Distributional Incentives in an Equilibrium Model of Domestic Sovereign Default

Pablo D'Erasmo
University of Maryland

Enrique G. Mendoza
University of Maryland and NBER

April 2, 2012
(Detailed Abstract and Methodological Notes)

1 Detailed Abstract

Reinhart and Rogoff (2008) discovered 68 dramatic episodes of outright, or de jure, default on domestic sovereign debt in a cross-country database going as far back as 1750. They also discovered that domestic public debt accounts for a large fraction of total government debt in the majority of countries (about $2/3^{\text{rds}}$ on average), and that domestic default events are associated with periods of severe financial turbulence, which often included defaults on external debt, banking system collapses and full-blown economic crises. This "forgotten history" of domestic sovereign default is thus an important feature of the economic history of most countries, and one that remains largely unexplained.

The ongoing European debt crisis also highlights the importance of default risk in domestic sovereign debt markets. If we view the European Union as a "domestic economy," it follows that the vast majority of the economy's public debt is held by domestic residents, and that default by one or more members of the EU is therefore more akin to a domestic sovereign default than to the standard external default that is the focus of most of the academic literature. Moreover, domestic public debt is large as a share of GDP for the EU as a whole, and very large for some of its members such as Italy or Greece.

Historically, domestic defaults in nation states occur via mechanisms such as forcible conversions, lower coupon rates, unilateral reductions of principal, and suspension of payments, all of them involving domestic public debt holders. In the case of the European Union, there are multiple nation states, but economically very highly integrated, sharing a common central bank, and with sovereign debt of the various members largely held within the EU, so that again what

may seem like an external default (as the recent default by Greece) is in fact more akin to a domestic default.

A feature common to both historical domestic defaults and the European debt crisis is the fact that sovereign default entails substantial redistribution across domestic agents, with all of these agents presumably entering in the payoff function of the sovereign. This is in sharp contrast with what standard models of sovereign default assume, because typically default is modeled as a decision not to repay debt by a government with a payoff equivalent to the utility of a representative home agent vis-a-vis a foreign lender who is disconnected from the economy where that government and home agent reside. The implications of the decision to default for the payoff of the government's creditors is irrelevant for the sovereign making the default decision, and both the costs and benefits of default affect all domestic agents in the same way (since the economy is inhabited by a single representative agent). These obervations suggest that standard models of sovereign default face serious limitations in explaining the forgotten history of domestic debt or in addressing key features of the European debt crisis, because in actual domestic defaults and in the European crisis, governments making default decisions take into account the implications of their default choice on the welfare of government creditors, and evaluate the different costs and benefits of default on various groups of domestic agents.

In this paper, we develop a two-period model with heterogeneous agents and non-insurable aggregate risk in which domestic default can be optimal for a utilitarian government that responds to distributional incentives. The government finances the gap between exogenous stochastic expenditures and endogenous lump-sum taxes by issuing non-state-contingent debt, retaining the option to default. The government evaluates the costs and benefits of default caring about the welfare of all domestic agents, and thus the social welfare function includes weights that are used to aggregate individual welfare. We consider a baseline utilitarian social welfare function in which these weights are given by the economy's initial wealth distribution, as well as alternatives with ad-hoc weights that can be justified by political economy or egalitarian considerations. In this environment, the distribution of public debt across private agents interacts with the government's optimal default, debt issuance and tax decisions. Default is optimal when repaying hurts relatively poor agents more than defaulting hurts relatively rich agents, and this happens when the ownership of public debt is sufficiently concentrated.

2 Methodological Notes

The following text provides a technical sketch of the model and an illustration of the mechanism that drives optimal default as a policy for redistribution. We plan to make the exposition in the actual paper less technical and more focused on the intuition of the mechanism, but at this

point we thought it would be useful to provide details showing the layout of the model and what we have done so far.

The economy exists for two periods and is inhabited by a continuum of agents with aggregate unit measure. Agents differ in their wealth position (i.e. their holdings of government debt). We take as given an initial distribution of wealth across households, but the distribution at the end of the first period is endogenous. The government is represented by a social planner with a utilitarian payoff that issues one-period, non-state-contingent debt, levies lump-sum taxes, and has the option to default. Government debt is the only asset available in the economy and is entirely held by domestic agents. There is aggregate risk in the form of changes in government outlays and default risk, and there is no idiosyncratic uncertainty. This setup is designed to highlight how the equilibrium determination of government bond prices, the level of future public debt, and the choices on default and taxes react to changes in the initial wealth distribution.

2.1 Preferences and Endowments

Preferences are given by:

$$u(c_0) + \beta E[u(c_1)]$$

where $\beta \in (0,1)$ is the discount factor and c_t for $t = 0, 1$ is individual consumption. The utility function is a standard, twice-continuously differentiable, strictly concave function that satisfies the Inada conditions.

All agents receives non-stochastic income y each period. We denote the aggregate supply of government bonds at $t = 0$ as B_0 and set the initial wealth distribution by assuming that initially a fraction γ of households are "low-wealth" (L) individuals who hold $b_0^L = 0$, and a fraction $(1-\gamma)$ are "high-wealth" (H) agents who hold $b_0^H = \frac{B_0 - \gamma b_0^L}{1-\gamma} \geq b_0^L$. b_0^H is the level of debt that is consistent with market clearing in the market of government bonds at time $t = 0$. Thus, the parameter γ summarizes the initial wealth distribution.

2.2 Government

At $t = 0$, the government has outstanding debt B_0 and can issue one-period, non-state contingent discount bonds $B_1 \in \mathcal{B} \equiv [0, \infty)$ at the price $q_0 \geq 0$. Each period it collects revenues from lump sum taxes τ_t and pays for exogenous stochastic expenditures equal to $g_t \in \{\underline{g}, \overline{g}\}$. The transition probabilities of government expenditures are given by $F(g_{t+1}, g_t)$.

We assume that the government repays the debt at $t = 0$ but has the option to default at $t = 1$. We denote the default decision by $d_1 \in \{0, 1\}$ where $d_1 = 0$ implies no default. The level of taxes is determined after the default decision.

