Large Bets and Stock Market Crashes

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Abstract

We examine five episodes of market dislocations and show that observed price declines are similar in magnitude to price impact of large sales documented during those events, as implied by market microstructure invariance. Flash crashes had larger than predicted price declines with rapid V-shaped recoveries. The 1929 crash had smaller decline than predicted. Reconciling predicted frequencies of crashes to observed frequencies requires the distribution of quantities sold either to have fatter tails than a log-normal or a larger variance than estimated from portfolio transitions data. These results have implications for design of an earlier warning system for identifying systemic risks.

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Once in a while, stock markets plummet and rattle financial markets, leaving stunned market participants, puzzled economists, and frustrated policymakers unable to explain why the crash or panic they just witnessed happened at all. In the aftermath of crashes and panics, it has typically emerged that specific market participants were engaged in heavy selling as market dislocations unfolded. This paper studies the following five stock market crashes, for which data on the magnitude of selling pressure became publicly available following official studies of the crashes:

- After the stock market crash of October 1929, it was documented that margin calls resulted in massive selling of stocks and reductions in loans to finance margin purchases.

- After the October 1987 stock market crash, the U.S. Presidential Task Force on Market Mechanisms (1988) (hereafter the "Brady Report") documented quantities of stock index futures contracts and baskets of stocks sold by portfolio insurers.

- After the futures market dropped by 20% at the open of trading three days after the 1987 crash, it was revealed that George Soros had executed a large sell order during the opening minutes and later sued his broker for an excessively expensive order execution.

- After the Fed cut interest rates by 75 basis points in response to a market plunge on January 21, 2008, it emerged that, around the same time, Société Générale liquidated billions of Euros in stock index future positions accumulated by rogue trader Jérôme Kerviel.

- After the flash crash of May 6, 2010, the Staffs of the CFTC and SEC (2010*b*,*a*) identified approximately $4 billion in sales of futures contracts by one entity as a trigger for the event.

Before the first two of these events—the crashes of 1929 and 1987—the size of potential selling pressure was widely known and publicly discussed, but market participants had different opinions concerning whether the selling pressure would have a significant effect on prices. Before the last three crash events—associated with the Soros trades, the Société Générale trades, and the flash crash trades—the sellers knew precisely the quantities they intended to sell, but they either estimated inaccurately or were willing to incur the much lower prices they received compared to prices before the trades were made.

The purpose of this paper is to examine these five crash events from the perspective of market microstructure invariance, a conceptual framework developed by Kyle and Obizhaeva (2011*b*). Our main result is that, given the information about the dollar magnitudes of potential selling pressure which existed before these crashes occurred, market microstructure invariance would have made it possible to generate reasonable predictions of the size of the future declines. Our results suggest that market microstructure invariance can be used as a practical tool to help quantify the systemic risks which result from sudden liquidations of speculative positions.

Market microstructure invariance is based on the intuition that the speed of trading varies across markets, i.e., "business time" passes more quickly in active markets than in inactive markets. When appropriate adjustment is made for the rate at which business time passes, market properties related to the dollar rate at which mark-to-market gains and losses are generated do not vary across markets. As discussed in more detail below, this implies specific formulas which relate the size distribution of bets and their price impact to trading volume and volatility.

Two features of microstructure invariance make practical predictions possible.

First, the invariance principle, by its very nature, implies that only a small number of parameter values need to be estimated, and these parameter values are the same for active markets and inactive markets, liquidations of large positions and liquidations of small positions. Thus, rather than attempting the statistically impractical task of estimating *ad hoc* "market crash" parameters from a historical database, including a presumably small number of rare crash events for specific markets, a small number of necessary parameters can be estimated from databases pooling a large number of typical transactions in many different markets, some active and some inactive. In this paper, we use the parameter estimates that Kyle and Obizhaeva (2011*a*) obtain from a database of more than 400,000 portfolio transition trades in individual stocks, typically executed on different days and under normal market conditions. In a portfolio transition, a third-party "transition manager" executes trades which convert a legacy institutional portfolio managed by an incumbent asset manager into a target portfolio managed by a new asset manager. Portfolio transition trades are well-suited for estimating the size and price impact of institutional trades because the sizes of the trades to be executed are objectively known in advance and are typical in size to other institutional trades.

Second, given parameter estimates, practical application of microstructure invariance requires limited market-specific data. To estimate the market impact of a given dollar amount of selling pressure, the only additional pieces of information required are estimates of expected dollar volume and expected returns volatility, both of which can be obtained from recent historical data, such as daily returns and dollar volume data for previous months. It is not necessary to have additional information, such as the extent of order shredding or characteristics of traders.

In a speculative market, price fluctuations occur as a result of some investors placing "bets" which move prices, while other traders attempt to profit by intermediating among the bets being placed. A bet is an "intended order" whose size is known in advance of trading. Large bets can result either from trading by one large entity or from correlated trades of multiple entities based on the same underlying motivation. Société Générale's liquidation of Kerviel's rogue trades, George Soros's large order to sell futures contracts, and the $4 billion sale during the flash crash are three examples of bets placed by one entity. In all three cases, the sellers intended to trade specific quantities before the trades were executed. The forced margin sales by numerous market participants during the 1929 crash and the correlated sales by investors following the strategy of portfolio insurance during the October 1987 crash are two examples of bets representing correlated trades by multiple traders acting for the same underlying reason.

Using dollar volume and returns volatility as its only inputs in addition to a single market depth parameter estimated from portfolio transitions data, the invariance hypothesis generates predictions about the size of the price impacts resulting from the innovations in order flows documented for these crash events. The invariance hypothesis also predicts how frequently market dislocations of these magnitudes are expected to happen. The frequency of crashes depends on the frequency with which bets are placed and the size distribution of the bets themselves. Kyle and Obizhaeva (2011*a*) find that portfolio transition trades follow a distribution similar to a log-normal distribution with variance of 2.50. This large variance implies that half the variance in returns results from fewer than 0.10% of bets and suggests significant kurtosis in returns, consistent with occasional market crashes.

Table 1 summarizes our results, using volume and volatility estimated from daily data over the month before the crash event. For each of the five crash events, the table gives the estimated size of the dollar amounts liquidated

(percent of daily volume and GDP), actual price decline (percent), predicted price decline (percent), and predicted frequency of occurrence of such large bets.

Table 1: Summary of Five Crash Events: Actual and Predicted Price Declines

	Actual	Predicted	%ADV	%GDP	Frequency
1929 Market Crash	25%	49.22%	241.52%	1.136%	once in 5,539 years
1987 Market Crash	32%-40%	19.12%	66.84%	0.280%	once in 716 years
1987 Soros's Trades	22%	7.21%—15.83%	2.29%	0.007%	once per month
2008 SocGén Trades	9.44%	12.37%	27.70%	0.401%	once in 819 years
2010 Flash Crash	5.12%	0.50%—1.91%	1.49%	0.030%	several per year

Table 1 shows the actual price changes, predicted price changes, orders as percent of average daily volume and GDP, and implied frequency.

Table 1 shows that three of the crash events involve much larger selling pressure than the other two. The 1929 crash, the 1987 crash, and the Société Générale trades of 2008 all involve sales of more than 25% of average daily volume the previous month or more than 25 basis points of GDP. By contrast, the sales by Soros in 1987 and the flash crash of 2010 both involve sales of only 2.29% and 1.49% of average daily volume the previous month. Overall, predicted price declines are similar to actual price declines. This suggests that microstructure invariance provides estimates of price impact which could have been useful to policymakers and traders alike.

For the 1929 stock market crash, the actual price decline of 25% was much smaller than the predicted decline of 49.22%. We hypothesize that the smaller than predicted price declines may have resulted from the efforts financial markets made in 1929 to spread the impact of margin selling out over several weeks rather than several days.

For the 1987 stock market crash, the actual decline of 32%–40% was larger than the predicted decline of 19.12%. At the time, academics, policymakers, and market participants were aware of the potential size of portfolio insurance trades, but market participants did not take the size of the potential price

declines seriously enough.

For liquidation of Jerome Kerviel's trades by Société Générale in January 2008, the predicted price decline of broad European index such as the Stoxx Europe Total Market Index was 12.37%, similar to the actual decline of 9.44%. Placed in the context of other market dislocations, this liquidation stands out as being comparable in size to sales of portfolio insurers in 1987. The large size of the potential price impacts suggest that if central banks in Europe and the U.S. had been warned before these trades were executed, they could have prepared a response in advance rather than responded to events *ex post*.

The actual plunges in prices associated with Soros's 1987 trades and the 2010 flash crash, 22% and 5.12% respectively, are larger than the predicted declines of 7.21%–15.83% and 0.50%–1.91% respectively. We hypothesize that both the large size of the price declines and the rapid recoveries followed these two crash events were the result of the speed with which these trades were executed. These were "flash-crash" events in which the trades were executed in minutes, not hours.

Extrapolating from the size distribution of portfolio transition trades, the magnitude of selling during the three large crash events were approximately 6-standard- deviation bet events while the two flash crashes were approximately 4.5-standard-deviation bet events.

Invariance predicts that the smaller 4.5-standard-deviation bets—the size of Soros's in 1987 and the flash crash of 2010—are expected to occur several times per year. We believe that such events probably would not have attracted much notice if their price impact had been reduced by spreading the trades out over hours instead of minutes.

Concerning the 6-standard-deviation crash events, assuming bet rates and a distribution of bet sizes extrapolated from portfolio transition data, crash events similar to the 1929 crash would be expected to occur once every 5,539 years, crash events like the 1987 crash once every 716 years, crash events like and the Société Générale liquidation as infrequently as once every 819 years. Obviously, the actual frequency of crashes is far higher than fitting a log-normal distribution to portfolio transition trades implies. To match actual frequencies of market dislocations, either the variance of the underlying log-normal distribution needs to higher than the value of 2.50 estimated from portfolio transition data in Kyle and Obizhaeva (2011*a*), or the tails of the empirical distribution need to be fatter for extremely large bets, such as would be the case with a power law rather than a log-normal distribution. It is entirely

reasonable to believe that the variance of bets is larger than estimated from portfolio transition data, because these estimates did not take into account the possibility of common bets correlated across asset managers. For example, increasing the standard deviation of the log-normal by only 20% would predict too many crashes, not too few. It would convert 6-standard-deviation events into 5-standard-deviation events, reducing their frequency by a factor of about 300, thus predicting 1929-magnitude crashes approximately once every 20 years and 1987 crashes or Société Générale crashes approximately once every 3 years.

If we think of the results in this paper as letting stock market crashes tell us something about whether portfolio transition trades are a good dataset for testing market microstructure invariance parameters rather than *vice versa*, we conclude that the price impact estimates from portfolio transitions data generalize reasonably well to stock market crashes, but the estimated size distribution of bets needs fatter tails or a higher variance.

In the rest of this paper, we have sections discussing more details about previous literature on market crashes, market microstructure invariance, particulars of each of the five crash events, the frequency of crashes, lessons learned, and concluding thoughts.

1 Conventional Wisdom, Animal Spirits, and Banking Crises

Since before the 1929 crash, economists and market participants have been puzzled by why market crashes occur in the apparent absence of fundamental information. There have long been two camps of thinking, which we will refer to as "conventional wisdom" and "animal spirits." Neither of these two theories offers an explanation for crash events.

Conventional Wisdom. Conventional wisdom holds that prices react to changes in fundamental information, not to the price pressure resulting from trades by individual investors. In the 1960s and 1970s, this conventional wisdom became associated with efficient markets hypothesis and the capital asset pricing model.

The efficient-market hypothesis is consistent with the idea that investors trade for various reasons including portfolio rebalancing motives and profiting

from private information. Since many investors are competing for information, however, one interpretation of the efficient markets hypothesis is that it would be highly unusual for investors to have private information of sufficient value so that the information content of their trades would move prices significantly. Competitive markets with minimal private information thus imply minuscule price impact. Consistent with this theory, Scholes (1972) reports that relatively large blocks may be sold at approximately prevailing market prices.

Conventional wisdom based on the CAPM implies that the demand for market indices is very elastic. It also implies that the demand for individual stocks is almost infinitely elastic. The quantities observed changing hands in the market are too small compared to trading volume or shares outstanding and their effect on prices can not explain dramatic plunges in market prices.

Views consistent with the conventional wisdom are shared by many prominent economists, including Miller (1991), for example, wrote the following about the 1987 crash: "Putting a major share of the blame on portfolio insurance for creating and overinflating a liquidity bubble in 1987 is fashionable, but not easy to square with all relevant facts... No study of price-quantity responses of stock prices to date supports the notion that so large a price increase (about 30 percent) would be required to absorb so modest (1 to 2 percent) a net addition to the demand for shares."

