Academic performance and college dropout: Using longitudinal expectations data to estimate a learning model

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We estimate a simple structural learning model of the college drop-out decision which takes advantage of unique data to reduce the assumptions that are traditionally necessary for identification. We find that approximately forty-five percent of the drop-out that occurs before the start of the third year can be attributed to student learning about academic performance. We find that students who perform poorly tend to learn that staying in school is not beneficial, not that they leave simply because they have lost the option to stay or believe they are more likely to lose the option in the future. As to why students find staying in school is no longer beneficial, the most important avenue is that performing poorly reduces how enjoyable it is to be in school. However, the reduction in the financial returns to graduating that accompanies poor performance would also be sufficient to create substantial drop-out.

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Section I. Introduction

The policy importance of understanding why many college students do not complete a degree has been widely recognized, with children from low income families receiving particular attention due to their high drop-out rates relative to other students (Bowen et al., 2009). Unfortunately, due in large part to the difficulty of obtaining ideal data, much remains unknown about how students make the drop-out decision.[1]

The desire to understand the underlying reasons for college drop-out highlights a fundamental tension present in empirical micro-economics; while structural models formed directly from economic theory represent a potentially powerful tool for understanding the mechanisms that underlie individual decision-making (and for providing pre-implementation evidence about the effects of possible policy changes), their practical usefulness will be undermined if concerns about the validity of central assumptions lead to concerns about the identification of model parameters. From a conceptual standpoint, the drop-out outcome is best viewed as the end result of a process in which a student learns about a variety of utility-influencing factors after arriving at school (Manski, 1989; Altonji, 1993). Then, empirical work that is closely tied to theory relies heavily on the characterization of individual-specific beliefs about these factors throughout the time a student is in school. Traditionally, researchers working with models which require beliefs have relied on assumptions that allow them to characterize beliefs indirectly. For example, a common assumption, often referred to as Rational Expectations, is that an individual's beliefs about a particular factor (e.g., grade performance) coincide with the actual distribution from which that factor is drawn (Das and van Soest, 2000). However, recent research such as Manski (2004) has stressed that these types of assumptions are arbitrary and untestable, in which case

[1]Differences in college drop-out by family income have been found to be at least as important as differences in college entrance by family income from the standpoint of creating differences in college degree attainment by family income (Manski and Wise, 1983; Manski, 1992; NCES, 2007).

Describing the traditional difficulties of understanding the underlying reasons for drop-out, Bowen and Bok (1998) write, "One large question is the extent to which low national graduation rates are due to the inability of students and their families to meet college costs, rather than to academic difficulties or other factors." Tinto (1975) suggests that drop-out is related to academic and social integration, but direct tests of this are scarce (Draper, 2005).

their use in the estimation of models of behavior may raise concerns about identification.

A natural alternative to indirect approaches for characterizing beliefs is to use carefully worded survey questions in order to elicit beliefs directly (Dominitz, 1998; Dominitz and Manski, 1996, 1997). Unfortunately, at present it is difficult to take full advantage of the potential benefits that this type of "expectations" data may have for the estimation of structural models. Recently, longitudinal surveys have begun to embrace the promise of eliciting beliefs directly (Manski, 2004). However, the reality that these surveys are designed to allow researchers from a variety of disciplines to study a broad range of topics limits the extent to which data collection can be driven by any one specific type of model. In addition, surveys that are administered annually or biennially are not ideal for the study of issues such as drop-out where it is important to characterize beliefs frequently and at rather specific times.

In this paper we estimate a simple structural learning model of the drop-out decision which takes advantage of a unique, longitudinal data collection effort motivated directly by the tension described above. The data come from the Berea Panel Study (BPS), a longitudinal survey of students at Berea College. The BPS represents a unique opportunity to estimate the structural model because our design of survey instruments was guided closely by theoretical models of learning. As such, in addition to its contribution to the substantive area of education, this paper makes a contribution by illustrating the benefits of collecting detailed longitudinal data with a very specific model of behavior in mind.[2] More specifically, the data contain two unique features that are of central importance given our interest in learning. First, the survey is unique among surveys of college students in its frequency of contact; each

[2]In terms of modeling, most similar are the dynamic, discrete choice models of Arcidiacono (2004) and Stange (forthcoming). These papers highlight the policy importance of incorporating learning into models of decision-making in higher education, but also identify the types of unavoidable issues that arise when conventional data sources must be used for the estimation of these types of models. For example, the findings in S&S (2009) suggest that certain relevant conclusions about the option value of schooling, of particular interest in Stange (2009), may be ruled out by the types of assumptions that are often used to construct beliefs (about academic factors in his case) when expectations data are not available. Further, while the choice of NELS-88 is natural for the detail it provides about the college period, it does not allow for one to model how beliefs about post-college earnings depend on grade performance even if one is willing to make standard assumptions - because the data do not contain information about post-college period earnings.

See Attanasio and Kaufmann (2009) for an example of the use of (income) expectations data in models of educational attainment

student was surveyed approximately twelve times each year while in school, with the first survey taking place immediately before the beginning of the student's freshman year. Second, taking advantage of recent methodological advances in the elicitation of beliefs (Dominitz, 1998; Dominitz and Manski, 1996, 1997), the BPS was perhaps the first sustained longitudinal survey to have a strong focus on the direct elicitation of beliefs.

Located in central Kentucky, Berea College operates with a mission of providing an education to students of "great promise but limited economic resources," and, as such, has a demographic focus that is desirable given our interests. This paper builds on previous background work which illustrated the benefits of conducting a detailed case study at this particular school. Stinebrickner and Stinebrickner, hereafter S&S, (2003a) documented that, similar to what is seen for students with comparable family income and college entrance scores elsewhere, between forty and fifty percent of the entering students at Berea fail to graduate (and few transfer) - even though the direct costs of schooling are zero (or perhaps negative) due to a full tuition subsidy and room and board subsidies for all students. Taking advantage of unique survey questions in the BPS, S&S (2008a) found that, while credit constraints do influence the decisions of a small number of students by making it difficult to smooth consumption between the schooling and working portions of their lives, they do not play a substantial role in determining the overall drop-out rate of students at Berea. Thus, our background work shows that factors unrelated to financial resources per se play the prominent role in the drop-out of these low income students. This motivates our current objective of examining the process through which these non-financial-resource factors may influence the drop-out decision.

We focus primarily on understanding the importance of the most widely recognized non-financial-resource explanation - that after entering college, students learn about how well they will perform academically. We find that approximately forty-five percent of the drop-out that occurs before the start of the third year can be attributed to this type of learning, with the importance of this type of learning falling to eleven percent for the third year. As a result, this paper contributes some of the

strongest direct evidence to a recent literature which recognizes the importance of learning in determining schooling outcomes (Manski, 1989; Altonji, 1993; Carneiro et al., 2005; Cunha et al., 2005, S&S, 2009; Stange, forthcoming).

In previous work (S&S, 2009) we examined drop-out between the end of the first year and the beginning of the second year using a reduced form model. The benefit of estimating a structural model is that, by conducting a series of simulations, we are able to distinguish between the possible avenues through which learning about academic performance can matter. We start by simulating a counter-factual scenario which results from removing the traditional institutional requirement that students must surpass semester-specific grade performance cutoffs in order to progress. We find that students who perform poorly tend to learn that staying in school is not beneficial, not that they leave simply because they have lost the option to stay in school or have learned that they are more likely (than they previously believed) to lose the option in the future. As to why students find staying in school is no longer beneficial, further simulations find that the most important avenue is that performing poorly reduces how enjoyable it is to be in school. However, important for policy reasons described in our conclusions, the reduction in the financial returns to graduating that accompanies poor performance would also be sufficient to create substantial drop-out.

Section II. The Berea Panel Study, the sample, and motivating descriptive statistics

Designed and administered by Todd Stinebrickner and Ralph Stinebrickner, the BPS is a longitudinal survey that takes place at Berea College and elicits information of relevance for understanding a wide variety of issues in higher education, including those related to drop-out, college major, time-use, social networks, peer effects, and transitions to the labor market. The BPS consists of two cohorts. Baseline surveys were administered to the first cohort (the 2000 cohort) immediately before it began its freshman year in the fall of 2000 and baseline surveys were administered to the second cohort (the 2001 cohort) immediately before it began its freshman year in the fall of 2001. In addition to collecting detailed background information, the baseline surveys were designed to take

advantage of recent advances in survey methodology (see, e.g., Barsky et al., 1997; Dominitz, 1998; and Dominitz and Manski, 1996, 1997) in order to directly elicit individual-specific expectations towards uncertain outcomes and the factors that might influence these outcomes. Substantial follow-up surveys that were administered at the beginning and end of each subsequent semester document how expectations change over time.

Because some survey questions of interest are not available for the 2000 cohort, we focus on the 2001 cohort. Approximately 88% of all students who entered Berea in the Fall of 2001 participated in the BPS survey. S&S (2009) found that few students who leave Berea transfer to other four year schools. We exclude students who transfer, but note that results change very little under a different treatment of these students. Our sample contains 341 students.

In order to obtain standard observable characteristics, X_i, the BPS survey data are linked to administrative data from Berea College. We focus primarily on a student's sex and his/her high school grade point average. The proportion of students that are male is 44.57% and the average (std. deviation) high school grade point average is 3.37 (.46). The academic credentials of students at Berea, including college entrance exam scores (average 23.35, std. deviation 3.60), are similar to those at the University of Kentucky and the University of Tennessee (S&S, 2008a). Our sample can be generally thought of as a group of students from low income families (average family income $26,000, std. deviation family income $17,000), and for most of what we do here we do not differentiate by family income within the sample.

