Mortgage Innovation and the Foreclosure Boom

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Abstract

How much of the recent rise in foreclosures can be explained by the large number of nontraditional, low-downpayment mortgage contracts originated between 2003 and 2006? We present a model where heterogeneous households select from a set of possible mortgage contracts and choose whether to default on their payments given realizations of income and housing price shocks. The set of contracts consists of traditional fixed rate mortgages as well as nontraditional mortgages with low downpayments and delayed amortization schedules. The mortgage market is competitive and each contract, contingent on household earnings and assets at origination as well as loan size, must earn zero expected profits. In this model, an unanticipated 25% housing price decline following a brief introduction of non-traditional mortgages causes an increase in foreclosure rates that closely approximates the 150% increase in the data between the first quarter of 2007 and the first quarter of 2009. The model also matches origination rates for non-traditional mortgages observed prior to the crisis. Since we do not select parameters to match these two key aspects of the recent data, these quantitative predictions provide support for our model. In a counterfactual experiment where new mortgages are not introduced, the same price shock causes foreclosure rates to increase by only 86%. Thus, the availability and popularity of nontraditional mortgages in the four years prior to the crisis can explain over 40% of the rise in foreclosures.

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1 Introduction

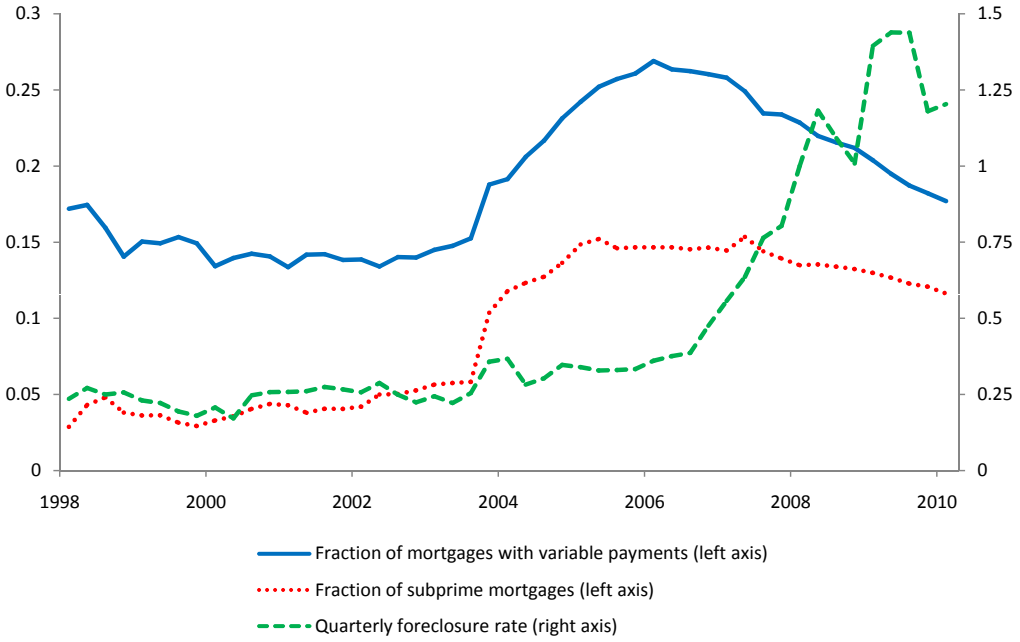
Between 2003 and 2006, the composition of the stock of outstanding residential mortgages in the United States changed in several important ways. The fraction of mortgages with variable payments relative to all mortgages increased from 15% to over 25% (see figure 1.) At the same time, the fraction of "subprime" mortgages (mortgages issued to borrowers perceived by lenders to be high-default risks) relative to all mortgages rose from 5% to nearly 15%. Recent work (see e.g. Gerardi et al., forthcoming, figure 3) has shown that many of these subprime loans are characterized by high loan-to-value (LTV) ratios and non-traditional amortization schedules.

Low downpayments and delayed amortization cause payments from the borrowers to the lender to be backloaded compared to standard loans. By lowering payments initially, these features made it possible for more households to obtain the financing necessary to purchase a house. At the same time however, because these contracts are characterized by little accumulation of home equity early in the life of the loan, they are prone to default when home prices fall. Not surprisingly then, (see Gerardi et al., 2009) mortgages issued between 2005 and 2006 with high leverage and non-traditional amortization schedules have defaulted at much higher frequency than other loans since home prices began their collapse in late 2006.

Our objective is to quantify the importance of non-traditional mortgages for the recent flare-up in foreclosure rates depicted in figure 1. Specifically, we ask the following question: How much of the rise in foreclosures can be attributed to the increased originations of non-traditional mortgages between 2003 and 2006?

To answer this question, we describe a housing model where the importance of non-traditional loans for mortgage default rates can be measured. The model predicts origination rates of non-traditional mortgages between 2003 and 2006 and a spike in foreclosure rates following the collapse in home prices in late 2006 that are both consistent with the relevant evidence. In the context of that model, we find that the popularity of non-traditional mortgages between 2003 and 2006 can account for 40% of the crisis.

Figure 1: Recent trends in the mortgage market



Source: National Delinquency Survey (Mortgage Bankers Association). Foreclosure rates are the number of mortgages for which a foreclosure proceeding are started in a given quarter divided by the initial stock of mortgages.

In our model economy, households move stochastically through three stage of life and make their housing and mortgage decisions in the middle stage. Two types of mortgages are available to households: a standard fixed-rate mortgage (FRM) with a 20% downpayment and fixed payments until maturity, and a mortgage with no-downpayment and delayed amortization which we call LIP for "low initial payment." We think of this second mortgage as capturing the backloaded nature of the mortgages that became popular after 2003 in the United States.

Mortgage holders can terminate their contract before maturity. We consider a mortgage

termination to be a foreclosure if it occurs in a state where the house value is below the mortgage's balance (that is, the agent's home equity is negative) or where the agent's income realization is such that they cannot make the mortgage payment they would owe for the period.[1] Foreclosures are costly for lenders because of the associated transactions costs and because they occur almost always when home equity is negative. As a result, intermediaries demand higher yields from agents whose asset and income position make foreclosure more likely. In fact, intermediaries do not issue loans to some agents because their default risk is too high or because the agents are too poor to make a downpayment. In particular, our model is consistent with the fact that agents at lower asset and income positions are less likely to become homeowners, face more expensive borrowing terms, and are more likely to default on their loan obligations.

We choose parameters so that when only FRMs are available the model matches pre-2003 homeownership and default rates, among other key features of housing markets. We then consider the impact of introducing the LIP option in such an economy, both in the long-run and on impact.

Quite intuitively, introducing LIP contracts causes a rise in steady state homeownership, default rates, and welfare. LIPs enable some households with low assets and income (those who could be interpreted as subprime) to become homeowners. At the same time, the availability of these contracts cause default rates to be higher for two complementary reasons which our environment makes explicit. First, high-default risk households select into homeownership. Second, these contracts are characterized by a much slower accumulation of home equity than FRMs, which makes default in the event of a home value shock much more likely, even at equal asset and income household characteristics.

While these steady state predictions are interesting, the available evidence suggests that the break in the composition of the mortgage stock occurred briefly before the collapse of

[1]Here we are assuming the default law is consistent with antideficiency statutes (as in Arizona and California for example) whereby the defaulting household is not responsible for the deficit between the proceeds from the sale of the property and the outstanding loan balance. In Section 5.7 we consider a variation in punishment following a foreclosure that resembles laws in states with recourse.

house prices in late 2006 and the spike in foreclosures. There is also growing evidence that the fraction of high-LTV, delayed amortization mortgages in originations has dwindled to a trickle since the collapse of prices.[2]

We simulate this course of events using a three-stage transition experiment. Specifically, we begin in a steady state of an economy with only FRMs calibrated to match key aspects of the US economy prior to 2003. We then introduce the nonstandard mortgage option for two periods, which represents four years in our calibration. The model predicts that when they become available, non-traditional mortgages are selected by a third of home-owners, which is consistent with estimates of the fraction in originations of mortgages with non-traditional features between 2003 and 2006. In fact, the experiment produces a pattern for the share of non-traditional mortgages in the mortgage stock that resembles the pattern shown in figure 1. In the third stage, we assume a surprise 25% collapse in home prices, remove the nonstandard mortgage option, and then let the economy transit to a new long-run steady state. This experiment causes foreclosure rates to rise by 148% during the first two years of stage 3. By comparison, in the data, the overall foreclosure rate increased by 150% between the first quarter of 2007 and the first quarter of 2009. We emphasize that the model's close match with data during stage 2 and 3 is not a consequence of our calibration, since the deep parameters are calibrated only to match stage 1. Hence one can view stage 2 and 3 as passing an informal test of the model.

To quantify the role of mortgage innovation in this foreclosure rate increase, we then run an experiment where the LIP mortgage option is not offered in the second stage. In this counterfactual, the increase in foreclosure rates caused by the price shock falls to 86%. Thus, the origination of nontraditional mortgages for two model periods can explain $\frac{148-86}{148} \approx 42\%$ of the rise in foreclosures. In another counterfactual, we find that lower downpayments account

[2]The Mortgage Bankers Association (MBA)'s mortgage origination survey suggests for instance that after falling to 50% of originations in 2005, traditional FRMs now account for 90% of originations. According to the same source, the fraction of interest-only mortgages in originations rose to nearly 20% in 2006, and has now fallen to below 5%. It is also estimated (see e.g. Harvard's "2008 State of the Nation's Housing") that subprime loans accounted for roughly 20% of originations between 2004 and 2006, up from less than 8% between 2000 and 2003. They now account for less than 5% of new mortgage issues.

for most of the contribution of non-traditional mortgages to the increase in foreclosure rates, while delayed amortization and payment spikes play a limited role.

Our paper is closely related to several studies of the recent evolution of the US housing market and mortgage choice.[3] In particular, our work complements the findings of Gerardi et al (2009) who run a counterfactual exercise similar in spirit to ours, but using a completely different methodology. Specifically, they estimate an econometric hazard model of sales or foreclosures using data from Massachusetts. In their model, the likelihood of default in a given contract period depends on the loan's leverage at origination, a proxy for current home equity, and local home price and economic conditions, among other determinants. Using this model, they calculate that had the loan issued in 2002 experience similar home price conditions as loans issued in 2005, they would have defaulted at very high rates as well, despite the fact that they were issued under much stricter underwriting standards, particularly in terms of leverage at origination. Those very high foreclosure rates, however, are about half of their counterparts for 2005 loans. This suggests that while loose underwriting standards alone cannot account for the foreclosure crisis, they did magnify the impact of the price collapse significantly.

One advantage of running a counterfactual inside a dynamic equilibrium model such as ours is that the resulting calculations capture the consequences of endogenous changes to the sample of borrowers caused by changes in underwriting standards. As our model illustrates, loosening underwriting standards clearly encourages the participation in mortgage markets

[3]There are numerous other housing papers which are a bit less closely related. Campbell and Cocco (2003) study the microeconomic determinants of mortgage choice but do so in a model where all agents are homeowners by assumption, and focus their attention on the choice between adjustable rate mortgages and standard FRMs with no option for default. Rios-Rull and Sanchez-Marcos (2008) develop a model of housing choice where agents can choose to move to bigger houses over time. A different strand of the housing literature (see e.g. Gervais (2002) and Jeske and Krueger (2005)) studies the macroeconomic effects of various institutional features of the mortgage industry, again where there is no possibility of default. Davis and Heathcote (2005) describe a model of housing that is consistent with the key business cycle features of residential investment. Our paper also builds on the work of Stein (1995) and Ortalo-Magné and Rady (2006) who study housing choices in overlapping generation models where downpayment requirements affect ownership decisions and house prices. Our framework shares several key features with those employed in these studies, but our primary concern is to quantify the effects of various mortgage options, particularly the option to backload payments, on foreclosure rates.

of agents prone to default.

Chambers et al. (2009) also study the effect introducing new mortgage options in a dynamic equilibrium model and argue that the development of mortgages with gradually increasing payments has had a positive impact on participation in the housing market. The idea that mortgage innovation may have implications for foreclosures is taken up in Garriga and Schlagenhauf (2009). They quantify the impact of an unanticipated aggregate house price decline on default rates where there is cross-subsidization of mortgages **within** but not **across** mortgage types. A key difference between our paper and theirs is that we consider a menu of different terms on contracts **both** within **and** across mortgage types. Effectively Garriga and Schlagenhauf (2009) apply the equilibrium concept in Athreya (2002) while we apply the equilibrium concept in Chatterjee et al. (2007). This enables us to build a model that is consistent with the fact that mortgage terms do vary even across households who opt for the same type of mortgage. More importantly, we present simulations that suggest that the two equilibrium concepts result in significantly different quantitative predictions.

Campbell and Coco (2010) also consider the household default decision problem with FRM, ARM, and interest-only contracts. They consider a richer set of stochastic processes for aggregate house prices, interest rates and inflation than we do which enable them to discuss the effects of various nominal risks on default behavior. On the other hand, they do not price mortgages individually as we do and do not quantify the role of looser underwriting standards for the recent foreclosure crisis, which is our primary question.

Along the separation dimension our paper is more closely related to Guler (2008) where intermediaries offer a menu of FRMs at endogenously chosen downpayment rates without cross-subsidization or Chatterjee and Eyigungor (2009) where intermediaries offer a menu of infinite maturity interest-only mortgage contracts in which borrowers do not accumulate home equity over time. Guler studies the impact of an innovation to the screening technology on default rates and Chatterjee and Eyigungor study the effect of an endogenous price drop arising out of an overbuilding shock.

Section 2 lays out the economic environment. Section 3 describes optimal behavior on

the part of all agents and defines an equilibrium. Section 4 provides our calibration. Section 5 describes our steady state results, with subsections which focus on: Selection, Default, Subprime Mortgages and the Distribution of Interest Rates, Separating vs. Pooling Contracts, the Welfare Implications of Innovation, and Recourse vs. Non-recourse Policies. Section 6 presents our main transition experiment. Section 7 concludes.

2 The Environment

Time is discrete and infinite. The economy is populated by a continuum of households and by a financial intermediary. Each period a mass one of households is born. Households move stochastically through four stages: youth (Y), mid-age (M), old-age (O), and death. At the beginning of each period, young households become mid-aged with probability ρ_M, mid-age households become old with probability ρ_O, and old households die with probability ρ_D and are replaced by young households. The population size is at its unique invariant value.

Each period when young or mid-aged, households receive earnings y_t denominated in terms of the unique consumption good. These shocks follow a three state Markov transition matrix. Specifically, earnings are drawn from $\{y_Y^L, y_Y^M, y_Y^H\}$ for young agents and from $\{y_M^L, y_M^M, y_M^H\}$ for mid-aged agents, with transition matrices π_Y and π_M, respectively, where $y_s^L < y_s^M < y_s^H$ for $s \in \{Y, M\}$. Earnings shocks obey a law of large numbers. Agents begin life at an income level drawn from the unique invariant distribution associated with π_Y. When old, agents earn a fixed, certain amount of income $y_O > 0$.

Households can save in one-period bonds $a_{t+1} \geq 0$ that earn rate $1 + r_t \geq 0$ at date t with certainty. When old, agents buy annuities that pay rate $\frac{1+r_t}{1-\rho_D}$.

Households value both consumption and housing services. They can obtain housing services from the rental market or from the owner-occupied market. On the first market, they can rent quantity $h^1 > 0$ of housing services at unit price R_t at date t. When they become mid-aged, agents can choose instead to purchase quantity $h_0 \in \{h^2, h^3\}$ of housing capital for unit price q_t, where $h^3 > h^2 > h^1$. We refer to this asset as a house.

Homeowners can choose to sell or foreclose on their house and become renters. When they do so, they must remain renters for at least one period. After this one period, mid-aged renters are given the option to buy a home with probability $\gamma \in [0, 1]$. Constraining agents to become renters for at least one period when they sell their home economizes on one choice variable by precluding house-to-house transitions. In addition, since the arrival of the buying option is exogenous, the expected value of renting summarizes all the information relevant to the agent's selling decision.