As the academics most associated with portfolio insurance, Leland and Rubinstein (1988) echo this argument: "To place systematic portfolio insurance in perspective, on October 19, portfolio insurance sales represented only 0.2 percent of total U.S. stock market capitalization. Could sales of 1 in every 500 shares lead to a decline of 20 percent in the market? This would imply a demand elasticity of 0.01—virtually zero—for a market often claimed to be one of the most liquid in the world."

The Brady Report compares the market crashes of 1929 and 1987 and comes to similar conclusions about the 1929 crash: "To account for the contemporaneous 28 percent decline in price, this implies a price elasticity of 0.9 with respect to trading volume which seems unreasonably high. As a percentage of total shares outstanding, margin-related selling would have been much smaller. Viewed as a shift in the overall demand for stocks, margin-related selling could have accounted realistically for no more than 8 percent of the value of outstanding stock. On this basis, the implied elasticity of demand is 0.3 which is beyond the bound of reasonable estimates."

Brennan and Schwartz (1989) note that portfolio insurance would have a minimal effect on prices, because most portfolio-consumption models imply elasticities of demand for stock more than 100 times the elasticities observed during the 1987 crash. Many observers of the 1987 stock market crash, including Miller (1988, p. 477) and Roll (1988), looked therefore to explanations other than the price pressure of the large quantities traded to explain the large changes in prices. For all five crash events, however, it is difficult to find new fundamental information shocks to which market prices would have reacted with the magnitude of price declines actually observed.

We disagree with the conventional wisdom on the grounds that the conventional wisdom of low price impact is empirically incorrect. Large trades, even those known to have no information content such as the margin sales of 1929 or the portfolio insurance sales in 1987, do have large effect of prices. Our examination of five historical episodes suggests that actual price changes are similar to those predicted by the invariance formula. Indeed, the invariance price-impact model performs well when extrapolating from portfolio transition trades in normal markets for individual stocks to unusually large bets in market indexes.

Animal Spirits. Animal spirits holds that price fluctuations occur as a result of random changes in psychology, which may not be based on information or rationality. The term "animal spirits" was coined in by Keynes (1936) who says that most of our decisions can be taken only as the result of "animal spirits—a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities." Akerlof and Shiller (2009) recently advocated for this theory again: "To understand how economies work and how we can manage them and prosper, we must pay attention to the thought patterns that animate people's ideas and feelings, their animal spirits." According to animal spirit theory, market crashes are times when decisions are driven by emotions and social-psychologically induced changes in mind set instead of rational calculations. Promtly after the 1987 crash, for example, Shiller (1987) surveyed traders and found that "most investors interpreted the crash as due to the psychology of other investors."

We disagree with animal spirits theory. Animal spirits suggests random unpredictable nature of stock market crashes. In reality, large crashes are neither random nor unpredictable. The possibility of crashes is often discussed before

crashes occur. For example, brokers were raising margins before the 1929 stock market crash in order to protect themselves from increases in expected market volatility. The SEC, worried that portfolio insurance made the market fragile, did the study of portfolio insurance before the 1987 crash happened. Smaller events like the sales of George Soros in 1987, the liquidation of fraudulent position by Société Générale, and the flash crash were unpredictable, but prices rapidly mean-reverted. Ironically, the more predictable the crash is, the more permanent the price impact it triggers.

Stock Market Crashes and Banking Crises. The five stock market crashes discussed in this paper differ from the long-lasting financial crises catalogued by Reinhart (2009). The crises examined by Reinhart (2009) include sovereign defaults, banking crises associated with collapse of the banking system, exchange rate crises associated with currency collapse, and bouts of high inflation. Reinhart (2009) document that it usually takes many years and significant changes in macroeconomic policies and market regulations for the affected economies to recover from these fundamental problems associated with insolvency of financial institutions underlying the economy.

In contrast, stock market crashes or panics triggered by large bets are likely to be short-lived if followed by appropriate government policy. For example, the Federal Reserve System implemented an appropriately loose monetary policy immediately after the 1929 crash, which calmed down the market by the end of 1929. The great depression of the 1930s resulted from subsequent deflationary policies associated with the gold standard, not the 1929 crash. In the liquidation of Jerome Kerviel's rogue trades in 2008, the immediate 75-basis point interest rate cut by the Fed probably prevented this event from immediately spiraling into a deeper financial crisis, but it did not prevent the collapse of Bear Stearns a few weeks later. It was the bursting of the real estate credit bubble, not the unwinding of Jerome Kerviel's fraud, that led to the deep and long-lasting recession which unfolded in 2008-2009.

2 Market Microstructure Invariance

The invariance hypothesis is based on the simple intuition that traders play trading games, the rules of these trading games are the same across stocks and across time, but the speed with which these games are played varies across

stocks based on levels of trading activity. Trading games are played faster if securities have higher levels of trading volume and volatility.

As discussed in Kyle and Obizhaeva (2011*b*), this intuition leads to simple formulas for market depth and bid-ask spread as functions of observable dollar trading volume and volatility. The expected percentage price impact from buying or selling X shares of a stock with a current stock price P dollars, expected trading volume V shares per calendar day, and daily percentage standard deviation of returns σ ("volatility"), is given by

$$\frac{\Delta P(X)}{P} = \exp\left[\bar{\lambda}/10^4 \cdot \left(\frac{P \cdot V}{40 \cdot 10^6}\right)^{1/3} \cdot \left(\frac{\sigma}{0.02}\right)^{4/3} \cdot \frac{X}{(0.01)V}\right] - 1. \qquad (1)$$

In this formula, the market impact parameter $\bar{\lambda}$ is scaled so that it measures the percentage market impact of trading $X = 1\%$ of expected daily volume V of a hypothetical "benchmark stock" with stock price of \$40 per share, expected daily volume of one million shares, and volatility of 2% per day. The formula shows how to extrapolate market impact for the benchmark stock to assets with different levels of dollar volume and volatility. Microstructure invariance also makes predictions about bid-ask spread costs. In the context of significant market dislocations, bid-ask spread costs are so small relative to impact costs that we ignore them in this paper.

We chose to consider continuously compounding returns rather than simple returns as in Kyle and Obizhaeva (2011*a*), because our analysis deals with very large orders, sometimes equal in magnitude to trading volume of several trading days. In contrast, Kyle and Obizhaeva (2011*b*) consider relatively small portfolio transition orders with the average size of about 3.90% of daily volume and median size of 0.59% of daily volume; for these orders, the distinction between continuous compounding and simple compounding is immaterial.

Kyle and Obizhaeva (2011*a*) estimate the parameter $\bar{\lambda} = 5.78$ basis points (standard error $2 \cdot 0.195$), using data on implementation shortfall of more than 400,000 portfolio transition trades. A portfolio transition occurs when one institutional asset manager is replaced by another. Trades converting the legacy portfolio into the new portfolio are typically handled by a professional transition manager. Implementation shortfall, as discussed by Perold (1988), is the difference between actual execution prices and prices based on transactions-cost-free "paper trading" at prices observed in the market when the order is placed. Portfolio transition trades are ideal for using implementation shortfall to estimate transactions costs because the known exogeneity of the size of the

trades eliminates selection bias.

Formula (1) describes market impact during both normal times and times of crash or panic, for individual stocks and market indices. Most of the events that we consider in this paper occurred in markets with high trading volume and during the times of significant volatility. For market with exceptionally high trading volume and volatility, the market impact implied by equation (1) is greater than the impact obtained from the conventional heuristics.

The conventional wisdom about market impact can be illustrated by a naive implementation of the the formula $\lambda = \sigma_V / \sigma_U$ from Kyle (1985). Under the assumptions that the standard deviation of fundamentals σ_V is proportional to price volatility $\sigma \cdot P$ and the standard deviation of order imbalances σ_U is proportional to dollar volume V, the price impact can be calculated as

$$\frac{\Delta P(X)}{P} = \exp\left[\bar{\lambda}/10^4 \cdot \left(\frac{\sigma}{0.02}\right) \cdot \frac{X}{(0.01)V}\right] - 1. \tag{2}$$

There are two main differences between the invariance hypothesis and conventional wisdom. First, according to the conventional wisdom in equation (2), increasing dollar volume by a factor of $1,000$—approximately consistent with dollar volume differences between a benchmark stock and stock index futures—the impact of executing an order equal to a given percentage of expected daily volume does not change. According to microstructure invariance, the same increase in dollar volume increases the price impact of trading a given percentage of average daily volume by a factor of $(1000)^{1/3} = 10$. The impact is ten times greater than conventional wisdom would predict. Second, according to conventional wisdom, doubling volatility doubles the market impact of trading a given percentage of expected daily volume. According to microstructure invariance, doubling volatility increases the price impact of trading a given percentage of expected daily volume by a factor of $2^{4/3} \approx 2.52$.

When the effects of volume and volatility are taken into account, as suggested by the invariance hypothesis, we conclude that the observed market dislocations could have been caused by selling pressure, because their effect on prices is much higher than conventional wisdom suggests. The execution of large bets—"small" relative to large overall trading volume—can lead to significant changes of market prices, especially during volatile times.

In the last section of the paper, we also examine whether the frequency of crashes and panics matches the predictions of invariance hypothesis.

Implementation Issues. In order to apply the model of market microstructure invariance to the data on observed market dislocations, several implementation issues need to be addressed.

First, it is necessary to identify the boundaries of the market, given that different securities and futures contracts, traded on various exchanges, may share the same fundamentals. For example, when a large order is placed in the S&P 500 futures market, should the market volume include only S&P 500 future volume, or should it also include volume in the 500 underlying stocks, stocks not part of the index, ETFs, index options, and other related markets? Since a bet in S&P 500 futures contracts is a bet about the entire U.S. economy, it should be related to the markets for all other securities though some factor structure. Thus, the volume and volatility inputs in our formulas should not be thought of as parameters of narrowly defined markets of a particular security in which the bet was placed, but rather as parameters from much broader markets. While at this time we do not have a definitive understanding of how to aggregate estimates across economically related markets in the context of the invariance hypothesis, this is an interesting issue for further research.

Second, it is likely that the price impact of an order—especially its transitory price impact— is related to the speed or aggressiveness with which this order is executed. Our market impact formula assumes that orders are executed at an appropriate speed in some "natural" units of time, with the speed proportional to the speed with which the trading game itself is being played. For example, a very large trade in a small stock may be executed over several weeks, while a large trade in the stock index futures market might be executed over several hours. If execution is speeded up relative to a natural flow of time, then our formula probably underestimates the expected cost. For unusually rapid execution of orders, we expect to see larger immediate price impact than implied by our estimate from portfolio transitions data; moreover, we expect much of this impact to be transitory, reversing itself soon after the trade is completed.

Third, our price impact estimates are based on assumptions about the expected volume and the expected volatilities prevailing during extreme events. We estimate volume and volatility based on historical data for recent months before the crash or panic event. During times of market stress, both volume and volatility can increase. If higher levels of contemporaneous volume and volatility are used to estimate price impact, the estimated impact will be higher as a result of the greater volatility and lower as a result of the greater

volume (holding the size of the order constant as a fraction of past volume). Whether unusually high volume or volatility at the time of order execution are associated with higher price impact is not well-understood. This is an interesting issue for future research.

Fourth, while our market impact formula predicts expected price changes, the actual price changes reflect not only sales by particular groups of traders placing large bets but also many other events occurring at the same time, including arrival of news and trading by other traders. Our identifying assumption is that the effect of these forces on prices is zero. We also provide a brief discussion of how other factors could have influenced market prices during the episodes we examine.

The remainder of this paper extrapolates the invariance model to examine several market dislocations.

3 The Stock Market Crash of October 1929

The October 1929 stock market crash is the most infamous crash in the history of the United States. The crash of 1929 became seared in the memories of many because it is associated with the even more extraordinary decline in stock prices which occurred from 1930 to 1932, bank runs, and the Great Depression.

In the 1920s, many Americans became heavily invested in stocks. In many dimensions, stock market speculation in the late 1920s was similar to stock speculation in the late 1990s. In the late 1920s, a significant portion of stock investments was made in margin accounts. After doubling in value during the two years prior to September 1929, the Dow Jones average fell by 9% from 336.13 to 305.85 during the week before Black Thursday, October 24, 1929, including a drop of 6.32% on Wednesday October 23. This steep price decline led to liquidations of stocks in margin accounts on the morning of Black Thursday. During the first few hours of trading, the Dow Jones average fell from the Wednesday closing value of 305.85 to 272.32, a decline of 11%. After a group of prominent bankers publicly announced steps to support the market with significant purchases, the decline began to reverse itself. By the Friday close, October 25, 1929, the index recovered to 301.22, but confidence was badly shaken.

