Catching falling knives: speculating on market overreaction*

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Abstract

Some agents on financial markets do not have private information about fundamentals only, but also about the market itself. Particular funds or high-frequency traders may be better at inferring information from past trades, and thus at identifying over- or undervalued assets. I study the dynamic trading behavior of such agents and show their impact is ambiguous: they decrease the likelihood to observe a large deviation of prices from fundamentals, but slow down price discovery in the long run. Their presence increases adverse selection, and they will eventually be excluded by too high spreads if their informational advantage is too low. Such agents insure the market against short-run crashes by "catching falling knives", a service for which they earn a positive profit on average, with large gains when prices bounce back, but losses otherwise. Their profit distribution displays a high variance and fat tails, it is thus likely that not enough agents acquire this type of information, so that insurance against crashes is under-provided.

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1 Introduction

In periods of financial turbulence, some agents have to sell large amounts of assets for reasons unrelated to their information about these assets' fundamental value. It then becomes difficult for other investors to keep a cool head as prices start to plunge, as there is both fundamental uncertainty and uncertainty about whether sales are informative or not. Accounts of famous market crashes typically involve this second source of uncertainty: uncertainty about the extent of portfolio insurance for the 1987 crash (Gennotte and Leland (1990)), speculation about whether the "flash crash" of May 6, 2010 was due to fundamental or non fundamental causes [1], unawareness of long/short equity fund managers of how crowded this category was during the "quant event" of August 2007 (Khandani and Lo (2007), Khandani and Lo (2011)).

In such stress situations, prices can diverge significantly from their fundamental value for some time. Why are there not enough agents to seize this profit opportunity and make the divergence disappear? In the case at hand, agents have an incentive to acquire information not only on the fundamentals, but also on whether the market is overreacting or not, which may be much easier. Such agents, able to identify whether observed sales are informative or not, can be important for market stability. I focus in my model on this particular type of information which, following Gennotte and Leland (1990), I call "supply-information".

I show that supply-informed traders correct short-run mispricings: they behave as contrarians when the market overreacts (as do the "opportunistic buyers" identified by Kirilenko *et al.* (2011)), and as positive feedback traders when the market underreacts to observed trades. This behavior however makes it more difficult for market-makers to learn supply-information themselves, so that the long-run impact of these speculators is negative. They can also be excluded from trading in the long-run if their information is not valuable enough, and their profit has fat tails: most of the time they are prevented from trading by high spreads, but with a small probability a sudden crash appears and they try to "catch a falling knife". On average they make a profit because they know the crash is likely to be driven by uninformed sales, but when this is actually not the case they make a loss.

These "supply-informed" traders can be hedge funds relying either on their staff's connections or on quantitative techniques to detect a stock's undervaluation due to liquidity events, and using typically long/short equity strategies, buying stocks they suspect to be affected

[1] See explanations 1-6 in Easley, Lopez de Prado, and O'Hara (2011), pp. 2-3, while Kirilenko *et al.* (2011) (p. 37) state that high frequency traders' contribution to high trading volumes may have been *mistaken* for liquidity by fundamental traders.

by fire sales, and selling similar stocks that are not. At a different frequency, some high-frequency traders use a lot of data on the order flow and seem to base their trading strategies on superior information about liquidity more than about fundamentals. This paper shows that analyzing these traders as traditional "value-informed" traders can be misleading.

To understand their behavior, I build an extension to the Glosten and Milgrom (1985) model with "positive feedback traders"[2], the number of which being uncertain, and supply-informed traders who know this number but have no private information about the asset's value. Supply-informed traders' impact is different from traditional value-informed traders, so that the latter category cannot be thought of as a theoretical shortcut to encompass these two different types of relevant private information.

For the exposition, assume a situation in which price drops trigger fire sales, much more than expected by market-makers. Since they overestimate the probability of an informed sale, market-makers update prices downwards too much after each sale. Supply-informed traders understand this is over-reaction, and thus buy and behave as contrarians (Proposition 4). Interestingly, in some situations this behavior implies that supply-informed trading does not affect trade imbalance, so that measures such as the PIN may not capture them (Remark 1). This behavior can also prevent a crash from happening when market-makers underestimate the amount of positive feedback trading[3] (Proposition 5), and thus believe sales to be more informative than they really are. When the asset's value is high supply-informed traders make a gain, otherwise they make losses by trying to "catch a falling knife". At the same time, due to this behavior market-makers need more time to learn the amount of positive feedback, which on average slows down price discovery (Corollary 2). In equilibrium, market-makers expect to face supply-informed traders, which increases adverse selection and the spread (Remark 5). If their informational advantage is low, supply-informed traders will be eventually excluded from the market by too high spreads (Lemma 2 and Corollary 1), thus they make most of their profit at the beginning of the trading period, when market-makers are uncertain about the amount of positive feedback trading. Finally, supply-informed traders make a positive profit with fat tails (Numerical result 1).

Identifying supply-informed trading with (some) high-frequency strategies yields two sets of testable hypotheses: the asymmetry between positive feedback sales and purchases should affect how HFT activity impacts the PIN measure (Testable hypothesis 1), and how likely

[2]Meaning here traders who buy after price increases and sell after decreases.

[3]As will be apparent below, I assume rational priors in this model: uninformed agents know ex ante the true probability that positive feedback trading is high. But they don't know which state of the world realizes and thus always under- or over-estimate positive feedback ex post.

a market is to underreact or overreact to observed trades should determine the number of HFT present on the market (Testable hypothesis 2).

I also show some phenomena that arise in this framework due to the two-dimensionality of uncertainty: *on average* (conditionally on positive feedback being underestimated) prices can diverge from the asset's value for some periods (Proposition 5), as in the "rational crashes" literature. Moreover, sales can contain a positive signal as they can lead market-makers to understand that many past sales were actually uninformative fire-sales (Remark 3), and observing no trade can increase or decrease the spread (Remark 4).

The remainder of this paper is organized as follows: the end of this section relates the paper to previous works in the literature, section 2 details the framework and derives the equilibrium, section 3 studies the trading pattern of supply-informed traders and their impact on market liquidity. Section 4 studies their impact on short term events and on long term convergence as well as their profit, section 5 concludes. References and proofs can be found in the Appendix.

Related literature

This work tries to bridge a gap between the literature on "non-fundamental uncertainty", and the literature on speculation. In the former, a dimension of uncertainty other than the value of the asset is introduced to investigate an anomaly: excess volatility of transaction prices in Easley and O'Hara (1992), the possibility of herding behavior in Avery and Zemsky (1998) (although Park and Sabourian (2011) show that they wrongly attribute the possibility of herding to the multi-dimensionality of uncertainty) or, more related to this paper, the possibility of non-fundamental crashes or bubbles in Gennotte and Leland (1990), Jacklin, Kleidon, and Pfleiderer (1992) or Romer (1993). I contribute to this literature by studying how speculation based on supply-information only can affect some of these anomalies.

Gennotte and Leland (1990) introduce "supply-informed" traders, but they assume them to be too few to play a role in preventing a crash from happening; since then the literature tends to underestimate this possibility. On the other hand, the literature on speculation (seminal works here include Hart and Kreps (1986) and Stein (1987)) focused on speculators with "fundamental" information or trying to use public information to infer the value of an asset. In De Long *et al.* (1990b) speculators are implicity informed about both positive feedback and the asset's fundamental value. But at least for certain types of market participants it is arguably easier to get information about the non-fundamental parameter than

about fundamentals, in troubled times supply-informed traders' behavior may actually be more more relevant.

In other words, this paper studies a dynamic framework where uninformed liquidity providers face a situation where both uncertainty and adverse selection are two-dimensional, as in the static framework of Gennotte and Leland (1990). Hong and Rady (2002) is to my knowledge the only example of a dynamic model of a financial market with two-dimensional adverse selection, but in their model market-makers are the agents informed about liquidity, thus no type of agent has to learn about both parameters. Brunnermeier and Pedersen (2005) study predatory trading by agents with information about distressed market participants, uncertainty is two-dimensional but not adverse selection, as there are no traders with fundamental information. The interaction between these two types of informed traders can be important however, as supply-informed trades can mistakenly be interpreted as giving information about fundamentals. On the other hand Jacklin, Kleidon, and Pfleiderer (1992) study a framework close to this paper's, but without the supply-informed traders of Gennotte and Leland (1990), so that in their model supply-information is imperfect but symmetric.

Does it make a difference? A preview of the results can be gained by looking at Figure 1[4]. In the first case 0.1% traders are supply-informed but public information gives a 5% probability that positive feedback trading is important, in the second there are 5% supply-informed traders but public information gives a 0.1% probability to the same event. In the latter case prices deviate much more from fundamental value, and for a longer time.

[Insert Fig. 1 here.]

A last theoretical strand of the literature that is important for the model is concerned with possible microfoundations of positive feedback trading. Scharfstein and Stein (1990) and Dasgupta and Prat (2006) show fund managers' incentives to mimic other managers for reputational motives, a behavior which can lead to positive feedback (and herding). In De Long *et al.* (1990b) "destabilizing speculation" can reinforce the effects of positive feedback trading instead of attenuating them. Chowdhry and Nanda (1998) study the effect of margin calls, which trigger uninformed sales. In Brunnermeier and Pedersen (2009) margin calls endogenously drive prices down, which increases margin requirements, such that "margin spirals" occur. I take as given a simplified form of positive feedback behavior, that may be partly rationalized by one of the papers above, and studies how supply-informed agents anticipating this behavior can help or hinder the aggregation of information on the market.

[4]Unless specified otherwise, simulations are all run 10.000 times over 100 periods using Scilab 5.2.1. All sets of parameters I use are reported in the Appendix A.3 and the figures in A.2.

In this sense I complement this literature by showing that trading on supply information will attenuate the short-term effects of positive feedback trading, but is very risky however and will remain limited.

Finally, as the framework of this paper is motivated by troubled period on financial markets, it is linked to papers investigating recent events such as the "flash crash" of May 2010 (Easley, Lopez de Prado, and O'Hara (2011), Kirilenko *et al.* (2011)) or the "quant event" of August 2007 (Khandani and Lo (2007), Khandani and Lo (2011)), as well as to empirical papers documenting the performance of different strategies that could be identified to supply-informed trading, which will be discussed in section 4.4.

2 Framework and equilibrium

2.1 Traders and trading mechanism

Assume an asset that gives a final payoff $v = 1$ with publicly known prior probability π, and $v = 0$ otherwise. There are four types of traders, two of them are informed and two uninformed. There are $3x_N$ *uninformed traders* of two types:

-*Noise traders* sell, buy, or hold, each with probability 1/3, regardless of the price in the current period or of past history. They can be seen as trading for exogenous liquidity motives and their behavior micro-founded as in Glosten and Milgrom (1985).

-*Positive feedback traders* have to sell after a sell order, to buy after a buy order.

A proportion $\alpha\sigma_T$ of uninformed traders will engage in positive feedback trading, where T is the direction of the trade in the previous period. Thus if there has been a purchase in the previous period a proportion $\alpha\sigma_1$ of uninformed traders want to buy. σ_1 and σ_{-1} are parameters of the model and are different, traders could have to sell after a price decrease but may not want to buy back after a price increase. I assume $\sigma_0 = 0$: if there has been no trade in the previous period, nobody is constrained in the current period. Finally, there is uncertainty about the constraints uninformed traders face: with prior probability λ_1 we have $\alpha = \alpha^+$, and with prior probability $1 - \lambda_1$ we have $\alpha = \alpha^-$, with $\alpha^+ > \alpha^-$. Thus, market-makers do not know if the proportion of orders coming from positive feedback traders is large and equal to $3\alpha^+\sigma_T x_N$ or low and equal to $3\alpha^-\sigma_T x_N$. This structure is similar to assuming the amount of noise trading is unknown, but is more general and gives more flexibility: depending on the parameters the market will overreact or underreact more or less strongly, and potentially differently after sales and purchases. Then we have two types of *informed traders*:

-*Value-informed traders*, in number x_I, know the true value v of the asset.

-*Supply-informed traders*, in number x_S, know the true value of α. They will hold a different expectation than market-makers about the asset's value, thus they may want to buy or sell depending on market-makers' prices.

Under these assumptions the total number of traders $x_S + x_I + 3x_N$ is common knowledge and normalized to 1. The following table sums up the partition of traders when the direction of last period's trade was $T \in \{-1, 0, 1\}$ and $\alpha \in \{\alpha^+, \alpha^-\}$:

Types		Proportion
Uninformed	Noise	$3(1 - \alpha\sigma_T)x_N$
	Positive feedback	$3\alpha\sigma_T x_N$
Informed	Value-informed	x_I
	Supply-informed	x_S

Trading mechanism: in each period perfectly competitive market-makers quote bid and ask prices, a trader is randomly selected and buys, sells or holds. Since the asset can only take the values 0 or 1, value-informed traders always buy or sell, hence a buy order always contains some positive information about the true value of the asset, and a sell order some negative information.[5] Note that each trader is "small" and cannot expect to trade again in the future.

Consider what market-makers know in period t. Having kept track of previous trades, they know what value of the asset supply-informed traders would have inferred from these trades, depending on their type. Denote by E_t^+, E_t^- the expected value of the asset at the beginning of period t, respectively from the point of view of α^+ and α^- supply-informed traders. From their prior probability λ and previous trades, market-makers in period t have inferred the probability, denoted λ_t, that supply-informed traders are of type α^+. Finally, all traders know the last order and thus the direction of positive feedback.

Market-makers usually have information about the order flow that allows them to learn about the supply. This happens in the model as market-makers will learn α over time. Supply-informed traders have an even more precise information (here, perfect), that may come from inside sources (liquidity needs of constrained institutions) or from access to/quick processing of data on order flow (high-frequency trading). Market-makers can also be interpreted as a modeling device to mirror uninformed liquidity providers more generally.

Denote by T_t the trade that takes place in period t, with $T_t = 1$ a purchase, -1 a sale, and 0 no trade. The variable O_t^f denotes the direction of the order feedback traders submit

[5] This assumption excludes the possibility of informational cascades, as value-informed traders always want to trade based on their perfectly accurate signal.

in period t, with the same values $-1, 0, 1$. Denoting I_t the public information available in period t we have[6]:

$$\begin{aligned} E_t^+ &= \Pr(v = 1 | I_t, \alpha = \alpha^+) \\ E_t^- &= \Pr(v = 1 | I_t, \alpha = \alpha^-) \\ \lambda_t &= \Pr(\alpha = \alpha^+ | I_t) \end{aligned}$$

2.2 Market-makers' prices and supply-informed traders' orders

I first show the existence of a unique equilibrium and derive how traders and market-makers update their beliefs from one period to the next. Section 2.4 will sum up the whole process.

At the beginning of each period, the directions of positive feedback and value-informed traders' orders are already determined and do not depend on quotes. Supply-informed traders (for instance of type α^+) buy in period t if $A_t < E_t^+$, sell if $B_t > E_t^+$, hold otherwise. A difficulty is that they may change sides depending on the arrival of information. I denote the direction in which supply-informed traders would like to trade by O_t^+ and O_t^- according to their type. Market-makers quote a bid B_t and an ask A_t in period t. Due to perfect competition between market-makers, these prices must be such that they make zero expected profit:

$$A_t = \Pr(v = 1 | I_t, O_t^+, O_t^-, O_t^f, T_t = 1) \quad (1)$$

$$B_t = \Pr(v = 1 | I_t, O_t^+, O_t^-, O_t^f, T_t = -1) \quad (2)$$

The quotes depend on the orders submitted by supply-informed traders, which in turn depend on the quotes. In equilibrium O_t^+, O_t^-, A_t, B_t are such that supply-informed traders do indeed behave as market-makers assumed they would do when setting their prices. This problem has a unique solution:

Proposition 1 (Existence and uniqueness)**.** *For a given vector $(E_t^+, E_t^-, O_t^f, \lambda_t)$ there is a unique vector (O_t^+, O_t^-, A_t, B_t) such that market-makers make zero expected profit (1 and 2 are satisfied), and their expectation of supply-informed traders' behavior is correct, that is:*

$$\forall i \in \{-, +\},\ O_t^i = 1 \Leftrightarrow E_t^i > A_t,\ O_t^i = 0 \Leftrightarrow A_t > E_t^i > B_t,\ O_t^i = -1 \Leftrightarrow E_t^i < B_t$$

[6] All notations used in the paper can be found for reference in the Appendix A.1

and moreover $O_t^+ = O_t^- \Rightarrow O_t^+ = O_t^- = 0$.

See the Appendix A.4.1 for the expression of bid and ask prices and the proof of this proposition. The intuition is simple: assume we have an equilibrium where $E_t^+ > A_t > E_t^-$, thus A_t is such that market-makers make zero profit in expectation on a sale, knowing α^+ traders may be buying. With another price A' in the same interval profit would be different from zero. If A' is higher than E_t^+ now the ask price is higher and adverse selection is lower since no supply-informed trader can buy, thus market-makers' profit would be strictly positive. Conversely it would be strictly negative with $A' < E_t^-$.

Notice that if there is a sale (resp. a purchase) in period t, we have

$$B_t(\text{ resp. } A_t) = \lambda_{t+1} E_{t+1}^+ + (1 - \lambda_{t+1}) E_{t+1}^- \tag{3}$$

The expected value of the asset knowing there has been a further sale is a weighted average of the ex-post expectations by both types of supply-informed traders. Thus market-makers cannot expect both types of supply-informed traders on the same side: if both types sold, market-makers would make a loss at the current bid.

