Feedback Effects and the Limits to Arbitrage*

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Abstract

This paper identifies a limit to arbitrage that arises from the fact that a firm's fundamental value is endogenous to the act of exploiting the arbitrage. Trading on private information reveals this information to managers and helps them improve their real decisions, in turn enhancing fundamental value. While this increases the profitability of a long position, it reduces the profitability of a short position – selling on negative information reveals that firm prospects are poor, causing the manager to cancel investment. Optimal abandonment increases firm value and may cause the speculator to realize a loss on her initial sale. Thus, investors may strategically refrain from trading on negative information, and so bad news is incorporated more slowly into prices than good news. The effect has potentially important real consequences – if negative information is not incorporated into stock prices, negative-NPV projects may not be abandoned, leading to overinvestment.

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1 Introduction

Whether financial markets are informationally efficient is one of the most hotly-contested debates in finance. Proponents of market efficiency argue that profit opportunities in the financial market will lead speculators to trade in a way that eliminates any mispricing. For example, if speculators have negative information about a stock, and this information is not reflected in the price, they will find it profitable to sell the stock. This will push down the price, causing it to reflect speculators' information. However, a sizable literature identifies various limits to arbitrage, which may deter speculators from trading on their information. (This notion of "arbitrage" is broader than the traditional textbook notion of risk-free arbitrage from trading two identical securities. Here, we use "arbitrage" to refer to investors trading on their private information.) For example, De Long, Shleifer, Summers, and Waldmann (1990) and Shleifer and Vishny (1997) show that the slow convergence of price to fundamental value may render arbitrage activities too risky. This in turn dissuades trading if the speculator has a short horizon, which may in turn arise from informational asymmetries with her own investors. Other explanations for limited arbitrage rely on market frictions such as short-sales constraints. All of these mechanisms treat the firm's fundamental value as exogenous to the arbitrage process and rely on market imperfections to explain why speculators will not drive the price towards fundamental value. Thus, as financial markets develop, these limits to arbitrage may weaken.

In this paper, we identify a quite different limit to arbitrage, which does not rely on exogenous forces but is instead generated endogenously as part of the arbitrage process. It stems from the fact that the *value of the asset being arbitraged is endogenous to the act of exploiting the arbitrage* – it depends on speculators' trading behavior and market prices. By trading, speculators cause prices to move, which in turn reveals information to real decision makers, such as managers, board members, corporate raiders, and regulators. These decision makers then take actions based on the information revealed in the price, and these actions change the underlying asset value. This may make the initial trading unprofitable, deterring it from occurring in the first place.

To fix ideas, consider the following example. Suppose that a firm (acquirer) announces the acquisition of a target. Also assume that some speculators conducted some analysis suggesting that this acquisition will be value-destructive. Traditional theory suggests that these speculators should sell the acquirer's stock. However, large-scale selling will convey to the acquirer that speculators believe the acquisition is a bad idea. As a result, the acquirer may end up cancelling the acquisition. In turn, cancellation of a bad acquisition will boost firm value, reducing the speculator's profit from her short position and in some cases causing her to suffer a loss. Put differently, the acquirer's decision to cancel the acquisition means that the negative information possessed by speculators is now less relevant, and hence they should not trade on it. Thus, the information ends up not being reflected in the price.

Our mechanism is based on the presence of a feedback effect from the financial market to real economic decisions. A common perception is that managers know more about their

own firms than outsiders (e.g. Myers and Majluf (1984)). While this is likely plausible for internal information about the firm in isolation, optimal managerial decisions also depend on external information (such as market demand for a firm's products, or potential synergies with a target) about which outsiders may be more informed. A classic example of how information from the stock market can shape managerial decisions is Coca-Cola's attempted acquisition of Quaker Oats. On November 20, 2000, the *Wall Street Journal* reported that Coca-Cola was in talks to acquire Quaker Oats. Shortly thereafter, Coca-Cola confirmed such discussions. The market reacted negatively, sending Coca-Cola's shares down 8% on November 20th and 2% on November 21st. Coca-Cola's board rejected the acquisition later on November 21st, potentially due to the negative market reaction. The following day, Coca-Cola's shares rebounded 8%. Thus, speculators who had short-sold on the initial merger announcement, based on the belief that the acquisition would destroy value, may have ended up losing money – precisely the effect modeled by this paper. In the same context, Luo (2005) provides large-sample evidence that acquisitions are more likely to be cancelled if the market reacts negatively to them, and that the effect is more pronounced when the acquirer is more likely to have something to learn from the market. More broadly, Chen, Goldstein, and Jiang (2007) show that the sensitivity of investment to price is higher when the price contains more private information not known to managers. Edmans, Goldstein, and Jiang (2011) demonstrate that a firm's market price affects the likelihood that it becomes a takeover target, which may arise because potential acquirers learn from the market price.

Moreover, our model can apply to corrective actions (i.e., actions that improve firm value upon learning negative information about firm prospects) undertaken by stakeholders other than the manager. Such stakeholders likely have less information than the manager and may be more reliant on information held by outsiders. Examples include managerial replacement (undertaken by the board, or by shareholders who lobby the board), a disciplinary takeover (undertaken by an acquirer), or the granting of a subsidy or a bail-out (undertaken by the government). We demonstrate a barrier to the feedback effect, that hinders decision makers from learning from the market.

An important aspect of our theory is that it generates asymmetry between trading on positive and negative information. The feedback effect delivers an equilibrium where speculators trade on positive news but do not trade on negative news. Yet, it does not give rise to the opposite equilibrium, where speculators trade on negative news only. The intuition is as follows. When speculators trade on information, they improve the efficiency of the firm's decisions – regardless of the direction of their trade. If the speculator has positive information on a firm's prospects, trading on it will reveal to the manager that investment is profitable. This will in turn cause the firm to invest more, thus increasing its value. If the speculator has negative information, trading on it will reveal to the manager that investment is unprofitable. This will in turn cause the firm to invest less, also increasing its value as contraction is the correct decision. When a speculator buys and takes a long position in a firm, she benefits further

from increasing its value via the feedback effect. By contrast, when she sells and takes a short position, she loses from increasing the firm's value via the feedback effect. Note that, for the speculator to lose from the feedback effect, she must end up with a short position. Thus, the model implies that investors are less likely to engage in short-sales than sales – even though the model contains no short-sale constraints.

Even though the speculator's trading behavior is asymmetric, in general it is not automatic that the impact on prices is asymmetric. The market maker is fully rational and takes into account the fact that the speculator buys on positive information and does not trade on negative information. Thus, he adjusts his pricing function accordingly. Therefore, it may seem that negative information will be impounded in prices to the same degree as positive information – even though it may lead to a neutral rather than negative order flow, the market maker knows that a neutral order flow can stem from the speculator having negative information but choosing not to trade, and may decrease the price accordingly. By contrast, we show that the asymmetry in trading behavior does translate into asymmetry in price impact. The crux is that the market maker cannot distinguish the case of a speculator who has negative information but chooses to withhold it, from the case in which the speculator is absent (i.e. there is no information). Thus, a neutral order flow does not lead to a large stock price decrease, and so negative information has a smaller effect on prices. Indeed, Hong, Lim, and Stein (2000) show empirically that bad news is incorporated in prices more slowly than good news. They speculate that this arises because it is firm management that possesses value-relevant information, and they will publicize it more enthusiastically for favorable than unfavorable information. Our paper presents a formal model that offers an alternative explanation. Here, key information is held by a firm's investors rather than its managers, who "publicize" it not through public news releases, but by trading on it. They also choose to disseminate good news more readily than bad news, but for different reasons from firm management, i.e., because of the feedback effect.

In standard models of underreaction, if bad news has a smaller effect on short-run returns (i.e. between $t = 0$ and $t = 1$) than good news, this must be counterbalanced by bad news generating a larger long-run drift (between $t = 1$ and $t = 2$) than good news. We show that this need not be the case in a model with feedback. It is indeed true that, if the state is bad and little bad news comes out in the short-run (due to the speculator not trading on it), there is more bad news to come out in the long-run. However, in a feedback model where firm value is endogenous, the manager can take a corrective action to mitigate the negative impact of the state on firm value. If the feedback effect is sufficiently strong, bad news has a smaller effect than good news in both the short-run and the long-run.

While the above considers the returns to good and bad news, the model also generates predictions regarding the returns to good and bad investment decisions. Naturally, the returns to investment are positive (both in the short-run and long-run) if the state is good and negative if the state is bad. More interestingly, we show that the returns to good investment are more front-loaded than the returns to a bad investment – i.e., a higher proportion of the returns

manifests at $t = 1$ than at $t = 2$. This result again stems from the asymmetry of the speculator's trading strategy. Even if the speculator is aware that the investment is bad at $t = 1$, she may not trade on this information due to the feedback effect. Thus, the value-destructiveness of the investment seeps out ex post at $t = 2$. This result is consistent with the negative long-run returns to M&A, documented by Agrawal, Jaffe, and Mandelker (1992) and Rau and Vermaelen (1998).

In addition to interesting effects on stock returns, the asymmetry of the speculator's trading strategy can also generate important real consequences. Since negative information is not incorporated into prices, it does not influence management decisions. Thus, while positive-NPV projects will be encouraged, some negative-NPV projects will not be canceled – even though there is an agent in the economy who knows with certainty that the project is negative-NPV – leading to overinvestment overall. In contrast to standard overinvestment theories based on the manager's private benefits (e.g., Jensen (1986), Stulz (1990), Zwiebel (1996)), here the manager is fully aligned with firm value and there are no agency problems. The manager wishes to maximize firm value by learning from prices, but is unable to do so since speculators refrain from revealing their information. Applied to M&A as well as organic investment, the theory may explain why M&A appears to be "excessive" and a large fraction of acquisitions destroy value (see, e.g., Andrade, Mitchell, and Stafford (2001).)

As mentioned above, the primary motivation for our paper is to identify a limit to arbitrage. Different authors have emphasized different factors that lead to limits on arbitrage activities. Campbell and Kyle (1993) focus on fundamental risk, i.e., the risk that firm fundamentals will change while the arbitrage strategy is being pursued. In their model, such changes are unrelated to speculators' arbitrage activities. De Long, Shleifer, Summers, and Waldmann (1990) argue that noise-trading risk, i.e., the risk that noise trading will increase the degree of mispricing, may render arbitrage activities unprofitable. Noise trading only affects the asset's market price and not its fundamental value, which is again exogenous to the act of arbitrage. Shleifer and Vishny (1997) show that, even if an arbitrage strategy is sure to converge in the long-run, the possibility that mispricing may widen in the short-term may deter speculators from trading on it, if they are concerned with redemptions by their own investors. Similarly, Kondor (2009) demonstrates that arbitrageurs may stay out of a trade if they believe that it may become more profitable in the future. Many authors (e.g., Pontiff (1996), Mitchell and Pulvino (2001), and Mitchell, Pulvino, and Stafford (2002)) focus on the transaction costs and holding costs that arbitrageurs have to incur while pursuing an arbitrage strategy. Others (Geczy, Musto, and Reed (2002), and Lamont and Thaler (2003)) discuss the importance of short-sales constraints. While these papers emphasize market frictions as the source of limits to arbitrage, our paper shows that limits to arbitrage arise when the market performs its utmost efficient role: guiding the allocation of real resources. Thus, while limits to arbitrage based on market frictions may attenuate with the development of financial markets, the effect identified by this paper may strengthen – as investors become more sophisticated, managers will learn from them to

a greater degree.[1] Our model deliberately shuts down the sources of the limits to arbitrage identified by prior theories: the speculator is risk-neutral, there are no agency problems due to portfolio delegation, and there is no exogenous friction on trading (other than a standard transaction cost), such as short-sale constraints. With all these forces switched off, a limit to arbitrage nevertheless arises because the speculator endogenously chooses not to trade due to the feedback effect.

Our paper is related to the literature exploring the theoretical implications of the feedback effects from market prices to real decision making. Several papers in this literature have shown that the feedback effect can be harmful for real efficiency. Most closely related is Goldstein and Guembel (2008), who show that it provides an incentive to uninformed speculators to short sell a stock, reducing its value by inducing a real decision based on false information. Their paper also highlights an asymmetry between buy-side and sell-side speculation, but only with respect to uninformed trading; here, we show that informed speculators are less likely to trade on bad news rather than good news, in turn generating implications for the speed of incorporation of news into prices.[2] Bond, Goldstein, and Prescott (2010), Dow, Goldstein, and Guembel (2010), and Goldstein, Ozdenoren, and Yuan (2011) also model complexities arising from the feedback effect. Overall, the point in our paper – that negatively informed speculators will strategically withhold information from the market, because they know that the release of negative information will lead managers to fix the underlying problem – is new in this literature.

This paper proceeds as follows. Section 2 presents the model. Section 3 contains the core analysis, demonstrating the asymmetric limit to arbitrage. Section 4 investigates the extent to which information affects beliefs and prices, Section 5 discusses potential applications of the model, and Section 6 concludes. Appendix A contains all proofs not in the main text.

2 The Model

The model has three dates, $t \in \{0, 1, 2\}$. There is a firm whose stock is traded in the financial market. The firm's manager needs to take a decision as to whether to continue or abandon an investment project. The manager's goal is to maximize expected firm value; since there are no agency problems between the manager and the firm, we will use these two terms interchangeably. At $t = 0$, a risk-neutral speculator may be present in the financial market. If present, she is informed about the state of nature θ that determines the profitability of continuing vs. abandoning the project. Trading in the financial market occurs at $t = 1$. In addition to the speculator, two other types of agents participate in the financial market: noise traders whose trades are unrelated to the realization of θ, and a risk-neutral market maker. The latter collects the orders from the speculator and noise traders, and sets a price at which he executes the orders

[1] Still, as will become clear in the model description, our mechanism does require a non-zero trading cost in addition to the feedback effect.

[2] In addition, while they require two trading periods to generate the asymmetry, our model is simpler and requires only a single period.

out of his inventory. At $t = 2$, the manager takes the decision, which may be affected by the trading in the financial market at $t = 1$. Finally, all uncertainty is resolved and payoffs are realized. We now describe the firm's investment problem and the trading process in more detail.

2.1 The Firm's Decision

Suppose that the firm has an investment project that can be either continued or abandoned at $t = 2$. We denote the firm's decision as $d \in \{i, n\}$, where $d = i$ represents continuing the investment and $d = n$ represents no investment (also referred to as "abandonment" or "correction"). The firm faces uncertainty over the realization of value under each possible action. In particular, there are two possible states $\theta \in \Theta \equiv \{H, L\}$ ("high" and "low"). We denote the value of the firm realized in $t = 2$ as $v = R^d_\theta$, which depends on both the state of nature θ and the manager's action d.

We assume that whether continuation or abandonment is desirable depends on the state of nature (i.e., there is no dominant action). Without loss of generality, we set:

$$R^i_H > R^n_H \tag{1}$$

$$R^n_L > R^i_L, \tag{2}$$

that is, continuation is optimal in state H, while abandonment is optimal in state L. We also set:

$$R^i_H > R^n_L, \tag{3}$$

that is, the highest possible firm value is achieved in state H, consistent with this being labeled as the "high" state. This assumption is also without loss of generality as, if it is not satisfied, the highest firm value is achieved in state L and we can simply reverse notations.

Note that equations (1) and (2) imply:

$$R^i_H - R^i_L > R^n_H - R^n_L. \tag{4}$$

Equation (4) is the driving force behind our results. It means that taking the corrective action reduces the negative effect of state L on firm value. Put differently, if the state is L rather than H, the reduction in firm value is lower if the manager has taken action n. In turn, equation (4) incorporates two cases, depending on whether firm value is monotonic in the underlying state:

Case 1: $R^n_H > R^n_L$. In this case, state H is better for firm value, no matter what action has been taken by the firm. Hence, the corrective action attenuates, but does not eliminate, the effect of the state on firm value. Abandonment reduces the volatility of firm value, i.e., the dependence of firm value on the state. For example, state H can represent high demand for the firm's products, while state L represents low demand. Whether the firm continues to invest in its production process or not, its value will be lower in state L, but the negative effect of state L is attenuated if the firm does not invest.

