Pledgability and Liquidity: A New Monetarist Model of Macro and Financial Activity*

Venky Venkateswaran† Randall Wright‡

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Abstract

When limited commitment precludes unsecured credit, to facilitate intertemporal transactions, assets can help by serving either as collateral (as in Kiyotaki-Moore), or as a direct payment instrument (as in Kiyotaki-Wright). We argue that these two roles for assets are often, but sometimes not, equivalent. Then, focusing on the collateral function, we develop a general framework where assets differ in their *pledgability* – the extent to which they can be used to secure loans – and apply it to a variety of substantive issues in macro and finance. Our setup nests standard growth and asset-pricing theories as special cases. Since we can price currency, as well as neoclassical capital, equity, real estate etc., we can analyze the effects of monetary policy on investment, real returns, housing markets etc., capturing as special cases classic results by Fisher, Mundell, Tobin et al. Moreover, monetary policy affects output and employment, and we can generate a standard Phillips curve.

Liquidity differentials across assets, along both extensive and intensive margins, also affect all of the above variables, which allows us to study the impact of financial developments on aggregate activity. We consider general mechanisms for determining the terms of trade in some markets, including axiomatic or strategic bargaining, as well as competitive Walrasian pricing, so we can discuss markups and holdup problems. Calibrated versions of the model are able to account reasonably well for the effects of monetary policy and financial innovation on the economy. In particular, the model allows for increases in inflation or nominal interest rates to realistically raise output, employment, investment, housing (prices and quantities) and the value of the stock market. This does not mean, however, that such policies are necessarily good for welfare. By contrast, some financial innovations can hurt output, employment, investment and stock values while increasing welfare.

PRELIMINARY AND INCOMPLETE - DO NOT CIRCULATE.

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†Pennsylvania State University (vuv10@psu.edu).

‡University of Wisconsin-Madison and NBER (rwright@bus.wisc.edu).

> Collateral is, after all, only good if a creditor can get his hands on it. Niall Ferguson, *The Ascent of Money*.

1 Introduction

Our goal is to develop a tractable theory of the role of assets in the exchange process. We begin with the premise that credit is hindered by various imperfections, or frictions, including limited commitment. Interacted with some notion of imperfect monitoring/memory, limited commitment implies that assets have a role in facilitating intertemporal trade, as formalized in modern monetary theory by, e.g., Kocherlakota (1998) and Wallace (2010). In our view, obvious requirements for a theory that is meant to take this seriously are: (1) it must be general equilibrium, in the sense of providing a complete and internally consistent description of an economic environment; (2) agents in this environment should trade with each other, not simply against their budget lines, as in classical competitive analysis. Only when we have such a theory, where agents trade with each other, can we can reasonably ask *how* they trade: Is exchange bilateral or multilateral? Do agents use barter, money, or secured/unsecured credit? Do they take the terms of trade as given, or do they bargain? It is from this vantage that we want to study asset pricing and the implications for macroeconomic activity.

By way of example, suppose that you want either a consumption or a production good from someone now, but you have neither consumption nor production goods that they want at the moment, so you cannot simply barter. Perhaps you will have something at a later date that they may want, maybe claims to commodities, or to general purchasing power, or any other asset or source of income. Consider promising that if they give you what you want now you will reciprocate by transferring something of value to them in the future. But they worry that you may renege on this pledge – this is what the lack-of-commitment friction means. We can then try to get you to honor your obligations using rewards and

punishments, such as allowing you to continue having access to credit in the future if you do not default, and excluding you from future credit if you do. But if agents are anonymous, or more generally, when monitoring or record keeping is imperfect, this may not work well. This leads to a role for assets in the facilitation of exchange.

There are two ways in which assets can help in this regard. First, if you want to acquire something, and have assets at hand, you can turn them over to a counterparty immediately. In this case, assets serve as a medium of exchange. Alternatively, you can give the seller the right to seize the assets in the event that you renege on your promise to deliver something of value in the future. In this case, the assets serve as collateral. Collateral is useful in the presence of commitment problems because it helps ensure compliance: if you fail to honor an obligation, the creditor gets the collateral. To the extent that you value the collateral, losing it constitutes a punishment that helps deter opportunistic misbehavior. For this to work it is not even necessary that the counterparty values the collateral; it can be enough that you do, since this makes your promise to repay debt more credible (of course, if the collateral is also valued by the counterparty, the situation can be all the better, since then he may not worry about the risk that you renege).

The two ways in which assets facilitate intertemporal exchange – serving as a medium of exchange and serving as collateral – are related.[1] To make this concrete, suppose that you have assets that are currently worth v, and will be worth v' later. If you want to pledge some of the assets, how big a pledge is credible? If no punishments are available except for seizing the collateral, the credit limit is v', since clearly you prefer honoring the obligation if and only if it is less than v'. It is equally clear that, instead of pledging the assets as collateral, you can turn them over now. At least, this is true without additional

[1] As David Andolfatto put it in a recent blog: "On the surface, these two methods of payment look rather different. The first entails immediate settlement, while the second entails delayed settlement. To the extent that the asset in question circulates widely as a device used for immediate settlement, it is called money (in this case, backed money). To the extent it is used in support of debt, it is called collateral. But while the monetary and credit transactions just described look different on the surface, they are equivalent in the sense that capital is used to facilitate transactions that might not otherwise have taken place."

complications, that many models do not specifiy explicity, implying a preference for either immediate or deferred settlement. In terms of the literature, as a canonical model of assets being used as a medium of exchange, we have in mind Kiyotaki and Wright (1989, 1993), and as a model of them being used as collateral, we have in mind Kiyotaki and Moore (1997, 2005). We discuss in more detail below the connection between these two types of models; for now, let us pursue the idea of assets serving as collateral.[2]

Given you want to use assets as collateral, what matters is *how much* you can use. In the language of Holmstrom and Tirole (2011), what matters is the *pledgability* of various assets. We propose a framework to study the implications of assets differing in pledgability, or liquidity, and put it to work on several applications in finance and macro. The setup nests standard growth and asset-pricing theories as special cases, and can be viewed as an extension of the *New Monetarist* approach recently surveyed by Williamson and Wright (2010) and Nosal and Rocheteau (2011). In the model, since we can price money as well as neoclassical capital, equity, real estate etc., we can analyze the effects of monetary policy on investment, real returns, housing markets etc., capturing as special cases classic results by Fisher, Mundell, Tobin et al. This allows us to clarify, e.g., how inflation affects the real and nominal returns on different assets as a function of pledgability. Monetary policy also affects output and employment, and thus can generate a standard (downward-sloping) Phillips curve. We can also study when currency is driven out by other assets, which has clear implications for issues in international economics, like dollarization.

Liquidity differentials across assets, along both extensive and intensive margins, also affect output, employment, investment, returns, etc. This allows us to study the macro

[2] We mention here two details. First, models along the lines of Kiyotaki and Wright (1989, 1993) are usually framed in terms of consumer loans, while Kiyotaki and Moore (1997, 2005) models are framed as producer loans. This may matter for some issues, but not for the big picture under consideration in these introductory remarks. Second, there are two main types of collateralized loans. A mortgage, e.g., is secured by the house bought with the loan – if you renege, they take your house. But if you renege on a home equity loan used to pay for a vacation, e.g., they don't reposess the vacation, they again take the house. Credit in Kiyotaki and Moore (1997, 2005) models looks more like the latter (i.e., home equity loans). There are also more complicated arrangements, e.g., repurchase agreements (see e.g. Monnet and Narajaba (2011)).

impact of financial developments. Also, as in the literature on over-the-counter markets in finance (see footnote 3), we consider general mechanisms for determining the terms of trade, including as special cases Nash, Kalai and other bargaining solutions, as well as Walrasian pricing. This allows us to analyze markups and holdup problems. Calibrated versions of the model are able to account reasonably well for the aggregate effects of monetary policy and financial innovation. In particular, increases in inflation or nominal interest rates realsitically raise output, employment, investment, housing (both prices and quantities) and the value of the stock market. This does not mean, however, that such policies are necesarily good for welfare. By contrast, some financial innovation actually looks bad for output, employment, investment and stock values, yet increases welfare. The rest of the paper involves making all this precise and studying the implications.[3]

2 The Model

There is a $[0, 1]$ continuum of infinitely-lived households. Each period in discrete time has two distinct markets. One is a frictionless centralized market, labeled AD for Arrow-Debreu, where households trade assets, labor and certain consumption goods. The other is a market where they trade different goods, subject to various frictions that impede credit, labeled KM for Kiyotaki-Moore. We assume KM convenes before AD, but no interesting results depend on this timing convention. All agents always participate in AD, while only a measure

[3] In terms of other work, similar in spirit is Lester et al. (2011), where differential liquidity is modeled using information frictions: some agents are unable to recognize certain assets' quality. That paper assumes agents either recognize quality perfectly or not at all, and in the later case reject assets outright. This avoids bargaining under asymmetric information, but means liquidity differs only along the extensive margin (acceptance by more or fewer counterparties), not the intensive margin. One can tackle bargaining under asymmetric information in these models, as in Rocheteau (2011), Li and Rocheteau (2008, 2009) and Li et al. (2011). Those papers deliver liquidity differentials along the intensive margin, but typically rely on special protocols (e.g., informed agents making take-it-or-leave-it offers, which eliminates any incentive for sellers to invest in information, the main focus in Lester et al. (2011)). Because our approach is based on pledgability and not recognizability – i.e., on commitment rather than information frictions – it is much easier, so we can study applications well beyond those papers. On commitment issues generally, see the large body of work eminating from Kehoe and Levine (1993, 2001) and Alvarez and Jermann (2000). Other related papers include Sanches and Williamson (2010), Gu and Wright (2010), Mattesini et al. (2009), and some work in finance on over-the-counter markets, including Duffie et al. (2005), Geromichalos et al. (2007), Ferraris and Watanabe (2008), Lagos and Rocheteau (2008, 2009), Lagos (2010, 2011) and Li and Li (2010). For more on the literature, see the above-mentioned surveys.

$2\sigma \leq 1$, chosen at random each period, participate in KM. By not participating, we mean that some households neither derive utility from, nor have an endowment of, KM goods. Of the measure σ participating in this marklet, we assume they all have an endowment $\bar{q}$, while $\sigma/2$ have utility function $u_b(Q)$ and $\sigma/2$ have utility function $u_s(Q)$, where $u_b^{'}(Q) > u_s^{'}(Q)$ for all Q, and the subscripts stand for buyer and seller. Buyers and sellers meet in KM, either bilaterally or multilaterally, as discussed below, and potentially trade because the former have higher marginal utility.[4]

If a seller in KM gives $q \leq \bar{q}$ of his endowment to a buyer, the cost to the former is $c(q) \equiv u_s(\bar{q}) - u_s(\bar{q} - q)$, while the gain for the latter is $u(q) \equiv u_b(\bar{q} + q) - u_b(\bar{q})$. Notice q is the transfer, while $Q = \bar{q} + q$ and $Q = \bar{q} - q$ are net consumption for buyers and sellers. This notation makes the setup look just like the baseline New Monetarist model (e.g., Lagos and Wright (2005)), except that instead of having sellers produce q, our KM market is a pure-exchange market, with transfers coming out of endowments. This is not crucial, but it allows us to interpret all production and employment as occurring in the AD market. In any case, for a seller to hand over q, it is obvious that he must get something in return. In many models of this type, the buyer hands over assets, as a medium of exchange, but as discussed above, here they promise a payment in the next AD market. This captures the notion, albeit in a stylized way, that households sometime want or need home improvements or repairs, vacations, medical treatment etc., for which they need loans.

The key friction making credit imperfect is limited commitment: buyers are free to renege on promised payments. If no punishments are available beyond seizing collateral, sellers will only issue credit up to some limit that generally depends on the value of one's assets. In reality, unsecured credit is not impossible, of course, and some purchases for home

[4] We can alternatively assume they have the same marginal utility but different endowments – we merely need some motive for trade.We can also assume, e.g., that q is a factor of production, and agents realize different productivity shocks in KM. In some presentations of related models, trade is motivated by random matching, with σ interpreted as the probability of meeting someone who produces a good that you like. Also, instead of having households trade with each other in KM, at some cost in terms of simplicity, we can alternatively have them trade with firms or with retailers (e.g., Aruoba et al. 2012).

repairs, medical treatment etc. can by put on one's credit card, but as long as there are limits, households sometimes still need collateralized loans. For now the limit on unsecured credit is exogenous, although one can endogenize it as in Kehoe and Levine (1993) and Alvarez and Jermann (2000). To keep things straight, in the benchmark model we use language suggesting that assets are used as collateral, not as a direct payment instrument. In some cases (e.g., home equity) this seems more natural than others (e.g., cash, or maybe government bonds), as we discuss in more detail below. Also, we mention when that while we frame the presentation as a theory of consumer credit, one can retool the model as one of producer or investor credit without much difficulty.

In AD, as in standard growth theory, there is a numeraire good x that can be used for consumption or investment, produced by firms using capital and labor according to a CRS technology $f(k,\ell) + (1-\delta_k)\,k$, where δ_k is the capital depreciation rate. We assume f is strictly increasing and concave, and as usual in macro, k and ℓ are complements in the sense that $f_{k\ell} > 0$. For firms, in the AD market, the usual profit-maximization conditions are

$$\omega = f_\ell(k,\ell) \text{ and } \rho = f_k(k,\ell), \tag{1}$$

where ω is the wage and ρ the rental rate on capital, in terms of x. This is all we need to say about firms for the rest of the analysis.

Households own the production capital k, as well as housing capital h with depreciation rate δ_h. Housing can be in fixed supply H, in which case we set $\delta_h = 0$, or produced endogenously, as discussed below. There is also an asset e that we interpret as equity in a Lucas tree, paying a dividend γ in every AD market in units of x. This asset is always in fixed supply, normalized to 1. There is also fiat money m, and the supply M evolves as a policy choice. Finally, there is a real bond b, with supply B also set by policy, that purchased are in one AD market and redeemed for a unit of numeraire in the next AD market. Define a portfolio $\mathbf{a} = (b, e, h, k, m)$ consisting of bonds, equity, housing, capital,

and money. Let $\boldsymbol{\phi} = (\phi_b, \phi_e, \phi_h, \phi_k, \phi_m)$ be the asset price vector, with the understanding that $\phi_k = 1$ since k and x are the same physical object.[5]

Households have quasi-linear utility over AD consumption, housing and labor, $U(x,h) - \ell$. They have discount factor $\beta \in (0,1)$ between the AD market and the next KM market, but do not discount between KM and AD, without loss of generality. A household in AD with portfolio $\mathbf{a}$ has net worth

$$y = y(\mathbf{a}) = b + (\gamma + \phi_e)\, e + (1 - \delta_h)\, \phi_h h + (\rho + 1 - \delta_k)\, k + \phi_m m - d + \Omega \tag{2}$$

in terms of numeraire, where d is debt (which could be negative) from the previous KM market, and Ω denotes other income including taxes or transfers. We assume KM debt d is settled every period in AD, which is without loss of generality, given quasi-linear utility.

