Feedback Effects of Commodity Futures Prices*

Michael Sockin† Wei Xiong‡

October 2012

Abstract

This paper presents a model to analyze feedback effects of commodity futures prices on commodity demand through an information channel. Our model builds on two key features: 1) futures prices of key industrial commodities such as oil and copper serving as key barometers of global productivities, and 2) complementarity in industrial producers' production decisions as a result of their need to trade produced goods. As a result of information frictions and production complementarity, increases in the futures prices, even if driven by non-fundamental factors, can lead to increased, rather than decreased, commodity demand and thus spot prices. This feedback effect contradicts two widely held arguments that speculators who trade only in futures markets cannot affect spot prices and that commodity price increases driven by non-fundamental reasons must be accompanied by reduced demand.

*We wish to thank Thierry Foucault, Lutz Kilian, Joel Peress, and seminar participants at Emory, HEC-Paris, INSEAD, and Princeton for helpful discussion and comments.

†Princeton University. Email: msockin@princeton.edu.

‡Princeton University and NBER. Email: wxiong@princeton.edu.

1 Introduction

The dramatic boom-and-bust cycle of commodity prices in recent years has stimulated intense debates in academic and policy circles regarding whether speculation in commodity futures markets might have affected commodity prices. Two basic economic arguments frequently appear in these debates (e.g., Krugman (2008a, 2008b), Hamilton (2009), Kilian and Murphy (2012)): First, speculators who trade only in futures markets cannot affect the spot prices of commodities because they do not affect the physical supply and demand of these commodities. Second, if commodity prices are driven up by non-fundamental factors, commodity demand must fall and commodity inventories must pile up. Indeed, based on these arguments and the observation that the large oil price increase from \$60 per barrel to over \$140 during January 2007-July 2008 was not accompanied by any significant increase in oil inventory, many conclude that trading in the oil futures market could not have been a driver of the oil price increase. Regardless of whether this conclusion holds true, the arguments on which it builds ignore potentially important feedback effects of commodity futures prices.

Futures prices of oil and copper are widely followed as barometers of global economic strength as these commodities are important industrial inputs. Consistent with this, Kilian (2009) provides evidence that oil prices are closely tied to global economic activity. Commodity markets are closely monitored by financial traders, central bankers, and economic policy makers as an important sector in the world economy. The annual and quarterly economic reports of IMF and BIS regularly cover commodity markets. Popular news carriers such as Wall Street Journal, Bloomberg, and CNN often list the futures price of oil, together with stock market indices and key interest rates, in headlines of daily financial market movements. When industrial producers across the world use futures prices of oil and copper as signals of the strength of the global economy, an increase in the futures prices, even if driven by non-fundamental factors, may lead industrial producers to increase, rather than decrease, commodity demand, which in turn can cause spot prices of these commodities to rise. Such feedback effects thus motivate a re-examination of the aforementioned economic arguments.

This paper studies the feedback effects of trading in futures markets on commodity demand and spot prices through the information channel. We first develop a basic model in the absence of speculators to illustrate the feedback mechanism and then extend the model to have a group of financial traders who trade only in the futures market. Our basic model features a set of markets related to a certain commodity, which one can interpret as either

oil or copper. The core of the model is a futures market. In this market, industrial producers across the world take long positions to acquire the commodity as an input to their production, while a group of intermediaries take short positions and purchase the commodity from decentralized spot markets to make delivery. Like the standard models of asset trading with asymmetric information (e.g., Grossman and Stiglitz (1980) and Hellwig (1980)), the futures market also allows the industrial producers to aggregate dispersed, private information about global productivity. A key deviation of our model from the standard models is the presence of complementarity in producers' commodity demands. This is a result of people's need to trade produced goods for consumption purposes. It is more desirable for one producer to produce more and thus demand more of the commodity when producers of other goods produce more. To capture such complementarity, we adopt a modified setting of Angeletos and La'O (2012) with producers on different islands producing different goods and trading with each other.

Despite the non-linearity in the industrial producers' production decisions, we derive a unique log-linear, noisy rational expectations equilibrium in closed form. In the equilibrium, each producer's commodity demand is a log-linear function of his private signal and the futures price, while the futures price is a log-linear function of the global productivity and noise that originates from the intermediaries' required return for their intermediation.

This tractable model allows us to characterize the feedback effects of the futures price. The futures price allows each producer to extract information about not only the global productivity but also other producers' production decisions. As a result, an increase in the futures price motivates each producer to increase his commodity demand. This effect counters the reduced demand driven by an increased cost. The information effect and the cost effect jointly determine the elasticity of the producers' aggregate commodity demand to the futures price. When the complementarity in producers' commodity demands is small, the cost effect always dominates the information effect and the producers' demand elasticity is negative. However, when the complementarity is sufficiently strong, the information effect can dominate and cause the producers' demand elasticity to become positive. That is, the producers increase their commodity demand in response to a rising futures price.

The presence of the feedback effects has important implications for structural modeling of commodity markets. While it is appealing to decompose the shocks in the economy into orthogonal shocks to the demand and supply sides (e.g., Kilian (2009)), the feedback effects

make it possible for a shock to the supply side to affect the demand side and vice versa. Furthermore, the feedback effects make it difficult to interpret and sign demand elasticities as they can be downward, flat, or even positive sloping. Taken together, our model motivates structural models to systematically account for the feedback effects.

To examine the effects of futures market speculation, we extend the basic model to incorporate a group of financial traders, who trade only at the long side of the futures market and who always close their positions before delivery. These traders are motivated by the increasing presence of commodity index traders in the commodity futures markets in the recent years. The uncertainty in these traders' positions introduces another random factor in the equilibrium futures price. As other market participants cannot separate heavy buying by financial traders from strong demand from producers, heavy buying by financial traders leads to a higher futures price. Under certain conditions, the higher futures price can in turn drive up commodity demand of producers and thus the spot price through the information channel.

Taken together, our analysis shows that it is possible for traders who trade only in futures markets to impact commodity demand and spot prices and that non-fundamental noise in the financial traders' position can cause commodity demand and spot prices to rise together. These results help clarify the two aforementioned economic arguments in evaluating the economic impacts of speculative trading in commodity markets. In particular, our results caution the use of empirical strategies that build on joint increases in commodity prices and inventories to identify the impact of speculative trading.

This paper is related to the emerging literature that analyzes whether the large inflow of financial investment to commodity futures markets might have affected commodity prices, e.g., Tang and Xiong (2010), Singleton (2011), Cheng, Kirilenko, and Xiong (2012), Hamilton and Wu (2012), Kilian and Murphy (2012), and Henderson, Pearson, and Wang (2012). See Singleton (2011) and Fattouh, Kilian, and Mahadeva (2012) for reviews of these and other related studies. This paper proposes a theoretical framework to highlight a specific information channel through which speculative trading in futures markets affects commodity demands and spot prices. This framework reinforces information frictions emphasized by Singleton (2011) and can help empirical researchers design sharper tests of the impact of financial traders in commodity markets.

Our model builds on the setting of Angeletos and La'O (2012) for the goods market

equilibrium to derive endogenous complementarity in producers' production decisions. As they focus on endogenous economic fluctuations as a result of the lack of centralized communication channels between different households, their model does not feature any centralized market trading. The centralized futures market and the feedback effects of futures prices thus differentiate our model from theirs.

The economics and finance literature has long pointed out that trading in financial markets helps aggregate information and the resulting prices of financial securities feed back to real world activities (e.g., Bray (1981) and Subrahmanyam and Titman (2001)). More recently, the literature emphasizes that such feedback effects can be particularly strong in the presence of strategic complementarity in agents' actions. Morris and Shin (2002) show that in such a setting, noise in public information has an amplified effect on agents' actions and thus on equilibrium outcomes. In our model, the commodity futures price serves such a role of public information and feeds back the effects of noise originated from financial traders to industrial producers' production decisions. Feedback effects of financial prices are also modeled in several other contexts, such as from stock prices to firm capital investment decisions and from exchange rates to policy choices of central banks (e.g., Angeletos, Lorenzoni and Pavan (2010) and Goldstein, Ozdenoren, and Yuan (2011, 2012)).

The paper is organized as follows. Section 2 describes the setting of our basic model, while Section 3 analyzes the equilibrium. We extend the model to include a group of financial traders who trade only in the futures market in Section 4. Section 5 concludes the paper. We provide the technical proofs to all of the propositions in the appendix.

2 The Model

We develop a model with three dates $t = 0, 1, 2$ to analyze the market equilibrium of a commodity. One can think of this commodity as oil or copper, which is used across the world as a key production input. We adopt a modified setup of Angeletos and La'O (2012) to model a continuum of islands of total mass 1. Each island produces a single good, which can either be consumed at "home" or traded for another good produced "away" by another island. Their trading of the produced goods determines the goods' prices.

In reality, spot markets of commodities are often decentralized, while centralized futures markets provide key platforms for different market participants to aggregate demands and supplies for commodities, and, more importantly, to aggregate private information about

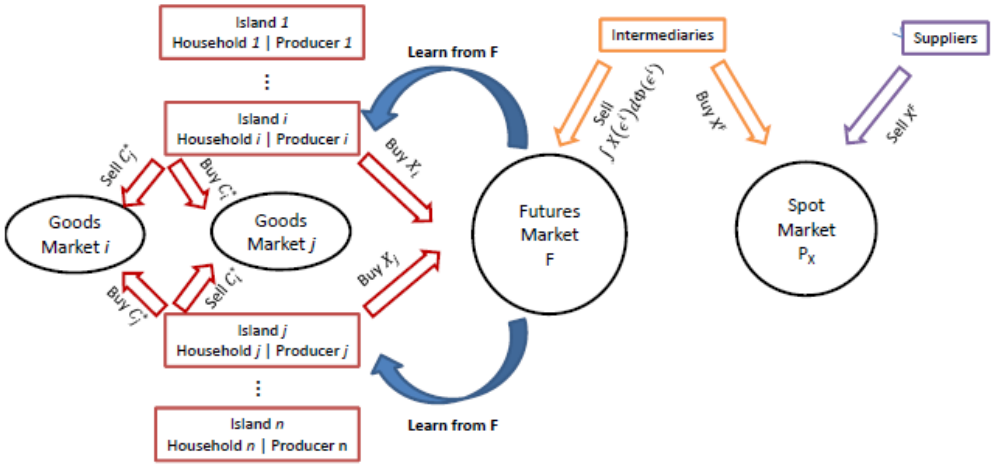


Figure 1: Structure of the Basic Model

global productivity. Our model features a centralized market for a futures contract written on the commodity, in addition to decentralized spot markets for the commodity and for the produced goods. The futures contract is traded at $t = 0$ for delivery at $t = 1$.[1]

Figure 1 illustrates the basic structure of the model. There are four types of agents: households on the islands, industrial producers on the islands, a group of commodity suppliers, and a group of intermediaries. The industrial producers trade only in the futures market at $t = 0$, and take delivery of the commodity for their production at $t = 1$. Their produced goods are distributed to the households on their respective islands at $t = 2$. The households then trade their goods with each other and consume. The commodity suppliers do not participate in the futures market and sell commodities in the spot markets to intermediaries, who intermediate between industrial producers and commodity suppliers by shorting the futures contract and purchasing the commodity in the spot markets. We will extend the model to include another group of financial traders who trade only in the futures market in Section 4.