A house of size $h_t \in \{h^2, h^3\}$ delivers h_t in housing services. Furthermore, agents enjoy a fixed ownership premium $\theta > 0$ as long as they own a quantity $h_t \in \{h^2, h^3\}$ of housing capital. Specifically, household utility is:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$$

where

$$u(c_t, h_t) = U(c_t, h_t) + \theta 1_{\{h_t \in \{h^2, h^3\}\}},$$

$c_t \geq 0$, $h_t \in \{h^1, h^2, h^3\}$, and U satisfies standard assumptions. We think of θ as capturing any enjoyment agents derive from owning rather than renting their home, but it also serves as a proxy for any monetary benefit associated with owning which we do not explicitly model.

Once agents purchase a house, housing capital follows a Markov Process over $\{h^1, h^2, h^3\}$ with transition matrix

$$P(h_{t+1}|h_t) = \begin{bmatrix} 1 & 0 & 0 \\ \lambda & 1-2\lambda & \lambda \\ 0 & \lambda & 1-\lambda \end{bmatrix},$$

where $\lambda > 0$. In other words, a fraction λ of agents who own a house of size h^2 see the quantity of capital go up to h^3, while a fraction λ of these agents see the value of their house fall to h^1, which is an absorbing state. Likewise, a fraction λ of agents who own a house of size h^3 see the quantity of capital they own fall to h^2.

We interpret these changes in the stock of housing an agent owns as uninsurable, idiosyncratic house value shocks. In the absence of such shocks, households would never find themselves with negative equity in a steady state equilibrium. There are several possible interpretations for these shocks. For instance, they could represent neighborhood shocks which make a house in a given location more or less valuable. Note that while devaluation shocks satisfy a law of large numbers we do not need to assume that these shocks are independent across households. Since devalued houses of size h^1 provide no advantage over rental units, no agent would strictly prefer to purchase a house of that size and all homeowners whose housing capital fall to that level are at least as well off selling their house and becoming renters as they would be if they keep their house.

Owners of a house of size $h \in \{h^1, h^2, h^3\}$ bear maintenance costs δh in all periods where $\delta > 0$. Maintenance costs must be paid in all periods by homeowners. Under that assumption, a house does not physically depreciate which maintains the low cardinality of the housing state space. We also assume that when agents become old, they sell their house and become renters for the remainder of their life.

The financial intermediary holds household savings and can store these savings at return $1+r_t$ at date t. It also holds a stock of housing capital. It can add to this stock by transforming the consumption good (i.e. deposits) into housing capital at a fixed rate $A_t > 0$. That is, it can turn quantity k of deposits into quantity $A_t k$ of housing capital at the start of any given period. The intermediary can rent out its housing capital and can sell part of it to new home owners. In any period, it can also reduce its stock of capital by turning quantity h of housing capital into quantity $\frac{h}{A_t}$ of the consumption good. The intermediary incurs maintenance cost δ on each unit of housing capital it rents. A unit of consumption good rented thus earns net return $R_t - \delta$.[4]

Households that purchase a house of size $h_0 \in \{h^2, h^3\}$ at a given date are constrained to finance this purchase with one of two possible types of mortgage contracts. The first

[4]Note that the fact that each agent's housing choice set is discrete does not impose an integer constraint on the intermediary since it deals with a continuum of households.

contract (which is designed to mimic the basic features of a standard fixed-rate mortgage, or FRM) requires a downpayment of size $\nu h_0 q_t$ at date t where $\nu \in (0,1)$ and stipulates a yield $r_t^{FRM}(a_0, y_0, h_0)$ that depends on the household wealth and income characteristics (a_0, y_0) and on the selected house size h_0 at origination of the loan in date t. Given this yield, constant payments $m_t^{FRM}(a_0, y_0, h_0)$ and a principal balance schedule $\{b_{t,n}^{FRM}(a_0, y_0, h_0)\}_{n=0}^{T}$ can be computed using standard calculations, where T is the maturity of the loan. Specifically, suppressing the initial characteristics for notational simplicity,

$$m_t^{FRM} = \frac{r_t^{FRM}}{1-(1+r_t^{FRM})^{-T}}(1-\nu)h_0 q_t$$

and, for all $n \in \{0, T-1\}$,

$$b_{t,n+1}^{FRM} = b_{t,n}^{FRM}(1+r_t^{FRM}) - m_t^{FRM},$$

where $b_{t,0}^{FRM} = (1-\nu)h_0 q_t$. Standard calculations show that $b_{t,T}^{FRM} = 0$.

The second contract, which we denote by LIP since it features **l**ow **i**nitial **p**ayments, stipulates a yield $r_t^{LIP}(a_0, y_0, h_0)$, no down-payment, constant payments $m_{t,n}^{LIP}(a_0, y_0, h_0) = h_0 q_t r_t^{LIP}(a_0, y_0, h_0)$ that do not reduce the principal for the first $n^{LIP} < T$ periods, and fixed-payments for the following $T - n^{LIP}$ periods with a standard FRM-like balance schedule $\{b_{t,n}^{LIP}(a_0, y_0, h_0)\}_{n=n^{LIP}}^{T}$. In other words,

$$m_{t,n}^{LIP} = \begin{cases} h_0 q_t r_t^{LIP} & if \quad n < n^{LIP} \\ \frac{r_t^{LIP}}{1-(1+r_t^{LIP})^{-(T-n^{LIP})}} h_0 q & if \quad n \geq n^{LIP} \end{cases}$$

and, for all $n \in \{0, T-1\}$,

$$b_{t,n+1}^{LIP} = b_{n,t}^{LIP}(1+r_t^{LIP}) - m_{t,n}^{LIP},$$

where $b_{t,0}^{LIP} = h_0 q_t$, and $b_{t,T}^{LIP} = 0$. Notice that for $n < n^{LIP}$, $b_{t,n+1}^{LIP} = b_{t,0}^{LIP}$ so that the principal remains unchanged for n^{LIP} periods.

LIPs, therefore, have two main characteristics: low downpayment and delayed amortization. These are two of the salient features of the mortgages that became highly popular in the United States around 2003 (see Gerardi et al., forthcoming.) Naturally, delayed amortization can take many forms. Subprime mortgages, for instance, often feature balloon payments rather than interest-only periods.

Mortgages are issued by the financial intermediary. The intermediary incurs service costs which we model as a premium $\phi > 0$ on the opportunity cost of funds loaned to the agent for housing purposes.

A mid-aged household can terminate the contract at the beginning of any period, in which case the house is sold. We will consider a termination to be a foreclosure when the outstanding principal exceeds the house value or when the agent's state is such that it cannot meet its mortgage payment in the current period. The next section will provide a formal definition of these events. In the event of foreclosure, fraction $\chi > 0$ of the sale value is lost in transaction costs (e.g. legal costs, costs of restoring the property to saleable conditions, etc.).[5] If the mortgage's outstanding balance at the time of default is $b_{t,n}$, the intermediary collects $\min\{(1-\chi)q_t h_t, b_{t,n}\}$, while the household receives $\max\{(1-\chi)q_t h_t - b_{t,n}, 0\}$.

Agents may also choose to sell their house even when they can meet the payment and have positive equity, for instance because they are borrowing constrained in the current period. Recall also that agents sell their house when they become old. Those contract terminations, however, do not impose transaction costs on the intermediary.

The timing in each period is as follows. At the beginning of the period, agents discover whether or not they have aged and receive a perfectly informative signal about their income draw. Mid-aged agents who own homes also observe the realization of their devaluation shock at the beginning of the period (hence the market value of their home). These agents then decide whether to remain home-owners or to become renters either by selling their house or through foreclosure. Renters discover whether or not home-buying is an option at the beginning of the period. Agents who just turned mid-aged get this option with probability one.

[5]For more discussion of these costs, see http://www.nga.org/Files/pdf/0805FORECLOSUREMORTGAGE.PDF.

Agents who get the home-buying option make their housing and mortgage choice decisions at the beginning of the period, after all uncertainty for the period is resolved. At the end of the period, agents receive their income, mortgage payments are made, and consumption takes place.

3 Equilibrium

We will initially study equilibria in which all prices are constant. To ease notation, we drop all time markers using the convention that, for a given variable x, $x_t \equiv x$ and $x_{t+1} \equiv x'$.

3.1 Household's problem

We state the household problem recursively. In general, the household value functions will be written as $V_{age}(\omega)$ where $\omega \in \Omega_{age}$ is the state facing an agent of $age \in \{Y, M, O\}$.

3.1.1 Old agents

For old agents, the state space is $\Omega_O = I\!R_+$ with typical element $\omega \equiv a \geq 0$. The value function for an old agent with assets $a \in I\!R_+$ solves

$$V_O(a) = \max_{a' \geq 0} \left\{ u\left(c, h^1\right) + \beta(1 - \rho_D) V_O(a') \right\}$$

s.t.

$$c = a\frac{(1+r)}{1-\rho_D} + y^O - h^1 R - a' \geq 0.$$

3.1.2 Mid-aged agents

For mid-aged agents, the state space is

$$\Omega_M = I\!R_+ \times \{y_M^L, y_M^M, y_M^H\} \times \{0,1\} \times \{h^1, h^2, h^3\} \times I\!N \times \{\{\{FRM, LIP\} \times I\!R_+ \times \{h^2, h^3\}\} \cup \{\emptyset\}\}$$

with typical element $\omega = (a, y, H, h, n; \kappa)$. Here, $H = 1$ means that the household begins the period as a homeowner, while $H = 0$ if it begins as a renter. Further, $h \in \{h^1, h^2, h^3\}$ is the quantity of housing capital the household owns at the start of a given period once the devaluation shock has been revealed.[6] We write $n \in \{0, 1, \ldots\}$ for the number of periods that have elapsed since the household last received the home-buying option. In particular, n measures the age of the mortgage for agents who have one. Furthermore, $n = 0$ means that the agent must choose whether or not to buy a home in the current period.

The final argument, κ, denotes the type of mortgage chosen by a homeowner - that is, $\kappa \equiv (\zeta, r^{\zeta}, h_0) \in \{FRM, LIP\} \times I\!R^{+} \times \{h^2, h^3\}$ which lists the agent's mortgage and house choice when they purchase their home. In equilibrium, the yield on a given loan will depend on the agent's wealth-income position (a_0, y_0) and house size choice h_0 at origination. For agents who enter a period as renters, the current house size and mortgage type arguments are undefined, so we write $\kappa = \emptyset$.

We begin with the case where the household does not need to make a home-buying decision.

Case 1: $n \geq 1$

If the household enters the period as renter (i.e. $H = 0$), the value function is:

$$\begin{aligned} V_M(a, y, 0, h^1, n; \emptyset) &= \max_{c, a'} u\left(c, h^1\right) \\ &+ \beta(1 - \rho_O) E_{y'|y} \left[\begin{array}{c} (1-\gamma) V_M(a', y', 0, h^1, n+1; \emptyset) \\ +\gamma V_M(a', y', 0, h^1, 0; \emptyset) \end{array} \right] \\ &+ \beta \rho_O V_O(a') \\ \text{s.t. } c + a' &= y + a(1 + r) - R h^1. \end{aligned}$$

Indeed, if the household remains mid-aged and receives the option to buy, n reverts to 0, while otherwise, n increases by to $n + 1$.

[6] We need both H and h to differentiate a renter from a homeowner whose size h^2 received a shock down to h^1.

Households who already own a home (i.e. $H = 1$) have to decide whether to remain homeowners or to become renters. We will write $H'(\omega) = 1$ if they choose to remain homeowners and $H'(\omega) = 0$ if they become renters. The event $H'(\omega) = 0$ entails a sale of the house and a termination of the mortgage contract. We think of that event as a foreclosure in two cases. First, if it is not budget feasible for the household to meet its mortgage payment $m_n(\kappa)$, that is if,

$$y + a(1 + r) - m_n(\kappa) - \delta h < 0, \tag{3.1}$$

the household is constrained to terminate its mortgage. Abusing language somewhat, we call this event an *involuntary default* and write $D^I(\omega) = 1$, while $D^I(\omega) = 0$ otherwise. A second form of default occurs when the household can meet their mortgage payment (i.e. (3.1) does not hold) but the household chooses nonetheless to become renters and

$$qh - b_n(\kappa) < 0, \tag{3.2}$$

i.e. home equity is negative. We call this event a *voluntary default* and write $D^V(\omega) = 1$.

If neither (3.1) nor (3.2) holds but the household decides to sell their house and become renters, we write $S(\omega) = 1$, while $S(\omega) = 0$ otherwise. In that case, the household pays their mortgage balance and their asset position is augmented by the value of their home equity. Note that

$$1 - H'(\omega) = S(\omega) + D^I(\omega) + D^V(\omega).$$

In other words, (S, D^I, D^V) classify a mortgage termination into three mutually exclusive events: a simple sale (in which the intermediary need not get involved), an involuntary default, or a voluntary default. For agents who become old in the period, we consider the associated sale to be a foreclosure only if $qh - b_n(\kappa) < 0$ and write $D^O = 1$ in that case, while $D^O = 0$ in all other cases. Equipped with this notation, we can now define the value function

of a homeowner (i.e. a household whose $H = 1$):

$$\begin{aligned} V_M(a, y, 1, h, n; \kappa) &= \max_{c \geq 0, a' \geq 0, (H', D^I, D^V, S) \in \{0,1\}^4} u(c, (1 - H')h^1 + H'h) \\ &+ (1 - H')\beta E_{y'|y} \left[\begin{array}{c} (1 - \rho_O)(1 - \gamma)V_M(a', y', 0, h^1, n + 1; \emptyset) \\ +(1 - \rho_O)\gamma V_M(a', y', 0, h^1, 0; \emptyset) + \rho_O V_O(a') \end{array} \right] \\ &+ H'\beta E_{(y',h')|(y,h)} \left[\begin{array}{c} (1 - \rho_O)V_M(a', y', 1, h', n + 1; \kappa) \\ +\rho_O V_O(a' + \max\left\{(1 - D^O\chi)qh - b_{n+1}(\kappa), 0\right\}) \end{array} \right] \end{aligned}$$

subject to:

$$\begin{aligned} c + a' &= y + (1 + r)(a + (1 - H')\max((1 - (D^I + D^V)\chi)qh - b_n(\kappa), 0)) \\ &\quad -H'(m_n(\kappa) + \delta h) - (1 - H')Rh^1 \\ D^I &= 1 \text{ if and only if (3.1) holds} \\ D^V &= 1 \text{ if } H' = 0 \text{ and (3.2) holds} \\ S &= 1 - H' - D^I - D^V \end{aligned}$$

There are several things to note in the statement of the household's problem. Starting with the objective, housing services depend on the household's housing status and the size of the house. Households who choose to sell must be renters for the period but may get the option to buy again in the following period. The right-hand side of the budget constraint depends on whether or not the household keeps its house. When households become renters, their asset position is increased by the value of the house net of their outstanding principal and in the event of default, net of transaction costs. Their housing expenses are the sum of mortgage and maintenance payments if they keep the house or the cost of rental otherwise. The final constraint states that selling the house without incurring default costs is only possible if the household is able to meet its mortgage obligations and has positive equity.

Case 2: $n = 0$ (The agent gets the option to buy a home)

Agents who receive the option to buy a home at the start of a given period must decide whether to exercise that option, and if they become homeowners, what mortgage to use to finance their house purchase. Write $K(\omega_0)$ for the set of mortgage contracts available to a household in state ω_0. The set $K(\omega_0)$ has typical element $\kappa = (\zeta, r^{\zeta}, h_0)$. The household's value function solves:

$$\begin{aligned} V_M(a, y, 1, h, 0; \emptyset) &= \max_{c \geq 0, a' \geq 0, H' \in \{0,1\}, \kappa \in K(\omega_0)} u(c, H'h_0 + (1 - H')h^1) \\ &+ (1 - H')\beta E_{y'|y} \left[\begin{array}{c} (1 - \rho_O)(1 - \gamma)V_M(a', y', 0, h^1, 1; \emptyset) + \\ (1 - \rho_O)\gamma V_M(a', y', 0, h^1, 0; \emptyset) + \rho_O V_O(a') \end{array} \right] \\ &+ H'\beta E_{(y',h')|(y,h_0)} \left[\begin{array}{c} (1 - \rho_O)V_M(a', y', 1, h', 1; \kappa) \\ +\rho_O V_O(a' + \max\{qh_0 - b_1(\kappa), 0\}) \end{array} \right] \end{aligned}$$

subject to:

$$\begin{aligned} c + a' &= y + (1 + r)(a - H'\nu 1_{\{\zeta = FRM\}} qh_0) - H'(m_0(\kappa) + \delta h_0) - (1 - H')Rh^1 \\ a &\geq H'\nu 1_{\{\zeta = FRM\}} qh_0 \end{aligned}$$

Households who choose to become homeowners ($H' = 1$) choose the contract $\kappa^* \in K(\omega_0)$ that maximizes their future expected utility. We will write $\Xi(\omega_0) = \kappa^*$ for this part of the household's choice, while $\Xi(\omega_0) = \emptyset$ if $H' = 0$. Note that included in the choice of the contract is the size of the house h_0.

