Unemployment and Business Cycles*

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Abstract

We develop and estimate a DSGE model that accounts for key business cycle properties of labor market variables. In sharp contrast to leading New Keynesian models, wages are not subject to exogenous nominal rigidities. Instead we embed an insight of Hall and Milgrom (2008) into our model about why real wages are relatively insensitive to the state of the economy. Our model outperforms the canonical Diamond-Mortensen-Pissarides model both in a statistical sense and in terms of the plausibility of the estimated structural parameter values. The model does at least as well empirically as conventional DSGE models that simply impose nominal wage stickiness.

*The views expressed in this paper are those of the authors and do not necessarily reflect those of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

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1. Introduction

A key challenge facing macroeconomists is how to account for the observed volatility of labor markets. Empirical New Keynesian models address this challenge by assuming that wages are sticky.[1] While many of these models fit the time series data well, they have been severely criticized on the grounds that they "invoke an inefficiency that intelligent actors could easily avoid" (see Hall (2005, p. 51).[2] The search-theoretic framework associated with Diamond (1982), Mortensen (1985) and Pissarides (1985) (DMP) is not subject to this critique. However, as Shimer (2005) stresses, those models have difficulty accounting for the observed volatility of labor markets. This shortcoming reflects the pronounced procyclicality of real wages and the strong counter cyclicality of vacancy-posting yields in the standard DMP model. Together these forces act to dampen firms' incentives to post vacancies in a boom.

In this paper we develop and estimate a New Keynesian model that accounts for key business cycle properties of labor market variables like wages, employment, job vacancies and unemployment. In contrast to leading empirical New Keynesian models, we do not assume that wages are sticky. Instead, we derive wage inertia as an equilibrium outcome.

Our analysis builds on an insight due to Hall and Milgrom (2008) (HM) about why wages can be relatively insensitive to the state of the economy.[3] According to HM, by the time workers and firms sit down to bargain, they know there is a surplus to be shared if they can come to terms. So rather than just going their separate ways in the wake of a disagreement, workers and firms continue to negotiate. This process introduces a delay in the time required to make a deal. During this delay, firms and workers suffer various costs. HM's key insight is that if these costs are relatively insensitive to the aggregate state of the economy, then negotiated wages will inherit that insensitivity.

[1] For example, Christiano, Eichenbaun and Evans (2005), Smets and Wouters (2003, 2007), Gali, Smets and Wouters (2012) assume that nominal wages are subject to Calvo frictions.

[2] Hall (2005) described one line of defense for sticky wages. He introduced sticky wages into the DMP framework in a way that satisfies the condition that no worker-employer pair has an unexploited opportunity for mutual improvement (Hall, 2005, p. 50). A sketch of Hall's logic is as follows. In a model with labor market frictions, there is a gap between the reservation wage required by a worker to accept employment and the highest wage a firm is willing to pay an employee. This gap, or bargaining set, fluctuates with the shocks that affect the surplus enjoyed by the worker and employer. When calibrated based on aggregate data the fluctuations in the bargaining set are sufficiently small and the width of the set is sufficiently wide that a constant wage rate can remain inside the set for an extended period of time. This idea has been further explored in the influential papers, Gertler and Trigari (2009) and Gertler, Sala and Trigari (2008). See also Christiano, Ilut, Motto and Rostagno (2008) and Christiano, Trabandt and Walentin (2011b). A concern about the DMP-based strategy for justifying sticky wages is that the shocks which move actual firms' bargaining sets are far more volatile than what the aggregate data suggest. As a result, it may be harder to rationalize sticky wages in the DMP framework than had initially been recognized.

[3] For a paper that pursues a reduced form version of HM in a calibrated real business cycle model, see Hertweck (2006).

When we incorporate this insight into our estimated business cycle model, negotiated wages are relatively insensitive to business cycle shocks.[4] In the wake of an expansionary shock, the rise in wages is relatively small, so that firms receive a substantial fraction of the rents associated with employment. Consequently, firms have a strong incentive to expand their labor force. In addition, the muted response of wages to aggregate shocks means that firms' marginal costs are relatively acyclical. This acyclicality enables our model to account for the inertial response of inflation even with modest exogenous rigidities in prices. Providing a plausible account for inflation inertia has been a long-standing challenge for business cycle models.

We estimate our model using a Bayesian variant of the strategy in Christiano, Eichenbaum and Evans (2005) that minimizes the distance between the dynamic response to three shocks in the model and the analog objects in the data. The latter are obtained using an identified vector autoregression (VAR) for 12 post-war quarterly U.S. times series that include key labor market variables. The particular Bayesian strategy that we use is the one developed in Christiano, Trabandt and Walentin (2011).

We contrast the empirical properties of our model with estimated versions of leading alternatives. These alternatives include the standard DMP model as well the standard New Keynesian sticky wage model of the labor market proposed in Erceg, Henderson and Levin (2000) (EHL).

We show that our model outperforms the DMP model in a statistical sense and in terms of the plausibility of the estimated structural parameter values. We also show that our model does as well as the EHL model for comparable variables. Of course, unlike our model, the EHL is simply silent about labor market variables like job vacancies and job flows. So we cannot compare the models along these dimensions.

Our paper is organized as follows. Section 2 describes the labor market of our model in isolation. In section 3 we integrate the labor market model into a simple New Keynesian model without capital. We use this model to exposit the intuition about how our model of the labor market works in a general equilibrium setting with sticky prices. Section 4 describes our medium sized DSGE model. Section 5 describes our econometric methodology. In section 6, we present our empirical results. Section 7 contains concluding remarks.

2. The Labor Market

In this section we discuss of our model of labor markets. The basic structure corresponds to the one in Ravenna and Walsh (2008) where each worker is matched with one firm, an assumption which we relax in our empirical model. The critical difference is in how we model

[4] A similar idea is pursued in an efficiency wage context by Alexopoulos (2004).

wage determination.

We assume there is a large number of identical and competitive firms that produce a homogeneous good using labor. Let ϑ_t denote the marginal revenue associated with an additional worker hired by a firm. In this section, we treat ϑ_t as an exogenous stochastic process. In the next section we embed the labor market in a general equilibrium model and determine the equilibrium process for ϑ_t.

In our benchmark specification we assume that at the start of period t a firm pays a fixed cost, κ, to meet a worker with probability one. We refer to this specification of the cost of meeting a worker as the *hiring cost specification*. Once a worker and firm meet they engage in bilateral bargaining. If bargaining results in agreement, as it always does in equilibrium, then the worker begins production immediately.

The number of workers employed in period t is denoted l_t. The size of the labor force is fixed at unity so that $1-l_t$ is the number of unemployed workers during the period. Towards the end of the period a randomly selected fraction, $1-\rho$, of employed workers is separated from their firm. These workers join the ranks of the unemployed and search for work. So, at the end of the period there are $1-\rho l_t$ workers searching for a job. In period $t+1$ a random fraction, f_{t+1}, of searching workers meets with a firm and the complementary fraction remain unemployed. Thus, each of the l_t employed workers in period t remains with the same firm in period $t+1$ with probability ρ, moves to another firm in period $t+1$ with probability $(1-\rho)f_{t+1}$ and is unemployment in period $t+1$ with probability $(1-\rho)(1-f_{t+1})$. Our measure of unemployment in period t is $1-l_t$. We think of the $(1-\rho)f_{t+1}$ workers that change jobs between t and $t+1$ as job-to-job movements in employment. With our specification, the job-to-job transition rate is substantial and procyclical, consistent with the data (see, e.g., Shimer, 2005a.)[5] While controversial, the standard assumption that the job separation rate is acyclical has been defended on empirical grounds (see Shimer, 2005a).[6] Finally, we think of the time period as one quarter. This time period is relatively long from the point of view of the US labor market, where the median duration of unemployment is around 7 weeks in our sample.[7] So we adopt the assumption that workers meet firms, bargain and start work in the same period.

The value to a firm of employing a worker at the equilibrium real wage rate, w_t, is denoted J_t which satisfies the following recursive relationship:

$$J_t = \vartheta_t - w_t + \rho E_t m_{t+1} J_{t+1}. \tag{2.1}$$

The wage, w_t, is the outcome of a bargaining process described below. Also, m_{t+1} is the

[5] For a recent paper that stresses the importance of job-to-job transitions, see Van Zandweghe (2010).

[6] For a different view, see Fujita and Ramey (2009).

[7] See the variable, LNU03008276, in the online database, Fred, provided by the Federal Reserve Bank of St. Louis.

discount factor which in this section we assume is an exogenous stochastic process. When we embed the labor market in a general equilibrium model, we determine the equilibrium process for m_t. The presence of ρ in (2.1) reflects that a worker matched with a firm in period t remains matched in $t+1$ with probability ρ. Because there is free entry, firm profits must be zero:

$$\kappa = J_t. \tag{2.2}$$

The value to a worker of being matched with a firm that pays w_t in period t is denoted V_t :

$$V_t = w_t + E_t m_{t+1} \left[\rho V_{t+1} + (1-\rho)\left(f_{t+1} V_{t+1} + (1-f_{t+1})\, U_{t+1}\right)\right]. \tag{2.3}$$

Here, f_{t+1} denotes the probability that a worker searching for a job in period t meets a firm in $t+1$. Also U_{t+1} is the value of being an unemployed worker in period $t+1$. The recursive representation of U_t is:

$$U_t = D + E_t m_{t+1} \left[f_{t+1} V_{t+1} + (1-f_{t+1})\, U_{t+1}\right]. \tag{2.4}$$

In (2.4), D denotes goods received by unemployed workers from the government.[8]

We denote the number of employed workers in period t by l_t which evolves according to,

$$l_t = (\rho + x_t)\, l_{t-1}. \tag{2.5}$$

Here x_t denotes the hiring *rate* so that the number of new hires in period t is equal to $x_t l_{t-1}$. Note that the job finding rate is given by,

$$f_t = \frac{x_t l_{t-1}}{1 - \rho l_{t-1}}. \tag{2.6}$$

Here the numerator is the number of workers that are newly-hired at the beginning of time t, while the denominator is the number of workers who are searching for work at the end of time $t-1$.

2.1. Wage Determination: Alternating Offer Bargaining

To ensure that our results are not driven by exogenous wage rigidities, we assume that all workers and firms bargain over wages every period. In bargaining over the current wage rate, a worker and a firm take as given the state-contingent wage process that will obtain in future periods as long as they are matched. Because hiring costs are sunk at the time of bargaining and because the expected duration of a match is independent of how long that match has

[8] One can also interpret D as the value of home production by unemployed workers, see Ravenna and Walsh (2008). See Aguiar, Hurst and Karabarbounis (2012) for evidence on the importance of home production by the unemployed.

already been in place, the bargaining problem of all workers is the same, regardless of how long they have been matched with a firm.

Consistent with Hall and Milgrom (20008), wages are determined according to the alternating offer bargaining protocol proposed in Rubinstein (1982) and Binmore, Rubinstein and Wolinsky (1986). When a firm and a worker meet, the firm makes a wage offer. The worker can accept the offer or reject it. If he accepts it, work begins immediately. If he rejects the offer, he can go to his outside option or he can make a counteroffer. In the latter case there is a probability, δ, that negotiations break down. In that case the firm and the worker revert to their outside options. For the worker, the outside option is unemployment, which has value U_t. For the firm, the outside option has a value of zero. We only study model parameterizations in which workers who reject an offer prefer to make a counteroffer rather than go to the outside option.

In a similar way, when a worker makes an offer, a firm can accept the offer, it can reject the offer and go to the outside option, or it can reject the offer and plan to make a counteroffer. In the latter case there is a probability, δ, that negotiations break down and no counteroffer is made. To actually make a counteroffer, the firm incurs a cost, γ. We only consider model parameterizations in which a firm chooses to make a counteroffer after rejecting an offer from the worker.

Let w_t denote the initial wage offered by the firm. We denote the worker's offer in the i^{th} bargaining round by $w_t^{l(i)}$, where i is odd. We denote the firm's offer in the i^{th} bargaining round by $w_t^{f(i)}$, where i is even and $w_t^{f(0)} \equiv w_t$. The sequence of offers across subsequent bargaining rounds is given by,

$$w_t, w_t^{l(1)}, w_t^{f(2)}, w_t^{l(3)}, w_t^{f(4)}, ... \tag{2.7}$$

If the horizon is finite, one can solve for this sequence by starting with the take-it-or-leave-it offer made by one of the parties in the last bargaining round and work backward to the first offer. In equilibrium the first offer, w_t, is accepted. However, the nature of the first offer is determined by the details of the later bargaining rounds in case agreement is not reached in the first bargaining round. When the $w_t^{f(i)}$ and $w_t^{l(i)}$ that solve a bargaining problem are functions of i, the solution to the bargaining problem is not stationary. Obviously, when the possible number of periods is finite, the solution to the bargaining problem is not stationary.

