Debt, Labor Markets and the Creation and Destruction of Firms*

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Abstract

We analyze the financing and liquidation decisions of firms that face a labor market with search frictions. In our model, debt facilitates the process of creative destruction (*i.e.*, the elimination of inefficient firms and the creation of new firms) but may induce excessive liquidation and unemployment; in particular, during economic downturns. Within this setting we examine policy interventions that influence the firms' financing and liquidation choices. Specifically, we consider the role of monetary policy, which can reduce debt burdens during economy-wide downturns, and tax policy, which can influence the incentives of firms to use debt financing.

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1 Introduction

Economic forecasters and policymakers have long recognized that financial structure at the corporate and household level can influence macro-economic conditions. The most recent economic crisis, which was triggered in part by the substantial leverage in the real estate and banking sectors, is perhaps the most visceral illustration of this point. While financial economists have produced a plethora of work (*e.g.*, Brunnermeier 2009) that consider policy issues that relate to the leverage of financial institutions, the more general issue of the interaction between corporate financing choices and macro policy has received scant attention.

To examine the interaction between corporate financing choices and macro policy we incorporate a canonical corporate finance model, in which debt plays a fundamental role to limit agency problems within the firm, into a canonical model taken from the macro/search literature in which production requires a suitable match between workers and firms. In particular, we follow Hart and Moore (1995) and assume that debt choices are made by investors to indirectly restrain managers who enjoy private benefits of control and insert this into a macro labor search model along the lines of Pissarides (2000). In the resulting framework, we explore how firms' capital structure choices, through their effects on liquidation, can affect the tightness of labor markets in both booms and recessions, and how these effects can in turn affect the emergence of new firms.

Our model includes two generations of firms. The first generation are established firms that may subsequently be liquidated and the second generation are potential entrants that may hire the workers that are liberated from the established firms that liquidate. Firms of both generations interact in a labor market which is subject to search frictions. Established firms continue their operations only when they are able to retain their labor force, which is in turn affected by labor market conditions. Entrants need to hire workers produce and such hiring ability is affected by the tightness in the labor market. Thus there is an interaction between firms of different generations since firm entry (*i.e.*, creation) affects the demand for labor, and hence the ability of established firms to retain workers, and firm liquidation (*i.e.*, destruction) affects the labor supply, and hence the process

of firm creation. Capital structure plays an important role in this interaction since debt choices by firms affect their liquidation decisions. Indeed, since firm liquidation decisions affect labor market conditions, there are potential externalities created by individual debt choices on workers as well as emerging new firms.

Within the context of this model we examine a number of policy issues that influence a firm's debt obligations. These include a tax policy that subsidizes the use of debt financing by firms,[1] and monetary policy which, by affecting the overall price level, can influence the real value of a firm's nominal debt obligations. We also examine how expectations about monetary policy affect the capital structure choices of firms and through this channel, how monetary policy influences the liquidation of established firms and the creation of new ventures.

Our analysis starts with the simplest form of our model that includes a fixed number of established firms and an unlimited number of ex-ante identical entrants that can potentially emerge. As we show, in this setting there is no externality associated with debt financing, so the optimal subsidy or tax on debt is zero. However, even within the context of this simple model there can be an important role for policies that influence firms' liquidation choices. Specifically, by generating inflation, a loose monetary policy can reduce the real value of debt during economy wide downturns and, as a result, reduce bankruptcies in bad times, when liquidations are socially costly. In addition, since such a policy leads to higher ex-ante debt ratios, it increases bankruptcies in good times, when there would otherwise be too few liquidations.

We next consider a setting where the potential entrants are heterogeneous. When this is the case, firm debt choices are not in general socially optimal, and can lead to either too much or too little liquidation depending on parameters. The deviation from the social optimum arises because of negative externalities imposed on unemployed workers in the event of liquidation (*i.e.*, liquidation causes the unemployed to have more workers to compete with for jobs), as well as positive externalities that benefit emerging new firms that need to hire labor. Depending on the magnitude of these two effects a social planner

[1]The issue of the desirability of debt subsidies has been periodically raised. For instance during the Clinton and Bush administrations, the Congressional Budget Office (1997, 2005) considered proposals to eliminate the unequal treatment of debt and equity.

may want to use tax policy to tilt firms towards either more or less debt financing.

In addition to Hart and Moore (1995) and Pissarides (2000), which provide the basis for our model, our analysis is related to a number of papers in the literature. These include several theoretical contributions that consider potential negative spillovers created by debt financing. For instance, bankruptcy induced fire-sales (as discussed in Shleifer and Vishny 1992 and more recently Lorenzoni 2008) which impose negative externalities on other firms by affecting their collateral constraints. In addition, our analysis of positive externalities of liquidations is related to Schumpeter's (1939) ideas on creative destruction, and to more recent work by Kashyap et al.(2008), which examines the inability of Japanese banks to shut down failing firms in the 1990s. Finally, a contemporaneous paper by He and Matvos (2012) considers a case where debt facilitates firm exit when companies compete for survival in a declining industry and concludes that firms use less than the socially optimal amount of debt financing. To our knowledge, however, we are the first to consider the influence of debt in an economy where externalities can be imposed on workers as well as emerging new firms, and therefore the first to analyze the effects of policies that influence the real value of debt obligations and the incentives to use debt financing on labor markets and on the process of firm creation and destruction.

The rest of the paper is organized as follows. Section 2 describes our framework and Section 3 considers the analysis of a particular case within our framework, *i.e.*, when potential entrants are identical in terms of entry costs (*i.e.*, homogenous entry case). Section 4 considers the policy implications that emanate from the homogenous entry case. Section 5 considers the analysis of the general case (*i.e.*, heterogeneous entry case) and examines the policy implications for this general case. Section **??** considers a number of extensions including issues related to the interaction between firms' debt choices and unemployment insurance and issues relative to consider policy implications in an economy with more than one productive sector. Section 6 concludes. Proofs and other technical derivations are relegated to the appendix.

2 The model

We consider a risk-neutral economy in which the discount rate is normalized to zero. The economy consists of three periods: two productive periods $t = 1, 2$ and an interim period in which existing firms can be liquidated and new firms can be created. Next, we describe the agents, technology, contracting environment and labor market.

2.1 Agents: workers and two generations of firms

2.1.1 Established firms

The economy starts in the first productive period, $t = 1$, with a continuum of mass one of workers who are each already employed by a firm. An established firm produces in period 1 and, provided that it retains its worker, produces in period 2 as well. An established firm that fails to retain its worker is liquidated and realizes a liquidation value which we normalize to zero.

When an established firm i produces in period t, it generates a cash-flow r_{it} that can be decomposed as follows:

$$r_{it} = s_t + \varepsilon_i. \tag{1}$$

The first component s_t is an aggregate productivity shock in period t that is common to all firms. Formally $\{s_t\}_{t=1,2}$ is a sequence of two binomial variables where

$$s_1 = \begin{cases} s_h & \text{with prob. } p \\ s_l & \text{with prob. } 1-p \end{cases} \tag{2}$$

and

$$\Pr(s_2 = s_h | s_1 = s_l) = p - \varphi \leq \Pr(s_2 = s_h | s_1 = s_h) = p + \varphi \tag{3}$$

with $0 \leq \varphi \leq \min\{1-p, p\}$, that is, s_1 and s_2 are positively correlated.[2]

The second cash-flow component ε_i is a firm-specific shock, which is independent across firms, constant over time and drawn from a uniform distribution:

$$\varepsilon_i \sim U[-\overline{\varepsilon}, +\overline{\varepsilon}]. \tag{4}$$

[2]This formulation ensures that the probabilities included in the distribution are well-defined.

2.1.2 Entrants

Entrants are firms that by incurring an entry cost can come into the economy in the interim period, between production periods 1 and 2. We assume that there is a large set of potential entrants which are ex-ante identical except for their entry costs. In particular, potential entrants can be ordered according to their entry costs,

$$k(j) = k + \phi j, \tag{5}$$

where $k > 0$ is the minimum entry cost, $\phi \geq 0$ is a parameter that measures the difference in entry costs among potential entrants (*e.g.*, if $\phi = 0$ all entrants have the same entry cost), and $j \geq 0$ indexes entrants according to the magnitude of their entry costs.

After incurring the entry cost, an entrant j needs to hire a worker in a labor market with search frictions. If an entrant j succeeds in hiring a worker, it becomes active and generates a cash-flow r_{j2} at the end of period 2. However, if it fails to hire a worker, firm j loses its investment $k(j)$ and remains inactive and generates no cash-flow. Analogous to established firms, the cash-flow of an active entrant j in period 2 is

$$r_{j2} = s_2 + \varepsilon_j \tag{6}$$

where s_2 is the aggregate productivity shock in period 2 common to all firms (entrants and established firms) in the economy and ε_j is a firm-specific shock, independent across firms, and uniformly distributed, *i.e.*, $\varepsilon_j \sim U[-\overline{\varepsilon}, +\overline{\varepsilon}]$.

2.2 Contracting environment

Firms are initially controlled by investors who subsequently transfer control to managers. Following Hart and Moore (1995), we assume that managers enjoy private benefits of control which, consequently, makes an established firm to continue its operations in period 2 as long as the manager has access to the necessary funds to retain the firm's worker. For simplicity, we also assume that any funds available beyond those needed to retain the worker are paid out to the investors and that control benefits are sufficient to entice managers to run the firms (i.e., we ignore the effects of pecuniary compensation on managerial behavior).

Before transferring control to the manager, investors set the firm's capital structure. In particular, firm i issues short-term debt with a face value $d_i \geq 0$ that matures at the end of period 1, just after the cash-flow r_{i1} is realized. We assume that short-term debt is a "hard claim" which cannot be renegotiated with creditors and, because of this, forces the firm to liquidate if it fails to meet its payment d_i. The short-term debt market also exists in period 1, so if the firms cannot repay repay its short-term debt from its period 1 cash-flow, r_{i1} it may be able to cover the shortfall by borrowing funds against its period 2 cash-flow, r_{i2}. We exclude any other financial contract (other than the residual equity owned by the investors) and in particular, we assume that debt cannot be made contingent on specific cash-flow components $\{\varepsilon_i, s_t\}$.