The government is benevolent with a payoff function in the class of utilitarian social welfare functions. The benchmark case is one with a standard utilitarian social welfare function in which the utility of each agent is weighed according to the economy's wealth distribution, i.e. "low-wealth" (L) individuals have weight γ and "high-wealth" (H) individuals have weight $(1-\gamma)$. Thus, this benchmark planner maximizes average welfare. Other utilitarian payoffs can aggregate individual utilities with arbitrary weights, which could be justified with political economy considerations, and can also be extended to incorporate egalitarian concerns (e.g. by introducing penalties defined relative to the utility of the poorest agents). The government, however, cannot discriminate a particular household or set of households when setting taxation, debt and default policies.

2.3 Bond Market and Budget Sets

At $t = 0$, households can hold positions in government bonds defined by $b_1 \in \mathcal{B} \equiv [0, \infty)$, so they are not allowed to borrow. At equilibrium, the discounted price of debt will be such that the following market clearing condition of the government bond market holds:

$$B_1 = \gamma b_1^L + (1-\gamma) b_1^H. \tag{1}$$

Hence, ex post the distribution of wealth at the end of the first period can differ from the initial wealth distribution, which will occur whenever the optimal savings plans of private agents imply $b_1^L > 0$.

At $t = 0$, the government budget constraint is

$$\tau_0 = g_0 + B_0 - q_0 B_1,$$

and households' budget constraints are

$$c_0^i + q_1 b_1^i = y + b_0^i - \tau_0, \quad i = L, H.$$

If, at the beginning of $t = 1$, the government does not default, each household receives back the corresponding b_1^i units of government debt. Taxes are then set to satisfy the following government budget constraint:

$$\tau_1^{d_1=0} = g_1 + B_1 - q_1 B_2, \tag{2}$$

and the households' budget constraints are:

$$c_1^i + q_2 b_2^i = y + b_1^i - \tau_1^{d_1=0}, \qquad i = L, H \tag{3}$$

Note that, because of the two-period horizon, even if the government repays, the final period's equilibrium debt and debt price will satisfy $B_2 = q_1 = 0$. However, in order to make disposable income at $t = 1$ comparable with that of an infinite horizon economy, we can set arbitrary terminal values $q_1 = \hat{q}$ and $B_2 = \hat{B}$.

If the government defaults at $t = 1$, the market for government bonds closes. In this case, taxes are

$$\tau_1^{d_1=1} = g_1, \tag{4}$$

and the households' budget constraints are all given by

$$c_1^i = y - \tau_1^{d_1=1}. \tag{5}$$

3 Equilibrium with Endogenous Policies

We start by characterizing the household problem and then we present how optimal government policies are determined. Since at $t = 0$ the level of debt and government expenditures are set before agents make decisions, and since we can derive the equilibrium level of taxes from the government budget constraint, it is sufficient to express the value function of the households and bond prices as a function of (B_1, γ) and the default decision in period $t = 1$ as a function of (B_1, g_1, γ).

3.1 Households' Problem

For a given value of B_1 and γ a household with initial debt holdings b_0^i for $i = L, H$ chooses b_1^i by solving this maximization problem:

$$\begin{aligned} v^i(B_1, \gamma) \;=\; & \max_{b_1^i} \Big\{ u(y + b_0^i - q_0(B_1, \gamma) b_1^i - \tau_0) + \\ & \beta E_{g_1} \Big[(1 - d_1(B_1, g_1, \gamma)) u(y + b_1^i - \tau_1^{d_1=0}) + d_1(B_1, g_1, \gamma) u(y - \tau_1^{d_1=1}) \Big] \Big\}. \end{aligned} \tag{6}$$

The continuation value in case of default does not depend on the level of household debt (b_1^i) since when it defaults, the government does not discriminate across households. Taxes in period 1 are affected by the government default decision.

An important feature of the individual agent's problem is that agents take into account the possibility of default in formulating their optimal choice of bond holdings. The first-order condition of this problem, evaluated at the equilibrium level of taxes, is:

$$-u'(c_0^i) q_0(B_1, \gamma) + \beta E_{g_1} \Big[u'(y - g_1 + b_1^i - B_1 + \hat{q}\hat{B})(1 - d_1(B_1, g_1, \gamma)) \Big] \leq 0, \; = 0 \;\; if \;\; b_1^i > 0$$

At a given γ and B_1, when the value of g_1 is such that the government chooses to default ($d_1(B_1, g_1, \gamma) = 1$), the marginal value of an extra unit of debt is zero.[1] Thus, conditional on B_1, a larger default set (i.e. a larger set of values of g_1 such that the government defaults), implies that the marginal benefit of an extra unit of savings decreases. This implies that, everything else equal, a higher default probability results in a lower demand for government bonds, a lower equilibrium bond price, and higher taxes. This has important redistributive implications, because by affecting the bond supply the government affects the expected probability of default and changes the equilibrium bond price. Note also that from the households' perspective, the individual bond decision has no marginal effect on $d_1(B_1, g_1, \gamma)$.

3.2 Government's Problem

We follow a recursive strategy by solving first the problem of the government in the final period, followed by the optimal debt choice at $t = 0$. At $t = 1$, when contemplating to default or not, the government solves:

$$\max_{d \in \{0,1\}} \left\{ W_1^{d=0}(B_1, g_1, \gamma), W_1^{d=1}(g_1, \gamma) \right\}, \tag{7}$$

where $W_1^{d=0}(B_1, g_1, \gamma)$ and $W_1^{d=1}(B_1, g_1, \gamma)$ denote social welfare at the beginning of period 1 in the case of no default and default respectively. Using the government budget constraint to substitute for the corresponding values of $\tau_1^{d=0}$ and $\tau_1^{d=1}$ and under the baseline utilitarian social welfare function that uses γ to weigh individual welfare, the government payoffs can be expressed as:

$$W_1^{d=0}(B_1, g_1, \gamma) = \gamma u(y - g_1 + b_1^L - B_1 + q_1 B_2) + (1 - \gamma) u(y - g_1 + b_1^H - B_1 + q_1 B_2) \tag{8}$$

and

$$W_1^{d=1}(g_1, \gamma) = u(y - g_1). \tag{9}$$

Combining the payoff functions under default and no default, if follows that the government defaults if:

$$\gamma \left[u(y - g_1 + \overbrace{(b_1^L - B_1)}^{\leq 0} + q_1 B_2) - u(y - g_1) \right] +$$

$$(1 - \gamma) \left[u(y - g_1 + \overbrace{(b_1^H - B_1)}^{\geq 0} + q_1 B_2) - u(y - g_1) \right] \leq 0$$

[1] The utility in the case of default equals $u(y - g_1)$, independent of b_1^i.