We suppose that the solution to the bargaining problem is well approximated (perhaps because there is a sufficiently large number of bargaining rounds) by a stationary sequence of offers and counteroffers. That is, the sequence of wages, (2.7), is well approximated by two numbers, w_t and w_t^l

$$w_t^l, w_t, w_t^l, w_t, w_t^l, w_t, ...$$

Suppose that it is the firm's turn to make an offer. The firm would like to propose the

lowest possible wage. However, there is no point for the firm to propose a wage that the worker would reject. So, the firm proposes a wage that just makes the worker indifferent between accepting it and rejecting it in favor of making a counteroffer. In the case of indifference, we assume that the worker agrees to the offer. So, the wage offered by the firm satisfies:

$$V_t = \delta U_t + (1-\delta)\frac{V_t^l}{1+r}, \tag{2.8}$$

where V_t is defined in (2.3). The object on the right hand side of (2.8) is the worker's disagreement payoff, i.e. what he receives in case he rejects the firm's offer and intends to make a counteroffer. The variable, r, is a within-period discount rate that captures the worker's impatience to enjoy the benefits of reaching agreement. Below, we make an analogous assumption about the firm's disagreement payoff. We assume, but always verify in practice, that the worker's disagreement payoff is no smaller than his outside option, U_t. The worker's disagreement payoff reflects our assumption that when a worker rejects an offer with the intention of making a counteroffer, there is a probability $\delta \in (0,1)$ that both parties revert to their outside options.

The object, V_t^l, denotes the value of employment to a worker who makes a counteroffer, w_t^l, that is accepted by the firm. We show below that there is no reason for the worker to consider the possibility that w_t^l will be rejected by the firm in the next bargaining round. The condition that defines V_t^l is:

$$V_t^l = w_t^l + E_t m_{t+1}\left[\rho V_{t+1} + (1-\rho)\left(f_{t+1}V_{t+1} + (1-f_{t+1})U_{t+1}\right)\right]. \tag{2.9}$$

The term after the first plus sign in (2.9) is the same as the corresponding term in (2.3).

Now consider the problem of a worker who makes a wage offer to a firm. The worker wants the highest possible wage. But, there is no point for the worker to propose a wage that the firm will reject. So, the worker proposes a wage that makes the firm just indifferent between accepting it and rejecting it in favor of making a counter offer. In the case of indifference, we assume that the firm agrees to take the offer. So, the wage offered by the workers satisfies:

$$J_t^l = \delta \times 0 + (1-\delta)\left[-\gamma + \frac{1}{1+r}J_t\right]. \tag{2.10}$$

Here J_t^l denotes the value of a match to the firm that employs a worker at wage w_t^l :

$$J_t^l = \vartheta_t - w_t^l + \rho E_t m_{t+1} J_{t+1}. \tag{2.11}$$

The right side of (2.10) is the firm's disagreement payoff, i.e. what the firm receives if it rejects the worker's offer and intends to make a counteroffer. The presence of J_{t+1} on the right side of (2.11) reflects our assumption that a firm which hires a worker at wage rate

w_t^l expects to employ him at the wage rate w_{t+1} if the match survives into period $t+1$. In (2.10), the 0 represents the surplus received by the firm if negotiations break down. In practice we must verify that the firm's disagreement payoff is no less than the value of its outside option, zero.

An equilibrium is a stochastic process for the following ten variables:

$$x_t, J_t, w_t, l_t, V_t, U_t, f_t, V_t^l, J_t^l, w_t^l, \tag{2.12}$$

that satisfy the ten equilibrium conditions, (2.1)-(2.6), (2.8), (2.9), (2.10), (2.11). We refer to such a stochastic process as an *alternating offer equilibrium*.

The equilibrium conditions exhibit a recursive structure that we exploit in our analysis. Equations (2.9) and (2.11) imply

$$V_t^l = V_t + w_t^l - w_t, \;\; J_t^l = J_t + w_t - w_t^l. \tag{2.13}$$

Use (2.13) to substitute out for V_t^l in (2.8) and for J_t^l in (2.10). The resulting two equations reduce to the following single equation, after solving out for $w_t - w_t^l$ and rearranging:

$$J_t = \frac{1+r}{1-\delta}\left[V_t - \alpha U_t - \omega\right], \tag{2.14}$$

where

$$\alpha \equiv 1 - r\frac{1-\delta}{r+\delta}, \;\; \omega \equiv \frac{(1-\delta)^2}{(r+\delta)(1+r)}\gamma. \tag{2.15}$$

We refer to (2.14) as an *alternating offer sharing rule*. We can use the seven equations (2.1)-(2.6) and (2.14) to determine the equilibrium values of the first seven variables in (2.12). The last three variables in (2.12) can then be determined using the two equations in (2.13) plus (2.8) and (2.10).

2.2. Alternating Offer Bargaining: Some Intuition

A key finding of this paper is that when wages are the outcome of an alternating offer bargaining process, negotiated wages will, for empirically plausible parameterizations, be relatively insulated from general economic conditions. This property is the key feature of our model that allows it to account for the observed volatility of labor market variables. For the purpose of developing intuition, in this subsection we use the value of unemployment, U_t, as the indicator of general economic conditions. Shocks that expand economic activity tend to simultaneously raise U_t. In what follows we provide intuition about how the paramaters governing the alternating offer sharing rule, δ, γ and r, influence the responsiveness of the wage to U_t. To do so, we consider a bargaining session between a given worker and a firm in partial equilibrium, i.e., taking all the variables outside their control as given. We also

suppose that we are in nonstochastic state, i.e a situation in which the aggregate shocks are fixed at their unconditional mean values, but the idiosyncratic worker-level uncertainty in the labor market remains.

Let w_U denote the elasticity of the negotiated wage, w, to U, in nonstochastic steady state. We seek to understand how w_U varies across alternative specifications of the bargaining environment. Formally,

$$w_U \equiv \frac{d\log w}{d\log U} = \frac{U}{w}W_U,\ W_U \equiv \frac{dw}{dU}. \tag{2.16}$$

Here, U and w denote the value of unemployment and the equilibrium wage in non-stochastic steady state. The derivative in (2.16) treats the rise in U as something experienced idiosyncratically by one worker-firm bargaining pair. The variables outside the worker-firm bargaining pair are taken as given in the following experiments.

2.2.1. A fall in γ

In this subsection we provide intuition about how a change in γ affects W_U and w_U . The basic argument is straightforward. As it turns out W_U is independent of γ. So the effect of a change in γ on w_U operates entirely through its effect on U/w. A fall in γ increases the disagreement payoff of the firm, thus putting the worker in a relatively weaker bargaining position. So, other things equal, the fall in γ leads to a fall in w. But the zero profit condition for firms implies that w is independent of all the bargaining parameters. So some other variable must change to ensure that the steady state wage rate remains equal to w. The only variable that can change is U. A rise in U which corresponds to a fall in the unemployment rate, increases the worker's disagreement payoff and therefore increases his bargaining power. Other things equal this increase leads to a rise in w, an effect that must exactly offset the downward pressure on w stemming from the rise in the disagreement payoff of the firm. Since w_U only depends on U, a fall in γ leads to a rise in w_U.

We now discuss more formally why a fall in γ increases w_U, the elasticity of the wage rate with respect to U_t. We begin by proving that W_U is unaffected by γ. Define $\tilde{V}_t$ and $\tilde{J}_t$ as follows:

$$\tilde{V}_t \equiv E_t m_{t+1}\left[\rho V_{t+1} + (1-\rho)\left(f_{t+1}V_{t+1} + (1-f_{t+1})U_{t+1}\right)\right] \tag{2.17}$$

$$\tilde{J}_t \equiv \vartheta_t + \rho E_t m_{t+1} J_{t+1}. \tag{2.18}$$

The variables, $\tilde{V}_t$ and $\tilde{J}_t$, are taken as given by an individual worker-firm pair. Using this notation, the indifference conditions, (2.8) and (2.10) can be written:

$$w_t = -\tilde{V}_t + \delta U_t + \frac{1-\delta}{1+r}\left(w_t^l + \tilde{V}_t\right) \tag{2.19}$$

$$w_t^l = \tilde{J}_t + (1-\delta)\gamma + \frac{1-\delta}{1+r}\left(w_t - \tilde{J}_t\right). \tag{2.20}$$

We interpret relation (2.19) as giving the wage offer w_t that a firm makes given it's view about w_t^l.We refer to (2.19) as the firm's best response function. Similarly, we interpret relation (2.20) as giving the wage offer w_t^l that a worker makes given it's view about w_t.So we refer to (2.20) as the worker's best response function. The equilibrium values of w_t and w_t^l are given by the intersection of the firm's and worker's best response functions.

In Figure 1, panel A we graph the best response functions, (2.19) and (2.20) with w_t on the vertical axis and w_t^l on the horizontal axis. The slope of the worker's best response function, taking into account that it is w_t^l that appears on the horizontal axis, is $(1+r)/(1-\delta) \geq 1$. The slope of the firm's best response function is $(1-\delta)/(1+r) \leq 1$. Note that the worker's best response function is steeper than the firm's.

Consider the effect of an increase in U, $\Delta U > 0$, on w depicted in panel A of Figure 1. Note that U only enters the firm's best response function, which shifts up in a parallel way by $\delta \Delta U$. The result is an increase in w. Totally differentiating the best response functions, setting $d\tilde{V}_t = d\tilde{J}_t = 0$ and evaluating the derivative in steady state, we obtain:

$$W_U = \frac{\delta (1+r)^2}{(r+\delta)(2+r-\delta)}. \tag{2.21}$$

It follows that γ has no impact on W_U, i.e. the impact on the level of the negotiated wage of a change in U does not depend on γ.

We now show that the steady state value of the wage rate w is also not affected by γ, or for that matter δ and r. Combine (2.1) and (2.2) to obtain:

$$\kappa = \frac{\vartheta - w}{1 - \rho\beta}. \tag{2.22}$$

Here, ϑ is the steady state value of what the worker produces. Expression (2.22) says that the cost of meeting a worker, κ, must equal the present value of what the worker brings into the firm. The present value expression takes into account discounting, β, and also the fact that the worker-firm match remains in place with probability ρ. From (2.22) we see that w does not depend on γ, δ or r.

Taken together the previous results imply that the sign of the impact of γ on w_U is completely determined by the sign of the impact of γ on U. To analyze how U depends on γ, consider Figure 1, Panel B which displays the worker's and firm's best response functions as well as a horizontal line at the steady state wage rate w implied by (2.22). In steady state all three equations must be satisfied so steady state corresponds to the points where the three lines intersect.

A decrease in γ shifts the worker response function, (2.20), to the left (see Figure 1, Panel B). Other things equal, this shift induces a fall in the wage rate. The situation is depicted by point b in Figure 1, Panel B where the two best response functions intersect at a point

below the horizontal line. But, a lower value of w is not consistent with the equilibrium condition, (2.22). To re-establish steady state, the firm's best response function must shift up so that the intersection of the two best response functions occurs on the horizontal line (see point c, Figure 1, Panel B). The only variable that can adjust to make this happen is U. Recall that U only enters the firm's best response function. From (2.19) we see that it does so by two channels. One is direct and the other operates via the steady state value of $\tilde{V}$. It turns out that for reasonable parameter values, the firm's best response function shifts up in response to a rise in U.[9] So U must rise in steady state after the fall in γ.

From (2.16) we conclude that a smaller value of γ is associated with a rise in w_U. So from the perspective of the alternating offer model, smaller values of γ are associated with more sensitivity of the wage rate to workers' outside options.

2.2.2. An increase in δ and r

We now establish that W_U is decreasing in r and increasing in δ. To understand the impact of δ on W_U it is useful to first consider the extreme case where $\delta = 0$. In this situation, there is no chance that workers and firms will be exogenously thrown to their outside options in the midst of negotiations. Here U simply doesn't enter the best response functions and $W_U = 0$.[10] More generally an increase in δ directly raises the importance of U in the worker's disagreement payoff, a force that makes W_U increasing in δ. The basic reason for why W_U is decreasing in r is that a rise in the intra-period discount rate reduces disagreement payoffs.

More formally, (2.21) implies

$$\begin{aligned} \frac{dW_U}{dr} &= -\frac{2(1-\delta)^2}{(r+1)(r+\delta)(2+r-\delta)} W_U < 0, \\ \frac{dW_U}{d\delta} &= \left(\frac{1+r}{(r+\delta)(2+r-\delta)}\right)^2 [r(2+r-\delta)+\delta(r+\delta)] > 0. \end{aligned} \tag{2.23}$$

[9] Substitute the steady state version of (2.4) into (2.3) and solve the result for V:

$$V = \frac{w + (1-\rho)(U-D)}{1-\beta\rho},$$

where $w = \vartheta - (1-\rho\beta)\kappa$ is defined by (2.22). Use the latter to express $\tilde{V} = \beta\rho V + (1-\rho)(U-D)$ as a function of U only. Substituting this into (2.19) and rearranging, the firm's best response function, (2.19) reduces to:

$$w_t = \frac{\Phi(r,\rho,\delta)}{(1+r)(1-\beta\rho)} U - \frac{r+\delta}{1+r}\beta\rho\frac{\vartheta-(1-\rho\beta)\kappa}{1-\beta\rho} + \frac{r+\delta}{1+r}\frac{1}{1-\beta\rho}(1-\rho)D + \frac{1-\delta}{1+r}w_t^l,$$

where

$$\Phi(r,\rho,\delta) \equiv \delta[(1+r)(1-\beta\rho)-(1-\rho)] - r(1-\rho).$$

The condition for an increase in U to shift the firm's best response function up is $\Phi(r,\rho,\delta) > 0$. This condition is satisfied in the parameterizations we work with in this paper.

[10] Of course V and V^l must exceed U for workers not to want to go to their outside option.

The effects of r and δ on $W_{U,t}$ are complicated because they shift and rotate the worker's and the firm's best response functions (see Figure 1, Panel A). Nevertheless, the sign of the effect of r and δ on W_U is definite in each case. It is easy to see graphically why W_U is decreasing in r. The increase in r makes the worker response function steeper which, other things the same, reduces W_U. In addition, the rise in r makes the firm response function flatter. Other things equal, that effect also reduces W_U (see Figure 1, Panel A).