2.3 Labor markets

To complete the description of the economy, we need to explain how the labor market operates. There are three instances in which firms and workers meet: (i) the initial allocation of workers to established firms when the economy starts at $t = 1$; (ii) the retention of workers by established firms in the interim period to continue production at $t = 2$; and (iii) the hiring of workers by entrants in the interim period to become active and produce at $t = 2$. In each case we assume that firms pay the workers before production takes place. The model includes search frictions (described below) in the reallocation of workers from established to new firms in the interim period but abstract from search frictions in period $t = 1$. For simplicity, however, we assume that workers are employed (and paid) by established firms when the economy starts.[3]

2.3.1 Labor market frictions

In the interim period, entrants face a labor market with search frictions that make it costly for firms and workers to find a suitable match (*e.g.*, Pissarides 2000). The labor market is formally described by a matching technology that specifies the likelihood of a match, and a sharing rule that describes how the matched parties share the surplus created by the newly formed relationship.

[3]The case in which established firms face a labor market with search costs does not qualitatively affect our results.

The matching technology is characterized by a constant-returns-to-scale Cobb-Douglas matching function:[4]

$$m(a, v) = \lambda a^{\alpha} v^{(1-\alpha)} \tag{7}$$

where m, the number of matches, is determined by a, the number of workers actively looking for jobs, and v, the number of entrants searching for workers. In this function, $0 < \alpha < 1$ is the elasticity of the number of matches to the number of workers looking for jobs, and $\lambda > 0$ is a measure of the efficiency of the matching technology.

Following the search literature, we refer to the ratio of firms (i.e., entrants) to workers $\theta \equiv \frac{v}{a}$ as the "labor market tightness". Given this matching technology, $q(\theta) \equiv m(1, \theta)$ is the probability that a worker finds a suitable position, and $q(\theta)/\theta$ is the probability that an entrant hires a suitable worker.[5] Notice that some entrants and workers will not find a match, that is, some entrants will fail to hire while some workers will remain unemployed. Total employment, *i.e.*, the mass of workers that look for a job during the interim period but cannot find one, is $u \equiv a - m(a, v)$.

When there is a match between an entrant and a worker, the surplus they create, $E(r_2|s_1)$, is allocated according to a sharing rule. In particular, when matched with an entrant, a worker receives a wage

$$w_2 = \gamma + \beta E(r_2|s_1) \tag{8}$$

where $\gamma \geq 0$ and $\beta \in [0, 1]$, which implies that the entrant's j expected cash-flow**,** gross of its entry cost $k(j)$, is

$$E(r_2|s_1) - w_2 = (1 - \beta)E(r_2|s_1) - \gamma.$$

This specification encompasses the case in which $\beta = 0$, where workers receive a fixed wage, as well as the case in which $\gamma = 0$, where workers and firms share the surplus generated by their relation, with β being the workers' share of the surplus, i.e., their

[4]Petrongolo and Pissarides (2001) justify the use of a Cobb-Douglas function with constant returns to scale on the basis of its success in empirical studies.

[5]Since $m(a, v)$ is characterized by constant returns-to-scale, it follows that $q(\theta) \equiv \frac{m(a,v)}{a} = m(1, \theta)$ and $\frac{q(\theta)}{\theta} \equiv \frac{m(a,v)}{v} = m\left(\frac{1}{\theta}, 1\right)$. Also, we assume an interior solution, which requires that λ is small enough such that, in equilibrium, probabilities are well defined, i.e., $m(a, v) < \min\{a, v\}$.

bargaining power. We assume that $E(r_2|s_1) - w_2 > k$ (i.e., $k < (1-\beta)E(r_2|s_1) - \gamma$) so that entry by new firms can sometimes be profitable.[6]

2.3.2 Workers' retention costs

We assume that an established firm must pay a worker his outside option to retain him. This payment corresponds to the expected compensation that the worker would receive if he quits his job and searches for an alternative job during the interim period, i.e., $U \equiv q(\theta)w_2$. This assumption, which implies that workers receive no rents when retained by established firms, simplifies the analysis since it implies that the investors' design of the firm's optimal capital structure is not influenced by the desire to reduce worker rents in period 2.[7]

2.4 Timing of events

There are two production periods and an interim period in which firm destruction and creation as well as worker reallocation take place. Specifically:

Period 1 ($t = 1$): A continuum of mass one of established firms employ one worker each. At the beginning of the period, each established firm i issues short-term debt d_i, and then transfers the control of its operations to a manager. At the end of the period, firm i produces a cash-flow r_{i1}, and its short-term debt d_i matures.

Interim period: Managers of established firms make their liquidation decisions and entrant j incur cost $k(j)$ to enter the market.

Period 2 ($t = 2$): Each entrant j attempts to hire an unemployed worker. If an entrant and a worker match, the firm becomes active. Established firms that are not liquidated and entrants that become active generate cash-flows $\{r_{i2}\}$ and $\{r_{j2}\}$ respectively.

The following time-line summarizes the relevant events:

[6]Note that this formulation assumes that, as indicated above, wages are paid at the beginning of the period, and hence, before the state is realized. This is without loss of generality since agents are risk neutral.

[7]Alternatively we could have considered a set-up in which workers earn rents in period 2, and hence are willing to accept a lower wage in period 1. However, because the excess wage in period 2 influences the capital structure and liquidation choices, ex ante firm values are lower when worker receives rents in period 2.

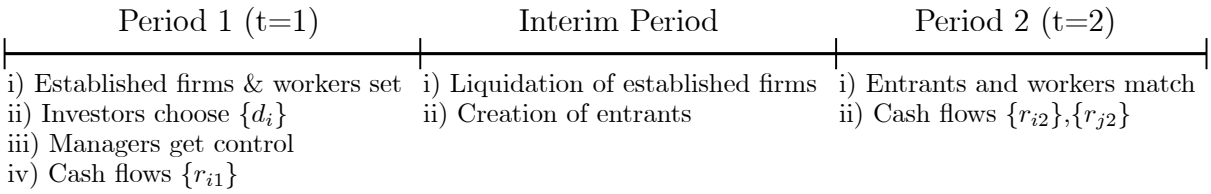


Figure 1: Timing of Events

3 Analysis of the model: Identical entrants

We start the analysis by considering the special case in which all potential entrants are identical in terms of their entry costs, *i.e.*, $\phi = 0$ so that $k(j) = k$ for all j. This case is particularly tractable because, as we show, the model can be solved recursively and produces many (but not all) of the results that are obtained from the analysis of the general case with heterogeneous entry costs, *i.e.*, $\phi \geq 0$, which we consider in Section 5.

The analysis starts in Section 3.1 with a characterization of the labor market conditions in the interim period assuming an exogenous number of job-seekers, *i.e.*, an exogenous labor supply. Then, in Section 3.2, we endogenize the labor supply by considering the firms' liquidation decisions in the interim period as a function of their debt choices in $t = 1$ (and taking into account the labor market effects derived in Section 3.1). Finally, in Section 3.3, we study the choice of debt by established firms at $t = 1$.

3.1 Labor market and entry

Consider the entry decision in the interim period $v(s_1)$ for a given labor supply $\bar{a} > 0$. Expressed in terms of market tightness, *i.e.*, $\theta_1 \equiv \theta(s_1) \equiv \frac{v(s_1)}{\bar{a}}$, the expected profit of each entrant is:

$$V(s_1) = -k + \frac{q(\theta_1)}{\theta_1}\left[(1-\beta)E(r_2|s_1) - \gamma\right], \tag{9}$$

where $\frac{q(\theta_1)}{\theta_1}$ is the probability of finding a worker and $[(1-\beta)E(s_2|s_1) - \gamma]$ is the expected cash-flow retained by the firm. Setting $V(s_1) = 0$ (since new firms enter the market until their expected profit from entering is zero) and using the matching function (7) the following lemma attains:

Lemma 1 *In equilibrium the labor market tightness is* $\theta^*(s_1) = \left(\frac{\lambda[(1-\beta)E(s_2|s_1)-\gamma]}{k}\right)^{1/\alpha}$ *and the workers' reservation utility is*

$$U^*(s_1) = \lambda\theta^*(s_1)^{(1-\alpha)}\left(\gamma + \beta E(s_2|s_1)\right). \tag{10}$$

From Lemma 1 there follow a number of implications. Specifically, Lemma 1 indicates that market tightness, $\theta^*(s_1)$, increases with the efficiency of the matching technology λ, and the expected surplus generated by a match $E(s_2|s_1)$, but decreases with the worker's share of this surplus (*i.e.*, decreases in both β and γ), and the fixed entry cost k. In addition, the workers' reservation utility, $U^*(s_1)$, increases with the efficiency of the matching technology λ, and the expected surplus $E(s_2|s_1)$, and decreases with the entry cost k. Parameters that affect worker's compensation conditioned on finding a job (i.e., β and γ) do not have a monotonic effect on $U^*(s_1)$. Intuitively, β and γ can reduce the worker's outside option $U^*(s_1)$ since higher worker compensation reduces firm entry, which in turn decreases the worker's probability of finding a job.

It is worth stressing that Lemma 1 also indicates that $\theta^*(s_1)$ and $U^*(s_1)$ are independent of the labor supply $\bar{a}$. Since the labor demand $v(s_1)$ is perfectly elastic –there is a unlimited number of potential entrants with identical entry costs– labor supply shocks, i.e., changes in $\bar{a}$, are fully offset by a corresponding change in firm entry until the equilibrium values $\theta^*(s_1)$ and $U^*(s_1)$ are achieved.