3.1.3 Young agents

For young agents, the state space is $\Omega_Y = I\!R^+ \times \{y^L, y^M, y^H\}$ with typical element $\omega = (a, y)$. The value function $V_Y : \Omega_Y \mapsto I\!R$ for a young agent with assets a and income y solves

$$\begin{aligned} V_Y(a,y) &= \max_{c\geq 0, a'\geq 0} \left\{ u\left(c, h^1\right) + \beta E_{y'|y}\left[(1-\rho_M)V_Y(a',y') + \rho_M V_M(a',y',0,h^1,0;\emptyset)\right]\right\} \\ \text{s.t. } c + a' &= y + a(1+r) - Rh^1. \end{aligned}$$

3.2 Intermediary's problem

All possible uses of loanable funds must earn the same return for the intermediary. This implies, first, that the unit price q of housing capital must equal $\frac{1}{A}$.[7] Otherwise, the intermediary would enjoy an unbounded profit opportunity. Arbitrage between renting and selling houses also requires that:

$$q = \sum_{t=1}^{\infty} \frac{R-\delta}{(1+r)^t} \Longleftrightarrow R = rq + \delta. \tag{3.3}$$

In particular, a change in q must be associated with a change in R in this environment. Finally, arbitrage requires that for all mortgages issued at a given date, the expected return on the mortgage net of expected foreclosure costs cover the opportunity cost of funds, which by assumption is the returns to storage plus the servicing premium ϕ.

To make this latter condition precise, given discount rate $r + \phi$, denote the expected present value to the intermediary of a mortgage contract κ held by a mid-aged agent in state $\omega \in \Omega_M$ by $W^\kappa(\omega)$. In order to define $W^\kappa(\omega)$, we need to consider several cases. First, if the

[7] Specifically, the intermediary chooses k to solve $\max \quad qAk - k$ which implies that $qA = 1$ must hold in equilibrium.

homeowner's mortgage is not paid off, so that $\omega = (a, y, 1, h, n; \kappa)$ with $n \in (0, T-1]$, then

$$\begin{aligned} W^{\kappa}(\omega) &= \left(D^{I}(\omega) + D^{V}(\omega)\right) \min\{(1-\chi)qh, b_n(\kappa)\} + S(\omega) b_n(\kappa) \\ &\quad + \left(1 - D^{I}(\omega) - D^{V}(\omega) - S(\omega)\right) \left(\frac{m_n(\kappa)}{1+r+\phi} + E_{\omega'|\omega} \left[\frac{W^{\kappa}(\omega')}{1+r+\phi} \right] \right). \end{aligned}$$

Second, if the household just became mid-aged and her budget set is not empty so that $\omega_0 = (a_0, y_0, 0, h^1, 0)$ and, for some contract κ,

$$y_0 + \left(a_0 - \nu q h_0 \cdot 1_{\{\zeta = FRM\}}\right)(1+r) - m_0(\kappa) - \delta h_0 \quad \geq \quad 0,$$

then

$$W^{\kappa}(\omega_0) = \frac{m_0(\kappa)}{1+r+\phi} + E_{\omega'|\omega_0} \left[\frac{W^{\kappa}(\omega')}{1+r+\phi} \right].$$

Finally, in all other cases, $W^{\kappa}(\omega) = 0$.[8]

The zero profit condition on a loan contract κ can then be written as

$$W^{\kappa}(\omega_0) - (1 - \nu 1_{\{\zeta = FRM\}}) q h_0 = 0. \tag{3.4}$$

where ω_0 is the borrower's state at origination. In equilibrium, the set $K(\omega_0)$ of mortgage contracts available to an agent who becomes mid-aged in state ω_0 is the set of contracts that satisfy condition (3.4).

3.3 Aggregate demand for housing capital

The household's problem yields decision rules for a given set of prices. In turn, these decision rules imply in the usual way transition probability functions across possible agent states.[9] In section 5 we study equilibria in which the distribution of agent states is invariant under those probability functions. Denote by μ_Y, μ_M, and μ_O these invariant distributions for agents in

[8] Specifically, this is the case when: (i) the agent just turned mid-aged and her budget set is empty; (ii) the agent is a renter; or (iii) the agent has been mid-aged for more than T periods.

[9] *Note: these distributions are formally derived in appendix C.*

each possible life stage defined respectively on Ω_Y, Ω_M, and Ω_O. For instance, $\mu_Y(a, y)$ is the steady state cross-sectional distribution of young agents over wealth and earnings.

Given these steady state distributions, the housing market capital clearing condition can be stated succintly:[10]

$$\int_{\Omega_M} h1_{\{H'=1, h(\omega)=h\}} d\mu_M \; - \int_{\Omega_M} h'1_{\{H'=1\}} P(h'|\omega) d\mu_M \; = Ak, \tag{3.5}$$

where k is the quantity of deposits the intermediary transforms into housing capital. This condition simply says that in equilibrium the production of new housing capital must equal the housing capital lost to devaluation. If houses tend to appreciate on average, market clearing requires instead that the intermediary transform part of its stock into the consumption good. Because $q = \frac{1}{A}$ holds in equilibrium, the intermediary is willing to accommodate any allocation of total housing capital across renters and owners.

3.4 Definition of a steady state equilibrium

Equipped with this notation we may now define an equilibrium. A steady-state equilibrium is a set $K : \Omega_M \mapsto \{FRM, LIP\} \times I\!R^+ \times \{h^2, h^3\}$ of mortgages available to households who receive the home-buying option, a pair of housing capital prices $(q, R) \geq (0, 0)$, a value $k > 0$ of investment in housing capital, agent value functions $V_{age} : \Omega_{age} \mapsto I\!R$ for $age \in \{Y, M, O\}$, saving policy functions $a'_{age} : \Omega_{age} \mapsto I\!R^+$, a mortgage choice policy function $\Xi : \Omega_M \mapsto K(\omega_0)$, a housing policy function $H' : \Omega_M \mapsto \{0, 1\}$, mortgage termination policy functions $D^I, D^V, S : \Omega_M \mapsto \{0, 1\}$, and distributions μ_{age} of agent states on Ω_{age} such that:

1. Household policies are optimal given all prices;
2. $q = \frac{1}{A}$;
3. The allocation of housing capital to the rental and the owner-occupied market is optimal for the intermediary. That is, condition (3.3) holds;

[10] *Note: this expression is derived in appendix D.*

4. The intermediary expects to make zero profit on all mortgages. In other words, condition (3.4) holds for all $\omega_0 \in \Omega_M$ and all mortgages in $K(\omega_0)$;

5. The market for housing capital clears every period (i.e. (3.5) holds);

6. The cross-sectional distribution of household states is invariant given pricing functions and agent policies.

4 Calibration

We choose our benchmark set of parameters so that a version of our economy with only FRM mortgages matches the relevant features of the US economy prior to 2003. As figure 1 shows, FRMs accounted for around 85% of mortgages and the fraction was mostly stable between 1998 and 2003. Furthermore, evidence available from the American Housing Survey (AHS) suggests that mortgages with non-traditional amortization schedules accounted for a small fraction of the 15% of non-FRMs prior to 2003. Traditional FRMs and traditional ARMs accounted for 95% of all mortgages in the AHS sample before then. At the same time, data available from the Federal Housing Finance Board for fully amortizing loans show no increase in average loan-to-value ratios between 1995 and 2003. These numbers suggest that high-LTV (low downpayment), delayed amortization mortgages accounted for a small fraction of the stock of mortgages and of originations before 2003.

We will also calibrate the benchmark economy under the assumption that $\gamma = 0$, i.e. that buying a home is a one-time-only option for computational tractability.[11] We experimented with various small values of γ and found that our steady state results change very little. For

[11] Forcing agents who have sold their home or defaulted to become renters for the rest of their life enables us to price mortgage contracts for each possible asset-income-house size position at origination independently from rates offered to borrowers with different characteristics since the value of renting is independent of the yield schedule in that case. When agents have the option to take another mortgage after they terminate their first contracts the value of renting depends in part on what terms are offered on contracts offered at positions different from their situation when they become mid-aged. Instead of solving one fixed point problem at a time, we need to jointly solve a high-dimensional set of fixed points. This problem is computationally demanding, even in steady state.

values of γ near or in excess of 1%, some recalibration becomes necessary. We emphasize, however, that even when $\gamma = 0$, home-buyers are not identical at origination. Since agents become mid-aged after a different numbers of period, the model generates an endogenous distribution of asset-holdings among potential home-buyers. As we will argue in the next section, this heterogeneity matters critically for the impact of changes in the mortgage menu. Likewise, the model generates an endogenous distribution of transitions to home-ownership by age (defined as the number of periods an agent has been alive) which captures important features of its empirical counterpart in the US.

We will think of a model period as representing 2 years. We specify some parameters directly via their implications for certain statistics in our model. These include the parameters governing the income and demographic processes. The other parameters will be selected jointly to match a set of moments with which we want our benchmark economy to be consistent.

We set demographic parameters to $(\rho_M, \rho_0, \rho_D) = (\frac{1}{7}, \frac{1}{15}, \frac{1}{10})$ so that, on average, agents are young for 14 years starting at 20, mid-aged for 30 years, and old aged (retired) for 20 years. The income process is calibrated using the Panel Study of Income Dynamics (PSID) survey. We consider households in each PSID sample whose head is between 20 and 34 years of age to be young while households whose head is between 35 and 64 years are considered to be mid-aged. Each demographic group in the 1999 and 2001 PSID surveys is then split into income terciles. The support for the income distribution is the average income in each tercile in the two surveys, after normalizing the intermediate income value for mid-aged agents to 1. This yields a support for the income distribution of young agents of {0.2937, 0.7855, 1.7452}, while the support for mid-aged agents is {0.3129, 1, 2.5164}. We assume that income in old age is 0.4. This makes retirement income 40% of median income among the mid-aged, which is consistent with standard estimates of replacement ratios. Thus, our process generates a hump shape in earnings across age.

We then equate the income transition matrix for each age group to the frequency distribution of transitions across terciles for households which appear in both the 1999 and the

2001 survey. The resulting transition matrix for young agents is:

$$\begin{bmatrix} 0.6828 & 0.2581 & 0.0591 \\ 0.2690 & 0.5103 & 0.2207 \\ 0.0481 & 0.2317 & 0.7202 \end{bmatrix}$$

while, for mid-aged agents, it is:

$$\begin{bmatrix} 0.8032 & 0.1804 & 0.0164 \\ 0.1545 & 0.6901 & 0.1554 \\ 0.0423 & 0.1295 & 0.8282 \end{bmatrix}$$

The economy-wide cross-sectional variance of the logarithm of income implied by the resulting distribution is near 0.68, while the autocorrelation of log income is about 0.75.[12] We let the (two-year) risk-free rate be $r = 0.08$ and choose the maintenance cost (δ) to be 5% in order to match the yearly gross rate of depreciation of housing capital, which is 2.5% annually according to Haring et al.(2007).

The terms of FRM contracts are set to mimic the features of common standard fixed-rate mortgages in the US. The down-payment ratio ν is 20% while the maturity T is 15 periods (or 30 years). The LIP contract we introduce assumes $n^{LIP} = 3$ and $T = 15$ so that agents make no payment toward principal for 6 years and make fixed payments for the remaining 12 contract periods (or 24 years) unless the contract is terminated before maturity.

Housing choices depend on the substitutability of consumption and housing services as well as the owner-occupied premium. We specify, for all $c > 0$ and $h \in \{h^1, h^2, h^3\}$,

$$U(c, h) = \psi \log c + (1 - \psi) \log h.$$

[12]Krueger and Perri (2005) report estimates for the cross-sectional variance of log yearly income of roughly 0.4 and for the autocorrelation of log income in the $[0.80 - 0.95]$ range. These numbers imply that log two-year income has an autocorrelation in the $[0.88 - 0.96]$ range and variance in the $[0.36 - 0.39]$ range. The details of the conversion from one-year to two-year numbers are available upon request. The difficulty is that aggregating an MA(1) process leads to an ARMA(1,1) process.

Preferences are fully described by (θ, ψ, β). We select these parameters in our joint calibration, to which we now turn.

Ten parameters remain: the owner-occupied premium (θ), the household discount rate (β), the utility weight on consumption (ψ), housing TFP (A), the rental unit size (h^1), house sizes (h^2, h^3), the mortgage service premium (ϕ), the foreclosure transaction cost (χ), and the house shock probability (λ). We select these parameters jointly to target: home-ownership rates, the average ex-housing assets-to-income ratio for mid-aged agents, the average loan-to-income ratio at mortgage origination, the ratio of rents (in imputed terms for home-owners) to personal consumption expenditures for all households, the average rent-to-income ratio for low-income renters, the ratio of housing spending to personal consumption expenditures for homeowners, the average yield on FRMs, the average loss severity rates on foreclosed properties, the average market discount on foreclosed houses, and the average foreclosure rate prior to 2003.

We now elaborate our approach to measuring target values. When $\gamma = 0$, agents only become homeowners when they become mid-aged. Correspondingly, we target the ownership rate among households whose head is between 35 and 44. The Census Bureau reports that rate is roughly $\frac{2}{3}$.[13] The model's counterpart to that number is the rate of ownership among agents who have been mid-aged for five periods or fewer. This is the rate we report throughout the paper.

The average non-housing assets to yearly income ratio we choose to target is based on Survey of Consumer Finance (SCF) data. The average ratio of non-housing assets to income among homeowners whose head age is between 35 and 64 in the 2001 survey is 1.86, which corresponds to a ratio of assets to two-years worth of income of 0.93.[14]

The mortgage loan size at origination is $(1 - \nu)hq$ for FRMs and hq for LPMs, where

[13] See http://www.census.gov/hhes/www/housing/hvs/annual08/ann08ind.html, table 17.

[14] Because agents only have one asset in our model besides a house, we interpret a as net assets. Our measure of net assets does not include housing-related assets or debts, such as home equity or mortgages. Since agents are not allowed to have negative assets in our model, households who have negative non-housing assets are assumed to have zero assets in the calculation.

$h \in (h^2, h^3)$ is the initial house size. Evidence available from the American Housing Survey (AHS) suggests that prior to 2003 the ratio of this original loan size to yearly income is around 2.72 on average, or 1.36 in two-year terms.[15]

According to the evidence available from the Bureau of Economic Analysis, the ratio of rents (in imputed terms for owners) to overall expenditures is near 15%, and we make this our fourth target. Turning to the rent-to-income ratio for poor renters, Green and Malpezzi (1993, p11) calculate that poor households who are renters spend roughly 40% of their income on housing. On the other hand, according to the 2001 Consumer Expenditure Survey, expenditures on owned dwellings account for 16.5% of the expenditures of home-owners.

Next, we choose to target an average FRM-yield of 7.2% yearly, or 14.5% over a two-year period. This was the average contract rate on conventional, fixed rate mortgages between 1995 and 2004 according to Federal Housing Finance Board data.

