How Important Is Mispricing?

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Abstract

We quantify the asset pricing and real investment distortions implied by behavioral theories in which information processing biases lead to predictable asset returns. Our benchmark model features economic agents who incorrectly process information about firm productivity. These agents determine both firms' asset prices *and* their investment decisions. We estimate the extent of information processing biases using observed asset pricing anomalies. Our findings indicate that the true magnitude of mispricing is several times larger than empirically observed anomalies suggest. The model also shows when and how information processing biases can cause substantial capital misallocation. Lastly, we evaluate alternative models in which managers are rational and find that these alternatives do not match observed return predictability as well as our benchmark model. Overall, our results suggest that the observed magnitude of return anomalies understates inefficiencies in firms' asset prices and investment decisions.

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There is surprisingly little research that tests the *quantitative* predictions of theories in behavioral finance in which errors in investors' expectations lead to asset return predictability.[1] As a result, we do not know how large the biases in agents' expectations of firms' cash flows must be in order to explain observed return predictability.[2] Moreover, we lack any sense of how poorly agents with such biased expectations would allocate capital across firms and which biases are most important for efficient allocation. To address these questions parsimoniously, we analyze a model in which observed return predictability comes from behavioral biases, rather than compensation for risk.

Our goal is to quantify the asset pricing and investment distortions implied by behavioral theories of information processing biases. To this end, we augment a neoclassical model of firm investment and valuation with agents who are subject to information processing biases. We estimate the extent of these biases using cross-sectional asset return predictability coming from firms' valuation ratios, investment rates, and cash flows.

The estimation and analysis of our model yields three qualitative insights. First, observed return anomalies are the tip of the iceberg: insofar as information processing biases cause observable anomalies, the unobservable asset mispricing and misallocation of capital are much larger than their observable proxies. Second, models in which managers are rational predict return predictability patterns that do not match empirical patterns as well as our benchmark

[1] Examples of behavioral finance theories in which errors in investors' expectations lead to return predictability include (Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subramanyam (1998), and Hong and Stein (1999)). Two studies with quantitative structural estimates of mispricing and investment are Chirinko and Shaller (2001) and Panageas (2005). Qualitative tests of models of mispricing include Baker, Stein, and Wurgler (2003), Gilchrist, Huberman, and Himmelberg (2005), and Polk and Sapienza (2009).

[2] Numerous studies examine stock return predictability in order to test asset pricing theories. Banz (1981), Bernard and Thomas (1989), and Fama and French (1992) show that stocks with certain characteristics—such as those with low market capitalizations, high recent earnings, and high ratios of book-to-market equity—experience higher average stock returns than conventional asset pricing theory predicts.

model in which managers have the same biases as investors. Third, the benchmark model shows when and how information processing biases can cause substantial misallocation of capital.

Our benchmark model features a firm with time-varying productivity and costly capital adjustment. Managers who determine firm investment decisions and investors who value firm assets have homogeneous beliefs and information. As each firm's productivity prospects change, its manager dynamically responds by increasing or decreasing firm's capital stock. Because capital adjustment is costly, the manager only partially adjusts to productivity prospects, which generates persistence in the firm's growth opportunities.

Current productivity is not observable, so agents must estimate it using two pieces of information: cash flow and a soft information signal. Cash flow is a noisy indicator of the firm's current productivity. The soft information signal captures all productivity-relevant information that is not reflected in cash flow. Real-world examples of observable soft information include analyst forecasts, management guidance, and firm-specific news stories.

Economic agents potentially suffer from two biases in processing information about productivity: overconfidence and excessive extrapolation. Overconfident agents believe the signal precision to be higher than it actually is, as Daniel, Hirshleifer, and Subrahmanyam (1998) and Scheinkman and Xiong (2003). Agents who excessively extrapolate believe the persistence of productivity to be higher than it actually is, as in Barberis, Shleifer, and Vishny (1998).[3] Previous research proposes that these two biases could explain why value stocks earn higher returns than growth stocks. Accordingly, we identify the magnitude of information processing biases by matching the empirical value anomaly, along with other asset return data.

[3] Our model also allows for biases in which the agent *under*estimates the signal precision and the persistence of productivity. Our empirical estimates, however, are consistent with overconfidence and excessive extrapolation.

We jointly estimate all parameters so that our model replicates the relevant features of the empirical data as closely as possible. We use the simulated method of moments (SMM) to find production function parameters such that firms in our model mimic real firms' values of investment, cash flow, and Tobin's Q. Our SMM procedure also ensures that the representative agent's information processing biases generate asset pricing anomalies in our model that mimic empirical asset return predictability. We measure each return anomaly using firms' total asset returns—not just equity returns—based on CRSP, CompuStat, and bond return data from 1968 to 2006. We analyze total asset returns because firms' capital structures do not affect the key economic variables in our model.

The analysis and estimation of the model produces several insights. First, the magnitude of mispricing is several times larger than empirically observed anomalies, such as the value premium, suggest. The reason is that empirical anomalies are poor proxies for errors in investors' beliefs, which determine firms' true expected returns. For example, although a firm's Tobin's Q is a common proxy for mispricing, only some the variation in Q comes from errors in investors' expectations. Much of the variation occurs for rational reasons, such as the anticipation of time-varying productivity. In the main specification, sorting by a firm's (true) expected return to a rational agent produces a top-to-bottom decile difference in expected returns of 20.4%. In contrast, sorting by Q produces a difference of just 6.9% in expected returns. An exploration of different model parameters reveals that the overconfidence bias, not the excessive extrapolation bias, explains why empirical anomalies are poor proxies for investors' beliefs. Overconfident agents' biases depend on an unobservable soft information signal, whereas extrapolative agents exhibit biases mainly depend on observable proxies for productivity, such as cash flow.

Another insight comes from considering an alternative model in which managers are rational, while investors remain biased. We find that the rational manager model incorrectly predicts that firm investment only weakly forecasts abnormal asset returns. However, our benchmark model in which managers share the same mistaken beliefs as investors matches empirical return predictability quite well. Based on this evidence and the benchmark model's tractability, we focus on this model in our analysis.

In the benchmark model, managers make costly mistakes in allocating capital because they invest too much (little) when investors are excessively optimistic (pessimistic) about firm productivity. We use structural estimates of the model's parameters to quantify the welfare cost of biased management. The aggregate value of invested capital is 9.5% lower than it would be if unbiased managers seized control of firms and determined real investment. The aggregate value *added* of invested capital in a steady state economy in which managers are biased is 64.5% lower than it is in a steady state economy in which managers are rational. Underlying these aggregate welfare costs of biased management, there is a considerable range of investment inefficiencies across firms: some firms actually reduce value below the cost of invested capital.

Our model also provides new insights into the roles of the excessive extrapolation and overconfidence biases in capital allocation and mispricing. In theory, an overconfident agent falsely believes that noise in his soft information signal is informative. As a result, an overconfident manager invests too much when his signal is high and too little when it is low. These deviations are costly, but they do not necessarily cause average investment to be too high or low. Consistent with this argument, our simulations show that an overconfident managers chooses a more volatile investment rate than a rational manager, but average investment is similar for both managers.

In contrast, a manager that excessively extrapolates productivity trends believes that productivity shocks are more persistent than they really are. A forward-looking extrapolative agent puts too much value on the firm's future growth opportunities, believing that she can capitalize on productivity shocks before they dissipate. Consequently, she increases investment today to lower expected adjustment costs. Consistent with this reasoning, our simulations indicate that extrapolative managers exhibit higher average investment rates than rational managers. Both of the individual biases contribute significantly to welfare costs, but overconfidence plays a somewhat greater quantitative role in most specifications.

In pricing assets, investors who are overconfident and those who extrapolate both have excessively volatile beliefs about asset values, producing a value anomaly. However, in estimating firm productivity, an overconfident agent pays more attention to his soft information signal than to cash flow, whereas an extrapolative agent places too much weight on cash flow. This discrepancy leads to opposing predictions for asset return predictability from cash flows. In simulations, we confirm that cash flows positively (negatively) predict returns when investors are overconfident (extrapolative). The empirically weak cash flow anomaly suggests that these two effects offset, implying both are similarly important.

This study's quantitative structural approach contrasts with the reduced form approach in several earlier studies that test the validity of their models' qualitative predictions. Influential examples of this approach include Baker, Stein, and Wurgler (2003), Gilchrist, Himmelberg, and Huberman (2005), and Polk and Sapienza (2009) among many others. These papers focus on the impact of inefficient asset prices on rational managers' investment decisions, which also depend on the cost of external finance and their incentives to maximize the current and future market

value. Our model focuses on the case in which financing is frictionless and managers maximize the firm's current market value.