Market conditions worsened the following week, with more heavy margin selling. On Black Monday, October 28, 1929, the Dow plummeted 13.47%, closing at 260.64. On Black Tuesday, October 29, 1929, the Dow fell an addi-

tional 11.73%, closing at 230.07. Thus, over one week, the Dow fell by about 25%. The slide continued for three more weeks, with prices reaching a temporary low point of 198.69 on November 13, 1929, about 48% below the high of 381.17 on September 3, 1929. During this period, the New York Fed bought government securities and cut its discount rate twice in an effort to restore confidence and provide liquidity to the financial system.

How much margin selling occurred during the week which included Black Thursday, Black Monday, and Black Tuesday? We follow the previous literature by trying to estimate margin selling indirectly from data on broker loans. Our research strategy is to use changes in broker loans during the fall of 1929 to infer the amount of margin selling of stocks. This research strategy in consistent with the way regulators and market participants looked at the situation in the 1920s.[1]

In the 1920s, there was rapid growth in credit used to finance ownership of equity securities and upward pressure on interest rates. Demand for stocks shifted the supply of funds to the debt market down, while demand for leverage to finance stock investment increased demand for credit. To finance their purchases, individuals and non-financial corporations relied either on bank loans collateralized by securities or on margin account loans at brokerage firms.

When individuals and non-financial corporations borrowed through margin accounts at brokerage firms, the brokerage firms financed a modest portion of the loans with credit balances from other customers. To finance the rest, brokerage firms pooled securities pledged as collateral by customers under the name of the brokerage firm (i.e., in "street name") and then "re-hypothecated" these pools by using them as collateral for broker loans. In some ways, the broker loan market of the 1920s played a role similar to the shadow banking system of the first decade of the 21st century. Similarities include the large size of the market, lack of its regulation, its perceived safety, and the large fraction of overnight or very short maturity loans.

High interest rates on broker loans—typically 300 basis points or higher than interest rates on loans on otherwise similar money market instruments—were attractive to lenders. Banks supplied their funds to the market, with New York banks frequently acting as intermediaries arranging broker loans for

[1]Our analysis is based on several documents: Board of Governors of the Federal Reserve System (1929, 1927-1931); Monetary Statistics Book ; Galbraith (1954); Senate Committee on Banking and Currency (1934); Friedman and Schwartz (1963) ; Smiley and Keehn (1988); Haney (1932)

non-New-York banks and non-bank lenders. Instead of investing in common stocks deemed to be overvalued, investment trusts, which played a role similar to closed end mutual funds today, also placed a large fraction of the new equity they raised into the broker loan market. Finally, as a result of growing earnings and proceeds of securities issuance, corporations possessed considerable cash balances. Attracted by high interest rates, some of them also invested a large portion of their funds in the broker loan market rather than in new plant and equipment.

The broker loan market was controversial during the 1920s, just as the shadow banking system was controversial during the period surrounding the financial crisis of 2008-2009. Some thought the broker loan market should be tightly controlled to limit speculative trading in the stock market on the grounds that lending to finance stock market speculation diverted capital away from more productive uses in the real economy. Others thought it was impractical to control lending in the market, because the shadow bank lenders would find ways around restrictions and lend money anyway. The New York Fed chose to discourage New York banks from increasing broker loans and other loans collateralized by securities, and as a result, loans to brokers by these banks declined after reaching a peak in 1927. This put upward pressure on broker loan rates and attracted even more non-bank and foreign bank lenders into the market. The non-bank lenders often bypassed the banking system entirely by making loans to brokerage firms directly.

Market participants in the late 1920s watched statistics on broker loans carefully, noting the tendency for broker loans to increase as the stock market rose. Markets were also aware that margin account investors were buyers with "weak hands," likely to be flushed out of their positions by margin calls if prices fell significantly. They thought deeply about who the buyers would be if a collapse in stock prices forced margin account investors out of their positions. Such discussions in 1929 mirrored similar discussions in 1987 concerning who would take the opposite side of portfolio insurance trades.

Data on Broker Loans. In the 1920s, data on broker loans came from two sources. First, the Fed collected weekly broker loan data from reporting member banks in New York City supplying the funds or arranging loans for others. Second, the New York Stock Exchange collected monthly broker loan data based on demand for loans by NYSE member firms. The Fed data do not include broker loans which non-banks made directly to brokerage firms without

using banks as intermediaries; such loans bypassed the Fed's reporting system. The broker loan data reported by the New York Stock Exchange do include some of these broker loans. Since loans unreported to the Fed were a significant source of broker loans and these loans fluctuated significantly around the 1929 stock market crash, we rely relatively heavily on the NYSE numbers in our analysis below, but also pay careful attention to the weekly dynamics of the Fed series for measuring selling pressure during the last week of October 1929.

Figure 1 shows the weekly levels of the Fed's broker loan series and the monthly levels of the NYSE broker loan series. Two versions of each series are plotted, one with bank loans collateralized by securities added and one without. In addition, the figure shows the level of the Dow Jones Industrial Average from 1926 to 1930. The time series on both broker loans and stock prices follow similar patterns, rising steadily from 1926 to October 1929 and then suddenly collapsing. According to Fed data, broker loans rose from \$3.141 billion at the beginning of 1926 to \$6.804 billion at the beginning of October 1929. According to NYSE data, the broker loan market rose from \$3.513 billion to \$8.549 billion during the same period. As more and more non-banks were getting involved in the broker loan market, the difference between NYSE broker loans and Fed broker loans steadily increased until the last week of October 1929, when the difference suddenly shrank as these firms pulled their money out of the broker loan market.

During the period 1926 to 1930, the weekly changes in broker loans were typically relatively small and often changed sign, as shown in the bars at the bottom of figure 1. The last week of October 1929, which marked the beginning of the stock market crash, and the first weeks of November 1929 were significant exceptions. During these weeks, there were huge negative changes almost twenty times larger than the average magnitude of changes during other weeks. During a period of several weeks, this huge deleveraging reduced the level of broker loans back to the beginning of 1928.

Figure 2 shows what happened between September 4, 1929, and December 31, 1929, in more detail. In the weeks leading up to the stock market crash during the last week of October 1929, the reported Fed numbers were stable: \$6.761 billion on September 25, \$6.804 billion on October 2, \$6.713 billion on October 9, \$6.801 billion on October 16, and \$6.634 billion on October 23. As the market crashed during the last week of October 1929, the quantity of broker loans reported by the Fed collapsed as well. Reported broker loans fell to \$5.538 billion on October 30, \$4.882 billion on November 6, \$4.172 billion on

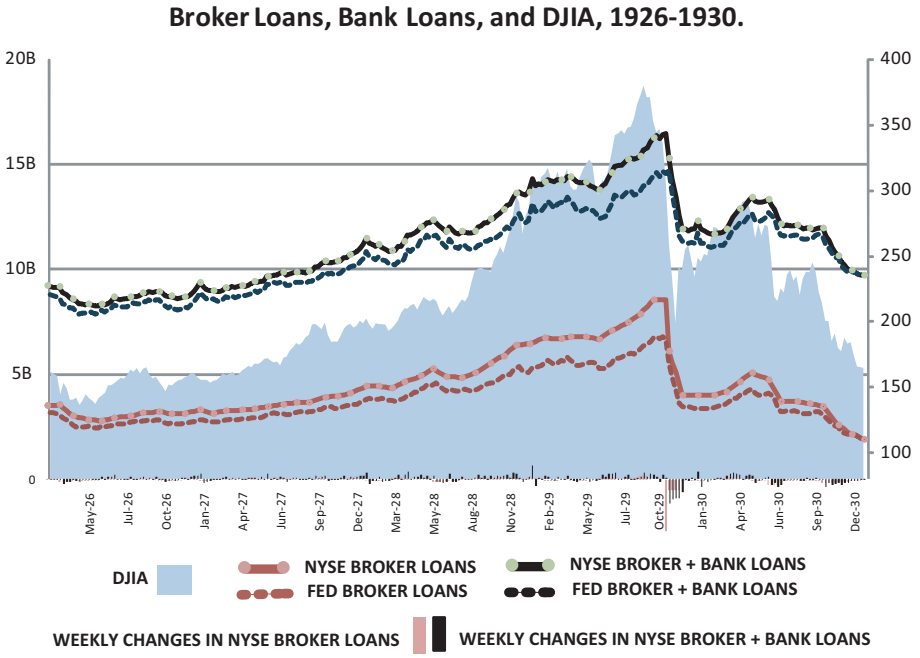


Figure 1: Broker Loans and 1929 Market Crash.

Figure shows the weekly dynamics of seven variables during January 1926 to December 1930: NYSE broker loans (red solid line), Fed broker loans (red dashed line), the sum of NYSE broker loans and bank loans (black solid line), the sum of Fed broker loans and bank loans (black dashed line), changes in NYSE broker loans (red bars), changes in the sum of NYSE broker loans and bank loans (black bars), and the Dow Jones averages (in blue). Monthly levels of NYSE broker loans are marked with markers. Weekly levels of NYSE broker loans are obtained using a linear interpolation from monthly data; except for October 1929, when all changes in NYSE broker loans are assumed to occur during the last week.

November 13, $3.587 billion on November 20, and $3.450 billion on November 27.

The monthly broker loans as reported by the NYSE were $8.549 billion on

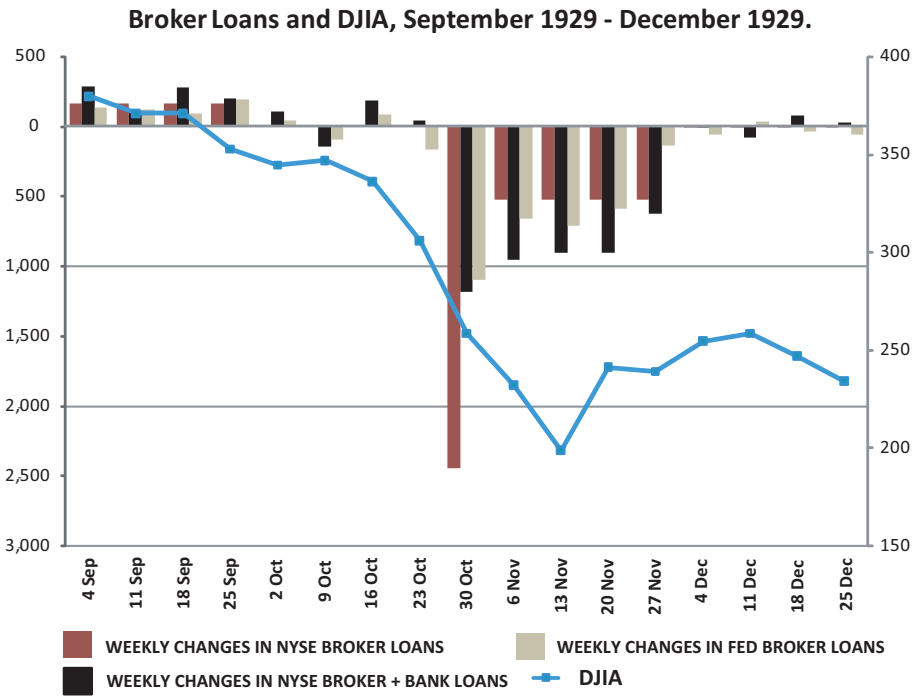


Figure 2: Broker Loans during September 1929 to December 1929.

Figure shows the dynamics during September 1929 to December 1929 of the Dow Jones averages (blue line), weekly changes in NYSE broker loans (red bars), weekly changes in the sum of NYSE broker loans and bank loans (black bars), and weekly changes in Fed broker loans (grey bars). Weekly levels of NYSE broker loans are obtained using a linear interpolation from monthly data; except for October 1929, when all changes in NYSE broker loans are assumed to occur during the last week.

September 30, $6.109 billion on October 31, $4.017 billion on November 30, 1929, and $3.990 billion on December 31. We estimate weekly values for the NYSE monthly time series by linearly interpolating values from the weekly Fed series, with the exception of the critical month of October 1929. Based on the patterns of weekly Fed numbers during that month, we assume that

the October decline in broker loans occurred entirely during the last week of October. Thus, as measured by the NYSE, broker loans fell by $2.340 billion during last week of October and then by an additional $2.092 billion in November 1929. From September 30 to December 31, the NYSE broker loans fell by $4.559 billion.

Immediately after the initial stock market break on Black Thursday, a group of prominent New York bankers had put together an informal fund of about $750 million to provide support to the market. According to press reports, the group did not intend to support prices at a particular floor, but rather intended to provide bids as prices fell, thus allowing the market to find a new level in an orderly manner. The group appears to have supported the market by allowing the positions of large under-margined stock investors to be liquidated gradually.

