The I Theory of Money*

Markus K. Brunnermeier† and Yuliy Sannikov‡

first version: Oct. 10, 2010
this version: September 14, 2012

Abstract

A theory of money needs a proper place for financial intermediaries. Intermediaries create money by taking deposits from savers and investing them in productive projects. The money multiplier depends on the size of intermediary balance sheets, and their ability to take risks. In downturns, as lending contracts and the money multiplier shrinks, the value of money rises. This leads to a Fisher disinflation that hurts borrowers and amplifies shocks. An accommodative monetary policy in downturns, focused on the assets held by constrained agents, can mitigate these destabilizing adverse feedback effects. We devote particular attention to interest rate cuts, and study the potential for such policies to create moral hazard.

*We are grateful to comments by seminar participants at Princeton, Bank of Japan, Philadelphia Fed, Rutgers, Toulouse School of Economics, Wim Duisenberg School, University of Lausanne, Banque de France-Banca d'Italia conference, University of Chicago, New York Fed, Chicago Fed, Central Bank of Chile, Penn State, Institute of Advanced Studies, Columbia University, University of Michigan, University of Maryland, Northwestern, Cowles General Equilibrium Conference, Renmin University, Johns Hopkins, Kansas City Fed, IMF, LSE, LBS, Bank of England and the Central Bank of Austria.

†Princeton University.

‡Princeton University.

1 Introduction

A theory of money needs a proper place for financial intermediaries. Financial institutions are able to create money, for example by lending to businesses and home buyers, and accepting deposits backed by those loans. The amount of money created by financial intermediaries depends crucially on the health of the banking system and the presence of profitable investment opportunities. This paper proposes a theory of money and provides a framework for analyzing the interaction between price stability and financial stability. It therefore provides a unified way of thinking about monetary and macroprudential policy.

Since intermediation involves taking on some risk, a negative shock to end borrowers also hits intermediary balance sheets. Intermediaries' individually optimal response is to lend less and accept fewer deposits. Hence, the amount of inside money in the economy shrinks. When the total demand for money as a store of value changes little, the value of outside money increases, leading to deflation.

Specifically, in our model the economy moves between two polar regimes. In one regime the the financial sector is well capitalized: it can overcome financial frictions and is able to channel funds from savers to productive projects. Through their monitoring role, financial institutions enable productive agents to issue debt and equity claims. Savers hold little outside money, as they mostly hold deposits with intermediaries. The value of outside money is low. In the other polar regime, in contrast, the financial sector is undercapitalized and savers choose to hold outside money. Whenever an agent becomes productive, he buys capital from less productive agents using money. Thus, the existence of money in equilibrium helps productive agents to get greater access to capital even without explicit borrowing and lending. Outside money plays the role as an important store of value. The value of money becomes high.

A negative shock to productive agents lowers the financial sector's risk bearing capacity and hence brings the economy closer to the second regime with high value of money. That is, a negative productivity shock leads deflation of Fisher (1933). Since financial institutions accept nominal demand deposits, they are hit on both sides of their balance sheets. On the asset side, they are exposed to productivity shocks. On the liabilities side, they are hurt by the Fisher deflation. This amplifies the initial shock even further, absent appropriate monetary policy. As financial institutions cut on lending and create less money, the money multiplier declines endogenously.

Monetary policy can work against the adverse feedback loops that precipitate crisis, by

affecting the prices of assets held by constrained agents and redistributing wealth. Since monetary policy softens the blow of negative shocks and helps the reallocation of capital to productive uses, this wealth redistribution is not a zero-sum game, but it can actually lead to improved welfare.

Simple interest rate cuts in downturns improve economic outcomes only if they stimulate prices of assets, such as long-term government bonds, held by intermediaries. Without such assets, these interest rate cuts would have no impact on real economic activity and only reduce inflation. However, wealth redistribution towards the constrained sector leads to a rise in economic activity and an increase in the price of capital. As the financial sector recovers, it creates more money and reverses the deflationary pressure. The appreciation of long-term bonds also mitigates money demand, as long-term bonds can be used as a store of value as well. Since long-term bonds provide intermediaries with a hedge against losses due to negative macro shocks, monetary policy can serve as an insurance mechanism against adverse events.

This "stealth recapitalization" of the financial system through monetary policy creates much less moral hazard than explicit bailouts of failing institutions. The reason is that monetary policy is a crude redistributive tool that helps the strong institutions more than the weak. The cautions institutions that bought long-term bonds as a hedge against downturns benefit more from interest rate cuts than the opportunistic institutions that increased leverage to take on more risk. In contrast, ex-post bailouts of the weakest institutions create strong risk-taking incentives ex-ante.

Related Literature. While in almost all papers, a negative productivity shock causes inflationary pressures, in our setting it induces deflationary pressure absent a monetary intervention. This is consistent with the empirical output-inflation patterns before 1960 under the (extended) Gold Standard, as e.g. documented by Cagan (1979). Like in monetarism (see e.g. Friedman and Schwartz (1963)), an endogenous reduction of money multiplier (given a fixed monetary base) leads to deflation in our setting. However, in our setting outside money is only an imperfect substitute for inside money. Intermediaries, either by channeling funds through or by underwriting and thereby enabling firms to approach capital markets directly, enable a better capital allocation and more economic growth. Hence, in our setting monetary intervention should aim to recapitalize undercapitalized borrowers rather than simply increase the money supply across the board. Another difference is that our approach focuses more on the role of money as a store of value instead of the transaction role of money. Overall, our approach is closer in spirit to banking channel literature, see

e.g. Patinkin (1965), Tobin (1970), Gurley and Shaw (1955), Bernanke (1983) Bernanke and Blinder (1988) and Bernanke, Gertler and Gilchrist (1999).[1] Another distinct feature of our setting is that our effects arise despite the fact that prices are fully flexible. This is in sharp contrast to the New Keynesian framework, in which a nominal interest rate cut also lowers real rates and thereby induces households to consume more. Recently, Cordia and Woodford (2010) introduced financial frictions in the new Keynesian framework. In contrast, our framework focuses on the redistributional role of monetary policy, a feature we share with Scheinkman and Weiss (1986). In Kiyotaki and Moore (2008) money is desirable as it does not suffer from a resellability constraint, unlike capital in their model.

Within a three-period framework, Diamond and Rajan (2006) and Stein (2010) also address the role of monetary policy as a tool to achieve financial stability. More generally, there is a growing macro literature which also investigated how macro shocks that affect the balance sheets of intermediaries become amplified and affect the amount of lending and the real economy. These papers include Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999), who study financial frictions using a log-linearized model near steady state. In these models shocks to intermediary net worths affect the efficiency of capital allocation and asset prices. However, log-linearized solutions preclude volatility effects and lead to stable system dynamics. Brunnermeier and Sannikov (2010) also study full equilibrium dynamics, focusing on the differences in system behavior near the steady state, and away from it. They find that the system is stable to small shocks near the steady state, but large shocks make the system unstable and generate systemic endogenous risk. Thus, system dynamics are highly nonlinear. Large shocks have much more serious effects on the real economy than small shocks. He and Krishnamurthy (2010) also study the full equilibrium dynamics and focus in particular on credit spreads. For a more detailed review of the literature we refer to Brunnermeier et al. (2010).

This paper is organized as follows. Section 2 informally describes the logical framework around which we construct our model. Section 3 frames the ideas from Section 2 into a basic model. Section 4 characterizes the equilibrium. Section 5 presents a computed example and discusses equilibrium properties, including capital and money value dynamics, the amount of lending through intermediaries, and the money multiplier. Section 6 introduces long-term bonds and studies the effect of interest-rate policies as well as open-market operations.

[1]The literature on credit channels distinguishes between the bank lending channel and the balance sheet channel (financial accelerator), depending on whether banks or corporates/households are capital constrained. Strictly speaking our setting refers to the former, but we a agnostic about it and prefer the broader credit channel interpreation.

Section 7 concludes.

2 The Model.

We build a dynamic but simple model of money and intermediation. The following model is a product of multiple iterations, streamlined to leave out inessential technical ingredients and to include several parameters of interest in a straightforward manner.

Agents. The economy is populated by households and intermediaries. Intermediaries take deposits from unproductive households (savers) and finance projects of productive households (entrepreneurs). For simplicity, we assume that the mass of entrepreneurs among households is zero, i.e. all household wealth is concentrated in the hands of savers. To run projects, entrepreneurs need physical capital. They can rent capital directly from savers, or buy it using financing through intermediaries.

Technology. Capital k_t produces output at rate $(a - \iota_t)k_t$ and grows deterministically according to

$$dk_t = (\Phi(\iota_t) - \delta)k_t \, dt,$$

where ι_t is the rate of investment and Φ is a standard investment function with adjustment costs, such that $\Phi(0) = 0$, $\Phi' > 0$ and $\Phi'' \leq 0$.

Shocks. Macro shocks hit the economy with Poisson intensity λ. In the event of a shock, some of the capital gets stolen. Capital is better monitored when financed by intermediaries than when rented out by saver households. Only fraction ϕ of capital financed by intermediaries gets stolen, as compared to the fraction $\underline{\phi} > \phi$ of capital rented by households.

In addition, while intermediaries are able to diversify across investments, households cannot. When a shock hits, each household loses its entire capital with probability $\underline{\phi}$. With probability $1 - \underline{\phi}$, no capital gets stolen.

Importantly, these macro shocks affect the net worth of intermediaries but not the total quantity of capital in the economy, which grows according to

$$dK_t/dt = (\Phi(\iota_t) - \delta)K_t. \tag{2.1}$$

Preferences. Both intermediaries and households have logarithmic utility. Households discount their utility at rate r, while intermediaries, at rate $\rho > r$.

Money. In the baseline model, there is a fixed amount of fiat money in the economy that pays zero interest. Money is backed by taxes. The government taxes all output at a fixed rate $\tau \in [0, 1)$, and uses the proceeds to purchase money at the current market price.

When we consider monetary policy in Section ..., we allow the central bank to pay interest on money deposits and introduce long-term government bonds.