This paper is motivated most generally by the reality that, consistent with what is seen for students from low income families elsewhere (S&S, 2008a, Manski, 1992), the drop-out rate at Berea is substantial. The outcome variable we examine here is whether a student leaves school for at least a semester at any point during the first 3.5 years of school. Nine percent, 18%, 26%, 34%, 39%, and 46%, respectively, of the students in our sample have left school as of the start of the second, third, fourth, fifth, sixth, and seventh semesters, respectively. The use of the term drop-out would be a misnomer to

the extent that students who leave Berea return and complete a degree in the future. However, this is quite rare. For example, only ten percent of the students who left school at any time before the start of the seventh semester subsequently returned to school and were still enrolled at the start of the eighth semester. We also find that leaving school is very rare for those who have not left as of the seventh semester. For example, only two percent of the individuals who were in school for the seventh semester were not in school for the eighth semester.

To further motivate the learning nature of our model, we examine responses to the following question which was administered at the time of college entrance:

Question B What is the percent chance that you will eventually graduate from Berea College?____

While more than 40% of students in our sample will fail to graduate, students, on average, believe that there is only a 14% chance that they will fail to graduate from Berea. A similar finding comes from a related question which asks "What is the percent chance that you will be enrolled at Berea in the fall semester of the next academic year." While 18% of students will not be enrolled, students, on average, believe that there is only a 9% chance that they will not be enrolled. The reality that the perceived drop-out rate at entrance is very low while the actual drop-out rate is substantial suggests that substantial learning may be taking place.[3]

Section III. A model of drop-out

III.A. Choices We consider a simple dynamic model of sequential decision-making under uncertainty. A student arrives at the beginning of his first semester with beliefs about a variety of factors that influence the costs and benefits of remaining in school. We simplify the discussion by assuming that

[3]From a theoretical standpoint, the presence of substantial amounts of drop-out does not necessarily imply that people have updated the mean of the distribution describing their beliefs about average grade performance. For example, when the labor market returns to good academic performance are non-linear, in theory, one might decide to enter school knowing that he will leave school even if the mean of his belief distribution does not change (i.e., he plans to leave if he does not find out he is better than expected). However, if a person believes initially that there is very little chance of dropping out, then an observed drop-out outcome likely implies that some type of learning about the mean of the distribution has taken place.

students make decisions at the beginning of each semester t, starting with the second semester.[4] At the end of each semester for which a student is still in school, the student first checks to see if he is being forced to leave college due to poor academic performance or because he has graduated. If he is not being forced to leave college, he uses information received during the previous semester to update the beliefs he held at the beginning of the semester and then decides whether to return to school (S) for semester t or whether to enter the labor force (N). Given our earlier findings that relatively few people return to school after leaving, we assume that N is a terminal state.

This paper's contribution is in its use of unique data to reduce the reliance on assumptions that otherwise would be necessary. In the spirit of trying to keep identification as transparent as possible, we have specified an extremely parsimonious choice set {S,N}, thereby avoiding a variety of assumptions that would accompany additional endogenous choices. The parsimonious choice set does imply that our model cannot be used to examine how students make other important decisions such as: how much to study, what major to choose, and whether to attend graduate school after college. However, as discussed in more detail in the next sections, our model is generally flexible enough to capture many of the costs and benefits that go along with these additional decisions that people are making in the background. Thus, taking into account the potential benefits of avoiding assumptions that would accompany additional endogenous choices, the parsimonious choice set may be, on net, advantageous for our primary objectives: 1) understanding the overall effect that learning about academic performance has on drop-out and 2) differentiating between several broad reasons for why this type of learning may matter.[5]

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[4]In our empirical work, a student is classified as leaving at t if he began semester t-1 and did not return for semester t. The choice of how to group students is not overly important given that the large majority of departures take place between semesters.

[5]To illustrate, with our model we will be able to examine whether learning about academic performance is important because performing poorly: 1) reduces how enjoyable it is to be in school or 2) reduces the financial returns to graduating. A model which included more endogenous choices might not be beneficial (and given additional assumptions could be less beneficial) for understanding the importance of these two reasons, but would potentially allow us to understand ***why*** learning about academic performance influences how enjoyable it is to be in school or ***why*** learning about academic performance influences the returns to schooling - e.g., does learning have its

We also do not model the decision of how many courses to take in a particular semester. As such, we are assuming that students who do not fail out of school make steady progress towards graduation, with this progress characterized by the number of semesters attended. This does not seem overly restrictive given that Berea requires full-time attendance and given that grade cutoffs for failing out of school are meant to identify those who are not progressing in a timely fashion. Given that roughly seventy-five percent of students who graduate do so in four years, we assume that students who choose to return for the eighth semester will graduate at the end of that year. Thus, referring to the start of semester t as "time t," with t=1 being the time of entrance and t=9 being the time of graduation, a student makes a choice from the set {S, N} at any of the times t=2, t=3,..., t=8 for which he is still in school.

III.B. Value Functions Our emphasis on understanding the importance of learning suggests the desirability of a dynamic, forward-looking model. The fundamental object needed for estimation is the discounted expected utility, or value, associated with the two options {S,N} that a person considers at each time t that he is still in school and has the option of continuing. In this subsection we describe the value functions in general terms. In subsequent subsections we describe the components of the value functions in more detail.

Let $U^N_t(\Omega_t)$ be the current period utility for a person who is in the workforce at time t with a state of Ω_t. Then, for a person who is still in school at the end of semester t-1, the value of entering the workforce (N) at time t is:

$$(1)\quad V^N_t(\Omega_t)=E\sum_{\tau=t}^{T*}\beta^{\tau-t}U^N_\tau(\Omega_\tau),$$

where T* is the end of a person's utility horizon, β is the discount factor, and, to be consistent with the reality that there are two semesters in each year, each period in the workforce represents six months.

Let $U^S_t(\Omega_t)$ be the current period utility for a person who is in school at t with a state of Ω_t. For a person who is still in school at the end of semester t-1 and is not forced to leave due to poor academic performance or graduation, the value of returning to school (S) for semester t is given by the Bellman

influence by changing decisions about, for example, how much to study?

equation:

(2) $V^{S}_{t}(\Omega_t) = EU^{S}_{t}(\Omega_t) + (PrFail_t)\ \beta V^{N}_{t+1}(\Omega_{t+1}) + (1 - PrFail_t)\ \beta Emax[V^{S}_{t+1}(\Omega_{t+1}), V^{N}_{t+1}(\Omega_{t+1})]$.

The first term is the expected current period reward of being in college at time t. The second term indicates that with probability $PrFail_t$ the student will fail out of school at the end of semester t, in which case he will be forced to enter the workforce permanently. The third term indicates that with probability 1-$PrFail_t$ the student will not fail out of school at the end of semester t, in which case he will have the option of returning to school for semester t+1 or entering the workforce.[6] The expected value in the third term is over all elements of Ω_{t+1} whose values are not known at time t given Ω_t and the choice of S at t.

We complete our description of the model by specifying the functions U^{N}_{t} and U^{S}_{t} in III.C and by describing the elements of Ω_t and how these elements evolve between t and t+1 in III.D.

III.C Current Period Utility We assume that $U^{N}_{t}(\bullet)$ is linear in C_t, a person's consumption at time t. Letting $\epsilon_{N,t}$ represent a period-specific, idiosyncratic shock to the utility derived from option N that is known to the individual but not the econometrician,

(3) $U^{N}_{t}(\bullet) = C_t + \epsilon_{N,t}$.

The assumption that the utility in Eq. (3) is linear in consumption facilitates an easy interpretation of model parameters (Section III.F), is convenient for characterizing expected future utility (Section III.D), and allows us to avoid estimating potentially hard-to-identify parameters associated with the curvature in the utility function. As in other recent work in this area (Stange, forthcoming) this assumption is made primarily for convenience. However, somewhat mitigating the effect of this assumption is that, as discussed in the next subsection, we do not explicitly model consumption during school, the period when consumption would be most likely to be at the low levels where differences between a linear and non-linear assumption for the utility function would be most important. Survey questions eliciting beliefs about minimum future income reveal little concern that post-college earnings might turn out to be close to zero in a particular year.

[6] This equation represents tjat case where the student does not graduate at the end of semester t (i.e., t+1<9)

One could assume that the function U^S_t is identical in form to the function U^N_t, in which case the (average) utility difference between a schooling period (S) and a non-schooling period (N) is simply the difference in a person's consumption between the two periods. However, such an approach is worrisome because: 1) even if the amount of his own money that a student spends on consumption while in school is observed, it may be difficult to measure actual consumption while in school because there are types of consumption that are provided free of charge on a college campus (e.g., computing resources, television, etc.) and 2) the potential for certain types of leisure activities on a college campus that may not be available outside of school suggests that the mapping from consumption to utility may be quite different in (S) and (N). Indeed, S&S (2008a) found some evidence that students believe that they are smoothing marginal current period utility between the schooling and working portions of their lives even when they have little of their own money to spend on consumption per se.

The general difficulty of understanding how much utility a person receives while in school motivated us at the beginning of each semester to use Survey Question A.1 (Appendix A) to directly measure the object of interest - how much a student enjoys being in school relative to the alternative of being in the workforce. Central to our construction of the current period utility function is the binary variable EN_t which has a value of one if a person reports at the beginning of time t that he believes that being in school is more enjoyable than being out of school (i.e., a person circles 1 or 2 on A.1).

