# Systemic Risk, International Regulation, and the Limits of Coordination*

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**Abstract**

This paper examines the incentives of national regulators to coordinate regulatory policies in the presence of systemic risk in global financial markets. In a two-country and three-period model, correlated asset fire-sales by banks generate systemic risk across national financial markets. Relaxing regulatory standards in one country increases both the cost and the severity of crises for both countries in this framework. In the absence of coordination independent regulators choose inefficiently low regulatory standards and systemic banking crises can occur in equilibrium. A central regulator internalizes the systemic risk and improves the welfare of cooperating countries. Also, common central regulation will voluntarily emerge only between sufficiently similar countries.

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# 1 Introduction

The underlying shocks that precipitated the financial crisis of 2007-2009 quickly spread across global financial markets and were amplified at an unprecedented scale. The strikingly global nature of the crisis has revived interest in the international coordination of financial regulation. Regulatory reforms and the strengthening of coordination between national financial regulators are prominent items on the international reform agenda. The Financial Stability Board (FSB), a new international regulatory body, was set up by the G-20 countries during the crisis to create guidelines for regulatory coordination and the supervision of systemic risk in the international financial system. This paper analyzes the incentives of national regulators towards international cooperation when there is systemic risk in global financial markets using a game-theoretic model.

In the model, systemic risk in financial markets is generated through asset fire-sales in the banking system. The model shows that in the absence of cooperation, independent regulators choose inefficiently low regulatory standards compared to regulation levels that would be chosen by a central regulator. A central regulator internalizes systemic risk and improves welfare in cooperating countries. The model also demonstrates that a common central regulation will voluntarily emerge only between sufficiently similar countries.

Key features of a third generation of bank regulation principles popularly known as Basel III, strengthens capital regulations and adds new elements to Basel bank regulation principles such as liquidity and leverage ratio requirements. With Basel III, the objective of regulation aimed creating a level playing field for international bank is supported by an objective of creating sound regulatory practices that will contain systemic risk in national and international financial markets, and prevent pro-cyclical amplification of these risks over time. In this paper, I revisit the issue of coordinating international financial regulation in light of recent developments in the international regulatory infrastructure. Acharya (2003, 2009) and Dell'Aricca and Marquez (2006) are notable studies in the literature on international financial regulation. Both studies focus on the level playing field objective of financial regulation, and examine the benefits to international coordination of financial regulation under externalities that operate through the competition in loan markets.

The paper diverges from the previous literature by focusing on systemic externalities across financial markets generated by fire-sales of assets in the banking system. I examine the effects of systemic externalities on the nature of international financial regulation in the absence of cooperation between regulators, as well as its effects on the incentives of national regulators towards cooperation. The paper also makes a contribution to the literature by studying the effects of structural differences across countries on the choice of regulatory standards when countries are linked through the externalities in international financial markets. I show that a common central regulation will voluntarily emerge only between sufficiently similar countries.

It is commonly observed that in times of distress, financial intermediaries reduce the size of their investment by selling assets. Selling assets is seen as a rational and prudent act from an individual

bank perspective because it allows banks to operate on a smaller scale but with a stronger balance sheet. An individual bank takes the market price as given and decides how much of its assets to sell to continue operating at an optimal scale. In normal times, the assumption that banks can sell assets as required at the current market price is innocuous. However, when numerous banks concurrently face the same type of shocks and sell assets simultaneously, asset prices can fall, forcing banks to sell additional assets. In a financially integrated world, banks from different countries sell assets in a global market to potentially the same set of buyers. Therefore, initial shocks that hit individual countries can be amplified in globally integrated financial markets, and this is the systemic externality considered in this paper.

I seek answers to the following questions: How do national regulators behave under this systemic externality if they act non-cooperatively? Would an individual regulator tighten or relax regulation if regulation is tightened in another country? Would national regulators relinquish their authority to a central international regulator who would impose the same set of regulatory standards across countries? How do asymmetries across countries affect the nature of regulatory standards and the incentives of national regulators towards international cooperation?

Briefly, I propose a three-period, two-good model that features two countries with independent regulators. In each country there is a continuum of banks. Banks are protected by limited liability, and there is deposit insurance in each country. Banks borrow consumption goods from local deposit markets and invest in a productive asset in the first period. The investment which can be retained by banks until the last period produces constant and positive net returns. A continuum of global investors with large resources in the second period can purchase productive assets in the second period to produce consumption goods in the third and final period. Assets in different countries are perfect substitutes for global investors. However, global investors are not as productive as the domestic banks in operating domestic assets and face decreasing returns to scale from these assets.

All uncertainty is resolved in the model at the beginning of the second period and one of the two states of the world is realized: a good or a bad state. In the good state there are no shocks and banks' investment produces net positive returns in the last period. However, in the bad state, banks' investments are distressed and they have to be restructured to produce the normal positive returns that are obtained in good times.

I solve the equilibrium of this model by backwards induction. Following the shocks in the interim period, banks need to sell some fraction of their assets in a global capital market to pay for the restructuring costs. An asset sale in the bad state is unavoidable because other domestic resources required to carry out the restructuring process are unavailable. The price of the productive asset is determined in a competitive market where banks from the two countries and global investors meet.

I show that a higher initial investment by banks in any of these countries will lead to a lower price for the productive asset in this market. If the asset price falls below a minimum threshold, the value of assets which can be retained by banks is lower than the value of the initial investment,

and banks become insolvent. Once it is known that banks are insolvent, the deposit insurance fund requires bank owners to manage the productive assets to realize the returns in the last period, then liquidates bank assets and makes payments promised by banks to depositors. In this case the deposit insurance fund faces a deficit. In the presence of deposit insurance, depositors in the model are passive and they always deposit their money with the bank. If the bank fails, the deposit insurance fund makes the necessary payment to depositors.

Regulation in this model can be interpreted as setting the maximum leverage ratio. Each regulator determines initial regulatory standard by taking into account the equilibrium in the asset market in the interim period. Due to limited liability, banks always leverage up to the maximum by borrowing funds from the local deposit market. In other words, the maximum leverage ratio always binds. Therefore, the initial investment level of banks in a given country is determined completely by the regulatory standard.

In the first period, regulators act simultaneously and choose the regulatory standard for their domestic banks by taking the regulatory standard in the other country as given. I show that when the countries are symmetric, there exists a unique symmetric Nash equilibrium of the game between the two regulators. Moreover, regulation levels in the two countries are strategic substitutes: if one regulator allows tightens the regulatory standard in its jurisdiction, the other regulator optimally loosens its regulatory standard. The intuition behind this result is as follows: when the first country reduces the maximum leverage level (i.e. tightens regulatory standards), the extent of fire-sale of assets in the bad state by banks in that country are reduced, and a higher price is realized for the assets sold by these banks. This increases the expected returns in the bad state, allowing the regulator in the other country to relax regulation levels.

I show that, due to the systemic risk discussed above, regulatory standards in equilibrium when regulators act non-cooperatively will be inefficiently lax compared to regulatory standards that would be chosen by a central regulator. A central regulator aims to maximize the total welfare of the two countries, and internalizes these externalities. I assume that the central regulator has to choose the same regulation levels in both countries due to political reasons. If the two countries are symmetric, I show that forming a regulatory union will increase welfare in both countries. Therefore, it is incentive compatible for the independent regulators of symmetric countries to relinquish their authority to a central regulator.

I also consider the incentives of regulators when there are asymmetries between countries. I focus on the asymmetries in the asset returns. In particular, I assume that banks in one country are uniformly more productive than the banks in the other country in terms of managing the long-term assets. I show that cooperation would voluntarily emerge only between sufficiently similar countries. In particular, the regulator in the high return country chooses lower regulatory standards in equilibrium, and is less willing to compromise on stricter regulatory standards.

Interest in the international coordination of financial regulation is not an entirely recent phe-

nomenon. Arguments in favor of coordination and harmonization of regulatory policies across countries were made in the 1988 Basel Accord (Basel I) which focused on credit risk and set minimum capital requirements for internationally active banks and was enforced in the G-10 countries in 1992.[1]

However, Basel I did not create an entirely level playing field for internationally active banks as countries maintained a significant degree of discretion about different dimensions of regulation. Furthermore rapid developments in financial markets, especially increasingly complex financial products brought about by financial innovation created significant differences about the stringency of capital regulations across countries in practice (Barth et al., 2008). These developments created a challenge for regulators and paved the way for Basel II.[2]

While actual progress on the implementation of Basel II was slower than expected, the global financial crisis led to a renewed urgency in increased cooperation and the better regulation of international financial markets as insufficient policy coordination between countries and deficiencies in Basel II regulatory mechanisms were blamed for the severe contagion of the crisis. However, international regulatory mechanisms proposed prior to the crisis emphasized the soundness of financial institutions individually (micro-prudential regulation), but mostly neglected regulatory standards that could enhance the stability of the financial system as a whole by taking into account systemic risks (macro-prudential regulation). The model in this paper focuses on the latter (macro-prudential regulation) in the context of regulating systemic risk in the international banking system.

The paper proceeds as follows. Section 2 is a brief summary of the related literature. Section 3 provides the basics of the model and presents the main results of the paper without resorting to a particular functional form. International financial regulation between asymmetric countries is considered in Section 4. In section 5, I investigate systemic failures for the symmetric countries case. Conclusions are presented in Section 6. The Appendix contains a closed form solution for the symmetric model for a particular functional form in order to aid intuition. All proofs are also provided in the appendix.

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[1]Basel I's intent was to strengthen the soundness and stability of the international banking system, and diminish competitive inequality among international banks by creating a level playing field (Basel 1988).

[2]Basel I was updated in 2004 with more sophisticated sets of rules and principles for capital regulation that intended to accommodate the developments in global financial markets. A survey done by Bank for International Settlements (BIS) in 2006 showed that 95 countries (comprising 13 BSBC member countries plus 82 non-BSBC jurisdictions) were planning to implement Basel II by 2005.

symmetric model for a particular functional form in order to aid intuition. All proofs are also provided in the appendix.

## 2 Literature Review

This paper belongs to the international financial regulation theory, which was being developed during the past decades. This paper is closest to Acharya (2003), Dell'Ariccia and Marquez (2006) and Acharya (2009). Dell'Ariccia and Marquez (2006) investigates the incentives of national regulators to form a regulatory union in a two-country banking model, where a single bank from each country competes for loans in both markets in a Bertrand differentiated products setup. Therefore, if one of the banks is allowed to expand its balance sheet, low average returns to bank loans will be realized in both markets. Banks in their model are also endowed with a costly monitoring technology. Low average returns reduces incentives of banks for monitoring, and hence undermines the stability of banks. They show that, under this externality, independent national regulators will implement lower capital requirements compared to capital requirements that would be implemented by a central regulator. They also show that symmetric countries definitely gain from cooperation, whereas a cooperation voluntarily emerges only between sufficiently similar asymmetric countries.

Although my paper is similar Dell'Ariccia and Marquez (2006), we have the following differences: Instead of externalities that arise through loan markets, I consider externalities between national financial markets that arise through asset markets and asset prices. This externality channel between countries is novel to the literature on international financial regulation. Moreover, in Dell'Ariccia and Marquez (2006) the asymmetry between countries is due to differences in regulator's tastes and preferences, whereas I focus on asymmetries due to structural differences across countries, such as the differences in productivity rates.

Acharya (2003) shows that convergence in international capital adequacy standards will not be effective unless it is accompanied by convergence in other aspects of banking regulation, such as closure policies. Externalities in his model are in the form of cost of investment in the risky asset. He assumes that the bank in one country increases costs of investment for itself and for the bank in the other country as it invest more in the risky asset, thereby creating externalities for the bank in the neighboring country.

Acharya (2009) is very similar to Acharya (2003). There are two sources of externalities between banks, one is positive and one is negative. Negative externality is that only a fraction of the failing bank's depositors migrates to the surviving bank. This reduces the total amount invested in the safe asset. He interprets the safe asset as capital goods leased to the corporate sector. Competition in capital markets implies that the rate of return on the safe asset is equal to the marginal product of capital. Therefore a lower investment in the safe asset due to the failure of one of the banks, increases the cost of capital for the surviving bank. The positive externality on the other hand, is a reduction in the cost of investment in the risky asset for the surviving bank. This captures the

fact that surviving bank might have strategic benefits such as obtaining human capital and trading desks of the failing bank. Acharya shows that if the negative externality dominates the positive externality, than there will be a collective/systemic risk shifting across banks in different regions. In other words, banks in different regions will choose their investments to be highly correlated compared to globally optimal correlation levels.

This paper differs from the papers above in terms of the sources of the externalities between the national financial markets. In this paper, the externalities between countries arise due to correlated asset choices and asset fire-sales in the times of distress. Moreover, papers by Acharya do not consider the possibility of a regulatory union between countries and its welfare effects.

There is also a branch of this literature that considers regulation of a multi-national bank that operates across two countries. Two notable studies are Dalen and Olsen (2003) and Holthausen and Rønde (2004). They focus on the tension between home and host country regulation of the multi-national bank and informational asymmetries are the driving force of the regulatory competition. My paper differs from these studies by focusing on a model where banks invest only in a single country and hence are regulated only by their home country, but interact with each other in global asset markets. The tension between regulators in my model arises from the externalities that banks in different countries create for each other in global asset markets during the times of distress.

This study can also be viewed as a part of the broader literature on macroeconomic policy coordination which was quite active especially from late 1970s through 1990s. Hamada (1974, 1976) are the pioneer studies in application of the game theoretic approach to study strategic interactions among national governments. Cooper (1985) and Persson and Tabellini (1995) provide extensive reviews of this literature.

Lastly, this paper is also related to the literature that features asset fire-sales. The common theme across these studies is that under certain conditions asset prices can move away from the fundamental values and assets can be traded in markets at fire-sale prices. Origins of this idea can be found in Williamson (1988) and Shleifer and Vishny (1992). These studies claim that fire-sales are unavoidable when major players in an industry face correlated shocks. In such a scenario a firm needs to sell assets to restructure and continue operations at a smaller scale; however it cannot sell its assets at the full value because other firms in the same industry are experiencing similar problems. Outside investors would buy and manage assets but they are not as sophisticated as the firms in the industry. Therefore, they would be willing to pay than full value of the assets to the distressed firms. Moreover, unsophisticated investors may face decreasing returns in the amount of assets they employ. This makes the situation even worse for distressed firms because if many of them try to sell assets to outside investors simultaneously, they have to accept higher discounts.

The reason for fire-sales takes different forms among different papers. In Allen and Gale (1994, 1998) and Acharya and Yorulmazer (2008), the reason for fire-sales is the limited amount of cash in the market that is available to buy long-term assets that are offered for sale by agents who

immediately need liquid resources. The scarcity of liquid resources leads to necessary discounts in asset prices. Therefore, this phenomenon is called the "cash-in-the-market pricing".

The closest papers from the fire-sales literature to mine are Lorenzoni (2008) and Gai et al. (2008). These two papers are essentially addressing the same question: how private banking contracts compare to the socially optimal banking contracts in the face of pecuniary externalities in financial markets generated through asset fire-sales? The reason for fire-sales in these studies is limited commitment on financial contracts and the fact that asset prices are determined in a spot market. They consider a single country, three period model with a continuum of banks. Banks borrow from consumers and offer them state contingent contracts. In the interim period banks are hit by shocks and need to sell assets in some states to restructure the distressed investment. These papers show that there is over-borrowing and hence over-investment in the risky asset in a competitive setting compared to the socially optimal solution. This result is obtained because banks do not internalize the externalities they create through fire sales. In other words, each bank treats the market price of assets as given when it makes borrowing and investment decisions in the initial period. On the other hand, the planner considers the fact that a higher investment will translate into lower prices for capital sold by banks during the times of distress. The main difference between my paper and these papers is that they focus on issues in a single country case, and therefore issues related to the international dimension of regulation are not considered by these studies.

# 3 Model

There are three periods, $t = 0, 1, 2$; and two countries, $i = A, B$. In each country there is a continuum of banks with a unit mass, and a financial regulator. One may interpret the regulator as a central bank. There is also a unit mass of global investors. All agents are risk neutral.

There are two goods in this economy: a consumption good and a capital good, the liquid and illiquid assets, respectively. The supply of consumption goods in each country at $t = 0$ is perfectly elastic, and at $t = 1, 2$ there is no supply of consumption goods.

Banks have a technology that converts consumption goods into capital goods one-to-one at $t = 0$ where these capital goods yield $R > 1$ consumption goods per unit at $t = 2$. In other words, at $t = 0$ banks in country $i$ choose the level of investment $n_i$ in the capital good. In period 2, this investment yields $Rn_i$ units of consumption goods. Consumption good is perishable, and the capital fully depreciates at $t = 2$. Capital good may never be converted into consumption goods.

Banks will borrow the funds necessary for the investment from the local deposit market. I assume that there is a deposit insurance fund operated by the regulator in each country. Therefore banks will be able to borrow at a constant and zero interest rate. I also assume that banks are protected by limited liability.[3]

[3]This assumption is imposed to match the reality and to simplify the analysis of the model. All results carry on to the case where there is no limited liability for banks.

Banks are subject to regulation in the form of maximum investment levels. Regulatory standards are set by the individual national regulators at the beginning of $t = 0$ independently. Regulator of country $i$ determines the maximum investment allowed for banks in its jurisdiction, $N_i$, while taking into account the regulation in the other country, $N_j$, as given. Regulatory standard in a country is chosen to maximize the expected domestic social welfare. Investment levels of banks in country $i$, have to satisfy $n_i \leq N_i$.

Banks may face shocks at the interim period that prevent them from realizing the full returns from this investment. All uncertainty in the model is resolved at the beginning of $t = 1$: a country lands into good times with probability $q$, and into bad times with probability $1 - q$. In order to simplify the analysis, I will also assume that shocks are perfectly correlated across countries. In good times no banks are hit with shocks, therefore no further actions is taken. Banks keep managing their capital goods, and they realize the full returns from their investment in the last period. However, in bad times, all projects of all banks in a given country are distressed. In case of distress, the investment has to be restructured in order to remain productive. Restructuring costs are equal to $c \leq 1$ units of consumption goods per unit of capital. If $c$ is not paid, capital is scrapped, i.e. it fully depreciates.

