The Determinants of Bank Capital Structure*

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Abstract

A bank is a financial institution that raises funds through deposits and makes loans. The corporate finance of banks differs from that of firms because they use deposits while firms do not. There is considerable evidence that the market for deposits is segmented from the market for equity and bankruptcy costs for banks are high. We develop a model based on these features and obtain a number of results. Despite everybody being risk neutral, the cost of equity capital is higher than the cost of deposits except in very special cases. When firms' returns are perfectly correlated and they compete with banks for capital in the equity market, the equilibrium allocation involves banks having no equity. If there are two types of firms that have independently distributed returns and firm and bank bankruptcy costs are high, banks do not diversify and lend to one sector only. When firm bankruptcy costs are low but bank bankruptcy costs are high then two-sector banks that diversify across sectors drive out one-sector banks.

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1 Introduction

The characterizing features of a bank are that it is an institution that accepts deposits and makes loans. If a financial intermediary does not take deposits, it is not a bank. Similarly, if it does not make loans. The role of deposits is what distinguishes the corporate finance of banks from the corporate finance of other firms. While both banks and firms use equity and bonds, only banks use deposits.

Although their role has varied over time, deposits remain an important source of funds for banks in all countries. Figure 1 shows the deposits as a proportion of bank liabilities for a number of countries from 1990-2009. It can be seen that Italy has the highest proportion with between 80 and 90 per cent of deposit finance for most of the period until the crisis when it drops sharply. Japan comes next with around 70 to 80 per cent. The proportion of deposit finance for Germany lies between 60 and 70 percent. The U.S. fell from 70 percent to a low of 55 per cent in 2000 but rose again to 65 per cent by the end of the period. France has steadily increased its share of deposit finance from under 55 per cent in 1990 to just under 70 per cent in 2009. In all these countries deposits are the major form of bank finance.

In most countries bank assets are at higher levels than stock market capitalization. Figure 2 shows bank assets over GDP, while Figure 3 shows stock market capitalization over GDP. It can be seen that the U.S. is an outlier with a high proportion of stock market capitalization and a low proportion of bank assets. At the other end of the spectrum, Japan has high level of bank assets and a moderate stock market capitalization. In Germany and Italy bank assets are significantly higher than stock market capitalization. In France this used to be true as well but in 2009 the two are more comparable.

Despite its empirical importance, deposit finance has played very little role in the theory of banking. The market for deposits is significantly segmented from other markets. While most people in developed countries have bank accounts, with the exception of the U.S., relatively few people own stocks or other types of financial assets either directly or

indirectly (see, e.g., Guiso, Halassios and Jappelli (2002)). The markets for deposits and equity thus appear to be highly segmented.

Our analysis also focuses on bankruptcy costs since there is considerable empirical evidence these are of first order importance. They are particularly high for financial institutions. For example, James (1991) finds that when banks are liquidated, bankruptcy costs are 30 cents on the dollar. In a sample of non-financial firms, Andrade and Kaplan (1998) find a range of 10-23 per cent for the ex post bankruptcy costs. Thus it appears that the costs of bankruptcy are higher in banks than in non-financial firms.

There are a number of issues that arise with the measurement of bankruptcy costs. Almeida and Philippon (2007) point out that bankruptcies typically occur in bad times when the marginal utility of consumption is high. This means that the ex ante risk adjusted cost of bankruptcy is significantly higher than the expected cost. For example, they estimate that for a BBB rated firm the costs of distress increase from 1.4 per cent of firm value to 4.5 per cent if risk is accounted for. Acharya, Bharath and Srinivasan (2007) investigate the relationship between industry distress and firm distress. They find that recovery rates are 10-15 per cent lower when the industry is in distress compared to when the firm is in distress alone. Losses due to fire sales of assets are a part of this increase in bankruptcy costs. Glover (2012) points out that firms with lower bankruptcy costs will use more debt and are more likely to be the ones that end up bankrupt. As a result, estimates of bankruptcy costs based on firms that go bankrupt are likely to be biased downward. In his sample, bankruptcy costs for bankrupt firms are 25 per cent of pre-distress value. Adjusting for the bias, he estimates the average cost of bankruptcy to be 45 per cent. Unfortunately, these effects of risk aversion, fire sales and leverage have only been estimated for non-financial firms. It seems likely that they will also be important for banks. Our standard assumption is therefore that the costs of bankruptcy are higher in banks.

In this paper we incorporate a segmented market for deposit finance into the theory of banking where the main role of capital is to reduce bankruptcy costs. In the basic model

banks finance themselves with equity capital and deposits. They invest the proceeds in a risky asset with an expected return above one. The providers of equity capital can invest directly in the risky asset, while the providers of deposits can only invest in an alternative opportunity with a return of one. For simplicity, both groups are risk neutral. There is a fixed supply of equity capital and deposits in the economy.

We start by considering the simplest case where there are full bankruptcy costs for banks so that liquidation proceeds are zero. Banks invest directly in the risky asset and therefore have a risky income stream. The role of equity capital in this case is to reduce bankruptcy costs. We show that the opportunity cost of equity capital in equilibrium is strictly above the expected return on the risky asset. All the equity capital is therefore invested in bank shares. None is invested in the risky asset. If the total supply of equity in the economy is scarce relative to the total amount of deposits, not all deposits will be used in the banking system. Some will be invested in depositors' alternative opportunity and the equilibrium return to depositors will be one. If capital is more plentiful, then all deposits will be used in the banking system and the return on deposits will be bid up above one but it remains below the expected return on the risky asset and hence below the return on equity.

The next version of the model considers the case where bankruptcy costs are intermediate. As bank bankruptcy costs fall, there are two effects. The first is that output is higher because of the lower bankruptcy costs. This means that, when some depositors are investing in their alternative opportunity, capital per bank goes down. This in turn implies that the number of banks increases, more deposits are used in the banking system and the productivity of the economy is higher. The extra output generated by these two channels goes entirely to the shareholders in the form of a higher opportunity cost of capital as the depositors continue to earn one. At the point where all deposits are used in the banking sector, the amount of capital per bank remains constant. The effect of lower bankruptcy costs is then to increase the return on deposits as well as on the return on capital. In this version of the model, it is again the case that the cost of equity capital is greater than the

return on the risky asset, which in turn is greater than the return on deposits. Thus our paper provides a rationale for why equity capital is an expensive form of finance. This is a standard assumption in the literature (see, e.g., Gorton and Winton (2003)).

One explanation for the high cost of equity capital is the problem of adverse selection when funds are raised using equity issues. However, this cannot explain why equity is costly in the long run since banks can always retain earnings until they reach the desired capital ratio. In contrast, our theory is consistent with equity being expensive in the long run. It is important to stress that the result of our theory is that it is expensive relative to deposits. If bonds were introduced into our model and the were not traded in a segmented market, equity would not be expensive relative to them.

In the extreme case where there are no bankruptcy costs, there is no advantage to equity capital in the banking system. This is the only case in our model where bank capital structure is indeterminate. The capital providers can invest directly in risky assets or indirectly through bank shares. Both provide an expected return equal to that on the risky asset. In addition, the depositors also earn an expected return equal to the return on the risky asset.

The next stage in the analysis is to introduce firms that own the assets and raise finance from the equity market and from bank loans. Now equity capital has the role of mitigating bankruptcy costs in the productive sector as well as in the banking sector. We maintain the assumption that bankruptcy costs are higher in the banking sector than in the productive sector. One might expect given this that capital would be higher in the banking sector than in the productive sector, but in fact the reverse is true. The reason is that there are now two stages of bankruptcy, one in firms and one in banks. Bank bankruptcy only occurs when firms default. For this reason it is optimal to put the capital in the firms rather than the banks. In equilibrium banks are conduits with no capital. They set the rate on deposits and the rate on loans equal and go bankrupt together with the firms. The only case where this is not true is where there are no bankruptcy costs in the productive sector. In this case, it is irrelevant whether the capital is held within banks

or firms. For every other level of bankruptcy costs, it is better to put the capital in the firms.

So far the model assumes that all firms' revenues are perfectly correlated. There is in effect one sector. We next consider the case where there are two independent sectors in the economy. These can be thought of as different geographic regions or different industries. Now banks have the possibility to diversify and reduce risk as an alternative to capital. We start by analyzing the case where banks hold assets directly. Here we show that diversification is always optimal. Banks hold a positive amount of capital and shareholders obtain higher returns.

We finally focus on the case where there are two publicly traded sectors where firms raise capital in the equity market and borrow from banks. The returns in these sectors are independent so banks now have the opportunity to diversify and reduce risk in that way. However, when there are full bankruptcy costs in both the banking and the productive sectors, banks do not diversify. We show that it is strictly better for them to specialize, act as conduit banks and lend to one sector only. The reason is that in this case they can align bankruptcy costs in the banking and in each productive sector as previously discussed. However, when there are no bankruptcy costs in the productive sector, it is optimal for banks to diversify and have positive capital. There is thus a critical level of bankruptcy costs in the productive sectors such that banks switch from diversifying across sectors to lending to one sector only. Thus the role of diversification in banking is complex.

The paper proceeds as follows. Section 2 develops the basic model. The equilibrium of this is considered in Section 3. Section 4 considers what happens when firms are publicly traded and compete with banks for capital. The role of diversification when there are two non-publicly traded sectors is considered in Section 5, and when there are two publicly traded sectors in Section 6. Finally, Section 7 contains concluding remarks.

2 A model of bank capital structure with a single non-publicly traded productive sector

In this section we develop a simple one-period model of financial intermediation where banks raise external funds through deposits and capital, and invest in a risky technology.

The risky technology is such that for each unit invested at date 0 there is a stochastic return r at date 1 uniformly distributed on the support $[0, R]$, with $Er = \frac{R}{2} > 1$.

Since there are constant returns to scale we normalize the size of every bank to 1. Each bank finances itself with an amount of capital k_B and an amount of deposits $1 - k_B$. The bank has limited liability. There are two groups of risk neutral investors, shareholders and depositors. The former supply capital to banks. The opportunity cost of capital in the bank equity market is ρ. Shareholders can also invest directly in the risky technology so that $\rho \geq R/2$. The latter supply deposits. The promised per unit rate from the bank is r_D. Depositors have an endowment of 1 each and the opportunity to store this with return 1 for each unit invested so that $u \geq 1$. The two markets are segmented in the sense that depositors do not have access to the equity market. The idea is that they have high participation costs which makes them unwilling to enter the equity market. The capital providers on the other hand have zero participation costs. The total supply of capital is denoted K. The total supply of deposits is D. The ratio of the two is

$$\frac{K}{D} = \eta > 0. \tag{1}$$

Since banks invest in a risky technology, deposits are risky. The bank repays the promised rate r_D if $r \geq \overline{r}_B = r_D(1 - k_B)$, and goes bankrupt otherwise. When it goes bankrupt, the proceeds from liquidation are $h_B r$ with $h_B \in [0, 1]$ and these are distributed pro rata to depositors. The bankruptcy costs are thus $(1 - h_B)r$.

Each bank maximizes its expected profits, which are distributed to the shareholders. There is free entry so that the banking sector is competitive. Depositors maximize their

expected utility and deposit in the bank(s) that give them the highest expected return.

3 The equilibrium with a single productive sector

In this section we analyze the equilibrium of the model. This requires the following:

1. Banks choose k_B and r_D to maximize expected profits.
2. Capital providers maximize expected utility.
3. Depositors maximize expected utility.
4. Banks make zero expected profits in equilibrium.
5. The equity market clears.
6. The deposit market clears.

We start by considering the individual bank's optimization problem:

$$\max_{k_B, r_D} E\Pi = \int_{\overline{r}_B}^{R} (r - r_D(1 - k_B)) \frac{1}{R} dr - \rho k_B \tag{2}$$

subject to

$$EU = \int_{0}^{\overline{r}_B} \frac{h_B r}{1 - k_B} \frac{1}{R} dr + \int_{\overline{r}_B}^{R} r_D \frac{1}{R} dr \geq u \tag{3}$$

$$E\Pi \geq 0 \tag{4}$$

$$0 \leq k_B \leq 1, \tag{5}$$

where

$$\overline{r}_B = r_D(1 - k_B). \tag{6}$$

The bank chooses k_B and r_D to maximize its expected profit net of the cost of funds. The first term is what the bank obtains from the investment after paying $r_D(1-k_B)$ to the depositors. This is positive only for $r > \overline{r}_B$, in which case it is distributed to the shareholders. If $r < \overline{r}_B$, the bank goes bankrupt, depositors receive a pro rata share $\frac{h_B r}{1-k_B}$ of the liquidation proceeds, and shareholders receive nothing. The second term ρk_B is the shareholders' opportunity cost of providing capital. The first constraint (3) requires that the expected utility of depositors is at least equal to their opportunity cost u. The first term is the payoff when the bank goes bankrupt, and the second term is the payment they receive when the bank remains solvent. The third constraint (4) is the requirement that the shareholders obtain their opportunity cost from providing capital to the bank. The last constraint (5) is a feasibility constraint on the amount of capital.

In equilibrium, since there is free entry into the banking market, each bank's expected profit must be zero. This means that ρ adjusts so that $E\Pi = 0$.

Capital providers can either supply equity to the banks for a return of ρ or invest in their alternative opportunity for a return $R/2$. The sum of these two investments must be equal to K for the equity market to clear. Capital providers will invest in bank equity alone if $\rho > R/2$. They will invest both in bank equity and in the alternative opportunity if $\rho = R/2$. In other words,

$$N_B k_B \leq K,$$

where there is an equality when $\rho > R/2$ and a strict inequality otherwise.