While, in theory, EN_t might capture all academic aspects of relevance for characterizing current period utility, in practice, there are reasons that this might not be the case. First, while, in theory, the effect of academic measures on enjoyability might be taken into account in answers to Question A.1, in practice, it is difficult to know exactly what students condition on when answering the question. For example, perhaps students think largely about the social part of schooling when answering A.1 or tend to consider the effect of grades in a best-case type scenario. Then, even after taking into account EN_t, $U^S_t(\bullet)$ may depend on a person's cumulative grade point average G_t at the beginning of t and his grade performance g_t in semester t; g_t potentially influences the utility of being in school in semester t because

school may be unenjoyable if a person has difficulty understanding course material and G_t may influence utility in semester t conditional on g_t because school may be particularly stressful if a person believes that he is close to failing out. Second, while our interest in understanding the full impact of learning about ability imples that $U^S_t(\bullet)$ should capture all non-earnings avenues through which poor academic performance may influence drop-out, it is not clear whether answers to A.1 would take into account, for example, that families may provide less encouragement to stay in school when grade performance is bad. Finally, a concern in certain policy circles is that students may have a knee-jerk reaction to bad outcomes. In this case, if A.1 is collected somewhat after a student leaves school, EN_t may not capture the entire effect that grades had on the exit decision. Motivated by this discussion, we specify $U^S_t(\bullet)$ as

(4) $U^S_t(\bullet)=\gamma_0+\gamma_1 EN_t + \gamma_2 G_t + \gamma_3 g_t + \epsilon_{S,t}$,

where $\epsilon_{S,t}$ is the analog to $\epsilon_{N,t}$. We define $\epsilon_t=\{\epsilon_{N,t}, \epsilon_{S,t}\}$. Our particular interest in academic issues motivated our inclusion of the grade variables in (4). However, some of arguments in the previous paragraph might also suggest that other factors, such as student health or whether a parent lost a job, might also not be fully captured by EN_t. Then, given the objective of quantifying the importance of learning about academic performance, one might wish to also include these factors explicitly in (4) if it is possible that they might be correlated with what a person learns about his grade performance. We do this as a robustness check in Section VI.

An examination of Eqs. (3) and (4) reveals how, as discussed in Section III.A, our model is flexible enough to capture many of the costs and benefits that accompany certain decisions that are not modeled explicitly. Our model is one where students learn about how much they will enjoy school and what their earnings will be in the future. For illustration, consider a student who decides to increase his study effort. Given our objectives, what is needed is for this change in effort to be reflected in our characterization of how enjoyable it is to be in school and our characterization of what students believe about earnings conditional on years of completion. With respect to the former, the term EN_t in Eq. (4) would account for decreases in current period utility associated with the reduction in current period

leisure, while the terms G_t and g_t in Eq. (4) would allow for the possibility that studying may lead to additional current period utility benefits not captured by EN_t through improved academic performance. With respect to the latter, an improvement in grades (that would accompany increased study effort) would influence a student's beliefs about consumption both by increasing the probability that the person graduates and, as discussed in more detail below, by influencing the future consumption a person receives conditional on graduation.[7] Then, while our model cannot provide direct information about issues related to studying, it does take into account the implications of studying that are important for our model of learning.

III.D State Variables The set of state variables at time t, $\Omega(t)$, includes all variables whose time t values provide information about $U^S_\tau(\bullet)$ and $U^N_\tau(\bullet)$, τ=t, t+1, t+2,...

State variables providing information about $U^S_\tau(\bullet)$, τ=t, t+1, t+2,...

We first consider the state variables whose time t values provide information about U^S for the current period t. Examining Eq. (4), G_t, EN_t, and $\epsilon_{S,t}$ are known to person i at the beginning of time t. A student's beliefs about g_t are constructed by censoring an underlying belief variable g_t^*. Specifically, assuming that g_t^* is normally distributed with an individual-specific mean μ_t and an individual-specific variance σ^2_t, a student's beliefs are given by:

(5) g_t=4.0 if g_t^*>4.0, g_t=0 if g_t^*<0, $g_t=g_t^*$ else, with $g_t^* \sim N(\mu_t,\sigma^2_t)$.

Then, G_t, EN_t, ϵ_t, μ_t, and σ_t are elements of $\Omega(t)$.

We next think about what time t information influences U^S in the future periods t+1, t+2,... . As described in the previous paragraph, when a person arrives at t+1 the variables that will provide information about U^S_{t+1} are G_{t+1}, EN_{t+1}, ϵ_{t+1}, μ_{t+1}, and σ_{t+1}. Then, given the recursive nature of the Bellman Equation in (2), what is necessary is to specify the process by which G, EN, ϵ, μ, and σ evolve between

[7]One can think of how the model would adjust for other "background decisions" as well. For example, a change to a new major may influence current period utility through both changes in grade performance (as captured by G_t and g_t) and changes in how much a person enjoys studying the new subject area (as captured by EN_t). Naturally, there are some limits to the ability of our model to adjust. For example, one concern would be that a change in major might influence a person's perceptions about the relationship between grades and earnings which is discussed later.

t and t+1.

ϵ_{t+1} is not known by person i at time t. Largely for computational reasons described below, we assume that $\epsilon_{N,t+1}$ and $\epsilon_{S,t+1}$ are each drawn at t+1 from an extreme value distribution, with ϵ_{t+1} independent of ϵ_t.

G_{t+1} is determined by the technical relationship between a person's cumulative grade point average (GPA) at the start of a semester and his current GPA in that semester. For example, under our implicit assumption in III.A that a person takes an equal number of courses each semester,

$$(6)\quad G_{t+1}=\frac{t}{t+1}G_t+\frac{1}{t+1}g_t.$$

We assume that the binary variable EN_{t+1} depends on EN_t, g_t, and other unobserved factors $v_{EN,t+1}$:

$$(7)\quad EN_{t+1}=1 \text{ iff } EN_{t+1}^*=\alpha_{EN,0}+\alpha_{EN,1}EN_t+\alpha_{EN,2}g_t+v_{EN,t+1}>0,$$

so that EN_{t+1} is determined at t+1 after g_t is observed and $v_{EN,t+1}\sim N(0,1)$ is drawn.

Finally, the process by which μ_t and σ^2_t evolve represents learning about academic performance in the model. As discussed in detail in Section IV, because we observe μ_t and μ_{t+1} we are not forced to assume that individuals update beliefs in any specific manner. Instead we estimate the parameters of a parsimonious updating equation:

$$(8)\quad \mu_{t+1}=\alpha_{\mu,0}+\alpha_{\mu,1}\mu_t+\alpha_{\mu,2}g_t+v_{\mu,t+1},$$

with $v_{\mu,t+1}\sim N(0,\sigma^2_\mu)$ drawn at t+1.

It is beliefs about person-specific grade performance at given times, rather than beliefs about academic "ability" per se, that is relevant for our particular model. Nonetheless, considering the simplest textbook learning setting in which students learn about a constant, person-specific academic "ability" illustrates issues related to Eq. (8).[8] Suppose grades are determined by $g_t=\mu+v_t$ with μ being the constant

[8]While beliefs about grade performance and beliefs about ability are closely related, focusing on grade performance makes things less complicated in certain important ways. For example, if one cares about learning about ability per se then it is necessary to disentangle whether changes in beliefs about grade performance are due to learning about ability or are due to other endogenous decisions or trends (e.g., how much to study in a semester or time-series trends in grades across semesters etc.). Perhaps most importantly, it is much easier to construct survey questions which allow students to describe their beliefs about the relationship between grade performance and future earnings than it is to construct survey questions which allow students to describe their beliefs about the relationship between ability and future earnings. Regardless, S&S (2009) find that the distinction between beliefs about grade

representing "ability" and ν_t representing transitory noise. Bayesian learning about μ would have the "posterior mean" as a weighted average of the "prior mean" and the "noisy signal" with the weights depending on both the amount of uncertainty at t about μ and the amount of variation in ν_t (i.e., the signal-to-noise ratio). Using survey questions which ascertain individual-specific beliefs related to the signal-to-noise ratio, S&S (2009) finds evidence of individual-specific heterogeneity in weights, but that the very large majority of explainable heterogeneity in μ_{t+1} arises because of heterogeneity in the observed values of μ_t and heterogeneity in the observed beliefs about g_t. Thus, we simplify matters here by assuming that the coefficients in (8) are constant across people. It is also natural to believe that the coefficients in Eq. (8) might change over time. For example, in the simple Bayesian model above, the signal-to-noise ratio would be expected to change over time as individuals resolve uncertainty about μ. Thus, in our empirical work we estimate different coefficients in Eq. (8) for different stages of college.

The update σ_{t+1} is given by

(9) $\sigma_{t+1} = \alpha_{\sigma,0} + \alpha_{\sigma,1}\sigma_t + v_{\sigma,t+1}$,

with $v_{\sigma,t+1} \sim N(0,\sigma^2_{\sigma})$ drawn at t+1 and the parameters again being allowed to vary across stages of college.

Eqs. (6)-(9) show that, from the perspective of a person at time t, G_{t+1}, EN_{t+1}, μ_{t+1} and σ_{t+1} are random variables whose means depend on the previously identified state variables G_t, EN_t, μ_t and σ_t that are known by the person at time t. Randomness in G_{t+1}, EN_{t+1}, μ_{t+1}, and σ_{t+1} is present due to uncertainty about g_t as characterized by μ_t, and σ_t, as well as uncertainty about the unobservables $v_{EN,t+1}$, $v_{\mu,t+1}$, and $v_{\sigma,t+1}$.

State variables influencing $U^N_t(\bullet)$

At time t, a person who is choosing between S and N must implicitly think about V_N from equation (1) for each possible time $t' \geq t$ at which he might choose to leave school. Under the linear

performance and beliefs about ability are not particularly important. Most of the learning that takes place about grade performance is due to learning about ability (as defined to be grade performance at a constant level of study effort and course difficulty).

assumption in equation (3), equation (1) becomes

$$(10)\quad V^{N}_{t'}(\Omega_{t'}) = \sum_{\tau=t'}^{T^*} \beta^{\tau-t'} E(C_\tau) + \beta^{\tau-t'} E(\epsilon_{N,\tau}).$$

Thus, for each possible exit time t′, a person must think about the average consumption that he would receive in each period τ after leaving. We assume that a student's beliefs about his average consumption at time τ will vary with: 1) t′-1, the number of semesters he completes before leaving, 2) $G_{t'}$, his cumulative GPA at the time he leaves, and 3) his age at τ. We write beliefs about average consumption at time τ for a student who leaves school at t′as the function $\bar{C}_\tau(t', G_{t'}, AGE(\tau))$. As discussed in Section IV, by directly eliciting information about the function $\bar{C}_\tau$ we are able to take into account that, for a variety of reasons, the function $\bar{C}_\tau$ may vary substantially across students. Given student i's individual-specific function $\bar{C}_\tau$, with t′ a choice variable and a person's age at τ known, the state variables at t that influence i's beliefs about the average consumption associated with N at a future time τ are those that are related to beliefs about $G_{t'}$: μ_t, σ^2_t, and G_t.