The only way for banks to restructure their investment is to sell some fraction of the investment to global investors in exchange of consumption goods. The capital sales of banks will carry the features of a fire-sale: the capital good will be traded below its fundamental value and the price will decrease as banks try to sell more capital. Banks in each country will be able to retain only a fraction of their assets after fire-sales. If the asset price falls below a threshold, value of assets which can be retained by banks will be lower than the value of the initial investment, and hence banks will become insolvent. I call this situation a "systemic crisis". Once it is known that banks are insolvent, deposit insurance fund requires the bank owners to manage the productive assets to realize the returns in the last period, and then liquidates banks and makes the payments promised by banks to depositors. In this case the deposit insurance fund faces a deficit. It might also be the case that after fire-sales banks still have enough assets to make the promised payments to the depositors, so that they continue to be solvent even though they make smaller profits compared to good times. The sequence of events are illustrated in the time line graph below.

### 3.1 Global Investors

Global investors are endowed with unlimited resources of consumption goods at $t = 1$.[4] They can purchase capital, $y$, from banks in each country at $t = 1$ and employ this capital to produce $F(y)$ units of consumption goods at $t = 2$. For global investors capital supplied by the banks in these

[4]The assumption that there are some global investors with unlimited resources at the interim period when no one else has, can be justified with reference to the empirical facts usually observed during financial crises. When countries are hit by shocks and their assets are distressed, there are still some investors with large liquid resources, e.g. Warren Buffet, who can buy assets from troubled institutions.

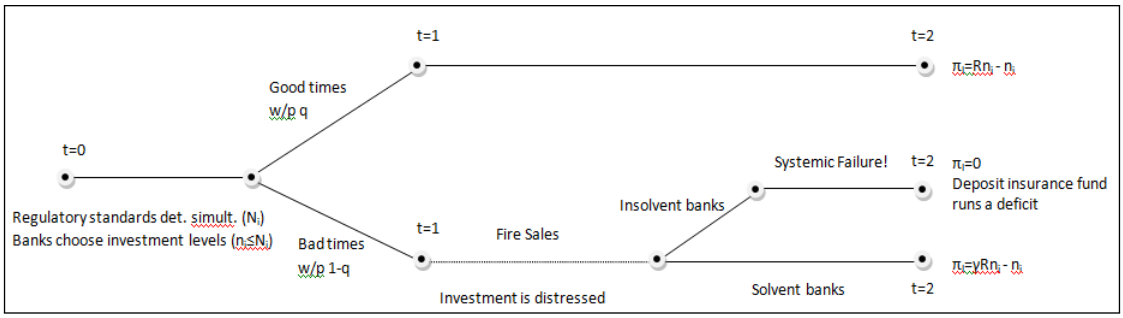



Figure 1: Timing of the Model

two countries are identical. Let $P$ denote the market price of the capital good at $t = 1$. [5] Since we have a continuum of global investors, each investor will treat the market price as given, and he will choose the amount of capital to purchase, $y$, to maximize net returns from investment at $t = 2$

$$\max_{y \geq 0} F(y) - Py \tag{1}$$

The amount of assets they optimally buy satisfies the following first order conditions

$$F'(y) = P \tag{2}$$

The FOC above determines global investors' (inverse) demand function for assets. Using this, we can define their demand function $D(P)$ as follows

$$y = F'(P)^{-1} \equiv D(P) \tag{3}$$

We need to impose some more structure on the return function of global investors and the model parameters in order to ensure that the equilibrium of this model is well-behaved.

### 3.2 Basic Assumptions

**Assumption 1** (CONC)**.**

$$F'(y) > 0 \text{ and } F''(y) < 0 \text{ for all } y \geq 0, \text{ with } F'(0) \leq R.$$

Assumption *CONC* says that global investors' return is strictly increasing the amount of capital employed, $F'(y) > 0$, however they face decreasing returns to scale in the production of consumption goods, $F''(y) < 0$, as opposed to banks which are endowed with a constant returns to scale technology as describe above. Concavity of the return function will imply that the demand func-

[5] Price of capital at $t = 0$ will be one as long as there is positive investment, and the price of capital at $t = 2$ will be zero since capital fully depreciates at this point.

tion of global investors for capital goods is downward slopping. Global investors will require higher discounts to absorb more capital that from distressed banks at $t = 1$ (see Figure 2).

Furthermore, $F'(0) \leq R$, implies that global investors are less productive than banks at each level of capital employed. This assumption intends to capture that foreign investors who buy and manage domestic assets in a country during fire-sales can be less productive than early domestic owners of these assets. There could be different reasons for this. Global investors could have less know-how in that particular industry. They may also not know as much about the institutional structure of the countries from which they buy assets.

**Assumption 2** (ELAST)**.**

$$\epsilon_{P,y} = -\frac{\partial y}{\partial P}\frac{P}{y} = -\frac{F'(y)}{yF''(y)} > 1 \quad \textit{for all } y \geq 0$$

Assumption *ELAST* says that the global investors' demand for the capital good is elastic. This assumption implies that the amount spent by global investors on asset purchases, $Py = F'(y)y$, is strictly increasing in $y$. Therefore we can write Assumption *ELAST* also as

$$F'(y) + yF''(y) > 0$$

If this assumption was violated, there would be multiple levels of asset sales that would raise a given amount of liquidity, and hence multiple equilibria in the asset market at $t = 1$ would be possible. This assumption is imposed by Lorenzoni (2008) and Korinek (2011) as well in order to rule out multiple equilibria under fire-sales.[6]

**Assumption 3** (REG)**.**

$$F'(y)F'''(y) - 2F''(y)^2 \leq 0 \quad \textit{for all } y \geq 0$$

Assumption *REG* holds whenever the demand function of global investors is log-concave.[7] In order to see this, let $\phi(y) \equiv F'(y)$ denote the (inverse) demand function of global investors. We can rewrite Assumption *REG* as

$$\phi(y)\phi''(y) - 2\phi'(y)^2 \leq 0.$$

Note that the demand function will be log-concave if and only if $\phi(y)\phi''(y) - \phi'(y)^2 \leq 0$. Log-concavity of demand function is a common assumption used in the Cournot games literature (see Amir (1996)), and it ensures the existence and uniqueness of equilibrium in a simple *n-player*

[6] Gai et al. (2008) is the leading example where this assumption is not imposed and hence multiple equilibria in the asset market is considered.

[7] A function is said to be log-concave if the logarithm of the function is concave.

Cournot game. Therefore, I call it a "regularity" assumption on $F(\cdot)$. Clearly Assumption *REG* will hold whenever the demand function is log-concave. However, Assumption *REG* is weaker than log-concavity, and it may also hold if the demand function is log-convex, i.e. if $\phi(y)\phi''(y) - \phi'(y)^2 \geq 0$.

Assumption *REG* will ensure that the objective functions of regulators are well-behaved and it will be crucial in showing that the equilibrium of this model exists and it is unique.

Many regular return functions satisfy conditions given by Assumptions *CONC*, *ELAST* and *REG*. Here are two examples that satify all three assumptions above:[8]

**Example 1** $F(y) = R\ln(1+y)$

**Example 2** $F(y) = \sqrt{y + (1/2R)^2}$

The following example satisfies Assumption *CONC*, but not Assumptions *ELAST* and *REG*.

**Example 3** $F(y) = y(R - 2\alpha y)$ where $2\alpha y < R$ for all $y \geq 0$.

**Assumption 4** (RANGE)**.**

$$1 + (1-q)c < R \leq 1/q$$

Assumption *RANGE* says that the return on investment for banks must not be too low because otherwise equilibrium investment levels will be zero, and it must not be too high, because otherwise equilibrium investment levels will be infinite. This assumption allows us to focus on the interesting cases where equilibrium investment levels are neither zero nor infinite.

### 3.3 Equilibrium with Symmetric Countries

In this section, I consider only symmetric countries. I will solve the model by backwards induction: first, I will analyze the equilibrium at the interim period in each state of the world, for a given set of investment levels; then I will proceed to solve for the game between the regulators at $t = 0$. Note that, if good times are realized $t = 0$, no further actions need to be taken by agents of our model. Therefore, we just need to analyze the equilibrium of the model for bad times at at $t = 1$.

I solve the model without resorting to some particular functional form. The results of this paper hold for any functional form and parameter set that satisfy Assumptions *CONC*, *ELAST*, *REG* and *RANGE*.

#### 3.3.1 Crisis and Fire-Sales

Consider the problem of a bank in country $i$ if bad times are realized at $t = 1$. A bank reaches to $t = 1$ with a level of investment equal to $n_i$ which was chosen at the initial period. At the beginning of $t = 1$, a bank in a crisis country $i$ decides what fraction of capital to restructure ($\chi_i$), and what

[8] Please see the appendix for derivations.

fraction of restructured capital to sell $(1-\gamma_i)$ to pay restructuring costs. Note that $\gamma_i$ will then be representing the fraction of capital bank keep after fire-sales.[9] Hence it chooses $\chi_i$ and $\gamma_i$ to maximize total returns from that point on

$$\max_{0\leq\chi_i,\gamma_i\leq 1} \pi_i = R\gamma_i\chi_i n_i + P(1-\gamma_i)\chi_i n_i - c\chi_i n_i \tag{4}$$

subject to the budget constraint

$$P(1-\gamma_i)\chi_i n_i - c\chi_i n_i \geq 0 \tag{5}$$

The first term in (4) is the total return that will be obtained from the unsold part of the restructured assets, $\chi_i n_i$. The second term is the revenue raised by selling a fraction $(1-\gamma_i)$ of the restructured assets, $\chi_i n_i$, at the given market price $P$. The last term, $c\chi_i n_i$, gives the total cost of restructuring. Budget constraint (5) says that revenues raised by selling capital must be greater than or equal to the restructuring costs.

By Assumption *CONC*, the equilibrium price of capital must satisfy $P \leq F'(0) \leq R$, otherwise global investors will not purchase any capital. Later on, we will show that in equilibrium we must also have $c < P$. Therefore for the moment we will assume that equilibrium price of assets satisfy

$$c < P \leq R \tag{6}$$

Let's consider the first order conditions of the maximization problem (1) while ignoring the constraints for the moment

$$\frac{\partial \pi_i}{\partial \chi_i} = [R\gamma_i + P(1-\gamma_i) - c]n_i \tag{7}$$

$$\frac{\partial \pi_i}{\partial \gamma_i} = (R-P)\chi_i n_i \tag{8}$$

From the latter of the FOCs it is obvious that $P \leq R$ implies that $\pi_i$ is increasing in $\gamma_i$. Since we indeed have $P \leq R$ from the discussion above, we know that budget constraint (5) will bind, and we will obtain that the fraction of assets retained by banks after fire-sales is

$$\gamma_i = 1 - \frac{c}{P} \tag{9}$$

[9] Following Lorenzoni (2008) and Gai et al. (2008), I assume that banks have to restructure an asset before selling it. Basically, this means that bank receive the asset price $P$ from global investors, use a part, $c$, to restructure the asset, and then they deliver the restructured assets to global investors. Therefore banks will sell assets only if $P$ is greater than the restructuring cost, $c$. We could instead assume that it is the responsibility of global investors to restructure the assets that they purchase without qualitatively changing our results. However, the model is more easily solved under the current story.

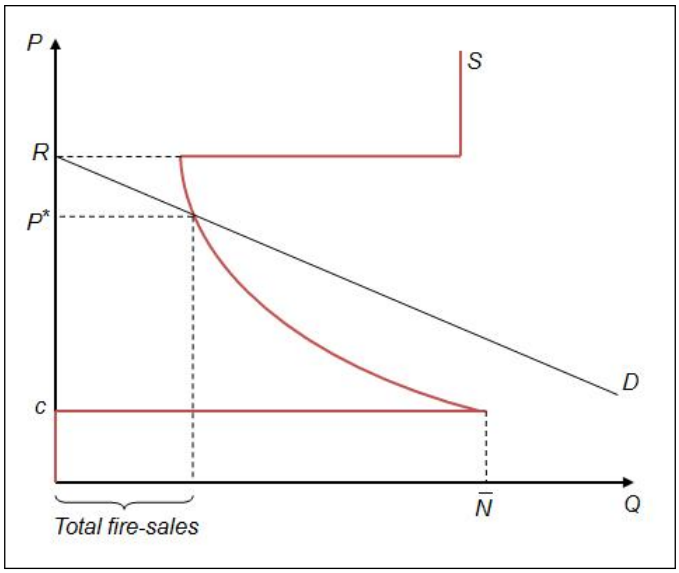



Figure 2: Equilibrium in the Capital Goods Market

Using (9) we can write the first of the FOCs as

$$\frac{\partial \pi_i}{\partial \chi_i} = R\gamma_i n_i \geq 0 \tag{10}$$

From (10) we see that revenues are increasing in $\chi_i$ at $t = 1$. Therefore, banks will optimally choose $\chi_i = 1$. In other words, banks will optimally choose to restructure the full fraction of the investment. Therefore, scrapping of capital will never arise equilibrium.

From (9) we can also obtain the total capital supply of a bank in country $i$ as

$$S_i(P, n_i) = (1 - \gamma_i)n_i = \frac{cn_i}{P} \tag{11}$$

for $c < P \leq R$ which is downward slopping and convex. A downward slopping supply curve is standard in the fire-sales literature. A negative slope implies that if there is a decrease in the price of assets banks have to sell more assets in order to pay for the restructuring costs. During a fire-sales, banks are selling a valuable investment, that they would not otherwise sell, due to an exogenous pressure (e.g. paying for the restructuring costs here).

Note that if the asset price is greater than $R$, banks want to sell all of the assets they have, because they can get at most $R$ per unit by keeping and managing these assets. On the other hand, if the asset price is lower than $c$, then they will optimally scrap all of their capital ($\chi_i = 0$). However, prices above $R$ and below $c$ will never arise in equilibrium as discussed above. The total asset supply curve of banks from the two countries is plotted in Figure 2 for an initial total investment in the two countries of $N_0$.

### 3.3.2 Equilibrium in the Capital Market at t=1

Equilibrium price of assets, $P^*$, will be determined by the market clearing condition

$$E(P^*, n_A, n_B) = D(P^*) - S(P^*, n_A, n_B) = 0 \tag{12}$$

The condition above says that the excess demand in the capital market, denoted by $E(P, n_A, n_B)$, is equal to zero at the equilibrium price. $D(P)$ in (12) is the demand function of global investors which was obtained from the FOCs of global investors' problem as shown by (3). Lastly, $S(P, n_A, n_B)$ is the total supply of projects given by

$$S(P, n_A, n_B) = \frac{c(n_A + n_B)}{P} \tag{13}$$

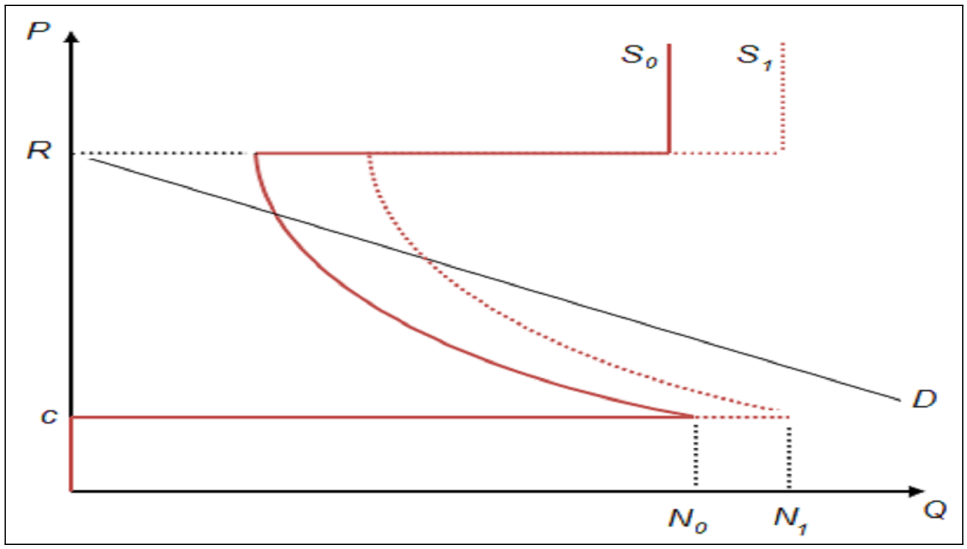



Figure 3: Capital Goods Market: Comparative Statics

This equilibrium is illustrated in Figure 3. Note that the equilibrium price of assets will be a function of the initial investment level in the two countries. Therefore, from the perspective of the initial period we will denote the equilibrium price as $P^*(n_A, n_B)$.

How does how a change in the initial investment level in one of the countries affect the price of capital at $t = 1$? Lemma 1 shows that if investment into the risky asset in one country increases at $t = 0$, a lower price for capital will be realized in the fire-sales state at $t = 1$.

**Lemma 1.** *$P^*(n_i, n_j)$ is decreasing in $n_i$ for $i = A, B$ under Assumptions CONC and ELAST.*

*Proof.* See the appendix. □

Lemma 1 implies that higher investment in the risky asset in one country (a higher $n_i$) increases the severity of the financial crisis for both countries by lowering the asset prices. This effect is

illustrated in Figure (3). Suppose that initial investment level in country $A$ increases, increasing the total investment in the two countries from $N_0$ to $N_1$. In this case, banks in country A will have to sell more assets at each price, as can be seen from individual supply function given by (11). Graphically, total supply curve will shift to the right, as shown by the dotted-line supply curve, causing a decrease in the equilibrium price of assets.

Next lemma shows that higher initial investment in the risky asset also causes more fire-sales at $1 = 1$ if the bad state is realized.

**Lemma 2.** *Equilibrium fraction of assets sold in each country,* $1 - \gamma^*(n_i, n_j)$*, is increasing in* $n_i$ *for* $i = A, B$ *under Assumptions CONC and ELAST.*

*Proof.* See the appendix. □

Suppose that initial investment in the risky asset in country $i$, increases at $t = 0$. By Lemma 1 we know that the equilibrium price of capital will be lower as a result. Lower asset prices induce more fire-sales by banks due to their downward slopping supply curve. Lemma 2 implies that increase in the risky investment in one country increases the cost of the financial crisis for both countries by leading to more fire-sales in the bad state. Therefore, we conclude that a higher investment in the risky investment in one country reduces the (ex-ante) welfare of the other country by making financial crises more severe (lower asset prices by Lemma 1) and more costly (more fire-sales by Lemma 2).

### 3.3.3 Banks' Problem

Banks borrow from the local deposit market, and investment in the risky asset at $t = 0$. Investment level of a bank in country $i$ has to satisfy $n_i \leq N_i$, where $N_i$ is the maximum investment level determined by the regulator of country $i$. In other words, banks choose how much to borrow from deposit markets and invest in the productive assets at $t = 0$ while taking the price in the capital market at $t = 1$ as given. There is a deposit insurance in each country and banks are protected by limited liability. Therefore, banks can borrow consumption goods from the local deposit market at zero interest rate at $t = 0$. Moreover, to simplify the analysis, I assume that banks have no initial endowments at $t = 0$. [10]

Each bank in country $i$ at $t = 0$ will choose the investment level, $n_i$, to maximize expected profits given by

$$\max_{0 \leq n_i \leq N_i} \Pi_i(n_i) = \; q(R-1)n_i + (1-q)\, \max\left[R\gamma n_i - n_i, 0\right] \tag{14}$$

where $\gamma$ is the equilibrium fraction of rescued assets by a bank in country $i$ which is equal to $1 - c/P$ as shown by (9).