The household AD problem is

$$W(y,h) = \max_{x,\ell,\hat{\mathbf{a}}} \{U(x,h) - \ell + \beta V(\hat{\mathbf{a}})\} \text{ s.t. } x = y + \omega\ell - \boldsymbol{\phi}\hat{\mathbf{a}},$$

where $V(\hat{\mathbf{a}})$ is the continuation value at market closing. The contol variables are AD consumption, employment and a new portfolio $\hat{\mathbf{a}}$. The state includes net worth and housing, because in contrast to other assets, h affects $U(\cdot)$ directly, not only through the budget equation. Notice one gets utility from h brought into the period, while $\hat{h}$ can only be enjoyed next period, just like $\hat{k}$ can only be used in production next period. Eliminating ℓ using the budget equation, we have

$$W(y,h) = \frac{y}{\omega} + \max_x \left\{ U(x,h) - \frac{x}{\omega} \right\} + \max_{\hat{\mathbf{a}}} \left\{ -\frac{\boldsymbol{\phi}\hat{\mathbf{a}}}{\omega} + \beta V(\hat{\mathbf{a}}) \right\}, \tag{3}$$

which immediately implies W is linear in net worth with slope $1/\omega$. Moreover, the choices of x and $\hat{\mathbf{a}}$ are independent of $y = y(\mathbf{a})$ – so, in particular, all households have the same

[5] We can easily extend the model to include nominal bonds, demand deposits, foreign currencies, and other assets, but to keep things manageable we stick to those listed in the text. We can also allow agents other than the government to issue bonds, but this will not happen in equilibrium – while one might imagine wanting to buy bonds issued by another household to use as KM collateral, this cannot be a good deal for both parties when they are intrinsically homogeneous.

portfolio exiting the AD market.[6]

The value function for a household entering the KM market with $\mathbf{a}$ is

$$V(\mathbf{a}) = W\left[y(\mathbf{a}), h\right] + \sigma\left[u\left(q\right) - d/\omega\right] + \sigma[\tilde{d}/\omega - c\left(\tilde{q}\right)], \tag{4}$$

where (q, d) denotes the terms of trade when one is a buyer, and $(\tilde{q}, \tilde{d})$ when one is a seller, comprised of a quantity q and a debt obligation d coming due in the following AD market. The first term on the RHS is one's payoff if one does not participate in KM. The second is the expected surplus from being a KM buyer, using $u\left(q\right) = u_b\left(\bar{q} + q\right) - u_b\left(\bar{q}\right)$ and the result that W is linear in wealth with slope $1/\omega$. The final term is the expected surplus from being a seller, using $c\left(q\right) = u_s\left(\bar{q}\right) - u_s\left(\bar{q} - q\right)$.

The total amount pledged – one's debt position – is

$$d = d\left(\mathbf{d}\right) = d_b + \left(\gamma + \phi_e\right) d_e + \left(1 - \delta_h\right)\phi_h d_h + \left(\rho + 1 - \delta_k\right) d_k + \phi_m d_m + d_u, \tag{5}$$

where $\mathbf{d} = (d_b, d_e, d_h, d_k, d_m, d_u)$ is a vector of promises to deliver quantities of each asset, plus unsecured debt d_u. Note from (5) that bond pledges are evaluated at face value, as are money and unsecured debt; pledges of equity are evaluated cum-dividend; pledges of capital are evaluated before factor markets convene. Pledges of home equity are evaluated at market prices after depreciation - this reflects a timing assumption. The creditor can seize the house if the debtor defaults, but this foreclosure occurs at the end of the period, i.e. after the current flow of housing services and depreciation have occurred. Of course, no one cares about the composition of payments in AD, only their real value d.

However, one's portfolio $\hat{\mathbf{a}}$ matters in KM due to the following *pledgability restrictions*:

$$d_j \leq D_j\left(a_j\right) \text{ for } j = b, e, h, k, m \quad , \tag{6}$$

$$d_u \leq D_u \tag{7}$$

[6] This generates a big gain in tractability, because we do not need to track an endogenous distribution across agents as a state variable – it is degenerate at the close of each AD market. In fact, the distribution is generally degenerate only if we condition on household type, but here there is only one type. If they were intrinsically heterogenous, households might choose different values of $\hat{\mathbf{a}}$, but these choices would still independent of $\mathbf{a}$ for each type. Tractability comes not from degeneracy *per se*, but from history independence.

Thus, unsecured debt is available only up to D_u, beyond which promises must be backed, using assets as collateral. We assume $D_j(0) = 0$, and $\partial D_j/\partial a_j \geq 0$. A special case is full pledgability, $D_j(a_j) = a_j$. We assume this is true for currency, $D_m(m) = m$, although we can also imagine situations where $D_m(m) < m$. Whenever $D_j(a_j) < a_j$ one is forced to take a *haircut* when using a_j as collateral. At the same time, if $D_u > 0$, one can be *leveraged*, which means getting credit beyond one's net worth. Given $\mathbf{a}$, the upper bound on debt comes from pledging all assets to the limit:

$$\bar{D}(\mathbf{a}) \equiv D_b(b) + (\gamma + \phi_e) D_e(e) + (1 - \delta_h)\phi_h D_h(h) + (\rho + 1 - \delta_k) D_k(k) + \phi_m m + D_u \quad (8)$$

In equilibrium, households honor their obligations – there is no default and no assets are ever seized – but the fact that households could default motivates our pledgability constraints. There are various ways to endogenize – or at least rationalize – such restrictions.[7] While it is obviously desirable to explore the microfoundations of imperfect credit at a deeper level, this paper is not about that. Instead, in this paper, we are interested in devloping the implications of pledgability constraints, taking them as given, and think one can learn something from the exerecise, even if we ultimately prefer more explicit descriptions of the frictions underlying imperfect credit.

The next step is to determine the terms of trade in the KM market using a generic mechanism. To see what this entails, suppose for now that KM trade is bilateral. Then in each buyer-seller meeting, a mechanism generates a trade (q, d) satisfying

$$d = \omega z(q) \text{ and } d \leq \bar{D}(\mathbf{a}), \quad (9)$$

[7] Kiyotaki and Moore (1997,2005) appeal to resource diversion, in the sense that after a default the creditor can seize only some fraction of the assets, while the debtor absconds with the rest. One can also imagine seizure using up resources (e.g., litigation costs), as discussed in Iacoviello (2005). Holmstrom and Tirole (2011) take pledgability as a primitive for investors, but provide a variety of economic stories to motivate it. A recent formalization based on asymmetric information is developed in Rocheteau (2011), Li and Rocheteau (2012) and Li et al. (2012), who assume assets can be counterfeited, at a fixed cost. Using a natural approach to bargaining under asymmetric information, they show how sellers will accept asset a_j only up to a limit $\bar{a}_j$, which depends on parameters, including the cost of counterfeiting. As a special case, Lester et al. (2012) assume 0 cost of counterfeiting. See Gertler and Kiyotaki (2010) for a survey with many other references.

for some funciton $z(q)$. Thus, to acquire q, a buyer must pledge a payment $\omega z(q)$ in numeraire, where $z(q)$ is in terms of utility, because each unit of numeraire translates into $1/\omega$ hours of leisure, each of which is worth 1 util given our preferences. A simple example is buyer-take-all bargaining: to acquire q a buyer must pledge $d = \omega c(q)$ to exactly compensate the seller for his cost, but any such pledge must respect the constraint $d \leq \bar{D}(\mathbf{a})$. For the buyer-take-all mechanism, it is easy to see the following: let q^* solve $u'(q^*) = c'(q^*)$ and $d^* = \omega c(q^*)$; if $d^* \leq \bar{D}(\mathbf{a})$ the outcome is $q = q^*$ and $d = d^*$; but if $d^* > \bar{D}(\mathbf{a})$, then $d = \bar{D}(\mathbf{a})$ and $q < q^*$ solves $\bar{D}(\mathbf{a}) = \omega c(q)$.

More generally, consider the proportional bargaining solution in Kalai (1977), where the buyer has bargaining power θ. It is easy to show the outcome is the same as buyer-take-all, except we replace $c(q)$ with

$$z(q) = \theta c(q) + (1-\theta) u(q). \tag{10}$$

Thus, let q^* solve $u'(q^*) = z'(q^*)$, which turns out to be the same as the solution to $u'(q^*) = c'(q^*)$, and $d^* = \omega z(q^*) = \theta c(q^*) + (1-\theta) u(q^*)$, which is greater than d^* implied by buyer-take-all unless $\theta = 1$. Then $d^* \leq \bar{D}(\mathbf{a})$ implies $q = q^*$ and $d = d^*$, and $d^* > \bar{D}(\mathbf{a})$ implies $d = \bar{D}(\mathbf{a})$ where $q < q^*$ solves $\bar{D}(\mathbf{a}) = \omega z(q)$. The outcome is also similar with generalized Nash bargaining, except[8]

$$z(q) = \frac{\theta u'(q)}{\theta u'(q) + (1-\theta) c'(q)} c(q) + \frac{(1-\theta) c'(q)}{\theta u'(q) + (1-\theta) c'(q)} u(q). \tag{11}$$

Other mechanisms that have been used in related models include strategic bargaining (see e.g. He et al. (2012)), pure mechanism design (Hu et al. (2009)), price posting with directed search (**?**; Rocheteau and Wright (2005)) or with undirected search (He et al.

[8] With nonlinear $u(\cdot)$ or $c(\cdot)$, for the same θ, Nash and Kalai give the same outcome when $d^* \leq \bar{D}(\mathbf{a})$, but *not* when $d^* > \bar{D}(\mathbf{a})$. Kalai bargaining has been used frequently in recent monetary theory because it is more tractable than Nash (see Lester et al. 2012 for references). Also, to be clear, with Nash bargaining agents may endogenously choose asset positions that make the contraint bind – e.g., in the pure courrency model of Lagos-Wright (2005) they choose $\hat{m}$ so that they cannot afford q^*. This is due to the fact that, with Nash, unless $\theta = 1$ an agent's surplus can decrease when his constraints are relaxed (see Aruoba et al. 2007). But it is still true that if the constraints are slack then Nash yields $q = q^*$ for all θ.

(2012)), and auctions (Galenianos and Kircher (2008); Dutu et al. (2009)). Some of these approaches, such as auctions, are more interesting or make more sense with multilateral trade. Once we depart from bilateral trade, it also makes sense to consider Walrasian pricing. Thus, suppose positive and (for simplicity) equal measures of buyers and sellers meet in the same location. Buyers solve $\max_q \{u(q) - d/\omega\}$, subject to $d = \phi_q q \leq \bar{D}(\mathbf{a})$, where they take parametrically ϕ_q, the price of q in terms of numeraire x. If $\phi_q q \leq \bar{D}(\mathbf{a})$ does not bind then $u'(q) = \phi_q/\omega$, and if it binds then $q = \bar{D}(\mathbf{a})/\phi_q$. Sellers solve $\max \{\phi_q q/\omega - c(q)\}$, which implies $\phi_q = \omega c'(q)$. Combining these results, $\omega c'(q^*) q^* \leq \bar{D}(\mathbf{a})$ implies $q = q^*$ and $d = \omega c'(q^*) q^*$, while $\omega c'(q^*) q^* > \bar{D}(\mathbf{a})$ implies $d = \bar{D}(\mathbf{a})$ and q solves $\omega c'(q) q = \bar{D}(\mathbf{a})$. This is qualitatively the same as bargaining, except now $z(q) = qc'(q)$.

A variety of standard solution concepts can therefore be considered special cases of a generic trading mechanism. Rather than take a stand on a particular one, we work with an abstract mechanism defined by a function $z(q)$, where $z(0) = 0$ and $z(q)$ is strictly increasing and differentiable almost everywhere, such that the following condition holds:

Assumption 1 *If* $\omega z(q^*) \leq \bar{D}(\mathbf{a})$ *then* $q = q^*$ *and* $d^* = \omega z(q^*)$*; but if* $\omega z(q^*) > \bar{D}(\mathbf{a})$ *then* $d = \bar{D}(\mathbf{a})$ *and* q *solves* $\bar{D}(\mathbf{a}) = \omega z(q)$.

A useful property of any such mechanism is that the outcome depends on a buyer's portfolio iff his constraint binds, and does not depend at all on the seller's portfolio. Hence, the last term in (4), the expected surplus from being a seller, does not depend on $\hat{\mathbf{a}}$.[9]

Given this, we now characterize the choice of $\hat{\mathbf{a}}$ in the portfolio problem (3). First form

[9] The result that a seller's surplus is independent of his portfolio may depend on quasi-linear utility, but even with this assumption, it does not hold for any possible mechanism – e.g., while it does hold for the mechanism in Hu et al. (2009), where the terms of trade depend on a buyer's portfolio directly and not only via his constraint, the result does not hold for a simple generalization where the terms of trade also depend on the seller's portfolio directly. But as the examples in the text make clear, the result holds for many standard mechanisms.

the Lagrangian

$$\begin{aligned}\mathcal{L} &= -\boldsymbol{\phi}\hat{\mathbf{a}}/\omega + \beta W\left[y(\hat{\mathbf{a}}), \hat{h}\right] + \beta\sigma\left[u\left(q\right) - z\left(q\right)\right] + \Sigma_j \lambda_j \left[D_j\left(\hat{a}_j\right) - d_j\right] + \lambda_u\left(D_u - d_u\right) \\ &\quad + \lambda_q\left[d_b + \left(\gamma + \phi_e^{'}\right) d_e + \left(1-\delta_h\right)\phi_h' d_h + \left(\rho' + 1 - \delta_k\right) d_k + \phi_m' d_m + p_d - \omega' z\left(q\right)\right].\end{aligned}$$

The constraints with multipliers λ_u and λ_j, $j = b, e, h, k, m$, say that unsecured pledges are limited by D_u and pledges secured by $\hat{a}_j$ are limited by $D_j\left(\hat{a}_j\right)$. The constraint with multiplier λ_q says $d = \omega' z\left(q\right)$ must be consistent with the trading mechanism, where $d = d(\mathbf{d})$ is the total value of pledges given in (5). Note that in this constraint we use the wage ω' and asset prices $\boldsymbol{\phi}'$ next period, since that is when the relevant KM trades occur, while $\hat{\mathbf{a}}$ is chosen this period at prices given by $\boldsymbol{\phi}$.

The FOC's are

$$\hat{a}_j \quad : \quad -\frac{\phi_j}{\omega} + \beta\frac{\partial W[y(\hat{\mathbf{a}}), \hat{h}]}{\partial \hat{a}_j} + \lambda_j \frac{\partial D_j(\hat{a}_j)}{\partial \hat{a}_j} \leq 0, \ = \ \text{if } \hat{a}_j > 0 \tag{12}$$

$$d_j \quad : \quad -\lambda_j + \lambda_q \frac{\partial d(\mathbf{d})}{\partial d_j} \leq 0, \ = \ \text{if } d_j > 0 \tag{13}$$

$$q \quad : \quad \beta\sigma\left[\frac{\partial u\left(q\right)}{\partial q} - \frac{\partial z\left(q\right)}{\partial q}\right] - \lambda_q \omega' \frac{\partial z\left(q\right)}{\partial q} \leq 0, \ = \ \text{if } q > 0. \tag{14}$$

In (13), $\partial d(\mathbf{d})/\partial d_j$ is the marginal value of a d_j pledge – e.g., $\partial d(\mathbf{d})/\partial d_e = \gamma + \phi_e$ given cum-dividend equity transfers. A solution to the household's problem is characterized by (12)-(14), plus $U_x(x, h) = 1/\omega$, which determines AD consumption x, and the budget equation, which determines employment ℓ.

2.1 Case 1: Liquid

Suppose the household has sufficient pledgable assets to aquire q^*. In this 'plain vanilla' case, as is standard, in equilibrium fiat money cannot be valued and $\phi_m = 0$. For every other $\hat{a}_j$, $j = b, e, h, k$, we have $\lambda_j = 0$ and the FOC (**??**), which must hold at equality in equilibrium by market clearing, becomes $\phi_j = \omega\beta\partial W/\partial\hat{a}_j$. Dreiving $\partial W/\partial\hat{a}_j$ using (2)-(3)

and simplifying, we get the asset-pricing conditions:

$$\begin{aligned}
\phi_b &= \frac{\beta\omega}{\omega'} && (15)\\
\phi_e &= \frac{\beta\omega}{\omega'}\left(\gamma+\phi_e'\right) && (16)\\
\phi_h &= \frac{\beta\omega}{\omega'}\left(1-\delta_h\right)\phi_h' + \beta\omega U_h\left(x',h'\right) && (17)\\
1 &= \frac{\beta\omega}{\omega'}(\rho'+1-\delta_k). && (18)
\end{aligned}$$

Since $\omega = 1/U_x(x,h)$, (15) says the bond price equals the MRS, $\beta U_x(x',h')/U_x(x,h)$. Similarly, (16)-(17) set the prices of h and k to the MRS times the assets' payoffs, while (18) is the standard capital Euler equation.