Obtaining an analytic expression for the response of w_U to δ and r is straightforward. Signing the response of w_U to these parameters is less straightforward. In numerical experiments we found that w_U is increasing in δ. So, the model has the intuitively appealing feature that bargaining is less responsive to general economic conditions when δ is smaller. We found that the sign of w_U to an increase in r is opposite to the sign of dW_U/dr. While the latter is negative, an increase in r raises U.

We conclude this subsection by emphasizing its static, partial equilibrium nature. Dynamic, general equilibrium considerations can in principle overturn the results in this section. As it turns out, in the estimated medium-sized DSGE model, w_U is decreasing in γ and r but increasing in δ.

3. Incorporating the Labor Market Model into a Simple Macroeconomic Framework

In this section we incorporate the labor market model of the previous section into the benchmark New Keynesian macroeconomic model. We use this framework to explore the intuition for how the alternating offer bargaining model of the labor market helps to account for the cyclical volatility of key labor variables. In contrast to the analysis in subsection (2.2), here we allow for dynamic general equilibirum effects.

3.1. Simple Framework

As in Merz (1996) and Andolfatto (1995) we assume that each household has a unit measure of workers. Each worker supplies labor inelastically to the labor force. Workers don't derive disutility from working and maximize the expected quantity of funds that they bring home, subject to their labor market opportunities. An employed worker brings home the real wage, w_t. An unemployed worker receives government provided unemployment compensation of D goods. The latter is financed via lump-sum taxation. Workers give households their labor income and unemployment benefits in exchange for perfect consumption insurance. Our assumptions about preferences implies that it is optimal for the household to allocate the

same level of consumption, C_t, to each worker, independent of their job market experience.[11]

The household maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln(C_t)$$

subject to the sequence of budget constraints:

$$P_t C_t + B_{t+1} \leq W_t h_t + (1 - h_t) P_t D + R_{t-1} B_t - T_t.$$

Here $0 \leq h_t \leq 1$ denotes the fraction of the household's workers that is employed. In addition, T_t denotes lump-sum taxes net of lump-sum profits and B_{t+1} denotes purchases of bonds in period t. Finally, R_{t-1} denotes the gross nominal interest rate on bonds purchased in the previous period.

A final homogeneous good, Y_t, is produced by competitive and identical firms using the following technology:

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}, \ \epsilon > 1. \tag{3.1}$$

The representative firm chooses the specialized inputs, $Y_{j,t}$, to maximize profits:

$$P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj,$$

subject to the production function. The firm's first order condition is:

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}} \right)^{\epsilon} Y_t. \tag{3.2}$$

Following Ravenna and Walsh (2008), we say that the j^{th} input good is produced by a monopolist *retailer*, with production function

$$Y_{j,t} = \exp(a_t) h_{j,t},$$

where $h_{j,t}$ is the quantity of the intermediate good purchased by the j^{th} producer. This intermediate good is purchased in competitive markets at the after-tax price $(1 - \nu) P_t^h$ from a *wholesaler*. Here, ν represents a subsidy (financed by a lump-sum tax on households) which has the effect of eliminating the monopoly distortion in the steady state. That is, $1 - \nu = (\epsilon - 1)/\epsilon$. In the retailer production function, a_t denotes a technology shock that has the law of motion:

$$a_t - (\tau_1 + \tau_2) a_{t-1} + \tau_1 \tau_2 a_{t-2} = \varepsilon_t,$$

[11] Because the representative household has a large number of workers it maximizes total household wage income by instructing each worker to maximize expected wage income.

where ε_t is the iid shock to technology and $|\tau_i| < 1, i = 1, 2$. For reasons discussed below, we adopt an AR(2) specification to allow for a hump-shaped response of technology to a shock. The monopoly producer of $Y_{j,t}$ sets $P_{j,t}$ subject to Calvo sticky price frictions. In particular,

$$P_{j,t} = \begin{cases} P_{j,t-1} & \text{with probability } \xi \\ \tilde{P}_t & \text{with probability } 1-\xi \end{cases}. \tag{3.3}$$

Here, $\tilde{P}_t$ denotes the optimal price set by the $1-\xi$ producers that have the opportunity to reoptimize.

Let

$$s_t = \frac{\vartheta_t}{\exp(a_t)} \tag{3.4}$$

where $\vartheta_t = P_t^h/P_t$ so that $(1-\nu)s_t$ denotes the retail firm's real marginal cost. Also, let

$$h_t = \int_0^1 h_{j,t} dj.$$

The wholesalers that produce h_t correspond to the perfectly competitive firms modeled in the previous section. Recall that they produce h_t using labor only and that labor has a fixed marginal productivity of unity. The total supply of the intermediate good is given by l_t which equals the total quantity of labor used by the wholesalers. So, clearing in the market for intermediate goods requires

$$h_t = l_t. \tag{3.5}$$

We adopt the following monetary policy rule:

$$\ln(R_t/R) = \alpha \ln(R_{t-1}/R) + (1-\alpha)\left[\phi_\pi \pi_t + \phi_y \log(l_t/l)\right] + \varepsilon_{R,t} \tag{3.6}$$

where $\pi_t = P_t/P_{t-1}$ denotes the gross inflation rate and $\varepsilon_{R,t}$ is a monetary policy shock.

3.2. Integrating the Labor Market into the Simple Framework

There are four points of contact between the model in this section and the one in the previous section. The first point of contact is the labor market in the wholesale sector where wages are determined as in section 2. The second point of contact is via ϑ_t in (3.4) which corresponds to the real price that appears in the previous section (see, e.g., (2.1)). The third point of contact occurs via the asset pricing kernel, m_{t+1}, which is now given by:

$$m_{t+1} = \frac{C_t}{C_{t+1}}. \tag{3.7}$$

The fourth point of contact is the resource constraint which specifies how the homogeneous good, Y_t, is allocated among the uses of goods in this economy. For our benchmark model, this constraint is given by

$$C_t + \kappa x_t l_{t-1} = Y_t, \tag{3.8}$$

where

$$Y_t = \exp(a_t)\, l_t. \tag{3.9}$$

Here $\kappa x_t l_{t-1}$ denotes the cost of generating new hires in period t. Expression (3.9) is the production function for the final good. The absence of price distortions in this expression reflects Yun's (1996) result that these distortions can be ignored in (3.9) when linearizing about a nonstochastic steady state in which price distortions are absent.

From the perspective of the model in this section, the prices in the previous section correspond to real prices. So, w_t and w_t^l are to be interpreted as real wages, where conversion to real is accomplished using P_t,i.e. firms and workers bargain over real wages using the protocol discussed in the previous section.

3.3. Quantitative Results in the Simple Model

This section displays the dynamic response of our simple model to monetary policy and technology shocks. In addition we discuss the sensitivity of these responses to δ, γ and r which govern the behavior of the alternating offer bargaining model. The first subsection below reports the values assigned to the parameters of the models. Impulse responses are presented in the second subsection.

3.3.1. Baseline Parameterization

Table 1 lists the baseline parameter values. We set the parameters of the monetary policy rule, (3.6), ϕ_π, ϕ_y, α equal to $1.5, 0.05$ and 0.75,respectively. For convenience we assume that the steady state inflation rate, π, is equal to zero. In addition we set the parameter that controls the degree of price stickiness, ξ, to 0.75. In addition, we assume that the parameter which governs the elasticity of substitution between retail consumption goods, ϵ, is equal to 6. This value implies a steady state markup of 1.2. We set the discount factor β to $(1.03)^{-.25}$.

We assume that the intra period discount rate, r, is equal to the daily value implied by β, i.e., $r = \beta^{-4/365} - 1$. This way of calibrating r is consistent with HM's assumption that the period between alternating offers is one day. As in Ravenna and Walsh (2008) we assume that $\rho = 0.9$ which implies a match survival rate that is consistent with both HM and Shimer (2012).[12] Finally, we set δ, the probability that negotiations break down after an offer is rejected, to 0.65 of one percent. This value is roughly the same as the one used by HM.

[12] Denote the probability that a worker separates from a job at a monthly rate by $1-\tilde{\rho}$. The probability that a person employed at the end of a quarter separates in the next three months is $(1-\tilde{\rho})+\tilde{\rho}(1-\tilde{\rho})+\tilde{\rho}^2(1-\tilde{\rho}) = (1-\tilde{\rho})\left(1+\tilde{\rho}+\tilde{\rho}^2\right)$. Shimer (2012) reports that $\tilde{\rho} = 1-0.034$, implying a quarterly separation rate of 0.0986. HM assume a similar value of 0.03 for the monthly separation rate. This value is also consistent with Walsh's (2003) summary of the empirical literature.

We calibrate the remaining model parameters, D, κ and γ to hit three steady state targets. The first target is a steady state unemployment rate, $1-h$, of 5 percent. The second target is a value of 1 percent for steady state hiring costs relative to gross output, i.e., $\kappa x h/Y = 0.01$. The third target is a steady state value of unemployment benefits relative to wages, D/w , equal to 0.4. The resulting values of D, κ and γ are 0.396, 0.1, and 0.0156,respectively.

Finally we assume the parameters, τ_1 and τ_2, which govern the law of motion for technology are equal to 0.9 and 0.80, respectively. This specification implies that the maximal response of a_t to a technology shock occurs with a delay. So the simple model can mimic an important property of our estimated DSGE model. According to our estimates, the growth rate of technology is positively autocorrelated, so that the maximal response of the level of technology also occurs with a delay.

Table 2 summarizes the steady state properties of the simple model. Note that in conjunction with the other parameter values, the calibrated value of γ is roughly equal to two day's output in the model.[13] HM use a value of γ that is roughly equal to one-quarter of a day's work. The estimated DSGE model in section (4) implies a value of γ that is roughly equal to four day's output in the model. So the value of γ that we use here is roughly half-way between HM's assumed value and our estimated value. In assessing these values, it would be useful to have evidence that bears directly on γ. We do not have such evidence. Of course, if our model was literally true, no offers would ever be rejected so there could never be direct evidence on the cost of preparing a counter offer to a rejected offer.

3.3.2. Impulse Responses

Figures 2 and 3 display the dynamic responses to monetary policy and technology shocks. We report results for the baseline parameterization. In addition, we display results for three other parameterizations, each of which changes the value of one parameter relative to the baseline case. In the first case, we lower γ to 0.0143. In the second case we raise δ to 0.0075.Finally, in the third case, we raise r to $(1.032)^{1/365} - 1$.

Figure 2 displays the dynamic responses of our baseline model and the three alternatives to a 25 basis points (on an annualized basis) negative shock to monetary policy, $\varepsilon_{R,t}$, in (3.6). In the baseline model, real wages respond by a very modest amount with the peak rise equal to 0.05 of one percent. Inflation also responds very modestly, with a peak rise of 0.04 of one percent (on annual basis.) At the same, there is a substantial increase in consumption, which initially jumps by 0.2 of one percent. Finally, the unemployment rate is also very responsive, dropping by 0.2 of one percent in the impact period of the shock.

We now consider the impact of a lower value of γ on the model economy. As reported in

[13]Daily output is one quarter's production divided by 120 days. Steady state quarterly output is 0.94. So the value of daily output is 0.94/120 or 0.007. The calibrated value of γ is roughly twice this amount.

Figure 2, steady state consumption rises while steady state unemployment falls. Steady state inflation and the real wage are unaffected. Figure 2 also shows that the dynamic responses of real wages and inflation to the monetary policy shock are stronger than in the baseline case. At the same time, consumption and unemployment respond by less than in the baseline case. The basic intuition here is the same as the one discussed in subsection (2.2.1), even though here we allow for dynamic general equilibrium effects.

Consider next the effect of raising either δ or r. In both cases, steady state consumption increases and unemployment falls relative to the baseline case. Consistent with the intuition in (2.2.2), a rise in δ increases the sensitivity of the real wage to the policy shock. As a result consumption and unemployment respond by less than in the baseline case while inflation responds by more. As we stressed above these effects reflect that a higher value of δ makes the disagreement payoff of workers more sensitive to the value of their outside option, U_t. The impact of a rise in r are qualitatively similar to the effects of a rise in δ.

Figure 3 displays the dynamic responses of our baseline model and the three alternatives to a 0.1 of a percent technology shock. In the baseline model, real wages rise but by a relatively modest amount. Inflation too falls by a modest amount, with a peak decline of about one-quarter of one percent (on an annualized basis). Notice that unemployment falls by a substantial amount in the impact period of the shock, declining by one-quarter of one percent. Clearly our model is not subject to the problems of the standard DMP model highlighted by Shimer (2005).

The effect of lowering γ is to make the real wage and inflation more responsive to the technology shock. While the response of consumption is not much affected, the decline in unemployment is muted relative to the baseline parameterization. As with the monetary policy shock, these results are broadly consistent with the intuition in subsection (2.2.1). Finally notice that the effect of raising δ is to exacerbate the impact of the technology shock on real wages, while muting its effect on the unemployment rate.

We conclude this section with an important caveat. The impact of perturbing γ and δ on the response of different variables to monetary policy and technology shocks in the model economy is quite robust. But it is easy to find examples in which dynamic general equilibrium considerations overturn the simple static intuition regarding changes in r highlighted in subsection (2.2.2). Indeed in Figures 2 and 3 a higher value of r is associated with a larger initial rise in in real wages and a marginally smaller decline in the unemployment rate after an expansionary monetary policy and technology shock, respectively.