III.E. More detail about value functions

Given the discussion in III.C and III.D, we can rewrite the value functions in Eqs. (10) and (2).

$$(11)\quad V^{N}_{t}(G_t) = \sum_{\tau=t}^{T^*} \beta^{\tau-t} \bar{C}_\tau(t, G_t, AGE(\tau)) + \beta^{\tau-t} E(\epsilon_{N,\tau})$$

$$(12)\quad V^{S}_{t}(G_t, EN_t, \mu_t, \sigma_t, \epsilon_t) = EU^{S}_{t}(G_t, EN_t, g_t, \epsilon_t) + Pr(G_{t+1} < F_{t+1})\ \beta V^{N}_{t+1}(G_{t+1})$$
$$+ Pr(G_{t+1} \geq F_{t+1})\ \beta Emax[V^{S}_{t+1}(G_{t+1}, EN_{t+1}, \mu_{t+1}, \sigma_{t+1}, \epsilon_{t+1}), V^{N}_{t+1}(G_{t+1})],$$

where we have rewritten $PrFail_t$ to make explicit that a person fails out of school if G_{t+1} is less than an institutional cumulative grade cut-off F_{t+1} at t+1.[9] With G_t, EN_t, and ϵ_t known, the first expectation in Eq. (12) involves a one-dimensional integral over a person's beliefs at time t about g_t as characterized by μ_t and σ_t. With ϵ_{t+1} not observed as of time t and randomness in G_{t+1}, EN_{t+1}, μ_{t+1}, and σ_{t+1} present due to uncertainty about g_t, $v_{EN,t+1}$, $v_{\mu,t+1}$, and $v_{\sigma,t+1}$, the second expectation involves a multi-dimensional integral

[9]The stated cutoffs at Berea were F_2 =0.0, F_3 =1.5, F_4 =1.67, F_5 =1.85, F_6 =2.0, F_7 =2.0, F_8 =2.0. However, the cutoffs were, in practice, somewhat lower because students were often able to successfully appeal suspension decisions. As a result, we choose to use empirical cutoffs constructed as the minimum value of G_t at which a person was observed remaining in school in our sample: F_2 =0.0, F_3 =1.17, F_4 =1.31, F_5 =1.82, F_6 =1.83, F_7 =1.89, and F_8 =2.0. Results depend little on which set is used.

over a person's beliefs about g_t (as characterized by μ_t and σ_t) and over the distribution of the random variables ϵ_{t+1} and $v_{t+1}=\{v_{EN,t+1},v_{\mu,t+1}, v_{\sigma,t+1}\}$.

III.F. Identification

The current period utility parameters in Eq. (4) are identified by observed choices. Eq. (3) shows that the deterministic portion of utility from being out of the workforce is a person's consumption. Normalizing the constant in Eq. (3) to be zero allows the coefficients in Eq. (4) to be interpreted as effects on utility measured in consumption dollars. This normalization also fixes the scale of the discrete choice problem so that it is possible to estimate the variance of ϵ_t. Assuming that ϵ has an Extreme Value distribution, we estimate the parameter τ where $Var(\epsilon_{N,t})=Var(\epsilon_{S,t})=\tau^2\pi^2/6$. The parameters of Eqs. (7-9) are identified because our unique data collection efforts imply that both dependent and independent variables in these equations are observed in each semester.

IV. Data

Section III indicates that solving the necessary value functions for person i (up to the value of ϵ_t) requires observing G_t, EN_t, μ_t, and σ^2_t for each semester that i is in school, and also requires knowledge of the person-specific function $\bar{C}_\tau$.[10] In addition, while it is *beliefs* about g_t (as given by μ_t and σ^2_t) that are used to compute value functions, it is *actual* values of g_t that will be used to estimate the parameters of the transition process in Eq. (8).

G_t and g_t are obtained for each t from administrative data. The first four rows of Table 1 show the sample mean (standard error of the sample mean) of g_t at t=1,2,...,7 for the full sample of students who were still enrolled as of t (Row 1) and for three subsamples created by stratifying the full sample on the basis of how long students remained in school (Rows 2-4). Looking across columns in Row 1 reveals that, for the full sample of students who were still enrolled as of t, there is a statistically significant increase in the mean of g_t across semesters. However, the sample mean in Row 2 for the composition-constant subset of students who were in school for all of the seven semesters changes very

[10]Also needed is the discount factor β. We assume a yearly discount factor of .95.

little across time, suggesting that the increase over time in the first row is due largely to the change in composition that arises as worse students leave school over time. Row 3 provides some evidence of this by showing that students who left school after completing four, five, or six semesters have sample average values of g_t that are somewhat lower than the sample average values in Row 2. Even stronger evidence of changes in composition appear in Row 4 which shows that students who left school after completing one, two, or three semesters have sample average values of g_t that are much lower than the sample average values in Row 2. Thus, the results suggest that grades are indeed likely to be important determinants of drop-out, especially among those that leave school early.

Moving away from administrative data, the unique feature of this project is that the BPS was designed to minimize the assumptions needed to characterize the individual-specific values of EN_t, μ_t, and σ^2_t for each t and the individual-specific function $\bar{C}_\tau(\bullet)$ for each τ. With respect to EN_t, as discussed earlier we use Question A.1 to elicit a direct measure of how much a person enjoys school relative to being out of school. Row (14) of Table 1 shows that students tend to enter school with a very positive outlook about the non-pecuniary utility of being in college; 89% of students in the sample believe that school will be somewhat more enjoyable or much more enjoyable than not being in college (i.e., EN_1=1). The next three rows show EN_t for those who were in school for all seven of the semesters, those who left school after completing between four and six semesters, and those who left school after completing between one and three semesters. The three groups entered school similarly optimistic. However, by the beginning of the third semester, the sample percentage of students with EN_t=1 decreased by only six percentage points for those who remained in school for all seven of the semesters (Row 15), by nine percentage points for those who left school after completing between four and six semesters (Row 16), but by twenty-five percentage points for those who left school after completing between one and three semesters (Row 17).

Motivating our approach of directly eliciting information about μ_t, σ^2_t, and $\bar{C}_\tau$, Manski (2004) describes why it is not possible to identify both beliefs about a factor that might influence a decision and

preferences about that factor solely from observed data on choices. Our desire to move away from traditional but untestable assumptions that allow one to characterize beliefs indirectly motivated the emphasis of the BPS on directly eliciting information using carefully worded survey questions.

For example, we administered Question A.2 (Appendix A) at the beginning of each semester t to elicit directly each student's subjective beliefs about the distribution of g_t. Paying close attention to methodological suggestions in Dominitz (1998) and Dominitz and Manski (1996, 1997), the question asks each student to report the "percent chance" that g_t will fall in each of a set of mutually exclusive and collectively exhaustive categories. Importantly, students who left school were sent exit surveys immediately after leaving school. This allows us to observe beliefs about g_t at the beginning of semester t both for those who decided to stay in school for semester t and for those who were in school for t-1 but did not return for semester t. Examining the first year of college, S&S (2009) found strong evidence of the usefulness of directly eliciting beliefs in this context; directly elicited beliefs were found to be inconsistent with the standard Rational Expectations (RE) assumption described in the introduction and were found to satisfy certain theoretical implications that were not satisfied by beliefs constructed under RE.

For descriptive purposes, we compute the approximate mean of the distribution describing beliefs about g_t from a person's answers to Question A.2 (Appendix A) by assuming that a person's beliefs are uniformly distributed within each of the grade categories. Rows 6-9 of Table 1 show the sample averages of these approximate means at times t=1,2,...,7 for the full sample of those who were still enrolled in school at the beginning of t (Row 6) and over subsamples generated by stratifying on the basis of how long students remained in school (Rows 7-9). Comparing the t=1 entry of Row 6 to the t=1 entry of Row 1, we find that, in the sample as a whole, students are, on average, substantially overoptimistic about their average grade performance at entrance. Comparing the t=2 entry of Row 6 to the t=1 entry of Row 6 we find that, in the full sample, students update their beliefs significantly between t=1 and t=2. Comparing the first four entries of Rows 7 and 8 to the first four entries of Row 9 we find that the

learning is concentrated largely in the subsample of students who left school in the first three semesters, a result that is consistent with the fact that Row 4 showed that this group had particularly low grades. Rows 10-13 show approximate standard deviations of the distribution describing beliefs about g_t averaged over the full sample and averaged over subsamples generated by stratifying on the basis of how long students remained in school. The results indicate that uncertainty decreases significantly over time, even under the composition-constant sample of students who remain in school for all semesters (Row 11).

Eq. (5) described our assumption, needed for estimation, that i's beliefs about g_t can be represented by censoring an underlying latent random variable $g_t^* \sim N(\mu_t,\sigma^2_t)$. At each time t, we obtain our person-specific measures of μ_t and σ_t by fitting the censored random variable to the person's self-reported probabilities from Question A.2. Specifically, for each person we choose μ_t and σ_t to minimize

$$(13)\quad \sum_{j=1}^{6} |PR_{observed}(g_t \in CAT_j) - PR_{model}(g_t \in CAT_j)| \ ,$$

where $CAT_1,...,CAT_6$ represent the grade categories [4.0,3.5), [3.5,3.0), [3.0,2.5), [2.5,2.0), [2.0,1.0), and [1.0,0], respectively, the first term in the difference is the self-reported perceived probability of category CAT_j from Question A.2 and the second term in the difference is the probability that the censored random variable produces a realization in category CAT_j. We find that a censored normal is able to fit the self-reported probabilities quite well. For example, for t=1 we find that the average value of $|PR_{observed}(g_t \in CAT_j) - PR_{model}(g_t \in CAT_j)|$ across all categories j and all sample members is .018.

