Buy-it-now or Take-a-chance: Price Discrimination through Randomized Auctions *

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Abstract

Online tracking technology allows platforms to offer advertisers targeted consumer demographics, improving match quality, but thinning markets. Bidding data from Microsoft Advertising Exchange exhibits a large gap in the top two bids, consistent with this intuition. This motivates our new mechanism. Bidders can "buy-it-now", or "take-a-chance" in an auction where the top $d > 1$ bidders are equally likely to win. The randomized allocation incentivizes high valuation bidders to buy-it-now. Running counterfactual simulations on our data, we find it improves revenue by 4.5% and consumer surplus by 11% compared to an optimal second-price auction.

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1 Introduction

Advertising technology is changing fast. Consumers can now be reached while browsing the internet, playing games on their phone or watching videos on YouTube. The large companies that control these new media — household names like Google, Facebook and Yahoo! — generate a substantial part of their revenue by selling advertisements. They also know increasing amounts of information about their users. This allows them to match advertisers to potential buyers with ever greater efficiency. While this matching technology generates surplus for advertisers, it also tends to create thin markets where perhaps only a single advertiser has a high willingness to pay. These environments pose special challenges for the predominant auction mechanisms that are used to sell online ads because they reduce competition among bidders, making it difficult for the platform to extract the surplus generated by targeting.

For example, a sportswear firm advertising on the New York Times website may be willing to pay much more for an advertisement placed next to a sports article than one next to a movie review. It might pay an additional premium for a local consumer who lives in New York City and an even higher premium if the consumer is known to browse websites selling sportswear. Each layer of targeting increases the sportswear firm's valuation for the consumer but also dramatically narrows the set of participating bidders to fellow sportswear firms in New York City. Without competition, revenue performance may be poor (Bergemann and Bonatti 2010, Levin and Milgrom 2010).

Consider a simple model: When advertisers "match" with users, they have high valuation; otherwise they have low valuation. Assume that match probabilities are independent across bidders, and sufficiently low that the probability that *any* bidder matches is relatively small. Then a second-price auction will typically get low revenue, since the probability of two "matches" occurring in the same auction is small. On the other hand, setting a high fixed price is not effective since the probability of zero "matches" occurring is relatively large and many impressions would go unallocated. Hence, allowing targeting creates asymmetries in valuations that can increase efficiency, but decrease revenue. Because of this, some have suggested it is better to create thicker markets by not disclosing information, thus "bundling" many different impressions together (Ghosh, Nazerzadeh and Sundararajan 2007, Even-Dar, Kearns and Wortman 2007, McAfee, Papineni and Vassilvitskii 2010). The question of how to optimally bundle is a subject of ongoing research (Bergemann, Bonatti and Said 2011).

Since targeting increases total surplus, platforms would like to allow targeting while still extracting the surplus this creates. This paper outlines a new and simple mechanism for doing so. We call it *buy-it-now or take-a-chance* (BIN-TAC), and it works as follows. Goods are auctioned with a buy-it-now price p, set relatively high. If a single bidder chooses buy-it-now, they get the good for price p. If more than one bidder takes the buy-it-now option, a second price auction is held between those bidders with reserve p. Finally, if no-one participates in buy-it-now, an auction is held in which the top d bidders are eligible to receive the good, and it is randomly awarded to one of them at the $(d+1)$-st price.

In this manner, we combine the advantages of an auction and a fixed price mechanism. When matches occur, advertisers can self-select into the fixed-price buy-it-now option, allowing for revenue extraction. Advertisers are incentivized to do take the "buy-it-now" option because in the event that they "take-a-chance" on winning via auction, there is a significant probability they will not win the impression, even if their bid is the highest. On the other hand, when no matches occur, the auction mechanism ensures the impression is still allocated, thereby earning revenue.

BIN-TAC is simple, both in that it is easy to explain to advertisers and in that it requires relatively little input from the mechanism designer: a choice of buy-it-now price, randomization parameter d and a reserve in the take-a-chance auction. As we show both analytically and through monte carlo simulation, BIN-TAC generally outperforms the two leading alternatives: a second price auction with reserve, or the "bundling" solution in which the platform withholds targeting information. At least in principle one could do better still by using the revenue-optimal mechanism suggested in Myerson (1981), which is considerably more complicated. We demonstrate via numerical simulations that BIN-TAC approximates the allocations and payments of the optimal mechanism, achieving a similar performance.

To analyze its performance in a real-world setting, we turn to historical data from the Microsoft Advertising Exchange. By estimating the distribution of advertiser valuations, we can simulate the effect of introducing the BIN-TAC mechanism. We also consider a bundling strategy in which all impressions on a given webpage browsed by a user located in a particular geographic region are sold as identical products. We find that the optimal BIN-TAC mechanism generates 4.5% more revenue than the optimal second-price auction, while at the same time improving consumer surplus by 11%. This is possible because the optimal second-price auction uses a high reserve to extract surplus from the long tail of valuations, whereas the BIN-TAC mechanism does this through a high buy-it-now price,

which avoids excluding low valuation bidders. By comparison, the second-price auction without any reserve generates 8.5% less revenue but 17.2% higher consumer surplus. All of them outperform the bundling strategy, although we cannot rule out better performance from an optimal bundling strategy.

We view the main contribution of our paper as introducing and analyzing a new and simple price discrimination mechanism that makes use of randomized auctions, and then testing its performance in a realistic environment. While our focus is on the display advertising market, we note that there are other markets in which randomized allocations are used as a screening tool. For example, Priceline offers users the choice between a hotel of their choice at a high fixed price, or the opportunity to bid for a random hotel room of certain guaranteed characteristics (e.g. location, star rating).

A secondary contribution of the paper is to document participation and bidding behavior in the display advertising market. While there has been theoretical work on this market (Muthukrishnan 2010, McAfee 2011), and empirical work on the search advertising market (Ostrovsky and Schwarz 2009, Athey and Nekipelov 2010), there has been little empirical work of this sort on display advertising. We document that there is a large gap between the highest and second highest valuations in these auctions, consistent with targeting creating thin markets. We also show that advertisers vary their bids based on the location of their users, using the user demographics provided by the platform to achieve better matches.

Related Work: Our work is related to the literature on price discrimination and screening. Here we consider a mechanism that treats all bidders symmetrically, and proceeds sequentially. Other papers have suggested sequential screening approaches. In one setting, the buyers themselves learn their type dynamically, in two stages (Courty and Li 2000). In this case, offering contracts after the first type revelation but before the second may be optimal; see Bergemann and Said (2010) for a survey on dynamic mechanisms. In the static setting, sequential screening and posted-price mechanisms can be used to design optimal (or near-optimal) mechanisms when the bidders have multi-dimensional private information (see for example Rochet and Chone (1998) and Chawla, Hartline, Malec and Sivan (2010)).

More generally, the question of whether sellers should provide information that allows buyers to target their bids arises in the analysis of optimal seller disclosure (see for example Lewis and Sappington (1994) and Bergemann and Pesendorfer (2007)). The idea of bundling goods together to take advantage of negative correlation in valuations — in this case the negative

correlation in the valuations from "match" or "no match" — dates back to Adams and Yellen (1976); see also McAfee, McMillan and Whinston (1989). Our paper is similar in style to Chu, Leslie and Sorensen (2011), who combine theory, simulations and empirics to argue that bundle-size pricing is a good approximation to the more complicated (but theoretically superior) mixed bundling pricing scheme for a monopolist selling multiple goods.

We focus on a private values setting, while Abraham, Athey, Babioff and Grubb (2010) consider an adverse selection problem that arises in a pure common value setting when some bidders are privately informed. This is motivated by the case when some advertisers are better able to utilize the user information provided by the platform. They show that asymmetry of information can sometimes lead to low revenue in this market. Our paper is also related to the literature on buy-now auctions (Budish and Takeyama 2001, Reynolds and Wooders 2009).

Finally from an empirical perspective, our paper contributes to the growing literature on online advertising and optimal pricing. Much of the work here is experimental in nature — for example, Lewis and Reiley (2011) ran a randomized experiment to test advertising effectiveness, while Ostrovsky and Schwarz (2009) used an experimental design to test the impact of reserve prices on revenues. There has also been recent empirical work on privacy and targeting in online advertising (Goldfarb and Tucker 2011a, Goldfarb and Tucker 2011b).

Organization: The paper proceeds in three parts. First, we give an overview of the market for display advertising. In the second part we introduce a stylized environment, and prove existence and characterization results for the BIN-TAC mechanism. We also provide analytic results concerning the revenue maximizing parameter choices, and compare our mechanism to others using both theory and monte carlo simulation. Finally, in the third part we provide an empirical analysis of a display advertising marketplace, including counterfactual simulations of our mechanism's performance. All proofs are contained in the appendix.

2 The Display Advertising Market

This paper proposes a new second degree price discrimination strategy for advertising platforms such as Microsoft, Google and Facebook. In these markets, advertisers care about the characteristics of the users they advertise to, but it is up to the platform to choose whether

or not to disclose what they know about their users. The online display advertising market is an example of such a market. Its organization is depicted in Figure 1. On one side of the market are the "publishers": these are websites who have desirable content and therefore attract Internet users to browse their sites. These publishers earn revenue by selling advertising slots on these sites.

The other side of the market consists of advertisers. They would like to display their advertisements to users browsing the publisher's websites. They are buying user attention. Each instance of showing an advertisement to a user is called an "impression". Advertiser demand for each impression is determined by which user they are reaching, and what the user's current desires or intent are. For example, a Ferrari dealer might value high income users located close to the dealership. A mortgage company might value people that are reading an article on "how to refinance your mortgage" more than those who are reading an article on "ways to survive your midlife crisis", while the dealership might prefer the reverse.

Some large publishers, primarily AOL, Microsoft and Yahoo!, sell directly to advertisers. Since the number of users browsing such publishers is extremely large (e.g. 1.5% of total worldwide Internet pageviews are on Yahoo![1]), they can predict with high accuracy their user demographics. Consequently, they think of themselves of having a known inventory, consisting of a number of products in well-defined buckets: for example, male 15-24 year olds living in New York City viewing the Yahoo! homepage. They can thus contract to sell 1 million impressions delivered to a target demographic to a particular advertiser. Provided they have the inventory, they should be able to fulfill the contract. Transactions of this kind are generally negotiated between the publisher and the advertiser.

Alternatively, content is sold by auction through a centralized platform called an advertising exchange. Examples of leading advertising exchanges include the Microsoft Advertising Exchange (a subset of which we examine in this paper), Google's DoubleClick, and Yahoo's RightMedia.[2] Advertising exchanges are a minor technological wonder. They work in real-time. When a user loads a participating publisher's webpage, a "request-for-content" is sent to the advertising exchange. This request will specify the type and size of advertisement to be displayed on the page, as well as information about the webpage itself (potentially including information about its content), and information about the user browsing the page.[3]

[1] Source: alexa.com

[2] "In Sept 2009, RightMedia averaged 9 billion transactions a day with hundreds of thousands of buyers and sellers." Muthukrishnan (2010)

[3] For example, it may include their IP address and cookies that indicate their past browsing behavior.

