Household Risk Management*

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Abstract

Households risk management, that is, households' insurance against adverse shocks to income, assets, and financing needs, is limited and often completely absent, in particular for poor households. We explain the limited extent and absence of risk management in an intertemporal model in which households have access to complete markets subject to collateral constraints and in which the financing needs for consumption and durable goods purchases override the risk management concerns. The absence of many markets for claims which allow household risk management is consistent with our model and should not be considered a puzzle. Risk management of the price risk of durable goods moreover depends on the sign of the hedging demand and on whether the household owns or rents the durable goods.

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1 Introduction

We argue that the absence of household risk management is due to the fact that households' financing needs exceed their hedging concerns. We provide a standard neoclassical model in which households' ability to promise to pay is limited by the need to collateralize such promises. Collateral constraints hence restrict both financing as well as risk management as both require households to issue promises to pay. Given this link, households limit their risk management and may completely abstain from hedging when financing needs are sufficiently strong. Thus, the absence of household risk management and the lack of markets that provide such insurance should not be considered a puzzle.

Households' primary financing needs are two: purchases of durable goods and the accumulation of human capital. First, households consume the services of durable goods, most importantly housing, and the purchase of such goods needs to be financed. Second, investment in education requires financing, and education and learning-by-doing imply an age-income profile which is upward sloping on average. The bulk of financing actually extended to households is for purchases of durable goods. Indeed, more than 90% of household liabilities are attributable to durable goods purchases, mainly real estate (around 80%) and vehicles (around 6%), and less than 4% of household liabilities are attributable to education purposes.[1] We study a model in which all household borrowing needs to be collateralized by households' stocks of durable goods. Since most household financing is comprised of such loans, our model is plausible empirically. While households are able to borrow for education only to a very limited extent, consistent with our model, education and learning-by-doing are nevertheless important as they result in age-income profiles which are upward sloping on average which means that households have an incentive to borrow against the future using other means, namely, by financing durable goods.

Shiller (1993) has argued that markets which allow households to manage their risks would significantly improve welfare and that the absence of such markets hence presents an important puzzle. For example, Shiller (2008) writes that "[t]he near absence of derivatives markets for real estate ... is a striking anomaly that cries out for explanation

[1] In the first quarter of 2009, data from the *Flow of Funds Accounts of the U.S.* suggests that home mortgages are 78% of household liabilities and consumer credit is about 19% and, according to the *Federal Reserve Statistical Release G.19*, 12% is non-revolving consumer credit (which includes automobile loans as well as non-revolving loans for mobile homes, boats, trailers, education, or vacations). Data from the *2007 Survey of Consumer Finances* on the purpose of debt suggests that in 2007, about 83% of household debt is due to the purchase or improvement of a primary residence or other residential property, about 6% is due to vehicle purchases, less than 4% is due to education, and about 6% is due to the purchase of goods or services which is not further broken out.

and for actions to change the situation." We provide a rationale why households may not use such markets even if they exist. And given this lack of demand from households, the absence of such markets may not be so puzzling after all. The explanation we provide is simple: households' primary concern is financing, that is, shifting funds from the future to today, not risk management, that is, not transferring funds across states in the future. Risk management would require households to make promises to pay in high income states in the future, but this would reduce households' ability to promise to pay in high income states to finance durable goods purchases today, because households' total promises are limited by collateral constraints. Our dynamic model of complete markets subject to collateral constraints allows an explicit analysis of the connection between financing and risk management, and shows that the cost of risk management may be too high.

Consistent with the view that financing needs may override risk management concerns, we discuss evidence on U.S. households which suggests that poor (and financially constrained) households are less well insured against many types of risks, such as health risks or flood risks, than richer (and less financially constrained) households. Furthermore, a similar positive relation between income and risk management has recently been documented for farmers in developing economies, and we summarize the pertinent evidence. In addition, there is evidence that firms' financial constraints affect corporate risk management. Similar to the household finance context analyzed in this paper, Rampini and Viswanathan (2010, 2011) argue that firms' financing needs may override their hedging concerns and thus severely constrained firms may abstain from risk management. This is in contrast to received theory, formalized by Froot, Scharfstein, and Stein (1993), which suggests that constrained firms should hedge. The extant results in the literature do not take into account firms' financing needs and this literature hence reaches a rather different conclusion. One important consequence of the absence of risk management by constrained households and firms is that such households and firms are then more susceptible to shocks.

Section 2 reviews the evidence on household and firm risk management. Section 3 provides the model of household risk management and shows that financing needs for durable goods and education may override hedging concerns. Durable goods price risk management is analyzed in Section 4, which shows that while the direct effect of durable goods prices on net worth is straightforward, the hedging demand effect depends on preferences and may in fact reduce the need for risk management. Section 5 considers households' ability to rent durable goods and the interaction between the rent vs. buy decision and risk management.[2] Section 6 concludes.

[2]The asset pricing implications of housing have recently been considered by Lustig and Van Nieuwer-

2 Stylized Facts on Household Risk Management

In this section we briefly survey evidence of what we consider a stylized fact, namely that poor (and more financially constrained) households are less well insured than richer (and less financially constrained) households. Indeed, we think this is part of a much broader pattern applying to entrepreneurial households and firms as well, and we briefly discuss evidence on risk management by Indian farmers and U.S. corporations suggesting that financial constraints reduce risk management substantially.

Among U.S. households, health insurance coverage varies considerably with income and age according to data from the U.S. Census Bureau. Panel A of Table 1 reports the percentage of people without health insurance in the U.S., which varies from 25% of people with income less than $25,000 to 8% of people with income exceeding $75,000. Similarly, Panel B of Table 1 reports variation by age; for adults, the fraction without health insurance decreases from 28% and 26% for age group 18-24 and age group 25-34, respectively, to 14% for age group 45-64, and to less than 2% for age group 65 and up. Brown and Finkelstein (2007) report that participation in long-term care insurance by individuals aged 60 and over also varies substantially by wealth, increasing from about 3% for the bottom wealth quartile to about 20% for the top quartile in U.S. data, as reported in Panel C of Table 1. Browne and Hoyt (2000) find that flood insurance coverage, both in terms of the number of policies per capita and the amount of coverage per capita, is positively correlated with disposable personal income per capita using U.S. state level data. Clearly, the extent to which households are insured hence varies substantially with households' income. And, assuming that individuals in age group 18-24 and age group 25-34 are more financially constrained, households' insurance level also seems to vary with financial constraints. This evidence is consistent with the view that there is an important connection between household risk management and households' financial constraints. That said, there are certainly other reasons why insurance participation varies with income, such as crowding out of private insurance by public programs, as stressed, for example, by Brown and Finkelstein (2008).