In sum, in this section we have shown that the alternating offer labor market model has the capacity to account for the cyclical properties of key labor market variables. In the next section we analyze whether it actually provides an empirically convincing accout of those properties. To that end we embed it in a medium-sized DSGE model which we estimate and

evaluate.

4. An Estimated Medium-sized DSGE Model

In this section, we describe a medium-sized DSGE model similar to one in CEE, modified to include our labor market assumptions. The first subsection describes the problems faced by households and goods producing firms. The labor market is discussed in the second subsection. The third subsection specifies the law of motion of the three shocks to agents' environment. These include a monetary policy shock, a neutral technology shock and an investment-specific technology shock. The last subsection briefly presents a version of the model corresponding to the standard DMP specification of the labor market, i.e. wages are determined by a Nash sharing rule and firms face vacancy posting costs. In addition, we also examine a version of the model with sticky wages as proposed in EHL. These versions of the model represent important benchmarks for comparison.

4.1. Households and Goods Production

The basic structure of the representative household's problem is the same as the one in section 3.2). Here we allow for habit persistence in preferences, time varying unemployment benefits, and the accumulation of physical capital, K_t.

The preferences of the representative household are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln \left(C_t - bC_{t-1} \right).$$

The parameter b controls the degree of habit formation in household preferences. We assume $0 \leq b < 1$. The household's budget constraint is:

$$P_t C_t + P_{I,t} I_t + B_{t+1} \leq \left(R_{K,t} \Lambda_t - a(\Lambda_t) P_{I,t} \right) K_t + \left(1 - h_t \right) P_t D_t + h_t W_t + R_t B_t - T_t \ . \quad (4.1)$$

As above, T_t denotes lump taxes net of firm profits and D_t denotes the unemployment compensation of an unemployed worker. In contrast to (2.4), D_t is exogenously time varying to ensure balanced growth. In (4.1), B_{t+1} denotes beginning-of-period t purchases of a nominal bond which pays rate of return, R_{t+1} at the start of period $t+1$, and $R_{K,t}$ denotes the nominal rental rate of capital services. The variable, $P_{I,t}$, denotes the nominal price of an investment good. Also, I_t denotes household purchases of investment goods.

The household owns the stock of capital which evolves according to

$$K_{t+1} = \left(1 - \delta_K \right) K_t + \left[1 - S\left(I_t / I_{t-1} \right) \right] I_t.$$

The function, $S\left(\cdot \right)$, is an increasing and convex function capturing adjustment costs in investment. We assume that S and its derivative are both zero along a steady state growth

path. We discuss this function below. Capital services are given by $\Lambda_t K_t$ where Λ_t denotes the utilization rate of capital. Analogous to CEE we assume that the household sells capital services in a perfectly competitive market. In (4.1), $R_{K,t}\Lambda_t K_t$ represents the household's earnings from supplying capital services. The increasing convex function $a(\Lambda_t)$ denotes the cost, in units of investment goods, of setting the utilization rate to Λ_t.

As in our simple macroeconomic model, we assume that a final good is produced by a perfectly competitive representative firm using the technology, (3.1). The final good producer buys the j^{th} specialized input, $Y_{j,t}$, from a retailer who uses the following technology:

$$Y_{jt} = (\Lambda_{j,t}K_{j,t})^{\alpha}(z_t h_{j,t})^{1-\alpha} - \phi_t. \tag{4.2}$$

Here $\Lambda_{j,t}K_{j,t}$ denotes the total amount of capital services that firm j purchases in a perfectly competitive market. Also, ϕ_t represents an exogenous fixed cost of production which grows in a way that ensures balanced growth. The fixed cost is calibrated so that, along the balanced growth path, profits are zero. In (4.2), z_t is a technology shock whose properties are discussed below. Finally, $h_{j,t}$ is the quantity of an intermediate good purchased by the j^{th} retailer. This good is purchased in competitive markets at the price P_t^h from a *wholesaler*, whose problem is discussed in the next subsection. Analogous to CEE, we assume that to produce in period t, the retailer must borrow $P_t^h h_{j,t}$ at the start of the period at the interest rate, R_t. The retailer repays the loan at the end of period t when it receives its sales revenues. The j^{th} retailer sets its price, $P_{j,t}$, subject to its demand curve, (3.2), and the Calvo sticky price friction:

$$P_{j,t} = \begin{cases} \tilde{\pi}_t P_{j,t-1} & \text{with probability } \xi \\ \tilde{P}_t & \text{with probability } 1-\xi \end{cases},$$

where $\tilde{\pi}_t = (\pi_{t-1})^{\kappa^f}\pi^{1-\kappa^f}$ with $\kappa^f \in (0,1)$. A value of κ^f equal to one corresponds to the case of full dynamic indexation considered in CEE. We define the gross inflation rate as $\pi_t \equiv P_t/P_{t-1}$ which has a nonstochastic steady state value of π.

4.2. Wholesalers and the Labor Market

Each wholesaler employs a measure of workers. Let l_{t-1} denote the representative wholesaler's labor force at the end of $t-1$. A fraction $(1-\rho)$ of these workers separate exogenously. So, the wholesaler has a labor force of ρl_{t-1} at the start of period t. At the beginning of period t the firm selects its hiring rate, x_t which determines the number of new workers that it meets at time t. For our empirical model, we follow Gertler, Sala and Trigari (2008) by assuming that the firm's cost hiring is an increasing function of the hiring rate,

$$\kappa_t x_t^2 l_{t-1}/2. \tag{4.3}$$

The cost is denominated in units of the final consumption good. Here κ_t is an exogenous process which grows in a way that ensures balanced growth. When the cost of hiring new workers is linear in the number of new workers that the firm meets, $x_t l_{t-1}$, the labor market equilibrium conditions coincide with the ones derived for the hiring cost specification in the model of section (3). Convexity in the cost of hiring has two effects. First, it implies that as employment rises in response to an expansionary shock, the marginal cost of hiring increases. Second, it enhances the value of a worker to the firm during wage negotiations because the firm has already paid the costs of meeting the worker. If the firm were to walk away from this worker, it would have to bear the cost of meeting another worker. Greater convexity makes this effect more important.

After setting x_t,the firm has access to l_t workers (see (2.5)). The firm then engages in bilateral bargaining with individual workers.[14] As above, the real wage rate w_t, i.e. W_t/P_t, denotes the equilibrium real wage that emerges from the bargaining process. As with the small model, we verify numerically that all bargaining sessions conclude successfully with the firm representative and worker agreeing to an employment contract, i.e. neither firms nor workers choose to reject an offer and go to their outside option. Thus, in equilibrium the representative wholesaler employs all l_t workers with which it has met, at wage rate w_t.

In what follows, we derive various value functions and an expression for the firm's hiring decision. We then discuss alternating offer bargaining in the medium-sized model.

4.2.1. Value Functions and Hiring Decision

To describe the bargaining process we must define the values of employed and unemployed workers, V_t and U_t. We must also define the value, J_t, assigned by the firm to employing a marginal worker that it is in contact with. We express each of U_t, V_t and J_t in units of the final good. The value of being an unemployed worker is given by (2.4) except that D is replaced by D_t.The job finding rate is given by (2.6) where x_{t+1} and l_t denote the average value of the corresponding wholesaler specific variables. Individual workers view x_{t+1} and l_t as being exogenous and beyond their control.[15] As in (2.3), V_{t+1} is the value of a worker that is employed at the equilibrium wage w_{t+1} in period $t+1$.

We now consider the value, J_t, assigned by the firm to employing the marginal worker in l_t at the wage rate, w_t :

$$J_t = \vartheta_t - w_t + E_t m_{t+1} F_{l,t+1}(l_t). \tag{4.4}$$

Here, $\vartheta_t \equiv P_t^h/P_t$ is the real price of the intermediate good produced by a worker. Thus, $\vartheta_t - w_t$ represents the time t flow profit associated with a marginal worker. The term, $F_{l,t+1}$,

[14] Alternatively we could assume that there is a union which represents all the workers that are in contact with the representative wholesaler and that it bargains with management over the wage rate.

[15] Since wholesalers are identical, x_{t+1} and l_t are equal to the values chosen by the representative wholesaler.

represents the contribution of a marginal worker to the wholesaler's time $t+1$ profit. The present discounted value of the representative wholesaler's profits beginning in $t+1$ is:

$$F_{t+1}(l_t) = \max_{x_{t+1}} \left\{ [\vartheta_{t+1} - w_{t+1}](\rho + x_{t+1}) l_t - 0.5\kappa_{t+1} x_{t+1}^2 l_t + E_{t+1} m_{t+2} F_{t+2}((\rho + x_{t+1}) l_t) \right\}. \tag{4.5}$$

Differentiating $F_{t+1}(l_t)$ with respect to l_t and taking the envelope condition into account we obtain:

$$\begin{aligned} F_{l,t+1}(l_t) &= (\vartheta_{t+1} - w_{t+1})(\rho + x_{t+1}) - 0.5\kappa_{t+1} x_{t+1}^2 + E_{t+1} m_{t+2} F_{l,t+2}((\rho + x_{t+1}) l_t)(\rho + x_{t+1}) \\ &= J_{t+1}(\rho + x_{t+1}) - 0.5\kappa_{t+1} x_{t+1}^2, \end{aligned} \tag{4.6}$$

where x_{t+1} is the hiring rate that solves the maximization problem in (4.5). The second equality in (4.6) makes use of (4.4). Using (4.6) to substitute out for $F_{l,t+1}(l_t)$, in (4.4) we conclude:

$$J_t = \vartheta_t - w_t + E_t m_{t+1} \left[J_{t+1}(\rho + x_{t+1}) - 0.5\kappa_{t+1} x_{t+1}^2 \right]. \tag{4.7}$$

This expression can be simplified using the first order condition for x_t. Maximizing the time t version of (4.5) with respect to x_t and using (4.6) we obtain:

$$\kappa_t x_t = \vartheta_t - w_t + E_t m_{t+1} \left[J_{t+1}(\rho + x_{t+1}) - 0.5\kappa_{t+1} x_{t+1}^2 \right]. \tag{4.8}$$

Combining (4.7) and (4.8), yields

$$J_t = \kappa_t x_t. \tag{4.9}$$

So, the value to a firm which has an initial labor of l_{t-1} of employing a marginal worker is equal to the marginal cost of hiring the a worker. Using (4.9) to substitute out for period $t+1$ adjustment costs in (4.7), we obtain a useful recursive representation expression for J_t :

$$J_t = \vartheta_t - w_t + E_t m_{t+1} J_{t+1}(\rho + 0.5 x_{t+1}). \tag{4.10}$$

4.2.2. Wage Bargaining

The equilibrium wage rate, w_t, is the outcome of a version of the alternating offer bargaining process described in the simple model. The only difference is that the cost to a firm of making a counteroffer to an offer that it rejects is given by γ_t instead of γ. The variable γ_t varies in an exogenous way to ensure that the model has a well defined balanced growth path. It is straightforward to show that we can summarize the outcome of the bargaining process with the analog to (2.14) where γ is replaced by γ_t:

$$J_t = \frac{1+r}{1-\delta} [V_t - \alpha U_t - \omega_t], \tag{4.11}$$

where

$$\alpha \equiv 1 - r\frac{1-\delta}{r+\delta},\ \omega_t \equiv \frac{(1-\delta)^2}{\delta+r}\gamma_t.$$

The five key equilibrium conditions related to the labor market and wholesalers are (4.10), (4.9), (4.11), (2.6) and (2.4) with D replaced by D_t. These equations reduce to the analogs of (2.2) and (2.1) when the cost of hiring new workers is linear in the number of new workers. In this case, the equilibrium conditions of the labor marketare identical to what they are in our simple model except that D and γ are replaced by D_t and γ_t.

4.3. Market Clearing, Monetary Policy and Functional Forms

The total amount of intermediate goods purchased by retailers from wholesalers is:

$$h_t \equiv \int_0^1 h_{j,t}dj.$$

Recall that the output of intermediate goods produced by wholesalers is equal to the the number of workers they employ. So the the supply of intermediate goods is l_t. It follows that as in the simple model, market clearing for intermediate goods requires $h_t = l_t$. Market clearing for final goods requires:

$$C_t + \frac{1}{\Psi_t}(I_t + a(\Lambda_t)K_t) + 0.5\kappa_t x_t^2 l_{t-1} + G_t = Y_t.$$

The right hand side of the previous expression denotes the quantity of final goods. The left hand side represents the various ways that final goods are used. Homogeneous output, Y_t, can be converted one-for-one into either consumption goods, goods used to hire workers or government goods, G_t. In addition Y_t is used to increase the cost of capital utilization. Finally, Y_t can be used to produce investment goods using a linear technology in which one unit of final goods is transformed into Ψ_t units of I_t. Perfect competition in the production of investment goods implies

$$P_{I,t} = \frac{P_t}{\Psi_t}.$$

It is convenient to substitute out for hiring costs in the resource constraint using (4.9):

$$C_t + \frac{1}{\Psi_t}(I_t + a(\Lambda_t)K_t) + 0.5J_t x_t l_{t-1} + G_t = Y_t. \tag{4.12}$$

The asset pricing kernel, m_{t+1}, is constructed using the marginal utility of consumption, which we denote by $u_{c,t}$:

$$u_{c,t} = \frac{1}{C_t - bC_{t-1}} - \beta b E_t \frac{1}{C_{t+1} - bC_t}.$$

Then,

$$m_{t+1} = \beta \frac{u_{c,t+1}}{u_{c,t}}.$$

We adopt the following specification of monetary policy:

$$\log R_t = \rho_R \log R_{t-1} + (1-\rho_R)\left[\log R + r_\pi \log\left(\pi_t/\pi\right) + r_y \log\left(\mathcal{Y}_t/\mathcal{Y}\right)\right] + \sigma_R \varepsilon_{R,t},$$

where variables without a time subscript correspond to values in non-stochastic steady state. Here, $\varepsilon_{R,t}$ is a unit variance, zero mean shock to monetary policy. We assume that the measure of output that the central bank looks at when setting R is given by,

$$\mathcal{Y}_t = C_t + \frac{I_t}{\Psi_t} + G_t.$$

We assume that G_t grows in an exogenous way that is consistent with balanced growth. In terms of shocks, we assume that $\log(\mu_{z,t}) \equiv \log\left(z_t/z_{t-1}\right)$ and $\log(\mu_{\Psi,t}) \equiv \left(\Psi_t/\Psi_{t-1}\right)$ are AR(1) processes. The parameters that control the autocorrelations and standard deviations of both processes are denoted by (ρ_z, ρ_Ψ) and (σ_z, σ_Ψ),respectively.