Similarly, depositors can either deposit their money in the banks for a promised return of r_D and an expected utility $EU = u \geq 1$, or use their alternative opportunity to store the funds with a return of 1 and an expected utility $EU = 1$. Again, these terms must sum to D for the deposit market to clear. The depositors will just deposit in banks and will not use their alternative opportunity if $u > 1$. They will both deposit and invest in the alternative opportunity if $u = 1$. It will be shown below that the form of the equilibrium

depends on whether the constraint (3) binds with $u = 1$ or $u > 1$. In other words,

$$N_B(1 - k_B) \leq D,$$

where there is an equality when $u > 1$ and a strict inequality otherwise.

3.1 Full bankruptcy costs in the banking sector ($h_B = 0$)

For simplicity, we assume $h_B = 0$ to start with. This corresponds to the case where liquidation proceeds are zero and depositors obtain nothing in the case the bank goes bankrupt. We have the following result.

Proposition 1 *The unique equilibrium with $h_B = 0$ is as follows:*

i) For $R < \overline{R} = \frac{4(1+\eta)}{1+2\eta} < 4, k_B = \frac{4}{R} - 1 \in (0,1), r_D = \frac{R}{2}, \rho = \frac{2}{4-R} > \frac{R}{2}, E\Pi = 0, EU = u = 1, \overline{r}_B = R - 2, N_B k_B = K$ and $N_B(1 - k_B) < D$.

ii) For $R \geq \overline{R}$, $k_B = \frac{\eta}{1+\eta} \in (0,1), r_D = \frac{R}{2}, \rho = \frac{1+4\eta(1+\eta)}{4\eta(1+\eta)}\frac{R}{2} > \frac{R}{2}, u = \frac{1+2\eta}{2+2\eta}\frac{R}{2} \in [1, \frac{R}{2})$, $E\Pi = 0, EU = u > 1$, $\overline{r}_B = \frac{1}{(1+\eta)}\frac{R}{2}, N_B k_B = K$ and $N_B(1 - k_B) = D$.

The proposition shows how the surplus generated from the banks' investments in the risky asset is split between the depositors and the shareholders. When $R \to 2$, $k_B \to 1$ and the surplus approaches zero. Shareholders obtain $\rho \to \frac{R}{2} \to 1$ and depositors put almost all of their funds in their storage alternative. This ensures that $u = 1$. Capital and deposits have a similar opportunity cost but equity allows bankruptcy costs to be avoided. As R increases, banks start to use a greater proportion of deposits. As long as some depositors remain unbanked, shareholders obtain all the surplus. This is higher the greater is R. To ensure banks' expected profits are zero in the competitive capital market, ρ is bid up. Capital is now more costly than with a lower R but the benefits from limiting bankruptcy are also higher because the bankruptcy threshold $\overline{r}_B = r_D(1 - k_B) = R - 2$ goes up as R increases and k_B decreases. Eventually, all deposits are used in the banking sector and no depositors use their alternative storage. The point at which this occurs is

$R = \overline{R}$. For $R > \overline{R}$, the supply of deposits to the banking sector no longer increases and the expected return on deposits u is bid up. Further increases in R also result in u being bid up. The shareholders and depositors now share the surplus. Both u and ρ continue to rise as R increases.

Next consider how the variation in the ratio of total capital to total deposits, η, affects the equilibrium. For $\eta \to 0$, $\overline{R} \to 4$. In the first region deposits are abundant and this ensures that some depositors have to invest in their alternative opportunity so $u \to 1$. In the second region with $R > \overline{R}$, $k_B \to 0$, $\rho \to \infty$, and $u \to 1$. For $\eta \to \infty$, $\overline{R} \to 2$ and the first region in the proposition becomes empty. In the second region, as $\eta \to \infty$, $k_B \to 1$, $\rho \to \frac{R}{2}$ and $u \to \frac{R}{2}$. In other words, as capital becomes more abundant, banks use more and more equity finance, bankruptcy risk falls and both ρ and u tend to $\frac{R}{2}$.

One important point is that all the capital is absorbed in the banking sector because the return ρ is higher there than investing directly in risky assets. In contrast, when $R < \overline{R}$, not all the funds of depositors are placed in banks. Some proportion is invested directly in the alternative opportunity. This is true in all cases below, so we omit the explicit statement in the following propositions. It can easily be seen that $\partial \overline{R}/\partial \eta < 0$. This means that the degree of financial inclusion increases with the amount of capital K in the economy for a given level of deposits D.

3.2 Intermediate bankruptcy costs in the banking sector $(0 < h_B < 1)$

We now extend the basic model to include partial bankruptcy costs in the banking sector, i.e., $h_B \in (0,1)$, so that depositors obtain $\frac{h_B r}{1-k_B}$ when the bank goes bankrupt. We obtain the following result, which is similar in spirit to that in Proposition 1, but algebraically more complex.

Proposition 2 *The unique equilibrium with $h_B \in (0,1)$ is as follows:*

i) For $R \leq \overline{R} = \frac{2(1+\eta)-(1-h_B)\left(1-h_B+\sqrt{4\eta(1+\eta)+(1-h_B)^2}\right)}{(1+\eta)h_B}, k_B = \frac{(2-h_B R)(2(2-h_B)-R)}{2(1-h_B)^2 R} \in (0,1), r_D = \frac{2(1-h_B)R}{2(2-h_B)-h_B R} < R, \rho = \frac{2-h_B R}{2(2-h_B)-R} > \frac{R}{2}, E\Pi = 0, EU = u = 1$ and $\overline{r}_B = \frac{R-2}{1-h_B}$.

ii) For $R > \overline{R}$, $k_B = \frac{\eta}{1+\eta} \in (0,1), r_D = \frac{2u(1-h_B)R}{2u(2-h_B)-h_BR} < R, \rho = \frac{u(2u-h_BR)}{2u(2-h_B)-R} > \frac{R}{2}$, $E\Pi = 0, EU = u = \frac{2(1+\eta)-(1-h_B)\left(1-h_B-\sqrt{4\eta(1+\eta)+(1-h_B)^2}\right)}{2(1+\eta)(2-h_B)}\frac{R}{2} \in (1, \frac{R}{2})$ *and* $\overline{r}_B = \frac{2u(1-h_B)R}{(1+\eta)(2u(2-h_B)-h_BR)}$.

The result is similar to that in Proposition 1. As before, the sharing of the surplus generated from the banks' investments in the risky asset and banks' capital structure depend on the size of the return R. When $R \to 2$, $k_B \to 1$ and $\rho \to \frac{R}{2} = 1$ for any $h_B < 1$. Depositors invest almost entirely in the storage technology and $u = 1$. As R increases, banks start using less capital and more deposits. The surplus from the banks' investment increases and goes entirely to the shareholders in the form of a higher ρ till R approaches $\overline{R}$. After that, deposits are used up entirely in the banking sector and the increasing surplus is split between consumers and shareholders.

Differently from before, banks' capital structure and the sharing of the surplus depend on the size of the bankruptcy proceeds as represented by h_B. For a given $R \leq \overline{R}$, the higher h_B the lower the amount of capital k_B at each bank and the higher the shareholders' return ρ. For a given $R > \overline{R}$, k_B keeps decreasing with h_B and both shareholders and depositors obtain higher returns ρ and u. The intuition is simple. As bankruptcy proceeds increase, capital becomes less necessary as a way to reduce bankruptcy costs and thus each bank uses less of it. This increases the number of banks $N_B = \frac{K}{k_B}$ in the economy, the output produced and the return to shareholders and to depositors when $R > \overline{R}$.

3.3 No bankruptcy costs in the banking sector ($h_B = 1$)

We next consider the case where there are no bankruptcy costs so that $h_B = 1$. The difference is that depositors receive the full return r when the bank goes bankrupt. This leads to the following result.

Proposition 3 *With* $h_B = 1$, *there are multiple equilibria. In any of these,* $\rho = \frac{R}{2}$, $E\Pi = 0$,*and* $EU = u = \frac{R}{2}$. *Banks can choose any level of* $k_B \in [0,1]$ *and for* $k_B < 1$, $r_D = \frac{(1-\sqrt{k_B})}{1-k_B}R$.

The multiplicity of equilibria in the case without bankruptcy costs arises because capital can be invested in banks or directly in the risky technology. Depositors always invest in banks. The mix between capital providers' investments in banks and directly into the risky assets does not affect the real allocation.

Since the equilibrium is unique with $0 \leq h_B < 1$ but there are multiple equilibria with $h_B = 1$, we next consider the limit equilibrium as $h_B -> 1$.

Proposition 4 *As $h_B \to 1$, the unique equilibrium has $\overline{R} \to 2$, $k_B \to \frac{\eta}{1+\eta}$, $r_D \to \frac{1+\eta}{1+\eta+\sqrt{\eta(1+\eta)}}R$, $\rho \to \frac{R}{2}$, $E\Pi = 0$,and $EU \to \frac{R}{2}$. This limit equilibrium is one of the equilibria in Proposition 3.*

This result shows that the multiplicity with $h_B = 1$ is not robust to small changes in h_B. As soon as there are bankruptcy costs of any magnitude, bank capital structure matters. Only when there are no bankruptcy costs, does a Modigliani-Miller type of result hold and bank capital structure is irrelevant.

4 A single publicly traded productive sector

So far we have assumed that banks invest directly in the risky technology. We now consider the case where a continuum of publicly traded firms in a productive sector hold the risky technology with return $r \sim U[0, R]$ as before. Since it is a single sector, firms' returns are perfectly correlated. We consider the case of multiple sectors with independent returns below in Section 6.

Each firm requires 1 unit of funds and finances this with equity k_F and loans from banks of $1 - k_F$. The opportunity cost of the capital in the firm is $\rho \geq \frac{R}{2}$. The promised per unit loan rate on bank loans is r_L. If $r < r_L(1 - k_F)$ the firm goes bankrupt. The liquidation proceeds are $h_F r$ with $h_F \in [0, 1]$ and are distributed pro-rata to the banks providing the $1 - k_F$ in loans.

Banks raise equity k_B and take deposits $1 - k_B$. They pay $\rho \geq \frac{R}{2}$ to the capital providers and r_D to depositors. Each bank lends a total of one unit to firms. The bank

receives the per unit loan rate r_L if $r \geq r_L(1-k_F)$. If $r < r_L(1-k_F)$, then firms go bankrupt and banks receive $h_F r$ for each $1-k_F$ loaned out. Thus, each bank receives $\frac{h_F r}{1-k_F}$ per unit loaned with bankruptcy. When the bank receives r_L, it remains solvent and repays $r_D(1-k_B)$ to its depositors. When the bank receives $\frac{h_F r}{1-k_F}$, it will go bankrupt if $\frac{h_F r}{1-k_F} < r_D(1-k_B)$ and depositors obtain $\frac{h_F h_B r}{(1-k_F)(1-k_B)}$. By contrast, if $\frac{h_F r}{1-k_F} \geq r_D(1-k_B)$, the bank remains solvent and pays each of its $1-k_B$ depositors the promised repayment r_D.

As before, the total supply of capital and deposits is denoted K and D, respectively, with

$$\frac{K}{D} = \eta > 0$$

being the ratio of the two.

4.1 The equilibrium with a single publicly traded productive sector

In addition to conditions 1-6 in Section 3, in equilibrium it must now be the case that

6. Banks choose r_L and k_F to maximize their expected profits subject to the constraint that firms' shareholders must earn at least their opportunity cost ρ.

7. Firms make zero expected profits in equilibrium.

As before, the equity and the deposit markets have to clear in equilibrium. Given the presence now of two sectors, the conditions for this to occur are slightly different. In particular, market clearing requires that

$$N_F k_F + N_B k_B = K \tag{7}$$

and

$$N_B(1-k_B) \leq D, \tag{8}$$

where N_F and N_B are the number of firms and banks respectively. Condition (7) requires that all available capital K is allocated between the productive and the banking sectors. Condition (8) requires that the total deposits $N_B(1-k_B)$ in the banking sector do not exceed the total supply D in the economy. As in the case with the banking sector only, depositors can either deposit their endowment in the banks only so that (8) holds with equality and depositors obtain $u > 1$ or they can also make use of their alternative technology, in which (8) holds with strict inequality and $u = 1$. Conditions (7) and (8) imply that also the loan market clears so that

$$N_F(1-k_F) = N_B. \tag{9}$$

This states that the total lending $N_F(1-k_F)$ needed by the firms equals the total resources available for lending at the N_B banks.

4.2 Bankruptcy costs in the banking and in the productive sector ($0 \leq h_B \leq h_F < 1$)

We start by considering the case where there are bankruptcy costs in both sectors. In line with the empirical evidence we assume that $h_B \leq h_F$ to indicate that these costs are higher in the banking sector than in the productive one. In this case, each firm's expected profit is given by

$$E\Pi^F = \int_{\overline{r}_F}^{R} (r - r_L(1-k_F)) \frac{1}{R} dr - \rho k_F \tag{10}$$

where

$$\overline{r}_F = r_L(1-k_F)$$

is the threshold value of r below which the firm goes bankrupt. The first term represents what the expected payoff to the firm from the investment in the risky technology after paying $r_L(1-k_F)$ to the bank. This is positive only for $r > \overline{r}_F$, and it is distributed to

the firm shareholders. The latter have an opportunity cost of providing capital equal to the second term ρk_F.