Among farmers in rural India, Giné, Townsend, and Vickery (2008) find that participation in rainfall insurance programs increases in wealth and decreases with measures of borrowing constraints. Cole, Giné, Tobacman, Topalova, Townsend, and Vickery (2009)

burgh (2005) and Piazzesi, Schneider, and Tuzel (2007) in economies with similar preferences over two goods, (nondurable) consumption and housing services. Both studies consider a frictionless rental market for housing unlike us, which reduces households' financing needs substantially. Lustig and Van Nieuwerburgh (2005) consider the role of solvency constraints similar to ours and Piazzesi, Schneider, and Tuzel (2007) study the frictionless benchmark.

provide evidence on the importance of credit constraints for the adoption of rainfall insurance using randomized field experiments in rural India. Farmers who are randomly surprised with a positive liquidity shock are much more likely to buy insurance. Moreover, the authors report that the most frequently stated reason for not purchasing insurance is "insufficient funds to buy insurance." Farmers might of course use other risk sharing mechanisms, including informal ones. To overcome the limitation of analyzing specific risk sharing mechanisms in isolation, Townsend (1994) studies data on Indian farmers' household consumption directly in a seminal paper. Townsend finds that, while the full insurance model provides a remarkably good benchmark, "[t]here is evidence that the landless are less well insured than their village neighbors in one of the three villages." That is, there is "a hint of a pattern by land class. Specifically, the landless and small farmers in Aurepalle and the small and medium farmers in Shirapur seem more vulnerable."

For U.S. corporations, Nance, Smith, and Smithson (1993) find that firms which do not hedge are smaller and pay lower dividends in survey data for large industrial firms. Similarly, Géczy, Minton, and Schrand (1997) find a strong positive relation between derivatives use and firm size among large U.S. firms.[3] The evidence on the relation between corporate risk management and other financial variables is more mixed (see, e.g., Tufano (1996) as well as the aforementioned studies).

3 A Model of Household Risk Management

This section first describes our model of household finance, then shows that for households with sufficiently low net worth financing needs override hedging concerns, and finally considers the additional financing needs for education purposes.

3.1 Household Finance with Durable Goods

We consider an environment with three dates, 0, 1, and 2, and S states at time 1, $s \in \mathcal{S}$ with probability $\pi(s) > 0$, and no further uncertainty. There are two goods in the economy, (non-durable) consumption and housing, which we use as the stand in for all durable goods. Households are risk averse and have separable preferences over

[3]Approximately 41% of the firms with exposure to foreign currency risk in their data use currency derivatives and 59% use any type of derivative. Across firm size quartiles, currency derivative use increases from 17% for the smallest quartile to 75% for the largest quartile and the use of any derivatives increases from 33% to 90%; see their table 2.

consumption and housing described by

$$E\left[\sum_{t=0}^{2}\beta^t\left\{u(c_t)+v(h_t)\right\}\right] \tag{1}$$

where $c_t(s)$ and $h_t(s)$ are consumption and housing at time t in state s, respectively, where $u(c)=\frac{1}{1-\gamma}c^{1-\gamma}$ and $v(h)=\psi\frac{1}{1-\gamma}h^{1-\gamma}$ with $\gamma>0$, and $\beta<1$ is the rate of time preference. Households have labor income e_0 at time 0 and $e_t(s)$ at time t in state s. At time 0, households moreover are endowed with an amount of housing h_0.

Housing is a durable good, depreciates at the rate $\delta\in(0,1)$, and has a stochastic price $q_t(s)$ which we take as exogenous. Households can adjust the amount of housing freely, but there is no rental market for housing and households have to purchase housing to consume housing services. Housing is also used as collateral. Households have access to complete (and competitive) markets in one period ahead state-contingent claims subject to collateral constraints. The returns on these state-contingent claims $R_t(s)$ are determined by the pricing kernel of a representative investor, which we take as given. Households can thus issue state-contingent promises to pay $R_1(s)b_1(s)$ at time 1 in state s and receive an amount of $\pi(s)b_1(s)$ at time 0; similarly, they can issue promises to pay $R_2(s)b_2(s)$ at time 2 in state s and receive an amount of $b_2(s)$ at time 1 in state s. These state-contingent promises can potentially be negative, which means that households can buy contingent claims. Households' promises to pay are subject to the collateral constraints

$$\theta q_1(s)h_1(1-\delta) \geq R_1(s)b_1(s), \tag{2}$$

$$\theta q_2(s)h_2(s)(1-\delta) \geq R_2(s)b_2(s), \tag{3}$$

which require that fraction $\theta\in(0,1)$ times the resale value of housing exceeds the total promised payment, for all t and s. Rampini and Viswanathan (2010) derive such collateral constraints from a model with dynamic limited enforcement in which borrowers can abscond with all income and fraction $1-\theta$ of collateral and cannot be excluded from borrowing in the future.

The only friction we add to the standard neoclassical environment is that claims need to be collateralized to enforce repayment. Moreover, we assume that there is no rental market for capital for now, but do consider households' ability to rent in Section 5. Importantly, our environment is one with full information. Thus, households are able to trade contingent claims on all states of nature, which allows them to engage in risk management.

The household chooses consumption c_0 and $c_t(s)$, housing h_1 and $h_2(s)$, and state-contingent borrowing $b_t(s)$, for all $s\in\mathcal{S}$ and $t=1,2$, to maximize its expected utility

given by (1) subject to the collateral constraints (2) and (3) and the budget constraints

$$\begin{aligned} w_0 + \sum_{s \in \mathcal{S}} \pi(s) b_1(s) &\geq c_0 + q_0 h_1 && (4) \\ e_1(s) + q_1(s) h_1 (1-\delta) + b_2(s) &\geq c_1(s) + q_1(s) h_2(s) + R_1(s) b_1(s) && (5) \\ e_2(s) + q_1(s) h_2(s) (1-\delta) &\geq c_2(s) + R_2(s) b_2(s), && (6) \end{aligned}$$

where the household's net worth at time 0 is $w_0 \equiv e_0 + q_0 h_0 (1-\delta) - R_0 b_0$, where initial debt b_0 is taken as given. Note that there is no need to impose non-negativity constraints on consumption and housing as these would be slack given our preference assumptions. The household maximizes a strictly concave objective subject to a convex constraint set. Hence, the problem's first order conditions are necessary and sufficient for optimality and there exists a unique solution. Moreover, the value function induced by the problem is strictly increasing and strictly concave.

3.2 Financing Needs Override Risk Management Concerns

We now show that if a household's financing needs are sufficiently strong, then financing needs override hedging concerns. To see this we define a household's net worth at time 1 in state s by $w_1(s) \equiv e_1(s) + q_1(s) h_1 (1-\delta) - R_1(s) b_1(s)$. The household's problem in state s at time 1 can be written using the household's net worth $w_1(s)$ and the household's housing stock h_1 as state variables. Given state s at time 1, the household chooses consumption levels $c_1(s)$ and $c_2(s)$, housing stock $h_2(s)$, and borrowing $b_2(s)$ to maximize

$$\sum_{t=1}^{2} \beta^t \left\{ u(c_t(s)) + v(h_t(s)) \right\} \tag{7}$$

subject to the budget constraints

$$w_1(s) + b_2(s) \geq c_1(s) + q_1(s) h_2(s) \tag{8}$$

and (6) and the collateral constraint (3). The value function induced by this problem can be written as

$$\mathcal{V}(w_1(s), h_1) \equiv V(w_1(s)) + v(h_1), \tag{9}$$

that is, is separable between net worth $w_1(s)$ and housing h_1 due to the separability of preferences in consumption and housing and the absence of adjustment costs to the housing stock.