The loss severity rate is the present value of all losses on a given loan as a fraction of the default date balance. As Hayre and Saraf (2008) explain, these losses are caused both by transaction and time costs associated with the foreclosure process, and by the fact that foreclosed properties tend to sell at a discount relative to other, similar properties. Using a dataset of 90,000 first-lien liquidated loans, they estimate that loss severity rates range from around 35% among recent mortgages to as much as 60% among older loans. Based on these numbers we choose parameters so that in the event of default and on average,

$$\frac{\min\{(1-\chi)qh, b)}{b} = 0.5$$

where b is the outstanding principal at the time of default and qh is the house value. In other

[15]The AHS is a panel of about 55,000 houses and apartments. The survey is carried out every other year. For each survey year between 1995 and 2003, we selected all households who moved in the three years preceding the interview, who own their home and who have a mortgage. We do not observe mortgages at origination, but the income and loan-size information of recent movers is likely to proxy fairly effectively for their counterparts at origination time. Looking at these recent movers leaves us with between 2 and 3 thousand mortgages in each survey. We calculated the average loan-to-income ratio for each survey between 1995 and 2003 and, finally, averaged the resulting value across surveys.

words, on average, the intermediary recovers 50% of the outstanding principal it is owed on defaulted loans.

We target a market discount on foreclosed properties of 75%. We define this discount to be the average price of foreclosed properties divided by the average price of regular home sales, after conditioning on size at origination.[16] Hayre and Saraf (2008) estimate that foreclosed properties selling prices range from 90% of their appraised value among properties with appraisal values over $180,000 to 55% of their appraised values among properties with appraisal values near $20,000. Other studies of foreclosure discounts (see Pennington-Cross, 2004, for a review) typically find discount rates near 75% (i.e. a loss of 25% over comparable properties).

Note that the average foreclosure discount and the average loss severity rates are related since part of the loss incurred by intermediaries in the event of default stems from the fact that foreclosed properties tend to be devalued properties. However, a loss in market value of 25% alone could not account for an average loss severity rate of 50%. In the data, this discrepancy reflects the transaction costs associated with foreclosure. Our transaction cost parameter χ proxies for these costs and we use this parameter in our calibration to bridge the gap between the foreclosure discount and the total loss associated with foreclosure.

Finally, we target a two-year default rate of 3% which is near the average **two-year** foreclosure rate among all mortgages during the 1990s in the Mortgage Bankers Association's National Delinquency survey. Table 1 summarizes our parameterization.

5 Steady state results

In this section we study the effects of a permanent introduction of nontraditional mortgages. In contrast, section 6 studies the effect of a brief period of availability of nontraditional mortgages that ends with an unanticipated collapse in house prices and compares the features of the resulting transition experiment to the patterns displayed in the actual data in figure 1.

[16] *Note: See Appendix F for the foreclosure discount calculation.*

Table 1: **Benchmark parameters**

Parameter	Description	Value	Target
Parameters determined independently			
ρ_M	Fraction of young agents who become mid-aged	1/7	14 years of earnings on average prior to home purchase
ρ_O	Fraction of mid-aged agents who become old	1/15	30 years on average between home purchase and retirement
ρ_D	Fraction of old agents who die	1/10	20 years of retirement on average
r	Storage returns	0.08	2-year risk-free rate
δ	Maintenance rate	5%	Residential housing gross depreciation rate
ν	Downpayment on FRMs	0.20	Average Loan-to-Value Ratio
T	Mortgage maturity	15	30 years
n^{LIP}	Interest-only period for LIPs	3	6-years interest-only
Parameters determined jointly			
θ	Owner-occupied premium	3.220	Homeownership rates
λ	Housing shock probability	0.120	Foreclosure rates
A	Housing technology TFP	0.571	Average Loan-to-income ratio at origination
β	Discount rate	0.833	Average ex-housing asset-to-income ratio
ϕ	Mortgage service cost	0.042	Average mortgage yields
χ	Foreclosing costs	0.440	Loss-incidence estimates
ψ	Utility share on consumption	0.800	Average housing spending share
h^1	Size of rental unit	0.640	Rent-to-income ratio for low-income agents
h^2	Size of regular house	0.850	Owner's housing spending share
h^3	Size of luxury house	1.300	Foreclosure discount

5.1 The impact of introducing LIPs

Table 2 presents key steady state statistics in the benchmark economy and in an economy in which LIPs are available.

Table 2: Steady state statistics

	Data	Benchmark	FRM +LIP
Homeownership rate	67.00	66.78	72.12
Avg. ex-housing asset-to-income ratio	0.93	0.96	0.94
Avg. loan-to-income ratio	1.36	1.36	1.51
Avg. housing expenditure share	0.15	0.14	0.14
Rents-to-income ratio for renters	0.40	0.39	0.39
Housing spending share for homeowners	0.17	0.17	0.19
Avg. mortgage yields (FRMs, LIPs)	(14.50,NA)	(14.35,NA)	(14.06,17.51)
Loss-incidence estimates	0.50	0.50	0.46
Foreclosure rates	3.00	2.97	3.70
Foreclosure discount	0.75	0.71	0.70

The table shows that the presence of LIPs has two main consequences on steady state statistics: home-ownership rates and average default rates become significantly higher. When only FRMs are available, a large number of agents are unable to become homeowners because they can't afford a large downpayment. The fraction of newly mid-aged agents who enter housing markets and buy smaller houses rises from 38% to 41% when the LIP option is introduced. In addition, the fraction of agents who buy large houses rises from 35% to 39% as a result of the looser financial requirements imposed by LIPs.

Default rates, for their part, are higher when LIPs are present as a result of two complementary factors. First, LIPs enable agents at the bottom of the asset and income distribution to select into homeownership. These are high-default risk agents because they are more likely to find themselves unable to meet their mortgage payments at some point over the life of the contract. Second, even at equal asset-to-income conditions at origination, LIPs are associated with higher default rates because agents build up home equity slower than with FRMs. The

next section makes these ideas precise.

5.2 Selection and equity effects

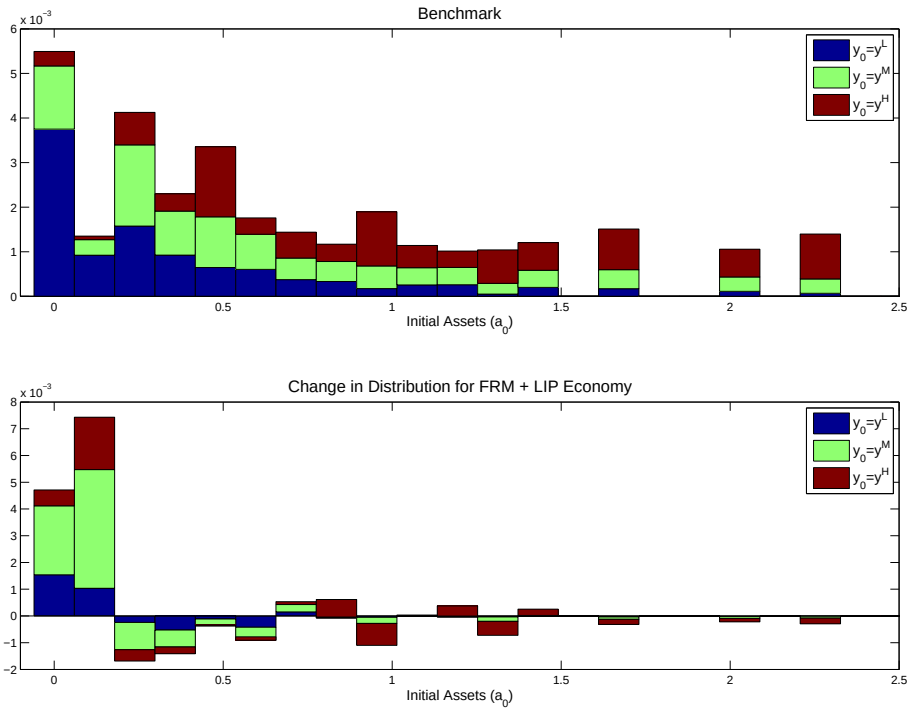
Recall that we have classified default into two categories. We call a default *voluntary* if the household can meet its mortgage payment but has negative net equity when it terminates its mortgage. If a household defaults because it cannot meet its mortgage payment, we call this event an *involuntary default*. LIP-holders are more prone to both events than FRM-holders because they tend to have low assets (selection effect) and because even holding borrower characteristics the same at origination, LIP holders are slower to accumulate equity in their homes (the equity effect.)

To understand the first effect, note that the stochastic nature of aging in our model implies that different agents become mid-aged after a different number of periods and with different levels of assets. Together with income levels when agents become mid-aged, this heterogeneity in asset holdings affects financing terms and housing decisions. Conversely, making LIPs available impacts the equilibrium distribution of wealth at purchase time, since a major incentive to save in the benchmark economy is the potential need to make a downpayment.

Figure 2 plots the endogenous distribution of assets among agents that just turned mid-aged. In the benchmark experiment, the upper panel shows that low income agents tend to have low assets, and vice-versa. While the largest mass of households is at the borrowing constraint, the second largest mass is at the downpayment amount for small houses ($0.30=\nu qh^2$). The lower panel shows how the distribution *changes* when LIPs are introduced. There is a noticeable shift to the left in the distribution as many agents anticipate that they may resort to the LIP option and no longer need to accumulate assets to meet downpayment requirements. In fact, the average level of assets of agents who just became mid-aged in the economy with LIPs is lower by 27% than its benchmark counterpart (0.55 vs. 0.75.)

Table 3 displays contract selection patterns in steady state. When LIPs are not available, many agents are constrained to rent because they cannot meet the downpayment imposed

Figure 2: Distribution of assets upon entering mid-age



by mortgages and/or cannot make the first payment (the distribution is left truncated at the downpayment level). Introducing LIPs enables some agents at the bottom of the asset distribution to become homeowners instead of renting, as the bottom panel of table 3 shows. LIPs also enable agents with high-income but low assets to buy bigger houses than they would without that option. These agents can afford high mortgage payments, but their assets are too low to meet high downpayment requirements for an h^3-size house.

Since the asset position at which agents become mid-aged depends on their income history when young and, in particular, on the length of that history, agents who turn mid-aged quickly accumulate assets over fewer periods, hence are more likely to find themselves with low assets when they are given the opportunity to purchase a house. As a result, our model predicts that agents who are young for fewer periods, hence, in a sense, are younger when they have to

Table 3: Rent-or-own decision rules by asset and income group

Contract	Rent	LIP		FRM	
House size	h^1	h^2	h^3	h^2	h^3
Benchmark					
y^L	$a_0 < 0.32$	–	–	$0.32 \leq a_0 < 3.14$	$3.14 \leq a_0$
y^M	$a_0 < 0.30$	–	–	$0.30 \leq a_0 < 1.24$	$1.24 \leq a_0$
y^H	$a_0 < 0.30$	–	–	$0.30 \leq a_0 < 0.53$	$0.53 \leq a_0$
FRM + LIP					
y^L	$a_0 < 0.20$	$0.20 \leq a_0 < 0.94$	–	$0.94 \leq a_0 < 3.14$	$3.14 \leq a_0$
y^M	–	$a_0 < 0.30$	–	$0.30 \leq a_0 < 1.24$	$1.24 \leq a_0$
y^H	–	–	$a_0 < 0.53$	–	$0.53 \leq a_0$

make a home buying decision, are more likely to remain renters. Likewise, young agents who do buy a home are more likely to buy a small house and to use a LIP to do so. Conversely, it predicts that newly mid-aged agents who choose to become renters tend to be younger, i.e. tend to be agents who became mid-aged after few periods of youth. These predictions are summarized in table 4.

Table 4: Mortgage Choice and Mean Age

Housing decision	FRM	LIP
Small house	30.83	30.92
Big house	32.70	27.17

Overall, agents who become home-owners do so after 5.2 periods of youth on average, i.e at an average age of 31 years old. By comparison, according to 2005 AHS data, first time home buyers are about 33 years old on average. More generally, our model implies an endogenous distribution of transitions into home-ownership by age. In our benchmark economy, 72% of transitions into ownership occur before age 40 (before 10 periods of life), while roughly 4% of these transitions are made by agents over age 60 (after more than 20 periods of life.) By way of comparison, consider for instance the subset of AHS respondents who changed residences

between 2003 and 2005 and report a transition from renting to owning their home.[17] Two-thirds of those transitions were made by households whose head was under 40 years of age, while under 5% of these transitions were made by households with heads over 60 years of age.

Whether because of bad income realizations or because they had fewer periods to save, agents who find themselves with low assets when home buying becomes an option are more likely to opt for a LIP than a FRM. These agents are much more likely to experience payment difficulties at some point in the life of the loan, hence are prone to involuntary default. This selection effect is compounded by the fact that opting for an LIP increases the likelihood of voluntary default by increasing the risk that a given home-owner will find themselves with negative equity.

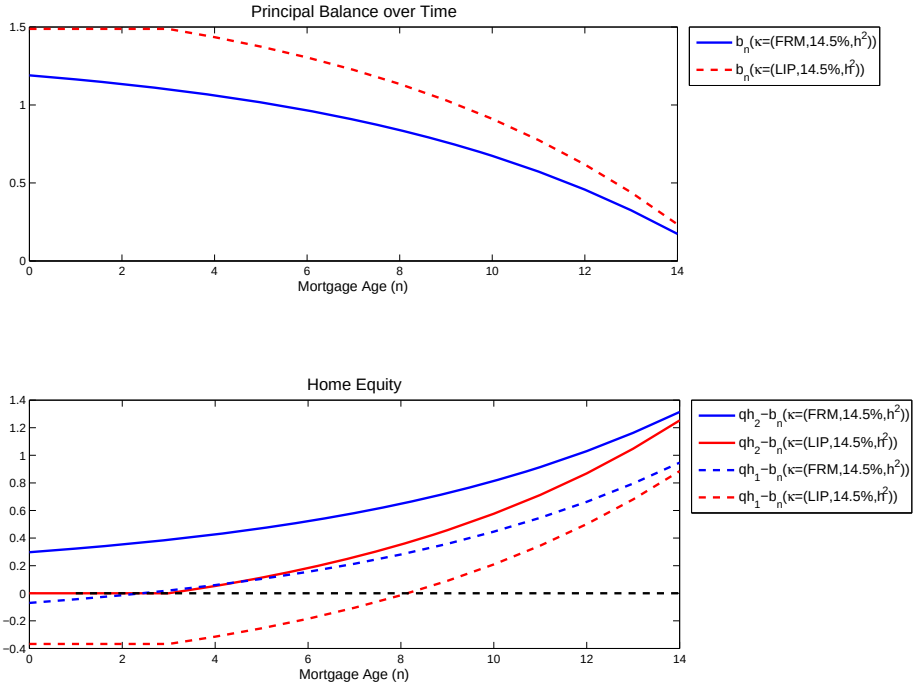
To illustrate this effect, Figure 3 displays the evolution of the principal balance and home equity as a function of maturity for both types of contract given an initial house size of h^2 and a contract rate of 14.5%. LIP-holders have less equity at all maturities for two reasons. First, LIPs require no down-payment. Second, while FRM contracts feature a progressive reduction of mortgage debt and a corresponding increase in home equity, LIP contracts only begin this process after three periods. It follows that a given devaluation shock is more likely to make net equity negative for LIP-holders than for FRM-holders, as the bottom panel of the figure illustrates. The dotted lines show home equity following a devaluation from h^2 to h^1 as a function of maturity. The shock causes equity to become negative on FRMs in the first three periods while for LIPs, equity become negative following the same shock for the first seven periods.

5.3 The determinants of default

Selection and equity effects combine to make default rates on LIPs much higher than on FRMs. Table 5 provides a breakdown of default frequencies by contract type across experiments. Each entry gives the fraction of mortgages of each type that go into default in steady state in each

[17] See http://www.jchs.harvard.edu/publications/rental/rh08_americas_rental_housing/rh08_americas_rental_housing.pdf, table A-4, for details.

Figure 3: Mortgage debt and home equity by contract type



of the two economies we consider.[18] When both contracts are offered, default rates are twice as high on LIPs as on FRMs. Since LIPs are characterized by much higher default rates than FRMs, they account for a disproportionate fraction of the overall default rate. LIPs account for nearly 47% of overall default rates even though they only represent 37% of all mortgages.