[10] This assumption is imposed to simplify the analysis. All results can be extended to the case where banks have some initial amount of costly capital which they have to invest in the risky asset. In this case, regulation can be defined as a capital adequacy requirement.

Let's look at banks' profit function in close detail. A bank borrows from the local deposit market at a constant zero interest and invests in the productive asset. With probability $q$ both countries are in good times, and hence the investment produces returns as expected $Rn_i$. Banks make the promised payments to depositors, $n_i$, leading to a net profit of $(R-1)n_i$. With probability $1-q$ both countries are in the bad state. Banks from the two countries face restructuring costs, and hence are forced to fire-sale their capital as discussed above. Since banks are price-takers in the capital market, the fraction of capital that they can save from the fire-sales, $\gamma = 1 - c/P$, will be exogenous to them. In other words, since each bank is small compared to market size, it will not take into account the effect of its investment choice at $t = 0$ on the equilibrium price $P$ and hence on the fraction of assets saved in equilibrium $\gamma$ (which is a function of $P$ as can be seen from (9)). This implies that each bank takes $\gamma$ as given when choosing $n_i$ at $t = 0$. This is also why the investment level of banks in country $j$ does not enter the objective function of banks in country $i$. Note also that in the current setup the only externality between the two financial markets is through the fire-sale market at $t = 1$.

Bank earn net positive returns for sure in the good state since $q(R-1) > 0$. Moreover, they never recieve negative profits in the bad state due to limited liability. Moreover, since banks do not internalize the effect of their investment on the stability of the financial system at $t = 1$, there is no effect to offset the positive returns from leveraging. Therefore, a bank's net expected return from risky investment at $t = 0$ will always be positive, and hence a bank will always be better off by leveraging. Hence, banks' optimal investment decision will always imply a maximum leverage, i.e. choosing $n_i$, the initial investment level, as high as possible. Therefore, whenever there exists a maximum investment requirement, it will bind. In other words, banks will always choose to leverage up to the maximum amount allowed by regulation.

If banks' projects are distressed at $t = 1$, they might receive zero profits, but even if this is the case (i.e. if they are insolvent), I assume banks are required by the law to perform the fire-sales and keep managing remaining projects until $t = 2$, and transfer the returns from assets to the deposit insurance fund. I also assume banks required to manage capital goods until the last period because there are no other sophisticated agents in our model domestic economy that could manage those assets. Furthermore, in practice, the dissolution process of insolvent banks usually does not happen immediately. It is a time consuming process because for example loans have to be called-off or sold to third parties to make payments to debtors. This assumption captures this time dimension of the dissolution process as well.

### 3.3.4 Regulators' Problem

Regulators of the two countries act first in the initial period, and they simultaneously determine the regulatory standards for the banks in their own jurisdiction. Regulation in each country $i = A, B$ takes the form of a maximum investment level allowed for domestic banks, $N_i \geq 0$. Banks in

country $i$ have to abide by the regulation by setting their investment levels as $n_i \leq N_i$. Regulators anticipate banks' reaction when they choose the regulatory standards. From banks' problem above, regulators know that, banks will always choose initial investment level as high as possible, i.e. that banks will set $n_i = N_i$.

The objective of a national financial regulator is to maximize the net expected social welfare of its own country. Social welfare is defined as the bank profits plus the cost of running the deposit insurance fund. Therefore, regulator $i$ chooses $N_i \geq 0$, to solve

$$\max_{N_i \geq 0} W_i(N_i, N_j) = \max_{N_i \geq 0} q(R-1)N_i + (1-q)[R\gamma^*(N_i, N_j)N_i - N_i] \tag{15}$$

while taking the regulatory standard in the other country, $N_j$, as given. Let $(\widehat{N}_A, \widehat{N}_B)$ denote the Subgame Perfect Nash Equilibrium (SPNE) of the game between the regulators at $t = 0$ whenever it exists.

Social welfare in (15) incorporates the fact that banks investment level, $n_i$, equals $N_i$, the regulatory standard, and it is the sum of two terms: with probability $q$, the good state will be realized in which the banks in country $i$ will obtain a net total return of $(R-1)N_i$. On the other hand, with probability $1-q$ both countries will lend into the bad state. Each regulator takes into account the effect of regulatory standards in both countries on the price of capital in the bad state. This is why $\gamma^*(N_i, N_j) = 1 - c/P^*(N_i, N_j)$, the fraction of assets that banks can keep after the fire-sales, is written as a function of regulatory standards in the two countries. In the bad state, banks will perform asset sales as described previously, and manage the remaining assets, $\gamma^*(N_i, N_j)N_i$ until $t = 2$ to obtain a gross return of $R$ per unit. Therefore net social returns in the bad state in country $i$ will be given by $R\gamma^*(N_i, N_j)N_i - N_i$. This term will be negative when banks are insolvent, reflecting the potential costs of providing the deposit insurance.

The following equation gives the first order conditions of regulator $i$'s problem in (15)

$$\frac{\partial W_i(N_i, N_j)}{\partial N_i} = qR + (1-q)R\left\{\frac{\partial \gamma^*(N_i, N_j)}{\partial N_i}N_i + \gamma^*(N_i, N_j)\right\} - 1 \tag{16}$$

We can write the FOC (16) as a sum of the marginal benefit and marginal cost of increasing $N_i$, the maximum investment level:

$$\frac{\partial W_i(N_i, N_j)}{\partial N_i} = \{qR + (1-q)R\gamma^*(N_i, N_j)\} + \left\{(1-q)R\frac{\partial \gamma^*(N_i, N_j)}{\partial N_i}N_i - 1\right\} \tag{17}$$

The first curvy brackets gives the expected gross marginal benefit from increasing $N_i$: with probability $q$ the good state will be realized, and a gross return of $qR$ will be obtained from the additional unit of investment. With probability $1-q$, the bad state will be realized, and a gross return of $R\gamma^*(N_i, N_j)$ will be obtained from the investment. Note that in the bad state, only a fraction of the full return can be obtained, and this fraction is a function of the investment level in the two

countries.

The second curvy brackets in (17) gives the expected marginal cost of increasing $N_i$ for regulator $i$: with probability $1-q$ the bad state will be realized, and an additional unit of investment will decrease the fraction of capital that can be retained by banks of country $i$ by $\partial\gamma^*(N_i, N_j)N_i$, causing a total loss of $(\partial\gamma^*(N_i, N_j)/N_i)N_i$. Lastly, "-1" in the second curvy bracket gives the marginal cost of funds required for the risky investment.

**An alternative formulation of regulators' objective function**

We can also write regulators' objective function in a way where returns to investments and costs of fire sales are explicitly given. In order to obtain an alternative expression for (15), we can start by substituting for $\gamma^*(\cdot)$ function using the derivation obtained in (9) to write it as

$$W_i(N_i, N_j) = q(R-1)N_i + (1-q)R\left\{\left(1 - \frac{c}{P^*(N_i, N_j)}\right)N_i - N_i\right\}$$

Rearrange the terms above, and then just add and subtract $cN_i$ terms to get

$$W_i(N_i, N_j) = q(R-1)N_i + (1-q)R\left\{(R-1)N_i - \frac{Rc}{P^*(N_i, N_j)}N_i + cN_i - cNi\right\}$$

Lastly, rearranging the terms inside the curvy brackets gives

$$W_i(N_i, N_j) = q(R-1)N_i + (1-q)\left\{(R-1)N_i - (R - P^*(N_i, N_j))\frac{cN_i}{P^*(N_i, N_j)} - cN_i\right\} \tag{18}$$

Consider this alternative objective function in detail. It is composed of two main terms as in (15): net expected returns in the good state and in the bad state. The first term, $q(R-1)N_i$, that gives the net expected return in the good state is clear, hence I focus on the latter. The first term inside the curvy brackets gives the net total return that could be obtained from the investment if there were no fire-sales. The second term is the cost of fire-sales: $cN_i \backslash P^*(N_i, N_j)$ is the amount of assets sold in fire-sales as given by (11), where banks receive $P^* < R$ from these assets instead of $R$. The last term inside the curvy brackets, $cN_i$, is the total cost of restructuring.

Since the two versions of the regulators' objective function are the same, I will use the first formulation in the rest of the paper for the sake of analytical convenience, even though the alternative formulation could be more intuitive.

**Analyzing the equilibrium with Symmetric Countries at t=0**

Having analyzed the problem of regulators, we can investigate the equilibrium of the game between regulators at $t = 0$. We would like to show that there exists a unique symmetric equilibrium of this game, and then perform comparative statics. I start by analyzing the properties of the best

response functions of regulators. The following proposition establishes that each regulator has a unique best response to each regulatory standard choice by the opponent country.

**Lemma 3.** *Under Assumptions CONC, ELAST and REG, each regulator's best response is unique valued.*

*Proof.* See the appendix. □

An interesting question in this setup concerns how that unique best response behaves as regulation in the opponent country changes. Suppose that the regulator of country B decides to tighten regulation, i.e. reduce $N_B$. How would the regulator of country A would react optimally? The next proposition shows that regulator A optimally chooses less regulation, i.e. a higher $N_A$, as regulator B imposes stricter regulations. In other words, *optimal investment levels in two countries are strategic substitutes.*

**Proposition 1.** *Under Assumptions CONC, ELAST and REG, optimal regulatory standards in the two countries are strategic substitutes.*

*Proof.* See the appendix. □

The intuition for this result is as follows. If regulator B reduces maximum investment level for its banks, there will be less distressed assets at $t = 1$ in the bad state, and hence a higher asset price, $P^*$, will be realized. Therefore, less assets will be sold in equilibrium as shown by Lemma 2, which means that banks in both countries will be able to retain a higher fraction of their initial investment at $t = 1$. This will increase the marginal return to investment in the bad state, and hence allow regulator A to optimally choose higher maximum investment level initially.

Next proposition shows that in order to have finite and strictly positive equilibrium investment level in the two countries, banks' return from investment $R$ should not be too low, or too high. The exact condition on $R$ is given by Assumption *RANGE*, which states that $1 + c(1 - q) < R \leq 1/q$.

**Lemma 4.** *The best responses of each regulator satisfy* $0 < N_i^* < \infty$ *for* $i = A, B$ *if Assumption RANGE holds, i.e. if* $1 + (1 - q)c < R \leq 1/q$.

*Proof.* See the appendix. □

Note that 1+(1-q)c is the net expected cost of the investment: each unit of investment requires one unit of consumption good initially, and with probability $1 - q$ bad times are realized in which case banks have to incur an extra restructuring cost of $c$ units of consumption good per unit of investment. If the return on the investment, $R$, is less then this expected cost, $1+(1-q)c$, countries will be better of by not investing at all. Therefore, if $R < 1 + (1 - q)c$ then equilibrium investment levels are zero in both countries. On the other hand if $R > 1/q$ then the expected return to the investment in the good state alone will be higher than initial cost of investment, which is 1. In

this case, even all the investment is expected to be lost in the bad state, net expected return to the risky investment will always be positive if sufficiently large resources are invested. In this case, we will obtain a corner where social welfare will be maximized by having infinite investment in the risky asset. Therefore for finite and strictly positive investment levels in equilibrium returns to investment must not be either too low or too high. In particular $R$ must be within the range given by Assumption *RANGE*.

Now, we are ready to talk about the existence of Nash equilibrium in the game between regulators. The nice features of the objective functions of regulators established above under the basic assumptions stated above help us to show that Nash equilibrium exists.

**Proposition 2.** *Under Assumptions CONC, ELAST, REG and RANGE, at least one pure strategy Nash Equilibrium exists in the game between two financial regulator at $t = 0$. Moreover there exists at least one symmetric pure strategy NE.*

*Proof.* See the appendix. □

Having shown that Nash equilibrium exists, the next natural question is whether there are multiple equilibria or there is a unique equilibrium. Fortunately, under the conditions stated previously, we have a unique symmetric equilibrium of the game between the regulators as shown by the following proposition.

**Proposition 3.** *Under Assumptions CONC, ELAST, REG and RANGE, there exists a unique symmetric NE of the game between the regulators at $t = 0$.*

*Proof.* See the appendix. □

### 3.3.5 Comparative Statics for the Symmetric Non-cooperative Equilibrium

What happens to the unique equilibrium investment levels in the two countries as good state becomes more likely, or if bank's per unit return from investment, $R$, increases? Next proposition shows that in both cases equilibrium investment levels in two countries increases.

**Proposition 4.** *Non-cooperative symmetric equilibrium investment levels are increasing in $q$ and $R$.*

*Proof.* See the appendix. □

This result is quite intuitive since when the good state becomes more likely, i.e. as $q$ increases, regulators will face a deficit of deposit insurance less often and they will allow more investment in equilibrium. On the other hand, as $R$ increases returns to investment in both good and bad states increase, making investment socially more profitable.

I conclude this section by showing that in equilibrium, price of assets must be greater than restructuring costs. I tentatively assumed this while discussing banks' optimal fire-sales decision

at $t = 1$ after they receive bad shocks. Now it is time to prove this claim formally. Under this result banks optimally restructure all assets in equilibrium as I have shown there. In other words, scrapping of capital never arises in equilibrium.

**Lemma 5.** *Under Assumptions CONC, ELAST, REG and RANGE, equilibrium price of assets satisfy* $P^* > c$.

*Proof.* See the appendix. □

Lemma 5 holds because if regulators allow the investment level in their country $N_i$ to be too high, they know that they will drive down the equilibrium price of assets to zero. Therefore, it is never optimal for any of the regulators to allow such high investment levels, independent of the choice of the competing regulator.

### 3.4 Centralized Regulation

Suppose that there is a higher authority, call it the central regulator, that determines optimal investment levels in these two countries. In practice this could be an international financial institution such as the International Monetary Fund or the Bank for International Settlements, or it could also be an institution created by a binding bilateral agreement between the two countries. I will assume that this central regulator must choose the same investment levels for both countries due to political reasons. The question that I address in this section is as follows: Suppose that at the beginning of $t = 0$ national regulators can either set regulatory standards independently, or instead they can relinquish their authority to the central regulator. Would they choose the latter?

In particular, I define the central regulator's problem as follows: it chooses the maximum investment levels in countries $A$ and $B$, $(N_A, N_B)$, to maximize the sum of expected social welfare of these countries given below

$$\max_{N_A,N_B\geq 0} GW(N_A, N_B) = \max_{N_A,N_B\geq 0} \sum_{i=A,B;j\neq i} \{q(R-1)N_i + (1-q)[R\gamma^*(N_i, N_j)N_i - N_i]\} \quad (19)$$

In other words, central regulator maximizes the sum of the objective functions of individual regulators. For symmetric countries, it is natural to assume that each country recives an equal weight in the central regulator's objective function.[11] I will denote the jointly optimal common investment levels by $(\widetilde{N}, \widetilde{N})$ and compare them to the investment levels in the uncoordinated symmetric equilibrium, $(\widehat{N}, \widehat{N})$.

There is another way to state the central regulator's problem for symmetric countries. We can think of the central regulator as choosing total investment level across the two countries,

[11] Note that the central regulator does not consider the welfare of the global investors. I am planning to allow this as as an extension later on.

$N = N_A + N_B$ to maximize the overall welfare of the two countries. After finding the optimal total investment level $\overline{N}$, it imposes $\widetilde{N}_i = \overline{N}/2$ for $i = A, B$ where

$$\overline{N} = \arg\max_{N \geq 0} \; q(R-1)N + (1-q)[R\widetilde{\gamma}^*(N)N - N] \tag{20}$$

It is easy to see that the two problems given by (19) and (20) are the same due to the symmetry of countries.

Now, we can compare the centrally optimal investment levels to investment levels that arise as a result of the strategic interaction between regulators. The following proposition shows that a central regulator will allow less investment in both countries compared to what would have been chosen by independent regulators.

**Proposition 5.** *$\widetilde{N} < \widehat{N}$, i.e. independent national regulators choose a higher investment in equilibrium compared to the centrally optimal common investment level.*

*Proof.* See the appendix. □

Proposition 5 shows that due to the systemic risk caused by asset fire-sales, regulatory standards when regulators act independently will be inefficiently low compared to regulatory standards that would be chosen by a central regulator. A central regulator will act to maximize the total welfare in the two countries, and hence it will internalize the externalities that arise from fire-sales. Central regulator takes into account the fact that allowing more investment in the risky asset in one country will reduce the welfare of the other country because of lower asset prices induced by higher initial investment.

### 3.4.1 Is voluntary cooperation possible?

We have shown that investment levels will be higher in both countries if regulators act strategically. Will countries ever benefit from a central regulation which would reduce investment levels in both countries? The following proposition shows that symmetric countries will always benefit from relinquishing their authority to a central regulator.

**Proposition 6.** *If the countries are symmetric then both regulators prefer to deliver their authority to a central systemic risk regulator, i.e. $W_i(\widetilde{N}_i, \widetilde{N}_j) \geq W_i(\widehat{N}_i, \widehat{N}_j)$ holds for $i = A, B$.*

*Proof.* See the appendix. □

When regulators act independently, each of them allows the investment into the risky asset up to the point where the expected marginal benefit from the risky asset is equal to the expected domestic marginal cost of the investment. However, at this level of investment, the marginal total cost across the two countries far exceeds the sum of the marginal benefits. This happens, because

each regulator does not consider the adverse effect of increasing investment level on the welfare of the other country. On the other hand, the central regulator can choose a total investment level in the risky asset where the total marginal benefit equals to the total marginal cost, and hence it can improve the overall welfare of the two countries.

# 4 Asymmetric Countries

In the previous section, we showed that regulators of symmetric countries are always better off by relinquishing their authority to a central regulator. Can we make a similar argument for countries that are asymmetric in some dimensions? In other words, if there are differences across countries, would national regulators still benefit from relinquishing their authority to a central regulator? In this section, I will answer this and the following questions that arise when there are asymmetries across countries: How would the asymmetries affect regulation levels in the two countries in equilibrium? How do central regulation levels compare to regulation levels chosen by national regulators independently? Which countries are more likely to accept a common central regulation?