Recall our model exhibits growth stemming from neutral and capital embodied technological progress. The variables $Y_t/\Phi_t, C_t/\Phi_t$, w_t/Φ_t and $I_t/(\Phi_t\Psi_t)$ converge to constants in nonstochastic steady state, where

$$\Phi_t = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t.$$

is a weighted average of the sources of technological progress. If objects like the fixed cost of production, the cost of posting a vacancy, the cost to a firm of preparing a counter offer.government purchases, and transfer payments were constant, they would become irrelevant over time. To avoid this implication it is standard in the literature to suppose that such objects grow at the same rate as output, which in our case is given by Φ_t. [16]. An unfortunate implication of this assumption is that technology shocks of both types immediately affect the vector of objects $[\phi, \kappa, \gamma, D, G]'$. It seems hard to justify such an assumption. To avoid this problem, we proceed as in Schmitt-Grohé and Uribe (2012) who assume that government purchases, G_t, are a distributed lag of unit root technology shocks, i.e. G_t is cointegrated with Y_t but has a smoother stochastic trend.

Proceeding as in Schmitt-Grohé and Uribe (2012), we assume that

$$[\phi_t, \kappa_t, \gamma_t, D_t, G_t]' = [\phi, \kappa, \gamma, D, G]' \, \Omega_t.$$

where Ω_t denotes the distributed lag of past values of Φ_{t-1} defined by

$$\Omega_t = \Phi_{t-1}^{\varsigma} \Omega_{t-1}^{1-\varsigma}. \tag{4.13}$$

[16] See for ACEL (2011) and Justiniano, Primiceri, and Tambalotti (2010).

Here $0 < \varsigma \leq 1$ is a parameter to be estimated. Note that Ω_t grows at the same rate as Φ_t.When ς is very close to zero, Ω_t is virtually completely unresponsive in the short-run to an innovation in either of the two technology shocks, a feature that we find very attractive on a priori grounds. Interesting, or point estimate of ς is close to zero so that the estimated model has this feature.

We assume that the cost of adjusting investment is assumed to take the following form:

$$S\left(I_t/I_{t-1}\right) = 0.5\exp\left[\sqrt{S''}\left(I_t/I_{t-1} - \mu_\Phi\mu_\Psi\right)\right] + 0.5\exp\left[-\sqrt{S''}\left(I_t/I_{t-1} - \mu_\Phi\mu_\Psi\right)\right] - 1.$$

Here, μ_Φ and μ_Ψ denote the unconditional growth rates of Φ_t and Ψ_t. The value of I_t/I_{t-1} in nonstochastic steady state is μ_Φ/μ_Ψ. In addition, S'' represents a model parameter that coincides with the second derivative of $S(\cdot)$, evaluated in steady state. It is straightforward to verify that $S\left(\mu_\Phi\mu_\Psi\right) = S'\left(\mu_\Phi\mu_\Psi\right) = 0$.

Finally we assume that the cost associated with setting capacity utilization is given by

$$a(\Lambda_t) = 0.5\sigma_b\sigma_a\left(\Lambda_t\right)^2 + \sigma_b\left(1-\sigma_a\right)\Lambda_t + \sigma_b\left((\sigma_a/2) - 1\right)$$

where σ_a and σ_b are positive scalars. We normalize the steady state value of Λ_t to one, which derermines the value of σ_b given an estimate of σ_a.

4.4. Alternative Labor Market Models

In this subsection we consider alternative labor market models that we include in our DSGE framework. The objective is to assess the relative empirical performance of these alternative models. First, we describe the canonical DMP model which is characterized by search costs and a particular Nash sharing rule. Second, we describe the canonical sticky nominal wage model of Erceg, Henderson and Levin (2000).

4.4.1. The Canonical DMP Model

To incorporate the Nash sharing rule into our DSGE model we simply set $\omega_t = 0$, $\alpha = 1$ and replace $(1+r)/(1-\delta)$ with $(1-\eta)/\eta$ in (4.11). Here, η is the share of total surplus given to workers. Doing so we obtain the *Nash sharing rule*:

$$J_t = \frac{1-\eta}{\eta}\left[V_t - U_t\right],\ 0 \leq \eta \leq 1. \tag{4.14}$$

We now discuss how to incorporate DMP-style search costs into our DSGE model. In particular we suppose that a firm pays a cost to post a vacancy in which case they meet workers with probability Q_t.The aggregate rate at which workers are hired, x_t, depends on aggregate vacancy rate, v_t

$$x_t = v_t Q_t. \tag{4.15}$$

Analogous to the hiring rate cost specification (4.3), we assume that the cost of setting the vacancy rate to v_t is given by[17]

$$0.5\kappa_t v_t^2 l_{t-1}. \tag{4.16}$$

The probability, Q_t, that a vacancy results in a meeting with a worker is determined by the matching function:

$$x_t l_{t-1} = \sigma_m \left(1-\rho l_{t-1}\right)^{\sigma} \left(l_{t-1} v_t\right)^{1-\sigma}. \tag{4.17}$$

Here $l_{t-1}v_t$ represents the number of vacancies posted at the start of period t, $1-\rho l_{t-1}$ denotes the number of workers searching for work at the start of period t. We refer to this structure defined by equations (4.15) - (4.17) as the *search cost specification*.

Four changes are required to incorporate the search cost specification into the medium-sized DSGE model. Recall that there are five labor market equilibrium conditions, (4.10), (4.9), (4.11), (2.6) and (2.4) with D replaced by D_t. First, replace (4.10) is replaced by

$$J_t = \vartheta_t - w_t + E_t m_{t+1} J_{t+1} \left(\rho + 0.5 v_{t+1} Q_{t+1}\right)$$

Second, replace (4.9) is be replaced by:

$$Q_t J_t = \kappa_t v_t. \tag{4.18}$$

So free entry and the zero-profit condition in the search cost specification imply that the expected return to posting a vacancy is equal to the marginal cost of doing so. Third, we add Q_t and v_t to the set of variables that must be solved and and add (4.15) and (4.17) to the list of equilibrium conditions used. The fourth change involves replacing the hiring cost term in (4.12) with the vacancy cost term (4.16) in the resource constraint. Doing so we obtain

$$C_t + \frac{1}{\Psi_t}(I_t + a(\Lambda_t)K_t) + 0.5\kappa_t v_t^2 l_{t-1} + G_t = Y_t. \tag{4.19}$$

We conclude by discussing an important feature of the search cost specification. Define labor market tightness as

$$\Gamma_t = \frac{v_t l_{t-1}}{1-\rho l_{t-1}}. \tag{4.20}$$

Relations (4.15) and (4.17) imply that Q_t is given by,

$$Q_t = \sigma_m \Gamma_t^{-\sigma}$$

It follows that the probability of filling a vacancy is decreasing in labor market tightness. As discussed below, this feature of the model is important in understanding the empirical shortcoming of the standard DMP model.

[17] At a slight cost of creating confusion, we simplify the notation by not distinguishing between the economy-wide average values of a variable and its value for a particular firm. In (4.15), v_t denotes the vacancy rate of the representative firm. At the same time, it is the economy-wide average values of v_t that define Q_t. The distinction between the economy-wide average value of a variable and its value for the representative firm is crucial when deriving the first order conditions associated with the firm's decisions.

4.4.2. The Canonical Sticky Wage Model

We now describe how to modify the medium-sized DSGE model to incorporate the sticky nominal wage framework of Erceg, Henderson and Levin (2000). We replace the wholesale production sector with the following environment. As in the medium-sized DSGE model, the final homogeneous good, Y_t, is produced by competitive and identical firms using the technology given by (3.1). The specialized inputs used in the production of Y_t are produced by retailers using capital services and a homogeneous labor input. The final good producer buys the j^{th} specialized input, $Y_{j,t}$, from a retailer who produces the input using the technology (4.2) As above the capital are purchased in competitive rental markers. In (4.2) h_{jt} refers to the quantity of a homogeneous labor input that firm j from'labor contractors'. These contractors produce the homogeneous labor input by combining a range of differentiated labor inputs, $h_{t,i}$, using the following technology:

$$h_t = \left[\int_0^1 (h_{t,i})^{\frac{1}{\lambda_w}} di\right]^{\lambda_w}, \ \lambda_w > 1. \tag{4.21}$$

Labor contractors are perfectly competitive and take the wage rate, W_t, of h_t as given. They also take the wage rate, $W_{t,i}$, of the i^{th} labor type as given. Profit maximization on the part of contractors leads to the labor demand curve:

$$h_{t,i} = \left(\frac{W_t}{W_{t,i}}\right)^{\frac{\lambda_w}{\lambda_w - 1}} h_t. \tag{4.22}$$

Substituting the (4.21) into (4.22) and rearranging, we obtain:

$$W_t = \left[\int_0^1 W_{t,i}^{\frac{1}{1-\lambda_w}} di\right]^{1-\lambda_w}. \tag{4.23}$$

Specialized labor inputs are supplied by a large number of identical households. The representative household has many members corresponding to each type, i, of labor and provides complete insurance to all of its members in return for their wage income. The household's budget constraint is given by (4.1) except that D_t is equal to zero. This constraint reflects our assumption that the household owns the capital stock, sets the utilization rate and makes investment decisions are described in the alternating offer bargaining specification of the model.

Given our assumptions, it is optimal for the household to assign an equal amount of consumption to each of its members. The household's utility is given by:

$$\log\left(C_t - bC_{t-1}\right) - A\int_0^1 \frac{h_{t,i}^{1+\psi}}{1+\psi} di. \tag{4.24}$$

Here $h_{t,i}$ denotes hours by the i^{th} member of the household. The wage rate of the i^{th} type of labor, $W_{t,i}$, is determined outside the representative household by a monopoly union that represents all i-type workers across all households.

In setting the wage rate the monopoly union face Calvo-type frictions. and . With probability $1-\xi_w$ the union can optimize the wage $W_{t,i}$ and with probability, ξ_w, it cannot. In the latter case, the nominal wage rate according to:

$$W_{t+1,i} = \pi_t \mu_\Phi W_{t,i}. \tag{4.25}$$

With this specification, the wage of each type i of labor is the same in steady state.

The union acts to maximize the welfare of its members. Because there is no state variable in the union's problem, all unions with the opportunity to re-optimize in a given period face the same problem. In particular, The union chooses the current value of the wage, $\tilde{W}_t$, to maximize:

$$E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \left[\upsilon_{t+s} \tilde{W}_t (\pi_{t+1} \cdots \pi_{t+s}) \mu_\Phi^s h_{t+s} - A \frac{(h_{t+s})^{1+\psi}}{(1+\psi)} \right]. \tag{4.26}$$

subject to its demand curve (4.22). For convenience we have dropped subscript i. Here, h_{t+s} denotes the quantity of hours worked in period $t+s$. The variable υ_{t+s} denotes the marginal value assigned by the representative household to the wage.[18] The union treats υ_t as an exogenous constant. In the above expression, ξ_w appears in the discounting because the union's period t decision only impacts on future histories in which it cannot re-optimize its wage. We refer the reader to Christiano, Trabandt and Walentin (2011) for a detailed exposition of this model.