In steady state, these reduce to

$$\begin{aligned}
\phi_b &= \frac{1}{1+r} && (19)\\
\phi_e &= \frac{\gamma}{r} && (20)\\
\phi_h &= \frac{\omega U_h(x,h)}{r+\delta_h} && (21)\\
\rho &= r+\delta_k && (22)
\end{aligned}$$

where $1+r = 1/\beta$ is the rate of time preference. The accounting return r_j on each asset, is defined as next period's payoff over the current price:

$$\begin{aligned}
1+r_b &= 1/\phi_b && (23)\\
1+r_e &= \left(\gamma+\phi_e'\right)/\phi_e && (24)\\
1+r_h &= \left[\left(1-\delta_h\right)\phi_h' + \omega U_h\left(x',h'\right)\right]/\phi_h && (25)\\
1+r_k &= \rho'+1-\delta_k. && (26)
\end{aligned}$$

From (15)-(18), these all equal $(1+r)\,\omega'/\omega$ in general, and $1+r$ in steady state. So, when liquidity is not scarce, all assets earn the same return. Of course, for housing, this requires measuring the payoff to include $U_h(x',h')$, not just the capital gain $(1-\delta_h)\,\phi_h'$.

The above results follow directly from the household problem. We now discuss macroeconomic equilibrium in two versions of the model: one with a fixed stock of housing H and $\delta_h = 0$; the other with an endogenous supply. In the first version, given initial conditions for k and h, equilibrium consists of time paths for: (i) AD consumption, capital investment, housing investment, and employment (x, k', h', ℓ) satisfying

$$\begin{aligned} 1 &= U_x(x, H) f_\ell(k, \ell) && (27) \\ U_x(x, H) &= \beta U_x\left(x', H\right)\left[f_k(k', \ell') + 1 - \delta_k\right] && (28) \\ h' &= H && (29) \\ x &= \gamma + f(k, \ell) - \left[k' - (1 - \delta_k) k\right]; && (30) \end{aligned}$$

(ii) KM consumption and (debt) payments $q = q^*$ and $d = f_\ell(k, \ell) z(q^*)$; and (iii) asset prices as described above. A steady state satisfies stationary versions of these conditions.[10]

It seems worth spending a little time on steady state in the benchmark situation, where liquidity is plentiful and money is not valued, before considering more complicated scenarios. To begin, it is routine to derive

$$\begin{aligned} D_1 \partial k/\partial \gamma &= f_\ell f_{k\ell} U_{xx} < 0 \\ D_1 \partial \ell/\partial \gamma &= -f_\ell f_{kk} U_{xx} < 0 \\ D_1 \partial x/\partial \gamma &= F U_x > 0, \end{aligned}$$

where $F = f_{kk} f_{\ell\ell} - f_{k\ell}^2 > 0$ and $D_1 = f_\ell \left[f_\ell f_{kk} - (f_k - \delta_k) f_{k\ell}\right] U_{xx} + F U_x > 0$. Thus, an increase in the dividend (the productivity of trees) reduces AD investment and employment, but increases consumption. This is pure a wealth effect. The net impact on total output, $\gamma + f(k, \ell)$, can be positive or negative, but is positive in the special case $U_{xx} = 0$.

[10] In case it is not obvious, (27) and (28) are the FOC's for x and $\hat{k}$, after inserting ω and ρ from (1); (29) clears the housing market, with ϕ_h adjusting to make that happen; and (30) clears the AD goods market, with consumption equal to output minus net investment, which as usual can be derived by aggregating individual budget equations. In these conditions ℓ denotes aggregate labor. Individual household labor depends on net wealth entering the AD market, which generally differs across households depending on their debt from the previous KM market, but we only need aggregate ℓ to define macro equilibrium.

Next, letting $A \approx B$ indicate that A and B take the same sign, we derive

$$\begin{aligned} D_1 \partial k/\partial H &= f_\ell f_{k\ell} U_{xh} \approx U_{xh} \\ D_1 \partial \ell/\partial H &= -f_\ell f_{kk} U_{xh} \approx U_{xh} \\ D_1 \partial x/\partial H &= f_\ell \left[(f_k - \delta_k) f_{k\ell} - f_\ell f_{kk} \right] U_{xh} \approx U_{xh}. \end{aligned}$$

Thus an increase in the housing stock increases AD investment, employment and consumption if x and h are complements in the sense that $U_{xh} > 0$, and decreases them if x and h substitutes in the sense $U_{xh} < 0$. These and other substantive results are summarized in Table 1a, including predictions about factor prices and the rental rate on housing, $r\phi_h$.[11] Most effects are unambiguous, except for those concerning housing – as in much work on home production (e.g., Aruoba et al. (2012)), these naturally depend on cross derivatives, with $+U_{xh}$ or $-U_{xh}$ indicating the effect takes the same or opposite sign as U_{xh}, and $?/-$ indicating it is ambigous in general but negative if $U_{xh} \geq 0$. Bute notice $\partial \phi_h/\partial H < 0$ is unambigous, which we interpret as a downward-sloping long-run demand for housing.

Table 1a: Parameter Effects in Case 1, liquid equilibrium (h exogenous)

	k	ℓ	x	ω	ρ	$(r+\delta_h)\,\phi_h$
γ	$-$	$-$	$+/0$	$+/0$	0	$+/0$
r	$-$	$?$	$-$	$-$	$+$	$-N_h$
δ_k	$*$	$?$	$-$	$-$	$+$	$-N_h$
H	$+U_{xh}$	$+U_{xh}$	$+U_{xh}$	$-U_{xh}/0$	0	$-N_x$

Table 1b: Parameter Effects in Case 1, liquid equilibrium (h endogenous)

	k	ℓ	x	ω	ρ	h
γ	$-$	$-$	$+$	$+$	0	$+U_{xh}$
r	$?/-$	$?$	$?/-$	$?/-$	$+$	$?/-$
δ_k	$?$	$?$	$-$	$+$	$+$	$-U_{xh}$
δ_h	$-U_{xh}$	$-U_{xh}$	$-U_{xh}$	$+U_{xh}$	0	$-$

Notes: $+U_{xh}$ means same sign as U_{xh} (similar for $-U_{xh}$); $+/0$ means positive for any concave f, and 0 for CRS, $-N_j$ means negative as long as good j is normal, $*$ means ambiguous but < 0 if ℓf_ℓ is inc in ℓ (which is true for Cobb-Douglas).

[11] Table 1 omits the effects of σ on the endogenous variables and the effects of the exogenous variables on q, because they are all 0 in Case 1 when liquidity is plentiful. And, of course, monetary policy has no effect here, because the equilibrium is nonmonetary.

This last result is useful when we endogenize the housing stock, by introducing into the AD market competitive home builders with convex cost function $g(\cdot)$. Thus, increasing h_1 to h_2 requires $g(h_2 - h_1)$ units of x, and the builders' FOC is $\phi_h = g'[h' - (1-\delta_h)h]$. Combined with the household's FOC for h', this yields the housing Euler equation

$$U_x(x,h)g'\left[h' - (1-\delta_h)h\right] = \beta U_x\left(x', H\right)\left\{(1-\delta_h)g'\left[h'' - (1-\delta_h)h'\right] + U_h\left(x', h'\right)\right\}.$$

Equilibrium with endogenous h uses this instead of (29), and

$$x = \gamma + f(k,\ell) - \left[k' - (1-\delta_k)k\right] - g\left[h' - (1-\delta_h)h\right]$$

instead of (30). In steady state the builders' FOC is $g'(\delta_h h) = \phi_h$, an upward-sloping long-run supply curve. Combined with downward-sloping demand, a unique (h, ϕ_h) clears the housing market. The effects of parameters with h endogenous are given in Table 1b.

Including housing in the model allows us to make several points. First, for reproducible assets, like h when it is endogenous, heuristically one can think of supply determining price and demand determining quantity, while for fixed $h = H$ demand simply pins down price. Second, although one can make a similar point by comparing k and e, the price of k is always $\phi_k = 1$, which is not true of ϕ_h. The fact that h, different from k and e, affects utility directly and not only via the budget equation has several implications – e.g., increasing in the supply of Lucas trees always makes liquidity less scarce, while increasing H makes liquidity either more or less scarce depending on the elasticity of housing demand (see He et al. (2012)). Also, empirically, there was a sizable increase in home equity loans since the turn of the millennium, and modeling this may be relevant for understanding the recent boom and bust in house prices (again see He et al. (2012)). Finally, housing provides a clear example where it may not be equivalent for an asset to serve as collateral, as in Kiyotaki-Moore, or as medium of exchange, as in Kiyotaki-Wright, a point we elaborate in section 3. However, having made these points, sometimes in what follows we omit h to simplify the presentation while focusing on other issues.

The last thing to do in Case 1 is to determine exactly when liquidity is plentiful. Assuming H fixed, and focusing on steady state, liquidity plentiful iff $\bar{D}(\mathbf{a}) > f_\ell(k,\ell)z(q^*)$, the amount needed to buy q^*, where $\bar{D}(\mathbf{a})$ is evaluated at the equilibrium portfolio:

$$\bar{D}(\mathbf{a}) = D_b(B) + \frac{(1+r)\gamma}{r} D_e(1) + \frac{U_h(x,H)}{r} D_h(H) + [f_k(k,\ell) + 1 - \delta_k] D_k(k) + D_u \quad (31)$$

When liquidity is not scarce, the exact form of payment is indeterminate – e.g., one can use b, e..., or any combination as long as one respects $d_j \leq D_j(a_j)$. We also mention that when liquidity is plentiful the model *dichotomizes*: the AD allocation (x, k', h', ℓ) is independent of the KM allocation q^*. One can break this by interacting q with (x, k', h', ℓ) either in preferences or in technology – e.g., drop separability between $U(x,h)$ and $u(q)$, or assume k is used in the production of q (Aruoba et al. (2012)). We instead break the dichotomy by assuming below that liquidity is scarce, so that AD and KM interact via financial considerations.

2.2 Case 2: Illiquid Non-monetary

We consider a nonmonetary equilibrium when (31) does not hold, so liquidity is too tight for buyers to get q^*. The FOC for q then implies $\lambda_q = \beta\sigma L(q)/\omega' > 0$, where

$$L(q) = \frac{u'(q) - z'(q)}{z'(q)} \quad (32)$$

will play a big role in what follows. Although not strictly neccesary, to ease the presentation we assume $L'(q) < 0$, noting that for many standard mechanisms, including Kalai and Walras, it holds automatically.[12] In any case, for all j other than m, the FOC for d_j implies

[12] Kalai and Walras yield

$$L(q) = \frac{\theta[u'(q) - c'(q)]}{\theta c'(q) + (1-\theta)u'(q)} \text{ and } L(q) = \frac{u'(q) - c'(q)}{c'(q)},$$

respectively, from which it easy to see $L'(q) < 0$ for all parameter values. While it does *not* necessarily hold for generalized Nash without additional assumptions – e.g., θ is close to 1; or, $c(q)$ is linear and $u(q)$ displays decreasing absolute risk aversion – one can dispense with the assumption $L' < 0$ at a cost of additional work (see Wright 2010 for the analysis in a different but related model). To avoid these complications we focus on specifications with $L'(q) < 0$.

$\lambda_j = \lambda_q \partial d/\partial d_j > 0$, and hence $d_j = D_j(\hat{a}_j)$. Therefore KM buyers borrow to the limit $D = \bar{D}(\hat{\mathbf{a}})$, and q solves $z(q)\omega' = \bar{D}(\hat{\mathbf{a}})$.

In terms of asset prices, the FOC's for $\hat{a}_j$ now imply $\phi_j = \omega\beta W_j + \omega\lambda_j D_j'(\hat{a}_j)$, and after inserting W_j, λ_j and market clearing we get

$$\phi_b = \frac{\beta\omega}{\omega'}\left[1 + D_b'(B)\sigma L(q)\right] \tag{33}$$

$$\phi_e = \frac{\beta\omega}{\omega'}\left(\gamma + \phi_e'\right)\left[1 + D_e'(1)\sigma L(q)\right] \tag{34}$$

$$\phi_h = \frac{\beta\omega}{\omega'}(1-\delta_h)\,\phi_h'\left[1 + D_h'(h')\sigma L(q)\right] + \beta\omega U_h\left(x', h'\right) \tag{35}$$

$$1 = \frac{\beta\omega}{\omega'}(\rho' + 1 - \delta_k)\left[1 + D_k'(k')\sigma L(q)\right]. \tag{36}$$

Compared to (15)-(18) from Case 1, each RHS[13] is now multiplied by $1 + D_j'(\hat{a}_j)\sigma L(q)$, which constitutes a *liquidity premium*. Using these pricing equations, the accounting return on each a_j is given by

$$1 + r_j = \frac{\omega'}{\omega}\frac{1+r}{1 + D_j'(\hat{a}_j)\sigma L(q)},$$

which is below the return on an illiquid asset whenever $D_j'(\hat{a}_j)\sigma L(q) > 0$, because $\hat{a}_j$ relaxes payment constraints.

Suppose $h = H$ is fixed and $\delta_h = 0$ (endogenous h can be handled as in Case 1). Then a Case 2 equilibrium consists of paths for: (i) the AD allocation (x, k', h', ℓ) satisfying

$$1 = U_x(x, H)\, f_\ell(k, \ell) \tag{37}$$

$$U_x(x, H) = \beta U_x\left(x', H\right)\left[f_k(k', \ell') + 1 - \delta_k\right]\left[1 + D_j'(k)\sigma L(q)\right] \tag{38}$$

$$h' = H \tag{39}$$

$$x = \gamma + f(k, \ell) + (1-\delta_k)k - k'; \tag{40}$$

(ii) the KM allocation (q, d) satisfying $d = \bar{D}(\hat{\mathbf{a}})$ and $z(q) = d/\omega'$; and (iii) asset prices as described above. The difference between the AD conditions here and in Case 1 is that (38)

[13] Except for housing, because of our timing assumption. The flow of housing services in the immediately following period $U(x, h)$ cannot be transferred or borrowed against, so it is not affected by the liquidity premium.

has $1+D_j'(k)\,\sigma L(q)$ on the RHS, since liquidity needs now affect investment. Steady state satisfies stationary versions of these conditions, including

$$\begin{aligned}\bar{D}(\hat{\mathbf{a}}) &= D_b(B)+\frac{(1+r)\,\gamma D_e(1)}{r-D_e'\sigma L(q)}+\frac{U_h(x,h)\,[1+D_h'\sigma L(q)]D_h(H)}{r-D_h'\sigma L(q)} \\ &\quad +[f_k(k,\ell)+1-\delta_k]\,D_k(k)+D_u,\end{aligned} \tag{41}$$

which we get by inserting into (8) the equilibrium values of b, e and h. This differs from (31) in Case 1 because asset prices (in the case of b, e and h) or quantities (in the case of k, and h when it is endogenous) generally depend on liquidity considerations.

We now derive the effects of parameter changes. To keep the presentation managable, for this exercise, we ignore bonds and housing and set $D_j(\hat{a}_j)=\mu_j\hat{a}$. Then, the steady state in this case, is summarized by (q,k,ℓ,x) satisfiying

$$\begin{aligned} z(q) &= \frac{(1+r)\,\gamma\mu_e}{r-\mu_e\sigma L(q)}+[f_k(k,\ell)+1-\delta_k]\,\mu_k k+D_u \\ 1+r &= [f_k(k,\ell)+1-\delta_k]\,[1+\mu_k\sigma L(q)] \\ x &= \gamma+f(k,\ell)-\delta_k k \\ 1 &= f_\ell(k,\ell)\,U_x(x).\end{aligned}$$

Differentiation yields

$$\begin{bmatrix} C_z & -C_k & -\mu_k k f_{k\ell} & 0 \\ C_f\mu_k\sigma L' & C_L f_{kk} & C_L f_{k\ell} & 0 \\ 0 & f_k-\delta & f_\ell & -1 \\ 0 & f_{k\ell}U_x & f_{\ell\ell}U_x & f_\ell U_{xx}\end{bmatrix}\begin{bmatrix} dq \\ dk \\ d\ell \\ dx\end{bmatrix}=\begin{bmatrix} \Gamma \\ dr-C_f L(\sigma d\mu_k+\mu_k d\sigma) \\ -d\gamma \\ 0\end{bmatrix}$$

where $\Gamma = dP_d + C_f k d\mu_k + C_\gamma d\gamma + C_{\mu_e} d\mu_e - C_r dr + C_\sigma d\sigma$ and

$$\begin{aligned}
C_z &= z' - \frac{(1+r)\,\gamma\mu_e^2\sigma}{(r-\mu_e\sigma L)^2}L', \\
C_f &= f_k + 1 - \delta_k, \\
C_k &= \mu_k\left(kf_{kk} + f_k + 1 - \delta_k\right), \\
C_L &= 1 + \mu_k\sigma L, \\
C_\gamma &= \frac{(1+r)\,\mu_e}{r-\mu_e\sigma L},\ C_{\mu_e} = \frac{(1+r)\,\gamma r}{(r-\mu_e\sigma L)^2}, \\
C_r &= \frac{(1+\mu_e\sigma L)\,\gamma\mu_e}{(r-\mu_e\sigma L)^2},\ C_\sigma = \frac{(1+r)\,\gamma\mu_e^2 L}{(r-\mu_e\sigma L)^2}.
\end{aligned}$$

Notice all the C's are positive, with the possible exception of C_k. Hence we impose a restriction:

Assumption 2 *(1)* $C_k f_{\ell\ell} \leq \mu_k k f_{k\ell}^2$*; equivalently,* $kF + (f_k + 1 - \delta_k)f_{\ell\ell} \geq 0$*, where we recall* $F = f_{kk}f_{\ell\ell} - f_{k\ell}^2$*. (2)* $C_k f_\ell \geq \mu_k k(f_k - \delta_k)f_{k\ell}$*; equivalently,* $(f_k + 1 - \delta_k)\,f_\ell \geq k\left[(f_k - \delta_k)f_{k\ell} - f_\ell f_{kk}\right]$.