Each bank makes expected profit

$$E\Pi^B = \int_{\overline{r}_B}^{\overline{r}_F} \left(\frac{h_F r}{1-k_F} - r_D(1-k_D) \right) \frac{1}{R} dr + \int_{\overline{r}_F}^{R} \left(r_L - r_D(1-k_D) \right) \frac{1}{R} dr - \rho k_B, \quad (11)$$

where

$$\overline{r}_B = \min \left(\frac{r_D(1-k_B)(1-k_F)}{h_F}, \ \overline{r}_F \right) \quad (12)$$

is the threshold value of r below which the bank goes bankrupt. This is lower than the firm's bankruptcy threshold if $\frac{r_D(1-k_B)(1-k_F)}{h_F} < \overline{r}_F$, and it is equal to $\overline{r}_F$ otherwise. The first term in (11) represents the expected payoff to the bank when firms go bankrupt but the bank remains solvent for $\overline{r}_B < r < \overline{r}_F$. In this case, the bank obtains the firms' liquidation proceeds $\frac{h_F r}{1-k_F}$ and retains the surplus after repaying depositors. By contrast, when $\overline{r}_B = \overline{r}_F$, the bank goes bankrupt whenever the firm does so, and the first term in (11) becomes zero. The second term is the expected payoff to the bank from lending one unit to firms after paying the total promised repayment $r_D(1-k_B)$ to its depositors. Note that $r_L \geq r_D(1-k_B)$ must hold, as otherwise the bank would always go bankrupt. The last term $\rho_B k_B$ is the opportunity cost for bank shareholders.

Depositors have an expected utility of

$$EU = \int_0^{\overline{r}_B} \frac{h_B h_F r}{(1-k_B)(1-k_F)} \frac{1}{R} dr + \int_{\overline{r}_B}^{R} r_D \frac{1}{R} dr \geq u \geq 1. \quad (13)$$

The first term is the payoff for $r < \overline{r}_B$ when the bank goes bankrupt and depositors obtain a share $\frac{h_B}{(1-k_B)}$ of the $\frac{h_F r}{(1-k_F)}$ resources available at the bank. The second term is depositors' payoff for $r \geq \overline{r}_B$, when the bank remains solvent and depositors obtain the promised repayment r_D.

We obtain the following result.

Proposition 5 *The unique equilibrium with* $0 \leq h_B \leq h_F < 1$ *is as follows:*

i) Banks hold $k_B = 0$ *and set* $r_D = r_L$.

ii) If $1 > h_F \geq h_B = 0$, *the equilibrium is as in Proposition 1 with the difference that firms hold the same capital as banks there, i.e.,* $k_F = \frac{4}{R} - 1$ *for* $R < \overline{R}$ *and* $k_F = \frac{\eta}{1+\eta}$ *for* $R \geq \overline{R}$.

iii) If $1 > h_F \geq h_B > 0$, *the equilibrium is as in Proposition 2 with the difference that firms hold the same capital as banks there, i.e.,* $k_F = \frac{(2-h_B R)(2(2-h_B)-R)}{2(1-h_B)^2 R}$ *for* $R < \overline{R}$ *and* $k_F = \frac{\eta}{1+\eta}$ *for* $R \geq \overline{R}$.

The proposition states that in equilibrium banks are simply a conduit between depositors and firms and hold no capital. This allows a reduction in the deadweight costs associated with the bankruptcy of firms and banks. The result is illustrated in Figure 4, which shows the output of a single firm as a function of the return r, and how this is split among shareholders and depositors. Consider first the case where both the bank and the firm hold positive capital and the firm goes bankrupt at a higher level of r than the bank, i.e., $\overline{r}'_F = r'_L(1-k'_F) > \overline{r}'_B = \frac{r'_D(1-k'_B)(1-k'_F)}{h_F}$. Region A represents the payoff to firm shareholders for $r \in (\overline{r}'_F, R]$, when the firm remains solvent and repays $r'_L(1-k'_F)$ to the bank. Region $B+C$ represents the payoff to the bank shareholders. For $r \in [\overline{r}'_F, R]$, bank shareholders receive the promised repayment $r'_L(1-k'_F)$. For $r \in [\overline{r}'_B, \overline{r}'_F)$, the firm goes bankrupt and the bank receives $\frac{h_F r}{1-k'_F}$. Region D represents the deadweight loss deriving from the bankruptcy of the firm. Region $E+F$ represents the payoff to bank depositors. For $r \in [\overline{r}'_B, R]$, the bank is solvent, and each depositor receives the promised repayment r'_D. Since there are $(1-k'_B)(1-k'_F)$ depositors per firm, they obtain $r'_D(1-k'_B)(1-k'_F)$ in total. For $r \in [0, \overline{r}'_B)$ the bank goes bankrupt. Each of the $(1-k'_B)$ depositor of the bank receives a share $\frac{h_B}{1-k'_B}$ of the resources $\frac{h_F r}{1-k'_F}$ that the bank has. Thus, the $(1-k'_B)(1-k'_F)$ depositors per firm obtain $h_B h_F r$ in total.

Consider now setting $k^*_B = 0$ and $r^*_D = r^*_L = r'_D$ so that the bank is a conduit with no capital and zero profit, and changing the level of firm capital to the level k^*_F so that $\overline{r}^*_F = r^*_L(1-k^*_F)$ equals $\overline{r}'_B = \frac{r'_D(1-k'_B)(1-k'_F)}{h_F}$. This means shifting the bankruptcy point of the firm $\overline{r}^*_F$ to the same level as the bankruptcy of the bank $\overline{r}'_B$. It is immediate to

see that this allows the deadweight loss in Region D to be eliminated and improves the allocation. Depositors are as well off as before, but shareholders are obtaining a higher return as they obtain the returns of Region $A + B + C + D$.

4.3 Bankruptcy costs in the banking sector only ($h_B \geq 0$ and $h_F = 1$)

We next consider the case where there are no bankruptcy costs in the productive sector ($h_F = 1$), while keeping some bankruptcy costs in the banking sector ($h_B \geq 0$). We have the following result.

Proposition 6 *With $h_F = 1$ and $h_B \geq 0$ there are multiple equilibria:*

i) If $h_B = 0$, the equilibrium values of r_D, ρ and u are as in Proposition 1.

ii) If $h_B > 0$, the equilibrium values of r_D, ρ and u are as in Proposition 2.

Banks and firms choose any level of k_B and k_F provided $\overline{r}_F = r_L(1 - k_F) \in [\overline{r}_B, R]$ such that

$$\overline{r}_B = \widehat{r}_D(1 - \widehat{k}_B) = \widehat{r}_D(1 - k_B)(1 - k_F), \tag{14}$$

where $\widehat{r}_D$ and $\widehat{k}_B$ come from Proposition 1 in i) and from Proposition 2 in ii). The value of r_L does vary in the range $[r_D, R]$ with the split of capital so that $E\Pi^B = E\Pi^F = 0$. For any $k_F > 0$, the number of firms is $N_F = \widehat{N}_B = \frac{K}{\widehat{k}_B}$ and the number of banks is $N_B = \frac{(1-\widehat{k}_B)}{(1-k_B)}\widehat{N}_B < \widehat{N}_B$. The amount of deposits used in the banking sector remain as one non-publicly traded productive sector.

The proposition shows that as in the case with a single non-traded productive sector, a Modigliani-Miller type of result holds when there are no bankruptcy costs in the traded productive sector. Depositors and shareholders make the same expected return in any of them as in Propositions 1 and 2 with a single non-traded productive sector, depending on the value of h_B. Bank and firm capital adjust in aggregate so as to guarantee those returns, but the specific capital structure of either banks or firms is irrelevant as long as (14) is satisfied. The loan rate adjusts to ensure that both bank and firm shareholders earn ρ. The multiplicity of equilibria and the irrelevance of capital structure composition do not

translate, however, to a multiplicity of allocations. Given that there are bankruptcy costs in the banking sector, the allocation is unique and it is the same as in Propositions 1 or 2 depending on whether h_B is equal to zero or positive. It is only the way the system reaches that allocation, that is how capital is split among banks and firms, that is irrelevant. The other variable in addition to the loan rate r_L that changes depending on the distribution of capital between banks and firms is the number of banks. If $k_F > 0$, the number of banks is reduced. The number of firms and the amount of deposits used in the banking sector remain the same.

5 Two non-publicly traded productive sectors

So far we have considered the case where there is a single productive sector where all firms' payoffs are perfectly correlated and banks lend to perfectly correlated firms only. In this section, we analyze the role of diversification on banks' capital structure. We assume that there are two sectors with independent returns r_1 and r_2, each uniformly distributed on the support $[0, R]$. We start with the simple case where banks invest in the two sectors directly similarly to Section 2, and we then analyze the case of two publicly traded sectors in the next section. As before, each bank raises k_B in capital and $1 - k_B$ in deposits, and invests one in total. Clearly, a diversified bank with two sectors will lend equally to each sector to maximize the benefit of diversification. The rest of the economy is as before. For simplicity, we focus on the case where $h_B = 0$ so that there are full bankruptcy costs from the bank failing.

As the bank invests in equal shares in two independent projects, the return of the bank's portfolio is equal to the weighted sum of the returns of each sector, that is $x = \frac{r_1}{2} + \frac{r_2}{2}$. If $x \geq r_D(1 - k_B)$, that is $r_1 + r_2 \geq \overline{x}_B = 2r_D(1 - k_B)$, then the bank remains solvent and repays depositors in full. To see when this occurs we consider the distribution of the sum of the returns r_1 and r_2 in Figure 5. The figure shows several areas depending on the values of r_1 and r_2. The bank goes bankrupt in Region A and remains solvent everywhere

else. Region B captures the case where $r_1 + r_2 \geq \overline{x}_B$ for $r_1 \in [\ \overline{x}_B - r_2, R]$ and $r_2 \in [0, \overline{x}_B]$. Region C represents the case where $r_1 + r_2 \geq \overline{x}_B$ for $r_1 \in [\ 0, R]$ and $r_2 \in [\overline{x}_B, R]$.

Given the return x and the areas of bank solvency as described above, the bank's maximization problem is now given by

$$\begin{aligned} \max_{k_B, r_D} E\Pi \quad = \quad & \int_0^{\overline{x}_B} \int_{\overline{x}_B - r_2}^{R} \left(\frac{r_1}{2} + \frac{r_2}{2} - r_D(1 - k_B) \right) \frac{1}{R^2} dr_1 dr_2 \\ & + \int_{\overline{x}_B}^{R} \int_0^{R} \left(\frac{r_1}{2} + \frac{r_2}{2} - r_D(1 - k_B) \right) \frac{1}{R^2} dr_1 dr_2 - \rho k_B \end{aligned} \tag{15}$$

subject to

$$EU = \int_0^{\overline{x}_B} \int_{\overline{x}_B - r_2}^{R} r_D \frac{1}{R^2} dr_1 dr_2 + \int_{\overline{x}_B}^{R} \int_0^{R} r_D \frac{1}{R^2} dr_1 dr_2 \geq u, \tag{16}$$

where

$$\overline{x}_B = 2r_D(1 - k_B), \tag{17}$$

and

$$E\Pi \geq 0 \text{ and } 0 \leq k_B \leq 1. \tag{18}$$

As before, the bank chooses k_B and r_D to maximize its expected profit subject to the depositors obtaining their opportunity cost $u \geq 1$ in expectation, but the problem is algebraically more complicated now. Expression (17) represents the bank's expected profit net of shareholders' opportunity cost ρk_B whenever it is solvent. The first term is what the bank obtains after paying $r_D(1 - k_B)$ to the depositors when the bank remains solvent in Region B for $r_1 \in [\overline{x}_B - r_2, R]$ and $r_2 \in [0, \overline{x}_B]$. The second term is what it obtains in Region C when $r_1 \in [0, R]$ and $r_2 \in [\overline{x}_B, R]$. Similarly, (16) represents the expected utility to depositors when the bank remains solvent. The last constraints in (18) are the non-negative profit condition for the bank and the feasibility constraint on the amount of capital.

We obtain the following result.

Proposition 7 *When $h_B = 0$, the unique two-sector bank equilibrium is as follows:*

i) For $R < \overline{R}, k_B = 1 - \frac{R\rho\sqrt{2(3\rho^2-4\rho+1)}}{(3\rho-1)^2} \in (0,1)$, $r_D = \frac{(3\rho-1)}{2\rho}$, $\rho = \rho_1(R), E\Pi = 0$, $EU = u = 1$.

ii) For $R \geq \overline{R}$, $k_B = \frac{\eta}{1+\eta} \in (0,1), r_D = \frac{u(3\rho-u)}{2\rho}, \rho = \rho_2(R), u = u(R), E\Pi = 0$ $, EU = u > 1$.

The expressions for $\overline{R}, \rho_1(R)$ and $\rho_2(R)$ are defined in the appendix.

The effect of diversification can be seen from Figure 5. Each outcome of returns (r_1, r_2) has equal probability $1/R^2$ so areas represent total probabilities. If a bank invests in one sector only, then it will go bankrupt at $\overline{r}_B$. The area below $\overline{r}_B$, i.e. $A2 + B2 + C2$, represents the probability of bankruptcy in this case. By splitting investment between two sectors, the probability of bankruptcy is now represented by $A1 + A2$ since in this case $r_1 + r_2 \leq \overline{x}_B$.