3.2 Liquidation decisions of established firms

So far we have characterized labor market tightness $\theta^*(s_1) \equiv \frac{v^*(s_1)}{\bar{a}}$ and workers' reservation utility $U^*(s_1)$ for a given labor supply $\bar{a}$. We now endogenize the labor supply $a(s_1)$ by noting that it corresponds to $l(s_1)$, *i.e.*, the number of workers employed at $t = 1$ by the established firms that are liquidated in the interim period:

$$a(s_1) = l(s_1). \tag{11}$$

Hence endogenizing $a(s_1)$ requires us to characterize the liquidation decisions of established firms, which in our setting are affected by managerial preferences. Specifically, since managers enjoy private benefits of control, an established firm is liquidated only

when its manager is unable to retain the firm's worker. Worker retention, in turn, requires firms to pay the workers' outside option, $U^*(s_1)$ with either internally generated or borrowed funds.[8]

3.2.1 Managerial liquidation choices

Since an established firm i produces a period 1 cash-flow $r_{1i} = s_1 + \varepsilon_i$ and a period 2 expected cash-flow $E(r_{2i}|r_{1i}, s_1) = E(s_2|s_1) + \varepsilon_i$, the firm is liquidated when:[9]

$$G(\varepsilon_i, s_1, d_i) \equiv 2\varepsilon_i + s_1 + E(s_2|s_1) - d_i - U^*(s_1) < 0. \tag{12}$$

Hence, for a given amount of debt, d_i, and an aggregate shock, s_1, firm i is liquidated when its idiosyncratic shock ε_i is smaller than $\varepsilon^{*1}_{d_i}$. This cut-off level, which makes $G(\varepsilon^{*1}_{d_i}, s_1, d_i) = 0$, is explicitly defined by:

$$\varepsilon^{*1}_{d_i} = \frac{1}{2}[d_i - s_1 - E(s_2|s_1) + U^*(s_1)]. \tag{13}$$

Since $\varepsilon_i \sim U[-\overline{\varepsilon}, +\overline{\varepsilon}]$, for a given realization of s_1 at $t = 1$, the probability that firm i is liquidated is

$$\Pr(\varepsilon_i < \varepsilon^{*1}_{d_i}) = \frac{1}{2} + \frac{d_i - s_1 - [E(s_2|s_1) - U^*(s_1)]}{4\overline{\varepsilon}}. \tag{14}$$

Intuitively, managers only liquidate their firms when they cannot raise the necessary funds to retain their workers, which is more likely to occur when firms have more debt, when the aggregate shock is less favorable, and when the conditions in the labor market make worker retention more costly.

[8]These observations illustrate the recursive nature of the model when $\phi = 0$. In this case liquidation decisions have no effect on labor market conditions (i.e., $\theta^*(s_1)$ and $U^*(s_1)$ are independent of $a(s_1)$), but labor market conditions do affect the wages paid by established firms, and thus affect liquidation decisions (i.e., $a(s_1)$ is affected by $\theta^*(s_1)$ and $U^*(s_1)$).

[9]Recall that for simplicity we have assumed that the firm pays the initial wage to the worker at the begining of period 1.

3.3 Debt choice of established firms

We now characterize the optimal capital structure, *i.e.*, the choice of debt, d_i, made by investors to maximize firm value. At the beginning of $t = 1$ firm i's investors solve:

$$\max_{d_i} \; p \int_{\varepsilon^{*h}_{d_i}}^{\overline{\varepsilon}} \frac{\varepsilon_i + E(s_2|s_h) - U^*(s_h)}{2\overline{\varepsilon}} d\varepsilon_i + (1-p) \int_{\varepsilon^{*l}_{d_i}}^{\overline{\varepsilon}} \frac{\varepsilon_i + E(s_2|s_l) - U^*(s_l)}{2\overline{\varepsilon}} d\varepsilon_i \tag{15}$$

where $\varepsilon^{*h}_{d_i}$ and $\varepsilon^{*l}_{d_i}$ correspond to $\varepsilon^{*1}_{d_i}$ when $s_1 = s_h$ and $s_1 = s_l$, respectively.

In the objective function (15), the first and second terms are the expected profits in period 2 when $s_1 = s_h$ and $s_1 = s_l$, respectively. These profits are affected by the debt choice because debt determines when the firm is liquidated, *i.e.*, it changes the liquidation cut-offs $\varepsilon^{*h}_{d_i}$ and $\varepsilon^{*l}_{d_i}$.

From (15) we can derive the following first order condition,

$$p[\varepsilon^{*h}_{d_i} + E(s_2|s_h) - U^*(s_h)] + (1-p)[\varepsilon^{*l}_{d_i} + E(s_2|s_l) - U^*(s_l)] = 0, \tag{16}$$

which can be rewritten as:

$$E[\varepsilon^{*1}_{d_i} + E(s_2|s_1) - U^*(s_1)] = 0. \tag{17}$$

Intuitively, as (17) indicates, the optimal amount of debt d^*_i is chosen so that the marginal firm that is liquidated has an expected value of zero. Using the definition of $\varepsilon^{*1}_{d_i}$ in (13) and since all established firms choose the same amount of debt (*i.e.*, $d^* = d^*_i$ for all i), equation (17) can be rewritten as:[10]

$$d^* = E(s_1) - E\left[E(s_2|s_1) - U^*(s_1)\right]. \tag{18}$$

Equation (18) shows that the optimal debt level d^* increases with the expected cash-flow in period 1, $E(s_1)$, as a higher expected cash-flow aggravates the managerial free cash-flow problem. Furthermore equation (18) also shows that d^* decreases with the expected value of the firm in period 2, $E\left[E(s_2|s_1) - U^*(s_1)\right]$, which is related to the severity of the debt overhang problem which, in turn it is related to the presence of search frictions in the labor market. Intuitively, search frictions make the period 2 relation

[10] Implicitly we are assuming that $E(s_1)$ is sufficiently large to ensure $d^* > 0$.

between established firms and their workers more valuable in expectation, and hence, reduce firms incentives to hold debt that may jeopardize this relationship. In terms of primitive parameters, $E\left[E(s_2|s_1) - U^*(s_1)\right]$ decreases with the efficiency of the matching technology λ, and increases with the entry cost k. In addition, worker compensation parameters (i.e., β and γ) have an ambiguous effect on the optimal debt choice. This is because, as discussed above, worker compensation has an ambiguous effect on the worker's reservation utility (i.e., $U^*(s_1)$) and since the optimal debt choice d^* depends on $E(U^*(s_1))$ the effect of worker's compensation on d^* is ambiguous as well. The following proposition states the main comparative statics that relate to the optimal debt choice.

Proposition 1 *The optimal amount of debt d^* increases with the firm's expected cash-flow in period 1, i.e., $E(s_1)$, and with the efficiency of the matching technology λ, and decreases with the entry cost k. Furthermore, the effect of worker compensation parameters (i.e., β and γ) in d^* is non-monotonic.*

Finally, notice that d^* determines $\varepsilon^{*1}_{d^*}$, which, in turn, determines the number of workers whose employers are liquidated at the end of the period:

$$l^*(s_1) = \Pr(\varepsilon_i < \varepsilon^{*1}_{d^*}), \tag{19}$$

Specifically, the above analysis produces a number of implications that relate to firms' debt choices and liquidation decisions. First, there are inefficiencies in firm liquidation. In particular, in good economic times, when $s_1 = s_h$, optimal debt choices lead a number of negative NPV firms to continue operating since managers have cash-flows generated in period 1 to continue operations, i.e., $\varepsilon^{*h}_{d^*} + E(s_2|s_h) - U^*(s_h) < 0$. In contrast, during recessions when $s_1 = s_l$, optimal debt choices lead a number of positive NPV firms to liquidate, i.e., $\varepsilon^{*h}_{d^*} + E(s_2|s_h) - U^*(s_h) < 0$ since the firm is unable to refinance its debt obligations. Indeed, as shown in (16), the optimal debt choice, d^*, equates p times the value saved from liquidating the (unprofitable) marginal firm, when $s_1 = s_h$, to $1-p$ times the value lost from liquidating the (profitable) marginal firm when $s_1 = s_l$. Second, the analysis implies that there is refinancing activity by firms in recessions but not in good times. Specifically, in recessions, the inframarginal firms have insufficient funds

to satisfy their debt obligations, d^*, but their positive future prospects allow them to borrow the necessary funds to pay d^*. In contrast, in good times, no additional borrowing occurs, since positive NPV firms have sufficient funds to repay d^* and only negative NPV firms lack the ability to raise funds. The following proposition summarizes the previous observations:

Proposition 2 *The optimal debt policy implies that: (i) in recessions, i.e., $s_1 = s_l$, the marginal firm that is liquidated has a positive NPV, and several inframarginal firms refinance their debt obligations which are repaid by borrowing additional funds; (ii) in good times, i.e., $s_1 = s_h$, the marginal firm liquidated has a negative NPV and no refinancing or additional borrowing occurs.*

We conclude this section by discussing how the two main elements of the model, namely the managerial agency conflict and labor market search frictions, interact to affect the liquidation decision of firms. In the absence of agency conflicts, search frictions reduce firms' incentives to liquidate in both recessions and good economic times (i.e., $U(s_1)$ decreases with λ). Adding managerial agency problems does not affect the unconditional (i.e., before s_1 is realized) probability of liquidation but changes the conditional one (i.e., after s_1 is realized). In particular, because of the agency conflict there is a higher probability of liquidation during recessions when the labor market conditions are worse, and a lower probability of liquidation in good economic times when the labor market conditions are better (i.e., $\theta^*(s_l) < \theta^*(s_h)$ and $U^*(s_l) < U^*(s_h)$). As a result, the use of debt by firms to address agency conflicts results in unemployment that is too high in recessions and too low in booms.[11]

4 Policy implications

The previous analysis considers the effect of debt on the creation and destruction of firms, and the resulting effect on the number of workers that are unemployed. Within this setting, we consider two sets of questions that relate to policy implications. First,

[11]Notice that the labor market conditions are pinned down by the zero profit condition (and hence, they are independent of firms' debt choices and liquidation decisions). As we will see in Section 5, this is not the case in the more general case in which entry is not perfectly elastic, i.e., $\phi > 0$.

we examine ex-ante policy interventions that change the amount of debt chosen by firms at $t = 1$. Second, we examine ex-post policy interventions that change the real value of firms' debt obligations in the interim period, after s_1 is realized.

In what follows, we consider a social welfare function that includes the sum of firm profits and wages, but excludes the private benefits of managerial control.[12] It is worth noting that the policy that maximizes this social welfare function does not minimize unemployment. Since what matters is the sum of firm profits and wages, from the social point of view it may be more desirable to have fewer but more efficient firms even if this results in higher unemployment. Considering an alternative social welfare function that includes managerial private benefits and/or an additional exogenous social cost of unemployment would reduce the socially desirable level of debt but does not qualitatively affect our results. More precisely, we will still obtain that private debt choices can affect search externalities in labor markets and that public policies that affect the incentives to use debt financing can have welfare implications.

4.1 Ex-ante policy interventions: Corporate tax policy

We first consider whether social welfare can increase by changing incentives for the use of debt at $t = 1$, for instance, through corporate tax policy. We abstract from the potential implementation costs associated with this policy and focus exclusively on how incentives to use more or less debt financing affects total production. We start by stating the following result.