2.1 Island households

Each island has a representative household. Following Angeletos and La'O (2012), we assume a particular structure for goods trading between households on different islands. Each island

[1] One can interpret this futures contract as a front-month or second-month futures contract traded in reality, which tends to attract much larger trading volume than more distant futures contracts.

is randomly paired with another island at $t = 2$. The households on the two islands trade their goods with each other and consume both goods produced by the islands. For a pair of matched islands, we assume that the preference of the households on these islands over the consumption bundle (C_i, C_i^*), where C_i represents consumption of the "home" good while C_i^* consumption of the "away" good, is determined by a utility function $U(C_i, C_i^*)$. The utility function increases with both C_i and C_i^*. This utility function specifies all "away" goods as perfect substitutes, so that the utility of the household on each island is well-defined regardless of the matched trading partner. The households on the two islands thus trade their goods to maximize each's utility. We assume that the utility function of the island households takes the Cobb-Douglas form

$$U(C_i, C_i^*) = \left(\frac{C_i}{1-\eta}\right)^{1-\eta} \left(\frac{C_i^*}{\eta}\right)^{\eta} \tag{1}$$

where $\eta \in [0, 1]$ measures the utility weight of the away good. A greater η means that each island values more of the away good and thus relies more on trading goods with other islands. Thus, η eventually determines the degree of complementarity in islands' goods productions.

2.2 Goods producers

Each island has a locally-owned representative firm to organize its goods production. We refer to each firm as a producer. The production requires the use of the commodity as an input. To focus on the commodity market equilibrium, we exclude other inputs such as labor from production. Each island has the following constant-returns-to-scale production function[2]:

$$Y_i = AX_i, \tag{2}$$

where Y_i is the output produced by island i, X_i is the commodity input, and A is the common productivity shared by all islands. For simplicity, we ignore the idiosyncratic component of each island's productivity. This simplification is innocuous for our qualitative analysis of how information frictions can affect commodity demand.

The productivity A is a random variable, which becomes only observable after the producers complete their productions at $t = 1$. We assume that A has a lognormal distribution:

$$\log A \backsim \mathcal{N}\left(\overline{A}, \tau_A^{-1}\right)$$

[2]One can also specify a Cobb-Douglas production function with both commodity and labor as inputs. The model remains tractable but the formulas become more complex and harder to interpret.

where $\overline{A}$ is the mean of $\log A$ and τ_A^{-1} is its variance. At $t = 0$, the goods producer on each island observes a private signal about $\log A$:

$$s_i = \log A + \varepsilon_i$$

where $\varepsilon_i \backsim \mathcal{N}(0, \tau_s^{-1})$ is random noise independent of $\log A$. τ_s is the precision of the signal. The signal allows the producer on the island to form his expectation of the global productivity, determine his production decision, and trade in the futures market to purchase the commodity input. The futures market serves to aggregate all of the private signals dispersed among the producers regarding $\log A$. As each producer's private signal is noisy, the information friction also motivates him to use the publicly observed futures price to form his expectation.

At $t = 0$, the producer on island i maximizes his expected profit by choosing his commodity input X_i:

$$\max_{X_i} \; E_i [P_i Y_i] - F X_i \tag{3}$$

where E_i is shorthand for $E[\cdot \mid \mathcal{I}_i]$, and P_i is the price of the good produced by the island. The producer's information set $\mathcal{I}_i = \{s_i, F\}$ includes his private signal s_i and the futures price F. The good price P_i is determined at $t = 2$ based on the matched trade with another island.

2.3 Commodity suppliers

There is a continuum of mass 1 of suppliers of the commodity. The suppliers do not participate in the futures market. Instead, they directly sell to intermediaries in decentralized spot markets at $t = 1$. We assume the suppliers face a convex cost

$$\frac{k}{1+k} e^{-\xi/k} \left(X^S\right)^{\frac{1+k}{k}}$$

in supplying the commodity, where X^S is the quantity sold, $k \in (0, 1)$ is a constant parameter, and ξ represents random noise. We assume that ξ has Gaussian distribution $\mathcal{N}\left(\bar{\xi}, \tau_\xi^{-1}\right)$ with $\bar{\xi}$ as its mean and τ_ξ^{-1} as its variance. Thus, given a spot price P_X, the suppliers face the following optimization problem:

$$\max_{X^S} P_X X^S - \frac{k}{1+k} e^{-\xi/k} \left(X^S\right)^{\frac{1+k}{k}}. \tag{4}$$

It is easy to determine the suppliers' optimal supply curve:

$$X^S = e^{\xi} P_X^k, \tag{5}$$

which shows ξ as uncertainty in the supply and k as the price elasticity of the commodity supply.[3]

2.4 Intermediaries

There is a continuum of identical intermediaries of mass 1. Each intermediary is a price taker. Intermediaries trade with island producers in the futures market at $t = 0$. They take short positions while island producers take long positions. They then purchase commodities from commodity suppliers in the spot markets at $t = 1$ to make delivery of their futures positions.

We assume that the intermediaries are risk-neutral and have a required return of ν from their positions. That is, they are willing to take infinitely large positions if $E\left[\log\left(F/P_X\right)|\,\mathcal{I}^F\right]$, the expected return from shorting futures and buying the commodity in the spot markets, is higher than ν. The required return is random and, in practice, depends on factors that determine the financial status and cost of capital of the intermediaries. We assume that ν has a random distribution:

$$\nu \sim \mathcal{N}\left(\bar{\nu}, \tau_\nu^{-1}\right)$$

where $\bar{\nu}$ is the mean of ν and τ_ν^{-1} is its variance. The intermediaries are aware of their required return at $t = 0$, but other participants of the markets do not observe ν. This random required return serves as a reduced form to capture noise in the futures market that originates from the intermediation sector.[4] Consequently, the intermediaries' information set at $t = 0$ is $\mathcal{I}^p = \{\nu, F\}$.

2.5 Joint equilibrium of different markets

Our model features a noisy rational expectations equilibrium of a number of markets: the goods markets between each pair of matched islands, the spot markets for the commodity, and the centralized commodity futures market. The equilibrium requires clearing of each of these markets:

[3] By letting the suppliers sell the commodity according to their marginal cost, our model ignores any potential feedback effect from the futures price to the supply side. In a more general setting with multiple rounds of spot market trading, suppliers and other agents may have incentives to store the commodity over time based on their expectations of future demands. Then, the futures price can feed back to these agents' storage decisions. We leave an analysis of such a feedback effect to the supply side to future research and instead focus on the feedback effect to the demand side.

[4] See Cheng, Kirilenko and Xiong (2012) for an empirical study of how during the post-crisis period in 2009-2011, financial distress affected the trading of financial intermediaries in commodity futures markets.

- At $t = 2$, for each pair of randomly matched islands $\{i, j\}$, the households of these islands trade their produced goods and clear the market of each good:

$$\begin{aligned} C_i + C_j^* &= AX_i, \\ C_i^* + C_j &= AX_j. \end{aligned}$$

- At $t = 1$, in the decentralized spot markets of the commodity, the intermediaries' demand equals the supply of the suppliers, $X^F = X_j^S$.

- At $t = 0$, in the futures market for the commodity, the aggregate long position taken by the goods producers equals the aggregate short position taken by the intermediaries:

$$\int_{-\infty}^{\infty} X_i\left(s_i, F\right) d\Phi\left(\varepsilon_i\right) = X^F\left(\nu, F\right),$$

where each producer's optimal short position $X_i\left(s_i, F\right)$ depends on his private signal s_i and the futures price F, and the intermediaries' optimal position $X^F\left(\nu, F\right)$ depends on his required return ν and the futures price F. The producers' short positions are integrated over the noise ε_i in their private signals.

3 The Equilibrium

3.1 Goods market equilibrium

We begin our analysis of the equilibrium with the goods markets at $t = 2$. For a pair of randomly matched islands, i and j, the representative household of island i possesses Y_i units of the good produced by the island while the representative household of island j holds Y_j units of the other good.[5] They trade the two goods with each other to maximize each's utility function given in (1). The following proposition describes the goods market equilibrium between these two islands.

Proposition 1 *For a pair of randomly matched islands, i and j, their representative households' optimal consumption of the two goods is*

$$C_i = (1-\eta) Y_i,\ C_i^* = \eta Y_j,\ C_j = (1-\eta) Y_j,\ C_j^* = \eta Y_i.$$

[5] Here we implicitly treat a representative household as representing different agents holding stakes in an island's good production, such as workers, managers, suppliers of inputs, intermediaries, etc. We agnostically group their preferences for the produced goods of their own island and other islands into the preferences of the representative household.

The price of the good produced by island i is

$$P_i = \left(\frac{Y_j}{Y_i}\right)^{\eta}. \tag{6}$$

Proposition 1 shows that each household divides its consumptions between the home and away good with fractions $1-\eta$ and η, respectively. When $\eta = 1/2$, the household consumes the two types of goods equally. The price of each good is determined by the relative output of the two matched islands. One island's good is more valuable when the other island produces more. This complementarity implies that each goods producer needs to take into account the production decisions of producers of other goods.

3.2 Producer's production decision

By substituting the production function in (2) into (3), the expected profit of the goods producer on island i, we obtain the following objective:

$$\max_{X_i} \; E\left[AP_iX_i \middle| s_i, F\right] - FX_i.$$

In a competitive goods market, the firm will produce to the level that the marginal revenue equals the marginal cost:

$$E\left[AP_i \middle| s_i, F\right] = F.$$

By substituting in P_i from Proposition 1, we obtain

$$X_i = \left\{\frac{E\left[AX_j^{\eta} \middle| s_i, F\right]}{F}\right\}^{1/\eta} \tag{7}$$

which depends on the producer's expectation $E\left[AX_j^{\eta} \middle| s_i, F\right]$ of the global productivity A and the production decision X_j of its randomly matched trading partner, island j. This expression demonstrates the complementarity in producers' production decisions. A larger η makes the complementarity stronger as the producer relies more on selling its goods to the other island.

The futures price F is an important source of information for the producer to form his expectation of $E\left[AX_j^{\eta} \middle| s_i, F\right]$, which serves as the main channel for the futures price to feed back into the producer's commodity demand and thus the commodity's spot price. The presence of complementarity strengthens this feedback effect relative to the standard model of asset trading with asymmetric information.

3.3 Intermediaries' short position

Intermediaries take short positions in the futures market and then purchase the commodity in the spot markets to make delivery. In equilibrium, they will trade so that the expected return from their positions exactly equals the required return ν

$$E\left[\log\left(\frac{F}{P_X}\right)\middle|\mathcal{I}^F\right]=\nu. \tag{8}$$

The clearing of the spot market requires that $X^F=X^S=e^{\xi}P_X^k$, which implies that

$$\log P_X=-\frac{1}{k}\xi+\frac{1}{k}\log X^F.$$

By substituting this spot price into (8), we obtain the following equation:

$$E\left[\log F+\frac{1}{k}\xi-\frac{1}{k}\log X^F\middle|\mathcal{I}^F\right]=\nu.$$

This equation gives the logarithm of the intermediaries' aggregate short position X_F :

$$\log X^F=k\log F-k\nu+\overline{\xi}, \tag{9}$$

which linearly increases with $\log F$ and decreases with ν.

As intermediaries are risk-neutral, they are able to perfectly insulate the futures market and thus the producers against uncertainty regarding the commodity supply in the spot markets. The intermediaries, however, also bring their own financial shock ν to the futures market.

3.4 Futures market equilibrium

By clearing the aggregate long position taken by island producers with the short position by intermediaries, we derive the futures market equilibrium. Despite the nonlinearity in the producer's production decision, we obtain a unique log-linear equilibrium in closed form. The following proposition summaries the futures price and positions taken by different participants in the equilibrium.