I focus on differences in returns to the risky investment across countries. In particular, I assume that banks in country $A$ are uniformly more productive than banks in country $B$. In terms of the parameters of the model, this assumption can be stated as $R_A > R_B$. This assumption can be justified if there is segregation between the investment markets of the two countries.

Furthermore, to simplify the analysis that follows, I also assume that $F'(0) = 1$.[12] Under this assumption, global investors will purchase capital only if the price of capital falls below one. This assumption will also rule out possible multiple equilibria in the capital goods market at $t = 1$ when there are return differences between countries. Note that capital goods in the two countries are still identical from global investors' perspective at $t = 1$.

The next proposition shows that when regulators act independently, regulator $A$ (regulator of the high return country) chooses lower regulatory standards (a higher $N$) in equilibrium. This result complies with Proposition 4 in the previous section where we have shown that equilibrium investment levels are increasing in the return to investments given by $R$.

**Proposition 7.** *If $R_A > R_B$, then $\widehat{N}_A > \widehat{N}_B$ in non-cooperative equilibrium.*

*Proof.* In the appendix. □

Now, we can compare common central regulatory standars to uncoordinated regulation levels when there are asymmetries between the countries. The next proposition shows that in order for a common central regulation to be acceptable by both regulators, it has to require stricter regulatory standards in both countries compared to the uncoordinated regulatory standards.

[12] This assumption simplifies the analysis by making the demand function of global investors independent of the return differences between the countries.

**Proposition 8.** *There exists no central regulation level* $N > \min\{\widehat{N}_A, \widehat{N}_B\}$.

*Proof.* See the appendix. □

The proof of Proposition 8 makes use of the envelope theorem to show that welfare of a country is decreasing in the investment level of the other country. In order to forgo the authority to independently and optimally choose the regulatory standards as a response to the regulatory standards chosen by the other country, each regulator must be compensated by stricter regulatory standard (a lower $N$) in the other country. Therefore, we can show that any common regulation level above $\widehat{N}_B$, which is the minimum of the two regulation levels given our assumption that $R_A > R_B$, will always be rejected by regulator $A$.

This discussion implies that if a common regulation level is accepted by regulator $A$, it will always be accepted by regulator $B$. This happens because a common regulation reduces investment levels in both countries as shown by Proposition 11. However, it reduces investment levels more in country $A$, compared to country $B$. Therefore, if regulator $A$ is willing to accept such a common regulation level, it will necessarily be accepted by regulator $B$ as shown by the following lemma.

**Lemma 6.** *For any common regulation level* $N$ *such that* $W_A(N, N) > W_A(\widehat{N}_A, \widehat{N}_B)$ *we have* $W_B(N, N) > W_B(\widehat{N}_B, \widehat{N}_A)$.

*Proof.* See the appendix. □

Lemma 6 allows us to focus on the welfare of country $A$ in search of an mutually acceptable common regulation levels. Let's define $N^m$ as the regulation level that would maximize the welfare of country A if it is uniformly imposed in both countries. Formally we define $N^m$ as follows.

**Definition 1.** $N^m \equiv \arg\max_N W_A(N, N)$

Given this definition, we can write the net maximum benefit from common central regulation to country $A$ as $W_A(N^m, N^m) - W_A(\widehat{N}_A, \widehat{N}_B)$. The next proposition shows that this net maximum benefit is decreasing in the differences between the countries.

**Proposition 9.** *Suppose that* $F'(0) = 1$. *Let* $s \equiv R_A - R_B > 0$. *Then for any* $R_A$, *there exists* $\widehat{s} \in (0, R_A - 1)$ *such that* $W_A(N^m, N^m) - W_A(\widehat{N}_A, \widehat{N}_B) \geq 0$ *if* $s \leq \widehat{s}$, *and* $W_A(N^m, N^m) - W_A(\widehat{N}_A, \widehat{N}_B) < 0$ *otherwise.*

*Proof.* See the appendix. □

Proposition 9 provides the main result of this section. It says that if the return differences between the two countries are above a threshold, then at least the high return country will be

worse off if a common regulation level is imposed across the two countries, even if the common regulation is chosen in a way to maximize the welfare of this country. Large return differences will imply that such common regulation level is too strict compared to the regulatory standard that would be chosen by the high-return country in the uncoordinated equilibrium. Therefore, welfare of the high return country will fall if they decide to accept common regulatory standards in the face of high return differences between countries. In other words, Proposition 9 shows that a voluntary cooperation can exists only between sufficiently similar countries. If the differences across countries are sufficiently high, then at least one of the countries will be worse off by accepting central regulation.

# 5 Systemic Failures in Regulated Economies

In this section I examine systemic failures when the two countries are symmetric. By systemic failures I refer to the fact that all banks in the two countries become insolvent after fire-sales. Systemic failures will occur if the asset prices in the crisis state is so low that the returns from investments that could be retained by banks after fire-sales are not enough to cover the promised return to depositors, which is simply equal to the initial value of the investment here. We might have systemic failures even in regulated economies. Since countries are symmetric and we assume perfectly correlated shocks across countries, systemic failures will happen in both countries at the same time if they occur. We can write the systemic failure condition in equilibrium as

$$R\gamma^*(\widehat{N},\widehat{N})\widehat{N} < \widehat{N} \tag{21}$$

where $\widehat{N}$ denotes symmetric equilibrium investment levels. The left hand side is the (expected) return from investments that could be retained by banks after the fire-sales, and the right hand side is the promised payments to depositors, which is simply the initial value of the investment.

For the rest of the analysis I will work with a particular functional form for which I can obtain closed form solution for equilibrium investment levels. The technology of the global investors is given by: $F(y) = R\ln(1+y)$. I solve the model for this particular functional form in the appendix. Using the closed form solution given in the appendix, we can show that the systemic failure condition given by (21) above is

$$\widehat{N} > \frac{aR}{2c}\left(\frac{R-1-c}{R-1}\right) \equiv \widehat{N}^c \tag{22}$$

where $\widehat{N}^c$ is defined as the critical equilibrium investment level beyond which banks fail in the bad state, i.e. if $\widehat{N} > \widehat{N}^c$ then banks in the two countries become insolvent in the bad state. We have already shown that $\widehat{N}$ is increasing in $q$. This helps us to prove the following result.

**Proposition 10.** *Let $F(y) = R\ln(1+y)$. If $1+c < R < \widehat{R}$ then there exists a $\widehat{q} \in (0, 1/R)$ such that for all $q \geq \widehat{q}$ we have that $\widehat{N}(q) \geq \widehat{N}^c$. In other words, for such $R$, if the probability of the good state is higher than $\widehat{q}$, banks fail in the bad state in non-competitive equilibrium. If $R \leq 1+c$ then banks always fail in the bad state, and if $R \geq \widehat{R}$ then banks never fail in the bad state where*

$$\widehat{R} \equiv \frac{1}{2}\left(2 + c + \sqrt{c}\sqrt{8+c}\right) \tag{23}$$

*Proof.* See the appendix. □

By Proposition 4 we have already shown that equilibrium investment level is increasing in $q$ and $R$. Proposition 10 shows that if $R$ is sufficiently high, then systemic failures do not occur. In order to prove this, I show that the difference $N^c - \widehat{N}$ is monotonically increasing in $R$, and this difference is positive for any value of $q$ if $R$ is sufficiently high. Remember that banks fail if $N^c < \widehat{N}$, which means that they will not fail as long as the difference $N^c - \widehat{N}$ is positive. On the other hand, if $R$ has moderate values, given by $1 + c < R < \widehat{R}$, then banks fail in the bad state only if the probability of good state, $q$, is sufficiently high. For moderate values of $R$, a sufficiently high $q$ leads to systemic failures because $\widehat{N}$ is increasing $q$, whereas $N^c$ is independent of $q$ as can be seen from (22). Hence, for any value of $R$ such that $1 + c < R < \widehat{R}$, there is a sufficiently high $q$ such that the difference $N^c - \widehat{N}$ is negative. And lastly, if $R$ is sufficiently low, given by $R < 1 + c$, then total return from maintained assets after fire-sales is never enough to cover the initial value of the investment since $1 + c$ is the marginal cost of funds for the investment, if the bad state is expected to occur with certainty. In order to prove this, I show that for these low values of $R$, the difference $N^c - \widehat{N}$ is negative for any value of $q$. Therefore systemic failures will happen for sure in the bad state in this case.

The region of parameters for which systemic failures occur in the bad state is illustrated in Figure 4. The horizontal axis in Figure 4 measures $q$, the probability of success, from 0 to 1, and the vertical axis measures $R$, the return to investment, from 1 to 2. Since we assume that $Rq \leq 1$, we should ignore the region where $Rq > 1$ in Figure 4. This region is shaded by grey. The blue region shows the set of $R, q$ pairs for a given $c$, for which systemic failures occur in the bad state. Technically, in blue region we have that $\widehat{N} > \widehat{N}^c$. There are two vertical red lines in Figure 4. The lower one shows $R = 1 + c$, and it is clear from the graph that banks fail for any value of $q$ if $R \leq 1 + c$. The higher red line shows $R = \widehat{R}$, and it is again clear from the graph that systemic failures never occur if $R \geq \widehat{R}$. Lastly, if $R$ is between the two red lines, i.e. if $1 + c < R < \widehat{R}$, then for any such $R$ there exists some $\widehat{q} \in (0, 1/R)$ such that systemic failures occur if $q \geq \widehat{q}$ as claimed in Proposition 10.

It is clear from the analysis above that systemic crises are more likely when the initial investment level is high. Since global regulation reduces investment levels in both countries, we can claim that moving to a central regulation can eliminate systemic failures. I conclude this section by stating

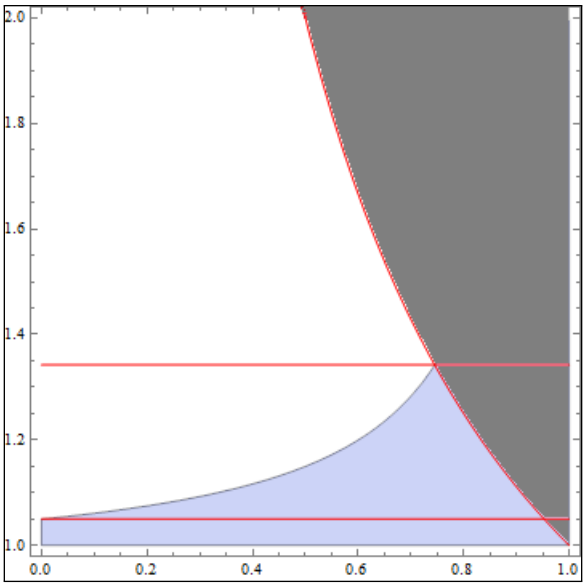



Figure 4: Systemic Failures

and (informally) proving this result.

**Corollary 1.** *Suppose that parameters of the model are such that in bad states there is a systemic failure, i.e. we have either*

*i.* $1 + c < R < \widehat{R}$ *and* $q \geq \widehat{q}(R)$, *or*

*ii.* $R \leq 1 + c$.

*Then, moving to a central regulation can eliminate systemic banking failures.*

*Proof.* We know that systemic crises happen when initial investment levels are high. If initial investment levels are close to the critical borders beyond which systemic crises occur, i.e. $\widehat{N}(\cdot)$ is slightly above $\widehat{N}^c(\cdot)$, then moving to global regulation can reduce investment levels in both countries to $\widetilde{N} < \widehat{N}^c$, and eliminate systemic failures in the bad state. □

## 6 Extensions: Discussion of Assumptions

In this section, I will discuss some of the assumptions I have imposed in the main model. I will examine whether my results are robust or not to changes in these assumption. I will consider assumptions of deposit insurance, limited liability for bank owners, and no initial equity capital for banks owners. I will also check the effects of different assumptions about the competition structure of the banking industry on the model results.

### 6.1 Deposit Insurance

I have assumed that there is a deposit insurance fund that protect depositors. Under this assumption banks were able to borrow at constant and zero net interest rate from depositors, since depositors will guaranteed that they will always be able to receive their dollar back. If the bad state arises and the bank has not sufficient funds to make the promised payments to depositor, the deposit insurance fund steps in, and funds the deficit between the promised payment and resources available to a bank.

What would happen if there was no deposit insurance? Our approach to answer this question will depend on the competition structure of the banking industry. First, suppose that banks are local monopolies in their local deposit market as in the main model. In this case, the interest on deposit contracts will be just enough to induce depositors to deposit their endowments with the banks. In technical terms, the individual rationality condition for depositors will bind. In this section, I will restrict attention to deposit contracts that are in the form of debt contracts.[13]

Consider the problem of a bank in country $i$ is choosing the amount to borrow and invest in the risky asset, $n_i$, and the interest rate on the deposits, $r$, subject to the depositors' individual rationality condition (IR), and the regulation that $n_i \leq N_i$:

$$\max_{r,n_i \geq 0} \quad q(R-r)n_i + (1-q)\max\{\gamma R n_i - r n_i, 0\} \tag{24}$$

subject to

$$qrn_i + (1-q)\min\{\gamma R n_i, r n_i\} \geq n_i \qquad (IR) \tag{25}$$

$$n_i \leq N_i \tag{26}$$

Note first that, $\gamma = 1 - c/P$ is the fraction of retained assets by banks at $t = 1$, which banks take as given as before. The problem of a bank is still linear. Therefore, banks' problem will still yield a corner solution, i.e. either there will be maximum investment ($n_i = N_i$), or no investment at all ($n_i = 0$). Note also that we can separate the choice of the borrowing/investment level ($n_i$) from the choice of deposit rate ($r$). Therefore, I will consider the choice of optimal $r$ for a given investment level. Analyzing banks' problem above shows for a given $\gamma$ that there are two cases to consider:

**Case 1** $R\gamma > 1$. In this case the returns in the bad state will be sufficient to cover the initial borrowing from depositors even in the bad state of the world. Therefore, banks will just offer zero net interest rate to depositors, i.e. they will set $r = 1$, and the *IR* will be satisfied. Banks will make net positive profits in both states of the world, therefore they will want to invest as much as possible. Hence, the regulatory requirement will bind ($n_i = N_i$).

[13] There are two justifications for this restriction. First, this assumption is realistic. The deposit contracts in practice are in the form of debt contracts. Secondly, from Townsend (1979) we know that in the case of costly state verification, debt contracts will be optimal. Therefore, debt contracts can be justified by assuming that depositors can observe the banks' asset returns only at a cost.

**Case 2** $R\gamma < 1$. In this case the returns in the bad state will not be sufficient to cover the initial borrowing from depositors in the bad state of the world. In the bad state, depositors of country $i$ will receive $R\gamma N_i < N_i$, *i.e.* less than their initial lending. Therefore, banks will have to offer positive net interest rate to depositors. For $IR$ condition to be satisfied, $r$ has to be such that

$$r \geq \frac{1-(1-q)R\gamma}{q} \equiv r^* \tag{27}$$

Banks will choose the lowest $r$ that satisfies (27) to maximize their profits, hence they will set $r = r^*$. We have to check banks' individual rationality condition as well. For such $r^*$, are they going to obtain net positive profits in equilibrium? Note that banks will be receiving zero profits in the bad state since $R\gamma^* < 1$. Therefore, from banks' problem given by (24), banks' expected profits will be $q(R-r^*)n_i$. Now, suppose that regulators choose standards assuming that banks will choose maximum allowed investment. We know that in this case there will be a unique symmetric equilibrium regulatory standards given by $(\widehat{N}, \widehat{N})$. Now, we have to check whether banks will indeed choose to make the maximum investment under these regulatory standards. For that to happen we need to have $R > r^*$ when $\gamma = \gamma(\widehat{N}, \widehat{N})$. Using the definition of $r^*$ from (27) this condition can be written as

$$\gamma(\widehat{N}, \widehat{N}) > \frac{1-qR}{(1-q)R} \tag{28}$$

In order to see that this is indeed the case, re-arrange the FOCs of the regulator's problem given by (16) to get

$$\gamma(\widehat{N}, \widehat{N}) = \frac{1-qR}{(1-q)R} - \frac{\delta\gamma(\widehat{N}, \widehat{N})}{\delta N_i} N_i > \frac{1-qR}{(1-q)R} \tag{29}$$

since $\delta\gamma(\widehat{N}, \widehat{N})/\delta N_i < 0$ as shown in the proof of Lemma 3. Therefore we conclude that we will have an equilibrium without deposit insurance with the same regulatory standards as before.

### 6.1.1 No deposit insurance and perfectly competitive deposit markets

Now, instead of assuming that each bank is a local monopoly in the deposit market, I will assume that deposit market is perfectly competitive, and analyze the robustness of the results to this change in the environment. If the deposit market is perfectly competitive banks will earn zero profits. Therefore each bank in country $i$ will choose the amount of investment in the risky asset $(n_i)$ to maximize the expected utility of a representative depositor:

$$\max_{0 \leq n_i \leq N_i 0} \quad qRn_i + (1-q)R\gamma n_i - n_i \tag{30}$$

Here, the assumption is that banks will transfer all resources generated from the investment to depositors. Therefore with probability $q$, the depositors will receive a net return of $(R-1)n_i$, and with probability $1-q$, they will receive the net return of $(\gamma R - 1)n_i$, which is net return the assets

retained by their bank after fire-sales. Note that for depositors choose to deposit their endowment with the bank, the expected net return on the deposits must be greater than zero. And if it is greater than zero, depositors will prefer maximum investment, i.e. setting $n_i = N_i$ will maximize (30). On the other hand, if regulators assume that banks will choose maximum investment, we know from the main section that there will be a unique set of regulatory standards given by $(\widehat{N}, \widehat{N})$. Lastly, we have to check whether banks will indeed chose maximum investment if $(N_i, N_j) = (\widehat{N}, \widehat{N})$. For $\gamma = \gamma(\widehat{N}, \widehat{N})$, re-arranging (30) shows that the expected net utility of a representative depositor will be greater than zero if

$$\gamma(\widehat{N}, \widehat{N}) > \frac{1 - qR}{(1-q)R} \tag{31}$$

We can again refer to the first order conditions of regulators' problem given by (16). Rearrange the FOCs to get

$$\gamma(\widehat{N}, \widehat{N}) = \frac{1 - qR}{(1-q)R} - \frac{\delta\gamma(\widehat{N}, \widehat{N})}{\delta N_i} N_i > \frac{1 - qR}{(1-q)R} \tag{32}$$

since $\delta\gamma(\widehat{N}, \widehat{N})/\delta N_i < 0$ as shown in the proof of Lemma 3. Therefore we conclude that we will have an equilibrium without deposit insurance with the same regulatory standards as before even when the deposit market is perfectly competitive.