Agents' Portfolios. Saver households and intermediaries choose to allocate their wealth between money and capital. We measure the values of all assets in the units of *output*. In these units, denote the equilibrium price of one unit of capital by q_t, and the price of money (per unit of capital) by p_t. Therefore, the total wealth in the economy is

$$q_t K_t + p_t K_t.$$

Prices of money and capital are fluctuating, and some capital may be stolen. Therefore, agents holding these assets experience capital gains (or losses). In addition, investors receive dividend from output produced by capital, and from taxes used to repurchase money.

Intermediaries may (and will in equilibrium) put a negative portfolio weight on money by accepting money deposits from households. They can borrow up to the amount that they can guarantee to repay for sure, even if hit by macro shock. Importantly, note that the deposits are denominated in money, i.e. intermediaries are obligated to repay households money, whatever is its market price.

The main source of financial frictions in this economy is that intermediaries can issue only debt and not equity, i.e. the must entirely absorb all losses from their projects.

2.1 Two Benchmark Cases.

In equilibrium, unproductive households can choose to invest in money or capital, but capital is risky. With intensity $\lambda\underline{\phi}$ each household loses its entire capital stake. Because a household would get an expected utility of $-\infty$ if it holds only capital, there is a demand for money.

A household can hold money either directly (outside money) or by making a deposit with an intermediary (inside money). The two types of money are equivalent, since intermediaries bound their leverage to make sure that the deposits are risk-free. The value of money depends on money supply (as well as demand), which is determined by the amount of inside money that intermediaries create by accepting deposits to invest in capital (projects). In turn, the intermediaries' ability and willingness to create money depends their risk-taking capacity, determined by their net worth.

Below we identify two important benchmarks. In equilibrium, the economy will fluctuate between the two. First, the economy is frictionless, and so money creation by intermediaries is unimpaired, if $\lambda = 0$. In this case, there are no macro shocks, and intermediaries do not need net worth buffers to absorb risk. Second, financial frictions become extreme if intermediaries cannot function because their net worth is zero. In this case, intermediaries create no inside money, and households hold only outside money or inefficiently rented capital.

For simplicity, we consider a degenerate investment function $\Phi : \{0\} \to \mathbb{R}$, under which capital in the economy grows at rate $g = \Phi(0) - \delta$ and produces output at rate a.

A Frictionless Economy. Denote by $\bar{p}$ and $\bar{q}$ the prices of money and capital at the steady state of the frictionless economy, with $\lambda = 0$. If so, then capital and money are both risk-free. Their returns must equal. Capital derives its return of

$$\frac{(1-\tau)a}{\bar{q}} + g$$

from its dividend yield and its capital gains rate. Because the value of money is proportional to the quantity of capital, the capital gains rate on money equals the growth rate on total capital, while the dividend yield depends on the tax rate. Thus, the return on money is

$$\frac{\tau a}{\bar{p}} + g.$$

Equating the two returns, we get

$$\frac{1-\tau}{\bar{q}} = \frac{\tau}{\bar{p}}.$$

Because households and intermediaries earn the same returns, but intermediaries consume at a higher rate as they are less patient, at the steady state the share of net worth that belongs to the intermediaries is $\bar{\eta} = 0$. Equating the households' demand for consumption goods, given their wealth of $(\bar{p} + \bar{q})K_t$, to aggregate supply, we get

$$r(\bar{p} + \bar{q})K_t = aK_t \quad \Rightarrow \quad \bar{p} + \bar{q} = \frac{a}{r}, \quad \bar{p} = \frac{\tau a}{r} \quad \text{and} \quad \bar{q} = \frac{(1-\tau)a}{r}. \tag{2.2}$$

Note that in this extreme case, money would be worthless if it were not backed by taxation, i.e. if $\tau = 0$.

If intermediaries had a small advantage over households in holding capital (e.g. if capital rented out by households generated a lower output rate $\underline{a} < a$), then all capital in the economy would be held by intermediaries. In this case, they would accept deposits in the

amount of

$$\frac{(1-\tau)a}{r}K_t$$

to finance their purchases of capital. As this is the value of inside money in the economy, and $\frac{\tau a}{r}K_t$ is the value of outside money, the money multiplier is $1/\tau$.

The Money Regime. If intermediaries have zero net worth but $\lambda > 0$, then all capital in the economy must be permanently held by households. Denote by $\underline{p}$ and $\underline{q}$ the prices of money and capital in this regime. Then households earn the return of

$$r_t^M = \frac{\tau a}{\underline{p}} + g$$

by holding money. From holding capital, they get a return of

$$r_t^H = \frac{(1-\tau)a}{\underline{q}} + g$$

until with intensity $\lambda\underline{\phi}$ they lose all capital. If a household puts weight $\underline{x}$ on capital and $1-\underline{x}$ on money, then the optimal choice of $\underline{x}$ maximizes

$$\underline{x}\, r_t^H + (1-\underline{x})\, r_t^M + \lambda\underline{\phi} \ln(1-\underline{x}),$$

because with intensity $\lambda\underline{\phi}$ the household loses a fraction $\underline{x}$ of wealth. Differentiating with respect to $\underline{x}$ and plugging in the expressions for r_t^M and r_t^H, we get

$$\left(\frac{(1-\tau)a}{\underline{q}} + g\right) - \left(\frac{\tau a}{\underline{p}} + g\right) - \lambda\underline{\phi}\frac{1}{1-\underline{x}} = 0.$$

Since $\underline{x} = \underline{q}/(\underline{q}+\underline{p})$ by the market-clearing condition for capital and $r(\underline{q}+\underline{p}) = a$ by the market-clearing condition for output, it follows that

$$\underline{q} = \frac{(1-\tau)a}{r+\lambda\underline{\phi}} \quad \text{and} \quad \underline{p} = \frac{a}{r}\frac{\tau r + \lambda\underline{\phi}}{r+\lambda\underline{\phi}}. \tag{2.3}$$

Money becomes more valuable when the intensity of shocks to individual capital $\lambda\underline{\phi}$ is greater. Even though money generates a lower dividend yield, households prefer to hold it because it is safe. Of course, the money multiplier in this extreme case is 1, as there is no inside money but only outside money.

Interestingly, the value of money is positive also if $\tau = 0$, and even if $\tau > -\lambda\underline{\phi}/r$ is

negative. That is, money can have value in equilibrium even if the government perpetually prints money to finance transfers to individual agents, as long as the rate of transfers is not too large. Of course, if $\tau \leq 0$, then there is also another equilibrium in which money is worthless.[2]

In the special case when $\lambda = 0$, formulas (2.3) reduce to (2.2).

In equilibrium the economy collapses towards the money regime after negative macro shocks. In the money regime, the price of capital falls to $\underline{q}$, the price of money rises to $\underline{p}$ and the money multiplier collapses to 1. As the economy recovers, intermediaries create more money, so the money multiplier expands and the value of outside money falls. Upon recovery, the economy starts looking like the frictionless benchmark described above, although that benchmark is never reached. In booms, intermediaries need net worth buffers to manage risks, and there is always a strictly positive chance of lapsing back into a crisis.

To characterize the equilibrium formally, we need to understand returns that households and intermediaries earn from capital and money, and combine these agents' portfolio optimization conditions with market clearing.

2.2 Returns and Equilibrium Conditions.

In equilibrium, asset prices p_t and q_t do not stay fixed - instead they change with time and with shocks. To characterize the equilibrium, we need to find prices and allocations, such that all agents maximize utility and markets clear. We have to find these quantities for any history $\{t_1 < t_2 \ldots < t_n \leq t\}$, where $t_1, \ldots t_n$ are the times of all previous shocks, and t is the current time. Let $(x_t, 1 - x_t)$, $x_t \geq 0$, the intermediaries' portfolio allocations to capital and money at time t, and by $(\underline{x}_t, 1 - \underline{x}_t)$, $x_t \in [0, 1]$, the households' portfolio allocations. Let N_t and $(p_t + q_t)K_t - N_t$ be the aggregate net worths of intermediaries and households, and C_t and $\underline{C}_t$, their aggregate consumption rates, respectively. Formally:

Equilibrium Definition. *An equilibrium is a map from histories* $\{t_1 < t_2 \ldots < t_n \leq t\}$ *to prices* p_t *and* q_t, *allocations* $(x_t, 1-x_t)$ *and* $(\underline{x}_t, 1-\underline{x}_t)$, *wealth levels* $(N_t, (p_t+q_t)K_t - N_t)$, *and consumption choices* $(C_t, \underline{C}_t)$ *such that*

(i) all markets, for capital, money and consumption goods, clear

(ii) all agents choose portfolio allocations and consumption rates to maximize utility

[2]Technically, the households would get utility $-\infty$ in this equilibrium, since they lose their entire wealth with positive probability. Thus, the equilibrium in which money is worthless is more meaningful in a model where households can diversify as intermediaries.

(iii) the agents' wealth levels satisfy their budget constraints

Returns. Denote by

$$\frac{dp_t/dt}{p_t} = \mu_t^p \quad \text{and} \quad \frac{dq_t/dt}{q_t} = \mu_t^q \tag{2.4}$$

the endogenous evolution of prices in the absence of shocks, and by $\breve{p}_t$ and $\breve{q}_t$ the new price levels in the event that a shock arrives at time t.

The returns that intermediaries and households earn on money and capital are as follows. Since the value of all money in the economy is $p_t K_t$, where K_t follows (2.1) and p_t follows (2.4), and since total payment flow to money holders equals $\tau a K_t$, the return on money is

$$r_t^M = \underbrace{\frac{\tau(a-\iota_t)}{p_t}}_{\text{dividend yield}} + \underbrace{\mu_t^p + \Phi(\iota_t) - \delta}_{\text{capital gains rate}}. \tag{2.5}$$

in the absence of shocks. Following a shock, the value of money changes by a factor of $\breve{p}_t/p_t$.

Likewise, an investment in capital of $q_t k_t$ earns the return of

$$r_t^K = \underbrace{\frac{(1-\tau)(a-\iota_t)}{q_t}}_{\text{dividend yield}} + \underbrace{\mu_t^q + \Phi(\iota_t) - \delta}_{\text{capital gains rate}}. \tag{2.6}$$

in the absence of shocks. Following a shock, the change in value of a capital stake depends on who holds it. The value of capital held by an intermediary changes by a factor of $(1-\phi)\breve{q}_t/q_t$. The value of capital held by a household changes by a factor of $\breve{q}_t/q_t$ with probability $1-\underline{\phi}$, and drops in value to zero with probability $\underline{\phi}$.