We can illustrate the effect of diversification with two examples, one for part i) and one for part ii). In each case we compare the allocation with diversification as given in Proposition 7 with the one without diversification as given in Proposition 1. We fix $K = 10, D = 100, \eta = 0.1$. This implies $\overline{R} = 3.36$ with diversification and $\overline{R} = 3.67$ without diversification. We then consider $R = 3$ for part i) and $R = 4$ for part ii).

Example 1: $R = 3$.

	Diversification	No diversification
k_B	0.21	0.33
r_D	1.3	1.5
ρ	2.59	2
u	1	1
$2\overline{x}_B$	2.06	
$\overline{r}_B$		1
Probability of bankruptcy	0.23	0.33
Total output per bank	1.34	1.33
N_B	46.85	30
Total deposits	36.85	20
Total economy output	125.89	120

The example illustrates that the main effect of diversification is to reduce the probability of bankruptcy from 0.33 to 0.23. Diversification implies that extreme outcomes where both returns are low or high become less likely, while moderate outcomes near the mean become more likely. This reduces the need for capital, which goes down from 0.33 to 0.21. Since the bank goes bankrupt less often, it can reduce its payment to depositors r_D from 1.5 to 1.3, while giving them the same expected utility. Also, the reduction in bankruptcy costs then means that the opportunity cost of capital is bid up from 2 to 2.59. Since less capital is needed per bank, more banks can be formed and more funds are used as deposits. Since the investment opportunities available to the bank are superior to those of individual investors, the greater use of deposit increases the efficiency of financial intermediation and of the economy. The economy total output goes from 120 to 125.89.

Example 2: $R = 4$.

	Diversification	No diversification
k_B	0.09	0.09
r_D	1.68	2
ρ	6.82	6.54
u	1.19	1.1
$2\overline{x}_B$	3.06	
$\overline{r}_B$		1.82
Probability of bankruptcy	0.29	0.45
Total output per bank	1.7	1.59
N_B	110	110
Total deposits	100	100
Total economy output	187.24	174.54

The main difference in Example 2 is that all the capital and all the deposits are already used in the banking sector when there is no diversification. Allowing diversification lowers the probability of bankruptcy from 0.45 to 0.29. This allows r_D to be reduced from 2 to 1.68 and depositors' expected utility u to be bid up from 1.1 to 1.19. Shareholders' opportunity cost of capital is also bid up from 6.54 to 6.82. Since all the resources are already used, there is no change in the number of banks and the amount of deposits used with diversification. However, the lower bankruptcy probability means that the total output per bank as well as total economy output are increased, respectively from 1.59 to 1.7, and from 174.54 to 187.24.

6 Two publicly traded sectors

In this section we turn to the case where there are two publicly traded sectors with independent returns r_1 and r_2, each uniformly distributed on the support $[0, R]$. As before, each firm requires 1 unit of funds and finances this with equity k_F and loans from banks of $1 - k_F$. The former has an opportunity cost $\rho \geq \frac{R}{2}$, while the latter has a promised per

unit rate of r_L. If $r_i > \overline{r}_F = r_L(1-k_F)$, firms in sector i are solvent and repay $\frac{r_L}{2}$ to the banks. If $r_i < \overline{r}_F$, firms in sector i go bankrupt and the liquidation proceeds $h_F r_i$ with $h_F \in [0,1]$ are distributed pro-rata to the banks providing the $1-k_F$ in loans. A bank diversifying across the two sectors raises k_B in capital and $1-k_B$ in deposits, and lends to each sector equally. The rest of the economy is as before. For simplicity, we maintain the assumption $h_B = 0$ throughout the section.

As before, the equilibrium requires banks choose the loan rate r_L, firm capital k_F, the deposit rate r_D, and the amount of bank capital k_B to maximize their expected profits, while guaranteeing depositors their opportunity cost u and firm shareholders' their opportunity cost ρ.

The bank's maximization problem depends on whether the bank goes bankrupt only when firms in both sectors go bankrupt or whether the bankruptcy of firms in one sector leads to the bank going bankrupt as well. The former corresponds to the case where

$$\frac{r_L}{2} > r_D(1-k_B). \tag{19}$$

The latter corresponds to the opposite case where

$$\frac{r_L}{2} < r_D(1-k_B). \tag{20}$$

The payments from the firms to a bank in the first case are illustrated in Figure 6. The bank goes bankrupt only when firms in both sectors go bankrupt for $\frac{h_F(r_1+r_2)}{2(1-k_F)} < r_D(1-k_B)$, i.e., for

$$r_1 + r_2 < \overline{x}_B = \frac{2r_D(1-k_B)(1-k_F)}{h_F}.$$

Note that (19) implies $\overline{x}_B < \overline{r}_F = r_L(1-k_B)$. Then the bank remains solvent in all regions except Region A. Region B captures the case where both firms go bankrupt and the bank receives $\frac{h_F(r_1+r_2)}{2(1-k_F)} > r_D(1-k_B)$. Region C and D correspond to where firms in only one sector go bankrupt and the bank obtains $\frac{h_F r_i}{2(1-k_F)} + \frac{r_L}{2}$. Finally, Region E is where all

firms remain solvent and the bank receives r_L in total. A two-sector bank's maximization problem is then given by:

$$\begin{aligned}
\underset{r_L,r_D,k_F,k_B}{Max} E\Pi^B &= \int_0^{\overline{x}_B}\int_{\overline{x}_B-r_2}^{\overline{r}_F}\left(\frac{h_F(r_1+r_2)}{2(1-k_F)}-r_D(1-k_B)\right)\frac{1}{R^2}dr_1dr_2 \\
&+\int_{\overline{x}_B}^{\overline{r}_F}\int_0^{\overline{r}_F}\left(\frac{h_F(r_1+r_2)}{2(1-k_F)}-r_D(1-k_B)\right)\frac{1}{R^2}dr_1dr_2 \\
&+\int_0^{\overline{r}_F}\int_{\overline{r}_F}^{R}\left(\frac{r_L}{2}+\frac{h_Fr_2}{2(1-k_F)}-r_D(1-k_B)\right)\frac{1}{R^2}dr_1dr_2 \\
&+\int_{\overline{r}_F}^{R}\int_0^{\overline{r}_F}\left(\frac{h_Fr_1}{2(1-k_F)}+\frac{r_L}{2}-r_D(1-k_B)\right)\frac{1}{R^2}dr_1dr_2 \\
&+\int_{\overline{r}_F}^{R}\int_{\overline{r}_F}^{R}\left(r_L-r_D(1-k_B)\right)\frac{1}{R^2}dr_1dr_2-\rho k_B,
\end{aligned} \tag{21}$$

subject to

$$EU=\int_0^{\overline{x}_B}\int_{\overline{x}_B-r_2}^{R}r_D\frac{1}{R^2}dr_1dr_2+\int_{\overline{x}_B}^{R}\int_0^{R}r_D\frac{1}{R^2}dr_1dr_2=u\geq 1 \tag{22}$$

$$E\Pi_i^F=\int_{\overline{r}_F}^{R}(r_i-r_D(1-k_B))\frac{1}{R}dr_i-\rho k_F\geq 0 \tag{23}$$

and the usual non-negativity constraints.

The first two terms in (21) represent the net payoff to the bank in Region B. The third and fourth terms are the expected payoff to the bank in Regions C and D where firms in only one sector remain solvent and the bank receives $\frac{r_L}{2}+\frac{h_Fr_i}{2(1-k_F)}$. The last term is the payoff in Region E. The last term ρ_Bk_B is the usual opportunity cost for bank shareholders.

Constraint (22) requires depositors' expected utility to be at least u. Depositors receive r_D whenever the bank is solvent. The first term represents the payoff to depositors in Regions B_1 and D_1. The second term groups together all other regions in the figure for $r_1\in[0,R]$ and $r_2\in[\overline{x}_B,R]$. Constraint (23) requires that the firms' shareholders obtain at least their opportunity cost ρ.

The payments to the bank in the second case where (20) holds are illustrated in Figure 7. The first difference is that now $\overline{x}_B > \overline{r}_F$. This means that the bank can now go bankrupt when one sector is solvent and returns in the bankrupt sector are sufficiently low, i.e., when $\frac{r_L}{2} + \frac{h_F r_i}{2(1-k_F)} < r_D(1-k_B)$ or, equivalently when

$$r_i < \widehat{r}_i = \frac{2r_D(1-k_B)(1-k_F)}{h_F} - \frac{r_F(1-k_F)}{h_F} = \overline{x}_B - \frac{\overline{r}_F}{h_F}.$$

This changes the shape of Region A where the bank goes bankrupt relative to Figure 6. The other main difference is that Regions C_1 and D_1 are new. These are bounded below by $\widehat{r}_i$ and above by r_i^*. The latter is the value of r_i where $\frac{h_F(r_i+r_j)}{2(1-k_F)} = r_D(1-k_B)$ and $r_j = r_L(1-k_F)$, i.e.,

$$r_i^* = \frac{2r_D(1-k_B)(1-k_F)}{h_F} - r_F(1-k_F) = \overline{x}_B - \overline{r}_F.$$

Using Figure 7, the bank's expected profit is now given by

$$\begin{aligned} E\Pi^B \;=\; & \int_{r_2^*}^{\overline{r}_F} \int_{\overline{x}_B - r_2}^{\overline{r}_F} \left(\frac{h_F(r_1+r_2)}{2(1-k_F)} - r_D(1-k_B) \right) \frac{1}{R^2} dr_1 dr_2 \\ & + \int_{\overline{r}_F}^{R} \int_{\widehat{r}_1}^{\overline{r}_F} \left(\frac{h_F r_1}{2(1-k_F)} + \frac{r_L}{2} - r_D(1-k_B) \right) \frac{1}{R^2} dr_1 dr_2 \\ & + \int_{\widehat{r}_2}^{\overline{r}_F} \int_{\overline{r}_F}^{R} \left(\frac{r_L}{2} + \frac{h_F r_2}{2(1-k_F)} - r_D(1-k_B) \right) \frac{1}{R^2} dr_1 dr_2 \\ & + \int_{\overline{r}_F}^{R} \int_{\overline{r}_F}^{R} \left(r_L - r_D(1-k_B) \right) \frac{1}{R^2} dr_1 dr_2 - \rho k_B. \end{aligned} \tag{24}$$

The first term represents the net payoff to the bank in Region B where both sectors go bankrupt. The second and third terms are the net payoff in Regions C and D, where either sector 1 or 2 goes bankrupt. Finally, the last term is the payoff in Region E where all firms remain solvent.

Depositors' expected utility is then given by

$$EU = \int_{\widehat{r}_2}^{r_2^*} \int_{\overline{r}_F}^{R} r_D \frac{1}{R^2} dr_1 dr_2 + \int_{r_2^*}^{\overline{r}_F} \int_{\overline{x}_B - r_2}^{R} r_D \frac{1}{R^2} dr_1 dr_2 \tag{25}$$

$$+ \int_{\overline{r}_F}^{R} \int_{\widehat{r}_1}^{R} r_D \frac{1}{R^2} dr_1 dr_2 = u \geq 1.$$

Again, depositors receive r_D whenever the bank is solvent. The first term represents the payoff to depositors in Region D_1; the second term corresponds to Regions B and D_2, while the last one groups Regions C and E.

We next solve the maximization problems. We start by analyzing the two extreme cases with $h_F = 0$ and $h_F = 1$ and then turn to the intermediate case with $h_F \in (0, 1)$.

6.1 Full bankruptcy costs in the banking and in the two publicly traded sectors ($h_B = h_F = 0$)

When $h_F = 0$, a two-sector bank only receives a positive payoff when firms do not go bankrupt, that is for $r_i > \overline{r}_F = r_L(1 - k_F)$. If firms in only one sector are solvent, then the bank receives $\frac{r_L}{2}$. This simplifies the bank's payoffs significantly, as shown in Figure 8. If $\frac{r_L}{2} \geq r_D(1 - k_B)$, the bank receives positive payments only in Regions C, D and E, and the bank's expected profit is then

$$E\Pi = \int_0^{\overline{r}_F} \int_{\overline{r}_F}^{R} \left(\frac{r_L}{2} - r_D(1 - k_B) \right) \frac{1}{R^2} dr_1 dr_2 \tag{26}$$

$$+ \int_{\overline{r}_F}^{R} \int_0^{\overline{r}_F} \left(\frac{r_L}{2} - r_D(1 - k_B) \right) \frac{1}{R^2} dr_1 dr_2 + \int_{\overline{r}_F}^{R} \int_{\overline{r}_F}^{R} (r_L - r_D(1 - k_B)) \frac{1}{R^2} dr_1 dr_2 - \rho k_B.$$

The terms have the usual meaning. Depositors have an expected utility of

$$EU = \int_0^{\overline{r}_F} \int_{\overline{r}_F}^{R} r_D \frac{1}{R^2} dr_1 dr_2 + \int_{\overline{r}_F}^{R} \int_0^{R} r_D \frac{1}{R^2} dr_1 dr_2 \geq u, \tag{27}$$

where again the terms have the usual meaning.

If $\frac{r_L}{2} < r_D(1 - k_B)$, the bank remains solvent only in Region E where firms in both sectors are solvent and repay the promised loan rate r_L. This implies that only the last

integral term in (26) remains positive and depositors expected utility is simply

$$EU = \int_{\overline{r}_F}^{R} \int_{\overline{r}_F}^{R} r_D \frac{1}{R^2} dr_1 dr_2.$$

We next show that when $h_B = h_F = 0$ two-sector banks will not survive in equilibrium.