Most studies using structural methods do not jointly model investment behavior and asset prices and even fewer allow for information processing biases. For example, Hennessy and Whited (2007) focus on the impact of corporate financing frictions on firms' investment decisions and do not model asset prices. Nearly all of the studies that model firms' asset returns and investment decisions, such as Gomes, Yaron, and Zhang (2006) and Liu, Whited, and Zhang (2009), do not consider behavioral biases. For example, Liu, Whited, and Zhang (2009) show that the average stock returns on portfolios formed on size and value could be consistent with the optimality conditions from rational firms' investment choices. Yet this consistency could occur whether stocks are priced efficiently based on differences in risk or inefficiently based on errors in investors' expectations. In the latter case, which we model, investment inefficiencies mirror stock pricing anomalies. Our contribution is to analyze these joint inefficiencies. Panageas (2005) is one of the few papers that models firms' stock prices and investment in the presence of behavioral biases. Panageas (2005) analyzes only pricing inefficiencies, which result from heterogeneity in investor beliefs and short-selling constraints. In contrast, we analyze investment inefficiencies, which do not arise in Panageas (2005), as well as pricing inefficiencies. In our model, homogeneous investors without constraints cause both types of inefficiencies.[4]

An overview of our paper follows. Section I introduces and solves our model of firm investment dynamics and stock prices. Section II describes the empirical identification strategy and methodology. Section III presents the estimation results and evaluates the predictions from

[4] Our paper is also related to Chirinko and Shaller's (2001) study of mispricing and investment in the aggregate Japanese stock market. They test the validity of investment Euler equations and argue that mispricing has a material impact on investment. We complement their study by modeling the source of mispricing and investment distortions and quantifying the effect of behavioral biases in the cross-section of US firms.

two model specifications against empirical benchmarks. Section IV analyzes the extent of mispricing and investment inefficiencies in simulated model economies under various specifications, including one in which managers are rational. Section V discusses and interprets the implications of our results for future research.

I. A Model of Investment and Asset Prices

A. Modeling Framework

This section presents the model that we empirically analyze in later sections. The model resembles the standard dynamic neoclassical framework, except that agents may exhibit information processing biases. There are two groups of agents in the economy: investors who price financial assets, and managers who make firms' investment decisions. All agents are risk-neutral and have the common discount rate of r. For most of the analysis, we assume that investors and managers have homogenous information and beliefs. In this case, a representative investor/manager effectively determines both asset prices and firms' investment policies.

All firms in the model share the same production technology, but their productivity at any given time may differ. Here we describe the model for an individual firm; later in our empirical analysis we consider a cross-section of such firms with differing productivity. Because we do not model tax or financing frictions, capital structure is irrelevant in the model. Without loss of generality, one can assume that investors finance all firms using only equity.

We denote the continuous passage of time by t. The firm pays a continuous flow of cash flows $d\pi_t$ given by

$$d\pi_t = (f_t dt + \sigma_a d\omega_t^a) m_t^{1-\alpha} K_t^{\alpha}. \tag{1}$$

The firm's capital stock is K_t, and $\alpha \in (0,1]$ is the returns-to-scale parameter. The capital stock

evolves according to

$$dK_t = I_t dt - \delta K_t dt, \tag{2}$$

where I_t denotes the firm's instantaneous investment rate and δ is the depreciation rate. To allow for variation in growth opportunities, we adopt the standard assumption that adjusting the capital stock is costly. Specifically, when the firm invests at rate I_t, in addition to the cost of the newly installed capital, it pays a quadratic adjustment cost of

$$\Psi(I_t, K_t) = \frac{\phi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t. \tag{3}$$

In Equation (3), the parameter ϕ measures the cost of adjusting the capital stock.

The variable m_t in Equation (1) denotes the economy-wide component of the cash flow process, which grows geometrically at an average rate of g according to

$$\frac{dm_t}{m_t} = g dt. \tag{4}$$

We define the firm-specific component of the cash flow process in Equation (1) as

$$da_t \equiv f_t dt + \sigma_a d\omega_t^a. \tag{5}$$

The term f_t in Equation (5) is the firm's *productivity*, which reverts to its long-term mean of $\overline{f}$ according to the law of motion

$$df_t = -\lambda(f_t - \bar{f})dt + \sigma_f d\omega_t^f. \tag{6}$$

The parameter λ measures the extent of mean reversion in productivity. We set the long-term mean of productivity $\overline{f}$ to one without loss of generality. The second term in Equation (5),

$\sigma_a d\omega_t^a$, represents serially uncorrelated noise in cash flows. The presence of this noise term implies that realized cash flows are an imperfect indicator of the firm's productivity.

Because agents in the economy do not observe productivity f_t directly, they estimate it using two available pieces of information. First, agents observe the realized cash flow rate da_t in Equation (5), which is informative about f_t. Second, agents observe a *soft information signal* s_t that is correlated with the innovations in f_t. The signal s_t summarizes all productivity-related information, such as analysts' forecasts, other than cash flow realizations. We assume that the signal s_t evolves according to

$$ds_t = \eta d\omega_t^f + \sqrt{1-\eta^2}\, d\omega_t^s. \tag{7}$$

The $d\omega_t^s$ term represents noise in the signal. The specification in Equation (7) ensures that the instantaneous variance of the signal is equal to one regardless of the value of η. To measure signal precision, we define

$$\theta \equiv \frac{\eta}{\eta + \sqrt{1-\eta^2}}. \tag{8}$$

A value of one (zero) for θ represents a fully revealing (uninformative) signal.[5] In summary, there are four jointly independent sources of randomness in the model: the standard Brownian motions $d\omega_t^m$ (the economy-wide innovation), $d\omega_t^f$ (the firm's productivity innovation), $d\omega_t^a$ (the cash flow noise), and $d\omega_t^s$ (the signal noise).

We model two types of information processing biases, *overconfidence* and *earnings extrapolation*. Prior studies model overconfidence as perceiving the precision of an informative signal to be higher than it actually is (Daniel, Hirshleifer, and Subrahmanyam (1998) and

[5] The modeling of the soft information signal follows Scheinkman and Xiong (2003).

Scheinkman and Xiong (2003)). Following these studies, we assume that the agents in the economy believe the parameter η to be $\eta_B > \eta$, and hence the signal precision to be $\theta_B > \theta$. The difference $\theta_B - \theta$ then measures how overconfident the agents are. Earnings extrapolation refers to the belief that earnings (i.e., cash flows in our model) are more persistent than they actually are (Barberis, Shleifer, and Vishny (1998)). We model this bias by assuming that agents believe the productivity mean reversion parameter in Equation (6) to be $\lambda_B < \lambda$. The difference $\lambda - \lambda_B$ then measures agents' excessive extrapolation of earnings.[6]

B. Information Processing

We now examine how agents use cash flow and signal realizations to estimate productivity f_t. Agents revise their beliefs about f_t using Bayes' rule, given their possibly incorrect beliefs about the productivity and signal processes—*i.e.*, given their behavioral parameters λ_B and η_B. Let $\hat{f}_t$ denote the conditional estimate of f_t given information available at time t. Then the law of motion of $\hat{f}_t$ is

$$d\hat{f}_t = -\lambda_B\left(\hat{f}_t - \overline{f}\right)dt + \sigma_f \eta_B \underbrace{ds_t}_{\equiv d\bar{\omega}_t^1} + \frac{\gamma_B}{\sigma_a}\underbrace{\frac{da_t - \hat{f}_t dt}{\sigma_a}}_{\equiv d\bar{\omega}_t^2}, \tag{9}$$

where $d\bar{\omega}_t^1 \equiv ds_t$ and $d\bar{\omega}_t^2 \equiv \left(da_t - \hat{f}_t dt\right)/\sigma_a$ are standard Brownian motions, and γ_B is the steady-state variance of the estimation error $\hat{f}_t - f_t$. The variance γ_B is the solution to

[6] In estimating the model, we do not impose the constraints $\theta_B > \theta$ and $\lambda_B < \lambda$, instead allowing for θ_B and λ_B to be higher or lower than θ and λ, respectively. As we show below, however, the empirical estimates indicate overconfidence ($\theta_B > \theta$) and earnings extrapolation ($\lambda_B < \lambda$).

$$\underbrace{\frac{\sigma_f^2}{2\lambda_B}}_{\text{variance of } f} = \underbrace{\frac{1}{2\lambda_B}\left[\left(\sigma_f \eta_B\right)^2 + \left(\frac{\gamma_B}{\sigma_a}\right)^2\right]}_{\text{variance of } \hat{f}} + \underbrace{\gamma_B}_{\text{variance of estimation error}}, \tag{10}$$

which gives

$$\gamma_B = \sigma_a\left(-\lambda_B\sigma_a + \sqrt{\left(\lambda_B\sigma_a\right)^2 + \left(1-\left(\eta_B\right)^2\right)\sigma_f^2}\right). \tag{11}$$

The learning process for a rational agent satisfies equations identical to (9) through (11), except that we replace η_B and λ_B with their rational counterparts, η and λ.[7]

C. The Firm's Optimization Problem

A manager who shares the same beliefs and information as investors determines the firm's investment policy. The manager sets investment to maximize firm value:

$$V(K_t, \hat{f}_t, m_t) = \max_I \; E_t\left(\int_{u=t}^{\infty} e^{-r(u-t)}\left[\hat{f}_u m_u^{1-\alpha} K_u^{\alpha} - I_u - \Psi\left(I_u, K_u\right)\right]du\right), \tag{12}$$

subject to the evolution of capital, productivity, and the state of the economy:

$$\begin{aligned} dK_u &= I_u du - \delta K_u du, \\ d\hat{f}_u &= -\lambda_B\left(\hat{f}_u - \overline{f}\right)du + \sigma_f \eta_B d\overline{\omega}_u^1 + \frac{\gamma_B}{\sigma_a} d\overline{\omega}_u^2, \\ \frac{dm_u}{m_u} &= g du. \end{aligned} \tag{13}$$

Firm value is the expected discounted value of future *net* cash flows—*i.e.*, cash flows