Proposition 2 *At $t=0$, the futures market has a unique log-linear equilibrium: 1) The futures price is a log-linear function of $\log A$ and ν:*

$$\log F=h_A\log A+h_\nu\nu+h_0,$$

with the coefficients $h_A \in [0,1]$, $h_\nu > 0$, *and* h_0 *given by*

$$\begin{aligned} h_A &= 1 - \left(\frac{1}{2}k^2\tau_\nu^{-1}\tau_s^{-2}\tau_A^3\eta^2\right)^{1/3} \cdot \\ &\quad \left(\sqrt[3]{1+\sqrt{1+\frac{4}{27}k^2\tau_\nu^{-1}\tau_s^{-2}\tau_A^3\eta^2}} + \sqrt[3]{1-\sqrt{1+\frac{4}{27}k^2\tau_\nu^{-1}\tau_s^{-2}\tau_A^3\eta^2}}\right), \end{aligned} \tag{10}$$

$$h_\nu = \sqrt{\tau_A^{-1}\tau_\nu h_A\left(1-h_A\right)}, \tag{11}$$

$$h_0 = \left(1-h_A\right)\left[\overline{A} + \frac{1}{2\tau_A} + \tau_s\tau_A^{-2}\left(1-h_A\right)\right] - h_\nu\bar{\nu}. \tag{12}$$

2) The long position taken by the producer on island i *is a log-linear function of his private signal* s_i *and* $\log F$*:*

$$\log X_i = l_s s_i + l_F \log F + l_0, \tag{13}$$

with the coefficients l_s, l_F, *and* l_0 *given by*

$$\begin{aligned} l_s &= \frac{1}{\eta}\frac{\tau_s}{\tau_A}\left(1-h_A\right), \\ l_F &= k\left(1-h_\nu^{-1}\right), \\ l_0 &= \overline{\xi} + \left(k-l_F\right)h_0 - \frac{1}{2}l_s^2\tau_s^{-1}. \end{aligned} \tag{14}$$

3) The short position taken by the intermediaries, which is equal to the producers' aggregate commodity demand, also has a log-linear form:

$$\log X^F = \left(l_s + l_F h_A\right)\log A + l_F h_\nu \nu + l_0 + l_F h_0 + \frac{1}{2}l_s^2\tau_s^{-1}. \tag{15}$$

Proposition 2 shows that each producer's logarithmic position $\log X_i$ is a linear function of his private signal and the futures price, while the logarithmic futures price aggregates the dispersed signals of the producers to partially reveal the global productivity A. The futures price does not depend on any producer's signal noise due to the aggregation across a large number of producers with independent noise. This feature is similar to Hellwig (1980). The futures price also depends on the intermediaries' financial shock ν, which serves the same role as noise trading in the standard models of asset trading with asymmetric information.

3.5 A benchmark without information frictions

To facilitate our discussion on the effects of information frictions on commodity demand, it is useful to establish a benchmark without any information friction. Suppose that the global

productivity A is observable by all market participants. Then, the producers can choose their optimal production decisions without any noise interference. The following proposition characterizes this benchmark.

Proposition 3 *When A is observable by all market participants, there is a unique, competitive equilibrium. In the equilibrium, 1) the producers have a symmetric commodity demand curve:*

$$\forall\ i \text{ and } j,\ X_i = X_j = \begin{cases} 0 & \text{if } F > A \\ [0,\infty) & \text{if } F = A \\ \infty & \text{if } F < A \end{cases};$$

2) the intermediaries' supply is

$$\log X^F = k \log F - k\nu + \bar{\xi};$$

3) the futures price is given by $\log F = \log A$ and the producers' equilibrium demand is $\log X_i = k \log A - k\nu + \bar{\xi}$. This competitive equilibrium is Pareto efficient and coincides with the limiting case of the equilibrium derived in Proposition 2 after letting $\tau_s \to \infty$.

Proposition 3 shows that when A is observable, there is a unique equilibrium despite the complementarity in the producers' production decisions. The uniqueness of the equilibrium comes from the competitiveness of the equilibrium. The perfect competition between producers leads to inelastic, downward demand for the commodity. That is, their demand is infinity if the futures price F is below A, zero if F is above A, and anywhere between zero and infinity if F equals A. This elastic demand curve dictates the equilibrium futures price to be A, but leaves the equilibrium commodity demand to be determined by the intermediaries' upward sloping supply curve. As a result, the complementarity between producers does not lead to multiple equilibria, in which producers coordinate on certain high or low demand levels. In the unique equilibrium, the producers' aggregate commodity demand increases with the global productivity $\log A$ with a coefficient of k and decreases with the intermediaries' required return ν with a coefficient $-k$.

This equilibrium is Pareto efficient. Thus, any deviation induced by informational frictions must at least hurt the welfare of some agents. This motivates us to use this equilibrium outcome as the benchmark to examine effects of information frictions on commodity demand.

3.6 Feedback effect of futures price

In the presence of information frictions, the futures price provides a channel for each producer to infer the global productivity in addition to his private signal. Through this channel, an increase in the futures price, even if driven by a shock to the supply side or by random noise in the futures market, can affect the producers' commodity demand beyond the usual cost effect.

3.6.1 Price elasticity of commodity demand

We first examine the price elasticity of an individual producer's demand for the commodity. Equation (7) reveals two forces: first, a higher futures price reduces the demand through a direct cost effect in the denominator; second, a higher price may increase the demand through the producer's learning from the price about the global productivity and other producers' production decisions, as captured by the term $E\left[AX_j^{\eta}\middle|\, s_i, F\right]$ in the numerator. As discussed before, the strength of this informational feedback effect increases with the complementarity parameter η.

The net effect of the direct cost effect and the indirect feedback effect determines a producer's price elasticity l_F given in Proposition 2. The following proposition proves that the elasticity is negative when η is sufficiently small but can be positive in certain region of η under certain sufficient conditions on other model parameters.

Proposition 4 *When η is sufficiently small, $l_F < 0$. The following sufficient, although not necessary, conditions ensure the existence of a range of η inside $[0,1]$ such that $l_F > 0$:*

$$4 < \frac{\tau_\nu}{\tau_A} < 2\sqrt{k\frac{\tau_\nu}{\tau_s}}.$$

The result that l_F is positive only when the complementarity effect is sufficiently strong highlights the key difference of our model to the standard models of asset trading with asymmetric information, e.g., Grossman and Stiglitz (1980) and Hellwig (1980). In these models, the price elasticity of traders' demand for financial assets is negative despite that they also extract information from the traded assets' prices regarding the assets' fundamentals.

The result that producers' demand curves can be upward sloping is in sharp contrast to the benchmark case without information frictions, and thus highlights the important effect of information frictions in commodity markets. Also note that despite that the demand curve

can be upward sloping, its slope l_F is always lower than the slope of the intermediaries' supply curve k. Thus, the equilibrium is stable.

3.6.2 Implications for structural models

An important consequence of the feedback effect is that the supply-side and demand-side shocks are no longer independent. That is, through the feedback effect of the futures price, a shock to the supply side can affect the demand side. In our model, the intermediaries' required return ν represents a shock to the supply side. Equation (9) shows that a higher ν shifts down the supply curve of the futures contract. By aggregating equation (13) across different producers, we obtain the following aggregate demand curve for the commodity:

$$\begin{aligned}\log\left[\int_{-\infty}^{\infty} X_i\left(s_i, F\right) d\Phi\left(\varepsilon_i\right)\right] &= l_s \log A + l_F \log F + l_0 + \frac{1}{2} l_s^2 \tau_s^{-1} \\ &= \left(l_s + l_F h_A\right) \log A + l_F h_\nu \nu + l_0 + \frac{1}{2} l_s^2 \tau_s^{-1} + l_F h_0.\end{aligned}$$

It is clear to see that ν affects the aggregate demand through the term $l_F h_\nu \nu$. This term emerges from the informational feedback effect of the futures price. The coefficient of this term $l_F h_\nu$ can be either positive or negative depending on the sign of l_F. Under the sufficient condition listed in Proposition 4, l_F is positive and thus ν shifts up the aggregate demand for the commodity.

This feedback channel has important implications for structural modeling of commodity markets. In particular, one needs to exercise caution when decomposing demand and supply shocks in commodity markets, such as in industrial metal and energy markets, into distinct structural shocks without controlling for the feedback effects from prices. For example, Kilian (2009) develops a widely used, structural VAR model for the dynamics of global oil production, global economic activity, and oil price. This framework decomposes shocks in the economy to three orthogonal sources: an oil supply shock, an aggregate demand shock, and an oil specific demand shock. To estimate these shocks, the framework imposes identification restrictions that the oil specific demand shock affects the price of oil but not global economic activity or oil supply.

While it is intuitive and appealing to treat the aforementioned shocks as independent, in reality market participants do not commonly observe these shocks and often have to rely on noisy measures. To the extent that they commonly observe futures prices and use the observed prices to form expectations of global economic fundamentals, the feedback effect

illustrated by our model becomes relevant and imposes challenges to the decomposition of shocks. In particular, the oil specific demand shock, by affecting the price of oil and thus people's expectations, can feed back to global economic activity. Furthermore, while our model setting treats the supply side as exogenous, the feedback effect could also affect the expectations of the intermediaries and suppliers and thus impact the supply side. These feedback effects induce violations to the identification restrictions of the oil specific demand shock.

Finally, it can be difficult to interpret or sign elasticities without taking into account the feedback effects. Demand curves can be downward, flat, or even upward sloping, despite the presence of the standard cost effect that decreases demand with price. In addition, shocks to supply can have a dampened or even an exacerbated effect on the equilibrium consumption of the commodity and thus the spot price due to the feedback effect through the futures price. Taken together, our model motivates structural models to systematically account for the feedback effects.

3.6.3 Some comparative statics

The coefficient $l_F h_\nu$ measures the magnitude of the feedback effect of ν on commodity demand, which depends on τ_ν, uncertainty about the intermediaries' required return, and τ_s, the precision of the producers' private signal on the global productivity. The following proposition characterizes the impact of τ_ν and τ_s on $l_F h_\nu$.

Proposition 5 *Fixing other model parameters, 1) $l_F h_\nu$ is increasing in τ_ν; 2) there exists a certain threshold τ_s^* for τ_s, above which $l_F h_\nu$ is decreasing with τ_s and below which $l_F h_\nu$ is increasing with τ_s.*

These comparative statics are mostly intuitive. As τ_ν increases, there is less uncertainty about the intermediaries' required return. The reduced noise component in the equilibrium futures price motivates the producers to put a greater weight on the futures price in their learning of the global productivity A. As a result, any given deviation of ν from its mean will have a greater feedback effect on producers' commodity demand and the spot price.

The comparative static of τ_s is more subtle due to the presence of two offsetting forces. On one hand, as τ_s increases, producers' private signals becomes more precise, which induces them to rely more on their private signals rather than the observed futures price in their learning of A. On the other hand, the more precise private signals also make the futures

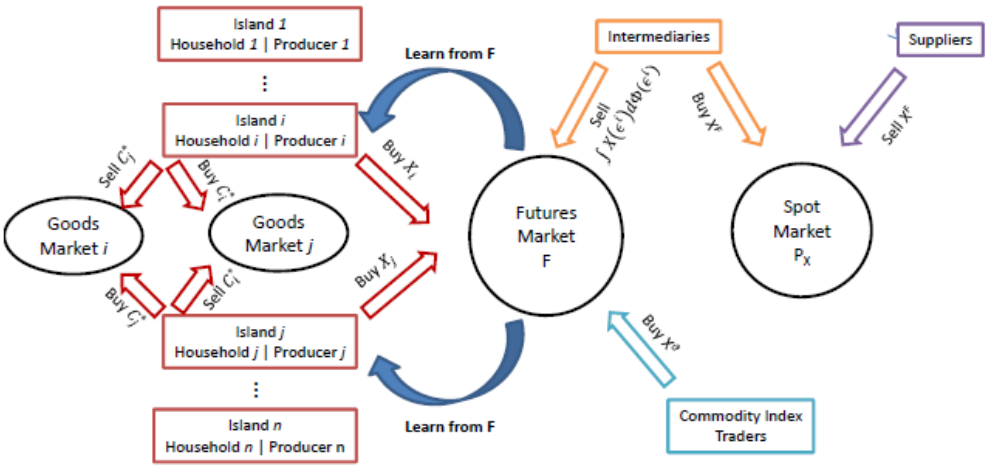


Figure 2: Structure of the Extended Model

price more informative of A, which, in turn, motivates producers to put a greater weight on the futures price in their learning. The latter force is dominating when the private signals are not sufficiently informative, while the former dominates otherwise.