2.3 Traders and market-makers' expectations

Supply-informed traders update their beliefs after each trade, thus $E_{t+1}^+ = \Pr(v = 1 | T_t, \alpha = \alpha^+, E_t^+)$, $E_{t+1}^- = \Pr(v = 1 | T_t, \alpha = \alpha^-, E_t^-)$. It will be convenient to use $H_t^j = \ln\left(\frac{E_t^j}{1-E_t^j}\right)$ for $j \in \{+, -\}$. Using Bayes' law it is straightforward to show that the traders' update is given by $H_{t+1}^j = H_t^j + \Delta H_t^j$, where the expression of ΔH_t^j is computed in the Appendix A.4.2.

Supply-informed traders infer nothing when there has been no trade in t; they always update their belief upwards after a purchase and downwards after a sale. Both types of traders do not update in the same way: they disagree about the extent of positive feedback trading, and a purchase is less informative if more positive feedback traders were trying to buy in the previous period. A further difference is that both types of supply-informed traders did not expect the same trading by supply-informed traders themselves in the previous period.

An order also gives information about the extent of positive feedback trading. Denoting $\Lambda_t = \ln\left(\frac{\lambda_t}{1-\lambda_t}\right)$, the market-makers' update is given by $\Lambda_{t+1} = \Lambda_t + \Delta\Lambda_t$, where the explicit expression of $\Delta\Lambda_t$ is given in the Appendix A.4.2. Market-makers infer information about λ_t from an observed trade for three reasons: observing two sales/purchases in a row is more likely when positive feedback is high; if α^i traders are expected to buy/sell, seeing a purchase/sale

is more likely when $\alpha = \alpha^i$; if $E_t^+ > E_t^-$, a purchase (resp. sale) is more likely when $\alpha = \alpha^+$ (resp. α^-) because there is a higher probability that value-informed traders buy (resp. sell).

λ_t makes jumps of different magnitudes, depending on how supply-informed and positive feedback traders are expected to trade. Each jump depends on the value taken at the beginning of the period by E_t^+ and E_t^-. Thus the process will never revert exactly to a previous state, and the order of trades will matter. If there is a purchase and then a sale it may have no direct influence on E_t^+ and E_t^-, but it will have one on λ_t, and hence on A_t and B_t. Thus it may change the orders supply-informed traders will submit and finally E_t^+ and E_t^-.

2.4 The stochastic process step by step

I sum up briefly how the process works: the market starts in period 1; the expectations of both types of supply-informed traders, the prior beliefs of market-makers about α, and the direction of positive feedback trading, ie. which trade took place in period "0", are given. Thus $E_1^+, E_1^-, \lambda_1, T_0$ are parameters of the model. I always assume $T_0 = 0$. Now consider $E_t^+, E_t^-, \lambda_t, T_{t-1}$ are known:

1. The trade in $t-1$ determines positive feedback in period t, namely:

$$O_t^f = T_{t-1}$$

2. As was shown in Proposition 1, the direction of positive feedback trading plus the beliefs of specialists and supply-informed traders uniquely determine the direction of supply-informed trading and the bid and ask prices:

$$(E_t^+, E_t^-, O_t^f, \lambda_t) \to (O_t^+, O_t^-, A_t, B_t)$$

3. The direction of positive feedback and supply-informed trading, plus the true type of supply-informed traders and value-informed traders, give the true probabilities that a buy order, a sell order or no order is received. The random draw of the next trader with these probabilities gives T_t. The true probability to observe T is just $\Pr(T_t = T | I_t, \alpha, v)$, the probability to observe T knowing the true value of α and the value of v.

4. The realized trade T_t together with supply-informed traders' previous expectations and the direction of positive feedback and supply-informed trading give supply-informed traders all the necessary information to update their beliefs according to the formulas of section 2.3.

Thus we have:

$$\begin{aligned} T_t, E_t^+, O_t^+, O_t^f &\rightarrow E_{t+1}^+ \\ T_t, E_t^-, O_t^-, O_t^f &\rightarrow E_{t+1}^- \end{aligned}$$

5. And finally market-makers can update their beliefs:

$$T_t, E_t^+, E_t^-, O_t^+, O_t^-, O_t^f, \lambda_t \rightarrow \lambda_{t+1}$$

Thus knowing the vector $(E_t^+, E_t^-, \lambda_t, T_{t-1})$ is enough to compute the other variables of interest in period t, the values $(E_{t+1}^+, E_{t+1}^-, \lambda_{t+1}, T_t)$ may take and the probability distribution on these values. Given $E_1^+, E_1^-, \lambda_1, T_0$ and the true values of v and α, the above transitions define a stochastic process. Denoting $S_t = (E_t^+, E_t^-, \lambda_t, T_{t-1})$ we have the following proposition:

Proposition 2.

$$\forall t \geq 1, \forall \{\hat{S}_i\}_{1 \leq i \leq t},\ \Pr(S_t = \hat{S}_t | S_i = \hat{S}_i, 1 \leq i \leq t-1, \alpha, v) = \Pr(S_t = \hat{S}_t | S_{t-1} = \hat{S}_{t-1}, \alpha, v)$$

The stochastic process defined above is a one-step Markov chain with an infinite denumerable state space. All states of the Markov chain are transient.

That the process has the Markov property should be obvious given the previous development. That the chain has an infinite denumerable state space and is transient follows directly from the fact that λ never "goes back" to a previous value (see section 2.3), hence the chain is almost never twice in the same state. The fact that supply-informed traders do not always trade in the same direction and that their trade does not depend only on the last trade prevents us from directly applying standard Markov tools to study the original process.

We can still define the transition matrix from one period to another between three states corresponding in this order to a purchase, no trade and a sale. Transition probabilities depend on the values of v and α and on the direction in which the supply-informed trade, which can depend on the period considered. Define first the following matrix, giving the impact of noise and positive feedback traders on the market:

$$M_N(\alpha) = \begin{pmatrix} 1+2\alpha\sigma_1 & 1-\alpha\sigma_1 & 1-\alpha\sigma_1 \\ 1 & 1 & 1 \\ 1-\alpha\sigma_{-1} & 1-\alpha\sigma_{-1} & 1+2\alpha\sigma_{-1} \end{pmatrix}$$

The behavior of value-informed traders can be characterized by the matrix $M_V(v) = (\mathbf{1}_{v=1}, 0, \mathbf{1}_{v=0}).I_3$:

value-informed traders buy if $v = 1$ and sell if $v = 0$. Similarly, if supply-informed traders are of type j their behavior is given by $M_{S,t} = (\mathbf{1}_{O_t^c=1}, \mathbf{1}_{O_t^c=0}, \mathbf{1}_{O_t^c=-1}).I_3$. For given values of v and α, the transition between periods t and $t+1$ obeys the following matrix:

$$M_t(v, \alpha) = x_N M_N(\alpha) + x_I M_V(v) + x_S M_{S,t} \tag{4}$$

2.5 Learning dynamics

$E_t^+, E_t^-, \lambda_t, A_t$ and B_t are bounded personal martingales, standard arguments show these beliefs converge to the truth (except for E_t^- when $\alpha = \alpha^+$ and E_t^+ when $\alpha = \alpha^-$, since these are probabilities based on priors giving 0 weight to the truth). It will be important in section 4 to know whether we can expect these quantities to come closer to the truth *in each period*, which I study here. To avoid discussing multiple cases, I introduce the following notations:

$$\begin{array}{rlll}
\tilde{H}_t^c = H_t^j & \text{if } \alpha = \alpha^j,\ v = 1, & -H_t^j \text{ if } \alpha = \alpha^j,\ v = 0 \\
\tilde{H}_t^u = H_t^j & \text{if } \alpha \neq \alpha^j,\ v = 1, & -H_t^j \text{ if } \alpha \neq \alpha^j,\ v = 0 \\
\tilde{\Lambda}_t = \Lambda_t & \text{if } \alpha = \alpha^+, & -\Lambda_t \text{ if } \alpha = \alpha^-
\end{array}$$

The $\tilde{}$ transformation simply ensures that a higher value of the quantity always corresponds to a belief closer to the truth. $\tilde{H}^c$ measures how close to the truth about the asset's value are *correct* supply-informed traders (i.e. who are actually on the market), and when $\tilde{H}^c \to +\infty$ they know the asset's value for sure. $\tilde{H}^u$ measures in the same way the hypothetical beliefs of *incorrect* supply-informed traders, for instance of α^- traders when $\alpha = \alpha^+$. Finally $\tilde{\Lambda}$ measures market-makers' beliefs on positive-feedback trading. Similarly I use the notations E_t^c and E_t^u.

Proposition 3. *In expectation $\tilde{H}_t^c$ goes up in each period and after all histories, while $\tilde{\Lambda}_t$ and p_t, the traded price at date t, do so only after a long enough time:*

$$\begin{array}{rcl}
\forall t \geq 0, \forall I_t, & E(\tilde{H}_{t+1}^c - \tilde{H}_t^c | I_0, \alpha, v) & > 0 \\
\lim\limits_{t \to +\infty} & E(\tilde{\Lambda}_{t+1} - \tilde{\Lambda}_t | I_0, \alpha, v) & > 0 \\
\lim\limits_{t \to +\infty} & E(|p_{t+1} - v| - |p_t - v| \ \ | I_0, \alpha, v) & < 0
\end{array}$$

which implies:

$$\forall K > 0, \quad \lim_{t \to +\infty} \Pr(\tilde{H}_t^c < K | I_0, \alpha, v) = 0$$
$$\forall K > 0, \quad \lim_{t \to +\infty} \Pr(\tilde{\Lambda}_t < K | I_0, \alpha, v) = 0$$

Thus E_t^c and p_t converge in probability to v. λ_t converges to 1 if $\alpha = \alpha^+$, 0 if $\alpha = \alpha^-$.

See the Appendix A.4.3 for the proof. In this Bayesian environment both supply-informed traders and market-makers eventually learn the truth about the values of v and α. Notice however that while supply-informed traders move closer to the truth in each period on average, this is the case for market-makers only *once supply-informed traders are close enough to the truth*. When supply-informed traders are far from the truth it may be the case that for some periods market-makers get more and more misled about α or about v on average (see section 4). In some sense, the convergence of market-makers' beliefs may take place only *after* the convergence of supply-informed traders' beliefs.

This proposition implies a lemma that will be extremely useful. The difficulty of this model is that transition probabilities between different trades are not constant over time due to supply-informed traders possibly changing their strategy. This problem disappears if $x_S \to 0$, or if at least after a long enough time supply-informed traders' behavior becomes constant, so that $M_{S,t}$ does not depend on t. Then trades follow a simple Markov chain with three states, and as supply-informed traders' updates after a given trade T_t do not depend on t their expected update in each period converges to some stationary limit. For market-makers' beliefs about α, notice that the observation of a given trade T_t may give rise to different updates depending on the values of E_t^+ and E_t^- (see equations 16 and 17). However, by Proposition 3 we know that E_t^c will converge to either 1 or 0, and if incorrect supply-informed traders' behavior is constant their beliefs will also converge to 1 or 0. Then in the limit the update of Λ_t after a given trade T_t will not depend on t, so that again the average update in each period will converge to a stationary limit. This gives the following lemma:

Lemma 1. *If α^j supply-informed traders' behavior is constant in the long run, ie. $\lim_{t \to +\infty} \Pr(\forall t' \geq t, O_{t'}^j = O_t^j \ | I_0, \alpha^j, v) = 1$, or if $x_S \to 0$, then $E(\Delta H_t^j | I_0, \alpha^j, v)$ converges to some limit denoted ΔH_∞^j. If this is true for both types of supply-informed traders, then $E(\Delta \Lambda_t^j | I_0, \alpha^j, v)$ converges to some limit denoted $\Delta \Lambda_\infty^j$. Moreover all limits can be computed in closed form.*

2.6 Discussion and examples

In this subsection I discuss my assumptions on traders' behavior and illustrate the implications for price movements and update of beliefs.

Positive feedback traders are close to "noise traders", but their behavior is completely determined by the previous trade.[7] The easiest way to understand them is behavioral: for some reason these traders believe that that there is momentum in prices, or think that market-makers systematically under-react to observed trades.

Rational traders could also exert positive feedback for many reasons: traders facing margin constraints, or with career concerns (Dasgupta and Prat (2006)), mutual funds committed to particular investment strategies (for instance the "cushion" technique), portfolio insurance may give rise to similar behavior, but not independently of history or of the current price. Strategic traders could also exert positive feedback in order to drive prices further down when distressed traders are selling, before buying at lower prices, as in Brunnermeier and Pedersen (2005), a possibility I excluded by assuming agents are small (the probability to trade again on the market is zero). To keep the model simple, I isolate from possibly more complicated behaviors the positive feedback component and take it as given and see what it implies for the aggregation of information in my setting. I now discuss the kind of price movements and inferences obtained in the model. Consider Fig. 2, which shows the probability tree of a given period.

[Insert Fig. 2 here.]

Assume for instance $\alpha = \alpha^-$, there has just been a sale, so that all traders know feedback traders will sell. Supply-informed traders correctly believe positive feedback to be lower than what market-makers think, thus they infer from the sales a higher probability that the asset is not valuable and want to sell it. Conversely, if $\alpha = \alpha^+$ after a buy order supply-informed traders tend to be more pessimistic than market-makers. Depending on the past history of prices, supply-informed traders may be more optimistic or more pessimistic than market-makers and may thus buy, sell, or stay inactive. This complex behavior captures the uncertainty prevailing on financial markets during critical times and the difficulty to know who just sold or bought, and why.

How the updating differs depending on the information about positive feedback trading can be best understood with an example. I assume there have been 10 consecutive purchases,

[7] I still need uninformed traders of the "noise" type however, otherwise all uncertainty would quickly disappear (after a series of sales for instance). An interesting feature is that even with a low proportion of noise traders, if there are enough positive feedback traders the convergence of prices can still be very low.

and 10 consecutive sales, x_S being zero for simplicity, and there was no trade in period 0.[8] I plot E_t^+ and E_t^- in each period on Figure 3. After the first purchase, both types of supply-informed traders draw the same inference, as none of them expected positive feedback traders would trade anyway. In the nine next periods positive feedback traders are expected to buy. The α^- supply-informed traders believe there are no feedback traders on the market and infer from this that in each period there is a 0.4 probability to observe a purchase if $v = 1$, against 0.3 if $v = 0$, thus they consider a purchase as a quite informative signal and update E^- quickly upwards. α^+ traders think that the second purchase may come from a positive feedback trader, and believe there was a 0.55 probability to observe a purchase against 0.45 for a sale. Thus the signal is less informative: they update E^+ upwards, but less strongly then α^- traders. In period 11 they observe the first sale. As there was a purchase before it cannot come from a positive feedback trader and is a strong signal that asset value may be low (0.225 probability to observe a sale if asset value is high against 0.325 if low). Further sales may come from positive feedback traders and are less informative (0.6 probability with high asset value against 0.7). Notice that these negative signals are even less informative than the positive signals at the beginning because there are more positive feedback sales than purchases ($\sigma_{-1} > \sigma_1$). This is why after 10 purchases and 10 sales α^+ traders have a higher expectation for the asset value than at the beginning, whereas α^- traders who believe there is no positive feedback have the same E^-.

[Insert Fig. 3 here.]

3 Supply-informed traders and liquidity provision

This section is devoted to studying the pattern of trading by the supply-informed, and the impact they have on market liquidity, mainly on trade imbalances and on the bid-ask spread.

3.1 Long-run behavior of supply-informed traders

I will now show results about the long-term behavior of supply-informed traders, in particular necessary conditions for them to remain active in the long run. These necessary conditions depend on the update of "incorrect" traders. There are no "incorrect" supply-informed on the market, but market-makers still need to know what is the expected value of the asset

[8] Other parameters: $3x_N = 0.9, x_I = 0.1, x_S = 0, \alpha^+ = 1, \alpha^- = 0, \sigma_1 = 0.25, \sigma_{-1} = 0.5, E_0^+ = E_0^- = 0.5$.

conditional on both values of α[9]. The difficulty is that the "incorrect" update depends on the behavior of "correct" supply-informed traders. When x_S is small and this problem disappears, the following result holds:

Proposition 4 (Supply-informed traders' behavior)**.** *If x_S is small enough,*

1. *for $\sigma_1 - \sigma_{-1}$ positive enough, $\lim_{t\to+\infty} \Pr(E_t^- > E_t^+ | I_0, \alpha, v) = 1$.*
2. *for $\sigma_1 - \sigma_{-1}$ negative enough, $\lim_{t\to+\infty} \Pr(E_t^+ > E_t^- | I_0, \alpha, v) = 1$.*

Thus, for $|\sigma_1 - \sigma_{-1}|$ large enough, if they are active in the long-run α^- (resp. α^+) traders trade in the same direction (resp. in the opposite direction) as most positive feedback trades.

See the Appendix A.4.4 for the proof of this result. When positive feedback is much more important after sales than after purchases, the main difference between both types of supply-informed traders is that α^+ traders believe most sales are due to positive feedback trading and not to value-informed trading, while α^- traders think these sales are very informative. After enough such observations, the former become more optimistic about the asset's value than the latter. The opposite is true when instead positive feedback affects purchases more than sales. In a market dominated by positive feedback sales (resp. purchases) this implies that α^+ traders would buy (resp. sell) while α^- traders would sell (resp. buy): α^+ traders think the market overreacts and tend to behave as contrarians, while α^- traders think the market underreacts and tend to behave as positive feedback traders.