While there was panic in the stock market during 1929 crash, there was no observable financial panic in the money markets. In this respect, the panic surrounding the 1929 stock market crash was entirely different from the panic surrounding the collapse of Lehman Brothers in 2008. From past experience pre-dating the establishment of the Fed in 1913, Wall Street was familiar with financial panics in which fearful lenders suddenly withdrew money from the money markets, short term interest rates spiked upwards, credit standards became more stringent, and weak borrowers were forced to liquidate collateral at distressed prices. In the last week of October 1929, interest rates actually fell and credit standards were relaxed by major banks, which cut margin requirements for stock positions. Some lenders abandoned the broker loan market because falling interest rates made lending in the broker loan market far less attractive than it used to be. The result was an unprecedented spike in demand deposits at New York banks, which rose from $13.314 billion to $15.110 billion during the last week in October. This increase in demand deposits conveniently gave the banks plenty of cash to use to finance increased loans on securities. The New York Fed encouraged easy credit by purchasing government securities, by cutting the discount rate, and by encouraging banks to expand loans on securities to support an orderly market.

As reported by Board of Governors of the Federal Reserve System (1927-1931) for 1929, bank loans on securities were relatively stable in the weeks leading up to the crash during the last week of October, ranging from $7.632 billion on September 4 to $7.920 billion on October 23. During the week of the crash beginning on October 23, the level of bank loans on securities

increased abruptly by $1.259 billion to $9.179 billion on October 30. The sudden increase in bank lending was unprecedented. It also turned out to be temporary. Loans on securities fell to $8.746 billion on November 6 and $8.369 billion on November 13. In the latter half of November, loans on securities fell to around $7.900 billion, similar to the level at the beginning of October, and stayed at that level until the end of 1929. On December 25, loans on securities were equal to $7.931 billion.

The large increase in loans on securities is consistent with the interpretation that bankers took the financing of some under-margined accounts out of the hands of brokerage firms and brought the broker loans onto their own balance sheets. The gradual reduction in these loans over several weeks suggests that the bankers were liquidating these positions gradually in order to avoid excessive price impact and thus contributed to a more orderly market. Instead of fire sale prices resulting from a credit squeeze, the picture was one of a sudden, brutal bursting of a stock market bubble financed by prudent margin lending to imprudent borrowers, with a rapid return to "normal" price levels in the stock market.

We define the time interval for the stock market crash of 1929 as the last week of October. The total reduction in brokerage loans during this week was approximately equal to $2.340 billion. The transfers of pledged collateral from brokerage firms to banks was equal to $1.259 billion. The amount of margin selling of stocks during that week can be therefore approximated by $1.181 billion ($2.340 billion minus $1.259 billion). From September 30 to December 31, the NYSE broker loans decreased by $4.559 billion from $8.549 billion to $3.990 billion. From September 25 to December 25, the bank loans on securities increased by $0.211 billion from $7.720 billion to $7.931 billion. The amount of margin selling of stocks during that last three weeks on 1929 can be therefore approximated by $4.348 billion, a total of about 4.18% of 1929 GDP of $104 billion

Our estimate assumes that every dollar in reduced margin lending represents a dollar of margin selling. In theory, it is possible for margin lending to fall for other reasons, including sales of bonds financed in margin accounts and cash transfers from bank accounts to margin accounts at brokerage firms. We doubt that bond sales or transfers from bank accounts were significant during the last week of October 1929 because the high interest rate spread between broker loan rates and interest rates on bonds and bank accounts would have made it non-economical for investors to finance bonds in margin accounts or to

maintain extra cash balances at banks while simultaneously holding significant margin debt.

Market Impact of Margin Selling. For the purposes of examining the implications of microstructure invariance, we define the 1929 crash period as the last week of October, during which stock prices fell 25%. Are forced margin calls of $1.181 billion in the last week of October 1929 massive enough to cause the observed downward spiral in stock prices? To apply the price impact equation (1) to the 1929 crash, we need to have estimates of dollar volume and volatility. We can then compare the market price decline implied by microstructure invariance with the historical price decline of 25% during the last week of October 1929. Of course, this exercise provides only rough estimates for price changes, since we must make a number of simplifying assumptions.

To convert 1929 dollars to 2005 dollars, we use the GDP deflator of 9.42. We use the year 2005 as a benchmark, because the estimates in Kyle and Obizhaeva (2011*a*) are based on the sample period 2001-2005, with more observations occurring in the latter part of that sample. In the month prior to the market crash, typical trading volume was reported to be $342.29 million per day in 1929 dollars, or almost $3.22 billion in 2005 dollars. Prior to 1935, the volume reported on the ticker did not include "odd-lot" transactions and "stopped-stock" transactions, which have been estimated to account for about 30 percent of the "reported" volume. We therefore adjust reported volume by multiplying it by the fraction 10/7. Historical volatility the month prior to October 1929 was about 2.00% per day. The total value of $1.181 billion traded during the last week of October is approximately equal to 242% of average daily volume in the previous month.

The price impact equation (1) implies that the forced margin-related sales of $1.181 billion triggered a price decline of 49.22%, calculated as

$$1-\exp\Big[-\frac{5.78}{10^4}\cdot\Big(\frac{488.98\cdot 10^6\cdot 9.42}{(40)(10^6)}\Big)^{1/3}\cdot\Big(\frac{0.0200}{0.02}\Big)^{4/3}\cdot\frac{1.181\cdot 10^9}{(0.01)(488.98\cdot 10^6)}\Big]. \tag{3}$$

As a robustness check, table 2 reports other estimates using historical trading volume and volatility calculated over the preceding N months, with $N = 1, 2, 3, 4, 6, 12$. The actual market drop in the the last week of October 1929 was 25%, significantly less that our predicted price declines ranging from 31.05% to 49.22%.

We also estimate implied margin sales during the entire period from September 25, 1929 to December 25, 1929. The change in broker loans netting out changes in bank loans financed by securities was equal to $4,348 billion during that period ($4.559 billion minus $0.211 billion). These transactions are expected to trigger a price decline ranging in size from 71.25% to 91.75%, which are much bigger than a contemporaneous price decline of 34% from 352.57 on September 25 to 234.07 on December 25. We believe that there are three reasons that the market crash of 1929 may have been so well contained.

First, there was clearly significant cash waiting on the sidelines to be invested in stocks in the event stock prices fell significantly. Some of this cash represented stock issuance by investment trusts and non-financial corporations.

Second, we believe that by spreading out the margin selling over a period of five weeks instead of a few days, the financial system of 1929 reduced the price impact which might otherwise have occurred.

Third, financial markets in 1929 may have been less integrated than today. For example, if we think of the stock market of 1929 as 125 separate markets in different stocks, invariance implies that price impact estimates would be reduced by a factor of $125^{1/3} = 5$.

Fourth, a legitimate question is whether the estimated price impact of 5.78 basis points calibrated for a benchmark stock using data on 2001-2005 can be extrapolated to earlier time periods. This is ultimately an empirical question, but a historical episode described in Wigmore (1985) implies that predictions of the invariance formula are not too far off, even when applied to other events in 1920s. Wigmore (1985, pp. 22-23) mentions that a sale of 189,760 shares of a tobacco company Webster-Eisenlohr at auction on November 6, 1929 resulted in share price falling from 7 1/8 to 2 1/8 by 70%. He further points out "rumors said the block was from the estate of Anthony Schneider, President, who died on October 30th in a fall from his apartment window". The average volume of Webster-Eisenlohr was about $55,000 per day. Since stock price plunged from 70 5/8 to 9 in the month prior to November 6, the historical volatility of 36% per day was certainly very high. If instead we assume expected volatility of 5 percent per day, the price impact equation (1) implies a price impact of 76 percent.

Nevertheless, one of the main lessons learned from applying market microstructure invariance to the 1929 crash is that financial markets in 1929 appear to be very resilient when compared with today's markets.

4 The Market Crash in October 1987

On October 14, 1987, the U.S. equity market began the most severe one-week decline in its history. The Dow Jones index dropped from about 2500 on the morning of Wednesday, October 14, 1987, to 1700 on Tuesday, October 20, 1987, a decline of 32%. Even worse, S&P 500 futures fell from 312 on the morning of October 14, 1987, to 185 at noon on October 20, 1987, a decline of about 40%.

Some market observers blamed portfolio insurance for this dramatic decrease in prices. Portfolio insurance was a trading strategy that replicated put option protection for portfolios by dynamically adjusting stock market exposure in response to market fluctuations. Since this strategy requires portfolio insurers to sell stocks when stock prices fall, following the strategy indeed generates large sales in falling markets, thus amplifying downward pressure on prices. Most portfolio insurers traded stock index futures contracts to implement the strategy. There has been a longstanding debate about the extent to which portfolio insurance trading contributed to the 1987 market crash.

An important question is therefore whether the size of sales by portfolio insurers was large enough to create price impact that explains the magnitude of price declines observed during the turbulent month of October 1987. Given estimates of the selling pressure exerted on the markets by portfolio insurance sales, we use the invariance formula to predict the price impact of portfolio insurance sales during the crucial days in October 1987. In order to calculate the predictions, we need to make several assumptions related to measurement of volume and volatility.

First, the stock market crash of 1987 occurred during chaotic market conditions, the primary symptom of which was a dramatic increase in volatility. The spirit of the invariance hypothesis is that the volatility in the price impact equation (1) represents the volatility that investors expect. This volatility determines the size of bets investors are willing to make and the degree of market depth they are willing to provide. We assume that the chaotic conditions surrounding the stock market crash of 1987 are captured by potentially high volatility estimates used as inputs into the formula. Note that dramatically different price impact estimates are possible, depending on whether volatility estimates are based on implied volatilities before the crash, implied volatilities during the crash, historical volatilities based on the crash period itself, or historical volatilities based on months of data before the crash.

Second, there have been numerous changes in market mechanisms between 1987 and 2005, including changes in order handling rules in 1998 affecting NASDAQ stocks, a reduction in tick size from 12.5 cents to one cent, and the migration of trading in stocks and futures from face-to-face trading floors to electronic platforms. While such changes may have lowered the bid-ask spread component of transactions costs, we assume that they have had little effect on market depth. This assumption makes it possible to apply market depth estimates for 2001-2005 to the 1987 experience.

Third, the NYSE and NASDAQ markets for individual stocks are connected to index futures contracts by arbitrage relationships. Trading by index arbitragers normally insures that the stock index futures market and the cash market move closely together. Most portfolio insurance strategies were implemented by trading futures contracts, providing "overlay" protection to an underlying portfolio of stocks. As heavy selling pressure in the futures markets pushed futures prices down relative to stock prices in the cash markets, normal arbitrage relationships broke down, and futures contracts became unusually cheap relative to the cash market. Many portfolio insurers abandoned their reliance on the futures markets and switched to selling stocks directly.

Given that markets for underlying stocks also exchange idiosyncratic risks, there is probably a more precise way to address the issue of how liquidity is aggregated across markets. A satisfactory theory should also address the issue of how liquidity is aggregated across correlated stock markets in different countries. During the stock market crash of 1987, stock indices fell in all major worldwide markets, indicating a systemic event of international proportions. The analysis of the 1987 crash by Roll (1988) identified the worldwide nature of the crash as an issue indicating some force at work other than the selling pressure of portfolio insurance in the U.S. market alone. Both the manner in which price pressure spreads among markets connected by strong arbitrage relationships, such as index futures and underlying stocks, and the manner in which price pressure spreads across correlated markets not connected by strong arbitrage relationships, such as U.S. and European stock markets, are important areas for theoretical and empirical research on market microstructure invariance. Currently, we do not have a detailed understanding concerning how to aggregate market depth measures across correlated markets. Consistent with the spirit of the Brady Report, we take the admittedly simplified approach of adding together cash and futures volume in the U.S., while ignoring stock markets in other countries.

Fourth, several news announcements on October 14, 1987, may have had a negative effect on prices. The filing of anti-takeover tax legislation induced risk arbitrageurs to sell stocks of takeover candidates. The announcement of poor numbers for the trade deficit for August 1987 also had a negative effect. These negative news announcements may themselves have sent prices lower, providing a trigger for the portfolio insurance sales which followed.

In the month prior to market crash, the typical daily volume in the S&P 500 futures market was equal to roughly $10.37 billion in 1987 dollars. The NYSE average daily volume was $10.20 billion in 1987 dollars. After adjusting these numbers for realized inflation by multiplying by the GDP deflator of 1.54 between 1987 and 2005, the trading volume in both markets was roughly equal to $15.97 billion and $15.71 billion in 2005 dollars, respectively. In the month prior to the crash, the historical volatility of S&P 500 futures returns was about 1.35% per day. Similar estimates can be obtained from the Brady Report.

