government spending over GDP averaged 14% between 1980 and 1990 (while we calibrate this variable to the post-1990 mean of 17%) and the absence of trade with the rest of the world.[11]

Section 3 relaxes the assumption of constant productivity growth and fiscal variables and studies their importance relative to demographic forces in accounting for the decline in the real rate and, more generally, the macroeconomic adjustments during Japan's lost decade.

3.2 The Effects of the Demographic Transition on the Real Interest Rate

The equilibrium under strict inflation targeting isolates the effects of the demographic transition on macroeconomic variables, hence constituting a useful benchmark without potential distortions induced by monetary policy on allocations and relative prices. The main consequence of the demographic transition associated with the increase in life expectancy is a progressive decrease of the real interest rate, by about two percentage points between 1990 and 2011. Figure 3) shows that the model captures remarkably well the downward trend in the real interest rate observed in the data over the sample period.

The longer life expectancy induces households to increase their asset holdings. Both workers and retirees correctly anticipate a longer retirement period. Absent other forms of compensation, households need to self-finance their consumption over an extended life time horizon. The higher demand for assets decreases the equilibrium real interest rate.

This effect manifests through a reduction in the marginal propensities to consume of both retirees and workers. An increase in the probability of surviving directly reduces the marginal propensity to consume of retirees. Furthermore, if $\sigma < 1$ (as in our calibration), the downward adjustment of the real interest rate reinforces the direct impact of the increase in life expectancy. Differentiating the marginal propensity to consume of retirees with respect to the probability of surviving at steady state

[10] This measure of real interest rate is ex-post and equals the lending rate (from the IMF International Financial Statistics database) minus the realized inflation rate of the GDP deflator (from the World Bank World Development Indicators).

[11] We abstract from open economy considerations throughout the analysis. While outside forces may have played some role, we follow the existing literature in focusing on domestic factors as potential explanations for the Japanese experience.

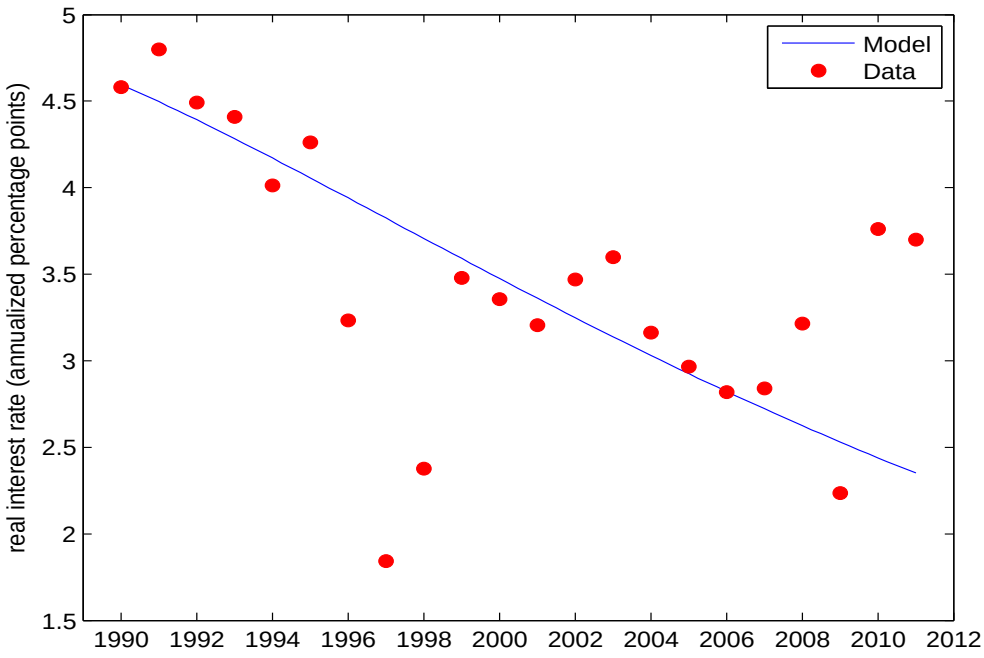


Figure 3: The continuous blue line is the annualized real interest rate in the model under strict inflation targeting. Each red dot corresponds to the annual ex-post realization of the real interest rate in the data (lending rate minus the inflation rate of the GDP deflator).

highlights these two effects

$$\frac{\partial \xi^r}{\partial \gamma} = -\beta^\sigma R^{\sigma-1}\left[\underbrace{1}_{\text{direct effect}} + \underbrace{(\sigma-1)\varepsilon_{R,\gamma}}_{\text{general equilibrium effect}}\right] < 0, \tag{41}$$

where $\varepsilon_{R,\gamma} \equiv (\partial R/R)/(\partial\gamma/\gamma) < 0$ is the elasticity of the real interest rate to the probability of surviving. Using the implicit function theorem, we can show that the steady state effect of an increase in life expectancy on the marginal propensity to consume of workers is also negative

$$\frac{\partial \xi^w}{\partial \gamma} = -\frac{\beta^\sigma(\Omega R)^{\sigma-1}\left[(\sigma-1)\varepsilon_{R,\gamma} - \left(\frac{\Omega-\omega}{\Omega}\right)\varepsilon_{\xi^r,\gamma}\right]}{\gamma\left[1+\beta^\sigma(\Omega R)^{\sigma-1}\left(\frac{\Omega-\omega}{\Omega}\right)\frac{1}{\xi^w}\right]} < 0,$$

where $\varepsilon_{\xi^r,\gamma} \equiv (\partial\xi^r/\xi^r)/(\partial\gamma/\gamma) < 0$ because of (41) above.

In addition to the effects of longer life expectancy, the decrease in the growth rate of the labor force acts as a negative shock to the growth rate of productivity by diminishing the efficiency use of capital. This effect reduces the expected return on capital, thus reinforcing the downward pressure on the real interest rate because of no arbitrage. At the same time, however, the incentives to invest also decrease because assets are relatively less attractive. On balance, the saving-for-retirement motive prevails for workers, whose consumption declines driven by the decrease in the marginal propensity to consume. Conversely, the decreased attractiveness of investment dominates for retirees, whose

consumption increases. The net aggregate effect is a reallocation of resources from consumption to investment. Interestingly, the changes in these variables as percentage of GDP over the simulation horizon are quite small, about 1.5 percentage points relative to the initial steady state.[12]

Contrary to the results in the model, investment and the saving rate decreased after 1990, suggesting that other forces may have played a more significant role than the demographic transition in the macroeconomic adjustment. In particular, Chen et al. (2006) show that the decline in TFP growth—which we keep constant in this section—can account for the decline in the Japanese saving rate, especially since 1990. In section 3, we show that the introduction time-varying productivity growth and fiscal variables better aligns our model with the data while confirming—in fact, reinforcing—the result for the real interest rate.

3.3 Interest Rate Policy and Persistent Deflation

Under strict inflation targeting, the nominal interest rate equals the real interest rate in every period. By replicating the flexible-price equilibrium, this policy internalizes the effects of the demographic transition on interest rates. The resulting equilibrium represents the non-stationary version of the result in the canonical New Keynesian model under optimal policy, according to which the nominal interest rate should exactly track the efficient real interest rate in each period (Clarida et al., 1999; Woodford, 2003).

In this section, we show that the failure to internalize the downward pressures of the demographic transition on the real interest rate may explain the persistent deflation experienced by Japan since 1990. For this purpose, we assume that the central bank follows a simplified version of the interest rate feedback rule originally proposed by Taylor (1993)

$$\frac{R_t}{R} = \pi_t^{\varphi_\pi}, \tag{42}$$

where $\varphi_\pi > 1$. In particular, we assume that $\varphi_\pi = 2$. Because the response of the nominal interest rate to inflation is symmetric and φ_π is fairly high relative to the standard value used in the literature (1.5), our explanation of the Japanese experience does not rely on a potentially weak or asymmetric response of the central bank to deflationary pressures (Bernanke and Gertler, 1999).