Case 2: $R_L^n > R_H^n$. In this case, if the corrective action is taken, firm value is higher in state L. Put differently, the corrective action is sufficiently powerful to overturn the effect of the state on firm value. Importantly, this second case does not require that abandonment reduces the volatility of firm value: it could be that $abs\left(R_H^n - R_L^n\right) > abs\left(R_H^i - R_L^i\right)$ so volatility is higher under correction. Instead, this case implies non-monotonicity of firm value in the state: one state does not dominate the other. For example, consider the case where continuation implies proceeding with a takeover decision, and abandonment implies keeping the cash for future opportunities. State H corresponds to a state in which current acquisition opportunities dominate future ones, and state L refers to the reverse. Under continuation, firm value is higher in state H, whereas if the firm chooses to postpone acquisitions, its value is higher in state L where future acquisition opportunities are superior. Another example is related to Aghion and Stein (2008): $d = i$ corresponds to a growth strategy, and $d = n$ corresponds to a strategy focused on current profit margins. Growth prospects are good if $\theta = H$ and bad if $\theta = L$. If the firm eschews the growth strategy ($d = n$), its value is higher in the low state where there are no growth opportunities, since in the high state, its rivals could pursue the growth opportunities, in turn worsening its competitive position.

The prior probability that the state is $\theta = H$ is $y = \frac{1}{2}$, which is common knowledge. We use q to denote the posterior probability the manager assigns to the case $\theta = H$. The manager's decision is conditioned on q, which in turn is calculated using information arising from trades in the financial market. Let γ denote the posterior belief that the state is H such that the manager is indifferent between continuation and abandonment, i.e.:

$$\gamma R_H^i + (1-\gamma) R_L^i = \gamma R_H^n + (1-\gamma) R_L^n. \tag{5}$$

The value of γ represents a "cutoff" that determines the manager's action. If and only if $q > \gamma$, he will continue the project. We will distinguish between two cases. The first case is where $\gamma < \frac{1}{2}$. Since the prior y is $\frac{1}{2}$, the manager would continue the investment without further information, i.e., ex ante, the investment has a positive net present value. The second case is where $\gamma > \frac{1}{2}$, and so the ex-ante net present value of the investment is negative.

2.2 Trade in the Financial Market

In $t = 0$, with probability $\lambda < 1$, a speculator arrives in the financial market. Whether the speculator is present or not is unknown to anyone else.[3] If the speculator is present, she sees the state of nature θ with certainty. We will use the term "positively-informed speculator" to describe a speculator who observes $\theta = H$, and "negatively-informed speculator" to describe a speculator who observes $\theta = L$. The variable λ is a measure of market sophistication or the informedness of outside investors and will be used to generate a number of comparative statics.

[3]Since private information is not public knowledge, its existence is also unlikely to be public knowledge. Chakraborty and Yilmaz (2004) also feature uncertainty on whether the speculator is present.

Trading in the financial market happens in $t = 1$. Always present is a noise trader, who trades $z = -1$, 0, or 1 with equal probabilities. If the speculator is present, she makes an endogenous trading choice $s \in \{-1, 0, 1\}$. Trading either -1 or 1 is costly for the speculator and entails paying a cost of κ. If the speculator is indifferent between trading and not trading (because her expected profits from trading exactly equal κ), we assume that she will not trade.

Following Kyle (1985), orders are submitted simultaneously to a market maker who sets the price and absorbs order flows out of his inventory. The orders are market orders and are not contingent on the price. The competitive market maker sets the price equal to expected asset value, given the information contained in the order flow. The market maker can only observe total order flow $X = s + z$, but not its individual components s and z. Possible order flows are $X \in \{-2, -1, 0, 1, 2\}$ and the pricing function is $p(X) = E(v|X)$. A critical departure from Kyle (1985) is that firm value here is endogenous, because the manager's action is based on information revealed during the trading process.

Specifically, the manager observes total order flow X, and uses the information in X to form his posterior q, which is then used in the investment decision. Allowing the manager to observe order flow X, rather than just the price p, simplifies the analysis without affecting its economic content. In the equilibria that we analyze, there is a one-to-one correspondence between the price and the order flow so it does not matter which variable the manager observes. Under the alternative assumption that the manager observes p, other, non-interesting, equilibria can arise, where the price is essentially uninformative. Since this paper's focus is to analyze the feedback effect, which requires the price to be informative, we do not analyze such equilibria here. It is also realistic to assume that managers have access to information about trading quantities in the financial market: first, market making is competitive and so there is little secrecy in the order flow; second, microstructure databases (such as TAQ) provide such information at a short lag – rapidly enough to guide investment decisions.

2.3 Equilibrium

The equilibrium concept we use is the Perfect Bayesian Nash Equilibrium. Here, it is defined as follows: (i) A trading strategy by the speculator: $S : \Theta \to \{-1, 0, 1\}$ that maximizes his expected final payoff $s(v - p) - |s|\kappa$, given the price setting rule, the strategy of the manager, and his information about the realization of θ. (ii) An investment strategy by the firm $D : \mathcal{Q} \to \{i, n\}$ (where $\mathcal{Q} = \{-2, -1, 0, 1, 2\}$), that maximizes expected firm value $v = R_\theta^d$ given the information in the order flow and all other strategies. (iii) A price setting strategy by the market maker $p : \mathcal{Q} \to \mathbb{R}$ that allows him to break even in expectation, given the information in the price and all other strategies. Moreover, (iv) the firm and the market maker use Bayes' rule in order to update their beliefs from the order they observe in the financial market. Finally, (v) all agents have rational expectations in that each player's belief about the other players' strategies is correct in equilibrium.

3 Feedback Effect and Asymmetric Limits to Arbitrage

In this section, we characterize the pure-strategy equilibria in our model. We demonstrate the emergence of asymmetric limits to arbitrage as a result of the feedback from market trading outcomes to the firm's investment decision. We consider Case 1 ($R_H^n > R_L^n$) first and then proceed to Case 2 ($R_H^n < R_L^n$).

3.1 Case 1: Firm Value is Monotone in the State: $R_H^n > R_L^n$

We start with the case where $\gamma < \frac{1}{2}$, i.e., without further information, the firm will choose to invest. Later, we will show that our main insight carries through to the case where $\gamma > \frac{1}{2}$. In our characterization, we make use of three different threshold levels of the cost of trading κ:

$$\kappa_1 \equiv \frac{1}{3}\left[\frac{1}{2}\left(R_H^i - R_L^i\right) + \frac{1-\lambda}{2-\lambda}\left(R_H^n - R_L^n\right)\right], \tag{6}$$

$$\kappa_2 \equiv \frac{1}{3}\left(\frac{1}{2} + \frac{1-\lambda}{2-\lambda}\right)\left(R_H^i - R_L^i\right), \tag{7}$$

$$\kappa_3 \equiv \frac{1}{3}\left(R_H^i - R_L^i\right), \text{ where} \tag{8}$$

$$\kappa_1 < \kappa_2 < \kappa_3, \text{ and} \tag{9}$$

$$\kappa_2 - \kappa_1 = \frac{1}{3}\frac{1-\lambda}{2-\lambda}\left[\left(R_H^i - R_L^i\right) - \left(R_H^n - R_L^n\right)\right] > 0. \tag{10}$$

The results also depend on whether order flow is sufficiently informative to overturn the decision to invest, which is the ex-ante optimal decision. Hence, we distinguish between two cases depending on whether the cutoff γ is higher or lower than $\frac{1-\lambda}{2-\lambda}$. As we will show, the quantity $\frac{1-\lambda}{2-\lambda}$ is relevant as, in some equilibria, it represents the posterior probability of state H under an order flow of $X = -1$. The first case is $\frac{1-\lambda}{2-\lambda} < \gamma$. Here, the probability λ that the speculator is present is sufficiently high that a negative order of $X = -1$ is sufficiently informative to deter the manager from investing. Thus, there is feedback from the market to real decisions for the case of $X = -1$.[4] Second, $\frac{1-\lambda}{2-\lambda} > \gamma$. Here, a negative order of $X = -1$ is not sufficiently informative to lead the manager to abandon the default plan of investing. Thus, there is no feedback effect for $X = -1$.

As we show, depending on the values of κ, four equilibrium outcomes can arise:

1. No Trade Equilibrium NT: the speculator does not trade,

2. Trade Equilibrium T: the speculator buys when she knows that $\theta = H$ and sells when she knows that $\theta = L$,

[4] While $X = -2$ is also a negative order flow, the firm's decision in this case is not relevant for equilibrium trading strategies as the speculator's information is fully revealed, as will become clear.

3. Partial Trade Equilibrium BNS (Buy - Not Sell): the speculator buys when she knows that $\theta = H$ and does not trade when she knows that $\theta = L$,

4. Partial Trade Equilibrium SNB (Sell - Not Buy): the speculator does not trade when she knows that $\theta = H$ and sells when she knows that $\theta = L$.

Proposition 1 provides the characterization of equilibrium outcomes.

Proposition 1 *(Equilibrium, firm value is monotone in the state, investment is ex-ante desirable). Suppose that $R_H^n > R_L^n$ and $\gamma < \frac{1}{2}$. Then the trading game has the following pure-strategy equilibria:*

When $\kappa < \kappa_1$, the only pure-strategy equilibrium is T.

When $\kappa_1 \leq \kappa < \kappa_2$: in the case of feedback ($\frac{1-\lambda}{2-\lambda} < \gamma$), the only pure-strategy equilibrium is BNS; in the case of no feedback ($\frac{1-\lambda}{2-\lambda} > \gamma$), the only pure-strategy equilibrium is T.

When $\kappa_2 \leq \kappa < \kappa_3$, there are two pure-strategy equilibria: BNS and SNB.

When $\kappa \geq \kappa_3$, the only pure-strategy equilibrium is NT.

That is, if and only if there is feedback ($\frac{1-\lambda}{2-\lambda} < \gamma$), there is a strictly positive range of parameter values ($\kappa_1 \leq \kappa < \kappa_2$) for which the BNS equilibrium exists but the SNB equilibrium does not exist. There is no range of parameter values for which the SNB equilibrium exists but the BNS equilibrium does not exist.

Proof. Given that firm value is always higher when $\theta = H$ than when $\theta = L$, it is straightforward to show that the speculator will never buy when she knows that $\theta = L$ and will never sell when she knows that $\theta = H$. Then, the only possible pure-strategy equilibria are NT, T, BNS, and SNB. Below, we identify the conditions under which each one of these equilibria holds. If an order flow of $X = -2$ ($X = 2$) is observed off the equilibrium path, we assume that the beliefs of the market maker and the manager are that the speculator knows that the state is L (H). Given that speculators always lose if they trade against their information, this is the only belief that is consistent with the intuitive criterion.

No Trade Equilibrium NT:

For a given order flow X, the posterior q, the manager's decision d and the price p are given by the following table (see Appendix A for the full calculations):

X	-2	-1	0	1	2
q	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
d	n	i	i	i	i
p	R_L^n	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	R_H^i

As shown in Appendix A, the profit for the negatively-informed speculator from deviating to selling is $\frac{1}{3}(R_H^i - R_L^i)$, and this is also the profit for the positively-informed speculator from deviating to buying. Thus, this equilibrium holds if and only if $\kappa \geq \kappa_3$.

Partial Trade Equilibrium SNB:

For a given order flow X, the posterior q, the manager's decision d and the price p are given by the following table:

X	-2	-1	0	1	2
q	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2-\lambda}$	1
d	n	i	i	i	i
p	R_L^n	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	$\frac{1}{2-\lambda}R_H^i + \frac{1-\lambda}{2-\lambda}R_L^i$	R_H^i

Calculating the profit for the negatively-informed speculator from deviating to not trading and for the positively-informed speculator from deviating to buying, we can see that this equilibrium holds if and only if $\kappa_2 \leq \kappa < \kappa_3$.

Partial Trade Equilibrium BNS:

For a given order flow X, the posterior q, the manager's decision d and the price p are given by the following table:

X	-2	-1	0	1	2
q	0	$\frac{1-\lambda}{2-\lambda}$	$\frac{1}{2}$	$\frac{1}{2}$	1
d	n	$\begin{cases} n & \text{if } \frac{1-\lambda}{2-\lambda} < \gamma \\ i & \text{if } \frac{1-\lambda}{2-\lambda} > \gamma \end{cases}$	i	i	i
p	R_L^n	$\begin{cases} \frac{1-\lambda}{2-\lambda}R_H^n + \frac{1}{2-\lambda}R_L^n & \text{if } \frac{1-\lambda}{2-\lambda} < \gamma \\ \frac{1-\lambda}{2-\lambda}R_H^i + \frac{1}{2-\lambda}R_L^i & \text{if } \frac{1-\lambda}{2-\lambda} > \gamma \end{cases}$	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	R_H^i

Calculating the profit for the negatively-informed speculator from deviating to selling and for the positively-informed speculator from deviating to not trading, we can see that this equilibrium holds if and only if $\kappa_2 \leq \kappa < \kappa_3$ for the case of no feedback ($\frac{1-\lambda}{2-\lambda} > \gamma$) and if and only if $\kappa_1 \leq \kappa < \kappa_3$ for the case of feedback ($\frac{1-\lambda}{2-\lambda} < \gamma$).

Trade Equilibrium T:

For a given order flow X, the posterior q, the manager's decision d and the price p are given by the following table:

X	-2	-1	0	1	2
q	0	$\frac{1-\lambda}{2-\lambda}$	$\frac{1}{2}$	$\frac{1}{2-\lambda}$	1
d	n	$\begin{cases} n & \text{if } \frac{1-\lambda}{2-\lambda} < \gamma \\ i & \text{if } \frac{1-\lambda}{2-\lambda} > \gamma \end{cases}$	i	i	i
p	R_L^n	$\begin{cases} \frac{1-\lambda}{2-\lambda}R_H^n + \frac{1}{2-\lambda}R_L^n & \text{if } \frac{1-\lambda}{2-\lambda} < \gamma \\ \frac{1-\lambda}{2-\lambda}R_H^i + \frac{1}{2-\lambda}R_L^i & \text{if } \frac{1-\lambda}{2-\lambda} > \gamma \end{cases}$	$\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$	$\frac{1}{2-\lambda}R_H^i + \frac{1-\lambda}{2-\lambda}R_L^i$	R_H^i

Calculating the profit for the negatively-informed speculator from deviating to not trading and for the positively-informed speculator from deviating to not trading, we can see that this

equilibrium holds if and only if $\kappa < \kappa_2$ for the case of no feedback ($\frac{1-\lambda}{2-\lambda} > \gamma$) and if and only if $\kappa < \kappa_1$ for the case of feedback ($\frac{1-\lambda}{2-\lambda} < \gamma$).

Thus, there is a range of κ for which the only equilibrium is BNS if $\kappa_1 < \kappa_2$ and $\frac{1-\lambda}{2-\lambda} < \gamma$. From (4) and (10), $\kappa_1 < \kappa_2$ requires $\lambda < 1$. In turn, $\frac{1-\lambda}{2-\lambda} < \gamma$ requires $\lambda > \frac{1-2\gamma}{1-\gamma}$. Thus, there exist values of λ that satisfy both of the above conditions if $\frac{1-2\gamma}{1-\gamma} < 1$, which always holds. ■

Proposition 1 demonstrates the sources of limits to arbitrage in our model, one of which is the feedback effect that is the focus of our paper. To understand the various forces, we start by describing the equilibrium outcomes in the case of no feedback, i.e., when $\frac{1-\lambda}{2-\lambda} > \gamma$. Here, an order flow of $X = -1$ may convey (depending on the equilibrium) negative information, but not sufficiently negative to deter the manager from abandoning the default plan of investing.[5] In this case, there are three regions of the parameter κ. When $\kappa < \kappa_2$, the only pure-strategy equilibrium is one where the speculator always trades on her information. When $\kappa_2 \leq \kappa < \kappa_3$, there are two pure strategy equilibria, exhibiting limited trade, one in which the speculator buys on good news but does not trade on bad news, and one in which she sells on bad news but does not trade on good news. When $\kappa \geq \kappa_3$, the only pure-strategy equilibrium entails no trade at all by the speculator.