However, unlike traders who have perfect information about the asset's value, supply-informed traders may be excluded by too high spreads. They may trade for some periods, then be excluded and trade again. The following lemma gives conditions under which their long run behavior is constant:

Lemma 2 (Exclusion and informational advantage)**.** *When $x_S \to 0$, correct supply-informed traders are always trading in the same direction in the long run, or not trading. In the former case (i),(ii) and (iii) below are equivalent, in the latter case (iv), (v) and (vi) are equivalent:*

(i). Supply-informed traders are active in the long run:
$\lim_{t\to+\infty} \Pr(\forall t' \geq t, O_{t'}^c = O_t^c \ | I_0, \alpha^j, v) = 1$ *with* $O_t^c \in \{-1, 1\}$.

(ii). $\Delta\tilde{H}_\infty^c > \Delta\tilde{\Lambda}_\infty + \max(\Delta\tilde{H}_\infty^u, 0)$.

[9]This is a difference with the "differences of opinions" literature, α^+ and α^- traders share common features with the "unresponsive"and "responsive" responsive traders of Harris and Raviv (1993), but are never present at the same time

(iii). $\lim_{t\to+\infty} \Pr(\tilde{H}^c_t > \tilde{\Lambda}_t + \max(\tilde{H}^u_t, 0)|I_0, \alpha, v) = 1.$

(iv). Supply-informed traders are inactive in the long run:
$\lim_{t\to+\infty} \Pr(\forall t' \geq t, O^c_{t'} = 0 \ |I_0, \alpha^j, v) = 1.$

(v). $\Delta\tilde{H}^c_\infty < \Delta\tilde{\Lambda}_\infty + \max(\Delta\tilde{H}^u_\infty, 0).$

(vi). $\lim_{t\to+\infty} \Pr(\tilde{H}^c_t < \tilde{\Lambda}_t + \max(\tilde{H}^u_t, 0)|I_0, \alpha, v) = 1.$

When x_S is too large, then if incorrect supply-informed traders' behavior is constant (i) still implies (ii) and (iii), while (iv) implies (v) and (vi).

See the Appendix A.4.4 for the proof. The proposition is lengthy but the intuition is simple: if supply-informed traders are active forever after a long enough time, then it must be the case that their informational advantage over market-makers increases over time. Conversely, if they are inactive forever, their informational advantage has to decrease. This implies the following corollary:

Corollary 1 (Exclusion of the supply-informed)**.** *For x_S small enough, supply-informed traders are inactive in the long-run if either:*

- $\sigma_1 - \sigma_{-1}$ *is positive enough, and* $(v, \alpha) = (1, \alpha^+)$ *or* $(0, \alpha^-)$.
- $\sigma_{-1} - \sigma_1$ *is positive enough, and* $(v, \alpha) = (1, \alpha^-)$ *or* $(0, \alpha^+)$.

The corollary follows quite directly from Proposition 4 and Lemma 2. Consider for instance the case where there are more feedback sales than purchases ($\sigma_{-1} - \sigma_1 > 0$), and $v = 0$. Many sales will be observed, however α^+ traders will think that some of them are positive feedback and will update more cautiously downwards than α^- traders. Thus after some time, even if $\alpha = \alpha^+$, we will have $E^- < E^+$ (Proposition 4), thus supply-informed traders' expectations are lagging behind market-makers': $\tilde{H}^c_t < \tilde{H}^u_t < \tilde{\Lambda}_t + \max(\tilde{H}^u_t, 0)$, and they will be excluded by too high spreads. Notice that both spreads and supply-informed traders' informational advantage converge to zero, what matters are the relative speeds at which both happen.

When supply-informed traders' behavior is constant in the long-run, trades follow a simple Markov chain with three states and it becomes possible to compute $(\mu_1(v, \alpha), \mu_0(v, \alpha), \mu_{-1}(v, \alpha))$ the stationary proportion of purchases, sales and periods without trades in the long-run conditional on v and α. It is possible in particular to compute a measure of order imbalance, here $|\mu_1(v, \alpha) - \mu_{-1}(v, \alpha)|$. A widely used measure of informed trading on financial markets is the

PIN (Easley *et al.* (1996), Easley *et al.* (2008)), which interprets order imbalance as coming from informed traders. Here the asymmetry between positive feedback sales and purchases can also create a trade imbalance, but the PIN is nonetheless increasing with value-informed trading. Is it also the case for supply-informed trading? To answer this question I do a comparative statics exercise on x_S[10]. Since the total number of traders needs to be constant, I assume that there are $3\hat{x}_N$ noise traders and $\hat{x}_I = 1 - 3\hat{x}_N$ value-informed traders to start with, and a small number $\hat{x}_S$ of additional supply-informed traders. The probability that a value-informed trader is selected to trade is thus $x_I = \hat{x}_I/(1 + \hat{x}_S) \simeq \hat{x}_I(1 - \hat{x}_S)$, and similarly the probabilities that a noise trader or a supply-informed trader are selected are close to $3\hat{x}_N(1 - \hat{x}_S)$ and $\hat{x}_S$. This comparative statics exercise applied to order imbalance yields the next result:

Remark 1. *Assume that x_I is high enough for $\mu_1(1,\alpha) - \mu_{-1}(1,\alpha)$ to be positive and $\mu_1(0,\alpha) - \mu_{-1}(0,\alpha)$ negative. Then supply-informed traders' impact on expected order imbalance is:*

- *positive if x_S is small and $|\sigma_1 - \sigma_{-1}|$ large enough.*
- *negative if one type of supply-informed traders trades in the same direction in the long-run for both realizations of v or is inactive, and this type is sufficiently likely ex ante.*

The first assumption means that there are enough value-informed traders for order imbalance to be a meaningful measure of informed trading in the first place: if x_I is very low and positive feedback trading much more important after sales it can be the case that sales are more likely than purchases even when $v = 1$. The first part follows from Corollary 1: when $|\sigma_1 - \sigma_{-1}|$ is large supply-informed traders trade in the same direction as value-informed traders in the long-run, or do not trade. Hence they strengthen order imbalance. Interestingly, this is the case only if they can be excluded from the market. For instance when $\sigma_{-1} - \sigma_1$ is positive and large, in the long-run we have $E_t^+ > E_t^-$, hence α^+ traders would like to buy, independently of v. When Corollary 1 holds, they will eventually be excluded from trading when $v = 0$ and will thus contribute positively to order imbalance. In the second part $|\sigma_1 - \sigma_{-1}|$ is lower so that the corollary does not hold and supply-informed traders are not excluded in the long-run. Then half of the time α^+ traders buy when the asset's value is high and add to order imbalance, half of the time they buy when the asset's value is low and reduce order imbalance. Their direct impact will be composed by the interaction

[10]Comparative statics results in this type of model have to be interpreted cautiously, since a higher number of informed traders may lead liquidity traders to trade less on this market(Dow (2004)). Strictly speaking, I compare a market with a higher proportion of supply-informed traders with a market with a lower proportion. This is not necessarily the same as studying the effect of "adding" supply-informed traders.

with positive feedback, but this indirect effect does not depend on whether v is high or low (the exact proof can be found in the Appendix A.4.5). Hence an additional supply-informed trader has exactly the same impact on the average order imbalance as an additional noise trader, which is at best null, and negative in this framework since it makes it less likely that a value-informed trader is selected.

3.2 Positive feedback, supply informed traders and the bid-ask spread

As usual in Glosten-Milgrom type models, the spread is a useful measure of adverse selection. A specificity here is that adverse selection is two-dimensional. This can easily be seen in the decomposition of the spread. Dropping time-subscripts, define E_i^+, E_i^-, λ_i as the updated values of E^+, E^-, λ after the observation of a trade in direction i. Using equation 3:

$$A - B = (\lambda_1 E_1^+ + (1-\lambda_1)E_1^-) - (\lambda_{-1}E_{-1}^+ + (1-\lambda_{-1})E_{-1}^-)$$

Assume a situation where $x_I \simeq 0$, such that $E_1^+ \simeq E_{-1}^+ = E^+, E_1^- \simeq E_{-1}^- = E^-$. Then

$$A - B = (\lambda_1 - \lambda_{-1})(E^+ - E^-)$$

if $E^+ > E^-$, market-makers know that if there is a purchase it's a signal that α^+ traders are correct and thus that the asset value is high, not only because value-informed traders can buy, but also because it means that all previous information must be interpreted by giving more weight to the possibility that $\alpha = \alpha^+$.

Remark 2 (Spread persistency)**.** *If in period t $E_t^+ \neq E_t^-$, the spread is different from zero even if $x_I = x_S = 0$.*

Thus we have a non-zero spread even when adverse selection disappears from the market. This shows that in contrast with standard Glosten-Milgrom models the spread does not come from adverse selection only: trades can be informative *per se*, even if they cannot be initiated by informed traders. Two purchases or two sales in a row imply that market-makers should move their expectations toward E^+, even if $x_I = x_S = 0$. But this implies that a purchase can contain some negative signal, and a sale some positive signal. In extreme situations, a sale can even be a more positive signal than a purchase:

Remark 3 (Abnormal price movements)**.** *For x_I and x_S small enough:*

- *If* $E_t^+ > E_t^-$ *and* $T_{t-1} = -1$ *then* $A_t < B_t$ *and* $B_t > B_{t-1}$*: the bid price goes up after a sale.*

- *If* $E_t^- > E_t^+$ *and* $T_{t-1} = 1$ *then* $A_t < B_t$ *and* $A_t < A_{t-1}$*: the ask price goes down after a purchase.*

See the Appendix A.4.6 for the proof. With the information structure of this model a purchase can be a negative signal when it leads market-makers to update downwards their belief that past sales were uninformative. An unappealing feature of the Glosten-Milgrom framework is that it implies that the spread can be sometimes negative. This is not a logical contradiction in the model as traders cannot buy and sell back to market-makers in the same period, the spread only comes from the fact that a purchase is a positive signal about the asset and a sale a negative signal. In other words the "spread" in the model is only the information component of the actual spread, the former can be sometimes negative here. I discuss this result in more details in the next subsection.

That the uncertainty about supply-information has an impact on the spread can also be seen by looking at the impact of a period without a trade on the bid-ask spread. If in period $t-1$ there is no trade, this is a signal that positive feedback is low at least when x_S is small enough. As a consequence λ_t is lower than λ_{t-1} (see equation 15). In Easley and O'Hara (1992) the absence of trade is a signal that no informative event happened on a given trading day and that the risk of adverse selection is low, which reduces the spread, while in Diamond and Verrecchia (1987) the absence of trade may be due to short sale prohibitions faced by a trader with negative information, which increases the spread. Here the effect of observing no trade can go in both directions:

Remark 4. *If* x_S *is small enough, observing no trade in period* $t-1$ *has a positive impact on the spread in period* t *if and only if:*

$$(E_t^+ - E_t^-)\left(\frac{1}{2} - (\lambda_t E_t^+ + (1-\lambda_t)E_t^-)\right) < 0$$

In particular, if in the long-run supply-informed traders disagree, $|E_t^+ - E_t^-| \to 1$*, this condition is equivalent to* $\lambda_t > 1/2$*.*

See the Appendix A.4.6 for the proof. Observing no trade makes market-makers update their beliefs closer to those of α^- traders (λ_t decreases), the effect on the spread depends on whether α^- traders hold higher expectations than α^+ traders or not, and on what market-makers already know about the amount of positive feedback. In the extreme case where

$E_t^+ \to 1$ and $E_t^- \to 0$, as typically happens when $\sigma_{-1} - \sigma_1$ is positive and large (see Proposition 4), getting the supply information is enough to know the true value of the asset. In a sense, market-makers are only uncertain about supply information. Observing no trade decreases λ, if $\lambda > 1/2$ this means that market-makers are more uncertain about supply-information, and thus the spread increases. Conversely, if λ is already low, observing no trade strengthens the market-makers' belief that positive feedback is low, and uncertainty decreases. Finally, remember that the market price p_t is equal to $\lambda_{t+1} E_{t+1}^+ + (1 - \lambda_{t+1}) E_{t+1}^-$ (equation 3), knowing v and the sign of $E_t^+ - E_t^-$ in the long-run is thus enough to predict the impact of no trade on the spread.

Finally, although the spread is positively affected by uncertainty about positive feedback, more supply-informed traders do not reduce the spread:

Remark 5. *Holding E_t^+, E_t^-, λ_t constant, an increase in x_S increases $A_t - B_t$.*

See the Appendix A.4.6 for the proof. The supply-informed traders' impact on the spread is thus similar to the value-informed traders': in a given period they are a source of adverse selection since traders of type α^j trade at the ask only if $E_t^j > A_t$ and at the bid only if $E_t^j < B_t$.

3.3 Discussion and examples

Supply-informed traders' behavior:

Proposition 4 characterizes how the two different types of supply-informed traders behave in the long-run [11]. The intuition is that α^+ type traders think the market is overreacting, while α^- traders think it is under-reacting. If there are more sales than purchases due to positive feedback, α^+ traders will after some time be more optimistic than α^- traders. When the market is dominated by sales α^+ traders thus always try to buy and α^- traders to sell, and the opposite if the market is dominated by purchases. Traders with only supply-information naturally tend to follow contrarian[12] strategies when they know the market is more noisy than expected, and positive-feedback strategies when they know it is less noisy. When the market underreacts they behave as the momentum traders in Hong and Stein (1999), when it overreacts they exert reversal.

For how long do supply-informed traders have a contrarian or positive feedback impact on the market depends on how long they are able to trade. Lemma 2 gives necessary conditions

[11] Under the sets of parameters I use in the next section, traders actually adopt this behavior from the beginning most of the time

[12] Contrarian behavior here is opposed to positive feedback, not to herding as in Park and Sabourian (2011).

for supply-informed traders to trade in the long-run: if they are able to trade it must be the case that supply-informed traders' informational advantage compared to market-makers grows over time, if they never trade this informational advantage must decrease over time.

Remember that the spread compensates market-makers for two adverse selection problems: the risks to trade with a value-informed trader or with a supply-informed trader. If the spread required to compensate the first risk is enough to exclude supply-informed traders, then they won't trade. In other words, if supply-informed traders' private information becomes not relevant enough compared to value-informed traders' private information, a low spread will be enough to exclude the former from trading. Thus supply-informed traders' activity in the long-run depends on a comparison between the speeds at which market-makers learn about v and about α. Remember from equation 3 that the quoted price is a weighted average between correct and incorrect traders' expectations. If incorrect traders do not converge to the true price, then in the end the price will be extremely close to the expectations of correct supply-informed traders. In order to be able to trade, supply-informed traders need to keep some informational advantage in the long-run: $\tilde{H_t}^c > \tilde{\Lambda}_t$ means that they update their own beliefs more quickly than market-makers update their beliefs about positive-feedback trading. Now if incorrect supply-informed traders also converge to the truth, market-makers' expectations will also be pushed towards the truth by incorrect traders' expectations ; in order to keep an informational advantage, supply-informed traders must update their beliefs more quickly than market-makers and incorrect supply-informed "combined", $\tilde{H_t}^c > \tilde{\Lambda}_t + \tilde{H_t}^u$. Notice that this will never be the case if incorrect traders update their beliefs more quickly than correct ones ($\tilde{H_t}^u > \tilde{H_t}^c$).

Thus, in some situations, market participants with supply-information will be actively trading only at the beginning of the trading-period, and eventually become inactive. They may help to correct public beliefs when these are "very wrong", but not have this role in the long-run.

Interestingly, Remark 1 shows that, when supply-informed traders are always active, a measure such as the PIN fails to identify them, because their trading direction in the long-run is independent of v. These traders still make profit however: market-makers' quotes are such that supply-informed traders make less losses when they buy and $v = 0$ than they make profit when they buy and $v = 1$, an information that order imbalance does not incorporate. In extreme situations supply-informed traders behave almost like value-informed traders in the long-run and are identified by measures based on order imbalance, while in less extreme situations their behavior can be uncorrelated to v and actually reduce order imbalance. This is

particularly interesting if some high-frequency trading strategies are akin to supply-informed trading: while such strategies do incorporate private information, they may be completely missed by the PIN measure. Remark 1 gives us the following testable hypothesis:

Testable hypothesis 1. *All else equal, an increase in the number of high-frequency traders on a given market should increase the PIN measure more when positive feedback is more asymmetric between purchases and sales.*

Supply information and the spread: The spread in this model does not come only from adverse selection, but also from the fact that the last trade gives information about how all past trades should be interpreted. For this second source of information, a purchase can be a positive or a negative signal about the asset's value. The latter case is not as counter-intuitive as it may seem. Consider for instance the "flash crash" of the Dow Jones on May 6, 2010. If the Dow Jones loses 5% in two minutes, it is a bad signal about the long-term value of this index. But if it loses 4 additional points in the next three minutes and there is still no news that could justify such a large drop, it is also a strong signal that the market does not function properly. It may imply that the first drop of 5% was probably not entirely driven by information either. Thus the latter sales can actually be interpreted as a positive signal, and probably this is part of the reason why prices bounced back so rapidly. Hence the possibility that prices go up after sales can make sense. The problem is that in a Glosten-Milgrom framework this implies a negative spread, which should be interpreted as a negative information component of the market spread. Figure 4 shows an example with 5 sales in a row, then 5 purchases, and the price always goes up.

[Insert Fig. 4 here.]