4.5. Econometric Methodology

In the following section, we conduct inference on medium-sized DSGE model of the previous section. Our methodology, taken from CTW, is described here. In particular, we pursue a Bayesian version of the impulse response matching procedure applied in CEE. To facilitate comparison, our analysis is based on the same vector autoregression (VAR) used in CTW and Christiano, Walentin and Trabandt (2011c). More precisely, CTW estimate a 14 variable VAR using quarterly data that are seasonally adjusted and cover the period 1951Q1 to 2008Q4. The CTW analysis applies 'Choleski identification' to identify a monetary policy shock and 'long-run identification' using a version of the strategy proposed in Fisher (2006) to identify two technology shocks. Specifically, to identify the monetary policy shock CTW assume that the only variable that the monetary policy shock affects contemporaneously is the Federal Funds Rate. They make two assumptions to identify the dynamic response to

[18] The object, υ_t, is the multiplier on the household budget constraint in the Lagrangian representation of the problem.

the technology shocks: (i) the only shocks that affect labor productivity in the long-run are the innovations to the neutral technology shock, $\log z_t$, and the innovation to the investment specific technology shock, $\log \Psi_t$; and (ii) the only shock that affects the price of investment relative to consumption in the long-run is the innovation to $\log \Psi_t$. These identification assumptions are satisfied in our model. An extensive general review of identification in VAR's appears in Christiano, Eichenbaum and Evans (1999). The specific technical details of how CTW compute impulse response functions imposing the shock identification are reported in ACEL.[19]

Our data set extends over a long range, while we estimate a single set of impulse response functions and model parameters. In effect, we suppose that there has been no parameter break over this long period. Whether or not there has been a break over this sample has been debated. For example, it has been argued that the parameters of the monetary policy rule have not been constant over this period. We do not review this debate here. Implicitly, our analysis sides with the conclusions of those that argue that the evidence of parameter breaks is not strong. For example, Sims and Zha (2006) argue that the evidence is consistent with the idea that monetary policy rule parameters have been unchanged over the sample. Christiano, Eichenbaum and Evans (1999) argue that the evidence is consistent with the proposition that the dynamic effects of a monetary policy shock have not changed during this sample. Standard lag-length selection criteria led CTW to work with a VAR with 2 lags.[20]

The 12 variables used to estimate the VAR include standard macroeconomic variables, as well as several labor market variables. The contemporaneous and 14 lagged responses to the three shocks, for a subset of the variables used in the VAR are stacked in a vector, $\hat{\psi}$. The subset of variables whose response we study are indicated in Figures 4-6. We assume that our model is true and we denote the true values of the model parameters by θ_0. Let $\psi(\theta)$ denote the model-implied mapping from a set of values for the model parameters to the impulse responses in $\hat{\psi}$. Thus, $\psi(\theta_0)$ denotes the true value of the impulse responses whose estimates appear in $\hat{\psi}$. According to standard classical asymptotic sampling theory,

[19] The identification assumption for the monetary policy shock by itself imposes no restriction on the VAR parameters. Similarly, Fisher (2006) showed that the identification assumptions for the technology shocks when applied without simultaneously applying the monetary shock identification, also imposes no restriction on the VAR parameters. However, ACEL showed that when all the identification assumptions are imposed at the same time, then there are restrictions on the VAR parameters. We found that the test of the overidentifying restrictions on the VAR fails to reject the null hypothesis that the restrictions are satisfied at the 5 percent critical level.

[20] CTW considered VAR specifications with lag length 1, 2, ..., 12. The Schwartz and Hannan-Quinn criteria indicate that a single lag in the VAR is sufficient. The Akaike criterion indicates 12 lags, though that result was discounted. See CTW for a sensitivity analysis with respect to e.g. the lag length of the VAR.

when the number of observations, T, is large, we have

$$\sqrt{T}\left(\hat{\psi}-\psi\left(\theta_0\right)\right) \overset{a}{\sim} N\left(0, W\left(\theta_0,\zeta_0\right)\right),$$

where ζ_0 denotes the true values of the parameters of the shocks in the model that we do not formally include in the analysis. Because our analysis is based on linearization, $\psi\left(\theta_0\right)$ is not a function of ζ_0. However, the sampling distribution of $\hat{\psi}$ is a function of ζ_0. We find it convenient to express the asymptotic distribution of $\hat{\psi}$ in the following form:

$$\hat{\psi} \overset{a}{\sim} N\left(\psi\left(\theta_0\right), V\left(\theta_0,\zeta_0,T\right)\right), \tag{4.27}$$

where

$$V\left(\theta_0,\zeta_0,T\right) \equiv \frac{W\left(\theta_0,\zeta_0\right)}{T}.$$

The estimation results reported in CTW provide $\hat{\psi}$ and a consistent estimator of $W\left(\theta_0,\zeta_0\right)$. We treat these objects as data for our purposes here. The elements in $\hat{\psi}$ are graphed in Figures 4-6. The solid lines in these figures are the numbers in $\hat{\psi}$. The gray areas are centered, two-standard error bands computed by bootstrap simulation. The matrix, $V\left(\theta_0,\zeta_0,T\right)$, is diagonal with the square of the standard errors on the diagonal (see CTW for details).

For the analysis in this paper, we conduct Bayesian inference in which we treat $\hat{\psi}$ as our observed data, we specify priors for θ, and we then compute the posterior distribution for θ given $\hat{\psi}$ using Bayes' rule. To use Bayes' rule, we require the likelihood of $\hat{\psi}$ given θ. Our asymptotically valid approximation of this likelihood is motivated by (4.27):

$$\begin{aligned} f\left(\hat{\psi}|\theta, V\left(\theta_0,\zeta_0,T\right)\right) &= \left(\frac{1}{2\pi}\right)^{\frac{N}{2}} \left|V\left(\theta_0,\zeta_0,T\right)\right|^{-\frac{1}{2}} \\ &\quad \times \exp\left[-\frac{1}{2}\left(\hat{\psi}-\psi\left(\theta\right)\right)' V\left(\theta_0,\zeta_0,T\right)^{-1}\left(\hat{\psi}-\psi\left(\theta\right)\right)\right]. \end{aligned} \tag{4.28}$$

Under these circumstances, the value of θ that maximizes the above function represents an approximate maximum likelihood estimator of θ. It is approximate for two reasons: (i) the central limit theorem underlying (4.27) only holds exactly as $T \to \infty$ and (ii) the value of $V\left(\theta_0,\zeta_0,T\right)$ that we use is guaranteed to be correct only for $T \to \infty$.

Treating the function, f, as the likelihood of $\hat{\psi}$, it follows that the Bayesian posterior of θ conditional on $\hat{\psi}$ and $V\left(\theta_0,\zeta_0,T\right)$ is:

$$f\left(\theta|\hat{\psi}, V\left(\theta_0,\zeta_0,T\right)\right) = \frac{f\left(\hat{\psi}|\theta, V\left(\theta_0,\zeta_0,T\right)\right) p\left(\theta\right)}{f\left(\hat{\psi}|V\left(\theta_0,\zeta_0,T\right)\right)}, \tag{4.29}$$

where $p\left(\theta\right)$ denotes the priors on θ and $f\left(\hat{\psi}|V\left(\theta_0,\zeta_0,T\right)\right)$ denotes the marginal density of $\hat{\psi}$:

$$f\left(\hat{\psi}|V\left(\theta_0,\zeta_0,T\right)\right) = \int f\left(\hat{\psi}|\theta, V\left(\theta_0,\zeta_0,T\right)\right) p\left(\theta\right) d\theta.$$

As usual, the mode of the posterior distribution of θ can be computed by simply maximizing the value of the numerator in (4.29), since the denominator is not a function of θ. The marginal density of $\hat{\psi}$ is required when we want an overall measure of the fit of our model and when we want to report the shape of the posterior marginal distribution of individual elements in θ. To compute the marginal likelihood, we use the standard Laplace approximation.

4.6. Results

Table 3 provides values for parameters that are not estimated in the medium-sized model. Moreover, table 4 shows the implied steady state of the alternating offer bargaining model with hiring costs at the estimated posterior mode. Table 5 presents the prior and posterior distributions for objects in our model (here, labelled 'Alternating Offers, Hiring'). Most of these objects are model parameters, but some are steady state properties such as the replacement ratio and the hiring-search to output ratio. For each of these steady state properties, some model parameter is made endogenous. We also report results for a version of our model with the search cost specification ('Alternating Offers, Search') and the search and hiring cost versions of the model with Nash bargaining. Finally, we also display results for the model with sticky wages, EHL. In variants of the model where particular parameters are not relevant, we have left the entries in Table 5 blank. We do not consider the analog of the CGG model studied in the small model without capital.

Note from Table 5 that the log, marginal likelihood favors our model over the variants by a substantial amount, at least 10 log points. These marginal likelihoods are all comparable. Figures 4, 5 and 6 provide a visual indication of the fit of the various models. The model is fit to all three sets of estimated impulse response simultaneously. The three figures display the point estimates of the VAR impulse responses to the indicated shocks. The grey areas represent 95 percent probability intervals. Note that our model does as well at matching (non, labor market) variables as the EHL model. Of course, our model also does reasonably well on the unemployment rate, vacancies and the job finding rate. The other models also do reasonably well at matching the responses to the shocks.

Although the models do roughly equally well by a visual metric (though, ours is substantially better according to the marginal likelihood), the good performance of the other models comes at a cost of relatively extreme parameter estimates. According to Table 5, the posterior modes for the replacement ratio are very large for the alternative variants of our labor market model. For the standard DMP formulation ('Nash Search Cost') the posterior mode is 0.97. This finding is reminiscent of the Hagedorn and Manovskii (2008) finding that a high replacement ratio has the potential to boost the volatility of unemployment and

vacancies in search and matching models.[21] The alternating offer bargaining model with search and the Nash model with hiring also imply high values for the replacement ratio. The posterior mode for the replacement ratio is substantially smaller in our model, at 0.74.

What the microeconomic data imply for the size of the replacement ratio is controversial. Gertler, Sala and Trigari (2008) summarize some of this literature and argue that a plausible range for the replacement ratio is given by the interval, 0.4 to 0.7. The lower bound of this range is based on studies of unemployment insurance benefits while the upper boundary as well as informal sources of insurance and possible utility enjoyed while unemployed. Recently Aguiar, Hurst and Karabarbounis (2012) find that the unemployed increase the amount of time that they spend on home production by roughly thirty percent. Taking this fact into account easily rationalizes a replacement rate of 0.74.

5. Conclusion

We have integrated alternating offer wage bargaining into an otherwise standard New Keynesian model of the business cycle. We conduct econometric inference on this model using a Bayesian version of impulse response matching. The result is a model that accounts well for aggregate dynamics. Significantly the model has no exogenous sources of wage stickiness and the parameter values can be defended based on the available studies of micro data. The model performs as well as standard sticky wage models on standard macroeconomic data and in addition it does well on aggregate labor market data.

[21] In an estimation setting that is more similar to ours than is Hagedorn and Manovskii (2008), Gertler, Sala and Trigari (2008) also find that absent frictions that create wage rigidity, a high value of the replacement ratio is required.

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Table 1: Calibrated Parameters in the Small Macro Model

Parameter	Value	Description
β	$1.03^{-0.25}$	Discount factor
ξ	0.75	Calvo price stickiness
$400(\pi - 1)$	0	Annual net inflation rate
ϵ	6	Elasticity, $\epsilon/(\epsilon - 1)$ = price markup
α	0.75	Taylor rule: interest rate smoothing
ϕ_π	1.5	Taylor rule: inflation coefficient
ϕ_y	0.05	Taylor rule: employment coefficient
h	0.95	Employment in steady state
ρ	0.9	Job survival probability
$\kappa x h/y$	0.01	Hiring cost to output ratio
D/w	0.4	Replacement ratio
$1 + r$	$\beta^{(-4/365)}$	Intra-period discounting
δ	0.0065	Prob. of bargaining session break-up
τ_1	0.9	Root 1 for AR(2) technology
τ_2	0.8	Root 2 for AR(2) technology

Table 2: Small Model Steady States and Implied Parameters

Variable	Model	Description
C	0.9405	Consumption
e^a	1	Steady state technology
s	1	Marginal cost of retailers
w	0.9893	Market wage
w^l	1.0055	Counteroffer wage
U	129.529	Value of unemployment
V	130.387	Value of worker at market wage
V^l	130.403	Value of worker at counteroffer wage
J	0.1	Firm value at market wage
J^l	0.0843	Firm value at worker counteroffer wage
u	0.05	Steady state unemployment rate
f	0.6552	Job finding rate
ω	2.3422	Parameter in sharing rule
D	0.3957	Unemployment benefits
κ	0.1	Hiring cost parameter
$90\gamma/y$	1.498	Days of lost production for firm

Table 3: Non-Estimated Parameters in the Medium-sized Model

Parameter	Value	Description
δ^k	0.025	Depreciation rate of physical capital
$400(\pi - 1)$	2.5	Annual net inflation rate
η_g	0.2	Government consumption to output ratio
β	0.99678	Discount factor
$\$/y$	0	Steady state profits to output ratio
Q	0.7	Vacancy filling rate
ρ	0.9	Job survival probability
u	0.055	Steady state unemployment rate
$1 + r$	$1.03^{1/365}$	Intra-period discounting (Alt. offer barg.)
λ^w	1.2	Steady state wage markup (EHL)
ξ^w	0.75	Wage stickiness (EHL)
$400\log(\mu_\Phi)$	1.7	Annual output per capita growth rate
$400\log(\mu_\Phi\mu_\Psi)$	2.9	Annual investment per capita growth rate

Table 4: Medium-sized Model Steady State at Posterior Mode in Alternating Offer/Hiring Cost Model

Variable	Value	Description
$p_{k'}k/y$	6.76	Capital to output ratio (quarterly)
c/y	0.58	Consumption to output ratio
i/y	0.22	Investment to output ratio
l	0.945	Steady state labor input
R	1.0138	Gross nominal interest rate (quarterly)
R^{real}	1.0075	Gross real interest rate (quarterly)
mc	0.704	Marginal cost (inverse markup)
σ_b	0.0355	Capacity utilization cost parameter
y	1.082	Output
ϕ/y	0.421	Fixed cost to output ratio
σ_m	0.662	Level parameter in matching function
f	0.632	Job finding rate
x	0.100	Hiring rate
J	0.118	Value of firm at market wage
J^l	0.078	Value of firm at workers counteroffer wage
V	261.57	Value of worker at market wage
V^l	261.61	Value of worker at counteroffer wage
U	261.24	Value of unemployment
v	0.131	Vacancy rate
w	0.853	Market wage
w^l	0.894	Counteroffer wage
$90\Gamma/y$	2.99	Days of lost production for firm