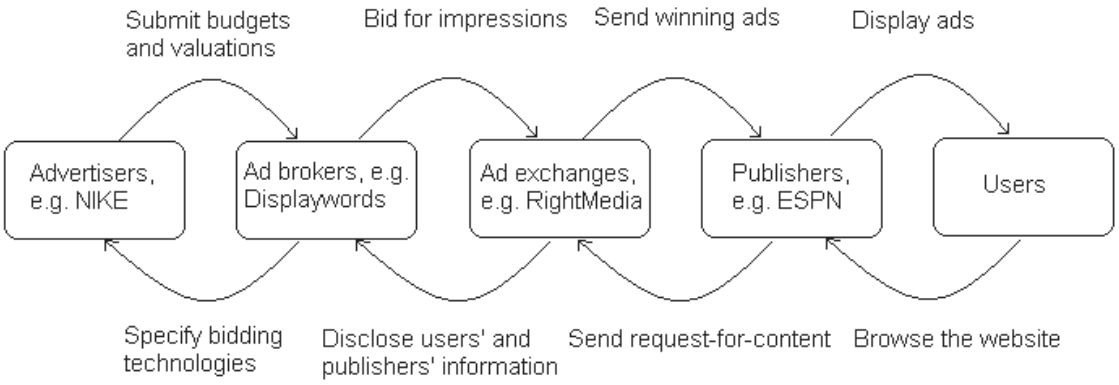


Figure 1: **The Display Advertising Market.**

The advertising exchange will then either allocate the impression to an advertiser at a previously negotiated price, or hold a second-price auction between participating advertisers. If an auction is held, all or some of the information about the webpage and user is passed along to ad brokers who bid on behalf of the advertisers. These ad brokers can be thought of as proprietary algorithms that take as input an advertiser's budget and preferences, and output decisions on whether to participate in an auction and how much to bid. The winning bidder's ad is then served by the ad exchange, and shown on the publisher's webpage.[4]

The bids placed in the auction are jointly determined by the preferences advertisers have, the ad broker interface and the disclosure policies of the ad exchanges or the publishers they represent. The ad brokers can only condition the bids they place on the information provided to them: if the user's past browsing history is not made available to them, they can't use it in determining their bid, even if their valuation would be influenced by this information. Similarly, the advertisers are constrained in expressing their preferences by the technology of the ad broker: if the algorithm doesn't allow the advertiser to specify a different willingness to pay based on some particular user characteristic, then this won't show up in their bids.

Ad exchanges have two main advantages over direct negotiation. First, they economize on transaction costs, by creating a centralized market for selling ad space. Second, they allow

[4]To make things yet more complicated, in some ad exchanges — though not Microsoft Advertising Exchange — two different pricing models coexist. The first is pay-per-impression, which is what we analyze in the current paper; the second is pay-per-click, where the payment depends on whether or not the user clicks on the advertisement. Ad exchanges use expected click through rates to compare these different bids through a single expected revenue number.

for very detailed products to be sold, such as the attention of a male 15-24 year old living in New York City viewing an article about hockey that has previously browsed articles about sports and theater. There is no technological reason why the products need to be sold in "buckets", as publishers tend to do when guaranteeing sales in advance. This "real-time" sales technology is often touted as the future of this industry, as it potentially improves the match between the advertiser and their target audience. We will focus on developing a real-time pricing mechanism for display advertising exchanges.

3 Model and Analysis

3.1 The Environment

A seller (publisher) has an impression to sell in real time, and they have information about the user viewing the webpage, summarized in a cookie.[5] The seller is considering one of two policies: either disclosing the cookie content to the advertiser (the "targeting" policy), or withholding it (the "bundling" policy). When they allow targeting, bidders know whether the user is a "match" for them or not. When a match occurs, the bidder has a high valuation. But the probability of a match is low and matches are assumed independent, so it is likely that everyone in the auction has a low valuation. Allowing targeting may make the market "thin" in the sense of bids being relatively low.

Instead the seller may choose to withhold the cookie, so that bidders are uncertain about whether the user is a match for them or not. The seller thus bundles good impressions with bad ones, so that bidders have intermediate valuations. This reduces match surplus, but also reduces the bidder's information rents and so may be good for revenue.

The formal model is as follows. There are n symmetric bidders who participate in an auction for a single good which is valued at zero by the seller. Bidders are risk neutral. They have value V_H for the good when a match occurs, and value V_L for the good if no match occurs, where $V_L \sim F_L$ and $V_H \sim F_H$. We assume that F_L has support $[\underline{\omega}_L, \overline{\omega}_L]$ and F_H has support $[\underline{\omega}_H, \overline{\omega}_H]$, and that these supports are disjoint (so $\overline{\omega}_L < \underline{\omega}_H$). We assume both F_L and F_H have continuous densities f_L and f_H. The Bernoulli random variable X indicates whether a

[5]Rougly speaking, a cookie refers to the information sent from the browser of the user to the website visited by user and can be used to store the state of the communication between them and other information about the user (RFC6265 2011).

match has occurred, and the event $X = 1$ occurs with probability $\alpha \in (0, 1)$.

The bidder type is a triple (X, V_L, V_H) is drawn identically and independently across bidders, so that a user who is a match for one advertiser need not be a match for the others. In the case with targeting, each advertiser's realized valuation $V = (1 - X)V_L + XV_H$ is private information, known only to the advertiser. Instead if the seller bundles all impressions, the advertiser knows V_L and V_H but does not know the realization of X, implying their expected valuation is $E[V] = (1 - \alpha)V_L + \alpha V_H$.

For simplicity of the presentation, we also make some technical assumptions on the *virtual valuations* $\psi(v) = v - \frac{1-F(v)}{f(v)}$.[6] We assume that $\psi(v)$ is continuous and increasing over the regions $[\underline{\omega}_L, \overline{\omega}_L]$ and $[\underline{\omega}_H, \overline{\omega}_H]$. We additionally assume that $\psi(v)$ single-crosses zero, that this intersection occurs in the low valuation region $[\underline{\omega}_L, \overline{\omega}_L]$, and that $\psi(\overline{\omega}_L) \leq \psi(\underline{\omega}_H)$. Overall, our environment is fully characterized by the tuple (n, α, F_L, F_H).

Discussion: We assume that the match random variables X and the valuations V_L and V_H are independent across bidders. We focus on independence for two reasons. First, it is an assumption that is often made in the screening and mechanism design literatures, and so is a natural starting point. Second, in the log data examined in this paper we observe little correlation in bids.[7]

We also will focus on environments where α is small, since this implies that the probability of zero or a single match is high. This is the interesting case, reflecting the industry concern that providing "too much" targeting information reduces competition and hurts revenues. In our data we often observe a large gap between the highest and second highest bid, which provides support for this focus.

3.2 Pricing Mechanisms

We propose using a randomized auction as a pricing mechanism. Our BIN-TAC mechanism works as follows. A *buy-it-now price* p is posted. Buyers simultaneously indicate whether they wish to *buy-it-now* (*BIN*). In the event that exactly one bidder elects to buy-it-now,

[6] Without these assumptions we would have to analyze multiple cases, which is straightforward but tedious.

[7] That bids are not positively correlated certainly does not prove that the underlying valuations are not positively correlated — it could just be that informational and technological constraints prevent advertisers from fully expressing their preferences — but it does provide some support for our assumption.

that bidder wins the auction and pays p. If two or more bidders elect to BIN, a second-price sealed bid auction with reserve p is held between those bidders. Bidders who chose to BIN are obliged to participate in this auction. Finally, if no-one elects to BIN, a sealed bid *take-a-chance* (TAC) auction is held between all bidders, with a reserve r. In that auction, one of the top d bidders is chosen uniformly at random, and if that bidder's bid exceeds the reserve, they win the auction and pay the maximum of the reserve and the $(d+1)$-th bid. Ties among d-th highest bidders are broken randomly prior to the random allocation. We call r the TAC-reserve, and d the randomization parameter.

To analyze the performance of BIN-TAC, it will be useful to have some benchmarks for comparison. A natural benchmark is the pricing mechanism that is most commonly used in practice, the second price auction (SPA). We distinguish between when an SPA is used and targeting is allowed (SPA-T), and when it is used with bundling (SPA-B).

A third benchmark is the revenue-optimal mechanism within the class of those that allow targeting (i.e. those that commit to reveal the cookie to all bidders for free).[8] Usually this mechanism is the second-price auction with an optimally chosen reserve price. However in this case the virtual valuations $\psi(v)$ are not increasing over the whole support of F — indeed they are (infinitely) negative over the region $(\overline{\omega}_L, \underline{\omega}_H)$. The optimal mechanism may require ironing (Myerson 1981).

In plain terms, ironing implies that sometimes the allocation will be randomized among bidders with different valuations. Just as in our TAC auction, the winner of the auction need not have the highest valuation. The difference is that in the optimal mechanism, the randomization only takes place when two or more bidders — including the highest valuation bidder — have valuations in a given "ironing" region. By contrast, in BIN-TAC this randomization occurs whenever no-one takes the BIN option. The differences will be clearer later when we compare the performance of the mechanisms. For now, we would like to present a simple example to illustrate why our BIN-TAC mechanism may be good at delivering both social surplus and revenue, as a motivation for the detailed equilibrium analysis that follows.

[8] A seller may potentially do better by withholding match information from everyone (bundling), or by giving different information to different bidders — see Bergemann and Pesendorfer (2007).

3.3 A Motivating Example

Consider a special case of our environment with just two bidders, and a match probability of 10%. Bidders have fixed symmetric valuations, equal to 10 if they match, and 1 otherwise. Now consider the expected outcomes of the two second-price auction mechanisms. With targeting, the allocation will be fully efficient. The probability that at least one bidder has a high valuation is $1 - (0.9)^2 = 0.19$, and so expected surplus is $10(0.19) + 1(0.81) = 2.71$. On the other hand, the probability that both bidders match is only 1%, so expected revenue is only $(0.01)10 + (0.99)1 = 1.09$.

Under bundling, the two bidders don't know if the impression is a match, and so value it at its expected value of $(0.1)10 + (0.9)1 = 1.9$. They bid identically, yielding expected revenues of 1.9. This is a significant improvement. But now the allocation may be ex-post inefficient, with the lower valuation bidder getting the impression. Expected surplus is equal to $(0.01)10 + (0.18)5.5 + (0.81)1 = 1.9$, much lower than before. Notice that the bundling strategy has eliminated all the buyers' information rents, so that the seller captures all the surplus as revenue.

Next, consider the BIN-TAC mechanism with a BIN price of 5.5, TAC reserve of 1, and randomization parameter 2. When a buyer matches, they will (weakly) take the BIN price, since their surplus on doing so is $10 - 5.5 = 4.5$, whereas if they take-a-chance, they have a 50% chance of getting the object, with expected payoff $(0.5)(10-1) = 4.5$. So if both buyers match, there will be an auction with revenue 10; if one buyer matches, the revenue will be 5.5; and if none match it will be 1. Adding this up gets an expected revenue of 1.9, as in the SPA-B case. But notice that the BIN-TAC allocation is fully efficient, and thus gets the same surplus as the SPA-T. So the BIN-TAC mechanism may improve revenues relative to the SPA-T, and welfare relative to the SPA-B.