Two sources of limits to arbitrage are present in the no-feedback case, both of which are common in the literature. The first source is the trading cost κ. As κ increases, we move to equilibria where speculators trade less on their information. Clearly, when speculators are subject to greater transaction costs, they have lower incentives to trade. The second source is the price impact that speculators exert when they trade on their information. In the intermediate region $\kappa_2 \leq \kappa < \kappa_3$, there are equilibria where the speculator trades on one type of information but not the other. There is symmetry in that both types of asymmetric equilibria are possible in exactly the same range of parameters. To understand the intuition behind these asymmetric equilibria, consider the BNS equilibrium without feedback (the case of the SNB equilibrium is analogous). Given that the market maker believes that the speculator buys on good news, a negative order flow is very revealing that the speculator is negatively informed and the price moves sharply to reflect this. Specifically, $X = -1$ is inconsistent with the speculator having positive information, and so the speculator only receives $\frac{1-\lambda}{2-\lambda}R^i_H + \frac{1}{2-\lambda}R^i_L$ from selling. Thus, the speculator makes little profit from selling on bad news; knowing this, she chooses not to trade on bad news. Conversely, given that the market maker believes that the speculator does not sell on bad news, a positive order flow is consistent with the speculator being negatively informed: $X = 1$ is consistent with the noise trader buying, and the speculator being negatively informed and choosing not to trade. Thus, the market maker sets a relatively low price of $\frac{1}{2}R^i_H + \frac{1}{2}R^i_L$, which allows the speculator to make high profits by buying. Thus, the equilibrium is sustainable. In sum, in both partial trade equilibria, the order flow in the direction in which the speculator does not trade becomes particularly informative, leading to larger price impact which reduces

[5]Note that an order flow of $X = -2$ always leads to no investment, but in this case the order flow is fully revealing and the speculator never makes a profit, and so this node is not relevant for determining the equilibrium trading strategies.

the potential trading profits. Thus, not trading in this direction is sustained in equilibrium. This force is symmetric in the absence of feedback.

We now move to the case of feedback, i.e., when $\frac{1-\lambda}{2-\lambda} < \gamma$. Here, an order flow of $X = -1$ provides enough negative information for the manager to abandon the investment. Abandonment is the optimal decision in state L; thus, improving the manager's decision reduces the speculator's profit in the node of $X = -1$ from $\frac{1-\lambda}{2-\lambda}(R_H^i - R_L^i)$ (in the case of no-feedback) to only $\frac{1-\lambda}{2-\lambda}(R_H^n - R_L^n)$. This reduced profit affects the speculator's equilibrium trading strategy and causes her not to sell on bad news if $\kappa_1 \leq \kappa$. Our main result is that the feedback effect introduces an additional limit to arbitrage that is distinct from those identified in prior literature – *arbitrage is limited because the value of the asset being arbitraged is endogenous to the act of arbitrage*. Unlike trading costs and price impact, the limit to arbitrage arising from the feedback effect is asymmetric: it reduces the extent of selling on bad news but not the extent of buying on good news. Indeed, the difference between equilibrium outcomes in the two cases of no-feedback and feedback is that in the range $\kappa_1 \leq \kappa < \kappa_2$, the Trade Equilibrium T is replaced with the Partial Trade Equilibrium BNS. However, there is no range of parameters where the SNB equilibrium exists but the BNS equilibrium does not exist.

The intuition behind the asymmetry of our effect is as follows. In the case of feedback, when the speculator sells on bad news, she may lead the manager to abandon a bad investment. By that, she improves firm value, since $R_L^n > R_L^i$. Since she is holding a short position, this increase in firm value reduces her profit. Hence, it deters the speculator from selling on bad news. On the other hand, the feedback effect does not deter the positively-informed speculator from buying on good news. Buying on good news may reveal to the manager that the state is good, which (weakly) causes him to increase investment; since investment is desirable in the good state, this augments firm value. The speculator will then profit from the increase in the value of her long position, which will further increase her incentive to trade.[6]

Overall, trading on her information in either direction – whether it is buying on positive information or selling on negative information – conveys information to the manager. This improves his decision making and thus fundamental firm value. Increased firm value augments the profitability of a long position but reduces the profitability of a short position. Hence, the feedback effect leads to an asymmetric limit to arbitrage that deters selling on bad news but not buying on good news. By contrast, the two limits to arbitrage studied in prior research are symmetric. A high trading cost κ leads to the NT equilibrium where there is no trading in either direction. Price impact leads to the two partial trade equilibria, BNS and SNB, but there is symmetry in that both equilibria are possible in exactly the same range of parameters. In particular, without feedback (i.e., if $\gamma > \frac{1-\lambda}{2-\lambda}$), there is no value of κ in which there is one

[6] In the case discussed so far ($\gamma < \frac{1}{2}$) the default option for the manager is to invest, and so positive news from the market does not change his decision and does not affect firm value. Hence we state that buying on good information causes the manager to weakly increase investment. As we will show later, if $\gamma > \frac{1}{2}$, buying on good news causes the manager to strictly increase investment, in turn strictly improving firm value. This effect is the driving force behind our results in the case of $\gamma > \frac{1}{2}$.

partial trade equilibrium but not the other.

The reason for why the feedback effect reduces trading profits is nuanced. Intuition may suggest that the market maker's pricing function can "undo" the feedback effect: the market maker is fully rational and takes into account the fact that the order flow will affect the manager's decision; since he is competitive, he sets a price that reflects this decision. Because the price that the speculator receives from selling will always reflect the action taken by the manager (be it continuation or investment), it may seem that the action does not matter. Such intuition turns out to be incorrect. The key to our result is that the source of the speculator's profits is not superior knowledge of the manager's action (since the action is always perfectly predicted by the market maker), but superior knowledge of the state. In turn, superior knowledge of the state results from fact that $\lambda < 1$, i.e., the speculator is not always present. To see this, consider the market maker's inference from seeing $X = -1$ in the BNS equilibrium. This order flow is consistent with either the speculator being absent (in which case the state may be H or L), or the speculator being present and negatively informed. If $\lambda = 1$, the first case is ruled out, and so the market maker knows for certain that $\theta = L$. Thus, the order flow of $X = -1$ is fully revealing: the market maker knows *both* that correction will occur, *and* that the state is L, and so sets price exactly equal to the fundamental value of R_L^n. Thus, the speculator makes zero profit. Indeed, if $\lambda = 1$, then $\kappa_1 = \kappa_2$ and there is no range of parameter values in which there is a BNS equilibrium only. By contrast, if $\lambda < 1$, the market maker predicts the manager's action but does not know the state. Since $X = -1$ can be consistent with the speculator being absent and the state being H, the market maker allows for the possibility that the state may be H and sets a price of $\frac{1-\lambda}{2-\lambda}R_H^d + \frac{1}{2-\lambda}R_L^d$. Because the speculator knows both the manager's action and the state is L, she makes a profit of $\frac{1-\lambda}{2-\lambda}\left(R_H^d - R_L^d\right)$, which in turn depends on the decision d. The source of her profit is her superior information on the state; since the state has a lower effect on firm value under correction, her profits are lower in this case. Put differently, the manager's action d (and thus the feedback effect on the manager's action) matters for the speculator's trading profits, not because the speculator's profits stem from superior knowledge of the manager's action, but because the action affects the value of the speculator's superior knowledge on the state.

We now wish to verify that the asymmetry between buy-side speculation and sell-side speculation, driven by the feedback effect, is not an artifact of the focus on the case of $\gamma < \frac{1}{2}$. The next proposition shows that when $\gamma > \frac{1}{2}$, i.e., when the default decision is abandonment, our results are qualitatively similar: without feedback, BNS and SNB equilibria occur over the same range of parameters, whereas with feedback, the BNS equilibrium occurs over a wider range than the SNB equilibrium. In the case of $\gamma < \frac{1}{2}$, the source of the limit to arbitrage was that the feedback effect reduces the profitability of a short position but does not affect the profitability of a long position, since positive order flow leads to investment but the investment would be undertaken in the absence of further information anyway. Here, the source is that the feedback effect increases the profitability of a long position but does not affect the prof-

itability of a short position, since abandonment would be undertaken in the absence of further information anyway. In both cases (for both $\gamma < \frac{1}{2}$ and $\gamma > \frac{1}{2}$), the intuition is the same: the feedback effect (weakly) increases the profitability of a long position and (weakly) decreases the profitability of short position, as discussed above.

For this proposition, define new threshold levels of the cost of trading κ:

$$\kappa_1' \equiv \frac{1}{3}\left[\frac{1}{2}\left(R_H^n - R_L^n\right) + \frac{1-\lambda}{2-\lambda}\left(R_H^i - R_L^i\right)\right], \tag{11}$$

$$\kappa_2' \equiv \frac{1}{3}\left(\frac{1}{2} + \frac{1-\lambda}{2-\lambda}\right)\left(R_H^n - R_L^n\right), \tag{12}$$

$$\kappa_3' \equiv \frac{1}{3}\left(R_H^n - R_L^n\right), \quad \textit{and} \tag{13}$$

$$\kappa_2' < \kappa_3', \ \kappa_2' < \kappa_1' \tag{14}$$

The cutoff for feedback effect to exist is also adjusted here. In some equilibria, $\frac{1}{2-\lambda}$ represents the posterior probability of state H if $X = 1$. If $\frac{1}{2-\lambda} > \gamma$, the probability λ that the speculator is present is sufficiently high that an order flow of $X = 1$ contains enough information to lead the manager to invest (as opposed to the default option of abandoning). Hence, there is feedback. If $\frac{1}{2-\lambda} < \gamma$, an order flow of $X = 1$ is not informative enough to lead the manager to invest. This is the case where there is no feedback.

The following proposition provides the characterization of equilibrium outcomes.

Proposition 2 *(Equilibrium, firm value is monotone in the state, investment is ex-ante undesirable). Suppose that $R_H^n > R_L^n$ and $\gamma > \frac{1}{2}$, then the trading game has the following pure-strategy equilibria:*

When $\kappa < \kappa_2'$, the only pure-strategy equilibrium is T.

When $\kappa_2' \leq \kappa < \kappa_3'$: in the case of no feedback ($\frac{1}{2-\lambda} < \gamma$), there are two pure-strategy equilibria, BNS and SNB; in the case of feedback ($\frac{1}{2-\lambda} > \gamma$), the BNS equilibrium always exists, whereas the SNB equilibrium exists only in the sub-range $\kappa_1' \leq \kappa < \kappa_3'$ or does not exist (if $\kappa_1' > \kappa_3'$).

When $\kappa \geq \kappa_3'$, the only pure-strategy equilibrium is NT.

That is, if and only if there is feedback ($\frac{1}{2-\lambda} > \gamma$), there is a range of parameter values for which the BNS equilibrium exists but the SNB equilibrium does not exist. If $\kappa_1' > \kappa_3'$, this range is $\kappa_2' \leq \kappa < \kappa_3'$; if $\kappa_1' < \kappa_3'$, this range is $\kappa_2' \leq \kappa < \kappa_1'$. There is no range of parameter values for which the SNB equilibrium exists but the BNS equilibrium does not exist.

Proof. The proof repeats similar steps to those in the proof of Proposition 1, and is thus omitted for brevity. ■

In the case of $\gamma < \frac{1}{2}$, the role of the feedback effect can be seen in the BNS equilibrium: it reduces the profits that the negatively-informed speculator would earn by deviating and selling, and so the BNS equilibrium is sustainable over a wider range of parameters than the

SNB equilibrium. Here, where $\gamma > \frac{1}{2}$, the feedback effect impacts the SNB equilibrium. Since buying improves the firm's fundamental value, the feedback effect increases the profit that the positively-informed speculator would earn by deviating and buying, and so the SNB equilibrium is sustainable over a narrower range of parameters than the BNS equilibrium (indeed, if $\kappa_1' > \kappa_3'$, it is not sustainable at all). In both cases (for $\gamma < \frac{1}{2}$ and $\gamma > \frac{1}{2}$), the end result is the same: the feedback effect increases the profits from informed buying and reduces the profits from informed selling, leading to the BNS equilibrium being sustainable over a wider range of transactions costs than the SNB equilibrium.

3.2 Case 2: Firm Value is Non-Monotone in the State: $R_H^n < R_L^n$

In this subsection, we consider the case where, if the firm does not invest, its value is higher in state $\theta = L$ ($R_H^n < R_L^n$). Hence, the corrective action is sufficiently powerful to outweigh the effect of the state on firm value and lead to a higher value in the low state. We start by characterizing equilibrium outcomes for the case where $\gamma < \frac{1}{2}$, i.e., without further information, the firm will choose to invest.

The analysis of equilibrium outcomes becomes more complicated in the case of non-monotonicity. In the previous subsection, where firm value is monotone in the state, a positively-informed speculator always loses money by selling and a negatively-informed speculator always loses money by buying, since firm value is always higher in state H than in state L. However, now that firm value may be higher in state L, a positively-informed speculator may find it optimal to sell and a negatively-informed speculator may find it optimal to buy. Hence, there are nine possible pure-strategy equilibria (each type of speculator – positively-informed and negatively-informed – may either buy, sell, or not trade). The following lemma simplifies the equilibrium analysis, moving us closer to the analysis conducted in the previous subsection.

Lemma 1 *Suppose that $R_H^n < R_L^n$ and $\gamma < \frac{1}{2}$, then:*

(i) The trading game has no pure-strategy equilibrium where the speculator sells when she knows that $\theta = H$.

(ii) The trading game has no pure-strategy equilibrium where the speculator buys when she knows that $\theta = L$.

Proof. (i) Suppose that the speculator sells when she knows that $\theta = H$, then $X \in \{-2, -1, 0\}$. In each one of these nodes, posterior probability q of state H is at least $\frac{1}{2}$ (given that these nodes are consistent with the action of the positively-informed speculator and may or may not be consistent with the action of the negatively-informed speculator, depending on her equilibrium action). Then, since $\gamma < \frac{1}{2}$, investment will occur, and so firm value is R_H^i. The price, however, will be between R_L^i and R_H^i, and so the speculator makes a loss from selling.

(ii) Suppose that the speculator buys when she knows that $\theta = L$, then $X \in \{0, 1, 2\}$. Given that the positively-informed speculator does not sell, the posterior probability q is $\frac{1}{2}$ at $X \in \{0, 1\}$. Hence, since $\gamma < \frac{1}{2}$, investment will occur, and so firm value is R_L^i. Since the price

is $\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$, the speculator will lose money on these nodes. When $X = 2$, there are two possibilities. If the positively-informed speculator buys in equilibrium, then the outcome is the same as on the other nodes. If she does not trade in equilibrium, then the negatively-informed speculator is revealed, buying a security worth R_L^n for a price of R_L^n, and again making a loss due to the transaction cost κ. ■

Following the lemma, there are four possible pure-strategy equilibria, just as in the previous subsection: NT, T, SNB, and BNS. However, the conditions for these equilibria to hold are now tighter. The reason that the positively-informed speculator never sells *in equilibrium* is that if the market maker and the manager believe that she sells, she cannot make a profit from selling. However, she still might be tempted to *deviate* to selling in any of the four equilibria mentioned above. When she sells, she misleads the market maker and the manager to think that the negatively-informed speculator is present, and so to abandon the investment. Since abandonment is suboptimal if $\theta = H$, this decision reduces firm value and causes the speculator to make a profit on her short position. Hence, for any of the above four equilibria to hold, an additional condition must be satisfied to ensure that the positively-informed speculator does not have an incentive to deviate to selling. Interestingly, the same issue does not arise with the negatively-informed speculator, as she never has an incentive to deviate to buying. If she does so, she misleads the market maker and the manager to think that the positively-informed speculator is present, and so to (incorrectly) take the investment. Again, this reduces firm value, but because the speculator has a long position, this causes her to make a loss. Goldstein and Guembel (2008) also derive conditions to ensure that the speculator does not deviate from the equilibrium to trade against her information.

Another issue that arises in this subsection is the specification of off-equilibrium beliefs. In Case 1, due to monotonicity, the only assumption that satisfied the intuitive criterion was that an off-equilibrium order flow of $X = 2$ is due to the positively-informed speculator (and so the posterior is $q = 1$), while an off-equilibrium order flow of $X = -2$ is due to the negatively-informed speculator (and so the posterior is $q = 0$). In this subsection, however, the intuitive criterion is not sufficient to rule out other off-equilibrium beliefs. Nevertheless, in the analysis that follows we retain this assumption regarding off-equilibrium beliefs, which is reasonable given the possible equilibria in our model. Our results remain the same for any other off-equilibrium beliefs that are monotone in the order flow.

The following proposition provides the characterization of equilibrium outcomes.

Proposition 3 *(Equilibrium, firm value is non-monotone in the state, investment is ex-ante desirable). Suppose that $R_H^n < R_L^n$ and $\gamma < \frac{1}{2}$, and suppose that the belief of the market maker and the manager is that an off-equilibrium order flow of $X = -2$ ($X = 2$) is associated with the negatively-informed (positively-informed) speculator. Then, if $\frac{\left(R_H^i - R_L^i\right)}{\left(R_L^n - R_H^n\right)}$ is sufficiently high, the characterization of equilibrium outcomes is identical to that in Proposition 1.*

Otherwise, the following ***additional*** *conditions are required for the various equilibria to hold:*

Equilibrium NT and SNB: $\kappa > \frac{1}{3}\left(-\left(R_H^i - R_L^i\right) + \left(R_L^n - R_H^n\right)\right)$;

Equilibrium BNS: in the case of feedback ($\frac{1-\lambda}{2-\lambda} < \gamma$), $\frac{6-3\lambda}{6-2\lambda}\frac{\left(R_H^i - R_L^i\right)}{\left(R_L^n - R_H^n\right)} > 1$; *in the case of no feedback* ($\frac{1-\lambda}{2-\lambda} > \gamma$), $\frac{8-3\lambda}{4-2\lambda}\frac{\left(R_H^i - R_L^i\right)}{\left(R_L^n - R_H^n\right)} > 1$.