The optimal equilibrium rate of investment ι_t maximizes the return on capital, i.e. it solves

$$\max_{\iota} \Phi(\iota) - \frac{(1-\tau)\iota}{q_t} \quad \Rightarrow \quad \Phi'(\iota_t) = \frac{1-\tau}{q_t}. \tag{2.7}$$

Optimal Portfolio Choice Conditions. With logarithmic utility, optimal portfolio weights x_t and $1-x_t$ that any intermediary puts on capital and money, solve

$$\max_{x} \underbrace{x\, r_t^K + (1-x)\, r_t^M}_{\text{return if no shocks}} + \lambda \underbrace{\log\left(x(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x)\frac{\breve{p}_t}{p_t}\right)}_{\text{utility drop if shock}}. \tag{2.8}$$

Thus, x_t must satisfy the first-order condition

$$r_t^K - r_t^M + \lambda \frac{(1-\phi)\frac{\breve{q}_t}{q_t} - \frac{\breve{p}_t}{p_t}}{x_t\,(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x_t)\,\frac{\breve{p}_t}{p_t}} = 0. \tag{2.9}$$

Note that the allocation to capital x_t that solves the intermediaries' problem automatically guarantees that their wealth stays positive after a negative shock, since their utility function is defined only on the positive domain.

Similarly, households choose portfolio weights $(\underline{x}_t, 1-\underline{x}_t)$ to solve

$$\max_{x\geq 0}\; x\, r_t^K + (1-x)\, r_t^M + \lambda(1-\underline{\phi})\, \log\left((1-x)\frac{\breve{p}_t}{p_t}\right) + \lambda \log\left(x\frac{\breve{q}_t}{q_t} + (1-x)\frac{\breve{p}_t}{p_t}\right) \tag{2.10}$$

Thus, $\underline{x}_t$ must satisfy the first-order condition

$$r_t^K - r_t^M + \lambda(1-\underline{\phi})\, \frac{\frac{\breve{q}_t}{q_t} - \frac{\breve{p}_t}{p_t}}{\underline{x}_t\,\frac{\breve{q}_t}{q_t} + (1-\underline{x}_t)\,\frac{\breve{p}_t}{p_t}} + \lambda\underline{\phi}\, \frac{-\frac{\breve{p}_t}{p_t}}{(1-\underline{x}_t)\,\frac{\breve{p}_t}{p_t}} = 0. \tag{2.11}$$

Market Clearing Conditions. Equating the total value of capital held by intermediaries and households to the value of all capital in the economy, we obtain the market-claring condition for capital

$$x_t N_t + \underline{x}_t((q_t+p_t)K_t - N_t) = q_t K_t \tag{2.12}$$

This condition automatically implies the market-clearing condition for money, because portfolio weights add up to 1.

Due to logarithmic utility, the utility-maximizing consumption rates of intermediaries and households are ρN_t and $r((q_t+p_t)K_t - N_t)$ respectively. Equating demand and supply, we get the market-clearing condition for output

$$\rho N_t + r((q_t+p_t)K_t - N_t) = (a-\iota_t)K_t. \tag{2.13}$$

The Budget Constraints. The law of motion of intermediaries' net worth, given their portfolio allocations and consumption choices, is

$$dN_t/dt = (x_t\, r_t^K + (1-x_t)\, r_t^M - \rho)\, N_t. \tag{2.14}$$

In the event of a shock, the intermediaries' net worth drops to

$$N_t' = \left(x_t(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x_t)\frac{\breve{p}_t}{p_t} \right) N_t. \tag{2.15}$$

Given (2.14) and (2.15), the net worth of households is automatically given by $(q_t+p_t)K_t - N_t$.

2.3 The State Variable that Drives Equilibrium.

Because our setting is scale-invariant, the severity of financial frictions in our economy is quantified by the fraction of wealth that belongs to intermediaries

$$\eta_t = \frac{N_t}{(q_t+p_t)K_t}. \tag{2.16}$$

That is, we expect two economies, in which η_t is the same, to look like scaled versions of one another. Therefore, we look for a Markov equilibrium with the state variable η_t, in which prices (p_t, q_t) and allocations $(x_t, \underline{x}_t)$ are functions of η_t.

Using (2.14) and (2.1), the law of motion of η_t in the absence of shocks can be written as

$$\frac{d\eta_t/dt}{\eta_t} = \underbrace{x_t\frac{(1-\tau)(a-\iota_t)}{q_t} + (1-x_t)\frac{\tau(a-\iota_t)}{p_t} + (x_t-\theta_t)(\mu_t^q - \mu_t^p) - \rho}_{\mu_t^\eta}, \tag{2.17}$$

where

$$\theta_t = \frac{q_t}{p_t+q_t} \tag{2.18}$$

is the fraction of the economy's wealth that is in capital. From (2.15), in the event of a shock, η_t drops to

$$\breve{\eta}_t = \eta_t \left(x_t(1-\phi)\frac{\breve{\theta}_t}{\theta_t} + (1-x_t)\frac{1-\breve{\theta}_t}{1-\theta_t} \right) \tag{2.19}$$

It is useful to summarize all equilibrium conditions in a proposition. Below, we express the equilibrium conditions in the form that includes only scaled variables such as η_t, q_t and x_t, and not aggregate variables such as N_t or K_t.

Proposition 1. *In a Markov equilibrium, the law of motion of the state variable η_t is given by (2.17) and (2.19), where $q_t = \theta_t(q_t+p_t)$, $p_t = (1-\theta_t)(q_t+p_t)$, and ι_t, q_t+p_t satisfy*

$$\theta_t(q_t+p_t)\Phi'(\iota_t) = 1 \quad \text{and} \quad (\rho\eta_t + r(1-\eta_t))(q_t+p_t) = a - \iota_t. \tag{2.20}$$

Functions $\theta_t = \theta(\eta_t)$, $x_t = x(\eta_t)$ *and* $\underline{x}_t = \underline{x}(\eta_t)$ *are jointly determined by the conditions*

$$x_t\eta_t + \underline{x}_t(1-\eta_t) = \theta_t,$$

$$\frac{(1-\tau)(a-\iota_t)}{q_t} - \frac{\tau(a-\iota_t)}{p_t} + \mu_t^q - \mu_t^p + \lambda\left((1-\phi)\frac{\breve{\theta}_t}{\theta_t} - \frac{1-\breve{\theta}_t}{1-\theta_t}\right)\frac{\eta_t}{\eta_t'} = 0 \quad \text{and}$$

$$\frac{(1-\tau)(a-\iota_t)}{q_t} - \frac{\tau(a-\iota_t)}{p_t} + \mu_t^q - \mu_t^p + \lambda(1-\underline{\phi})\frac{\frac{\breve{\theta}_t}{\theta_t} - \frac{1-\breve{\theta}_t}{1-\theta_t}}{\underline{x}_t\frac{\breve{\theta}_t}{\theta_t} + (1-\underline{x}_t)\frac{1-\breve{\theta}_t}{1-\theta_t}} - \lambda\underline{\phi}\,\frac{1}{1-\underline{x}_t} \leq 0,$$

with equality if $\underline{x}_t > 0$, *where it is convenient to express*

$$\mu_t^q - \mu_t^p = \frac{d\theta_t}{d\eta_t}\frac{\mu_t^\eta\eta_t}{\theta_t(1-\theta_t)}. \tag{2.21}$$

Proof. The formulas follow directly from (2.7), (2.13), (2.12), (2.9), (2.11), and the definitions of η_t and θ_t. Regarding (2.21), note that

$$\mu_t^q = \left(\frac{\theta'(\eta)}{\theta(\eta)} + \frac{p'(\eta)+q'(\eta)}{p(\eta)+q(\eta)}\right)\mu_t^\eta\eta_t \quad \text{and} \quad \mu_t^p = \left(\frac{-\theta'(\eta)}{1-\theta(\eta)} + \frac{p'(\eta)+q'(\eta)}{p(\eta)+q(\eta)}\right)\mu_t^\eta\eta_t,$$

so

$$\mu_t^q - \mu_t^p = \left(\frac{\theta'(\eta)}{\theta(\eta)} + \frac{\theta'(\eta)}{1-\theta(\eta)}\right)\mu_t^\eta\eta_t = \frac{d\theta/d\eta}{\theta_t(1-\theta_t)}\mu_t^\eta\eta_t$$

□

Proposition 1 implies a first-order ordinary differential equation for the function $\theta(\eta)$, which can be solved numerically to find equilibrium for any parameter values. The derivative $d\theta/d\eta$ depends on the values of function θ on the interval $[0,\eta]$. In the Appendix, we outline the algorithm that we use to compute examples.

Next we provide several examples to explain equilibrium dynamics, and to explore how it depends on model parameters. The Appendix provides details of the algorithm we used to compute solutions.

3 Examples; the Impact of Parameters on Equilibrium.

First, let us discuss the general properties of equilibria. Figure 1 provides information about the equilibrium with a degenerate investment function $\Phi : \{0\} \to \mathbb{R}$ that leads to growth $g = \Phi(0) - \delta = 4\%$, and parameters $a = 0.1$, $r = 5\%$, $\rho = 7\%$, $\lambda = 0.8$, $\phi = 0.01$, $\underline{\phi} = 0.3$ and $\tau = 0.1$.