3.4 Equilibrium Analysis

Returning to the general environment, we proceed by backward induction to characterize equilibrium strategies under BIN-TAC. If multiple players choose to BIN, the allocation mechanism reduces to a second-price auction with reserve p. Thus, it is weakly dominant for

players to bid their valuations.[9] Truth-telling is also weakly dominant in the TAC auction. The logic is standard: if a bidder with valuation v bids $b' > v$, it can only change the allocation when the maximum of the d-th highest rival bid and the reserve price is in $[v, b']$. But whenever this occurs, the resulting price of the object is above the bidder's valuation and if she wins she will regret her decision. Alternatively, if she bids $b' < v$, when she wins the price is not affected, and her probability of winning will decrease. Thus, overall, BIN-TAC is Bayes-Nash incentive compatible; both BIN and TAC are dominant-strategy incentive compatible, and the choice of which of the two to participate can be maximized based on other agents expected behavior.

Taking these strategies as given, we turn to the decision of which auction, buy-it-now or take-a-chance, an agent will choose. Intuitively, the BIN option should be more attractive to higher types: they have the most to lose from either random allocation (they may not get the good even if they are willing to pay the most) or from rivals taking the BIN option (they certainly do not get the good). This suggests that in a symmetric equilibrium, the BIN decision takes a threshold form: $\exists\ \overline{v}$ such that types with $v \geq \overline{v}$ elect to BIN, and the rest do not. This is in fact the case.

Prior to stating a formal theorem, we introduce the following notation. Let the random variable Y^j be the j-th highest draw in an i.i.d sample of size $n-1$ from F (i.e., the j-th highest rival valuation) and let Y^* be the maximum of Y^d and the TAC reserve r.

Proposition 1 (Equilibrium Characterization) *Assume $d > 1$. Then there exists a unique symmetric pure strategy Bayes-Nash equilibrium of the game, characterized by a threshold $\overline{v}$ satisfying:*

$$\overline{v} = p + \frac{1}{d} E\left[\overline{v} - Y^* | Y^1 < \overline{v}\right] \tag{1}$$

where types with $v \geq \overline{v}$ take the BIN option; and all types bid their valuation in any auction that may occur.

Equation (1) is intuitive: Which type is indifferent between the BIN and TAC options? If strategies are increasing, the only time the choice is relevant is when there are no higher valuation bidders (since otherwise those bidders would BIN and win the resulting auction).

[9] Since participation is obligatory at this stage, the minimum allowable bid is p, but no bidder would take the BIN option unless they had a valuation of at least p. Note, however, that even if a bidder's value is above p, they may not choose to take the BIN auction.

So if a bidder has the highest value and chooses to BIN, they get a surplus of $v-p$. Choosing to TAC gives $\frac{1}{d}E\left[v - Y^*|Y^1 < v\right]$, since they only win with probability $\frac{1}{d}$, although their payment of Y^* is on average much lower. Equating these two to find the indifferent type $\overline{v}$ yields Equation (1).[10]

Now we consider the revenue-maximizing choices of the design parameters: the BIN price p, the TAC reserve r and the randomization parameter d. It is hard to characterize the optimal d, as it is an integer programming problem which does not admit standard optimization approaches. However for a given d, the optimal BIN price and TAC reserve can be characterized using first order conditions:

Proposition 2 (Optimal Buy Price and Reserve) *For any randomization parameter d, the revenue-maximizing TAC reserve r^* is either equal to $\underline{\omega}_H$ or is the unique solution of*

$$r = \frac{1 - F(r)}{f(r)} \tag{2}$$

The optimal BIN price is given by $p(\overline{v}^*, r)$ where $p(\overline{v}, r) = \overline{v} - \frac{1}{d}E\left[\overline{v} - Y^*|Y^1 < \overline{v}\right]$ and $\overline{v}^*$ is either equal to $\underline{\omega}_H$ or is a solution of the equation below:

$$d - \frac{\partial E[\overline{v} - Y^*|Y^1 < \overline{v}]}{\partial \overline{v}} = \left(\frac{(d-1)f(\overline{v})}{1 - F(\overline{v})} + \frac{(n-1)f(\overline{v})}{F(\overline{v})}\right) E\left[\overline{v} - Y^*|Y^1 < \overline{v}\right] \tag{3}$$

Equation (2) is familiar: the optimal TAC reserve is exactly the standard reserve in Myerson (1981), ensuring that no types with negative virtual valuation are ever awarded the object. This is a little surprising, since BIN-TAC is not the optimal mechanism. But the usual incentive compatibility trade-offs apply, as the TAC reserve is relevant for the BIN choice. Raising the TAC reserve lowers the surplus from participating in the TAC auction, and so the seller can also raise the BIN price while keeping the indifferent type $\overline{v}$ constant. So the trade-off is the usual one: raising the TAC reserve increases expected payments from types above r^* — even those who take the BIN option — at the cost of losing revenue from the marginal type. This is why we get the usual solution.

On the other hand, the implicit equation for the optimal BIN price is new. Notice that the BIN price in some sense sets a reserve at $\overline{v}$. If two bidders meet the reserve, the seller gets

[10]The above theorem does not require our mixture distribution environment: it remains true for an IPV environment with arbitrary absolutely continuous F.

the second highest bid; if only one, the BIN price; and if none, he gets the TAC revenue. So a marginal increase in the threshold has three effects. First, if the highest bidder has valuation exactly equal to the threshold, following an increase she will shift from BIN to TAC. This costs the seller the difference between the BIN price and the expected revenue from the TAC auction (which is lower). Second, if the second highest bidder has valuation equal to the threshold, an increase will knock her out of the BIN auction, and the seller's revenue falls by $\overline{v} - p(\overline{v}, r^*)$. Finally, if the highest bidder is above the reserve and the second highest is below, an increase gains the seller $\frac{\partial p(\overline{v},r^*)}{\partial \overline{v}}$. Working out the probabilities of these various events, expanding $\frac{\partial p(\overline{v},r^*)}{\partial \overline{v}}$ and equating expected costs and benefits, we get the result.

We cannot rule out a corner solution for the buy-price, where it is set equal to $\underline{\omega_H}$. This can easily occur if the value of a match is high (i.e. $\underline{\omega}_H \gg \overline{\omega}_L$). In this case it is not profitable to randomize the allocation for any of the high types: the BIN price is set at $p(\underline{\omega}_H, r^*)$ so that the lowest high type at $\underline{\omega}_H$ elects to BIN.

3.5 Performance Comparisons

We are interested in comparing the BIN-TAC mechanism to the benchmarks in terms of both revenue and total welfare. For any mechanism M with parameters θ, define a payoff function $\pi(M, \theta, \beta)$ as follows (suppressing the dependence on the environment):

$$\pi(M, \theta, \beta) = ER(M, \theta) + \beta ECS(M, \theta) \tag{4}$$

where ER denotes expected revenue and ECS expected consumer surplus. Notice that when $\beta = 0$, the platform objective is just to maximize revenue as in the usual optimal mechanism design problem. Similarly, when $\beta = 1$ the objective aligns with the social planner problem of maximizing welfare. We say that mechanism M strictly dominates M' if $\max_\theta \pi(M, \theta, \beta) \geq \max_\theta \pi(M', \theta, \beta)$ for all $\beta \in [0, 1]$ and for all environments (n, α, F_L, F_H), with strict inequality for some environment and β. Strict dominance means that regardless of whether the platform is maximizing revenue, joint welfare, or some combination of the two, and regardless of the environment, mechanism M is better able to achieve that objective than M'.

Proposition 3 (Mechanism Performance) *(i) BIN-TAC strictly dominates SPA-T. (ii) When F_H and F_L are degenerate with atoms at V_H and V_L respectively, BIN-TAC strictly*

dominates SPA-B.

The formal proof is in the appendix, but we provide some intuition here. The first result follows by showing that SPA-T is just a special case of BIN-TAC, and therefore any performance achievable by SPA-T is also achievable by BIN-TAC. The idea is to turn the TAC auction into an SPA, by setting the randomization parameter $d = 1$ and the BIN-price p so high that the BIN option is never taken.

For the second part, notice that BIN-TAC will obviously be better than SPA-B for high β, since the bundling solution suppresses the information needed to ensure good match outcomes. A harder test then is whether BIN-TAC performs better even when the platform is only interested in revenue maximization ($\beta = 0$). We show that this is true whenever F_L and F_H are degenerate, so that the only source of private information is the match variable X. In that case bundling removes all private information, eliminating all information rents; and yet, BIN-TAC still achieves higher revenues. This is because running a TAC auction causes no distortion in the allocations at the bottom (all bidders have valuations V_L), and so BIN-TAC is able to extract most of the rents it creates by revealing the match information. Unfortunately. as we show in Monte Carlo simulations below, this result does not extend to the full environment, though BIN-TAC generally outperforms bundling.

By definition, BIN-TAC will have (weakly) worse revenue performance than the revenue-optimal mechanism. The question then is how close BIN-TAC gets. We will show, informally, that it gets very close indeed, using graphs and simulations. At this point it will be useful to describe the revenue-optimal mechanism in detail. Because it is hard to solve for analytically, to economize on space we derive it in the supplementary appendix. There we also show that the interesting case occurs when $\alpha\underline{\omega}_H < r^*(1 - F(r^*))$, where r^* is the optimal reserve of equation (2).[11] In that case, define the ironed virtual valuations as follows:

$$\phi(v) \quad = \quad \begin{cases} 0 & v \in [\underline{\omega}_L, r^*) \\ \psi(v) & v \in [r^*, \widetilde{v}] \\ \psi(\widetilde{v}) & v \in (\widetilde{v}, \underline{\omega}_H) \\ \psi(v) & v \in [\underline{\omega}_H, \overline{\omega}_H], \end{cases} \tag{5}$$

[11] When this condition fails, ironing is not required and the optimal mechanism is just a second price auction with $r^* \in [\underline{\omega}_H, \overline{\omega}_H]$, which can be implemented as a BIN-TAC auction with $d = 1$.