4 An Extension

We now extend the basic model to include a group of financial traders who trade only in the commodity futures market and who do not get involved in either taking or making delivery. This extension is motivated by the large inflow of investment into commodity futures markets in the recent years. Since mid-2000s, more and more institutional investors such as pension funds and insurance companies have started to treat commodities as a new asset class like stocks and bonds. These institutional investors regularly allocate a fraction of their portfolios to investing in futures and swap contracts written on commodities. They take only long positions, and typically close out their futures positions before maturity and roll into more distant futures contracts (i.e., replacing maturing contracts by more distant ones). As mentioned in the introduction, a widely held argument is that since these investors trade only in the futures markets, their trading does not affect the physical supply and demand of commodities, and thus one should not expect them to affect the spot market equilibrium. Our extension aims to examine this argument.

Specifically, suppose that in addition to the producers and intermediaries in the futures market, there is another group of traders, which we label commodity index investors (CITs).

CITs take only long positions in the futures market at $t = 0$ and unwind their positions at $t = 1$ before delivery. For simplicity, we assume that the aggregate position of CITs and producers is given by the producers' aggregate position multiplied by a factor e^{θ}:

$$e^{\theta} \int_{-\infty}^{\infty} X_i \left(s_i, F \right) d\Phi \left(\varepsilon_i \right),$$

where e^{θ} represents the contribution of CITs. We assume that θ is random and is independent of other sources of uncertainty in the model.[6] It has a normal distribution

$$\theta \backsim \mathcal{N} \left(\overline{\theta}, \tau_{\theta}^{-1} \right)$$

with a mean of $\overline{\theta}$ and variance of τ_{θ}^{-1}. The uncertainty in CITs' positions is realistic as in practice market participants cannot observe others' positions and thus cannot directly measure the size of CITs' positions.[7] As a result, the presence of CITs introduces an additional source of uncertainty to the long side of the futures market, as both goods producers and intermediaries cannot observe θ at $t = 0$.

At $t = 1$, CITs unwind their positions with intermediaries, who then need to deliver only $\int X \left(\varepsilon_i \right) d\Phi \left(\varepsilon_i \right)$ units of the commodity to goods producers.

The addition of CITs to the futures market does not directly affect the equilibrium of the goods markets at $t = 2$, which is derived in Proposition 1. Therefore, the commodity demand of each goods producer is still determined by (7) based on his expectation of the global productivity and other producers' production decisions. However, the presence of CITs affects the futures price and thus can indirectly feed back to the producers' decisions, which is the core of our analysis.

Intermediaries continue to take short positions in the futures market and purchase the commodity in the spot markets to make delivery to island producers. Like before, they still require an expected return ν on their positions as specified in (8). The clearing of the spot markets now requires that after removing the CITs' long positions from the intermediaries'

[6] This assumption treats CITs' trading as independent of supply and demand of the commodity. This feature is realistic as investment decisions of institutional investors in commodities are often motivated by portfolio diversification rather than fundamental information regarding a particular commodity. See Tang and Xiong (2010) for more information on trading of commodity index traders.

[7] Despite that large traders need to report their futures positions to the CFTC on weekly basis, ambiguity in trader classification and netting of positions taken by traders who are involved in different lines of business nevertheless makes the aggregate positions provided by the CFTC's public Commitment of Traders Report imprecise. See Cheng, Kirilenko, and Xiong (2012) for more detailed discussion of the trader classification and netting problems in the CFTC's Large Trader Reporting System and a summary of positions taken by commodity index traders.

aggregate short position, the remaining aggregate short position is purchased in the spot markets: $X^F/e^\theta = X^S = e^\xi P_X^k$. This implies that

$$\log P_X = -\frac{1}{k}\xi + \frac{1}{k}\log X^F - \frac{1}{k}\theta. \tag{16}$$

By substituting this logarithmic spot price into equation (8), we obtain that the intermediaries' aggregate short position in the futures market is determined by

$$\log X^F = k\log F - k\nu + \overline{\xi} + E\left[\theta|\mathcal{I}^F\right]. \tag{17}$$

Different from the corresponding equation (9) in the basic model, this equation contains an additional term $E\left[\theta|\mathcal{I}^F\right]$, the intermediaries' expectation of CITs' position. That is, the intermediaries anticipate that a fraction of their short positions will be unwound by the CITs. This also means that when CITs take a higher than expected long position (i.e., a large realization of θ), it would affect the equilibrium futures price.

By clearing the long positions, now taken by both producers and CITs, and the short positions by intermediaries, we obtain the following proposition on the futures market equilibrium, which is the analogue to Proposition 2.

Proposition 6 *At $t = 0$, the futures market has a unique log-linear equilibrium: 1) The futures price is a log-linear function of $\log A$, ν, and θ:*

$$\log F = h_A \log A + h_\nu \nu + h_\theta \theta + h_0, \tag{18}$$

with the coefficient $h_A \in [0,1]$ determined by the unique root of the following equation

$$(1-h_A)^3 = \left\{k^2\tau_\nu^{-1}\left[1 + \frac{\eta^2\tau_s^{-2}\tau_A^3}{\tau_\theta(1-h_A)^2}\right]^2 + \tau_\theta^{-1}\right\}\eta^2\tau_s^{-2}\tau_A^3 h_A, \tag{19}$$

and $h_\nu > 0$, $h_\theta > 0$, and h_0 given by

$$\begin{aligned}
h_\nu &= k\eta\tau_s^{-1}\tau_A\frac{h_A}{1-h_A}\left(1 - \frac{\eta^2\tau_s^{-2}\tau_A^3}{\tau_\theta(1-h_A)^2 + \eta^2\tau_s^{-2}\tau_A^3}\right)^{-1},\\
h_\theta &= \eta\tau_s^{-1}\tau_A\frac{h_A}{1-h_A},\\
h_0 &= (1-h_A)\left[\overline{A} + \frac{1}{2\tau_A} + \tau_s\tau_A^{-2}(1-h_A)\right] - h_\nu\bar{\nu} - h_\theta\bar{\theta}.
\end{aligned}$$

2) The long position taken by the goods producer on island i, which is also his commodity demand, is a log-linear function of his private signal s_i and $\log F$:

$$\log X_i = l_s s_i + l_F \log F + l_0, \tag{20}$$

with the coefficients l_s, l_F, and l_0 given by

$$\begin{aligned} l_s &= \frac{1}{\eta}\frac{\tau_s}{\tau_A}\left(1-h_A\right), \\ l_F &= k\left(1-h_\nu^{-1}\right), \\ l_0 &= \overline{\xi}+\left(\tau_\theta+\frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1}\tau_\theta\bar{\theta}+\left(k-l_F\right)h_0-\frac{1}{2}l_s^2\tau_s^{-1}. \end{aligned}$$

3) The short position taken by the intermediaries also has a log-linear form:

$$\log X^F=\left(l_s+l_Fh_A\right)\log A+l_Fh_\nu\nu+\left(1+l_Fh_\theta\right)\theta+l_0+l_Fh_0+\frac{1}{2}l_s^2\tau_s^{-1}. \tag{21}$$

Proposition 6 shows that θ becomes another source of uncertainty in the equilibrium futures price, in addition to $\log A$ and ν. It is easy to verify that as $\tau_\theta \to \infty$ (i.e., θ becoming observable to the market participants), the equilibrium futures price and each producer's commodity demand converge to those in the absence of CITs (Proposition 2).

4.1 Real consequence of futures market speculation

As CITs trade only in the futures market and do not take or make any delivery, their trading is a form of pure speculation in the futures market. Proposition 6 shows that in the presence of uncertainty about θ, a larger θ leads to a higher futures price as $h_\theta > 0$. Furthermore, it also feeds back to each producer's commodity demand and thus the spot price through the information channel of the futures price. By substituting equation (18) into (20), we have a producer's commodity demand:

$$\log X_i=l_ss_i+l_Fh_A\log A+l_Fh_\nu\nu+l_Fh_\theta\theta+l_Fh_0+l_0.$$

Then, the aggregate demand of the producers is

$$\log\left[\int_{-\infty}^{\infty}X_i\left(s_i,F\right)d\Phi\left(\varepsilon_i\right)\right]=l_Fh_\theta\theta+\left(l_s+l_Fh_A\right)\log A+l_Fh_\nu\nu+l_0+l_Fh_0+\frac{1}{2}l_s^2\tau_s^{-1}. \tag{22}$$

By substituting equation (21) into (16), we obtain the spot price

$$\log P_X=\frac{1}{k}\left[\left(l_s+l_Fh_A\right)\log A+l_Fh_\nu\nu+l_Fh_\theta\theta+l_0+l_Fh_0+\frac{1}{2}l_s^2\tau_s^{-1}\right]-\frac{1}{k}\xi.$$

Taken together, a larger θ not only leads to a higher futures price but also affects producers' demand and the spot price. This result contradicts the argument mentioned in the introduction that speculators who trade only in commodity futures markets are irrelevant to commodity demand and spot prices.

The impact of θ on the producers' aggregate demand and the spot price are both determined by the coefficient $l_F h_\theta$. As $h_\theta > 0$, $l_F > 0$ ensures that θ has a positive impact on the aggregate demand and the spot price. The following proposition provides a sufficient condition on the model parameters to ensure $l_F > 0$.

Proposition 7 *The following sufficient, although not necessary, condition ensures that* $l_F > 0$ *in the presence of CITs:*

$$\frac{k^3 \eta \tau_s^{-1}}{\left(1 + k\eta\tau_s^{-1}\tau_A\right)^2} > k^2 \tau_\nu^{-1} \left[1 + k^{-2}\tau_A \tau_\theta^{-1} \left(1 + k\eta\tau_s^{-1}\tau_A\right)^2\right]^2 + \tau_\theta^{-1}.$$

Under the condition listed in Proposition 7, a higher value of θ leads not only to a higher futures price but also to greater commodity demand from the producers and thus a higher spot price. This result thus contradicts the other argument mentioned in the introduction that commodity price increases driven by non-fundamental factors must be accompanied by reduced demand and, consequently, inventory spikes.

The key here is that commodity demand is not necessarily always driven by economic fundamentals. To the extent that noise in futures markets can affect futures prices, it can affect producers' expectations and thus their commodity demand. Therefore, it is important not to blindly attribute increases in commodity demands to improved economic fundamentals. Instead, one should be cautious in separating fundamental and noise-driven demands. This empirical identification problem is challenging but central in evaluating whether speculation in futures markets affects spot market dynamics.

4.2 Implications for market transparency

Equation (22) shows that the logarithmic aggregate demand is a linear function of $\log A$, ν, and θ, with the feedback effect of θ captured by the term $l_F h_\theta \theta$. We can measure the economic impact of θ by the the uncertainty induced by θ to the aggregate demand:

$$V_\theta = l_F^2 h_\theta^2 \tau_\theta^{-1}.$$

The following proposition characterizes the dependence of V_θ on uncertainty in θ.

Proposition 8 *Under the sufficient condition listed in Proposition 7, the uncertainty induced by* θ *to the producers' commodity demand decreases with* τ_θ.