We reconstruct selling pressure induced by portfolio insurers from tables in the Brady Report, pp. 197-198. From Figure 13 and Figure 14, we obtain the dollar volume of the largest traders on the NYSE: $257 million sold and $201 million bought on October 15, $566 million sold and $161 million bought on October 16, $1,748 million sold and $449 million bought on October 19, and $698 million sold and $863 million bought on October 20. From Figure 15 and Figure 16, we obtain the dollar volume of the largest traders in S&P 500 futures on the CME: $534 million sold and $71 million bought on October 14, $968 million sold and $109 million bought on October 15, $2,123 million sold and $109 million bought on October 16, $4,037 million sold and $113 million bought on October 19, and $2,818 million sold and $505 million bought on October 20. Total sell order imbalances, defined as sell orders minus buy orders and aggregated over the time span from October 14, 1987, to October 20, 1987, were $9.51 billion in the S&P 500 futures and $1.60 billion in the NYSE stocks in 1987 dollars. In 2005 dollars, these numbers correspond to $14.65 billion and $2.46 billion, respectively.

Some of the market participants classified as portfolio insurers in the Brady Report abandoned their portfolio insurance strategies in mid-October 1987. Instead of selling index futures and the NYSE stocks, as suggested by their trading portfolio insurance strategies, they switched to buying these securities with the hope of making profits by providing liquidity to others in clearly distressed financial markets. We think that for the purpose of analyzing the effect of "true" portfolio insurers on the market prices, it is more appropriate

to leave aside purchases of portfolio insurers during that period and consider only sales generated by this group, which amounted to $10.48 billion in the S&P 500 index futures and $3.27 billion in the NYSE stocks in 1987 dollars. The aggregate selling pressure of these sales—equal to about 67% of one day's volume in both markets during the previous month—is predicted to lead to a price decline of 19.12%, obtained from equation (1) as

$$1-\exp\Big[-\frac{5.78}{10^4}\cdot\bigg(\frac{(10.37+10.20)\cdot 10^9\cdot 1.54}{40\cdot 10^6}\bigg)^{1/3}\cdot\Big(\frac{0.0135}{0.02}\Big)^{4/3}\cdot\frac{(10.48+3.27)}{(0.01)(10.37+10.20)}\Big] \tag{4}$$

Table 3 reports, for robustness, other estimates based on historical trading volume and volatility calculated over the preceding N months, with $N = 1, 2, 3, 4, 6, 12$. We also report separately price impact based on portfolio insurers' gross sell orders and net sell order imbalances calculated as sales minus purchases. The table shows that the estimated price impact of portfolio insurers' order imbalances ranges from 11.13% to 15.75%. The estimated price impact of portfolio insurers' sales ranges from 13.59% to 19.12%.

For robustness, we also calculate price changes for S&P 500 futures and NYSE stock markets separately, but we have to adjust quantities sold in these markets by transactions of index arbitrageurs, who spread out effects of portfolio insurers' sales across markets. From Figure 13 and Figure 14 of the Brady Report, we obtain the dollar volume of index arbitrageurs in S&P 500 futures market: $717 million sold and $407 million bought on October 15, $1,592 million sold and $394 million bought on October 16, $1,774 million sold and $110 million bought on October 19, and $128 million sold and $32 million bought on October 20. In total, index arbitrageurs sold $3,238 billion in NYSE market and probably bought equal amount in S&P 500 futures market.

For the S&P 500 futures market, we adjust $10.48 billion sold in that market by portfolio insurers by subtracting $3,238 billion provided by index arbitrageurs and, using volatility and volume of S&P 500 futures, find that the effect of these transactions on S&P 500 futures prices varies from 11.58% to 16.12%. For the NYSE stock market, we adjust $3.27 billion sold in that market by portfolio insurers by adding $3,238 billion sold by index arbitrageurs and, using dollar volume of NYSE market with volatility of S&P 500 futures, find that the effect of these transactions on NYSE prices varies from 10.43% to 14.87%. As expected, these estimates are smaller than our estimates for aggregated markets. Conceptually, it is more appropriate to perform these

calculations for aggregated markets, because arbitrage trading ties the two markets together.

Regardless of the estimation method, our implied price impact is somewhat smaller than the astonishing price drops of 32% in the cash equity market and 40% in the S&P 500 futures market observed during the 1987 market crash. The 1987 market appears to be less resilient than the 1929 market. The general similarity between predicted and observed values, however, is consistent with our hypothesis that heavy selling induced by program trading played a dominant role in the tumultuous events of October 1987. The price impact may have been amplified by negative information which triggered the crash and aggravated by breakdowns in the market mechanism documented in the Brady Report.

5 Trades of George Soros on October 22, 1987

People know George Soros as a prominent philanthropist and a successful speculator. One of his most famous trades was shorting the British Pound and "breaking the Bank of England" in September 1992, when he made almost $2 billion. Not many people remember, however, that George Soros also had less cheerful days. One of them was Thursday, October 22, 1987, just three days after the historic market crash of 1987. On that day, George Soros decided to sell a large number of S&P 500 futures contracts. The sale has been attributed to pessimistic predictions by Robert Prechter, who forecasted further declines in market prices based on similarities between market patterns during the 1929 crash and the 1987 crash. This transaction turned out to be so costly that it made George Soros think about withdrawing from active management of Quantum Fund.

The Commodity Futures Trading Commission (1988) issued a report describing the events of October 22, 1987, without mentioning Soros by name. The December S&P 500 futures contract closed at 258 on October 21, 1987. Approximately two minutes before the opening bell at 8:28 a.m. on October 22, a customer of the clearing member submitted a 1,200-contract sell order at a limit price of 200, more than 20% below the previous day's close. The price plummeted all the way to 200 over the first minutes of trading, at which point the sell order was executed. At 8:34 a.m., a second identical limit order for 1,200 contracts from the same customer was delivered to the same broker and also executed. It was mentioned that these transactions liquidated a long

position acquired on the previous day at a loss of about 22 percent, or about $60 million in 1987 dollars. Within minutes, S&P futures prices rebounded and, over the next two hours, recovered to the levels of the previous day's close. Those traders who bought futures contracts providing liquidity minutes earlier could have quickly sold the contracts for enormous profits.

Within several days, the Quantum Fund, managed by George Soros, sued the broker Shearson Lehman Hutton for tipping off other traders in the Chicago Mercantile Exchange's S&P 500 futures pit about a big sell order. According to the claim, traders agreed to keep prices at artificially low levels while Quantum Fund's large sell order was executed, and this allegedly illegal conspiracy made the fund lose $60 million. We use the market microstructure invariance to examine whether a 20% price impact is reasonable for execution of Soros' order.

Two other events may have exacerbated the decline in prices in the morning of October 22. First, when the broker executed the second order, he mistakenly sold 651 more contracts than the order called for. The oversold contracts were taken into the clearing firm's error account and liquidated at a significant loss to the broker. Second, the Commodity Futures Trading Commission (1988) says that the same clearing firm also entered and filled four large sell orders for a pension fund customer between 9:34 a.m. and 10:45 a.m., with a total of 2,478 contracts sold at prices ranging from 230 to 241. Remarkably, these additional orders are for almost exactly the same size as Soros's orders, a fact which suggests information leakage or coordination regarding the size of these unusually large orders.

Since the S&P 500 contract size is equal to 500 multiplied by the value of the S&P 500 index, one contract represented ownership of about $129,000 with S&P level of 258. Soros sold 2400 contracts, which corresponds to about $309.60 million in 1987 dollars. The historical volatility in the previous month was 8.63% per day and the average daily volume in S&P 500 futures market was $13.52 billion. These estimates are very high because of the increase in realized volatility and volume during the preceding week, which included the crash day of October 19, 1987, as well as other days with unusually high returns volatility. Note that in contrast to the analysis of the 1987 crash, our estimates are based only on what has happened in the S&P 500 futures market, because we assume that the futures market and cash market became disconnected during the short period when these orders were executed, as reflected by the basis falling to about -60 points during that period. The

price impact equation (1) predicts that the sale of 2,400 contracts—equal to 2.29% of average daily volume during the previous month—would trigger price impact of 7.21%, obtained as

$$1-\exp\Big[-\frac{5.78}{10^4}\cdot\Big(\frac{13.52\cdot 10^9\cdot 1.54}{40\cdot 10^6}\Big)^{1/3}\cdot\Big(\frac{0.0863}{0.02}\Big)^{4/3}\cdot\frac{309.60\cdot 10^6}{(0.01)(13.52\cdot 10^9)}\Big]. \quad (5)$$

Table 4 presents implied price impact of (A) Soros's sell order of 2,400 contracts; (B) Soros's sell order of 2,400 contracts and 651 contracts of error trades, i.e., 3,051 contracts in total; and (C) Soros's sell order of 2,400 contracts, 651 contracts of error trades, and the sell order of 2,478 contracts by the pension fund, i.e., 5,529 contracts in total. The estimates are based on the historical trading volume and volatility of S&P 500 futures contracts calculated over the preceding N months, with $N = 1, 2, 3, 4, 6, 12$.

The implied price impact triggered by 2,400 contracts sold by George Soros ranges from 1.93% to 7.21%. If the error trade of 651 contracts is taken into account, then the implied price impact ranges between 2.45% and 9.07%. If both the error trade of 651 contracts and 2,478 contracts sold by the pension fund are taken into account, then the implied price impact ranges from 4.40% to 15.23%. We believe that the last estimate is the most reasonable one, since it accounts for all sell orders and relies on one-month historical market data that better reflects significant uncertainty during that day. This estimate is similar to but somewhat smaller than the price decline of 22% observed on the morning of October 22. Perhaps the expected volatility was higher than the one-month historical volatility. Perhaps the price decline was exacerbated to the extent that some traders were indeed informed about existing requests to execute sale orders in the S&P 500 futures market and profited from front-running based on this information. Also, the large price impact may be explained by the particular execution strategy of George Soros, who placed a limit sell order with a limit price of 200, substantially below the previous market close, signalling to the market the possibility that significant private information was behind the order.

6 The Liquidation of Jérôme Kerviel's Rogue Trades by Société Générale during January 21-23, 2008

On January 24, 2008, the Société Générale issued a press release stating that the bank had "uncovered an exceptional fraud." In the following weeks and months, Société Générale (2008*a*); Société Générale (2008*c*,*b*) issued further statements revealing how a trader, Jérôme Kerviel, had used "unauthorized" trading to take massive long positions, far beyond his limits in futures markets on major European indices.

According to the report of the investigation committee, Kerviel had established positions in index futures worth €50 billion: a €30 billion position in futures on the Euro STOXX 50, a €18 billion position in futures on DAX, and a €2 billion in futures on the FTSE 100. He has mostly acquired these positions between January 2 and January 18, 2008 and concealed his fraud using fictitious trades, forged documents, and emails that misleadingly suggested that all of his positions were hedged. The fall in index values in the first half of January led to losses on these directional bets. The fraudulent positions were discovered on January 18. Their subsequent liquidation between January 21 and January 23 resulted in huge losses of €6.4 billion which, taking into account the €1.5 billion profit as of December 31, 2007, were reported as a global loss of €4.9 billion.

The Société Générale shareholders, who were already concerned with losses due to the bank's exposure to subprime mortgages, were especially disappointed with additional losses incurred while unwinding these fraudulent positions. The bank officials blamed unfavorable market conditions, not the market impact associated with liquidating the trades themselves, for the large losses associated with liquidating the rogue positions.

We examine whether the losses associated with price impact predicted by our invariance model are consistent with actual reported losses and observed declines in prices. Answering this question involves extrapolating our invariance model, estimated from portfolio transition trades in individual stocks, to the market for stock indices. Due to existence of strong no-arbitrage relations between European markets, we do our analysis under the assumption that all European stock and futures markets are perfectly integrated. We aggregate data across the largest European exchanges and the most actively traded index

futures at that time.

We aggregate data across seven largest European exchanges: NYSE Euronext (Europe) with market capitalization of $2.10 trillion in 2008, London Stock Exchange with market capitalization of $1.87 trillion, Deutsche Börse with market capitalization of $1.11 trillion, BME Spanish Exchanges with market capitalization of $0.95 trillion, SIX Swiss Exchange with market capitalization of $0.86 trillion, NASDAQ OMX Nordic Exchange with market capitalization of $0.56 trillion, and Borsa Italiana with market capitalization of $0.52 trillion. Other European exchanges had market capitalization lower than $0.15 trillion. Based on the data from the World Federation of Exchanges, we find that in the month preceding January 18, 2008, the historical daily volume for seven largest European exchanges was $106.11 billion or €69.51 billion, given the exchange rate of $1.4690 per Euro on January 17, 2008.

We also add data on ten most actively traded index futures based on the historical average daily volume in the month preceding January 18, 2008: Euro Stoxx 50 with daily volume of €55.19 billion, DAX with daily volume of €32.40 billion, FTSE 100 with daily volume of €9.92 billion, CAC 40 with daily volume of €8.73 billion, IBEX 35 with daily volume of €4.14 billion, AEX with daily volume of €3.92 billion, Swiss Market Index (SMI) with daily volume of €3.27 billion, FTSE MIB with daily volume of €2.84 billion, OMX Stockholm 30 with daily volume of €0.32 billion, and Stoxx 50 Euro with daily volume of €0.26 billion. In total, the average daily volume on ten major index futures was €110.98 billion. The total daily volume in both European cash and futures markets was equal to €180.49.