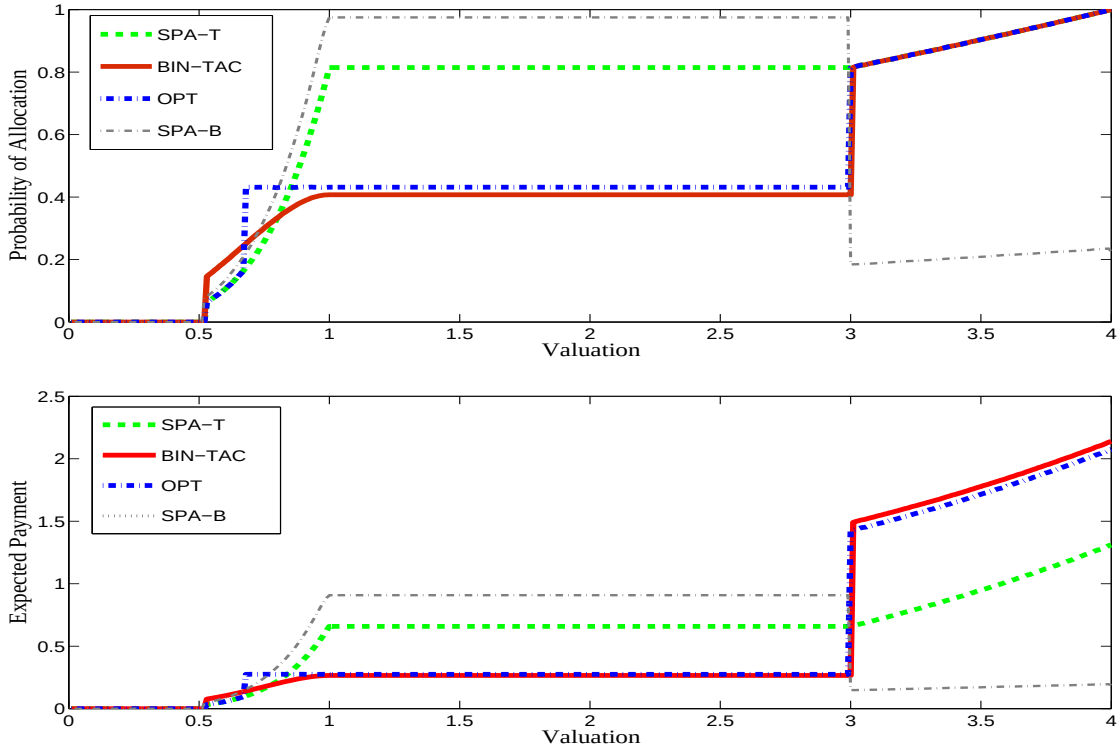


Figure 2: **Comparison of Allocations and Payments.** Allocation probabilities (top panel) and expected payments (bottom panel) for the OPT, SPA-T, SPA-B and BIN-TAC mechanisms when the distributions F_L and F_H are uniform. The x-axis corresponds to the bidder's valuation (unknown to them under bundling).

where $\widetilde{v}$ is the solution of:

$$-F(\widetilde{v})^2 + (2-\alpha)F(\widetilde{v}) + \alpha(\underline{\omega}_H - \widetilde{v})f(\widetilde{v}) = 1-\alpha.$$

The allocation procedure works as follows: award the good to the bidder with the highest ironed virtual valuation, breaking ties at random, provided the virtual valuation is positive. Notice that all types between $\widetilde{v}$ and $\underline{\omega}_H$ get the same ironed virtual valuations, and therefore if they tie, the winner is selected at random. Like BIN-TAC, this is inefficient, but allows additional revenue extraction from higher types.

Having obtained this characterization, we can compare BIN-TAC with the optimal mechanism. For now, let us focus on a simple environment, where F_L is uniform over $[0,1]$ and F_H is uniform over $[\Delta, \Delta+1]$. Figure 2 shows the interim allocation probabilities (top panel) and expected payments by type (bottom panel) as a function of bidder type, in the case where

$\Delta = 3$, $\alpha = 0.05$ and $n = 5$ (with optimal parameter choices). The optimal mechanism has a discontinuous jump in the allocation probability at $\widetilde{v} = 0.676$, and then irons until the high valuation region on $[3, 4]$. As you can see, BIN-TAC is able to approximate the discontinuous increase in allocation probability at $\widetilde{v}$ with a smooth curve, by randomizing the allocation in that region using the TAC auction. By contrast, the slope of the SPA-T allocation schedule is steep on this region and so the SPA-T cannot extract revenue from the high types (who could easily pretend to be a lower type while barely changing their probability of winning). This is clear from the bottom panel.

For SPA-B, the figures depict the allocation probabilities and expected payments as the function of the *realized* valuations of the agent (which are unknown to the agent under bundling). For instance, for a given realized high valuation v_H (i.e. on $[3, 4]$), the allocation probability plotted on the y-axis is equal to the average allocation probability across all types $(1 - \alpha)V_L + v_H$, who will bid their expected valuations in the second-price auction under bundling. As you can see, this implies that the allocation probability is no longer monotone: since matches are ex-ante unlikely ($\alpha = 0.05$), the most aggressive bidders in the bundling auction are those who have high valuations even without matching (high v_L), and therefore they are most likely to get the object. This makes the trade-off clear: the bundling mechanism raises expected payments when there is no match (because bidders don't know they haven't matched), and substantially lowers them in the case of a match.

Table 1 compares the expected revenue and welfare obtained by all the mechanisms. The performance of BIN-TAC is close to the optimal mechanism (about 96% of OPT), much better than the optimal SPA-T (85%). The table also shows that SPT-B performs less well than both BIN-TAC and OPT, especially in terms of expected consumer surplus. This is because it often fails to match advertisers and users correctly.

Table 1: Revenue Comparison: Uniform Environment

Mechanism	OPT	SPA-T	BIN-TAC (d=2)	BIN-TAC (d=3)	SPA-B
Expected Revenue	0.89	0.76	0.85	0.83	0.81
Expected Consumer Surplus	0.51	0.67	0.48	0.40	0.16

Expected revenue and welfare under different mechanisms, for the uniform environment with $\Delta = 3$, $\alpha = 0.05$ and the number of bidders $n = 5$.

3.6 Monte Carlo Simulations

We would like to test our mechanism against the benchmarks in a variety of other settings. We drop the assumption that F_L and F_H have disjoint support. The optimal BIN-TAC mechanism remains easy to calculate. Nothing in the proof of Theorem 2 required the disjoint supports for determining r^* and p^*, and so these can be solved for numerically for each d. Thus the optimization problem reduces to a one dimensional discrete optimization problem in the randomization parameter d, which can be quickly solved. Finding the optimal mechanism is more challenging, but can be done using standard optimization techniques.

For our simulations, we restrict ourselves to location families where the distribution $F_H(\cdot) = F_L(\cdot - \Delta)$ for some shift-parameter Δ, as in the uniform case above. Δ is the difference in mean valuation between the high and low groups, which we call the "match increment". We consider two location families: one where V_L is normal, and another where V_L is log normally distributed. In both cases V_L has mean 1 and standard deviation 0.5. We allow Δ, n and α to vary across experiments, and compute r^*, p^* and d^* as discussed. The default parameters we consider are $n = 10$, $\Delta = 5$, and $\alpha = .05$, and we vary one parameter at a time. Each experiment is repeated for 100000 impressions, and we calculate the average revenues.

The results are presented in Figures 5, 6 and 7. In all cases, on the y-axis we plot the revenue as a fraction of the revenue from the optimal mechanism. Recall that BIN-TAC generalizes SPA-T, so its performance is always at least as good, and often significantly better. In all cases, the BIN-TAC extracts at least 90% of the optimal revenue, compared to a worst-case performance of around 82% for the SPA-T. Consistent with our discussion of Theorem 3, the SPA-B in some cases does even better than OPT (when there are very few bidders), but its performance sharply degrades as the probability or value of a match gets large.

We see this in Figures 5 and 6. The expected number of matches is αn, and so as either α or n increases, the performance of the mechanisms that allow targeting improves relative to the SPA-B. Over some range, BIN-TAC also significantly outperforms the SPA-T, but as the number of bidders or the probability of match get sufficiently high, both converge to the OPT mechanism (which is itself an SPA with high reserve).

Figure 7 shows the dependence on the gap Δ. As expected, the performance of BIN-TAC increases while that of SPA-T falls as Δ gets larger, over some range. Since there is more revenue to be gained from high-valued bidders, BIN-TAC can only perform better with a large Δ. For sufficiently high Δ though, both BIN-TAC and the SPA set high reserves,

"throwing away" low-valued impressions and extracting all their revenue from matches, with equal revenue performance.

Overall, the performance of BIN-TAC is very good, at least for the distributions and parameters chosen. The main caveats are that it doesn't perform well with very few bidders (when bundling is preferable), and has little to recommend it when matches are highly probable or very valuable (a second-price auction would do as well). Its niche is in markets with relatively large numbers of bidders but low match probabilities, so that markets are "thin" in the sense of having relatively low matches in expectation.

4 Empirical Application

Our theoretical analysis has shown that there are cases in which BIN-TAC performs well. We now test our mechanism's performance in a real-world setting. We have historical data from Microsoft Advertising Exchange, one of the world's leading ad exchanges. Our data comes from a single large publisher's auctions on this exchange and consists of a 0.1% random sample of a week's worth of auction data from this publisher, sampled within the last two years. This publisher sells multiple "products", where a product is a URL-ad size combination (e.g. a large banner ad on the sports landing page of the New York Times).

The data includes information from both the publisher and the advertiser. On the publisher side, we see the url of the webpage the ad will be posted on, the size of the advertising space and the IP address of the user browsing the website. We form a unique identifier for the url-size pair, and call that a product. We determine which US state the user IP originates from, and call that a region. We use controls for product and region throughout the descriptive regressions. Unfortunately, we don't have more detailed information on the product or the user, as the tags and cookies passed by the publisher to the ad exchange were not stored.

On the advertiser side, we see the company name, the ad broker they employed, a variable indicating the ad they intend to show, and their bid.[12] We observe who won each auction and the final price. We drop auctions in which the eventual allocation was determined by biased bids and modifiers.[13] We also restrict attention to impressions that originate in the

[12]In the overwhelming majority of cases there is a single ad for each company, but some larger firms have multiple ad campaigns simultaneously. We treat these as being a single ad campaign in what follows because each firm should have the same per impression valuation across campaigns.

[13]When the advertiser has a technologically complex kind of ad to display, their bid is modified down

US, and where the publisher content is in English. Finally, we restrict only to reasonably frequently sold products, those with at least 100 sales in the dataset. This leaves us with a sample of 83515 impressions.

The dataset is summarized in Table 2. For confidentiality reasons, bids have been rescaled so that the average bid across all observations is equal to 1 unit. Bids are very skew, with the median bid being only 0.57 units. Perhaps as a consequence of this skewness, the winning bid — which is more heavily sampled from the right tail of the bid distribution — is much higher at 2.96 units. There are on average 6 bidders per auction, but there is considerable variation in participation, with a standard deviation of nearly 3. Bids are not strongly correlated: as the table shows, the correlation between a randomly selected pair of bids from each auction is only 0.01. This is not statistically significant at 5% (p-value 0.116, $N = 15827$).

The advertisers are themselves quite active in the market. On average they bid on 0.7% of all impressions, and win nearly 40% of those they bid on. But participation is quite skew, and the median advertiser is far less active, bidding on only 0.02% of impressions; while the most active advertiser participates in nearly 90% of auctions. Some advertisers choose to participate in relatively few auctions, but tend to bid quite highly and therefore win with relatively high probability. Others bid lower amounts in many auctions, and win with lower probability. The first strategy is followed by companies who want to place their advertisements only on webpages with specific content or to target specific demographics, while the latter strategy is followed by companies whose main aim is brand visibility.

4.1 Descriptive Evidence

Before proceeding to the main estimation and simulations, we provide some evidence that advertisers bid differently on different users (i.e. there is matching on user demographics). We also show that the platform is doing poorly in extracting this match surplus as revenue.

Our first piece of evidence is that leading advertisers vary their bids on the same product over short periods of time. Figure 3 shows re-scaled bids in 50 auctions by five large advertisers for the most popular webpage slot sold by this publisher. The advertisers were chosen at

(for allocation purposes) and up (for payment purposes). When the advertiser has a previously negotiated contract with the platform, their bid may be biased (usually upward for allocation purposes, and downward for payment). It is hard to know how to treat these auctions without taking some kind of stand on whether valuations vary with the kind of ad being displayed, what contract each bidder has, what other bidders know about these contracts etc.