Proposition 8 shows that greater uncertainty about the CITs' futures position (i.e., smaller τ_θ) leads to a greater impact of their trading on the aggregate commodity demand and thus the spot price. This is because greater uncertainty makes it more difficult for producers and intermediaries to separate the CITs' futures position from the producers' position in determining the actual demand for the commodity. This result thus motivates financial policies that enhance transparency in commodity futures markets so that market participants can more precisely measure trading due to actual commodity demand as opposed to speculative trading.

4.3 Empirical implications and further discussion

The key implication of our model is that non-fundamental factors originating from futures markets can feed back to commodity demand and spot prices through the informational channel of futures prices. Constructing a valid measure of non-fundamental factors that affect futures prices is a challenge. The large inflow of financial investment to commodity futures markets in the recent years is a potential candidate. Several considerations, however, complicate the direct use of the positions of hedge funds and commodity index traders (as released by the weekly reports of the Commodity Futures Trading Commission). First, these financial traders might possess private information about demand and supply of commodities. As a result, while their trading is treated as exogenous in our model, it may be correlated with economic fundamentals in reality. Thus, it is necessary to either control for the fundamental-related investment flow or ideally find an instrument to extract investment flow that is unrelated to economic fundamentals. Second, financial traders may trade to either fulfill their own investment purposes or to facilitate the trading needs of other market participants. In the former case, the changes of their futures positions are positively correlated with the changes of futures prices, while in the latter case, their trades provide liquidity and are negatively correlated with price changes.[8] This consideration also cautions against blindly using financial traders' positions as a measure of non-fundamental factors in commodity futures markets.

Our model is most relevant for industrial commodities whose demands are closely tied to global economic strength. Crude oil and copper are examples of such commodities. On the other hand, there are also other commodities, such as cocoa and coffee, whose demands

[8] See Cheng, Kirilenko and Xiong (2012) for an analysis of these two types of trades in the commodity futures markets during the financial crisis.

are less correlated with economic growth. We expect the feedback effects of futures prices to be stronger for the commodities in the first group than in the latter. This contrast provides ground for potential cross-sectional tests of feedback effects.

We view prices of the front-month or second-month futures contracts as the most relevant futures prices for empirical tests of feedback effects. This is because in reality front-month and second-month contracts tend to attract most trading and thus fulfill most of the information aggregation role in commodity markets. Since these front contracts are only a few weeks away from maturity, their prices are close to the spot price. It is useful to note that the prices of more distant futures contracts often deviate from the front contracts. The price spread between the front and distant contracts reflects confounding factors not considered in our model, such as the convenience yield from holding physical commodities and cost for arbitrageurs to arbitrage mispricing on the futures price curve.[9]

The feedback effects illustrated by our model originate from information frictions about global economic fundamentals. While noise in commodity futures markets can distort people's expectations for a certain period of time, we expect them to eventually correct their expectations with updated information. Thus, the feedback effects are likely to be relevant only for short- and medium-terms. A systematic evaluation of the horizons of feedback effects requires a dynamic model beyond our current framework, which we leave for future research.

A dynamic framework can also help address another issue ignored by our model—commodity storage. In practice, commodity suppliers and other speculators can choose to stock up based on their expectations of commodity prices in the future. To the extent that futures prices may affect the expectations of these agents, another feedback effect may arise through their decisions to store commodities over time. This feedback effect complements the feedback effect through producers' commodity demand and can further exacerbate the impacts of non-fundamental factors in commodity markets.

[9] In a recent empirical study, Alquist and Kilian (2010) summarize evidence that the spread between the price of distant oil futures contracts (such as 3-month, 6-month, and 9-month contracts) and the spot price does not provide much predictive power for future spot prices and that the prices of distant futures contracts do a worse job in predicting future spot prices than the current spot price because the futures prices tend to be more volatile. Their findings are consistent with the presence of the aforementioned confounding factors in the price spread between the front and distant contracts.

5 Conclusion

This paper develops a model to examine an information channel for commodity futures prices to feed back to commodity demand and spot prices. When goods producers extract information about the global productivity and other goods producers' production decisions from the futures price of a commodity, an increase in the futures price does not simply represent a higher cost of production, but also signals a strong global economy and more trading opportunities for their produced goods. We characterize the conditions for the indirect information effect to dominate the direct cost effect and thus for producers to increase their demand in response to the increased futures price. Through this information channel and under certain sufficient conditions, our model also shows that by driving up the futures price, a shock to the supply side or noise induced by financial traders who trade only in futures markets can cause both commodity demand and the spot price to rise. These results caution the widely used structural models of commodity markets, which ignore the feedback effects of futures prices. They also generate contradictions to the widely held arguments that speculators who trade only in futures markets are irrelevant to spot prices and that commodity price increases driven by non-fundamental factors must be accompanied by inventory spikes. Taken together, our model shall serve as a conceptual framework for designing sharper empirical tests of the impact of speculative trading on commodity markets.

Appendix Proofs of Propositions

A.1 Proof of Proposition 1

Consider the maximization problem of the household on island i:

$$\max_{C_i, C_i^*} \left(\frac{C_i}{1-\eta}\right)^{1-\eta} \left(\frac{C_i^*}{\eta}\right)^{\eta}$$

subject to the budget constraint

$$P_i C_i + P_j C_i^* = P_i Y_i. \tag{23}$$

The two first order conditions with respect to C_i and C_i^* are

$$\left(\frac{C_i^*}{C_i}\right)^{\eta}\left(\frac{1-\eta}{\eta}\right)^{\eta} = \lambda_i P_i \tag{24}$$

$$\left(\frac{C_i}{C_i^*}\right)^{1-\eta}\left(\frac{\eta}{1-\eta}\right)^{1-\eta} = \lambda_i P_j \tag{25}$$

where λ_i is the Lagrange multiplier for his budget constraint. Dividing equations (24) and (25) leads to

$$\frac{\eta}{1-\eta}\frac{C_i}{C_i^*} = \frac{P_j}{P_i}$$

which is equivalent to $P_j C_i^* = \frac{\eta}{1-\eta} P_i C_i$. By substituting this equation back to the household's budget constraint in (23), we obtain $C_i = (1-\eta)\, Y_i$.

The market clearing of the island's produced goods requires $C_i + C_j^* = Y_i$, which implies that $C_j^* = \eta Y_i$.

The symmetric problem of the household of island j implies that $C_j = (1-\eta)\, Y_j$, and the market clearing of the goods produced by island j implies $C_i^* = \eta Y_j$.

The first order condition in equation (24) also gives the price of the goods produced by island i. Since the household's budget constraint in (23) is entirely in nominal terms, the price system is only identified up to λ_i, the Lagrange multiplier. Following Angeletos and La'O (2012), we normalize λ_i to 1. Then,

$$P_i = \left(\frac{C_i^*}{C_i}\right)^{\eta}\left(\frac{1-\eta}{\eta}\right)^{\eta} = \left(\frac{\eta Y_j}{(1-\eta)\, Y_i}\right)^{\eta}\left(\frac{1-\eta}{\eta}\right)^{\eta} = \left(\frac{Y_j}{Y_i}\right)^{\eta}.$$

A.2 Proof of Proposition 2

We first conjecture that the futures price and each island producer's long position take the following log-linear forms:

$$\log F = h_0 + h_A \log A + h_\nu \nu \tag{26}$$

$$\log X_i = l_0 + l_s s_i + l_F \log F \tag{27}$$

where the coefficients h_0, h_A, h_ν, l_0, l_s, and l_F are determined by the equilibrium conditions.

Define

$$z \equiv \frac{\log F - h_0 - h_\nu \bar{\nu}}{h_A} = \log A + \frac{h_\nu}{h_A}(\nu - \bar{\nu})$$

which is a sufficient statistic of information contained in F. Then, conditional on observing his private signal s_i and the futures price F, the island producer i's expectation of $\log A$ is

$$E\left[\log A \mid s_i, \log F\right] = E\left[\log A \mid s_i, z\right] = \frac{1}{\tau_A + \tau_s + \frac{h_A^2}{h_\nu^2}\tau_\nu}\left(\tau_A \overline{A} + \tau_s s_i + \frac{h_A^2}{h_\nu^2}\tau_\nu z\right),$$

and his conditional variance of $\log A$ is

$$Var\left[\log A \mid s_i, \log F\right] = \left(\tau_A + \tau_s + \frac{h_A^2}{h_\nu^2}\tau_\nu\right)^{-1}.$$

According to equation (7),

$$\log X_i = \frac{1}{\eta}\left\{\log\left(E_i\left[AX_j^\eta\right]\right) - \log F\right\}. \tag{28}$$

By substituting in equation (27) for $\log X_j$ into the expression above, we obtain

$$\begin{aligned} E_i\left[AX_j^\eta\right] &= E_i\left\{\exp\left[\log A + \eta\left(l_0 + l_s s_j + l_F \log F\right)\right]\right\} \\ &= \exp\left[\eta\left(l_0 + l_F \log F\right)\right] \cdot E_i\left\{\exp\left[\log A + \eta l_s s_j\right]\right\}. \end{aligned}$$

We first derive the conditional expectation in the above equation

$$\begin{aligned} & E\left[\exp\left(\log A + \eta l_s s_j\right)|s_i, \log F\right] = E\left[\exp\left(\left(1+\eta l_s\right)\log A + \eta l_s \varepsilon_j\right)|s_i, \log F\right] \\ = \; & \exp\left\{\left(1+\eta l_s\right)E_i\left[\log A\right] + \frac{\left(1+\eta l_s\right)^2}{2}Var_i\left[\log A\right] + \frac{\eta^2 l_s^2}{2}Var_i\left[\varepsilon_j\right] + \left(1+\eta l_s\right)\eta l_s Cov_i\left[\varepsilon_j \log A\right]\right\}. \end{aligned}$$

By recognizing that $Cov_i\left[\varepsilon_j \log A\right] = 0$ and substituting in the expressions of $E_i\left[\log A\right], Var_i\left[\log A\right]$, and $Var_i\left[\varepsilon_j\right]$, we obtain that

$$\begin{aligned} & \log E\left[\exp\left(\log A + \eta l_s s_j\right)|s_i, \log F\right] \\ = \; & \left(1+\eta l_s\right)\left(\tau_A + \tau_s + \frac{h_A^2}{h_\nu^2}\tau_\nu\right)^{-1}\left(\tau_A \overline{A} + \tau_s s_i + \frac{h_A^2}{h_\nu^2}\tau_\nu z\right) \\ & + \frac{\left(1+\eta l_s\right)^2}{2}\left(\tau_A + \tau_s + \frac{h_A^2}{h_\nu^2}\tau_\nu\right)^{-1} + \frac{\eta^2 l_s^2}{2}\tau_s^{-1}. \end{aligned}$$

Consequently, equation (28) gives

$$\begin{aligned} \log X_i &= \frac{1}{\eta}\left\{\log\left(E_i\left[AX_j^\eta\right]\right) - \log F\right\} \\ &= \left(l_0 + l_F \log F\right) + \frac{1}{\eta}\log E_i\left[\exp\left(\log A + \eta l_s s_j\right)\right] - \frac{1}{\eta}\log F \\ &= l_0 + \left(l_F - \frac{1}{\eta}\right)\log F + \left(\frac{1+\eta l_s}{\eta}\right)\left(\tau_A + \tau_s + \frac{h_A^2}{h_\nu^2}\tau_\nu\right)^{-1} \cdot \\ & \quad \left(\tau_A \overline{A} + \tau_s s_i + \frac{h_A^2}{h_\nu^2}\tau_\nu \frac{\log F - h_0 - h_\nu \bar{\nu}}{h_A}\right) + \frac{\left(1+\eta l_s\right)^2}{2\eta}\left(\tau_A + \tau_s + \frac{h_A^2}{h_\nu^2}\tau_\nu\right)^{-1} + \frac{\eta l_s^2}{2}\tau_s^{-1}. \end{aligned}$$