The determinant of the sqaure matrix is

$$\begin{aligned}
D_2 &= C_z C_L F U_x + C_z C_L f_\ell\left[f_\ell f_{kk} - (f_k - \delta_k)f_{k\ell})\right]U_{xx} \\
&\quad + C_f\mu_k\sigma L'\left(C_k f_{\ell\ell} - \mu_k k f_{k\ell}^2\right)U_x \\
&\quad + C_f\mu_k\sigma L' f_\ell\left[-C_k f_\ell + \mu_k k\left(f_k - \delta_k\right)f_{k\ell}\right]U_{xx}
\end{aligned}$$

The first and second terms on the RHS are positive. Under Assumption 2 (1) and (2), the third and fourth terms are, too. Hence Assumption 2 implies $D_2 > 0$. However, even without Assumption 2, if μ_k small then D_2 is close to $C_z C_L F U_x + C_z C_L f_\ell\left[f_\ell f_{kk} - (f_k - \delta_k)f_{k\ell})\right]U_{xx} > 0$, without additional assumptions.

We first derive the effects of γ:

$$\begin{aligned}
D_2\partial q/\partial\gamma &= C_\gamma C_L F U_x + C_\gamma C_L f_\ell \left[f_\ell f_{k\ell} - (f_k - \delta_k) f_{kk}\right] U_{xx} - C_L f_\ell f_{k\ell} \left(-C_k + \mu k f_{kk}\right) U_{xx} \\
D_2\partial k/\partial\gamma &= -C_f C_\gamma \mu_k \sigma L' f_{\ell\ell} U_x + f_\ell \left(C_z f_\ell + C_f \mu_k^2 \sigma L' k f_{k\ell} - C_\gamma C_f \mu_k \sigma L' f_\ell\right) U_{xx} \\
D_2\partial \ell/\partial\gamma &= C_f C_\gamma \mu_k \sigma L' f_{\ell k} U_x + C_f \mu_k \sigma L' f_\ell \left[C_\gamma \left(f_k - \delta_k\right) - A C_z C_L f_{kk}\right] U_{xx} \\
D_2\partial x/\partial\gamma &= C_z C_L F U_x + C_f \mu_k \sigma L' \left(C_k f_{\ell\ell} - \mu_k k f_{k\ell}^2\right) U_x
\end{aligned}$$

The first expression is ambiguous: due to the usual wealth effect k falls, which reduces q, as one can see by setting $C_\gamma = 0$; but when $C_\gamma > 0$ and $U_{xx} = 0$, one can see a positive effect, resulting from the fact that e is more pledgable. Similarly, k is ambiguous, but decreases with γ if $U_{xx} = 0$, because k is not as important when the value oif e increases. Like k, ℓ also decreases with γ if $U_{xx} = 0$. But x unambiguously increases with γ by Assumption 2.

Next we have

$$\begin{aligned}
D_2\partial q/\partial r &= -C_r C_L F U_x + \left(C_k f_{\ell\ell} - \mu_k k f_{k\ell}^2\right) U_x - C_r C_L f_\ell \left[f_\ell f_{kk} - \left(f_k - \delta_k f_{k\ell}\right)\right] U_{xx} \\
&\quad + f_\ell \left[C_k f_\ell - \mu_k k \left(f_k - \delta_k\right) f_{k\ell}\right] U_{xx} \\
D_2\partial k/\partial r &= \left(C_z + C_r C_f \mu_k \sigma L'\right) \left(f_\ell^2 U_{xx} + f_{\ell\ell} U_x\right) \\
D_2\partial \ell/\partial r &= \left(C_z + C_r C_f \mu_k \sigma L'\right) \left[f_\ell \left(f_k - \delta_k\right) U_{xx} - f_{k\ell} U_x\right] \\
D_2\partial x/\partial r &= \left(C_z + C_r C_f \mu_k \sigma L'\right) \left[\left(f_k - \delta_k\right) f_{\ell\ell} - f_\ell f_{k\ell}\right] U_x
\end{aligned}$$

The first expression is negative by Assumption 2, so q falls with r because the value of e, and possibly also the level of k, fall. The effect on k is ambiguous since, as is standard, higher r reduces investment, but now with the value of e falling there is an incentive to invest more for collateral reasons. Then ℓ and x both increase iff k increases.

In terms of the financial parameters, startying with the probability one needs a loan,

we have

$$\begin{aligned}
D_2\partial q/\partial\sigma &= C_\sigma C_L F U_x + C_f \mu L \left(-A f_{\ell\ell} + \mu k f_{kk}^2\right) U_x + C_\sigma C_L f_\ell \left[f_\ell f_{kk} - (f_k - \mu_k) f_{k\ell}\right] U_{xx} \\
&\quad + C_f \mu_k L f_\ell \left[-C_k f_\ell + (f_k - \delta_k)\mu_k k f_{k\ell}\right] U_{xx} \\
D_2\partial k/\partial\sigma &= -\left(C_z L + C_\sigma \sigma L'\right)\left(f_\ell^2 U_{xx} + f_{\ell\ell} U_x\right) \\
D_2\partial \ell/\partial\sigma &= \left(C_z L + C_\sigma \sigma L'\right) C_f \mu_k \left[f_{k\ell} U_x + (f_k - \delta_k) f_\ell U_{xx}\right] \\
D_2\partial x/\partial\sigma &= \left(C_z L + C_\sigma \sigma L'\right) C_f \mu_k \left[f_\ell f_{k\ell} - (f_k - \delta_k) f_{\ell\ell}\right] U_x
\end{aligned}$$

Notice that $\partial q/\partial\sigma > 0$ as long as Assumption 2 holds: when the probability σ increases, agents respond by being more liquid. One can see this in the second term, $\partial k/\partial\sigma > 0$, using the result that $C_z L + C_\sigma \sigma L' = L z' > 0$. For equity in Lucas trees, which are in fixed supply, e cannot increase, but the price ϕ_e does (CHECK). The effect on employmenht is ambiguous, because on the one hand agents build more k, but on the other they are richer due to equity prices rising and hence want more leisure, unless $U_{xx} = 0$ in which case $\partial\ell/\partial\sigma$. And in any case, $\partial x/\partial\sigma > 0$.

Now considering the unsecured credit limit P_d, we derive

$$\begin{aligned}
D_2\partial q/\partial P_d &= C_L F U_x + C_L f_\ell \left[f_\ell f_{kk} - (f_k - \delta_k) f_{k\ell})\right] U_{xx} > 0 \\
D_2\partial k/\partial P_d &= -C_f \mu_k \sigma L' \left(f_\ell^2 U_{xx} + f_{\ell\ell} U_x\right) < 0 \\
D_2\partial \ell/\partial P_d &= C_f \mu_k \sigma L' \left[f_{k\ell} U_x + (f_k - \delta_k) f_\ell U_{xx}\right] \lesseqgtr 0, \ < 0 \text{ if } U_{xx} = 0 \\
D_2\partial x/\partial P_d &= -C_f \mu_k \sigma L' \left[(f_k - \delta_k) f_{\ell\ell} - f_\ell f_{k\ell}\right] U_x < 0
\end{aligned}$$

When P_d increases, naturally, q increases. Then, since k is not so critical as collateral, investment falls. Employment may go up or down, but it falls if $U_{xx} = 0$. And x falls in any case.

In terms of the pledgability parameters, first, the effects of μ_e are qualitiatively just like those of P_d – both increase pledgability. However, these is nothing individuals can do at the margin to take further advantage of this, the way they can increase investment in k when

it becomes more pledgable due to an increas in μ_k (see below). However, the market in a sense does it for them – when μ_e increases individuals would like to hold more equity in Lucas trees, but since the supply is fixed, the price goes up. Yet this is enough to increase the value of equity for the representative agent. So there is a multiplier effect when μ_e increases that is not there when P_d increases, but otherwise the effects are qualitatively the same.

Next we have

$$\begin{aligned}
\frac{D_2}{C_f}\partial q/\partial \mu_k &= kC_L F U_x + \sigma L\left(-C_k f_{\ell\ell} + \mu_k k f_{k\ell}^2\right) \\
&\quad + kC_L f_\ell \left[f_\ell f_{kk} - \left(f_k - \delta_k\right) f_{k\ell}\right] U_{xx} + \sigma L f_\ell \left[-C_k f_\ell + \mu_k k \left(f_k - \delta_k\right) f_{k\ell}\right] U_{xx} \\
\frac{D_2}{C_f}\partial k/\partial \mu_k &= -\left(C_z \sigma L + k C_f \mu_k \sigma L'\right)\left(f_\ell^2 U_{xx} + f_{\ell\ell} U_x\right) \\
\frac{D_2}{C_f}\partial \ell/\partial \mu_k &= -\left(C_z \sigma L + k C_f \mu_k \sigma L'\right)\left[f_\ell \left(f_k - \delta_k\right) U_{xx} + f_{\ell k} U_x\right] \\
\frac{D_2}{C_f}\partial x/\partial \mu_k &= -\left(C_z \sigma L + k C_f \mu_k \sigma L'\right)\left[\left(f_k - \delta_k\right) f_{\ell\ell} - f_\ell f_{k\ell} U_x\right]
\end{aligned}$$

The first expression is positive, and hence q increases with the pledgability of k, by Assumption 2. The second is ambiguous: on the one hand, when k becomes more pledgable, a substitution-like marginal effect says agents should invest more; on the other hand, a wealth-like effect says they do not need to invest so much since the same amount of k goes further. Since the effect on k is ambiguous, so is the effect on ℓ – and even if k goes up we cannot be sure that ℓ also goes up unless $U_{xx} \simeq 0$. We can however be sure that x goes up iff k goes up.

Parameter Effects: Case 2

	q	k	ℓ	x	ω	ρ	
γ							
r							
σ							
P_d							
μ_k							
μ_e							

The last thing we do here is check when Case 2 obtains as the only equilibrium – i.e., when is liquidity scarce, but money still not valued? The answer is

$$f_\ell(k,\ell)\, z(q^*) > \bar{D}(\hat{\mathbf{a}}) \geq f_\ell(k,\ell)\, z\left(q^i\right). \tag{42}$$

The first inequality in (42) says that agents cannot borrow enough to get q^*, which means that liquidity is scarce. The second says the following: as will become clear below, in monetary equilibrium households choose real balances so that they can purchase q^i, where $\sigma L\left(q^i\right) = i$; if this is less than they can purchase using only credit, they choose to hold no currency.

2.3 Case 3: Monetary

We now characterize a monetary equilibrium. As in Case 2, $\lambda_q = \beta\sigma L(q) > 0$ and $\lambda_j = P'(p_j)\,\lambda_q > 0$, so all constraints bind, and buyers borrow to the limit

$$\bar{D}(\hat{\mathbf{a}}) = D_b(B) + \left(\gamma + \phi_e'\right) D_e(1) + (1-\delta_h)\,\phi_h' D_h(\hat{h}) + \left(\rho' + 1 - \delta_k\right) D_k(\hat{k}) + \phi_m'\hat{m} + D_u, \tag{43}$$

which now includes real balances $\phi_m'\hat{m}$. The asset pricing equations for (b, e, h, k) are the same as (33)-(36), plus a new condition for pricing currency:

$$\phi_m = \frac{\beta\omega}{\omega'}\phi_m'\left[1 + \sigma L(q)\right]. \tag{44}$$

As above, an equilibrium is defined as a path satisfying the relevant conditions for: (i) the AD allocation; (ii) the KM allocation; and (iii) asset prices. However, having money valued has major implications for how the economy functions.

To see this, first rearrange (44) as

$$\frac{\phi_m}{\phi_m'}\frac{\omega'}{\beta\omega} = 1 + \sigma L(q).$$

The LHS is the inflation rate ϕ_m/ϕ_m' times the real interest rate $\omega'/\beta\omega$, which, by the no-arbitrage condition known as the Fisher equation, equals the gross nominal interest rate,

$1+i$.[14] Assuming that monetary policy sets i, this pins down $q = q^i$ as the solution to $\sigma L\left(q^i\right) = i$. A steady state can now be summarized by an allocation satifying the same conditions we had in Case 2, except we replace the condition $\omega z\left(q\right) = \bar{D}\left(\hat{\mathbf{a}}\right)$ with $i = \sigma L\left(q\right)$, since at the margin q depends on monetary policy and not the real supply of pledgable assets.

The key observation here is that q and hence the liquidity premium $L\left(q\right)$ are pinned down by i. Indeed, substituting $\sigma L\left(q\right) = i$, we can write (**??**) as

$$1 + r = \left[f_k\left(k, \ell\right) + 1 - \delta_k\right]\left[1 + D_k'(\hat{k})i\right],$$

to show the direct effect of monetary policy on investment. Changes in pledgability $D_k(\hat{k})$ will change this condition, but since in equilibrium $q = q^i$ does not change, there is a direct effect on k but no indirect effect via q. Similarly, there is a direct effect on other assets' prices ϕ_j and returns

$$1 + r_j = \frac{\omega'}{\omega}\frac{1+r}{1 + D_j'\left(\hat{a}_j\right)i},$$

but no indirect effect through q, as long as we keep the same i.