[7] We assume that initial productivity f_0 is normally distributed with mean $\overline{f}$ and variance $\sigma_f^2/(2\lambda)$. There is also an initial signal $s_0 = f_0 + \varepsilon_0$, where ε_0 is normally distributed with a mean of zero and variance of $v_0 = \gamma\sigma_f^2/\left(\sigma_f^2 - 2\gamma\lambda\right)$. To ensure the model starts at the steady state, we choose the initial signal variance such that the initial value of rational productivity estimate $\hat{f}_0^{rational} = \overline{f} + \frac{\sigma_f^2}{\sigma_f^2 + 2\lambda v_0}\left(s_0 - \overline{f}\right)$ has estimation error variance of γ. The biased agents believe that the initial signal variance is $v_0^B = \gamma_B\sigma_f^2/\left(\sigma_f^2 - 2\gamma_B\lambda_B\right)$. Thus, their perceived initial estimation error variance matches their perceived steady-state estimation error variance γ_B.

minus investment and adjustment costs—according to the manager's beliefs and information. The Hamilton-Jacobi-Bellman equation for this maximization is

$$rV(K_t, \widehat{f}_t, m_t) = \max_I \left\{ \begin{array}{l} \widehat{f}_t m_t^{1-\alpha} K_t^{\alpha} - I - \Psi\left(I_t, K_t\right) + \left(I - \delta K_t\right) V_K \\ -\lambda_B \left(\widehat{f}_t - \overline{f}\right) V_f + g m_t V_m + \frac{1}{2}\left(\left(\sigma_f \eta_B\right)^2 + \left(\frac{\gamma_B}{\sigma_a}\right)^2 \right) V_{ff} \end{array} \right\}. \tag{14}$$

Substituting the adjustment cost function from Equation (3) and maximizing the right-hand side with respect to I yields

$$\frac{I_t}{K_t} = \delta + \frac{V_K - 1}{\phi}. \tag{15}$$

Equation (15) is the standard neoclassical investment policy: the firm invests at a rate that exceeds the replacement of depreciated capital if and only if the *marginal q*, V_K, exceeds one. The rate of adjustment of the capital stock is inversely related to adjustment costs φ.

Substituting Equation (15) into Equation (14) and suppressing the t subscripts to simplify notation, we obtain a partial differential equation that characterizes the firm's value V:

$$\begin{aligned} rV = {} & \widehat{f} m^{1-\alpha} K^{\alpha} - \delta K + \frac{1}{2\varphi}\left(V_K - 1\right)^2 K \\ & -\lambda_B \left(\widehat{f} - \overline{f}\right) V_f + g m V_m + \frac{1}{2}\left(\left(\sigma_f \eta_B\right)^2 + \left(\frac{\gamma_B}{\sigma_a}\right)^2 \right) V_{ff}. \end{aligned} \tag{16}$$

We solve for the firm value V in Equation (16) and the resulting investment policy I in Equation (15) using numerical methods described in the Appendix.

II. Empirical Estimation of the Model

A. Identification and Moment Selection

We employ a combination of calibration and estimation to determine the values of the 11 key model parameters. We calibrate three parameters—g, r, and δ—to match the exact values of three empirical moments and estimate the other eight parameters. We use value-weighted moments to calibrate economy-wide parameters and equal-weighted moments to calibrate firm-level parameters. The discount rate (r) is equal to the sum of the annualized averages of the real risk-free rate of interest (1.26%) and the equal-weighted excess return on firm assets (5.91%). We use the yield on one-month nominal Treasury Bills deflated by the Consumer Price Index (CPI) as the basis for the risk-free rate. We set the average growth rate of economy-wide dividends (g) to match the average of value-weighted growth in firm assets (1.40%). The depreciation rate of firm assets (δ) matches the equal-weighted depreciation rate on firm assets (2.96%). We provide detailed definitions of firm asset returns, depreciation rates, and other empirical quantities in Section II.B.

We select 15 additional moments to ensure that they collectively provide sufficient information about the values of the other eight model parameters. Strictly speaking, each of the technology and information processing parameters affects all of our moments, implying that all of our moments are informative about all of the parameters. Ten moments based on firms' production processes and cash flows primarily help us identify firms' six perceived technology parameters: α, ϕ, σ_a, σ_f, θ_B, and λ_B. If all agents were rational, these ten moments would be sufficient for identifying the six parameters above. Five additional moments based on differences in firms' expected asset returns mainly help us identify the magnitude of the two overreaction biases, which arise from the discrepancy between θ and θ_B and between λ and λ_B. Our point

estimates of the eight model parameters above are the values that produce model behavior that most closely matches the 15 cross-sectional moments described below.

We use the averages of firms' Tobin's Q and cash flows to identify the return-to-scale parameter (α). The extent to which alpha is below one increases firms' average cash flows and the value of their growth opportunities. We infer the adjustment cost parameter (ϕ) from the coefficient on Q in a regression of investment on Q and cash flow. High adjustment costs cause firm investment to respond less to growth opportunities. The volatility of firm asset returns, cash flow, investment, and Q jointly provide important information about the volatility of productivity and cash flows in the model (σ_f and σ_a). A key determinant of the perceived informativeness of the signal about productivity (θ_B) is the response of firms' asset returns to their cash flows. The degree of persistence in firms' cash flows is useful for estimating the true and perceived rate of mean reversion in firm productivity (λ and λ_B).

We use differences in firms' average asset returns to identify the differences between the true and perceived productivity and signal processes. Recall that the difference λ and λ_B measures the excessive extrapolation of trends in productivity, while the difference between θ_B and θ captures the magnitude of overconfidence in the precision of the soft productivity signal. An increase in either of these biases produces an increase in the value anomaly. We measure the short-term (long-term) value anomaly using the difference between the one-year (years two and three) average returns of the firms in the top and bottom Q deciles. When forming portfolios based on Q and other accounting variables, we use data from the firms' most recently ended fiscal year, allowing for a three-month reporting lag, and rebalance portfolios monthly.

To distinguish which of the two biases produces the observed value anomaly, we exploit their conflicting predictions about return predictability coming from firms' cash flows. If

investors excessively extrapolate from current cash flow information, they will be too optimistic about firms that have high cash flows, implying such firms will have low future returns. In contrast, if investors are overconfident in their soft productivity signal and place too little weight on cash flow information, they will be too pessimistic about firms with high cash flows, implying such firms will have high future returns. Thus, the extent to which firms with high current cash flows exhibit high future returns reveals the relative importance of the extrapolation and overconfidence biases. To quantify this, we compute the difference between the one-year average returns of firms in the top and bottom deciles of cash flows.

One test of the model is whether it can replicate the observed relationships between managers' real investment decisions and investors' valuations of firms. In some specifications, we use two additional return predictability moments as over-identifying restrictions to test this aspect of the model's performance. These two moments are the difference between the one-year (years two and three) average returns of the firms in the top and bottom investment deciles.

B. Empirical Methodology for Estimating Moments

Our goal in constructing an empirical sample is to provide an appropriate testing ground for the model in Section I. The basis for the sample is the universe of all public US firms from 1968 through 2006. We restrict the sample to firms that have common stocks trading on the NYSE, Nasdaq, or AMEX and those with accounting information.

Because the model considers a firm in its steady state, all sample firms must have at least 60 months of continuous stock price data. The accounting information necessary for the identification of key model parameters includes data on market equity, book equity, book debt, cash, property, plant, and equipment (PP&E), cash flows, investment, sales, and total assets for

each of the last two years. The sample excludes firms with negative values of market equity, book debt, PP&E, sales, and non-cash assets. It also excludes public utilities, regulated firms, and financial firms—those with Standard Industry Classification codes in the ranges 4900-4999, 9000-9999, or 6000-6999—because our model of the capital stock may not apply to these firms. We also exclude firms with stock prices less than $1 and those without trading volume in the most recent month because these firms' stock returns may contain significant measurement error.

Computing the empirical moments requires two types of raw inputs: accounting variables and asset market variables. Each firm's accounting variables include book equity, book debt, total assets, Tobin's Q, investment, cash flows, and depreciation. We define book equity as in Davis, Fama, and French (2000), book debt as in Kaplan and Zingales (1997), total assets using the non-cash asset definition in Cooper, Gulen, and Schill (2008). We do not include cash in firm assets because firms in the model in Section I do not require any cash. Thus, we also subtract cash from the numerator in the Q computation, which is market equity plus book equity minus cash divided by (non-cash) assets. Market equity is shares outstanding times the stock price at the end of the firm's fiscal year. Investment is the annual growth rate of assets. Cash flow is after-tax operating income before depreciation and before deducting research and development expenses, again divided by assets. That is, we treat research and development as an investment, not an expense. Depreciation is the annual change in the difference between gross capital and PP&E, divided by assets.

We compute monthly asset returns for each firm as the weighted average of equity and debt returns, using the firm's capital structure weights. Measuring the equity return is straightforward using standard CRSP data. A lack of firm-specific bond return data presents a challenge for measuring debt returns. For the years 1973 through 2006, we use the monthly

return on the Lehman Brothers investment grade bond index as a proxy for firms' debt returns. For the years 1968 to 1972, when the Lehman index is unavailable, we use the equal-weighted monthly return on US Treasuries with maturities between five and 10 years from the CRSP US Monthly Treasury Database. From 1973 to 2006, a regression of the investment grade returns on the Treasury index return yields a coefficient of 1.00 with a standard error of 0.03 and an R^2 of 83%. The correlation of 0.91 and regression coefficient near 1.0 suggest that movements in risk-free rates between five and 10 years explain the majority of investment grade bond returns.