In less extreme situations however the spread remains positive in the model, but can be reduced by this phenomenon. A negative spread occurs only with extreme differences of E^+ and E^- and beliefs of market-makers about α far away from the truth, which happens with a very small probability (if $E_1^+ = E_1^-$, if E_t^+ and E_t^- diverge over time it is very likely that λ_t converges quickly to the truth), or with a very small x_I, in which case a Glosten-Milgrom framework makes less sense. Among the several thousands of simulations used in the next section, the spread has not been negative once. Hence under reasonable parameters, the message to draw from Remark 3 is the following: when the market is very noisy and market-makers still do not know whether past trades were informative or not, the spread can be high even if adverse selection is low. Conversely, if given the past pattern of trades a sale would be a good signal, the spread can be low but adverse selection high.

Remark 4 shows that uncertainty about non-fundamental information increases the spread, which implies in particular that the observation of no trade can increase or decrease the spread depending on market-makers' beliefs about positive feedback. This does not imply however that adding supply-informed traders reduces the spread, as shows Remark 5: in a given period, more supply-informed traders may give more information about positive feedback, but they are a source of adverse selection. Of course they may also speed up the convergence of prices, such that their dynamic impact on the spread is negative, as is the case for value-informed traders. But this is not always the case, as shown in the next section.

4 Supply-informed traders and market stability

In this section I analyze further the role of supply-informed traders by looking at the impact of increasing their number on the price-discovery process and the likelihood of crashes.

4.1 Supply-informed traders and short-run crashes/bubbles

Proposition 3 shows that in the long-run private information about the true value of the asset and about positive feedback trading will be fully disclosed. I will show however that, as market-makers have to update two beliefs at the same time, they may be expected to get more and more misled about one of the two parameters for some periods. This is an important step toward understanding the consequences of the supply-informed investors' informational advantage.

Lemma 3. *For x_S small enough, $E(\tilde{H}^u_{t+1} - \tilde{H}^u_t | I_0, \alpha, v) < 0$ if either:*

- *α^- is small enough, and $(v, \alpha, T_t) = (1, \alpha^+, -1)$ or $(0, \alpha^+, 1)$.*
- *$\alpha^+\sigma_{-1}$ and $\alpha^+\sigma_1$ are high enough, and $(v, \alpha, T_t) = (1, \alpha^-, 1)$ or $(0, \alpha^-, -1)$.*

For x_S small enough, $E(\tilde{\Lambda}_{t+1} - \tilde{\Lambda}_t | I_0, \alpha, v) < 0$ if (E^c, E^u) is close enough to $(1-v, v)$ and α^+ is close enough to α^-.

See the Appendix A.4.7 for the complete proof. This remark means that for some parameters "incorrect" traders' beliefs about v and market-makers' beliefs about α can diverge from the truth *on average* (conditionally on v and α) for some periods, even if market-makers will converge to the truth in the end. Notice that these are sufficient conditions, not necessary ones. The underlying idea is to put the agents in the worst possible situation to learn about the parameter and find sufficient conditions for them to diverge. For "incorrect"

supply-informed traders of type α^- this is done by assuming they expect very little positive feedback while positive feedback is actually widespread. Trades due to positive feedback are misinterpreted as informative trades, and the "incorrect" expectation goes up on average if there was a purchase the period before, even if $v = 0$. For incorrect supply-informed traders of type α^+, I assume another kind of extreme scenario where if $\alpha = \alpha^+$ then almost all uninformed traders are positive feedback. Then observing a sale after a purchase is almost a sure signal that $v = 0$ for these "incorrect" traders, and their expectations can diverge from the truth when v is actually high.

The case of market-makers is more interesting. They infer information about α from two sources: if the observed trade can be a positive feedback trade, this is a signal that $\alpha = \alpha^+$; but if the past history implies for instance that $E_t^+ > E_t^-$, seeing a purchase is more likely if $\alpha = \alpha^+$ because then value-informed traders are more likely to buy. The sufficient condition simply assumes a case where the first component of the signal is absent, and the second one is misleading because supply-informed traders are still far from the truth. When at least correct supply-informed traders are closer to the true value of the asset, market-makers will be expected to converge in every period by Proposition 3.

Fig. 5 gives an example of a market with many informed traders who sell because the asset's value is low[13]. α^+ supply-informed traders are far away from the truth and in the beginning market-makers are almost certain that $\alpha = \alpha^+$. Then they expect to see many purchases because as the α^+ traders they believe that the asset's value is high. Yet they observe many sales on average and, as there is not much positive feedback trading, it becomes more and more likely that these sales come from informed traders who know the asset's value is low, but this is unlikely if indeed $\alpha = \alpha^+$. Thus these sales are interpreted as signals that $\alpha = \alpha^-$ and λ goes *down* on average.

[Insert Fig. 5 here.]

Finally, due to the first part of Lemma 3, I find in this model an anomaly already present in the "rational crashes" literature, for instance in Jacklin, Kleidon, and Pfleiderer (1992): conditional on the asset's value and the amount of positive feedback trading being high, prices can *on average* diverge from the asset's value for a finite number of periods.

Proposition 5 (Divergence from fundamental value)**.** *Assume* $E_1^+ = E_1^-$, $\alpha = \alpha^+$, $3\alpha^+ x_N$ *large,* λ_1 *and* α^- *low,* $|\sigma_1 - \sigma_{-1}|$ *large enough:*

[13] I consider the following parameters: $v = 0, 3\alpha^+ x_N = 0.05, 3\alpha^- x_N = 0.005, x_N = 0.2, x_I = 0.2, \sigma_1 = 0, \sigma_{-1} = 1, E_1^+ = 0.9, E_1^- = 0.05, \lambda_1 = 0.95, \alpha = \alpha^+$.

- *If $v = 1$ and $\sigma_1 < \sigma_{-1}$, then $E(p_t|I_0, \alpha, v)$ is decreasing in t for $t \in [1, t_1], t_1 > 1$.*
- *If $v = 0$ and $\sigma_1 > \sigma_{-1}$, then $E(p_t|I_0, \alpha, v)$ is increasing in t for $t \in [1, t_1], t_1 > 1$.*

In both cases more supply-informed traders reduce the expected divergence between p_t and v.

As we know that in the end prices will converge to the true asset's value, this proposition means that for some parameters expected prices will exhibit reversal. The condition $E_1^+ = E_1^-$ is not necessary (actually the proposition would be trivial for extreme differences of E_1^+ and E_1^-) but makes the exposition clearer. To prove the first part notice that when λ_1 is low enough market-makers' beliefs can be arbitrarily close to E_t^- for a finite number of periods, and moreover for a high enough $3\alpha^+ x_N$ there will be almost only sales in the market if $\sigma_{-1} - \sigma_1$ is positive and large. Since according to Lemma 3 E_t^- will diverge in expectation from the true value after each sale, prices will diverge as well as long as λ remains low enough. The reasoning is symmetric for the second case. In both situations, with a low λ the α^- traders will be excluded from trade by high spreads (see equations 11 and 13). α^+ traders will buy in the case $\sigma_1 < \sigma_{-1}$ and sell if $\sigma_1 > \sigma_{-1}$. Thus, increasing x_S makes a purchase more likely to occur when $v = 1$, and a sale more likely when $v = 0$, without affecting α^- traders' inferences (since they wrongly assume supply-informed do not trade), as a result E_t^- diverges less from v in expectation.

Fig. 6 illustrates Proposition 5: incorrect α^- traders get more and more misled by positive feedback sales and update their expectations downwards on average, while correct α^+ traders update them upwards. If at the beginning market-makers give more weight to the uncorrect traders (low prior probability λ_1) the average prices will follow incorrect expectations at and go down at first, then over time market-makers will learn about α and go back to the correct traders' expectations.

[Insert Fig. 6 here.]

4.2 Supply-informed traders and long-run convergence

I now study the impact of an increase in x_S in the long-run. Firstly, when conditions of Lemma 1 are met, the speed of convergence has a simple equivalent:

Proposition 6. *If both types of supply-informed traders' behavior is constant in the long run or if $x_S \to 0$, then $E(|p_t - v| \ |I_0, \alpha, v)$ is equivalent when $t \to \infty$ to a constant times $e^{-t(\Delta\tilde{\Lambda}_\infty + \max(0, \Delta\tilde{H}^u_\infty))}$ (resp. $e^{-t\Delta\tilde{H}^c_\infty}$) if correct supply-informed traders are active (resp. inactive).*

See the Appendix A.4.8 for the proof. This proposition shows that there are two very different scenarios depending on the model's parameters: if uncertainty about positive feedback is mild, then by Lemma 2 supply-informed traders will be inactive in the long-run, which means that λ_t will converge faster than E_t^c. Thus the price will be equivalent to E_t^c in the long run, and the speed of convergence will be the same as the speed of convergence of E_t^c, as in a standard Glosten-Milgrom framework. Conversely, when uncertainty about positive feedback is higher, supply-informed traders are active in the long run and E_t^c converges faster than λ_t. In the case $v = 1, \alpha = \alpha^+$ for instance prices become equivalent to $\lambda_t + (1-\lambda_t)E_t^-$ in the long run, and hence the speed of convergence is determined by market-makers' updating about positive feedback.

I now focus on the most interesting case in which supply-informed traders are trading against a short-run crash as shown in Proposition 5, and thus have a positive short-run effect on the market. The next corollary shows that this trading behavior can have a negative impact in the long-run:

Corollary 2. *If $v = 1, \alpha = \alpha^+$, x_S is small and $\sigma_{-1} - \sigma_1$ is large enough for supply-informed traders to be active in the long-run, then adding supply-informed traders has a negative long-run impact on $E(\tilde{H}_{t+1}^c - \tilde{H}_t^c)$ if $T_t = -1$, and also on $E(\tilde{\Lambda}_{t+1} - \tilde{\Lambda}_t)$ if x_I is small enough. When $v = 1, \alpha = \alpha^+$, and x_S and x_I are small enough, if E_t^- converges to 0 adding supply-informed traders slows down price discovery after each sale in the long run.*

See the Appendix A.4.8 for the proof. The most interesting effect is on $\tilde{\Lambda}_t$: if there was a sale in the previous period, $\alpha^+ - \alpha^-$ and σ_{-1} are large and x_I is small, adding people trading on supply-information makes it more difficult for market-makers to learn this type of information. With such parameters, after a sale there is an excellent opportunity to acquire information about α. If $\alpha = \alpha^+$ there are strong chances to observe a further sale, since positive feedback is high, whereas if $\alpha = \alpha^-$ chances are much lower since positive feedback is weak[14]. Now if x_S increases, if $\alpha = \alpha^+$ supply-informed traders buy, while if $\alpha = \alpha^-$ they sell. Thus the likelihood ratios $\Pr(T = -1|\alpha = \alpha^+)/\Pr(T = -1|\alpha = \alpha^-)$ and $\Pr(T = 1|\alpha = \alpha^-)/\Pr(T = 1|\alpha = \alpha^+)$ decrease: additional supply-informed traders jam the signal due to their contrarian behavior.

This can be compared to what Smith and Sorensen (2000) call "confounded learning": when there are many positive feedback traders and they sell more than they buy, supply-informed traders buy because they know that market-makers over-react to sales, but by doing

[14]This is where the assumption that x_I is small is needed, otherwise a sale could be a signal that $\alpha = \alpha^-$, since value-informed traders are more likely to sell according to α^- traders.

so they make it more difficult for market-makers to infer information about positive feedback trading. In a more general model, it may be possible that with positive probability the market reaches a state where market-makers cannot infer anything about positive feedback trading from observed transactions.

By Proposition 6, if E_t^- converges to 0 then $p_t - v$ becomes proportional to $e^{-t\Delta\tilde{\Lambda}_\infty}$ in the long run and the negative impact on the update of λ_t leads to a slower price discovery. Supply-informed traders are jamming the update of precisely the parameter that remains the most uncertain in the long run. Finally, the proposition shows this effect only after a sale, the opposite effect may happen after purchases. If there are enough sales on the market however, that is if $3x_N\alpha^+$ and $\sigma_1 - \sigma_{-1}$ are high enough, the former effect will dominate. The next subsection will illustrate this point using simulations.

4.3 Simulations: "normal" and "turbulent" times, and supply-informed traders' profit

That a higher x_S may lead to a lower average price conditional on $\alpha = \alpha^+, v = 1, T_{t-1} = -1$ does not mean that more supply-informed traders slow down price discovery unconditionally on a specific realization of (v, α). To go further, I simulate the model to estimate the unconditional expectation of $|p_t - v|$ as a function of x_S. I will use two main sets of parameters.

Parameters for "normal" and "turbulent" times: the "Baseline" parameters are meant to illustrate a balanced case where the realizations of α and v happen with equal probability, such that agents never start with priors far from the truth: $\lambda_1 = 0.5$, $E_0^+ = E_0^- = 0.5$. The proportions of different types of traders are chosen to be "reasonable" and also not too different from their counterparts in Gennotte and Leland (1990) and Jacklin, Kleidon, and Pfleiderer (1992): I use $x_I = 5\%$, which is enough to observe some convergence without simulating thousands of periods (reason for which Jacklin, Kleidon, and Pfleiderer (1992) use 20%), and at the same time not too far from the upper bound for the proportion of funds owned by value-informed investors, around 2% according to Gennotte and Leland (1990).

In the baseline case I set $x_S = 5\%$, more than in Gennotte and Leland (1990) since I assume a less extreme form of supply-information[15]. Thus we have 90% of uninformed traders, a certain proportion of which are positive feedback traders. The proportion of the market covered by insurance portfolio before the 1987 crash was between 3% and 5%, however other more informal strategies like stop-loss strategies can have similar positive feedback effects. It

[15] In Gennotte and Leland (1990) supply-informed traders know the exact amount of the supply.

is quite hard to estimate the importance of such strategies, Gennotte and Leland (1990) use first a percentage of 5% of "hedgers" when they consider only insurance portfolio, and then 15% to include more informal strategies. In the baseline case I will use $\alpha^+ = 0.15$, $\frac{\alpha^-}{\alpha^+} = 10\%$, $\sigma_1 = 0.8, \sigma_{-1} = 1$. Thus the proportion of positive feedback traders will be 13.5% after a sale and 10.8% after a purchase. It can be interpreted as modeling hedging strategies when markets are not completely liquid: not all hedgers are able to sell when they want to, thus they won't all buy back when prices go up again.

The second important set of parameters is the "Turbulence" scenario. The parameters are of the same orders of magnitude as in the previous case, but there are much more positive feedback sales than purchases, more value-informed traders, and a low value of λ_1 (0.05, to compare with the 0.9999 probability that no informational event occurs in Avery and Zemsky (1998)). This set of parameters may represent a market in troubled times, with many constrained sales, the number of which is underestimated at the beginning. Finally I use other sets of parameters, mainly consisting in taking one of the two first sets of parameters and changing only one parameter to study its impact. All sets of parameters are summed up in the Appendix A.3.

Supply-informed traders and prices: Figure 7 shows the average $|p_{100} - v|$ in both scenarios as a function of x_S, with confidence intervals at the 95% confidence level. Under these parameters supply-informed traders slow down long-term price discovery unconditionally on v and α, even with the "Turbulence" parameters. The same picture emerges when considering convergence after a smaller number of periods.

I then look at the average of $|p_t - v|$ over time, but conditionally on $v = 1, \alpha = \alpha^+$ with the "Turbulence" parameters, see Figure 8. Prices move away from fundamental value on average in the ten first periods, which means that many "rational crashes" are happening in the simulations. Average prices first go downwards as expected from Proposition 5, then bounce back and converge to v. This anomaly however is less and less pronounced when x_S increases, as expected from Proposition 5: supply-informed traders act as support-buyers and help avoiding crashes. But we also see that even conditionally on $v = 1, \alpha = \alpha^+$ more supply-informed traders imply less price-discovery in the long-run, as expected from Corollary 2: supply-informed traders always jam the updating process of correct supply-informed. Since in the long-run market-makers almost have the same beliefs as the correct supply-informed, the effect of x_S on long-run price discovery is negative.

[Insert Fig. 7 and 8 here.]

Supply-informed traders' profit Finally, I study the profit supply-informed traders make on their activity. Given a history $\{A_t, B_t, T_t, O_t^+, O_t^-, O_t^f, \lambda_t, E_t^+, E_t^-\}_{t=1...100}$ I measure supply-informed traders' ex-post profits by:

$$\Pi = \sum_{t=1}^{100} \mathbf{1}_{O_t^c=1}(v - A_t) + \mathbf{1}_{O_t^c=-1}(B_t - v) \tag{5}$$

I can compute the average of this measure in the baseline scenario, both unconditional and conditional on specific realizations of (v, α), and for several values of x_S. Figures 9 and 10 show my estimates, as well as the 95% confidence level interval for the estimate of unconditional expected profit. Under the baseline parameters supply-informed traders' profit is not statistically different from zero (we know the theoretical expected profit is positive). Under these parameters, supply-informed traders' informational advantage over market-makers is too small: most of the time they just don't trade. When they do, it means that market-makers have been misled about the true value of α and hold expectations far enough from E_t^c for supply-informed traders to be able to trade. But since their information is not much better than market-makers', there is a high probability that supply-informed traders try to buy an asset with low value, or to sell an asset with high value. This can be seen in the averages conditional on (v, α).