Proposition 3 *When entrants have identical entry costs, i.e., when $\phi = 0$, investors choose the socially optimal amount of debt to fund their firms at $t = 1$.*

Proposition 3 indicates that there is no need for ex-ante public interventions or, in other words, that incentives or subsidies that distort the use of debt financing by

[12]Ignoring managerial private benefits is consistent with a political system in which managers have a negligible influence in the outcome of political elections. Alternatively, it is also consistent with a situation in which the marginal benefits of the last dollar of managerial compensation are negligible in comparison with the benefits of the marginal dollar of worker and investor compensation. See Hart (1995) pp. 126-130 for an insightful discussion of the conditions in which ignoring managerial private benefits for the analysis of capital structure can be justified.

firms would reduce social welfare. This finding may appear somewhat surprising since, as already established in the search literature, entry and liquidation decisions create externalities in the presence of search frictions and the resulting market tightness is in general either too high or too low from a social point of view.[13] However, in the case considered up to now, with identical entry costs, i.e., $\phi = 0$, any tax incentive that affects debt choice (and hence firm liquidation) is fully offset by changes in firm entry and does not affect labor market tightness θ_1^*. Put differently, while θ_1^* may be socially suboptimal, it cannot be affected by firms' leverage choices and thus any policy that influences those choices is ineffective. Moreover, since tax incentives do not correct inefficiencies in the labor market and, on the contrary, distort firms' liquidation policies, tax incentives that distort debt choices reduce welfare relative to the case in which no incentives are provided. Therefore, as a consequence, when $\phi = 0$ the (ex-ante) social and private choices of leverage coincide.

4.2 Ex-post policy interventions

We now consider the possibility of "monetary" interventions that affect the real value of debt obligations after period 1. We abstract from institutional details of implementation and simply consider the social welfare implications of changes in the real value of the firms' debt obligations after s_1 is realized (but before the liquidation decisions are made).

We model monetary policy as a technology that modifies the real value of firms' debt obligations from d to $d-\tau$ at the expense of a social cost $c(\tau)$ where $c(0) = c'(0) = 0$, and $c'' > 0$. We refer to the monetary policy as inflationary when $\tau > 0$ and as deflationary when $\tau < 0$. We assume that the marginal cost of an inflationary policy is smaller than the cost of an equivalent deflationary policy (i.e., $c(\tau) \leq c(-\tau)$ and $c'(\tau) \leq c'(-\tau)$ for any τ) which is consistent with the fact that, in general, countries exhibit some degree of inflation, albeit, in most cases moderate.[14]

To gain intuition we analyze first the case where firms do not anticipate monetary interventions. Specifically, we examine the socially optimal monetary policy as a function

[13] Specifically, a firm's liquidation creates a negative externality for unemployed workers and a positive externality for firms with vacancies.

[14] For instance the cost function $c(\tau) = \tau^2$ for $\tau \geq 0$ and $c(\tau) = \varphi\tau^2$ for $\tau < 0$ where $\varphi \geq 1$.

of the realization of the aggregate shock s_1, i.e., $\tau(s_1) \equiv \tau_1$, taking as given the equilibrium debt choice d^*. Formally the policymaker faces the following problem:

$$\max_{\tau_1} \int_{\varepsilon(\tau_1)}^{\overline{\varepsilon}} \frac{\varepsilon_i + E(s_2|s_1) - U^*(s_1)}{2\overline{\varepsilon}} d\varepsilon_i - c(\tau_1) \tag{20}$$

$$\text{s.t.} \quad \varepsilon(\tau_1) = \frac{1}{2}[d^* - \tau_1 - s_1 - E(s_2|s_1) + U^*(s_1)], \tag{21}$$

whose solution is characterized by the next proposition.

Proposition 4 *When monetary policy is not anticipated, it is optimal to follow an inflationary monetary policy during recessions, i.e., $\tau_l < 0$, and a restrictive policy during booms, i.e., $\tau_h > 0$.*

As shown in Proposition 2, during economic booms there is less liquidation than would be socially desirable (due to the managerial desire for continuation which makes some negative NPV firms continue their operations) and during recessions there is more firm liquidation than would be socially optimal (due to debt overhang which forces managers to liquidate positive NPV firms). Thus, a policy that increases the real value of debt during booms and decreases it during recessions helps to make firm liquidation closer to the social optimum.

We now consider the case where firms foresee that an active monetary policy is followed, and adjust their debt choices at $t = 1$ accordingly. Proposition 5 characterizes the optimal policy and how an anticipated monetary policy affects the firms' choice of debt at $t = 1$.

Proposition 5 *When monetary policy is anticipated, the optimal monetary policy $\tau_1^* = \{\tau_l^*, \tau_h^*\}$ is inflationary during recessions and deflationary during booms, i.e., $\tau_l^* > 0 > \tau_h^*$ and firms increase their debt choices at $t = 1$ beyond d^*, i.e., $d^*_{\tau_1^*} \geq d^*$. Moreover, the choice of debt by firms $d^*_{\tau_1^*}$ is ex-ante socially optimal and the monetary policy τ_1^* is time consistent.*

From the previous proposition a number of observations arise.[15] First, as in the case in which monetary policy is unanticipated, τ_1^* is chosen to offset the effects on liquidation

[15] See the proof of Proposition 5 for a full characterization of the optimal monetary policy τ_1^*, and the optimal debt choices by firms, $d^*_{\tau_1^*}$.

of too much debt in recessions, i.e., $s_1 = s_l$, and too little debt in booms, i.e., $s_1 = s_h$. Second, firms anticipating an active monetary policy adjust their debt choices at $t = 1$ in a socially optimal manner. To be more specific, whether firms end up increasing or decreasing their debt obligations depends on whether the government has an inflationary or deflationary bias in its monetary policy, which, in turn, depends on the relative cost of inflation and deflation. In our case, since $c'(\tau) \leq c'(-\tau)$ for $\tau > 0$, firms increase their debt obligation to $d^*_{\tau_1^*} \geq d^*$ in anticipation of a policy that is biased toward inflation.

Finally, Proposition 5 states that the optimal monetary policy is time consistent, i.e., the government can carry out the optimal policy without committing to it in advance. Intuitively, the optimal policy is time consistent since, in this setting, liquidation decisions that maximize firm values also maximize social welfare. Roughly speaking, this implies that policymakers have no incentive to mislead firms when they announce their policy objectives, implying that policymakers' ability to commit is not necessary to implement the optimal policy. As we show in the analysis of Section 5, this is not the case when entrants can be heterogeneous.

5 The general case

5.1 Debt choices with heterogeneous entrants

We now consider the case in which entrants differ in their entry costs, i.e., $\phi > 0$. As in the case of identical entrants, i.e., $\phi = 0$, solving the model requires us to characterize: (i) in period 1, the firms' debt choices $\{d_i\}$, which are identical, i.e., $d_i = d$; and (ii) for each $s_1 = \{s_l, s_h\}$ in the interim period, the labor market conditions, $\theta(s_1)$ and $U(s_1)$, and the firm creation and liquidation choices, $v(s_1)$ and $l(s_1)$. In particular, the equilibrium is characterized by the following conditions:

1. Firms enter the market until the *marginal* entrant is indifferent between incurring its entry cost and staying out (i.e., zero profit condition for the marginal entrant). Hence, if $v(s_1)$ is the number of entrants, for the marginal entrant the following

expression holds

$$k + \phi v(s_1) = \frac{q(\theta_1)}{\theta_1} \left[(1-\beta) E(r_2|s_1) - \gamma \right]. \tag{22}$$

2. Workers' outside option $U(s_1)$ depends on the number of liquidated firms $l(s_1)$, and the number of entrants $v(s_1)$:

$$U(s_1) = \lambda \theta_1(s_1)^{(1-\alpha)} (\gamma + \beta E(s_2|s_1)) \tag{23}$$

where $\theta_1(s_1) = \frac{v(s_1)}{l(s_1)}$.

3. Firm liquidation $l(s_1)$ depends on $U(s_1)$ and the firms' period 1 debt choice, d:

$$l(s_1) = \frac{d + \bar{\varepsilon} - s_1 - E(s_2|s_1) + U(s_1)}{4\bar{\varepsilon}}. \tag{24}$$

4. In period 1, firms' debt choices depend on $U(s_1)$ and are given by:

$$d = E(s_1) - E\left[E(s_2|s_1) - U(s_1) \right]. \tag{25}$$

Notice that whereas expressions for $v(s_1)$, $U(s_1)$, $l(s_1)$ and d resemble the corresponding expressions (9), (10), (18) and (19), there is an important difference. While with identical entrants the model is recursive (i.e., $\theta(s_1)$ and $U(s_1)$ affect but are not affected by $v(s_1)$, $l(s_1)$ and d) with heterogeneous entrants the model is not recursive, i.e., all endogenous variables have effects on each other. Specifically, the zero profit condition (22) does not pin down $\theta(s_1)$ and $U(s_1)$ and instead (22), (23), (24) and (25) must be solved simultaneously. In particular, debt choices influence the labor market conditions as described by $\theta(s_1)$ and $U(s_1)$.

As shown below, heterogeneity in entrant types leads to some policy implications which are in sharp contrast to those obtained with identical entrants. In particular, debt choices affect labor market conditions and produce social welfare distortions. Therefore, with heterogeneous entrants, there are a number of policy choices which, by influencing firms' debt choices, affect not only firm liquidation and unemployment but also social welfare. We discuss these implications next.

5.2 Policy implications

As in Section 4, we consider policy choices that either influence the firms' capital structures choices, i.e., ex-ante interventions, or alternatively, affect the real value of debt obligation, i.e., ex-post interventions. To perform the policy analysis it is useful to denote the worker's share of the surplus that is produced when the worker matches with a newly created firm as

$$\tilde{\beta}(s_1) \equiv \frac{w_2(s_1)}{E\left(s_2|s_1\right)} = \frac{\gamma}{E\left(s_2|s_1\right)} + \beta. \tag{26}$$

As we show, how the surplus created by new ventures is divided between workers and entrants, i.e., $\tilde{\beta}(s_1)$, affects some optimal policy choices investigated below.