### 6.2 Limited Liability

In the main text, I assumed that banks are protected by limited liability. Limited liability assumption means that bank profits are always (weakly) positive. If a bank's assets falls short of its liabilities, then the bank owners receive only zero profits. Banks always wanted to make unlimited investment in the risky asset asset under this assumption. Now instead suppose that bank owners have some wealth, or endowment at the last period that can be seized by depositors if the returns on assets are not enough to cover the promised payments to depositors.

If there is no limited liability a bank in country $i$ will be choosing $0 \leq n_i \leq N_i$ to maximize the expected profits:

$$\max_{0 \leq n_i \leq N_i 0} \quad qRn_i + (1-q)R\gamma n_i - n_i \tag{33}$$

After substituting for $\gamma = 1 - c/P$ using (9) we can write (33) as

$$\max_{0 \leq n_i \leq N_i 0} \quad qRn_i + (1-q)R(1 - c/P)n_i - n_i \tag{34}$$

where P is the price of capital in the fire-sale market in the bad state at $t = 1$. Each bank takes this price as given. Note that the problem technically becomes essentially the same as the case where there is no deposit insurance and the deposit market is perfectly competitive which is analyzed in Section 6.1.1. This can be seen by comparing problems (24) and (33).

The first order conditions of the banks' problem above will be given by

$$\frac{\partial \pi}{\partial n_i} = qR + (1-q)R(1-c/P) - 1 \tag{35}$$

By re-arranging the terms it is easy to see that first order condition will be positive if and only if

$$P > \frac{c(1-q)R}{R-1} \equiv \underline{P} \tag{36}$$

In other words, as long as $P \geq \underline{P}$, banks will still want to make unlimited investment in the risky asset. On the other hand if regulators expect banks to set $n_i = N_i$, they will choose the same set of regulatory standards as in the case with limited liability. Note that, assuming that regulator's still care about the losses due to fire-sales, the regulators' objective function will not change. Therefore, in order to show that equilibrium regulatory standards do not change when the limited liability assumption is removed, we have to check whether the price of capital in the uncoordinated equilibrium satisfies $P^*(N_A, N_B) \geq \underline{P}$. This can be checked using the first order conditions of the regulator's problem given by (16), which I reproduce here after substituting for $\gamma^*(N_A, N_B) = 1 - c/P^*(N_A, N_B)$:

$$\frac{\partial W_i(N_i, N_j)}{\partial N_i} = qR + (1-q)R\left\{1 - \frac{cP^*(N_i, N_j) - c\frac{\partial P^*(N_i,N_j)}{\partial N_i}N_i}{P^*(N_i, N_j)^2}\right\} - 1 = 0 \tag{37}$$

By rearranging the terms above we can obtain

$$\frac{1}{P^*(N_i, N_j)} - \frac{R-1}{c(1-q)R} = \frac{c\frac{\partial P^*(N_i,N_j)}{\partial N_i}N_i}{P^*(N_i, N_j)^2} < 0 \tag{38}$$

The negativity of the left hand side follows from Lemma 1 which showed that $\partial P^*(N_i, N_j)/\partial N_i < 0$. On the left hand side we have $1/P^* - 1/\underline{P}$. Therefore, we obtain that we indeed have $P^*(N_A, N_B) > \underline{P}$ in equilibrium. We conclude that the equilibrium obtained under the limited liability assumption will prevail when remove this assumption.

### 6.3 Initial Bank Equity Capital

In the main text, I also assumed that banks have no initial endowment that they can invest in the risky asset. Banks raised necessary funds for investment from the deposit market. Hence, liability side of their balance sheet contained only debt, but not any equity capital.[14]

In this section, I will assume that banks have an initial endowment of $E$ units of consumption

[14]The term equity capital here should not be confused with the capital good. Any initial endowment of bank owners will still be in the form of consumption good. I use the term "equity capital" to refer to the bank owners' own endowment that they invest in the bank.

good which they have to invest in the risky asset. The equity is costly: the opportunity cost of equity to bank owners, $\rho$, is greater than one, the cost of insured deposits. These two assumptions are a common way of introducing equity capital to a banking model (See Dell'Ariccia and Marquez (2006), Hellmann et al. (2000), Repullo (2004) among others). The assumption that the equity capital is fixed captures the fact that it is difficult for banks to raise equity capital at short notice.

When there is bank equity capital, regulation will take the form of minimum capital ratio requirement. Let $k_i = E/n_i$ be the actual capital ratio of a bank in country $i$. Then, regulation will require banks to have $k_i \geq K_i$ where $K_i$ is the capital adequacy requirement in country $i$.

Given its equity, and the price of capital goods in the bad state of $t = 1$, each bank chooses how much to invest in the risky asset, $n_i$, as before to maximize expected profits:

$$\max_{0 \leq n_i \leq N_i 0} \quad q(Rn_i - (n_i - E)) + (1-q)max\left\{R\gamma n_i - (n_i - E), 0\right\} - \rho E \tag{39}$$

subject to the capital regulation

$$k_i = \frac{E}{n_i} \geq K_i \tag{40}$$

Note that $n_i - E$ is the amount of funds borrowed from the local deposit market. We can write the capital ratio requirement condition as

$$n_i \leq \frac{E}{K_i} \equiv N_i \tag{41}$$

This analysis shows that there is one-to-one mapping from capital regulations to the form of regulation used in the main text. Banks' problem do not change. Banks still want to invest in the risky asset as much as possible as long as the net expected return is positive. Therefore, banks problem will yield that the minimum capital requirement binds, i.e. banks will choose $n_i = E/K_i$ in an equilibrium with positive investment levels.

Consider the regulators' problem after equity is introduced to the model. Regulators will anticipate that for a given capital ratio requirement, $K_i$, banks will choose their total investment level such that this requirement binds: $n_i = E/K_i$. Banks will raise $E/K_i - E = (1 - K_i)E/K_i$ units of consumption goods from the local deposit market. Therefore we can write regulators' objective function as

$$W_i(K_i, K_j) = q\left[R\frac{E}{K_i} - \frac{E}{K_i}\right] + (1-q)\left[R\widehat{\gamma}\left(\frac{E}{K_i}, \frac{E}{K_j}\right)\frac{E}{K_i} - \frac{E}{K_i}\right] \tag{42}$$

The function $\widehat{\gamma}(\cdot)$ is the same as the function $\gamma(\cdot)$ except that it is defined over the minimum capital ratios $(K_i, K_j)$, not over the total investment levels. If we define $N_i \equiv \frac{E}{K_i}$ we can express

the objective function above as

$$\max_{N_i \geq 0} W_i(N_i, N_j) = \max_{N_i \geq 0} q(R-1)N_i + (1-q)[R\gamma^*(N_i, N_j)N_i - N_i] \tag{43}$$

This objective function is exactly the same as the regulators' problem in the main text. Therefore all qualitative results of the main section will carry on when we introduce costly bank equity and redefine regulation as a minimum capital ratio requirement.

Note that when we introduce costly equity, the net expected return to the risky investment must be sufficiently large to cover the opportunity cost of internal bank equity, $\rho E$, for banks. Otherwise, banks will choose not to invest in the risky asset at all. Therefore, the set of parameters where we have strictly positive investment in equilibrium is smaller under costly equity.

# 7 Conclusion

I examine the incentives of individual national regulators towards cooperation when there is systemic risk in global financial markets. I consider a two-country and three-period model. Banks invest in long-term projects in the initial period. They may face negative shocks in the interim period which forces them to sell assets. Asset sales of banks feature the characteristics of a fire-sale: the higher the number of assets sold, the lower the market price of assets is. Due to this externality between national financial markets, a systemic risk is generated. If regulation level is relaxed in one country, banks in this country invest more in the risky asset in the initial period. If the bad state arises in the interim period, these banks are forced to sell more assets, causing the asset price to fall further. A lower asset price will increase the cost of distress for the banks in the other country as well. Banks may even default in equilibrium if the asset prices fall below a threshold. I show that, in the absence of cooperation, independent regulators choose inefficiently low regulation compared to regulation levels that would be chosen by a central regulator. A central regulator takes the systemic risk into account and improves welfare in cooperating countries by imposing higher regulatory standards. Therefore, I show that it is incentive compatible for national regulators of symmetric countries to relinquish their authority to a central regulator.

I also consider the incentives towards cooperation between the regulators of asymmetric countries. In particular, I consider differences in returns to the risky investment across countries. I show that the regulator of the high return country chooses lower regulatory standards in equilibrium. Moreover, a common regulation, if it exists, necessarily raises regulatory standards in both countries. This implies that a common regulation will be accepted by the regulator of the low return country, if it would be accepted by the regulator of the high return country. Based on these results, we are able to prove that a central regulation will only emerge between countries that are sufficiently similar. If the difference in returns across countries is too high, then a common regulation level will be rejected at least by the regulator of the high return country.

# 8 Appendix

In this appendix, I will present functional form examples for the global investors' technology and I will solve the model for one of these functional forms. I will also provide the proofs of the propositions and lemmas in the main text.

## 8.1 Functional Form Examples

**Example 1** $F(y) = R\ln(1+y)$
For this return function we obtain the (inverse) demand function as

$$P = F'(y) = \frac{R}{1+y} \text{ and hence } y = F'^{-1}(P) = \frac{R-P}{P} \equiv D(P) \tag{8.1}$$

This demand function is clearly downward slopping and convex as seen below

$$D'(P) = -\frac{R}{P^2} < 0 \text{ and } D''(P) = \frac{2R}{P^3} > 0$$

$F(\cdot)$ satisfies Assumption *CONC* since

$$F''(y) = -\frac{R}{(1+y)^2} < 0 \text{ and since } F'(0) = R$$

Let's check whether this functional form satisfies the conditions given by *Assumption CONC and Assumption REG*, respectively.

$$F'(y) + yF''(y) = \frac{R}{1+y} - y\frac{R}{(1+y)^2} = \frac{Ra}{(1+y)^2} > 0$$

Clearly *Assumption ELAST* is satisfied. Below we see that this function satisfies *Assumption REG* as well:

$$F'(y)F'''(y) - 2F''(y)^2 = \frac{R}{(1+y)}\frac{2R}{(1+y)^3} - 2\left(\frac{R}{(1+y)^2}\right)^2 = 0$$

From above we can also see that this return function induces a log-convex demand function since we will have $F'(y)F'''(y) - F''(y)^2 > 0$

**Example 2** $F(y) = \sqrt{y+a^2}$ For this example the demand function will be obtained as

$$P = F'(y) = \frac{1}{2\sqrt{y+a^2}} \text{ and hence } y = F'^{-1}(P) = \frac{1}{4P^2} - a^2 \equiv D(P)$$

This demand function is also downward slopping and convex as seen below

$$D'(P) = -\frac{1}{2P^3} < 0 \text{ and } D''(P) = \frac{3}{2P^4} > 0$$

*Assumption CONC* is satisfied since

$$F''(y) = -\frac{1}{4\left(y+a\right)^{3/2}} < 0 \text{ and } F'(0) = R \text{ implies that } \frac{1}{2a} = R \text{ or } a = \frac{1}{2R}$$

We can easily show that *Assumption ELAST* is satisfied:

$$F'(y) + yF''(y) = \frac{1}{2\left(y+a\right)^{\frac{1}{2}}} - y\frac{1}{4\left(y+a\right)^{\frac{3}{2}}} = \frac{y+2a}{4\left(y+a\right)^{\frac{3}{2}}} > 0$$

Likewise we can show that this function satisfies *Assumption REG* as well:

$$\begin{aligned} F'(y)F'''(y) - 2F''(y)^2 &= \frac{1}{2\left(y+a\right)^{\frac{1}{2}}}\frac{3}{8\left(y+a\right)^{\frac{5}{2}}} - 2\left(\frac{1}{4\left(y+a\right)^{\frac{3}{2}}}\right)^2 \\ &= \frac{-1}{16\left(y+a\right)^3} < 0 \end{aligned}$$

Note that in contrast to the first example this functional form induces a log-concave demand function since we can show that $F'(y)F'''(y) - F''(y)^2 < 0$

## 8.2 Symmetric Countries: An example

In this section, I will obtain closed form solutions for non-cooperative equilibrium regulation levels and regulation levels under cooperative benchmark for the particular functional form choice for the global investors' technology given by Example 1 above. I will also verify some of the main results and comparative statics in the main text for this particular functional form.

For analytical convenience suppose that the technology of global investors is given by the following logarithmic function as investigated by Example 1 above

$$F(y) = A\ln(a+y) \tag{8.2}$$

where the amount of assets the global investors optimally buy satisfies the following first order conditions

$$F'(y) = \frac{A}{a+y} = P \tag{8.3}$$

which will induce a downward slopping demand function

$$y = F'(P)^{-1} = \frac{A-aP}{P} \equiv D(P) \tag{8.4}$$

Imposing *Assumption CONC* on this functional form gives

$$F'(0) = R \Rightarrow \frac{A}{a} = R \quad \text{or} \quad A = aR \tag{8.5}$$

It is shown in the previous section that this functional form satisfies the conditions given by *Assumptions ELAST and REG*. Since this functional form satisfies all sufficient conditions, we can proceed with solving for the equilibrium. We start solving the model backwards as in the previous

section. Therefore we first find the equilibrium price at $t = 1$ using the market clearing condition.

$$D(P) = S(P, N_A + N_B) \Rightarrow \frac{A - aP}{P} = \frac{c(N_A + N_B)}{P} \tag{8.6}$$

Hence, we get the equilibrium price of assets at $t = 1$ as

$$P^* = \frac{A - c(N_A + N_B)}{a} \tag{8.7}$$

which is clearly decreasing in the investment levels in both countries. Note also that equilibrium price is determined only by the total investment level in the two countries. Exact division of the total investment between the countries will not affect $P^*$. This property of the equilibrium price of assets is very helpful in the analysis of the model.

We can also obtain equilibrium fraction of assets retained by banks after fire-sales as a function of initial investment levels in each country by plugging the equilibrium price given by equation (8.7) into equation (9) that defines this fraction as a ratio of market price, which will give that

$$\gamma^*(N_A, N_B) = 1 - \frac{c}{P^*(N_A, N_B)} = 1 - \frac{ac}{A - c(N_A + N_B)} \tag{8.8}$$

Remember that regulator $i'$s objective function is

$$\max_{N_i \geq 0} W_i(N_i, N_j) = q(R-1)N_i + (1-q)[R\gamma^*(N_i, N_j)N_i - N_i] \tag{8.9}$$

Substituting for $\gamma^*(N_i, N_j)$ from (8.8) gives

$$\max_{N_i \geq 0} W_i(N_i, N_j) = q(R-1)N_i + (1-q)\left[R\left(1 - \frac{ac}{A - c(N_i + N_j)}\right)N_i - N_i\right] \tag{8.10}$$

where FOCs can be obtained as

$$\frac{\partial W_i(N_i, N_j)}{\partial N_i} = qR + (1-q)R\left\{1 - \frac{ac(A - cN_j)}{[A - c(N_i + N_j)]^2}\right\} - 1 = 0 \tag{8.11}$$

Solving for $N_i$ gives the best response function of regulator $i$

$$N_i^*(N_j) = \frac{A - cN_j - \sqrt{\sigma ac(A - cN_j)}}{c} \tag{8.12}$$

where we define

$$\sigma \equiv \frac{(1-q)R}{R-1} \tag{8.13}$$

We can use the best response functions to solve for the symmetric equilibrium investment level. After some algebra we can obtain that for $i = A, B$

$$\widehat{N}_i = \frac{4A - \sigma ac - \sqrt{8\sigma Aac + (\sigma ac)^2}}{8c} \tag{8.14}$$

Note that by Assumption *CONC* we impose that $A = aR$. Substituting for $A$ using this identity

gives

$$\widehat{N}_i = \frac{a\left(4R - \sigma c - \sqrt{8R\sigma c + (\sigma c)^2}\right)}{8c} \tag{8.15}$$

### 8.2.1 Central Regulation

Let's consider the central regulator's problem where the central regulator chooses the total investment level in both countries.

$$\max_{N \geq 0} W(N) = q(R-1)N + (1-q)[R\gamma^*(N)N - N] \tag{8.16}$$

Denote the solution to this global problem by $\overline{N}$. Central regulator will impose maximum investment level in each country as $\widetilde{N}_i = \overline{N}/2$ as discussed before. $\overline{N}$ will be characterized by the FOCs of the problem above which we could derive as

$$\frac{\partial W(N)}{\partial N} = qR + (1-q)R\left\{1 - \frac{acA}{(A - c\overline{N})^2}\right\} - 1 = 0 \tag{8.17}$$

Solving for $\overline{N}$ and substituting $\widetilde{N}_i = \overline{N}/2$ gives the globally regulated investment level in each country as

$$\widetilde{N}_i = \frac{A - \sqrt{\sigma Aac}}{2c} \tag{8.18}$$

### 8.2.2 Cooperative vs non-cooperative regulatory standards

We can verify Proposition 5 by comparing regulation levels in cooperative and non-cooperative settings. Remember that Proposition 5 has claimed that $\widetilde{N}_i < \widehat{N}_i$ for $i = A, B$. In order to see that multiply numerator and denominator of $\widetilde{N}_i$ given by equation (8.18) to write it equivalently as

$$\widetilde{N}_i = \frac{4A - \sqrt{16\sigma Aac}}{8c} \tag{8.19}$$

Now comparing this and $\widehat{N}_i$ given by (8.15), we see that $\widehat{N}_i > \widetilde{N}_i$ if and only if $\sqrt{16\sigma Aac} > \sigma ac + \sqrt{8\sigma Aac + (\sigma ac)^2}$ which is true if

$$\sqrt{16\sigma Aac} > \sqrt{8\sigma Aac + (\sigma ac)^2} \tag{8.20}$$

since $\sigma ac > 0$. Taking square both sides of (8.20) and rearranging gives

$$8\sigma Aac > (\sigma ac)^2 \tag{8.21}$$

After substituting for $A = aR$ from Assumption *CONC* and using the definition of $\sigma$ from (8.13) this is equivalent to having

$$c < \frac{8(R-1)}{1-q} \tag{8.22}$$

We can show that this inequality is true by Assumption *ELAST*. To see that rearrange (8.22) to write it as

$$1 + c(1-q) < 8R - 7 \tag{8.23}$$

by Assumption *ELAST* we know that $1+c(1-q) < R$, hence it remains to show that $R < 8R-7$. This is true since we have initially assumed that $R > 1$. Therefore we can conclude that $\widetilde{N}_i < \widehat{N}_i$.