Proposition 8 *When $h_B = h_F = 0$, one-sector banks strictly dominate two-sector banks. The equilibrium is then the same as in part i) and ii) of Proposition 5.*

The equilibrium involves specialized banks lending to one sector only. The proposition can be easily understood with the use of Figure 8. In both sectors, the firm goes bankrupt for $r_i < \overline{r}_F = r_L(1-k_F)$. Since $h_F = 0$, the bank receives zero and goes bankrupt as well. For $\frac{r_L}{2} > r_D(1-k_B)$, the payoff to the bank is positive in Regions C, D and E.

In the one-sector bank, with the firms having the same capital and loan rate as with two-sector banks, the bank receives r_L in both areas D and E. The total output is the same as for a two-sector bank since Regions C and D are equal. With one-sector banks all of the bank's receipts are channeled to depositors. In the two-sector banks, they go to both depositors and shareholders. Since shareholders earn more than depositors, the depositors in one-sector banks are better off. Therefore, it is possible to lower r_D and earn positive profits.

Diversification does not improve the allocation. The reason is that banks' and firms' bankruptcy cannot be fully aligned with two-sector banks. It is true that firms and banks go bankrupt together if firms in both sectors go bankrupt. However, in areas C and D the two-sector banks survive when only firms in one sector are bankrupt if $\frac{r_L}{2} \geq r_D(1-k_B)$. In contrast, with one-sector banks, firms and their banks are always bankrupt together. If $\frac{r_L}{2} < r_D(1-k_B)$, firms and banks are fully aligned in terms of bankruptcy with diversification, but bank bankruptcy occurs in Regions A, C and D instead of only in Region A when there are one-sector banks for each sector.

6.2 No bankruptcy costs in the productive sectors ($h_B = 0$ and $h_F = 1$)

When $h_F = 1$, the bank receives the promised r_L when firms do not go bankrupt, that is for $r > \overline{r}_F = r_L(1 - k_F)$, and a share $\frac{r_i}{2(1-k_F)}$ when firms in sector i go bankrupt. In contrast to the previous subsection where $h_F = 0$, diversification is optimal since there are no bankruptcy costs. Similarly to Section 4.3, there is also multiplicity of equilibria in banks' and firms' capital structure, but not in terms of allocation of resources.

Each firm's expected profit remains as in (10). Banks' expected profits and depositors' utility are as in (21), (22) when $\frac{r_L}{2} > r_D(1-k_B)$ and as in (24) and (25) $\frac{r_L}{2} < r_D(1-k_B)$ after substituting $h_F = 1$. Note that for the case where $\frac{r_L}{2} < r_D(1-k_B)$, r_i^* equates to $\widehat{r}_i$ and Regions C_1 and D_1 disappear.

Proposition 9 *With $h_B = 0$ and $h_F = 1$, two-sector banks strictly dominate one-sectors bank and there are multiple equilibria. The equilibrium values of r_D, ρ and u are as in Proposition 7. Banks and firms choose any level of k_B and $k_F \in [0, \overline{k}_F]$ such that*

$$\overline{x}_B = 2\widehat{r}_D(1 - \widehat{k}_B) = 2\widehat{r}_D(1 - k_B)(1 - k_F), \tag{28}$$

where $\overline{k}_F$ is such that $\overline{x}_B = \overline{r}_F$, and $\widehat{r}_D$ and $\widehat{k}_B$ come from Proposition 7. The value of r_L does vary in the range $[r_D, R]$ with the split of capital so that $E\Pi^B = E\Pi^F = 0$. For any $k_F > 0$, the number of firms is $N_F = \widehat{N}_B = \frac{K}{\widehat{k}_B}$ and the number of banks is $N_B = \frac{(1-\widehat{k}_B)}{(1-k_B)}\widehat{N}_B < \widehat{N}_B$. The amount of deposits is as in the case of two non-publicly traded sectors.

We illustrate the proposition with the usual example where $R = 3$ and $u = 1$. The values for r_D and ρ are as in Example 1 above in the diversification column, i.e., $r_D = 1.31$ and $\rho = 2.59$. With $r_L = R = 3$, $k_F = 0$ and $k_B = 0.21$. Increasing k_F to 0.05, for example, reduces k_B to 0.172 and r_L to 2.23. This allows firms' and banks' shareholders to earn their opportunity cost of capital $\rho = 2.59$ and depositors to obtain $u = 1$ with $r_D = 1.31$. In terms of Figure 6, $\overline{r}_F = r_L(1 - k_F)$ moves from $R = 3$ to the interior point 2.12 but

still above $\overline{x}_B = 2.06$. As k_F keeps on rising, it comes to the threshold value $\overline{k}_F = 0.057$ where $r_L = 2.18, k_B = 0.167$ and $\overline{x}_B = \overline{r}_F = 2.056$.

For a level of $k_F > \overline{k}_F$, banks make negative expected profit with $\rho = 2.59$. To see this, if we hold r_D constant at 1.31, an increase of k_F beyond $\overline{k}_F$ implies that $\frac{r_L}{2}$ falls below $r_D(1 - k_B)$ and we are in the situation represented in Figure 7. This involves $r_i^* > \widehat{r}_i > 0$ and a larger Region A where the bank goes bankrupt as shown in Figure 7 compared to Figure 6. The greater loss of output implies that the bank now makes negative expected profits. In order to reduce the loss in output, it is optimal to adjust r_D so that $\overline{x}_B = \overline{r}_F$, but profits remain negative. For example, at $k_F = 0.06$, $r_L = 2.16, r_D = 1.30$ and $k_B = 0.167$ and $\Pi^B = -0.0002$. In the range $k_F \in [0, \overline{k}_F]$ the value of returns $r_1 + r_2 = \overline{x}_B$ at which the bank goes bankrupt remains constant. In contrast, for $k_F > \overline{k}_F$, r_D goes down and k_B needs to increase to ensure depositors remain at $u = 1$. This increases the effective use of capital and is costly so that banks' profits are negative. As k_F increases further, r_D continues to adjust to ensure $\overline{x}_B = \overline{r}_F, k_B$ keeps increasing and bank profits become increasingly negative. Within the equilibrium, it is optimal for banks to choose any level of $k_F \in [0, \overline{k}_F]$ and adjust appropriately to satisfy (28). This also shows that there cannot be another equilibrium with two sectors with $k_F > \overline{k}_F$ because the equilibrium ρ would be lower and banks could always increase profits by choosing $k_F < \overline{k}_F$.

It is also easy to see from Example 1 with no diversification that one-sector banks cannot enter without earning losses in competing with two-sector banks. This is because at the optimal levels of k_F, r_L, k_B and r_D, the most that can be offered to shareholders is $\rho = 2$.

6.3 Intermediate bankruptcy costs in the productive sectors ($h_B = 0$ and $h_F < 1$)

In Subsection (6.1) we showed that one-sector banks dominated two-sector banks when $h_F = 0$. In Subsection (9) we showed that two-sector banks dominated when $h_F = 1$. This suggests that there is a critical value of $h_F \in (0, 1)$ denoted $\overline{h}_F$ at which the two types

of banks are equivalent in terms of ρ and u provided to shareholders and depositors but are different in terms of their capital structure and interest rates on loans and deposits. Above the critical level $\overline{h}_F$ two-sector banks dominate, while below one-sector banks do.

7 Concluding remarks

The standard assumption in the literature is that bank equity capital is costly. This paper provides a foundation for that assumption. It is important to stress though that it is relative to deposit finance that bank capital is costly, not bond finance. This result arises because the deposit market is segmented from the equity market.

Our analysis stresses the role of bankruptcy costs. One of the key issues in this case is whether it is better to put capital in firms or banks. Banks will only go bankrupt when firms go bankrupt and hence it can be optimal in some circumstances for firms to use all the capital as this reduces firm bankruptcy and bank bankruptcy. In such cases banks are simply conduits and have very low capital.

It is usually argued that diversification is a good way to reduce risk and hence bankruptcy costs. While this is true in many circumstances in our model, it is not always true. In particular, it can be optimal for banks to specialize in particular sectors or regions of the economy and not to diversify. The reason is that if they specialize they can act as conduit banks, whereas if they diversify this is not possible.

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A Proofs

Proof of Proposition 1: Solving (3) with equality for k_B after setting $h_B = 0$, we find

$$k_B = 1 - \frac{(r_D - u)}{r_D^2} R. \tag{29}$$

Substituting this into (2), differentiating with respect to r_D, and solving for r_D gives

$$r_D = \frac{u(2\rho - u)}{\rho}. \tag{30}$$

Using (30) in (2), equating to zero since $E\Pi = 0$ in equilibrium, and solving for ρ gives

$$\rho = \frac{2u^2}{4u - R}. \tag{31}$$

Substituting (31) into (30) leads to

$$r_D = \frac{R}{2},$$

and together with (29), this implies

$$k_B = \frac{4u}{R} - 1. \tag{32}$$

If depositors use their alternative opportunity then $u = 1$. This will occur if

$$\frac{k_B}{1 - k_B} > \eta.$$

In this case, banks will be formed until all the capital is used up. However, there will still be deposits left over, which depositors will put in their alternative opportunity, so $u = 1$. Using (32) in this gives

$$R < \overline{R} = \frac{4(1 + \eta)}{1 + 2\eta}.$$

Putting $u = 1$ in (31) and (32) gives $\rho = \frac{2}{4-R}$ and $k_B = \frac{4}{R} - 1$. It can easily be checked that $\rho > \frac{R}{2}$ and $k_B \in (0, 1)$. Substituting the expressions for r_D and k_B into (6) gives $\overline{r}_B = R - 2$. Given $\rho > \frac{R}{2}$ and $u = 1$, we have $N_B k_B = K$ and $N_B(1 - k_B) < D$. This gives the first part of the proposition.

For $R \geq \overline{R}$, deposits are in short supply and in this case $u > 1$. The equilibrium level

of u is found from

$$\frac{k_B}{1-k_B} = \eta,$$

where k_B is given by (32). Solving gives

$$u = \frac{1+2\eta}{2+2\eta}\frac{R}{2}.$$

Using this in (31) and (32) gives $\rho = \frac{1+4\eta(1+\eta)}{4\eta(1+\eta)}\frac{R}{2}$and $k_B = \frac{\eta}{1+\eta}$. It can be checked that $\rho > \frac{R}{2}$, $k_B \in (0,1)$ and $u \in [1, \frac{R}{2})$. Substituting the expressions for r_D and k_B into (6) gives $\overline{r}_B = \overline{r}_B = \frac{1}{(1+\eta)}\frac{R}{2}$. Given $\rho > \frac{R}{2}$ and $u > 1$, we have $N_B k_B = K$ and $N_B(1-k_B) = D$. This gives the second part of the proposition. □

Proof of Proposition 2: Solving (3) with equality for k_B and $h_B > 0$, we find

$$k_B = 1 - \frac{2(r_D - u)}{(2-h_B)r_D^2}R. \tag{33}$$

Substituting this into (2), differentiating with respect to r_D, and solving for r_D gives

$$r_D = \frac{2u\left((2-h_B)\rho - u\right)}{(2-h_B)\rho - uh_B}. \tag{34}$$

Using (34) in (2), equating to zero since $E\Pi = 0$ in equilibrium, and solving for ρ gives

$$\rho = \frac{u\left(2u - h_B R\right)}{2u(2-h_B) - R}. \tag{35}$$

Substituting (35) into (34) leads to

$$r_D = \frac{2u(1-h_B)R}{2u(2-h_B) - h_B R}, \tag{36}$$

and together with (33), this implies

$$k_B = \frac{(2u - h_B R)(2u(2-h_B) - R)}{2u(1-h_B)^2 R}. \tag{37}$$

Similarly to the case with $h_B = 0$, depositors use their alternative opportunity and thus $u = 1$ when

$$\frac{k_B}{1-k_B} > \eta.$$

Using (37) in this gives

$$R < \overline{R} = \frac{2(1+\eta) - (1-h_B)\left(1 - h_B + \sqrt{4\eta(1+\eta) + (1-h_B)^2}\right)}{(1+\eta)h_B}. \tag{38}$$

Putting $u = 1$ in (37), (36) and (35) gives $k_B = \frac{(2-h_BR)(2(2-h_B)-R)}{2(1-h_B)^2R}$, $r_D = \frac{2(1-h_B)R}{2(2-h_B)-h_BR}$ and $\rho = \frac{2-h_BR}{2(2-h_B)-R}$. To show that r_D, ρ and k_B are positive, we start by showing that $2 - h_BR > 0$ and $2(2-h_B) - R > 0$ for any $R < \overline{R}$. Substituting (38) into $2 - h_BR$, we obtain

$$\begin{aligned} 2 - h_B\overline{R} &= \frac{2(1+\eta)h_B - h_B\left(2(1+\eta) - (1-h_B)^2 - (1-h_B)\sqrt{4\eta(1+\eta) + (1-h_B)^2}\right)}{(1+\eta)h_B} \\ &= \frac{(1-h_B)\left(1 - h_B + \sqrt{4\eta(1+\eta) + (1-h_B)^2}\right)}{(1+\eta)} > 0. \end{aligned} \tag{39}$$

Then, substituting (38) into $2(2-h_B) - \overline{R}$, we obtain

$$\begin{aligned} 2(2-h_B) - \overline{R} &= \frac{(4-2h_B)(1+\eta)h_B - \left(2(1+\eta) - (1-h_B)\left(1 - h_B + \sqrt{4\eta(1+\eta) + (1-h_B)^2}\right)\right)}{(1+\eta)h_B} \\ &= (1-h_B)\frac{\sqrt{4\eta(1+\eta) + (1-h_B)^2} - (1+2\eta)(1-h_B)}{(1+\eta)h_B}. \end{aligned}$$

The sign of the numerator is the same as the sign of

$$4\eta(1+\eta) + (1-h_B)^2 - (1+2\eta)^2(1-h_B)^2.$$

This simplifies to

$$4\eta\,(1+\eta)\left(1 - (1-h_B)^2\right) > 0,$$

so that

$$2(2-h_B) - \overline{R} > 0. \tag{40}$$

This implies that r_D is positive and less than R as $h_B < 1$ and

$$R - r_D = \frac{(2-h_BR)R}{(2(2-h_B) - h_BR))} > 0 \text{ for } R \leq \overline{R}.$$

Finally, it can be seen that $\rho > \frac{R}{2}$, as

$$\rho - \frac{R}{2} = \frac{(R-2)^2}{2(2(2-h_B)-R)} > 0 \text{ for } R \leq \overline{R}.$$

It follows from (39) and (40) that $k_B > 0$. Also, $k_B < 1$ since, using the expression for k_B in the proposition, we have

$$2(1-h_B)^2R - (2-h_BR)(2(2-h_B)-R) = (R-2)(2(2-h_B)-h_BR) > 0 \text{ for } R < \overline{R}.$$

Substituting then the expressions for k_B and r_D into $\overline{r}_B = r_D(1-k_B)$ gives $\overline{r}_B = \frac{R-2}{1-h_B}$. This completes the first part of the proposition.