5.3.1 Default hazards

The high propensity of LIPs to default is also reflected in comparatively high hazard rates. Hazard rates measure the fraction of mortgages of a given type that default at each possible maturity conditional on having survived that long. In order to display equilibrium hazard rates, we generated a random sample of 50,000 mortgages drawn from the invariant distribu-

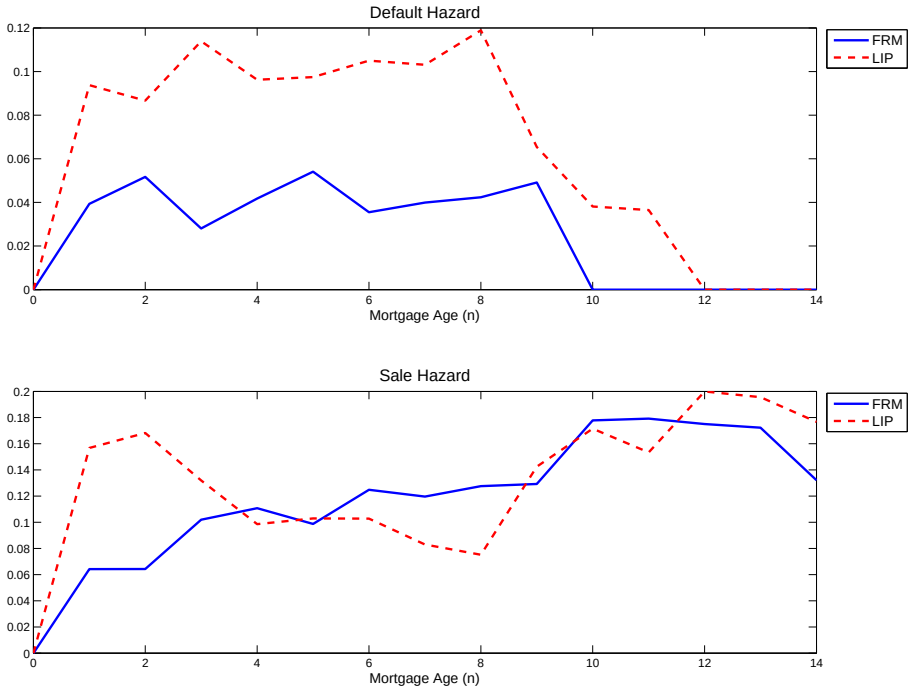
[18]Note: See Appendix F for the default rate calculations.

Table 5: Default rates by mortgage type

	voluntary	involuntary	total
Benchmark			
FRM	2.96	0.00	2.97
FRM + LIP			
FRM	2.78	0.01	2.79
LIP	5.61	0.22	5.83

tion in the economy where both types of mortgages are offered. Figure 4 plots average hazard rates associated with sales or default for each mortgage type.

Figure 4: Average hazard rates



Default hazards follow a hill-shaped pattern. They peak after two to three periods but eventually fall as borrowers accumulate equity in their home. Hazard rates are uniformly

higher for LIPs than for FRMs due to the selection and equity effects we have discussed. Sale hazard rates display a similar pattern for young loans but show no decline after they reach their peak.[19]

The representative sample of mortgages we generated also enables us to study the effect of loan and borrower characteristics at origination on hazard rates using a standard econometric approach. Specifically, we estimated a competing risk model with the following covariates for borrower $i \in \{1, \ldots, 50000\}$: 1) mortgage type ($1_{LIP_i} = 1$ if the borrower selected a LIP, 0 otherwise), 2) loan-to-income ratio at origination (LTY_i), and 3) asset-to-loan ratio at origination (ATL_i).

Define $\gamma^e_{n,i}$ to be the hazard rate at loan age n for homeowner i due to event $e \in \{D, S\}$, where D stands for default (of either type) while S stands for sale. We adopt a standard Cox proportional hazard specification for hazard rates, namely:

$$\gamma^e_{n,i} = H\left(\gamma^e_n \times \exp\left\{\beta^e_{LIP} 1_{LIP,i} + \beta^e_{LTY} LTY_i + \beta^e_{ATL} ATL_i\right\}\right),$$

where $H(x) = \exp(-\exp x)$ for all $x > 0$, and γ^e_n is the baseline hazard rate at loan age n. The six coefficients can then be estimated via maximum likelihood, together with estimates of the corresponding standard errors.[20]

Table 6 shows the result of this estimation. As expected, estimated hazard rates into default are significantly higher for LIPs than for FRMs, rise with loan-to-income at origination, and fall with the asset-to-loan ratio. These coefficients suggest that hazard rates are

[19]To understand why, recall that a termination is recorded as a sale provided the borrower has positive home equity. As figure 3 makes clear, early in the life of a mortgage, devaluation shocks cause equity to be negative and many terminations, therefore, are due to default rather than sales. For older loans, equity is likely to remain positive even if the house devalues. Another reason why sale hazards generally rise over the life of the loan is that it takes several shocks for borrowers to become exposed to the risk of involuntary default or to reach a point where consumption would become suboptimally low if they held on to their home.

[20]Most of the empirical literature on default rates adopts a version of this proportional hazard specification. It is difficult to compare our results directly to their outcomes because these studies usually control for covariates that have no clear counterpart in our model, and usually lack detailed information on borrower assets. Gerardi et al., 2009, for instance, include proxies for county-level economic and state-wide house price conditions, but do not control for assets at origination. Still, theirs and most other papers find as we do that high LTVs at origination and high debt-to-income ratios have a significant effect on default rates.

noticeably different across different sets of covariates. For instance, holding other covariates the same, simple calculations show that hazard rates on LIPs are twice as high as on FRMs. Because LIPs are more prone to default, they are less prone to sales relative to the baseline hazard, since the two termination risks are competing risks.

Table 6: Determinants of mortgage termination

Covariate	Default	Sale
LIP indicator	0.6812*** (0.0243)	-0.2515*** (0.0154)
Loan to Income Ratio	0.1182*** (0.0064)	0.3287*** (0.0050)
Assets to Loan Ratio	-0.2147*** (0.0358)	-0.6309*** (0.0215)

*Notes: Standard errors are in parenthesis; Log Likelihood : -76943.461; *** significant at 1% level*

5.3.2 Negative equity: necessary but not sufficient

In both the benchmark economy and the economy where LIPs are available, the vast majority (98.5%) of defaults involve negative equity. The key reason for this is that agents who have positive equity in their house and foresee that they may find themselves in an involuntary default situation tend to sell rather than run the risk of losing their equity to transaction costs. Several steady state statistics illustrate this behavior. Consider for instance the set of households who, should they choose to keep their house, face a positive probability of being in an involuntary default situation in the next period.[21] Almost 92% of these high-risk households choose to sell their house, while selling rates are around 10% among other mortgage holders.

While almost all foreclosures involve negative home equity, many households (roughly 70.7%) with negative home equity choose to keep their house and continue meeting their mortgage obligations. While defaulting would entail a net worth gain for these households, they would be forced to rent a smaller housing unit when $\gamma = 0$ and hence would forego the ownership premium.

[21] *Note: See Appendix F for the calculation of this probability.*

These model predictions are consistent with the empirical literature on the determinants of foreclosure (see, e.g., Gerardi et al., 2007). Available data suggest that most foreclosures involve negative equity but that, at the same time, most households with negative equity choose not to foreclose. Our model, in this sense, captures the fact that most foreclosures involve a combination of negative equity and adverse income shocks.

5.4 The endogenous distribution of interest rates

5.4.1 Yield schedules

A distinguishing feature of our model is that mortgage terms depend not only on mortgage types but also on the initial asset and income position of borrowers as well as the size of the loan. Figure 5 plots the menu of equilibrium FRM and LIP (2 year) interest rate offerings agents can obtain from the intermediary when they become mid-aged, depending on the house size they opt for and their asset-income position at origination.[22] Note that these are equilibrium offerings and only some points on these menus will actually be selected in equilibrium.

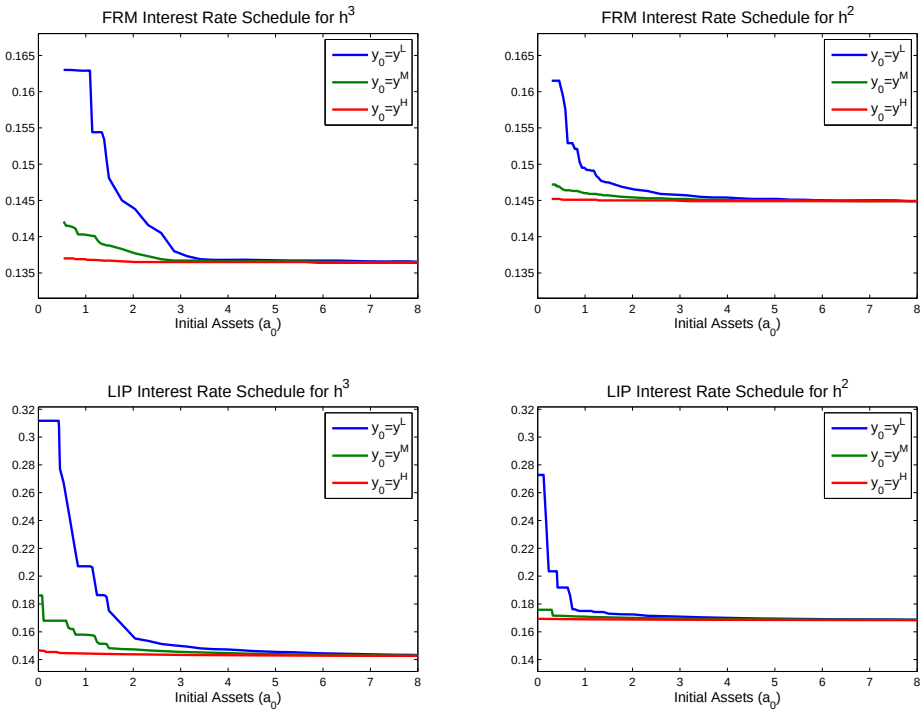
All interest rate schedules in Figure 5 are left-truncated because agents whose income and assets are too low do not get a mortgage in equilibrium. The left truncation can be thought of as an endogenous borrowing constraint associated with different borrower characteristics.[23] Some agents become mid-aged with asset and income characteristics such that the intermediary could not break even on a mortgage issued to them, which may occur even when the agents have the means to finance the initial downpayment.

Among agents who do receive a mortgage offer, yields fall both with assets and income. This prediction accords with the well-documented mortgage industry practice of including

[22] Yields offered on FRMs are the same in the benchmark and FRM+LIP economies. This is because the house price is unchanged and there are no externalities.

[23] The rate is truncated since the household default probability is too high for the bank to break-even at any mortgage rate. This happens when the mortgage payment in the first period is so high that the budget set is empty (i.e., for $m_0(\zeta, r^\zeta, h_0) > y_0 + (a_0 - \nu q h_0 \cdot 1_{\{\zeta = FRM\}})(1+r)$ with $c = a' = 0$). Since $m_0(\zeta, r^\zeta, h_0)$ is strictly increasing in r^ζ, we know there is an interest rate $\overline{r}^\zeta$ that depends on borrower characteristics (y_0, a_0, h_0) such that for any $r > \overline{r}^\zeta(y_0, a_0, h_0)$ the bank cannot break even.

Figure 5: Equilibrium yield schedules for the FRM+LIP economy

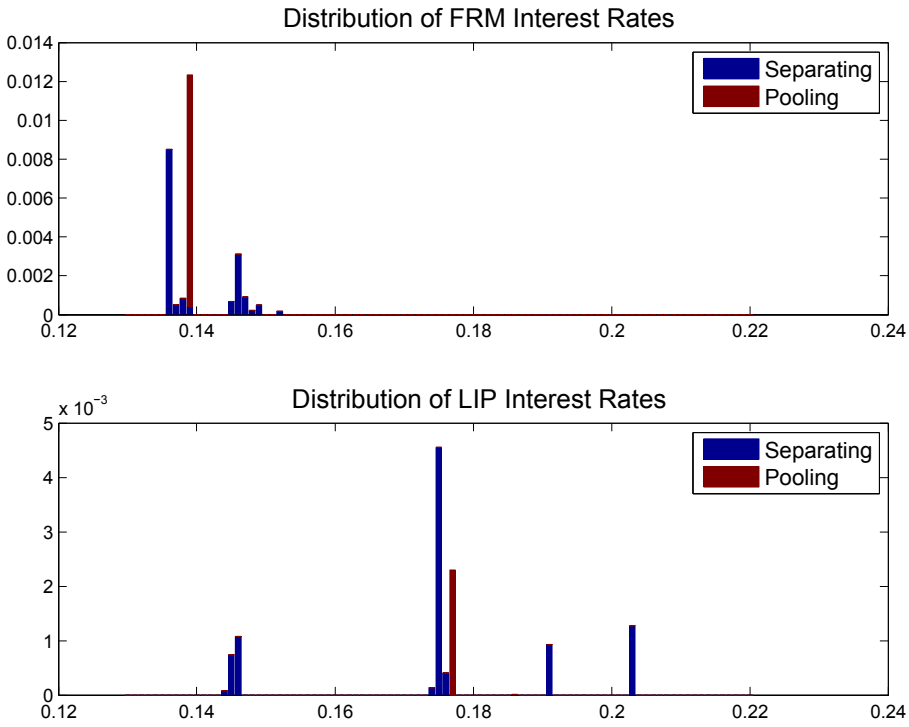


overall debt-to-income ratios in their rate sheets. It is also borne out by the statistical evidence available from the Survey of Consumer Finance, as we will discuss below. A glance at the vertical scale of the figure reveals that LIP rates exceed FRM rates at all possible asset-income positions.

Figure 6 graphs the equilibrium distribution of interest rates by mortgage type when both mortgages are available. There is relatively little variation in FRM rates, but the distribution of LIP yields displays a fair amount of dispersion.[24] The figure also shows that some agents

[24]By way of comparison, statistical evidence available from the Survey of Consumer Finance suggests that traditional (FRM) mortgages exhibit less variation than other mortgages. To compute the relevant moments in the data, we looked at all the mortgages issued within the two years prior to the 2004 survey. We restrict the sample to recently issued mortgages so that current income and assets are reasonable proxies for their counterparts at origination time. We also restrict our attention to households whose head age is between 30 and 45 since mortgages are only issued at the mid-age stage in our model. We define net worth as liquid assets,

Figure 6: Distribution of equilibrium interest rates for FRM + LIP model



who take advantage of the nontraditional mortgage to become home-owners incur (2 year) interest rates in excess of 20%. On the other hand, agents who switch from FRMs to LIPs either to purchase bigger homes or because they find it optimal to delay payments receive rates not unlike those offered on FRMs, because their default risk does not rise much as a result of the change.

Table 7 shows that much of the equilibrium variation in yields can be accounted for by loan

CDs, stocks, bond, vehicles, primary residence, real estate investment, business interest minus housing debts, credit card, installment debts, and line of credits. This notion of net worth includes housing equity because we observe agents shortly after the mortgage origination. Housing equity, at that time, reflects mainly the down-payment made at origination by the borrower. That downpayment, in turn, was part of assets prior to the origination. Relative to these data benchmarks, our model understates the variation in yields suggested and overstates the degree to which income and yields are correlated. A key reason for both findings is that the SCF sample of both FRMs and other mortgages are characterized by much heterogeneity in maturity and initial loan-to-value ratios which we do not model and for which SCF data do not enable one to control. This heterogeneity raises the volatility of yields and reduces the correlation with asset and income for reasons which our model cannot replicate.

Table 7: Determinants of log mortgage yield

Covariate	Coefficient
Assets at origination	-0.0867*** (0.0004)
Income at origination	-0.1184*** (0.0005)
Loan size	0.1089*** (0.0013)

*Notes: Standard errors are in parenthesis; $R^2 = 0.7231$; *** significant at 1% level*

and borrower characteristics at origination. Once again using our representative sample of 50,000 mortgages to regress log mortgage yields on assets, income and loan size at orgination yields an R^2 of nearly three-quarters. Furthermore, higher assets and income are associated with lower yields since they reduce the likelihood of default. A higher loan size at origination, however, is associated with a higher yield as a result of the negative correlation in our model between loan size and downpayments.