We estimate each empirical moment using the time series average of cross-sectional estimates following Fama and MacBeth (1973). For the five return predictability moments, such as the one-year predictability from Tobin's Q, we use monthly cross-sectional regressions. The value of Q in each month is based on the values of book debt, book equity, and market equity in the most recently ended fiscal year. We use the same timing for monthly firm cash flows and investment. We compute the averages and standard deviations of variables such as Q, cash flows, investment, and returns using monthly cross-sectional estimates. We winsorize Q, cash flow, and investment at the 5% level to reduce the influence of outliers. For returns, we winsorize at the 5% level only in computing volatility and not for any other return moments.

The point estimate for each regression coefficient is the time series average of the cross-sectional estimates. We use the time series autocorrelation and standard deviation of the coefficients as the basis for computing standard errors. We aggregate the monthly estimates of return predictability moments into annual estimates. For all moments, we model the time series of coefficients as a first-order autoregressive process. When the autoregressive coefficient is significantly different from zero, we use the delta method for computing the moment's standard error. For all other moments, we use Newey-West (1987) standard errors with the Newey-West

(1994) recommendation for choosing the number of yearly lags. We estimate the covariance matrix (W) using the element-wise product of the 15 by 15 correlation matrix of the 15 yearly moments and the outer product of the 15 by 1 standard error vector.

C. Estimation Procedure

The goal of the SMM estimation is to select model parameters that produce simulated model moments that are as close as possible to the empirical moments above. We compute the distance between the model and empirical moments using an SMM criterion function, which assumes a quadratic form. Let m and d be the 15 by 1 vectors of simulated model and empirical data moments, respectively. We select model parameters to minimize the SMM criterion function, $(m - d)^{\mathrm{T}} W^{-1}(m - d)$. This function uses the inverted covariance matrix as weighting for each moment, which is efficient under the conditions given in Cochrane (2000). We use a nonlinear least squares routine to find the parameters that minimize this SMM criterion function.

Section II.B summarizes the empirical computation of d and W. The first step in computing the vector of simulated model moments (m) is to obtain numerical approximations of the firm value and policy functions, using methods described in detail in the Appendix. Next, we apply these numerical approximations over discrete time intervals of 40 periods per year to generate simulated sample paths of model firms. Specifically, we generate simulated data for a cross section of 1,000 firms for 500 years. For each firm, we draw the initial values of true productivity and agents' productivity estimate from their respective steady-state distributions.[8] To allow the capital stock to reach its steady-state distribution, we discard the first 100 years of

[8] For convenience, we assume productivity shocks are independent across firms. Because there are many firms in this economy that exhibit predictable and independent asset returns, a rational arbitrageur could obtain an infinitely high risk-free return. Of course, there are no rational agents in our model. Moreover, introducing productivity shocks that are correlated across firms, even if they are not priced, would eliminate the possibility of risk-free arbitrage and would not affect the qualitative conclusions below.

data. Finally, we use the remaining 400 years of data to compute the model counterparts of empirical data moments and the resulting SMM criterion. Because we have 39 years of empirical data, using 400 years of simulated data corresponds to more than 10 simulated samples.

III. Estimation Results

This section reports our model's ability to match the empirical moments described in Section II.B. Our first estimation relies on 13 moments, excluding the two moments measuring return predictability from firm investment. We use all moments in estimating our 15-moment model.

A. Empirical Moments

The first two columns in Table 1, Panel A show the estimates and standard errors of all 15 empirical moments. The average Tobin's Q of 1.58 and average cash flow of 13.2% suggest that the average firm has diminishing returns to scale and growth opportunities with significant value. The cross-sectional standard deviation of Q of 0.937 may seem high relative to average Q. More than 5% of firms have Q values exceeding 4.38, which increases the dispersion in Q. The persistence of cash flow is 0.790, which is considerably higher than we would obtain if we used fixed effects. The investment sensitivities to Q and cash flow of 0.067 and 0.660 are similar to the estimates in other studies. The Q coefficient is slightly higher than the values reported in Kaplan and Zingales (1997) because we use a univariate specification to measure this coefficient.

[Insert Table 1 here.]

The last five rows in Panel A of Table 1 report the difference in asset returns between the top and bottom decile portfolios sorted on cash flow, Q, and investment at different time horizons. The significantly negative Q and investment anomalies are consistent with the well-

known value, investment, and asset growth effects in equity returns found in Fama and French (1992), Titman, Wei, and Xie (2004), and Cooper, Gulen, and Schill (2008), respectively. The annualized return differences from sorts on Q and investment are -6.38% and -6.71%, respectively, one year after portfolio formation. These anomalies decline somewhat to annualized averages of -4.56% and -2.42% in years two and three after portfolio formation. The positive but statistically insignificant one-year return predictability of 1.39% from cash flow shows that post-earnings announcement drift is not large in our sample. Nevertheless, this coefficient is informative about the relative importance of the two information processing biases, as discussed in Section II.A. Some anomaly estimates appear slightly smaller than standard estimates because we measure anomalies using firms' asset returns, not just their equity returns.

Table 1 presents the results from minimizing of the SMM criterion function in the 13-moment estimation and the 15-moment estimation that includes two investment anomaly moments. The columns in Panel B show the parameter estimates that produce the best fit to the data, along with their standard errors. The third and fourth columns in Panel A compare empirical moments to the model's predictions. The last two rows in these columns summarize how the two models fit the data, using χ^2 statistics with degrees of freedom of five and seven. The $\chi^2(5)$ value of 8.88 for the 13-moment model shows that most of the simulated model moments match the 13 empirical moments quite closely. Formally, the p-value of 0.114 implies that we cannot reject the hypothesis that the five over-identifying restrictions are valid. The $\chi^2(7)$ value of 16.27 and p-value of 0.023 for the 15-moment model suggests a slightly worse fit, implying that the we can reject the over-identifying restrictions at the 5% level. In both estimations, the standard errors on all eight parameters in Panel B are small relative to the parameter values, indicating that there is enough statistical power to identify each parameter.

Figure 1 graphically summarizes the fit of the model in the 13-moment and 15-moment estimations. The height of each vertical bar represents the deviation of the model's predicted moment from each empirical moment, as a percentage of the empirical moment. In general, the model matches the technology moments more closely because the estimates of the return predictability moments are less precise. Visually, this implies that the ten leftmost bars in Figure 1 are smaller in magnitude than the five rightmost bars.

[Insert Figure 1 here.]

The model is able to match most moments quite well in both the 13-moment and 15-moment estimations. For the ten technology moments across both estimations, excluding the standard deviation of investment, the largest percentage model error is 16%. The two prediction errors for the volatility of firm investment are -42% and -43%, meaning that the models predict too little investment volatility. One possible reason is that we do not allow for any fixed adjustment costs, which could produce lumpy and hence more volatile firm investment.

The 13-moment and the 15-moment models both qualitatively match the empirical return anomalies well, but there are economically significant quantitative differences in some cases. These model errors are generally much less significant in a statistical sense because the standard errors on return anomalies tend to be large relative to the standard errors of technology moments. As a result, the SMM procedure sacrifices the model's fit to return anomalies in exchange for a better fit to the technology moments. The 13-moment model predicts a one-year value anomaly is a close match to the data and a one-year investment anomaly that is too large by 61%.

By design, the 15-moment estimation matches the investment anomaly better, which is the main goal of this estimation. However, the 15-moment model fails to reproduce most of the empirical value anomaly, so we view this model as a conservative estimate of the importance of

mispricing. Although neither estimation matches the cash flow anomaly well in percentage terms, both model errors are statistically insignificant at even the 10% level because the empirical anomaly magnitude is small relative to its standard error.

The parameter estimates of technology moments reported in Panel B of Table 1 are reasonable given the empirical data. The estimates of returns to scale (α) of 0.849 and 0.852 in the two models are both significantly below one. The adjustment cost parameters (ϕ) of 2.90 and 2.88 are somewhat smaller than the estimates in Gomes, Yaron, and Zhang (2006), who use aggregated investment data which tends to be more smooth. The two estimates of mean reversion in productivity (λ) of 0.230 and 0.163 are low because cash flow is quite persistent. The third and fourth columns in Panel A of Table 1 show that both models match this feature of the data well.

Several of the information processing parameter estimates in Panel B are notable. The perceived quality of the soft information signal (θ_B) is higher at 0.936 in the 13-moment model than its value of 0.810 in the 15-moment model. These high precisions of the soft signal are necessary for the models to match the weak response of market returns to cash flows, which both models do quite well. The actual quality of the soft signal (θ) is 0.451, which is statistically and economically significantly lower than the perceived precisions in both models. This implies that agents overreact to the soft information signal, especially in the 13-moment estimation. The model endogenously generates this result in order to match the negative return predictability from Tobin's Q, which it does well in the 13-moment model. Because there is less overconfidence in the 15-moment model, this model predicts a smaller investment anomaly. This occurs at the expense of matching the value anomaly.

Another way for the model to produce negative return predictability from Q is to set the perceived mean reversion in productivity (λ_B) to a value lower than the actual mean reversion (λ).

Indeed, the parameter estimates in both models satisfy this excessive extrapolation condition. The overconfidence bias is strong relative to the extrapolation bias in both models in order to match the positive empirical estimate of post-earnings announcement drift. In fact, both models produce slightly too much return predictability from cash flow, though the prediction error is not statistically significant. The extrapolation bias in the 15-moment model is smaller than the 13-moment model's bias in order to more closely match the investment anomaly. Overall, though, most of the parameter estimates in the two models are quite similar.