For the above equation to match the conjectured equilibrium position in equation (27), the constant term and the coefficients of s_i and $\log F$ have to be identical. We thus obtain

the following equations for determining the coefficients in (27):

$$\begin{aligned} l_0 &= l_0 + \left(\frac{1+\eta l_s}{\eta}\right)\left(\tau_A + \tau_s + \frac{h_A^2}{h_\nu^2}\tau_\nu\right)^{-1}\left(\tau_A \overline{A} - \frac{h_A}{h_\nu^2}\tau_\nu\left(h_0 + h_\nu \bar{\nu}\right)\right) \\ &\quad + \frac{\left(1+\eta l_s\right)^2}{2\eta}\left(\tau_A + \tau_s + \frac{h_A^2}{h_\nu^2}\tau_\nu\right)^{-1} + \frac{\eta l_s^2}{2}\tau_s^{-1}, \end{aligned} \tag{29}$$

$$l_s = \left(\frac{1+\eta l_s}{\eta}\right)\left(\tau_A + \tau_s + \frac{h_A^2}{h_\nu^2}\tau_\nu\right)^{-1}\tau_s, \tag{30}$$

$$l_F = l_F - \frac{1}{\eta} + \left(\frac{1+\eta l_s}{\eta}\right)\left(\tau_A + \tau_s + \frac{h_A^2}{h_\nu^2}\tau_\nu\right)^{-1}\frac{h_A}{h_\nu^2}\tau_\nu. \tag{31}$$

By substituting equation (31) into (30), we have

$$l_s = \frac{1}{\eta}\frac{\tau_s}{\tau_\nu}\frac{h_\nu^2}{h_A}. \tag{32}$$

Substituting this expression into equation (30), we also have that

$$h_\nu^2 = \tau_A^{-1}\tau_\nu h_A\left(1 - h_A\right). \tag{33}$$

For h_ν to be real, it is necessary that $h_A \in [0,1]$, a condition we will verify later.

From equation (29) and our expressions for l_s and h_ν, we obtain h_0 given in equation (12).

We now use the market clearing condition for the futures market to determine three other equations for the coefficients in the conjectured log-linear futures price and log-linear producer position. Aggregating equation (27) gives the aggregate long position of the producers:

$$\begin{aligned} \int_{-\infty}^{\infty} X_i\left(\varepsilon_i\right) d\Phi\left(\varepsilon_i\right) &= \int_{-\infty}^{\infty} \exp\left[l_0 + l_s s_i + l_F \log F\right] d\Phi\left(\varepsilon_i\right) \\ &= \int_{-\infty}^{\infty} \exp\left[l_0 + l_s\left(\log A + \varepsilon_i\right) + l_F\left(h_0 + h_A \log A + h_\nu \nu\right)\right] d\Phi\left(\varepsilon_i\right) \\ &= \exp\left[\left(l_s + l_F h_A\right)\log A + l_F h_\nu \nu + l_0 + l_F h_0\right]\int_{-\infty}^{\infty} \exp\left[l_s \varepsilon_i\right] d\Phi\left(\varepsilon_i\right) \\ &= \exp\left[\left(l_s + l_F h_A\right)\log A + l_F h_\nu \nu + l_0 + l_F h_0 + \frac{1}{2} l_s^2 \tau_s^{-1}\right]. \end{aligned} \tag{34}$$

Equation (9) implies that $\log X^F = k \log F - k\nu + \overline{\xi}$

$$\begin{aligned} \log X^F &= k\left(h_0 + h_A \log A + h_\nu \nu\right) - k\nu + \overline{\xi} \\ &= k h_A \log A + k\left(h_\nu - 1\right)\nu + \overline{\xi} + k h_0. \end{aligned}$$

Then, the market clearing condition $\log\left[\int_{-\infty}^{\infty} X_i(\varepsilon_i)\, d\Phi(\varepsilon_i)\right] = \log X^F$ requires that

$$\begin{aligned} l_s + l_F h_A &= k h_A, && (35)\\ l_F h_\nu &= k(h_\nu - 1), && (36)\\ l_0 + l_F h_0 + \frac{1}{2} l_s^2 \tau_s^{-1} &= \overline{\xi} + k h_0. && (37) \end{aligned}$$

Equation (36) directly implies that

$$l_F = k\left(1 - h_\nu^{-1}\right). \tag{38}$$

Substituting equations (32) and (38) into equation (35) yields

$$h_A^2 = \frac{1}{k\eta}\frac{\tau_s}{\tau_\nu} h_\nu^3. \tag{39}$$

We see that $h_\nu \geq 0$ is necessary for h_A to be real and that $h_A = 0$ whenever $h_\nu = 0$. Given the expression for h_ν^2 in (33), we arrive at

$$(1 - h_A)^3 = \frac{k^2\eta^2\tau_A^3}{\tau_s^2\tau_\nu} h_A. \tag{40}$$

This equation has a unique real root $h_A \in [0, 1]$. First note that the LHS of equation (40) is monotonically decreasing in h_A and the RHS is monotonically increasing in h_A. Thus, if there is a root, the root must be unique. Next, note that at $h_A = 0$ the LHS is above the RHS while at $h_A = 1$ the LHS is below the RHS. Then, according to the intermediate value theorem, there must be a root inside $[0, 1]$.

We can directly solve for the unique real root of equation (40). Define $H_A = 1 - h_A$. Then, the equation becomes

$$H_A^3 + k^2\eta^2\tau_s^{-2}\tau_\nu^{-1}\tau_A^3 H_A - k^2\eta^2\tau_s^{-2}\tau_\nu^{-1}\tau_A^3 = 0.$$

This is a depressed cubic polynomial of the form $x^3 + px + q = 0$, which has one real and two complex roots. Following Cardano's method, the one real root is given by

$$H_A = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}.$$

Since $q = -p$, the above simplifies to

$$H_A = p^{1/3}\left(\sqrt[3]{\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{p}{27}}} + \sqrt[3]{\frac{1}{2} - \sqrt{\frac{1}{4} + \frac{p}{27}}}\right).$$

Substituting in $p = k^2\eta^2\tau_s^{-2}\tau_\nu^{-1}\tau_A^3$, we arrive at the expression for h_A in equation (10).

By substituting the expression for h_A back into equation (33), we obtain a quadratic equation for h_ν. We take the positive root given in equation (11) to fulfill the necessary condition for h_A to be real. With h_A and h_ν determined, l_s is then given by (32), l_F by (38), h_0 by (29), and l_0 by (37).

The intermediary's short position is given by equation (34).

A.3 Proof of Proposition 3

We keep the same setting outlined in the main model, except letting A be observable by all market participants. We first derive the equilibrium. In this setting, each producer's private signal s_i becomes useless as A is directly observable. We can still use equation (7) to derive producer i's optimal commodity demand:

$$X_i = \frac{A^{1/\eta}}{F^{1/\eta}} \left\{ E\left[X_j^\eta | A, F, s_i \right] \right\}^{1/\eta}.$$

As the producers now share the same information about A, they must have the same expectation about their future trading partners' production decisions. As a result, $X_i = X_j$ for any i and j. Then, the above equation implies that in equilibrium $F = A$. Each producer has an inelastic demand whenever $F \leq A$. That is,

$$X_i = \begin{cases} 0 & \text{if} \quad F > A \\ [0, \infty) & \text{if} \quad F = A \\ \infty & \text{if} \quad F < A \end{cases}.$$

Market clearing of the futures market requires that the producers' aggregate commodity demand equals the short position of intermediaries, i.e., $X_i = X^F$. Equation (9) thus gives the producers' aggregate commodity demand: $\log X^F = k \log A - k\nu + \bar{\xi}$. It is clear that this equilibrium is unique.

It is also easy to verify that this equilibrium coincides with the limiting case of Proposition 2 after letting $\tau_s \to \infty$.

We now use the First Fundamental Theorem of Welfare to show that the competitive equilibrium derived in this setting is Pareto efficient. We can rewrite the optimization problem of the representative household on island i as

$$\max_{C_i, C_i^*} \left(\frac{C_i}{1-\eta} \right)^{1-\eta} \left(\frac{C_i^*}{\eta} \right)^{\eta}$$

subject to the following budget constraint with transfers:

$$P_i C_i + P_j C_i^* = W_i^f + W_i^s.$$

where $W_i^f = P_i Y_i - F X_i$ is the profit from the producer on the island and $W_i^s = F X_i$ is the net profits returned from the intermediaries and commodity suppliers to the representative household on the island. The intermediaries are passive investors in the futures market, requiring a rate of return ν from shorting the futures contract and buying the commodity at the spot price P_X. By netting out the intermediaries with the suppliers, we can view them together as one group of competitive firms in the economy that solves the following profit maximization problem to supply the commodity to island households by shorting the futures contract:

$$\max_{X^F} F e^{-\nu} X^F - \frac{k}{1+k} e^{-\bar{\xi}/k} \left(X^F\right)^{\frac{1+k}{k}}.$$

The first order condition of this optimization leads to the following supply curve of commodity futures:

$$\log X^F = k \log F - k\bar{\nu} + \bar{\xi},$$

which is identical to equation (9) derived earlier. Since $F = A$, we arrive at

$$\log X = k \log A - k\bar{\nu} + \bar{\xi}.$$

We have thus constructed a competitive price equilibrium with transfers, where the transfers to each island household are W_i^f from the island's producer and W_i^s from the intermediaries and commodity suppliers. Competitive firms maximize profits, there is an efficient use of social resources, the derived consumption bundle $\{C_i, C_i^*\}$ also maximizes each household's utility. Since the household's utility is locally non-satiated, we can apply the First Fundamental Theorem of Welfare to say that the allocation in the no information friction benchmark is Pareto Efficient.

A.4 Proof of Proposition 4

Using equations (33) and (38), we can rewrite l_F as

$$l_F = k\left(1 - \sqrt{\frac{\tau_A}{\tau_\nu h_A \left(1 - h_A\right)}}\right).$$

Since $k \in (0,1)$ by assumption and $h_A \in [0,1]$, $l_F > 0$ if and only if

$$h_A \left(1 - h_A\right) > \frac{\tau_A}{\tau_\nu}.$$

Substituting for h_A using equation (40), this condition is equivalent to

$$(1-h_A)^4 > \frac{\tau_A}{\tau_\nu}\frac{k^2\eta^2\tau_A^3}{\tau_s^2\tau_\nu} = \frac{k^2\tau_A^4}{\tau_s^2\tau_\nu^2}\eta^2$$

which is equivalent to

$$h_A < h^* \equiv 1 - \frac{k^{1/2}\tau_A}{\tau_s^{1/2}\tau_\nu^{1/2}}\eta^{1/2}.$$

As h_A is determined by the unique root of equation (40), this condition is further equivalent to the LHS of equation (40) being lower than the RHS, which is

$$(1-h^*)^3 < \frac{k^2\tau_A^3}{\tau_s^2\tau_\nu}\eta^2 h^*.$$

By substituting in h^*, we obtain

$$\frac{k\tau_A}{\tau_s}\eta - \frac{k^{1/2}\tau_\nu^{1/2}}{\tau_s^{1/2}}\eta^{1/2} + 1 < 0,$$

which is equivalent to

$$\phi(\eta) \equiv k\frac{\tau_A}{\tau_s}\left(\eta^{1/2} - \frac{1}{2}\frac{k^{-1/2}\tau_\nu^{1/2}\tau_s^{1/2}}{\tau_A}\right)^2 + 1 - \frac{1}{4}\frac{\tau_\nu}{\tau_A} < 0. \tag{41}$$

Thus, $l_F > 0$ if and only if the condition in (41) holds true.