Notice that all households lose g_1 of their income to government absorption regardless of the default choice, and that utility under default is the same for all agents and given by $u(y - g_1)$.

The distributional effects of a default are implicit in the above default condition. Given that low- and high-wealth agents satisfy $(b_1^L - B_1) \leq 0$ and $(b_1^H - B_1) \geq 0$ respectively, it follows that, for a given B_1, the payoff under repayment allocates (weakly) lower welfare for L agents and higher for H agents, and the gap between the latter and the former is larger the larger B_1. Moreover, since the default payoffs are the same for both types of agents, this is also true of the *difference* in welfare under repayment v. default: It is higher for H agents than for L agents and it gets larger as B_1 rises. To induce default, however, is necessary not only that L agents have a smaller difference in the payoffs of repayment v. default, but that the difference is negative (i.e. they must attain lower welfare under repayment than under default), which requires $B_1 - \hat{q}\hat{B} > 0$. In turn, the government budget constraint implies that this holds if taxes are higher in the repayment state ($\tau_1^{d=0} = g_1 + B_1 - q_1 B_2$ under repayment vs. $\tau_1^{d=1} = g_1$ under default).[2] In summary, if repayment requires higher taxes than default, default is always preferable than repayment for L agents and viceversa for H agents.

Since we can also write the consumption allocations under repayment as $c_1^L = y - \tau_1^{d=0} + b_1^L$ and $c_1^H = y - \tau_1^{d=0} + b_1^H$, the distributional effects of default can also be interpreted in terms of how the changes in taxes and wealth caused by a default affect each agent's consumption (and hence utility). First, since $b_1^H > b_1^L$, default wipes out the net worth of H agents but has a smaller effect on the net worth of L agents (or no effect if $b_1^L = 0$), thus reducing the welfare of the former without affecting the latter (or more generally, hurting L agents less if we simply assume $b_1^H > b_1^L$, without assuming $b_1^L = 0$). Second, with regard to taxes, we established that for default incentives to exist τ_1 must be higher under repayment than under default, but both types of agents face the same tax rate. This still has distributional implications, however, because marginal utility is higher for L agents, and thus they suffer more if taxes rise under repayment.

The distribution of wealth determines the weight the utilitarian planner assigns to the gains and losses that default imposes on the different agents. As γ increases, the fraction of L agents is larger, and thus the value of repayment for the government decreases because it weights more the welfare loss that L agents endure under repayment. Hence, with sufficient wealth inequality default becomes optimal, because the utilitarian planner attains higher social welfare under default. Note that differences in γ also affect the date-0 bond decision rules b_1^L and b_1^H and hence the market price of bonds q_0, even for a given supply of bonds B_1.

The government also needs to choose its optimal debt level. This is done at $t = 0$. On this

[2]This condition can also be stated in terms of the primary fiscal balance. For repayment to be welfare-reducing for the poor, there must be a primary surplus in the repayment state (i.e. $\tau_1 - g_1 > 0 \rightarrow B_1 - q_1 B_2 > 0$).

date, taking prices as given, the government solves a problem similar to a Ramsey problem in the sense that the government chooses its debt policy internalizing all the price and quantity effects we just described. More specifically, the government chooses B_1 to maximize the "indirect" social welfare function:

$$W_0(\gamma) = \max_{B_1} \left\{ \gamma v^L(B_1, \gamma) + (1-\gamma) v^H(B_1, \gamma) \right\}. \tag{10}$$

To provide some intuition about the solution, it is illustrative to rearrange the government first-order condition as follows:

$$\begin{aligned} q_0 \left[\gamma u'(c_0^L) + (1-\gamma) u'(c_0^H))\right] - \beta E \left[(1-d_1)[\gamma u'(c_1^L) + (1-\gamma) u'(c_1^H)]\right] \\ + \underbrace{\frac{\partial q_0}{\partial B_1}}_{\leq 0} \left[\gamma u'(c_0^L) \underbrace{(B_1 - b_1^L)}_{\geq 0} + (1-\gamma) u'(c_0^H) \underbrace{(B_1 - b_1^H)}_{\leq 0} \right] \\ + \beta E_{g_1} \left\{ \Delta d_1 \left[\gamma \underbrace{[u(y-g_1) - u(c_1^L)]}_{>\text{ or } <0} + (1-\gamma) \underbrace{[u(y-g_1) - u(c_1^H)]}_{<0} \right] \right\} \quad \leq \quad 0. \end{aligned}$$

This expression can be broken into four terms. The first two are identical to the negative of the first-order conditions of the household. Since they are evaluated from the perspective of the social planner, they represent the social marginal benefit and cost of one more unit of debt at a given debt price and "default propensity" of the government. The planner takes into account that, as the level of debt increases, all agents pay less taxes today but pay more taxes in the following period, and H agents postpone more consumption today to buy debt. The third term corresponds to the effect of issuing more debt on the equilibrium price of government bonds. Since L agents are net borrowers ($(b_1^L - B_1) \leq 0$) and H agents are net savers ($(b_1^H - B_1) \geq 0$), a lower value of q_0 has a differential effect. Net savers receive a higher return on the bond as the total stock of government debt increases. The final term corresponds to the change in the default propensity of the government in the following period. A default is a cost for H agents who hold higher debt than the average and possibly a benefit for L agents with low or zero debt (recall we showed earlier that repayment causes a welfare loss for L agents that do not hold debt if $B_1 - \hat{q}\hat{B} > 0$, or equivalently if at $t = 1$ taxes in the repayment state are higher than in the default state).