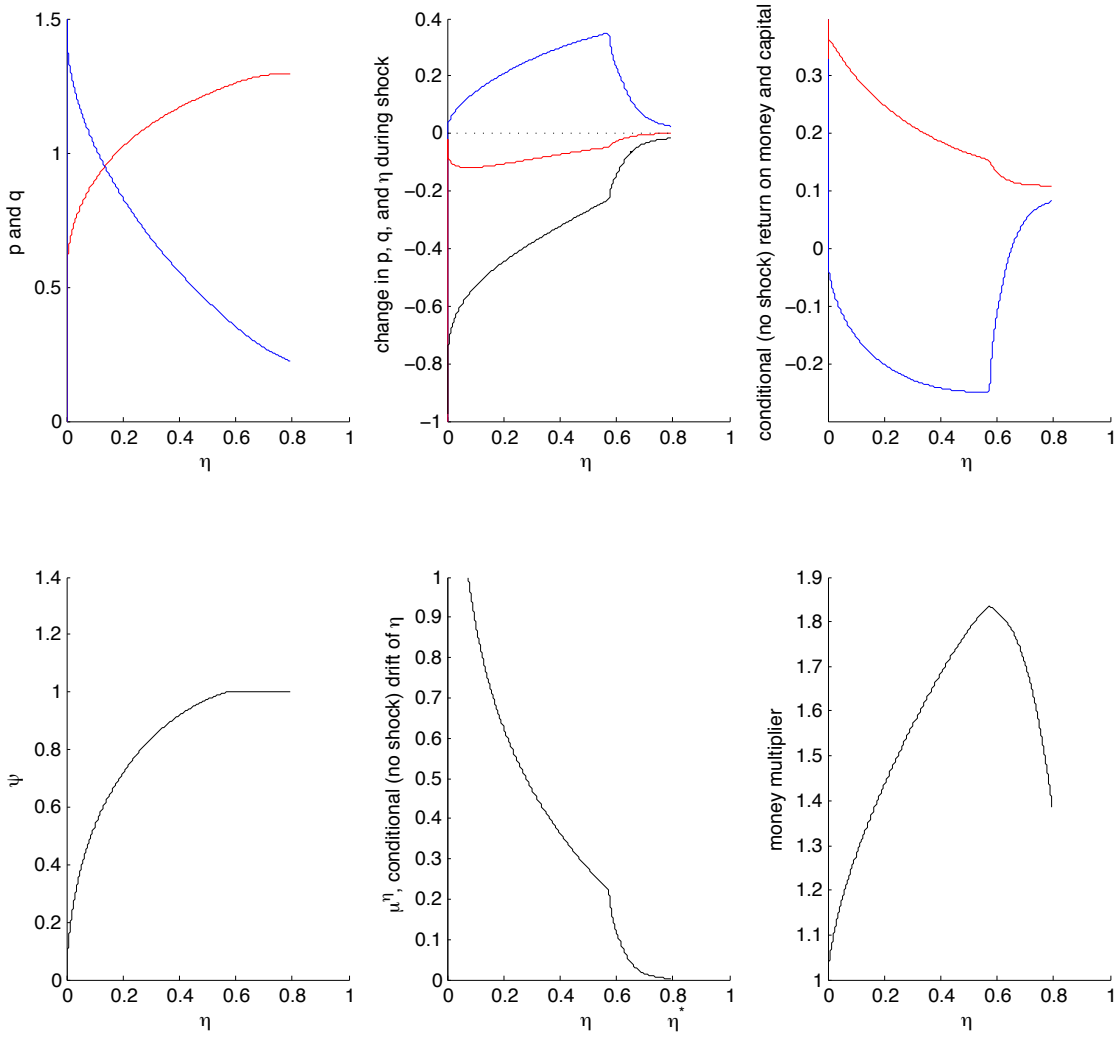


Figure 1: Equilibrium: quantities related to prices are in blue, and capital, in red.

In the absence of shocks, the equilibrium state variable η_t has drift μ_t^η given by (2.17) and shown in the bottom middle panel. The drift is decreasing monotonically over the interval $[0, \eta^*]$ towards 0. Point η^* plays the role of the stochastic steady state: when the state

variable η_t is at η^*, then it stays there when there are no shocks. If a shock arrives, η_t jumps down, as some of capital financed by intermediaries gets stolen and due to amplification through prices.

The model exhibits the traditional amplification channel on the asset side of the intermediary balance sheets: as the price of capital shown on the top left panel of Figure 1 in blue drops. In addition, shocks hurt borrowers on the liability sides of the balance sheet through the Fisher deflationary spiral. As we can observe from blue price graph on the top left panel the figure, money appreciates following a negative shock.

The reason for the deflationary pressure after a shock is as follows. As intermediaries suffer losses, they contract their balance sheets. Thus, they take fewer deposits and create less inside money. The total supply of money (inside and outside shrinks), but the demand for money does not change significantly since households still want to allocate a portion of their savings to safe money. As a result, the value of money goes up. The bottom right panel of Figure 1 shows that the money multiplier, the ratio of total money to outside money, decreases towards 1 as η decreases to 0.[3]

Top middle panel shows the reaction of money and capital to a shock. While money (blue) appreciates in value, capital (red) falls. Because intermediaries are borrow money to buy capital (some of which is also stolen), their net worth and η_t drop (black). Top right panel shows the returns on money (blue) and capital in the absence of shocks.

The model exhibits nonlinear amplification effects. Near the steady state intermediaries are least constrained: they hold all capital in the economy and prices respond little to shocks. As a result, there is little endogenous risk near η^*. In contrast, when intermediaries sell some of their capital holdings to households to reduce risk, prices become much more sensitive to shocks. Households are much worse at monitoring capital, and they cannot diversify, so they attach a much lower valuation to capital than intermediaries. The deeper the crisis, the lower the households' speculative motive to hold capital temporarily until the economy recovers, the less the price of capital. The top middle panel of Figure 1 shows clearly that capital and money both change in value much more when households invest in capital than near the steady state. Bottom left panel shows the equilibrium allocation $\psi_t = \eta_t x_t/\theta_t$ of capital to intermediaries.

Next, we explore how model parameters affect equilibrium. In particular, we want to

[3]It may look strange that the money multiplier decreases in η_t near η^*. This happens because when intermediaries already finance all capital in the economy, i.e. $\psi_t = 1$, the amount of money they create is decreasing in their need for outside funding, i.e. decreasing in η.

understand conceptually the effects of liquidity, shocks, and fiscal policy on equilibrium.

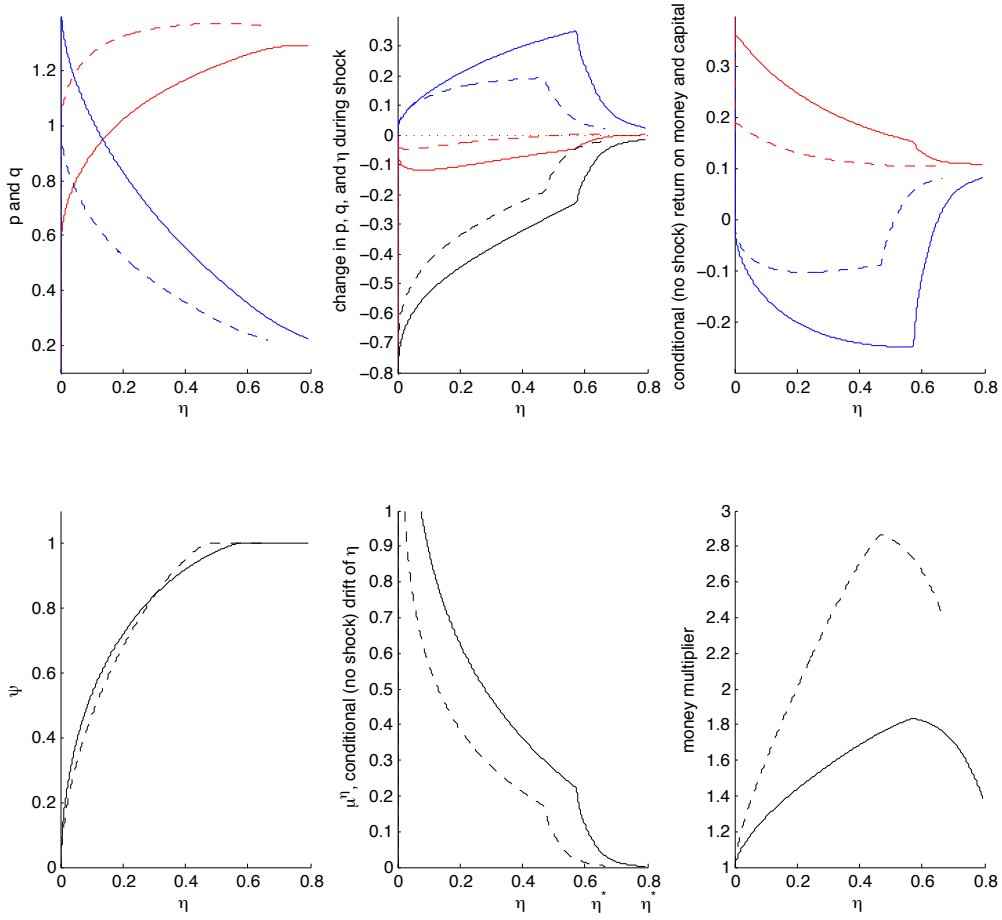


Figure 2: Higher capital liquidity ($\underline{\phi} = 0.1$, dashed) and lower liquidity ($\underline{\phi} = 0.3$, solid).

Liquidity. Market liquidity in our setting is measured by the difference between households' and intermediaries valuations of capital. Lower market liquidity gives rise to higher endogenous risk.

Figure 2 illustrates comparative statics on $\underline{\phi}$, which leads to lower capital valuation by households. Capital is more liquid when $\underline{\phi}$ is lower, as the second-best (households') valuation of capital is higher and closer to first best. It can be seen from the top middle panel that both money and capital react to shocks less when capital liquidity is higher. This leads to a higher price of capital and a lower price of money at the steady state, and higher equilibrium leverage. As the intermediary sector is less afraid of liquidity problems, the money multiplier is higher. At the same time, the risk premia on the top right panel are lower.