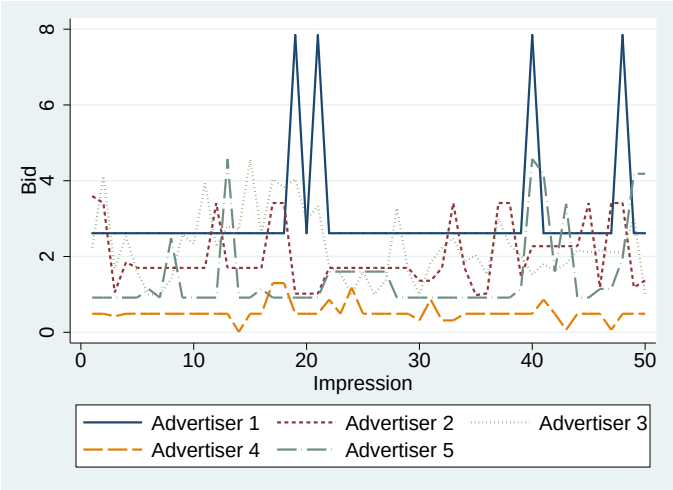


Figure 3: **Bids over Time.** The figure shows the (rescaled) bids of five advertisers in our data, selected at random from the top 50 advertisers (ranked by purchases) on 50 randomly chosen successive impressions of the most popular product. Note that the set of impressions differs across bidders (there are no impressions on which all 5 participate).

random from the top 50 advertisers in our dataset (ranked by purchases). The 50 auctions are chosen to be consecutive for each bidder.[14] The bids exhibit considerable variation, even though all of these impressions were auctioned within a 3-hour period. While this could in principle be driven by decreases in the advertisers' available budget, since the bids go both up and down it seems more likely that this variation arises from matching on user demographics.

A more direct test of advertiser-user matching is to look for the significance of advertiser-user fixed effects in explaining bids. Specifically, we estimate an unrestricted model where the dependent variable is bids and the controls are advertiser-user dummies, versus a restricted model with just advertiser and user fixed effects, but not their interaction. The restricted model is overwhelmingly rejected by the data (p-value ≈ 0). This points towards matching on demographics.

Proving that this matching is motivated by economic considerations is a little more difficult. The only user demographic we observe is the user region, and it is hard to know a priori what the advertisers' preferences over regions are. To get a handle on this, we turn to another proprietary dataset that indicates how often an advertiser's webpage was viewed by internet

[14]Since the same set of bidders don't participate in every auction, the impression number on the x-axis corresponds to different impressions for different bidders.

users in different regions of the country during the calendar month prior to the auction.[15] Our intention is to proxy for the advertisers' geographic preferences (insofar as these exist) using this pageview data. The idea is that firms who operate in only a few regions probably attract all their pageviews from those regions, and also only want to advertise in those regions. If this is right, advertisers who attract a large fraction of their pageviews from a particular region should participate more frequently and bid higher on users from those regions.[16] We normalize the pageviews from a particular state by the state population to get a per capita pageview measure, and then construct the fraction of normalized pageviews each region receives, calling this the "pageview ratio".

In Table 3, we present results from regressions of auction participation (a dummy equal to one if the advertiser participated), and bid (conditional on participation) on the pageview ratio, as well as a number of fixed effects. Because the sheer size of our dataset makes it difficult to run the fixed effect regressions, we run this on a subsample consisting of the top 10% of advertisers.[17] The first column shows participation as a function of the pageview ratio, as well as product-region fixed effects, and time-of-day fixed effects (since participation and bids may vary with the user's local time). We find a positive but insignificant effect. But when we include advertiser fixed effects to control for different participation frequencies across advertisers, we find a much bigger and now highly significant effect. All else equal, an advertiser is 3.3% more likely to bid on a user from a state that contributes 10% of the population-weighted pageviews for their site. This is a large increase, as the average probability of participation is only around 1%.

Turning to the bids, we find similar estimates and significance levels from the specifications with and without advertiser fixed effects. We find that firms bid higher on users from more relevant regions, although this effect is relatively modest in economic terms. Given that our proxy for advertiser preferences is relatively crude, it is notable that we find these effects. This provides some evidence that the matching is surplus increasing, in that advertisers are able to target regions where their most valuable customers are.

Next we ask whether the platform is able to extract most of the consumer surplus. There

[15] For example, if these auctions were in May, the pageview data would be taken from April.

[16] Because the pageview data dates from a period before our exchange data we are not worried about reverse causality (i.e. advertisers who win more impressions from region X later get more views from region X).

[17] Fortunately since participation is highly skewed, these advertisers account for 90% of the bids. With only bidder fixed effects we could use a within transformation to reduce the computational burden; but unfortunately this is not possible with multiple non-interacting fixed effects.

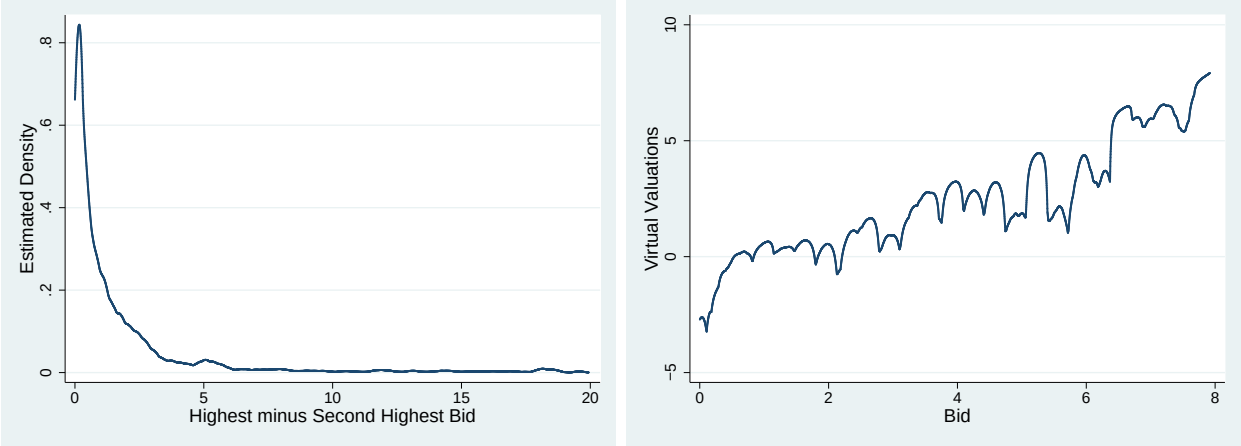


Figure 4: **Bidding Gap and Virtual Valuations.** The left panel shows a kernel density estimate of the probability density function of the (normalized) gap between the highest and second highest (rescaled) bids in auctions for the product with the highest sales volume in our dataset. The right panel shows the estimated virtual valuations as a function of bids.

is often a substantial gap between the highest and second highest bid in the auction, which suggests the answer is no. To see this, we look at the product with the highest sales volume in the data (over 38% of all impressions). The left panel of Figure 4 shows a kernel density estimate of the gap. The average bid in an auction is 0.88, while the mean gap is much larger at 1.89, indicating that there is a lot of money left on the table by a second-price mechanism (see Table 2 for other summary statistics). That gap itself is extremely skewed.

Assuming bids are equal to valuations — an assumption we will motivate in the next section — the right panel shows the virtual valuations $\psi(v)$ as a function of the bids. Although the virtual valuations are never infinitely negative, as in our stylized model, they are certainly non-monotone. This implies that BIN-TAC may be able to extract more revenue than a second price auction. We test this in the next section.

4.2 Estimation and Counterfactual Simulations

Our theoretical model is of a single auction rather than a whole market, and so in order to provide micro-foundations for our simulation approach, we need to enrich the model. We make the following assumptions for the estimation. There is a fixed set of N bidders who are always present in the market. As in the text, the model is symmetric independent private values. Each bidder draws their valuations for each impression identically, independently and privately according to some distribution F_j supported on $[0, \infty)$ (where j indexes products).

So bidder valuations are independent both across bidders and within a bidder over time. From the summary statistics we know that participation varies across advertisers. We assume that participation costs are zero, so that when a bidder does not participate it must be because they have zero valuation for the impression (since with any positive valuation there is weakly positive surplus from bidding). Finally, we assume that bidders bid their valuations, which is a weakly dominant strategy in a second-price auction.

Given these assumptions, we are able to make the following inference from the second-price auction data. Letting $i = 1 \dots I$ index bidders and $t = 1 \dots T$ index auctions, if bidder i makes a bid of $b_{i,t}$ in auction t, their valuation is $b_{i,t}$. If bidder i did not participate in auction t, their valuation for that particular impression must have been zero. Then since there is a one-to-one mapping from the distribution of bids and participation to the valuations, F_j is non-parametrically identified. We could therefore estimate the valuation density for each product using non-parametric methods. But, as we will show below, the counterfactual simulations will never require estimating more than some conditional moments of order statistics (e.g. the expected value of the d-th highest valuation when the highest valuation is less than $\overline{v}$). So instead we estimate these moments by the corresponding sample average.

Discussion: The simulations that follow lean heavily on the above assumptions. In particular, there are two assumptions that really drive the counterfactual results. The first is that bidders bid their valuations in the second-price auctions. This is probably a pretty good approximation for what is a vastly more complicated process, involving different advertiser objective functions and budget constraints, and different ad broker algorithms and interfaces.

The second is that participation costs are zero, and therefore non-participation implies zero valuation. In fact advertisers may find it costly to fully express their preferences, and therefore issue instructions not to bid on certain less desirable user groups, rather than precisely delineating how much less those groups are worth. We assume they have zero valuation for those users, but in fact their valuations may just be less than the participation costs.

How bad is the zero participation cost assumption? Well, the 5th percentile of bids in our data is equal to 0.013 — tiny in real terms, with an almost zero probability of winning, and even lower surplus. So at least some bidders have participation costs closely approximated by zero, and so this may not be a bad assumption to make.

The other assumptions on bidder symmetry and independence are made mainly for computa-

tional purposes. To address the concern that the symmetry and independence assumptions are driving our results, we will do some robustness tests based on different informational assumptions in a later subsection.