5.4.2 Subprime mortgages

Figure 1 illustrates the large increase in subprime mortgages as a fraction of all mortgages starting between 2003 and 2006. If we define subprime as the bottom 30% of households who obtained the highest mortgage interest rates in our model when both FRMs and LIPs are available, the average return is 14.09% on prime mortgages and 18.14% on subprime mortgages. Moreover, the model average return is 14.03% on prime FRMs and 15.13% on subprime FRMs, while the average return is 14.60% on prime LIPs and 18.27% on subprime LIPs. These interest rates reflect default likelihoods. The default rate is 2.01% on prime FRMs and 3.32% on prime LIPs, while the default rate is higher at 3.35% on subprime FRMs and 5.96% on subprime LIPs.

5.5 Pooling versus separating contracts

Allowing mortgage contracts to "separate" borrowers on the basis of observable characteristics like their income and wealth at origination seems paramount in a model designed to

understand default patterns and produces a model that is consistent with the fact that terms vary a lot across contracts even among mortgages of the same type. At the same time, doing so requires pricing mortgages individually for each possible combination of asset, income and loan size characteristics, which greatly complicates our computations. An alternative assumption, which is computationally less demanding, is to impose that all mortgages of a given type offer the exact same terms independent of borrower characteristics. This "pooling" assumption has been implemented by Garriga and Schlagenhauf (2009). Low-risk borrowers in such an equilibrium subsidize high-risk borrowers. In particular, the intermediary issues contracts to some borrowers on which it expects to lose money. Such cross-subsidization is unlikely to survive in competitive environments since another intermediary can simply offer a contract with lower interest rate to households with observable high income and/or assets and skim those good customers away from the pooled contract.

In this section, we conduct a counterfactual experiment to examine the quantitative importance of allowing intermediaries to offer mortgage contracts that separate households on observables at origination like income, asset, and loan size characteristics rather than offering "pooling" FRM and LIP contracts. Table 8 compares key steady state statistics for the two equilibrium concepts in an economy where LIPs and FRMs are available, while Table 9 displays selection patterns.

In the pooling case, low-income agents with assets between 0.08 and 0.20 buy a house using a LIP. In the separating case, these agents are constrained to rent because their likelihood of default is too high to cover financing costs at any yield. On the other hand, low income agents with assets between 0.68 and 0.94 opt for an FRM instead of a LIP in order to avoid paying higher yields at origination. Finally, some of the agents who use LIPs to purchase a big house in the separating equilibrium opt instead for an FRM and a small house in the pooling equilibrium. As a result, the pool of LIP borrowers is of bad credit quality in the pooling equilibrium. The foreclosure rate, correspondingly, rises by almost 40% in the pooling equilibrium and as a consequence, LIP interest rates rise in Table 8. The difference between the equilibrium distributions of interest rates in the separating and pooling contracts

Table 8: The role of separation

	FRM+LIP	FRM+LIP, pooling
Homeownership rate	72.12	75.47
Ex-housing asset/income ratio	0.94	1.04
Loan to income ratio	1.51	1.66
Avg. housing expenditure share	0.14	0.13
Rents to income ratio for renters	0.39	0.39
Housing spending share for homeowners	0.19	0.18
Avg. mortgage yields (FRMs, LIPs)	(14.06,17.51)	(13.99,17.75)
Loss-incidence estimates	0.46	0.49
Foreclosure rates	3.70	5.16
Foreclosure discount	0.70	0.69

Table 9: Rent-or-own decision rules in pooling and separating equilibria

Contract	Rent	LIP		FRM	
House size	h^1	h^2	h^3	h^2	h^3
FRM + LIP					
y^L	$a_0 < 0.20$	$0.20 \leq a_0 < 0.94$	–	$0.94 \leq a_0 < 3.14$	$3.14 \leq a_0$
y^M	–	$a_0 < 0.30$	–	$0.30 \leq a_0 < 1.24$	$1.24 \leq a_0$
y^H	–	–	$a_0 < 0.53$	–	$0.53 \leq a_0$
FRM + LIP, pooling					
y^L	$a_0 < 0.08$	$0.08 \leq a_0 < 0.68$	–	$0.68 \leq a_0 < 3.41$	$3.41 \leq a_0$
y^M	–	$a_0 < 0.30$	–	$0.30 \leq a_0 < 1.34$	$1.34 \leq a_0$
y^H	–	–	$a_0 < 0.53$	–	$0.53 \leq a_0$

is plotted in Figure 6, which makes clear the differences in observed heterogeneity between the two equilibrium concepts.

In summary, pooling mortgage contracts have a large impact on the model's steady state default statistics and do not generate the heterogeneity in mortgage rates within contract type observed in the data.

5.6 The welfare implications of innovation

The introduction of LIPs unambiguously improves the welfare of all agents because they provide households with a new financing option without altering house prices given our linear housing technology. This provides a rationale for why we might see such contract innovation. However, the welfare consequences of innovation are bound to differ across agents. Agents whose homeownership prospects at birth are not significantly improved by the introduction of LIPs will not benefit much, while agents whose ownership prospects do rise significantly are likely to see their welfare rise markedly. This section quantifies the gains.

To determine the gains, we consider two measures. First, we calculate an ex-ante measure; we ask what fraction of an agent's lifetime consumption an agent would be willing to give up at birth in the benchmark economy to be born instead in an economy where both mortgages are offered.[25] Letting (k^L, k^M, k^H) be this consumption equivalent measure for agents born with income (y_Y^L, y_Y^M, y_Y^H), respectively, we find that $(k^L, k^M, k^H) = (3.80\%, 3.01\%, 0.35\%)$ which yields an average ex-ante welfare gain associated with the availability of the LIP option of around 2.39% in consumption-equivalent terms. Since income is quite persistent, households who are born poor know that they are most likely to remain renters their entire life in the benchmark economy due to the downpayment requirement of FRMs. Hence, they have the most to gain from the introduction of the LIP option. Likewise, households born rich know that they are most likely to opt for an FRM when they become mid-aged because they carry lower interest rates. Hence, they have the least to gain from the introduction of the LIP option. We should emphasize that these magnitudes would differ in a model where housing prices (both on the owner-occupied and rental markets) respond endogenously to the available mortgage types.[26]

Second, we consider an ex-post welfare measure starting from the cross-sectional earnings and wealth distribution from our benchmark economy and ask each agent how much they

[25] *Note: this welfare measure is formally defined in appendix E.*

[26] In an earlier version of this paper where we considered a strictly concave housing technology and prices varied endogenously, we still found there was an ex-ante gain to the introduction of LIPs.

Table 10: Ex-post consumption equivalent welfare for innovation

Age	k^L_{age}	k^M_{age}	k^H_{age}	Overall
Y	0.68%	0.30%	0.04%	0.34%
M(n=0)	0.05%	0.35%	0.37%	0.26%
Total	0.73%	0.65%	0.41%	0.60%

would be willing to pay to switch to an economy with both FRMs and LIPs. Since the benchmark parameterization assumes that agents only have a one-time purchase chance when they turn mid-aged (i.e. $\gamma = 0$) and mid-aged agents who already have an FRM cannot refinance into a LIP mortgage contract, only young agents and newly mid-aged agents in our benchmark economy can experience a welfare change when LIPs become available. Integrating out wealth using the cross-sectional distribution of young and newly mid-aged agents yields welfare measures conditional on income levels of (0.73%,0.65%,0.41%) with an average of 0.60%. Table 10 shows welfare results by income and by age. While these ex-post numbers are much lower than their ex-ante counterparts, they remain significant.

5.7 Policy experiment: the role of recourse

So far we have maintained the assumption that, in the event of default, the borrower's liability is limited to their home. In several states – known as anti-deficiency or non-recourse states – the law does in fact make it difficult for mortgage lenders to pursue deficiency judgments. The list of such states varies but generally includes Arizona, California, Florida (and sometimes Texas).[27] There are other states, known as "one-action" states, that allow the holder of the claim against the household to only file one lawsuit to either obtain the foreclosed property or to sue to collect funds. The list of such "nearly non-recourse" states includes Nevada and New York. Even in states where deficiency judgments are legal, conventional wisdom is that the costs associated with these judgments are so high, and the expected returns are so low,

[27]See, for instance, http://www.helocbasics.com/list-of-non-recourse-mortgage-states-and-anti-deficiency-statutes

that recourse is seldom used.

Some empirical studies (see Ghent and Kudlyak , 2009) find that recourse decreases the probability of default when there is a substantial likelihood that a borrower has negative home equity. In this subsection we quantify the role of the recourse assumption for equilibrium statistics. Table 11 compares steady state statistics in our FRM+LIP economy (which assumes no recourse) to their counterparts in the same economy with recourse. In particular, in the event of default by a borrower with assets $a \geq 0$ and house size h, the intermediary collects $\min\{(1-\chi)qh + a, b\}$ with recourse (as opposed to $\min\{(1-\chi)qh, b\}$ without recourse), while the household retains $\max\{(1-\chi)qh + a - b, 0\}$ with recourse (as opposed to $\max\{(1-\chi)qh - b, 0\}$ without recourse). In other words, in the recourse economy, any asset the household owns at the time of default can be claimed as collateral by the lender. Thus, recourse imposes a harsher punishment on borrowers, thus lowering the extensive default margin, and the higher repayment by borrowers lowers the intensive loss incidence margin to the lender. Both effects on these margins lead to lower interest rates.

Table 11: The role of recourse

	FRM+LIP (no recourse)	Full recourse
Homeownership rate	72.12	80.50
Ex-housing asset/income ratio	0.94	0.93
Loan to income ratio	1.51	1.68
Avg. housing expenditure share	0.14	0.14
Rents to income ratio for renters	0.39	0.39
Housing spending share for homeowners	0.17	0.18
Avg. mortgage yields (FRMs)	(14.06,17.51)	(12.48,16.03)
Loss-incidence estimates	0.54	0.33
Foreclosure rates	3.70	2.76
Foreclosure discount	0.70	0.73

As the table shows, this change in the environment greatly reduces average loss incidence rates for obvious reasons. At the same time, this makes default much more costly for households and, as a result, foreclosure rates fall by 28%. By comparison, Ghent and Kudlyak

(2009) estimate that at average borrower characteristics, the likelihood of default is about 20% higher in antideficiency states than in recourse states. Note that we assume that liquid assets are collected without any transaction costs, which raises the equilibrium impact of recourse. In that sense, our results should be considered an upper bound on the potential effect of recourse. An interesting aspect of this experiment is that allowing for recourse actually raises home-ownership rates. This is because mortgage rates fall when default risk decreases, making homeownership a cheaper option ex-ante.

Table 12: Ex-post consumption equivalent welfare for recourse

Age	k^L_{age}	k^M_{age}	k^H_{age}	Overall
Y	0.11%	0.15%	0.09%	0.12%
M(n=0)	0.05%	0.04%	0.01%	0.03%
M(n>0)	-0.07%	-0.11%	-0.13%	-0.10%
Total	0.09%	0.07%	-0.03%	0.04%

What are the normative consequences of the large positive economic changes in home-ownership and default associated with implementation of a recourse policy? In terms of an ex-ante measure, the consumption equivalent welfare gains to the recourse regime are given by $(k^L, k^M, k^H) = (0.34\%, 0.69\%, 0.45\%)$ for a total welfare gain of approximately one-half of one percent.

In terms of an ex-post measure, we start from the equilibrium stationary earnings/wealth distribution associated with our FRM+LIP non-recourse economy and ask each agent in the cross-sectional distribution how much they would be willing to pay to switch to an economy with recourse. The gains for the young and newly mid-aged agents who do not yet have a house arise from the lower interest rates they may pay. Again, since agents who already own houses cannot switch contracts when $\gamma = 0$ they do not receive the interest rate benefit and only receive the welfare loss associated with harsher punishment (attachment to their assets) in event of default. As Table 12 shows, while young and newly mid-aged agents receive a welfare gain of 0.15% owing to lower mortgage rates in an economy with full recourse, mid-

aged homeowners receive a welfare loss of 0.10% since they are subject to more repayment obligations in the event of foreclosure. These competing effects yield an economywide welfare gain of only 0.04% from moving to an economy where recourse is implemented. Since there are both winners and losers in this experiment, we also calculate the fraction of the population who favor the proposed change to recourse. Our model predicts that 21.33% of the population prefers recourse, 8.74% of the population opposes recourse, and the rest (69.93%) are indifferent. Thus, there would be little opposition to implementing recourse policies.

6 Transitional effects of mortgage innovation

The previous section shows that the introduction of nontraditional mortgages has significant long-run effects on foreclosure rates. We will now describe our main quantitative experiment designed to evaluate the role of these new mortgages in the foreclosure crisis depicted in Figure 1. Figure 1 suggests that the course of events leading up to the collapse of house prices and the foreclosure crisis can be decomposed into three basic stages. Prior to 2003, the composition of the mortgage stock is stable and traditional mortgages are the dominant form of home financing. Around 2003, the composition of the mortgage stock changes noticeably as nontraditional mortgages start accounting for a high fraction of originations. At the start of 2007, prices start collapsing and the flow of traditional mortgages begins rising once again as originations of non-traditional mortgages slow to a trickle.[28]

We will use our model to simulate this course of events and quantify the role of nontraditional mortgages using a three-stage experiment. In the first stage (the pre-2003 period), the economy is in our benchmark, FRM-only steady state. In the second stage of the experiment, we introduce the option for newly mid-aged agents to finance their house purchase with a LIP mortgage. We assume that this introduction is unanticipated by agents, but perceived as permanent once it is made. Two periods later, in the third stage, we shock the economy

[28]According to the Mortgage Origination Survey data, the share of traditional FRMs in originations had risen to nearly 90% by the second quarter of 2008.

with an unanticipated 25% aggregate price decline and take away the LIP option. This shock is meant to approximate the widely unanticipated collapse of house prices since the start of 2007.[29] In terms of our model, we cause home prices to fall by assuming that A rises in the third stage.[30] This drop in prices catches agents as a complete surprise so that, at the time of the shock, the distribution of states across agents is the one implied by the first two stages of the experiment. It is important to emphasize that we calibrated our parameters only to match the first stage of our experiment, so that the model predictions for the second two stages can be thought of as an informal test of the model.

Because the intermediary is also surprised by the price shock, it experiences unforeseen revenue and capital losses. A formal definition of the intermediary's net profit on a given mortgage is provided in appendix B. In steady state, those profits are zero. However, the unexpected drop in prices causes net-equity to turn negative on many houses. Thus, default rates rise unexpectedly which reduces revenues and raises foreclosure losses for the intermediary, causing profits to become negative until contracts written before the price shock disappear. One has to be explicit about who bears these losses. We assume that constant lump-sum taxes are imposed on all agents following the price shock in such a way as to exactly cover the intermediary's losses in present value terms. Computationally, this involves guessing a value for the constant and permanent tax, solving for the new steady state equilibrium and the transition to this new state, evaluating the present value of the intermediary's transitory losses, and updating the permanent tax level until losses and tax revenues match.[31]

Figures 7 and 8 and Table 13 show the outcome of this three-stage experiment. Once they become available, LIPs account for 33% of all originations, raising the fraction of LIPs

[29]The Case-Shiller home price index fell by 33% in real terms between the first quarter of 2007 and the first quarter of 2009, while the home price index computed by the Federal Housing Finance Agency house price index fell by 14% during the same period.

[30]Of course, this exogenous unanticipated rise in A is only intended to provide a model-based explanation of the housing price drop and should not be interpreted as a theory for the behavior of house prices since 2003. Just what caused the home price to collapse in 2003 is likely to remain an open question for a long time.

[31]Note: see appendix A for details. There are obviously many possible ways to redistribute the intermediary's losses. Per capita losses are negligible in practice and barring extremely concentrated tax schemes, their exact distribution will not have large effects on the results we present.