Next we gauge the combined economic magnitude and plausibility of the two biases. The last three rows in Panel B of Table 1 compare the volatility of the error in estimating productivity for a rational agent and for a biased agent, along with the standard deviation of true productivity. In the 13-moment (15-moment) model, the biased agent's perceived estimation error is 78% (49%) lower than a rational agent's perceived error, suggesting the biases are large. However, the difference in the two perceived volatilities of less than 0.1961 is only 26% of the magnitude of the volatility in true productivity, suggesting such biases may be difficult to detect and correct.

IV. Analysis of the Simulated Economy

This section analyzes the properties of simulated model economies that mimic real-world firm behavior and asset return predictability, using parameters such as those in Table 1. Using simulated data allows us to explicitly measure many features of the economy that econometricians cannot typically observe. Our discussion focuses on the 13-moment estimation because it produces a simulated value anomaly that is closer to the empirical value anomaly. We view this anomaly as critical for identification, whereas the investment data and thus the investment anomaly may be noisy, as argued earlier.

We divide the analysis into four subsections. First, we analyze firms' expected returns and asset mispricing from the perspective of a rational investor. This investor evaluates firms' expected cash flows, knowing that managers with information processing biases select firm investment and that biased investors determine asset prices. Second, to explore how return predictability depends on managerial behavior, we simulate an economy in which rational managers choose firm investment but biased investors still determine asset prices. Third, we measure investment inefficiencies by contrasting economic outcomes, such as firm value and capital allocation, when biased and rational managers choose firm investment. Fourth, we distinguish the individual impacts of the two information processing biases, overconfidence and excessive extrapolation, on capital misallocation and mispricing.

A. Analysis of Mispricing with Biased Investors and Managers

Empirical asset pricing research often uses the predictability in average *realized* returns as a proxy for mispricing. Instead, we use two direct measures with natural economic interpretations that are unavailable to empiricists. First, we analyze the predictability in firms' *true* expected returns in excess of the risk-free rate.[9] Second, we compare the level of firms' actual asset prices to the *true* value of the firm. The firm value from the perspective of a rational agent is the present value of the firm's cash flows discounted at the appropriate rate (r).[10]

[Insert Table 2 here.]

[9] We estimate firms' expected returns for up to 10 years from the perspective of a rational agent by computing the firms' average returns, including both price changes and dividends, based on 5,000 simulated sample paths of the economy. Although there is no aggregate risk in our model, we use abnormal returns in these computations to facilitate interpretation. We obtain a firm's abnormal return from its raw return by subtracting the risk-free rate.
[10] True firm value is a solution to a partial differential equation analogous to Equation (16), except that it includes an extra state variable representing the rational estimate of productivity. We solve this equation numerically using the same method described in the Appendix.

Panel A in Table 2 reports summary statistics of true one-year expected returns and mispricing. We define mispricing as the firm's value under rational expectations divided by its market value, so that a ratio below one implies price exceeds value. This definition makes the market efficiency notion proposed in Black (1986) directly observable: "we might define an efficient market as one in which price is within a factor of 2 of value" (p. 533). In his view, efficiently priced firms are ones with mispricing values between 0.5 and 2.0. The summary statistics in Panel A reveal that the standard deviations of one-year expected returns and mispricing are both higher in the 13-moment estimation (5.8% and 17.3%) than in the 15-moment estimation (4.8% and 11.2%), implying less efficient pricing in the 13-moment model. In both models, average value-weighted returns are lower than equal-weighted returns, indicating that larger firms have lower expected returns, as explained below.

Panel A shows that the equal-weighted average mispricing (*i.e.*, value-to-price) ratio is 0.7444 in the 13-moment model, meaning that the average firm is substantially overpriced. This occurs because investors and managers overestimate the value of firms' growth options. Both information processing biases cause agents to think their information about future productivity is more precise than it actually is. Consequently, they expect firms to make better-informed investment decisions than they actually will, which leads agents to overprice firms' assets.

Panels B and C in Table 2 show the distribution of true one-year expected returns and mispricing for portfolios sorted on firm characteristics. The first column in Panel B shows that, in simulations based on the 13-moment model, there is large variation in expected returns: the bottom decile of firms have expected returns of 20.4% less than the top decile. The distribution is smooth and positively skewed, much like the empirical data on firms' realized returns. The second column in Panel B shows a similar distribution for the 15-moment model, which exhibits

a decile spread of 16.1%. Equally important, columns three to eight show that the *majority* of the differences in portfolios' expected returns remain unexplained by Q, investment, and cash flow in both models. The investment anomaly in the 13-moment model is the most able to explain differences in expected returns, but this occurs because the investment anomaly is counterfactually large in this model.

Panel C shows that market prices deviate widely from firm values. The top-to-bottom decile spread in mispricing is 60.7% in the 13-moment estimation, which matches empirical return anomalies, and it is 39.3% even in the 15-moment estimation, which predicts a value anomaly that is too small. The 13-moment estimates show that the average firm in the bottom mispricing decile is inefficiently overpriced even by Black's (1986) loose criterion. Interestingly, in the 13-moment model, Q does a better job in explaining mispricing than it does in explaining expected returns. The reason is that Q reflects a long-term component of mispricing that is not eliminated in the next year.

[Insert Table 3 here.]

Table 3 display estimates from cross-sectional regressions predicting one-year expected returns in the 13-moment and 15-moment simulations in Panels A and B, respectively. The regressors include five observable firm characteristics: Q, investment, market capitalization (size), cash flow, and one-year asset returns. The bottom rows in the two panels report the R^2 statistics from the regressions.

In the simulated data from both models, the return predictability coefficients on firm size in the univariate Regression (4) and the multivariate Regression (8) are negative. Although none of the estimations include moments relating to return predictability from firm size, the model predicts a negative size premium, which is consistent with the empirical findings in Banz (1981)

and Fama and French (1992). In our models, size is a noisy proxy for investor beliefs, which are excessively volatile and exhibit mean reversion, leading to a negative size premium. The negative return predictability coefficients on Q and investment are not surprising and are consistent with the values reported in Tables 1. The positive coefficient on cash flow is also consistent with these earlier results.

The contemporaneous coefficient on past one-year returns in predicting one-year true expected returns is close to zero in Panels A and B in Table 3, showing that the models do not predict positive return momentum of the sort identified empirically in Jegadeesh and Titman (1993). This is interesting because there is "earnings momentum" in the model, meaning that cash flow positively predicts returns, because overconfident agents underreact to cash flow signals. This effect alone would seem to produce return momentum, too, because cash flow is positively associated with returns. There is, however, a significant countervailing force that arises from the correlation between innovations in the soft information signal and returns. Agents overreact to the soft signal, thereby leading to mean reversion in returns. The two effects roughly offset at the empirical parameter estimates in these specifications. Naturally, including moments that measure return momentum in the estimations could produce model parameters that would lead to positive momentum.

The bottom rows in both panels in Table 3 reveal that the ability of each of the individual firm characteristics to explain true expected returns is surprisingly low. The R^2s from the univariate specifications in Panel A range from 0.0023 to 0.2313. The main reason for the low R^2s is that the econometrician cannot directly measure the soft information signal to which investors overreact. This is why using data from the simulated economy is necessary to measure the true extent of mispricing. Regression (8) shows that the joint explanatory power of the five

firm characteristics is much greater than the sum of their univariate explanatory power. Mechanically, this occurs because cash flow and investment are strongly negatively correlated and predict returns with opposite signs. Intuitively, only the joint specification is able to separately identify the two distinct information processing biases, which have different implications for post-earnings announcement drift but similar implications for the investment anomaly. The success of this joint model in our simulations may have an empirical counterpart: Chen, Novy-Marx, and Zhang (2010) argue that an asset pricing model with factors representing firm investment, cash flow, and market risk outperforms the widely used Fama-French (1993) three-factor model.

In Figures 2 and 3, we compare firms' true expected returns to expected returns conditional on two observable variables and one unobservable variable at horizons up to 10 years. We use Tobin's Q and investment rates as the two observable variables and mispricing as the unobservable variable. The eight lines in each figure represent expected returns for firms ranked in the top and bottom deciles according to their true three-month expected returns, mispricing, Q values, and investment rates.[11] The top and bottom lines depict sorts on true expected returns. The next two outermost lines represent sorts on mispricing, while the four innermost lines show sorts on investment and Q. Because Q and investment negatively predict returns, firms in the top deciles of these two observable characteristics are in the bottom deciles of expected returns. Figures 2 and 3 are identical except that Figure 2 uses parameter estimates from the 13-moment model, whereas Figure 3 uses estimates from the 15-moment model.

[Insert Figures 2 and 3 here.]

The main result in both figures is that the sorts based on true three-month expected returns produce large differences in future returns in absolute terms and in comparison to the two

[11] We use three months as a realistic short investment horizon.

sorts based on the observable variables, Q and investment. In the 13-moment shown in Figure 2, the difference between firms' one-year expected returns is 20.4% using sorts based on three-month expected returns, whereas the difference is only 6.9% (10.7%) using sorts based on Q (investment). In this sense, the observable return anomalies vastly understate the true differences in expected returns. The 15-moment estimates in Figure 3 are qualitatively similar. The two main differences in Figure 3 are its reduced scale and the fact that observable characteristics do an even worse job of explaining true expected returns. Remarkably, in both figures, differences in firms' expected returns of more than 1% per year persist beyond the 10-year investment horizon. Clearly, standard empirical analyses face serious practical and statistical challenges in detecting return predictability at such long horizons.