We are interested in comparing the "optimal" BIN-TAC mechanism to other leading mechanisms. For simplicity, we restrict attention throughout to the class of mechanisms that make the same parameter choices for all products (e.g. we rule out different reserves or randomization parameters by product or user-region). In each case we find these optimal parameters by maximizing the revenue functions defined in equations (6) and (7) below, using standard optimization methods.[18] To get standard errors on our revenue and consumer surplus estimates, we bootstrap the estimation sample and re-run the simulation procedure, holding the parameter choices fixed.[19]

Mechanisms with Targeting: The two policies we want to compare here are the second price auction with targeting and BIN-TAC. The two SPA mechanisms are easiest. For example, with a reserve of r, the expected revenue depends on the joint distribution of the top two valuations: since bidders bid their valuations, the item sells if the highest valuation exceeds r, and then the revenue is the maximum of the second highest bid and r. Letting the k-th highest bid in an auction t be $b_t^{(k)}$, our estimate is then given by the sample average across the T auctions:

$$\text{Revenue}^{\text{SPA}}(r) = \frac{1}{T}\sum_{t=1}^{T} 1(b_t^{(1)} > r)\max\{b_t^{(2)}, r\} \tag{6}$$

BIN-TAC is harder, as a bidder's equilibrium decision to take the BIN option depends on their beliefs about the distribution of rival valuations. From the model, advertiser behavior is characterized by a threshold value $\overline{v}_j = \overline{v}_j(p, d, r)$ for each product, above which they will take the BIN option, and below which they will TAC. From Theorem 1, this threshold solves the implicit equation $\overline{v}_j - p = \frac{1}{d}E[\overline{v} - Y^*|Y^1 < \overline{v}_j]$, where $Y^* = \max\{Y^d, r\}$ and Y^1 and Y^d are the 1st and d-th order statistics of rival bids on product j. To solve this equation for fixed (p, d, r), we need to estimate the expected TAC payment $E[Y^*|Y^1 < s]$ for varying s.

Under symmetry, the joint distribution of valuations is exchangeable, and so the joint distri-

[18] This raises an over-fitting concern, in that the parameters are optimized for this specific realization of the data generating process. However given our sample size, the bias this introduces is likely to be small.

[19] We use 100 bootstrap samples (i.e. samples of T impressions drawn randomly with replacement).

bution of rival bids is exactly the same as the joint distribution of $N-1$ randomly selected bids. So our estimate of the TAC payment conditional on winning on product j is given by:

$$\text{TAC Payment}(s,r) = \frac{1}{T}\sum_{t=1}^{T}\frac{\sum_k 1(b_t^{(1)} < s)\max\{b_t^{(d)}, r\}}{\sum_k 1(b_t^{(1)} < s)}$$

where k indexes the N choices of $N-1$-length bid vectors for each auction, including zeros for bidders that didn't participate and restricting the sample only to product j.[20] We can then solve for the equilibrium $\overline{v}(p,d,r)$ for each set of BIN-randomization parameters (p,d,r), and get a revenue estimate as follows:

$$\begin{aligned}\text{Revenue}^{\text{BIN-TAC}}(p,d,r) = &\frac{1}{T}\sum_{t=1}^{T} 1(b_t^{(2)} \geq \overline{v}(p,d,r))b_t^{(2)} + \frac{1}{T}\sum_{t=1}^{T} 1(b_t^{(1)} \geq \overline{v}(p,d,r) > b_t^{(2)})p \\ &+ \frac{1}{T}\sum_{t=1}^{T} 1(b_t^{(1)} < \overline{v}(p,d,r))\sum_{j=1}^{d} 1(b_t^{(j)} \geq r)\max\{b^{(d+1)}, r\}\end{aligned} \tag{7}$$

Bundling Mechanisms: As we do not observe all the impression characteristics provided to advertisers in this market, we cannot consider the optimal bundling strategy. But we can examine bundling by product and user region, where the platform strips away all user characteristics except for the region, so that advertisers are buying a random impression of a given size on a given website viewed by a user from a particular US state. This is unlikely to be optimal, but provides a lower bound on the revenues from the bundling strategy.

For this analysis, we allow for bidder valuations to be asymmetric and vary by both product and region. Our estimate of a bidder's willingness to pay for this "bundled" impression is just their average bid across all auctions of this product-region combination, taking their implicit bids when they didn't participate as equal to zero. Given that participation costs are zero and all bidders have strictly positive valuations for a bundled impression, all bidders will participate in all auctions. We assume that these impressions are sold by second-price auction without reserve.[21].

[20] It is correct to include the non-participating bidders, as in principle all N bidders are present in every period and so the distribution of rival bids drops only one of them — probably a bidder who would not have participated in any case.

[21] The platform could in principle set a reserve to extract all the consumer surplus (since the willingness to pay of each advertiser for a bundled impression can be calculated here), but this doesn't seem realistic.

Robustness to Informational Assumptions: The above theory and structural estimation follows the empirical auctions literature in treating bidder's valuations as private information.[22] A different modeling approach was suggested in an influential paper by Edelman, Ostrovsky and Schwartz (2007). They proposed a complete information model of sponsored search auctions. Their logic was that since these players compete with high frequency and can potentially learn each others' valuations, a complete information model may be a better approximation to reality than an incomplete information model.

Following this intuition, we also consider counterfactual simulations under complete information. The only model this affects is the BIN-TAC model, as under weak refinements the SPA equilibria under incomplete and complete information coincide. However in the BIN-TAC model we unfortunately now have multiple equilibria.[23]

To see this, consider a case where the bidder with the highest valuation is going to take the BIN option. Then the remaining bidders are indifferent between BIN and TAC, since in either case they will lose the auction and get payoff 0. We employ a trembling hand perfection refinement to eliminate this multiplicity. Specifically, for any probability $\epsilon > 0$ that the highest bidder will take the TAC option instead, the second-highest bidder faces a non-trivial choice between BIN and TAC. Applying this logic restores a generically unique equilibrium prediction.[24] We can therefore solve for the unique trembling hand perfect equilibrium of each auction, and estimate the expected revenues from the average sample revenues at any parameter vector.

We also perform a worst-case analysis over all rationalizable beliefs about rival strategies. From the point of view of revenue, the worst-case for BIN-TAC occurs when bidders are least inclined to take the BIN option: specifically, when they believe that all other bidders will choose to TAC and then bid zero. This implies that incentives to take the BIN option must be provided directly by the design, through the randomization parameter d and the reserve price r in the TAC auction. Since these beliefs are identical across all auctions, we can compute the indifference threshold $\overline{v}(p, d, r)$ implied by these beliefs, and then calculate revenue in exactly the same way as in the incomplete information case.

[22] See for example Laffont and Vuong (1996). See also Athey and Nekipelov (2010) for a model of sponsored search models in this tradition.

[23] This arises also in the generalized second price auction — see Edelman et al. (2007) and Varian (2007).

[24] We prove this in the supplementary appendix.

4.3 Results

The results are in Tables 4 and 5. We find that the optimal reserve when running a second price auction is high: nearly twice as high as the second highest bid. By contrast, BIN-TAC always uses relatively low reserves (all well below the average bid), and instead offers a high buy price (which is close in magnitude to the optimal SPA reserve). To make this buy price attractive, the platform threatens to randomize between the top three bidders in each auction (four in the worst-case scenario), which is significant given that there are only six bidders in an average auction.

The welfare performance of these mechanisms is detailed in Table 5. The SPA without reserve earns revenue of 0.98 per auction, and leaves substantial consumer surplus — on average 1.97 per auction.[25] Adding the large optimal reserve improves revenue slightly (to 1.03 per auction), but hurts consumer surplus substantially (it falls to 1.44).

BIN-TAC does better than both of these mechanisms in terms of revenue. Interestingly, it also does better on consumer surplus than the SPA-T. This happens because the optimal SPA-T reserve price is very high — to extract revenue from the long right tail — and so many impressions are not sold, resulting in inefficiency and lower total welfare. By contrast, BIN-TAC has the BIN price to extract this revenue, and so the reserve is much lower, and more impressions are sold. Even accounting for distortions owing to the TAC auction, this is a welfare improvement.

By contrast, the bundling strategy underperforms. Revenues are much lower than the SPA-T, and consumer surplus falls even more dramatically. This is because there is considerable variation in match surplus across impressions even after conditioning on product and region, and so bundling along only these two dimensions destroys a lot of surplus.

Finally, the BIN-TAC results in the bottom part of the table show that the revenue estimates are relatively robust to how we model the information structure. However in models where the bidders are more informed, or dubious about the BIN option, consumer surplus is lower. In those cases the BIN decision is taken less often, thereby increasing the distortion from TAC auctions.

[25] The per auction revenue of 0.98 is lower than the average second highest bid of 1.07 in Table 2 because of a small fraction (2.3%) of auctions with only a single bidder, which will realize zero revenue in an SPA without reserve.

5 Conclusion and Future Work

We have introduced the BIN-TAC mechanism, designed to allow sellers to capture the surplus created by providing match information. This mechanism outperforms the second-price auction mechanism in this setting, and is preferable to bundling goods together by withholding information, at least when there is a reasonable size population of potential bidders. Moreover, we demonstrated through an example that the mechanism can closely approximate Myerson's optimal mechanism with ironing, despite its relative simplicity.

Our analysis of the exchange marketplace revealed that it has many features that make it a good place to apply our mechanism: large differences between the highest and second highest bid, and evidence of matching on user characteristics that the platform has chosen to make available to advertisers. Although the market does not fit our stylized model, we found that the BIN-TAC mechanism would nonetheless improve revenues and consumer surplus relative to the existing mechanism, a second price auction with reserve.

Due to data limitations we were not able to compare our mechanism to an optimal bundling strategy. Instead, we looked at what would happen if the platform only provided advertisers with product and user location information, rather than more detailed demographics. This bundling strategy performed poorly, but it is an interesting and open research question as to whether switching mechanisms to BIN-TAC is in fact better than retaining the SPA with a more thoughtful bundling strategy.

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Appendix

Proof of Theorem 1

Let a be a binary choice variable equal to 1 if the bidder takes BIN and zero if TAC. Fix a player i, and fix arbitrary measurable BIN strategies $a_j(v)$ for the other players. Let q be the probability that no other bidder takes the BIN option, equal to $\prod_{j\neq i}\left(\int 1(a_j(v)=0)dF(v)\right)$.

Let $\pi(a,v)$ be the expected payoff of action a for type v given that the bidder bids their valuation in any auction that follows. Then we have that $\frac{\partial}{\partial v}\pi(1,v)\geq q$, as a marginal increase in type increases the payoff by the probability of winning, which is lower bounded by q when taking the BIN option. Similarly we have that $\frac{\partial}{\partial v}\pi(0,v)\leq \frac{q}{d}$, as the probability of winning when taking-a-chance is bounded above by q/d. Then $\pi(a,v)$ satisfies the strict single crossing property in (a,v); it follows by Theorem 4 of Milgrom and Shannon (1994), the best response function must be strictly increasing in v, which in this case implies a threshold rule. It follows that any symmetric equilibrium must be in symmetric threshold strategies. So fix an equilibrium of the form in the theorem, and let the payoffs to taking taking BIN be $\pi_B(v)$ and to TAC be $\pi_T(v)$. They are given by:

$$\pi_B(v) = E\left[1(v>Y^1>\overline{v})(v-Y^1)\right] + E\left[1(Y^1<\overline{v})(v-p)\right]$$
$$\pi_T(v) = \mathbb{E}\left[1(Y^1<\overline{v})1(Y^*<v)\frac{1}{d}(v-Y^*)\right]$$

The threshold type $\overline{v}$ must be indifferent, so

$$\begin{aligned}\pi_B(\overline{v}) &= \mathbb{E}\left[1(Y^1 < \overline{v})(\overline{v} - p)\right] \\ &= \mathbb{E}\left[1(Y^1 < \overline{v})\frac{1}{d}(\overline{v} - Y^*)\right] = \pi_T(\overline{v}).\end{aligned} \tag{8}$$

Next, we show a $\overline{v}$ satisfying Eq. (1) exists and is unique. The right hand side of Eq. (1) is a function of $\overline{v}$ with first derivative $\frac{1}{d}(1 - \frac{\partial}{\partial \overline{v}}\mathbb{E}[Y^*|Y^1 < \overline{v}]) < 1$. Since at $\overline{v} = 0$ it has value $p > 0$ and globally has slope less than 1, it must cross the 45° line exactly once. Thus there is exactly one solution to the implicit Eq. (1).