Figure 7: Fraction of LIPs in mortgage stock during transition

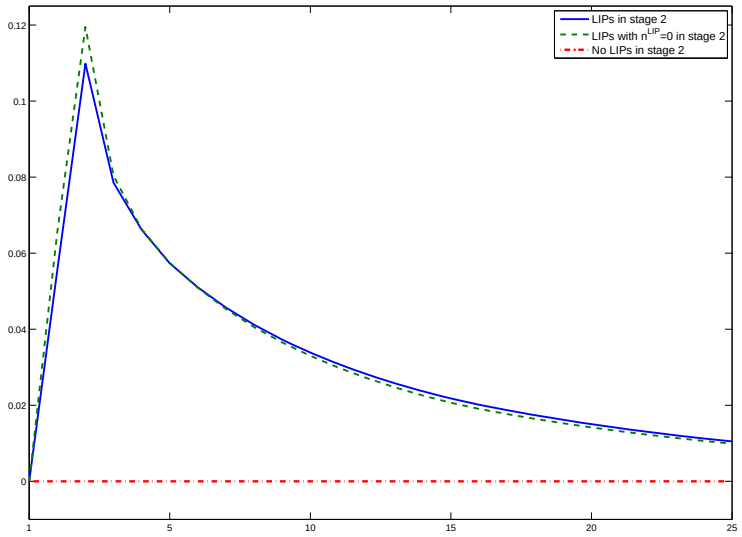


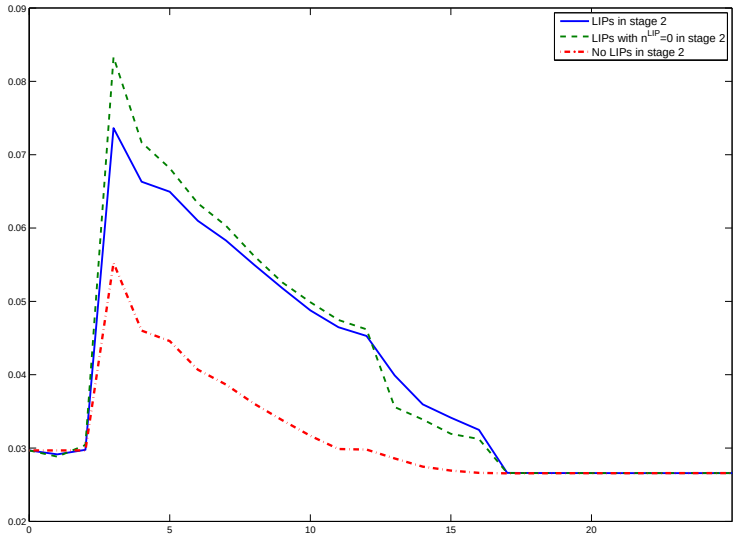
Table 13: Summary of transition results

	Data	LIPs in stage 2	No LIPs in stage 2	$n^{LIP}=0$ in stage 2
Fraction of LIPs in originations in stage 2	[20-35%]	33%	0%	37%
Total increase in foreclosures 2007Q1-2009Q1	150%	148%	86%	189%

Notes: Our data counterpart for LIP originations is a rough estimate based on available estimates of subprime mortgage originations, and the fact that the fraction of traditional FRM mortgages in origination fell to 50% in 2005, from a peak of 85%.

in the mortgage stock from 0 to 11% in two model periods (meant to capture the 4 years between 2003 and 2006). While it is difficult to find precise data on the exact share of mortgages with very low down-payments between 2003 and 2006, the high popularity of non-traditional mortgages between 2003-06 accords well with the available evidence. According to the Mortgage Origination Survey, traditional FRMs accounted for only 50% of all originations in 2005 (the first year of the survey.) Of course, not all other mortgages issued were of the low-

Figure 8: Foreclosure rates during transition



initial-payment type. However, the fact that FRMs account for a stable 85% of the mortgage *stock* prior to 2003 suggests that they accounted for roughly 85% of originations before then. It seems safe to assume that the 35% drop in FRM originations owes in significant part to the popularity of high-LTV, non-traditional amortization loans. It is estimated that subprime loans alone accounted for a quarter of all originations in 2005, and the Loan Performance data discussed by Gerardi et al. (forthcoming) suggests that the majority of these mortgages featured high leverage and/or delayed amortization. Thus it seems reasonable to assume that loans with high-leverage and/or other non-traditional features accounted for between 20% and 35% of originations at that time. Once again, we want to emphasize that the model's ability to fall within originations of nontraditional mortgages witnessed in the data in Table 13 is a strong test of the model.

The homeownership rate also rises as more agents are able to purchase homes thanks to mortgage innovation. Because we do not give home-buyers the option to default in the first period of their life and the number of originations rises in stage 2, average default rates fall

slightly in the first period of stage 2, but they rise in the second period of stage 2 as LIPs issued in the first period now become subject to default.

Once the price shock strikes in stage 3, foreclosure rates double to over 7.36% in one period. The aggregate shock pushes a number of agents with contracts written prior to the shock (when houses were expensive) into a negative net equity position. The price shock thus causes foreclosure rates to rise by around 148% over the first two years of the crisis. According to the data displayed in Figure 1 and Table 13, foreclosure rates rose by about 150% between the first quarter of 2007 and the first quarter of 2009. Once again, since none of the parameters were chosen to match moments in the third stage of the experiment, this is a strong test of the model.

To answer the question posed at the beginning of the paper - how much of the rise in foreclosures can be attributed to the increased originations of non-traditional mortgages between 2003 and 2006? - we also need to run a counterfactual experiment where, in stage 2, LIPs are not offered but we shock the economy with the same price decrease in stage 3. The result is shown in the dotted lines on figure 8. The increase in foreclosure rates become noticeably smaller, because the price shock now strikes an economy where agents have more equity in their homes. Specifically, as shown in table 13, the increase in foreclosure rates on impact falls to 86% when LIPs are not introduced. Thus, the origination of nontraditional mortgages for two model periods can explain $\frac{148-86}{148} \approx 42\%$ of the rise in foreclosures.

We can also quantify the relative importance of low downpayments and delayed amortization – the two key features of our LIPs – by running a third experiment where in stage 2, LIPs feature a zero downpayment, but no interest only period. As figure 7 shows, LIPs become slightly more popular in stage 2 in part because, without an interest only period, the default risk on those mortgages falls as do, therefore, yields. Figure 8 shows, more importantly, that the impact on foreclosure rates of the price shock in stage 3 is very similar to what we obtained in the first experiment (even somewhat higher because LIPs are more popular in stage 2), suggesting that the fact that agents with LIPs enter their contract with zero equity is the principal factor behind their role in the foreclosure increase.

7 Conclusion

The calculations we present in this paper suggest that the popularity of non-traditional mortgages between 2003 and 2006 almost doubled the magnitude of the foreclosure crisis. Put another way, innovation in mortgage practices during that time period made the US economy much more sensitive to price shocks. Our calculations also suggest that lower downpayments account for most of the contribution of non-traditional mortgages to the increase in foreclosure rates, while delayed amortization played a limited role.

These quantitative findings have several implications for how one should interpret current events. Mortgage innovation can raise welfare by expanding the range of choices for a number of households. The nature of these innovations, however, does make an increase in default rates likely.

Our findings also raise at least two natural questions. First, did mortgage innovation directly contribute to the run-up and eventual collapse in home-prices? Second, what caused the marked change in mortgage practices circa 2003?

Our transition experiment takes the path of home prices as given. If non-traditional mortgages contributed to the price collapse directly as well, their contribution to the crisis may be even greater than our numbers suggest. For instance, the availability of these mortgages may have led to some form of overbuilding as in Chatterjee and Eyigungor (2009). Their presence may also have contributed to the fragility and eventual freeze of the financial system, leading to a collapse of demand for housing, hence of housing prices. Formalizing and quantifying these ideas are promising avenues for future work, and should reinforce our main message: mortgage innovation played a very significant role in the recent foreclosure boom.

As for what caused the break in the composition of mortgage originations around 2003, several explanations have been proposed. For instance, many view it as the natural consequence of the US government's effort to promote home-ownership over the past two decades, which included a loosening of the restrictions on what mortgages government-sponsored agencies could back or purchase. Our experiments take the timing of the break as given. Further

research into what explains the timing of the burst of innovation in mortgage practices at the beginning of this decade will greatly enhance our understanding of the current foreclosure crisis.

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Appendix (not intended for publication)

A Computation

A.1 Steady State Equilibrium

1. The asset space consists of twenty equally spaced asset grid points between 0 and $\nu q h^2$, twenty equally spaced asset grid points between $\nu q h^2$ and $\nu q h^3$, twenty equally spaced asset grid points between $\nu q h^3$ and and $q h^2$, and another sixty equally spaced asset grid points from $\nu q h^2$ to wherever the asset choice decisions do not bind.

2. We use value function iteration to find $V_O(a)$ on the asset grid from which we obtain decision rules $a'_O(a)$ for old agents. The value functions are approximated by using linear interpolation.

3. Obtain a candidate value function of mid-aged renters $V_M(a, y, 0, \cdot)$ by value function iteration under the assumption $\gamma = 0$.

4. Given the value functions for old agents and the candidate for mid-aged renters, use value function iteration to find $V_M(a, y, 1, h, n \geq T; \kappa)$ on the asset grid from which we obtain asset choice decision rules $a'_M(a, y, 1, h, n \geq T; \kappa)$ and homeownership decisions $H'(a, y, 1, n > T; \kappa)$ for mid-aged homeowners who have paid off their mortgage for each (y, h). The value functions are approximated using linear interpolation.

5. To avoid calculating value functions for contracts which are not feasible for a given (a_0, y_0, h_0), check if the household in that state has enough assets to make a downpayment. If so, use backward induction starting from $V_M(a, y, 1, h, n = T - 1; \kappa)$ to $V_M(a, y, 1, h, n = 0; \kappa)$ using the risk-free rate as the initial guess for the mortgage interest rate for each contract (to compute the mortgage payment). Calculate $W^\kappa(\omega_0)$ according to the resulting decision rules where $\omega_0 = (a_0, y_0, 1, h_0, 0)$.If this present value is less than the initial loan size, increase the interest rate and repeat this step. Otherwise, the equilibrium interest rate is found. The value functions are approximated using linear interpolation.

6. Once we obtain the value functions for new homeowners, we can update the values of mid-aged renters by using the true γ. Repeat from step 4 and iterate until the value function for mid-aged renters converges.

7. Given the value functions for old and mid-aged agents, use value function iteration to find $V_Y(a, y)$ and obtain decision rules $a'_Y(a, y)$ and contract selection decisions (ζ, r^ζ, h_0). Because of the potential discontinuity caused by the downpayment requirements, the value functions for young agents are solved for by grid search.

8. Solve for the equilibrium stationary distribution μ given the implied law of motion.

A.2 Transition Dynamics

1. Using price q_{ss} start with the initial steady state equilibrium from the above algorithm with zero lump-sum taxes.

2. Start with an initial guess for lump-sum taxes $\tau_t = \tau = 0$ over the entire transition $t = 0, 1, ...$

3. Solve for the final steady state with the new house price $q^n = 0.75 * q_{ss}$ using the lump-sum tax τ from step 2.

4. Solve the optimization problems for homeowners who have purchased the house before the unanticipated house price shock occurs by backward induction. If a household chooses to sell its home, they sell at the new price q^n. If a household chooses to remain a homeowner, they have to follow the original mortgage terms given in $m_n(\kappa^o)$ and $b_n(\kappa^o)$ where κ^o denotes the old contract.

   - If the agent is a homeowner but it is not budget feasible for her to make her mortgage payment $m_n(\kappa^o)$ obtained before the unanticipated price shock (i.e. $y + a(1+r) - m_n(\kappa^o) - \delta h - \tau < 0$,), then the value function solves:

$$V_M^{H'=0}(a, y, 1, h, n; \kappa^o) = \max_{c,a'} u\left(c, h^1\right) + \beta E_{y'|y}\left[(1-\rho_O)V_M(a', y', 0, \ldots) + \rho_O V_O(a')\right]$$
$$\text{s.t. } c + a' = y + a(1+r) + \max\left\{(1-\chi)q^n h - b_n(\kappa^o), 0\right\} - R^n h^1 - \tau.$$

   - If it is budget feasible for a homeowner to make her mortgage payment, then if the household chooses to sell its house and start to rent (so that $H' = 0$), define the value function by

$$V_M^{H'=0}(a, y, 1, h, n; \kappa^o) = \max_{c,a'} u\left(c, h^1\right) + \beta E_{y'|y}\left[(1-\rho_O)V_M(a', y', 0, \ldots) + \rho_O V_O(a')\right]$$
$$\text{s.t. } c + a' = y + a(1+r) + \max\{q^n h - b_n(\kappa^o), 0\} - R^n h^1 - \tau.$$

   - If the agent is able to meet her current mortgage payment and chooses to keep her house ($H' = 1$), define the value function by

$$V_M^{H'=1}(a, y, 1, h, n; \kappa^o) = \max_{c,a'} u\left(c, h\right) + \beta E_{(y',h')|(y,h)}\Big[(1-\rho_O)V_M(a', y', 1, h', n+1; \kappa^o) + \rho_O V_O(a' + \max\left\{q^n h - b_{n+1}(\kappa^o), 0\right\})\Big]$$

   s.t. $c + a' = y + a(1+r) - m_n(\kappa^o) - \delta h - \tau$.

5. Select a large integer N to be the number of periods during the transition. In the first period of the transition, start the economy with the initial steady state distribution.

Starting from the second period along the transition path, apply the decision rules to solve for the distribution one period ahead. For renters, use the decision rules solved in the final steady state. For homeowners, if they purchase the house before the transition starts, use the optimization problems solved in step 4. If they purchase the house after the transition starts, use the decision rules from the final steady state. Young agents who turn mid-aged during the transition purchase houses at the new price q^n. Continue to solve for the distribution in every period of the transition path.

6. Given the decision rules and distribution along the transition path, calculate the discounted present value of the net profits for the financial intermediary over the transition path. Update the lump-sum tax τ such that τ/r is equal to the discounted present value of net profits. Return to step 3 and repeat using the updated τ until the discounted present value of net profits equals the discounted present value of the lump-sum tax.

7. Check if the distribution converges to the final steady state in N periods. If not, increase N and repeat all the steps above.

B Aggregate profits on mortgage activity

This appendix derives an aggregate net profit expression. We use it in the transition experiment as well as for the pooling contract experiment. For simplicity but without any loss of generality we do so in the case where $T = 2$. Since breaking down default by type is irrelevant for these calculations, we will also write D for $D^I + D^V$ throughout the derivation. Also without any loss of generality, we will focus on the economy with FRMs only, drop mortgage type superscripts (κ) everywhere, and write $n(\omega)$ for the mortgage period associated with state $\omega \in \Omega_M$. In particular, for newly born agents, $n(\omega) = 0$, while for agents in the second period of their mid-age, $n(\omega) = 1$.