### 8.2.3 Comparative Statics for the Non-cooperative Equilibrium

We can analytically check how non-cooperative equilibrium investment levels behave as main parameters of the model, $q, R$, and $c$ change for this functional form. For comparative statics with respect to $q$ we can directly take partial derivative of equilibrium investment level $\widehat{N}_i$ with respect to $q$. Note that $q$ appears only implicitly inside $\sigma$ given by (8.13), hence we can write

$$\frac{\partial \widehat{N}_i}{\partial q} = \frac{\partial \widehat{N}_i}{\partial \sigma}\frac{\partial \sigma}{\partial q} = -\frac{a}{8c}\left\{1 + \frac{8Rc + 2c\sigma}{\sqrt{8R\sigma c + (\sigma c)^2}}\right\}\left\{-\frac{R}{R-1}\right\} > 0 \tag{8.24}$$

We see from the derivative above that equilibrium investment levels in both countries increase as the probability of the good state, $q$, rises.

For comparative statics with respect to $R$, it is analytically more convenient to take use Implicit Function Theorem on the first order conditions to obtain the sign of the derivative. First order conditions implicitly define $\widehat{N}_i$ as a function of $q, R, c$ (and other parameters of the model). Let's denote this function by $\Psi(\widehat{N}_i; q, R, c)$. Using the fact that $\widehat{N}_i = \widehat{N}_j = \widehat{N}$ for $i \neq j$ in equilibrium, we can write the first order conditions given by equation (8.11) as

$$\Psi(\widehat{N}; q, R, c) = qR + (1-q)R\left\{1 - \frac{ac(aR - c\widehat{N})}{(aR - 2c\widehat{N})^2}\right\} - 1 = 0 \tag{8.25}$$

Now, using the IFT we have that

$$\frac{d\widehat{N}}{dR} = -\frac{\partial \Psi(\cdot)/\partial R}{\partial \Psi(\cdot)/\partial \widehat{N}} \tag{8.26}$$

I evaluate each derivative separately and show that $\partial\Psi(\cdot)/\partial R$ is positive and $\partial\Psi(\cdot)/\partial\widehat{N}$ is negative, and hence $d\widehat{N}/dR$ is positive.

$$\begin{aligned}\frac{\partial \Psi(\cdot)}{\partial R} &= q + (1-q)\left\{1 - \frac{ac(aR - c\widehat{N})}{(aR - 2c\widehat{N})^2}\right\} \\ &= -(1-q)R\left\{\frac{a^2c(aR - 2c\widehat{N})^2 - 2a^2c(aR - c\widehat{N})(aR - 2c\widehat{N})}{(aR - 2c\widehat{N})^4}\right\}\end{aligned} \tag{8.27}$$

The first two terms are clearly positive. Let's consider the last term inside the curvy brackets. We can simplify that term as

$$\frac{a^2c(aR - 2c\widehat{N})(-aR)}{(aR - 2c\widehat{N})^4} < 0 \tag{8.28}$$

which implies that the third term in (8.27) is also positive, and hence $\partial\Psi(\cdot)/\partial R$ is positive. Secondly

consider

$$\frac{\partial \Psi(\cdot)}{\partial \widehat{N}} = -(1-q)R\left\{\frac{-ac^2(aR-2c\widehat{N})^2 + 2ac^2(aR-c\widehat{N})(aR-2c\widehat{N})}{(aR-2c\widehat{N})^4}\right\} \tag{8.29}$$
$$= -(1-q)R\left\{\frac{-ac^2(aR-2c\widehat{N})(-aR)}{(aR-2c\widehat{N})^4}\right\} < 0$$

Putting these results we obtain that $d\widehat{N}/dR$ is positive, or in words equilibrium investment level in each country increases as banks' per unit return from investment, $R$, increases.

Lastly, for comparative statics with respect to the cost of restructuring capital, $c$, I directly take derivative of $\widehat{N}$ with respect to $c$ to obtain

$$\frac{\partial \widehat{N}_i}{\partial c} = \frac{a\left(-\sigma - \frac{(8R\sigma+2\sigma c)}{\sqrt{8R\sigma c+(\sigma c)^2}}\right)(8c) - 8a\left(4R - \sigma c - \sqrt{8R\sigma c+(\sigma c)^2}\right)}{(8c)^2} \tag{8.30}$$

The derivative above is clearly negative and tells us that equilibrium investment levels are decreasing in the restructuring costs.

## 8.3 Proofs Omitted in the Text

### 8.3.1 Proofs for the Symmetric Countries Case

**Lemma 1.** *$P^*(n_i, n_j)$ is decreasing in $n_i$ for $i = A, B$ under Assumptions CONC and ELAST.*

*Proof.* Applying the IFT on the MC condition gives

$$\frac{dP^*}{dn_i} = -\frac{\partial E(\cdot)/\partial n_i}{\partial E(\cdot)/\partial P} = \frac{\partial S(P^*, n_i, n_j)/\partial n_i}{D'(P^*) - \partial S(P^*, n_i, n_j)/\partial P} \tag{8.31}$$

First, note that using the expression for the total supply function given by (13) we obtain

$$\frac{\partial S(\cdot)}{\partial n_i} = \frac{c}{P} > 0 \tag{8.32}$$

Hence, we can write the derivative in (8.31) as

$$\frac{dP^*}{dn_i} = \frac{c}{P^* D'(P^*) - P^*\left[\partial S(P^*, n_i, n_j)/\partial P\right]} \tag{8.33}$$

The following equivalence will help us to write this derivative using only return function of global investors' return function, $F(\cdot)$, and its derivatives

$$P^*\frac{\partial S(P^*, n_i, n_j)}{\partial P} = P^*\frac{-c(n_i+n_j)}{P^{*2}} = \frac{-c(n_i+n_j)}{P^*} = -S(P^*, n_i, n_j) \tag{8.34}$$

Using this equivalence we can express the derivative given by (8.33) as

$$\frac{dP^*}{dn_i} = \frac{c}{P^* D'(P^*) + S(P^*, n_i, n_j)} \tag{8.35}$$

Let $y^* \equiv S(P^*, n_i, n_j)$ denote the total volume of equilibrium fire-sales. In equilibrium we will have $P^* = F'(y^*)$ from the demand curve. Therefore we can obtain

$$D'(P^*) = \frac{1}{F''(y^*)} \tag{8.36}$$

where we make use of the fact that $D'(P) \equiv F'(P)^{-1}$ as given by (3). Hence, we can rewrite the denominator of the expression (8.35) above as

$$P^* D'(P^*) + S(P^*, n_i, n_j) = \frac{F'(y^*)}{F''(y^*)} + y^* \tag{8.37}$$

which we can write equivalently as

$$P^* D'(P^*) + S(P^*, n_i, n_j) = \frac{F'(y^*) + y^* F''(y^*)}{F''(y^*)} < 0 \tag{8.38}$$

This expression is negative since $F''(y) < 0$ by Assumption *CONC* and

$$F'(y) + yF''(y) > 0 \tag{8.39}$$

by Assumption *ELAST*. Therefore we conclude that $dP^*/dn_i < 0$. □

**Lemma 2.** *Equilibrium fraction of assets sold in each country, $1 - \gamma^*(n_i, n_j)$, is increasing in $n_i$ for $i = A, B$ under Assumptions CONC and ELAST.*

*Proof.* Using (9) we can write banks' asset sales in equilibrium as

$$1 - \gamma^*(n_i, n_j) = \frac{c}{P^*(n_i, n_j)} \tag{8.40}$$

Note that

$$\frac{\partial \gamma^*}{\partial n_i} = \frac{\partial \gamma^*}{\partial P^*} \frac{dP^*}{dn_i} < 0 \tag{8.41}$$

since $\delta\gamma/\delta P = c/P^2 > 0$ from (9) and by Lemma 1 we have that $dP^*/dn_i < 0$ for $i = A, B$. Therefore, equilibrium fraction of assets rescued after fire-sales ($\gamma^*$) is decreasing in $n_i$ for $i = A, B$.

Since equilibrium fraction of assets sold in each country is given by $1 - \gamma^*(n_i, n_j)$, we obtain that this fraction is increasing in $n_i$ for $i = A, B$. □

**Lemma 3.** *Under Assumptions CONC, ELAST and REG, each regulator's best response is unique valued.*

*Proof.* For this proof I refer to the conditions given by Assumptions *CONC*, *ELAST* and *REG*. I show that if the global investors' return function, $F(\cdot)$ satisfies these conditions, then the objective functions of independent regulators are concave. This also tells us that first order conditions of the regulators problem, which also implicitly defines their best response functions, is monotone and decreasing. Therefore, there is a unique solution to these first order conditions or in other words best response of each regulator is unique valued.

Let's reproduce regulators' objective function here for convenience

$$\max_{N_i \geq 0} \; W_i(N_i, N_j) = \max_{N_i \geq 0} \; q(R-1)N_i + (1-q)[R\gamma^*(N_i, N_j)N_i - N_i] \tag{8.42}$$

FOCs for regulator of country $i$'s problem will be given by

$$\frac{\partial W_i(N_i, N_j)}{\partial N_i} = qR + (1-q)R\left\{\frac{\partial \gamma^*(N_i, N_j)}{\partial N_i}N_i + \gamma^*(N_i, N_j)\right\} - 1 \tag{8.43}$$

Let's define the following function for convenience

$$v^i(N_i, N_j) \equiv \frac{\partial \gamma^*(N_i, N_j)}{\partial N_i}N_i + \gamma^*(N_i, N_j) \tag{8.44}$$

Hence, we can write the FOCs simply as

$$\frac{\partial W_i(N_i, N_j)}{\partial N_i} = qR + (1-q)Rv^i(N_i, N_j) - 1 \tag{8.45}$$

We will show that Under Assumptions *CONC*, *ELAST* and *REG* we have $v_1^i(N_i, N_j) < 0$, hence the objective function is concave. This also means that the best response functions are unique-valued. Note that in Lemma 1 we have obtained

$$\frac{dP^*}{dN_i} = \frac{c}{P^*D'(P^*) + S(P^*, N_i, N_j)} \tag{8.46}$$

which is negative as we have shown there. Since $D(P^*) = S(P^*, N_i, N_j)$ by the market clearing condition, we can also express this derivative as

$$\frac{dP^*}{dN_i} = \frac{c}{P^*D'(P^*) + D(P^*)} \tag{8.47}$$

We will use this expression in the derivative of $\gamma^*$ with respect to $N_i$ below

$$\begin{aligned} \frac{\partial \gamma^*(N_i, N_j)}{\partial N_i} &= \frac{\partial \gamma^*(N_i, N_j)}{\partial P^*}\frac{dP^*}{dN_i} \\ &= \left(\frac{c}{P^*}\right)^2 \frac{1}{D(P^*) + P^*D'(P^*)} < 0 \end{aligned} \tag{8.48}$$

Hence we can obtain the second derivative as

$$\frac{\partial^2 \gamma^*(N_i, N_j)}{\partial N_i^2} = -\left(\frac{c}{P^*}\right)^2 G(P^*)\frac{dP^*}{dN_i} \tag{8.49}$$

where we define

$$G(P^*) \equiv \frac{2D(P^*) + 4P^*D'(P^*) + P^{*2}D''(P^*)}{P^*[D(P^*) + P^*D'(P^*)]^2} \tag{8.50}$$

Note that the derivative of $v^i(\cdot)$, which was defined by (8.44), with respect to the first argument is

equal to

$$v_1^i(N_i, N_j) \equiv \frac{\partial v^i(N_i, N_j)}{\partial N_i} = \frac{\partial^2 \gamma^*(N_i, N_j)}{\partial N_i^2} N_i + 2\frac{\partial \gamma^*(N_i, N_j)}{\partial N_i} \tag{8.51}$$

Put the findings above together to get this derivative as

$$v_1^i(\cdot) = -\left(\frac{c}{P^*}\right)^2 \left[G(P^*)N_i - \frac{2}{c}\right] \frac{dP^*}{dN_i} \tag{8.52}$$

again where $G(P^*)$ is as defined by (8.50) above.We will show that $G(P^*)$ is negative under Assumptions *CONC*, *ELAST* and *REG*, and hence $v_1^i(\cdot) < 0$.

Note that using $D(P) \equiv F'(P)^{-1}$ and $P = F'(y)$ we can obtain

$$(i)\ D(P) = y, \quad (ii)\ D'(P) = \frac{1}{F''(y)} \quad \text{and} \quad (iii)\ D''(P) = -\frac{F'''(y)}{F''(y)^3} \tag{8.53}$$

Hence, we can write the expression in the numerator of $G(P^*)$ as

$$2D(P^*) + 4P^*D'(P^*) + P^{*2}D''(P^*) = 2y^* + \frac{4F'(y^*)}{F''(y^*)} - \frac{F'(y^*)^2F'''(y^*)}{F''(y^*)^3} \tag{8.54}$$

Re-arranging the RHS of (8.54), we obtain

$$\frac{2y^*F''(y^*)^3 + 4F'(y^*)F''(y^*)^2 - F'(y^*)^2F'''(y^*)}{F''(y^*)^3} \tag{8.55}$$

Note that the denominator of the last expression is negative by *Assumption CONC*. Re-arrange the numerator to write it as

$$2F''(y^*)^2\underbrace{\left[y^*F''(y^*) + F'(y^*)\right]}_{(+)\ by\ ELAST} - F'(y^*)\underbrace{\left[F'(y^*)F'''(y^*) - 2F''(y^*)^2\right]}_{(-)\ by\ REG} > 0 \tag{8.56}$$

The expression in (8.56) is positive under Assumption *ELAST* and *REG* as shown above. This implies that

$$2D(P^*) + 4P^*D'(P^*) + P^{*2}D''(P^*) < 0 \tag{8.57}$$

i.e. the numerator of $G(P^*)$ which was given by (8.54) is negative. Putting these results together we obtain that $v_1^i(N_i, N_j) < 0$.

The analysis above shows that FOCs of each regulator is monotone and decreasing. Therefore, we conclude that their best response functions are unique valued as desired. □

**Proposition 1.** *Under Assumptions CONC, ELAST and REG, optimal regulatory standards in the two countries are strategic substitutes.*

*Proof.* Optimal regulatory standards in the two countries are strategic substitutes if and only if the best response functions of regulators are downward slopping. We can apply the Implicit Function Theorem (IFT) on the first order conditions to obtain the sign of the slope of best response functions. This sign is shown to be equal to the sign of the cross derivative of the objective function due to the results in Proposition 1. In order to show that the sign of the cross derivative of the objective function is negative, I again refer to the technical conditions given by Assumptions *CONC*, *ELAST*

and *REG*. I show that for any (induced) demand function that satisfies Assumptions *CONC* to *REG*, this sign is negative and hence optimal investment levels are strategic substitutes.

If $N_i$ and $N_j$ are strategic substitutes we must have $\partial^2 W_i(N_i, N_j)/\partial N_i \partial N_j < 0$. Remember that

$$\frac{\partial W_i(N_i, N_j)}{\partial N_i} = qR + (1-q)Rv^i(N_i, N_j) - 1 \tag{8.58}$$

where

$$v^i(N_i, N_j) \equiv \frac{\partial \gamma^*(N_i, N_j)}{\partial N_i} N_i + \gamma^*(N_i, N_j) \tag{8.59}$$

hence

$$\begin{aligned} \frac{\partial^2 W_i(N_i, N_j)}{\partial N_i \partial N_j} &= (1-q)R \frac{\partial v^i(N_i, N_j)}{\partial N_j} \\ &= (1-q)R \left\{ \frac{\partial^2 \gamma^*(N_i, N_j)}{\partial N_i \partial N_j} N_i + \frac{\partial \gamma^*(N_i, N_j)}{\partial N_j} \right\} < 0 \end{aligned} \tag{8.60}$$

The sing of equation (8.60) is negative since

(i) $\frac{\partial \gamma^*(N_i, N_j)}{\partial N_j} < 0$ as shown by eq (8.48) and

(ii) for the cross derivative of $\gamma^*(N_i, N_j)$ we know that

$$\frac{\partial^2 \gamma^*(N_i, N_j)}{\partial N_i \partial N_j} = \frac{\partial^2 \gamma^*(N_i, N_j)}{\partial N_i^2} \tag{8.61}$$

since $\gamma^*(N_i, N_j)$ is determined only by the sum of the two investment levels, not by their individual values. Therefore we get an equation similar to the one obtained in the proof of Lemma 3

$$v_2^i(\cdot) \equiv \frac{\partial v^i(N_i, N_j)}{\partial N_j} = -\left(\frac{c}{P^*}\right)^2 \left[ G(P^*) N_i - \frac{1}{c} \right] \frac{dP^*}{dN_i} \tag{8.62}$$

which is negative under Assumptions 1 to 3 as shown in the proof of Lemma 3. Hence, the best response functions are downward slopping which implies that $N_i$ and $N_j$ are strategic substitutes. □

**Lemma 4.** *The best responses of each country satisfy* $0 < N_i^* < \infty$ *for* $i = A, B$ *if Assumption RANGE holds, i.e. if* $1 + c(1-q) < R \leq 1/q$.

*Proof.* **Part 1** If $R \leq 1/q$ then $N_i^* < \infty$.
Let's first define $M = N_A + N_B$ such that

$$P^*(M) = c \tag{8.63}$$

Fix some $N_j$ and consider two exhaustive cases:

*Case 1* $N_j < M$.
Define

$$\overline{N}_i = M - N_j \tag{8.64}$$

Consider regulators objective function as $N_i \to \overline{N}_i$

$$\lim_{Ni \to \overline{N}_i} W_i(N_i, N_j) = \lim_{Ni \to \overline{N}_i} q(R-1)N_i + (1-q)[R\gamma^*(N_i, N_j)N_i - N_i] \tag{8.65}$$

Now note that

$$\lim_{Ni \to \overline{N}_i} \gamma^*(N_i, N_j) = \lim_{Ni \to \overline{N}_i} \left(1 - \frac{c}{P^*(N_i, N_j)}\right) = 0 \tag{8.66}$$

since $\lim_{Ni, N_j \to M} P^*(N_i, N_j) = c$ by definition in (8.63). Therefore

$$\lim_{Ni \to \overline{N}_i} W_i(N_i, N_j) = \lim_{Ni \to \overline{N}_i} (qR-1)N_i \leq 0 \tag{8.67}$$

since $qR \leq 1$ by Assumption *RANGE*. Hence, it is never optimal to choose $N_i^* \geq \overline{N}_i$ in this case.

*Case 2* $N_j \geq M$.