The top panel of figure 4 compares nominal (continuous blue line) and real (dashed red line) interest rates under the Taylor rule (42). As mentioned, under inflation targeting, the nominal interest rate exactly tracks the real interest rate because inflation is equal to zero in every period. Conversely, rule (42) does not respond to movements in the real interest rate associated with the demographic transition. Holding inflation constant, the Taylor rule would tend to set the nominal interest rate systematically too high in an environment of declining real rates. The failure to internalize the consequences of the demographic transition on the real interest rate in the monetary policy process creates a perverse general equilibrium effect. The systematic overshooting of the nominal interest rate for given inflation rate depresses demand, hence generating deflation. The Taylor rule endogenously

[12] As the number of workers decreases and retirees live longer, the dependency ratio ψ_t (the number of retirees relative to the number of workers) increases. Consequently, the distribution of wealth shifts in favor of retirees. This change is large: The fraction of assets held by retirees doubles, increasing from 15 to 30% over the simulation horizon.

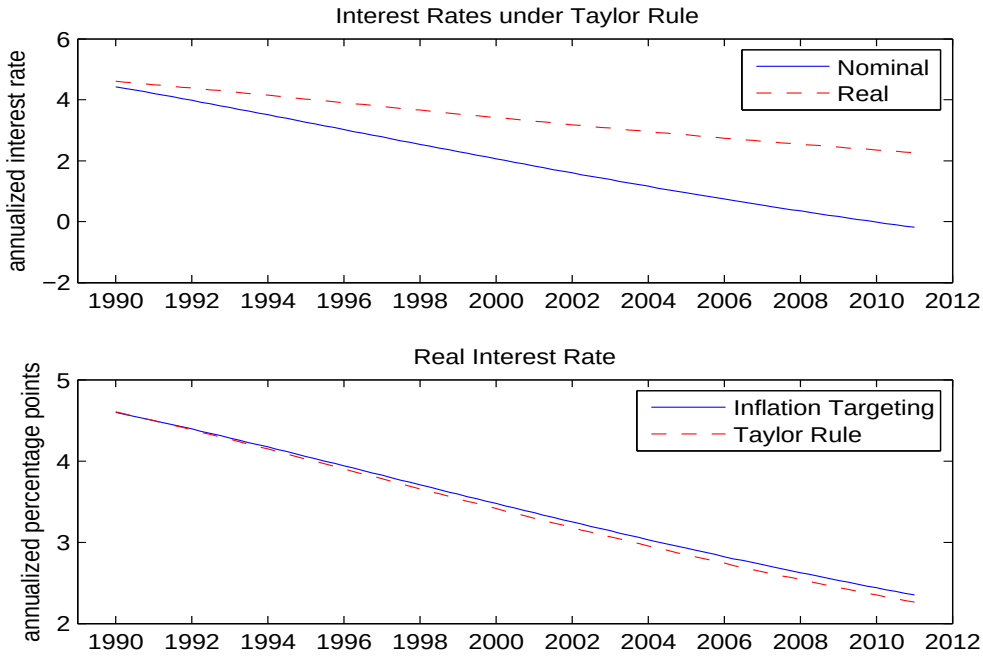


Figure 4: Top panel: Nominal (continuous blue line) and real (dashed red line) interest rate under Taylor rule. Bottom panel: Real interest rate under inflation targeting (continuous blue line) and Taylor rule (dashed red).

responds to the deflationary pressures forcing a lower nominal interest rate in equilibrium than in case of inflation targeting.

Interestingly, the bottom panel of figure 4 shows that the resulting real interest rate is almost identical in the two cases. Therefore, the model continues to capture well the downward trend in real interest rates observed in the data also when the central bank follows the Taylor rule. However, under the interest rate rule regime, the composition of the real interest rate is different than in the case of inflation targeting. The Taylor rule achieves approximatively the same real rate with a decisively lower nominal rate and deflation, as opposed to zero inflation and a nominal interest rate that exactly tracks the real interest rate. Figure 5 shows that the magnitude of deflation obtained in the model is remarkably comparable to the deviations of CPI inflation from an implicit inflation target of 1.5% (Leigh, 2009) in the data.

The macroeconomics adjustment under the Taylor rule is very similar to the baseline case described in the previous section. Quantitatively, the differences are minimal. Again, section 3 provides a richer assessment of the macroeconomic outlook for Japan post-1990 in which productivity growth and fiscal variables also play a role. Before moving to that analysis, the next section presents evidence on one key implication of the model, that is the forecastability of real interest rate and inflation.

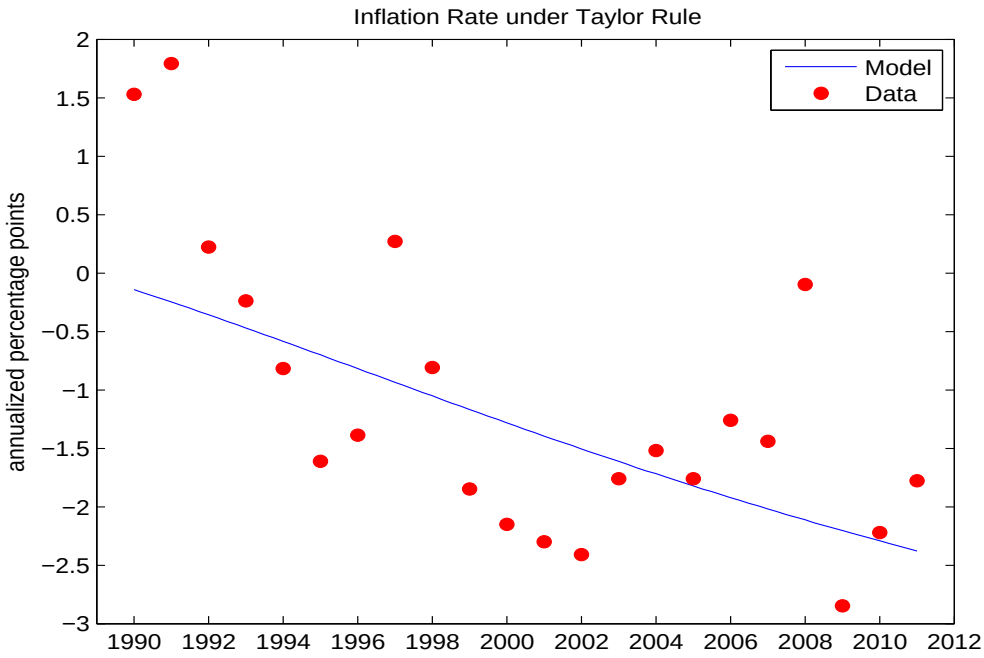


Figure 5: The continuous blue line is the annualized inflation rate in the model under the Taylor rule. Each red dot corresponds to the annual CPI inflation rate in the data minus the implicit inflation target, assumed to be 1.5%.

4 Were Declining Real Interest Rates Forecastable?

Although subject to revisions, demographic trends are typically slow moving and projections are usually available over long horizons. Our model suggests that declining real interest rates are the consequence of the demographic transition, independently of the monetary policy rule.[13] Therefore, a key implication of our analysis is that agents should have been able to forecast the downward pressure on the real interest rate. In addition, if agents had anticipated that the central bank would not internalize the downward pressure on real interest rates, deflation would also have been forecastable. In this section, we provide two types of evidence in support of this hypothesis, the first based on descriptive survey evidence while the second relying on a statistical forecasting model.

4.1 Survey Evidence

[To be completed]

4.2 Forecasting Model

[To be completed]

[13] Conversely, deflation would have been forecastable only if agents had anticipated that the central bank would not internalize the downward pressure on real interest rates. In this sense, the result in Erceg et al. (2002) that deflation was not forecastable is consistent with our finding under the assumption that agents perceived the central bank would have internalized the effects of the demographic transition on the real interest rate.