For $R > \overline{R}$, deposits are in short supply and in this case $u > 1$. The equilibrium level of u is found from

$$\frac{k_B}{1-k_B} = \eta, \tag{41}$$

where k_B is given by (37). Solving gives

$$u = \frac{2(1+\eta) - (1-h_B)\left(1-h_B+\sqrt{4\eta(1+\eta)+(1-h_B)^2}\right)}{2(1+\eta)(2-h_B)}\frac{R}{2}. \tag{42}$$

We know from the definition of $\overline{R}$ that $u = 1$ at $R = \overline{R}$. It follows from (42) that $u > 1$ for $R > \overline{R}$. Also, from (40) it follows that $u < \frac{R}{2}$ since $(2(2-h_B)-h_BR) > 0$ and thus $\frac{h_B\overline{R}}{2(2-h_B)} < 1$.

From (41) we have $k_B = \frac{\eta}{1+\eta}$. Closed form solutions for r_D and ρ can be found by using (42) in the expressions (36) and (35). To check that $r_D < R$, we calculate

$$R - r_D = \frac{(2u-h_BR)R}{(2u(2-h_B)-R)}.$$

Substituting for u from (42), the numerator becomes

$$\begin{aligned} 2u - h_BR &= \left(\frac{2(1+\eta) - (1-h_B)\left(1-h_B-\sqrt{4\eta(1+\eta)+(1-h_B)^2}\right)}{(1+\eta)(2-h_B)} - h_B\right)R \\ &= \left(\frac{1+\eta h^2 + (1-h_B)\left(2\eta+\sqrt{4\eta(1+\eta)+(1-h_B)^2}\right)}{(1+\eta)(2-h_B)}\right)R > 0. \end{aligned}$$

Substituting the expression for u from (42), the denominator becomes

$$\begin{aligned} 2u(2-h_B)-R &= \left(\frac{2(1+\eta)-(1-h_B)\left(1-h_B-\sqrt{4\eta(1+\eta)+(1-h_B)^2}\right)}{(1+\eta)}-1\right)R \\ &= \left(\frac{1+\eta-(1-h_B)\left(1-h_B-\sqrt{4\eta(1+\eta)+(1-h_B)^2}\right)}{(1+\eta)}\right)R>0. \end{aligned}$$

This implies that $r_D < R$ for $R > \overline{R}$. Moreover, it is easy to see that $\rho > \frac{R}{2}$, since

$$\rho - \frac{R}{2} = \frac{(R-2u)^2}{2\left(2u(2-h_B)-R\right)} > 0.$$

Substituting then the expressions for k_B and r_D into $\overline{r}_B = r_D(1-k_B)$ gives $\overline{r}_B = \frac{2u(1-h_B)R}{(1+\eta)(2u(2-h_B)-h_BR)}$. This completes the second part of the proposition. □

Proof of Proposition 3: Since there are no bankruptcy costs, there are no efficiency gains from having capital in the banks. This means it is always possible to set up a bank with $r_D = R$ and $k_B = 0$ such that

$$EU = \int_0^R r\frac{1}{R}dr = \frac{R}{2}.$$

Thus, in equilibrium depositors must always receive $EU = \frac{R}{2}$. Since capital providers can always invest directly in the risky technology, they receive at least $\frac{R}{2}$ as well. Since total output with no bankruptcy costs is $\frac{R}{2}$ for each unit invested, the capital providers will earn exactly $\frac{R}{2}$. So one equilibrium involves all depositors using banks with no capital and all capital providers investing in their alternative opportunity. However, there exist many other equilibria. In these, banks choose a level of capital k_B and r_D such that $E\Pi = 0$ and $EU = \frac{R}{2}$. Substituting $\rho = \frac{R}{2}$ in (2) and solving $E\Pi = 0$ with respect to r_D gives r_D as in the proposition. □

Proof of Proposition 4: Using the expression for $\overline{R}$ from Proposition 2, it can be seen that $\underset{h\to 1}{Lim}\overline{R} \to 2$. For $R \leq \overline{R}$, substituting $\overline{R}$ into the expression for k_B in part i) of Proposition 2 and taking the limit for $h_B \to 1$ gives $k_B \to \frac{\eta}{1+\eta}$. Similarly, it can be shown that at $R = \overline{R}$, $\underset{h_B\to 1}{Lim}\, r_D \to \frac{1+\eta}{1+\eta+\sqrt{\eta(1+\eta)}}$, $\underset{h_B\to 1}{Lim}\,\rho \to \frac{R}{2} \to \frac{\overline{R}}{2}$ and $\underset{h_B\to 1}{Lim}\, EU \to \frac{\overline{R}}{2}$. The case for $R > \overline{R}$ can be shown similarly. The limit equilibrium with $h_B \to 1$ is one of the multiple equilibria in Proposition 3. It is the one with $k_B = \frac{\eta}{1+\eta}$ and $r_D = \frac{(1-\sqrt{k_B})}{1-k_B}R =$

$\frac{1+\eta}{1+\eta+\sqrt{\eta(1+\eta)}}R$. □

Proof of Proposition 5: Suppose there exists a candidate equilibrium, defined as X, with

$$k'_B > 0, k'_F > 0, r'_L > r'_D, \rho' \geq \frac{R}{2}, u' \geq 1,\ \overline{r}'_F = r'_L(1-k'_F) > \overline{r}'_B = \frac{r'_D(1-k'_B)(1-k'_F)}{h_F}. \tag{43}$$

We are able to show that this cannot be an equilibrium by transferring the capital of the bank to the firm and reducing the bankruptcy costs of the firm in the range $(\overline{r}'_B, \overline{r}'_F]$. To see this, consider the following deviation, which we denote Z, where we choose k^*_F to align the bankruptcy threshold of the firm $\overline{r}^*_F$ to the one of the bank $\overline{r}'_B$ in equilibrium X:

$$k^*_B = 0, k^*_F = k'_B(1-k'_F) + k'_F, r^*_D = r^*_L = r'_D, \overline{r}^*_F = \overline{r}^*_B = r^*_L(1-k^*_F) = \overline{r}'_B. \tag{44}$$

We can then calculate the total net output going to the bank's shareholders per firm and the firm's shareholders in the candidate equilibrium X as

$$Y_X = (1-k'_F)\left[\int_{\overline{r}'_B}^{\overline{r}'_F}\left(\frac{h_F r}{1-k'_F} - r'_D(1-k'_B)\right)\frac{1}{R}dr + \int_{\overline{r}'_F}^{R}(r'_L - r'_D(1-k'_B))\frac{1}{R}dr\right] + \tag{45}$$

$$+\int_{\overline{r}'_F}^{R}(r - r'_L(1-k'_F))\frac{1}{R}dr.$$

The firm borrows $(1-k'_F)$. The first term in the square bracket represents the firm payment to the bank's shareholders per unit loan when the firm goes bankrupt and the second term is the payment per unit loan when the firm remains solvent. The last term is the payment to the firm's shareholders when the firm is solvent.

The total net output in the proposed deviation Z is

$$Y_Z = \int_{\overline{r}^*_F}^{R}(r - r^*_L(1-k^*_F))\frac{1}{R}dr. \tag{46}$$

Given $\overline{r}^*_F = \overline{r}'_B$ and that $k^*_F = k'_B(1-k'_F) + k'_F$ implies $r^*_L(1-k^*_F) = r'_D(1-k'_B)(1-k'_F)$, we can rewrite the output in the equilibrium Z as follows:

$$Y_Z = (1-k'_F)\left[\int_{\overline{r}^*_F}^{\overline{r}'_F}(r - r'_D(1-k'_B))\frac{1}{R}dr + \int_{\overline{r}'_F}^{R}(r'_L - r'_D(1-k'_B))\frac{1}{R}dr\right]$$

$$+\int_{\overline{r}'_F}^{R}(r-r'_L(1-k'_F))\frac{1}{R}dr.$$

It can be seen that with $h_F = 1$, the output in the candidate equilibrium X and in the deviation equilibrium Z is the same. Since $h_F < 1$, $Y_Z > Y_X$ so that shareholders are better off with the deviation. Given that depositors receive the same expected utility in the candidate equilibrium and in the deviation, the latter represents a Pareto improvement. This implies that any equilibrium must have banks be conduits, that is $k_B = 0$ and $r_D = r_L$ so that banks make zero expected profits. □

Proof of Proposition 6: It can be seen that an equilibrium exists for part i) (part ii)) as in Proposition 1(2) with $k_B = \widehat{k}_B$ and $k_F = 0$. Choose any non negative k_B and k_F such that (14) is satisfied and $\overline{r}_F$ lies in the range $[\overline{r}_B, R]$. Then it follows from the proof of Proposition 5 that, since $h_F = 1$, $Y_Z = Y_X$, the shareholders of the banks and firms will have the same expected return as in Proposition 1(2). Although r_D, ρ and u remain constant, r_L adjusts to ensure banks' and firms' shareholders earn ρ.

To find N_F and N_B, recall that from Proposition 1 it holds $\widehat{N}_B = \frac{K}{\widehat{k}_B}$ and from (14) it is

$$\widehat{k}_B = 1-(1-k_B)(1-k_F)$$

or

$$k_F = 1-\frac{1-\widehat{k}_B}{1-k_B}. \tag{47}$$

Then, substituting (9) and (47) into (7), we obtain

$$N_F\left(1-\frac{1-\widehat{k}_B}{1-k_B}\right)(1-k_B)+N_F k_B = K,$$

which simplifies to $N_F\widehat{k}_B = K$ and thus

$$N_F = \widehat{N}_B = \frac{K}{\widehat{k}_B}.$$

Substituting this and (47) into (9) gives

$$N_B = \frac{1-\widehat{k}_B}{1-k_B}\frac{K}{\widehat{k}_B} < \widehat{N}_B$$

since $\widehat{k}_B > k_B$. This completes the proposition. □

Proof of Proposition 7: Solving (16) equal to u, we find

$$k_B = 1 - \frac{\sqrt{r_D(r_D - u)}}{\sqrt{2}r_D^2}R.$$

Substituting this into (17), differentiating with respect to r_D and solving for r_D gives

$$r_D = \frac{u(3\rho - u)}{2\rho}. \tag{48}$$

Putting this back in the expression for k_B gives

$$k_B = 1 - \frac{\rho R\sqrt{2(3\rho^2 - 4\rho u + u^2)}}{u(3\rho - u)^2}. \tag{49}$$

Substituting (48) and (49) in (17) and solving it equal to zero with respect to ρ after setting $u = 1$ gives one real positive solution as follow

$$\rho_1(R) = \frac{3 + 9R - 3R^2}{(27 - 2R^2)} + \frac{\sqrt{3\,(12 - 36R + 83R^2 - 40R^3 + 6R^4)}\left[Cos(\frac{t}{3}) + \sqrt{3}\,\mathrm{Si}\,n(\frac{t}{3})\right]}{(27 - 2R^2)},$$

where

$$t = Arctan\left(\frac{R(R-2)(27-2R^2)\sqrt{6(R-2)(-8+36R-6R^2+3R^3}}{(-72+324R+954R^2-1557R^3+960R^4-258R^5+24R^6)}\right).$$

The derivation of this solution makes use of the de Moivre's Theorem and it is available from the authors upon request. Depositors make use of their alternative investment opportunity and $u = 1$ if

$$\frac{k_B}{1 - k_B} > \eta. \tag{50}$$

Substituting the solution $\rho = \rho_1(R)$ into (49), and solving (50) with equality with respect to R gives

$$\overline{R} = \frac{(3\overline{\rho} - 1)^2}{\overline{\rho}(1 + \eta)\sqrt{2(3\overline{\rho}^2 - 4\overline{\rho} + 1)}},$$

where $\overline{\rho} = \rho_1(\overline{R})$. Putting $u = 1$ in (**??**) and (49) completes the first part of the proposition.