As $\phi(0) = 1$ and $\phi(\eta)$ is a continuous function, when η is sufficiently small (41) is violated, and $l_F < 0$.

As $\phi(\eta)$ reaches its minimum point at $\eta^{1/2} = \frac{1}{2}\frac{k^{-1/2}\tau_\nu^{1/2}\tau_s^{1/2}}{\tau_A}$ with a value of $1 - \frac{1}{4}\frac{\tau_\nu}{\tau_A}$. The following sufficient, although not necessary, conditions ensure there exists a region of η inside $[0,1]$ such that (41) holds true:

$$\frac{1}{2}\frac{k^{-1/2}\tau_\nu^{1/2}\tau_s^{1/2}}{\tau_A} < 1, \text{ and } 1 - \frac{1}{4}\frac{\tau_\nu}{\tau_A} < 0.$$

These conditions are equivalent to the conditions listed in Proposition 4.

A.5 Proof of Proposition 5

From equations (11) and (38), one can rewrite $l_F h_\nu$ as

$$l_F h_\nu = k\left(1 - h_\nu^{-1}\right)h_\nu = k\left(h_\nu - 1\right) = k\left(\sqrt{\tau_A^{-1}\tau_\nu h_A(1-h_A)} - 1\right),$$

which is monotonic with respect to $g \equiv \tau_\nu h_A(1-h_A)$. From equation (40), we have that

$$g = \frac{\tau_s^2\tau_\nu^2}{k^2\eta^2\tau_A^3}(1-h_A)^4.$$

We focus on how g moves with τ_ν and τ_s.

By applying the Implicit Function Theorem to equation (40), we have

$$\frac{\partial h_A}{\partial \tau_\nu} = \frac{\frac{k^2\eta^2\tau_A^3}{\tau_s^2\tau_\nu^2}h_A}{3\left(1-h_A\right)^2 + \frac{k^2\eta^2\tau_A^3}{\tau_s^2\tau_\nu}} \geq 0,$$

as $h_A \in [0,1]$. Then,

$$\begin{aligned}\frac{\partial g}{\partial \tau_\nu} &= 2\frac{\tau_s^2}{k^2\eta^2\tau_A^3}\tau_\nu\left(1-h_A\right)^3\left[\left(1-h_A\right) - 2\tau_\nu\frac{\partial h_A}{\partial \tau_\nu}\right] \\ &\propto \frac{\frac{k^2\eta^2\tau_A^3}{\tau_s^2\tau_\nu}h_A + \frac{k^2\eta^2\tau_A^3}{\tau_s^2\tau_\nu}\left(1-h_A\right)}{3\left(1-h_A\right)^2 + \frac{k^2\eta^2\tau_A^3}{\tau_s^2\tau_\nu}} > 0.\end{aligned}$$

Thus, $l_F h_\nu$ is increasing with τ_ν.

Again by applying the Implicit Function Theorem to equation (40), we have

$$\frac{\partial h_A}{\partial \tau_s} = \frac{2\frac{k^2\eta^2\tau_A^3}{\tau_s^3\tau_\nu}h_A}{3\left(1-h_A\right)^2 + \frac{k^2\eta^2\tau_A^3}{\tau_s^2\tau_\nu}} \geq 0,$$

which suggests that h_A is monotonically increasing in τ_s. Then,

$$\begin{aligned}\frac{\partial g}{\partial \tau_s} &= 2\frac{\tau_\nu^2}{k^2\eta^2\tau_A^3}\tau_s\left(1-h_A\right)^3\left[\left(1-h_A\right) - 2\tau_s\frac{\partial h_A}{\partial \tau_s}\right] \\ &\propto \frac{3\left(1-h_A\right)^3 + \frac{k^2\eta^2\tau_A^3}{\tau_s^2\tau_\nu}\left(1-h_A\right) - 4\frac{k^2\eta^2\tau_A^3}{\tau_s^2\tau_\nu}h_A}{3\left(1-h_A\right)^2 + \frac{k^2\eta^2\tau_A^3}{\tau_s^2\tau_\nu}} \\ &= \frac{k^2\eta^2\tau_A^3}{\tau_s^2\tau_\nu}\frac{\left(1-2h_A\right)}{3\left(1-h_A\right)^2 + \frac{k^2\eta^2\tau_A^3}{\tau_s^2\tau_\nu}},\end{aligned}$$

which is either above or below zero depending on whether h_A is below or above $1/2$. As h_A is monotonically increasing in τ_s, there exists a certain threshold τ_s^*, above which $l_F h_\nu$ is decreasing with τ_s and below which $l_F h_\nu$ is increasing with τ_s.

A.6 Proof of Proposition 6

The proof proceeds much like the proof of Proposition (2). We provide the arguments for where it changes. We now conjecture that the futures price takes the following log-linear form:

$$\log F = h_0 + h_A \log A + h_\nu \nu + h_\theta \theta \tag{42}$$

and each island producer's long position $\log X_i$ still takes the log-linear form in (27), where the coefficients h_0, h_A, h_ν, h_θ, l_0, l_s, and l_F are again determined by the equilibrium conditions.

Define

$$z \equiv \frac{\log F - h_0 - h_\nu \bar{\nu} - h_\theta \bar{\theta}}{h_A} = \log A + \frac{h_\nu}{h_A}\left(\nu - \bar{\nu}\right) + \frac{h_\theta}{h_A}\left(\theta - \bar{\theta}\right)$$

which is a sufficient statistic of information contained in F. Then, conditional on observing his private signal s_i and the futures price F, the island producer i's expectation of $\log A$ is

$$\begin{aligned} E\left[\log A \mid s_i, \log F\right] &= E\left[\log A \mid s_i, z\right] \\ &= \frac{1}{\tau_A + \tau_s + h_A^2\left(h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}\right)^{-1}}\left(\tau_A\overline{A} + \tau_s s_i + h_A^2\left(h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}\right)^{-1} z\right), \end{aligned}$$

and his conditional variance of $\log A$ is

$$Var\left[\log A \mid s_i, \log F\right] = \left(\tau_A + \tau_s + h_A^2\left(h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}\right)^{-1}\right)^{-1}.$$

By again substituting in equation (27) for $\log X_j$ into (28), we obtain

$$\begin{aligned} E_i\left[AX_j^\eta\right] &= E_i\left\{\exp\left[\log A + \eta\left(l_0 + l_s s_j + l_F \log F\right)\right]\right\} \\ &= \exp\left[\eta\left(l_0 + l_F \log F\right)\right] \cdot E_i\left\{\exp\left[\log A + \eta l_s s_j\right]\right\}. \end{aligned}$$

We first derive the conditional expectation in the above equation

$$\begin{aligned} & E\left[\exp\left(\log A + \eta l_s s_j\right) | s_i, \log F\right] = E\left[\exp\left(\left(1+\eta l_s\right)\log A + \eta l_s \varepsilon_j\right) | s_i, \log F\right] \\ = \; & \exp\left\{\left(1+\eta l_s\right) E_i\left[\log A\right] + \frac{\left(1+\eta l_s\right)^2}{2} Var_i\left[\log A\right] + \frac{\eta^2 l_s^2}{2} Var_i\left[\varepsilon_j\right] + \left(1+\eta l_s\right)\eta l_s Cov_i\left[\varepsilon_j \log A\right]\right\}. \end{aligned}$$

By recognizing that $Cov_i\left[\varepsilon_j \log A\right] = 0$ and substituting in the expressions of $E_i\left[\log A\right], Var_i\left[\log A\right]$, and $Var_i\left[\varepsilon_j\right]$, we obtain that

$$\begin{aligned} & \log E\left[\exp\left(\log A + \eta l_s s_j\right) | s_i, \log F\right] \\ = \; & \left(1+\eta l_s\right)\left(\tau_A + \tau_s + h_A^2\left(h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}\right)^{-1}\right)^{-1}\left(\tau_A\overline{A} + \tau_s s_i + h_A^2\left(h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}\right)^{-1} z\right) \\ & + \frac{\left(1+\eta l_s\right)^2}{2}\left(\tau_A + \tau_s + h_A^2\left(h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}\right)^{-1}\right)^{-1} + \frac{\eta^2 l_s^2}{2}\tau_s^{-1}. \end{aligned}$$

Consequently, equation (28) gives

$$\begin{aligned}
\log X_i &= \frac{1}{\eta}\left\{\log\left(E_i\left[AX_j^{\eta}\right]\right) - \log F\right\} \\
&= (l_0 + l_F \log F) + \frac{1}{\eta}\log E_i\left[\exp\left(\log A + \eta l_s s_j\right)\right] - \frac{1}{\eta}\log F \\
&= l_0 + \left(l_F - \frac{1}{\eta}\right)\log F + \frac{(1+\eta l_s)^2}{2\eta}\left(\tau_A + \tau_s + h_A^2\left(h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}\right)^{-1}\right)^{-1} \\
&\quad + \frac{\eta l_s^2}{2}\tau_s^{-1} + \left(\frac{1+\eta l_s}{\eta}\right)\left(\tau_A + \tau_s + h_A^2\left(h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}\right)^{-1}\right)^{-1} \cdot \\
&\quad \left(\tau_A\overline{A} + \tau_s s_i + h_A^2\left(h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}\right)^{-1}\frac{\log F - h_0 - h_\nu\bar{\nu} - h_\theta\bar{\theta}}{h_A}\right).
\end{aligned}$$

For the above equation to match the conjectured equilibrium position in equation (27), the constant term and the coefficients of s_i and $\log F$ have to be identical. We thus obtain the following equations for determining the coefficients in (27):

$$\begin{aligned}
l_0 &= l_0 + \frac{(1+\eta l_s)^2}{2\eta}\left(\tau_A + \tau_s + h_A^2\left(h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}\right)^{-1}\right)^{-1} + \frac{\eta l_s^2}{2}\tau_s^{-1} + \left(\frac{1+\eta l_s}{\eta}\right)\cdot \qquad (43) \\
&\quad \left(\tau_A + \tau_s + h_A^2\left(h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}\right)^{-1}\right)^{-1}\left(\tau_A\overline{A} - h_A^2\left(h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}\right)^{-1}\left(h_0 + h_\nu\bar{\nu} + h_\theta\bar{\theta}\right)\right), \\
l_s &= \left(\frac{1+\eta l_s}{\eta}\right)\left(\tau_A + \tau_s + h_A^2\left(h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}\right)^{-1}\right)^{-1}\tau_s, \qquad (44) \\
l_F &= l_F - \frac{1}{\eta} + \left(\frac{1+\eta l_s}{\eta}\right)\left(\tau_A + \tau_s + h_A^2\left(h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}\right)^{-1}\right)^{-1} h_A\left(h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}\right)^{-1}. \qquad (45)
\end{aligned}$$

By substituting equation (45) into (44), we have

$$l_s = \frac{1}{\eta}\left(h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}\right)\frac{1}{h_A}\tau_s. \qquad (46)$$

Substituting this expression into equation (44), we also have that

$$h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1} = \tau_A^{-1}h_A\left(1 - h_A\right). \qquad (47)$$

For h_ν and h_θ to be real, it is necessary that $h_A \in [0,1]$, a condition we will verify later.

From equation (43) and our expressions for l_s and $h_\nu^2\tau_\nu^{-1} + h_\theta^2\tau_\theta^{-1}$, we obtain h_0 given in Proposition 6.