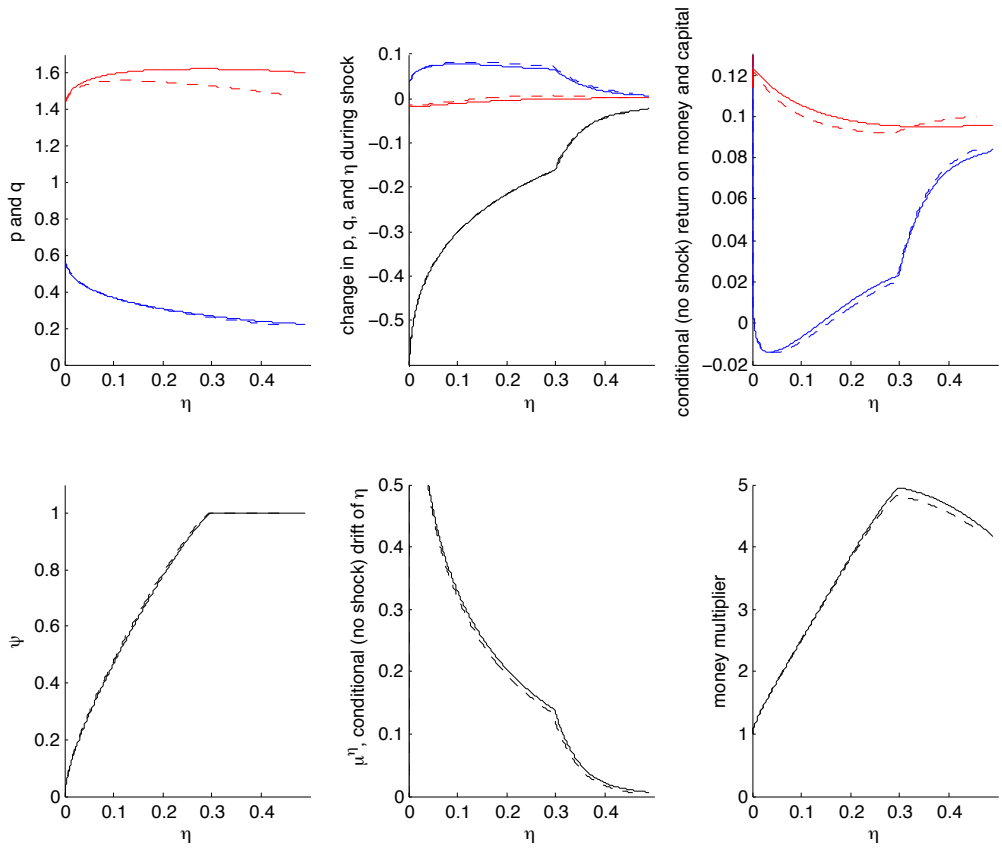


Figure 3: Equilibrium with $\underline{\phi} = 0.03$, $r = 5\%$, and $\rho = 7\%$ (dashed) and 6% (solid).

Intermediary Impatience. Figure 3 illustrates comparative statics on ρ (for higher capital liquidity than in our baseline model). Naturally, a higher value of ρ shifts down the location of the stochastic steady state, as intermediaries choose to pay out their net worth sooner. The price of capital is also higher with more patient intermediaries. These effects of ρ on equilibrium dynamics are intuitive.

What is noteworthy (and a bit surprising) is that the effect of the difference between the discount rates of intermediaries and households $\rho - r$ on equilibrium dynamics is so small, even as the difference $\rho - r$ changes by a factor of 2. The reaction of capital and money to shocks, the dynamics of η_t, as well as the allocation of capital ψ as a function of η, are almost the same. While some readers may question the assumption that $\rho > r$, as it may be difficult to interpret the value of the difference $\rho - r$, fortunately this difference has a small effect on equilibrium dynamics. The expected amount of time it takes, starting from the steady state η^*, to reach the point where intermediaries sell capital to households may be a

bit lower when ρ is higher.

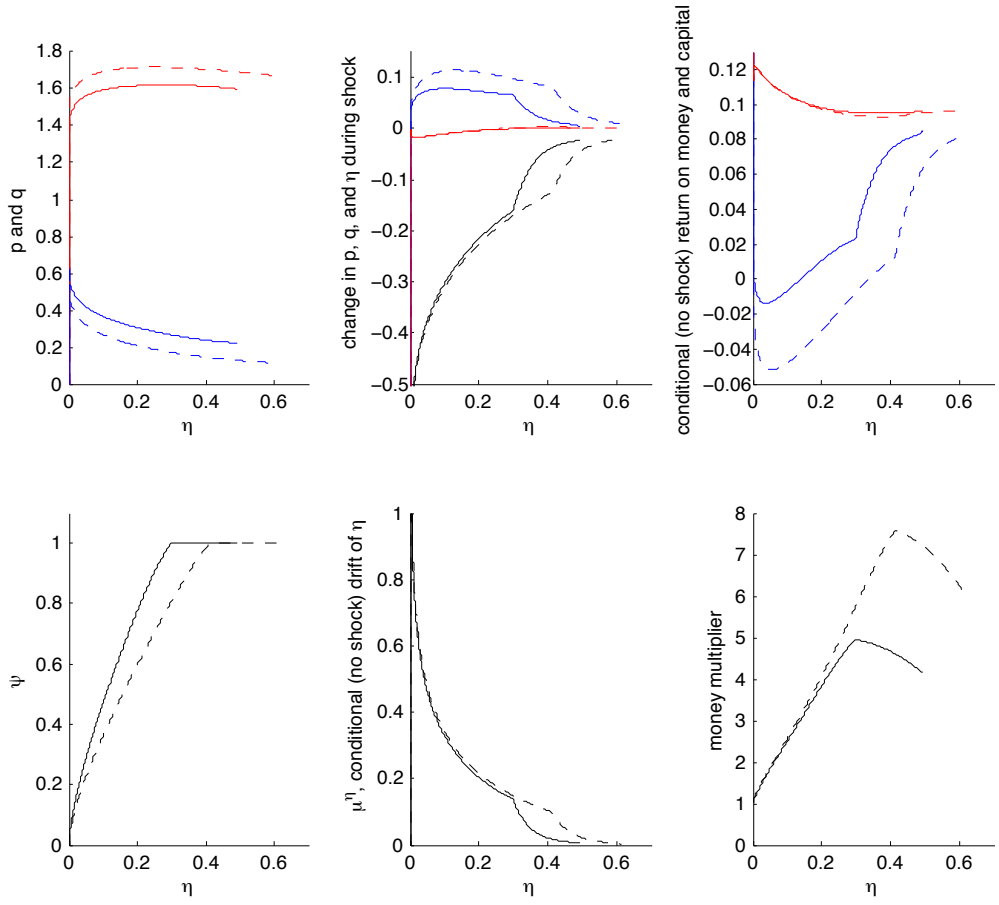


Figure 4: Equilibrium with $\underline{\phi} = 0.03$, and $\tau = 10\%$ (solid) and 5% (dashed).

Fiscal Backing of Money. As the tax rate τ goes to 0, the value of money in the frictionless economy benchmark goes to 0, while its value in the "money regime" without intermediaries stays positive. As a result, lower values of τ lead to higher amplification on the liabilities side of the intermediaries' balance sheets, through the Fisher deflationary spiral. When money is backed future tax revenues, the equilibrium is much more stable. Otherwise, money is a pure "flight to safety" asset, which has value only because it appreciates and serves a role in severe downturns.[4]

[4]This logic suggests that gold, which does not produce any cash flows, appreciates much more in downturns than government debt, which is backed by future tax revenue. Note also that monetary policy can mitigate, or even reverse the Fisher deflationary spiral, as we discuss in the next sections.

Figure 4 shows that money reacts to shocks much more in an environment where the fundamental value of money in the frictionless economy is lower. As a result, intermediaries accumulate more wealth at the steady state, and start deleveraging (by selling capital to households) at a higher level of η_t. Moreover, when the fundamental value of money in the frictionless economy is lower, then the money multiplier is higher.

4 Macro-prudential and Monetary Policy.

Financial frictions lead to a number of inefficiencies in equilibrium. They limit risk sharing, as intermediaries and households who rent out capital are forced to absorb the fundamental risks of their projects. They lead to resource misallocation, away from natural buyers of capital who have a superior monitoring technology. Also, importantly, prices and deflationary pressures amplify shocks create endogenous risk on both sides of the intermediaries' balance sheets. Endogenous risk calls for higher net worth cushions to be maintained to perform any given level of financing, and limits the level of economic activity. In downturns, endogenous risk redistributes wealth away from productive sectors towards less productive agents.

Policy has the potential to mitigate some of these inefficiencies. It can undo some of the endogenous risk by redistributing wealth towards compromised sectors. It can control the creation of endogenous risk by affecting the path of deleveraging. It can also work to prevent the build-up of systemic risk in booms.

Policies affect the equilibrium in a number of ways, and can have unintended consequences. Here is a list of useful questions. How does a policy affect equilibrium leverage? Does the policy create moral hazard? Does the policy inflated asset prices in booms? What happens to endogenous risk? How does the policy affect the frequency of crises, i.e. episodes characterized by resource misallocation and loss of productivity?

Since the focus of this paper is money, we focus on several monetary policies in this section. These policies can be divided in several categories. Traditional monetary policy sets the short-term interest rate. It affects the yield curve through the expectation of future interest rates, as well as through the expected path of the economy, with the supply and demand of credit. When the zero lower bound for the short-term policy rate becomes a constraint, forward guidance is an additional policy tool that is often employed in practice. The use of this tool depends on credibility issues, as it ties the central bank's hands in the future and leaves it less room for discretion. The distinction between short-term interest rates and forward guidance becomes immaterial when the central bank can commit to monetary

policy, as we consider in this paper.

Several non-tranditional policies have also been employed. The central bank can directly purchases assets to support prices or affect the shape of the yield curve. The central bank can lend to financial institutions, and choose acceptable collateral as well as margin requirements and interest rates. Some of these programs work by transferring tail risk to the central bank, as it suffers losses (and consequently redistributes them to other agents) in the event that the value of collateral becomes insufficient and the counterparty defaults. Other policies include direct equity infusions into troubled institutions. At some point, the distinction between monetary, macro-prudential and fiscal policies becomes fuzzy.

In the next section, we focus on policies that affect interest rates and redistribute wealth through the prices of nominal long-term assets. The redistributive effects of monetary policy depend on who holds these assets. In turn, asset allocation depends on the anticipation of future policy, as well as the demand for insurance. We introduce perpetual long-term bonds, and allow the monetary authority to both set the interest rate on short-term money, and affect the composition of outstanding government liabilities (money and long-term bonds) through open-market operations.