Similarly, at the time of college entrance, for some combinations of possible exit times t', possible exiting grade point averages $G_{t'}$, and possible future years $\tau \geq t'$, we also utilized survey questions to directly elicit the beliefs about the expected future yearly earnings that determine the individual-specific function $\bar{C}_\tau(t', G_{t'}, AGE(\tau))$.[11] With respect to t', we collected information about

[11]The survey question (full question not shown) informed respondents that "when reporting incomes take into account the possibility that you will work full-time, the possibility that you will work part-time, and (for the hypothetical scenarios which involve graduation) the possibility that you will attend graduate or professional school. When reporting income you should ignore the effects of price inflation."

leaving college immediately, after one full year of school, after three full years of school, and at the time of graduation. With respect to $G_{t'}$, we collected information about leaving school with a GPA of 3.75, 3.0, and 2.0. With respect to τ, we collected information about earnings in the first year out of school, at the age of 28, and at the age of 38. We further reduced the number of possible combinations by assuming that $G_{t'}$ does not influence future earnings if a person leaves school without graduating. Thus, we collected beliefs about the expected earnings that would be received at three future points in time (first year out of school, age 28, and age 38) for each of six schooling scenarios (leave school immediately, leave school after one year, leave school after three years, graduate with a 2.0 GPA, graduate with a 3.0 GPA, and graduate with a 3.75 GPA).

Figure 1 shows the sample mean of $\bar{C}_\tau$ at each of the three points in time for each of six schooling scenarios. With respect to the premium to completing a degree, the first set of bars shows that students believe that, in the first year out of school, the premium of graduating with a 3.0 grade point average would be $25,000 (130%) relative to the scenario of leaving immediately, would be $23,000 (103%) relative to the scenario of leaving after finishing one year, and would be $14,000 (46%) relative to the scenario of leaving after finishing three years. The second and third sets of bars show that these premiums remain quite similar at the age of 28 and at the age of 38. With respect to the premium to performing well academically conditional on graduating, the first set of bars shows that students believe that, in the first year out of school, the premium of graduating with a 3.75 GPA would be $10,000 (25%) relative to the scenario of graduating with a 2.0 GPA and would be $5,000 (11%) relative to the scenario of graduating with a 3.0 GPA. The third set of bars shows that students believe that, at age 38, the premium of graduating with a 3.75 GPA would be be $16,000 (28%) relative to the scenario of graduating with a 2.0 GPA and would be $7,000 (11%) relative to the scenario of graduating with a 3.0 GPA.

V. Solving value functions and Estimation

V.A. Solving Value Functions

Computing $V^N_{t'}$. Given our assumption that a student's GPA does not influence his future earnings if he does not graduate, it is necessary to compute $V^N_{t'}$ for 1) for every possible time t' that a person might leave school under the scenario that he does not graduate (t'=2,3,...,8) and 2) for each possible value of $G_{t'}$ that a person could have under the scenario in which he graduates (t'=9). Then, Eq. (11) implies: 1) for each t'=2,3,...,8, $\bar{C}_\tau$ is needed for all $\tau \geq t'$ and 2) for t'=9, $\bar{C}_\tau$ is needed for all $\tau \geq t'$ for each possible value of G_9. Section IV discussed the combinations of t', G_t and τ at which we elicited $\bar{C}_\tau$ directly. Our approach is to use these directly elicited combinations to interpolate $\bar{C}_\tau$ for all other necessary combinations.[12] Figure 2 shows the sample mean value of $V^N_{t'}()$ for the six schooling scenarios from Section IV. As expected given Figure 1, there exist sizeable lifetime premiums both for completing more years of schooling and for having a higher GPA at graduation.

Solving $V^S_{t'}$. The expected value in the last term of Eq. (12) is present because uncertainty exists at t about ϵ_{t+1}, EN_{t+1}, G_{t+1}, μ_{t+1}, and σ_{t+1}. The assumption that $\epsilon_{N,t+1}$ and $\epsilon_{S,t+1}$ have extreme value distributions implies that the Emax has a well-known closed form solution conditional on the realizations of EN_{t+1}, G_{t+1}, μ_{t+1}, and σ_{t+1}. Then, evaluating the last expected value involves summing the closed form solution over the probability function of the binary random variable EN_{t+1} and integrating over the densities of the continuous random variables G_{t+1}, μ_{t+1}, and σ_{t+1}. Appendix B describes the simulation approach that we take to evaluate this integral. This simulation approach takes into account that uncertainty about G_{t+1}, μ_{t+1}, σ_{t+1}, and EN_{t+1} is driven primarily by uncertainty about g_t.

The recursive formulation of value functions in Eq. (12) motivates a backwards recursion solution process of the general type that is standard in finite horizon, dynamic, discrete choice models.

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[12]We use a straightforward interpolation approach under the following assumptions: 1) to deal with the fact that values of $\bar{C}$ is only observed directly for the time a person leaves school, at age 28, and at age 38, we assume that $\bar{C}$ is linear between the time a person leaves school and the age of 28, is linear between the age of 28 and 38, and is constant after the age of 38; 2) to deal with the fact that values of $\bar{C}$ are only observed at the drop-out times t'=1, t'=3, and t'=7, we assume that $\bar{C}$ is linear between t'=1 and t'=3, is linear between t'=3 and t'=7, and is the same at t'=8 as it is at t'=7; 3) to deal with the fact that, for t'=9 (graduation), values of $\bar{C}$ are only observed for the values of G_9=2.0, G_9=3.0, and G_9=3.75, we assume that V^N_9 is linear between G_9=2.0 and G_9=3.0 and is linear between G_9=3.0 and G_9=4.0 (with the slope being identified by the values of V^N_9 at G_9=3.0 and G_9=3.75 and this slope being used to extrapolate values of V^N_9 between G_9=3.75 and G_9=4.0).

The most basic property of the algorithm is that, in order to solve all necessary value functions at time t, it is necessary to know value functions at time t+1 for each combination of the state variables in $\Omega(t+1)$ that could arise at time t+1. In Appendix B we discuss computational issues that arise when implementing the backwards recursion solution process in our particular application, including the modification that is needed to deal with the fact that we have multiple continuous, serially correlated state variables, G_{t+1}, μ_{t+1}, and σ_{t+1}.

V.B Estimation We estimate the parameters of the model by Maximum Likelihood. The likelihood contribution for person i is the joint probability of observing his schooling decisions and all values of EN_t, μ_t, and σ_t that are reported after t=1. The likelihood terms associated with the reported values of μ_t and σ_t involve density evaluations with the densities determined by Eqs. (8) and (9). The likelihood term associated with the reported values of EN_t involve probability calculations as described by Eq. (7). With respect to schooling choices, we examine decisions from whether to return for the second semester (t=2) through whether to return for the fourth year (t=7). For a person who chooses to return to school in each semester through the seventh semester, the likelihood contribution associated with his observed choices is the probability that he chooses S in t=2, t=3, ..., and t=7. For a person who chooses to leave school at some time $t' \leq 7$, the likelihood contribution associated with his observed choices is the probability that he chooses S in t=2, t=3, ..., t=t′-1 and chooses N in t=t′. For a person who is forced out of school due to bad academic performance at time t′, the likelihood contribution associated with his observed choices is the probability that he chooses S in t=2, t=3, ..., t=t′-1. At each time t, the probability of choosing S is given by $PR(V_S>V_N)$. With value functions solved up to ϵ_t and the components of ϵ_t having Extreme Value distributions, this probability has the standard closed form logit solution. The Maximum Likelihood approach is also conducive to dealing with missing data. For example, if a person does not answer a survey at time t, then EN_t, μ_t, and σ_t will be missing. We construct the joint distribution of the missing data from Eqs. (7)-(9)

and compute the choice probability at t by using simulation methods to integrate the choice probability conditional on EN_t, μ_t, and σ_t over the constructed distributions.

VI. Results

The parameters to be estimated are those that appear in Eq. (4) and Eqs. (7-9). Estimates are shown in Table 2.

VI.A. Estimates of parameters related to the evolution of EN_t, μ_t, and σ_t

Estimates of the parameters of Eq. (7) are shown in the second panel of Column 1. By far the most important determinant of whether someone likes school in period t+1, $EN_{t+1}=1$, is whether the person liked school in t, $EN_t=1$ (t-statistic 13.16). $EN_{t+1}=1$ is also influenced in a significant manner by the student's grades in t (t-statistic 3.82).

Estimates of the parameters of the updating Eqs. (8) and (9) are shown in the third and fourth panels of Column 1. The strength of our approach in estimating Eqs. (8) and (9) is that we directly observe μ_{t+1}, μ_t, σ_{t+1}, σ_t, and g_t. Given the discussion in Section III.D that the coefficients in Eqs. (8) and (9) may vary over time, we estimate Eqs. (8) and (9) separately for updates that take place after the first and second semesters, updates that take place after the third and fourth semesters, and updates that take place during the remaining time in school. Focusing on Eq. (8), Rows 11-22 show that, for all updates, both μ_t and g_t play an important role in determining the update μ_{t+1}. The results show evidence that the relative influence of μ_t in determining μ_{t+1} does increase over time, as would be expected if uncertainty is resolved over time. For example, Rows 11-22 show that the ratio of the estimated effect of μ_t to the estimated effect of g_t (i.e., $\alpha_{\mu,1}/\alpha_{\mu,2}$) is .320/.255=1.25 for the first two updates (t=1,2), is 2.18 for the next two updates (t=3,4), and is .558/.172=3.24 for the remainder of the updates (t>4).

VI.B. Estimates of utility parameters

Like much previous work, we find a strong reduced-form correlation between grade

performance and drop-out. Estimating the Logit model that results from setting $\beta=0$, $\tau=1$, and allowing only G_t to enter current period utility (Eq. 4), we find in Column 2 that the coefficient on G_t has a t-statistic of approximately 8.5. The usefulness of estimating the model in this paper is that it allows an opportunity to understand *why* grade performance is consistently found to be so strongly correlated with drop-out in reduced form specifications. Specifically, our model allows us to differentiate between three broad avenues through which poor academic performance could lead to drop-out: 1) poor performance causes students to fail out of school immediately or causes the value of continuing in school to decrease because it increases the probability of failing out in the future, 2) poor performance reduces how enjoyable it is to be in school, and 3) poor performance reduces the value of staying in school by reducing the earnings that a person will receive in the future if he does graduate.