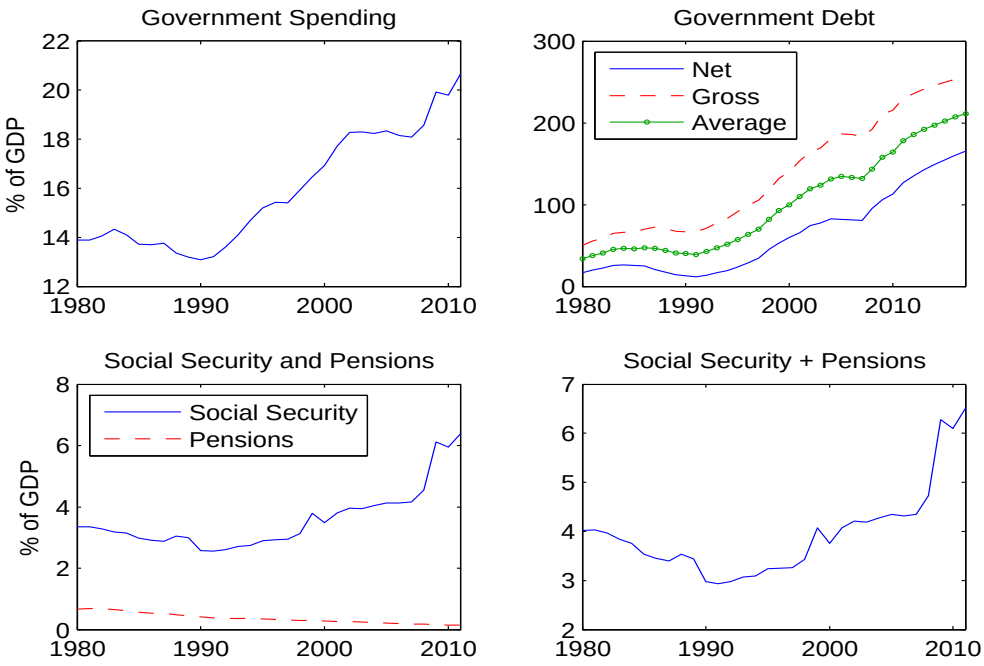


Figure 6: Ratio of government spending (top-left), debt (top-right), social security and pensions (bottom) to GDP in Japan. Source: DLX/Haver and IMF World Economic Outlook Database April 2012.

5 Do Productivity Growth and Fiscal Policy Change the Picture?

So far, we have fixed productivity growth and fiscal policy variables in percent of GDP at their post-1990 averages. While this approach has been useful to highlight the main mechanism we want to emphasize in this paper, these variables have changed substantially over the last three decades.

On the productivity front, Hayashi and Prescott (2002) argue that the slowdown in TFP growth was the main cause of Japan's lost decade. Indeed, productivity growth averaged 4.01% annualized between 1980 and 1990 before slowing down to an average rate of 0.56% (the value that we have used in our baseline calibration) between 1990 and 2011.

Figure 6 shows the evolution of government spending, debt, social security benefits and pensions, as a fraction of GDP. Even before the recent recession, government spending increased from about 14% to 18% of GDP. Perhaps most notably, over the same period government debt ballooned from relatively moderate values to the highest level among industrialized economies.[14] Conversely, social security and pensions have remained fairly stable, around 3.8% and 0.27%, respectively, with the exception of the increase in benefits during the last recession.

In this section, we ask if the developments in productivity growth and fiscal policy can substantially alter our main results about the real interest rate and deflation. For this purpose, on top of the demographic variables, we also feed the model with exogenous processes for labor-augmenting

[14] As figure 6 shows, the increase varies from a factor of 10 for net debt (from 13% in 1990 to 127% in 2011) to a factor of 3.5 for gross debt (from 67% in 1990 to 229% in 2011). The IMF projections display no sign of consolidation for the five years ahead.

productivity growth x_t as well as government spending and debt in percent of GDP (g_t and b_t) that match the paths observed in the data. Furthermore, the next section extends the model to include social security benefits for retirees, which we calibrate to post-1990 sample average.

5.1 Introducing Social Security Benefits

We capture social security benefits as simple lump-sum transfers to individuals who leave the labor force. As a consequence, the only modification in the household problem is that the budget constraint for a retiree becomes

$$C_t^r(j,\tau) + A_t^r(j,\tau) = \frac{R_{t-1}A_{t-1}^r(j,\tau)}{\gamma_t \pi_t} + E_t^r, \tag{43}$$

where E_t^r denotes social security benefits and is not indexed by either j or τ because of the lump-sum nature of the transfer. Appendix B.1 shows that the consumption function that solves a generic retiree's optimization problem is

$$C_t^r(j,\tau) = \xi_t^r \left[\frac{R_{t-1}A_{t-1}^r(j,\tau)}{\gamma_t \pi_t} + S_t^r \right], \tag{44}$$

where S_t^r represents the present discounted value of social security benefits for a retiree

$$S_t^r = E_t^r + \frac{S_{t+1}^r}{R_t/(\gamma_{t+1}\pi_{t+1})}. \tag{45}$$

While the budget constraint for a worker does not change, the presence of social security transfers alters her consumption function (see again Appendix B.1 for the derivations) because workers anticipate the extension of benefits after retirement

$$C_t^w(j) = \xi_t^w \left[\frac{R_{t-1}A_{t-1}^w(j)}{\pi_t} + H_t^w + S_t^w \right], \tag{46}$$

where S_t^w is the present discounted value of social security benefits for a worker after retirement

$$S_t^w = \frac{(\Omega_{t+1} - \omega_{t+1})S_{t+1}^r}{\Omega_{t+1}R_t/\pi_{t+1}} + \frac{\omega_{t+1}S_{t+1}^w}{\Omega_{t+1}R_t/\pi_{t+1}} \tag{47}$$

Importantly, the presence of social security benefits leaves unchanged the expressions for the marginal propensities to consume of both retirees and workers. Thus, aggregation remains feasible and follows the same steps as as in the baseline model (see Appendix B.3 for the details). The only other relevant change in the model is that the government budget constraint now also accounts for social security as an additional expenditure item

$$\frac{B_t}{P_t} = \frac{R_{t-1}}{\pi_t}\frac{B_{t-1}}{P_{t-1}} + G_t + E_t - T_t. \tag{48}$$

We keep the same calibration as in the baseline experiment, except for productivity growth and fiscal variables. The production function (29) implies a series for the level of labor-augmenting pro-

ductivity according to

$$X_t = \left[\frac{Y_t}{(N_t^w)^{\alpha} K_{t-1}^{1-\alpha}} \right]^{\frac{1}{\alpha}}. \tag{49}$$

We use data on GDP and labor force times hours worked as our measures of output (Y_t) and labor input (N_t^w), respectively. We construct the capital input (K_t) with the perpetual inventory method, using the capital stock in 1981 as initial value. Productivity growth averages 4.01% before 1990 and 0.56% thereafter. We feed the model with the realized values of productivity growth between 1990 and 2011. After 2011, productivity growth converges to its post-1990 average over three years.

We calibrate the exogenous processes for government spending and debt as a fraction of GDP based on figure 6. In the initial steady state, government spending accounts for 14% of GDP while debt amounts to 43% of annualized GDP. We feed the model with exogenous processes for these two variables such that their dynamics match the evolution in the data over the period 1990-2011 and 1990-2017, respectively. We calibrate the ratio of social security benefits and pensions to a constant fraction of GDP, equal to 4%. As mentioned, the rationale for this assumption is that the sum of social security benefits and pensions has remained roughly stable until the recent recession. As figure 6 shows, social security benefits, not pensions, drive the recent increase which is likely to be temporary and of countercyclical nature.