To go into more detail, again writing $D_j'\left(a_j\right) = \mu_j$, we derive

$$\left[\begin{array}{cccc} -\sigma L' & 0 & 0 & 0 \\ C_f\mu_k\sigma L' & C_L f_{kk} & C_L f_{k\ell} & 0 \\ 0 & f_k - \delta_k & f_\ell & -1 \\ 0 & f_{\ell k}U_x & f_{\ell\ell}U_x & f_\ell U_{xx} \end{array}\right]\left[\begin{array}{c} dq \\ dk \\ d\ell \\ dx \end{array}\right] = \left[\begin{array}{c} Ld\sigma - di \\ dr + C_L d\delta_k - C_f L(\sigma d\mu_k + \mu_k d\sigma) \\ kd\delta_k - d\gamma \\ 0 \end{array}\right]$$

where the determinant of the square matrix is

$$D_3 = -\sigma L' C_L\left\{FU' + f_\ell\left[f_\ell f_{kk} - f_{k\ell}\left(f_k - \delta_k\right)\right]U_{xx}\right\} > 0.$$

Consider first the effect of monetary policy in terms of the nominal interest rate i. We have

$$\partial q/\partial i = 1/\sigma L'\left(q\right) < 0,$$

[14] One can interpret i as the return on an illiquid nominal bond, which need not actually trade in equilibrium, because it is illiquid. Monetary policy can peg the rate of growth in M or the inlfation rate, since they are equal in steady state, or they can peg i, and let M evolve endogenously.

which is the usual Friedman effect. Similarly,

$$D_3\partial k/\partial i = C_f\mu_k\sigma L'\left(f_\ell^2 U_{xx} + f_{\ell\ell}U_x\right) > 0,$$

as long as $\mu_k > 0$, which is the Tobin effect (obviously $\mu_k = 0$ implies $\partial k/\partial i = 0$). Also,

$$D_3\partial\ell/\partial i = -C_f\mu_k\sigma L'\left[f_{k\ell}U_x + \left(f_k - \delta_k\right)f_\ell U_{xx}\right],$$

which is ambiguous in sign: the first term inside the square bracket is positive, but the second is negative, so the net effect depends on U_{xx} and U_x. Hence, the long-run Phillips curve can slope up or down. The intuition stems from opposing wealth and substitution effects resulting from a higher level of steady state capital. With CRRA utility and a Cobb-Douglas production function, e.g., one can show $\partial\ell/\partial i > 0$ iff risk aversion is below a threshold. Specifically, with $U(x) = \frac{x^{1-\eta}}{1-\eta}$Cobb-Douglas $f(k,\ell) = k^{\alpha\ell 1-\alpha}$, we have

$$\begin{aligned}
U_x f_{kl} + U_{xx} f_l\left(Af_k - \delta\right) &= x^{-\eta}\alpha(1-\alpha)k^{\alpha-1\ell-\alpha} - \eta x^{-\eta-1}(1-\alpha)k^\alpha\ell^{-\alpha}\left(\alpha k^{\alpha-1}\ell^{1-\alpha} - \delta\right) \\
&= (1-\alpha)x^{-\eta-1}k^\alpha\ell^{-\alpha}k^{-1}\left[\alpha x - \eta\left(\alpha k^\alpha\ell^{1-\alpha} - \delta k\right)\right] \\
\partial\ell/\partial i &> 0 \Longleftrightarrow \alpha x - \eta\left(\alpha k^\alpha\ell^{1-\alpha} - \delta k\right) > 0 \\
&\Longleftrightarrow \frac{\alpha k^\alpha\ell^{1-\alpha} - \alpha\delta k}{\alpha k^\alpha\ell^{1-\alpha} - \delta k} > \eta
\end{aligned}$$

Thus, the substitution effect from a monetary policy change dominates the income effect, i.e. CM employment increases with inflation, if the curvature of the utility function is sufficiently low. Note that the left hand side is greater than 1. Although the effect on ℓ is ambiguous, in general, AD consumption definitely increases with i.

$$D_3\partial x/\partial i = C_f\mu_k\sigma L'\left[\left(f_k - \delta_k\right)f_{\ell\ell} - f_\ell f_{k\ell}\right]U_x > 0,$$

It is also easy to determine the effects on prices. For factor prices,

$$\begin{aligned}
D_3\partial\omega/\partial i &= C_f\mu_k\sigma L' f_\ell\left[f_{\ell k} - \left(f_k - \delta_k\right)f_{\ell\ell}\right]U_{xx} > 0 \\
D_3\partial\rho/\partial i &= -C_f\mu_k/\left(1 + \mu_k\sigma L\right) < 0,
\end{aligned}$$

Hence, the wage rate ω rises while the rental rate ρ falls with i. For the return to equity, we have

$$\partial R_e/\partial i = (\partial R_e/\partial q)(\partial q/\partial i) > 0,$$

since $\partial R_e/\partial q = -R_e\mu_e/(1+\mu_e\sigma L) < 0$ and we showed above that $\partial q/\partial i < 0$. This is the Mundell effect: increasing inflation makes people want to switch their portfolios out of m and into e, but since $e = 1$ is in fixed supply, the result is to raise the price ψ and lower the return $R_e = 1 + \gamma/\psi$.

In terms of the impact of σ, we derive

$$D_3\partial q/\partial\sigma = -L/\sigma L' > 0,$$

again simplifying. Hence, q goes up with σ, because agents carry more real balances when the probability they need them goes up – i.e., one can easily check $\partial\phi/\partial\sigma > 0$. Notice that $\partial q/\partial\sigma = -L\partial q/\partial\sigma$, so the elasticity of q wrt σ is the negative of the elasticity wrt i. But, different from i, σ neither affects the other quantities, $\partial k/\partial\sigma = \partial\ell/\partial\sigma = \partial x/\partial\sigma = 0$, nor the prices, $\partial R_e/\partial\sigma = \partial\omega/\partial\gamma = \partial\rho/\partial\gamma = 0$.

In terms of the impact of the dividend γ, the productivity of Lucas trees, we have

$$\begin{aligned}
\partial q/\partial\gamma &= 0 \\
D_3\partial k/\partial\gamma &= -C_L\sigma L' f_{k\ell} f_\ell U_{xx} < 0 \\
D_3\partial\partial\ell/\partial\gamma &= C_L\sigma L' f_{kk} f_\ell U_{xx} < 0 \\
D_3\partial x/\partial\gamma &= C_L\sigma L' F U_x > 0.
\end{aligned}$$

The dividend does not affect q, even though equity is used for transaction purposes, since q is pinned down by $\sigma L(q) = i$. An increase in γ reduces investment and employment, and increases consumption x. This is due to a wealth effect: even if we shut down decentralized trade, by setting $\sigma = 0$, e.g., we still get $\partial k/\partial\gamma < 0$, $\partial\ell/\partial\gamma < 0$ and $\partial x/\partial\gamma > 0$. Also, one can easily check that the effects on k and ℓ net out to leave wage and rental rates unchanged:

$\partial\omega/\partial\gamma = \partial\rho/\partial\gamma = 0$. Also, given $\partial q/\partial\gamma = 0$, we have $\partial R_e/\gamma = 0$. Since $R_e = 1 + \gamma/\psi$, this means that the elasticity of equity prices wrt dividends is $(\partial\psi/\partial\gamma)(\gamma/\psi) = 1$, as it would if equity were priced fundamentally, even though here it bears a premium $\psi > \gamma/r$.

From the constraint $z(q) = d/\omega$, where $d = (\gamma + \phi_e)\mu_e + (\rho + 1 - \delta_k)\mu_k k + \phi m$, if we ignore bonds, housing and unsecured debt, we derive

$$0 = \frac{(\gamma + \phi_e)\mu_e}{\gamma} + C_f \mu_k \frac{\partial k}{\partial \gamma} + \frac{\partial \phi m}{\partial \gamma}$$

The first term is positive – i.e., one gets more for an equity pledge when γ increases. The second term is negative, since $\partial\rho/\partial\gamma = 0$ and $\partial k/\partial\gamma < 0$, so one now gets less for a pledge backed by capital. The net effect on real balances is unclear – e.g., ϕm definitely falls with γ when $\mu_e > 0 = \mu_k$, and rises when $\mu_e = 0 < \mu_k$. But in any case, in monetary equilibrium, since q is pinned down, net pledgeable wealth does not change with γ – the pledgability of capital and/or real balances get completely crowded out.

In terms of the pledgability parameters, we have

$$\begin{aligned}
\partial q/\partial \mu_k &= 0 \\
D_3 \partial k/\partial \mu_k &= C_f \sigma^2 L L' \left(f_\ell^2 U_{xx} + f_{\ell\ell} U_x\right) > 0 \\
D_3 \partial \ell/\partial \mu_k &= -C_f \sigma^2 L L' \left[(f_k - \delta_k) f_\ell U_{xx} + f_{k\ell} U_x\right] \approx \partial \ell/\partial i \\
D_3 \partial x/\partial \mu_k &= C_f \sigma^2 L L' \left[f_{\ell\ell}(f_k - \delta_k) - f_\ell f_{\ell k}\right] U_x > 0,
\end{aligned}$$

Again, and increase in μ_k does not affect q, but since it does make k better in payments, it increases investment. It has an ambiguous effect on employment, in the same way $\partial\ell/\partial i$ is ambiguous, but the net effect on x is positive. In terms of prices,

$$\begin{aligned}
\partial\omega/\partial\mu_k &= \Phi(f_k + 1 - \delta)\sigma^2 L L' f_\ell \left[f_\ell f_{\ell k} - (f_k - \delta) f_{\ell\ell}\right] U_{xx} > 0 \\
\partial\rho/\partial\mu_k &= -(f_k + 1 - \delta)\sigma L/(1 + \mu_k \sigma L) < 0,
\end{aligned}$$

after simplification. And again, since q does not change with μ_k, $\partial R_e/\partial\mu_k = 0$.

Moving from μ_k to μ_e, it is clear that the effects on (q, k, ℓ, x) are nil, and hence so are the effects on (ω, ρ). But μ_e does affect the price of equity: $\partial R_e/\partial \mu_e = -R_e \sigma L/\left(1 + \mu_e \sigma L\right) < 0$. Intuitively, when it is better in payments, demand for e and hence the price ψ go up, reducing the dividend-price ratio. The reason q does not change is that the increased pledgability of equity completely crowds out real balances. Also note that for an asset in fixed supply, like e, increasing pledgability raises the price, while for a reproducible asset, like k, where the price in terms of x is 1, increasing pledgability raises the quantity.

Since r and μ_k enter only the second equilibrium condition, with the opposite sign, the effects of r on the allocation (q, k, ℓ, x) take the opposite sign to the effects of μ_k, at least assuming monetary policy pegs i so that q does not change (it would be different if we were to peg inflation, say, since then i would rise with r). The same is true for factor prices. The effect on R_e of an increase in r is positive, since $R_e\left[1 + \sigma L\left(q\right)\right] = 1 + r$, given i and hence q do not change. In terms of δ, we have

$$\begin{aligned}
\partial q/\partial \delta_k &= 0 \\
D_3 \partial k/\partial \delta_k &= -C_L \sigma L' \left[\left(f_\ell - k f_{k\ell}\right) f_\ell U_{xx} + f_{\ell\ell} U_x\right] \\
D_3 \partial \ell/\partial \delta_k &= -C_L \sigma L' \left\{\left[k f_{kk} - \left(f_k - \delta_k\right)\right] f_\ell U_{xx} - f_{k\ell} U_x\right\} \\
D_3 \partial x/\partial \delta_k &= C_L \sigma L' F k U_x < 0 \\
D_3 \partial \omega/\partial \delta_k &= -C_L \sigma L' f_\ell \left[F k U' + f_\ell f_{k\ell} - \left(f_k - \delta_k\right) f_{\ell\ell}\right] U_{xx} < 0 \\
\partial \rho/\partial \delta_k &= 1.
\end{aligned}$$

Notice higher δ_k has ambiguous effects on k and ℓ, although again we can sign the effects on factor prices. Again, since q does not depend on δ_k, neither does R_e. We summarize the main results as follows:

Table 3: Parameter Effects in Case 3

	q	k	ℓ	x	ω	ρ	R_e
i	$-$	$+$	?	$+$	$+$	$-$	$+$
σ	$+$	0	0	0	0	0	0
γ	0	$-$	$-$	$+$	0	0	0
μ_k	0	$+$	?	$+$	$+$	$-$	0
μ_e	0	0	0	0	0	0	$-$
r	0	$-$	?	$-$	$-$	$+$	0
δ_k	0	?	?	$-$	$-$	$+$	0

2.4 Numerical Results (Preliminary)

We paramatereize the model as follows. Preferences and technology are assumed to take standard functional forms.

$$\begin{aligned} U(x,h) &= \ln\ x + \psi\frac{h^{1-\varepsilon}}{1-\varepsilon} \\ f(k,l) &= k^{\alpha}l^{1-\alpha} \\ u(q) &= 2Bq^{1-\eta} \\ c(q) &= Bq \end{aligned}$$

Terms of trade in the KM are determined through proportional bargaining as in Kalai (1977), with θ indexing the share of the buyer.

$$z(q) = (1-\theta)u(q) + \theta c(q)$$

The maximum amount of physical capital, housing and equities that can be pledged are

$$\begin{aligned} D_k(k) &= \mu_k(\rho' + 1 - \delta)k \\ D_e(e) &= \mu_e(\gamma + \phi'_e)e \\ D_h(h) &= \mu_h(1-\delta_h)\phi'_h h \end{aligned}$$

We begin with the version with a fixed level of housing stock, normalized to 1, i.e..$H = 1$ and $\delta_h = 0$ The baseline choices for the other parameters are as follows. Using a year as a time period, we choose $\beta = 0.96$ and $\delta_k = 0.06$. The parameters governing utility from housing services are set to $\psi = 0.02$ and $\varepsilon = 2$. Dividend from equity is equal to about 5% of steady state output, with $\gamma = 0.05$. The probability of a needing a loan and the buyer's share are both set[15] to 0.5 KM preference and production parameters are $B = 10, \eta = 0.5$. The pledgability coefficients, μ_j, are all set equal to 0.3.

While not (yet !) a full-fledged calibration, they serve the purpose of illustrating some of the theoretical results Figure 1 plots the dependence of various endogenous variables on policy, i.e. the nominal interest rate i (or equivalently as we discussed before, on inflation). Increases in interest rate make holding money balances, costly increasing the demand for alternative sources of liquidity, leading to higher prices for equity and real estate and a higher level of steady state capital. Given our parameter choces, the substitution effect on labor supply is stronger than the income effect, causing employment to rise. If interest rates are high enough, we no longer have a monetary equilibrium, reflected in the flat sections of the graphs.

The next figure repeats this analysis for a change in μ_h, the degree of pledgability of housing. When μ_h is sufficiently low, the economy has a monetary equilibrium. In this region, any changes in liquidity provided by housing is offset by adjustments in the real value of money holdings, leaving liquidity premia on the other sources of liquidity (viz. capital and equity) unchanged. In other words, when the supply of housing fixed, μ_h has no effect on output, employment, investment and asset prices in a monetary equilibrium. In the non-monetary region (i.e. for μ_h high enough), more liquidity from housing reduces the liquidity role of capital and equity, driving down investment and stock market values. The non-monotonic effects on home prices (and home equity) stem from the fact that as

[15] See Lagos and Wright (2005).