Estimates of the current period utility parameters associated with being in school (Eq. 4) for the full model are shown in the first panel of Column 1. The results indicate that the second avenue above is very relevant. Both G_t and g_t have a significant effect on the current period utility of being in school with the estimated effects having t-statistics of 3.73 and 1.78, respectively. In addition, in Section VI.A we found that g_t influences the measure EN, which is itself seen in Column 1 to have a significant effect on utility (t-statistic=2.68). With income/consumption measured in hundreds of thousands of dollars, the coefficients imply that a person with G_t =4.0, g_t=4.0, and EN_t=1 would receive, on average, about the same amount of current period utility as a person who is out of school with an annual income of roughly the average expected annual income of someone who leaves school at the beginning of college ($20,000). Each .50 reduction in cumulative grade point average reduces current period utility of school by the consumption equivalent of .50*.292*$100,000=$14,600.

The current period utility specification for Eq. (4) used in Column 1 is very parsimonious.

It is worth examining whether it is useful to add to Eq. (4) student characteristics that have been consistently found to be related to drop-out in other work. Repeating the exercise in Column 2 after replacing G_t with an indicator of whether a student is male, we find in Column 3 that males are significantly more likely to drop-out (t-statistic≈-2.00). Repeating the exercise in Column 2 after replacing G_t with a student's high school grade point average, we find in Column 4 that students with higher high school grade point average are significantly less likely to drop-out (t-statistic=3.31). However, when we add the male and HSGPA variables to the full, dynamic model in Column 1, neither is statistically significant (t-statistics of .43 and .82, respectively, full results not shown). Thus, the evidence suggests that the effect of these two variables in the reduced-form arises primarily because the variables are related to grade performance or beliefs about future grade performance. We also find that, as in S&S (2009), results change very little when we add to the current period utility Eq. (4) variables measuring a student's health, family income, and whether his parent lost a job in the last period.[13]

From the estimates in Column 1 alone, it is not possible to quantify the importance of the second avenue in determining the drop-out decision or to get any sense of whether avenues (1) and (3) above are also relevant. As a result, in the next section we use simulations to quantify the overall importance of learning and the relative importance of the three avenues described above.

VI.C. Simulations

For several counterfactual scenarios, which imply various changes to G_t, g_t, EN_t, μ_t, and σ_t, we use the estimates from Column 1 of Table 2 to compute the proportion of students that would drop out by the beginning of the second year (T=3), by the beginning of the third year (T=5), and by the beginning of the fourth year (T=7). For each student, the probability of dropping out at or before the start of the Tth semester is given by $1-\prod_{t=2}^{T}\Pr(V^S_t>V^N_t)$. Section V described the

[13]

techniques that we use during estimation to compute the probabilities that appear in this expression. Here we require additional simulations to incorporate the changes to G_t, EN_t, μ_t, and σ_t. However, these simulations are straightforward extensions of our methods for dealing with missing data as described in Section V.

We begin with a baseline scenario in which nothing is changed. For this scenario, we wish to use actual values of G_t, EN_t, μ_t, and σ_t, so that the additional use of simulation is only necessary because, for someone who leaves school at the start of semester t', actual values of G_t, g_t, EN_t, μ_t, and σ_t are not observed in the data for $t>t'$.[14] Our baseline calculation finds that .193 of students drop out before the start of the second year (T=3, compared to .18 actual), that .355 of students drop out before the start of the third year (T=5, compared to .34 actual), and that .483 of students drop out before the start of the fourth year (T=7, compared to .46 actual).

To quantify the overall importance of learning, we next simulate the drop-out proportion under a no-learning counterfactual scenario in which a person's beliefs about grade performance do not change after the time of entrance and actual grades g_t are drawn from this perceived grade distribution. Specifically, for all t: 1) $\mu_t=\mu_1$ and $\sigma_t=\sigma_1$, 2) the distribution of actual grades g_t is determined by Eq. (5) given parameters μ_1 and σ_1 and 3) $PR(EN_t=1)$ is determined from Eq. (7) based on EN_1 and by g_1, g_2,..., g_{t-1}. Under this no-learning scenario, we find that .106 of students would drop out by the start of the second year, .194 of students would drop out before the start of the third year, and that .309 of students would drop out before the start of the fourth year. Thus, .45=(.193-.106)/.193 of the drop-out in the first year, .45=(.355-.194)/(.355) of the drop-out in the first two years, and .36=(.483-.309)/(.483) of the drop-out in the first three years can be attributed to what students learn about their academic performance. These cumulative numbers imply that .45

[14]Recall that, for someone who leaves school at the beginning of semester t, we collect information about beliefs at t using an exit survey.

of drop-out in the first year, .45 of drop-out in the second year, but only .11 of drop-out in the third year is caused by learning about academic performance. That learning about academic performance plays a bigger role earlier in college is consistent with the descriptive statistics in Section IV.

Finally, we perform three additional simulations to provide evidence about the quantitative importance of the three broad avenues in Section VI.B through which learning about grade performance could cause drop-out. To examine the first avenue (that poor academic performance causes students to fail out of school immediately or causes the value of continuing in school to decrease because it increases the probability of failing out in the future), we repeat the baseline simulation, but remove the institutional rule that students are forced to leave school due to poor academic performance. We find that the percentage of students who would drop out would decrease only trivially, from .483 to .463. Thus, the results suggest that students who perform poorly tend to learn that staying in school is not beneficial, not that they leave simply because they have lost the option to stay or believe they are more likely to lose the option in the future.[15]

Differentiating between the remaining two avenues above is a matter of understanding why students find that it is not beneficial to be in school if they have performed poorly. Maintaining the assumption that students cannot fail out, we first examine the importance of avenue 3 (that poor performance reduces the value of staying in school by reducing the earnings that a person will receive in the future if he does graduate) by simulating the model under the counterfactual assumption that a person's beliefs about his earnings upon graduation are determined by his beliefs about grade performance at the start of college rather than by what he learns about his actual grade

[15]We do not observe beliefs about earnings for final GPA's of less than 2.0. For this simulation, we assume that a student's beliefs about the earnings associated with GPA's less than 2.0 is the same as his beliefs about the earnings associated with a GPA of 2.0. Thus, if anything, the true effect of removing the possibility of failing out would be even smaller.

performance during college.[16] We find that .386 of students would drop out under this counterfactual scenario, so that approximately .50=(.463-.386)/(.463-.309) of the drop-out that can be attributed to learning about academic performance would disappear under this scenario. Finally, continuing to maintain the assumption that students cannot fail out, we examine the importance of avenue 2 (that performing poorly makes it less enjoyable to be in school) by simulating the model under the counterfactual assumption that a person's non-pecuniary utility during school corresponds to the utility that would have been received if the student's perceptions about grade performance were correct at the time of entrance.[17] We find that .344 of students would drop out under this scenario, so that approximately .77 =.(463-.344)/(.463-.309) of the drop-out that can be attributed to learning about academic performance would disappear under this counterfactual.

VII. Conclusion

We find that learning about academic performance plays a very important role in determining college drop-out, with this type of learning being substantially more important for explaining attrition that occurs in the first two years of college than for explaining attrition that occurs later in college. We find that students who perform poorly tend to learn that staying in school is not worthwhile, not that they fail out or learn that they are more likely (than they previously believed) to fail out in the future.

The most important avenue through which school becomes less worthwhile is that performing poorly reduces how enjoyable it is to be in school relative to being out of school, with our simulations showing that 77% of the attrition that is due to learning would disappear under the hypothetical scenario

[16]Specifically, we set a person's beliefs about earnings upon graduation equal to what he would expect if he were to graduate with G_9 equal to the mean of the distribution describing his beliefs about grades at the time of entrance (i.e., the approximate mean from the t=1 response to Question A.2).

[17]When computing the current period utility in Eq. (4), we characterize G_t, g_t, and EN_t under the assumption that a person's grades in each period equal to the mean of the distribution describing his beliefs about grades at the time of entrance (i.e., the approximate mean from the t=1 response to Question A.2).

that poor performance does not influence current period utility. As discussed in Section III.C., it is difficult to know exactly why poor academic performance is found to have such a large effect on current period utility. Given the set of possibilities, it seems at least possible that increasing social support for students who have performed poorly might be beneficial. However, even if these types of interventions would make students feel happier in school at a given level of grade performance, we find that the reduction in the financial returns to graduating that accompanies poor performance is itself sufficient to create over half of the drop-out due to learning about ability.

Therefore, our results suggest that substantial changes in drop-out may require changing grade performance itself. The model in this paper cannot examine the potential benefits of college policies which would, for example, encourage more study effort. However, the finding in S&S (2009) that learning about grade performance should be attributed primarily to learning about academic ability (i.e., grade performance at a given level of effort and course difficulty) rather than learning about effort. Then, given that increasing effort may be difficult and would come at the expense of leisure, our results here generally suggest the importance of having students arrive at school better prepared for the academic challenges of college. To this end, improvements in the quality of elementary and secondary schools would seemingly be helpful, but ensuring that pre-college students have correct perceptions about what level of preparation is necessary to succeed in college may also be important.