Consider then an agent in state ω at origination with initial loan size b_0 and initial house size $h > 0$, with mortgage yield r^Y. Then:

$$W(\omega) = \frac{m}{1+r+\phi} + \int \left\{ D(\omega') \frac{\max((1-\chi)qh(\omega'), b(1))}{1+r+\phi} + S(\omega') \frac{b(1)}{1+r+\phi} + (1 - D(\omega') - S(\omega')) \frac{m}{(1+r+\phi)^2} \right\} P(\omega'|\omega) d\omega', \tag{B.1}$$

where, per standard fixed mortgage payment algebra:

$$m = b_0(1+r^Y) - b_1 \tag{B.2}$$

$$\text{and } m = b_1(1+r^Y) \tag{B.3}$$

Note that to economize on notation we do not make explicit the fact that b_0, b_1 and r^Y are functions of the agent's state. In this context:

$$\text{Net expected profits on agent of type } \omega \text{ at origination } \equiv W(\omega) - b_0. \tag{B.4}$$

Plugging expression (B.1,B.2,B.3) into (B.4) gives:

$$W(\omega) - b_0 = \frac{m}{1+r+\phi} +$$
$$\int_{\omega'} \left\{ D(\omega') \frac{\max((1-\chi)qh(\omega'), b(1))}{1+r+\phi} + S(\omega') \frac{b(1)}{1+r+\phi} + (1 - D(\omega') - S(\omega')) \frac{m}{(1+r+\phi)^2} \right\} P(\omega'|\omega) d\omega' \quad - \quad b_0$$
$$= \frac{b_0(1+r^Y) - b_1}{1+r+\phi} +$$
$$\int_{\omega'} \left\{ D(\omega') \frac{\max((1-\chi)qh(\omega'), b(1))}{1+r+\phi} + S(\omega') \frac{b(1)}{1+r+\phi} + (1 - D(\omega') - S(\omega')) \frac{b_1(1+r^Y)}{(1+r+\phi)^2} \right\} P(\omega'|\omega) d\omega' \quad - \quad b_0$$
$$= \frac{b_0(r^Y - (r+\phi))}{1+r+\phi} +$$
$$\int_{\omega'} \left\{ D(\omega') \frac{\max((1-\chi)qh(\omega'), b(1)) - b(1)}{1+r+\phi} + (1 - D(\omega') - S(\omega')) \frac{b_1(r^Y - (r+\phi))}{(1+r+\phi)^2} \right\} P(\omega'|\omega) d\omega'$$

The last equality uses the fact that $b(1) = b(1)[D(\omega') + S(\omega') + (1 - D(\omega') - S(\omega'))]$ for all ω'. Integrating it over all possible ω such that $n(\omega) = 0$ yields:

$$\text{Aggregate intermediary profits on mortgages in steady state } \equiv$$
$$\int \omega) \left(1 - D^I(\omega) - D^V(\omega) - S(\omega)\right) \frac{(r^\zeta(\omega) - (r+\phi))}{(1+r+\phi)^{n(\omega)+1}} d\mu_M(\omega)$$
$$- \int \left(b(\omega) - \min\{(1-\chi)qh(\omega), b(\omega)\}\right) (D^I(\omega) + D^V(\omega)) \frac{(1+r+\phi)}{(1+r+\phi)^{n(\omega)+1}} d\mu_M(\omega) \tag{B.5}$$

after observing, first, that $D(\omega) + S(\omega) = 0$ when $n(\omega) = 0$ since we do not allow agents to sell or default in the first period of the mortgage and, second, that for any integrable function $g : \Omega_m \mapsto I\!R$,

$$\int_{\Omega_M} \left(\int_{\{\omega' \in \Omega_M | n(\omega') = 1\}} g(\omega') P(\omega'|\omega) d\omega' \right) d\mu_M(\omega) = \int_{\{\omega' \in \Omega_M | n(\omega') = 1\}} g(\omega') d\mu_M(\omega').$$

This last expression says that the mass of agents who reach a given node is the probability of reaching that node from a given ω at origination. In other words, integrating the expected present value expression over all possible origination state amounts to computing a cross-sectional average in steady state. Expression B.5 thus gives the intermediary's aggregate profits on its mortgage activities in steady state. While the argument in this appendix has assumed $T = 2$, it extends unchanged to the general case.

The first integral in the profit expression gives the net return on active mortgages that are

not terminated in the current period, while the second term is the cost (direct capital losses and opportunity cost) associated with the capital lost in the event of foreclosure.

C Steady state distributions of agent states

The transition matrix across ages is given by:

$$\begin{bmatrix} (1-\rho_M) & \rho_M & 0 \\ 0 & (1-\rho_O) & \rho_O \\ \rho_D & 0 & 1-\rho_D \end{bmatrix}$$

since the old are immediately replaced by newly born young people. Let (η_Y, η_M, η_O) be the corresponding invariant distribution of ages. The invariant mass of agents born each period is then given by

$$\mu_0 \equiv \eta_O \rho_D.$$

With this notation in hand, we can define invariant distributions over possible states at each demographic stage.

The young

The invariant distribution μ_Y on Ω_Y solves, for all $y \in \{y^L, y^M, y^H\}$ and $A \subset I\!R_+$:

$$\mu_Y(A, y) = \mu_0 1_{\{0 \in A\}} \pi^*(y) + (1-\rho_M) \int_{\omega \in \Omega_Y} 1_{\{a'_Y(\omega) \in A\}} \Pi(y|\omega) \mu_Y(d\omega)$$

where $\pi^*(y)$ is the mass of agents born with income y (in other words, π^* denotes the invariant distribution associated with our Markov process for income), $a'_Y : \Omega_Y \mapsto I\!R_+$ is the saving decision rule for young agents, and, abusing notation somewhat, $\Pi(y|\omega)$ is the likelihood of income draw $y \in \{y^L, y^M, y^H\}$ in the next period given current state $\omega \in \Omega_Y$.

The mid-aged

The invariant distribution for mid-aged households μ_M on Ω_M solves, for all $y \in \{y^L, y^M, y^H\}$, $A \subset I\!R_+$ and $(H, h, n; \kappa) \in \{0,1\} \times \{h^1, h^2, h^3\} \times I\!N \times \{\{\{FRM, LIP\} \times I\!R_+ \times \{h^2, h^3\}\} \cup \{\emptyset\}\}$:

$$
\begin{aligned}
\mu_M(A,y,H,h,n;\kappa) &= \rho_M \int_{\Omega_Y} 1_{\{(H,h,n)=(0,h^1,0)\}} 1_{\{a'_Y(\omega)\in A\}} \Pi(y|\omega)\mu_Y(d\omega) \\
&+ (1-\rho_0)\gamma \int_{\Omega_M} 1_{\{(H=n=H'(\omega)=0,a'_M(\omega)\in A\}} \Pi(y|\omega)\mu_M(d\omega) \\
&+ (1-\rho_0)(1-\gamma) \int_{\Omega_M} 1_{\{n(\omega)=n-1,H=H'(\omega)=0,a'_M(\omega)\in A\}} \Pi(y|\omega)\mu_M(d\omega) \\
&+ (1-\rho_0) \int_{\Omega_M} 1_{\{(H=1,H'(\omega)=1,n(\omega)=n-1,\kappa=\kappa(\omega),a'_M(\omega)\in A\}} \Pi(y|\omega)P(h|\omega)\mu_M(d\omega) \\
&+ (1-\rho_0) \int_{\Omega_M} 1_{\{(H=1,H'(\omega)=1,n(\omega)=0=n-1,\kappa=\Xi(\omega),a'_M(\omega)\in A\}} \Pi(y|\omega)P(h|\omega)\mu_M(d\omega)
\end{aligned}
$$

where $a'_M : \Omega_M \mapsto I\!R_+$ is the optimal saving policy for mid-aged agents, $n(\omega)$ extracts the contract age argument of ω, $\kappa(\omega)$ extracts the contract type argument of ω, and $P(h|\omega)$ is the likelihood of a transition from state ω to a state where the house size is h.

The first term corresponds to agents who age from young to mid-aged. The second and third term corresponds to agents who choose to be renters at state ω. Those agents either receive the option to buy a home in the subsequent period, or do not. The final two terms are for agents that choose to remain or become homeowners. All told, these integrals capture the five distinct ways an agent can lend into a given state $(A,y,H,h,n;\kappa)$.

The old

The invariant distribution μ_O on $\Omega_O \equiv I\!R_+$ solves, for all $A \subset I\!R_+$:

$$
\mu_O(A) = (1-\rho_D) \int_{\Omega_O} 1_{\{a'_O(\omega)\in A\}} \mu_O(d\omega) + \rho_O \int_{\Omega_M} 1_{\{a'_M(\omega)+\max\{H'(\omega)[qh(\omega)-b_{n+1}(\kappa)],0\}\in A\}} \mu_M(d\omega)
$$

where, for $\omega \in \Omega_M$, $h(\omega)$ extracts the house size argument of ω, while $b(n+1,\kappa)$ is the principal balance on a mortgage of type κ after $n+1$ periods. Recall that we assumed that housing sales due to the old age shock do not entail foreclosure costs.

D Housing market clearing

The housing market capital clearing condition can be stated in simple terms, after some algebra. The total demand for housing capital (whether rented or owned) in each period is

given by:

$$\int_{\Omega_Y} h^1 d\mu_Y \; + \int_{\Omega_O} h^1 d\mu_O + \int_{\Omega_M} h^1 1_{\{H'=0\}} d\mu_M \; + \int_{\Omega_M} h 1_{\{H'=1, h(\omega)=h\}} d\mu_M$$

The first two terms give the demand for housing by the young and old agents, who, by assumption, are renters. The third term is demand from mid-aged agents who choose to be renters. The last integral captures mid-aged agents who choose to be homeowners. Their use of housing capital depends on the size of the home that they own.

Similarly, the total quantity of housing available in a given period is the sum of the housing agents carry over from the past period and of the new capital produced by the intermediary. It can be stated formally as:

$$Ak + \int_{\Omega_Y} h^1 d\mu_Y \; + \int_{\Omega_O} h^1 d\mu_O + \int_{\Omega_M} h^1 1_{\{H=0\}} d\mu_M \; + \int_{\Omega_M} h 1_{\{H=1, h(\omega)=h\}} d\mu_M$$

But the laws of motion for agent states in our economy imply that:

$$\int_{\Omega_M} h 1_{\{H=1, h(\omega)=h\}} d\mu_M \; = \int_{\Omega_M} h' 1_{\{H'=1\}} P(h'|\omega) d\mu_M \tag{D.1}$$

where $P(h'|\omega)$ is the likelihood that the agent's house size will be $h' \in \{h^1, h^2, h^3\}$ in the next period given current state $\omega \in \Omega_M$.

It follows that the market for housing capital clears provided

$$\int_{\Omega_M} h 1_{\{H'=1, h(\omega)=h\}} d\mu_M \; - \int_{\Omega_M} h' 1_{\{H'=1\}} P(h'|\omega) d\mu_M \; = Ak, \tag{D.2}$$

where k is the quantity of deposits the intermediary transforms into housing capital each period.

E Welfare calculations

E.1 Mortgage innovation

Consider agents born with income y^i for $i \in \{L, M, H\}$ in the benchmark economy and let $U^{bench}(y^i)$ and $U^{FRM+LIP}(y^i)$ denote the lifetime utility they expect at birth in the two different economies. Denote the optimal consumption and housing service plans in the benchmark economy by $\left\{c_t^{bench}, s_t^{bench}\right\}$ for an agent born with initial income y^i. For the ex-ante welfare measure, let $1 + k^i$ be the multiple one has to apply to the consumption of agents born with income y^i in the benchmark economy to make their welfare equal to that which they would

receive in an FRM+LIP economy. That is, k^i solves:

$$\begin{aligned} U^{FRM+LIP}(y^i) &= E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t^{bench}(1+k^i), h_t^{bench})\right] \\ &= E_0\left[\sum_{t=0}^{\infty}\beta^t \left\{ \begin{array}{l} \psi\ln(c_t^{bench}) + \psi\ln(1+k^i) \\ +(1-\psi)\ln(h_t^{bench}) + \theta 1_{\{h_t^{bench}\in\{h^2,h^3\}\}} \end{array} \right\}\right] \\ &= U^{bench}(y^i) + \frac{\psi\ln(1+k^i)}{(1-\beta)} \end{aligned}$$

It follows that:

$$\begin{aligned} & (1-\beta)\left[U^{FRM+LIP}(y^i) - U^{bench}(y^i)\right] = \psi\ln(1+k^i) \\ \Longrightarrow \quad & k^i = \exp\left(\frac{(1-\beta)}{\psi}[U^{FRM+LIP}(y^i) - U^{bench}(y^i)]\right) - 1 \end{aligned}$$

To calculate the ex-post welfare measure, let $U^{bench}(\omega_{age})$ and $U^{FRM+LIP}(\omega_{age})$ denote lifetime utility of an agent with $age \in \{Y, M, O\}$ in state $\omega_{age} \in \Omega_{age}$ in the benchmark and FRM+LIP economies. Let $1+\tilde{k}(\omega_{age})$ be the multiple one has to apply to the consumption of an agent in state ω_{age} in the benchmark economy to make his welfare equal to that when LIPs are also available. Then $\tilde{k}(\omega_{age})$ is given by

$$\tilde{k}(\omega_{age}) = \exp\left(\frac{(1-\beta)}{\psi}[U^{FRM+LIP}(\omega_{age}) - U^{bench}(\omega_{age})]\right) - 1$$

The contribution for agents of each age status and income level is given by

$$k^i_{age} = \frac{\int_{\omega_{age}\in\Omega_{age}} 1_{\{y=y^i\}}\tilde{k}(\omega_{age})\mu_{age}(d\omega_{age})}{\eta^i}, \quad \text{for } age \in \{Y, M, O\},\ i \in \{L, M, H\}$$

where η^i is the measure of agents with earnings i in the invariant distribution (here just 1/3). Note that $k_O = 0$ since old agents don't have any welfare change associated with the regime change. In this case, the overall welfare gain/loss is given by

$$\sum_{i\in\{L,M,H\}}\sum_{age\in\{Y,M,O\}} k^i_{age}\eta^i.$$

E.2 Recourse experiment

Denote lifetime utility that an agent born with income y in the FRM+LIP economy with full recourse as $U^{RC}(y)$. For the ex-ante measure, let $1+k^i$ be the multiple one has to apply to the consumption of agents born with income y^i in the FRM+LIP economy without recourse

to make their welfare equal that which they would receive in the full recourse FRM+LIP economy. That is, k^i solves:

$$k^i = \exp\left(\frac{(1-\beta)}{\psi}[U^{RC}(y) - U^{FRM+LIP}(y)]\right) - 1$$

To calculate the ex-post welfare measure, we denote $U^{FRM+LIP}(\omega_{age})$ and $U^{RC}(\omega_{age})$ as the lifetime utility agents with $age \in \{Y, M(n), O\}$ in state $\omega_{age} \in \Omega_{age}$ in FRM+LIP economies without and with recourse respectively. Let $1 + \tilde{k}(\omega_{age})$ be the multiple one has to apply to the consumption of an agent in state ω_{age} in the economy without recourse to make his welfare equal to the economy with full recourse. Then $\tilde{k}(\omega_{age})$ is given by

$$\tilde{k}(\omega_{age}) = \exp\left(\frac{(1-\beta)}{\psi}[U^{RC}(\omega_{age}) - U^{FRM+LIP}(\omega_{age})]\right) - 1$$

The contribution for agents of each age status and income level is given by

$$k^i_{age} = \frac{\int_{\omega_{age}\in\Omega_{age}} 1_{\{y=y^i\}}\tilde{k}(\omega_{age})\mu_{age}(d\omega_{age})}{\eta^i}, \quad \text{for } age \in \{Y, M(n), O\},\ i \in \{L, M, H\}.$$

In this case, the overall welfare gain/loss is given by

$$\sum_{i\in\{L,M,H\}} \sum_{age\in\{Y,M(n),O\}} k^i_{age}\eta^i.$$

F Moment Calculations

- We define the Foreclosure Discount as FD $\equiv m(h^2)f(h^2) + m(h^3)f(h^3)$ where for each initial house size, $h_0 \in \{h^2, h^3\}$, $m(h_0) \equiv \frac{\int_{\{\Omega_M:h_0(\omega)=h_0\}}(D^I(\omega)+D^V(\omega))d\mu_M}{\int_{\{\Omega_M\}}(D^I(\omega)+D^V(\omega))d\mu_M}$ is the share of contracts in default, and $f(h_0) \equiv \left(\frac{\int_{\{\Omega_M:h_0(\omega)=h_0\}}(D^I(\omega)+D^V(\omega))qhd\mu_M}{\int_{\{\Omega_M:h_0(\omega)=h_0\}}(D^I(\omega)+D^V(\omega))d\mu_M}\right) / \left(\frac{\int_{\{\Omega_M:h_0(\omega)=h_0\}}S(\omega)qhd\mu_M}{\int_{\{\Omega_M:h_0(\omega)=h_0\}}S(\omega)d\mu_M}\right)$ is the average house value for defaulters relative to the average house value for sellers.
- Involuntary default rates on FRM contracts are given by $\frac{\int_{\Omega_M} D^I(\omega)1_{\{\zeta=FRM,n<T,H=1\}}d\mu_M(\omega)}{\int_{\Omega_M} 1_{\{\zeta=FRM,n<T,H=1\}}d\mu_M(\omega)}$. The expression is similar for involuntary default rates, and for LIPs.
- Let $\gamma(\omega)$ be the probability of homeowners facing a positive probability of being in an involuntary default situation next period if they stay in their house (i.e. $\gamma(\omega) = E_{\omega'|\omega,H'=1}\left[1_{\{D^I(\omega')=1\}}\right]$). Using this definition, the probability that a household sells its

house when $\gamma(\omega) > 0$ is then given by

$$\frac{\int_{\Omega_M} 1_{\{S(\omega)=1,\gamma(\omega)>0,n<T,H=1\}} d\mu_M(\omega)}{\int_{\Omega_M} 1_{\{\gamma(\omega)>0,n<T,H=1\}} d\mu_M(\omega)}.$$