We adjust the individual discount factor so that the real interest rate in the initial steady state corresponds to the baseline experiment (i.e. 4.6%).[15]

We summarize the macroeconomic adjustment in response to demographic forces, variations of productivity growth, and changes in fiscal variables, by comparing the saving rate in the model with the data (figure 7). Following Chen et al. (2006), we define the saving rate as

$$sr_t \equiv \frac{y_t - c_t - g_t - \delta k_{t-1}}{y - \delta k_{t-1}} = \frac{k_t - k_{t-1}}{y - \delta k_{t-1}}. \tag{50}$$

Using the neoclassical growth model, these authors show that productivity growth explains the decline in the Japanese saving rate between 1955 and 2000. Other factors, such as time-varying depreciation rates, capital income taxes and government spending, play a less important role. The introduction of time-varying productivity growth and fiscal variables in our model leads to a similar conclusion. The simulation captures fairly well the downward trend in the saving rate observed in the data.[16]

In our model, on average, workers continue to save (as in the baseline case) due to the prospects of a longer retirement period. However, in periods of particularly low productivity growth, households tend to increase their consumption temporarily. This effect is particularly strong for retirees, whose lifetime horizon is comparatively shorter than for workers. Contrary to the baseline model, consumption for retirees (as a percentage of GDP) now increases throughout the simulation horizon. In addition, the trend increase in government spending constitutes an additional force that depresses the aggregate

[15] We retain the assumption of perfect foresight also throughout this part of the analysis. While this approach may be less than desirable for fiscal policy and especially for productivity growth, we compare our results to contributions in the literature (e.g. Chen et al., 2006) that have employed the same methodology.

[16] Our results are not directly comparable to Chen et al. (2006), as we use the most recent vintage of data (93SNA), available from 1980. Yet, the broad message is the same.

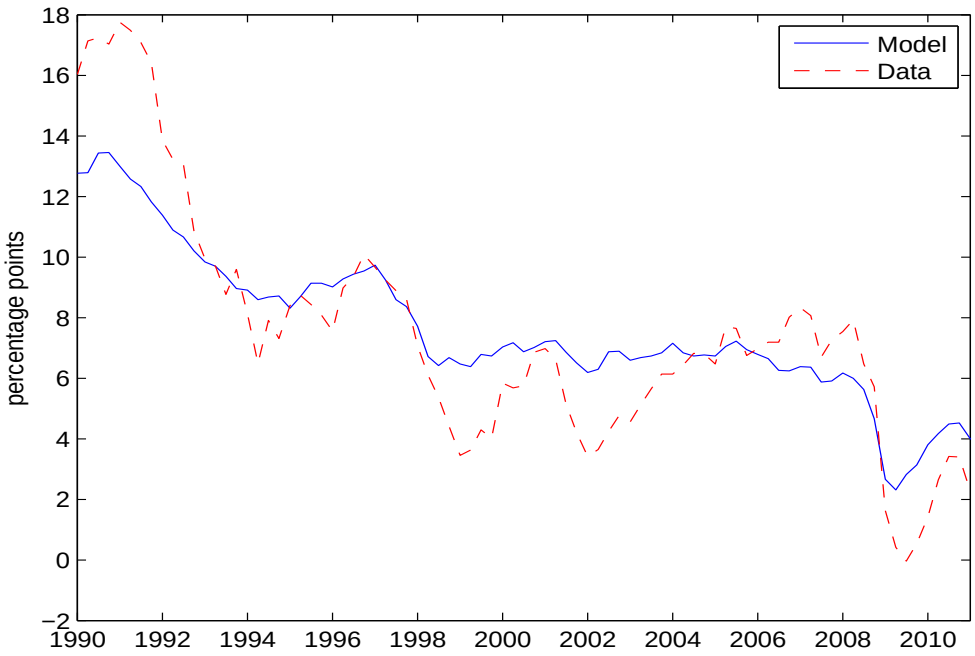


Figure 7: The Japanese saving rate in the model (continuous blue line) and in the data (dashed red line) between 1990 and 2011.

saving rate.

Figure 8 shows that, in spite of the declining saving rate, the real interest rate continues to decrease over the simulation horizon when not only demographic forces but also productivity growth and fiscal variables are time-varying (marked blue line), as in the baseline experiment. Perhaps not surprisingly then, under a Taylor rule, inflation also declines. For both variables, the magnitudes are very much comparable to the case in which productivity and fiscal variables are constant at their 1990-2011 average.

The demographic transition continues to be key to obtain a declining profile of real interest rates and inflation as in the data. Figure 8 shows that productivity growth and fiscal variables alone would imply a slightly increasing path for both variables. As mentioned, the intuition is that on average workers—who are the marginal savers—continue to be willing to save given the longer expected lifetime horizon. The presence of time-varying productivity growth introduces some short run fluctuations on this trend, while the increase in government spending only partially crowds out private consumption, as Ricardian equivalence fails due to the life-cycle structure of the model In particular, retirees are not very sensitive to the increase in government consumption and treat government bonds partly as net wealth. Conversely, the increase in public expenditure affects workers more directly, thus reinforcing the effect on the real interest rate.

In sum, the explicit consideration of the dynamics of productivity growth and fiscal variables confirms the main findings of the baseline simulations. The demographic transition exerts downward pressure on the real interest rate. If the monetary authority does not internalize this effect, deflation emerges in equilibrium.

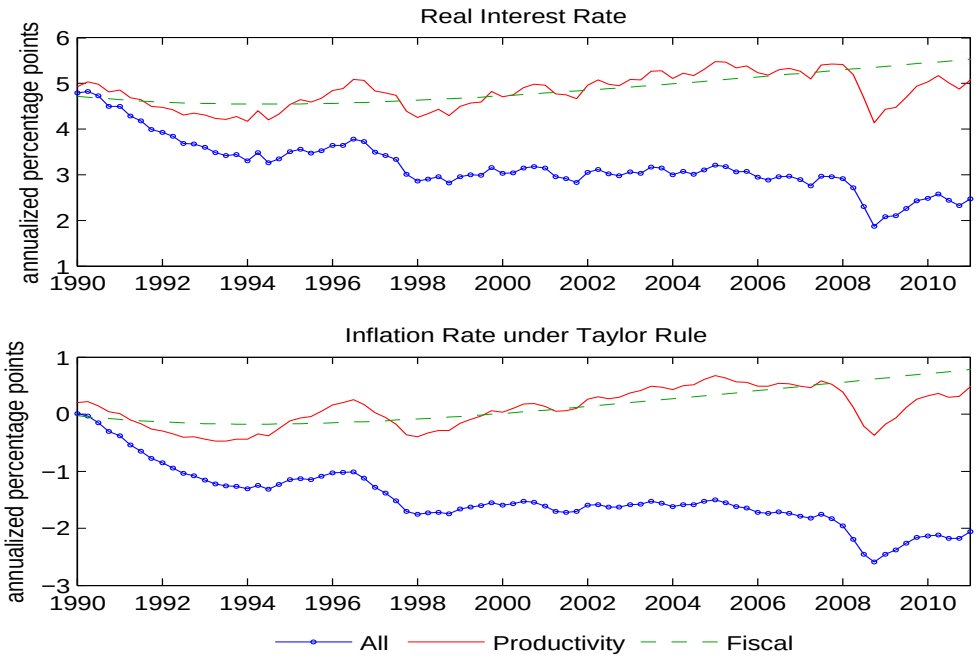


Figure 8: Top panel: Real interest rate. Bottom panel: Inflation rate under Taylor rule. The marked blue line is the simulation with time-varying demographic, productivity and fiscal variables. The continuous red line is the simulation with time-varying productivity only. The dashed green line is the simulation with time-varying fiscal variables only.

6 Conclusion

[To be completed].