Money and Long-Term Bonds. We extend our baseline model in two ways: we allow money to pay the floating rate interest and we introduce perpetual bonds, which pay interest at a fixed rate in money. Monetary policy sets interest $r_t \geq 0$ on money and controls the value $b_t K_t$ of all perpetual bonds outstanding. These policies are independent of fiscal policy: the fiscal authority taxes output at rate τ as before, and uses it to redeem money and long term bonds. The monetary authority takes deposits of outside money, and finances interest payments as well as open market operations by printing money.

We now denote by $p_t K_t$ the supply of all outstanding nominal assets: outside money and perpetual bonds. Also, let B_t be the endogenous equilibrium price of the perpetual bond, which follows

$$\frac{dB_t}{dt} = \mu_t^B \, B_t \tag{4.1}$$

in the absence of shocks, and jumps to $\breve{B}_t$ when a shock arrives. Note that r_t and b_t are policy instruments, while B_t is the endogenous equilibrium process.

Returns. Capital earns return of the same form as before: it follows (2.6) in the absence of shocks, and jumps in value by $(1-\phi)\breve{q}_t/q_t$ for intermediaries and $\breve{q}_t/q_t$ or 0 for households in the event of a shock.

The return on money and bonds can be derived as follows. First, someone who borrows

a dollar of money to invest in bonds earns

$$r_t^B - r_t^M = \frac{1}{B_t} + \mu_t^B - r_t,$$

since he gets the current yield and appreciation of the bond (relative to money) and has to pay nominal interest r_t to borrow. Second, the world portfolio of money and bonds earns

$$(1 - b_t/p_t)\, r_t^M + b_t/p_t\, r_t^B = \frac{\tau a}{p_t} + \mu_t^p + \Phi(\iota_t) - \delta.$$

While the above expressions hold in the absence of shocks, a shock causes the value of the bond relative to money to change by $\breve{B}_t/B_t$, and the value of the world portfolio of money and bonds, by $\breve{p}_t/p_t$. Therefore, we get the returns on money and bonds that are summarized in the table below[5]

	no shock	shock
money	$r_t^M = \frac{\tau(a-\iota_t)}{p_t} + \mu_t^p + \Phi(\iota_t) - \delta - \frac{b_t}{p_t}\left(\frac{1}{B_t} + \mu_t^B - r_t\right)$	$\breve{M}_t \equiv \frac{\breve{p}_t}{p_t} \frac{1}{1 - \frac{b_t}{p_t} + \frac{b_t}{p_t}\frac{\breve{B}_t}{B_t}}$
bonds	$r_t^B = \frac{\tau(a-\iota_t)}{p_t} + \mu_t^p + \Phi(\iota_t) - \delta + \left(1 - \frac{b_t}{p_t}\right)\left(\frac{1}{B_t} + \mu_t^B - r_t\right)$	$\frac{\breve{B}_t}{B_t}\breve{M}_t$

Equilibrium Conditions. Intermediaries solve

$$\max_{x,y}\ x\, r_t^K + (1-x)\, r_t^M + y(r_t^B - r_t^M) + \lambda \log\left(x(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x)\,\hat{M}_t + y\left(\frac{\breve{B}_t}{B_t} - 1\right)\breve{M}_t\right).$$

The first-order conditions with respect to portfolio weights x_t and y_t are

$$r_t^K - r_t^M + \lambda \frac{(1-\phi)\frac{\breve{q}_t}{q_t} - \breve{M}_t}{x_t\,(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x_t)\,\breve{M}_t + y_t\left(\frac{\breve{B}_t}{B_t} - 1\right)\breve{M}_t} = 0, \tag{4.2}$$

$$\frac{1}{B_t} + \mu_t^B - r_t + \lambda \frac{\breve{M}_t\left(\frac{\breve{B}_t}{B_t} - 1\right)}{x_t\,(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x_t)\,\breve{M}_t + y_t\left(\frac{\breve{B}_t}{B_t} - 1\right)\breve{M}_t} = 0. \tag{4.3}$$

[5]Note that the change in value of long-term bonds in the event of a shock is typically not equal to $\breve{b}_t/b_t$, as this fraction takes into account the change in value of *outstanding* bonds due to open-market operations immediately following the shock.

Likewise, the households' first-order condition with respect to $\underline{x}$ is

$$r_t^K - r_t^M + \lambda(1-\underline{\phi})\,\frac{\frac{\breve{q}_t}{q_t} - \breve{M}_t}{\underline{x}_t\,\frac{\breve{q}_t}{q_t} + (1-\underline{x}_t)\,\hat{M}_t} + \lambda\underline{\phi}\,\frac{-\breve{M}_t}{(1-\underline{x}_t)\,\breve{M}_t} \geq 0 \tag{4.4}$$

with equality if $\underline{x}_t \geq 0$. Note that we set household allocation to long term bonds $\underline{y}_t = 0$, as we focus on policies under which only intermediaries choose to hold long-term bonds to insure themselves against shocks.[6] The market-clearing conditions for output and capital are the same as before,

$$\rho\eta_t + r(1-\eta_t) = \frac{a-\iota_t}{p_t+q_t} \quad \text{and} \quad x_t\eta_t + \underline{x}_t(1-\eta_t) = q_t.$$

The market-clearing condition for bonds is

$$y_t\eta_t = \frac{b_t}{p_t+q_t}. \tag{4.5}$$

The Law of Motion of η_t. In the absence of shocks,

$$\frac{dN_t/dt}{N_t} = x_t\,r_t^K + (1-x_t)\,r_t^M + y_t\,(r_t^B - r_t^M) - \rho$$

and

$$\frac{d((p_t+q_t)K_t)/dt}{(p_t+q_t)K_t} = \Phi(\iota) - \delta + (1-\theta_t)\mu_t^p + \theta_t\mu_t^q.$$

Therefore,

$$\frac{d\eta_t/dt}{\eta_t} = x_t\,r_t^K + (1-x_t)\,r_t^M + y_t\,\left(\frac{1}{B_t} + \mu_t^B - r_t\right) - \rho - (\Phi(\iota) - \delta + (1-\theta_t)\mu_t^p + \theta_t\mu_t^q). \tag{4.6}$$

In the event of a shock, N_t and $(q_t+p_t)K_t$ change by factors of

$$x_t\,(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x_t)\,\hat{M}_t + y_t\left(\frac{\breve{B}_t}{B_t} - 1\right)\breve{M}_t \qquad \text{and} \qquad \theta_t\frac{q_t'}{q_t} + (1-\theta_t)\frac{p_t'}{p_t}$$

[6] Households refrain from holding long-term bonds if

$$\frac{1}{B_t} + \mu_t^B - r_t + \lambda(1-\underline{\phi})\,\frac{\breve{M}\left(\frac{\breve{B}_t}{B_t} - 1\right)}{\underline{x}_t\,\frac{\breve{q}_t}{q_t} + (1-\underline{x}_t)\,\breve{M}} + \lambda\underline{\phi}\,\frac{\breve{M}\left(\frac{\breve{B}_t}{B_t} - 1\right)}{(1-\underline{x}_t)\,\breve{M}} \leq 0.$$

respectively. Therefore, η_t drops to

$$\breve{\eta}_t = \eta_t \frac{x_t\,(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x_t)\,\breve{M}_t + y_t\left(\frac{\breve{B}_t}{B_t}-1\right)\breve{M}_t}{\theta_t\frac{\breve{q}_t}{q_t} + (1-\theta_t)\frac{\breve{p}_t}{p_t}} \tag{4.7}$$

Lemma 1 provides an alternative convenient expression for the law of motion of η_t.

Lemma 1. *The law of motion of η_t is*

$$\frac{d\eta_t/dt}{\eta_t} = (r-\rho)(1-\eta_t) + \lambda\left((1-\phi\breve{\theta}_t)\frac{\eta_t}{\breve{\eta}_t} - 1\right) \quad \text{in the absence of shocks and} \tag{4.8}$$

$$\frac{\breve{\eta}_t}{\eta_t} = x_t\,(1-\phi)\frac{\breve{\theta}_t}{\theta_t} + (1-x_t)\,\tilde{M}_t + y_t\left(\frac{\breve{B}_t}{B_t}-1\right)\tilde{M}_t, \quad \text{where} \quad \tilde{M}_t = \hat{M}\frac{q_t+p_t}{\breve{q}_t+\breve{p}_t}. \tag{4.9}$$

Equation (4.8) is instructive. It suggests that the impact of the policy on equilibrium dynamics works primarily by moderating the impact of shocks on the intermediary sector, i.e. lowering $\eta_t/\breve{\eta}_t$. While the economy becomes more resilient to shocks, the speed of the recovery $d\eta_t/dt$ gets lower as risk premia decrease, particularly if the policy stimulates the price of capital relative to money θ_t.

Proof. First, multiplying the numerator and denominator of (4.7) by $\frac{q_t+p_t}{\breve{q}_t+\breve{p}_t}$, we get (4.9).