5.2.1 The case without aggregate uncertainty

To gain intuition about the implications of heterogeneous entry, it is useful to start with the case without aggregate uncertainty in $t = 1$, i.e., $s_l = s_h = \bar{s}_1$. In this case, firms, which are still exposed to idiosyncratic shocks, can perfectly foresee macroeconomic conditions including the conditions in the labor market. Without aggregate uncertainty, expression (26) becomes

$$\tilde{\beta}(\bar{s}_1) \equiv \frac{\gamma}{E\left(s_2\right)} + \beta \equiv \bar{\beta}, \tag{27}$$

and the following result holds:

Proposition 6 *Without aggregate uncertainty at $t = 1$ i.e., $s_l = s_h = \bar{s}_1$, there exists a debt choice $\bar{d}^*$ that implements the firms' privately optimal liquidation policy.*

To see why Proposition 6 holds notice that from (25) it follows that without aggregate uncertainty the optimal debt choice is

$$\bar{d}^* = \bar{s}_1 - E(s_2) + U^*(\bar{s}_1). \tag{28}$$

Intuitively, $\bar{d}^*$ is set to remove any funds in excess of those needed to undertake positive NPV investments at $t = 1$. Specifically, notice that a firm with an idiosyncratic shock ε' such that $\varepsilon' + E(s_2) = U^*(\bar{s}_1)$ has a zero continuation value, i.e., the cash-flow from

continuing operations equals the firm's labor costs. In contrast, the manager of firm i continues operations whenever firm i's idiosyncratic shock ε_i is such that $\varepsilon_i + E(s_2) > U^*(\bar{s}_1) - [\bar{s}_1 + \varepsilon_i - d_i]$, that is, whenever the cash-flow from continuing operations is sufficient to cover labor costs net of the firm's first period cash-flows (i.e., $[\bar{s}_1 + \varepsilon_i - d_i]$.) Thus by setting $d_i = \bar{s}_1 - \varepsilon_i' = \bar{d}^*$ investors allow firms with $\varepsilon_i > \varepsilon'$ to continue operations while forcing firms with $\varepsilon_i < \varepsilon'$ to liquidate. As stated in Proposition 6 this implies that firms' liquidation decisions are privately optimal (i.e., there is no privately inefficient liquidation or continuation in the interim period).

The public policy question that remains is whether privately optimal debt choices $\bar{d}^*$ correspond to the socially optimal debt choices, i.e., the debt choice that maximizes social welfare d_W^*. The following proposition shows that this is not generally the case:

Proposition 7 *With heterogeneous entrants $\phi > 0$ and no aggregate uncertainty at $t = 1$, i.e., $s_l = s_h = \bar{s}_1$, private debt choices are generally socially suboptimal. Specifically, $\bar{d}^* < d_W^*$ (resp. $\bar{d}^* > d_W^*$) whenever $\bar{\beta} < \alpha$ (resp. $\bar{\beta} > \alpha$). Private debt choices are socially optimal, i.e., $\bar{d}^* = d_W^*$, only when $\bar{\beta} = \alpha$.*

In contrast to the homogenous entry case ($\phi = 0$) discussed in Proposition 3, with heterogenous entrants private and social choices are in general different. Intuitively this occurs because when $\phi > 0$ firm liquidations are not fully offset by firm entry and thus debt choices, which affects firm liquidation, have an effect on labor market tightness.

Proposition 7 can be directly related to several results in the search literature that consider whether firm liquidation and exit affects social welfare by creating externalities on either unemployed workers or on entrants. In particular, in a market with search frictions, a firm's exit imposes a negative externality on job-seekers (i.e., a "thick market externality") and a positive externality on other potential employers (i.e., a "congestion externality"). Hence, in a setting like ours, in which debt choices determine firms' exits, the optimal trade-off between these two externalities is directly connected to debt choices.

a) Ex-ante interventions: Corporate tax policy

Having established the conditions that relate private and social debt choices, we now

discuss the ex-ante tax policy interventions that can affect firms' capital structure choices. An immediate corollary from Proposition 7 is the following:

Corollary 1 *In the conditions of Proposition 7 the socially optimal corporate tax policy should promote (discourage) debt financing whenever* $\bar{\beta} < \alpha$ *(resp.* $\bar{\beta} > \alpha$*).*

Intuitively, the conditions stated in Proposition 7 show that the policymaker would like to set the number of firm liquidations to the point where the marginal benefit in terms of additional matches with new firms is equal to the cost that it imposes on the unemployed workers. Specifically, the thick market and congestion externalities exactly offset each other only when $\bar{\beta} = \alpha$, that is, when the worker's share of the surplus $\bar{\beta}$ is equal to the elasticity of the matching function with respect to unemployment α. This is the familiar Hosios (1990) condition that appears in a number of applications in search theory including the model considered here.[16]

Further intuition on the optimal tax policy can be gained by considering the limiting cases relative to the parameters $\bar{\beta}$ and α that govern the comparative statics. First, when $\bar{\beta}$ is large (small), the size of the positive congestion externality created by liquidation is small (large)–since entrants get a small (large) surplus from matches–and the size of the negative thick market externality on workers is large (small)–since unemployed workers lose the opportunity of finding a highly paid job. Second, when α is large (small), the size of the positive congestion externality created by liquidation in other firms is large (small)–since entrants' chances of finding a worker are strongly (weakly) affected by liquidations–and the size of the negative thick market externality on workers is small (large)–since in the constant returns to scale matching technology is such that the effect of the liquidating firm (i.e., $1 - \alpha$ is reduced).[17]

[16]The policymaker would like to increase the number of vacancies to the point where the marginal benefit in terms of additional matches is equal to the cost. In equilibrium, firms enter until the marginal benefits in terms of their bargained returns is equal to the cost. So if $(1 - \bar{\beta})$ is low (and hence, $\bar{\beta}$ is too high), they are getting too small a fraction of the return, and they will not enter enough. If $(1 - \bar{\beta})$ is too high, then they are getting too much of the surplus, so there will be excess entry. The right value of $(1 - \bar{\beta})$ turns out to be the one that is equal to the elasticity of the matching function with respect to number of vacancies $(1 - \alpha)$, that is, when $\bar{\beta} = \alpha$.

[17]It is also worth noting that the relation between $\bar{\beta}$ and α does not depend on the number of entrants, $v(\bar{s}_1)$. That is $v(\bar{s}_1)$ affects both the market tightness in equilibrium and the socially optimal market tightness but not whether one is smaller or larger than the other.

Given the relationship between wages and aggregate productivity Corollary 2 can be stated:

Corollary 2 *Given the conditions of Proposition 7, an increase in future productivity (i.e., $E(s_2)$) increases the socially optimal amount of debt.*

An increase in the expected aggregate productivity, i.e., $E(s_2)$, decreases $\bar{\beta}$, and tends to make the labor market too tight from the social point of view. This occurs because, when there are real wage rigidities, increases in $E(s_2)$ do not translate into a proportional wage increases $w_2(\bar{s}_1)$. Thus real wage rigidities tend to make the labor market too tight during periods of expected economic prosperity (that is, there are too many firms looking for workers relative to the number of unemployed workers) and too loose during periods of lousy economic prospects (that is, too many unemployed workers looking for jobs relative to the number of job vacancies). As stated in Corollary 2, when good economic times are expected at $t = 2$, the positive externalities that liquidation creates on new firms looking for workers are greater than the negative externalities that liquidation has on other unemployed workers and therefore debt should be promoted at $t = 1$. When bad economic times are expected (*i.e.*, when $E(s_2)$ is low) firms can easily find workers, and hence, additional liquidations do not help these firms much while it hurts the unemployed workers who already a small probability of finding a job. Thus when $E(s_2)$ is low public policy should discourage the use of debt in firms' capital structures at $t = 1$.

b) Ex-post policy interventions

We now discuss the effects of ex-post interventions throughout an active (and anticipated) monetary policy. We start by describing the results when monetary policy is the only policy tool (i.e., when an active corporate tax policy at $t = 1$ is not available) and later consider the optimal mix of monetary and tax policy.

Proposition 8 *With no aggregate uncertainty at $t = 1$, i.e., $s_l = s_h = \bar{s}_1$, and heterogenous entrants $\phi > 0$, anticipated monetary interventions τ_1^* generate costly inflation when $\bar{\beta} > \alpha$ and costly deflation when $\bar{\beta} < \alpha$ and lead to worse outcomes relative to the case in which policymakers commit not to intervene.*

With heterogenous entry costs policymakers face the problem of time inconsistency in monetary policy interventions (as described by Kydland and Prescott, 1977). This time inconsistency problem arises from the differences in the social and private benefits of liquidation choices. If policymakers prefer that firms have lower (higher) debt obligations they will choose to inflate (deflate) ex-post. However, this gives firms an incentive to choose a higher (lower) debt obligation ex-ante. In the absence of aggregate uncertainty, these choices lead to a situation with high (low) nominal debt obligations and high (deflation) inflation which render policy interventions costly for the government and ineffective to affect firm liquidation.

From the previous discussion there follow two main conclusion. First, when monetary interventions have a social cost, the government would like to commit to not intervene to use monetary interventions to correct firms capital structures. Second, when both corporate tax policy and monetary interventions are available, the optimal policy will rely exclusively on tax policy interventions. This conclusion depends on the assumption that tax policy interventions have no social cost and needs to be qualified if social costs of taxes are considered. With social tax policy costs then the optimal policy mix would consider tax policy and monetary interventions. In this case, nevertheless, monetary interventions, which would still be subject to time inconsistency, would lead to additional social costs by inducing costly monetary adjustments and by interfering with the optimal provision of tax incentives to corporate debt.

5.2.2 The general case with aggregate uncertainty

We now consider the optimal policy interventions in the general case of heterogenous entry with aggregate uncertainty. To facilitate the presentation we compare the general results with those obtained in the absence of aggregate uncertainty.

a) Ex-ante interventions

When there is aggregate uncertainty, corporate tax incentives can help for two reasons. As in the case of aggregate uncertainty debt choices affect the magnitude of the search externalities that can arise in the labor market. Furthermore, *in addition to the previously*

identified distortion, debt choices can create an additional externality related to a general equilibrium effect. This additional externality occurs because firms' debt choices affect the workers' outside option in the interim period, $U(s_1)$, which in turn, affects the firms' liquidation in period 1.[18,19] While the specific conditions to promote and/or discourage debt choices are given in the appendix, it is important to note that, in general, private debt choices are socially suboptimal and that labor market conditions have an effect on the optimal tax policy. For example, if $\tilde{\beta}(s_h) < \alpha < \tilde{\beta}(s_l)$ the socially optimal amount of debt trades-off the possibility of ending up with too much debt in recessions against too little debt during booms.

b) Ex-post interventions

From an ex-post point of view, there are now two reasons to increase (*reduce*) the value of debt during good economic times, that is to resolve the free-cash low (*debt overhang*) problem and to increase (*decrease*) the tightness of the labor market.