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A Model Solution

A.1 Retirees

The first-order conditions with respect to capital, government bonds and shares are, respectively,

$$\begin{aligned}
\left(C_t^r\left(j,\tau\right)\right)^{\rho-1} &= \beta\gamma_{t+1}\left(V_{t+1}^r\left(j,\tau\right)\right)^{\rho-1}\frac{\partial V_{t+1}^r\left(j,\tau\right)}{\partial K_t^r\left(j,\tau\right)}\\
\left(C_t^r\left(j,\tau\right)\right)^{\rho-1} &= \beta\gamma_{t+1}\left(V_{t+1}^r\left(j,\tau\right)\right)^{\rho-1}\frac{\partial V_{t+1}^r\left(j,\tau\right)}{\partial\left(B_t^r\left(j,\tau\right)/P_t\right)}\\
P_{Ft}\left(C_t^r\left(j,\tau\right)\right)^{\rho-1} &= \beta\gamma_{t+1}\left(V_{t+1}^r\left(j,\tau\right)\right)^{\rho-1}\frac{\partial V_{t+1}^r\left(j,\tau\right)}{\partial x_{Ft}^r\left(j,\tau\right)}.
\end{aligned}$$

The envelope conditions, used to obtain the Euler equations for the retiree's problem, are

$$\begin{aligned}
\frac{\partial V_t^r\left(j,\tau\right)}{\partial K_{t-1}^r\left(j,\tau\right)} &= \left(V_t^r\left(j,\tau\right)\right)^{1-\rho}\left(C_t^r\left(j,\tau\right)\right)^{\rho-1}\frac{R_t^K+(1-\delta)}{\gamma_t}\\
\frac{\partial V_t^r\left(j,\tau\right)}{\partial\left(B_{t-1}^r\left(j,\tau\right)/P_{t-1}\right)} &= \left(V_t^r\left(j,\tau\right)\right)^{1-\rho}\left(C_t^r\left(j,\tau\right)\right)^{\rho-1}\frac{R_{t-1}}{\gamma_t\pi_t}\\
\frac{\partial V_t^r\left(j,\tau\right)}{\partial x_{Ft-1}^r\left(j,\tau\right)} &= \left(V_t^r\left(j,\tau\right)\right)^{1-\rho}\left(C_t^r\left(j,\tau\right)\right)^{\rho-1}\frac{P_{Ft}+D_{Ft}}{\gamma_t}.
\end{aligned}$$

Combining the two sets of optimality conditions yields the standard Euler equations for bonds, capital and equity

$$1=\beta\frac{R_t}{\pi_{t+1}}\left[\frac{C_{t+1}^r(j,\tau)}{C_t^r(j,\tau)}\right]^{-\frac{1}{\sigma}}=\beta\left[R_{t+1}^K+(1-\delta)\right]\left[\frac{C_{t+1}^r(j,\tau)}{C_t^r(j,\tau)}\right]^{-\frac{1}{\sigma}}=\beta\frac{P_{Ft+1}+D_{Ft+1}}{P_{Ft}}\left[\frac{C_{t+1}^r(j,\tau)}{C_t^r(j,\tau)}\right]^{-\frac{1}{\sigma}}, \tag{51}$$

To solve the problem of retirees, guess that consumption is a fraction of total wealth

$$C_t^r\left(j,\tau\right)=\xi_t^r\left(\frac{R_{t-1}A_{t-1}^r(j,\tau)}{\gamma_t\pi_t}\right). \tag{52}$$

Substitution into the Euler equation (51) yields a law of motion for the marginal propensity to consume of a retiree ξ_t^r

$$\xi_{t+1}^r\left(\frac{R_tA_t^r(j,\tau)}{\gamma_{t+1}\pi_{t+1}}\right)=\left(\frac{\beta R_t}{\pi_{t+1}}\right)^{\sigma}\xi_t^r\left(\frac{R_{t-1}A_{t-1}^r(j,\tau)}{\gamma_t\pi_t}\right). \tag{53}$$

Substitution of the guess (52) into the budget constraint of a retiree (10) leads to the expression below for the dynamics of asset holdings

$$A_t^r(j,\tau)=\left(1-\xi_t^r\right)\frac{R_{t-1}A_{t-1}^r(j,\tau)}{\gamma_t\pi_t}.$$

Combining the last expression with the law of motion for the marginal propensity to consume of a retiree (53) yields the first-order non-linear difference equation for ξ_t^r (12) in the text.

Moreover, conjecture that the value function is linear in consumption

$$V_t^r(j,\tau) = \Delta_t^r C_t^r(j,\tau). \tag{54}$$

Then, from (5), it must be the case that

$$(\Delta_t^r C_t^r(j,\tau))^\rho = (C_t^r(j,\tau))^\rho + \beta\gamma_{t+1}\left(\Delta_{t+1}^r C_{t+1}^r(j,\tau)\right)^\rho.$$

Substituting for consumption at $t+1$ from (51) and simplifying yields

$$(\Delta_t^r)^\rho = 1 + \gamma_{t+1}\beta^\sigma\left(\frac{R_t}{\pi_{t+1}}\right)^{\sigma-1}\left(\Delta_{t+1}^r\right)^\rho.$$

From (12), it then follows that the proportionality term in (54) is

$$\Delta_t^r = (\xi_t^r)^{\frac{\sigma}{1-\sigma}}.$$

A.2 Workers

The first-order condition for workers' asset holdings is

$$(C_t^w(j))^{\rho-1} = \beta\left[\omega_{t+1}V_{t+1}^w(j) + (1-\omega_{t+1})V_{t+1}^r(j,t+1)\right]^{\rho-1} \left[\omega_{t+1}\frac{\partial V_{t+1}^w(j)}{\partial A_t^w(j)} + (1-\omega_{t+1})\frac{\partial V_{t+1}^r(j,t+1)}{\partial A_t^w(j)}\right].$$

The envelope conditions are for the worker's problem are

$$\frac{\partial V_t^w(j)}{\partial A_{t-1}^w(j)} = (V_t^w(j))^{1-\rho}(C_t^w(j))^{\rho-1}\frac{R_{t-1}}{\pi_t}$$

and

$$\frac{\partial V_t^r(j,t)}{\partial A_{t-1}^w(j)} = \frac{\partial V_t^r(j,t)}{\partial A_{t-1}^r(j,t)}\frac{\partial A_{t-1}^r(j,t)}{\partial A_{t-1}^w(j)} = \frac{\partial V_t^r(j,t)}{\partial A_{t-1}^r(j,t)},$$

where the last equality follows from (7) – i.e., from the initial conditions of the retirees' optimization problem.

Combining the first order condition with the envelope conditions yields an Euler equation for workers of cohort j

$$(C_t^w(j))^{\rho-1} = \frac{\beta R_t}{\pi_{t+1}}\left[\omega_{t+1}V_{t+1}^w(j) + (1-\omega_{t+1})V_{t+1}^r(j,t+1)\right]^{\rho-1} \left[\omega_{t+1}\left(V_{t+1}^w(j)\right)^{1-\rho}\left(C_{t+1}^w(j)\right)^{\rho-1} + (1-\omega_{t+1})\left(V_{t+1}^r(j,t+1)\right)^{1-\rho}\left(C_{t+1}^r(j,t+1)\right)^{\rho-1}\right]. \tag{55}$$

To solve the problem of workers, conjecture that their value function has the same form as (54)

$$V_t^w(j) = \Delta_t^w C_t^w(j). \tag{56}$$

Substituting this guess back into the Euler equation (55) together with (54) leads to

$$\left(C_t^w(j)\right)^{\rho-1} = \frac{\beta R_t}{\pi_{t+1}}\left[\omega_{t+1}\Delta_{t+1}^w C_{t+1}^w(j) + (1-\omega_{t+1})\Delta_{t+1}^r C_{t+1}^r(j,t+1)\right]^{\rho-1} \left[\omega_{t+1}\left(\Delta_{t+1}^w\right)^{1-\rho} + (1-\omega_{t+1})\left(\Delta_{t+1}^r\right)^{1-\rho}\right]. \quad (57)$$

Defining the adjustment term $\Omega_t \equiv \omega_t + (1-\omega_t)\left(\frac{\Delta_t^r}{\Delta_t^w}\right)^{1-\rho}$, the Euler equation becomes

$$\omega_{t+1}C_{t+1}^w(j) + (1-\omega_{t+1})\left(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w}\right)C_{t+1}^r(j,t+1) = \left(\frac{\beta\Omega_{t+1}R_t}{\pi_{t+1}}\right)^{\sigma} C_t^w(j). \quad (58)$$

The guess for the decision rule of a worker is

$$C_t^w(j) = \xi_t^w\left(\frac{R_{t-1}A_{t-1}^w(j)}{\pi_t} + H_t^w\right), \quad (59)$$

where human wealth H_t^w is defined in (22).