Second, justifying (4.8) takes some algebra. Taking $\theta_t - x_t$ times (4.2) and subtracting $y_t - (1-\theta_t)\frac{b_t}{p_t}$ times (4.3), we get

$$(\theta_t - x_t)(r_t^K - r_t^M) - \left(y_t - (1-\theta_t)\frac{b_t}{p_t}\right) + \tag{4.10}$$

$$\lambda\,\underbrace{\frac{(\theta_t-x_t)(1-\phi)\frac{\breve{q}_t}{q_t} - (\theta_t-x_t)\breve{M}_t - \left(y_t-(1-\theta_t)\frac{b_t}{p_t}\right)\breve{M}_t\left(\frac{\breve{B}_t}{B_t}-1\right)}{x_t\,(1-\phi)\frac{\breve{q}_t}{q_t} + (1-x_t)\,\breve{M}_t + y_t\left(\frac{\breve{B}_t}{B_t}-1\right)\breve{M}_t}}_{\frac{\theta_t(1-\phi)\frac{\breve{q}_t}{q_t}+(1-\theta_t)\breve{M}_t+(1-\theta_t)\frac{b_t}{p_t}\breve{M}_t\left(\frac{\breve{B}_t}{B_t}-1\right)}{x_t\,(1-\phi)\frac{\breve{q}_t}{q_t}+(1-x_t)\,\breve{M}_t+y_t\left(\frac{\breve{B}_t}{B_t}-1\right)\breve{M}_t}-1} = 0,$$

Furthermore, since $\breve{M}_t\left(1+\frac{b_t}{p_t}\left(\frac{\breve{B}_t}{B_t}-1\right)\right)=\frac{\breve{p}_t}{p_t}$, we have

$$\frac{\theta_t(1-\phi)\frac{\breve{q}_t}{q_t}+(1-\theta_t)\breve{M}_t+(1-\theta_t)\frac{b_t}{p_t}\breve{M}_t\left(\frac{\breve{B}_t}{B_t}-1\right)}{x_t\left(1-\phi\right)\frac{\breve{q}_t}{q_t}+\left(1-x_t\right)\breve{M}_t+y_t\left(\frac{\breve{B}_t}{B_t}-1\right)\breve{M}_t}=\frac{\theta_t(1-\phi)\frac{\breve{q}_t}{q_t}+(1-\theta_t)\frac{\breve{p}_t}{p_t}}{x_t\left(1-\phi\right)\frac{\breve{q}_t}{q_t}+\left(1-x_t\right)\breve{M}_t+y_t\left(\frac{\breve{B}_t}{B_t}-1\right)\breve{M}_t}$$

$$=\frac{(1-\phi)\breve{\theta}_t+1-\breve{\theta}_t}{x_t\left(1-\phi\right)\frac{\breve{\theta}_t}{\theta_t}+\left(1-x_t\right)\tilde{M}_t+y_t\left(\frac{\breve{B}_t}{B_t}-1\right)\tilde{M}_t}=(1-\phi\breve{\theta}_t)\frac{\eta_t}{\breve{\eta}_t},$$

where we multiplied both the numerator and denominator by $\frac{q_t+p_t}{\breve{q}_t+\breve{p}_t}$.

Adding (4.10) to (4.6), we get

$$\frac{d\eta_t/dt}{\eta_t}=\theta_t\frac{(1-\tau)(a-\iota_t)}{q_t}+(1-\theta_t)\frac{\tau(a-\iota_t)}{p_t}+\lambda\left((1-\phi\breve{\theta}_t)\frac{\eta_t}{\breve{\eta}_t}-1\right)-\rho$$

Furthermore, since $\rho\eta_t+r(1-\eta_t)=(a-\iota_t)/(p_t+q_t)$ and $\theta_t=q_t/(p_t+q_t)$, we have

$$\frac{d\eta_t/dt}{\eta_t}=(r-\rho)(1-\eta_t)+\lambda\left((1-\phi\breve{\theta}_t)\frac{\eta_t}{\breve{\eta}_t}-1\right).$$

□

5 Monetary Policy: An Example.

This section provides an example of how monetary authority can affect asset prices, in particular prices of long-term bonds, through monetary policy. Without long-term bonds, interest rate policy alone would not have any real effect on equilibrium in our model: it affects only the nominal return on money, and the interest rate that intermediaries pay to depositors. Likewise, with constant interest rate, bonds and money are perfect substitutes as the price of bonds in terms of money is fixed (and so open market operations alone do not have any effect on the economy).

In the following example, the central bank sets the short-term interest rate to $\rho_t = 0.5\%+\eta_t*10\%$, and target the ratio of outstanding bond value to total value of nominal assets to be $b_t/p_t = 0.5$. Note that the short-term rate is lowered when η_t drops, causing long-term bonds to appreciate (as shown in the bottom middle panel). This recapitalizes intermediaries, who hold long-term bonds, and mitigates the impact of shocks on the economy. Figure 5 applies this policy to our baseline example, and compares equilibria with and without the

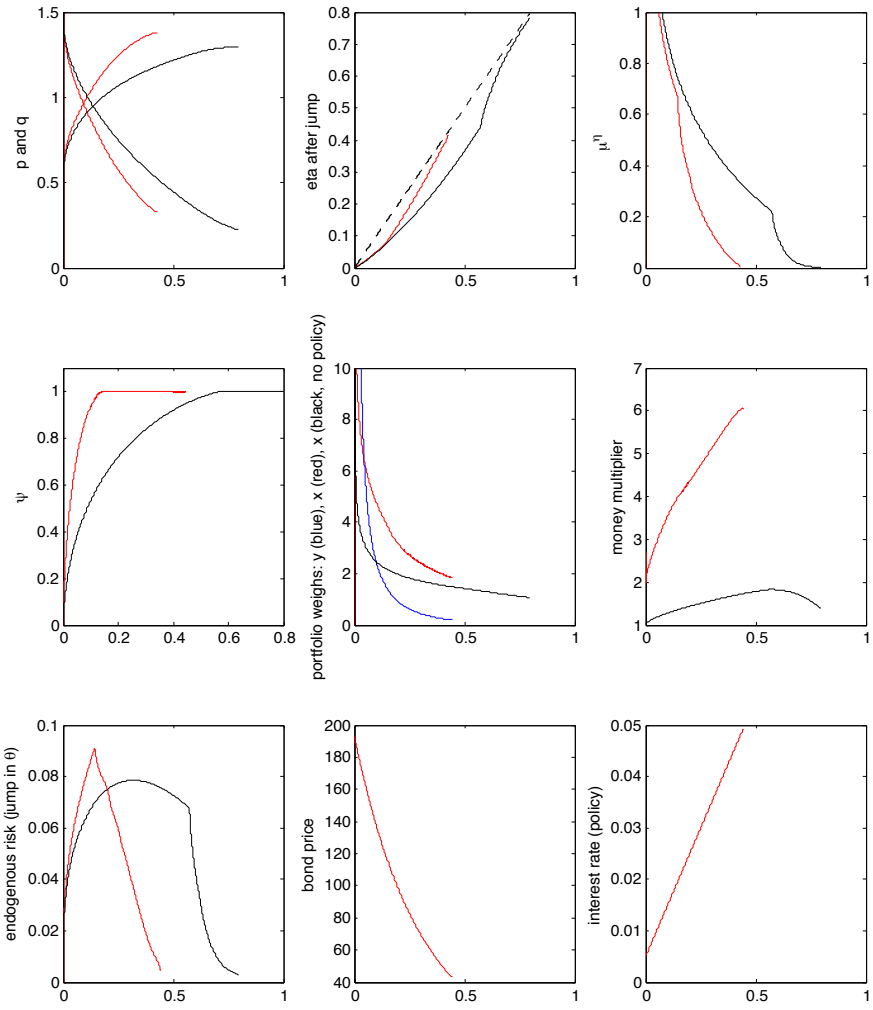


Figure 5: Equilibrium in the baseline model without policy (black) and with (red).

policy.

The policy works through multiple channels. First, it mitigates the deflationary spiral: for any level of η_t, the value of money is higher with the policy. As bonds appreciate when η_t goes down, they fill in some of the demand for money, and as a result outside money appreciates less. Intermediaries can also create more inside money in downturns by borrowing against bonds.[7] Second, bonds provide intermediaries with a partial hedge against shocks. Because

[7]Note that the graph for p on the top left panel of Figure 5 may be deceptive: the rise in p_t in downturns reflects for the most part the appreciation of bonds rather than deflation. Note that bonds appreciate by a factor of about 4 in downturns, while p_t appreciates by a factor of only 3 (and bonds comprise one half of the value of all nominal assets).

bonds, which intermediaries hold on the asset side of balance sheets, appreciate after shocks, intermediary net worth η_t drops by less with the policy than without.

The location of the steady state η^* is endogenous, and it drops significantly with the introduction of the policy. As risk premia driven by endogenous risk become lower, intermediaries' earnings fall and their wealth at the steady state becomes lower. The top right panel shows the drift of η_t with and without policy. Despite lower net worth, intermediaries are able to finance all capital near the steady state, as the risk on the liabilities side of their balance sheets is diminished.

Even though the policy leads to higher equilibrium leverage, it does not necessarily create moral hazard. The reason is that the "stealth recapitalization" through bond prices does not reward the weak institutions that took risk and suffered losses, but the cautious institutions that bought bonds to insure themselves against downturns. Indeed, endogenous risk of capital is comparable to that without policy, while the endogenous risk of money is significantly lower.

The money multiplier is much greater with the policy, as intermediaries can create money by borrowing against bonds, which also serve as a natural hedge.

6 Conclusion

We consider an economy in which some agents are savers who look for ways to store wealth, while others are entrepreneurs who look to borrow against profitable projects. There is a role for financial intermediation and a role for money. Savers have the option to hold *outside* money or make deposits with intermediaries, i.e. hold *inside* money. Thus, intermediation and money are complementary. A shock to intermediaries causes them to shrink balance sheets and create less inside money; such a shock leads to a rising demand for outside money, i.e. deflation. This deflationary spiral amplifies shocks, as it hurts borrowers who owe nominal debt. It works on the liabilities side of the intermediary balance sheets, while the liquidity spiral that hurts the price of capital works on the asset side. Importantly, in this economy the money multiplier is endogenous: it depends on the health of the intermediary sector.

Endogenous risk, manifested in the deflationary and liquidity spirals, exacerbates the effect of shocks and leads to inefficiencies. Endogenous risk depresses the price of productive assets particularly when they are illiquid, i.e. their value differs significantly in first and second-best use. Endogenous risk in money is particularly large when the money multiplier

in boom is large, i.e. the intermediary sector provides significant amounts of credit.

Monetary policy can work against the deflationary spiral. For example, interest rate cuts in downturns can lead to appreciation of long-term nominal assets, recapitalizing institutions that hold these assets and increasing the supply of the safe asset. Such a policy also, indirectly, can reduce endogenous risk due to the liquidity spiral, as it makes the system more stable.

Of course, any policy that provides insurance against downturns creates some moral hazard. However, these drawbacks are mitigated by the crudeness of the policy: it does not redistribute towards the weakest failing institutions, but rather to stronger and more cautions institutions that took care to hedge the downturn risk.

7 Bibliography

Bernanke, B. (1983) "Non-Monetary Effects of the Financial Crisis in the Propagation of the Great Depression", *American Economic Review*, Vol. 73, 257-276.

Bernanke, B. and A. Blinder (1989) "Credit, Money, and Aggregate Demand", *American Economic Review, Papers and Proceedings*, Vol. 78, No. 2, 435-439.