Things become more interesting in the turbulence scenario, where now supply-informed traders have a more important informational advantage. Average profits are still low: again if trades are such that market-makers learn quickly about α, supply-informed traders won't be able to trade on their private information. More interestingly, their profit has a high variance compared to the mean, and is very skewed: supply-informed traders earn a lot when $\alpha = \alpha^+, v = 1$, that is in the case where they prevent the formation of a crash, but this happens with a low probability. Moreover, sometimes they buy because they think the market is overreacting to sales, whereas the asset's value is actually low, then they make important losses. When $\alpha = \alpha^-$ their informational advantage is small and again supply-informed traders are most of the time excluded by too high spreads. Note that the signs of average profit estimates conditional on (v, α) are consistent with Proposition 4. Finally, Figure 11 shows the quantile function of realized profits in the turbulence scenario: most of the time supply-informed traders make a profit close to zero, but the first and last deciles show important losses and profits, respectively.

[Insert Fig. 9, 10 and 11 here.]

Numerical result 1 (Supply-informed traders' profit)**.** *Based on the simulations, under the baseline parameters supply-informed traders are mostly inactive and their profit is thus small. Otherwise, under the turbulence parameters:*
-supply-informed traders make a higher profit on average, but with a high variance.
-they are active for a long time only when positive feedback is important, otherwise they almost always remain inactive.
-the profit distribution is both skewed and fat-tailed: they make a high profit 2.5% *of the time, moderate losses* 2.5% *of the time, and close to zero profit otherwise.*

These simulations give us a clearer picture of what traders with supply-information are doing: they are most of the time waiting for the market to be misled enough about the amount of positive feedback trading, in which case they will have a short-lived profit opportunity. Once in a while it happens that positive feedback trading is important and widely underestimated by the market, then they know the market underestimates the value of the asset and buy for long periods, generating high profits if the asset's value was indeed high, and losses otherwise. This pattern implies a risky activity, that can generate high losses but also very high profits.

Assume now that supply-information can be acquired at some cost c. A marginal agent considering acquiring supply-information and becoming a supply-informed trader would expect to get a profit proportional to the expectation of equation 5. Numerical results show that this expectation is decreasing in x_S, thus there is an equilibrium number of supply-informed traders such that marginal profit is equal to c. Identifying supply-informed traders to some HFT strategies again gives the following testable hypotheses:

Testable hypothesis 2. *All else equal, a higher HFT activity should be caused by: an increase in noise trading, more uncertainty about whether the market is overreacting or underreacting to observed trades, a higher asymmetry between positive feedback after sales and purchases.*

Indeed, when x_I increases information is revealed faster, supply-information is outdated more quickly and supply-informed traders are sooner excluded from trading. When λ_1 is closer to $1/2$ and $\alpha^+ - \alpha^-$ is high, uncertainty about supply is higher and supply information is more valuable. Finally, a higher asymmetry between sales and purchases implies higher mispricings and thus profits for supply-informed traders.

4.4 Discussion

The trading behavior and the profit of supply-informed traders in this model have various empirical counterparts depending on how one interprets "supply-information". Most of the time (ie. when $\alpha = \alpha^-$) these traders follow momentum strategies, which give a positive return because conditionally on $\alpha = \alpha^-$ prices under-react to observed trades. In the model, a trader without private information who would follow a naive momentum strategy would not make any profit, as he would be exposed to the "momentum crashes" evidenced by Daniel and Moskowitz (2011): with a small probability a period of market stress arises ($\alpha = \alpha^+$) and a momentum strategy brings high losses on average.

When $\alpha = \alpha^+$, supply-informed traders are supplying liquidity by trading in the direction opposite to price pressure. Coval and Stafford (2007) show that investors short selling stocks likely to be affected by flow-induced selling by mutual funds and buying ahead of forced purchases earn an average abnormal return over 10%, which is an example of supply-informed trading. For troubled periods, Cella, Ellul, and Giannetti (2011) show for a number of crisis episodes[16] that stocks mainly held by institutions with a short trading horizon experience larger price drops than the others, and then experience larger reversals. A long/short equity strategy consisting in buying stocks largely held by short horizon institutions and short selling similar stocks with more long term investors is a typical example of what is defined in this paper as supply-informed trading.

Numerical result 1 illustrates clearly the risk of a trading strategy based on catching falling knives. In practice this risk is magnified by the possibility of a very slow reversal: the supply-informed trader can report a trading loss for a very long time if too many noise trades go in the wrong direction ("noise trader risk" as in De Long *et al.* (1990a), an effect amplified here by positive feedback), or if other supply-informed traders are slow to arbitrage the mispricing ("synchronization risk"in Abreu and Brunnermeier (2002)). Supply-informed traders could thus trade less than they do in this model, or become positive feedback traders themselves if they hit funding constraints, and papers such as Easley, Lopez de Prado, and O'Hara (2011) suggest that natural liquidity providers actually became liquidity consumers during the flash crash of 2010.

More generally, it would be interesting for future research to see how allowing supply-informed traders to time their trade would interact with two-dimensional adverse selection and the spread. Selling strategically to depress prices further before buying as in Brunner-

[16]Mainly the aftermath of Lehman Brothers' bankruptcy in 2008, but also October 1987, the Russian default of 1998, the quant event of 2007, the bailout of Bear Sterns in 2008.

meier and Pedersen (2005) requires important market power from the trader, while this paper implicitly assumes competitive traders. Selling at the beginning of the period and trying to buy at the trough is another possible strategy, but it should be heavily discouraged by the spread, and indeed transaction costs increase a lot during stress periods. There is promising research on the role of "timing" in Glosten-Milgrom type models (see Malinova and Park (2009)), but a tractable framework that could be applied in complex settings as the one I use here is still lacking. This section shows that, even in the optimistic case where supply-informed traders always buy stocks they see as undervalued and sell stocks they consider to be overvalued, their positive short-run impact also implies a negative impact on long term price discovery.

5 Conclusion: is there enough supply information?

This paper offers a simple variation on the Glosten-Milgrom model allowing for the presence of positive feedback trading in a stylized but also flexible way, and of traders with private information about the extent to which such positive feedback trading takes place. These supply-informed traders could be identified to various funds (for instance in the long/short equity category) and market participants more informed than average traders, typically about the financial situation of big players, who could exert price pressure on the market if financially distressed. At a different frequency, some high-frequency trading strategies rely on a lot of data about order flow and thus partly on information about liquidity, not fundamentals.

Interesting anomalies arise in such a framework because of two-dimensional asymmetric information. Besides "rational crashes", these anomalies include prices going up after a sale or the spread remaining positive even when value-informed traders don't trade any more. The main contribution however is to study how supply-informed traders can affect market stability in uncertain times.

I show that the role of agents with supply-information is ambiguous: when uncertainty about positive feedback is important they slow down price discovery in the long-run, jam the market-makers' update on the amount of positive feedback trading, and add a second dimension to adverse selection, which widens the spread. Their only, but possibly crucial, clear positive effect on the market is that they help prevent large deviations due to underestimation of positive feedback trading. Such agents can be seen as providing the market with an insurance against crashes (or bubbles) caused by misinterpretation of past trades, the premium to pay being larger spreads and slower long-run price-discovery.

Interestingly, in markets where positive feedback is such that non-fundamental crashes can be expected to happen, profits from supply-informed trading can be large, but they also have a high variance, the risks associated with being wrong being important ("catching a falling knife"), and are positively correlated with the asset's value. The short term volatility implied by unanticipated fire sales is thus attenuated by supply informed traders. Not eliminated however, as the number of market participants acquiring such information should remain modest, due to the risks associated with supply informed trading.

A Appendix

A.1 Notations

v	value of the asset, equals 0 or 1
x_N	$3x_N$ is the number of uninformed traders
x_I	number of value-informed traders
x_S	number of supply-informed traders
α	$3\alpha\sigma_T x_N$ is the true number of positive feedback traders
α^j,	$j \in \{+, -\}$, two possible values of α
c, u	are superscripts denoting correct and incorrect supply-informed traders
σ_T,	$3\alpha\sigma_T x_N$ is the true number of positive feedback traders
T_t	direction of the trade in period t
O_t^j	with $j \in \{+, -, c, u, f\}$, direction of trade of α^+, α^-, correct, incorrect s.i. and feedback traders resp.
A_t	ask price in period t
B_t	bid price in period t
p_t	price of the transaction in period t
E_t^j	with $j \in \{+, -, c, u\}$ denotes the expected value of the asset for type j supply-informed traders
H_t^j	with $j \in \{+, -, c, u\}$ is equal to $\ln\left(\frac{E_t^j}{1-E_t^j}\right)$
$\tilde{H}_t^j$	is equal to H_t^j if $v = 1$, $-H_t^j$ otherwise
λ_t	probability market-makers assign to $\alpha = \alpha^+$
Λ_t	is equal to $\ln\left(\frac{\lambda_t}{1-\lambda_t}\right)$
$\tilde{\Lambda}_t$	is equal to Λ_t if $\alpha = \alpha^+$, $-\Lambda_t$ otherwise.

A.2 Figures

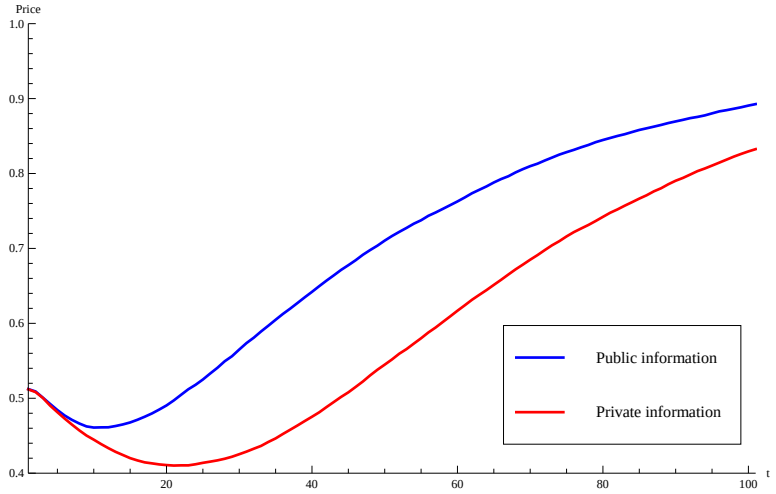


Figure 1: Evolution of prices depending on whether supply information is private or public.

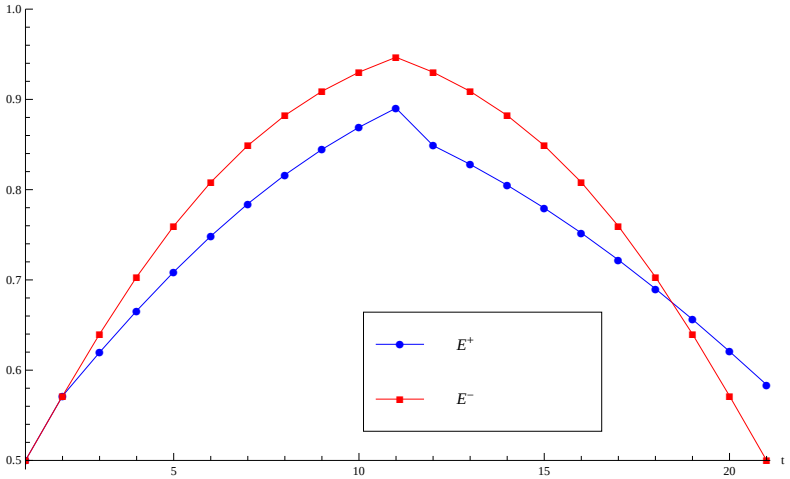


Figure 3: Supply informed traders' expectations. 10 purchases, 10 sales.

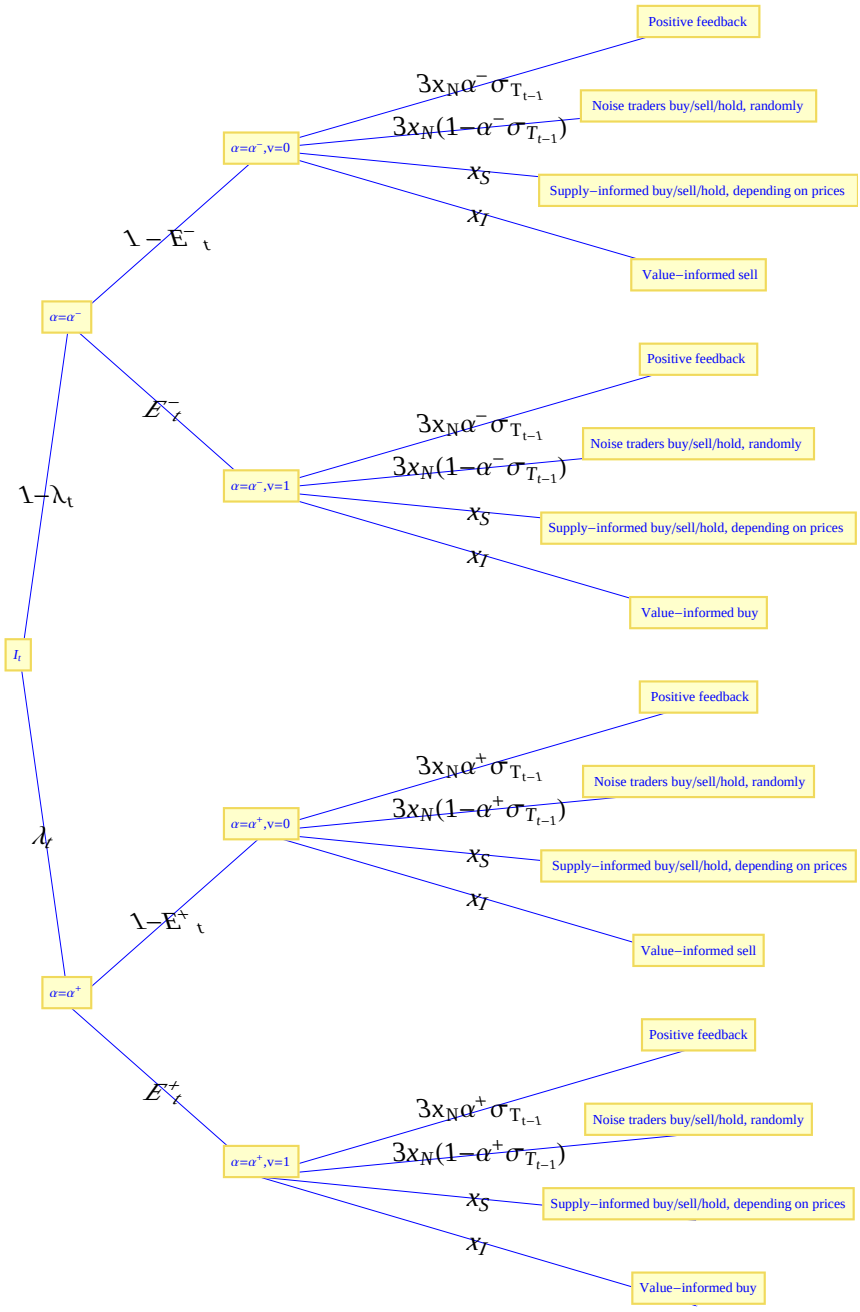


Figure 2: Probability tree in period t.

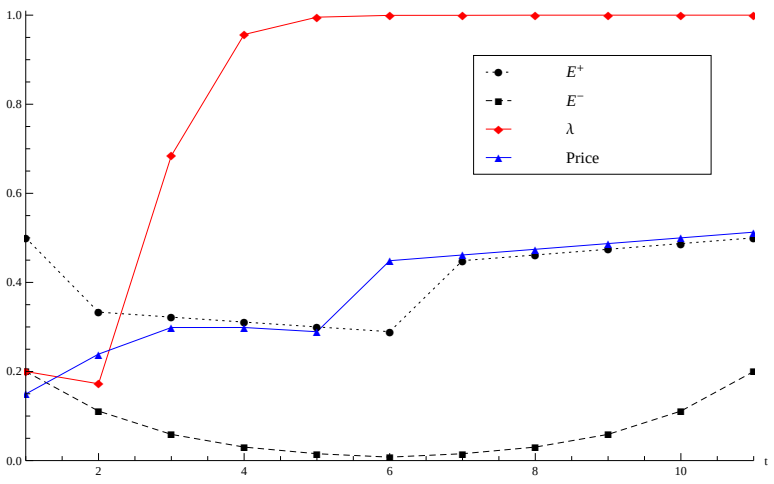


Figure 4: Expectations and prices. 5 sales, 5 purchases.

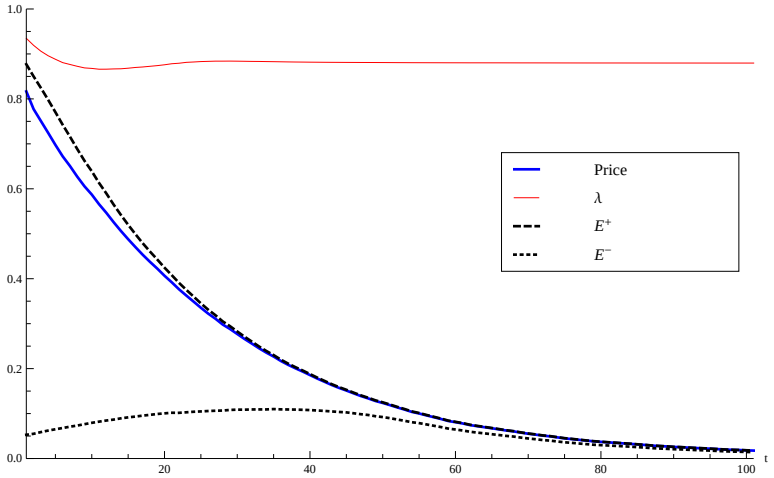


Figure 5: The market learning about v, and then about λ.