The household's problem at time 0 is then to choose c_0, h_1, and $b_1(s)$, $\forall s \in \mathcal{S}$, to maximize

$$u(c_0) + v(h_0) + \beta E \left[V(w_1) + v(h_1) \right], \tag{10}$$

subject to (4) and (2) given the definitions of V in (9) and $w_1(s)$ above.

Noting that the time 1 budget constraints (5) bind in all states and that state-contingent promises satisfy (2), we know that net worth at time 1 in state s is bounded below, namely,

$$w_1(s) \geq e_1(s) + q_1(s)h_1(1-\theta)(1-\delta) \geq e_1(s),$$

and, similarly, we can show that $c_2(s) \geq e_2(s)$.

Households' limited ability to promise implies that their net worth at time 1 in all states is bounded below. But this means that the household must be collateral constrained against all states at time 1 if the household's net worth at time 0 is sufficiently low, since the marginal value of net worth at time 1 must be bounded above.

Proposition 1 (Financing needs override risk management concerns) *If a household's net worth w_0 is sufficiently low, the household is constrained against all states at time 1, and hence does not engage in risk management.*

The proof is in the Appendix. Households' limited ability to credibly promise repayment means that households cannot pledge future income and households' net worth has to be at least future labor income. Moreover durable goods purchases require some down payment per unit of capital from the household and hence implicitly force households to shift additional net worth to time 1. Both these aspects imply that if household net worth is relatively low at time 0, the household shifts resources to time 0 to the extent possible, that is, borrows as much as possible against durable goods.

3.3 Financing Education

Age-income profiles are upward sloping on average partly because of economic growth and partly presumably because of learning by doing, that is, skill accumulation with experience. These properties of the labor income process give households further incentives to borrow as much as they can against their durable goods, that is, housing, and thus exhaust their debt capacity and abstain from risk management.

Suppose moreover that households are able to invest in education. For simplicity we consider investment in education at time 0 only. An amount i_0 invested in education at time 0, which includes both foregone labor income and direct costs, results in increased income at time 1 in state s of $e_1(s)f(i_0)$, where f is strictly increasing and strictly concave, $f(0) = 1$, and $\lim_{i \to 0} f'(i) = +\infty$. Note that households in our model cannot borrow against future labor income and can only borrow against durable goods. In the

household's problem at time 0 the budget constraint (4) is replaced by

$$w_0 + \sum_{s \in \mathcal{S}} \pi(s) b_1(s) \geq c_0 + q_0 h_1 + i_0 \tag{11}$$

and the definition of net worth at time 1 in state s changes to $w_1(s) \equiv e_1(s) f(i_0) + q_1(s) h_1 (1 - \delta) - R_1(s) b_1(s)$. The household now chooses its investment in education i_0 as well. Note that the household's problem is still well behave, that is, the constraint set convex.

Proposition 2 *In the problem with investment in education, if a household's net worth w_0 is sufficiently low, the household is constrained against all states at time 1, and hence does not engage in risk management.*

The household's Euler equation for investment in education can be written as

$$\begin{aligned} 1 &= \sum_{s \in \mathcal{S}} \pi(s) \frac{\beta V'(w_1(s))}{\mu_0} e_1(s) f'(i_0) \\ &\geq \pi(s) \frac{\beta V'(w_1(s))}{\mu_0} e_1(s) f'(i_0), \quad \forall s \in \mathcal{S}, \end{aligned}$$

where μ_0 is the multiplier on the time 0 budget constraint and all other first order conditions are as in the proof of Proposition 1. The budget constraint (11) implies that $w_0 \geq i_0$ and hence as w_0 goes to zero, so does i_0 implying that $f'(i_0)$ goes to ∞. But then $\beta V'(w_1(s))/\mu_0$ must go to zero, $\forall s \in \mathcal{S}$, using the Euler equation for investment in education, and, using equation (17), $\lambda_1(s)/\mu_0$ must go to $R_1^{-1}(s)$ implying that the multipliers on the collateral constraints $\lambda_1(s) > 0$, $\forall s \in \mathcal{S}$. The intuition is that if the household's net worth is sufficiently low, then the household's investment in education decreases so much that the marginal rate of transformation on the investment in education eventually exceeds the return on saving net worth for state s, $R_1(s)/\pi(s)$, for all states.

Investment in education is an additional reason why households are likely to have higher net worth later in life, giving them further incentives to finance as much of their durable goods purchases as they can, rather than using their limited ability to pledge to shift funds across states later on.

4 Durable Goods Price Risk Management

Durable goods price risk affects households in two ways. First, the price of durable goods that households own has a direct effect on households' net worth. In addition, the price of durable goods has an indirect effect because it affects households' consumption

opportunities going forward. This second effect in fact can mitigate households' hedging demand, that is, may further reduce the need for household risk management. The economic intuition is straightforward. When housing prices are low, housing is cheap, which in turn may reduce the need for net worth. We provide conditions under which this will indeed be the case.

To address how the price of housing affects households going forward, we consider the value function of a household at time 1 in state s defined in equation (9). The marginal value of net worth is $V'(w_1(s))$. The housing price $q_1(s)$ affects households' hedging demand by affecting households' marginal value of net worth. Thus, we study how the marginal value of net worth varies with the housing price $q_1(s)$, that is, we evaluate $\partial V'(w_1(s))/\partial q_1(s)$ taking net worth $w_1(s)$ as well as the price of housing at time 2, $q_2(s)$, as given.

We find that the effect of the price of housing on the marginal value of net worth depends on preferences.

Proposition 3 **(i)** *If the household is unconstrained at time 1 in state s, then the marginal value of net worth increases with the housing price if $\gamma > 1$ and decreases if $\gamma < 1$ (and is independent of the housing price for logarithmic preferences).* **(ii)** *If the household is constrained at time 1 in state s and the endowment at time 2 $e_2(s) = 0$, then the marginal value of net worth increases with the housing price if $\gamma > 1$ and decreases if $\gamma < 1$ (and is independent of the housing price for logarithmic preferences).*

The proof is in the Appendix. Thus, when households' coefficient of relative risk aversion exceeds unity, lower housing prices reduce the marginal value of net worth. With sufficient curvature, the income effect of lower housing prices dominates the substitution effect, and thus low housing prices actually reduce the marginal value of net worth. Vice versa, high housing prices raise the marginal value of net worth in that case. This means that this effect lowers households need for durable goods price risk management when γ is larger than 1.

This result is reminiscent of the results in the consumption based asset pricing literature that show that investors' hedging demand in the presence of expected return variation depends in a similar way on the coefficient or relative risk aversion; investors hedge states with low expected returns when the coefficient of relative risk aversion exceeds 1 and otherwise hedge high expected returns.

When $\gamma > 1$ there is thus a sense in which lower housing prices are goods news, because they lower the marginal value of wealth, but of course the direct effect of lower housing prices on households' net worth likely overwhelms the indirect effect.

5 Risk Management and the Buy vs. Rent Decision

In the analysis so far we have not considered households' ability to rent durable goods. If there were a frictionless rental market, ownership of a durable good and the use of its services would be separable. The need to collateralize claims might still limit risk sharing,[4] but tenure choice would not affect households' portfolio choice. Moreover, households' demand for housing services would not induce a substantial financing need in that case.