As expected volatility, we use the historical volatility of 0.0011 for the Stoxx Europe Total Market Index (TMI) which represents the Western Europe region as a whole.

According to the price impact equation (1), the liquidation of a €50 billion Kerviel's position—equal to about 27.70% of the average daily volume in aggregated stock and futures markets—is expected to trigger a price decline of 12.37% in European markets,

$$1 - \exp\Big[- \frac{5.78}{10^4} \cdot \left(\frac{180.49 \cdot 1.4690 \cdot 0.92 \cdot 10^9}{40 \cdot 10^6} \right)^{1/3} \left(\frac{0.0011}{0.02} \right)^{4/3} \frac{50}{(0.01)180.49} \Big], \tag{6}$$

In this equation, we use an exchange rate of $1.4690 per Euro to convert Euro volume into U.S. dollar volume and a GDP deflator of 0.92 to convert 2008 dollars into 2005 dollars. The predicted values are similar to observed price

changes. The Stoxx TMI index fell by 9.44% from the market close of 316.73 on January 18, 2008, to its lowest level of 286.82 on January 21, 2008.

Table 5 shows the estimates of price impact based on historical trading volume and volatility of futures on European indices calculated over the preceding N months, with $N = 1, 2, 3, 4, 6, 12$.

The official release reports losses of €6.30 billion. We assume that market predicted impact costs are equal to half of predicted price impact since, assuming no leakage of information about the trades, the trader can theoretically walk the demand curve, trading only the last contracts at the worst expected prices. Thus, invariance predicts the total cost of unwinding Kerviel's position to be equal to 6.39% of the initial €50 billion position, i.e., €3.19 billion.

Since the official losses are calculated using the end of year 2007 prices as benchmarks, we also adjust our estimates for losses on fraudulent positions between December 31, 2007 and January 18, 2008, because the levels of futures prices dropped significantly during that period. From December 28, 2007, to January 18, 2008, the futures on the Euro STOXX 50 fell by 9.18% from 4,435 to 4,028, the futures on the DAX fell by 9.40% from 8,144 to 7,379, and the futures on the FTSE 100 fell by 8.68% from 6,455 on December 31, 2007, to 5,895 on January 18, 2008.

If we assume that Kerviel acquired his positions gradually at average prices from December 31, 2007, to January 18, 2008, then we have to add additional losses of 9.18%/2 for Euro STOXX 50 futures, 9.40%/2 for DAX futures, and 8.68%/2 for FTSE 100 futures, i.e., €2.31 billion in total ("adjustment A"). The total loss of €3.19 billion adjusted for an additional €2.31 billion loss will then amount to about €5.50 billion, which is close to the reported losses of €6.30 billion. If, however, we assume that Kerviel acquired his position on December 31, 2007, then the additional loss will be equal to 2· €2.31 billion, i.e., €4.62 billion ("adjustment B"). The total loss of €3.19 billion adjusted for additional the €4.62 billion loss reaches €7.81 billion, even higher than reported losses.

Table 5 reports estimates of total losses with both adjustments. The total implied trading costs range from €3.13 billion to €3.85 billion. The total loss with "adjustment A" ranges from €5.44 billion to €6.16 billion. The total loss with "adjustment B" varies from €7.75 billion to €8.47 billion. Since these numbers are consistent with the losses of €6.30 billion reported by Société Générale (2008*a*); Société Générale (2008*c*,*b*) during liquidation of Kerviel's positions, we conclude that these losses are consistent with the

price impact of liquidating the trades and should not be attributed to adverse market movements coincidentally occurring during the same period between January 21 and January 23, 2008.

We can also do our analysis under the assumption that the Euro STOXX 50, the DAX, and the FTSE 100 futures markets are distinct markets. In the month preceding January 18, 2008, historical volatility per day was 98 basis points for futures on the Euro STOXX 50, 100 basis points for futures on the DAX, and 109 basis points for futures on the FTSE 100. The estimates of historical daily volume was €55.19 billion for Euro STOXX 50 futures, €32.40 billion for DAX futures, and £7.34 billion for FTSE 100 futures. Given the exchange rate of €1.3440 for £1 on January 17, 2008, Kerviel's positions of €30 billion in Euro STOXX 50 futures, €18 billion in DAX futures, and €2 billion in FTSE 100 futures represented about 54%, 56%, and 20% of daily trading volume in these contracts, respectively.

Our calculations show that the liquidation of a €30 billion position in Euro STOXX 50 futures is expected to trigger a price impact of 14.34%, liquidation of €18 billion in the DAX futures market has an estimated price impact of 12.75%, and liquidation of €2 billion in FTSE futures has an estimated price impact of 4.81%. The predicted values are similar to observed price changes. Indeed, Euro STOXX 50 futures fell by 10.50% from 4,028 to 3,605, the DAX futures fell by 11.91% from 7,379 to 6,500, and the FTSE 100 futures fell by 4.65% from 5,895 to 5,621 during the period between January 18 and January 23, 2008, from close to close. Comparing the market values of underlying indices at close on January 18, 2008, to the lowest values they reached during January 21 through January 23, 2008, we find price declines of similar magnitudes. The Euro Stoxx 50 index fell by 11.67% from 3,995 on January 18 to its lowest value of 3,529 on January 21, 2008. The DAX index fell by 12.71% from 7,314 on January 18 to its lowest value of 6,384 on January 23, 2008. The FTSE 100 index fell by 9.54% from 5,902 on January 18 to its lowest value of 5,339 on January 22, 2008.

The implied trading losses range from €3.13 billion to €3.85 billion. The total loss with "adjustment A" ranges from €5.44 billion to €6.16 billion. The total loss with "adjustment B" varies from €7.75 billion to €8.47 billion. These numbers are very similar to our estimates for integrated markets and actual reported losses.

The week of liquidation of Kerviel's positions was characterized by extreme volatility and price declines in other European markets, not affected directly

by the sales of Euro Stoxx 50 futures, DAX futures, and FTSE 100 futures. For example, the Spanish index IBEX 35 dropped by 7.54% from 13,655 on the close of January 18, 2008, to the lowest point of 12,626 on January 21, 2008, the biggest one-day fall in the history of the index since its inauguration in 1992. Next day January 23, 2008, IBEX 35 declined further by 5.45% to 11,937 followed by a rapid recovery to 13,107 by the close of January 24, 2008. The Italian index FTSE MIB fell by 10.11% from 35,753 on January 18, 2008, to the lowest point of 32,140 on January 22, 2008, and then recovered to 34,237 on January 24, 2008. The Swedish index OMX Stockholm 30 fell by 8.63% from 957.63 on January 18, 2008, to the lowest point of 875.02 on January 22, 2008, and then recovered to 958.55 on January 24, 2008. The French index CAC 40 fell by 11.53% from 5,092 on January 18, 2008, to the lowest point of 4,505 on January 22, 2008, and then recovered to 4,843 by the end of the same day. The Dutch index AEX fell by 10.80% from 450.08 on January 18, 2008, to the lowest point of 401.45 on January 22, 2008 and then recovered to 444.23 by the end of January 24, 2008. The Swiss Market Index fell by 9.63% from 7,692 on January 18, 2008, to the lowest point of 6,951 on January 22, 2008 and then fully recovered to 7,691 by the end of January 24, 2008. Similar patterns were documented during the 1987 crash, when not only U.S. markets but also many major world markets experienced severe declines, as emphasized in Roll (1988). This supports our strategy of looking at the price effects on the aggregated markets rather than thinking about markets as isolated pools of liquidity. This also suggests an important lesson that execution of a large bet in one market may echo in other related markets and lead to significant declines in their price levels.

There are two concerns that may affect our analysis. First, January 21, 2008, was a holiday in the United States. In 2007, the futures markets had only one third of the typical volume on days when U.S. markets were closed. Lower trading volume on January 21 could have reduced market liquidity, making the unwinding of Kerviel's positions more expensive. Second, the Fed unexpectedly announced a substantial 75-basis point cut in interest rates on January 22, 2008, several days before its regularly scheduled meeting. This announcement had a positive effect on stock markets around the world and should have helped Société Générale to obtain more favorable execution prices on some portion of its trades.

7 The Flash Crash of May 6, 2010

Not all market crashes happen in the United States in October, and not all of them last for a long time. The flash crash of 2010 occurred on May 6 and lasted for only twenty minutes.

May 6, 2010, started with uncertainty concerning the debt crisis in Europe, elections in the United Kingdom, and an upcoming jobs report in the United States. These worries led to growing uneasiness in the financial markets. The S&P 500 declined by three percent during the first half of that day. In the afternoon, something bizarre happened. The E-mini S&P 500 futures contract suddenly fell from 1,113 at 2:40 p.m. to 1,056 at 2:45 p.m., a decline of 5.12% over a five-minute period. Pre-programmed circuit breakers built into the CME's Globex electronic trading platform stopped futures trading for five seconds. Over the next ten minutes, the market rose by about 5%, recovering all of the earlier losses. This several-standard-deviation event happened so quickly that it could have been missed by those who stepped away from their desks for a cup of coffee.

Shaken market participants began a search for guilty culprits. "Fat finger" errors and a cyber attack were theories quickly discarded. Many accused algorithmic traders of failing to provide liquidity during the collapse of market prices.

After the flash crash, the Staffs of the CFTC and SEC (2010*b*,*a*) issued a joint report. The report highlighted the fact that an automated execution algorithm sold 75,000 S&P 500 E-mini futures contracts between 2:32 p.m. and 2:51 p.m. ET on the CME's Globex platform. The period of execution corresponded precisely with the V-shaped flash crash. The E-mini contract represents exposure of 50 times the S&P 500 index, one tenth the multiple of 500 for the older but otherwise similar contract sold by portfolio insurers in 1987. Given the S&P 500 index values , the program sold S&P 500 exposure of approximately $4.37 billion. The joint report did not mention the name of the seller, but journalists identified the large seller as Waddell & Reed.

Many people did not believe the report's suggestion that selling 75,000 contracts could have triggered a price drop of 5%. Indeed, the order represented only 3.75% of the daily trading volume of about 2,000,000 contracts per day in S&P 500 E-mini futures market and was at least twice smaller if comparing to volume available in both future and stock markets. A legitimate question is whether the execution of such an order could have resulted in a flash crash.

We first need to make assumptions about expected trading volume and volatility. Given a multiplier of 50, one S&P 500 E-mini contract represents ownership of about $58,200, with an S&P level of 1,164 on May 5, 2010. During the preceding month, the average trading volume in E-mini contracts was about $132 billion per day in 2010 dollars. The average volume in the stock market was about $161 billion per day in 2010 dollars. Not surprisingly, the trading volume was much higher on May 6, 2010. The historical volatility was about 1.07% per day. Since volatility on May 6 was higher than usual due to the European debt crisis, we also use a rough estimate of expected volatility equal to 2.00% per day. Given a GDP deflator of 0.90 between 2005 and 2010, equation (1) implies that the order—equal to about 3.31% of average daily volume in S&P500 E-mini futures market in the previous month or 1.49% for futures and stock market aggregated—is expected to trigger a price decline of 0.70%,

$$1-\exp\Big[-\frac{5.78}{10^4}\cdot\bigg(\frac{(132+161)\cdot 0.90\cdot 10^9}{40\cdot 10^6}\bigg)^{1/3}\cdot\Big(\frac{0.0107}{0.02}\Big)^{4/3}\cdot\frac{75,000\cdot 50\cdot 1,164}{0.01\cdot(132+161)\cdot 10^9}\Big].$$

Table 6 shows additional estimates based on historical trading volume and volatility of S&P 500 E-mini futures contracts calculated over the preceding N months, with $N = 1, 2, 3, 4, 6, 12$, as well as both historical volatility and volatility of 2% per day.

The predicted price impact is smaller than the actual decline in prices of 5.12%. The estimates based on historical volatility range from 0.50% to 0.84%. The estimates based on two-percent volatility range from 1.60% to 1.91%. If we do not treat the cash market and the futures market as one market but focus only on the futures market, then the estimates range from 0.88% to 1.49% for historical volatility and from 2.71% to 3.35% for volatility of two percent (not reported).

Note that our estimates may underestimate actual price changes, since the price impact parameter estimated from portfolio transition trades assumes that trades are executed at a natural speed consistent with the manner in which portfolio transition trades are executed. We believe that realized market depth is influenced by the speed of trading as, for example modeled in Kyle, Obizhaeva and Wang (2012). The joint report indicates that the speed of execution of this large order over such a short period of time was unusually fast for the the S&P 500 E-mini futures contract. We believe that the unusually rapid execution of this order probably led to much higher price

impact than predicted from the invariance hypothesis. A rapid price recovery following a severe price decline is consistent with this hypothesis.