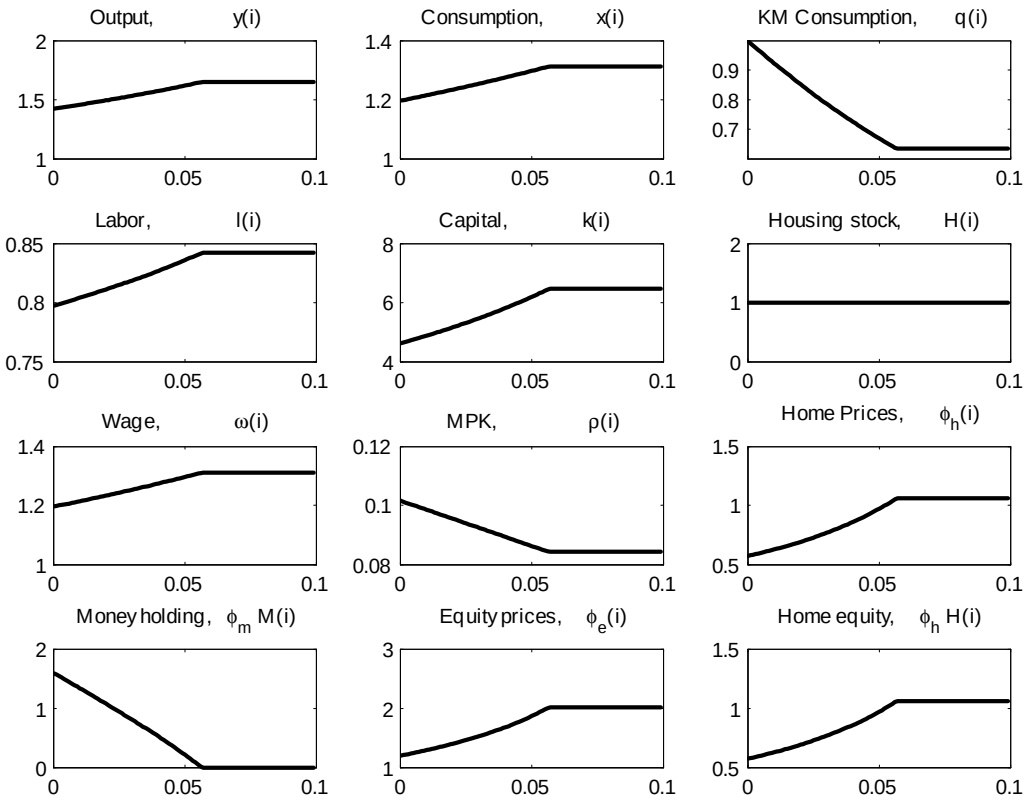


Figure 1: Effect of the nominal interest rate

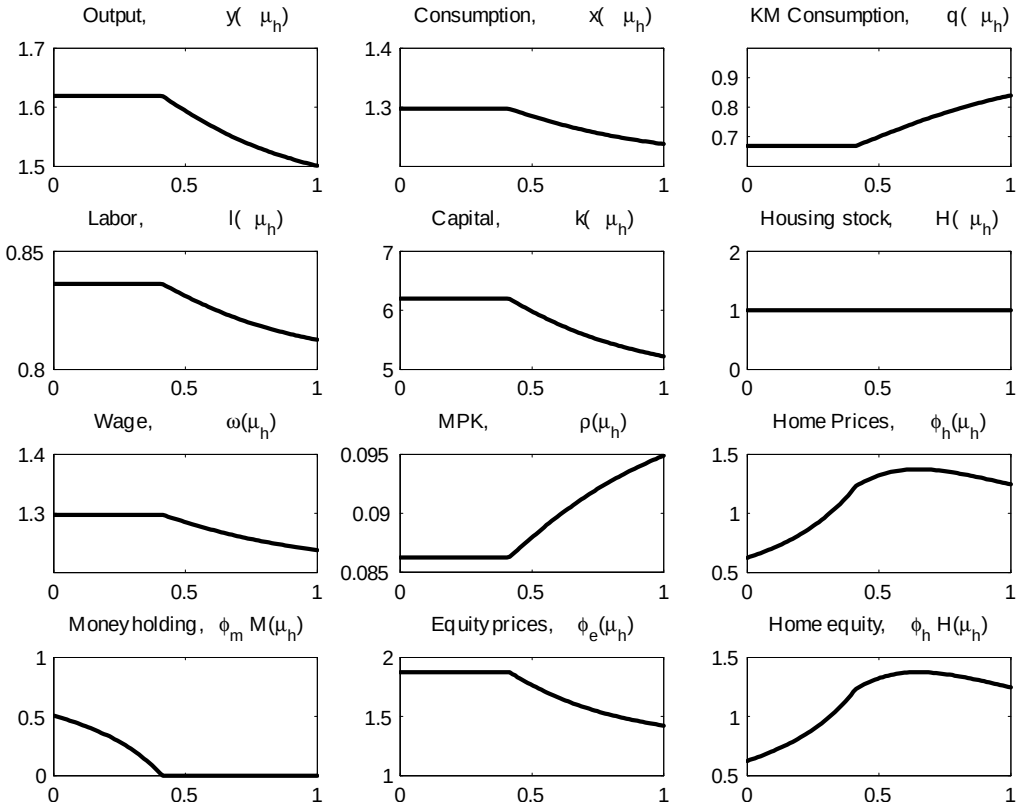


Figure 2: Effect of μ_h, the degree of housing pledgability

liquidity becomes more abundant, the premium attached to liquid assets declines, which can more than offset the upward effect on demand (and prices) because of greater pledgability.

The Appendix contains similar figures for the other parameters of interest (e.g. μ_k, μ_e, H).

Nest step: a full calibration !

3 Means of Exchange or Collateral

3.1 Housing

The analysis so far assumes that payment in the KM occurs only through collateralized borrowing arrangements. As mentioned earlier, an alternative way in assets can faciliate trade is as a medium of exchange, i.e. the buyer can hand over the assets to the seller during the KM interaction, as in Kiyotaki and Wright. We now turn to a deeper analysis

of whether and when this distinction matters,

First, consider housing. The timing in our baseline analysis is as follows - only the person who buys the house in the previous AD gets to enjoy the current flow of housing services, i.e. $U(x, h)$. If a creditor seizes the house following a default, she can can keep it or sell it, but the current flow of housing services still accrues to the original owner. On the other hand, if the house is transfered during the intervening KM, neither party gets to enjoy the housing services in the following period. Then, a mortgage (or a home equity loan) dominates transfering a part of the house as payment, because it preserves the value of the current housing services. But one could alternatively assume housing is *fungible*. Thus, $U(x, h)$ now depends on the end-of-period position in housing (after trading in the AD). Then, buyer is indifferent between immediate and deferred settlement of her obligations to the seller because she can adjust her trades in the market and enjoy the same utility. So, depending on assumptions, it may or may not matter whether h is used as collateral or a payment instrument.

For other assets - bonds, equity or physical capital, - timing of the transfer is relevant only if it has an effect on the fraction of the asset's value that can be pledged. For example, a lender might be unwilling to lend credit upto the full value of the collateral because the debtor can 'divert'a fraction of the asset before its seizure by the creditor. If this risk of diversion can be eliminated by the immediate transfer of posession, then the asset will provide more liquidity as means-of-exchange rather than as collateral. Alternatively, if source of limited pledgability is asymmetric information (as in Li and Rocheteau), the seller's unwillingness to accept assets beyond a certain amount does not change with the timing of that transfer, i.e. the two modes of payment are equivalent. In the following subsection, we consider a model where moral hazard limits pledgability and show that heterogeneity can also induce a strict preference for immediate versus deferred settlement.

3.2 Heterogeneity

For simplicity, we focus on a case with only 2 assets - equity and money. We also assume that the source of limited pledgability is moral hazard - the tree requires maintenance to retain its full value. Agents in possession of the tree *after* trading (both centralized and decentralized) choose a maintenance level $n \in (0,1)$ at a utility cost[16] per tree of χn. A tree maintained at level n is equivalent to a fraction n of a fully-maintained asset (i.e. one with $n = 1$). Now, a debtor might choose to neglect to care for an asset that is pledged to a creditor, preferring to forfeit it instead of expending resources on its maintenance. In the Appendix, we show the inability of debtors to commit to maintaining collateral leads to endogenous limits to pledgability of the form

$$D_e(e) = \mu e$$

where $\mu = 1 - \frac{\chi\omega'}{\gamma+\phi_e}$. The Appendix also shows that when all agents face the same maintenance cost χ, then the liquidity value of the asset is the same whether it is used as a medium of exchange or pledged as collateral.

This is no longer when true when the cost of maintaining the asset differs across agents. In particular, suppose there are two types of agents with cost levels $\chi^1$and χ^2. Without loss of generality, we let $\chi^1 < \chi^2$. The (commonly known) fraction[17] of type-1 agents in the economy is denoted κ. We consider 2 cases. In the first, labeled ex-ante heterogeneity, an agent's type is perfectly observed during decentralized trade. In the other case, the agent's type is the outcome of a shock which is realized only after decentralized trade and is privately observed. In the first case, collateralized borrowing will be strictly preferred over borrowing in some meetings, while the assets will be transferred in others. The intuition is simple - if one agent is commonly known to be better equipped to hold the asset, then the optimal

[16] In the Appendix, we analyze an alternative specification, where the cost of maintenance is denominated in consumption goods.

[17] This could be explicitly modeled as the endogenous outcome of an ex-ante choice of agents to invest in a maintenance technology.

arrangement will feature possession by that agent. On the other hand, if differences between agents emerge *after* decentralized trading and are privately observed, immediate transfer can lead to assets facilitating trade to a higher degree than arrangements with deferred settlement. Intuitively, private information limits the extent to which ex-post liquidity can be credibly pledged. Thus, assets characterized by observable differences in agents' abilities to maintain them are more likely to used as collateral than assets where such differences are not apparent.

3.2.1 *Ex-ante* Heterogeneity

Define

$$\mu^j = 1 - \frac{\chi^j}{\frac{1}{\omega}(\gamma + \phi_e)} \qquad\qquad j = 1, 2$$

Note that there are now 4 types of meetings in this economy. When both the seller and the buyer are of the same type, deferred settlement and immediate transfers lead to the same outcome. However, if a type-1 buyer meets a type-2 seller, then she is better off retaining the asset and using it as collateral (because that allows her to pledge upto μ^1 of the asset's value in the following AD market) instead of handing it over to the seller (who values it at a fraction $\mu^2 < \mu^1$). The opposite is true when a type-2 buyer holding an asset meets a type-1 seller.

As before, depending on parameter values, the equilibrium can be liquid or illiquid, monetary or non-monetary. In addition, equilibria can also differ, depending on the portfolio choices whether both types of agents hold both assets and money. For example, there are 3 types of stationary monetary equilibria:

- Case I: Both types hold money
- Case II: Type-2 agents hold only money and type-1 agents only assets
- Case III: Both types hold assets, only type-2 hold money

We describe only the first case in detail here (the other two are discussed in the Appendix). Since both types hold money, the level of KM activity is pinned down by the nominal interest rate, i.e. by (57). However, type-2 agent's asset holdings will be zero. To see this, note that the optimality condition for type-1 agents' asset holdings implies

$$\begin{aligned}\phi_e &= \beta\mu^1(\gamma+\phi_e)\left[1+\sigma\left(\frac{u'(q)}{z'(q)}-1\right)\right] \\ &= \beta\mu^1(\gamma+\phi_e)\left[1-\sigma+\sigma\frac{u'(q)}{z'(q)}\right] \\ &> \beta\mu^1(\gamma+\phi_e)\left[(1-\sigma)\frac{\mu^2}{\mu^1}+\sigma\frac{u'(q)}{g'(q)}\left(\kappa+(1-\kappa)\frac{\mu^2}{\mu^1}\right)\right]\end{aligned}$$

The term on the right hand side in the second line is the marginal value of the asset for the type 2 agent. The inequality obtains because $\frac{\mu^2}{\mu^1}<1$.

Type-1 and type-2 agents, however, differ in the composition of their liquidity holdings. The real value of money holdings of type-j agents, $\hat{m}^j$, are given by

$$\begin{aligned}\frac{\phi_m}{\omega}\hat{m}^1 &= z(q)-\frac{B}{\omega'}(\gamma+\phi_e)\frac{A}{\kappa}\mu^1 \\ \frac{\phi_m}{\omega}\hat{m}^2 &= z(q)\end{aligned}$$

where

$$\mu^j = 1-\frac{\chi^j\omega}{\gamma+\phi_e}$$

3.2.2 *Ex-post* Heterogeneity

Now, suppose differences between agents arise ex-post, i.e. after decentralized trade. In particular, assume that the agent-specific maintenance cost χ is a random from $\{\chi^1,\chi^2\}$, with associated probabilities $\{\kappa,1-\kappa\}$. As before, let $\chi^1<\chi^2$. Importantly, the realization of the cost becomes known only after decentralized trade occurs[18]. Again, we restrict

[18] What is crucial is that private information makes state-contingent collateralized borrowing infeasible. The timing assumption, i.e. the fact that shocks are realized only after decentralized trade, serves to eliminate any issues relating to asymmetric information.

attention to stationary liquidity-constrained equilibria. This requires that it is optimal to fully maintain an unencumbered asset at both cost levels.

If one unit of the asset is transferred to the seller during exchange, she evaluates it at

$$\frac{1}{\omega'}(\gamma + \phi'_e) - \bar{\chi}$$

where $\bar{\chi}$ is the expected cost. Defining $\bar{\mu} = 1 - \frac{\bar{\chi}}{\frac{B}{\omega'}(\gamma+\psi')}$, the per-unit value of the asset to the seller can be written as

$$\bar{\mu}\frac{1}{\omega'}(\gamma + \phi'_e)$$

Thus, the seller discounts the asset by the expected cost she has to incur in maintaining the asset. What about under collateralized borrowing ? Since the cost is privately observed, the promise cannot be conditioned on the realization of the buyer's χ. If the following condition holds

$$\kappa\mu^1 \leq \ \mu^2$$

the value to the seller is maximized by setting the promised payment equal to a fraction $\mu^2$of the asset's value in the following AD market. With this promised payment, the buyer's maintenance constraint binds if her cost realization is high and is slack otherwise. Therefore, the liquidity value of the asset as collateral is

$$\mu^2\frac{1}{\omega'}(\gamma + \phi'_e)$$

which is strictly less than the value attached to each unit of the asset by the seller. In other words, an immediate transfer provides more liquidity than a collateralized borrowing arrangement. This leads to the following result.

Proposition 1 *The stationary monetary equilibria of an economy with only collateralized borrowing features a lower asset price $\phi_e$and KM activity q, compared to an economy where the buyer can transfer the asset to the seller during decentralized trade.*

4 Other Applications and Extensions

Here, we extend our baseline model from section 2 in several directions - endogenous liquidity choice, risk, additional markets for trading assets.

4.1 Endogenous Liquidity

Limits to pledgability can also arise when making assets fully pledgable is costly. For example, a seller might need to We demonstrate this here with an extension that makes an asset's liquidity the result of an endogenous choice. For expositional simplicity, we focus on a monetary equilibrium with a single asset in fixed supply.

4.1.1 Specification I: Fixed cost

We assume that all agents have access to two technologies for enforcement of debt contracts collateralized by claims on this asset. The first is free and allows sellers to appropriate a fraction of μ_1 of the asset holdings of a defaulting debtor. The second involves incurring a fixed cost in the preceding CM, but the enables the appropriation of a greater fraction $\mu_2 > \mu_1$ of the asset upon default. The cost is agent-specific, i.e. agent $s \in [0,1]$ needs to invest $\kappa_s \in [\underline{\kappa}, \bar{\kappa}]$ to be able to take advantage of the superior enforcement technology.

Let ξ denote the (endogenous) fraction of agents who choose to invest in the μ_2-technology. Let σ denote the probability that any given agent will be a seller (and symmetrically, a buyer) in the KM. Conditional on being a seller, her type is determined by her decision to invest in the superior enforcement technology in the preceding CM. Conditional on being a buyer, χ also represents the fraction of her meetings that are of type 2, i.e. with pledgability μ_2. Then, from the buyer's perspective, the probability of a type 1 meeting, α_1, is given by $\alpha(1-\xi)$ and that of a type 2 meeting is $\alpha\xi$.

Let $\Delta(\xi)$ denote the benefit to the seller from having access to the superior enforcement technology, where we emphasize the dependence of the benefit on ξ, the fraction of sellers

with access to the μ_2 technology.

$$\Delta \equiv (z(q_2) - c(q_2)) - (z(q_1) - c(q_1))$$

where q_1, q_2 represent KM production levels at the two types of meetings.

Without loss of generality, we order agents so that κ_s is increasing in s and let $H(\cdot)$ denote the corresponding cdf, i.e. $H(s)$ is the fraction of agents with a cost less than or equal to κ_s. Then, in equilibrium, the fraction of agents who chose to invest in the μ_2 technology, denoted ξ, is given by

$$\xi = 0 \qquad \text{if} \qquad \beta\sigma\Delta < \underline{\kappa} \tag{45}$$

$$= H(s) \qquad \text{if} \qquad \beta\sigma\Delta = \kappa_s \tag{46}$$

$$= 1 \qquad \text{if} \qquad \beta\sigma\Delta > \bar{\kappa} \tag{47}$$

Define

$$T(\xi) = H(\ \max\{s : \kappa_s \leq \beta\sigma\Delta(\xi)\ \ \}\ )$$

The characterization of the equilibrium with endogenous enforcement capacity is completed by the following fixed point relationship:

$$\xi = T(\xi) \tag{48}$$

The following assumption lists conditions which guarantee that there is at least one solution to (48).

Condition 1 *In the region* $(0, q^*)$*, the function* g *satisfies both the following conditions:*

1. $g(q) - c(q)$ *is increasing.*

2. $\frac{g(q_2)}{g(q_1)} \leq \frac{c(q_2)}{c(q_1)}$ *for any* $q_1 < q_2$.

These conditions are met, for example, under proportional bargaining *a la* Kalai Kalai (1977). In this case, the seller's net surplus from a type j meeting is simply $(1-\theta)\cdot(u(q_j) -$

$c(q_j))$, where θ, a parameter, indexes the share of the total surplus appropriated by the buyer. The next lemma establishes monotonicity of Δ when Assumption 1 holds.

Lemma 1 *$\Delta(\xi)$ is increasing.*

Proof. In the Appendix. ■

This result allows us to apply Tarski's fixed point theorem to (48) and thus, ensures existence of an equilibrium. Note that, in general, there can be multiple solutions to (48). In particular, depending on the support of κ_s, we can have equilibria where no one invests in the μ_2-technology (i.e. $\xi = 0$) or everyone does ($\xi = 1$). There could be multiple interior fixed points as well.

4.1.2 Specification II: Continuous cost

Here, we consider an alternative technology for investing in pledgability. Instead of a fixed cost, the ex-ante investment required is now described by a continuous function, $\Gamma(\mu)$, increasing and convex. Note that the marginal value to a buyer of increasing μ is simply $\psi + \delta$. Then, an equilibrium with an interior choice for μ is characterized by the following fixed point problem:

$$\Gamma'(\mu) = \beta(\phi_e(\mu) + \gamma)i \tag{49}$$

where we make explicit the dependence of asset prices on μ. Note that, in a monetary equilibrium, asset prices are strictly increasing in the degree of pledgability. A sufficient condition for the existence of an equilibrium with $\mu > 0$ is

$$\Gamma'(0) < \beta(\frac{\gamma}{r} + \gamma)i$$

As with the fixed cost specification, we can have multiple equilibria.