For $R > \overline{R}$, $u > 1$ and all deposits are used in the banking sector. The equilibrium level of u is found again from

$$\frac{k_B}{1 - k_B} = \eta,$$

where k_B is given by (49). Rearranging the expression gives

$$\frac{u(3\rho-u)^2}{\rho R\sqrt{2(3\rho^2-4\rho u+u^2)}}=1+\eta \tag{51}$$

Substituting then r_D as in (48) and k_B as in (49) into (17) gives

$$\Pi(\rho,u)=\frac{6\rho u(u-3\rho)+R(9\rho u-3u^2+2(\rho-u)\sqrt{2(3\rho^2-4\rho u+u^2)}}{6u(3\rho-u)}. \tag{52}$$

Solving (51) and (52) equal to zero with respect to u and ρ gives the solutions $u=u(R)$ and $\rho=\rho_2(R)$. □

Proof of Proposition 8: Suppose first that there exists a two-sector bank with $\frac{r'_L}{2}\geq r'_D(1-k'_B)$ that pays $\rho'\geq\frac{R}{2}$ per unit of capital to its shareholders and gives $EU=u'<\frac{R}{2}$ per unit of funds to its depositors. Given that in any equilibrium banks must make zero expected profits, $k'_B>0$ must hold. Otherwise, $k'_B=0$, banks would have positive expected profits since they obtain $\frac{r_L}{2}$ from both sectors with positive probability. Consider then a conduit one-sector bank with $r^*_D=r^*_L=r'_L, k^*_B=0$ and $k^*_F=k'_F$. Firms are indifferent between borrowing from two-sector and one-sector banks because their shareholders earn ρ' per unit of capital in both cases. The total expected payment from firms to two-sector banks is

$$\int_0^{\overline{r}'_F}\int_{\overline{r}'_F}^{R}\frac{r'_L}{2}\frac{1}{R^2}dr_1dr_2+\int_{\overline{r}'_F}^{R}\int_0^{\overline{r}'_F}\frac{r'_L}{2}\frac{1}{R^2}dr_1dr_2+\int_{\overline{r}'_F}^{R}\int_{\overline{r}'_F}^{R}r'_L\frac{1}{R^2}dr_1dr_2=\frac{r'_L\left(R-r'_L(1-k'_F)\right)}{R}.$$

The total expected payment from firms to one-sector banks is

$$\int_{\overline{r}^*_F}^{R}r^*_L\frac{1}{R}dr_1=\frac{r^*_L\left(R-r^*_L(1-k^*_F)\right)}{R}=\frac{r'_L\left(R-r'_L(1-k'_F)\right)}{R}.$$

With one-sector banks the total expected payment is received by depositors in return for the one unit of funds that they supply. With two-sector banks the one unit of funds is supplied partially by depositors and partially by shareholders because $k'_B>0$. Since the shareholders in the two-sector banks receive a return $\rho'\geq\frac{R}{2}$, the depositors in the one-sector bank must be strictly better off than the depositors in the two-sector bank. Therefore it is possible for the one-sector bank to reduce r^*_D so that positive profits are made. Hence, there cannot be two-sector banks in equilibrium.

Consider now the case where $\frac{r'_L}{2}<r'_D(1-k'_B)$ and the bank remains solvent only when

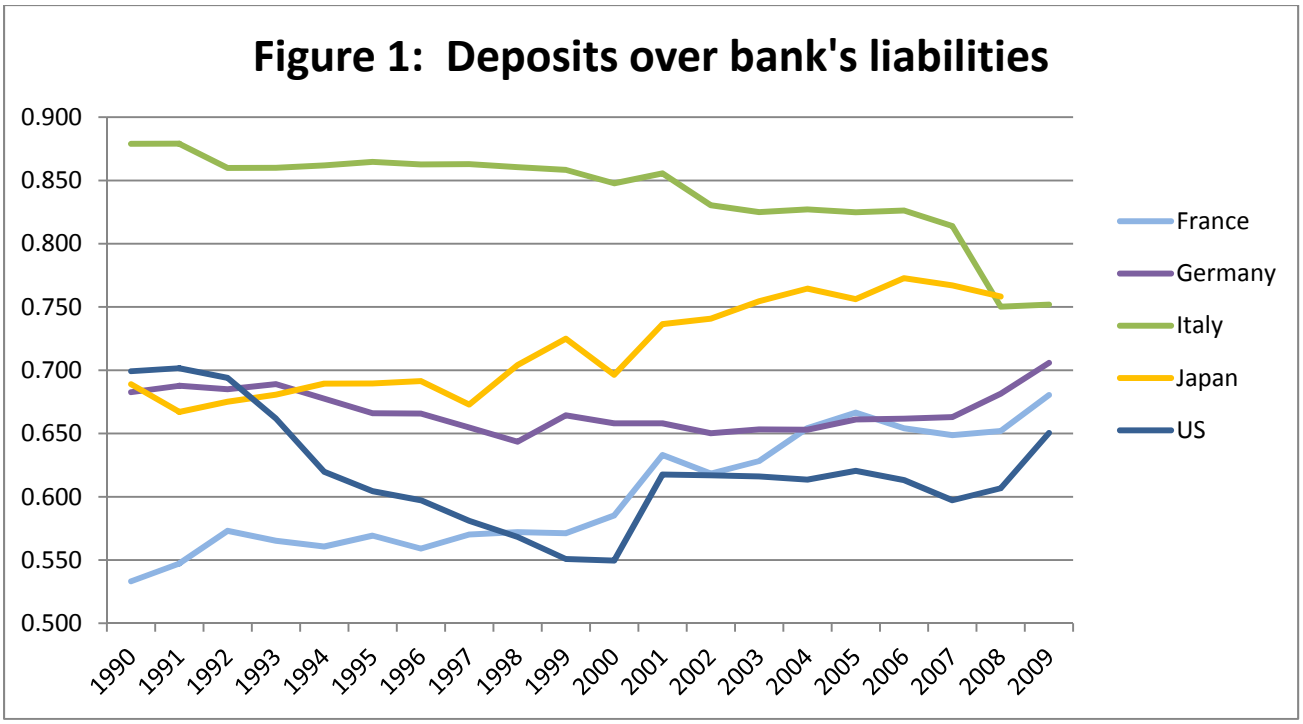
firms in both sectors repay r_L. Here the total expected payment received by depositors and shareholders in the two-sector banks is less than the total expected payments to the depositors in one-sector banks,

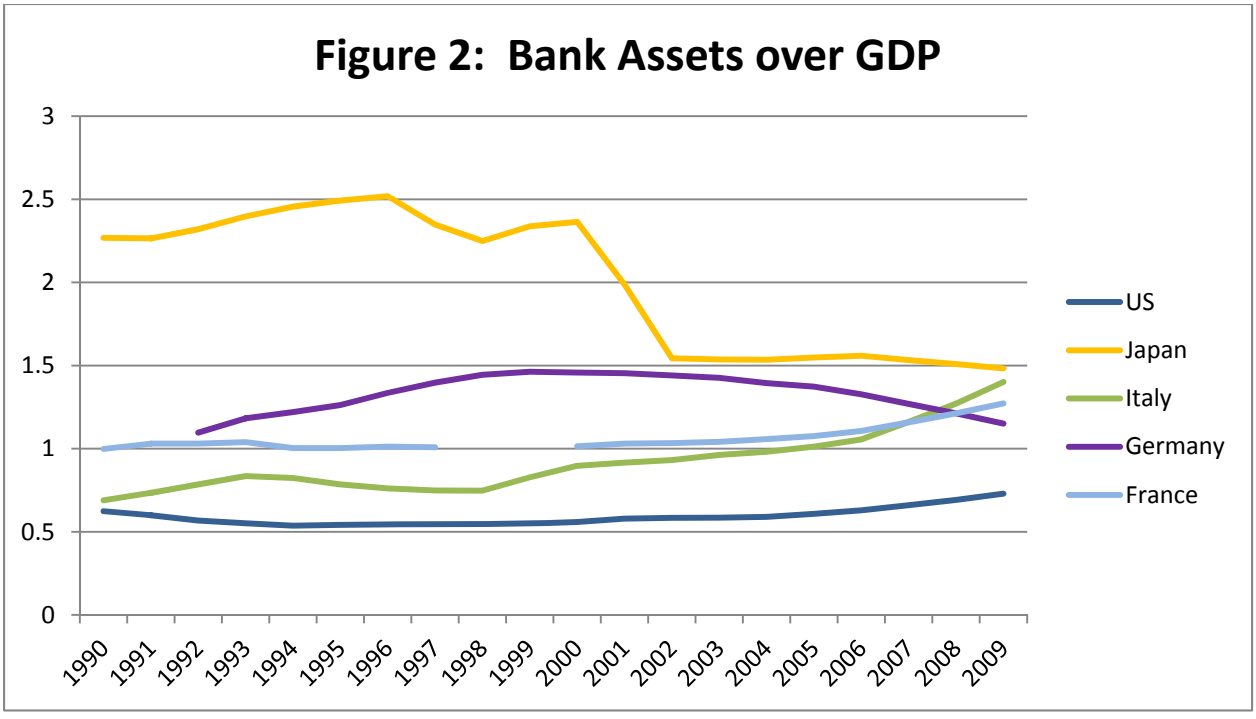
$$\int_{\overline{r}'_F}^{R}\int_{\overline{r}'_F}^{R} r'_L \frac{1}{R^2} dr_1 dr_2 = \frac{r'_L\left(R - r'_L(1-k'_F)\right)^2}{R^2} < \frac{r^*_L\left(R - r^*_L(1-k^*_F)\right)}{R},$$

since $r^*_L = r'_L$ and $k^*_F = k'_F$. This implies that one-sector banks again dominate.

Consider next the one-sector bank equilibrium as in Proposition 5. A similar argument to above can be used to show that two-sector banks cannot compete. Two-sector banks must offer contracts to firms that are at least as attractive as those offered by the one-sector banks. Suppose they offer the same contract. Then the output of two-sector banks is either equal or less depending on whether $\frac{r'_L}{2}$ is greater or smaller than $r'_D(1-k'_B)$. In the former case, it is not possible to make depositors and shareholders as well off since output is the same and depositors in one-sector banks receive it all while they do not in two-sector banks. In the latter case, when output is strictly less, it is again clear that depositors cannot be made as well off in two-sector banks. □

Proof of Proposition 9: It follows similarly to the proof of Proposition 6.


Figure 1: Deposits over bank's liabilities
0.900
0.850
0.800
0.750
0.700
0.650
0.600
0.550
0.500
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
France
Germany
Italy
Japan
US


Figure 2: Bank Assets over GDP
3
2.5
2
1.5
1
0.5
0
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
US
Japan
Italy
Germany
France

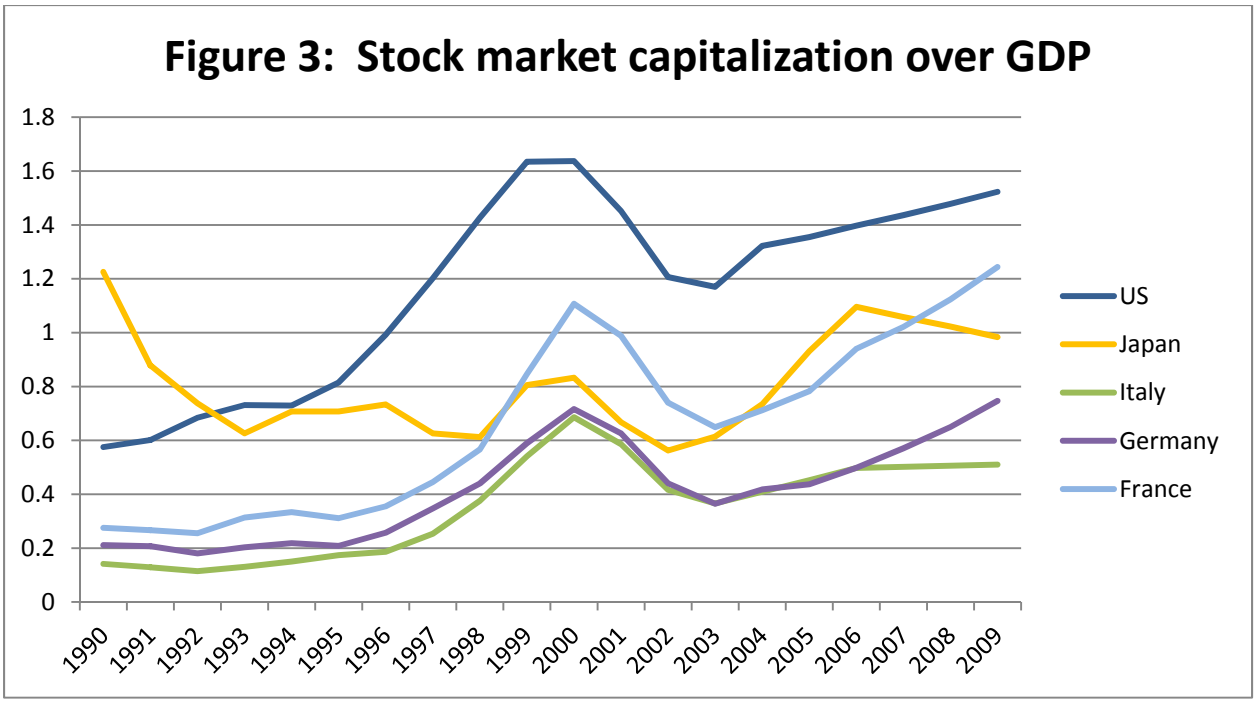

Figure 3: Stock market capitalization over GDP
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
US
Japan
Italy
Germany
France