8 The Frequency of Market Crashes

Two questions immediately come to mind of worried investors and regulators. How unusual are large bets in financial markets? How often do they lead to crashes and mini-crashes?

Large orders may be placed into the marketplace more often than traders usually think. Using portfolio transitions data, Kyle and Obizhaeva (2011*a*,*b*) find that distributions of buy and sell order sizes can be closely approximated by a log-normal with the mean that depends on the dollar volume and volatility of the stock. The distribution of order sizes $\tilde{X}$ of a security with a security price P dollars, trading volume V shares per calendar day, and daily returns volatility σ, can be approximated as,

$$\ln\left(\frac{|\tilde{X}|}{V}\right) = -5.69 - 2/3 \cdot \ln\left(\frac{\sigma \cdot P \cdot V}{(0.02)(40)(10^6)}\right) + \sqrt{2.50} \cdot \tilde{Z}, \qquad (7)$$

where $\tilde{Z}$ is a standardized normal variable. For the benchmark stock with trading volume of \$40 million per day and volatility 2% per day, the mean of the underlying normal variable is estimated to be equal to -5.69, and its variance is equal to 2.50. Since the median of a log-normal distribution is the mean of the underlying normal, a median order size is approximately 0.34% of daily volume, or $\$136,000$ for the benchmark stock. Kyle, Obizhaeva and Tuzun (2010) confirm this finding for the sample of trades reported in the Trades and Quotes dataset, where the median of trade sizes is smaller compared to the median of portfolio transition orders (due to order shredding), but the variance is similar in magnitude.

These bets are estimated to arrive with the expected Poisson arrival rate of γ bets per day,

$$\ln(\gamma) = \ln(85) + 2/3 \cdot \ln\left(\frac{\sigma \cdot P \cdot V}{(0.02)(40)(10^6)}\right), \qquad (8)$$

where the number of bets for the benchmark stock is estimated to be 85 bets per day.

Using these formulas, it is easy to calculate how unusual are orders that we identify as a source of market dislocations. The margin selling in October 1929 represents a 6.16-standard deviation event in the stock market that is expected to happen once every $5,539$ years (given $2,010$ bets per day). The sales of portfolio insurers in 1987 represent a 6.00-standard deviation event in the stock and futures markets that is expected to happen only once in 716 years (given $5,606$ bets per day). The George Soros's sell order of S&P 500 futures contracts represent a 4.47-standard deviation event in the stock market that is expected to happen about once in a month (given $14,579$ bets per day). The sales of Kerviel's positions of major European index futures represent a 6.22-standard deviation event in the European market that is expected to happen once in 819 years (given $19,059$ bets per day). The sale of S&P 500 E-mini futures triggered the flash crash in 2010 represent a 4.65-standard deviation event that is expected to occur several times during a year (given almost $29,912$ bets per day in the aggregated futures and cash market).

These calibrations suggest that large bets associated with crashes appear more frequently than the log-normal distribution with the estimated variance of 2.50 would imply. There may be several explanations of why the right tail of bet size distribution is fatter than predicted.

First, the calibrated standard deviation of bets from portfolio transitions data may be too small due to unique features of portfolio transitions. Other institutional trades may be better approximated by a log-normal distribution with bigger variance. Note, however, that a normal distribution implies that a 5-standard deviation event is 291 times more likely than a 6-standard deviation event. Increasing the estimated standard deviation of $2.50^{1/2}$ or 1.58 by 20%, so that 6-standard deviation events would become 5-standard deviation events, would imply that market crashes occur far too frequently. Thus, the true standard deviation may be higher by about 15%.

Second, the far right tail of the distribution of bets may be better described by a power law than a log-normal. Indeed, Kyle and Obizhaeva (2011*a*) find a slight deviation from the log-normal distribution for the very largest portfolio transitions orders in most active stocks.

Thinking more about these alternatives is an important agenda for future research in market microstructure.

9 Lessons Learned

It is, of course, impossible to infer from only five data points definitive conclusions about the ability of microstructure invariance to predict the price impact of liquidations of large quantities of stock. Each of the crash events involves event-specific features which make it difficult to estimate the size of the positions liquidated, long-term volume and volatility, and the effects of other contemporaneous events that may have affected market conditions. Application of microstructure invariance concepts to intrinsically infrequent historical episodes therefore requires an exercise in judgement to extract appropriate lessons learned. Nevertheless, the five cases we have examined suggest important lessons, both for policymakers interested in measuring and predicting crash events of a systemic nature and for asset managers interested in minimizing market impact costs associated with execution of large trades that might potentially disrupt markets.

Price Impact is Large Liquid Markets. For the five crash events examined in this paper, the predicted declines are large and similar in magnitude to what actually occurred. This is consistent with the interpretation that microstructure invariance may apply not only to individual stocks but also to stock index futures markets or a combination of futures and cash markets. Microstructure invariance implies that liquidating a given fraction of average daily volume has larger price impact in markets with high volume and high volatility than in markets with low volume and low volatility.

Furthermore, in contrast to models and empirical literature suggesting that price impact is concave in trade size, microstructure invariance is based on the maintained assumption that price impact is linear, even for very large trades. Both of these features of microstructure invariance tend to magnify the estimated price impact of liquidations of large quantities of stock when crash events occur. The large predicted magnitudes suggest that both increasing price impact as volume increases (using a cube root) and linear price impact are reasonable assumptions to apply to the stock market as a whole, not just individual stocks.

Speed of Liquidation Magnifies Short-term Price Effects. It is probable that the speed of execution influences the way in which orders affect prices. Rapid execution of trades results in large temporary price impact, but does

not increase permanent impact. This implies that trades executed rapidly are likely to generate a V-shaped price path, with price first plunging sharply and then rapidly recovering. Similar patterns arise in the equilibrium model of Kyle, Obizhaeva and Wang (2012). For the V-shaped price path not to make it easy for others to profit from "front-running" the trades, it is also necessary for the execution of such trades to be accompanied by a dramatic, transitory increase in price volatility.

Both the 1987 Soros episode and the 2010 flash crash are consistent with the idea that very rapid execution of trades incurs transaction costs higher than predicted by invariance and generates high volatility and rapid mean reversion in prices. In contrast, the 1987 portfolio insurance trades and the 2008 Société Générale trades were executed quickly, over a few days, but not over weeks, and thus the actual price changes during these episodes are similar to predicted ones. As we explained above, measures implemented by bankers and regulators in the last week of October 1929 smoothed the margin selling out over a period of five weeks rather than a few days, and the 1929 margin-related trades triggered price decline smaller than predicted by invariance.

The main lesson is that slower execution of large trades would lessen temporary price impact and reduce transitory volatility. One obvious reason is that slow execution separates trades based on short-half-life private information from liquidity trades. For example, trades in the 1929 crash, the 1987 crash and during liquidation of Société Générale's trades in 2008 were certainly liquidity trades. While the 2010 flash crash trades may be thought to have private information, this information probably was not be based on a 15-minute rumor. Soros explained his trades in 1987 by referring to Prechter's Elliot Wave Theory, something which takes place over a long period of time.

The Financial System in 1929 Was Remarkably Resilient. The 1987 portfolio insurance trades of $13 billion were equal to about 0.28% of GDP in that year (1987 GDP was $4.7 trillion). Execution of these trades triggered a price impact of 32% in the "cash" stock market and 40% in the futures market. The 1929 margin-related sales of $1.181 billion during the last week of October were equal to 1% of GDP in that year (1929 GDP was $104 billion). These margin-related sales were followed by additional sales equal to 3% of GDP in subsequent weeks. Even though the sales of 1% of GDP were almost four times larger than the 1987 portfolio insurance trades, their execution changed prices by only 24%. The financial system in 1929 was remarkably resilient.

As market prices were falling in October 1929, several bankers got together to formulate a common plan to stabilize market prices. The bankers quickly assembled a fund of $750 million to buy securities in order to support prices. Their decisions were publicized, and the panic stopped. These meetings were not unprecedented. Similar actions, for example, were undertaken by J.P. Morgan and other bankers who stabilized the market in 1907 crash. If such meetings were to happen today, they would likely raise anti-trust concerns.

The New York Fed also acted prudently in 1929. In the 1920s, bankers and their regulators were aware that if non-bank lenders suddenly withdrew funds from the broker loan market, there would be pressure on the banking system to make up the difference. By discouraging banks from lending into the broker loan market prior to the 1929 crash, the New York Fed increased the ability of banks to support the broker loan market after the stock market crashed. During the last week of October 1929, the New York Fed wisely reversed its course and encouraged banks to provide bank loans on securities to their clients as a substitute for broker loans. At the same time, many brokers cut margins from 40% to 20% on their loans to avoid immediate execution.

These actions created a space for the financial system to adjust over a period of about five weeks rather than a few days. One possible effect was to reduce the short-term impact of liquidations of margin loans by spreading the execution of the trades out over time. Consistent with this hypothesis, we find that during the 1929 crash, the actual price impact of 24% was small relative to predicted price changes of 49.22%. The stock market prices stabilize by the end of 1929. There were no major failures of banks or brokerage firms.

Early Warning Systems May Be Useful and Practical. Two events—1929 margin sales and 1987 portfolio insurance—involved aggregated trades by numerous entities. In both cases, data was publicly available before the crash event. Data on broker loans was published by the Federal Reserve System and the NYSE. Estimates of assets under management by portfolio insurers were available before the 1987 crash. In both cases, potential price impacts of liquidations were topics of conversation among policy makers and market participants.

A good example is the 1987 stock market crash. The debate about the extent to which portfolio insurance trading contributed to the 1987 market crash started before the crash itself occurred. The term "market meltdown," popularized by then NYSE chairman John Phelan, was used in the year or

so before the stock market crash to describe a scenario of cascading portfolio insurers' sell orders resulting in severe price declines and posing systemic risks to the economy . In the summer of 1987, the SEC conducted Division of Market Regulation (1987) a study of a cascading meltdown scenario before the crash itself. After describing in some detail what a market crash would look like, the study dismissed the risk of a crash as a remote possibility, in agreement with conventional Wall Street wisdom at the time.

Conventional wisdom has held that, given the substantial trading volume in the U.S. equity markets, and especially the index futures market, there was enough liquidity available to accommodate sales of portfolio insurers without any major downward adjustment in stock prices. During hearings before the House Committee on Energy and Commerce (1987), Hayne E. Leland defended portfolio insurance: "We indicated that average trading will amount to less than 2% of total stocks and derivatives trading. On some days, however, portfolio insurance trades may be a greater fraction... In the event of a major one-day fall (e.g., 100 points on the Dow Jones Industrial Average), required portfolio insurance trades could amount to $4 billion. Almost surely this would be spread over 2-3 day period. In such a circumstance, portfolio insurance trades might approximate 9-12% of futures trading, and 3-4% of stock plus derivatives trading."

If regulators had used our simple and easily implemented methodology prior to the market crash of 1987, they would find that size of portfolio insurance industry was close at that time to a tipping point. Plugging historical estimates of volume and volatility in July 1987 into our formula yields that estimated sales of $4 billion by portfolio insurers in response to a 100-point drop in the Dow Jones (i.e., about 4%) would lead to a 4-percent change in price, which may further trigger a cascading effect and a meltdown of the market. We believe that the impact of events like the crashes and panics discussed in this paper can be reduced, if regulators systemically monitor market conditions to create awareness of the possibility that large bets are imminent and if market participants learn to appreciate that large bets will inevitably lead to large price impact.

Nowadays, a similar analysis may be warranted to assess the effect of leveraged ETFs on financial markets, an interesting topic for further research.

10 Conclusion

Financial markets are vulnerable to large plunges resulting from the execution on unusually large bets. This point is consistent with the observations of Black (1971), who believed that the execution of large orders will always exert an impact on prices, regardless of technological advances in market microstructure and methods of execution.

Our use of market microstructure invariance to examine five stock market crashes suggests that market depth is less than many traders think, especially for the largest and deepest markets such as stock indices. Depth is influenced by market conditions and also by the speed of trading. Our analysis suggests that when large orders are executed faster than the natural speed appropriate for the trading games being played, these executions may induce even sharper changes in market prices, probably with transitory impact. The probability of large market dislocations in the future can be lessened if traders use more reasonable trading algorithms and, in general, understand better how market depth is related to the volume and volatility of the markets in which they participate.

References

Akerlof, George A., and Robert J. Shiller. 2009. *Animal Spirits: How Human Psychology Drives the Economy, and Why It Matters for Global Capitalism.* Princeton University Press.

Black, Fischer. 1971. "Towards a Fully Automated Exchange, Part I." *Financial Analysts Journal*, 27: 29–34.