There are two more noteworthy features of Figures 2 and 3. First, using the two observable variables, one can forecast a significant amount of positive return predictability but very little negative return predictability. Even the firms ranked in the top Q decile, as represented by the fifth line from the top in both figures, have expected abnormal returns of only -1%. In contrast, the firms with the lowest three-month expected returns have one-year expected returns of less than -8%. Firms ranked in the bottom Q decile exhibit substantially positive returns of more than 6% per year, which is closer to the 12% expected return for firms ranked in the top decile of three-month expected returns. This asymmetry in observable return predictability applies to the investment anomaly, too.

Second, sorting on true mispricing does not predict future returns as well as sorting on true expected returns. This occurs because mispricing in some firms disappears more rapidly than in other firms. The expected rate of convergence to fundamentals depends on past realizations of the soft signal and cash flows.

B. Analysis of Mispricing with Rational Managers and Biased Investors

This subsection investigates whether a model with rational managers and biased investors has the potential to match the empirical data. We assume that a rational manager selects investment to maximize true firm value, rather than investors' expectations of firm value. A rational manager uses the true mean reversion rate of productivity and the true signal quality to estimate future productivity. As a result, this estimate differs from investors' estimates. The manager considers only her productivity estimate when determining the firm's investment policy according to Equation (15). Biased investors are aware that the manager is pursuing a different investment policy from their ideal policy and think that this reduces firm value. The differing beliefs of investors and managers imply that the firm value function in Equation (16) depends on an additional state variable that represents the rational managers' beliefs. We solve for the value function using the polynomial approximation described in the Appendix.[12]

We analyze the steady state properties of the economy with the rational manager in Panels A and B in Tables 4. Panel A compares the simulated empirical moments for the two rational manager economies to the simulated moments in otherwise identical economies where the managers exhibit biases. The third and sixth columns in Panel A compute the percentage changes in each steady state moment in moving from the irrational manager economy to the rational manager economy.

[Insert Table 4 here.]

The most interesting result in Panel A is that the one-year value anomaly increases by more than 50% (102% or 57%) while the one-year investment anomaly decreases by more than 50% (97% or 63%) in the economy with rational managers. The value anomaly arises when

[12] The approximation now includes a third state variable with an polynomial approximation degree of 10.

investors are too optimistic about productivity, leading to a high Q value and low future returns. When a rational manager runs a firm with excessively optimistic investors, she does not exercise as many growth options as investors would like. Thus, an even greater proportion of the firm's asset price comes from overpriced growth opportunities when the manager is rational. As a result, the rationally managed firm's future return is even more negative, as realized productivity disappoints the biased investors.

The investment anomaly, however, arises when high managerial investment coincides with excessive investor optimism. These two events coincide the most in an economy where managers and investors share the same optimism. This general intuition and our simulations all suggest that the rational manager model produces an investment anomaly that is much smaller than the value anomaly. Empirically, we observe value and investment anomalies of comparable size. Accordingly, aside from the analysis in Table 4, we focus on the specifications where both managers and investors have the same beliefs, which seem to fit the return predictability data.

The bottom four rows in Panel B in Table 4 provide further insights into mispricing in the rational manager and biased manager economies. The equal-weighted mean of mispricing is much closer to one in the economy where the rational manager runs the firm—*e.g.*, 0.875 vs. 0.753 in the 13-moment model. This reduction in overpricing occurs because biased investors think that the rational manager chooses a suboptimal investment policy, whereas a biased manager selects the optimal policy. For the same reason, changes in the degree of disagreement between the rational manager and biased investors produce volatility in mispricing. This is why the volatility of mispricing is higher in the rational manager economy—*e.g.*, 0.246 vs. 0.173.

C. Analysis of Investment Inefficiencies

We now examine which firms make value-reducing investment decisions in our model. We quantify the magnitude of the investment inefficiencies using several measures that are unavailable to empirical researchers. The basis for these measures is the comparison of firm values, capital stocks, and investment rates in two situations: when a manager with information processing biases selects investment and when a rational manager with no biases selects investment. From the earlier simulations, we know firm values, capital stock, and investment rates in the biased economy. We simulate and compute these quantities in the hypothetical rational economy by setting both information processing biases equal to zero, while holding all other model parameters constant. That is, we set the perceived mean reversion in productivity (λ_B) equal to the actual mean reversion (λ) and the perceived quality of the productivity signal (θ_B) equal to the actual quality of the signal (θ). We refer to the economic variables, such as investment rates, from these two sets of simulations as "biased" and "rational" variables.

Table 5 reports several comparisons between the biased and rational economies in our 13-moment and 15-moment estimations. To compare firm value in the two economies, we adopt the perspective of a rational agent who properly computes the expected discounted value of future cash flows. We define value added as the difference between the firm's value (V) and the cost of its capital stock (K). The firm value per unit of capital is V/K. This ratio corresponds to average Q in the rational economy, but it differs from average Q in the biased economy because market prices differ from rational firm value.

[Insert Table 5 here.]

Table 5 demonstrates that behavioral biases reduce the value added of capital by a large amount. In the 13-moment estimation, the aggregate value added in the biased economy is only

35.5% as large as value added in the rational economy, as shown in the top row in Table 6. In the 15-moment estimation in which behavioral biases are smaller, the aggregate value added in the biased economy is larger at 92.0% of the rational economy's value added.

In both estimations, the value per unit of capital (V/K) is significantly lower and the capital stock is too large in the biased economy. The intuition is that biased managers perceive future investment opportunities to be greater than they are, so they prefer to hold a larger capital stock to minimize future adjustment costs. This logic relies on a reasonable assumption about the production function. With a Cobb-Douglas production function and a multiplicative productivity shock, the impact of a positive productivity shock is greater than the impact of an equal-sized negative shock. Because growth options have more upside than downside, biased managers accumulate too much capital in anticipation of growth. Because a greater proportion of a small firm's value consists of growth opportunities, which are the source of agents' biases, biases reduce the value of small firms by a larger percentage.

Next we compare the marginal investment rates chosen by a biased versus a rational manager. To facilitate this comparison, we analyze the investment choices of both managers immediately after they assume control of a firm whose capital stock has been set by a biased manager. This comparison differs somewhat from the steady state comparisons above because adjustment costs prevent a rational manager from instantly changing the capital stock and reaching a new steady state.

Table 6 reports univariate statistics and regressions designed to explain the difference between biased and rational investment rates. The positive values in the top row of Panel A in Table 6 show that rational managers would invest much less than biased managers, on average, which is consistent earlier findings. In the 13-moment model, the equal-weighted average of

overinvestment is 0.138%. The standard deviation of overinvestment is large at 6.92% relative to the mean, implying that many firms do not invest enough in any given time period. Both overinvestment and underinvestment are common because managers react too much to the soft information signal, which could be higher or lower than true firm productivity.

[Insert Table 6 here.]

Panels B and C in Table 6 display the results from regressions of overinvestment on Q, investment, size, cash flow, and one-year returns. By far, the best predictor of overinvestment is a firm's observable investment rate. In the 13-moment (15-moment) model, a firm's investment rate accounts for 44% (8%) of the variation in overinvestment. The intuition is that managerial overreaction to information, from overconfidence and excessive extrapolation, makes investment decisions too volatile. As a result, firms that invest a lot tend to invest too much. This is why the coefficient on investment is highly positive in Regression (2) predicting overinvestment shown in Table 6. Similarly, biased managers of firms with high Q values and valuable growth opportunities overreact to the positive information about productivity and invest too much relative to a rational manager. This explains the positive coefficient on Q in Regression (1).

D. Distinguishing the Impact of Overconfidence and Extrapolation

This subsection analyzes the separate impact of the two information processing biases, overconfidence and excessive extrapolation, for capital misallocation and mispricing. Using the parameter estimates from the 13-moment model in Table 1 as a starting point, we consider two parameter changes.[13] First, we eliminate overconfidence in all agents by setting perceived signal quality equal to actual signal quality ($\theta_B = \theta = 0.458$), while retaining all other parameter values,

[13] In unreported analyses, we consider variations in parameters starting from the parameter estimates in the 15-moment model in Table 1 and obtain qualitatively similar findings.

including the excessive extrapolation parameters ($\lambda_B = 0.143$ and $\lambda = 0.230$). Second, we eliminate excessive extrapolation in all agents by setting $\lambda_B = \lambda = 0.230$, while retaining all other parameter values, including overconfidence parameters ($\theta = 0.936$ and $\theta_B = 0.458$). We simulate the behavior of economies with these two sets of parameter estimates to isolate the impact of the two individual biases.

Table 7 reports the moments from Simulation 1 in which agents excessively extrapolate, Simulation 2 in which agents are overconfident, and a fully rational economy without both biases. For comparison purposes, the last two columns in Table 7 redisplay the moments in the 13-moment model with both biases and the empirical moments. One notable aspect of Table 7 is that the many of the real moments are quite similar in the biased and rational economies. As shown below, this fact does *not* imply that biases have no real effects, but it does mean that most real moments play a small role in identifying biases. The exception to this is Tobin's Q, which differs significantly in the biased and rational economies because Q depends on asset prices.

[Insert Table 7 here.]

Tables 8 and 9 report the mispricing and capital misallocation properties of both one-bias simulations. Table 8 shows the equal-weighted mispricing (rational value / market price) and expected returns for portfolios consisting of firms sorted into deciles using firm characteristics. All sorting characteristics, such as Q and cash flow, are observable, except true mispricing. Our analysis focuses on top-to-bottom decile spreads in mispricing and expected returns.

[Insert Table 8 here.]