We now use the market clearing condition for the futures market to determine three other equations for the coefficients in the conjectured log-linear futures price and log-linear producer position. By substituting equation (42) into (27) and then aggregating equation

(27), we have that

$$e^{\theta}\int_{-\infty}^{\infty}X_i\left(\varepsilon_i\right)d\Phi\left(\varepsilon_i\right)=\exp\left[\left(1+l_Fh_\theta\right)\theta+\left(l_s+l_Fh_A\right)\log A+l_Fh_\nu\nu+l_0+l_Fh_0+\frac{1}{2}l_s^2\tau_s^{-1}\right]. \tag{48}$$

The market clearing of the futures market requires

$$e^{\theta}\int_{-\infty}^{\infty}X_i\left(\varepsilon_i\right)d\Phi\left(\varepsilon_i\right)=X^F. \tag{49}$$

The intermediaries' net short position is given equation (17), which contains $E\left[\theta|\log F,\nu\right]$. Define

$$z^\theta\equiv\frac{\log F-h_0-h_\nu\nu}{h_\theta}=\theta+\frac{h_A}{h_\theta}\log A$$

which is a sufficient statistic of information contained in F to financial intermediaries. Thus,

$$\begin{aligned}E\left[\theta\mid\log F,\nu\right] &= E\left[\log\theta\mid z^\theta\right]\\ &= \left(\tau_\theta+\frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1}\left(\tau_\theta\bar{\theta}+\frac{h_\theta^2}{h_A^2}\tau_Az^\theta\right)\\ &= \left(\tau_\theta+\frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1}\left[\tau_\theta\bar{\theta}+\frac{h_\theta^2}{h_A^2}\tau_A\left(\frac{\log F-h_0-h_\nu\nu}{h_\theta}\right)\right].\end{aligned}$$

This implies that

$$\begin{aligned}\log X^F &= \left(k+\left(\tau_\theta+\frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1}\frac{h_\theta}{h_A^2}\tau_A\right)\log F-\left(k+\left(\tau_\theta+\frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1}\frac{h_\theta}{h_A^2}\tau_Ah_\nu\right)\nu\\ &\quad+\overline{\xi}+\left(\tau_\theta+\frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1}\left(\tau_\theta\bar{\theta}-\frac{h_\theta}{h_A^2}\tau_Ah_0\right)\end{aligned} \tag{50}$$

Since $\log F=h_0+h_A\log A+h_\nu\nu+h_\theta\theta$, equation (50) implies that

$$\begin{aligned}\log X^F &= \left(k+\left(\tau_\theta+\frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1}\frac{h_\theta}{h_A^2}\tau_A\right)\left(h_A\log A+h_\theta\theta\right)\\ &\quad+k\left(h_\nu-1\right)\nu+\overline{\xi}+\left(\tau_\theta+\frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1}\tau_\theta\bar{\theta}+kh_0\end{aligned}$$

Then, the market clearing condition in (49) requires that

$$l_s+l_Fh_A = \left(k+\left(\tau_\theta+\frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1}\frac{h_\theta}{h_A^2}\tau_A\right)h_A, \tag{51}$$

$$l_Fh_\nu = k\left(h_\nu-1\right), \tag{52}$$

$$1+l_Fh_\theta = \left(k+\left(\tau_\theta+\frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1}\frac{h_\theta}{h_A^2}\tau_A\right)h_\theta \tag{53}$$

$$l_0+l_Fh_0+\frac{1}{2}l_s^2\tau_s^{-1} = \overline{\xi}+\left(\tau_\theta+\frac{h_\theta^2}{h_A^2}\tau_A\right)^{-1}\tau_\theta\bar{\theta}+kh_0. \tag{54}$$

From (52), it again immediately follows that

$$l_F = k\left(1 - h_\nu^{-1}\right). \tag{55}$$

From (51) and (53), it follows that

$$l_s = h_\theta^{-1} h_A. \tag{56}$$

Substituting (47) into (46) and applying (56), we arrive at

$$h_\theta = \eta \tau_s^{-1} \tau_A \frac{h_A}{1 - h_A}. \tag{57}$$

Substituting (55), (56), and (57) into (51), we arrive at

$$h_\nu = k\eta\tau_s^{-1}\tau_A \frac{h_A}{1-h_A}\left(1 - \frac{\eta^2\tau_s^{-2}\tau_A^3}{\tau_\theta\left(1-h_A\right)^2 + \eta^2\tau_s^{-2}\tau_A^3}\right)^{-1}. \tag{58}$$

Substituting (57) and (58) into (47), we arrive at

$$\left(1-h_A\right)^3 = \left\{k^2\tau_\nu^{-1}\left[1 + \frac{\eta^2\tau_s^{-2}\tau_A^3}{\tau_\theta\left(1-h_A\right)^2}\right]^2 + \tau_\theta^{-1}\right\}\eta^2\tau_s^{-2}\tau_A^3 h_A.$$

This equation also has a unique real root $h_A \in [0,1]$ by similar arguments given for (40) in the proof of Proposition 2.

A.7 Proof of Proposition 7

By Equation (55), for $l_F \geq 0$, we must have $h_\nu \geq 1$. We therefore require rewrite this condition, using (58) and that $h_A \in [0,1]$, as

$$\frac{k\eta^3\tau_\theta^{-1}\tau_s^{-3}\tau_A^4}{\left(1-h_A\right)^2} \geq \frac{1-h_A}{h_A} - k\eta\tau_s^{-1}\tau_A.$$

The LHS is always positive. For this condition to be satisfied, a sufficient condition is that $\frac{1-h_A}{h_A} \leq k\eta\tau_s^{-1}\tau_A$, or

$$h_A \geq h^* = \frac{1}{1 + k\eta\tau_s^{-1}\tau_A}.$$

As h_A is the unique root of equation (19), this condition is equivalent to the following condition on the two sides of equation (19):

$$LHS(h^*) > RHS(h^*).$$

By substituting h^* into the two sides, we obtain the following condition:

$$\frac{k^3\eta\tau_s^{-1}}{\left(1 + k\eta\tau_s^{-1}\tau_A\right)^2} > k^2\tau_\nu^{-1}\left[1 + k^{-2}\tau_A\tau_\theta^{-1}\left(1 + k\eta\tau_s^{-1}\tau_A\right)^2\right]^2 + \tau_\theta^{-1}.$$

A.8 Proof of Proposition 8

By using equation (55), (57), and (58), we rewrite

$$V_\theta = l_F^2 h_\theta^2 \tau_\theta^{-1} = \left(k\eta\tau_s^{-1}\tau_A \frac{h_A}{1-h_A} - \frac{\tau_\theta (1-h_A)^2}{\tau_\theta (1-h_A)^2 + \eta^2 \tau_s^{-2}\tau_A^3} \right)^2 \tau_\theta^{-1}.$$

Then, direct calculation leads to

$$\begin{aligned} \frac{\partial V_\theta}{\partial \tau_\theta} &= \tau_\theta^{-1} l_F h_\theta \left[2 \left(\frac{k\eta\tau_s^{-1}\tau_A}{(1-h_A)^2} + \left(\frac{\tau_\theta (1-h_A)^2}{\tau_\theta (1-h_A)^2 + \eta^2 \tau_s^{-2}\tau_A^3} \right)^2 \frac{2\eta^2\tau_s^{-2}\tau_A^3}{\tau_\theta (1-h_A)^3} \right) \frac{\partial h_A}{\partial \tau_\theta} \right. \\ &\quad \left. -2 \left(\frac{\tau_\theta (1-h_A)^2}{\tau_\theta (1-h_A)^2 + \eta^2 \tau_s^{-2}\tau_A^3} \right)^2 \frac{\eta^2\tau_s^{-2}\tau_A^3}{\tau_\theta^2 (1-h_A)^2} - \tau_\theta^{-1} l_F h_\theta \right] \end{aligned}$$

Note that $h_A \in (0,1)$ and that

$$\frac{\partial h_A}{\partial \tau_\theta} = -\frac{2k^2\eta^2\tau_\nu^{-1}\tau_s^{-2}\tau_A^3\tau_\theta^{-2}\frac{h_A}{(1-h_A)^2}\left(1+\frac{\eta^2\tau_s^{-2}\tau_A^3}{\tau_\theta(1-h_A)^2}\right) + \tau_\theta^{-2}\eta^2\tau_s^{-2}\tau_A^3 h_A}{\left[\begin{array}{c} 3(1-h_A)^2 + k^2\eta^2\tau_A^3\tau_s^{-2}\tau_\nu^{-1} + k^2\eta^2\tau_\nu^{-1}\tau_s^{-2}\tau_A^3. \\ \left(1+\frac{\eta^2\tau_s^{-2}\tau_A^3}{\tau_\theta(1-h_A)^2} + 2h_A\frac{\eta^2\tau_s^{-2}\tau_A^3}{\tau_\theta(1-h_A)^3}\right)\left(1+\frac{\eta^2\tau_s^{-2}\tau_A^3}{\tau_\theta(1-h_A)^2}\right) \end{array} \right]} < 0.$$

Then, $l_F h_\theta > 0$ is sufficient for $\frac{\partial V_\theta}{\partial \tau_\theta}$ to negative.

References

Alquist, Ron and Lutz Kilian (2010), What do we learn from the price of crude oil futures?, *Journal of Applied Econometrics* 25, 539-573.

Angeletos, Marios and Jennifer La'O (2012), Sentiment, Working paper, MIT.

Angeletos, George-Marios, Guido Lorenzoni, and Alessandro Pavan (2010), Beauty contests and irrational exuberance: A neoclassical approach, Working paper, MIT.

Bray, Margaret (1981), Futures trading, rational expectations, and the efficient markets hypothesis, *Econometrica* 49, 575-596.

Cheng, Ing-haw, Andrei Kirilenko, and Wei Xiong (2012), Convective risk flows in commodity futures markets, Working paper, Princeton University.

Fattouh, Bassam, Lutz Kilian, and Lavan Mahadeva (2012), The role of speculation in oil markets: What have we learned so far?, Working paper, University of Michigan.

Goldstein, Itay, Emre Ozdenoren, and Kathy Yuan (2011), Learning and complementarities in speculative attacks, *Review of Economic Studies* 78, 263-292.

Goldstein, Itay, Emre Ozdenoren and Kathy Yuan (2012), Trading frenzies and their impact on real investment, *Journal of Financial Economics*, forthcoming.

Grossman, Sanford and Joseph Stiglitz (1980), On the impossibility of informationally efficient markets, *American Economic Review* 70, 393-408.

Hamilton, James (2009), Causes and consequences of the oil shock of 2007-08, *Brookings Papers on Economic Activity*, 215-261.

Hamilton, James and Cynthia Wu (2012), Effects of index-fund investing on commodity futures prices, Working paper, University of California, San Diego.

Hellwig, Martin (1980), On the aggregation of information in competitive markets, *Joural of Economic Theory* 22, 477-498.

Henderson, Brian, Neil Pearson, and Li Wang (2012), New evidence on the financialization of commodity markets, Working paper, University of Illinois at Urbana-Champaign.

Kilian, Lutz (2009), Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market, *American Economic Review* 99, 1053-1069.

Kilian, Lutz and Daniel Murphy (2012), The role of inventories and speculative trading in the global market for crude oil, Working paper, University of Michigan.

Krugman, Paul (2008a), More on oil and speculation, *New York Times*, May 13.

Krugman, Paul (2008b), Speculative nonsense, once again, *New York Times*, June 23.

Singleton, Kenneth (2011), Investor flows and the 2008 boom/bust in oil prices, Working paper, Stanford University.

Subrahmanyam, A and Sheridan Titman (2001), Feedback from stock prices to cash flows, *Journal of Finance* 56, 2389-2413.

Tang, Ke and Wei Xiong (2010), Index investment and financialization of commodities, Working paper, Princeton University.