Equilibrium T: in the case of feedback ($\frac{1-\lambda}{2-\lambda} < \gamma$), $\frac{3-2\lambda}{3-\lambda}\frac{\left(R_H^i - R_L^i\right)}{\left(R_L^n - R_H^n\right)} > 1$; *in the case of no feedback* ($\frac{1-\lambda}{2-\lambda} > \gamma$), $2\frac{\left(R_H^i - R_L^i\right)}{\left(R_L^n - R_H^n\right)} > 1$.

Proof. The calculations of the posterior q, the manager's decision d and the price p for different order flows X in the various possible equilibria are identical to those provided in the proof of Proposition 1. Hence, the conditions for the positively-informed speculator to choose between buying and not trading and for the negatively-informed speculator to choose between selling and not trading are identical to those derived in the proof of Proposition 1. Analyzing the possible trading profits for the negatively-informed speculator from deviating to buying in each of the four possible equilibria, it is straightforward to see that she always loses from buying and hence will never deviate. Appendix A calculates the possible trading profits for the positively-informed speculator from deviating to selling in each of the four possible equilibria, which yields obtain the additional conditions stated in the body of the proposition. These conditions are binding only when $\frac{\left(R_H^i - R_L^i\right)}{\left(R_L^n - R_H^n\right)}$ is not sufficiently high. ■

As the proposition demonstrates, the main force identified in the previous subsection for the case where $R_H^n > R_L^n$, exists also in the case where $R_H^n < R_L^n$. That is, the feedback effect deters the negatively-informed speculator, but not the positively-informed speculator, from trading on her information. In this subsection, this force is even stronger because the range of transaction costs κ between κ_1 and κ_2, in which the BNS equilibrium exists due to feedback but the SNB equilibrium does not exist, is higher when $(R_H^n - R_L^n)$ is negative: see equation (10). A strong feedback effect, in which correction not only mitigates the effect of the low state but also overturns it, implies that the negatively-informed speculator can make a loss – even before transaction costs – when selling on bad news. This result is in contrast to standard informed trading models where a speculator can never make a loss (before transactions costs) if she trades in the direction of her information. This loss occurs at the $X = -1$ node; again, the key to this result is $\lambda < 1$. Even though both the speculator and market maker know that abandonment will occur if $X = -1$, they have differing views on firm value conditional on abandonment. The speculator knows that the corrective action will be taken, *and* that correction is desirable for firm value (since she knows that $\theta = L$), and so firm value is R_L^n. In contrast, the market maker knows the corrective action will be taken but is not certain that correction is desirable for firm value, because she is unsure of the underlying state θ. Order flow $X = -1$ is consistent with a negatively-informed speculator, but also with an absent speculator and selling by noise traders. Hence, it is possible that $\theta = H$, in which case the manager's corrective action is undesirable, leading to firm value of R_H^n. Therefore, the price set by the market maker is only $\frac{1-\lambda}{2-\lambda}R_H^n + \frac{1}{2-\lambda}R_L^n$, since he puts weight on the fact that correction may be undesirable, and so

the speculator loses $\frac{1-\lambda}{2-\lambda}\left(R_H^n - R_L^n\right)$ before transaction costs.

However, the proposition also shows that another force that arises due to feedback exists in this subsection, and that this force has implications on the characterization of equilibrium outcomes. This is the desire of the positively-informed speculator to deviate from her equilibrium behavior and manipulate the price by selling, even though she has good news. She can potentially profit from leading the manager to take the wrong decision, which enables her to profit from her short position. The manipulation incentive is not strong enough to interfere with equilibrium conditions as long as $\frac{\left(R_H^i - R_L^i\right)}{\left(R_L^n - R_H^n\right)}$ is sufficiently high. In this case, the loss from trading against good news (which is proportional to $(R_H^i - R_L^i)$) is high relative to the benefit from manipulation (which is proportional to $(R_L^n - R_H^n)$). Otherwise, there are additional conditions for the various possible equilibria, making it relatively more difficult to obtain the BNS equilibrium due to feedback.

Finally, we analyze the case where $\gamma > \frac{1}{2}$. It turns out that this case is the exact mirror image of the case where $\gamma < \frac{1}{2}$. Now, effectively, $\theta = H$ represents bad news and $\theta = L$ represents good news. This is because the default decision is to abandon the investment; under this decision, firm value is lower in state H than in state L. Thus, the speculator now sells if $\theta = H$ and buys if $\theta = L$. The next lemma is the mirror image of Lemma 1:

Lemma 2 *Suppose that $R_H^n < R_L^n$ and $\gamma > \frac{1}{2}$, then:*

(i) The trading game has no pure-strategy equilibrium where the speculator sells when she knows that $\theta = L$.

(ii) The trading game has no pure-strategy equilibrium where the speculator buys when she knows that $\theta = H$.

Proof. The proof is symmetric to the proof of Lemma 1 and hence is not repeated here. ■

Hence, the possible pure-strategy equilibria here are:

1. No Trade Equilibrium NT: the speculator does not trade,
2. Trade Equilibrium T': the speculator buys when she knows that $\theta = L$ and sells when she knows that $\theta = H$,
3. Partial Trade Equilibrium BNS' (Buy - Not Sell): the speculator buys when she knows that $\theta = L$ and does not trade when she knows that $\theta = H$.
4. Partial Trade Equilibrium SNB' (Sell - Not Buy): the speculator does not trade when she knows that $\theta = L$ and sells when she knows that $\theta = H$.

The characterization of equilibrium outcomes in the following proposition is symmetric to that in Proposition 3:

Proposition 4 *(Equilibrium, firm value is non-monotone in the state, investment is ex-ante undesirable). Suppose that* $R_H^n < R_L^n$ *and* $\gamma > \frac{1}{2}$*, and suppose that the belief of the market maker and the manager is that an off-equilibrium order flow of* $X = -2$ *(*$X = 2$*) is associated with the positively-informed (negatively-informed) speculator. Then, the characterization of equilibrium outcomes is symmetric to that in Proposition 3: parameters* R_H^i*,* R_L^i*,* R_H^n*,* R_L^n *are replaced with parameters* R_L^n*,* R_H^n*,* R_L^i*,* R_H^i*, respectively, and equilibria* T*,* BNS*,* SNB *are replaced with equilibria* T'*,* BNS'*,* SNB'*, respectively.*

Proof. The proof is symmetric to the proof of Proposition 3 and hence is not repeated here. ■

Overall, the result is identical to that of the case of $\gamma < \frac{1}{2}$. Due to feedback, the speculator is deterred from selling when she has bad news, but not from buying when she has good news. The only difference is that now, bad news entails $\theta = H$ and good news entails $\theta = L$.

In Case 1, where $\gamma > \frac{1}{2}$, the role of the feedback effect is seen in the SNB equilibrium: the feedback effect increases the profits that the positively-informed speculator would earn by deviating to buying, and so the SNB is sustainable over a narrower range of parameters. In the current scenario of $\gamma > \frac{1}{2}$ within Case 2, just as in the scenario of $\gamma < \frac{1}{2}$ (for both Case 1 and Case 2), the role of the feedback effect is seen in the BNS equilibrium: it deters the speculator from deviating to sell on bad news ($\theta = H$ in this case).

3.3 Summary and Discussion of Assumptions

The above analysis has shown that the presence of feedback from market trading to firms' decisions creates a wedge between buy-side speculation and sell-side speculation, discouraging speculators from selling on bad news, but encouraging them to buy on good news. Several assumptions play a key role in generating this result. These assumptions in turn lead to empirical predictions, since they demonstrate the conditions under which the asymmetric limit to arbitrage will exist.

First, the trading in the market has to contain sufficient information to influence the manager's decision. For example, consider the result in Proposition 1: for the wedge to arise, we require $\frac{1-\lambda}{2-\lambda} < \gamma$. Hence, it is important that the probability λ that the speculator is present is sufficiently high that the order flow is sufficiently informative to change managerial decisions. In turn, $\frac{1-\lambda}{2-\lambda} < \gamma$ is more likely to be satisfied the closer γ is to $\frac{1}{2}$, i.e. the closer the NPV of the project is to 0. When γ is close to $\frac{1}{2}$, the desirability of the investment is sufficiently uncertain that the manager's decision will be influenced by the trading in the financial market. If γ is very low, the ex-ante NPV of the project is so high that the manager will almost always undertake the investment, regardless of order flow.

Second, another important assumption is that $\lambda < 1$, so there is uncertainty on whether there is an informed speculator in the market. To see this, consider again the result in Proposition 1. The range between κ_1 and κ_2, in which a BNS equilibrium arises and a SNB equilibrium

does not arise, shrinks to zero if $\lambda = 1$. This is because the difference in expected profit from buying on good information and selling on bad information stems entirely from the difference in profit in the node where the speculator is partially revealed ($X = -1$ or 1). If $\lambda = 1$, the speculator is fully revealed in these nodes and her profits are zero, leading to no asymmetry. Where $\lambda < 1$, the speculator is not fully revealed and makes a positive profit from her private information about the state; the value of this information depends on the manager's decision (as this affects the dependence of firm value on the state) and thus the feedback effect. We would achieve the same result by instead assuming that the speculator is always present and informed, but can only trade with probability λ – for example, if with probability $1 - \lambda$ she receives a liquidity shock that prevents her from trading.[7]

Third, the reason that the speculator loses from increasing the firm's value is that she ends up with a short position. Hence, it is important that the speculator short sells rather than just sells stocks she previously owned, which in turn requires the speculator's initial position to be zero and short-sales to be possible. Thus, the model delivers the result that investors are more likely to engage in sales rather than short-sales, even in the absence of a short-sales constraint. However, if the speculator maximizes returns relative to other speculators rather than absolute returns (e.g. she is a mutual fund benchmarked against the performance of other mutual funds), then our limit to arbitrage may exist even if her initial position is strictly positive. For example, if she sells half of her portfolio, she increases the value of the remaining half, but increases the value of the entire portfolio held by her competitors, and so loses in relative terms. In this case, the limit to arbitrage identified by this paper may exist even in the presence of short-sales constraints. While short-sales constraints do not deter selling to a non-negative final position, the feedback effect can deter such selling if the speculator maximizes relative performance. Therefore, our model also predicts that investors who are evaluated according to absolute returns are more likely to sell on negative information than those who are benchmarked to their peers. Indeed, hedge funds appear to sell (not just short-sell) more readily than mutual funds, who are sometimes seen as "closet indexers."[8]

Fourth, the real decision is a corrective action in that it improves firm value in the low state. This is a natural assumption if the decision maker is the firm's manager who attempts to maximize firm value via an investment decision; another potential application is to a board of directors which chooses whether to fire an underperforming manager in the bad state. The model does not apply to amplifying actions that worsen firm value in the low state: for example, capital providers may withdraw their investment in the low state, reducing firm value further

[7] An alternative assumption would be that the speculator is always present, but sometimes she is uninformed. This, however, may introduce other complications, as the uninformed speculator may choose to trade to manipulate the price and the firm's decision, as in Goldstein and Guembel (2008).

[8] Note that the existence of benchmarking alone is typically insufficient to explain the reluctance of mutual funds to deviate from their benchmark, since the gains from beating one's benchmark (by deviating) equal the losses from underperforming one's benchmark. Thus, existing explanations typically rely on the asymmetry in in the flow-performance relation: the inflows from beating one's benchmark are lower than the outflows from underperforming one's benchmark. Our explanation for benchmarked investors' unwillingness to sell does not require such asymmetry.

(as in Goldstein, Ozdenoren, and Yuan (2011)), or customers or employees could terminate their relationship with a troubled firm (Subrahmanyam and Titman (2001)). Then, our model will have different implications: the speculator will no longer be reluctant to sell on bad news, since the information will reduce firm value further, enabling her to profit more on her short position.

4 Effect of Information on Beliefs and Prices

The previous section demonstrated that the feedback effect gives rise to an equilibrium where a speculator buys on good news and does not trade on bad news. In this section, we study the implications coming out of this equilibrium. The analysis that follows focuses on the BNS equilibrium where investment is ex-ante desirable ($\gamma < \frac{1}{2}$) and there is feedback ($\frac{1-\lambda}{2-\lambda} < \gamma$), and considers both Case 1 and Case 2 together. Section 4.1 calculates the effect of good and bad news on the state on the posterior beliefs q, in order to study the extent to which information reaches the manager and affects real decisions. Section 4.2 analyzes the impact of news on prices to generate stock return predictions.

4.1 Beliefs

Since the manager uses the posterior belief q to guide his investment decision, q measures the extent to which information reaches the manager and affects his actions. In a world in which no agent observed the state, or in which the manager did not learn from prices or order flow, the posterior q would equal the prior $y = \frac{1}{2}$. Conversely, in a world of perfect information transmission, $q = 1$ if $\theta = H$ and $q = 0$ if $\theta = L$. Our model, where information is partially revealed through prices, lies in between these two polar cases. The absolute distance between q and $\frac{1}{2}$ measures the extent to which information reaches the manager.

Thus far, we have shown that positive news received by the speculator has a different impact on her trades (and thus total order flow) than negative news. However, it is not obvious that this will translate into a differential impact on the manager's beliefs. The manager is rational and takes into account the fact that the speculator does not sell on negative information: he updates his beliefs using the asymmetric equilibrium trading strategy. In the BNS equilibrium in the proof of Proposition 1, the manager recognizes that $X = 1$ could be consistent with a negatively-informed speculator who chooses not to trade, and so $q(1)$ is no higher than $q(0)$ (where $q(X)$ denotes the posterior at $t = 1$ upon observing order flow X). Thus, even though bad news can lead to a positive order flow of $X = 1$, the manager knows that such an order flow can stem from a negatively-informed and non-trading speculator, and will decrease his posterior accordingly. Put differently, although negative information does not cause a negative order flow (on average), it can still have a negative effect on beliefs and be fully conveyed to the manager. Thus, it may seem still possible for good and bad news to be conveyed symmetrically to the

manager – by taking into account the speculator's asymmetric trading strategy, he can "undo" the asymmetry. Indeed, we start by showing that, if we do not condition on the presence of the speculator, the effects on beliefs of the good and bad states being realized are symmetric. This is a direct consequence of the law of iterated expectations: the expected posterior belief must be equal to the prior.