One striking fact in Panel A in Table 8 is that Simulation 1 with just excessive extrapolation produces a spread in true mispricing of 25.3% and one observable variable, Tobin's Q, captures virtually all of this spread (25.2%). In contrast, Simulation 2 with just

overconfident agents produces a much larger spread in true mispricing (50.8%) and no observable variable, such as Tobin's Q or investment, captures even half of the spread in mispricing. The reason is that overconfident agents' errors in expectations depend on their soft information signal, which is not easily observable, whereas one can partly infer the extrapolative agents' mistaken beliefs based on Tobin's Q, which summarizes past productivity trends.

Panel B in Table 8 shows that cash flows negatively predict returns with extrapolative investors in Simulation 1 and positively predict returns with overconfident investors in Simulation 2. In the 13-moment model in Table 1, these two effects roughly offset, producing a small positive cash flow anomaly. An overconfident investor ignores public cash flow signals in favor of his soft information signal, whereas an extrapolative investor puts too much weight on past trends in cash flow, thinking that such trends will continue.

[Insert Table 9 here.]

Table 9 summarizes the impact of each bias on real investment. The bottom row shows that the overconfidence parameter values in Simulation 2 produce more volatile biases in investment rates at 5.1% than the excessive extrapolation parameter values in Simulation 1 (2.7%). The unobserved volatility in the soft information signal must be large to produce the observable value anomaly in the data because the noise in the signal is only partially correlated with Tobin's Q. The extrapolation bias does not need to be as large because past productivity trends are more directly related to Q.

Table 9 also reveals that extrapolative managers exhibit higher average investment rates than rational managers by 1.5%. As noted earlier, an extrapolative manager puts too much value on the firm's future growth opportunities, believing that she can act on productivity shocks before they dissipate. Thus, she increases investment today to lower expected adjustment costs.

In contrast, there is no average bias in an overconfident manager's investment rate. The aggregate value added in the economy with only overconfidence (extrapolation) is 81.9% (90.7%) of the rational economy's value added. Overall, both biases contribute significantly to capital misallocation and mispricing, but overconfidence usually plays a larger role.

V. Concluding Discussion

Our study is one of the first attempts to quantify the asset pricing and investment distortions implied by behavioral theories of information processing biases. This paper proposes one framework for answering these questions. One of our main findings is that the observed magnitude of asset return anomalies understates inefficiencies in firms' asset prices and investment decisions. The reason is that econometricians cannot accurately measure investors' information sets. Summers (1986) and Hansen and Richard (1987) argue that this makes the efficient markets hypothesis difficult or impossible to test. Conversely, any observed evidence of asset return anomalies could represent the tip of an iceberg of true mispricing. A dynamic structural model allows us to quantify the size of this iceberg. The model and analysis shows that the overconfidence bias causes unobserved mispricing to be larger than observable return anomalies. The mechanism is that overconfident investors have incorrect beliefs that depend on an unobservable signal, making it difficult to uncover the true extent of mispricing. Other information processing biases, such as excessive extrapolation, do not share this feature.

We also find that a model with rational managers and irrational investors is difficult to reconcile with the return predictability patterns in the data. Specifically, the empirical investment anomaly is simply too large. This suggests that models in which exhibit information processing biases, such as our benchmark specification in which managers' and investors' biases are

identical, are promising to explore. Future work could test whether more nuanced versions of managerial biases fit the empirical data significantly better than our benchmark model.

Our model generates return predictability from behavioral biases alone, whereas in fact some return predictability may come from risk premiums. This paper does not test the relative merits of these explanations. Rather, our goal is to understand the implications of the behavioral explanation. Future models could combine both return predictability mechanisms. Although many of the qualitative insights here would apply to such models, there could be novel and unanticipated interactions between behavioral and risk-based channels.

One need not interpret the parameter estimates from our model as fixed and constant over time. It is possible or even likely that the asset return predictability patterns in the next 40 years will be quite different from predictability patterns observed in the past 40 years. As the economy's latent structural parameters change, behavioral biases may dissipate or be replaced by new biases. Furthermore, rational agents may learn only gradually about such changes, generating return patterns that are predictable only in hindsight. Under these alternative interpretations, our model still provides a useful ex-post description of the underlying mispricing and investment inefficiencies that occurred over the past 40 years. This description of the past may or may not apply to the future. Thus, our main contribution is not the specific parameter estimates but the modeling framework and the general insights it provides.

Appendix: Numerical Solution of the Model

In this Appendix, we solve for the firm's value function V. The partial differential equation in Equation (16) characterizes firm value as a function of three state variables: capital stock K_t, productivity estimate $\widehat{f}_t$, and state of the economy m_t. Now we introduce a variable transformation that reduces the number of state variables in Equation (16) to two. We define

$$X \equiv m^{-1}K, \tag{A1}$$

so that Equation (16) simplifies to

$$\begin{aligned}(r-g)J = \widehat{f}X^{\alpha} - \delta X + \frac{1}{2\varphi}\left(J_X - 1\right)^2 X - gXJ_X \\ -\lambda_B\left(\widehat{f} - \overline{f}\right)J_f + \frac{1}{2}\left(\left(\sigma_f \eta_B\right)^2 + \left(\frac{\gamma_B}{\sigma_a}\right)^2\right)J_{ff},\end{aligned} \tag{A2}$$

where

$$J\left(X, \widehat{f}\right) = m^{-1}V\left(K, \widehat{f}, m\right). \tag{A3}$$

Thus, one can solve for the function J in Equation (A2) and then use Equation (A3) to obtain the value function V. By differentiating both sides of Equation (A3) with respect to K, one obtains

$$V_K = J_X. \tag{A4}$$

Equation (A4) states that marginal q, V_K, equals the derivative of J with respect to X.

We use a dual numerical approximation strategy to solve for functions J and J_X. First, we approximate J_X. Differentiating both sides of Equation (A2) with respect to X, we obtain

$$rJ_X = \alpha \hat{f} X^{\alpha-1} - \delta + \frac{1}{2\varphi}\left(J_X - 1\right)^2 + \left[\frac{J_X - 1}{\varphi} - g\right] X J_{XX}$$
$$-\lambda_B\left(\hat{f} - \overline{f}\right) J_{fX} + \frac{1}{2}\left(\left(\sigma_f \eta_B\right)^2 + \left(\frac{\gamma_B}{\sigma_a}\right)^2\right) J_{ffX}. \tag{A5}$$

We solve for J_X in Equation (A5) using a Chebyshev polynomial approximation. We provide only a brief overview of this procedure—see Judd (1998) for details. Chebyshev polynomials are a family of orthogonal functions that constitute a basis for the space of continuous functions.

We approximate J_X as a sum of products of Chebyshev polynomials:

$$J_X\left(X, \hat{f}\right) = \sum_{m=0}^{M}\sum_{n=0}^{N} a_{m,n} T_m\left(2\frac{X - X_L}{X_H - X_L} - 1\right) T_n\left(2\frac{\hat{f} - f_L}{f_H - f_L} - 1\right). \tag{A6}$$

In Equation (A6), M and N are integers that specify approximation degrees, $\{a_{m,n}\}$ is a set of polynomial coefficients, and $\{T_m\}$ and $\{T_n\}$ are Chebyshev polynomials.[14] The approximation applies over intervals $X \in [X_L, X_H]$ and $\hat{f} \in [f_L, f_H]$, which we choose such that the realized state variables X and $\hat{f}$ attain values outside the bounds with less than 1% probability. To solve for the unknown polynomial coefficients $\{a_{m,n}\}$, we write Equation (A5) in terms of the Chebyshev approximation and its partial derivatives. Then we minimize the sum of squared differences between the right- and left-hand sides of the equation over a grid of $\left(X, \hat{f}\right)$ points using a nonlinear least squares routine.

Next, we use a similar polynomial approximation for the function J:

$$J\left(X, \hat{f}\right) = \sum_{m=0}^{M}\sum_{n=0}^{N} b_{m,n} T_m\left(2\frac{X - X_L}{X_H - X_L} - 1\right) T_n\left(2\frac{\hat{f} - f_L}{f_H - f_L} - 1\right). \tag{A7}$$

[14] We use approximation degrees $M = 80$ and $N = 10$.

Equation (A7) differs from Equation (A6) only in terms of the polynomial coefficients $\{b_{m,n}\}$. The partial differential equation that characterizes J in Equation (A2) is linear in J and its partial derivatives except for the term that includes $(J_X - 1)^2$. Because we already know the numerical approximation for J_X, we can use this function to compute $(J_X - 1)^2$. Finally, using the computed value of J_X and the Chebyshev approximations for J and its partial derivatives in Equation (A7), we express Equation (A2) as a linear system of equations in coefficients $\{b_{m,n}\}$, which we can easily solve by matrix inversion.