Finally, it should be noted that taxes and subsidies on debt play a role that can influence the incentives to inflate. In particular, by having a second channel for influencing a firm's debt choice, the analysis suggests complementary roles for tax and monetary policies.[20] Specifically, policy makers can use taxes and subsidies to induce firms to choose the socially optimal debt ratio ex-ante, and then use monetary policy to accommodate the realization of the aggregate shock in the interim period.[21]

[18] See the appendix for the formal derivations.

[19] In the case without aggregate uncertainty (Section 5.2.1) the firms debt choices also affect $U(s_1)$, which in turn, affects firms' liquidation in period 1. In that case, however, the NPV of the marginal firm liquidated is zero (liquidation choices are privately optimal) and hence so is the value of this general equilibrium externality in equilibrium.

[20] Again, if there are social costs associated with firm exits, then the social planner would make debt financing more costly, which would in turn lead to less inflation in an equilibrium without central planner commitment. However, if there are social benefits associated with firm exits, then the existence of debt subsidies will negate the incentive to implement a deflationary policy.

[21] Under some conditions, this combination allows for the implementation of the optimal monetary policy without the need for commitment.

6 Concluding remarks

Since the seminal work of Modigliani and Miller (1958), economists have examined the costs and benefits of financial leverage from the perspective of firms seeking financing. In this paper, we extend this analysis and examine how corporate financing choices influence the aggregate economy. In particular, we consider a setting where financial leverage can increase the probability of a firm liquidating following economic shocks, and within this setting we consider potential externalities. For example, corporate liquidations can have negative externalities during economic recessions, if they contribute to excess slack in the labor markets. In contrast, liquidations may have positive externalities during economic booms, if they facilitate the emergence of more productive startup companies.

The framework we develop provides intuition about the economic effects of policies that influence the magnitude of firm debt obligations. In particular, we consider inflation policy, which affects the real value of debt obligations, and show that in some situations an active policy that decreases debt obligations during economy-wide downturns can improve ex ante firm values. In addition, we identify conditions under which welfare can be improved with subsidies or taxes that alter the firms' use of debt financing.

While we do not explore this in our paper, there are a number of other policy choices that may be evaluated within the framework of our model. For example, the U.S. government provides subsidized debt for emerging industries that may create positive externalities, like renewable energy, as well as for failing industries, like automobiles, that might otherwise create negative spillovers. Since a primary motivation for these initiatives is to create and save jobs, a model, such as ours, that explicitly considers the effect of financing on the labor market might be relevant. In addition to calculating the relevant spillovers that are a direct consequence of the policy, an evaluation of these policies should also consider alternatives, like subsidized equity, as well as these policies on future decisions by the firm that may also influence the job market and the creation of new firms.

There are a number of important aspects of our analysis that merit further attention. In addition to considering the study of a richer set of policy tools, future research should also extend the scope of the model. For instance, we consider very limited dynamics

(firms interact in a single period) and therefore ignore how these policy choices influence business cycles and the growth rate of the economy. An analysis of the optimal debt policy and public interventions in a more dynamic (and more complicated) setting is a challenge that is left to future work.

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APPENDIX

Proof of Lemma 1

Note that $E(r_2|s_1) = E(s_2|s_1)$ and hence, the free-entry $V(s_1) = -k + \frac{q(\theta_1)}{\theta_1}[(1-\beta)E(r_2|s_1) - \gamma] = 0$ implies: $\theta_1^* = (\frac{\lambda[(1-\beta)E(s_2|s_1)-\gamma]}{k})^{1/\alpha}$ and

$$U^*(s_1) = q(\theta_1^*)(\gamma + \beta E(s_2|s_1)) = \lambda^{1/\alpha}(\frac{(1-\beta)E(s_2|s_1) - \gamma}{k})^{\frac{1-\alpha}{\alpha}}(\gamma + \beta E(s_2|s_1))\blacksquare$$

Proof of Proposition 2

From the social point of view firm i should be liquidated when its expected cash-flow in period 2, $E(r_{i2}|r_{i1}, s_1)$, is lower than the workers' outside option, $U^*(s_1)$. (Note that $U^*(s_1)$ is pinned down by the free-entry condition and hence is not affected by the liquidation decision or the amount of debt.)

$$H(\varepsilon_i, s_1) \equiv E(r_{i2}|r_{i1}, s_1) - U^*(s_1) = \varepsilon_i + E(s_2|s_1) - U^*(s_1) < 0. \tag{29}$$

In equilibrium the marginal firm liquidated is (see equation (13)):

$$\varepsilon_{d_i}^{*1} = \frac{1}{2}[d^* - s_1 - E(s_2|s_1) + U^*(s_1)] \tag{30}$$

which implies that:

$$H(\varepsilon_{d_i}^{*1}, s_1) = \frac{1}{2}[d^* - s_1 + E(s_2|s_1) - U^*(s_1)]. \tag{31}$$

There is too much liquidation in recessions (debt overhang) if the marginal firm liquidated in recession has a positive social value (that is, if $H(\varepsilon_d^{*l}, s_1) > 0$). Symmetrically, there is too little liquidation in recessions (a free cash-flow problem) if the marginal firm liquidated during booms has a negative social value (that is, $H(\varepsilon_d^{*h}, s_1) < 0$). Since

$$d^* = E(s_1) - E\left[E(s_2|s_1) - U^*(s_1)\right], \tag{32}$$

then $H(\varepsilon_d^{*l}, s_1) > 0$ and $H(\varepsilon_d^{*h}, s_1) < 0$ if and only if:

$$s_h - E(s_2|s_h) + U^*(s_h) > s_l - E(s_2|s_l) + U^*(s_l). \tag{33}$$

Since $U^*(s_h) > U^*(s_l)$, and since $s_h - E(s_2|s_h) \geq 0 \geq s_l - E(s_2|s_l)$, it follows that $H(\varepsilon_d^{*l}, s_1) > 0$ and $H(\varepsilon_d^{*h}, s_1) < 0\blacksquare$

Proof of Proposition 4. The government solves:

$$\max_{\tau_1} \int_{\varepsilon(\tau_1, s_1)}^{\overline{\varepsilon}} \frac{H(\varepsilon_i, s_1)}{2\overline{\varepsilon}} d\varepsilon_i - c(\tau_1) \tag{34}$$

$$\text{s.t.:} \quad \varepsilon(\tau_1, s_1) = \frac{1}{2}[d^* - \tau_1 - s_1 - E(s_2|s_1) + U^*(s_1)] \tag{35}$$

where

$$H(\varepsilon_i, s_1) \equiv E(r_{i2}|r_{i1}, s_1) - U^*(s_1) = \varepsilon_i + E(s_2|s_1) - U^*(s_1). \tag{36}$$

The problem yields the following f.o.c.:

$$\frac{1}{4\overline{\varepsilon}} H(\varepsilon(\tau_1), s_1) - c'(\tau_1) = 0. \tag{37}$$

Consider first the case in which $s_1 = s_h$. If $\tau_1 = 0$, then $\varepsilon(1, s_h) = \varepsilon_d^{*h}$, and from the proof of Proposition 2 above we know that $H(\varepsilon_d^{*h}, s_1) < 0$. In that case, the f.o.c. evaluated at $\tau_1 = 0$ has a positive sign and hence there is incentives to increase τ_1 beyond 0. (Note that $H(\varepsilon(\tau_1), s_1)$ is linear on τ_1 and hence the problem is well defined.)
Consider now the case in which $s_1 = s_l$. If $\tau_1 = 0$, then $\varepsilon(1, s_l) = \varepsilon_d^{*l}$, and from the proof of Proposition 2 we know that $H(\varepsilon_d^{*l}, s_1) > 0$. In that case, the f.o.c. evaluated at $\tau_1 = 0$ has a negative sign and hence there is incentives to decrease τ_1 below 0■

Proof of Proposition 3
The social planner solves the following optimization problem:

$$\max_d E[\int_{\varepsilon_{d_i}^{*1}}^{\overline{\varepsilon}} \frac{\varepsilon_i + E(s_2|s_1) - U(s_1)}{2\overline{\varepsilon}} d\varepsilon_i] + E[U(s_1)] \tag{38}$$

s.t.

$$\varepsilon_{d_i}^{*1} = \frac{1}{2}[d_i - s_1 - E(s_2|s_1) + U^*(s_1)]. \tag{39}$$

Since the reservation utility $U^* = q_2(\theta_2^*)(\gamma + \beta E(s_2|s_1))$ and $\theta^* = (\frac{\lambda[(1-\beta)E(s_2|s_1)-\gamma]}{k})^{1/\alpha}$, then $E(U(s_1))$ does not depend on d and hence the solution the above problem coincides with the private optimum, that is: $d^* = E[s_1 - E(s_2|s_1) + U(s_1)]$■

Proof of Proposition 5
Each firm i solves for the optimal $d_{\tau_1^c}^{i*}$ before s_1 occurs. This choice under a conjectured monetary policy $\tau_1^c = \{\tau_l^c, \tau_h^c\}$ which be implemented after the shock $s_1 = \{s_l, s_h\}$ is realized. Since each firm is infinitesimal, firm i chooses its debt choice and ignores any effect it may have on the monetary policy that is later implemented (i.e., firms take τ_1^c as given):

$$\max_{d_{\tau_1^c}^i} E[\int_{\varepsilon(\tau_1^c, s_1)}^{\overline{\varepsilon}} \frac{\varepsilon_i + E(s_2|s_1) - U(s_1)}{2\overline{\varepsilon}} d\varepsilon_i]$$

$$\text{s.t.} \quad : \quad \varepsilon(\tau_1^c, s_1) = \frac{1}{2}[d_{\tau_1}^i - \tau_1^c - s_1 - E(s_2|s_1) + U(s_1)]. \tag{40}$$