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Table 1 sample mean (std. error of sample mean)

	t=1	t=2	t=3	t=4	t=5	t=6	t=7
(1) g_t all observations	2.81 (.04)	2.83 (.04)	2.85 (.04)	2.97 (.04)	2.96 (.04)	3.02 (.04)	3.07 (.04)
(2) g_t completed ≥ 7 semesters	3.05 (.04)	3.05 (.04)	2.96 (.04)	3.04 (.04)	3.01 (.04)	3.06 (.04)	3.07 (.04)
(3) g_t completed 4,5,6 semesters	2.95 (.08)	2.88 (.08)	2.76 (.10)	2.77 (.12)	2.72 (.16)	2.71 (.219)	
(4) g_t completed 1,2,3 semesters	2.11 (.118)	1.85 (.15)	1.95 (.29)				
(5) G_{t+1} all observations	2.81 (.04)	2.82 (.04)	2.90 (.03)	2.97 (.03)	3.00 (.03)	3.01 (.03)	3.03 (.03)
(6) belief $E(g_t)$ all observations	3.21 (.01)	3.10 (.02)	3.14 (.02)	3.13 (.02)	3.16 (.02)	3.21 (.02)	3.12 (.02)
(7) belief $E(g_t)$ completed >7 semesters	3.21 (.02)	3.14 (.02)	3.19 (.02)	3.16 (.02)	3.17 (.02)	3.19 (.02)	3.14 (.02)
(8) belief $E(g_t)$completed 4,5,6 semesters	3.21 (.03)	3.14 (.04)	3.22 (.04)	3.19 (.05)	3.14 (.05)	3.21 (.05)	
(9) belief $E(g_t)$ completed 1,2,3 semesters	3.20 (.03)	2.96 (.04)	2.84 (.04)	2.53(.15)			
(10) belief std. dev. (g_t) all observations	.53 (.01)	.48 (.01)	.45 (.01)	.43 (.01)	.42 (.01)	.38 (.01)	.39 (.01)
(11) belief std. dev (g_t) completed >7 semesters	.52 (.01)	.47 (.01)	.47 (.01)	.44 (.01)	.44 (.01)	.43 (.01)	.40 (.01)
(12) belief std dev. (g_t) completed 4,5,6 semesters	.53 (.02)	.48 (.02)	.44 (.02)	.41 (.02)	.42 (.02)	.34 (.02)	
(13) belief std dev.(g_t) completed 1,2,3 semesters	.53 (.02)	.51 (.02)	.51 (.03)	.51 (.09)			
(14) EN_t all observations	.89 (.01)	.77 (.02)	.79 (.02)	.78 (.02)	.81 (.02)	.81 (.02)	.81 (.02)
(15)EN_t completed >7 semesters	.90 (.02)	.83 (.02)	.84 (.02)	.82 (.03)	.82 (.03)	.81 (.03)	.81 (.03)
(16) EN_t completed 4,5,6 semesters	.82 (.04)	.73 (.05)	.73 (.05)	.75 (.05)	.79 (.05)	.78 (.07)	
(17) EN_t completed 1,2,3 semesters	.89 (.03)	.67 (.05)	.64 (.07)	.46 (.13)			

Table 2 Estimates of structural model: Estimate (std. error)

		1	2	3	4
Utility Parameters (Eq. 4)					
γ_0 -Constant	1	-2.786 (.434)**	-.819 (.375)**	2.403 (.119)**	.225 (.609)
γ_1 -Coefficient on EN	2	.479 (.179)**			
γ_2 -Coefficient on G	3	.292 (.078)**	1.14 (.135)**		
γ_3 -Coefficient on g	4	.304 (.171)*			
-Coefficient Male	5			-.297 (.150)**	
-Coefficient HSGPA	6				.607 (.183)**
τ - variance $\epsilon_{N,t}$, $\epsilon_{S,t}$ is $\tau^2\pi^2/6$	7	.986 (.049)**	1.0 (normalized)	1.0 (normalized)	1.0 (normalized)
Evolution of EN_t (Eq. 7)					
$\alpha_{EN,0}$ -Constant	8	-.288 (.207)**			
$\alpha_{EN,1}$ -Coefficient on EN	9	1.130 (.085)**			
$\alpha_{EN,2}$ -Coefficient on g	10	.212 (.055) **			
Determinants of of μ_{t+1} (Eq. 8)					
t=1 and t=2					
$\alpha_{\mu,0}$ -Constant	11	1.423 (.125)**			
$\alpha_{\mu,1}$ -Coefficient on μ	12	.320 (.038)**			
$\alpha_{\mu,2}$ -Coefficient on g	13	.255 (.014)**			
Var($v_{\mu,t+1}$)	14	.095 (.005)**			
t=3 and t=4					
$\alpha_{\mu,0}$ -Constant	15	1.055 (.119)**			
$\alpha_{\mu,1}$ -Coefficient on μ	16	.463 (.033)**			
$\alpha_{\mu,2}$ -Coefficient on g	17	.212 (.021)**			
Var($v_{\mu,t+1}$)	18	.069 (.004)**			
t>4					
$\alpha_{\mu,0}$ -Constant	19	.902 (.005)**			
$\alpha_{\mu,1}$ -Coefficient on μ	20	.558 (.031)**			
$\alpha_{\mu,2}$ -Coefficient on g	21	.172 (.015)**			
Var($v_{\mu,t+1}$)	22	.071 (.004)**			
Determinants of σ_{t+1} (Eq. 9)					
t=1 and t=2					
$\alpha_{\sigma,0}$ -Constant	23	.245 (.027)**			
$\alpha_{\sigma,1}$ -Coefficient on σ	24	.265 (.042)**			
Var($v_{\sigma,t+1}$)	25	.056 (.003)**			
t=2 and t=3					
$\alpha_{\sigma,0}$ -Constant	26	.133 (.037)**			
$\alpha_{\sigma,1}$ -Coefficient on σ	27	.494 (.067)**			
Var($v_{\sigma,t+1}$)	28	.052 (.003)**			
t>3					
$\alpha_{\sigma,0}$ -Constant	29	.066 (.031)**			
$\alpha_{\sigma,1}$ -Coefficient on σ	30	.614 (.050)**			
Var($v_{\sigma,t+1}$)	31	.039 (.003)**			
Log Likelihood		-759.648			

Figure 1 Earnings expectations

Different scenarios and different ages

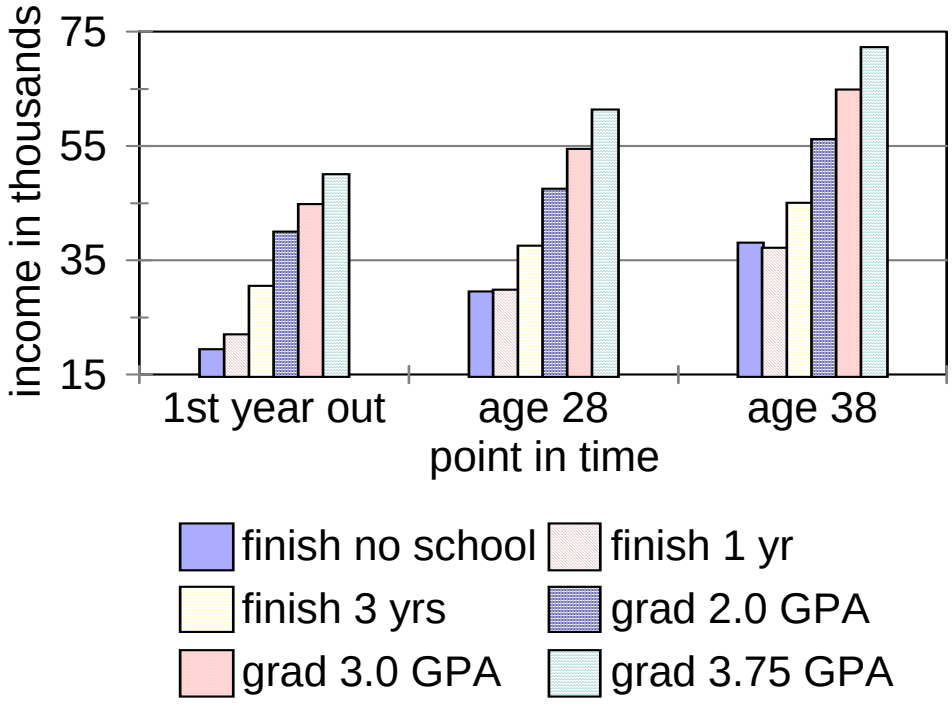
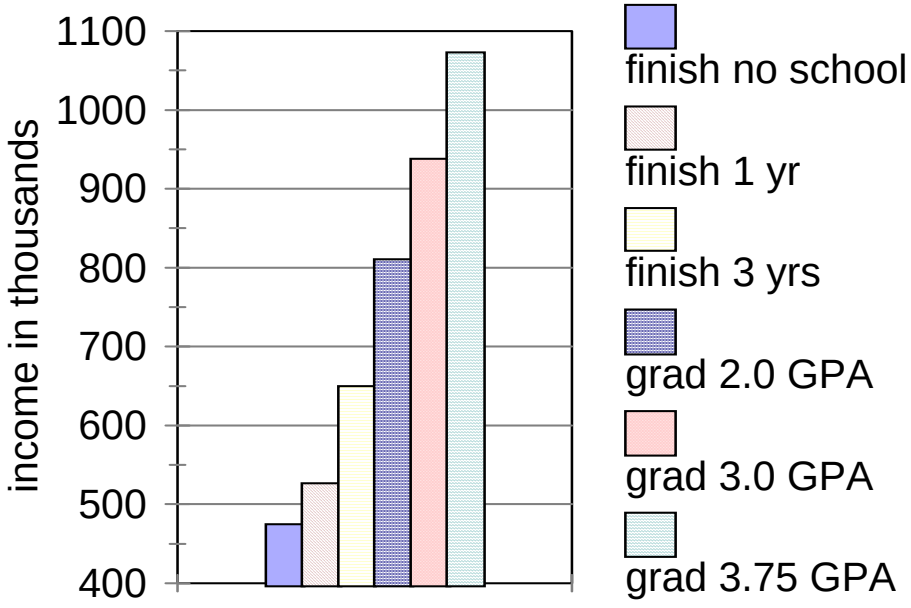


Figure 2 Discounted Expected Lifetime Earnings, VN(t')



Appendix A.