Lemma 3 *Consider the BNS equilibrium where* $\gamma < \frac{1}{2}$ *and* $\frac{1-\lambda}{2-\lambda} < \gamma$ *(i.e., there is feedback). (i) If* $\theta = H$*, the manager's expected posterior probability of the good state is* $q^H = \frac{(1-\lambda)^2}{6-3\lambda} + \frac{1}{3} + \frac{\lambda}{3}$ *and is increasing in* λ*. (ii) If* $\theta = L$*, the manager's expected posterior probability of the good state is* $q^L = \frac{1-\lambda}{6-3\lambda} + \frac{1}{3}$ *and is decreasing in* λ*. (iii) We have* $\frac{q^H+q^L}{2} = \frac{1}{2}$*: thus, the realization of state H has the same absolute impact on beliefs as the realization of state L.*

Proof. See Appendix A. ■

Of greater interest is to study the effect of the state realization conditional upon the speculator being present. We use the term "good news" to refer to $\theta = H$ being realized and the speculator being present, since in this case there is an agent in the economy who directly receives news on the state; "bad news" is defined analogously. While the above analysis studied the effect of the state being realized (regardless of whether the state is learned by any agent in the economy), this analysis studies the impact of the speculator receiving information about the state. The goal is to investigate the extent to which the speculator's good and bad news is conveyed to the manager at $t = 1$. The results are given in Proposition 5 below:

Proposition 5 *(Asymmetric effect of positive and negative news on beliefs at* $t = 1$*.) Consider the BNS equilibrium where* $\gamma < \frac{1}{2}$ *and* $\frac{1-\lambda}{2-\lambda} < \gamma$ *(i.e., there is feedback). (i) If* $\theta = H$ *and the speculator is present, the manager's expected posterior probability of the good state is* $q^{H,spec} = \frac{2}{3}$ *and is independent of* λ*. (ii) If* $\theta = L$ *and the speculator is present, the manager's expected posterior probability of the good state is* $q^{L,spec} = \frac{1-\lambda}{6-3\lambda} + \frac{1}{3}$ *and is decreasing in* λ*. (iii) We have*

$$\frac{q^{H,spec} + q^{L,spec}}{2} = \frac{1 + \frac{1-\lambda}{6-3\lambda}}{2}, \tag{15}$$

which is decreasing in λ*. Since* $\frac{1+\frac{1-\lambda}{6-3\lambda}}{2} > \frac{1}{2}$*, (15) implies that* $abs\left(q^{H,spec} - y\right) - abs\left(q^{L,spec} - y\right) > 0$*, i.e. the absolute increase in the manager's posterior if the speculator receives good news exceeds the absolute decrease in his posterior if the speculator receives bad news. The difference is decreasing in* λ*.*

Proof. See Appendix A. ■

Proposition 5 shows that the impact on beliefs of the speculator being present and positively informed is greater in absolute terms than the impact of the speculator being present and negatively informed. The asymmetry is monotonically decreasing in the frequency of the speculator's presence λ. This result holds even though the manager is rational and takes into

account the fact that the speculator trades asymmetrically when using the order flow to update his prior. The source of the result is that, even though the manager is rational, he is unable to distinguish the case of a negatively-informed speculator from that of an absent speculator (i.e. no information) – both of these cases lead to the order flow being $\{-1, 0, 1\}$ with uniform probability. Thus, negative information has a smaller effect on his belief. By contrast, if the speculator is always present, the manager has no such inference problem and there is no asymmetry. This can be seen by plugging $\lambda = 1$ into equation (15), in which case the average posterior equals the prior of $\frac{1}{2}$ and so we have $abs\left(q^{H,spec} - y\right) = abs\left(q^{L,spec} - y\right)$. Just as $\lambda < 1$ was a necessary condition for the asymmetric feedback equilibrium to be the only equilibrium in the first place, it is a necessary and sufficient condition for bad news to have a smaller effect on the manager's belief than good news.

The above analysis considered the change in the manager's posterior at $t = 1$. At $t = 2$, the state is realized and the posterior becomes either 1 (if $\theta = H$) or 0 (if $\theta = L$). Since bad news is conveyed to the manager to a lesser extent at $t = 1$, it seeps out to a greater extent ex post, between $t = 1$ and $t = 2$. Thus, bad news causes a greater change in the posterior between $t = 1$ and $t = 2$ than good news. This result is stated in Corollary 1 below:

Corollary 1 *(Asymmetric effect of high and low state realization on beliefs at $t = 2$). Consider the BNS equilibrium where $\gamma < \frac{1}{2}$ and $\frac{1-\lambda}{2-\lambda} < \gamma$ (i.e., there is feedback). The absolute impact on beliefs between $t = 1$ and $t = 2$ of the realization of the state is greater for the low state $\theta = L$ than for the high state $\theta = H$, i.e.*

$$abs\left(0 - q^{L,spec}\right) - abs\left(1 - q^{H,spec}\right) > 0.$$

The asymmetry is monotonically decreasing in the frequency of the speculator's presence λ.

Proof. Follows from simple calculations ■

The smaller effect of bad news on the posterior at $t = 1$ is counterbalanced by its larger effect at $t = 2$. As we will show in Section 4.2, surprisingly this result need not hold when we examine the effect of news on prices rather than posteriors.

4.2 Stock Returns

We now calculate the impact of the state realization and news on prices, in order to generate stock return implications. We study short-run stock returns between $t = 0$ and $t = 1$, and long-run drift between $t = 1$ and $t = 2$. While this analysis is similar to Section 4.1 but studying prices rather than beliefs, we will show that not all the results remain the same.

4.2.1 Short-Run Stock Returns

Even though the speculator trades asymmetrically, this does not necessarily imply that realizations of the good and bad states will have a differential price impact, since the market maker

takes into account the speculator's trading strategy when devising his pricing function. Lemma 4 is analogous to Lemma 3 and shows that, unconditionally, the good and bad states have the same absolute impact on prices. Let p_0 denote the "ex ante" stock price at $t = 0$, before the state has been realized.

Lemma 4 *Consider the BNS equilibrium where* $\gamma < \frac{1}{2}$ *and* $\frac{1-\lambda}{2-\lambda} < \gamma$ *(i.e., there is feedback):*

(i) The stock price impact of the good state being realized is $p_1^H - p_0 = \frac{\lambda}{6}\left[p\left(2\right) - p\left(-1\right)\right] > 0$.

(ii) The stock price impact of the bad state being realized is $p_1^L - p_0 = \frac{\lambda}{6}\left[p\left(-1\right) - p\left(2\right)\right] < 0 = -\left(p_1^H - p_0\right)$.

Proof. See Appendix A. ■

We have $p_1^H - p_0 = -\left(p_1^L - p_0\right)$: the negative effect of the bad state equals the positive effect of the good state. Thus, the unconditional expected return is zero. This is an inevitable consequence of market efficiency. The price at $t = 0$ is an unbiased expectation of the $t = 1$ expected price in the high state and the $t = 1$ expected price in the low state. Since both states are equally likely, the absolute effect of the good state must equal the absolute effect of the bad state. An uninformed investor cannot trade the stock at $t = 0$ and expect a non-zero average return at $t = 1$.

Proposition 6 is analogous to Proposition 5 and shows that, conditional on the speculator being present, good news has a greater effect than bad news:

Proposition 6 *(Asymmetric effect of positive and negative news on returns between* $t = 0$ *and* $t = 1$*.) Consider the BNS equilibrium where* $\gamma < \frac{1}{2}$ *and* $\frac{1-\lambda}{2-\lambda} < \gamma$ *(i.e., there is feedback):*

(i) If $\theta = H$ *and the speculator is present, the average return between* $t = 0$ *and* $t = 1$ *is* $p_1^{H,spec} - p_0 = \frac{1}{3}\left(1 - \frac{\lambda}{2}\right)\left(p\left(2\right) - p\left(-1\right)\right) > 0$.

(ii) If $\theta = L$ *and the speculator is present, the average return between* $t = 0$ *and* $t = 1$ *is* $p_1^{L,spec} - p_0 = \frac{\lambda}{6}\left(p\left(-1\right) - p\left(2\right)\right) < 0$.

(iii) The difference in the absolute average returns between the speculator learning $\theta = H$ *and* $\theta = L$ *is given by:*

$$abs\left(p_1^{H,spec} - p_0\right) - abs\left(p_1^{L,spec} - p_0\right) = \frac{1}{3}\left(1 - \lambda\right)\left(p\left(2\right) - p\left(-1\right)\right) > 0, \tag{16}$$

i.e. the stock price increase upon positive news exceeds the stock price decrease upon negative news. This difference is decreasing in λ.

(iv) The average return, conditional on the speculator being present, is positive:

$$p_1^{spec} - p_0 = \frac{1}{3}\frac{1-\lambda}{2}\left(p\left(2\right) - p\left(-1\right)\right) > 0. \tag{17}$$

This difference is decreasing in λ.

Proof. See Appendix A. ■

Proposition 6 states that the average return, *conditional* on the speculator being present, is positive – i.e. the stock price increase upon positive information exceeds the stock price decrease upon negative information (part (iii)). Put differently, positive information is impounded into prices to a greater degree than negative information. Since good and bad news are equally likely, this means that the average return, conditional on the speculator being present, is positive (part (iv)). As with Proposition 5, the key to this result is that, even though the market maker is rational, he is unable to distinguish the case of a negatively-informed speculator from that of an absent speculator (i.e. no information). If $\lambda = 1$, equations (16) and (17) become zero and there is no asymmetry; the asymmetry is monotonically decreasing in λ. Note that the positive average return given in part (iv) is not inconsistent with market efficiency, because it is conditional upon the speculator being present. An uninformed investor cannot buy the stock at $t = 0$ and expect to earn a positive return at $t = 1$ because she will not know whether the speculator is present.

4.2.2 Long-Run Drift

We now move from short-run returns to calculating the long-run drift of the stock price, to analyze the stock return analog of Corollary 1, i.e., the impact of the state realization on prices between $t = 1$ and $t = 2$. Corollary 1 showed that the smaller effect of bad news on beliefs at $t = 1$ is counterbalanced by a larger effect on beliefs at $t = 2$. Corollary 2 below shows that this need not be the case for returns: it is possible for bad news to have a smaller effect than good news at both $t = 1$ and $t = 2$.

Corollary 2 *(Asymmetric effect of positive and negative news on returns between $t = 1$ and $t = 2$). Consider the BNS equilibrium where $\gamma < \frac{1}{2}$ and $\frac{1-\lambda}{2-\lambda} < \gamma$ (i.e., there is feedback):*

(i) If $\theta = H$ and the speculator is present, the average return between $t = 1$ and $t = 2$ is $p_2^{H,spec} - p_1^{H,spec} = \frac{1}{3}\left(R_H^i - R_L^i\right) > 0$.

(ii) If $\theta = L$ and the speculator is present, the average return between $t = 1$ and $t = 2$ is

$$p_2^{L,spec} - p_1^{L,spec} = -\frac{1}{3}\left(R_H^i - R_L^i\right) - \frac{1}{3}\left(\frac{1-\lambda}{2-\lambda}\left(R_H^n - R_L^n\right)\right), \tag{18}$$

which is negative in Case 1, but can be positive or negative in Case 2.

(iii) If (18) < 0, the difference in the absolute average returns between the speculator learning $\theta = H$ and $\theta = L$ is given by:

$$abs\left(p_2^{H,spec} - p_1^{H,spec}\right) - abs\left(p_2^{L,spec} - p_1^{L,spec}\right) = \frac{1}{3}\left(\frac{1-\lambda}{2-\lambda}\left(R_L^n - R_H^n\right)\right),$$

which is positive in Case 2 and negative in Case 1. The magnitude of the difference is decreasing in λ.

(iv) Expected firm value at $t = 2$, conditional upon the speculator being present, is :

$$p_2^{spec} = \frac{1}{2}R_H^i + \frac{1}{3}R_L^i + \frac{1}{6}R_L^n,$$

and the average return between $t = 1$ and $t = 2$ if the speculator is present is:

$$p_2^{spec} - p_1^{spec} = \frac{1}{6}\frac{1-\lambda}{2-\lambda}\left(R_L^n - R_H^n\right),$$

which is positive in Case 2 and negative in Case 1. The magnitude of the difference is decreasing in λ.

Proof. See Appendix A. ■

Corollary 1 showed that, if the speculator is present, good news has a larger effect on beliefs at $t = 1$ than bad news, because she trades on the former but not the latter; thus, the expected change in beliefs between $t = 0$ and $t = 1$ is positive. Since bad news has a smaller effect at $t = 1$, it must have a larger effect at $t = 2$ (since the truth about the state comes out at $t = 2$), and so the average increase in beliefs between $t = 0$ and $t = 1$ is reversed by an average decrease in beliefs between $t = 1$ and $t = 2$. Corollary 2 shows that this need not be the case when we study prices rather than beliefs: the speculator's presence can lead to positive average returns in both the short-run (between $t = 0$ and $t = 1$) and also in the long-run (between $t = 1$ and $t = 2$). This is because the stock price depends not only on the beliefs about the state, but also the manager's action. Thus, there is an additional effect of the speculator on prices that does not exist in the analysis of beliefs: not only does she convey information about the state, but also this information improves the manager's decision-making and enhances firm value – the essence of the feedback effect. In Case 2 ($R_H^n < R_L^n$), this feedback effect is sufficiently strong to turn the average return between $t = 1$ and $t = 2$ positive. In state L, little bad news emerges about the state between $t = 0$ and $t = 1$, which means that there is a large amount of bad news to come out between $t = 1$ and $t = 2$; this in turn leads to the large downward revision in beliefs in Corollary 1. However, the effect on prices in Corollary 2 is muted because the damage to firm value caused by state L can be mitigated by taking the corrective action. Thus, the negative effect of bad news is smaller than the positive effect of good news between $t = 1$ and $t = 2$ as well as between $t = 0$ and $t = 1$ due to the feedback effect. Indeed, if the feedback effect is sufficiently strong, i.e. R_L^n is much higher than R_H^n, the return to bad news between $t = 1$ and $t = 2$ can be positive ($(18) > 0$). By contrast, in Case 1, ($R_H^n > R_L^n$), the long-run drift to the bad state is larger in magnitude, analogous to Corollary 1. Since state L is bad for firm value regardless of whether the manager takes the corrective action or not, the realization of state L at $t = 2$ leads to a large decrease in prices.

The analysis thus far has considered the impact of *news* on prices at $t = 1$ and $t = 2$. We now consider the impact of *investment* (a real variable) on prices; specifically, the extent to which it is impounded into prices at $t = 1$ or at $t = 2$. While Section 4.2.1 showed that good

news received by the speculator has a greater short-run price impact than bad news, Proposition 7 now demonstrates a related result: the proportion of the total returns to an investment that is realized in the short-run (at $t = 1$) rather than the long-run (at $t = 2$) is greater for a good investment ($\theta = H$) than a bad investment ($\theta = L$). In other words, the price impact of a good investment is more front-loaded than for a bad investment.

Proposition 7 *(Faster incorporation into prices of good investment than bad investment.) Consider the BNS equilibrium where* $\gamma < \frac{1}{2}$ *and* $\frac{1-\lambda}{2-\lambda} < \gamma$ *(i.e., there is feedback):*

(i) If investment is undertaken in state H:

(ia) The average return between $t = 0$ *and* $t = 1$ *is*

$$p_1^{i,H} - p_0 = \frac{1}{6(2+\lambda)}[(2+2\lambda-\lambda^2)R_H^i + (2-2\lambda)R_L^i - (2-\lambda-\lambda^2)R_H^n - (2+\lambda)R_L^n] > 0. \quad (19)$$

(ib) The average return between $t = 1$ *and* $t = 2$ *is*

$$R_H^i - p_1^{i,H} = \frac{R_H^i - R_L^i}{2+\lambda} > 0. \quad (20)$$

(ii) If investment is undertaken in state L:

(iia) The average return between $t = 0$ *and* $t = 1$ *is*

$$\frac{1}{6}[(1-\lambda)\left(R_H^i - R_H^n\right) + R_L^i - R_L^n] < 0. \quad (21)$$

(iib) The average return between $t = 1$ *and* $t = 2$ *is*

$$R_L^i - p_1^{i,H} = -\frac{1}{2}(R_H^i - R_L^i) < 0. \quad (22)$$

(iii) The proportion of total returns that manifests at $t = 1$ *rather than* $t = 2$ *is greater for a good investment than a bad investment, i.e.* $abs\left((22) - (21)\right) > abs\left((20) - (19)\right)$*. Thus, the returns to a good investment manifest more rapidly than the returns to a bad investment.*

Parts (i) and (ii) of Proposition 7 show that investing in the high state leads to both positive short-run returns between $t = 0$ and $t = 1$ and also positive long-run drift between $t = 1$ and $t = 2$. Investing in the low state leads to negative short-run returns and negative long-run drift. Part (iii) demonstrates that the proportion of total returns that manifests at $t = 1$ rather than $t = 2$ is greater for a good investment than a bad investment. Thus, the returns to a good investment manifest more rapidly than the returns to a bad investment, i.e., are more front-loaded. To our knowledge, this prediction has not yet been tested.

The intuition behind the asymmetry is different from Proposition 6. In both Propositions 6 and 7, the asymmetry occurs because the low state has a lesser impact on prices than the high state. In Proposition 6, this arises from the fact that $\lambda < 1$, which means that the market maker cannot distinguish the case of a negatively-informed speculator from that of an

uninformed speculator. Here, the intuition is as follows. If the investment is bad, the negative returns cannot manifest too strongly at $t = 1$, otherwise the decline in the stock price will have led to the investment being canceled. Thus, the negative returns must manifest predominantly at $t = 2$. Put differently, there are bad investments that do not lead to a sharply negative reaction at $t = 1$ because the speculator did not trade on the bad news. Instead, the value-destructiveness of the investment seeps out ex post.

5 Summary of Implications

This section discusses several implications of our model. The first is that this paper identifies a limit to arbitrage which, in contrast to alternative explanations, is likely to persist over time even as markets evolve and investors become more sophisticated. One existing source of limited arbitrage is market frictions such as short-sales constraints, which will likely diminish with the development of financial markets. A second is that investors in professional money managers make their allocation decisions based on short-run measures of performance, which leads to mutual funds avoiding arbitrage trading that will only converge in the long run (Shleifer and Vishny (1997)). Such behavior can either be irrational over-extrapolation, or rational if they have limited information on the fund manager's quality but instead must infer it imperfectly from short-run performance. Either way, if investor sophistication and information improve over time, this force will also diminish.