The f.o.c. from the previous problem characterizes $d_{\tau_1^c}^{i*}$, i.e., the optimal debt choice for firm i for a given conjecture τ_1^c

$$d_{\tau_1^c}^{i*} = E[\tau_1^c] + E[s_1 - E(s_2|s_1) + U(s_1)]. \tag{41}$$

which is the same for all firms $d^{i*}_{\tau^c_1} = d^*_{\tau^c_1}$. This follows since firms, are are ex-ante identical and under the condition (later verified) that all firms have identical conjectures about monetary policy.
Taken firms' idential debt choices as given $\bar{d}^*_{\tau^c}$, the policy maker solves for the policy $\tau^*_1 = \{\tau^*_l, \tau^*_h\}$ that maximizes welfare. Thus, in each state $s_1 = \{s_l, s_h\}$, the policy maker solves:

$$\max_{\tau_1} \int_{\varepsilon(\tau_1,s_1)}^{\overline{\varepsilon}} \frac{\varepsilon_i + E\left(s_2|s_1\right) - U^*\left(s_1\right)}{2\overline{\varepsilon}} d\varepsilon_i - c\left(\tau_1\right)$$

$$\text{s.t.:} \qquad \varepsilon\left(\tau_1, s_1\right) = \frac{1}{2}[\bar{d}^*_{\tau^c} - \tau_1 - s_1 - E\left(s_2|s_1\right) + U\left(s_1\right)] \tag{42}$$

which yields

$$\frac{\bar{d}^*_{\tau^c} - \tau^*_1 - s_1 + E\left(s_2|s_1\right) - U\left(s_1\right)}{4\overline{\varepsilon}} - c'\left(\tau^*_1\right) = 0. \tag{43}$$

In a rational expectations equilibrium $\tau^c_1 = \tau^*_1$ and $d^*_{\tau^c} = d^*_{\tau^*_1}$ and ths $(\tau^*_1, d^*_{\tau^*_1})$ are obtained by solving (41) and (43) simultaneously. Comparing (41) with (18), notice that, as stated in Proposition 5), $d^*_\tau = E(\tau_1) + d^*$ i.e., $d^*_\tau > d^* \Leftrightarrow E\left(\tau_1\right) > 0$.

The previous equilibrium $\{d^*_{\tau^*_1}, \tau^*_1\}$ in which firms and policymakers make privately optimal choices can be compared with the socially optimal equilibrium choices which are obtained by solving for $\{d, \tau_1\}$ in a simultaneous and coordinated manner:

$$\max_{\{d,\tau_1\}} E\Big(\int_{\varepsilon(\tau_1,s_1)}^{\overline{\varepsilon}} \frac{\varepsilon_i + E\left(s_2|s_1\right) - U^*\left(s_1\right)}{2\overline{\varepsilon}} d\varepsilon_i - c\left(\tau_1\right) \Big)$$

$$\text{s.t.:} \qquad \varepsilon\left(\tau_1, s_1\right) = \frac{1}{2}[d - \tau_1 - s_1 - E\left(s_2|s_1\right) + U\left(s_1\right)]. \tag{44}$$

Previous problem yields the following first order conditions:

$$d = E(\tau_1) + E[s_1 - E(s_2|s_1) + U(s_1)] \tag{45}$$

$$\frac{d - \tau_1 - s_1 + E(s_2|s_1) - U(s_1)}{4\overline{\varepsilon}} - c'(\tau_1) = 0 \text{ for } s_1 \in \{s_l, s_h\}. \tag{46}$$

Notice that conditions (45) and (46) are identical to (41) and (43) which characterize for decentralized private optimum which implies that the optimal monetary policy is time consistent■

Proof of Proposition 7
Assume that there is no aggregate uncertainty at $t = 1$ that is s_1 equals either s_h or s_l with probability one. (Note that still $E(s_2|s_1 = s_h) \geq E(s_2|s_1 = s_l)$.) Then we have the following equilibrium:

$$d^* = s_1 - E(s_2|s_1) + U^*(s_1) = s_1 - E(s_2|s_1) + \lambda(\theta^*_1)^{1-\alpha}\left[\gamma + \beta E(s_2|s_1)\right] \tag{47}$$

The social planner solves the following problem

$$\max_{d} \int_{\varepsilon_d^1}^{\overline{\varepsilon}} \frac{\varepsilon_i + E(s_2|s_1) - U(s_1)}{2\overline{\varepsilon}} d\varepsilon_i + U(s_1) + \\ + \underbrace{v(s_1)\frac{q(\theta_1)}{\theta_1}\left[(1-\beta)E(s_2|s_1) - \gamma\right]}_{\text{(Total) Entrants cash-flows}} - \underbrace{\left[kv(s_1) + \phi\frac{v^2(s_1)}{2}\right]}_{\text{(Total) Entry costs}} \tag{48}$$

where

$$U(s_1) = \lambda(\theta_1)^{1-\alpha}\left[\gamma + \beta E(s_2|s_1)\right] \quad ; \quad \varepsilon_d^1 = \frac{1}{2}[d - s_1 - E(s_2|s_1) + U^*(s_1)] \tag{49}$$

$$k + \phi v(s_1) = \frac{q(\theta_1)}{\theta_1}\left[(1-\beta)E(r_2|s_1) - \gamma\right]$$

$$\frac{q(\theta_1)}{\theta_1} = \lambda(\theta_1)^{-\alpha} \quad ; \quad \theta_1 = \frac{v(s_1)}{l(s_1)} \tag{50}$$

$$l(s_1) = \Pr(\varepsilon_i < \varepsilon_d^1) = \frac{d + 2\bar{\varepsilon} - s_1 - E(s_2|s_1) + U^*(s_1)}{4\bar{\varepsilon}} \tag{51}$$

The derivative of the social planner's objective function (SPOF) w.r.t. debt is:

$$\begin{aligned} \frac{\partial \text{SPOF}}{\partial d} &= \frac{-1}{4\overline{\varepsilon}}(\varepsilon_d^1 + E(s_2|s_1) - U(s_1))\left(1 + \frac{\partial U(s_1)}{\partial d}\right) \\ &+ \underbrace{\left(1 - \Pr\left(\varepsilon_i \geq \varepsilon_d^1\right)\right)}_{l(s_1)}\frac{\partial U(s_1)}{\partial d} \\ &+ \lambda\frac{\partial(\theta_1)^{-\alpha}}{\partial d}v(s_1)\left[(1-\beta)E(s_2|s_1) - \gamma\right] \\ &+ \frac{\partial v(s_1)}{\partial d}\underbrace{\left[\frac{q(\theta_1)}{\theta_1}(1-\beta)E(s_2|s_1) - \gamma - k - \phi v(s_1)\right]}_{=0 \text{ because of the zero profit entry condition}} \end{aligned} \tag{52}$$

Since

$$\frac{\partial U(s_1)}{\partial d} = (1-\alpha)\lambda\left(\theta_1\right)^{-\alpha}\frac{\partial \theta_1}{\partial d}\left[\gamma + \beta E\left(s_2|s_1\right)\right] \tag{53}$$

then we can rewrite $\frac{\partial \text{SPOF}}{\partial d}$ as:

$$\begin{aligned} \frac{\partial \text{SPOF}}{\partial d} &= \frac{-1}{4\overline{\varepsilon}}\left[1 + \frac{\partial U(s_1)}{\partial d}\right](\varepsilon_d^1 + E\left(s_2|s_1\right) - U(s_1)) \\ &+ l(s_1)\lambda\left(\theta_1\right)^{-\alpha}\frac{\partial \theta_1}{\partial d}\left[(1-\alpha)\left[\gamma + \beta E\left(s_2|s_1\right)\right] - \alpha\left[(1-\beta)E\left(s_2|s_1\right) - \gamma\right]\right] \end{aligned}$$

At the private optimum

$$d^* = s_1 - E(s_2|s_1) + U^*(s_1) \tag{54}$$

and hence,

$$\varepsilon_d^{1*} = \frac{1}{2}[d^* - s_1 - E\left(s_2|s_1\right) + U^*(s_1)] = U^*(s_1) - E\left(s_2|s_1\right). \tag{55}$$

Hence evaluating $\frac{\partial \text{SPOF}}{\partial d}$ at the private optimum (PO):

$$\left|\frac{\partial \text{SPOF}}{\partial d}\right|_{\text{PO}} = l^*(s_1)\lambda\left(\theta_1^*\right)^{-\alpha}\left|\frac{\partial \theta_1}{\partial d}\right|_{\text{PO}}\left[(1-\alpha)\left[\gamma + \beta E\left(s_2|s_1\right)\right] - \alpha\left[(1-\beta)E\left(s_2|s_1\right) - \gamma\right]\right] \tag{56}$$

(Note: We use $|\circ|_{\text{PO}}$ to denote that the expression is evaluated at the private optimum.) Next we show that $\frac{\partial \theta_1}{\partial d} < 0$:

$$\frac{\partial \theta_1}{\partial d} = \frac{1}{\left(l(s_1)\right)^2}[l(s_1)\frac{\partial v(s_1)}{\partial \theta_1}\frac{\partial \theta_1}{\partial d} - v(s_1)\frac{\partial l(s_1)}{\partial d}]$$

Hence

$$\begin{aligned}\frac{\partial \theta_1}{\partial d}\frac{1}{\theta_1}[l(s_1) - \frac{\partial v(s_1)}{\partial \theta_1}] &= -\frac{\partial l(s_1)}{\partial d} = \frac{-1}{4\bar{\varepsilon}}\left(1 + \frac{\partial U(s_1)}{\partial d}\right) = \\ &= \frac{-1}{4\bar{\varepsilon}}\left[1 + (1-\alpha)\lambda\theta_1^{-\alpha}\frac{\partial \theta}{\partial d}\left[\gamma + \beta E\left(s_2|s_1\right)\right]\right]\end{aligned}$$

and solving for $\frac{\partial \theta_1}{\partial d}$:

$$\frac{\partial \theta_1}{\partial d} = \frac{-1}{\frac{4\bar{\varepsilon}}{\theta_1}[l(s_1) - \frac{\partial v(s_1)}{\partial \theta_1}] + (1-\alpha)\lambda\theta_1^{-\alpha}\left[\gamma + \beta E\left(s_2|s_1\right)\right]} < 0 \tag{57}$$

[Note that

$$k + \phi v(s_1) = \lambda(\theta_1)^{-\alpha}\left[(1-\beta)E(r_2|s_1) - \gamma\right] \Longrightarrow \frac{\partial v(s_1)}{\partial \theta_1} < 0.]$$