If all households are unconstrained in their savings decisions, so that their Euler equations hold with equality, and $\Delta d_1 = 0$, then the above optimality condition simplifies to:

$$\frac{u'(c_0^L)}{u'(c_0^H)} = \frac{-(B_1 - b_1^H)(1-\gamma)}{(B_1 - b_1^L)\gamma},$$

Hence, under those assumptions we obtain the intuitive result that the social planner would want to set the ratio of date-0 marginal utilities of the two agents equal to their weighted relative savings ratio. Moreover, if L agents are constrained (i.e. $b_1^L = 0$) and $\Delta d_1 = 0$, then the optimality condition yields this result:

$$\underbrace{\gamma\left[q_0 u_0'^L) - \beta E\left[(1-d_1)u'(c_1^L)\right]\right]}_{>0} + \frac{\partial q_0}{\partial B_1}\left[\gamma u'(c_0^L)B_1 + (1-\gamma)u'(c_0^H)(B_1 - b_1^H)\right] = 0.$$

Thus, when the borrowing constraint is binding for some agents, the optimal level of debt issued by the government increases, because at the debt level consistent with the unconstrained optimality condition the planner would have $u'(c_0^L)/u'(c_0^H) > -(1-\gamma)(B_1 - b_1^H)/(\gamma B_1)$, and hence the marginal benefit of borrowing more to reduce τ_0 and allocate more consumption to L agents exceeds the cost of making H agents save more to buy the debt.

The equilibrium bond price is the value of $q_0(B_1, \gamma)$ that clears the debt market. That is, at $q_0(B_1, \gamma)$, the following condition needs to hold

$$B_1 = \gamma b_1^L(B_1, \gamma) + (1-\gamma) b_1^H(B_1, \gamma), \tag{11}$$

where B_1 in the left-hand-side of this expression represents the public bonds supply, and the right-hand-side is the government bond demand.

3.3 Competitive Equilibrium with Endogenous Policies

At initial B_0 and for each γ, a **Competitive Equilibrium with Endogenous Policies** is a household value, household decision rules, a bond price $q_0(B_1, \gamma)$ and government policies $d(B_1, g_1, \gamma)$, $B_1(\gamma)$, $\tau_0(B_1, \gamma)$, $\tau_1^d(B_1, g_1\gamma)$ such that:

1. Given prices and policies, $v^i(B_1, \gamma)$ and $b_1^i(B_1, \gamma)$ solve the households' problem.
2. The price function $q_0(B_1, \gamma)$ satisfies the market-clearing condition (equation (11)).
3. The government default decision $d(B_1, g_1, \gamma)$ solves problem (7).
4. Taxes $\tau_0(B_1, \gamma)$ and $\tau_1^d(B_1, g_1\gamma)$ are consistent with the government budget constraint.
5. The government debt policy $B_1(\gamma)$ is consistent with the solution of problem (10).

To solve for the price and default function, at a given B_1 and γ, we iterate on $q_0(B_1, \gamma)$, $b_1^i(B_1, \gamma)$ and the default decision rule $d_1(B_1, g_1, \gamma)$ until the bond market clears and the bond and default decisions are consistent with government's optimization.

4 Numerical Exercise

In this section, we parametrize the model and present results for a case where there is equilibrium default. Preferences are given by $u(c) = c^{1-\sigma}/(1-\sigma)$, with $\sigma = 2$ and $\beta = 0.93$. Income is equal to $y = 1$ for every household. Government expenditures are stochastic only in the second period. In particular, we set $g_0 = 0.10$, $g_1 \in \{\underline{g}, \overline{g}\}$ with $\underline{g} = 0.10$, $\overline{g} = 0.25$ and $\Pr(g_1 = \underline{g}) = 0.98$.

Initial government debt is set at $B_0 = 0.10$ and debt in period $t = 1$ can take values on $B_1 \in \{0, \ldots, 0.40\}$.[3] We use 21 equally spaced values of γ in the set $\{0.10, \ldots, 0.90\}$. Note that, because of the two-period horizon, even if the government repays, the final period's debt and debt price should satisfy $B_2 = q_1 = 0$. However, in order to make disposable income at $t = 1$ comparable with that of an infinite horizon economy, we set arbitrary terminal values $q_1 = \hat{q}$ and $B_2 = \hat{B}$ (parametrized to $\hat{q} = 0.65$ and $\hat{B} = 0.01$ respectively).

As in the characterization of the equilibrium, it is more intuitive to present the results by starting in period 1 and going backwards. First, we analyze the government's default decision at $t = 1$ for a given value of B_1. We then present the implications of the default decision on households' decision rules, prices and taxes. Finally, we describe the optimal government debt choice at period 0.

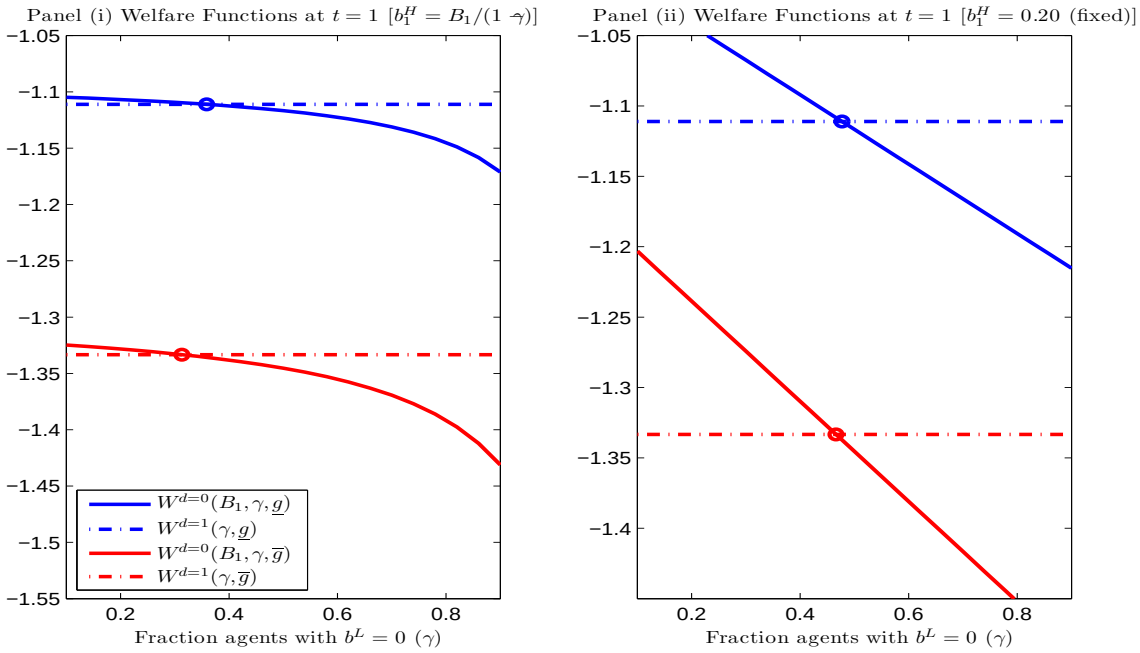
To illustrate the mechanics of the distributional incentives of default at work, Panel (i) of Figure 1 plots the welfare functions $W^{d=0}(B_1, g_1, \gamma)$ and $W^{d=1}(g_1, \gamma)$ as functions of γ for values of $g_1 \in \{\underline{g}, \overline{g}\}$ and a given $B_1 = 0.10$. In this chart, $W^{d=0}(B_1, g_1, \gamma)$ does not consider households' decision rules about b_1^i. Instead, in Panel (i) we assume that b_1^H adjusts with γ to clear the government bond market ($b_1^H = \frac{B_1}{1-\gamma}$) and in Panel (ii) we simply fixed $b_1^H = 0.20$. In both cases, we set $b_1^L = 0$