In this section we consider a rental market which is not frictionless. Renting durable goods is possible, albeit costly, but relaxes collateral constraints as landlords or lessors can more easily repossess rented durables. A similar market for rented capital has been analyzed in a corporate finance context by Eisfeldt and Rampini (2009) and Rampini and Viswanathan (2011). Sufficiently constrained households choose to rent, which affects their risk management or portfolio choice. Because renting housing is costly, households will continue to have a strong incentive to own housing and hence face considerable financing needs for housing. We are able to characterize the interaction between risk management and home ownership since in our model markets are complete, although subject to collateral constraints. In contrast the literature typically studies the interaction of the risk of home ownership and portfolio choice under the assumption that markets are incomplete. Sinai and Souleles (2005) argue that both home ownership and renting are risky when households do not have access to complete markets.

[TO BE COMPLETED.]

6 Conclusion

An explicit analysis of household risk management is provided in which households have access to complete markets subject to collateral constraints. Durable goods, specifically and most importantly housing, is used as collateral. In the absence of a frictionless rental market, households' demand for the services of consumer durables results in substantial financing needs. We show that if these financing needs are sufficiently strong, they override hedging concerns, which explains the almost complete absence of household risk management. In our view, proposals to introduce new markets providing household risk management tools are hence unlikely to be successful, as households may not use such markets even if they exist.

The fact that household risk management may require collateral in the form of mar-

[4] See, e.g., Lustig and Van Nieuwerburgh (2005).

gins has been recognized in the literature, but not explicitly analyzed. For example, Athanasoulis and Shiller (2000) write that "[m]argin requirements might deal with this [collection] problem, but only for people who have sufficient assets as margin. We will disregard these kinds of ... problems." Our work, in contrast, suggests that collateral constraints, together with households' financing needs, are at the heart of the explanation why household risk management is limited.

Appendix

Proof of Proposition 1. Consider the household's problem in state s at time 1 in equations (7), (8), (6), and (3). Denoting the multipliers on constraints (8), (6), and (3) by $\mu_1(s)$, $\mu_2(s)$, and $\lambda_2(s)$, respectively, the first order conditions are

$$\begin{aligned} u'(c_1(s)) &= \mu_1(s) && (12)\\ \beta u'(c_2(s)) &= \mu_2(s) && (13)\\ \mu_1(s) &= (\mu_2(s)+\lambda_2(s))R_2(s) && (14)\\ \mu_1(s)q_1(s) &= (\mu_2(s)+\lambda_2(s)\theta)q_2(s)(1-\delta)+\beta v'(h_2(s)). && (15)\end{aligned}$$

The marginal value of net worth is, using the envelope condition and (12), $V'(w_1(s)) = \mu_1(s) = u'(c_1(s))$. As noted in the text, $c_2(s) \geq e_2(s)$ and using (13) $\mu_2(s) \leq \beta u'(e_2(s)) < \infty$. If $\lambda_2(s) = 0$, then $\mu_1(s) = R_2(s)\mu_2(s) < \infty$. Thus, in this case the marginal value of net worth is bounded above. Suppose instead that $\lambda_2(s) > 0$. Substituting for $\lambda_2(s)$ in (15) using (12) through (14), we obtain

$$u'(c_1(s))(q_1(s) - R_2^{-1}(s)\theta q_2(s)(1-\delta)) = \beta u'(c_2(s))q_2(s)(1-\theta)(1-\delta) + \beta v'(h_2(s)),$$

which implies that if $c_1(s)$ goes to zero, then so must $h_2(s)$. But noting that (8) implies that $c_1(s) + h_2(s)(q_1(s) - R_2^{-1}(s)\theta q_2(s)(1-\delta)) = w_1(s)$ it is not possible that both $c_1(s)$ and $h_2(s)$ go to zero since $w_1(s) \geq e_1(s)$, and thus $V'(w_1(s))$ is bounded above.

Considering the household's time 0 problem in equations (10), (4), and (2), and denoting the multipliers on constraints (4) and (2) by μ_0, and $\pi(s)\lambda_1(s)$ (where we are slightly abusing notation), respectively, the first order conditions are

$$\begin{aligned} u'(c_0) &= \mu_0 && (16)\\ \mu_0 &= (\beta V'(w_1(s)) + \lambda_1(s))R_1(s) && (17)\\ \mu_0 q_0 &= \sum_{s\in\mathcal{S}} \pi(s)\left\{(\beta V'(w_1(s)) + \lambda_1(s)\theta)q_1(s)(1-\delta) + \beta v'(h_1)\right\}. && (18)\end{aligned}$$

Using (4) and (2), we have

$$w_0 \geq w_0 - h_1\left(q_0 - \sum_{s\in\mathcal{S}} \pi(s)R_1^{-1}(s)\theta q_2(s)(1-\delta)\right) \geq w_0 - h_1\left(q_0 - \sum_{s\in\mathcal{S}} \pi(s)b_1(s)\right) \geq c_0,$$

and thus as w_0 goes to zero, c_0 goes to zero and μ_0 goes to ∞. But since $V'(w_1(s))$ is bounded, (17) implies that $\lambda_1(s) > 0$ for w_0 sufficiently small, $\forall s \in \mathcal{S}$. □

Proof of Proposition 3. Part (i): Suppose $\lambda_2(s) = 0$ in first order conditions (12) through (15). Then these equations together with (8) and (6) imply that $c_2(s) = c_1(s)(\beta R_2(s))^{1/\gamma}$, $h_2(s) = c_1(s)(\beta\psi/(q_1(s) - R_2^{-1}(s)q_2(s)(1-\delta)))^{1/\gamma}$, and

$$w_1(s) + R_2^{-1}(s)e_2(s) = c_1(s) + R_2^{-1}(s)c_2(s) + \left(q_1(s) - R_2^{-1}(s)q_2(s)(1-\delta)\right)h_2(s).$$

Solving for $c_1(s)$, $c_2(s)$, and $h_2(s)$ and substituting into the value function we obtain

$$V(w_1(s)|q_1(s)) = \Phi_u(q_1(s))u(w_1(s) + R_2^{-1}(s)e_2(s))$$

where

$$\Phi_u(q_1(s)) \equiv \left(1 + \beta^{1/\gamma} R_2(s)^{(1-\gamma)/\gamma} + (\beta\psi)^{1/\gamma} \left(q_1(s) - R_2^{-1}(s)q_2(s)(1-\delta)\right)^{-(1-\gamma)/\gamma}\right)^{\gamma}.$$

Therefore,

$$\frac{\partial V'(w_1(s))}{\partial q_1(s)} \propto -\frac{1-\gamma}{\gamma} < (>)0$$

depending on whether $\gamma < (>)1$.