By contrast, the limit to arbitrage analyzed by this paper stems from firm value being endogenous to the act of arbitrage. This is a fundamental force that does not rely on short-sale constraints, investor irrationality or investor limited information, and so may continue to persist over time. (The only market imperfection that our model requires is trading costs, which exist even in developed financial markets). There is no exogenous friction preventing the arbitrageur from trading; instead, she endogenously chooses not to trade because of the feedback effect. All agents in the model act with full rationality: the market maker takes into account the manager's learning when setting the price, and this in turn affects the speculator's decision to trade; the market maker knows that the speculator is pursuing an asymmetric trading strategy. If anything, the limit to arbitrage may *increase* with investor sophistication, as this augments the extent to which speculators have value-relevant information which the manager attempts to learn by observing the price.

The second main category of applications stems from the fact that the limit to arbitrage is asymmetric. While the speculator buys on good information, she does not sell on bad information. This prediction has implications for trading volume, suggesting that volume should be higher upon good investments (such as M&A or capital expenditure) than bad investments. Such a relation is consistent with the well-documented positive correlation between trading volume and stock returns (see, e.g., Karpoff (1987)). Moreover, even though the market maker takes the asymmetric trading volume into account, Proposition 6 shows that negative informa-

tion will enter into prices more slowly, as found empirically by Hong, Lim, and Stein (2000). While Hong, Lim, and Stein's results are consistent with the Hong and Stein (1999) model that news travels more slowly in small firms with low analyst coverage, Hong and Stein do not predict an asymmetry between good and bad news.[9] Hong, Lim, and Stein speculate that the asymmetry arises because key information is held by the firm's managers, and they disseminate favorable information more enthusiastically than unfavorable information because they are evaluated according to the stock price. Our theoretical model offers a potential alternative explanation. Key information is held by a firm's investors, who disseminate information not through public news releases, but by trading on it. Their reluctance to disseminate bad news is not because they are evaluated according to the stock price, but due to the limit to arbitrage created by the feedback effect. Another difference is that, in an underreaction model, if bad news has a smaller effect on short-run returns than good news, it must be counterbalanced by a larger long-run drift. By contrast, Corollary 2 shows that bad news can generate a smaller effect on returns in both the short-run and long-run. Even though less bad news is transmitted to the market at $t = 1$, meaning that there is more to come out at $t = 2$, stock returns depend not only on the state, but also the manager's decision. In our feedback model, the manager can take a corrective action to mitigate the negative impact of the state on firm value, so the effect of bad news on returns is lower at $t = 2$ as well as at $t = 1$. While the above results are unconditional on investment occurring, the model also generates implications for the short- and long-run returns to investment. Proposition 7 shows that the returns to good investment are more front-loaded than the returns to a bad investment, because the speculator trades more readily on good news than bad news. Thus, the value-destructiveness of a bad investment seeps out to a greater extent ex post, leading to negative long-run returns as documented by Agrawal, Jaffe, and Mandelker (1992) and Rau and Vermaelen (1998).

Moreover, the feedback effect means that the lack of negative information in prices will have further consequences on real decisions. In particular, if speculators choose not to trade on negative information, then such negative information does not become incorporated into stock prices and fails to influence the manager's behavior. Thus, some negative-NPV projects will not be optimally abandoned, leading to overinvestment – even though there is an agent who knows with certainty that the investment is undesirable, it still takes place. In the model, even if $\theta = L$, we have $d = i$ if the noise trader does not sell. Critically, overinvestment does not occur because the manager is pursuing private benefits, as in the standard theories of Jensen (1986), Stulz (1990) and Zwiebel (1996). In contrast, the manager is fully aligned with firm value and there are no agency problems. The manager wishes to maximize firm value by learning from prices, but is unable to do so since speculators refrain from impounding their information into prices. Note that overinvestment occurs even though the manager is fully aware that the speculator does not trade on negative information and takes this into account.

[9] Note that our paper focuses on the effect of news on stock prices. It does not address the predictability of future returns from past returns, which is another component of the Hong, Lim, and Stein (2000) findings.

The above overinvestment result can apply to M&A as well as organic expansion. Luo (2005) shows that managers sometimes use the market reaction to announced M&A deals to guide whether they should cancel the acquisition. While he finds that some transactions are canceled in equilibrium, our model suggests that there are other negative-NPV deals that should optimally be canceled but are not because speculators do not impound their negative views into prices. This may explain why a large proportion of M&A deals destroy value (see, e.g., Andrade, Mitchell and Stafford (2001).)

6 Conclusion

This paper has modeled a limit to arbitrage that stems from the fact that firm value is endogenous to the act of exploiting the arbitrage. Even if a speculator has negative information on the state, she may strategically refrain from trading on it, because doing so conveys her information to the manager. The manager may then take a corrective action that improves firm value but reduces the profits from her short position below the cost of trading, and sometimes causes her to realize a loss. There are several important differences between the feedback-driven limit to arbitrage that we study, and the limits to arbitrage identified by prior literature. First, unlike limits to arbitrage based on fundamental risk, noise trader risk, or holding costs, our effect is asymmetric. Trading in either direction impounds information into prices, which improves the manager's decision-making and increases fundamental value. This increases the profitability of a long position but reduces the profitability of a short position, thus encouraging buying on good news but discouraging selling on bad news. Second, unlike limits to arbitrage based on short-sale constraints, holding costs or portfolio delegation, our model does not rely on exogenous forces or agency problems; instead, the effect is generated endogenously as part of the arbitrage process.

The asymmetry of our effect has implications for both stock returns and real investment. In terms of stock returns, bad news has a smaller effect on short-run prices than good news, even though the market maker is rational and takes the speculator's trading strategy into account when devising his pricing function. Interestingly, in contrast to underreaction models, the smaller short-run reaction to bad news may also coincide with smaller long-run drift, since the manager can take a corrective action to attenuate the negative effect of the state on firm value. In addition, the returns to a good investment are more front-loaded than the returns to a bad investment – since the speculator does not trade on negative information, the value-destructiveness of a bad investment seeps out ex post. In terms of real investment, the manager may overinvest in negative-NPV projects, even though there are no agency problems and he is attempting to learn from the market to take the efficient decision. Even though there is an agent in the economy who knows with certainty that the investment is undesirable, and the manager is aware of the speculator's asymmetric trading strategy, this information is not conveyed to the manager and so the project is not abandoned.

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A Proofs

Proof of Proposition 1

This proof only provides supplementary material to what is in the text.

No Trade Equilibrium NT. The order flows of $X = -2$ and $X = 2$ are off the equilibrium path and the posteriors are given by 0 and 1, respectively, as these are the only posteriors that satisfy the Intuitive Criterion (as stated in the main proof). The order flows of $X \in \{-1, 0, 1\}$ are observed on the equilibrium path and so the posteriors can be calculated by Bayes' rule:

$$\begin{aligned} q(X) &= \Pr(H|X) \\ &= \frac{\Pr(X|H)}{\Pr(X|H) + \Pr(X|L)}. \end{aligned}$$

We thus have:

$$\begin{aligned} q(-1) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} \\ &= \frac{1}{2}, \end{aligned}$$

and $q(0)$ and $q(1)$ are calculated in exactly the same way. Sequential rationality leads to the decisions d and prices p as given by the Table.

We now turn to calculating trading profits. If the positively-informed speculator chooses to deviate from not trading to buying:

- With probability (w.p.) $\frac{1}{3}$, $X = 2$ and she is fully revealed. Thus, trading profits are zero.
- W.p. $\frac{1}{3}$, $X = 1$ and she pays $\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$ per share. The fundamental value of each share is R_H^i, and so her profit is $\frac{1}{2}\left(R_H^i - R_L^i\right) > 0$.
- W.p. $\frac{1}{3}$, $X = 0$ and she pays $\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$ per share. The fundamental value of each share is R_H^i, and so her profit is $\frac{1}{2}\left(R_H^i - R_L^i\right) > 0$.

Thus, her expected gross profit is given by:

$$\frac{1}{3}\frac{1}{2}\left(R_H^i - R_L^i\right) + \frac{1}{3}\frac{1}{2}\left(R_H^i - R_L^i\right) = \frac{1}{3}\left(R_H^i - R_L^i\right) = \kappa_3. \tag{23}$$

A similar calculation shows that, if a negatively-informed speculator sells, her gross profit is also given by (23). Thus, if and only if $\kappa \geq \kappa_3$, the no-trade equilibrium is sustainable.

Partial Trade Equilibrium SNB. The order flow of $X = 2$ is off the equilibrium path and

the posterior is given by 1. The posteriors of the other order flows are given as follows:

$$\begin{aligned}
q(-2) &= \frac{0}{\lambda\,(1/3)} = 0 \\
q\,(-1) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} = \frac{1}{2} \\
q\,(0) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} = \frac{1}{2} \\
q(1) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + (1-\lambda)(1/3)} = \frac{1}{2-\lambda}.
\end{aligned}$$

Under this equilibrium, the negatively-informed speculator sells.

- W.p. $\frac{1}{3}$, $X = -2$ and she is fully revealed. Thus, trading profits are zero.
- W.p. $\frac{1}{3}$, $X = -1$ and she receives $\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$ per share. The fundamental value of each share is R_L^i, and so her profit is $\frac{1}{2}\left(R_H^i - R_L^i\right) > 0$.
- W.p. $\frac{1}{3}$, $X = 0$ and she receives $\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$ per share. The fundamental value of each share is R_L^i, and so her profit is $\frac{1}{2}\left(R_H^i - R_L^i\right) > 0$.

Thus, her expected gross profit is given by:

$$\frac{1}{3}\left(R_H^i - R_L^i\right) = \kappa_3.$$

If the positively-informed speculator deviates to buying:

- W.p. $\frac{1}{3}$, $X = 2$ and she is fully revealed. Thus, trading profits are zero.
- W.p. $\frac{1}{3}$, $X = 1$ and she pays $\frac{1}{2-\lambda}R_H^i + \frac{1-\lambda}{2-\lambda}R_L^i$ per share. The fundamental value of each share is R_H^i, and so her profit is $\frac{1-\lambda}{2-\lambda}\left(R_H^i - R_L^i\right) > 0$.
- W.p. $\frac{1}{3}$, $X = 0$ and she pays $\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$ per share. The fundamental value of each share is R_H^i, and so her profit is $\frac{1}{2}\left(R_H^i - R_L^i\right) > 0$.

Thus, her expected gross profit is given by:

$$\frac{1}{3}\left(\frac{1}{2} + \frac{1-\lambda}{2-\lambda}\right)\left(R_H^i - R_L^i\right) = \kappa_2.$$

Thus, the SNB equilibrium is sustainable if and only if $\kappa_2 \leq \kappa < \kappa_3$.

Partial Trade Equilibrium BNS. The order flow of $X = -2$ is off the equilibrium path and

the posterior is given by 0. The posteriors of the other order flows are given as follows:

$$\begin{aligned}
q(-1) &= \frac{(1-\lambda)(1/3)}{(1-\lambda)(1/3)+\lambda(1/3)+(1-\lambda)(1/3)} = \frac{1-\lambda}{2-\lambda} \\
q(0) &= \frac{\lambda(1/3)+(1-\lambda)(1/3)}{\lambda(1/3)+(1-\lambda)(1/3)+\lambda(1/3)+(1-\lambda)(1/3)} = \frac{1}{2} \\
q(1) &= \frac{\lambda(1/3)+(1-\lambda)(1/3)}{\lambda(1/3)+(1-\lambda)(1/3)+\lambda(1/3)+(1-\lambda)(1/3)} = \frac{1}{2} \\
q(2) &= \frac{\lambda(1/3)}{\lambda(1/3)} = 1.
\end{aligned}$$

There are two sub-cases to consider. In the sub-case of no feedback ($\frac{1-\lambda}{2-\lambda} > \gamma$), decision $d = i$ is taken for all of the order flows on the equilibrium path. Thus, analogous to the SNB equilibrium, informed trading (in this case, buying on good information) yields profits of κ_3; if the negatively-informed speculator deviates to selling, she earns profits of κ_2. Hence, this equilibrium is sustainable if and only if $\kappa_2 \leq \kappa < \kappa_3$. For the sub-case of feedback ($\frac{1-\lambda}{2-\lambda} < \gamma$), the manager now takes decision $d = n$ upon observing order flow $X = -1$. The profits from trading on positive information are unchanged. The profits from deviating to selling on negative information are now given as follows:

- W.p. $\frac{1}{3}$, $X = -2$ and she is fully revealed. Thus, trading profits are zero.
- W.p. $\frac{1}{3}$, $X = -1$ and she receives $\frac{1-\lambda}{2-\lambda}R_H^n + \frac{1}{2-\lambda}R_L^n$ per share. The fundamental value of each share is R_L^n because the manager is now taking the corrective action, and so her profit is $\frac{1-\lambda}{2-\lambda}(R_H^n - R_L^n) > 0$.
- W.p. $\frac{1}{3}$, $X = 0$ and she receives $\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$ per share. The fundamental value of each share is R_L^i, and so her profit is $\frac{1}{2}(R_H^i - R_L^i) > 0$.

Thus, her expected gross profit is given by:

$$\frac{1}{3}\left(\frac{1}{2}\left(R_H^i - R_L^i\right) + \frac{1-\lambda}{2-\lambda}\left(R_H^n - R_L^n\right)\right) = \kappa_1.$$

Thus, this equilibrium is sustainable if and only if $\kappa_1 \leq \kappa < \kappa_3$.

Trade Equilibrium T. All order flows are on the equilibrium path and so the posteriors are

given as follows:

$$\begin{aligned}
q(-2) &= \frac{0}{\lambda\,(1/3)} = 0 \\
q\,(-1) &= \frac{(1-\lambda)(1/3)}{(1-\lambda)(1/3)+\lambda(1/3)+(1-\lambda)(1/3)} = \frac{1-\lambda}{2-\lambda} \\
q\,(0) &= \frac{\lambda(1/3)+(1-\lambda)(1/3)}{\lambda(1/3)+(1-\lambda)(1/3)+\lambda(1/3)+(1-\lambda)(1/3)} = \frac{1}{2} \\
q(1) &= \frac{\lambda(1/3)+(1-\lambda)(1/3)}{\lambda(1/3)+(1-\lambda)(1/3)+(1-\lambda)(1/3)} = \frac{1}{2-\lambda} \\
q(2) &= \frac{\lambda\,(1/3)}{\lambda\,(1/3)} = 1.
\end{aligned}$$

The profits from buying on positive information are given by κ_2, as in the SNB equilibrium. For the profits from selling on negative information, there are two sub-cases to consider, which correspond to the two sub-cases in the BNS equilibrium. Without feedback ($\frac{1-\lambda}{2-\lambda} > \gamma$), the profits are given by κ_2 and so the equilibrium is sustainable if and only if $\kappa < \kappa_2$. With feedback ($\frac{1-\lambda}{2-\lambda} > \gamma$), the profits are given by κ_1 and so the equilibrium is sustainable if and only if $\kappa < \kappa_1$.

Proof of Proposition 3

This proof only provides supplementary material to what is in the text. As discussed in the main text, it is straightforward to show that the negatively-informed speculator will not deviate to buying. Here we calculate the profits made if the positively-informed speculator deviates to selling, to derive the necessary conditions to prevent such a deviation.

No Trade Equilibrium NT.

If the positively-informed speculator deviates to selling:

- W.p. $\frac{1}{3}$, $X = -2$ and she receives R_L^n for a share that is worth R_H^n, which yields a profit of $(R_L^n - R_H^n) > 0$.
- W.p. $\frac{1}{3}$, $X = -1$ and she receives $\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$ for a share that is worth R_H^i, which yields a profit of $\frac{1}{2}\left(R_L^i - R_H^i\right) < 0$.
- W.p. $\frac{1}{3}$, $X = -1$ and she receives $\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$ for a share that is worth R_H^i, which yields a profit of $\frac{1}{2}\left(R_L^i - R_H^i\right) < 0$.

Thus, her overall profits are given by

$$\frac{1}{3}\left(R_L^n - R_H^n\right) + \frac{1}{3}\left(R_L^i - R_H^i\right).$$

For the positively-informed speculator not to deviate to selling, we require

$$\frac{1}{3}\left(-\left(R_H^i - R_L^i\right) + \left(R_L^n - R_H^n\right)\right) < \kappa.$$

The calculations for the Partial Trade Equilibrium SNB are identical.