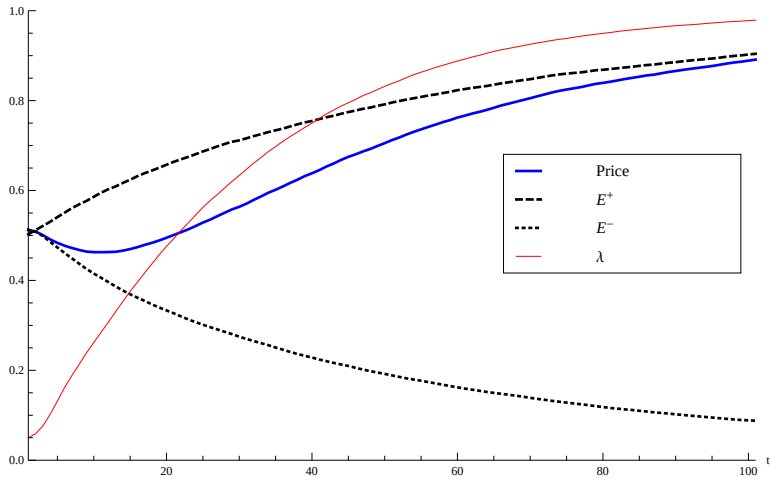


Figure 6: Beliefs and prices in the "Turbulence" scenario.

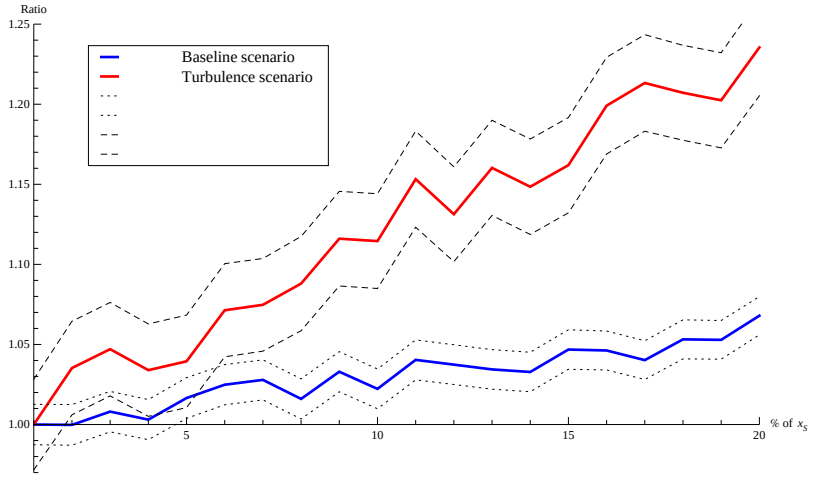


Figure 7: Average $|v - p|$ in period 100, as a function of x_S, normalized by the value for $x_S = 0$.

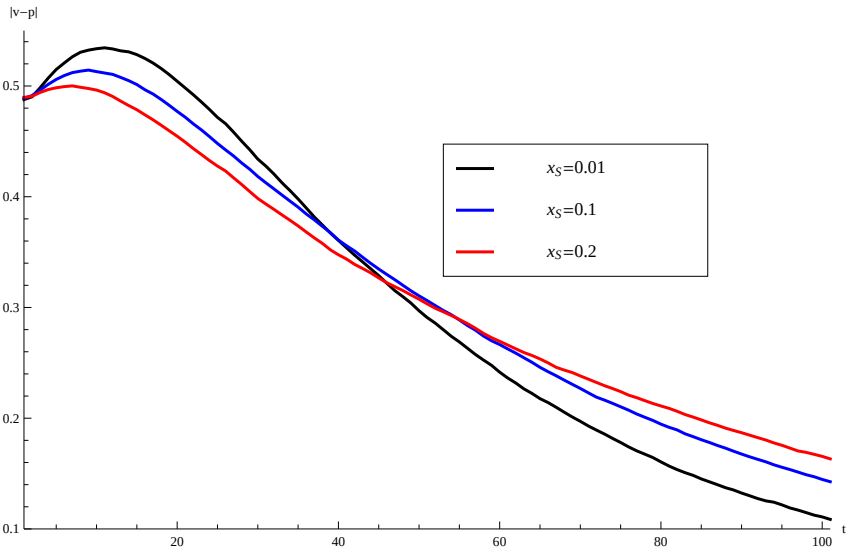


Figure 8: Average difference between price and fundamental value for different x_S, as a function of time. Conditional on $v = 1, \alpha = \alpha^+$.

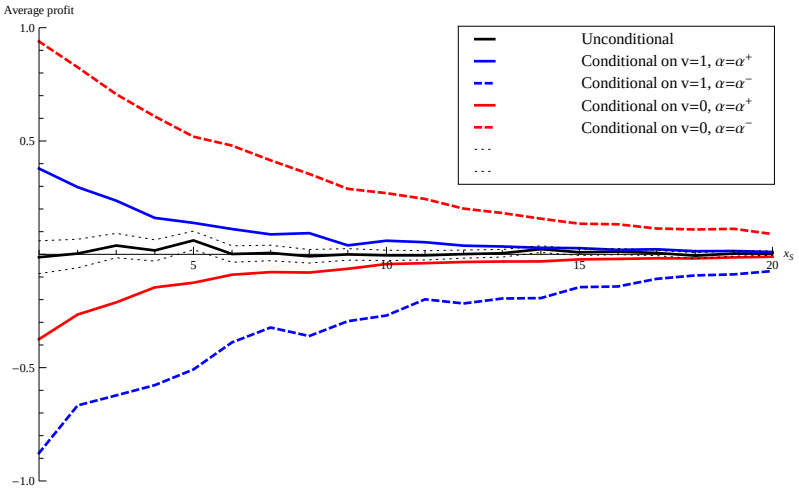


Figure 9: Supply-informed traders' average (conditional and unconditional) profits over 100 periods. Baseline case.

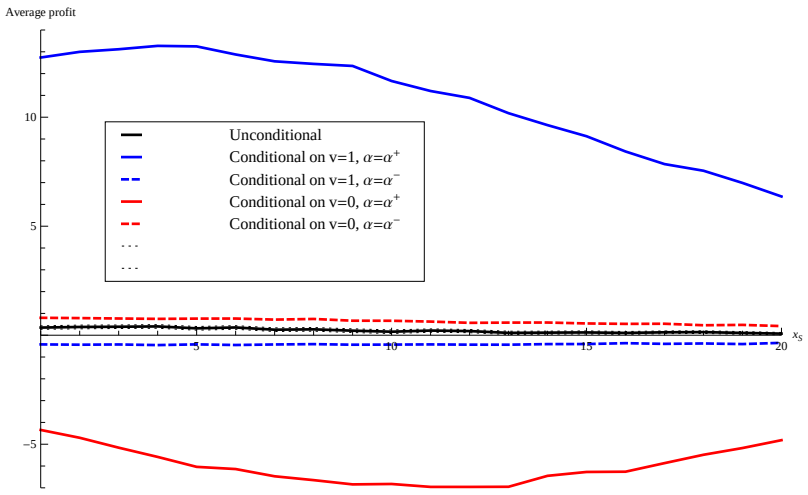


Figure 10: Supply-informed traders' average (conditional and unconditional) profits over 100 periods. Turbulence case.

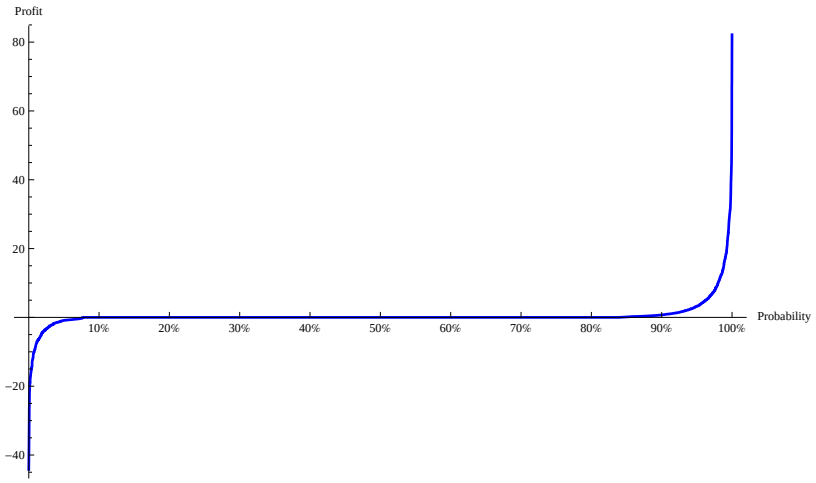


Figure 11: Quantile function of supply-informed traders' profits over 100 periods. Turbulence case, unconditional.

A.3 Parameters used in the figures

Main sets of parameters:

Scenario	α^+	α^-	$3x_N$	x_I	x_S	σ_{-1}	σ_1	E_0^+	E_0^-	λ_1
Baseline	0.15	0.015	0.9	0.05	0.05	1	0.8	0.495	0.505	0.5
Turbulence	0.8	0.08	0.89	0.1	0.01	1	0.1	0.495	0.505	0.05
Public information	0.8	0.08	0.899	0.1	0.001	1	0.1	0.495	0.505	0.05
Private information	0.8	0.08	0.85	0.1	0.05	1	0.1	0.495	0.505	0.001

Corresponding figures:

Fig.1: Public information and Private information parameters.

Fig.3: $x_N = 0.3, x_I = 0.1, x_S = 0, \alpha^+ = 1, \alpha^- = 0, \sigma_1 = 0.25, \sigma_{-1} = 0.5, E_0^+ = E_0^- = 0.5$.

Fig.5: $v = 0, 3\alpha^+ x_N = 0.05, 3\alpha^- x_N = 0.05, x_N = 0.2, x_I = 0.2, \sigma_1 = 0, \sigma_{-1} = 1, E_1^+ = 0.9, E_1^- = 0.05, \lambda_1 = 0.95, \alpha = \alpha^+$.

Fig.6: Turbulence parameters, $\alpha = \alpha^+, v = 1$.

Fig.4: id. with $E_0^+ = 0.5, E_0^- = 0.2, \lambda_1 = 0.2$.

Fig.7: Turbulence and Baseline parameters, different values of x_S.

Fig.8: Turbulence parameters, $v = 1, \alpha = \alpha^+$, different values of x_S.

Fig.9 and 10: Turbulence and Baseline parameters, different values of x_S.

Fig.11: Turbulence parameters.

A.4 Proofs

A.4.1 Proof of Proposition 1

I first show the following result: if $E_t^+ > E_t^-$ there are only four possible values for (O_t^+, O_t^-): $(1,0), (1,-1), (0,0), (0,-1)$. Conversely if $E_t^- > E_t^+$– this random vector can only take the following values: $(0,1), (0,0), (-1,0), (-1,1)$. The two types of supply-informed traders cannot both buy or both sell in the same period. It is straightforward to see that supply-informed traders with the highest expectation cannot sell while the other does nothing or buys. Remember that if there is a sale (resp. a purchase) in period t, we have

$$B_t(\text{ resp. } A_t) = \lambda_{t+1} E_{t+1}^+ + (1 - \lambda_{t+1}) E_{t+1}^-$$

If both types of supply-informed traders sell we have $E_t^+ < B_t, E_t^- < B_t$, but if there is a sale in period t we also have $E_{t+1}^+ < E_t^+, E_{t+1}^- < E_t^-$, so $B_t < \lambda_{t+1} E_t^+ + (1 - \lambda_{t+1}) E_t^-$, which means that either E_t^+ or E_t^- is larger than B_t, and thus that one type at least does not sell. The same demonstration can be followed to show both types cannot buy. Now, I use Bayes' law to compute the bid and ask prices, assuming the orders submitted by supply-informed traders to be known. All probabilities are conditional on I_t:

$$A_t = \frac{\Pr(v = 1 \cap T_t = 1)}{\Pr(v = 1 \cap T_t = 1) + \Pr(v = 0 \cap T_t = 1)}$$

I use the following notations with $j \in \{+,-\}, T \in \{-1,0,1\}$:

$$\begin{aligned} u^j_{t,T} &= \Pr(\text{ Observed trade } = T | I_t, v=1, \alpha = \alpha^j) \\ &= (1-\alpha^j\sigma^t)x_N + \mathbf{1}_{O^j_t=T}x_S + 3\alpha^j\sigma_T\mathbf{1}_{O^f_t=T}x_N + x_I\mathbf{1}_{T=1} \quad (6) \\ v^j_{t,T} &= \Pr(\text{ Observed trade } = T | I_t, v=0, \alpha = \alpha^j) \\ &= u^j_{t,T} + x_I(\mathbf{1}_{T=-1} - \mathbf{1}_{T=1}) \quad (7) \end{aligned}$$

where σ^t is the relevant σ in period t, that is σ_1 if there was a purchase in $t-1$, σ_{-1} if there was a sale, 0 otherwise. Now we can reexpress the bid and the ask prices:

$$A_t = \frac{\lambda_t E^+_t u^+_{t,1} + (1-\lambda_t)E^-_t u^-_{t,1}}{\lambda_t(u^+_{t,1} - (1-E^+_t)x_I) + (1-\lambda_t)(u^-_{t,1} - (1-E^-_t)x_I)} \quad (8)$$

$$B_t = \frac{\lambda_t E^+_t u^+_{t,-1} + (1-\lambda_t)E^-_t u^-_{t,-1}}{\lambda_t(u^+_{t,-1} + (1-E^+_t)x_I) + (1-\lambda_t)(u^-_{t,-1} + (1-E^-_t)x_I)} \quad (9)$$

Denoting $\Omega_t = x_I(1-\lambda E^+_t - (1-\lambda)E^-_t)$ the probability of trading with a value-informed trader and the asset value being low, direct calculation shows these necessary and sufficient conditions:

$$E^+_t < B_t \Leftrightarrow \Omega_t < (1-\lambda_t)\left(\frac{E^-_t}{E^+_t} - 1\right)u^-_{t,-1} \quad (10)$$

$$E^-_t < B_t \Leftrightarrow \Omega_t < \lambda_t\left(\frac{E^+_t}{E^-_t} - 1\right)u^+_{t,-1} \quad (11)$$

$$E^+_t > A_t \Leftrightarrow \Omega_t < (1-\lambda_t)\left(1 - \frac{E^-_t}{E^+_t}\right)u^-_{t,1} \quad (12)$$

$$E^-_t > A_t \Leftrightarrow \Omega_t < \lambda_t\left(1 - \frac{E^+_t}{E^-_t}\right)u^+_{t,1} \quad (13)$$

Finally, assuming $E^+_t > E^-_t$ (interverting λ_t and $1-\lambda_t$, E^+_t and E^-_t, α^+ and α^-, O^+_t and O^-_t,the same result holds by symmetry when this is not the case) and denoting $K_t = \lambda_t(\frac{E^+_t}{E^-_t} - 1)(u^+_{t,-1}), L_t = (1-\lambda_t)(1-\frac{E^-_t}{E^+_t})(u^-_{t,-1})$ we have

$$\begin{aligned} (O^+_t, O^-_t) = (1,0) &\Leftrightarrow E^+_t > A_t > E^-_t > B_t &\Leftrightarrow K_t < \Omega_t < L_t \\ (O^+_t, O^-_t) = (1,-1) &\Leftrightarrow E^+_t > A_t > B_t > E^-_t &\Leftrightarrow \Omega_t < \min(K_t, L_t) \\ (O^+_t, O^-_t) = (0,0) &\Leftrightarrow A_t > E^+_t > E^-_t > B_t &\Leftrightarrow \Omega_t > \max(K_t, L_t) \\ (O^+_t, O^-_t) = (0,-1) &\Leftrightarrow A_t > E^+_t > B_t > E^-_t &\Leftrightarrow L_t < \Omega_t < K_t \end{aligned}$$

Thus for any values taken by Ω_t, K_t, L_t there is one and only one possible vector (O^+_t, O^-_t).

A.4.2 Supply-informed traders and market-makers' updates

For supply-informed traders' beliefs, with the notations introduced in A.4.1 we have by Bayes' law:

$$H^+_{t+1} = H^+_t + \Delta H^+_t \tag{14}$$

with $\Delta H^+_t = \ln v^+_{t,T_t} - \ln u^+_{t,T_t}$ and symmetrically for E^-_t.

For market-makers' beliefs on positive feedback trading we can write similarly:

$$\Lambda_{t+1} = \Lambda_t + \Delta\Lambda_t \tag{15}$$

with $\Delta\Lambda_t = \ln y_{t,T_t} - \ln z_{t,T_t}$ and

$$y_{t,T} = \Pr(\text{ Observed trade } = T | I_t, \alpha = \alpha^+) = E^+_t u^+_{t,T} + (1 - E^+_t) v^+_{t,T} \tag{16}$$

$$z_{t,T} = \Pr(\text{ Observed trade } = T | I_t, \alpha = \alpha^-) = E^-_t u^-_{t,T} + (1 - E^-_t) v^-_{t,T} \tag{17}$$

A.4.3 Proof of Proposition 3

Assuming $v = 1$ and $\alpha = \alpha^+$, and dropping time subscripts for the $u^j_{t,T}, v^j_{t,T}$ we can write the expected "jump" between H^+_t and H^+_{t+1} as

$$E(\Delta H^+_t | I_0, \alpha, v) = u^+_1 \ln\left(\frac{u^+_1}{v^+_1}\right) + u^+_{-1} \ln\left(\frac{u^+_{-1}}{v^+_{-1}}\right) + (1 - u^+_1 - u^+_{-1}) \ln\left(\frac{1 - u^+_1 - u^+_{-1}}{1 - v^+_1 - v^+_{-1}}\right) \tag{18}$$

Simple analysis shows this expression is always positive, thus we know that H^+_t goes up on average. If v were 0, the true probabilities would be the vs, not the us, and the formula would be negative. Considering H^-_t is symmetric. This implies that $E(\tilde{H}^c_{t+1} - \tilde{H}^c_t | I_0, \alpha, v) > 0$ for any t, and as a consequence $\lim_{t \to +\infty} \Pr(\tilde{H}^c_t < K | I_0, \alpha, v) = 0$ for any $K > 0$.