Part (ii): Suppose $\lambda_2(s) > 0$ and hence (3) holds with equality. This equation together with (8) and (6) and (15) rewritten as

$$u'(c_1(s)) \left(q_1(s) - R_2^{-1}(s)\theta q_2(s)(1-\delta)\right) = \beta u'(c_2(s))q_2(s)(1-\theta)(1-\delta) + \beta v'(h_2(s))$$

can be solved for $c_1(s)$, $c_2(s)$, and $h_2(s)$ in the case where $e_2(s) = 0$. Substituting into the value function we obtain

$$V(w_1(s)|q_1(s)) = \Phi_c(q_1(s))u(w_1(s))$$

where

$$\Phi_c(q_1(s)) \equiv \left(1 + \beta \left(q_1(s) - R_2^{-1}(s)\theta q_2(s)(1-\delta)\right)^{-(1-\gamma)/\gamma} \left(\psi + (q_2(s)(1-\theta)(1-\delta))^{1-\gamma}\right)^{1/\gamma}\right)^{\gamma}.$$

Therefore,

$$\frac{\partial V'(w_1(s))}{\partial q_1(s)} \propto -\frac{1-\gamma}{\gamma} < (>)0$$

depending on whether $\gamma < (>)1$. □

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Table 1: Evidence on Households' Insurance Coverage Across Wealth and Age

This table reports the data on insurance coverage across households with different wealth and age from various sources. Panels A and B present data on people without health insurance coverage by income and age, respectively, from Table 6 of the U.S. Census Bureau's Report on *Income, Poverty, and Health Insurance Coverage in the United States: 2007.* Panel C presents data on private long-term care insurance ownership rates among individuals aged 60 and over from the *2000 Health and Retirement Survey* as reported by Brown and Finkelstein (2007), Table 1.

Panel A: People without Health Insurance Coverage by Income (in Thousands)					
	Total	$\leq$ \$25	\$25-\$49	\$50-\$74	$\geq$ \$75
Percentage uninsured	15.3	24.5	21.1	14.5	7.8

Panel B: People without Health Insurance Coverage by Age (in Years)							
	Total	$\leq$ 18	18-24	25-34	35-44	45-64	$\geq$ 65
Percentage uninsured	15.3	11.0	28.1	25.7	18.3	14.0	1.9

Panel C: Private Long-term Care Insurance Coverage Rates by Wealth Quartile					
	Total	Bottom	Third	Second	Top
Coverage rate (%)	10.5	2.8	6.0	11.3	19.6

Additional section for: Household Risk Management
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3 Household Income Risk Management

In this section we consider household income risk management in an endowment economy. We show that optimal household income risk management is incomplete and monotone increasing in the households' net worth, that is, richer households are better insured. Moreover, we show that "the poor cannot afford insurance."

3.1 Household Finance in an Endowment Economy

Consider household income risk management in an endowment economy. Time is discrete and the horizon is infinite. Households have preferences $E\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right]$ where we assume that $\beta\in(0,1)$ and $u(c)$ is strictly increasing, strictly concave, and satisfies $\lim_{c\to 0}u_c(c)=\infty$ and $\lim_{c\to\infty}u_c(c)=0$. Households' income $y(s)$ follows a Markov chain on state space $s\in S$ with transition matrix $\Pi(s,s')$ describing the transition probability from state s to state s', and $\forall s, s_+,\ s_+>s$, $y(s_+)>y(s)$. We use the shorthand $y'\equiv y(s')$ for income in state s' next period wherever convenient and analogously for other variables. Moreover, let $\underline{s}=\min\{s:s\in S\}$ and $\bar{s}=\max\{s:s\in S\}$ and analogously for $\underline{y}$ and $\bar{y}$ and let S also denote the cardinality of S in a slight abuse of notation.

Lenders are risk neutral and discount the future at rate $R^{-1}>\beta$, that is, are patient relative to the households, and have deep pockets and abundant collateral in all dates and states; lenders are thus willing to provide any state contingent claim at an expected return R.[1]

Enforcement is limited as follows: households can abscond with their income and cannot be excluded from markets for state contingent claims in the future. Extending the results in Rampini and Viswanathan (2010, 2011) to this environment, one can show that the optimal dynamic contracting with limited enforcement can be implemented with complete markets in one-period ahead Arrow securities subject to short sale constraints (which are a special case of collateral constraints).

We consider Markov chains which exhibit the following notion of positive persistence:

Definition 1 *A Markov chain $\Pi(s,s')$ displays* ***first order stochastic dominance (FOSD)*** *if $\forall s, s_+, \hat{s}',\ s_+>s$, $\sum_{s'\leq\hat{s}'}\Pi(s_+,s')\leq\sum_{s'\leq\hat{s}'}\Pi(s,s')$.*

This definition requires that the distribution of states next period conditional on current state s_+ first order stochastically dominates the distribution conditional on current state s for all $s_+>s$. A Markov chain which is independent over time, that is, satisfies $\Pi(s,s')=\Pi(s')$, $\forall s\in S$, exhibits FOSD. For a symmetric two-state Markov chain, FOSD

[1] We discuss the case in which $\beta=R$ below.

is equivalent to assuming that $\Pi(\bar{s},\bar{s}) = \Pi(\underline{s},\underline{s}) \equiv p \geq 1/2$, that is, that the autocorrelation ρ is positive, as $\rho = 2p - 1 \geq 0$.

3.2 Household's Income Risk Management Problem

The household solves the following recursive problem by choosing (non-negative) consumption c and a portfolio of Arrow securities h' for each state s' (and associated net worth w') given the exogenous state s and the net worth w (cum current income),

$$v(w,s) \equiv \max_{c,h',w' \in \mathbb{R}_+ \times \mathbb{R}^{2S}} u(c) + \beta E[v(w',s')|s] \tag{1}$$

subject to the budget constraints for the current and next period, $\forall s' \in S$,

$$\begin{aligned} w &\geq c + E[R^{-1}h'|s], & (2)\\ y' + h' &\geq w', & (3) \end{aligned}$$

and the short sale constraints, $\forall s' \in S$,

$$h' \geq 0. \tag{4}$$

Since the return function is concave, the constraint set convex, and the operator defined by the program in (1) to (4) satisfies Blackwell's sufficient conditions, there exists a unique value function v which solves the Bellman equation. The value function v is strictly increasing and strictly concave. Denoting the multipliers on the budget constraints (2) and (3) by μ and $\beta\Pi(s,s')\mu'$, respectively, and the multipliers on the short-sale constraints (4) by $\beta\Pi(s,s')\lambda'$, the first order conditions are

$$\begin{aligned} \mu &= u_c(c), & (5)\\ \mu' &= v_w(w',s'), & (6)\\ \mu &= \beta R\mu' + \beta R\lambda'. & (7) \end{aligned}$$

We have ignored the non-negativity constraints on consumption since they are not binding. The envelope condition is $v_w(w,s) = \mu$.