Panel (i) shows how $W^{d=0}$ changes in response to both the change in weights of the utilitarian planner and the changes in households' bond holdings with the assumed decision rules. Social welfare in the default state is lower with higher g_1 and, as explained, it is independent of γ. In contrast, social welfare under repayment decreases with γ because taxes needed for repayment represent a larger portion of income to L agents and as γ increases the weight assigned to L agents also increases. The circles in the plot mark the threshold $\hat{\gamma}$ such that the government chooses to default at $t = 1$ for any $\gamma > \hat{\gamma}$.

To separate the effects of the changes in weights and changes in households debt holdings, Panel (ii) shows again the welfare functions but now evaluated at $b_1^H = 0.20$ which is generally not the market clearing value ($b_1^H = 0.20$ only clears the debt market when $\gamma = 0.50$). Clearly, if the debt holdings of agents are kept unchanged, the welfare function loses its curvature and

[3] Results correspond to a case where there are 25 equally spaced values of B_1 and the upper bound is not a binding restriction.

Figure 1: Welfare Default vs No-Default $W_1^{d=0}(B_1, g_1, \gamma)$ and $W_1^{d=1}(g_1, \gamma)$

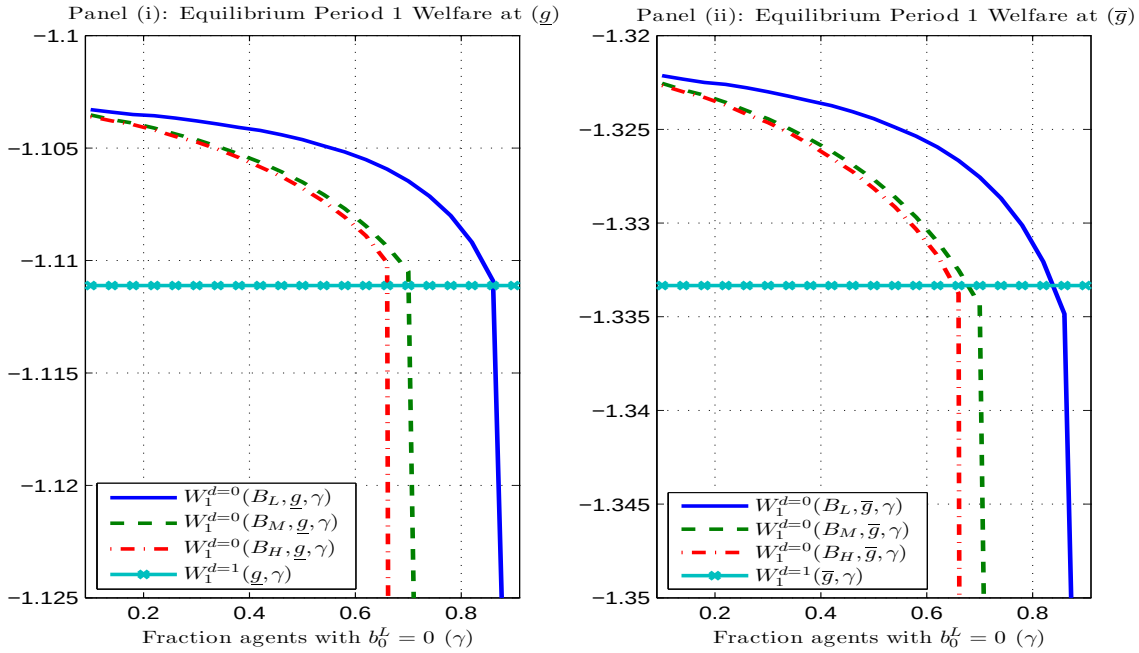


is simply the linear combination of two constants. Comparing Panels (i) and (ii), it is evident that $\hat{\gamma}$ is higher when we ignore the response of b_1^H to the change in γ. This occurs because the welfare loss of each individual H agent in the event of a default is larger when $b_1^H = 0.20$ than when $b_1^H = \frac{B_1}{1-\gamma}$ and $\gamma < 0.50$, and hence it takes a higher value of γ to make the welfare gain of the L agents offset the larger loss of H agents in the case of default.

Now we study the results for the "recursive" equilibrium of this two-period model, for which we have solved optimal decision rules, default and debt choices of the government, as well as bond prices and taxes. Panel (i) of Figure 2 shows date-1 equilibrium welfare functions for given values of government debt $B_1 = \{B_L, B_M, B_H\} = \{0.033, 0.05, 0.20\}$ at $g_1 = \underline{g}$. Panel (ii) present the same functions at $g_1 = \overline{g}$. In this chart, the distributions of household debt in period 1 are "recursive" equilibrium distributions, i.e. for each B_1 we solved for the price and private decision rules that clear the market contingent on the default decision.