Still assuming $\alpha = \alpha^+, v = 1$, consider now the update of Λ_t. We have $\Lambda_{t+1} = \Lambda_t + \Delta\Lambda_t$ thus this update depends on the $y^j_{t,T}, z^j_{t,T}$, which depend respectively on E^+_t and E^-_t. Again I will drop time subscripts and write $y_T(E^+_t), z_T(E^-_t)$. Denoting $\mathcal{E}_t$ the set of possible values for (E^+_t, E^-_t) at time t, we have:

$$E(\Delta\Lambda_t | I_0, \alpha, v) = \sum_{(E^+_t, E^-_t) \in \mathcal{E}_t} \sum_{T \in \{-1,0,1\}} \Pr(E^+_t, E^-_t) \times u^+_T \ln\left(\frac{y_T(E^+_t)}{z_T(E^-_t)}\right)$$

For $\epsilon > 0$, this expression can be rewritten as:

$$\begin{aligned} E(\Delta\Lambda_t|I_0,\alpha,v) &= \Pr(E_t^+ < 1-\epsilon|I_0,\alpha,v) \sum_{\substack{(E_t^+,E_t^-)\in\mathcal{E}_t \\ E_t^+<1-\epsilon}} \sum_{T\in\{-1,0,1\}} \Pr(E_t^+,E_t^-|I_0,\alpha,v) \times u_T^+ \ln\left(\frac{y_T(E_t^+)}{z_T(E_t^-)}\right) \\ &+ \Pr(E_t^+ \geq 1-\epsilon|I_0,\alpha,v) \sum_{\substack{(E_t^+,E_t^-)\in\mathcal{E}_t \\ E_t^+\geq 1-\epsilon}} \sum_{T\in\{-1,0,1\}} \Pr(E_t^+,E_t^-|I_0,\alpha,v) \times u_T^+ \ln\left(\frac{y_T(E_t^+)}{z_T(E_t^-)}\right) \end{aligned}$$

since we have $\lim_{t\to+\infty} \Pr(H_t^+ < K|I_0,\alpha,v) = 0$ for any $K > 0$, we also have $\lim_{t\to+\infty} \Pr(E_t^+ < 1-\epsilon|I_0,\alpha,v) = 0$ for any $\epsilon > 0$. Denoting $\mathcal{E}_\infty^-$ the set of all possible values for E_t^- as t goes to infinity and with ϵ going to zero we have:

$$\lim_{t\to+\infty} E(\Delta\Lambda_t|I_0,\alpha,v) = \sum_{E^-\in\mathcal{E}_\infty^-} \sum_{T\in\{-1,0,1\}} \Pr(E^-) \times u_T^+ \ln\left(\frac{y_T(1)}{z_T(E^-)}\right)$$

notice that $y_T(1) = u_T^+$. Thus, for each possible E_t^- the expression above is a weighted average of sums of three terms such as:

$$u_1^+ \ln\left(\frac{u_1^+}{z_1(E^-)}\right) + u_{-1}^+ \ln\left(\frac{u_{-1}^+}{z_{-1}(E^-)}\right) + (1-u_1^+-u_{-1}^+)\ln\left(\frac{1-u_1^+-u_{-1}^+}{1-z_1^+(E^-)-z_{-1}(E^-)}\right)$$

and such a sum is always positive as already noted for equation 18. The cases $v = 0$ and $\alpha = \alpha^-$ are symmetric. This shows that after a long enough time $\tilde{\Lambda}_t$ will go up in expectation as claimed in the proposition, and from this we deduce that $\lim_{t\to+\infty} \Pr(\tilde{\Lambda}_t < K|I_0,\alpha,v) = 0$, for any $K > 0$. Using equation 3, since p_t is an average of E_{t+1}^+ and E_{t+1}^- weighted by λ_{t+1}, market-makers' beliefs about v will become infinitely close to E_{t+1}^c and will share its properties, which implies that p_t after a long enough time p_t will become closer to v on average in each period and will eventually converge to v.

A.4.4 Proof of Proposition 4 and Lemma 2

Consider first Proposition 4. If x_S is close to zero, by Lemma 1 trades follow a simple Markov chain with three states and it is possible to compute $\mu(v,\alpha) = (\mu_1(v,\alpha),\mu_0(v,\alpha),\mu_{-1}(v,\alpha))^T$ the stationary measure associated with the matrix $M(v,\alpha)$. After a sale $\sigma^t = \sigma_{-1}$, by equations 6, 7, 16 and 17 α^+ traders will update their beliefs less downwards then α^- traders after a sale, and more upwards after a purchase (both types don't update their beliefs if there is no trade), thus $H_t^+ - H_t^-$ increases in $t+1$ if there is a sale in t. Conversely, if there is a purchase in t then $H_t^+ - H_t^-$ will decrease in $t+1$, and if there is no trade in t then the difference $H_t^+ - H_t^-$ stays the same. In general, whether H_t^+ is larger or smaller than H_t^- in the long run thus depends on how many purchases and sales there are on average. When $\sigma_1 - \sigma_{-1}$ is large however, σ_{-1} is small, thus

$\alpha^+\sigma_{-1} \simeq \alpha^-\sigma_{-1}$ and only the difference after purchases matters, hence in the long-run we will have $H_t^+ < H_t^-$. Conversely, when $\sigma_{-1} - \sigma_1$ is large then $\alpha^+\sigma_1 \simeq \alpha^-\sigma_1$ and only the difference after sales matters, hence in the long-run we have $H_t^+ > H_t^-$.

This result is useful to prove Lemma 2. If $x_S \to 0$, by Lemma 1 we can define $\Delta H_\infty^+, \Delta H_\infty^-$ and $\Delta\Lambda_\infty$ the average update of H^+, H^-, Λ in each period. In the long run E^+, E^- and λ thus behave as $e^{t\Delta H_\infty^+}/(1+e^{t\Delta H_\infty^+})$, $e^{t\Delta H_\infty^-}/(1+e^{t\Delta H_\infty^-})$ and $e^{t\Delta\Lambda_\infty}/(1+e^{t\Delta\Lambda_\infty})$ on average. Consider the case $v=1, \alpha=\alpha^+$, and assume $\Delta H_\infty^+ > \Delta H_\infty^-$. Then if supply-informed traders are active they have to buy, thus equation 12 has to be satisfied, a condition that can be written as:

$$\frac{(1-\lambda)(E^+-E^-)}{E^+(1-E^+)} = \frac{1+e^{H^+}}{1+e^{H^-}} \times \frac{1}{1+e^{\Lambda}} \times \left(1 - \frac{e^{H^-}}{e^{H^+}}\right) > \frac{x_I}{u_1^- - E^+ x_I}$$

The dominant term in the numerator is e^{2H^+}, while the dominant term in the denominator is $e^{H^+ + \Lambda + \max(0,H^-)}$. Thus if $\Delta H_\infty^+ > \Delta\Lambda_\infty + \max(\Delta H_\infty^-, 0)$ the left-hand side goes to infinity and is larger than the right-hand side which goes to a positive constant, if the opposite then the left-hand side goes to zero and is smaller than the left-hand side. If on the contrary $\Delta H_\infty^+ < \Delta H_\infty^-$ then if supply informed traders are active in the long run they have to sell using equation 10 and a similar reasoning, minus the former left-hand side must now be larger than a positive constant, but since $H^- > H^+$ the dominant term in the numerator is now $e^{H^+ + H^-}$ and in the denominator $e^{H^+ + H^- + \Lambda}$, so that supply-informed traders cannot be active, as stated in the proposition since $\Delta H_\infty^+ < \Delta H_\infty^-$ implies *a fortiori* that $\Delta H_\infty^+ < \Lambda + \max 0, \Delta H_\infty^-$.

All other cases are dealt with symmetrically, and I can sum up the results using $\tilde{H}^c, \tilde{H}^u, \tilde{\Lambda}$ as in the proposition. When x_S is not close to zero it is not warranted that $\Delta H_\infty^+, \Delta H_\infty^-$ and $\Delta\Lambda_\infty$ exist. However if at least incorrect supply-informed traders' behavior is constant in the long run, then (i) or (iv) implies that both types of traders have a constant behavior in the long run, the limits exist and we can apply the same reasoning as above.

A.4.5 Proof of Remark 1

I prove the second part of the remark, where a type of supply-informed traders is supposed to always trade in the same direction (including 0) in the long-run independently of v. As will be apparent below and to avoid considering too many cases, there is no loss of generality in assuming that this assumption holds for both types of supply-informed traders, if this is not the case and only type α satisfies the assumption then one has to assume that λ_1 is high enough if $\alpha = \alpha^+$, and low enough if $\alpha = \alpha^-$.

If supply-informed traders are in a negligible number or always trade in the same direction after a long enough time, trades follow a simple Markov chain with three states. Assuming as in

the remark that there is a probability $3\hat{x}_N(1-\hat{x}_S)$ to select a noise trader, $(1-3\hat{x}_N)(1-\hat{x}_S)$ a value-informed and $\hat{x}_S$ a supply-informed trader, I can then rewrite the transition matrix $M(v,\alpha)$ defined in equation 4 as:

$$M(v,\alpha) = \hat{x}_N(1-\hat{x}_S)M_N(\alpha) + (1-3\hat{x}_N)(1-\hat{x}_S)M_V(v) + \hat{x}_S M_S$$

It is now possible to compute $\mu(v,\alpha) = (\mu_1(v,\alpha), \mu_0(v,\alpha), \mu_{-1}(v,\alpha))^T$ the stationary measure associated with the matrix $M(v,\alpha)$. The goal here is to analyze the average trade imbalance, that is $|\mu_1(v,\alpha) - \mu_{-1}(v,\alpha)|$. I will show that, regardless of a given type of supply-informed traders' trading direction, their impact is always negative. Consider first the case in which α traders always buy in the long-run. Then we have:

$$\begin{aligned}
\mu_1(1,\alpha) - \mu_{-1}(1,\alpha) &= \frac{1-\hat{x}_N(1-\hat{x}_S)(3-\alpha(\sigma_1-\sigma_{-1}(2-3\hat{x}_N(1-\hat{x}_S))))}{1-\alpha\hat{x}_N(1-\hat{x}_S)(2\sigma_{-1}+\sigma_1(2-3\alpha\sigma_{-1}\hat{x}_N(1-\hat{x}_S)))} \\
\mu_1(0,\alpha) - \mu_{-1}(0,\alpha) &= \frac{-1+\hat{x}_N(1-\hat{x}_S)(3-\alpha(\sigma_{-1}-\sigma_1(2-3\hat{x}_N(1-\hat{x}_S))))+\hat{x}_S(2-\alpha(\sigma_1+\sigma_{-1})\hat{x}_N(1-\hat{x}_S))}{1-\alpha\hat{x}_N(1-\hat{x}_S)(2\sigma_{-1}+\sigma_1(2-3\alpha\sigma_{-1}\hat{x}_N(1-\hat{x}_S)))}
\end{aligned}$$

Under the assumption that $v=0$ and $v=1$ are equally likely and that there are enough value-informed traders to obtain more purchases than sales when $v=1$ and more sales than purchases when $v=0$ (notice that this will be the case for x_N low enough, that is, since $\hat{x}_I + 3\hat{x}_N = 1$, for $\hat{x}_I$ high enough) and after some rearranging, we have:

$$\frac{1}{2}|\mu_1(1,\alpha)-\mu_{-1}(1,\alpha^+)|+\frac{1}{2}|\mu_1(0,\alpha)-\mu_{-1}(0,\alpha)| = \frac{1}{2}\frac{(1-\hat{x}_S)(1-3\hat{x}_N)(2-\alpha(\sigma_1+\sigma_{-1}\hat{x}_N(1-\hat{x}_S)))}{1-\alpha\hat{x}_N(1-\hat{x}_S)(2\sigma_{-1}+\sigma_1(2-3\alpha\sigma_{-1}\hat{x}_N(1-\hat{x}_S)))} \tag{19}$$

Differentiating with respect to x_S gives:

$$-\frac{2(1-3\hat{x}_N)(1-\alpha(\sigma_1+\sigma_{-1})\hat{x}_N(1-\hat{x}_S)+\alpha^2\hat{x}_N^2(1-\hat{x}_S)^2(\sigma_1^2+\sigma_{-1}^2-\sigma_1\sigma_{-1}))}{(1-\alpha\hat{x}_N(1-\hat{x}_S)(2\sigma_{-1}+\sigma_1(2-3\alpha\sigma_{-1}\hat{x}_N(1-\hat{x}_S))))^2}$$

since $1 > 3\hat{x}_N$ and $\alpha(\sigma_1+\sigma_{-1}) \leq 2$ all terms in the numerator are positive and thus the derivative is negative: increasing $\hat{x}_S$ has a negative impact on expected order imbalance in this case.

Consider now the case in which α traders always sell in the long-run. Then:

$$\begin{aligned}
\mu_1(1,\alpha) - \mu_{-1}(1,\alpha) &= \frac{1-\hat{x}_N(1-\hat{x}_S)(3-\alpha(\sigma_1-\sigma_{-1}(2-3\hat{x}_N(1-\hat{x}_S))))-\hat{x}_S(2-\alpha(\sigma_1+\sigma_{-1})\hat{x}_N(1-\hat{x}_S))}{1-\alpha\hat{x}_N(1-\hat{x}_S)(2\sigma_{-1}+\sigma_1(2-3\alpha\sigma_{-1}\hat{x}_N(1-\hat{x}_S)))} \\
\mu_1(0,\alpha) - \mu_{-1}(0,\alpha) &= \frac{-1+\hat{x}_N(1-\hat{x}_S)(3-\alpha(\sigma_{-1}-\sigma_1(2-3\hat{x}_N(1-\hat{x}_S))))}{1-\alpha\hat{x}_N(1-\hat{x}_S)(2\sigma_{-1}+\sigma_1(2-3\alpha\sigma_{-1}\hat{x}_N(1-\hat{x}_S)))}
\end{aligned}$$

the average of both terms in absolute values is the same as in equation 19, hence again this derivative is negative. Finally, when α traders are inactive in the long-run we get:

$$\begin{aligned}
\mu_1(1,\alpha) - \mu_{-1}(1,\alpha) &= \frac{(1-\hat{x}_S)(1-\hat{x}_N(3+\alpha(\sigma_1-\sigma_{-1}(2-3\hat{x}_N))))}{1-\alpha\hat{x}_N(1-\hat{x}_S)(2\sigma_{-1}+\sigma_1(2-3\alpha\sigma_{-1}\hat{x}_N(1-\hat{x}_S)))} \\
\mu_1(0,\alpha) - \mu_{-1}(0,\alpha) &= \frac{(1-\hat{x}_S)(-1+\hat{x}_N(3+\alpha(\sigma_{-1}-\sigma_1(2-3\hat{x}_N))))}{1-\alpha\hat{x}_N(1-\hat{x}_S)(2\sigma_{-1}+\sigma_1(2-3\alpha\sigma_{-1}\hat{x}_N(1-\hat{x}_S)))}
\end{aligned}$$

this gives the same expected order imbalance as before, its derivative with respect to $\hat{x}_S$ is again negative, which completes the proof that the expected average order imbalance is negatively affected by an increase in $\hat{x}_S$.

A.4.6 Proof of Remarks 3, 4 and 5

I first prove Remark 3. I assume $x_I = x_S = 0, T_{t-1} = 1, E_t^+ > E_t^-$ and show that $B_t > A_t$. By continuity the result will hold for small enough values of x_I and x_S. Finally I drop the time subscripts for clarity. Since $x_I = 0$ we have by definition:

$$A = \frac{\lambda E^+ u_1^+ + (1-\lambda)E^- u_1^-}{\lambda u_1^+ + (1-\lambda)u_1^-},\ B = \frac{\lambda E^+ u_{-1}^+ + (1-\lambda)E^- u_{-1}^-}{\lambda u_{-1}^+ + (1-\lambda)u_{-1}^-}$$

Then the spread is negative if and only if:

$$\begin{aligned} B > A &\Leftrightarrow (\lambda E^+ u_1^+ + (1-\lambda)E^- u_1^-)(\lambda u_{-1}^+ + (1-\lambda)u_{-1}^-) < (\lambda E^+ u_{-1}^+ + (1-\lambda)E^- u_{-1}^-)(\lambda u_1^+ + (1-\lambda)u_1^-) \\ &\Leftrightarrow E^+(u_1^+ u_{-1}^- - u_1^- u_{-1}^+) < E^-(u_1^+ u_{-1}^- - u_1^- u_{-1}^+) \\ &\Leftrightarrow u_1^+ u_{-1}^- - u_1^- u_{-1}^+ \text{ since } E^+ > E^- \end{aligned}$$

And finally we just have to compute the value of $u_1^+ u_{-1}^- - u_1^- u_{-1}^+$ when $x_I = x_S = 0$:

$$u_1^+ u_{-1}^- - u_1^- u_{-1}^+ = (1-\alpha^+\sigma_{-1})x_N \times (1+2\alpha^-\sigma_{-1})x_N - (1-\alpha^-\sigma_{-1})x_N \times (1+2\alpha^+\sigma_{-1})x_N = -3x_N^2(\alpha^+ - \alpha^-) < 0$$

The proof is symmetric when $E^- > E^+, T_{t-1} = -1$. The implication on prices is straightforward: if $T_{t-1} = -1$ then market-makers' prior belief about the asset's value at time t is just B_{t-1}. Among the updated beliefs A_t, B_t one has to be higher than B_{t-1} and the other lower, if $B_t > A_t$ it must be the case that $B_t > B_{t-1}$.