4.2 Liquidity Risk

In this subsection, we extend the deterministic model by introducing risk along two important dimensions. The first extension makes asset dividends stochastic, while the second introduces heterogeneity and uncertainty about meetings. We show that liquidity risk is an independent source of variation in returns across assets. Intuitively, assets which provide liquidity in states when it is needed the most command a higher premium (and therefore, earn a lower rate of return).

4.2.1 Stochastic Dividends

Suppose the dividend stream γ is an iid random variable, with mean $\bar{\delta}$. The realization of this variable is known during KM trade. Then, in a stationary monetary equilibrium, the average return on an asset are given by

$$1 + r_a \equiv \frac{\phi_a + \bar{\gamma_a}}{\phi_a} = \frac{1+r}{1+i\mu_a} - \frac{\mu_a \text{Cov}\left(\lambda_q, \frac{\gamma_a}{\phi_a}\right)}{1+i\mu_a} \tag{50}$$

The first term is the standard liquidity adjustment from the deterministic case studied earlier. The second term is an adjustment for liquidity risk. The asset earns a lower return if it has higher dividend payouts in states when liquidity is most valuable (λ_q is high).

We can also use (50) to analyze return differentials in an environment with multiple assets. Consider an extension of the benchmark model with 2 assets. The first asset, denoted b, is 'riskless', i.e. pays off the same dividend in every state and is fully pledgable, i.e. $\mu_b = 1$. The second, denoted e, has a risky dividend stream and is illiquid, i.e. $\mu_e < 1$. Then, the excess return earned by the risky asset consists of both a liquidity premium and a liquidity *risk* premium, as the following decomposition reveals:

$$r^e - r^b = \underbrace{\frac{(1+r)i}{1+i}\left(\frac{1-\mu_e}{1+i\mu_e}\right)}_{\text{Liquidity Premium}} - \underbrace{\frac{\mu_e \text{Cov}\left(\lambda_0, \frac{\gamma_e}{\phi_e}\right)}{1+i\mu_e}}_{\text{Liquidity } \textit{Risk} \text{ Premium}}$$

4.2.2 Risky Meetings

Now suppose there are 2 types of meetings and 2 types of assets. Both assets have the same average liquidity i.e. $\mathbb{E}\mu_A^s = \mathbb{E}\mu_B^s$, where the expectation is taken over meetings. But, asset A has the same liquidity in both meetings, i.e. $\mu_A^1 = \mu_A^2$ while asset B has higher liquidity in one of the meetings, i.e. is 'risky'. Then, it trades at a lower liquidity premium (or equivalently, its accounting rate of return is lower),

$$r_B \geq r_A$$

5 Conclusion

TO BE ADDED.

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Appendix A Numerical Results

The following graphs plot various endogenous variables as function of exogenous variables, given the parameterization strategy in Section **??**.

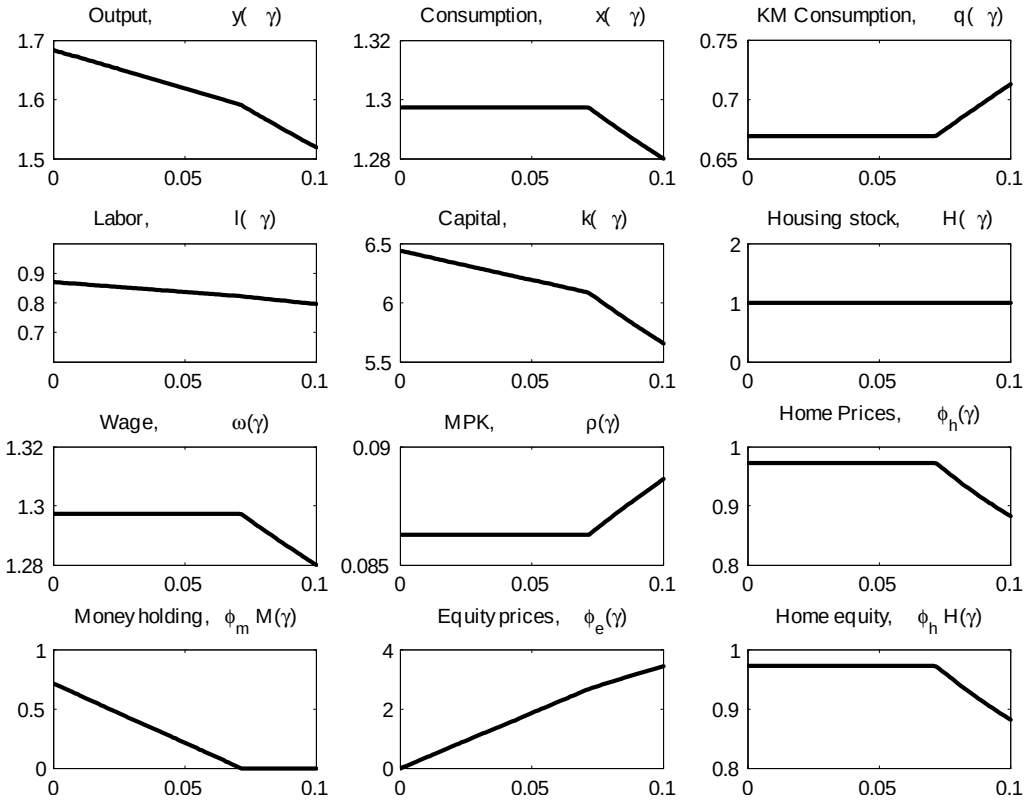


Figure 3: Effect of equity dividend, γ

In a non-monetary equilibrium, higher equity dividend leads to higher equity values (and therefore, more liquidity), leading to lower investment and home prices. In a monetary equilbirium, however, factor and asset prices are pinned down by policy, leaving consumption in both markets unchanged (in the AD market, through the household labor-leisure choice)

An exogenous increase in the supply of homes lower their relative price and therefore, the amount of liquidity they provide. As before, whether this leads to higher liquidity premia on comepting providers of liquidity depends on whether we are in Case 2 or 3.

The effects of higher equity pledgability on other assets are similar to the housing pledgability case discussed in the text. Notice the non-monotonic effects on stock market valuations.

The non-monotonicity referred to earlier (in the discussion related to μ_e and μ_h) shows up here on quantities (because the relative price of capital is fixed, by assumption, at 1). Also, for sufficiently high μ_k, the available liquidity is more than enough to purchase q^*, eliminating all liquidity premia.

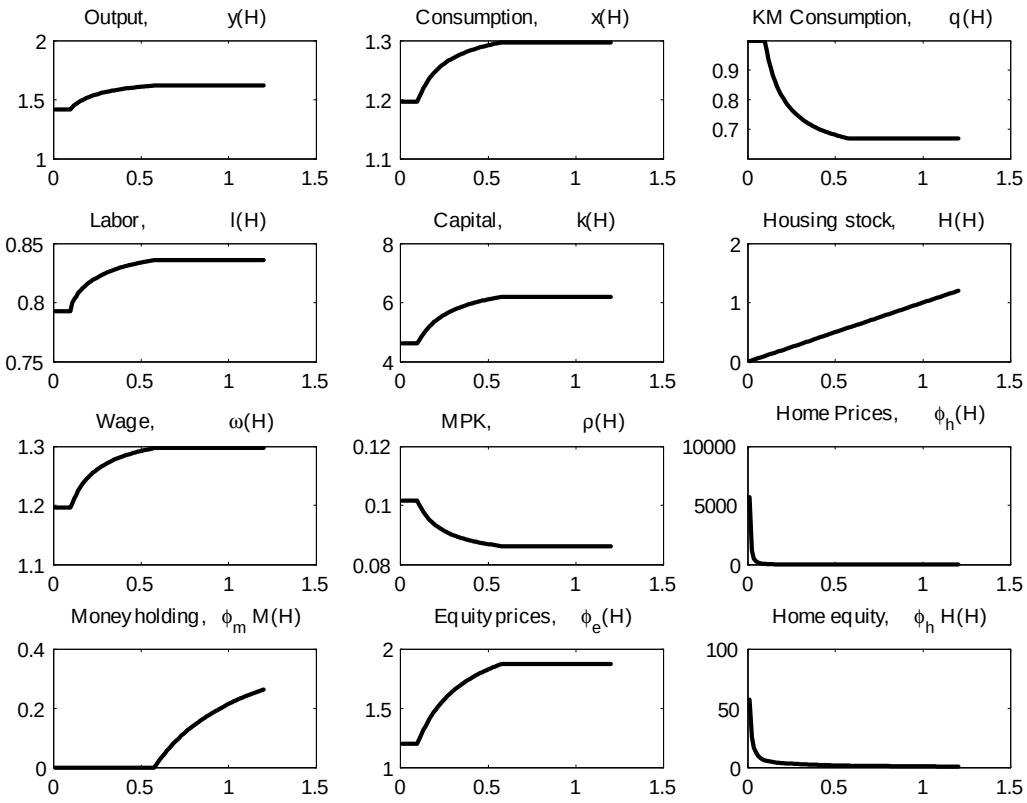


Figure 4: Effect of housing stock, H

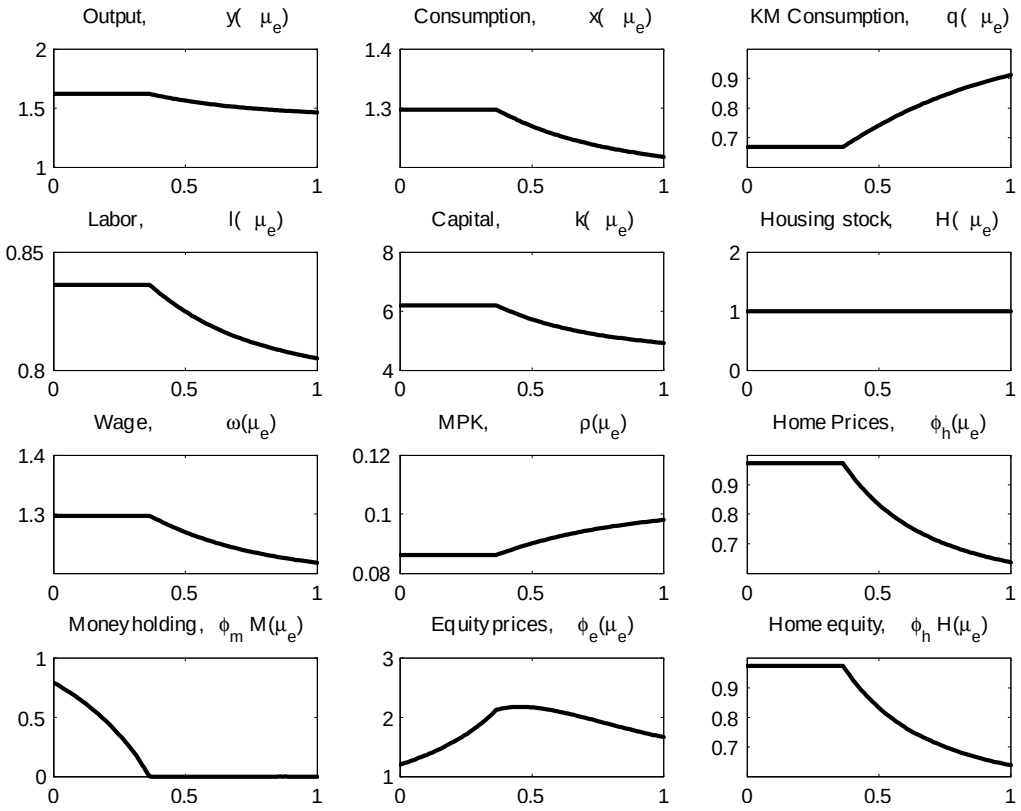


Figure 5: Effect of equity pledgability

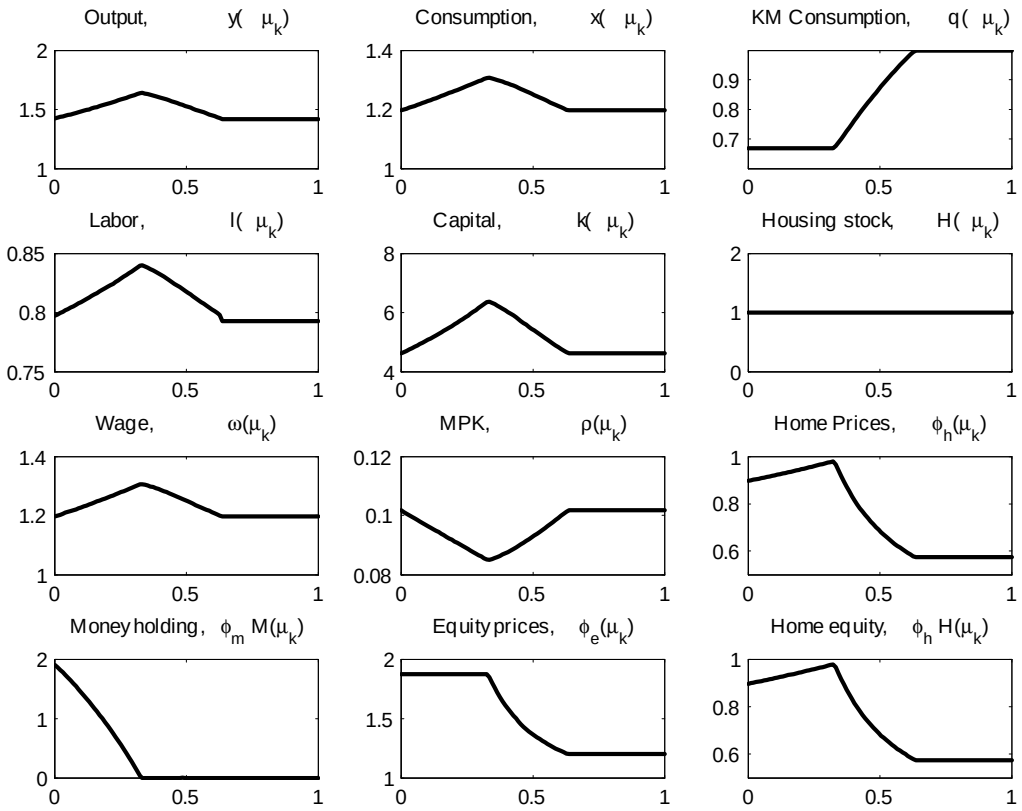


Figure 6: Effect of capital pledgability, μ_k

Appendix B Pledgability with moral hazard

This section studies an alternative source of moral hazard, one that arises when assets require maintenance to retain their full value. In such an environment, a debtor might choose to neglect to care for an asset that is pledged to a creditor, preferring to forfeit it instead of expending resources on its maintenance. We will show the inability of debtors to commit to maintaining collateral leads to endogenous limits on the extent of liquidity provided by assets. The form of these limits is very similar to the ones that result from the threat of diversion, but the nature of the moral hazard problem here leads to additional effects in response to policy changes.

For simplicity, we focus on a case without physical capital, i.e. there are only 2 assets - the Lucas tree and money. The key feature is that the trees have to be maintained between periods i.e. *after* trading - both centralized and decentralized - ends. Agents in possession of the tree choose a maintenance level $n \in (0,1)$ at a utility cost (later, we consider a specifcation with resource costs) per tree of χn. A tree maintained at level n is equivalent to a fraction n of a fully-maintained asset (i.e. one with $n = 1$) and so yields a dividend of $n\gamma$ and trades at a price $n\phi'_e$ in the following AD market.

As before, in the KM, an agent can use the asset as collateral, but, importantly, she cannot commit to a maintenance level. If she defaults on the obligation, the creditor seizes the asset, but has no other recourse.