The relation between wealth inequality or debt dispersion (measured by γ) and default in Figure 2 is similar to that observed in Figure 1. These figures show that there is a negative relation between the level of debt and default. At higher levels of debt, lower values of γ trigger a default. For example, when the debt is high ($B_1 = 0.20$)) and $g_1 = \overline{g}$, if $\gamma > 0.62$ households are better off (in terms of the utilitarian average) if the government defaults since debt repayment would result in higher taxes. What it is also visible from this figure is that the default threshold value of γ is lower when government expenditures are $g_1 = \overline{g}$ than when $g_1 = \underline{g}$. This is also

Figure 2: Equilibrium Welfare $W_1^{d=0}(B_1, g_1, \gamma)$ and $W_1^{d=1}(g_1, \gamma)$



reflected in the government default decision, presented in Figure 3.

Consistent with the previous plots, Figure 3 shows again that the government has higher incentives to default when government debt is higher or when the distribution of debt holdings is more unequal (higher γ).

Figure 4 presents the value of γ such that the government is indifferent between defaulting and repaying in period 1. We denote this threshold by $\hat{\gamma}(B_1, g_1)$.

This figure shows that the threshold is decreasing in B_1 (i.e. less dispersion is needed to induce the government to default when debt is higher) and that is decreasing in g_1 (when government expenditures are higher the government has stronger incentives to default). This last feature of $\hat{\gamma}$ is very important to generate default in equilibrium. If, for a given value of B_1, the threshold $\hat{\gamma}$ is the same for both values of g_1, the probability of default equals one, so the equilibrium price has to equal zero and the government has not access to that debt level (i.e. there is a zero probability of default in equilibrium). On the other hand, if, for a given B_1, the threshold is different across values of g_1, the probability of default will be positive but strictly less than one for $\gamma \in \left[\hat{\gamma}(B_1, \underline{g}), \hat{\gamma}(B_1, \overline{g})\right)$, so the price can be positive and the government has access to this debt level (i.e there is default with positive probability).

Figure 5 shows the equilibrium price function $q_0(B_1, \gamma)$ as well as the bond price if the government were not allowed to default on the debt $q_0^{RF}(B_1, \gamma)$ (i.e. the "risk free" price).

The equilibrium bond price is positive only for combinations of γ and B_1 such that the

Figure 3: Equilibrium Government Default Policy

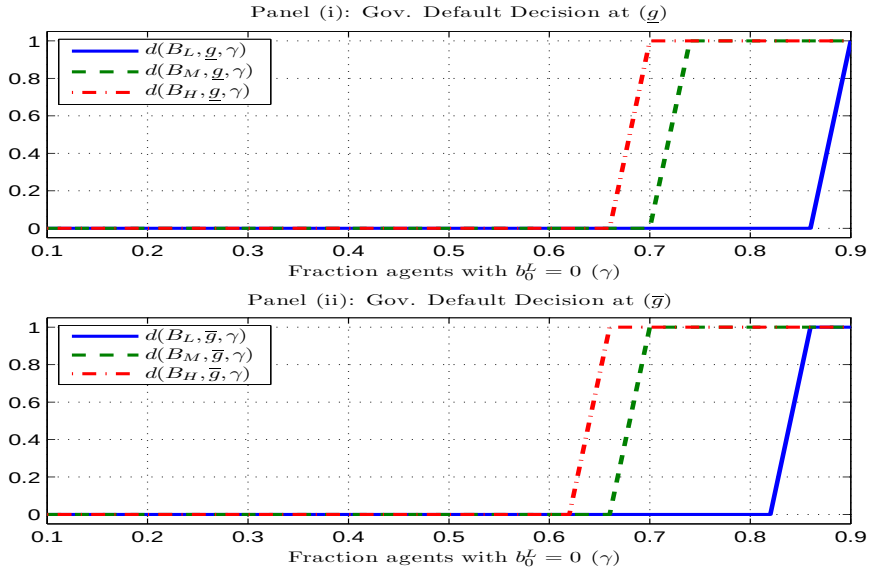


Figure 4: Default Threshold $\hat{\gamma}(B_1, g_1)$

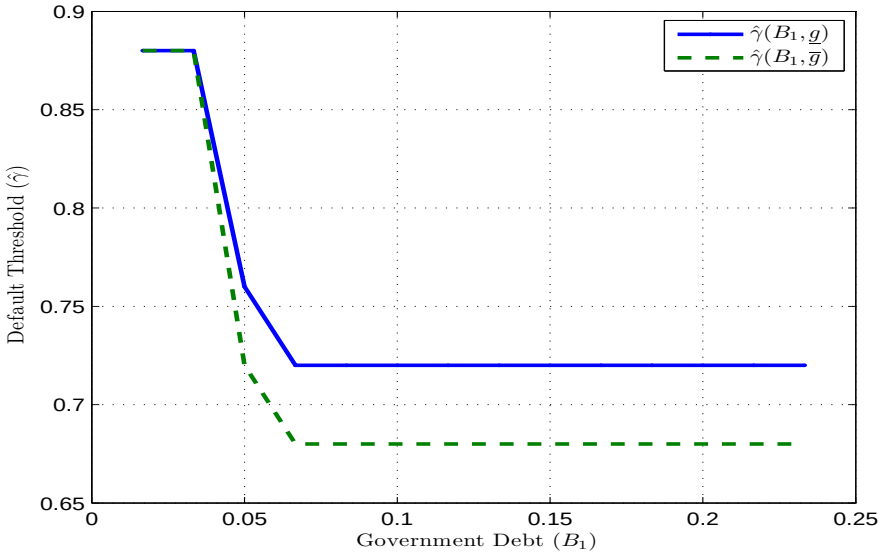
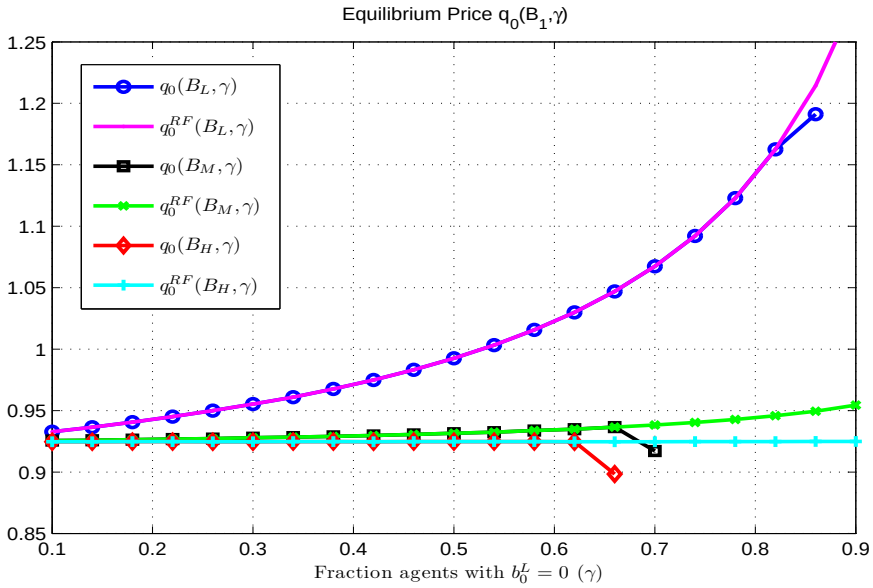