Consider now Remark 4. As shown in the text, the observation of no trade in period $t-1$ has a negative impact on λ_t. Since there hasn't been any trade in period $t-1$, both types of supply-informed traders have the same belief that there will be no positive feedback, since moreover x_S is assumed very small we have $u_{t,T}^+ = u_{t,T}^-$ and $v_{t,T}^+ = v_{t,T}^-$ for any trade direction T. I thus write u and v for both u^+, u^- and v^+, v^-. Using equations 8 and 9, and dropping for simplicity the time subscripts (all equal to t), we have:

$$\frac{\partial A}{\partial \lambda} = \frac{(E^+ - E^-)u_1 v_1}{\Pr(T=1)^2}, \frac{\partial B}{\partial \lambda} = \frac{(E^+ - E^-)u_{-1} v_{-1}}{\Pr(T=-1)^2}$$

Since we also have $u_1 = v_{-1} = x_N + x_I$ and $u_{-1} = v_1 = x_N$, this gives:

$$\frac{\partial (A-B)}{\partial \lambda} = \frac{(E^+ - E^-)u_1 v_1 (\Pr(T=-1)^2 - \Pr(T=1)^2)}{\Pr(T=1)^2 \Pr(T=-1)^2}$$

reexpressing the difference between the two squares as a product we finally get:

$$\frac{\partial(A-B)}{\partial\lambda} = \frac{(E^+ - E^-)u_1 v_1 (u_1+v_1) x_I (0.5 - \lambda E^+ - (1-\lambda)E^-)}{\Pr(T=1)^2 \Pr(T=-1)^2}$$

which proves the remark.

Remark 5 considers again the case where $x_N = \hat{x}_N(1-\hat{x}_S), x_I = (1-3\hat{x}_N)(1-\hat{x}_S)$ and $x_S = \hat{x}_S$. It is useful to rewrite equations 6 and 7 (dropping time subscripts again) to define:

$$\hat{u}_T^j = (1-\alpha^j\sigma^{t-1})\hat{x}_N + (1-3\hat{x}_N)\mathbf{1}_{T=1} + 3\alpha^j\sigma_T\hat{x}_N\mathbf{1}_{T=T_{t-1}} \quad (20)$$

$$\hat{v}_T^j = u_T^j + (1-3\hat{x}_N)(\mathbf{1}_{T=-1} - \mathbf{1}_{T=1}) \quad (21)$$

Since $\hat{v}$ and $\hat{u}$ do not depend on $\hat{x}_S$ I will use these auxiliary variables each time I will need to differentiate an expression with respect to $\hat{x}_S$. If in the current period no type of supply-informed trades then using equations 8 and 9 it is immediate that $\hat{x}_S$ has no impact on the ask or the bid price and hence on the spread. Otherwise, assume that α^+ traders buy, and thus by Proposition 1, that α^- traders do not (as will be apparent below, the role of both types of traders is entirely symmetric). We can then write:

$$u_1^+ = (1-\hat{x}_S)\hat{u}_1^+ + \hat{x}_S, \quad u_1^- = (1-\hat{x}_S)\hat{u}_1^-$$
$$v_1^+ = (1-\hat{x}_S)\hat{v}_1^+ + \hat{x}_S, \quad v_1^- = (1-\hat{x}_S)\hat{v}_1^-$$

The ask price is equal to $A = \frac{\Pr(v=1\cap T=1)}{\Pr(v=1\cap T=1)+\Pr(v=0\cap T=1)}$, with:

$$\begin{aligned}
\Pr(v=1\cap T=1) &= \lambda E^+((1-\hat{x}_S)\hat{u}_1^+ + \hat{x}_S) + (1-\lambda)E^-(1-\hat{x}_S)\hat{u}_1^- \\
\Pr(v=0\cap T=1) &= \lambda(1-E^+)((1-\hat{x}_S)\hat{v}_1^+ + \hat{x}_S) + (1-\lambda)(1-E^-)(1-\hat{x}_S)\hat{v}_1^-
\end{aligned}$$

Differentiating these expressions with respect to $\hat{x}_S$:

$$\begin{aligned}
\frac{\partial \Pr(v=1\cap T=1)}{\partial \hat{x}_S} &= \lambda E^+ - (\lambda E^+\hat{u}_1^+ + (1-\lambda)E^-\hat{u}_1^-) \\
&= \frac{1}{1-\hat{x}_S}(\lambda E^+ - \Pr(v=1\cap T=1)) \\
\frac{\partial \Pr(v=0\cap T=1)}{\partial \hat{x}_S} &= \lambda(1-E^+) - (\lambda(1-E^+)\hat{v}_1^+ + (1-\lambda)(1-E^-)\hat{v}_1^-) \\
&= \frac{1}{1-\hat{x}_S}(\lambda(1-E^+) - \Pr(v=0\cap T=1))
\end{aligned}$$

This gives:

$$\begin{aligned}\frac{\partial A}{\partial \hat{x}_S} &= \frac{(\lambda E^+ - \Pr(v=1\cap T=1))\Pr(T=1) - (\lambda - \Pr(T=1))\Pr(v=1\cap T=1)}{(\Pr(T=1))^2(1-\hat{x}_S)} \\ &= \frac{\lambda(E^+ - A)}{\Pr(T=1)(1-\hat{x}_S)}\end{aligned}$$

Since α^+ traders buy, E^+ is larger than A and the derivative is positive, the result is of course the same if α^- traders buy. The reasoning for the bid is exactly similar, the $\hat{u}_1, \hat{v}_1$ are replaced with $\hat{u}_{-1}, \hat{v}_{-1}$ and in the end $\hat{x}_S$ has a negative impact on the bid if and only if $E^j < B$, which is true when type j traders sell. This proves that under all possible configurations an increase in $\hat{x}_S$ has a positive impact on the bid-ask spread.

A.4.7 Proof of Lemma 3

For the two first parts there are four sets of sufficient conditions under which $E(\tilde{H}^u_{t+1} - \tilde{H}^u_t | I_0, \alpha, v) < 0$:

$$\begin{aligned}\alpha = \alpha^+, \alpha^- \simeq 0, x_S \simeq 0, \quad & v=1, \quad T_t = -1, 3\alpha^+\sigma_{-1}x_N > x_I \\ \text{or} \quad & v=0, \quad T_t = 1, 3\alpha^+\sigma_1 x_N > x_I \\ \alpha = \alpha^-, \alpha^- > 0, x_S \simeq 0, \quad & v=1, \quad T_t = 1, \alpha^+\sigma_1 \simeq 1 \\ \text{or} \quad & v=0, \quad T_t = -1, \alpha^+\sigma_{-1} \simeq 1\end{aligned}$$

Consider the case $v=1, \alpha=\alpha^+, T_t=-1$. Assuming $x_S=0$, we can write the expected update of incorrect supply-informed traders as:

$$\begin{aligned}E(H^-_{t+1} - H^-_t | I_0, \alpha=\alpha^+, v=1) &= u^+_1 \ln\left(\frac{u^-_1}{v^-_1}\right) + u^+_{-1}\ln\left(\frac{u^-_{-1}}{v^-_{-1}}\right) \\ &= (x_I + (1-\alpha^+\sigma_{-1})x_N)\ln\left(\frac{x_I + (1-\alpha^-\sigma_{-1})x_N}{(1-\alpha^-\sigma_{-1})x_N}\right) \\ &+ (1+2\alpha^+\sigma_{-1})x_N \ln\left(\frac{(1+2\alpha^-\sigma_{-1})x_N}{(1+2\alpha^-\sigma_{-1})x_N + x_I}\right) \\ &= \ln\left(\frac{x_I + x_N}{x_N}\right)(x_I - 3\alpha^+\sigma_{-1}x_N) \text{ if } \alpha^- = 0\end{aligned}$$

Thus $E(H^-_{t+1} - H^-_t | I_0, \alpha=\alpha^+, v=1) < 0$ if $3\alpha^+\sigma_{-1}x_N > x_I$, which shows this part of the remark by continuity. The symmetric case is dealt with similarly. When incorrect supply-informed traders are of type α^+ the proof is even more direct, since when $\alpha^+\sigma_T$ tends to 1 the logarithm corresponding to the "wrong" signal tends to infinity.

Consider now the update of market-makers about α, for instance in the case where $\alpha=\alpha^+, v=$

$1, E^+ \simeq 0, E^- \simeq 1$. Using the notations given in A.4.2 and dropping time subscripts, if $\alpha^+ \simeq \alpha^-$ and $x_S \simeq 0$ we have for $i = \{0, 1\}, u_i^+ \simeq u_i^-$ such that:

$$E(\Lambda_{t+1} - \Lambda_t | I_0, \alpha = \alpha^+, v = 1) \simeq u_1^+ \ln\left(\frac{v_1^+}{u_1^+}\right) + u_0^+ \ln\left(\frac{v_0^+}{u_0^+}\right) + (1 - u_1^+ - u_0^+)\left(\frac{1 - v_1^+ - v_0^+}{1 - u_1^+ - u_0^+}\right)$$

an expression that can only be negative.

A.4.8 Proof of Proposition 6 and Corollary2

Under the assumptions of the Proposition, Lemma 1 and Lemma 2 apply, so that we know under which conditions supply-informed traders are active or inactive in the long run. Assume $v = 1$, the reasoning would of course be exactly symmetric if $v = 0$. Dropping time subscripts and rewriting equation 8 or 9 gives:

$$1 - p = \frac{\lambda(1 - E^+)v^+ + (1 - \lambda)(1 - E^-)v^-}{\lambda(v^+ + E^+ x_I + (1 - \lambda)(v^- + E^- x_I)}$$

with $(v^+, v^-) = (v_1^+, v_1^-)$ for the ask price and $(v^+, v^-) = (v_{-1}^+, v_{-1}^-)$ for the bid price. Using the definition of H^+, H^-, Λ this gives:

$$1 - p = \frac{e^\Lambda(1 + e^{H^-})v^+ + (1 + e^{H^+})v^-}{(1 + e^{H^+})(1 + e^{H^-})(e^\Lambda v^+ + v^-) + x_I(e^{H^+}(1 + e^{H^-}) + e^{H^-}(1 + e^{H^+}))}$$

Since H_t^+, H_t^-, Λ behave as $t \times \Delta H_\infty^+, t \times \Delta H_\infty^-, t \times \Delta\Lambda$, $1 - p$ is equivalent when t goes to infinity to the ratio of the dominant terms in the numerator and denominator. When $\alpha = \alpha^+$ both H^+ and Λ are positive in the long run. If α^+ traders are active then according to Lemma 2 we have $H^+ > \Lambda + \max(0, H^-)$ so that the dominant term in the numerator is e^{H^+} and the dominant term in the denominator is the greater of $e^{\Lambda + H^+ + H^-}$ and $e^{\Lambda + H^+}$, hence the ratio is equivalent to a constant times $e^{-\Lambda - \max(0, H^-)}$. If α^+ traders are inactive then $H^+ > \Lambda + \max(0, H^-)$, the dominant term in the numerator is thus e^Λ or $e^{\Lambda + H^-}$ while the dominant term in the denominator is $e^{\Lambda + H^+}$ or $e^{\Lambda + H^+ + H^-}$, whether H^- is positive or negative the ratio is equivalent to a constant times e^{-H^+}. If $\alpha = \alpha^-$ the same analysis can be repeated using the notation $\tilde{\Lambda}$, and if $v = 0$ the same analysis can be repeated by writing p instead of $1 - p$ and using $\tilde{H}^+$ and $\tilde{H}^-$.

For the corollary, consider the case $\alpha = \alpha^+, v = 1$. I drop the time subscripts, expectations are all conditional on $I_0, \alpha, v, O^+ = 1, O^- = -1$ (under the assumptions of the proposition, α^+ traders will buy in the long-run while E_t^- will go to zero, thus α^- traders will sell). Using the variables $\hat{u}_T^j, \hat{v}_T^j$ defined in equations 20 and 21, the expected update of H_t^+ after a sale can be written as:

$$E(\Delta H^+) = (\hat{u}_1^+(1 - \hat{x}_S) + \hat{x}_S) \ln\left(\frac{\hat{u}_1^+(1 - \hat{x}_S) + \hat{x}_S}{\hat{v}_1^+(1 - \hat{x}_S) + \hat{x}_S}\right) + \hat{u}_{-1}(1 - \hat{x}_S) \ln\left(\frac{\hat{u}_{-1}^+(1 - \hat{x}_S)}{\hat{v}_{-1}^+(1 - \hat{x}_S)}\right)$$

Differentiating with respect to $\hat{x}_S$ gives:

$$\begin{aligned}\frac{\partial E(\Delta H^+)}{\partial \hat{x}_S} &= (1-\hat{u}_1^+)\ln\left(\frac{u_1^+}{v_1^+}\right) + (1-\hat{u}_1^+) - (1-\hat{v}_1^+)\frac{u_1^+}{v_1^+} \\ &- \hat{u}_{-1}^+\ln\left(\frac{u_{-1}^+}{v_{-1}^+}\right) - \hat{u}_{-1}^+ + \hat{v}_{-1}^+\frac{u_{-1}^+}{v_{-1}^+}\end{aligned}$$

Using the definition of $\hat{u}^+$ and $\hat{v}^+$ and after rearranging:

$$\frac{\partial E(\Delta H^+)}{\partial \hat{x}_S} = \frac{1}{1-\hat{x}_S}\left(-E(\Delta H^+) + 1 + \ln\left(\frac{u_1^+}{v_1^+}\right) - \frac{u_1^+}{v_1^+} - u_1^+\left(1-\frac{u_1^+}{v_1^+}\right) - x_I\frac{u_1^+}{v_1^+}\right)$$

All terms inside the bracket are negative, in particular $E(\Delta H^+)$ has been shown to be positive in A.4.3, and $1+\ln x - x$ is always negative. The derivative is thus negative.

In the long-run, as $E_t^+ \to 1$ and $E_t^- \to 0$, using equation 15 the expected update of Λ_t can be written as:

$$E(\Delta\Lambda) = u_1^+\ln\left(\frac{u_1^+}{u_1^-}\right) + u_0^+\ln\left(\frac{u_0^+}{u_0^-}\right) + u_{-1}^+\ln\left(\frac{u_{-1}^+}{u_{-1}^-}\right)$$

Using equations 20 and 21, this can be rewritten as:

$$E(\Delta\Lambda) = ((1-\hat{x}_S)\hat{u}_1^+ + \hat{x}_S)\ln\left(\frac{(1-\hat{x}_S)\hat{u}_1^+ + \hat{x}_S}{(1-\hat{x}_S)\hat{v}_1^-}\right) + (1-\hat{x}_S)\hat{u}_0^+\ln\left(\frac{(1-\hat{x}_S)\hat{u}_0^+}{(1-\hat{x}_S)\hat{v}_0^-}\right) + (1-\hat{x}_S)\hat{u}_{-1}^+\ln\left(\frac{(1-\hat{x}_S)\hat{u}_{-1}^+}{(1-\hat{x}_S)\hat{v}_{-1}^- + \hat{x}_S}\right)$$

This gives:

$$\begin{aligned}\frac{\partial E(\Delta\Lambda)}{\partial \hat{x}_S} &= (1-\hat{u}_1^+)\ln\left(\frac{u_1^+}{v_1^-}\right) - \hat{u}_0^+\ln\left(\frac{u_0^+}{v_0^-}\right) - \hat{u}_{-1}^+\ln\left(\frac{u_{-1}^+}{v_{-1}^-}\right) \\ &+ u_1^+\left(\frac{1-\hat{u}_1^+}{u_1^+} + \frac{\hat{v}_1^-}{v_1^-}\right) - u_{-1}^+\left(\frac{\hat{u}_{-1}^+}{u_1^+} + \frac{1-\hat{v}_{-1}^-}{v_{-1}^-}\right)\end{aligned}$$

and after rearranging:

$$\frac{\partial E(\Delta\Lambda)}{\partial \hat{x}_S} = -E(\Delta\Lambda) + \ln\left(\frac{u_1^+}{v_1^+}\right) + \frac{1}{1-\hat{x}_S}\left(1-\frac{u_{-1}^+}{v_{-1}^-}\right)$$

$E(\Delta\Lambda)$ is positive in the long-run (Proposition 3). Since the last trade was a sale, when x_I is low enough a further sale is a signal that $\alpha = \alpha^+$ and a purchase that $\alpha = \alpha^-$, hence $u_1^+ < u_1^-$, $u_{-1}^+ > v_{-1}^-$, and the derivative is thus negative.

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