Question A.1 Circle the one answer that describes your beliefs at this time: (**Beginning of first year)**

1. I believe that being in college at Berea will be much more enjoyable than not being in college.
2. I believe that being in college at Berea will be somewhat more enjoyable than not being in college.
3. I believe that I will enjoy being in college at Berea about the same amount as I would enjoy not being in college.
4. I believe that being in college at Berea will be somewhat less enjoyable than not being in college.
5. I believe that being in college at Berea will be much less enjoyable than not being in college.

Question A.1 Circle the one answer that describes your beliefs at this time: (**Beginning of other semesters)**

1. I believe that being in college at Berea is much more enjoyable than not being in college.
2. I believe that being in college at Berea is somewhat more enjoyable than not being in college.
3. I have enjoyed being in college at Berea about the same amount as I would have enjoyed not being in college.
4. I believe that being in college at Berea is somewhat less enjoyable than not being in college.
5. I believe that being in college at Berea is much less enjoyable than not being in college.

Question A.2. We realize that you do not know exactly how well you will do in classes. However, we would like to have you describe your beliefs about the grade point average that you expect to receive in the first semester.

Given the amount of study-time you indicated above, please tell us the percent chance that your grade point average will be in each of the following intervals. That is, for each interval, write the number of chances out of 100 that your final grade point average will be in that interval.

Interval	Percent Chance (number of chances out of 100).
[3.5, 4.00]	_____
[3.0, 3.49]	_____
[2.5, 2.99]	_____
[2.0, 2.49]	_____
[1.0, 1.99]	_____
[0.0, .99]	_____

Note: A=4.0, B=3.0, C=2.0, D=1.0, F

Appendix B

The primary burden of computing value functions involves the computation of the expected future utility (Emax) of the option (S) in equation (2).

We assume that students believe that they will update μ_{t+1} and σ_{t+1} according to the predicted values from equations (8) and (9):

(A.1) $\mu_{t+1}=\alpha_{\mu,0}+\alpha_{\mu,1}\mu_t+\alpha_{\mu,2}g_t$

(A.2) $\sigma_{t+1}=\alpha_{\sigma,0}+\alpha_{\sigma,1}\sigma_t$.

Then, uncertainty about G_{t+1} and μ_{t+1} comes from uncertainty about g_t and uncertainty about EN_{t+1} comes from uncertainty about g_t and $v_{EN,,t+1}$. Letting $EMAX^*(EN_{t+1}, G_{t+1}, \mu_{t+1}, \sigma_{t+1})$ represent the well-known closed form that exists for the expected value of the maximum (conditional on EN_{t+1}, G_{t+1}, μ_{t+1}, and σ_{t+1}) when $\epsilon_{S,t+1}$ and $\epsilon_{N,t+1}$ have Extreme Value distributions,

(A.3) $Emax[V^S_{t+1}(\bullet),V^N_{t+1}(\bullet)] =$
$\int\int EMAX^*(EN_{t+1}(g_t,v_{EN,t+1}),G_{t+1}(g_t)\ \mu_{t+1}(g_t),\ \sigma_{t+1})\ f(g_t)\ h(v_{EN,t+1})\ dg_t\ dv_{EN,t+1}$

where f is the censored normal distribution in Eq. (5) which describes beliefs about g, and, as seen in Eq. (7), h is a standard normal random variable. A.3 can be rewritten as:

(A.4)

$$
\begin{aligned}
&PR(g_t=0)*\\
&\quad [Pr(EN_{t+1}=1|g_t=0)*\ EMAX^*(EN_{t+1}=1,G_{t+1}(0),\ \mu_{t+1}(0),\ \sigma_{t+1})\\
&\quad +Pr(EN_{t+1}=0|g_t=0)*\ EMAX^*(EN_{t+1}=0,G_{t+1}(0),\ \mu_{t+1}(0),\ \sigma_{t+1})]\\
&+PR(g_t=4.0)*\\
&\quad [Pr(EN_{t+1}=1|g_t=4)*\ EMAX^*(EN_{t+1}=1,G_{t+1}(4),\ \mu_{t+1}(4),\ \sigma_{t+1})\\
&\quad +Pr(EN_{t+1}=0|g_t=4)*\ EMAX^*(EN_{t+1}=0,G_{t+1}(4),\ \mu_{t+1}(4),\ \sigma_{t+1})]\\
&+PR(0<g_{t+1}<4.0)*\\
&\quad \int [Pr(EN_{t+1}=1|g_t)*\ EMAX^*(EN_{t+1}=1,G_{t+1}(g_t),\ \mu_{t+1}(g_t),\ \sigma_{t+1})\\
&\quad +Pr(EN_{t+1}=0|g_t)*\ EMAX^*(EN_{t+1}=0,G_{t+1}(g_t),\ \mu_{t+1}(g_t),\ \sigma_{t+1})]\ f(g_t|(0<g_t<4.0)dg_t.
\end{aligned}
$$

The integral in the last term of A.4 is simulated as the average value of the integrand over N draws from the conditional distribution $f(g_t|(0<g_t<4.0)$.

The most basic property of the standard solution algorithm for value functions is that, in order to solve all necessary value functions at time t, it is necessary to know value functions at time t+1 for each combination of the state variables in $\Omega(t+1)$ that could arise at time t+1.

Observable characteristics X are not burdensome because they are assumed to be exogenous and predetermined. This implies that value functions at time t+1 need to be solved only for the observed value of these variables. Similarly, ϵ_{t+1} is not computationally burdensome because it is assumed to be serially independent. In this case, ϵ_{t+1} influences $V^S_{t+1}()$ and $V^N_{t+1}()$ only through its effect on current period (t+1) utility. In general contexts, this would imply that, given $V^S_{t+1}()$ and $V^N_{t+1}()$ for some value ϵ_{t+1}, $V^S_{t+1}()$ and $V^N_{t+1}()$ could be obtained in a trivial manner for any other value ϵ_{t+1} by simply recalculating U^S_{t+1} and U^N_{t+1}. In the specific case here, where $\epsilon_{S,t+1}$ and $\epsilon_{N,t+1}$ have Extreme Value distributions, $V^S_{t+1}()$ and V^N_{t+1} do not have to be computed explicitly for different values of ϵ_{t+1} since the integration over $V^S_{t+1}()$ and V^N_{t+1} in the Emax leads to the well-known closed form solution represented by Emax* above.

The burden of solving value functions comes primarily from the variables EN_{t+1}, G_{t+1}, μ_{t+1}, and $\sigma_{t+1.}$ For each of these variables, the computational burden arises because: 1) there are multiple values for which value functions are needed at time t+1 and 2) the current period value of the variable provides information about both current and future utility. The latter characteristics implies that, in order to compute $V^S_{t+1}()$ for any particular

combination of these variables, it is necessary to recompute the computationally demanding Emax in time t+2.

EN_{t+1} is a discrete (binary) variable so it can take on only two particular values at time t+1. However, the remaining variables are serially correlated continuous variables, and this causes well-known difficulties for the backwards recursion solution methods. As discussed in detail in Bound et al. (2010), Keane and Wolpin (1994), Rust (1997), and Stinebrickner (2000), quadrature or simulation methods are a useful tool for addressing the difficulties of serially correlated, continuous variables because, in effect, they served to discretize the state space - an obvious necessity given that the backwards recursion process requires that value functions be solved for all combinations of state variables. Unfortunately, while finite, the number of possible combinations of G_{t+1}, μ_{t+1}, and σ_{t+1} is in practice very large so that it is infeasible to solve value functions using standard methods for all possible combinations of EN_{t+1}, G_{t+1}, μ_{t+1}, and σ_{t+1} that could arise.

We address this issue by implementing a modified version of the backwards solution process. The first step is to determine the range of possible values that each of the variables G_{t+1}, μ_{t+1}, and σ_{t+1} could have in each time period for which the individual is making decisions. The modified backwards recursion process can then take place. At each time t in the backwards recursion process, rather than solving value functions for all possible values of G_t, μ_t, and σ_t, value functions, V^S is solved for the largest possible values of each of these variables, the smallest possible values for each of these variables, and some subset of the possible values in between the largest and smallest possible values for each of these variables. We refer to a combination of values of G_t, μ_t, and σ_t for which V^S_t is solved as a grid point. The simulation of A.4 implies that solving the value functions associated with the grid points at time t requires knowledge of value functions V^S at time t+1 for various combinations of G_{t+1}, μ_{t+1}, and σ_{t+1}. The reality that these needed combinations will not correspond to the time t+1 grid point (for which value functions were actually solved at t+1) necessitates a value function approximation. Specifically, we interpolate the t+1 value function associated with a particular combination G_{t+1}', μ_{t+1}', and σ_{t+1}' as the weighted average of the value functions associated with the eight "surrounding" grid points, where the weight associated with a particular grid point is determined by the euclidian distance between the grid point and G_{t+1}', μ_{t+1}',and σ_{t+1}'.[18] This nonparametric interpolation approach using surrounding grid points has the virtue that the interpolated value function for (G_{t+1}', μ_{t+1}', σ_{t+1}') converges to the true value function as the number of grid points increases (i.e., as the grid points used in the weighted average become close to (G_{t+1}', μ_{t+1}', σ_{t+1}') .

[18]The "surrounding" grid points are defined to be the eight possible combinations of $\{G_{t+1}^H, G_{t+1}^L\}$, $\{\mu_{t+1}^H, \mu_{t+1}^L\}$, and $\{\sigma_{t+1}^H, \sigma_{t+1}^L\}$,where G_{t+1}^H is smallest value greater than G_{t+1}' for which value functions were solved at time t+1, G_{t+1}^L is largest value less than G_{t+1}' for which value functions were solved at time t+1, μ_{t+1}^H is smallest value greater than μ_{t+1}' for which value functions were solved at time t+1, μ_{t+1}^L is largest value less than μ_{t+1}' for which value functions were solved at time t+1,σ_{t+1}^H is smallest value greater than σ_{t+1}' for which value functions were solved at time t+1, σ_{t+1}^L is largest value less than σ_{t+1}' for which value functions were solved at time t+1. Then, the surrounding grid points form a cube around the point (G_{t+1}', μ_{t+1}', σ_{t+1}').