Table 5: Priors and Posteriors of Parameters for the Medium-sized Model

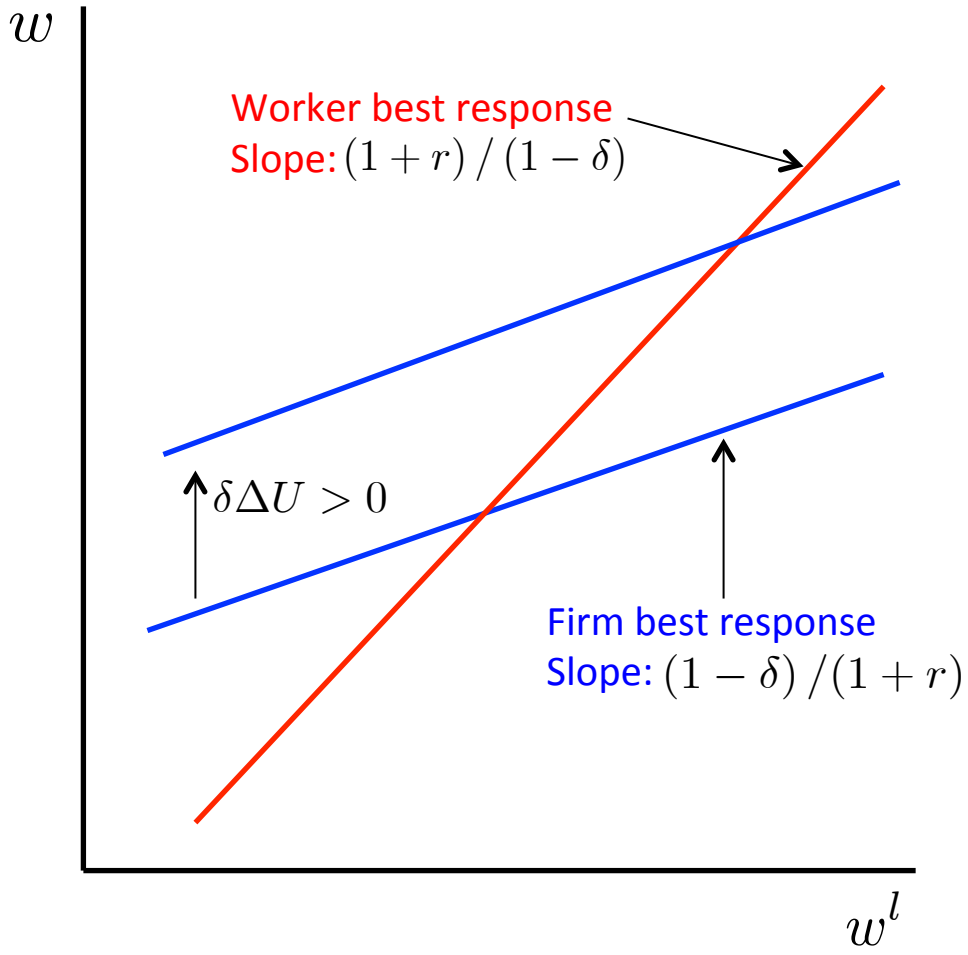
		Prior		Posterior Mode				
Model				**Alternating Offers**		**Nash Sharing**		**EHL**
Cost Specification				**Hiring**	**Search**	**Hiring**	**Search**	**-**
		Distr.	Mean,Std	Mode,Std	Mode,Std	Mode,Std	Mode,Std	Mode,Std
Price Setting Parameters								
Price Stickiness	ξ	B	0.50,0.10	**0.69**,0.03	**0.8**,0.03	**0.75**,0.03	**0.83**,0.02	**0.73**,0.03
Price Markup	λ_f	G	1.20,0.05	**1.42**,0.04	**1.42**,0.04	**1.46**,0.04	**1.43**,0.05	**1.31**,0.04
Price Indexation	κ_f	B	0.50,0.10	**0.70**,0.08	**0.76**,0.07	**0.74**,0.08	**0.66**,0.11	**0.72**,0.08
Monetary Authority Parameters								
Taylor Rule: Smoothing	ρ_R	B	0.75,0.10	**0.87**,0.01	**0.87**,0.01	**0.85**,0.01	**0.85**,0.02	**0.85**,0.01
Taylor Rule: Inflation	r_π	G	1.50,0.10	**1.35**,0.09	**1.40**,0.09	**1.34**,0.08	**1.34**,0.09	**1.36**,0.09
Taylor Rule: GDP	r_y	G	0.10,0.05	**0.05**,0.02	**0.05**,0.02	**0.03**,0.01	**0.03**,0.02	**0.07**,0.02
Preferences and Technology								
Consumption Habit	b	B	0.50,0.15	**0.84**,0.01	**0.83**,0.01	**0.83**,0.01	**0.82**,0.02	**0.76**,0.02
Capacity Util. Adj. Cost	σ_a	G	0.50,0.30	**0.26**,0.08	**0.25**,0.10	**0.25**,0.09	**0.09**,0.10	**0.07**,0.04
Investment Adj. Cost	$S^{''}$	G	8.00,2.00	**13.91**,1.8	**14.22**,1.9	**15.61**,2.1	**16.56**,2.6	**7.91**,1.03
Capital Share	α	B	0.30,0.02	**0.24**,0.01	**0.24**,0.02	**0.25**,0.02	**0.25**,0.02	**0.28**,0.01
Technology Diffusion	ς	B	0.20,0.10	**0.003**,0.003	**0.014**,0.009	**0.035**,0.013	**0.06**,0.04	**0.02**,0.01
Labor Market Parameters								
Prob. of Barg. Breakup	δ	G	0.05,0.025	**0.055**,0.01	**0.086**,0.02	**-**	**-**	-
Replacement Ratio	D/w	B	0.40,0.15	**0.74**,0.03	**0.80**,0.03	**0.90**,0.01	**0.97**,0.01	-
Hiring-Search Cost/Y	s_h	G	0.50,0.30	**0.52**,0.14	**0.04**,0.02	**0.16**,0.09	**0.18**,0.11	-
Match. Function Param.	σ	B	0.50,0.20	**0.55**,0.04	**0.53**,0.04	**0.58**,0.03	**0.54**,0.05	-
Inv. Labor Supply Elast.	ψ	G	0.30,0.20	-	-	-	-	**2.03**,0.43
Shocks								
Std. Monetary Policy	σ_R	G	0.65,0.05	**0.57**,0.03	**0.59**,0.03	**0.62**,0.03	**0.61**,0.03	**0.61**,0.03
Std. Neutral Technology	σ_{μ_z}	G	0.20,0.05	**0.11**,0.01	**0.12**,0.02	**0.12**,0.01	**0.1**4,0.02	**0.17,**0.04
Std. Invest. Technology	σ_Ψ	G	0.20,0.05	**0.14**,0.02	**0.14**,0.02	**0.13**,0.02	**0.12**,0.03	**0.16**,0.02
AR(1) Neutral Technology	ρ_{μ_z}	B	0.20,0.10	**0.13**,0.08	**0.15**,0.08	**0.18**,0.09	**0.28**,0.14	**0.36**,0.15
AR(1) Invest. Technology	ρ_Ψ	B	0.75,0.15	**0.63**,0.07	**0.65**,0.07	**0.66**,0.07	**0.71**,0.09	**0.57**,0.07
Memo Items								
Log Data Density (Laplace, 12 Variables):				**-631.1**	**-643.8**	**-650.8**	**-654.4**	**-**
Implied Worker Surplus/Total Surplus:				0.73	0.96	0.78	0.46	-

Notes: s_h denotes the steady state hiring or search cost to gross output ratio (in percent).

Figure 1: Alternating Offer Bargaining

Panel A: Best response functions

Effect of an increase in U



Panel B: Labor Market in Steady State

Effects of a fall in γ

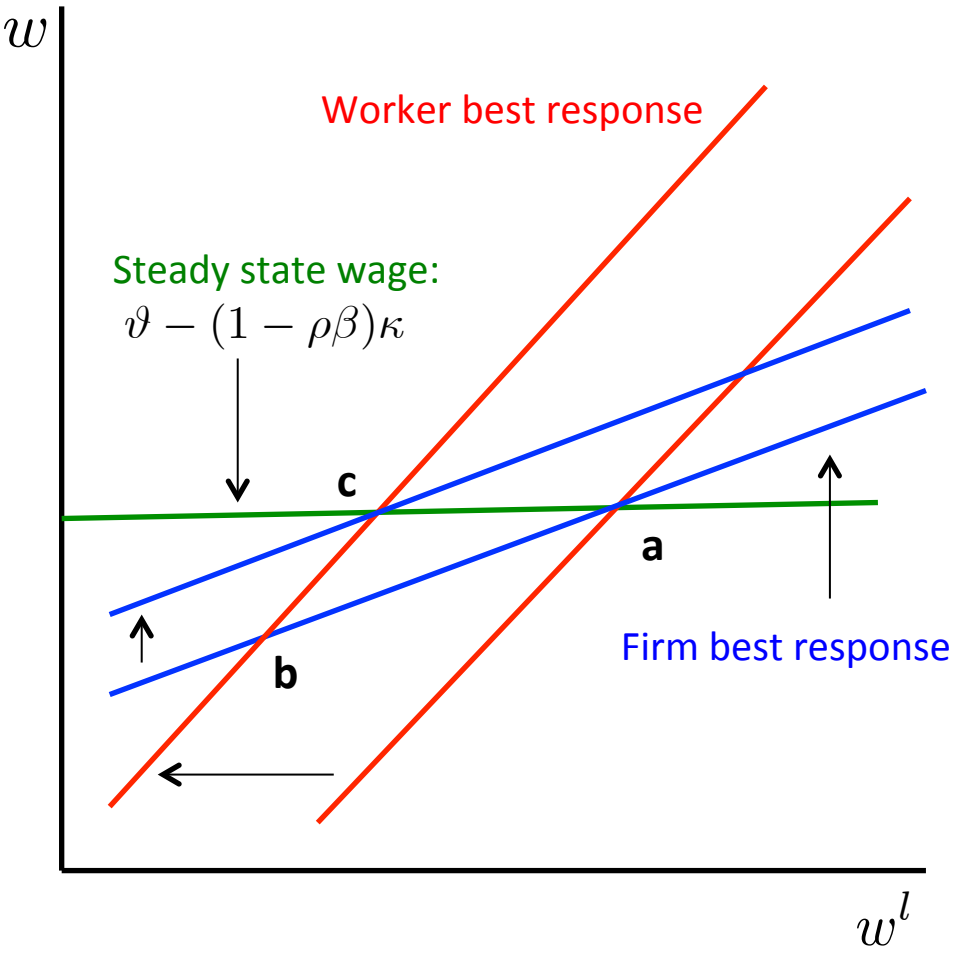


Figure 2: Simple Macro Model Responses to a 25 ABP Monetary Policy Shock

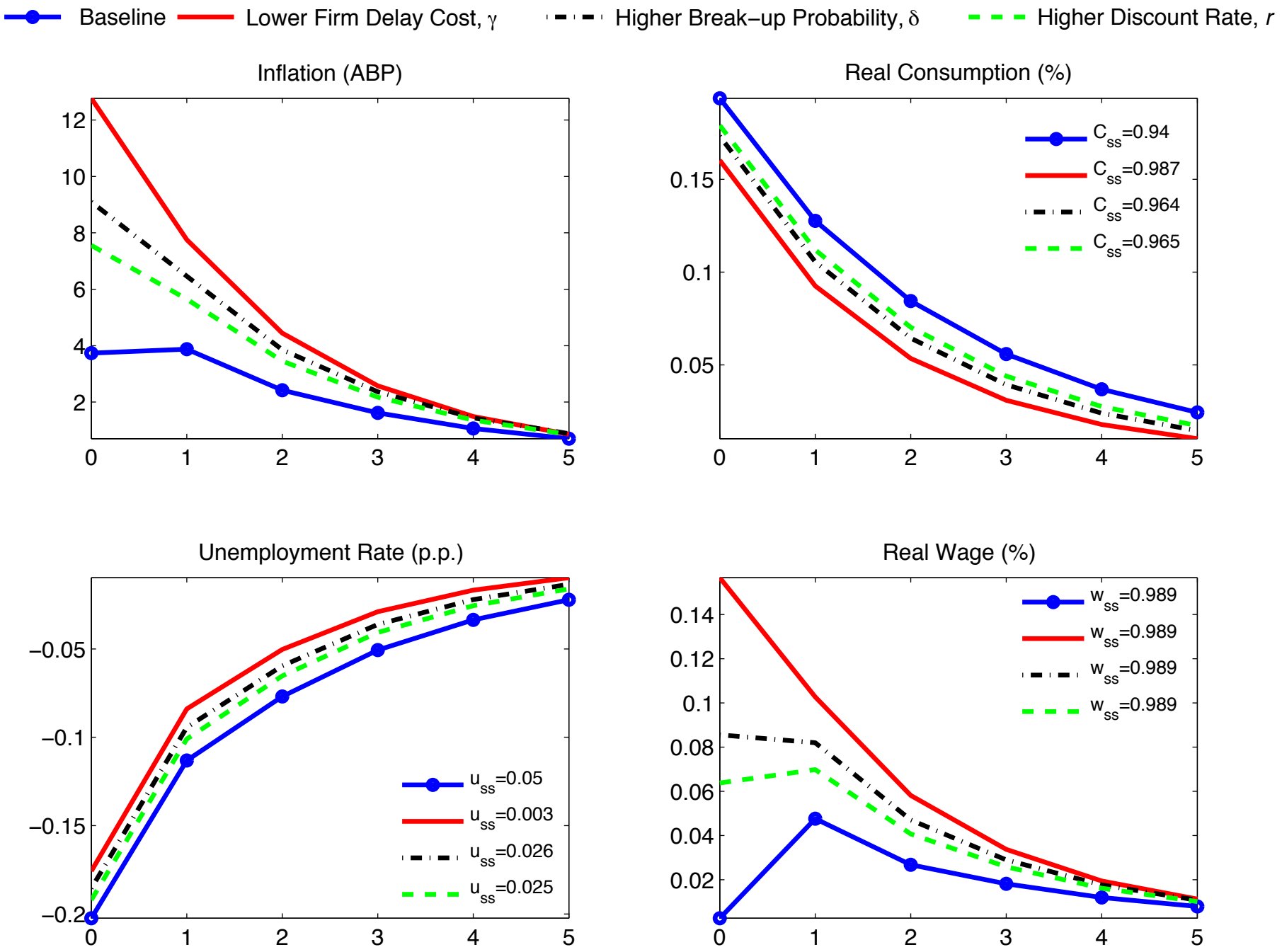


Figure 3: Simple Macro Model Responses to a 0.1 Percent Technology Shock, AR(2)