3.3 Characterization of Household Income Risk Management

Proposition 1 (Monotone household risk management) ***(i)*** *The set of states that the household hedges $S_h \equiv \{s' \in S : h(s') > 0\}$ is increasing in net worth w given the current state s, $\forall s \in S$.* ***(ii)*** *For $w_+ > w$ and denoting the net worth next period associated with w_+ (w) by w'_+ (w'), we have $w'_+ \geq w'$, $\forall s' \in S$, that is, w'_+ statewise dominates and hence FOSD w'; moreover, $h'_+ \geq h'$, $\forall s' \in S$, and $E[h'_+|s] \geq E[h'|s]$, $\forall s \in S$.*

Proof of Proposition 1. Part (i): Suppose $\exists \hat{s}' \in S_h$ such that $\hat{s}' \notin S_{h+}$. Using (7), (6), the envelope condition, and strict concavity of the value function we have

$$\beta R v_w(w(\hat{s}'), \hat{s}') = v_w(w, s) > v_w(w_+, s) \geq \beta R v_w(w_+(\hat{s}'), \hat{s}'),$$

implying, again by strict concavity of the value function, that $w(\hat{s}') < w_+(\hat{s}')$. But $w(\hat{s}') = y(\hat{s}') + h(\hat{s}') > y(\hat{s}') = w_+(\hat{s}')$, a contradiction.

Part (ii): Note that $w'_+ \geq w$, $\forall s' \in S$, implies that $y' + h'_+ = w'_+ \geq w' = y' + h'$, that is, $h'_+ \geq h'$, $\forall s' \in S$, and hence $E[h'_+|s] \geq E[h'|s]$. To see that $w'_+ \geq w$, $\forall s' \in S$, suppose not, that is, suppose $\exists \hat{s}' \in S$, such that $w_+(\hat{s}') < w(\hat{s}')$, i.e., $h_+(\hat{s}') < h(\hat{s}')$. Proceeding as in part (i), since $h(\hat{s}') > 0$, $\beta R v_w(w(\hat{s}'), \hat{s}') = v_w(w, s) > v_w(w_+, s) \geq \beta R v_w(w_+(\hat{s}'), \hat{s}')$, implying that $w_+(\hat{s}') > w(\hat{s}')$, a contradiction. □

Proposition 2 (Decreasing marginal value of net worth) *The marginal value of net worth $v_w(w, s)$ is decreasing in the state s.*

Proof of Proposition 2. Define the operator T as

$$Tv(w, s) \equiv \max_{c, h', w' \in \mathbb{R}_+ \times \mathbb{R}^{2S}} u(c) + \beta E[v(w', s')|s]$$

subject to equations (2) through (4). We show that if v has the property that $\forall s, s_+$, $s_+ > s$, $v_w(w, s_+) \leq v_w(w, s)$, then Tv inherits this property. Since the space of functions which have this property is closed, it follows that the fixed point has the property, too.

Suppose v has the property that $\forall s, s_+$, $s_+ > s$, $v_w(w, s_+) \leq v_w(w, s)$. For given w and s, suppose $\exists s'_+ > s'$ such that $h(s'_+) > h(s')$. Then $\beta R v_w(w(s'_+), s'_+) = v_w(w, s) \geq \beta R v_w(w(s'), s')$, implying, given the assumed property, that $w(s'_+) \leq w(s')$. But $w(s'_+) = y(s'_+) + h(s'_+) > y(s') + h(s') = w(s')$, a contradiction. Therefore, $h(s'_+) \leq h(s')$, $\forall s'_+ > s'$, that is, the household hedges lower income realizations (weakly) more.

Let $S = \{s_1, s_2, \ldots, s_S\}$. Denote the set of states that the household hedges by $S_h \equiv \{s' \in S : h(s') > 0\}$ and the highest state that the household hedges by $s_h = \max\{s' : s' \in S_h\}$. Take $w_+ > w$ and let S_{h+} and s_{h+} be associated with w_+ (and similarly for other variables). Suppose $\exists s_+ > s$, such that $Tv_w(w, s_+) > Tv_w(w, s)$ and, using the envelope condition, $c_+ < c$.

If $S_{h+} = \emptyset$, then equation (2) implies that $c_+ = w \geq c$, a contradiction. If $S_{h+} = \{s_1\}$, then

$$\beta R v_w(w_+(s'_1), s'_1) = Tv_w(w, s_+) > Tv_w(w, s) \geq \beta R v_w(w(s'_1), s'_1),$$

and therefore $w_+(s'_1) < w(s'_1)$ and $h_+(s'_1) < h(s'_1)$ since $y(s'_1) + h_+(s'_1) = w_+(s'_1) < w(s'_1) = y(s'_1) + h(s'_1)$. Using equation (2) and FOSD we have

$$w = c_+ + \Pi(s_+, s'_1) R^{-1} h_+(s'_1) < c + \Pi(s, s'_1) R^{-1} h(s'_1) \leq w,$$

a contradiction.

If $S_{h+} = \{s_1, s_2\}$, then proceeding as above we moreover have $w_+(s'_2) < w(s'_2)$ and $h_+(s'_2) < h(s'_2)$. Using equation (2), FOSD, and the fact that $h(s'_1) \geq h(s'_2)$, we have

$$\begin{aligned}
w &= c_+ + \Pi(s_+, s'_1)R^{-1}h_+(s'_1) + \Pi(s_+, s'_2)R^{-1}h_+(s'_2) \\
&< c + \Pi(s_+, s'_1)R^{-1}h(s'_1) + \Pi(s_+, s'_2)R^{-1}h(s'_2) \\
&\leq c + [\Pi(s, s'_1) + (\Pi(s_+, s'_1) - \Pi(s, s'_1))]R^{-1}h(s'_1) + [\Pi(s, s'_1) + \Pi(s, s'_2) - \Pi(s_+, s'_1)]R^{-1}h(s'_2) \\
&= c + \Pi(s, s'_1)R^{-1}h(s'_1) + \Pi(s, s'_2)R^{-1}h(s'_2) + (\Pi(s_+, s'_1) - \Pi(s, s'_1))R^{-1}(h(s'_1) - h(s'_2)) \\
&\leq c + \Pi(s, s'_1)R^{-1}h(s'_1) + \Pi(s, s'_2)R^{-1}h(s'_2) \leq w,
\end{aligned}$$

a contradiction.

If $S_{h+} = \{s_1, s_2, \ldots, s_{h+}\}$, then we have $w_+(s') < w(s')$ and $h_+(s') < h(s')$, $\forall s' \in S_{h+}$. Using equation (2), FOSD, and the fact that $h(s') \geq h(s'_+)$, $\forall s'_+ > s'$, we have