Figure 4

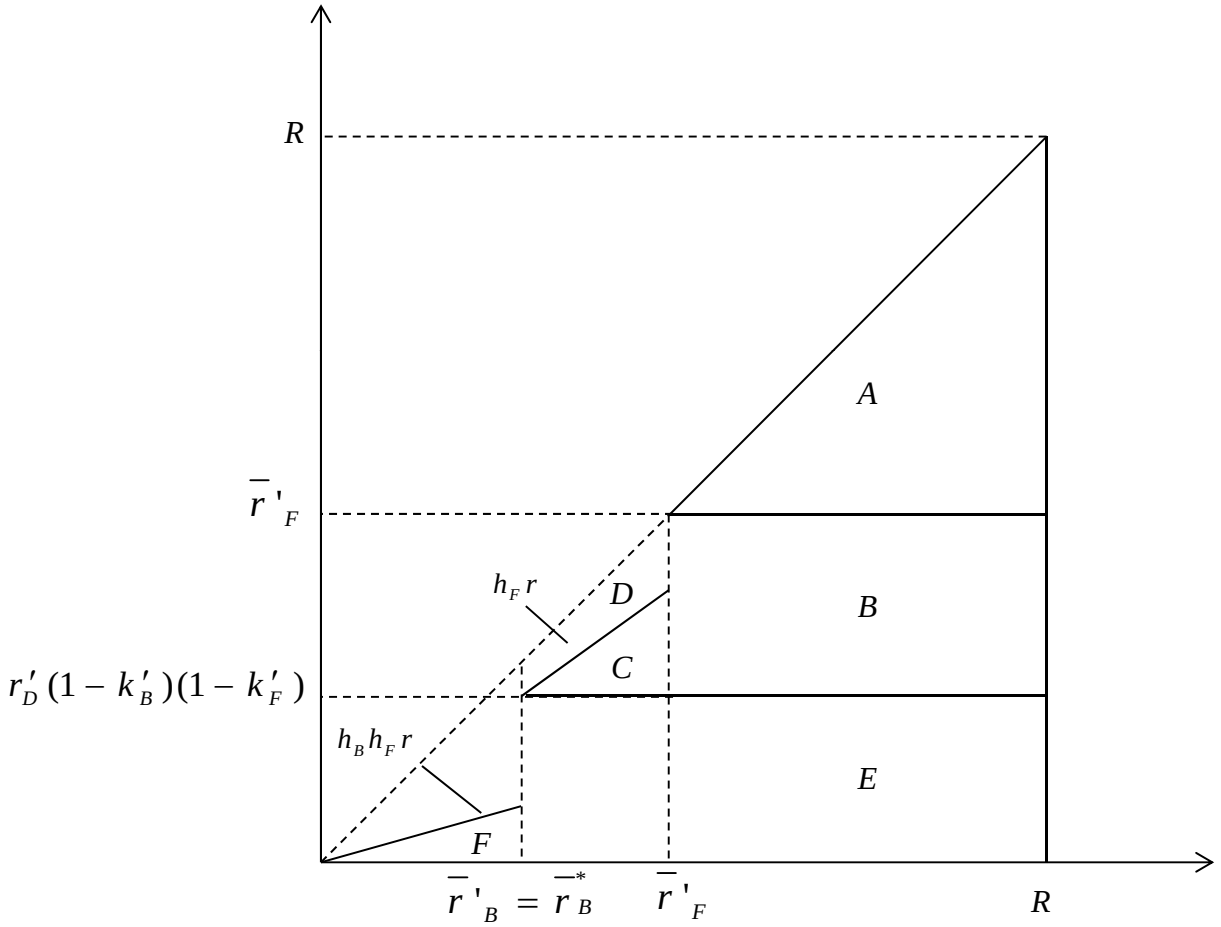


Figure 5

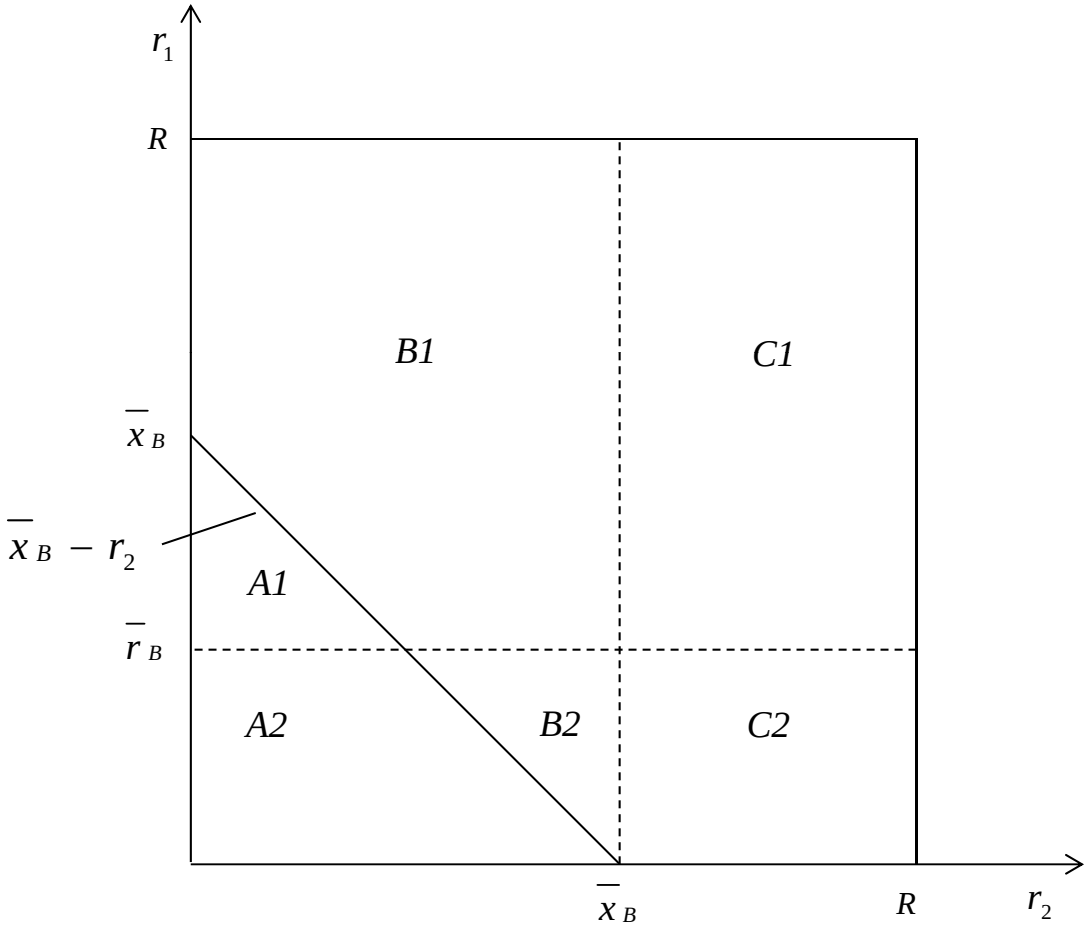


Figure 6

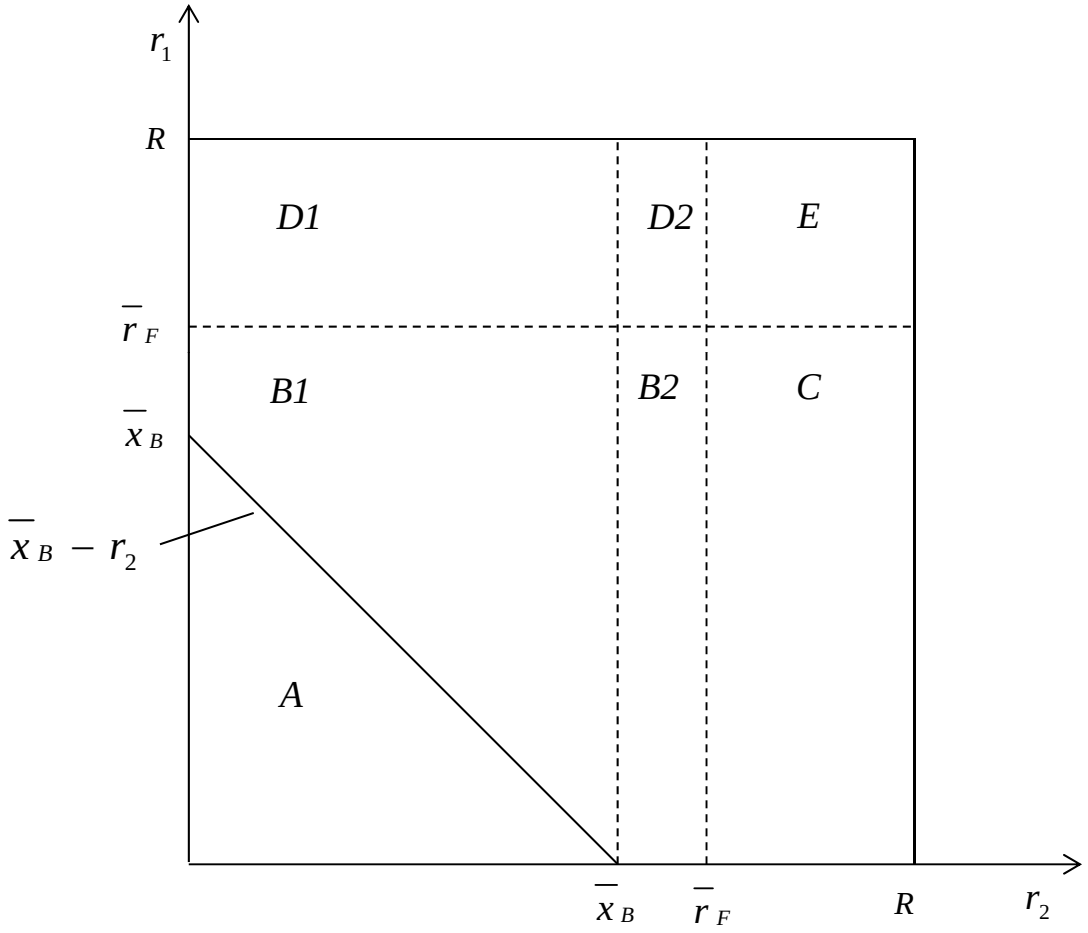


Figure 7

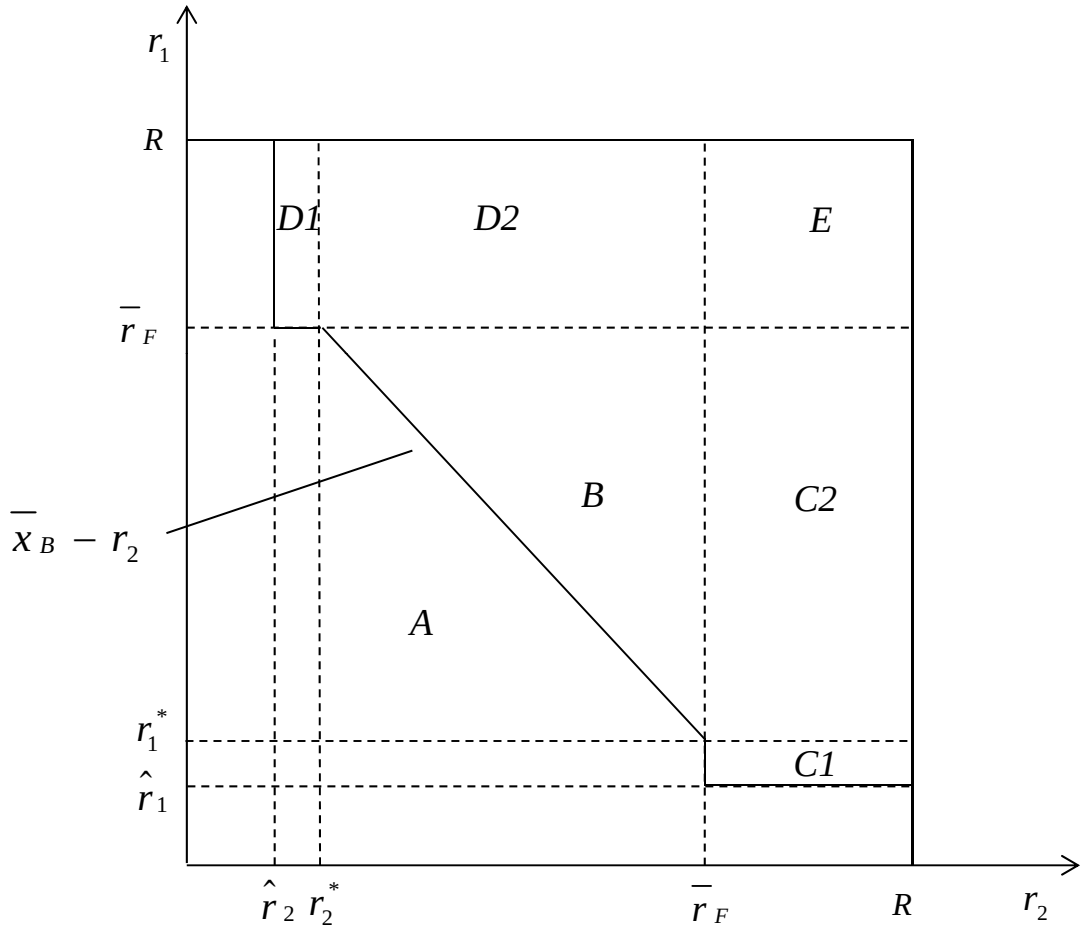


Figure 8