Hence

$$\begin{aligned}\left|\frac{\partial \text{SPOF}}{\partial d}\right|_{\text{PO}} &> 0 \Leftrightarrow (1-\alpha)\left[\gamma + \beta E\left(s_2|s_1\right)\right] - \alpha\left[(1-\beta)E\left(s_2|s_1\right) - \gamma\right] < 0 \\ &\Leftrightarrow (1-\alpha)\widetilde{\beta}E\left(s_2|s_1\right) - \alpha(1-\widetilde{\beta})E\left(s_2|s_1\right) < 0 \Leftrightarrow \widetilde{\beta} < \alpha\blacksquare\end{aligned}$$

AGGREGATE UNCERTAINTY & LIMITED ENTRY

Equilibrium Debt
A worker's outside option at $t = 2$ if he quits and looks for a job at $t = 2$ is:

$$U\left(s_1\right)=\lambda(\theta_1)^{1-\alpha}\left[\gamma+\beta E\left(s_2|s_1\right)\right]\lambda(\frac{v_1}{l_1})^{1-\alpha}\left[\gamma+\beta E\left(s_2|s_1\right)\right] \tag{58}$$

where

$$l_1\equiv l(s_1)\ \ ;\ v_1\equiv v(s_1) \tag{59}$$

The marginal new firm entering the market is:

$$k+\phi v_1=\frac{q(\theta_1)}{\theta_1}\left[(1-\beta)E(r_2|s_1)-\gamma\right]$$

The firm liquidates if the manager cannot retain the worker, that is, if

$$G(\varepsilon_i,s_1,d_i)\equiv 2\varepsilon_i+s_1+E(s_2|s_1)-d_i-U(s_1)<0 \tag{60}$$

which implies that the marginal firms liquidated, $\varepsilon_{d_i}^{*1}$, is determined by the following equation,

$$G(\varepsilon_{d_i}^{*1},s_1,d_i)\equiv 0 \tag{61}$$

which boils down to

$$\varepsilon_{d_i}^{*1}=\frac{1}{2}[d_i-s_1-E(s_2|s_1)+U(s_1)]. \tag{62}$$

and since there is a continuum [0,1] of firms at $t=1$, then:

$$l_1=\Pr(\varepsilon_i<\varepsilon_{d_i}^{*1})=\frac{d_i+2\bar{\varepsilon}-s_1-E(s_2|s_1)+U(s_1)}{4\bar{\varepsilon}} \tag{63}$$

The debt choice at $t=0$ solves:

$$\max_{d_i}E\int_{\varepsilon_{d_i}^{*1}}^{\bar{\varepsilon}}\frac{\varepsilon_i+E(s_2|s_1)-U(s_1)}{2\bar{\varepsilon}}d\varepsilon_i \tag{64}$$

which yields the following f.o.c.:

$$\frac{-1}{4\bar{\varepsilon}}E[\varepsilon_{d_i}^{*1}+E(s_2|s_1)-U^*(s_1)]=0, \tag{65}$$

which yields the following amount of debt

$$d^*=E\left[(s_1)-E(s_2|s_1)+E\left(U^*(s_1)\right)\right]. \tag{66}$$

Social Optimum Debt
The social planner solves the following problem:

$$\max_d E[\int_{\varepsilon_{d_i}^{*1}}^{\bar{\varepsilon}}\frac{\varepsilon_i+E(s_2|s_1)-U(s_1)}{2\bar{\varepsilon}}d\varepsilon_i]+E(U(s_1))+$$
$$+E\left[v_1\frac{q(\theta_1)}{\theta_1}\left[(1-\beta)E(s_2|s_1)-\gamma\right]\right]-E\left[kv(s_1)+\phi\frac{v^2(s_1)}{2}\right] \tag{67}$$

s.t.

$$\varepsilon_d^{*1} = \frac{1}{2}[d - s_1 - E(s_2|s_1) + U_2^*(s_1)]. \tag{68}$$

$$U(s_1) = \lambda(\theta_1)^{1-\alpha}\left[\gamma + \beta E\left(s_2|s_1\right)\right] \tag{69}$$

$$k + \phi v_1 = \frac{q(\theta_1)}{\theta_1}\left[(1-\beta)E(r_2|s_1) - \gamma\right]$$

$$\frac{q(\theta_1)}{\theta_1} = \lambda\theta_1^{-\alpha} \;\; ; \;\; \theta_1 = \frac{v_1}{l_1} \tag{70}$$

$$l_1 = \Pr(\varepsilon_i < \varepsilon_d^{*1}) \tag{71}$$

Note that unlike individual firms the social planner internalizes the effect that the choice of debt has on θ_1, and hence on ε_d^{*1} and $U(s_1)$. Deriving the social planner's objective function (SPOF) w.r.t. debt:

$$\begin{aligned}\frac{\partial \text{SPOF}}{\partial d} &= -E[[1+\frac{\partial U\left(s_1\right)}{\partial d}]\frac{\varepsilon_d^{*1}+E\left(s_2|s_1\right)-U^*\left(s_1\right)}{4\overline{\varepsilon}}] + E[\frac{\partial U\left(s_1\right)}{\partial d}l_1] \\ &\quad +E[\frac{\partial(\frac{q(\theta_1)}{\theta_1})}{\partial d}v_1((1-\beta)E\left(s_2|s_1\right)-\gamma)] \\ &\quad +E\{\frac{\partial v_1}{\partial d}\underbrace{[\frac{q(\theta_1)}{\theta_1}(1-\beta)E(s_2|s_1) - \gamma - k - \phi v(s_1)]}_{=0 \text{ because of the zero profit entry condition}}\}\end{aligned} \tag{72}$$

Notice that

$$\frac{\partial U(s_1)}{\partial d} = -(1-\alpha)\left[\gamma + \beta E(s_2|s_1)\right]\lambda(\theta_1)^{-\alpha}\frac{\partial \theta_1}{\partial d} \tag{73}$$

and

$$\frac{\partial\left(\frac{q(\theta_1)}{\theta_1}\right)}{\partial d} = -\alpha\lambda\theta_1^{-\alpha-1}\frac{\partial\theta_1}{\partial d} \tag{74}$$

so we can rewrite $\frac{\partial \text{SPOF}}{\partial d}$ as:

$$\begin{aligned}\frac{\partial \text{SPOF}}{\partial d} &= -E\left[\left[1 + \frac{\partial U(s_1)}{\partial d}\right]\frac{\varepsilon_d^{*1} + E(s_2|s_1) - U^*(s_1)}{4\overline{\varepsilon}}\right] + \\ &\quad +E\left[\left[l_1\lambda(\theta_1)^{-\alpha}E(s_2|s_1)\frac{\partial\theta_1}{\partial d}\right]\left[(1-\alpha)\widetilde{\beta}(s_1) - \alpha(1-\widetilde{\beta}(s_1))\right]\right]\end{aligned}$$

which evaluated at the private optimal (PO), that is,

$$\frac{-1}{4\overline{\varepsilon}}E[\varepsilon_{d_i}^{*1} + E(s_2|s_1) - U^*(s_1)] = 0, \tag{75}$$

gives:

$$\left|\frac{\partial \text{SPOF}}{\partial d}\right|_{\text{PO}} = -E\left[\left|\frac{\partial U(s_1)}{\partial d}\right|_{\text{PO}}\left(\frac{\varepsilon_d^{*1}+E(s_2|s_1)-U^*(s_1)}{4\overline{\varepsilon}}\right)\right]+$$
$$+E\left[\left[l_1^*\lambda(\theta_1^*)^{-\alpha}E(s_2|s_1)\left|\frac{\partial\theta_1}{\partial d}\right|_{\text{PO}}\right]\left[(1-\alpha)\widetilde{\beta}(s_1)-\alpha(1-\widetilde{\beta}(s_1))\right]\right]$$

(Note: As before, we use $|\circ|_{\text{PO}}$ to denote that the expression is evaluated at the private optimum.) Hence there are two effects determining if at the equilibrium the social planner has incentives to increase or decrease debt, that is, whether $\left|\frac{\partial U(s_1)}{\partial d}\right|_{\text{PO}}$: the "feedback effect" (in the first line) and the "search externality effect" (in the second line).

(1) The "*feedback effect*":

$$-E\left[\left|\frac{\partial U(s_1)}{\partial d}\right|_{\text{PO}}\left(\frac{\varepsilon_d^{*1}+E(s_2|s_1)-U^*(s_1)}{4\overline{\varepsilon}}\right)\right] \tag{76}$$

When firms choose the amount of debt, d^*, they do not take into account that d^* affects $U^*(s_1)$ and hence ε_d^{*1}. The marginal firm destroyed in good times has a value:

$$\varepsilon_d^{*h}+E(s_2|s_h)-U_2^*(s_h) \tag{77}$$

and the marginal firm destroyed in bad times has a value:

$$\varepsilon_d^{*l}+E(s_2|s_l)-U_2^*(s_l) \tag{78}$$

Since d^* affects $U^*(s_1)$ and hence ε_d^{*1} the net effect depends on whether d moves the outside option more in good or bad times multiplied by the value of the marginal firm destroyed, $\varepsilon_d^{*1}+E(s_2|s_1)-U_2^*(s_1)$, which is positive in bad times and negative in good times..
(2) The "*search externality effect*":

$$E\left[\left[l_1^*\lambda(\theta_1^*)^{-\alpha}E(s_2|s_1)\left|\frac{\partial\theta_1}{\partial d}\right|_{\text{PO}}\right]\left[(1-\alpha)\widetilde{\beta}(s_1)-\alpha(1-\widetilde{\beta}(s_1))\right]\right] \tag{79}$$

First notice that $\frac{\partial\theta_1}{\partial d}<0$ (see the proof in the case of no aggregate uncertainty). Hence the sign depends on $\left[(1-\alpha)\widetilde{\beta}(s_1)-\alpha(1-\widetilde{\beta}(s_1))\right]$. For instance consider the case in which $\widetilde{\beta}(s_h)<\widetilde{\beta}(s_l)<\alpha$. In that case the search externality would tend push things towards increasing debt since the labor market would tend to be too tight. Alternatively, if $\alpha<\widetilde{\beta}(s_h)<\widetilde{\beta}(s_l)$, the search externality would tend push things towards decreasing debt. Finally, if $\widetilde{\beta}(s_h)<\alpha<\widetilde{\beta}(s_l)$ the search externality induces increasing debt in good time and decreasing debt in bad time, hence , ex-ante, whether search externality induces an increase or decrease in debt depends on which one of the two effects dominates.