By definition of $M$ this implies that $P^*(N_i, N_j) < c$ for any $N_i > 0$. In this case banks optimally discard all capital at $t = 1$, i.e. $\chi^* = 0$ which implies that $\gamma^*(N_i, N_j) = 0$. Hence social welfare in country $i$ will be given by

$$W_i(N_i, N_j) = (qR-1)N_i \leq 0 \tag{8.68}$$

since $qR \leq 1$ by Assumption *RANGE*. Hence, $N_i^* = 0$ in this case. Therefore we conclude proof of Part 1 by showing that $N_i^* < \infty$ for $i = A, B$ as long as $qR \leq 1$.

**Part 2** For the second part of the proof I will show that welfare in country $i$ is always decreasing in $N_i$ when $R < 1 + c(1-q)$, and hence the best responses will be given by $N_i^* = N_j^* = 0$. Remember

$$W_i(N_i, N_j) = q(R-1)N_i + (1-q)[R\gamma^*(N_i, N_j)N_i - N_i] \tag{8.69}$$

Note that the highest value of $\gamma^*(N_i, N_j)$ will be obtained as $N_i, N_j \to 0$. Therefore

$$\text{If } W_i(0,0) < 0 \text{ then } W_i(N_i, N_j) < 0 \text{ for all } N_i, N_j > 0 \tag{8.70}$$

Consider first

$$\lim_{Ni, N_j \to 0} \gamma^*(N_i, N_j) = \lim_{Ni, N_j \to 0} \left(1 - \frac{c}{P^*(N_i, N_j)}\right) = 1 - \frac{c}{R} \tag{8.71}$$

since $\lim_{Ni, N_j \to 0} P^*(N_i, N_j) = R$. Using this we can write

$$\lim_{Ni, N_j \to 0} W_i(N_i, N_j) = \left\{q(R-1) + (1-q)[R\left(1 - \frac{c}{R}\right) - 1]\right\} N_i \tag{8.72}$$

from which we can obtain $\lim_{Ni, N_j \to 0} W_i(N_i, N_j) < 0$ as long as

$$q(R-1) + (1-q)[R\left(1 - \frac{c}{R}\right) - 1] < 0 \tag{8.73}$$

Re-arranging this inequality gives

$$qR + (1-q)R + (1-q)c - 1 < 0 \tag{8.74}$$

where further simplification implies $R < 1 + c(1-q)$.

Hence, we conclude that $N_i^*(N_j) = 0$ for $i = A, B$ as long as $R < 1 + c(1-q)$. □

**Proposition 2.** *Under Assumptions CONC, ELAST, REG and RANGE, at least one pure strategy Nash Equilibrium exists in the game between two financial regulator at $t = 0$. Moreover there exists at least one symmetric pure strategy NE.*

*Proof.* For this proof, I make use of a theorem due to Debreu (1952) which states that *"Suppose that for each player the strategy space is compact and convex and the payoff function is continuous and quasi-concave with respect to each player's own strategy. Then there exists at least one pure strategy NE in the game."*

I establish below that this game satisfies all three conditions stated in this theorem.

(i) Following Lemma 4 we can restrict strategy space for each regulator to $[0, M]$ which is compact and convex.

(ii) Continuity of the objective function is obvious.

(iii) For concavity we evaluate the second derivative of the objective function with respect to the own action:

$$\frac{\partial^2 W_i(N_i, N_j)}{\partial N_i^2} = (1-q)R\frac{\partial v^i(N_i, N_j)}{\partial N_i} < 0 \tag{8.75}$$

as shown by Lemma 3 above. Hence Nash Equilibria equilibria exist. Existence of a symmetric Nash Equilibrium equilibrium is implied by the symmetry of the game. □

**Proposition 3.** *Under Assumptions CONC, ELAST, REG and RANGE, there exists a unique symmetric NE of the game between the regulators at $t = 0$.*

*Proof.* I will make use of the theorem that states: *"If the best response mapping is a contraction on the entire strategy space, there is a unique Nash Equilibrium in the game."*

In two-player games best response functions are contraction everywhere if the absolute value of their slopes are less than one everywhere. In order to show this, I make use of the nice feature of equilibrium price function that it is determined only by the sum of the investment levels in the two countries.

$$\left|\frac{\partial N_i^*(N_j)}{\partial N_j}\right| = \left|-\frac{\frac{\partial^2 W_i(N_i,N_j)}{\partial N_i \partial N_j}}{\frac{\partial^2 W_i(N_i,N_j)}{\partial N_i^2}}\right| < 1 \tag{8.76}$$

which can be equivalently stated as

$$\left|\frac{\partial^2 W_i(N_i, N_j)}{\partial N_i \partial N_j}\right| < \left|\frac{\partial^2 W_i(N_i, N_j)}{\partial N_i^2}\right| \tag{8.77}$$

Using the expressions for these derivatives given before this corresponds to

$$\left|\frac{\partial^2\gamma^*(N_i,N_j)}{\partial N_i\partial N_j}N_i+\frac{\partial\gamma^*(N_i,N_j)}{\partial N_j}\right|<\left|\frac{\partial^2\gamma^*(N_i,N_j)}{\partial N_i^2}N_i+2\frac{\partial\gamma^*(N_i,N_j)}{\partial N_i}\right| \tag{8.78}$$

Note that derivative on the right hand side is negative by Lemma 3 and the derivative on the left hand side is negative by Proposition 1. Moreover, $\gamma^*(N_i,N_j)$ and its derivatives are determined only by the sum of the two investment levels, not by their individual values. This implies that

$$\frac{\partial^2\gamma^*(N_i,N_j)}{\partial N_j\partial N_i}=\frac{\partial^2\gamma^*(N_i,N_j)}{\partial N_i^2}\text{ and }\frac{\partial\gamma^*(N_i,N_j)}{\partial N_j}=\frac{\partial\gamma^*(N_i,N_j)}{\partial N_i} \tag{8.79}$$

which implies that $|LHS|<|RHS|$ in (8.78) and hence the slope of best response functions is less than one everywhere on the domain. □

**Proposition 4.** *Non-cooperative symmetric equilibrium investment levels are increasing in $q$ and $R$.*

*Proof.* Using Cramer's rule on FOCs we get

$$\frac{\partial\widehat{N}_i}{\partial\alpha}=-\frac{\frac{\partial^2W_i}{\partial N_i\partial\alpha}\frac{\partial^2W_j}{\partial N_j^2}-\frac{\partial^2W_i}{\partial N_i\partial N_j}\frac{\partial^2W_j}{\partial N_j\partial\alpha}}{\frac{\partial^2W_i}{\partial N_i^2}\frac{\partial^2W_j}{\partial N_j^2}-\frac{\partial^2W_i}{\partial N_i\partial N_j}\frac{\partial^2W_j}{\partial N_j\partial N_i}} \tag{8.80}$$

where $\alpha\in\{q,R,c\}$ is a parameter of the model. First note that

$$\frac{\partial^2W_i}{\partial N_i^2}<0,\text{ and }\frac{\partial^2W_j}{\partial N_j\partial N_i}<0\text{ for }i=A,B \tag{8.81}$$

by Lemma 3 and Proposition 1. Moreover in the proof of Proposition 3 we have shown that

$$\left|\frac{\partial^2W_i(N_i,N_j)}{\partial N_i\partial N_j}\right|<\left|\frac{\partial^2W_i(N_i,N_j)}{\partial N_i^2}\right| \tag{8.82}$$

which implies that the sign of the denominator above is positive. Moreover, in a symmetric equilibrium we will have

$$\frac{\partial^2W_i}{\partial N_i\partial\alpha}=\frac{\partial^2W_j}{\partial N_j\partial\alpha}\quad\text{and}\quad\frac{\partial^2W_j}{\partial N_j^2}=\frac{\partial^2W_i}{\partial N_i^2} \tag{8.83}$$

which allows us to write the derivative as

$$\frac{\partial\widehat{N}_i}{\partial\alpha}=-\frac{\frac{\partial^2W_i}{\partial N_i\partial\alpha}\overbrace{\left[\frac{\partial^2W_i}{\partial N_i^2}-\frac{\partial^2W_i}{\partial N_i\partial N_j}\right]}^{(-)}}{\underbrace{\frac{\partial^2W_i}{\partial N_i^2}\frac{\partial^2W_j}{\partial N_j^2}-\frac{\partial^2W_i}{\partial N_i\partial N_j}\frac{\partial^2W_j}{\partial N_j\partial N_i}}_{(+)}} \tag{8.84}$$

the term inside the brackets in the numerator is again negative by the inequality (8.82). Therefore

the sing of the derivative above will be equal to the sign of $\partial^2 W_i/\partial N_i \partial \alpha$. To obtain this sign consider the FOCs of regulators' problem

$$\frac{\partial W_i(N_i, N_j)}{\partial N_i} = qR + (1-q)Rv^i(N_i, N_j) - 1 \tag{8.85}$$

which will imply that in equilibrium

$$v^i(\widehat{N}_i, \widehat{N}_j) = \frac{1 - qR}{(1-q)R} \tag{8.86}$$

Therefore we can obtain that

$$\begin{aligned} \frac{\partial W_i(N_i, N_j)}{\partial N_i \partial R} &= q + (1-q)v^i(N_i, N_j) \\ &= q + \frac{1-qR}{R} = \frac{1}{R} > 0 \end{aligned} \tag{8.87}$$

in equilibrium, using equation (8.45). Hence we conclude that $\partial \widehat{N}_i/\partial R > 0$.

For comparative statics with respect to $q$ consider

$$\frac{\partial W_i(N_i, N_j)}{\partial N_i \partial q} = R - Rv^i(N_i, N_j) \tag{8.88}$$

which in equilibrium, using equation (8.45) we can write as

$$\frac{\partial W_i(\widehat{N}_i, \widehat{N}_j)}{\partial N_i \partial q} = R\left[1 - \frac{1-qR}{(1-q)R}\right] \tag{8.89}$$

$$= \frac{R-1}{1-q} > 0 \tag{8.90}$$

hence we can also conclude that

$$\frac{\partial \widehat{N}_i}{\partial q} > 0 \text{ for } i = A, B$$

i.e. equilibrium investment levels in both countries are increasing as the probability of good state rises. □

**Lemma 5.** *Under Assumptions CONC, ELAST, REG and RANGE, equilibrium price of assets satisfy $P^* > c$.*

*Proof.* By Proposition 4 we have established that equilibrium investment levels are increasing in both $q$ and $R$. Since Assumption *RANGE* restricts $qR \leq 1$, for a given set of other parameters we will obtain the highest investment in equilibrium if $qR = 1$. Consider the FOCs of regulators'

problem in this case

$$\begin{aligned}\frac{\partial W_i(N_i,N_j)}{\partial N_i} &= qR + (1-q)Rv^i(N_i,N_j) - 1 = 0 \\ &= (R-1)v^i(N_i,N_j) = 0 \\ &= \frac{\partial \gamma^*(N_i,N_j)}{\partial N_i}N_i + \gamma^*(N_i,N_j) = 0\end{aligned} \tag{8.91}$$

Lemma 3 has shown that

$$\frac{\partial \gamma^*(N_i,N_j)}{\partial N_i} < 0$$

Therefore for the FOCs above to hold we need

$$\gamma^*(N_i,N_j) = 1 - \frac{c}{P^*(N_i,N_j)} > 0 \tag{8.92}$$

which implies that $P^*(N_i,N_j) > c$ as needed. □

**Proposition 5.** $\widetilde{N} < \widehat{N}$, *i.e. independent national regulators choose a higher investment in equilibrium compared to the cooperative benchmark.*

*Proof.* For this proof I use the alternative formulation of central regulator's problem which I reproduce here for convenience

$$\max_{N\geq 0} W(N) = q(R-1)N + (1-q)[R\widetilde{\gamma}^*(N)N - N] \tag{8.93}$$

The FOCs for this problem will be given by

$$\left.\frac{\partial W(\cdot)}{\partial N}\right|_{\overline{N}} = qR + (1-q)R\widetilde{v}(\overline{N}) - 1 = 0 \tag{8.94}$$

where $\overline{N}$ is the optimal total investment level in the two countries. Rearranging the FOCs gives that

$$\widetilde{v}(\overline{N}) \equiv \widetilde{\gamma}^{*\prime}(\overline{N})\overline{N} + \widetilde{\gamma}^*(\overline{N}) = \frac{1-qR}{(1-q)R} \tag{8.95}$$

where $\widetilde{v}(\cdot)$ is similar to the function $v^i(N_i,N_j)$ except that it is defined over the total investment level in the two countries. The same is true for the function $\widetilde{\gamma}^*(\cdot)$. Now using the fact that globally optimal investment level in each country will satisfy $\widetilde{N} = \overline{N}/2$ we can write

$$\widetilde{v}(\overline{N}) = \widetilde{v}(2\widetilde{N}) = \gamma'(2\widetilde{N})(2\widetilde{N}) + \gamma(2\widetilde{N}) \tag{8.96}$$

Note that equilibrium fraction of rescued assets, $\widetilde{\gamma}^*(\cdot)$ , is determined by the sum of the investment levels in the two countries. In other words, exact division of global investment between the two countries do not affect $\widetilde{\gamma}^*(\cdot)$ and hence its derivatives. This property allows us to write $\widetilde{v}(\overline{N})$

as (with a slight abuse of notation)

$$\begin{aligned}\widetilde{v}(\overline{N}) &= \widetilde{v}(2\widetilde{N}) = \widetilde{\gamma}^{*\prime}(2\widetilde{N})(2\widetilde{N}) + \widetilde{\gamma}^{*}(2\widetilde{N}) \\ &= 2\gamma_1^*(\widetilde{N},\widetilde{N})\widetilde{N} + \gamma^*(\widetilde{N},\widetilde{N}) \\ &= v(\widetilde{N},\widetilde{N}) + \gamma_1^*(\widetilde{N},\widetilde{N})\widetilde{N} \end{aligned} \tag{8.97}$$

On the other hand, remember that each independent regulator's FOCs will give us

$$\left.\frac{\partial W_i(\cdot)}{\partial N_i}\right|_{(\widehat{N}_i,\widehat{N}_j)} = qR + (1-q)Rv^i(\widehat{N}_i,\widehat{N}_j) - 1 = 0 \tag{8.98}$$

from which we get that in equilibrium

$$v^i(\widehat{N}_i,\widehat{N}_j) = \frac{1-qR}{(1-q)R} \tag{8.99}$$

Comparing (8.99) and (8.95) we see that $\widetilde{v}(\overline{N}) = v^i(\widehat{N},\widehat{N})$ where $(\widehat{N},\widehat{N})$ are the symmetric Nash equilibrium investment levels. Using this together with equality (8.97) we can write

$$\begin{aligned} v(\widetilde{N},\widetilde{N}) &= \widetilde{v}(\overline{N}) - \gamma_1(\widetilde{N},\widetilde{N})\widetilde{N} \\ &= v^i(\widehat{N},\widehat{N}) - \gamma_1 = (\widetilde{N},\widetilde{N})\widetilde{N} \end{aligned} \tag{8.100}$$

we have previously shown that $\gamma'(N_i,N_j) < 0$ which implies that $v(\widetilde{N},\widetilde{N}) > v(\widehat{N},\widehat{N})$. Lemma 3 has shown that $v^i(N_i,N_j)$ is decreasing in $N_i$, and Proposition 1 has shown that $v^i(N_i,N_j)$ is decreasing in $N_j$. Since $v^i(N_i,N_j)$ is decreasing in both arguments, we can conclude that $\widetilde{N} < \widehat{N}$. □

**Proposition 6.** *If the countries are symmetric then both regulators prefer to deliver their authority to a central systemic risk regulator, i.e.* $W_i(\widetilde{N}_i,\widetilde{N}_j) \geq W_i(\widehat{N}_i,\widehat{N}_j)$ *holds for* $i = A, B$.

*Proof.* First, let's define $\widetilde{W}_i \equiv W_i(\widetilde{N}_i,\widetilde{N}_j)$ as the welfare of country $i = A, B$ under central regulation and $\widehat{W}_i \equiv W_i(\widehat{N}_i,\widehat{N}_j)$ as the welfare of country $i = A, B$ under the symmetric non-cooperative equilibrium. Note that by symmetry we have

$$\widetilde{W}_A = \widetilde{W}_B \text{ and } \widehat{W}_A = \widehat{W}_B$$

Since central regulation levels, $(\widetilde{N}_A,\widetilde{N}_B)$, maximize the total welfare of the two countries, $W_A+W_B$, by definition we know that

$$\widetilde{W}_A + \widetilde{W}_B \geq \widehat{W}_A + \widehat{W}_B$$

which implies that $\widetilde{W}_i \geq \widehat{W}_i$ for $i = A, B$. □

### 8.3.2 Proofs for the Asymmetric Countries Case

**Proposition 7.** *If* $R_A > R_B$, *then* $\widehat{N}_A > \widehat{N}_B$ *in non-cooperative equilibrium.*

*Proof.* Remember the FOCs of regulator's problem

$$\frac{\partial W_i(N_i,N_j)}{\partial N_i} = qR + (1-q)Rv^i(N_i,N_j) - 1 \tag{8.101}$$

which will imply that in equilibrium we have

$$v^i(N_i, N_j) = \frac{1 - qR_i}{(1-q)R_i} \equiv \beta_i \tag{8.102}$$

Note that $\beta_i$ is decreasing in $R$. Since $R_A > R_B$ by assumption, we will have that $\beta_A < \beta_B$. Remember that by $v^i(N_i, N_j)$ is defined as

$$v^i(N_i, N_j) \equiv \frac{\partial \gamma^*(N_i, N_j)}{\partial N_i} N_i + \gamma^*(N_i, N_j) \tag{8.103}$$

$\beta_A < \beta_B$ implies that $v^A(\widehat{N}_A, \widehat{N}_B) < v^B(\widehat{N}_B, \widehat{N}_A)$. Using the expression above this is equivalent to

$$\left.\frac{\partial \gamma^*(N_A, N_B)}{\partial N_A}\right|_{\widehat{N}_A, \widehat{N}_B} \widehat{N}_A + \gamma^*(\widehat{N}_A, \widehat{N}_B) < \left.\frac{\partial \gamma^*(N_B, N_A)}{\partial N_B}\right|_{\widehat{N}_B, \widehat{N}_A} \widehat{N}_B + \gamma^*(\widehat{N}_B, \widehat{N}_A) \tag{8.104}$$

Since $\gamma^*()$, and its derivatives determined only by the sum of the regulation levels, we will have

$$\gamma^*(\widehat{N}_A, \widehat{N}_B) = \gamma^*(\widehat{N}_B, \widehat{N}_A) \tag{8.105}$$

and

$$\left.\frac{\partial \gamma^*(N_A, N_B)}{\partial N_A}\right|_{\widehat{N}_A, \widehat{N}_B} = \left.\frac{\partial \gamma^*(N_B, N_A)}{\partial N_B}\right|_{\widehat{N}_B, \widehat{N}_A} \tag{8.106}$$