From (11), the decision rule for a retiree born in period j who just left the labor force is

$$C_t^r(j) = \xi_t^r\frac{R_{t-1}A_{t-1}^w(j)}{\pi_t}.$$

Substituting the last expression into the Euler equation yields

$$\omega_{t+1}\left(A_t^w(j) + \frac{H_{t+1}^w}{R_t/\pi_{t+1}}\right) + (1-\omega_{t+1})\left(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w}\right)\epsilon_{t+1}A_t^w(j) = (\beta\Omega_{t+1})^{\sigma}\left(\frac{R_t}{\pi_{t+1}}\right)^{\sigma-1}\frac{\xi_t^w}{\xi_{t+1}^w}\left(\frac{R_{t-1}A_{t-1}^w(j)}{\pi_t} + H_t^w\right),$$

where $\epsilon_t \equiv \xi_t^r/\xi_t^w$. Using the definition of Ω_t, the last expression becomes

$$A_t^w(j) + \frac{\omega_{t+1}H_{t+1}^w}{\Omega_{t+1}R_{t+1}/\pi_{t+1}} = \beta^{\sigma}\left(\frac{\Omega_{t+1}R_t}{\pi_{t+1}}\right)^{\sigma-1}\frac{\xi_t^w}{\xi_{t+1}^w}\left(\frac{R_{t-1}A_{t-1}^w(j)}{\pi_t} + H_t^w\right). \quad (60)$$

Moreover, from the budget constraint of a worker and the guess (59), we obtain the law of motion for worker j's wealth

$$A_t^w(j) + \frac{\omega_{t+1}H_{t+1}^w}{\Omega_{t+1}R_t/\pi_{t+1}} = (1-\xi_t^w)\left(\frac{R_{t-1}A_{t-1}^w(j)}{\pi_t} + H_t^w\right).$$

Substituting this result back into the Euler equation (60), it follows that the marginal propensity to consume for a worker evolves according to expression (18) in the text.

Finally, the original guess of the value function (19) is valid if

$$\left(\Delta_t^w C_t^w(j)\right)^{\rho} = \left(C_t^w(j)\right)^{\rho} + \beta\left[\omega_{t+1}\Delta_{t+1}^w C_{t+1}^w(j) + (1-\omega_{t+1})\Delta_{t+1}^r C_{t+1}^r(j,t+1)\right]^{\rho}.$$

The last equation, combined with (58), yields

$$(\Delta_t^w)^\rho = 1 + \beta^\sigma \left(\frac{\Omega_{t+1} R_t}{\pi_{t+1}}\right)^{\sigma-1} \left(\Delta_{t+1}^w\right)^\rho .$$

Expression (18) then implies that

$$\Delta_t^w = (\xi_t^w)^{\frac{\sigma}{1-\sigma}}.$$

Hence, the ratio of the proportionality factors in the value function is $\Delta_{t+1}^r / \Delta_{t+1}^w = \epsilon_t^{-(1/\rho)}$ and

$$\Omega_t = \omega_t + (1-\omega_t)\, \epsilon_t^{\frac{1}{1-\sigma}}.$$

A.3 List of Equilibrium Conditions

1. Dependency ratio:

$$(1+n_t)\,\psi_t = (1-\omega_t) + \gamma_t \psi_{t-1}. \tag{61}$$

2. Aggregate consumption for retirees:

$$C_t^r = \xi_t^r \left(\frac{R_{t-1}\lambda_{t-1}A_{t-1}}{\pi_t}\right). \tag{62}$$

3. Marginal propensity to consume for retirees:

$$\xi_t^r = 1 - \gamma_{t+1}\beta^\sigma \left(\frac{R_t}{\pi_{t+1}}\right)^{\sigma-1} \frac{\xi_t^r}{\xi_{t+1}^r}. \tag{63}$$

4. Retirees' welfare:

$$V_t^r = (\xi_t^r)^{\frac{\sigma}{1-\sigma}} C_t^r. \tag{64}$$

5. Aggregate consumption for workers:

$$C_t^w = \xi_t^w \left(\frac{R_{t-1}A_{t-1}^w}{\pi_t} + H_t\right). \tag{65}$$

6. Marginal propensity to consume for workers:

$$\xi_t^w = 1 - \beta^\sigma \left(\frac{\Omega_{t+1}R_t}{\pi_{t+1}}\right)^{\sigma-1} \frac{\xi_t^w}{\xi_{t+1}^w}. \tag{66}$$

7. Workers' welfare:

$$V_t^w = (\xi_t^w)^{\frac{\sigma}{1-\sigma}} C_t^w. \tag{67}$$

8. Aggregate value of human wealth:

$$H_t = \frac{W_t N_t^w}{P_t} - T_t + \frac{\omega_{t+1} H_{t+1}}{(1+n_{t+1})\,\Omega_{t+1} R_t/\pi_{t+1}}. \tag{68}$$

9. Adjustment factor:

$$\Omega_t = \omega_t + (1 - \omega_t)\, \epsilon_t^{\frac{1}{1-\sigma}}. \tag{69}$$

10. Ratio of marginal propensities to consume:

$$\epsilon_t = \xi_t^r / \xi_t^w. \tag{70}$$

11. Aggregate consumption:

$$C_t = \xi_t^w \left[(1 - \lambda_{t-1}) \frac{R_{t-1} A_{t-1}}{\pi_t} + H_t + \epsilon_t \lambda_{t-1} \frac{R_{t-1} A_{t-1}}{\pi_t} \right]. \tag{71}$$

12. Distribution of wealth

$$\lambda_t A_t = (1 - \omega_{t+1})\, A_t + \omega_{t+1}\, (1 - \xi_t^r)\, \lambda_{t-1} \frac{R_{t-1} A_{t-1}}{\pi_t}. \tag{72}$$

13. Retirees assets:

$$\frac{A_t^r}{A_t} = \lambda_t. \tag{73}$$

14. Workers assets:

$$\frac{A_t^w}{A_t} = 1 - \lambda_t. \tag{74}$$

15. No arbitrage:

$$\frac{R_t}{\pi_{t+1}} = R_{t+1}^K + (1 - \delta) = \frac{P_{Ft+1} + D_{Ft+1}}{P_{Ft}}. \tag{75}$$

16. Aggregate welfare:

$$V_t = \frac{(\xi_t^w)^{\frac{\sigma}{1-\sigma}} \left(C_t^w + \psi_t \epsilon_t^{\frac{\sigma}{1-\sigma}} C_t^r \right)}{1 + \psi_t}. \tag{76}$$

17. Real wage:

$$\frac{W_t}{P_t} = mc_t \frac{\alpha Y_t}{N_t^w}. \tag{77}$$

18. Rental rate:

$$R_t^K = mc_t \frac{(1 - \alpha) Y_t}{K_{t-1}}. \tag{78}$$

19. Marginal cost:

$$mc_t = \frac{\left(\frac{W_t / P_t}{X_t} \right)^{\alpha} (R_t^K)^{1-\alpha}}{\alpha^{\alpha} (1 - \alpha)^{1-\alpha}}. \tag{79}$$

20. Phillips curve:

$$(\pi_t - 1)\, \pi_t = \frac{\theta - 1}{\phi_P} \left(\frac{\theta}{\theta - 1} mc_t - 1 \right) + \frac{1}{R_t / \pi_{t+1}} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - 1)\, \pi_{t+1}. \tag{80}$$

21. Government budget constraint:

$$\frac{B_t}{P_t} = \frac{R_{t-1}}{\pi_t}\frac{B_{t-1}}{P_{t-1}} + G_t - T_t. \tag{81}$$

22. Fiscal rule:

$$\frac{B_t}{P_t} = bY_t. \tag{82}$$

23. Government consumption:

$$G_t = gY_t.$$

24. Monetary policy rule:

$$\pi_t \left[\frac{Y_t/Y_{t-4}}{X_t N_t^w/(X_{t-4}N_{t-4}^w)}\right]^{\varphi_y} = 1. \tag{83}$$

25. Aggregate assets:

$$A_t = K_t + \frac{B_t}{P_t} + P_{Ft}. \tag{84}$$

26. Profits of intermediate goods producers:

$$D_{Ft} = \left[1 - mc_t - \frac{\phi_P}{2}\left(\pi_t - 1\right)^2\right] Y_t. \tag{85}$$

27. Law of motion of capital:

$$K_t = (1-\delta)K_{t-1} + I_t. \tag{86}$$

28. Resource constraint:

$$\left[1 - \frac{\phi_P}{2}\left(\pi_t - 1\right)^2\right] Y_t = C_t + I_t + G_t. \tag{87}$$

A.3.1 Detrending

Lower case letters denote the stationary counterpart of the relevant variable: $s_t \equiv S_t/(X_t N_t^w)$. The equilibrium in terms of detrended variables is characterized by the following list of equations.