Proof of Theorem 2

To prove the first claim, let us fix d and $\overline{v}$. For any reserve price r, let $p(\overline{v}, r)$ denote the BIN-TAC price for threshold $\overline{v}$. By Equation (1) we have

$$p(\overline{v}, r) = \frac{d-1}{d}\overline{v} + \frac{1}{d}E\left[Y^*|Y^1 < \overline{v}\right] \tag{9}$$

Using first order conditions, we consider the effects of the marginal increase in reserve r on the revenue of BIN-TAC mechanism denoted by $\text{Rev}_{\text{BIN-TAC}}$. There are three cases:

- *The item is allocated via BIN:* If there are two bidders above $\overline{v}$, then increase of $\overline{v}$ does not change the revenue. But the revenue increases by $\frac{\partial p(\overline{v},r)}{\partial r}$ if a bidder wins the item at the buy-it-now price. This happens with probability $nF(\overline{v})^{n-1}(1 - F(\overline{v}))$. Hence the marginal increase in revenue from BIN auctions is equal to

$$\begin{aligned}& nF(\overline{v})^{n-1}(1 - F(\overline{v})) \times \frac{\partial p(\overline{v}, r)}{\partial r} \\ =\ & nF(\overline{v})^{n-1}(1 - F(\overline{v}))\frac{1}{d}\Pr\left(Y^d \le r | Y^1 < \overline{v}\right) \\ =\ & nF(\overline{v})^{n-1}(1 - F(\overline{v}))\frac{1}{d}\left(\frac{1}{F(\overline{v})^{n-1}}\left(\sum_{k=0}^{d-1}\binom{n-1}{k}(F(\overline{v}) - F(r))^k F(r)^{n-k}\right)\right) \\ =\ & (1 - F(\overline{v}))\left(\sum_{k=1}^{d}\frac{n}{d}\binom{n-1}{k-1}(F(\overline{v}) - F(r))^{k-1}F(r)^{n-k}\right)\end{aligned} \tag{10}$$

- *The item is allocated via TAC:* The revenue of TAC changes only if the price is equal to r. In the event of a bidder winning an item at TAC and then paying the reserve price r, the revenue increases by the marginal increase of r. Observe that if there are k ($1 \leq k \leq d$) bidders with valuation between r and $\overline{v}$ (and no bids above $\overline{v}$), then the revenue of the auction is equal to r with probability $\frac{k}{d}$. By this observation, the probability that the revenue is equal to r (and hence the marginal increase in the revenue) is given by

$$\begin{aligned} & \left(\sum_{k=1}^{d} \frac{k}{d}\binom{n}{k}(F(\overline{v})-F(r))^k F(r)^{n-k}\right) \\ = & \left(\sum_{k=1}^{d} \frac{n}{d}\binom{n-1}{k-1}(F(\overline{v})-F(r))^k F(r)^{n-k}\right) \\ = & (F(\overline{v})-F(r))\left(\sum_{k=1}^{d} \frac{n}{d}\binom{n-1}{k-1}(F(\overline{v})-F(r))^{k-1} F(r)^{n-k}\right) \end{aligned} \tag{11}$$

- *The item is not allocated:* In the event that the bidder chosen by TAC cannot receive the item because his valuation was equal to r (before marginal increase) the revenue decreases by r. In this case, the marginal decrease in the revenue is equal to

$$\begin{aligned} & r\left(\sum_{k=0}^{d-1} \frac{n}{d} f(r)\binom{n-1}{k}(F(\overline{v})-F(r))^k F(r)^{n-k-1}\right) \\ = & rf(r)\left(\sum_{k=1}^{d} \frac{n}{d}\binom{n-1}{k-1}(F(\overline{v})-F(r))^{k-1} F(r)^{n-k}\right) \end{aligned} \tag{12}$$

Summing up expressions (10), (11), and (12) we have

$$\frac{\partial \mathrm{Rev}_{\text{BIN-TAC}(d,p(\overline{v},r))}}{\partial r} = (1-F(r)-rf(r))\left(\sum_{k=1}^{d} \frac{n}{d}\binom{n-1}{k-1}(F(\overline{v})-F(r))^{k-1} F(r)^{n-k}\right)$$

Therefore, the derivative is equal to zero for the solution of $r = \frac{1-F(r)}{f(r)}$ denoted by r^*. By assumption, $\psi(v)$ single-crosses zero exactly once from below (in the region $[\underline{\omega}_L, \overline{\omega}_H]$), and so r^* is unique. Hence, for any $\overline{v}$, the optimal reserve is either r^*, or is one of the boundaries $\underline{\omega}_L$, $\overline{\omega}_L$, $\underline{\omega}_H$, and $\overline{\omega}_H$. Note that the derivative is positive at $\underline{\omega}_L$. Also, for reserve equal to $\overline{\omega}_H$, the item never sells. Moreover, observe that the reserve equal to $\overline{\omega}_L$ is dominated by

reserve equal to $\underline{\omega}_H$ since no low-type bidder would receive the item. Therefore, the optimal reserve is either equal to r^* or $\underline{\omega}_H$.

Again, we use first order conditions again to find the optimal choice of $\overline{v}$. There are three effects on the revenue of the mechanisms by marginally increasing $\overline{v}$.

- *The highest valuation bidder now declines to take BIN:* This reduces revenue by $p(\overline{v}, r) - E\left[Y^* | Y^1 < \overline{v}\right] = \frac{d-1}{d} E\left[\overline{v} - Y^* | Y^1 < \overline{v}\right]$, and happens with probability $nf(\overline{v})F(\overline{v})^{n-1}$.
- *The second highest valuation bidder now declines to take BIN:* This decreases revenue by $\overline{v} - p(\overline{v}, r) = \frac{1}{d} E\left[\overline{v} - Y^* | Y^1 < \overline{v}\right]$, and happens with probability $n(n-1)f(\overline{v})(1 - F(\overline{v}))F(\overline{v})^{n-2}$.
- *Only the highest bidder takes BIN, and pays slightly more:* With probability $n(1 - F(\overline{v}))F(\overline{v})^{n-1}$, the highest bidder may have valuation above $\overline{v}$ and the second highest below it. In this case revenue increases by $\frac{\partial p(\overline{v}, r)}{\partial \overline{v}} = \frac{d-1}{d} + \frac{1}{d} \frac{\partial E[Y^* | Y^1 < \overline{v}]}{\partial \overline{v}}$

Therefore, we have:

$$\begin{aligned}
\frac{\partial \text{Rev}_{\text{BIN-TAC}(d, p(\overline{v}, r))}}{\partial \overline{v}} &= n(1 - F(\overline{v}))F(\overline{v})^{n-1} \times \\
&\quad \left(-\frac{f(\overline{v})}{1 - F(\overline{v})} \frac{d-1}{d} E\left[\overline{v} - Y^* | Y^1 < \overline{v}\right] - \frac{f(\overline{v})}{F(\overline{v})} \frac{n-1}{d} E\left[\overline{v} - Y^* | Y^1 < \overline{v}\right] \right. \\
&\quad \left. + \left(\frac{d-1}{d} + \frac{1}{d} \frac{\partial E[Y^* | Y^1 < \overline{v}]}{\partial \overline{v}} \right) \right) \\
&= \frac{n}{d}(1 - F(\overline{v}))F(\overline{v})^{n-1} \times \\
&\quad \left(-\left(\frac{(d-1)f(\overline{v})}{1 - F(\overline{v})} + \frac{(n-1)f(\overline{v})}{F(\overline{v})} \right) E\left[\overline{v} - Y^* | Y^1 < \overline{v}\right] \right. \\
&\quad \left. + \left(d - 1 + \frac{\partial E[Y^* | Y^1 < \overline{v}]}{\partial \overline{v}} \right) \right)
\end{aligned}$$

Re-arranging terms, the optimal choice for $\overline{v}$ is either at the boundaries or is a solution of the equation below

$$d-1+\frac{\partial E[Y^*|Y^1<\overline{v}]}{\partial \overline{v}}=\left(\frac{(d-1)f(\overline{v})}{1-F(\overline{v})}+\frac{(n-1)f(\overline{v})}{F(\overline{v})}\right)E\left[\overline{v}-Y^*|Y^1<\overline{v}\right]$$

Similar to the previous argument, it is easy to see that the only boundary that would be optimal is $\underline{\omega}_H$.

Proof of Theorem 3

We prove the two results in turn. For part (i), we construct a BIN-TAC mechanism that achieves exactly the same outcomes as SPA-T for any type realization. Let the SPA-T have optimal reserve r^*, and let the BIN-TAC mechanism have TAC reserve r^*, randomization parameter $d=1$ and BIN price $p=\overline{\omega}_H$. Then no type will take the BIN option in equilibrium (it is strictly dominated), and so the TAC auction will always occur. Since $d=1$, this is just an SPA with reserve r^*. Since for this particular choice of parameters BIN-TAC does as well as SPA-T, in general BIN-TAC dominates SPA-T. Strict dominance follows from the uniform example given in the text.

For part (ii), fix the degenerate environment and consider a BIN-TAC mechanism with $d=n$, and a reserve of v_L. The highest BIN price that makes electing BIN optimal for high types is $p=v_H-\frac{(v_H-v_L)}{n}=\frac{n-1}{n}v_H+\frac{1}{n}v_L$. Then for any $\beta\in[0,1]$, we have:

$$\begin{aligned}\max_\theta \pi(\text{BIN-TAC},\theta,\beta) &\geq \left(1-(1-\alpha)^n-n\alpha(1-\alpha)^{n-1}\right)v_H+n\alpha(1-\alpha)^{n-1}\left(\beta v_H+(1-\beta)p^{BIN}\right)\\ &\quad+(1-\alpha)^n v_L\\ &=\left(1-(1-\alpha)^n-n\alpha(1-\alpha)^{n-1}\right)v_H+n\alpha(1-\alpha)^{n-1}\left((1-\frac{1-\beta}{n})v_H+\frac{1-\beta}{n}v_L\right)\\ &\quad+(1-\alpha)^n v_L\\ &\geq \left(1-(1-\alpha)^n-\alpha(1-\alpha)^{n-1}\right)v_H+\left(\alpha(1-\alpha)^{n-1}+(1-\alpha)^n\right)v_L\end{aligned}$$

where the last step uses the fact that the minimum of the function occurs at $\beta=0$. Under the SPA-B, all types bid the same, there is no consumer surplus, and revenue is equal to:

$$\max_\theta \pi(\text{SPA-B},\theta,\beta)=\alpha v_H+(1-\alpha)v_L$$

The final line of both expressions is a weighted average of v_L and v_H; it suffices to show that the mass on v_L is lower under BIN-TAC. This requires $(\alpha(1-\alpha)^{n-1} + (1-\alpha)^n) < (1-\alpha)$. After a bit of simple algebra, this is equivalent to showing $(1-\alpha)\left(1-\alpha(1-\alpha)^{n-2} - (1-\alpha)^{n-1}\right) \geq 0$, which holds by binomial expansion of 1 with equality for $n = 2$ and strictly for $n > 2$. This proves dominance; strict dominance follows by noting that the BIN-TAC payoff is strictly higher for $\beta > 0$.