Partial Trade Equilibrium BNS.
If the positively-informed speculator deviates to selling:

- W.p. $\frac{1}{3}$, $X = -2$ and she receives R_L^n for a share that is worth R_H^n, which yields a profit of $(R_L^n - R_H^n) > 0$.
- W.p. $\frac{1}{3}$, $X = -1$. In the case of feedback ($\frac{1-\lambda}{2-\lambda} < \gamma$), she receives $\frac{1-\lambda}{2-\lambda}R_H^n + \frac{1}{2-\lambda}R_L^n$ for a share that is worth R_H^n, which yields a profit of $\frac{1}{2-\lambda}(R_L^n - R_H^n) > 0$. In the case of no feedback ($\frac{1-\lambda}{2-\lambda} > \gamma$), she receives $\frac{1-\lambda}{2-\lambda}R_H^i + \frac{1}{2-\lambda}R_L^i$ for a share that is worth R_H^i, which yields a profit of $\frac{1}{2-\lambda}(R_L^i - R_H^i) < 0$.
- W.p. $\frac{1}{3}$, $X = 0$ and she receives $\frac{1}{2}R_H^i + \frac{1}{2}R_L^i$ for a share that is worth R_H^i, which yields a profit of $\frac{1}{2}(R_L^i - R_H^i) < 0$.

In the case of feedback, her overall profits are given by

$$\begin{aligned}&\frac{1}{3}(R_L^n - R_H^n) + \frac{1}{3}\frac{1}{2-\lambda}(R_L^n - R_H^n) + \frac{1}{3}\left(\frac{1}{2}\left(R_L^i - R_H^i\right)\right)\\&= \frac{1}{3}\left(\frac{3-\lambda}{2-\lambda}(R_L^n - R_H^n)\right) - \frac{1}{3}\left(\frac{1}{2}\left(R_H^i - R_L^i\right)\right).\end{aligned}$$

For the positively-informed speculator to choose buying over selling, her profits must be greater under the former. This requires:

$$\begin{aligned}&\frac{1}{3}\left(R_H^i - R_L^i\right) > \frac{1}{3}\left(\frac{3-\lambda}{2-\lambda}(R_L^n - R_H^n)\right) - \frac{1}{3}\left(\frac{1}{2}\left(R_H^i - R_L^i\right)\right)\\&\frac{3}{2}\left(R_H^i - R_L^i\right) > \frac{3-\lambda}{2-\lambda}(R_L^n - R_H^n).\end{aligned}$$

The first term is the "fundamental" effect, which represents the profits from trading in the direction of one's private information. The second term is the "feedback" effect, which arises because selling manipulates the order flow and causes the manager to take the wrong decision. This yields the condition $\frac{6-3\lambda}{6-2\lambda}(R_H^i - R_L^i) > 1$ in the Proposition.

In the case of no feedback, her overall profits are given by

$$\begin{aligned}&\frac{1}{3}(R_L^n - R_H^n) + \frac{1}{3}\frac{1}{2-\lambda}\left(R_L^i - R_H^i\right) + \frac{1}{3}\left(\frac{1}{2}\left(R_L^i - R_H^i\right)\right)\\&= \frac{1}{3}(R_L^n - R_H^n) - \frac{1}{3}\left(\frac{4-\lambda}{4-2\lambda}\left(R_H^i - R_L^i\right)\right).\end{aligned}$$

For the positively-informed speculator to choose buying over selling, we require:

$$\frac{1}{3}\left(R_H^i - R_L^i\right) > \frac{1}{3}\left(R_L^n - R_H^n\right) - \frac{1}{3}\left(\frac{4-\lambda}{4-2\lambda}\left(R_H^i - R_L^i\right)\right)$$
$$\frac{8-3\lambda}{4-2\lambda}\frac{R_H^i - R_L^i}{R_L^n - R_H^n} > 1.$$

Trade Equilibrium T.

If the positively-informed speculator deviates to selling, the calculations are exactly the same as in the Partial Trade Equilibrium BNS. However, the profits from buying (that we need to compare against the profits from selling) are different.

In the case of feedback, for the positively-informed speculator to choose buying over selling, we require:

$$\frac{1}{3}\left(\frac{1}{2} + \frac{1-\lambda}{2-\lambda}\right)\left(R_H^i - R_L^i\right) > \frac{1}{3}\left(\frac{3-\lambda}{2-\lambda}\left(R_L^n - R_H^n\right)\right) - \frac{1}{3}\left(\frac{1}{2}\left(R_H^i - R_L^i\right)\right)$$
$$\frac{3-2\lambda}{3-\lambda}\frac{R_H^i - R_L^i}{R_L^n - R_H^n} > 1.$$

In the case of no feedback, for the positively-informed speculator to choose buying over selling, we require:

$$\frac{1}{3}\left(\frac{1}{2} + \frac{1-\lambda}{2-\lambda}\right)\left(R_H^i - R_L^i\right) > \frac{1}{3}\left(R_L^n - R_H^n\right) - \frac{1}{3}\left(\frac{4-\lambda}{4-2\lambda}\left(R_H^i - R_L^i\right)\right)$$
$$2\frac{R_H^i - R_L^i}{R_L^n - R_H^n} > 1.$$

Proof of Lemma 3

For part (i), if $\theta = H$, the expected posterior is given by:

$$\begin{aligned} q^H &= (1-\lambda)\left[\frac{1}{3}q(-1) + \frac{1}{3}q(0) + \frac{1}{3}q(1)\right] + \lambda\left(\frac{1}{3}q(0) + \frac{1}{3}q(1) + \frac{1}{3}q(2)\right) \\ &= \frac{1-\lambda}{3}q(-1) + \frac{1}{3}q(0) + \frac{1}{3}q(1) + \frac{\lambda}{3}q(2) \\ &= \frac{(1-\lambda)^2}{6-3\lambda} + \frac{1}{3} + \frac{\lambda}{3}. \end{aligned} \tag{24}$$

We have:

$$\begin{aligned}\frac{\partial q^H}{\partial \lambda} &= \frac{1}{3} + \frac{1}{3}\left[\frac{-2(1-\lambda)(2-\lambda)+(1-\lambda)^2}{(2-\lambda)^2}\right] \\ &= \frac{1}{3}\left[1 + \left(\frac{1-\lambda}{2-\lambda}\right)^2 - 2\frac{1-\lambda}{2-\lambda}\right] \\ &= \frac{1}{3}\left[1 - \left(\frac{1-\lambda}{2-\lambda}\right)\right]^2 \\ &> 0.\end{aligned}$$

The expected posterior is increasing in λ: if the speculator is more likely to be present, she is more likely to impound her information into prices by trading.

Moving to part (ii), if $\theta = L$, we have:

$$\begin{aligned}q^L &= \frac{1}{3}\left(q\left(-1\right) + q\left(0\right) + q\left(1\right)\right) \\ &= \frac{1-\lambda}{6-3\lambda} + \frac{1}{3}.\end{aligned} \tag{25}$$

This is decreasing in λ. Even though the speculator does not trade upon $\theta = L$ if she is present, her information is still partially incorporated into prices. With $\theta = L$, there is a $\frac{1}{3}$ probability that the order flow is $X = -1$. This is consistent with the speculator being absent (in which case the state may be either H or L) or her being present and observing $\theta = L$; it is not consistent with the speculator observing $\theta = H$. The greater the likelihood that the speculator is present, the greater the likelihood that $X = -1$ stems from $\theta = L$, and thus the greater the decrease in the market maker's posterior. Part (iii) follows from simple calculations.

Proof of Proposition 5

For parts (i) and (ii), we have:

$$\begin{aligned}q^{H,spec} &= \frac{1}{3}\left(q\left(0\right) + q\left(1\right) + q\left(2\right)\right) \\ &= \frac{2}{3},\end{aligned} \tag{26}$$

$$\begin{aligned}q^{L,spec} &= \frac{1}{3}\left(q\left(-1\right) + q\left(0\right) + q\left(1\right)\right) \\ &= \frac{1-\lambda}{6-3\lambda} + \frac{1}{3}.\end{aligned} \tag{27}$$

Note that $q^{H,spec}$ is independent of λ, but $q^{L,spec}$ is decreasing in λ. The variable λ can affect the expected posterior in two ways: first, it can change the relative likelihood of the different order flows, and second, it can change the actual posterior given a certain order flow. Since we are conditioning on the speculator being present, the first channel is ruled out: conditional on the speculator being present and $\theta = H$, $X \in \{0, 1, 2\}$ with uniform probability regardless of λ;

conditional on the speculator being present and $\theta = L$, $X \in \{-1, 0, 1\}$ with uniform probability regardless of λ. Turning to the second channel, the only posterior that depends on λ is $q(-1)$: since $X = -1$ is inconsistent with the speculator being present and seeing $\theta = H$, it has a particularly negative impact on the likelihood of $\theta = H$ if the speculator is more likely to be present. By contrast, $X \in \{-2, 2\}$ is fully revealing and so the posterior is independent of λ; $X \in \{0, 1\}$ is completely uninformative and so the posterior is again independent of λ. Since $X = -1$ can only occur in the presence of a speculator if she has received bad news, only $q^{L,spec}$ depends on λ but $q^{L,spec}$ does not. Part (iii) follows from simple calculations.

Proof of Lemma 4

We start by calculating p_0. With probability $\frac{1}{2}$, the state will be $\theta = L$ and there is no trade, regardless of whether the speculator is present. Thus, $X \in \{-1, 0, 1\}$ with equal probability. With probability $\frac{1}{2}$, the state will be $\theta = H$. If the speculator is absent (w.p. $(1 - \lambda)$), there is no trade and we again have $X \in \{-1, 0, 1\}$. If the speculator is present, $X \in \{0, 1, 2\}$. Letting $p(X)$ denote the stock price set by the market maker after observing order flow X at $t = 1$, the price at $t = 0$ will be the expectation over all possible future prices at $t = 1$, and is given as follows:

$$\begin{aligned} p_0 &= \frac{\lambda}{2}\left(\frac{1}{3}p(0) + \frac{1}{3}p(1) + \frac{1}{3}p(2)\right) + \left(1 - \frac{\lambda}{2}\right)\left(\frac{1}{3}p(-1) + \frac{1}{3}p(0) + \frac{1}{3}p(1)\right) \\ &= \frac{1}{3}\left(\left(1 - \frac{\lambda}{2}\right)p(-1) + p(0) + p(1) + \frac{\lambda}{2}p(2)\right) \\ &= \frac{1}{6}\left[(1-\lambda)R_H^n + R_L^n + (2+\lambda)R_H^i + 2R_L^i\right]. \end{aligned} \tag{28}$$

Even though the initial belief y is independent of λ, the initial stock price p_0 is increasing in λ, because the speculator provides information to improve the manager's decision. Moreover, $\frac{\partial p_0}{\partial \lambda}$ is increasing in $(R_H^i - R_H^n)$, the increase in firm value from taking the efficient continuation decision in the high state. This is intuitive. If the speculator is present, she always buys in state H, which guarantees that $X \geq 0$ and the investment is undertaken. If she is absent, there is a possibility that $X = -1$. This leads the manager to take the suboptimal corrective action, reducing firm value by $(R_H^i - R_H^n)$. By contrast, $\frac{\partial p_0}{\partial \lambda}$ is independent of R_L^i and R_L^n, the firm values in the low state. This is because, if the low state is realized, the speculator's presence does nothing to help the manager's decision, since she does not trade.

For part (i), if $\theta = H$ is realized, the expected price at $t = 1$ is given by:

$$\begin{aligned} p_1^H &= (1-\lambda)\left[\frac{1}{3}p(-1) + \frac{1}{3}p(0) + \frac{1}{3}p(1)\right] + \lambda\left(\frac{1}{3}p(0) + \frac{1}{3}p(1) + \frac{1}{3}p(2)\right) \\ &= \frac{1-\lambda}{3}p(-1) + \frac{1}{3}p(0) + \frac{1}{3}p(1) + \frac{\lambda}{3}p(2). \end{aligned} \tag{29}$$

Note that:

$$\begin{aligned}\frac{\partial p_1^H}{\partial \lambda} &= \frac{1}{3}p(2) - \frac{1}{3}p(-1) + \frac{1-\lambda}{3}\frac{\partial p(-1)}{\partial \lambda} \\ &= \frac{1}{3}\left[R_H^i - \frac{1}{(2-\lambda)^2}R_L^n - \left(1 - \frac{1}{(2-\lambda)^2}\right)R_H^n\right] \\ &> \frac{1}{3}[R_H^i - R_L^n] \\ &> 0,\end{aligned}$$

i.e. p_1^H is increasing in λ, since the speculator impounds information about the high state into prices.

Turning to part (ii), if $\theta = L$ is realized, the expected price at $t = 1$ is given by:

$$p_1^L = \frac{1}{3}\left(p\left(-1\right) + p\left(0\right) + p\left(1\right)\right). \tag{30}$$

We have $\frac{\partial p_1^{L,spec}}{\partial \lambda} = \frac{R_L^n - R_H^n}{3(2-\lambda)^2}$. If the speculator is more likely to be present, then $X = -1$ is more likely to result from $\theta = L$. Thus, the price is higher if and only if firm value is higher in this state, i.e., $R_L^n > R_H^n$ (Case 2).

The calculations of $p_1^H - p_0$ and $p_1^L - p_0$ follow automatically.

Proof of Proposition 6

For part (i), if the speculator receives positive information, she will buy one share and so the expected price becomes:

$$p_1^{H,spec} = \frac{1}{3}\left(p\left(0\right) + p\left(1\right) + p\left(2\right)\right). \tag{31}$$

Unlike p_1^H (equation (29)), this is independent of λ, for the same reasons that $q^{H,spec}$ (equation (26)) is independent of λ. The stock return realized when the speculator receives good information is thus given by:

$$\begin{aligned}p_1^{H,spec} - p_0 &= \frac{1}{3}\left(p\left(0\right) + p\left(1\right) + p\left(2\right)\right) - \frac{1}{3}\left(\left(1 - \frac{\lambda}{2}\right)p\left(-1\right) + p\left(0\right) + p\left(1\right) + \frac{\lambda}{2}p\left(2\right)\right) \\ &= \frac{1}{3}\left(1 - \frac{\lambda}{2}\right)\left(p\left(2\right) - p\left(-1\right)\right) \\ &= \frac{1}{3}\left(1 - \frac{\lambda}{2}\right)\left(R_H^i - \frac{1-\lambda}{2-\lambda}R_H^n - \frac{1}{2-\lambda}R_L^n\right) > 0,\end{aligned} \tag{32}$$

and we have

$$\frac{\partial\left(p_1^{H,spec} - p_0\right)}{\partial \lambda} = \frac{1}{6}[R_H^n - R_H^i] < 0.$$

Equation (32) is decreasing in λ, whereas the stock return not conditioning on the specula-

tor's presence, (32), was increasing in λ. This is because p_0 is increasing in λ, but $p_1^{H,spec}$ is independent of λ.

For part (ii), if the speculator is present and receives negative information, we have:

$$p_1^{L,spec} = \frac{1}{3}\left(p\left(-1\right) + p\left(0\right) + p\left(1\right)\right) = p_1^L, \tag{33}$$

and

$$\begin{aligned} p_1^{L,spec} - p_0 &= \frac{1}{3}\left(p\left(-1\right) + p\left(0\right) + p\left(1\right)\right) - \frac{1}{3}\left(\left(1 - \frac{\lambda}{2}\right)p\left(-1\right) + p\left(0\right) + p\left(1\right) + \frac{\lambda}{2}p\left(2\right)\right) \\ &= \frac{\lambda}{6}\left(p\left(-1\right) - p\left(2\right)\right) = p_1^L - p_0 < 0. \end{aligned}$$

Parts (iii) and (iv) follow from simple calculations.

Dropping constants, both equation (16) (the asymmetry between the price impact of good and bad news) and equation (17) (the average return, conditional on the speculator being present) become:

$$\left(1-\lambda\right)\left(R_H^i - \frac{1-\lambda}{2-\lambda}R_H^n - \frac{1}{2-\lambda}R_L^n\right).$$

Differentiating with respect to λ gives:

$$\begin{aligned} &- R_H^i + \frac{1-\lambda}{2-\lambda}R_H^n + \frac{1}{2-\lambda}R_L^n + \left(1-\lambda\right)\left[\frac{1}{\left(2-\lambda\right)^2}R_H^n - \frac{1}{\left(2-\lambda\right)^2}R_L^n\right] \\ &= -R_H^i + \frac{3-4\lambda+\lambda^2}{\left(2-\lambda\right)^2}R_H^n + \frac{1}{\left(2-\lambda\right)^2}R_L^n \end{aligned}$$

The coefficients of R_H^n and R_L^n are positive and add up to one. That is, we have a convex combination of R_H^n and R_L^n, which is smaller that R_H^i, since $R_H^i > R_H^n$ and $R_H^i > R_L^n$. Thus, both equations (16) and (17) are decreasing in λ.