1. Dependency ratio:

$$(1+n_t)\psi_t = (1-\omega_t) + \gamma_t\psi_{t-1}. \tag{88}$$

2. Aggregate consumption for retirees:

$$c_t^r = \xi_t^r \frac{R_{t-1}}{\pi_t} a_{t-1}^r. \tag{89}$$

3. Marginal propensity to consume for retirees:

$$\xi_t^r = 1 - \gamma_{t+1}\beta^\sigma \left(\frac{R_t}{\pi_{t+1}}\right)^{\sigma-1} \frac{\xi_t^r}{\xi_{t+1}^r}. \tag{90}$$

4. Retirees' welfare:

$$v_t^r = (\xi_t^r)^{\frac{\sigma}{1-\sigma}} c_t^r. \tag{91}$$

5. Aggregate consumption for workers:

$$c_t^w = \xi_t^w \left(\frac{R_{t-1}}{\pi_t} a_{t-1}^w + h_t \right). \tag{92}$$

6. Marginal propensity to consume for workers:

$$\xi_t^w = 1 - \beta^\sigma \left(\frac{\Omega_{t+1} R_t}{\pi_{t+1}} \right)^{\sigma-1} \frac{\xi_t^w}{\xi_{t+1}^w}. \tag{93}$$

7. Workers' welfare:

$$v_t^w = (\xi_t^w)^{\frac{\sigma}{1-\sigma}} c_t^w. \tag{94}$$

8. Aggregate value of human wealth:

$$h_t = (w_t - \tau_t) + \frac{(1 + x_{t+1}) \omega_{t+1} h_{t+1}}{\Omega_{t+1} R_t / \pi_{t+1}}, \tag{95}$$

where $w_t \equiv W_t/(X_t P_t)$ and $\tau_t \equiv T_t/(X_t N_t^w)$.

9. Adjustment factor:

$$\Omega_t = \omega_t + (1 - \omega_t) \epsilon_t^{\frac{1}{1-\sigma}}. \tag{96}$$

10. Ratio of marginal propensities to consume:

$$\epsilon_t = \xi_t^r / \xi_t^w. \tag{97}$$

11. Aggregate consumption:

$$c_t = \xi_t^w \left[(1 - \lambda_{t-1}) \frac{R_{t-1}}{\pi_t} a_{t-1} + h_t + \epsilon_t \lambda_{t-1} \frac{R_{t-1}}{\pi_t} a_{t-1} \right]. \tag{98}$$

12. Distribution of wealth

$$(1 + x_{t+1} + n_{t+1})[\lambda_t - (1 - \omega_t)] a_t = \omega_t (1 - \xi_t^r) \lambda_{t-1} \frac{R_{t-1}}{\pi_t} a_{t-1}. \tag{99}$$

13. Retirees assets:

$$\frac{a_t^r}{a_t} = \lambda_t. \tag{100}$$

14. Workers assets:

$$\frac{a_t^w}{a_t} = 1 - \lambda_t. \tag{101}$$

15. No arbitrage:

$$\frac{R_t}{\pi_{t+1}} = R^K_{t+1} + (1-\delta) = (1 + x_{t+1} + n_{t+1})\frac{p_{Ft+1} + d_{Ft+1}}{p_{Ft}}. \tag{102}$$

16. Aggregate welfare:

$$v_t = \frac{(\xi^w_t)^{\frac{\sigma}{1-\sigma}} \left(c^w_t + \psi_t \epsilon_t^{\frac{\sigma}{1-\sigma}} c^r_t \right)}{1 + \psi_t}. \tag{103}$$

17. Real wage:

$$w_t = mc_t \alpha y_t. \tag{104}$$

18. Rental rate:

$$R^K_t = mc_t \frac{(1-\alpha) y_t}{k_{t-1}} \tag{105}$$

19. Marginal cost:

$$mc_t = \frac{(w_t)^\alpha (R^K_t)^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}. \tag{106}$$

20. Phillips curve:

$$(\pi_t - 1)\pi_t = \frac{\theta - 1}{\phi_P}\left(\frac{\theta}{\theta - 1} mc_t - 1 \right) + (1 + x_{t+1} + n_{t+1}) \frac{1}{R_t/\pi_{t+1}} \frac{y_{t+1}}{y_t} (\pi_{t+1} - 1)\pi_{t+1}. \tag{107}$$

21. Government budget constraint:

$$(1 + x_{t+1} + n_{t+1}) b_t = \frac{R_{t-1}}{\pi_t} b_{t-1} + g_t - \tau_t, \tag{108}$$

where $b_t \equiv B_t/(X_{t+1} N^w_{t+1} P_t)$.

22. Fiscal rule:

$$(1 + x_{t+1} + n_{t+1}) b_t = b y_t. \tag{109}$$

23. Government consumption

$$g_t = g y_t.$$

24. Monetary policy rule:

$$\pi_t \left(\frac{y_t}{y_{t-4}} \right)^{\varphi_y} = 1. \tag{110}$$

25. Aggregate assets:

$$a_t = k_t + b_t + p_{F,t}, \tag{111}$$

26. Profits of intermediate goods producers:

$$d_{Ft} = \left[1 - mc_t - \frac{\phi_P}{2} (\pi_t - 1)^2 \right] y_t. \tag{112}$$

27. Law of motion of capital:

$$(1 + x_{t+1} + n_{t+1})k_t = (1 - \delta)k_{t-1} + i_t. \tag{113}$$

28. Resource constraint:

$$\left[1 - \frac{\phi_P}{2}(\pi_t - 1)^2\right] y_t = c_t + i_t + g_t. \tag{114}$$

A.4 Numerical solution

[Add description of shooting algorithm]

B Model Solution with Social Security Benefits

B.1 Retirees

The first-order conditions with respect to capital, government bonds and shares and the envelope conditions are unchanged. The guess for consumption of a retirees becomes

$$C_t^r(j,\tau) = \xi_t^r \left[\frac{R_{t-1}A_{t-1}^r(j,\tau)}{\gamma_t \pi_t} + S_t^r\right], \tag{115}$$

where S_t^r is the present discounted value of social security benefits for a retiree

$$S_t^r \equiv \sum_{v=0}^{\infty} \frac{E_{t+v}^r}{\prod_{s=1}^{v}\left(\frac{R_{t+s-1}}{\gamma_{t+s}\pi_{t+s}}\right)} = E_t^r + \frac{S_{t+1}^r}{R_t/(\gamma_{t+1}\pi_{t+1})}. \tag{116}$$

Substitution into the Euler equation (51) yields a law of motion for the marginal propensity to consume of a retiree ξ_t^r

$$\xi_{t+1}^r \left(\frac{R_t A_t^r(j,\tau)}{\gamma_{t+1}\pi_{t+1}} + S_{t+1}^r\right) = \left(\frac{\beta R_t}{\pi_{t+1}}\right)^{\sigma} \xi_t^r \left(\frac{R_{t-1}A_{t-1}^r(j,\tau)}{\gamma_t \pi_t} + S_t^r\right). \tag{117}$$

Substitution of the guess (115) into the modified budget constraint of a retiree leads to the expression below for the dynamics of asset holdings

$$A_t^r(j,\tau) = (1 - \xi_t^r)\frac{R_{t-1}A_{t-1}^r(j,\tau)}{\gamma_t \pi_t} - \xi_t^r S_t^r + E_t^r.$$

Combining the last expression with the law of motion for the marginal propensity to consume of a retiree (117) yields the same first-order non-linear difference equation for ξ_t^r (12) as in the case without social security benefits. Similarly, the value function for retirees also takes the same form as in the case without social security benefits.