Figure 5: Equilibrium Bond Price

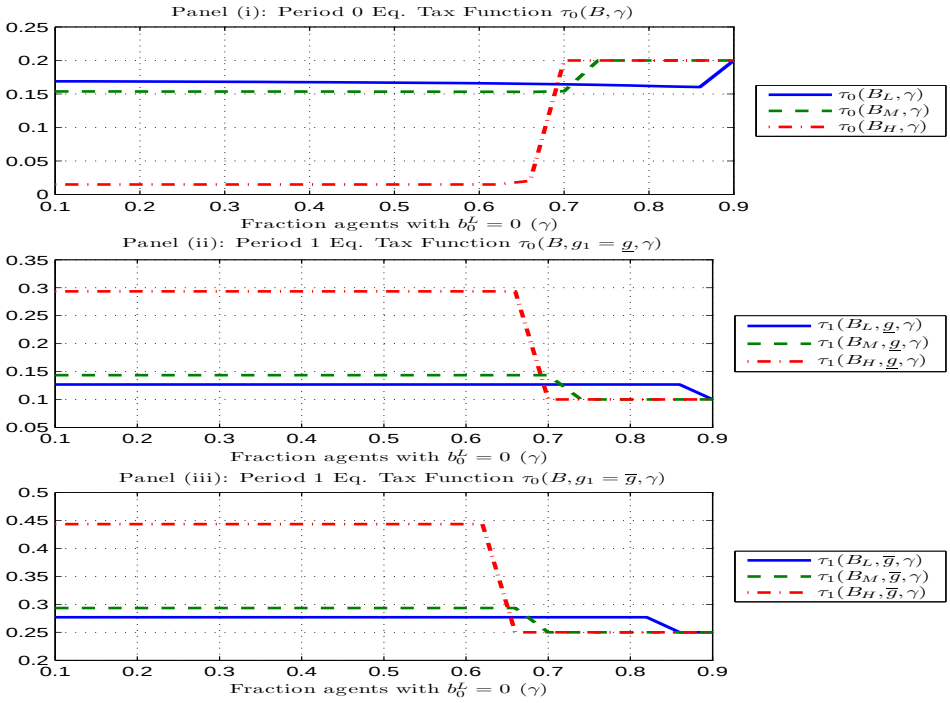


probability of default is less than one. For example, when $B_1 = 0.05$, the default probability equals 1 for values of $\gamma > 0.7$, so the bond price goes to zero. Three important characteristics are worth mentioning about this figure: (**i**) the equilibrium price is decreasing in B_1 (shifts across lines). For any given γ, as the government borrows more, the interest rate needed to create the incentives for households to demand a larger amount of debt increases. For values where the risk free rate coincides with the equilibrium interest rate, this change is driven exclusively by supply and demand factors. More specifically, as we show below, in many cases $b_1^L = 0$, so the marginal investor (i.e. the one determining the bond price) corresponds to an H household. Then, the equilibrium price is the one that is consistent with their first-order condition. For any value of b_0^H, as B_1 increases, lower values of q_0 are needed to generate the corresponding increase in b_1^H that clears the bond market. (**ii**) default risk reduces the price even further (i.e. increases interest rates). This can be seen by comparing the equilibrium bond price with the risk free price. For example, when $\gamma = 0.7$ and $B_1 = 0.05$ the probability of default equals 2% since the government will choose to default in case $g_1 = \overline{g}$. The equilibrium price equals zero when the default probability equals 1 and the distance between the risk free rate and the equilibrium price is maximized. That is, for cases where the government would prefer to default for any value of future government expenditures the equilibrium interest rate becomes infinity. (**iii**) for regions with no default risk, the bond price is increasing in γ. The rationale for the rise in debt prices as γ rises is more subtle than the effect of debt level on prices, and results from changes in the wealth distribution. A larger value of γ implies a more dispersed wealth distribution (i.e. higher b_0^H). Hence, a smaller fraction of the population demands a larger amount of debt and

this pushes prices up.

Figure 6 presents the resulting tax policy functions in period 0 and period 1. They are a function of B_1, g_1, the equilibrium price and the corresponding default decision.