$$\begin{aligned}
w &= c_+ + \Pi(s_+, s'_1)R^{-1}h_+(s'_1) + \cdots + \Pi(s_+, s'_{h+})R^{-1}h_+(s'_{h+}) \\
&< c + \Pi(s_+, s'_1)R^{-1}h(s'_1) + \cdots + \Pi(s_+, s'_{h+})R^{-1}h(s'_{h+}) \\
&= c + [\Pi(s, s'_1) + (\Pi(s_+, s'_1) - \Pi(s, s'_1))]R^{-1}h(s'_1) + [\Pi(s, s'_2) + (\Pi(s_+, s'_2) - \Pi(s, s'_2))]R^{-1}h(s'_2) \\
&\quad + \Pi(s_+, s'_3)R^{-1}h(s'_3) + \cdots + \Pi(s_+, s'_{h+})R^{-1}h(s'_{h+}) \\
&\leq c + \Pi(s, s'_1)R^{-1}h(s'_1) + \Pi(s, s'_2)R^{-1}h(s'_2) \\
&\quad + [\Pi(s_+, s'_1) + \Pi(s_+, s'_2) - \Pi(s, s'_1)) - \Pi(s, s'_2)]R^{-1}h(s'_2) \\
&\quad + \Pi(s_+, s'_3)R^{-1}h(s'_3) + \cdots + \Pi(s_+, s'_{h+})R^{-1}h(s'_{h+}) \\
&\leq c + \Pi(s, s'_1)R^{-1}h(s'_1) + \cdots + \Pi(s, s'_{h+-1})R^{-1}h(s'_{h+-1}) \\
&\quad + [\Pi(s_+, s'_1) + \cdots + \Pi(s_+, s'_{h+-1}) - \Pi(s, s'_1)) - \cdots - \Pi(s, s'_{h+-1})]R^{-1}h(s'_{h+-1}) \\
&\quad + \Pi(s_+, s'_{h+})R^{-1}h(s'_{h+}) \\
&\leq c + \Pi(s, s'_1)R^{-1}h(s'_1) + \cdots + \Pi(s, s'_{h+})R^{-1}h(s'_{h+}) \\
&\quad + [\Pi(s_+, s'_1) + \cdots + \Pi(s_+, s'_{h+}) - \Pi(s, s'_1)) - \cdots - \Pi(s, s'_{h+})]R^{-1}(h(s'_{h+-1}) - h(s'_{h+})) \\
&\leq c + \Pi(s, s'_1)R^{-1}h(s'_1) + \cdots + \Pi(s, s'_{h+})R^{-1}h(s'_{h+}) \leq w,
\end{aligned}$$

a contradiction. Thus Tv inherits the property that $\forall s, s_+,\ s_+ > s$, $Tv_w(w, s_+) \leq Tv_w(w, s)$. □

When the income process is independent over time, we have a more explicit characterization of optimal income risk management:

Proposition 3 (Incomplete risk management) *Suppose* $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$. ***(i)*** *The household hedges a lower interval of states, if at all, given net worth* w*, that is,* $S_h \equiv \{s' \in S : h(s') > 0\} = \{\underline{s}', \ldots, s'_h\}$ *and* $w(s') = w_h$, $\forall s' \in S_h$*, where the dependence of* S_h*,* s'_h*, and* w_h *on the current state* w *is suppressed for convenience.* ***(ii)*** *At net worth* $w = \underline{y}$*, the household does not hedge at all, i.e.,* $\lambda' > 0$, $\forall s' \in S$*, and* $S_h = \emptyset$*.* ***(iii)*** *At net worth* $w = \bar{y}$*, the household does not hedge the highest state next period, that is,* $\lambda(\bar{s}') > 0$ *and* $S_h \subsetneq S$*.*

Proof of Proposition 3. Part (i): To be completed. Part (ii): To be completed. Part (iii): At net worth $w = \bar{y}$, using (7) and the envelope condition, we have $v_w(\bar{y}) =$

$\beta R v_w(w(\bar{s}')) + \beta R\lambda(\bar{s}')$ which implies that $\lambda(\bar{s}') > 0$ since $w(\bar{s}') \geq \bar{y}$ and hence, by strict concavity of v, $\beta R\lambda(\bar{s}') = v_w(\bar{y}) - \beta R v_w(w(\bar{s}')) \geq (1 - \beta R)v_w(\bar{y}) > 0$. □

The interpretation of part (ii) is that "the poor can't afford insurance." Richer households are moreover better insured in the following sense:

Proposition 4 (Monotone variance of net worth and consumption) *Suppose* $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$. *The variance of net worth* w' *and consumption* c' *next period is decreasing in current net worth* w.

Proof of Proposition 4. Take $w_+ > w$ and denote with a subscript + the optimal policy associated with w_+. Let $\tilde{w}' \equiv w' - E[w']$, $\tilde{w}_h \equiv w_h - E[w']$, (and $\tilde{y}' = y' - E[w']$,) and analogously for $\tilde{w}'_+$, $\tilde{w}_{h+}$, and $\tilde{y}'_+$. We need to show that $var(\tilde{w}'_+) \leq var(\tilde{w}')$. Note that $\tilde{w}' = \max\{\tilde{w}_h, \tilde{y}'\}$ and analogously for $\tilde{w}'_+$. If $\tilde{w}_{h+} = \tilde{w}_h$, then $\tilde{w}'_+ = \tilde{w}'$ and the result is obvious. Assume instead that $\tilde{w}_{h+} > \tilde{w}_h$, w.l.o.g., and hence $E[w'_+] > E[w']$. Moreover, $\tilde{w}'_+ < \tilde{w}'$, $\forall s' \in S$ such that $\tilde{w}' > 0$ and $E[\tilde{w}'_+ | \tilde{w}'_+ > 0] < E[\tilde{w}' | \tilde{w}' > 0]$. Let $\hat{\tilde{w}}'_+ \equiv \max\{\tilde{w}'_+, 0\}$, $\forall s' \in S$ such that $\tilde{w}' > 0$ and $\hat{\tilde{w}}'_+ \equiv \max\{\hat{\tilde{w}}_h, \tilde{y}'\}$ otherwise, where $\hat{\tilde{w}}_h$ such that $E[\hat{\tilde{w}}'_+] = 0$. Note that $\exists \hat{\tilde{w}}_h \in (\tilde{w}_h, \tilde{w}_{h+}]$ since $E[\tilde{w}' | \tilde{w}' > 0] > E[\tilde{w}'_+ | \tilde{w}'_+ > 0] = E[\hat{\tilde{w}}' | \tilde{w}' > 0]$ and thus $E[\tilde{w}'_+ | \tilde{w}'_+ \leq 0] = E[\hat{\tilde{w}}'_+ | \tilde{w}' \leq 0] > E[\tilde{w}' | \tilde{w}' \leq 0]$. Since $|\hat{\tilde{w}}'_+| \leq |\tilde{w}'|$, $\forall s' \in S$, with strict inequality for some $s' \in S$, $var(\hat{\tilde{w}}'_+) < var(\tilde{w}')$. Moreover, $E[\hat{\tilde{w}}'_+ - \tilde{w}'_+] = 0$ and $\hat{\tilde{w}}'_+$ is a mean preserving spread of $\tilde{w}'_+$, that is, $var(\tilde{w}'_+) < var(\hat{\tilde{w}}'_+) < var(\tilde{w}')$. Moreover, consumption c is monotone and strictly increasing in net worth w. □

Proposition 5 (Household risk management under the stationary distribution) *Suppose* $\Pi(s, s') = \pi(s')$, $\forall s, s' \in S$. ***(i)*** *There exists a unique stationary distribution of net worth.* ***(ii)*** *The support of the stationary distribution is* $[\underline{w}, \bar{w}]$ *where* $\underline{w} = \underline{y}$ *and* $\bar{w} = \bar{y}$. ***(iii)*** *Under the stationary distribution, household risk management is monotone, incomplete with probability 1, and completely absent with strictly positive probability.*

Proof of Proposition 5. Part (i): To be completed. Part (ii): Denoting the net worth at the upper bound of the stationary distribution by $\bar{w}$ and using (7) and the envelope condition, we have $v_w(\bar{w}) = \beta R v_w(\bar{w}) + \beta R\lambda(\bar{s}')$, implying that $\lambda(\bar{s}') > 0$ and hence $w(\bar{s}') = \bar{y}$. Suppose net worth w_t at time t is such that $w_t > \bar{y}$. Any path which reaches a state s_{t+n} against which the household is constrained in $t+n$ periods results in a household net worth $w(s_{t+n}) = y(s_{t+n}) \leq \bar{y}$ and indeed net worth is bounded above by $\bar{y}$ from then onwards. Consider a path along which the household is never constrained; since $v_w(w) = \beta R v_w(w')$ along such a path, $\exists n < \infty$, such that $v_w(w_{t+n}) = (\beta R)^{-n} v_w(w_t) > v_w(\bar{y})$ and hence again $w_{t+n} < \bar{y}$ at time $t + n$ and thereafter. Thus, net worth levels above $\bar{y}$ are transient. Since (3) holds with equality and using (4), $w' \geq y' \geq \underline{y}$; levels of net worth $w < \underline{y}$ are therefore transient.