Bernanke, B. and M. Gertler (1989) "Agency Costs, Net Worth, and Business Fluctuations", *American Economic Review*, Vol. 79, No. 1, 14-31.

Bernanke, B., M. Gertler and S. Gilchrist (1999) "The Financial Accelerator in a Quantitative Business Cycle Framework", *Handbook of Macroeconomics* (Chapter 21), 1341-1393.

Bewley, T. (1980) "The Optimum Quantity of Money", in *Models of Monetary Economics*, eds. J. Kareken and N. Wallace. Minneapolis, Minnesota: Federal Reserve Bank.

Brunnermeier, M. K., T. Eisenbach, and Y. Sannikov (2010) "Macroeconomics with Financial Frictions: A Survey", in *Proceedings of the Econometric Society World Congress 2010 in Shanghai*, eds. E. Dekkel, Cambridge University Press.

Brunnermeier, M. K. and Y. Sannikov (2010) "A Macroeconomic Model with a Financial Sector", working paper, Princeton University.

Cagan, P. (1979) *Persistent Inflation: Historical and Policy Essays*, Columbia university Press.

Cordia, V. and M. Woodford (2010) "Credit Spreads and Monetary Policy", working paper, Columbia University.

Diamond, D. W. (1984) "Financial Intermediation and Delegated Monitoring", *The Review of Economic Studies*, 51(3), 393414.

Diamond, D. and R. Rajan (2006) "Money in a Theory of Banking",' *American Economic Review*, March 2006.

Friedman, M. and A. J. Schwartz. (1963) *A Monetary History of the United States, 1867-1960*, Princeton University Press.

Gurley, J and E. Shaw (1955) "Financial Aspects of Economic Development", *American Economic Review*, September 2006, 515-538.

He, Z., and A. Krishnamurthy (2010) "Intermediary Asset Prices", working paper, Northwestern University.

He, Z., and A. Krishnamurthy (2010) "A Model of Capital and Crises", forthcoming in *Review of Economic Studies.*

Holmstrom, B., and J. Tirole (1997): "Financial Intermediation, Loanable Funds, and the Real Sector", *The Quarterly Journal of Economics*, 112(3), pp.663691.

Kiyotaki, N. and J. Moore (1997) "Credit Cycles", *Journal of Political Economy*, Vol. 105, No. 2, 211-48.

Kiyotaki, N. and J. Moore (2008) "Liquidity, Business Cycles, and Monetary Policy", working paper, Princeton University.

Patinkin D. (1965) *Money, Interest and Prices: An Integration of Monetary and Value Theory*, Harper & Row NY.

Samuelson, P. A. (1958) "An exact consumption loan model of interest, with or without the social contrivance of money" *Journal of Political Economy* 66, 467-82.

Stein, J. (2010) "Monetary Policy as Financial-Stability Regulation", working paper, Harvard University.

Scheinkman, J. A. and L. Weiss (1986) "Borrowing Constraints and Aggregate Economic Activity", Econometrica, Vol. 54, 23-45

Tobin, J. (1969) "A General Equilibrium Approach to Monetary Theory", Journal of Money, Credit and Banking, Vol. 1, No. 1, 15-29.

A Numerical Procedure to find Equilibrium.

Computation to solve the differential equations and characterize equilibrium poses several challenges. First, the location of the steady state η^* is unknown. Second, the equations have singularities at $\eta = 0$ and η^*. Third, the derivative $d\theta(\eta)/d\eta$ depends on the value of the function $\theta(\eta)$ not only at point η, but at other points of the domain (in particular, $\breve{\eta} \in (0, \eta)$, to which η_t transitions in the event of a macro shock).

To resolve these challenges, we start by defining

$$\theta(\eta) = \underline{\theta} + \epsilon_1$$

on an interval $[0, \epsilon_2]$. Since ϵ_1 and ϵ_2 are small constants, we match the boundary condition $\theta(0) = \underline{\theta}$ approximately. We fix ϵ_1, but search for ϵ_2 to match an appropriate boundary condition at the steady state η^*. Since we always know the value of the function θ on the entire interval $[0, \eta]$ when we compute the derivative $d\theta(\eta)/d\eta$, we are always able to find $\breve{\eta}$ to find this derivative.

Formally, the procedure we employ is as follows. We use the Euler method. This method can be refined to a higher-order method, such as Runge-Kutta, if necessary. We set $\epsilon_L = 0$, $\epsilon_R = 0.4$, define $\epsilon_1 = (\epsilon_L + \epsilon_R)/2$ and set

$$\theta(\eta) = \theta(0) + \epsilon_1, \quad \text{for } \eta = \{\eta(1) = 0,\ \eta(2),\ \dots\ \eta(m) = \epsilon_2\}.$$

Then to solve the differential equation for $\eta(n)$, $n = m, m+1, \dots$, we perform the following sequence of steps. We set $\breve{n} = 1$ for $n = m$.

(i) For each $n \geq m$, we want to identify the value of $\breve{\eta}$ to which η jumps from $\eta(n)$ in the event of a shock. To identify $\breve{\eta}$, we try $\breve{\eta} = \eta(k)$ for $k = \breve{n},\ \breve{n}+1, \dots$ and in each case perform the following sequence of steps.

We calculate intermediary leverage x required to sink η to $\breve{\eta}$ using formula (2.19), which implies

$$x = \frac{\frac{1-\breve{\theta}_t}{1-\theta_t} - \frac{\breve{\eta}_t}{\eta_t}}{\frac{1-\breve{\theta}_t}{1-\theta_t} - \frac{\breve{\theta}_t}{\theta_t}(1-\phi)}, \tag{A.1}$$

where $\theta_t = \theta(n)$ and $\breve{\theta}_t = \theta(k)$. Note that a higher level of $\breve{\eta}_t$ corresponds to lower leverage, i.e. lower x.

Then compute

$$\psi(k) = \frac{x\eta_t}{\theta_t} \quad \underline{x} = \frac{1-\psi(k)}{1-\eta_t}\theta_t, \quad q_t = \theta_t \frac{a-\iota(q_t)}{r}. \tag{A.2}$$

To check if intermediary leverage x is appropriate, note that (2.9) and (2.11) (together with (2.19)) imply that

$$A(k) = \left(\frac{\breve{\theta}_t}{\theta_t}(1-\phi) - \frac{1-\breve{\theta}_t}{1-\theta_t}\right)\frac{\eta_t}{\breve{\eta}_t} + \underline{\phi}\frac{1}{1-\underline{x}} - (1-\underline{\phi})\frac{\frac{\breve{\theta}_t}{\theta_t} - \frac{1-\breve{\theta}_t}{1-\theta_t}}{\underline{x}\frac{\breve{\theta}_t}{\theta_t} + (1-\underline{x})\frac{1-\breve{\theta}_t}{1-\theta_t}} \geq 0,$$

with strict inequality only if $\psi = 1$. Therefore, if $A(k) < 0$ (or $A(k) > 0$ but $\psi(k) > 1$) then intermediary leverage is still too large and $\breve{\eta}$ too low. Therefore, we keep raising k until $A(k) > 0$ and $\psi(k) < 1$, and then we stop. We know at this point that $\breve{\eta}$ has to be between $\eta(k)$ and $\eta(k-1)$ and proceed to step (ii).

(ii) If $\psi(k-1) > 1$, then we identify $\breve{\eta} \in [\eta(k-1), \eta(k)]$ that leads to $\psi = 1$ according to a linear interpolation formula.[8] Let

$$w(n) = \frac{\psi(k-1) - 1}{\psi(k-1) - \psi(k)},$$

and

$$\breve{\eta} = w(n)\eta(k) + (1 - w(n))\,\eta(k-1), \quad \breve{\theta} = w(n)\,\theta(k) + (1 - w(n))\,\theta(k-1). \tag{A.3}$$

If $\psi(k-1) < 1$, then we want to identify $\breve{\eta} \in [\eta(k-1), \eta(k)]$ that leads to $A = 1$. Let

$$w(n) = \frac{A(k-1)}{A(k-1) - A(k)},$$

and then set $\breve{\eta}_t$ and $\breve{\theta}_t$ again according to (A.3).

Thereafter, compute x, ψ_t, $\underline{x}$ and q_t using (A.1) and (A.2), and let $p_t = (1-\theta_t)/\theta_t q_t$. To find $d\theta/d\eta$, we proceed as follows.

[8] Technically, we need $A > 0$ at $\breve{\eta}$, but the error from ignoring this issue is small since $A(k) > 1$.

Use (2.9) to calculate

$$\mu_t^q - \mu_t^p = \frac{\tau(a-\iota)}{p_t} - \frac{(1-\tau)(a-\iota)}{q_t} - \lambda\left(\frac{\breve{\theta}_t}{\theta_t}(1-\phi) - \frac{1-\breve{\theta}_t}{1-\theta_t}\right)\frac{\eta_t}{\breve{\eta}_t}.$$

Use (2.17) to calculate

$$\mu_t^\eta = x\frac{(1-\tau)(a-\iota)}{q_t} + (1-x)\frac{\tau(a-\iota)}{p_t} + (x-\theta_t)(\mu_t^q - \mu_t^p) - r.$$

Then

$$\frac{d\theta_t}{d\eta_t} = \theta_t(1-\theta_t)\frac{\mu_t^q - \mu_t^p}{\mu_t^\eta \eta_t}.$$

Let[9]

$$\theta(n+1) = \theta(n) + \frac{d\theta_t}{d\eta_t}(\eta(n+1) - \eta(n)).$$

(iii) Perform several checks. First, it could be that $\theta(n+1) > 1$. If so, then our guess of ϵ_1 was too high - we go back to the beginning to revise ϵ_1 down by setting $\epsilon_R = \epsilon_1$. Second, it could be that $d\theta_t/d\eta_t < 0$ at some point ϵ_t. If so, we revise ϵ_1 up by setting $\epsilon_L = \epsilon_1$. We also interrupt integration and revise ϵ_1 down if $\mu_t^\eta(n+1) > \mu_t^\eta(n)$, a condition that has been identified experimentally.

We then go back to step (i), and iterate until convergence.

[9]This is the Euler method for solving the ODE. It can be replaced by a more precise Runge-Kutta method, if necessary.

References