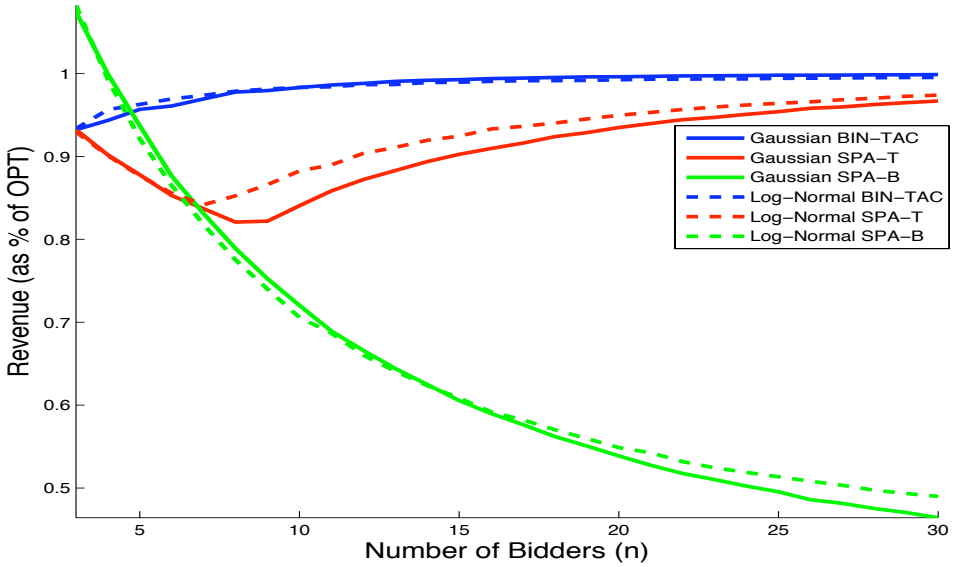


Figure 5: **Revenue Performance vs Number of bidders.** Simulated expected revenues for different mechanisms as the number of bidders n varies, in an environment where F_L has mean 1 and standard deviation 0.5, the match probability is 0.05 and the match increment is 5.

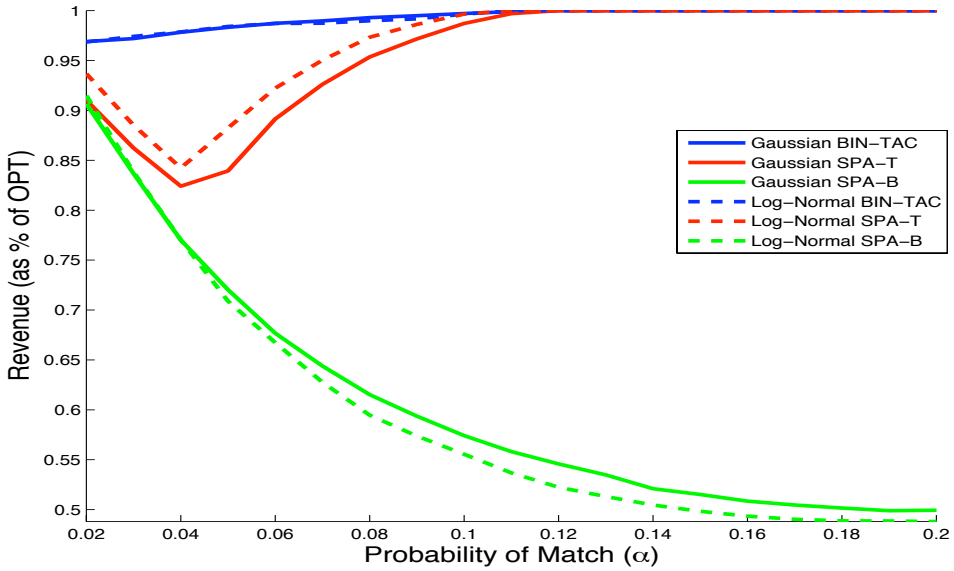


Figure 6: **Revenue Performance vs Match Probability.** Simulated expected revenues for different mechanisms as the probability of a match α varies, where F_L has mean 1 and standard deviation 0.5, the number of bidders is 10 and the match increment is 5.

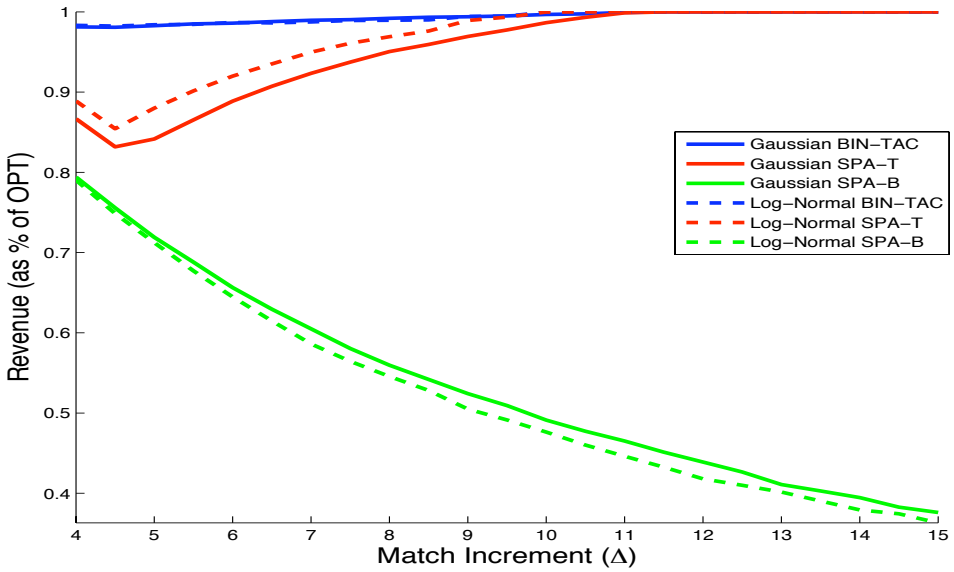


Figure 7: **Revenue Performance vs Match Increment.** Simulated expected revenues for different mechanisms as the match increment Δ varies, where F_L has mean 1 and standard deviation 0.5, the match probability is 0.05 and the number of bidders is 10.

Table 2: Summary Statistics: Microsoft Advertising Exchange Display Ad Auctions

	Mean	Median	Std. Dev.	Min	Max
Bid-Level Data					
Average bid	1.000	0.565	2.507	0.0000157	130.7
Number of bids	508036				
Auction-Level Data					
Winning bid	2.957	1.614	5.543	0.00144	130.7
Second highest bid	1.066	0.784	1.285	0.00132	39.22
Number of bidders	6.083	6	2.970	1	15
Bid correlation	0.01				
Number of auctions	83515				
Advertiser-Level Data					
% of auctions participated in (p1)	0.697	0.0251	4.641	0.00120	88.28
% of auctions won if participated (p2)	38.90	29.59	35.50	0	100
Correlation of (p1,p2)	-0.09				

Summary statistics for the full dataset, which is a 0.1 percent sample of a week's worth of auction data sampled within the last two years. An observation is a bid in the top panel; an auction in the middle panel; and an advertiser in the last panel. Bids have been normalized so that their average is 1, for confidentiality reasons. The bid correlation is measured by selecting a pair of bids at random in every auction with at least two bidders, and computing the correlation coefficient.

Table 3: Matching on Region

	Participation		Bids	
Advertiser Website Pageview Ratio	0.029	0.329***	0.264***	0.286***
	(0.022)	(0.015)	(0.052)	(0.053)
Time-of-Day Fixed Effects	yes	yes	yes	yes
Product-Region Fixed Effects	yes	yes	yes	yes
Advertiser Fixed Effects	no	yes	no	yes
N	5581749	5581749	417557	417557
R^2	0.02	0.34	0.04	0.26

Results from OLS Regressions. In the first two columns, the dependent variable is a dummy for participation. The sample used in the regressions consists of all auction-bidder pairs, limited to the 10% of bidders who participate most often. In the last two columns, the dependent variable is the bid. The sample used in the regressions only includes bids from the 10% of bidders who bid most often. The independent variable is the population-weighted fraction of pageviews of the advertiser's website that come from the region the user is in. Time-of-day fixed effects refer to a dummy for each quarter of the day, starting at midnight. Product-region fixed effects are dummies for the page-group advertised on, and the state the user is located in. Standard errors are robust. Significance levels are denoted by asterisks (* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$).

Table 4: Optimal Parameter Choices

Policy	p	d	r
SPA-T	-	-	1.96
BIN-TAC	2.60	3	0.43
BIN-TAC (complete information)	1.95	3	0.65
BIN-TAC (rationalizable worst case)	2.10	4	0.65

Revenue-maximizing parameter choices. For each of the above mechanisms, we find these by maximizing the revenue functions defined in the main text over the available parameters numerically using a grid search. The two modified BIN-TAC mechanisms are robustness checks, varying the informational assumptions made for BIN-TAC. In the complete information case, bidders know the valuations of the other participants, and made BIN decisions accordingly. In the rationalizable worst-case model, bidders assume they will only have to pay the reserve price in TAC auction, and therefore take the BIN option more rarely.

Table 5: Counterfactual Revenues and Welfare

Policy	Revenue	Consumer Surplus	Total Welfare
SPA-T (no reserve)	0.983	1.974	2.957
	(0.004)	(0.019)	(0.019)
SPA-T (optimal reserve)	1.028	1.471	2.499
	(0.005)	(0.018)	(0.020)
BIN-TAC	1.075	1.633	2.708
	(0.005)	(0.018)	(0.020)
SPA-B	0.644	0.730	1.374
(bundling by product-region)	(0.006)	(0.016)	(0.015)
Robustness to Informational Assumptions			
BIN-TAC (complete information)	1.072	1.589	2.661
	(0.005)	(0.018)	(0.020)
BIN-TAC (rationalizable worst case)	1.066	1.530	2.596
	(0.005)	(0.018)	(0.020)

Counterfactual simulations of average advertiser revenues, consumer surplus and total welfare (sum of producer and consumer surplus). All statistics reported outside parentheses are averages across impressions; those in parentheses are standard errors computed by bootstrapping the full dataset (i.e. they reflect uncertainty over the true DGP). Six different simulations are run. The first is of a second price auction without reserve, while the second is of a second price auction with optimal (revenue-maximizing) reserve. The third is of the BIN-TAC mechanism, under the incomplete information structure outlined in the text. The fourth is a bundling counterfactual where the impressions are bundled according to the product (i.e. URL and ad size) and user region, and sold by second-price auction. The last two are robustness checks, varying the informational assumptions made for BIN-TAC. In the complete information case, bidders know the valuations of the other participants, and made BIN decisions accordingly. In the rationalizable worst-case model, bidders assume they will only have to pay the reserve price in TAC auction, and therefore take the BIN option more rarely. Where applicable, the parameters used are the optimal parameters from Table 4.