Part (iii): Household risk management is monotone by Proposition 1 and incomplete with probability 1 since the stationary distribution of net worth is bounded above by

$\bar{y}$ and risk management is monotone and incomplete at $w = \bar{y}$ by Part (iii) of Proposition 3. By Part (ii) of that proposition, risk management is completely absent at $\underline{w} = \underline{y}$; by continuity, $\exists \varepsilon > 0$ such that for $w > \bar{w}$ with $|w - \bar{w}| < \varepsilon$, $v_w(w) > \beta R v_w(\underline{y})$, which means that the household does not hedge at all in this neighborhood. Clearly, $\bar{w}$ has positive probability under the stationary distribution since household income $\bar{y}$ has positive probability under the stationary distribution of income. If the household does not hedge $\underline{s}'$ at $\bar{w}$, then $\underline{w}$ has strictly positive probability. Consider instead a path along which the household continues to hedge the lowest income realization the following period, then $\exists n < \infty$ such that $v_w(w_{t+n}) = (\beta R)^{-n} v_w(\bar{w}) > v_w(\underline{y})$and hence $w_{t+n} < \underline{y}$, which is not possible. So the household must stop hedging the lowest state after a finite sequence of lowest income realizations, that is, the household does not hedge at all with positive probability under the stationary distribution. □

3.4 Stationary Distribution of Net Worth for $\beta R = 1$

Consider the limit of the above economy where $\beta R = 1$. We show that the economy displays full insurance under the stationary distribution in the limit and that household net worth is bounded in the limit. These results are related to results for the classic class of income fluctuations problems studied by Yaari (1976), Schechtman (1976), Bewley (1980), and others, in which households solve a consumption savings problem with non-contingent debt and borrowing constraints, that is, incomplete markets. Our results are similar in that there is complete consumption insurance in the limit, but they are rather different in that net worth is bounded in the limit whereas it grows without bound in these related papers.

We emphasize that for net worth levels below $\bar{w}$, household risk management is incomplete and monotone in current net worth even when $\beta R = 1$, although such levels of net worth are transient. The main result of our paper hence obtains even in this case, albeit in the transition only.

When income is independent over time, we know from equation (7) and the envelope condition that $v_w(w) = v_w(w') + \lambda'$ and therefore $v_w(w)$ is non-increasing. Denoting the upper bound of net worth under the stationary distribution by $\bar{w}$, we hence have $v_w(\bar{w}) \geq v_w(w')$, but by strict concavity $v_w(\bar{w}) \leq v_w(w')$, and thus $\bar{w} = w'$, $\forall s' \in S$, that is, $\bar{w}$ is absorbing. Note that for $w < \bar{y}$, $\lambda(\bar{s}') > 0$ and $w(\bar{s}') = \bar{y}$. Moreover, suppose $\exists s' \in S$, such that $w' > \bar{y}$, then $v_w(\bar{y}) > v_w(w')$ and $\lambda' > 0$, that is, $w' = y' \leq \bar{y}$, a contradiction. Therefore, $w' = \bar{y}$, for all $s' \in S$. Thus, the stationary net worth distribution collapses to unit mass at $\bar{w} = \bar{y}$.

The full insurance result is general:

Proposition 6 (Full insurance under the stationary distribution in the limit) *When $\beta R = 1$, the household engages in full insurance under the stationary distribution.*

Proof of Proposition 6. From equation (7) and the envelope condition that $v_w(w, s) = v_w(w', s') + \lambda'$ and therefore $v_w(w, s)$ is non-increasing. Consider the marginal value of

net worth at the upper bound of the stationary distribution for some state s, $v_w(\bar{w}(s), s)$; suppose there exists some state, say, w.l.o.g., next period, such that $v_w(\bar{w}(s), s) > v_w(w', s')$. But $v_w(w', s') \geq v_w(w'', s'')$, $\forall s'' \in S$, including $s'' = s$. But then, by concavity, $v_w(\bar{w}(s), s) \leq v_w(w'', s)$, a contradiction. Thus, $v_w(w, s) = v_w(w', s')$, $\forall (w, s), (w', s')$ in the support of the stationary distribution. $\square$

In the case of a symmetric two state Markov chain for income, we can solve for the stationary distribution of net worth in closed form. Specifically, say $S = \{s_L, s_H\}$ with $s_L < s_H$, and $\Pi(s_H, s_H) = \Pi(s_L, s_L) \equiv p$. We use subscripts L and H where convenient. Using the fact that the stationary distribution of y is $(1/2, 1/2)$ and that $w_H = y_H$ (why?), $w_L = y_L + h_L$, $c_H = w_H - (1-p)R^{-1}h_L$, $c_L = w_L - pR^{-1}h_L$, and $c_H = c_L$, that is, full insurance, we obtain

$$h_L = \frac{R}{R-\rho}(y_H - y_L), \quad w_L - w_H = \frac{\rho}{R-\rho}(y_H - y_L), \quad c_H = c_L \equiv c = E[y] + 1/2\frac{r}{R-\rho}(y_H - y_L)$$

where $\rho = 2p - 1 \geq 0$. When income is independent over time, $p = 1/2$ and $\rho = 0$, we have $h_L = y_H - y_L$, $w_H = w_L = y_H$, and $c = E[y] + 1/2r/R(y_H - y_L)$.

Note that $w_L \geq w_H$ and that the difference $w_L - w_H$ is increasing in the persistence ρ. So net worth as we defined it is higher in the low state than in the high state. To see why this is, denote the present value of income (ex current income), that is, human capital, by PV_s, and note that

$$\begin{aligned} PV_H &= R^{-1}(p(y_H + PV_H) + (1-p)(y_L + PV_L)) \\ PV_L &= R^{-1}((1-p)(y_H + PV_H) + p(y_L + PV_L)) \end{aligned}$$

which implies that $PV_H - PV_L = w_L - w_H$ or $w_H + PV_H = w_L + PV_L$, that is, total wealth, (financial) net worth plus human capital, is constant across states. When the household has low current income, his (financial) net worth is high to compensate for the reduction in present value of future labor income. When income is independent over time, the present value of future labor income is constant across states and so is the household's (financial) net worth.