Proof of Corollary 2

We start with part (i). If the speculator receives good news, she will buy and so the project will always be undertaken. We thus have $p_2^{H,spec} = R_H^i$. This yields:

$$\begin{aligned} p_2^{H,spec} - p_1^{H,spec} &= R_H^i - \frac{1}{3}\left(p\left(0\right) + p\left(1\right) + p\left(2\right)\right) \\ &= \frac{1}{3}\left(R_H^i - R_L^i\right). \end{aligned}$$

Moving to part (ii), if the speculator receives bad news, she will not trade. The project will be cancelled if the noise trader sells, else it will be continued. We thus have $p_2^{L,spec} = \frac{2}{3}R_L^i + \frac{1}{3}R_L^n$.

This yields:

$$\begin{aligned} p_2^{L,spec} - p_1^{L,spec} &= \frac{2}{3}R_L^i + \frac{1}{3}R_L^n - \frac{1}{3}\left(p\left(-1\right) + p\left(0\right) + p\left(1\right)\right) \\ &= -\frac{1}{3}\left(R_H^i - R_L^i\right) - \frac{1}{3}\left(\frac{1-\lambda}{2-\lambda}\left(R_H^n - R_L^n\right)\right), \end{aligned}$$

which can be positive or negative. Part (iii) follows from simple calculations. For part (iv), we first calculate the expected firm value at $t = 2$ if the speculator is present, not conditioning on the state. If $\theta = H$, the project is always undertaken, regardless of the order flow at $t = 1$, and so $v = R_H^i$. If $\theta = L$, whether the project is undertaken depends on the order flow: if $X = -1$, we have $d = n$ and so $v = R_L^n$; if $X \in \{0, 1\}$, we have $d = i$ and so $v = R_L^i$. Expected firm value at $t = 2$ is thus given by:

$$p_2^{spec} = \frac{1}{2}R_H^i + \frac{1}{3}R_L^i + \frac{1}{6}R_L^n,$$

and so we have

$$p_2^{spec} - p_1^{spec} = \frac{1}{6}\frac{1-\lambda}{2-\lambda}\left(R_L^n - R_H^n\right),$$

which is positive if we are in Case 2 and negative if we are in Case 1.

Proof of Proposition 7

We start with some preliminary results that will be useful in the main proof. From $\frac{1-\lambda}{2-\lambda} < \gamma$ we have

$$\lambda > 1 - \frac{\gamma}{1-\gamma},$$

which implies that $\lambda \in (1 - \frac{\gamma}{1-\gamma}, 1)$. Let $\lambda_{min} = 1 - \frac{\gamma}{1-\gamma}$ and $\lambda_{max} = 1$.

Define $[\tilde{R}_H^i \ \tilde{R}_L^i \ \tilde{R}_L^n] = [R_H^i - R_H^n \ R_L^i - R_H^n \ R_L^n - R_H^n]$. Note that $\tilde{R}_H^i > 0$ from equation (1). Equation (5) thus yields:

$$\tilde{R}_L^n = (1 - \lambda_{min})\tilde{R}_H^i + \tilde{R}_L^i. \tag{34}$$

For part (i), we first calculate $p_1^{i,H}$, the expected stock price at $t = 1$ if investment has been undertaken and the state is good. We have $\theta = H$ w.p. $\frac{1}{2}$. W.p. λ, the speculator is present and buys, so $X \in \{0, 1, 2\}$ with uniform probability and the investment is always undertaken. W.p. $1 - \lambda$ the speculator is absent, so there is no trade, which yields $X \in \{-1, 0, 1\}$. If $X = -1$, we have $d = n$ so we exclude this case. We therefore have:

$$\begin{aligned} p_1^{i,H} &= \frac{\frac{\lambda}{2}\frac{1}{3}\left(p(0) + p(1) + p(2)\right) + \frac{1-\lambda}{2}\frac{1}{3}\left(p(0) + p(1)\right)}{\frac{1}{2}\left(\lambda + (1-\lambda)\frac{2}{3}\right)} \\ &= \frac{p(0) + p(1) + \lambda p(2)}{2 + \lambda}. \end{aligned}$$

This is increasing in λ, because if the speculator is present, she will trade on her positive signal and impound it into prices. Note that this argument did not apply to $q^{H,spec}$ (equation (26)) and

$p_1^{H,spec}$ (equation (31)) because those quantities are conditional on the speculator being present, so $X = \{0, 1, 2\}$ were all equally likely. Here, $X = 2$ is particularly likely if the speculator is present.

For part (ia), the short-run return to an investment ($d = i$) in the high state is given by:

$$\begin{aligned} p_1^{i,H} - p_0 &= \frac{\lambda p(2) + p(1) + p(0)}{2+\lambda} - \frac{1}{3}\left(\left(1 - \frac{\lambda}{2}\right) p(-1) + p(0) + p(1) + \frac{\lambda}{2} p(2)\right) \\ &= \frac{4\lambda - \lambda^2}{6(2+\lambda)} p(2) + \frac{1-\lambda}{3(2+\lambda)} p(1) + \frac{1-\lambda}{3(2+\lambda)} p(0) - \left(\frac{1}{3} - \frac{\lambda}{6}\right) p(-1) \\ &= \frac{1}{6(2+\lambda)}[(2 + 2\lambda - \lambda^2) R_H^i + (2 - 2\lambda) R_L^i - (2 - \lambda - \lambda^2) R_H^n - (2+\lambda) R_L^n]. \end{aligned} \tag{35}$$

The sign of (35) is the same as the sign of

$$\begin{aligned} &(2 + 2\lambda - \lambda^2)\left(R_H^i - R_H^n\right) + (2 - 2\lambda)\left(R_L^i - R_H^n\right) - (2 - \lambda - \lambda^2)\left(R_H^n - R_H^n\right) - (2+\lambda)\left(R_L^n - R_H^n\right) \\ &= (2 + 2\lambda - \lambda^2)\tilde{R}_H^i + (2 - 2\lambda)\tilde{R}_L^i - (2+\lambda)\tilde{R}_L^n. \end{aligned} \tag{36}$$

Equation (36) is quadratic and concave in λ, and so its minimum occurs at either λ_{min} or λ_{max}. Thus, to prove that (36) > 0, it is sufficient to prove that it is positive at both λ_{min} and λ_{max}. At $\lambda = \lambda_{max}$, equation (36) reduces to $3\tilde{R}_H^i - 3\tilde{R}_L^n > 0$. At $\lambda = \lambda_{min}$, it reduces to $3\lambda_{min}(\tilde{R}_H^i - \tilde{R}_L^i) > 0$. Thus, equation (35) is positive.

For part (ib), the long-run drift is given by:

$$\begin{aligned} R_H^i - p_1^{i,H} &= R_H^i - \frac{\lambda p(2) + p(1) + p(0)}{2+\lambda} \\ &= \frac{R_H^i - R_L^i}{2+\lambda} > 0. \end{aligned} \tag{37}$$

For part (ii), we first calculate $p_1^{i,L}$, the expected stock price at $t = 1$ if investment has been undertaken and the state is bad. Regardless of whether the speculator is present, the order flow will be $X \in \{-1, 0, 1\}$ with uniform probability. If $X = -1$, we have $d = n$ so we exclude this case. We therefore have:

$$p_1^{i,L} = \frac{\frac{1}{2}\left(\frac{1}{3}p(0) + \frac{1}{3}p(1)\right)}{\frac{1}{2} \times \frac{2}{3}} = \frac{1}{2}\left(p(0) + p(1)\right).$$

This is independent of λ since the presence of the speculator does not change the order flow. Unlike in the earlier cases of p_1^L and $p_1^{L,spec}$ which did depend on λ, here we are conditioning upon the investment being undertaken. This rules out the case of $p(-1)$ which is the only price that depends on λ.

For part (iia), the short-run return to an investment in the low state is:

$$\begin{aligned} p_1^{i,L} - p_0 &= \frac{1}{2}\left(p(0)+p(1)\right) - \left[\frac{\lambda}{6}p(2) + \frac{1}{3}p(1) + \frac{1}{3}p(0) + \left(\frac{1}{3} - \frac{\lambda}{6}\right)p(-1)\right] \\ &= -\frac{\lambda}{6}p(2) + \frac{1}{6}p(1) + \frac{1}{6}p(0) - (\frac{1}{3} - \frac{\lambda}{6})p(-1) \\ &= \frac{1}{6}[(1-\lambda)\left(R_H^i - R_H^n\right) + R_L^i - R_L^n]. \end{aligned} \tag{38}$$

The sign of (38) is the same as the sign of:

$$\begin{aligned} &(1-\lambda)\left(R_H^i - R_H^n - (R_H^n - R_H^n)\right) + R_L^i - R_H^n - (R_L^n - R_H^n) \\ &= (1-\lambda)\tilde{R}_H^i + \tilde{R}_L^i - \tilde{R}_L^n. \end{aligned} \tag{39}$$

Equation (39) is decreasing in λ, since $\tilde{R}_H^i > 0$. This implies that

$$\begin{aligned} (1-\lambda)\tilde{R}_H^i + \tilde{R}_L^i - \tilde{R}_L^n &< (1-\lambda_{min})\tilde{R}_H^i + \tilde{R}_L^i - \tilde{R}_L^n \\ &= 0, \end{aligned}$$

from equation (34). Thus, equation (38) is negative.

Since $p_1^{i,L}$ is independent of λ, and p_0 is increasing in λ (because the speculator improves the manager's decisions), the absolute return is decreasing in λ.

For part (iib), the long-run drift is given by:

$$\begin{aligned} R_L^i - p_1^{i,L} &= R_L^i - \frac{1}{2}(p(0)+p(1)) \\ &= -\frac{1}{2}(R_H^i - R_L^i) < 0 \end{aligned} \tag{40}$$

and independent of λ, since $p_1^{i,L}$ is independent of λ.

For part (iii), we first calculate the difference between the long-run drift and the short-run return to a good investment, i.e. (37)-(35). This yields:

$$\begin{aligned} &\frac{1}{2+\lambda}(\tilde{R}_H^i - \tilde{R}_L^i) - \frac{1}{6(2+\lambda)}[(2+2\lambda-\lambda^2)\tilde{R}_H^i + (2-2\lambda)\tilde{R}_L^i - (2+\lambda)\tilde{R}_L^n] \\ &= \frac{1}{6(2+\lambda)}[(4-2\lambda+\lambda^2)\tilde{R}_H^i - (8-2\lambda)\tilde{R}_L^i + (2+\lambda)\tilde{R}_L^n]. \end{aligned}$$

In order to calculate $abs\left((37)-(35)\right)$, we must first sign (37)-(35). To prove this is positive, we must prove that

$$(4-2\lambda+\lambda^2)\tilde{R}_H^i - (8-2\lambda)\tilde{R}_L^i + (2+\lambda)\tilde{R}_L^n \tag{41}$$

is positive. If $\tilde{R}_L^i \leq 0$, it is automatic that $(41) > 0$. Suppose $\tilde{R}_L^i > 0$. Evaluating (41) at λ_{min}

and λ_{max}, respectively, yields

$$\begin{aligned}\lambda = \lambda_{min} :&(4 - 2\lambda_{min} + \lambda_{min}^2)\tilde{R}_H^i - (8 - 2\lambda_{min})\tilde{R}_L^i + (2 + \lambda_{min})[(1 - \lambda_{min})\tilde{R}_H^i + \tilde{R}_L^i] \\ &= (6 - 3\lambda_{min})\tilde{R}_H^i - (6 - 3\lambda_{min})\tilde{R}_L^i > 0. \\ \lambda = \lambda_{max} :&3\tilde{R}_H^i - 6\tilde{R}_L^i + 3\tilde{R}_L^n > 0.\end{aligned}$$

If (41) is monotonic in λ in its feasible range, it follows that (41) > 0. Suppose that (41) is not monotonic in λ. Then its derivative with respect to λ must have a root $\lambda_0 \in (\lambda_{min}, \lambda_{max})$. The derivative of (41) is equal to

$$-(2 - 2\lambda)\tilde{R}_H^i + 2\tilde{R}_L^i + \tilde{R}_L^n.$$

Then $\lambda_0 = 1 - \frac{2\tilde{R}_L^i + \tilde{R}_L^n}{2\tilde{R}_H^i}$. The condition $\lambda_0 > \lambda_{min}$, together with (41), yields

$$\begin{aligned}&1 - \frac{2\tilde{R}_L^i + \tilde{R}_L^n}{2\tilde{R}_H^i} > 1 - \frac{\tilde{R}_L^n - \tilde{R}_L^i}{\tilde{R}_H^i} \\ &\Rightarrow \frac{2\tilde{R}_L^i + \tilde{R}_L^n}{2\tilde{R}_H^i} < \frac{\tilde{R}_L^n - \tilde{R}_L^i}{\tilde{R}_H^i} \\ &\Rightarrow 4\tilde{R}_L^i < \tilde{R}_L^n,\end{aligned}$$

which implies

$$\begin{aligned}(41) &> (4 - 2\lambda + \lambda^2)\tilde{R}_H^i - (8 - 2\lambda)\tilde{R}_L^i + 4(2 + \lambda)\tilde{R}_L^i \\ &= (4 - 2\lambda + \lambda^2)\tilde{R}_H^i\tilde{R}_H^i + 6\lambda\tilde{R}_L^i \\ &> 0 \quad \text{for } \tilde{R}_L^i > 0.\end{aligned}$$

We now calculate the difference between the long-run drift and the short-run return to a good investment, i.e. (40) − (38). This yields:

$$\begin{aligned}&-\frac{1}{2}(R_H^i - R_L^i) - \frac{1}{6}[(1 - \lambda)R_H^i + R_L^i - R_L^n] \\ &= -\frac{1}{6}[(4 - \lambda)R_H^i - 2R_L^i - R_L^n].\end{aligned}$$

In order to calculate $abs\,((40) - (38))$, we must first sign (40) − (38). To prove this is negative, we must prove that

$$(4 - \lambda)\tilde{R}_H^i - 2\tilde{R}_L^i - \tilde{R}_L^n \tag{42}$$

is positive. At $\lambda = \lambda_{max}$, equation (42) becomes $3\tilde{R}_H^i - 2\tilde{R}_L^i - \tilde{R}_L^n > 0$. Since (42) is decreasing in lambda, we have (42) > 0.

Finally, we wish to show that $abs\,((40) - (38)) > abs\,((37) - (35))$. The sign of $abs\,((40) - (38))-$

$abs\,((37)-(35))$ is equal to the sign of:

$$\begin{aligned}
&(2+\lambda)[(4-\lambda)\tilde{R}^i_H - 2\tilde{R}^i_L - \tilde{R}^n_L] - [(4-2\lambda+\lambda^2)\tilde{R}^i_H - (8-2\lambda)\tilde{R}^i_L + (2+\lambda)\tilde{R}^n_L] \\
=&[(8+2\lambda-\lambda^2)\tilde{R}^i_H - (4+2\lambda)\tilde{R}^i_L - (2+\lambda)\tilde{R}^n_L] - [(4-2\lambda+\lambda^2)\tilde{R}^i_H - (8-2\lambda)\tilde{R}^i_L + (2+\lambda)\tilde{R}^n_L] \\
=&2(2+2\lambda-\lambda^2)\tilde{R}^i_H + 4(1-\lambda)\tilde{R}^i_L - 2(2+\lambda)\tilde{R}^n_L. \qquad (43)
\end{aligned}$$

(43) is quadratic and concave in λ. Then its minimum occurs at either λ_{min} or λ_{max}. At $\lambda = \lambda_{max}$, (43) reduces to $6(\tilde{R}^i_H - \tilde{R}^n_L) > 0$. For $\lambda = \lambda_{min}$,

$$\begin{aligned}
(43) &= 2(2+2\lambda_{min}-\lambda^2_{min})\tilde{R}^i_H + 4(1-\lambda_{min})\tilde{R}^i_L - 2(2+\lambda_{min})[(1-\lambda_{min})\tilde{R}^i_H + \tilde{R}^i_L] \\
&= 6\lambda_{min}(\tilde{R}^i_H - \tilde{R}^i_L) > 0.
\end{aligned}$$

Thus, (43) is always positive, and so $abs\,((40)-(38)) - abs\,((37) > (35))$.