Using these two equalities in (8.104) gives

$$\left.\frac{\partial \gamma^*(N_A, N_B)}{\partial N_A}\right|_{\widehat{N}_A, \widehat{N}_B} \left(\widehat{N}_A - \widehat{N}_B\right) < 0 \tag{8.107}$$

From Lemma 3 we know that

$$\left.\frac{\partial \gamma^*(N_A, N_B)}{\partial N_A}\right|_{\widehat{N}_A, \widehat{N}_B} < 0 \tag{8.108}$$

Therefore, for inequality (8.107) to be true we need to have $\widehat{N}_A - \widehat{N}_B > 0$, or equivalently $\widehat{N}_A > \widehat{N}_B$. □

**Proposition 8.** *There exists no central regulation level* $N > \min\{\widehat{N}_A, \widehat{N}_B\}$.

*Proof.* By Envelope Theorem we will have that

$$\frac{dW_i(N_i^*(N_j), N_j)}{dN_j} = \left.\frac{\partial W_i(N_i, N_j)}{\partial N_j}\right|_{N_i = N_i^*(N_j)} < 0 \tag{8.109}$$

By assumption we have $\widehat{N}_A > \widehat{N}_B$. Consider a central regulation $N = \widehat{N}_B$. From above we get that $W_B(\widehat{N}_B, \widehat{N}_B) > W_B(\widehat{N}_B, \widehat{N}_A)$. However, by definition $W_A(\widehat{N}_B, \widehat{N}_B) < W_A(\widehat{N}_A, \widehat{N}_B)$, i.e. a central regulation with $N = \widehat{N}_B$ is rejected by Regulator A. Now consider $N > \widehat{N}_B$. By Envelope Theorem we have that

$$W_A(N_A^*(N), N) < W_A(\widehat{N}_A, \widehat{N}_B) \tag{8.110}$$

Moreover, by definition

$$W_A(N,N) < W_A(N_A^*(N),N) \tag{8.111}$$

Hence, any common regulation such that $N > \widehat{N}_B$ will also be rejected by regulator A. □

**Lemma 6.** *For any common regulation level $N$ such that $W_A(N,N) > W_A(\widehat{N}_A,\widehat{N}_B)$ we have $W_B(N,N) > W_B(\widehat{N}_B,\widehat{N}_A)$.*

*Proof.* Suppose that $W_A(N,N) > W_A(\widehat{N}_A,\widehat{N}_B)$ for some $N$. From Proposition 8 we know that such $N$ must satisfy $N < N_B$. Note that we can obtain

$$W_A(\widehat{N}_A,\widehat{N}_B) > W_A(N_A^*(\widehat{N}_A),\widehat{N}_A) > W_A(\widehat{N}_B,\widehat{N}_A) \tag{8.112}$$

where the first inequality is follows from Envelope Theorem given by (8.110), and the second is by definition of optimality. Remember that

$$W_i(N_i,N_j) = qR_iN_i + (1-q)R_i\gamma^*(N_i,N_j)N_i - N_i \tag{8.113}$$

Hence, $W_A(N,N) - W_A(\widehat{N}_B,\widehat{N}_A)$ will be given by

$$qR_A[N-\widehat{N}_B] + (1-q)R_A[\gamma^*(N,N)N - \gamma^*(\widehat{N}_B,\widehat{N}_A)\widehat{N}_B] - [N-\widehat{N}_B] > 0 \tag{8.114}$$

Which we can re-arrange and write as

$$(1-qR_A)[\widehat{N}_B - N] + (1-q)R_A[\gamma^*(N,N)N - \gamma^*(\widehat{N}_B,\widehat{N}_A)\widehat{N}_B] \tag{8.115}$$

Now consider $W_B(N,N) - W_B(\widehat{N}_B,\widehat{N}_A)$ which will be equal to

$$(1-qR_B)[\widehat{N}_B - N] + (1-q)R_B[\gamma^*(N,N)N - \gamma^*(\widehat{N}_B,\widehat{N}_A)\widehat{N}_B] \tag{8.116}$$

Now, let's compare (8.115) and (8.116). First note that the first terms are positive in both of them. $\widehat{N}_B - N > 0$ as argued above and it receives a higher weight in (8.116) since $1-qR_B > 1-qR_A$ due to our assumption that $R_A > R_B$. Now, if the second terms are also positive, (8.116) will be positive and we are done. However, if second terms are negative, we know that this second term receives a higher weight in (8.115). Hence, if (8.115) is positive in spite of this higher weight on the negative term, (8.116) will necessarily be positive, since it carries a lower weight on the negative term. □

**Proposition 9.** *Suppose that $F'(0) = 1$. Let $s \equiv R_A - R_B > 0$. Then for any $R_A$, there exists $\widehat{s} \in (0, R_A - 1)$ such that $W_A(N^m,N^m) - W_A(\widehat{N}_A,\widehat{N}_B) \geq 0$ if $s \leq \widehat{s}$, and $W_A(N^m,N^m) - W_A(\widehat{N}_A,\widehat{N}_B) < 0$ otherwise.*

*Proof.* Let's see how this difference changes with $s$

$$\frac{d[W_A(N^m,N^m) - W_A(\widehat{N}_A,\widehat{N}_B)]}{ds} \tag{8.117}$$

this derivative will be given by

$$W_{A1}\frac{\partial N^m}{\partial s} + W_{A2}\frac{\partial N^m}{\partial s} + \left.\frac{\partial W_A}{\partial s}\right|_{N^m,N^m} - W_{A1}\frac{d\widehat{N}_A}{ds} - W_{A2}\frac{d\widehat{N}_B}{ds} - \left.\frac{\partial W_A}{\partial s}\right|_{\widehat{N}_A,\widehat{N}_B} \tag{8.118}$$

Applying envelope theorem reduces this to

$$\left.\frac{\partial W_A}{\partial s}\right|_{N^m,N^m} - W_{A2}\frac{d\widehat{N}_B}{ds} - \left.\frac{\partial W_A}{\partial s}\right|_{\widehat{N}_A,\widehat{N}_B} \tag{8.119}$$

Now note that since

$$W_A(N_A,N_B) = q(R_A-1)N_A + (1-q)[R_A\gamma^*(N_A,N_B)N_A - N_A] \tag{8.120}$$

we will have that

$$\left.\frac{\partial W_A}{\partial s}\right|_{N^m,N^m} = 0 \text{ and } \left.\frac{\partial W_A}{\partial s}\right|_{\widehat{N}_A,\widehat{N}_B} = 0 \tag{8.121}$$

Therefore

$$\frac{d[W_A(N^m,N^m) - W_A(\widehat{N}_A,\widehat{N}_B)]}{ds} = -W_{A2}\frac{d\widehat{N}_B}{ds} \tag{8.122}$$

First note that

$$W_{A2} \equiv \frac{\partial W_A(N,N)}{\partial N_B} = (1-q)R\frac{\partial\gamma^*(N_A,N_B)}{\partial N_B}N_A < 0 \tag{8.123}$$

Moreover, by Proposition 4 we have shown that $\widehat{N}_i$ is decreasing in $R$ for $i = A,B$. This will imply that as $s$ increases (or $R_2$ decreases), regulator 2 will choose a lower $\widehat{N}_B$ in equilibrium, i.e. $d\widehat{N}_B/ds < 0$.Therefore,

$$\frac{d[W_A(N^m,N^m) - W_A(\widehat{N}_A,\widehat{N}_B)]}{ds} = -W_{A2}\frac{d\widehat{N}_B}{ds} < 0 \tag{8.124}$$

Hence, the benefit from cooperating is decreasing in the difference between the two countries. Therefore if $W_A(N^m,N^m) - W_A(\widehat{N}_A,\widehat{N}_B) = 0$ for some $\widehat{s}$ it will be negative for $s \geq \widehat{s}$.

We will show that such an $\widehat{s}$ exists. It is clear that for $s = 0$ we have $W_A(N^m,N^m) - W_A(\widehat{N}_A,\widehat{N}_B) > 0$. On the other hand, as $R_B \to 1 + c(1-q)$, we can argue that $\widehat{N}_B$ will necessarily become zero. From Proposition 11 we know that for central regulation be acceptable by regulator 1, we must have $N^m < \widehat{N}_B$. However, as $\widehat{N}_B \to 0$, it won't be possible to reduce $\widehat{N}_B$ sufficiently to compensate regulator 1. Therefore, we will have $W_A(N^m,N^m) - W_A(\widehat{N}_A,\widehat{N}_B) < 0$ for sufficiently large $s$. By continuity an $\widehat{s}$ such that $W_A(N^m,N^m) - W_A(\widehat{N}_A,\widehat{N}_B) = 0$ must exist. □

### 8.3.3 Proof of the Systemic Failure Exercise

**Proposition 10.** *Let $F(y) = R\ln(1+y)$. If $1+c < R < \widehat{R}$ then there exists a $\widehat{q} \in (0,1/R)$ such that for all $q \geq \widehat{q}$ we have that $\widehat{N}(q) \geq \widehat{N}^c$.In other words, if the probability of the good state is higher than $\widehat{q}$, banks fail in the bad state in non-competitive equilibrium. If $R \leq 1+c$ then banks always fail in the bad state, and if $R \geq \widehat{R}$ then banks never fail in the bad state where $\widehat{R}$ is given by*

$$\widehat{R} \equiv \frac{1}{2}\left(2 + c + \sqrt{c}\sqrt{8+c}\right) \tag{8.125}$$

*Proof.* By Proposition 4 we have already shown that equilibrium investment level is increasing in $q$ and $R$. Fix some $R$. Note that, if $\widehat{N}(R,1/R,c) < \widehat{N}^c$, then $\widehat{N}(R,q,c) < \widehat{N}^c$ for all $q \in [0,1/R]$ since $\widehat{N}$ is increasing in $q$.

I will first show that when $q = 1/R$, the difference $\widehat{N}^c - \widehat{N}$ is monotonically increasing in $R$. Moreover, it is negative when $R$ is small and positive for sufficiently high $R$. Therefore, we will establish using Intermediate Value Theorem that there exists some $\widehat{R}$ such that $\widehat{N}^c(\widehat{R}, c) - \widehat{N}(\widehat{R}, 1/\widehat{R}, c) = 0$. Using definitions of $\widehat{N}^c$ and $\widehat{N}$ we can write this difference as

$$\widehat{N}^c - \widehat{N} = \frac{aR}{2c}\left(\frac{R-1-c}{R-1}\right) - \frac{a\left(4R - c\sigma - \sqrt{8c\sigma R + (c\sigma)^2}\right)}{8c} \tag{8.126}$$

where

$$\sigma = \frac{(1-q)R}{R-1}$$

as defined before. Evaluating this difference at $q = 1/R$ gives that

$$\widehat{N}^c - \widehat{N} = \frac{aR}{2c}\left(\frac{R-1-c}{R-1}\right) - \frac{a\left(4R - c - \sqrt{8cR + c^2}\right)}{8c} \tag{8.127}$$

We can determine the behavior of this difference as $R$ changes by looking at the derivative with respect to $R$ which will be given by

$$\begin{aligned}\frac{\partial\left(\widehat{N}^c - \widehat{N}\right)}{\partial R} &= \frac{aR}{2c}\left(\frac{R-1-c}{R-1} + \frac{cR}{(R-1)^2} - 1 + \frac{c}{\sqrt{8cR+c^2}}\right) \\ &= \frac{aR}{2c}\left(\frac{(R-1)^2 + c}{(R-1)^2} - 1 + \frac{\sqrt{8cR+c^2}}{c}\right) > 0\end{aligned} \tag{8.128}$$

This derivative is clearly positive, which means that the difference is monotonically increasing in $R$. Moreover, note that given $q = 1/R$

$$\lim_{R \to 1+c}\left\{\widehat{N}^c(R,c) - \widehat{N}(R,q,c)\right\} < 0 \tag{8.129}$$

This follows since $\widehat{N}^c(R,c) \to 0$ as $R \to 1+c$, which can be clearly seen from the definition of $\widehat{N}^c(R,c)$. We know that $\widehat{N}(R,q,c) > 0$ as $R \to 1+c$ since by Proposition 3 we have established that equilibrium investment levels are positive as long as $R > 1 + c(1-q)$. To complete the argument, I will show that when $q = 1/R$

$$\lim_{R \to \infty}\left\{\widehat{N}^c(R,c) - \widehat{N}(R,q,c)\right\} > 0 \tag{8.130}$$

In order to see this we can expand the difference $\widehat{N}^c - \widehat{N}$ given by (8.127) to write it as

$$\begin{aligned}\widehat{N}^c - \widehat{N} &= \frac{aR}{2c}\left(\frac{R-1-c}{R-1} - 1 - \frac{c}{4} + \frac{\sqrt{8cR+c^2}}{4}\right) \\ &= \frac{aR}{2c}\left(\frac{-c}{R-1} - \frac{c}{4} + \frac{\sqrt{8cR+c^2}}{4}\right)\end{aligned} \tag{8.131}$$

Now, note that as $R$ becomes large the first term inside the parenthesis goes to zero whereas the last term goes to infinity. Therefore this difference is definitely positive for large $R$. Now, using the Intermediate Value Theorem we can conclude that there exists some $\widehat{R} > 1 + c$ such that $\widehat{N}(\widehat{R}, 1/\widehat{R}, c) = \widehat{N}^c(\widehat{R}, c)$. Using the definitions of $\widehat{N}$, and $\widehat{N}^c$ from equations (22) and (22) respectively we can solve for $\widehat{R}$ as follows

$$\widehat{R} = \frac{1}{2}\left(2 + c + \sqrt{c}\sqrt{8 + c}\right) \tag{8.132}$$

**Case 1** $R \geq \widehat{R}$

The analysis above implies that for $R \geq \widehat{R}$ we have that $\widehat{N}(R, 1/R, c) \leq \widehat{N}^c(R, c)$. Moreover, since for any given $R$, the highest value of $\widehat{N}$ is obtained when $q = 1/R$, we will also have that for $R \geq \widehat{R}$ we have $\widehat{N}(R, q, c) \leq \widehat{N}^c(R, c)$ for all $q \in [0, 1/R]$. In other words, if $R \geq \widehat{R}$, banks never fail in the bad state for any value of $q \in [0, 1/R]$.

**Case 2** $1 + c < R < \widehat{R}$.

In this case we can show that

$$\lim_{q \to 0} \widehat{N}(R, q, c) < \widehat{N}^c < \lim_{q \to 1/R} \widehat{N}(R, q, c) \tag{8.133}$$

Hence by the Intermediate Value Theorem we will conclude that there exists $\widehat{q} \in (0, 1/R)$ such that $\widehat{N}(\widehat{q}) = \widehat{N}^c$. Therefore, banks will fail in the bad state if $q \geq \widehat{q}$, and they will survive otherwise. First, I will consider the first part of the inequality in (8.133). It will be useful to note that

$$\lim_{q \to 0} \sigma = \frac{R}{R-1} \text{ and } \lim_{q \to 1/R} \sigma = 1 \tag{8.134}$$

Therefore let's consider $\widehat{N}$ when $q = 0$ and check whether it is less than $\widehat{N}^c$

$$\frac{a\left(4R - \frac{cR}{R-1} - \sqrt{8Rc\frac{R}{R-1} + \left(\frac{R}{R-1}\right)^2}\right)}{8c} <^? \frac{aR}{2c}\left(\frac{R-1-c}{R-1}\right) \tag{8.135}$$

where left hand side of the inequality is $\widehat{N}$ evaluated when $q = 0$, and right hand side is $\widehat{N}^c$ given by (22). Simplifying both sides reduces this comparison to

$$4aR - \frac{acR}{R-1} - a\sqrt{8Rc\frac{R}{R-1} + \left(\frac{R}{R-1}\right)^2} <^? 4aR - \frac{4acR}{R-1} \tag{8.136}$$

simplifying further yields

$$\frac{3cR}{R-1} <^? \sqrt{8Rc\frac{R}{R-1} + \left(\frac{cR}{R-1}\right)^2} \tag{8.137}$$

taking square of both sides yields

$$9c^2\left(\frac{R}{R-1}\right)^2 <^? 8Rc\frac{R}{R-1} + c^2\left(\frac{R}{R-1}\right)^2 \tag{8.138}$$

rearranging gives

$$8c^2\left(\frac{R}{R-1}\right)^2 <^? 8Rc\frac{R}{R-1} \tag{8.139}$$

and after cancelling same terms of both sides we get

$$\frac{c}{R-1} <^? 1 \tag{8.140}$$

which is true as long as $1+c<R$ as we proposed for this case.

For the second part of the inequality (8.133) consider $\widehat{N}$ when $q=1/R$ and check when it is greater than $\widehat{N}^c$

$$\frac{a\left(4R-c-\sqrt{8Rc+c^2}\right)}{8c} >^? \frac{aR}{2c}\left(\frac{R-1-c}{R-1}\right) \tag{8.141}$$

where the left hand side of the inequality is $\widehat{N}$ evaluated when $q=1/R$, and right hand side is $\widehat{N}^c$ given by (22). Simplifying reduces this comparison to

$$4aR-ac\ -a\sqrt{8Rc+c^2} >^? 4aR-\frac{4acR}{R-1} \tag{8.142}$$

simplifying further yields

$$c\left(-1+\frac{4R}{R-1}\right) >^? \sqrt{8Rc+c^2} \tag{8.143}$$

solving for $R$ yields that this inequality is true as long as

$$R<\frac{1}{2}\left(2+c+\sqrt{c}\sqrt{8+c}\right)\equiv\widehat{R} \tag{8.144}$$

Therefore we can conclude that if $1+c<R<\widehat{R}$,there exists $\widehat{q}\in(0,1/R)$ such that $\widehat{N}(q)>\widehat{N}^c$ for $q\geq\widehat{q}$. In words, when $1+c<R<\widehat{R}$ banks fail systemically in the bad state if $q\geq\widehat{q}\,(R)$, and they survive if $q<\widehat{q}\,(R)$.

**Case 3** $\quad 1+c(1-q)<R\leq 1+c$

We know that when $1+c(1-q)<R\leq 1+c$ we will have $\widehat{N}^c(R,c)=0$ by definition and $\widehat{N}(R,q,c)>0$ from Proposition 3. Therefore, we will have that $\widehat{N}(R,q,c)\geq\widehat{N}^c$ whenever $1+c(1-q)<R\leq 1+c$ for all $q\in[0,1/R]$. Therefore banks always fail in the bad state in this case. □

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