Board of Governors of the Federal Reserve System. 1927-1931. *Annual Report of the Board of Governors of the Federal Reserve System.* 13th through 17th Annual Reports Covering Operations for the Years 1926 through 1930.

Board of Governors of the Federal Reserve System. 1929. *Federal Reserve Bulletin.*

Brennan, Michael J., and Eduardo S. Schwartz. 1989. "Portfolio Insurance and Financial Market Equilibrium." *Journal of Busines*, 62: 455–472.

Commodity Futures Trading Commission, Division of Economic Analysis and Division of Trading and Markets. 1988. *Final Report on Stock Index Futures and Cash Market Activity During October 1987 to the U.S. Commodity Futures Trading Commission.*

Division of Market Regulation, Securitie and Exchange Commission. 1987. *The Role of Index-Related Trading in the Market Decline of September 11 and 12, 1986.*

Friedman, Milton, and Anna Jacobson Schwartz. 1963. *A Monetary History of the United States, 1867-1960. NBER Book Series Studies in Business Cycles*, Princeton University Press.

Galbraith, John Kenneth. 1954. *The Great Crash, 1929.* Boston:Houghton Mifflin Co.

Haney, H. Lewis. 1932. "Brokers' Loans." *Journal of the American Statistical Association*, 27: 65–72.

House Committee on Energy and Commerce, U.S. Congress. 1987. *Program Trading: Hearing before the Subcommittee on Telecommunications and Finance of the Committee on Energy and Commerce.* Vol. 100th Cong., 1st Sess.

Kyle, Albert S. 1985. "Continuous Auctions and Insider Trading." *Econometrica*, 53(6): 1315–1335.

Kyle, Albert S., and Anna Obizhaeva. 2011*a*. "Market Microstructure Invariants: Empirical Evidence from Portfolio Transitions." Working Paper, University of Maryland.

Kyle, Albert S., and Anna Obizhaeva. 2011*b*. "Market Microstructure Invariants: Theory and Implications of Calibration." Working Paper, University of Maryland.

Kyle, Albert S., Anna Obizhaeva, and Tugkan Tuzun. 2010. "Trading Game Invariance in the TAQ Dataset." Working Paper, University of Maryland.

Kyle, Albert S., Anna Obizhaeva, and Yajun Wang. 2012. "Smooth Trading with Overconfidence and Market Power." Working Paper, University of Maryland.

Leland, Hayne E., and Mark Rubinstein. 1988. "Comments on the Market Crash: Six Months After." *The Journal of Economic Perspectives*, 2: 45–50.

Miller, Merton H. 1988. "Explaining the Events of October 1987." In *After the Crash: Linkages between Stocks and Futures. AEI Studies*, , ed. Robert J. Mackay. American Enterprise Institute for Public Policy Research, University Press of America.

Miller, Merton H. 1991. *Financial Innovations and Market Volatility.* Cambridge, MA:Blackwell.

Perold, André. 1988. "The Implementation Shortfall: Paper vs. Reality." *Journal of Portfolio Management*, 14(3): 4–9.

Reinhart, Carmen M., Kenneth Rogoff. 2009. *This Time Is Different: Eight Centuries of Financial Folly.* Princeton University Press.

Roll, Richard. 1988. "The International Crash of October 1987." *Financial Analysts Journal*, 44(5): 19–35.

Scholes, Myron S. 1972. "The Market for Securities: Substitution Versus Price Pressure and the Effects of Information on Share Prices." *The Journal of Business*, 45(2): 179–211.

Senate Committee on Banking and Currency, U.S. Congress. 1934. *Stock Exchange Practices: Report Of The Committee On Banking And Currency Pursuant To S.Res. 84 (72nd Congress) and S.Res. 56 and S.Res. 97 (73rd Congress).* Vol. 73rd Congress, Second Session, U.S. Government Printing Office. "The Pecora Commission Report".

Shiller, Robert J. 1987. "Investor Behavior in the October 1987 Stock Market Crash: Survey Evidence." NBER Working Paper 2446.

Smiley, Gene, and Richard H. Keehn. 1988. "Margin Purchases, Brokers' Loans and the Bull Market of the Twenties." *Business and Economic History*, 17: 129–142.

Société Générale. 2008*a*. "Explanatory note about the exceptional fraud." Paris, January 27, 2008.

Société Générale, General Inspection Department. 2008*b*. "Mission Green, Summary Report." May 20, 2008.

Société Générale, General Inspection Department. 2008*c*. "Mission Green, Summary Report: Interim conclusions as of February 20, 2008."

Staffs of the CFTC and SEC. 2010*a*. *Findings Regarding the Market Events of May 6, 2010.* Report of the Staffs of the CFTC and SEC to the Joint Advisory Committee on Emerging Regulatory Issues. September 30, 2010. "Final Report".

Staffs of the CFTC and SEC. 2010*b*. *Preliminary Findings Regarding the Market Events of May 6, 2010.* Report of the Staffs of the CFTC and SEC to the Joint Advisory Committee on Emerging Regulatory Issues. May 18, 2010. "Preliminary Report".

U.S. Presidential Task Force on Market Mechanisms. 1988. *Report of the Presidential Task Force on Market Mechanisms.* Washington:U.S Government Printing Office. The "Brady Report".

Wigmore, Barry. 1985. *The Crash and its Aftermath: A History of Securities Markets in the United States, 1929-1933.* Westport, CN:Greenwood Press.

Table 2: 1929 Stock Market Crash: Implied Price Impact of Margin Sales.

	Months Preceding 24 October 1929					
N:	1	2	3	4	6	12
ADV (in 1929-$M)	488.98	507.08	479.65	469.45	4425.47	429.06
Daily Volatility	0.0200	0.0159	0.0145	0.0128	0.0119	0.0111
Sales 10/24-10/30 (%ADV)	242%	233%	246%	252%	278%	275%
Price Impact 10/24-10/30	49.22%	38.67%	36.05%	32.04%	31.05%	28.72%
Sales 9/25-12/25 (%ADV)	1270%	1225%	1295%	1323%	1460%	1448%
Price Impact 9/25-12/25	91.75%	83.47%	80.71%	75.87%	74.56%	71.25%

Table 2 shows the implied price impact of $1.181 billion of margin sales during the week of 10/24-1/30 and $4.343 billion of margin sales during 9/25-12/25 given a GDP deflator adjustment which equates $1 in 1929 to $9.42 in 2005, along with average daily 1929 dollar volume and average daily volatility for $N = 1, 2, 3, 4, 6, 12$ months preceding October 24, 1929, based on a sample of all CRSP stocks with share codes of 10 and 11. The actual price decline was 25% during 10/24-10/30 and 34% during 9/25-12/25.

Table 3: 1987 Stock Market Crash: Effect of Portfolio Insurance Sales.

	Months Preceding 14 October 1987					
N:	1	2	3	4	6	12
S&P 500 Fut ADV (1987-$B)	10.37	11.29	11.13	10.12	10.62	9.85
NYSE ADV (1987-$B)	10.20	10.44	10.48	10.16	10.04	9.70
Daily Volatility	0.0135	0.0121	0.0107	0.0102	0.0112	0.0111
Sell Orders (% ADV)	66.84%	63.28%	63.65%	67.82%	66.53%	70.33%
Price Impact/Imbalances	15.75%	13.30%	11.47%	11.13%	12.39%	12.80%
Price Impact/Sell Orders	19.12%	16.20%	14.00%	13.59%	15.10%	15.60%
Price Impact/S&P 500 Sales	16.12%	13.36%	11.58%	11.47%	12.52%	13.11%
Price Impact/NYSE Sales	14.87%	12.80%	10.97%	10.43%	11.83%	12.07%

Table 3 shows the implied price impact triggered by portfolio insurers' net order imbalances in S&P 500 market and NYSE market ($9.51 billion in S&P 500 futures market and $1.60 billion on the NYSE market), portfolio insurers' sell orders only in both markets ($10.48 billion in S&P 500 futures market and $3.27 billion on the NYSE market), portfolio insurers' sales in S&P 500 market adjusted for purchases of index arbitrageurs ($10.48 billion minus $3.27 billion in S&P 500 futures market), and portfolio insurers' sales in NYSE market adjusted for sales of index arbitrageurs of $3.27 billion ($3.27 billion plus $3.27 billion) in 1987 dollar during the market crash in 1987, given an inflation adjustment converting $1 in 1987 to $1.54 in 2005, average daily dollar volume and average daily volatility based on N months preceding October 14, 1987, with $N = 1, 2, 3, 4, 6, 12$, for the S&P 500 futures contracts and the sample of all CRSP stocks with share codes of 10 and 11. The actual price decline was 32% in NYSE market and 40% in S&P 500 futures market.

Table 4: October 22, 1987: Effect of Soros's Trades.

	Months Preceding 22 October 1987					
N:	1	2	3	4	6	12
S&P 500 Fut ADV (1987-$B)	13.52	11.72	11.70	10.99	10.75	10.04
Daily Volatility	0.0863	0.0622	0.0502	0.0438	0.0365	0.0271
2,400 contracts as %ADV	2.29%	2.64%	2.65%	2.82%	2.88%	3.08%
Price Impact A	7.21%	5.18%	3.92%	3.42%	2.73%	1.93%
Price Impact B	9.07%	6.54%	4.96%	4.32%	3.45%	2.45%
Price Impact C	15.83%	11.53%	8.80%	7.70%	6.17%	4.40%

Table 4 shows the implied price impact of (A) Soros's sell order of 2,400 contracts; (B) Soros's sell order of 2,400 contracts and 651 contracts of error trades, i.e., 3,051 contracts in total; and (C) Soros's sell order of 2,400 contracts, 651 contracts of error trades, and sell order of 2,478 contracts by the pension fund, i.e., 5,529 contracts in total. The calculations assume a GDP deflator adjustments which equates $1 in 1987 to $1.54 in 2005, average daily 1929 dollar volume and average daily volatility for $N = 1, 2, 3, 4, 6, 12$ months preceding October 22, 1987 for the S&P 500 futures contracts. The actual price decline in the S&P 500 futures market was 22%.

Table 5: Effect of Liquidating Kerviel's Positions by Société Générale in January 2008

	Months Preceding January 18, 2008					
N:	1	2	3	4	6	12
Stk Mkt ADV (2008-€B)	69.51	66.51	67.37	67.01	66.73	66.32
Fut Mkt ADV (2008-€B)	110.98	114.39	118.05	117.46	127.17	121.26
Daily Volatility	0.0110	0.0125	0.0121	0.0117	0.0132	0.0111
Order as %ADV	27.70%	27.64%	26.97%	27.11%	25.79%	26.66%
Price Impact	12.37%	14.48%	13.67%	13.21%	14.79%	12.14%
Total Losses (2008-€B)	3.19	3.76	3.54	3.42	3.85	3.13
Losses/Adj A (2008-€B)	5.50	6.07	5.85	5.73	6.16	5.44
Losses/Adj B (2008-€B)	7.81	8.38	8.16	8.04	8.47	7.75

Table 5 shows the predicted losses of liquidating Kerviel's positions of €30 billion in STOXX 50 futures, €18 billion in DAX futures, and €2 billion in FTSE 100 futures under the assumption that the major European cash and futures markets are integrated, given an inflation adjustment of $1 in 2008 equal to $0.92 in 2005, average daily volume of the major European stock exchanges and index futures as well as daily volatilities of Stoxx Europe TMI, based on N months preceding January 18, 2008, with $N = 1, 2, 3, 4, 6, 12$. The actual price decline in the Stoxx Europe TMI was 9.44%.

Table 6: Flash Crash of May 6, 2010: Effect of 75,000 Contract Futures Sale.

	Months Preceding 6 May 2010					
N:	1	2	3	4	6	12
S&P500 Fut ADV (2010 $B)	132.00	107.49	109.54	112.67	100.65	95.49
Stk Mkt ADV (2010 $B)	161.41	146.50	142.09	143.03	132.58	129.30
Daily Volatility	0.0107	0.0085	0.0078	0.0090	0.0089	0.0108
Order as %ADV	1.49%	1.72%	1.73%	1.71%	1.87%	1.94%
Price Impact (hist σ)	0.70%	0.57%	0.50%	0.61%	0.63%	0.84%
Price Impact ($\sigma = 2\%$)	1.60%	1.76%	1.77%	1.75%	1.86%	1.91%

Table 6 shows the predicted price impact of 75,000 S&P 500 E-mini futures contracts, given an inflation adjustment equating $1 in 2010 to $0.90 in 2005, average daily volume and volatility of the S&P 500 E-mini futures based on N months preceding January 18, 2008, with $N = 1, 2, 3, 4, 6, 12$. The actual price decline in the S&P 500 E-mini futures market was 5.12%.