In the aggregate, the present discounted value of social security benefits for retirees is

$$S_t = E_t + \frac{\psi_t S_{t+1}}{(1+n_{t+1})\psi_{t+1}R_t/(\gamma_{t+1}\pi_{t+1})}, \tag{118}$$

where $S_t \equiv N_t^r S_t^r$ and $E_t \equiv N_t^r E_t^r$. The detrended version of the last equation is

$$s_t = e_t + \frac{\psi_t s_{t+1}}{(1+x_{t+1})\psi_{t+1}R_t/(\gamma_{t+1}\pi_{t+1})}. \tag{119}$$

B.2 Workers

The first-order condition for workers' asset holdings and the envelope conditions are the same as in the case without social security benefits. We also conjecture the same value function. The guess for the consumption function becomes

$$C_t^w(j) = \xi_t^w \left[\frac{R_{t-1}A_{t-1}^w(j)}{\pi_t} + H_t^w + S_t^w\right], \tag{120}$$

where S_t^w is defined as the present discounted value of social security benefits for a worker after retirement

$$S_t^w \equiv \sum_{v=0}^{\infty} \frac{\frac{(\Omega_{t+v+1}-\omega_{t+v+1})S_{t+v+1}^r}{\Omega_{t+v+1}R_{t+v}/\pi_{t+v+1}}}{\prod_{s=1}^{v}\left(\frac{\Omega_{t+s}R_{t+s-1}}{\pi_{t+s}}\right)} = \frac{(\Omega_{t+1}-\omega_{t+1})S_{t+1}^r}{\Omega_{t+1}R_t/\pi_{t+1}} + \frac{\omega_{t+1}S_{t+1}^w}{\Omega_{t+1}R_t/\pi_{t+1}} \tag{121}$$

From (115), the decision rule for a retiree born in period j who just left the labor force is now

$$C_t^r(j) = \xi_t^r\left(\frac{R_{t-1}A_{t-1}^w(j)}{\pi_t} + S_t^r\right).$$

Substituting the guesses (for the value function and for the decision rule) and the last expression into the Euler equation yields

$$\omega_{t+1}\left(A_t^w(j) + \frac{H_{t+1}^w}{R_t/\pi_{t+1}} + \frac{S_{t+1}^w}{R_t/\pi_{t+1}}\right) + (1-\omega_{t+1})\left(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w}\right)\epsilon_{t+1}\left(A_t^w(j) + \frac{S_{t+1}^r}{R_t/\pi_{t+1}}\right) =$$
$$(\beta\Omega_{t+1})^\sigma\left(\frac{R_t}{\pi_{t+1}}\right)^{\sigma-1}\frac{\xi_t^w}{\xi_{t+1}^w}\left(\frac{R_{t-1}A_{t-1}^w(j)}{\pi_t} + H_t^w + S_t^w\right),$$

where $\epsilon_t \equiv \xi_t^r/\xi_t^w$. Using the definition of Ω_t, the last expression becomes

$$A_t^w(j) + \frac{\omega_{t+1}H_{t+1}^w}{\Omega_{t+1}R_{t+1}/\pi_{t+1}} + \frac{\omega_{t+1}S_{t+1}^w}{\Omega_{t+1}R_{t+1}/\pi_{t+1}} + \frac{(\Omega_{t+1}-\omega_{t+1})S_{t+1}^r}{\Omega_{t+1}R_{t+1}/\pi_{t+1}} =$$
$$\beta^\sigma\left(\frac{\Omega_{t+1}R_t}{\pi_{t+1}}\right)^{\sigma-1}\frac{\xi_t^w}{\xi_{t+1}^w}\left(\frac{R_{t-1}A_{t-1}^w(j)}{\pi_t} + H_t^w + S_t^w\right). \tag{122}$$

Moreover, from the budget constraint of a worker and the guess (120), we obtain the law of motion

for worker j's wealth

$$A_t^w(j) + \frac{\omega_{t+1} H_{t+1}^w}{\Omega_{t+1} R_t / \pi_{t+1}} + S_t^w = (1 - \xi_t^w) \left(\frac{R_{t-1} A_{t-1}^w(j)}{\pi_t} + H_t^w + S_t^w \right).$$

Substituting this result back into the Euler equation (122) yields the same expression for the marginal propensity to consume for a worker as in the case without social security benefits. Furthermore, as for retirees, the value function also coincides with the baseline case.

In the aggregate, the present discounted value of social security benefits for workers is

$$(1 + n_{t+1}) \tilde{S}_t = \frac{(\Omega_{t+1} - \omega_{t+1}) S_{t+1} / \psi_{t+1}}{\Omega_{t+1} R_t / \pi_{t+1}} + \frac{\omega_{t+1} \tilde{S}_{t+1}}{\Omega_{t+1} R_t / \pi_{t+1}}, \tag{123}$$

where $\tilde{S}_t \equiv N_t^w S_t^w$. The detrended version of the last equation is

$$\tilde{s}_t = \frac{(\Omega_{t+1} - \omega_{t+1}) s_{t+1} / \psi_{t+1}}{\Omega_{t+1} R_t / \pi_{t+1}} + \frac{\omega_{t+1} \tilde{s}_{t+1}}{\Omega_{t+1} R_t / \pi_{t+1}}, \tag{124}$$

B.3 Aggregate Consumption, Distribution of Wealth, Government Budget Constraint and Resource Constraint

In this section we derive the aggregate and detrended conditions that change in presence of social security benefits.

The steps for aggregation are the same as in the baseline case. Aggregate consumption becomes

$$C_t = \xi_t^w \left[\frac{R_{t-1}}{\pi_t} (1 - \lambda_{t-1}) A_{t-1} + H_t + \tilde{S}_t \right] + \xi_t^r \left(\frac{R_{t-1}}{\pi_t} \lambda_{t-1} A_{t-1} + S_t \right) \tag{125}$$

The distribution of wealth evolves according to

$$[\lambda_t - (1 - \omega_{t+1})] A_t = \omega_{t+1} \left[(1 - \xi_t^r) \frac{R_{t-1}}{\pi_t} \lambda_{t-1} A_{t-1} + E_t - \xi_t^r S_t \right]. \tag{126}$$

Finally, the government budget constraint becomes

$$\frac{B_t}{P_t} = \frac{R_{t-1}}{\pi_t} \frac{B_{t-1}}{P_{t-1}} + G_t + E_t - T_t. \tag{127}$$

The detrended version of the last three equations are

$$c_t = \xi_t^w \left[\frac{R_{t-1}}{\pi_t} (1 - \lambda_{t-1}) a_{t-1} + h_t + \tilde{s}_t \right] + \xi_t^r \left(\frac{R_{t-1}}{\pi_t} \lambda_{t-1} a_{t-1} + s_t \right) \tag{128}$$

$$(1 + x_{t+1} + n_{t+1}) [\lambda_t - (1 - \omega_{t+1})] a_t = \omega_{t+1} \left[(1 - \xi_t^r) \frac{R_{t-1}}{\pi_t} \lambda_{t-1} a_{t-1} + e_t - \xi_t^r s_t \right] \tag{129}$$

$$b_t = \frac{R_{t-1}}{\pi_t} b_{t-1} + g_t + e_t - \tau_t \tag{130}$$