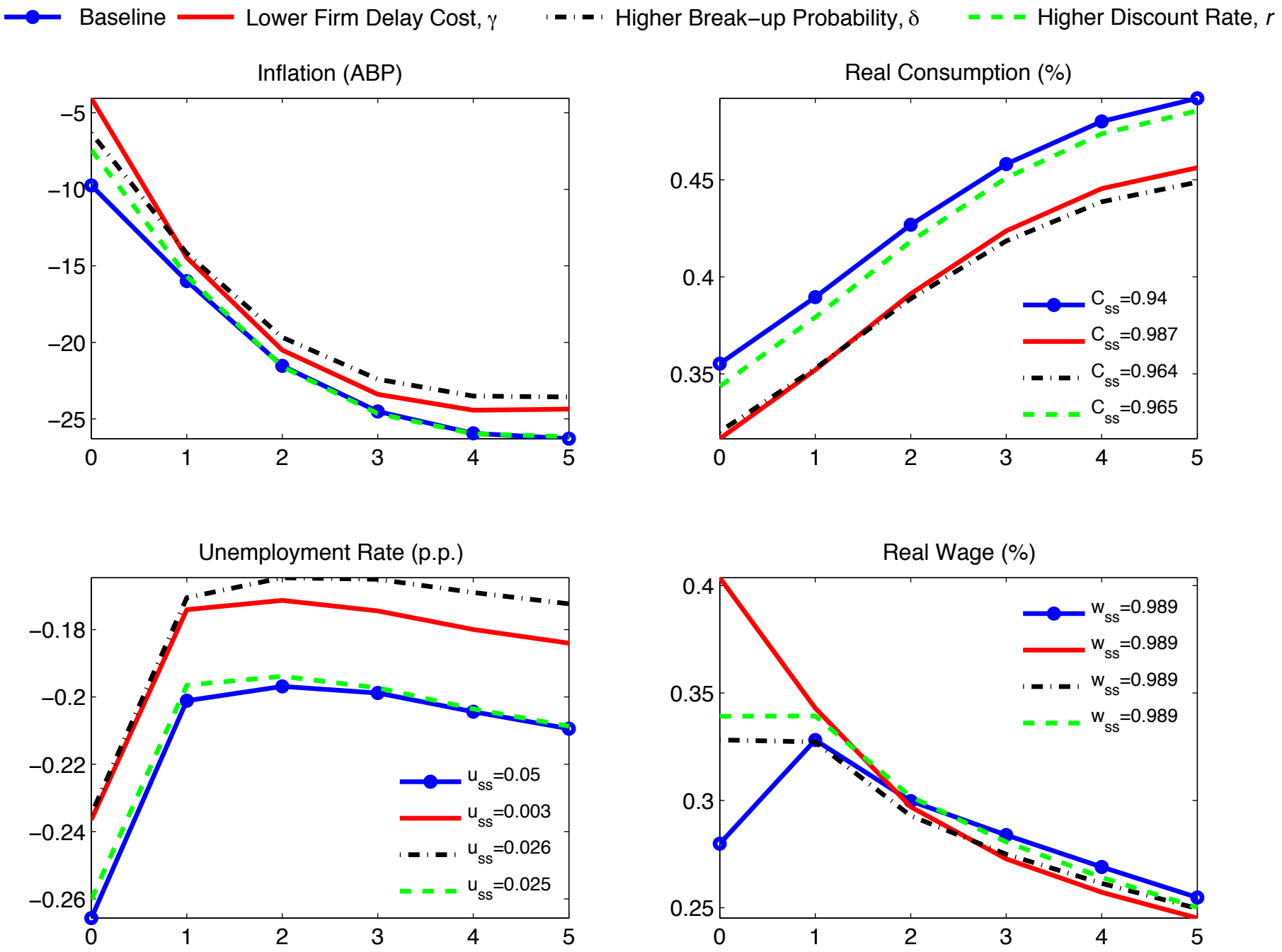


Figure 4: Medium–Sized Model Impulse Responses to a Monetary Policy Shock

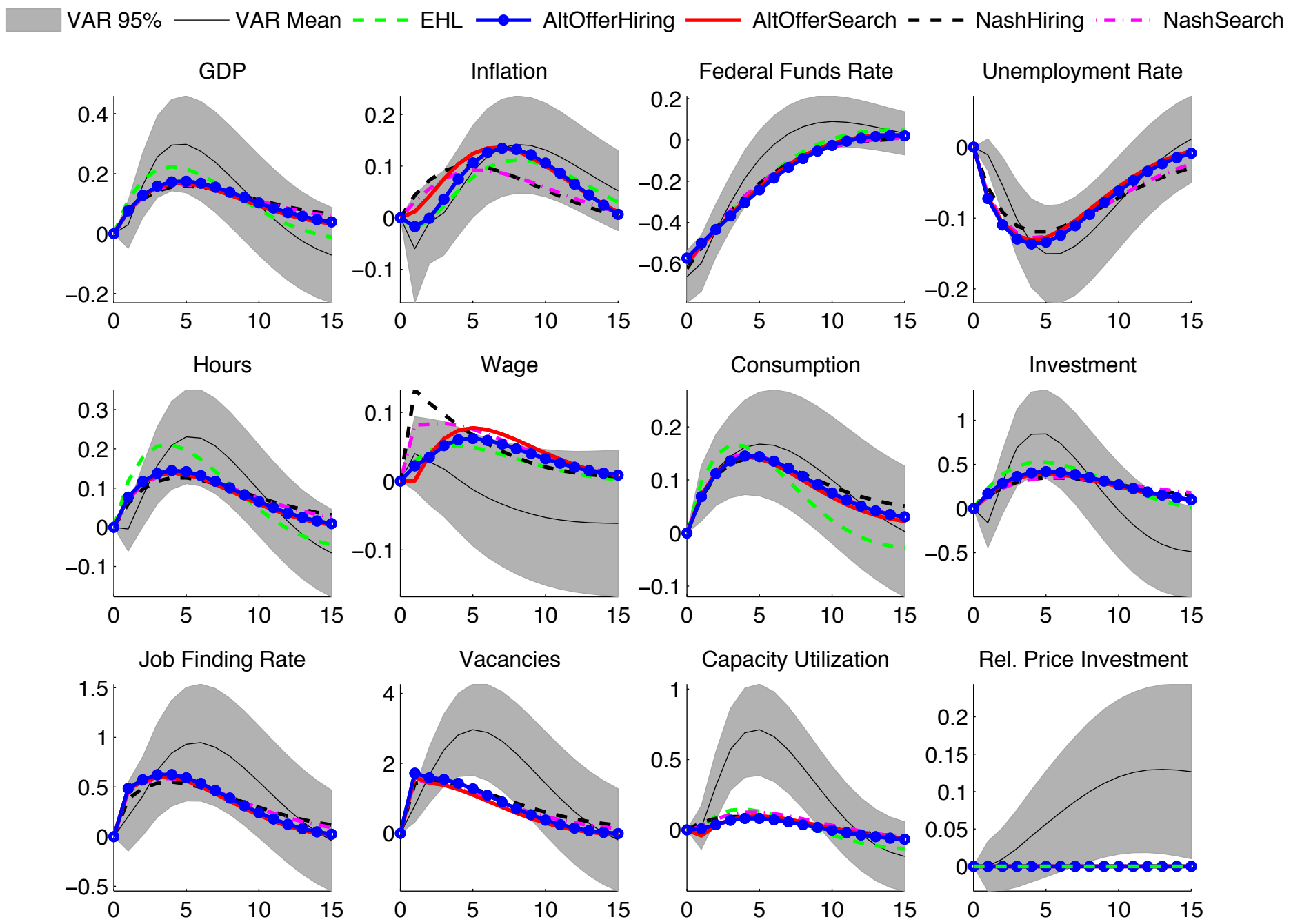


Figure 5: Medium–Sized Model Impulse Responses to a Neutral Technology Shock

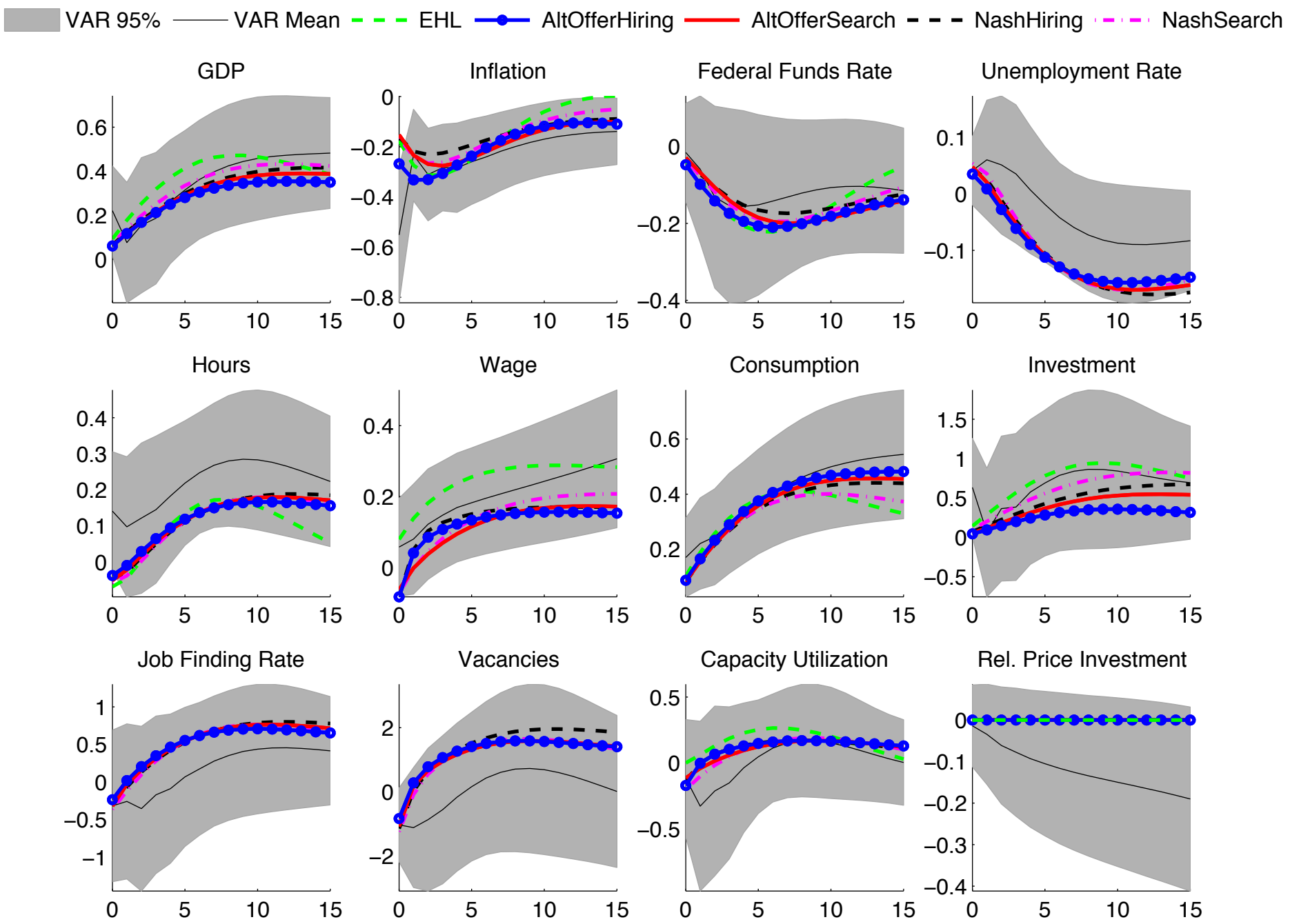


Figure 6: Medium–Sized Model Responses to an Investment Specific Technology Shock