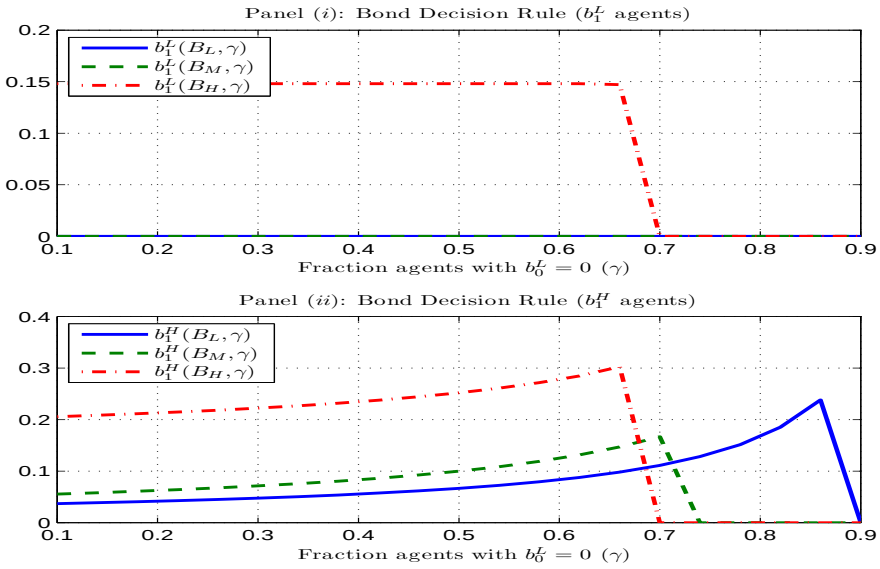
Figure 6: Equilibrium Tax Policy



Panel (i) presents τ_0 as a function of B_1 (new debt issuance). In regions where the government finds optimal not to default ($\gamma < 0.70$), higher debt results in lower τ_0. On the other hand, in regions where the probability of a government default is positive, τ_0 increases since the government has to pay a higher interest rate or it cannot issue new debt at all (values of γ such that τ_0 does not vary across debt levels). Panels (ii) and (iii) present τ_1 as a function of B_1, for g_1 equal to $\underline{g}$ and $\overline{g}$ respectively. When the government does not default, taxes are increasing in the level of debt (i.e. higher debt service results in higher taxes). If the government defaults, taxes are equal to g_1 for any value of debt. Thus, in Figure 6 taxes drop once γ is high enough for default to be optimal.

At equilibrium, the households' optimal bond decision rules capture the effects of the equilibrium set of taxes and prices. These decision rules are presented in Figure 7. This figure shows that in many cases $b_1^L = 0$ and $b_1^H = \frac{B_1}{1-\gamma}$, so the H-type households are effectively the "marginal" investor. In particular, poor agents continue to choose zero debt except in the case that $B_1 = B_H$, in which they choose a positive amount of debt holdings for states where the

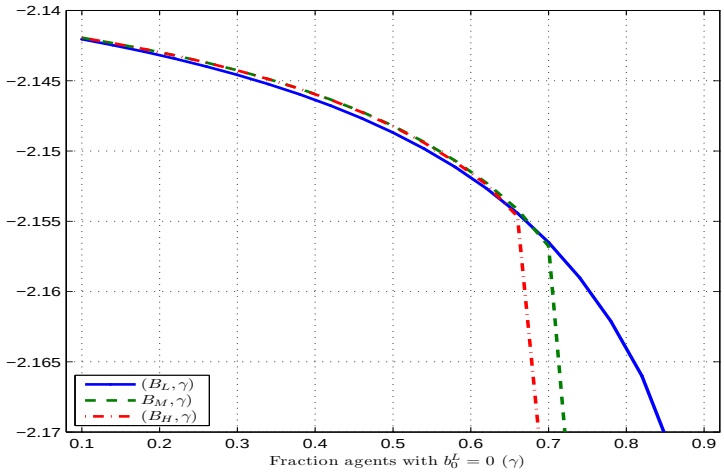
Figure 7: Equilibrium HH bond decision rules b_1^i



government does not default. The shape of b_1^H is derived from our assumption on the initial distribution of debt holdings (recall that $b_0^H = 0$ and $b_0^H = \frac{B_0}{1-\gamma}$).

Now, having examined the date 1 features of the model as well as the price, taxes and households' decision rules in period 0, we can complete the presentation of the solution of the model and show the optimal debt choice of the government in period 0 (equation (10)). Figure 8 shows the indirect social welfare function evaluated as to $t = 0$ as a function of γ and for the three values of B_1 considered earlier:

Figure 8: Equilibrium Initial Welfare as a function of B_1

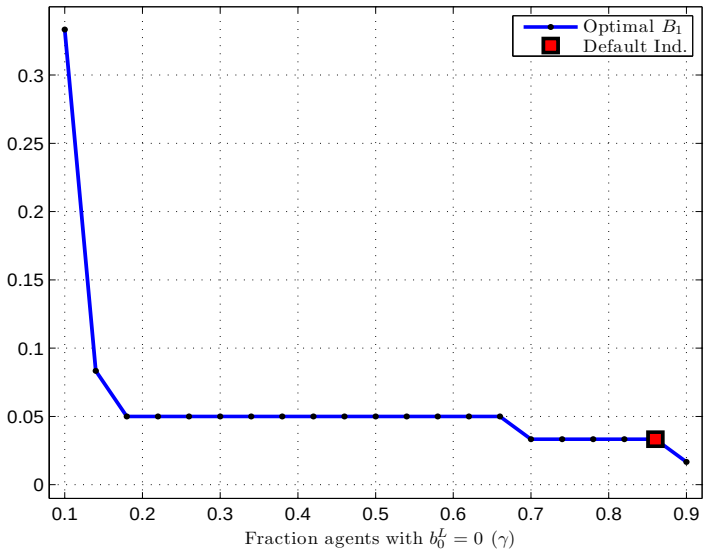


This figure shows that the planner attains a higher equilibrium payoff with a lower level of

debt choice as γ rises. The reason is that, a higher level of debt induces a reduction in τ_0 and increases τ_1 (see Figure 6). For low γ, this tax reallocation makes little difference for the welfare of both agents (in fact, for $\gamma < 0.25$ the welfare of L agents is actually slightly higher with more debt). On the other hand, as γ rises above 0.4, the welfare of L agents is significantly lower as the expected value of taxes increases.

Finally, Figure 9 shows the optimal debt level chosen by the government as a function of γ. The debt level identified by a square is an optimal debt choice with a positive probability of default.

Figure 9: Equilibrium Optimal Government Debt Policy



In line with the results and the intuition provided above, the optimal debt level falls as γ increases. Thus, this simple framework yields the interesting prediction that a higher concentration of wealth (i.e. a larger fraction of low-wealth individuals) supports a lower level of public debt, taking into account the possibility of domestic default driven by distributional incentives. Figure 9 shows that default is a zero probability event unless wealth concentration is sufficiently high. More specifically, we found an equilibrium with default at $\gamma = 0.86$ where at $t = 0$ the government chooses $B_1 = 0.033$ and defaults in $t = 1$ when g_1 realizes to be equal to $\overline{g}$. At higher values of γ, the government prefers an even lower level of debt and there is no default.

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