Consider the maintenance choice problem of an agent whose tree holdings are given by e and has an obligation D, denominated in the AD consumption good, collateralized by her asset holdings.

$$\max_n \quad \left\{ \max \quad \frac{1}{\omega'} \left[\, n(\gamma + \phi'_e)e - D, \; 0 \, \right] - n\chi e \right\}$$

Again, we restrict attention to stationary equilibria, which requires that the effective supply of the asset stays constant over time. A necessary condition is $\frac{1}{\omega'}(\gamma + \phi'_e) > \chi$. If this holds, conditional on deciding to maintain the asset, the agent will choose the maximum level $n = 1$. The optimal policy is then characterized by

$$\begin{aligned} n^* &= 1 \qquad \text{if} \qquad \frac{1}{\omega'} D \leq \frac{1}{\omega'} e(\gamma + \phi'_e) - e\chi \\ &= 0 \qquad \text{otherwise} \end{aligned}$$

Let μ be defined by

$$\chi = \frac{B}{\omega'}(\gamma + \phi'_e)(1 - \mu) \qquad \Rightarrow \qquad \mu = 1 - \frac{\chi \omega'}{\gamma + \phi'_e} \, . \tag{51}$$

If $n^* = 0$, the asset is worthless and the creditor gets nothing. Therefore, the limit on the amount that can be credibly promised with a units of the asset as collateral is

$$D_e(e) = \mu e$$

Thus, the inability to commit to maintenance leads to a pledgability restriction to the portfolio problem of the buyer (which is formally stated in following subsection). Note that

μ is taken as given by both the buyer and the seller, but it is an equilibrium object (and therefore, not immune to policy).

The Portfolio Choice Problem

The buyer's problem is given by

$$\max_{\hat{e},\hat{m},q,d_e,d_m} \quad -\frac{1}{\omega}\phi_e\hat{e} - \frac{1}{\omega}\phi\hat{m} - \beta\chi\hat{e} + \beta\sigma[u(q) - z(q)] + \beta\frac{1}{\omega'}\left[(\gamma + \phi'_e)\hat{e} + \phi'_m\hat{m}\right]$$

subject to

$$\begin{aligned} z(q) &= \frac{1}{\omega'}\left[(\gamma + \phi'_e)d_e + \phi' d_m\right] \\ \frac{1}{\omega'}(\gamma + \phi'_e)d_e &\leq \frac{1}{\omega'}(\gamma + \phi'_e)\mu\hat{e} \\ \frac{1}{\omega'}\phi' d_m &\leq \frac{1}{\omega'}\phi'\hat{m} \end{aligned}$$

This yields the following optimality conditions:

$$\lambda_0 = \beta\alpha\left(\frac{u'(q)}{g'(q)} - 1\right) \tag{52}$$

$$\lambda_a = \lambda_0 \tag{53}$$

$$\lambda_m \geq \lambda_0 \qquad \text{with equality if} \quad d_m > 0 \tag{54}$$

$$\frac{1}{\omega}\phi_e = \beta\mu\frac{1}{\omega'}(\gamma + \phi'_e)\left[1 + \sigma\left(\frac{u'(q)}{z'(q)} - 1\right)\right] \tag{55}$$

$$\frac{1}{\omega}\phi \geq \beta\frac{1}{\omega'}\phi'\left[1 + \sigma\left(\frac{u'(q)}{z'(q)} - 1\right)\right] \qquad \text{with equality if} \quad \hat{m} > 0 \tag{56}$$

Types of equilibria

Case I: (Liquid) If the asset does not facilitate exchange at the margin, then

$$\begin{aligned} &\frac{1}{\omega}\phi_e + \beta\chi = \beta\frac{1}{\omega}(\gamma + \phi_e) \\ \Rightarrow \qquad &\phi_e = \beta\frac{\frac{1}{\omega}\gamma - \chi}{\frac{1}{\omega}(1-\beta)} = \frac{\beta\mu\gamma}{1 - \beta\mu} \\ \Rightarrow \qquad &\mu = \frac{\frac{1}{\omega}\gamma - \chi}{\frac{1}{\omega}\gamma - \beta\chi} \end{aligned}$$

where μ is obtained by plugging the asset price in (51).

In this equilibrium, asset returns are

$$1 + r_a \equiv \frac{\gamma + \phi_e}{\phi_e} = \frac{1}{\beta\mu}$$

i.e. the measured rate of return on the asset includes an adjustment for the cost of maintaining the asset.

Case II: (Illiquid non-monetary) The expressions for asset prices and returns are the solution to the following system of equations:

$$\begin{aligned}
\phi_e &= \tilde{\beta}\frac{\frac{1}{\omega}\gamma - \chi}{\frac{1}{\omega}(1-\beta)} = \frac{\tilde{\beta}\mu\gamma}{1-\tilde{\beta}\mu} \\
\mu &= \frac{\frac{1}{\omega}\gamma - \chi}{\frac{1}{\omega}\gamma - \tilde{\beta}\chi} \\
r_e &= \frac{1}{\tilde{\beta}\mu} - 1 \\
z(q) &= \frac{1}{\omega}\left[(\gamma + \phi_e)\mu\right] \\
\tilde{\beta} &= \beta\left\{1 + \sigma\left[\frac{u'(q)}{z'(q)} - 1\right]\right\}
\end{aligned}$$

Thus, insufficient liquidity leads to an upward adjustment to the discount factor (i.e. $\tilde{\beta} > \beta$). Note that we need this adjusted discount factor $\tilde{\beta}$ to be less than one (i.e. the shortfall in liquidity cannot be too severe) for μ to be well-defined.

Case III: (Monetary) Now, the nominal interest rate pins down the level of activity in the KM

$$i = \sigma\left[\frac{u'(q)}{z'(q)} - 1\right] \tag{57}$$

The expressions for asset prices and returns take the same form as in Case II,

$$\begin{aligned}
\phi_e &= \tilde{\beta}\frac{\frac{1}{\omega}\gamma - \chi}{\frac{1}{\omega}(1-\beta)} = \frac{\tilde{\beta}\mu\gamma}{1-\tilde{\beta}\mu} \\
\mu &= \frac{\frac{1}{\omega}\gamma - \chi}{\frac{1}{\omega}\gamma - \tilde{\beta}\chi} \\
r_e &= \frac{1}{\tilde{\beta}\mu} - 1 \\
\tilde{\beta} &= \beta(1+i)
\end{aligned}$$

It is easy to see that μ increases[19] with $\tilde{\beta}$ and therefore, with i. Intuitively, tighter monetary policy increases asset prices and therefore, makes it easier to provide the right incentives for maintenance. This in turn allows a greater degree of the asset's value to be pledged credibly. This effect on pledgability amplifies the response of asset prices and returns to monetary policy (i.e. relative to a world where μ is invariant to policy as in Section **??**).

Proposition 2 *(The Fisher effect:) In a stationary monetary equilibrium, an increase in i reduces the measured accounting rate of return, r_e.*

[19] This need not be true if capital also provides liquidity value. In that case, an increase in i also raises the capital-labor ratio (and therefore the steady-state ω), which has a negative effect on μ. This effect is absent if the maintenance cost is denominated in terms of consumption goods.

B.1 Moral Hazard with Resource Cost of Maintenance

Suppose maintenance requires χ units of the consumption good. The analysis is very similar, except that μ is now given by

$$\mu = 1 - \frac{\chi}{(\delta + \psi')}$$

i.e. the degree of pledgability no longer depends on the marginal utility of consumption. This specification might make it easier to deal with liquid productive capital. If the maintenance cost is in labor units, then many results will depend on the effect on the marginal rate of substitution between consumption and leisure.

B.2 Equilibria under ex-ante heterogeneity:

Here, we characterize the complete set of stationary monetary equilibria when the types of agents is commonly known. In addition to the case considered in the main text, there are two other types of equilibria:

Case II: (Type-2 agents hold only money and type-1 agents only assets)

Now, we have 2 levels of KM production: q^1 for meetings with type-1 buyers and q^2 for those with type-2 buyers. The latter is determined as usual by (57). The former is given by the solution of the system of equations that characterize the illiquid monetary equilibrium above, with a per-buyer asset supply of $\frac{A}{\kappa}$ (instead of A). For this to be a valid equilibrium, type-2 (resp. type-1) agents should not have any incentive to hold assets (resp. money), i.e.

$$\begin{aligned} \frac{1}{\omega}\psi &\geq \beta\mu^1(\gamma+\phi_e)\left[(1-\sigma)\frac{\mu^2}{\mu^1} + \sigma\frac{u'(q)}{z'(q)}\left(\kappa + (1-\kappa)\frac{\mu^2}{\mu^1}\right)\right] \\ i &\geq \sigma\left[\frac{u'(q^1)}{z'(q^1)} - 1\right] \end{aligned}$$

Combined with the optimality conditions for asset and money holdings, the second condition implies that $q^1 \geq q^2$, while the first one says that q^1 cannot be too high relative to q^2. To see this, note that the first condition can be written as

$$1 + \sigma\left(\frac{u'(q^1)}{z'(q^1)} - 1\right) \geq \frac{\mu^2}{\mu^1} + \sigma\left(\kappa + (1-\kappa)\frac{\mu^2}{\mu^1}\right)\left(\frac{u'(q^2)}{z'(q^2)} - 1\right)$$

This bounds, from above, the shadow value of liquidity for type-2 agents $\left(\frac{u'(q^2)}{z'(q^2)} - 1\right)$ relative to that of type-1 agents$\left(\frac{u'(q^1)}{z'(q^1)} - 1\right)$.

Case III (Both types hold assets, only type-2 hold money)

In this case, the equilibrium is characterized by the solution to the following system of equations:

$$\begin{aligned}
i &= \sigma\left[\frac{u'(q^2)}{z'(q^2)}-1\right] \\
\frac{1}{\omega}\phi_e &= \beta\mu^1(\gamma+\phi_e)\left[(1-\sigma)\frac{\mu^2}{\mu^1}+\sigma\frac{u'(q)}{z'(q)}\left(\kappa+(1-\kappa)\frac{\mu^2}{\mu^1}\right)\right] \\
\frac{1}{\omega}\phi_e &= \beta\mu^1\frac{1}{\omega}(\gamma+\phi_e)\left[1+\sigma\left(\frac{u'(q^1)}{z'(q^1)}-1\right)\right] \\
\mu^1 &= 1-\frac{\omega\chi^1}{(\gamma+\phi_e)} \\
\mu^2 &= 1-\frac{\omega\chi^2}{(\gamma+\phi_e)}
\end{aligned}$$

The asset and real balance holdings are then given by plugging in quantities and prices from the above system into the following and solving for $\{\hat{a}^1, \hat{a}^2, \hat{m}^2\}$.

$$\begin{aligned}
z(q^1) &= \frac{1}{\omega'}(\gamma+\phi'_e)\hat{a}^1\mu^1 \\
z(q^2) &= \hat{m}^2+\frac{1}{\omega'}(\gamma+\phi'_e)\hat{a}^2\mu^2 \\
1 &= \hat{a}^1\kappa+\hat{a}^2(1-\kappa)
\end{aligned}$$

As with Case II, we need to make sure that type-1 agents do not have any incentive to hold money, i.e.

$$i \;\geq\; \alpha\left[\frac{u'(q^1)}{z'(q^1)}-1\right]$$

Appendix C Proof of Lemma 1

We consider 3 cases

- Case I: $q_1 = q_2 = q^*$
- Case II: $q_1 < q_2 = q^*$
- Case III: $q_1 < q_2 < q^*$

Obviously, in case I, $\Delta = 0$ for all ξ. In case II, note that

$$z(q_1) = \frac{\delta(1+r)\mu_1}{r-\beta\sigma(1-\xi)\tilde{L}(q_1)\mu_1}$$

where $\tilde{L}(q) = \frac{u'(q)}{z'(q)}-1$. Then, q_1 decreases when more sellers have the superior enforcement technology. Suppose otherwise, i.e. $q_1 \uparrow$ Then, $z(q_1) \uparrow \Rightarrow \frac{\delta(1+r)\mu_1}{r-\beta\sigma(1-\xi)\tilde{L}(q_1)\mu_1} \uparrow \Rightarrow \beta\sigma(1-$

$\xi)\tilde{L}(q_1)\mu_1 \uparrow \Rightarrow \tilde{L}(q_1) \uparrow \Rightarrow q_1 \downarrow$, contradicting the original claim. Therefore, an increase in ξ leads to an decrease in q_1. Since $z(q) - c(q)$ is increasing by assumption, it is easy to see that $\Delta = (z(q^*) - c(q^*)) - (z(q_1) - c(q_1))$ rises with ξ.

In case III, the quantities q_1 and q_2 are the solution to the following system of equations

$$i = \alpha(1-\xi)\tilde{L}(q_1) + \alpha\xi\tilde{L}(q_2) \tag{58}$$

$$z(q_2) - z(q_1) = \frac{\delta(1+r)(\mu_2 - \mu_1)}{r - \sigma(1-\chi)\tilde{L}(q_1)\mu_1 - \sigma\xi\tilde{L}(q_2)\mu_2} \tag{59}$$

where $\tilde{L}(q) = \frac{u'(q)}{z'(q)} - 1$. Taking the derivative with respect to χ, and rewriting in matrix notation,

$$\begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \begin{pmatrix} \frac{\partial q_1}{\partial \xi} \\ \frac{\partial q_2}{\partial \xi} \end{pmatrix} = \begin{pmatrix} \tilde{L}(q_1) - \tilde{L}(q_2) \\ \frac{z(q_2)-z(q_1)}{r - \mathbb{E}\mu\tilde{\lambda}} \left(\tilde{L}'(q_1)\mu_1 - \tilde{L}'(q_2)\mu_2 \right) \end{pmatrix}$$

where

$$\begin{aligned}
\Omega_{11} &= (1-\chi)\tilde{L}'(q_1)\sigma \\
\Omega_{12} &= \chi\tilde{L}'(q_2)\sigma \\
\Omega_{21} &= z'(q_1) + \frac{z(q_2) - z(q_1)}{r - \mathbb{E}\mu\tilde{L}}\sigma(1-\xi)\tilde{L}'(q_1)\mu_1 \\
\Omega_{22} &= -z'(q_2) + \frac{z(q_2) - z(q_1)}{r - \mathbb{E}\mu\tilde{L}}\alpha\xi\tilde{L}'(q_2)\mu_2
\end{aligned}$$

It is straightforward to show that the determinant $\mathbb{D}$ of the coefficient matrix on the left-hand side is positive. Solving,

$$\mathbb{D}\ \frac{\partial q_1}{\partial \xi} = -z'(q_2)\left(\tilde{L}(q_1) - \tilde{L}(q_2)\right) + \frac{z(q_2) - z(q_1)}{r - \mathbb{E}\mu\tilde{L}}\sigma\xi\tilde{L}'(q_2)\tilde{L}(q_1)(\mu_2 - \mu_1) \tag{60}$$

$$\mathbb{D}\ \frac{\partial q_2}{\partial \xi} = -z'(q_1)\left(\tilde{L}(q_1) - \tilde{L}(q_2)\right) - \frac{z(q_2) - z(q_1)}{r - \mathbb{E}\mu\tilde{L}}\sigma(1-\xi)\tilde{L}'(q_1)\tilde{L}(q_2)(\mu_2 - \mu_1) \tag{61}$$

Next, we note that

$$\frac{\partial \Delta}{\partial \xi} = (z'(q_2) - c'(q_2))\frac{\partial q_2}{\partial \xi} - (z'(q_1) - c'(q_1))\frac{\partial q_1}{\partial \xi}$$

Substituting,

$$\frac{\partial \Delta}{\partial \xi} = \left(z'(q_1)c'(q_2) - z'(q_2)c'(q_1)\right)\ \left(\tilde{L}(q_1) - \tilde{L}(q_2)\right) \tag{62}$$

$$- (z'(q_2) - c'(q_2))\frac{z(q_2) - z(q_1)}{r - \mathbb{E}\mu\tilde{L}}\sigma(1-\xi)\tilde{L}'(q_1)\tilde{L}(q_2)(\mu_2 - \mu_1) \tag{63}$$

$$- (z'(q_1) - c'(q_1))\frac{z(q_2) - z(q_1)}{r - \mathbb{E}\mu\tilde{L}}\sigma\xi\tilde{L}'(q_2)\tilde{L}(q_1)(\mu_2 - \mu_1) \tag{64}$$

Now, $\tilde{L}'(q) < 0$. Also, by the first statement in Assumption 1, the seller's surplus is increasing in q, i.e. $z'(q) - c'(q) > 0$, making the second and third terms positive. The second statement of Assumption 1 makes the first term positive as well, completing the proof.