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Table 1: The Fit of the Benchmark Model to the Data

Panel A: Comparing Empirical Moments and Simulated Moments in Two Estimations

Moments	Empirical Estimate	Standard Error	13-Moment Model	15-Moment Model
Mean Tobin's Q	1.5801	0.3696	1.6818	1.6317
Mean Cash Flow	0.1317	0.0333	0.1129	0.1102
Std Dev of Q	0.9370	0.2987	0.9166	0.9649
Std Dev of Abnormal Return	0.2108	0.0252	0.2029	0.1997
Std Dev of Cash Flow	0.0940	0.0206	0.0879	0.1040
Std Dev of Investment	0.1686	0.0327	0.0976	0.0968
Persistence of Cash Flow	0.7901	0.0170	0.8018	0.8066
Abn Ret Sensitivity to Cash Flow	1.7569	0.1485	1.7742	1.7593
Investment Sensitivity to Q	0.0666	0.0213	0.0690	0.0635
Investment Sensitivity to Cash Flow	0.6603	0.0544	0.6409	0.6551
Cash Flow – Year 1	0.0139	0.0143	0.0176	0.0408
Tobin's Q – Year 1	-0.0638	0.0235	-0.0663	-0.0265
Tobin's Q – Years 2 and 3	-0.0456	0.0205	-0.0576	-0.0169
Investment – Year 1 *	-0.0671	0.0139	-0.1077	-0.0353
Investment – Years 2 and 3 *	-0.0242	0.0040	-0.0880	-0.0225
χ^2 *statistic*			8.88	16.27
χ^2 *p-value*			(0.114)	(0.023)

* Not included in estimation in the 13-moment model

Table 1: The Fit of the Benchmark Model to the Data (Continued)

Panel B: Two Sets of Parameter Estimates in the Benchmark Model

	13-Moment Estimation		15-Moment Estimation	
Parameters	Estimate	Standard Error	Estimate	Standard Error
Returns-to-Scale α	0.8493	0.0244	0.8515	0.0088
Adjustment Cost φ	2.8968	0.3998	2.8839	0.4119
Productivity Shock σ_f	0.5200	0.0581	0.5180	0.1106
Noise in Earnings σ_a	0.1780	0.0491	0.2667	0.0349
Mean Reversion λ	0.2302	0.0393	0.1629	0.0309
Biased Mean Reversion λ_B	0.1434	0.0235	0.1508	0.0245
Signal Quality θ	0.4576	0.0672	0.4908	0.0512
Biased Quality θ_B	0.9364	0.0629	0.8095	0.0394
Std Dev of Estimation Error, Rational	0.2527		0.2976	
Std Dev of Estimation Error, Biased	0.0566		0.1507	
Std Dev of Productivity	0.7664		0.9076	

Table 2: The Cross Section of Expected Returns and Mispricing

Panel A: Summary Statistics for Expected Returns and Mispricing

Estimation	Variable	Value-Weighted Mean	Equal-Weighted Mean	Median	Std Dev
13-Moment	One-Year Expected Returns	-0.0133	0.0067	0.0006	0.0582
15-Moment	One-Year Expected Returns	-0.0027	0.0029	0.0015	0.0484
13-Moment	Mispricing	0.7444	0.7528	0.7321	0.1731
15-Moment	Mispricing	0.9553	0.9227	0.9207	0.1120

Panel B: Comparing True and Observed One-Year Expected Returns

Sorting Variable	1-Yr Expected Return		Q		Investment		Cash Flow	
Moments in Model	13	15	13	15	13	15	13	15
Lowest Decile	-0.0811	-0.0789	0.0611	0.0244	0.0776	0.0259	-0.0056	-0.0199
2	-0.0498	-0.0458	0.0228	0.0109	0.0368	0.0136	0.0024	-0.0090
3	-0.0331	-0.0297	0.0121	0.0054	0.0207	0.0083	0.0034	-0.0042
4	-0.0189	-0.0165	0.0033	0.0034	0.0102	0.0056	0.0055	-0.0016
5	-0.0058	-0.0044	0.0010	0.0006	0.0037	0.0020	0.0051	0.0046
6	0.0075	0.0073	-0.0044	-0.0008	-0.0051	-0.0002	0.0086	0.0056
7	0.0222	0.0197	-0.0064	-0.0026	-0.0099	-0.0028	0.0101	0.0071
8	0.0396	0.0335	-0.0068	-0.0036	-0.0158	-0.0059	0.0109	0.0115
9	0.0634	0.0517	-0.0081	-0.0041	-0.0211	-0.0076	0.0123	0.0133
Highest Decile	0.1231	0.0918	-0.0074	-0.0051	-0.0298	-0.0105	0.0149	0.0210

Table 2: The Cross Section of Expected Returns and Mispricing (Continued)

Panel C: Comparing True and Observed Mispricing

Sorting Variable	Mispricing		Q		Investment		Cash Flow	
Moments in Model	13	15	13	15	13	15	13	15
Lowest Decile	0.4866	0.7318	1.0027	0.9912	0.9490	0.9464	0.7084	0.8251
2	0.5817	0.8106	0.8763	0.9620	0.8614	0.9459	0.7569	0.8878
3	0.6348	0.8487	0.8272	0.9471	0.8187	0.9403	0.7634	0.9068
4	0.6778	0.8796	0.7879	0.9381	0.7859	0.9355	0.7681	0.9171
5	0.7176	0.9072	0.7617	0.9265	0.7634	0.9282	0.7628	0.9319
6	0.7576	0.9338	0.7295	0.9184	0.7331	0.9224	0.7676	0.9365
7	0.8010	0.9618	0.6994	0.9080	0.7132	0.9157	0.7670	0.9410
8	0.8535	0.9935	0.6686	0.8983	0.6828	0.9069	0.7580	0.9507
9	0.9239	1.0356	0.6266	0.8858	0.6455	0.9000	0.7499	0.9561
Highest Decile	1.0935	1.1245	0.5483	0.8517	0.5758	0.8850	0.7269	0.9733

Table 3: Cross-Sectional Regressions of One-Year Expected Returns on Firm Characteristics Using Simulated Data

Panel A: Simulated Data based on the 13-Moment Estimates

Regressions	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Q	-0.1020					0.0062	0.0098	-0.0066
Investment		-0.0280				-0.0313	-0.0713	-0.0659
Cash Flow			0.0056				0.0532	0.0614
Size				-0.0169				-0.0275
Past One-year Return					-0.0016			-0.0085
R^2	0.0323	0.2313	0.0105	0.0854	0.0023	0.2397	0.6454	0.8091

Panel B: Simulated Data based on the 15-Moment Estimates

Regressions	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Q	-0.0016					0.0003	0.0067	-0.0002
Investment		-0.0098				-0.0098	-0.0575	-0.0739
Cash Flow			0.0116				0.0572	0.0846
Size				-0.0036				-0.0287
Past One-year Return					0.0003			-0.0116
R^2	0.0024	0.0418	0.0583	0.0065	0.0009	0.0419	0.5167	0.7721

Table 4: Summarizing the Rational Manager Model's Fit to the Data

Panel A: Comparing Simulated Moments in the Rational and Biased Manager Models

Simulated Model	13-Moment			15-Moment		
Manager Type	Rational Manager	Biased Manager	% Change	Rational Manager	Biased Manager	% Change
Mean Tobin's Q	1.7167	1.6818	2.1%	1.6445	1.6317	0.8%
Mean Cash Flow	0.1176	0.1129	4.2%	0.1117	0.1102	1.4%
Std Dev of Q	0.5560	0.9166	-39.3%	0.7641	0.9649	-20.8%
Std Dev of Abnormal Return	0.2693	0.2029	32.7%	0.2853	0.1997	42.9%
Std Dev of Cash Flow	0.0905	0.0879	2.9%	0.1048	0.1040	0.7%
Std Dev of Investment	0.0640	0.0976	-34.4%	0.092	0.0968	-5.0%
Persistence of Cash Flow	0.7972	0.8018	-0.6%	0.8042	0.8066	-0.3%
Abn Ret Sensitivity to Cash Flow	1.8432	1.7742	3.9%	1.8553	1.7593	5.5%
Investment Sensitivity to Q	0.0751	0.0690	8.9%	0.0722	0.0635	13.7%
Investment Sensitivity to Cash Flow	0.5647	0.6409	-11.9%	0.6839	0.6551	4.4%
Cash Flow – Year 1	0.0092	0.0176	-47.7%	0.046	0.0408	12.9%
Tobin's Q – Year 1	-0.1339	-0.0663	101.9%	-0.0291	-0.0265	10.0%
Tobin's Q – Years 2 and 3	-0.1208	-0.0576	109.8%	-0.0155	-0.0169	-8.5%
Investment – Year 1	-0.0037	-0.1077	-96.6%	0.0237	-0.0353	-167.2%
Investment – Years 2 and 3	-0.0353	-0.0880	-59.9%	0.0093	-0.0225	-141.3%

Table 4: Summarizing the Rational Manager Model's Fit to the Data (Continued)

Panel B: Comparing Expected Returns and Mispricing in the Rational and Biased Manager Models

Simulated Model	13-Moment		15-Moment	
Manager Type	Rational Manager	Biased Manager	Rational Manager	Biased Manager
Value-weighted Mean Expected Return	-0.0090	-0.0133	-0.0019	-0.0027
Equal-weighted Mean Expected Return	0.0068	0.0067	0.0021	0.0029
Median Expected Return	0.0012	0.0006	0.0007	0.0015
Std Dev of Expected Returns	0.0730	0.0582	0.0592	0.0484
Value-weighted Mean Mispricing	0.8250	0.7444	0.9558	0.9553
Equal-weighted Mean Mispricing	0.8751	0.7528	0.9560	0.9227
Median Mispricing	0.8331	0.7321	0.9460	0.9207
Std Dev of Mispricing	0.2456	0.1731	0.1490	0.1120

Table 5: Real Effects of Information Processing Biases

Model	13-Moment Estimation	15-Moment Estimation
Aggregate value added V – K, biased / rational	0.355	0.920
Capital-weighted value per unit of capital V/K, biased	1.077	1.304
Capital-weighted value per unit of capital V/K, rational	1.385	1.375
Median-firm value per unit of capital V/K, biased	1.139	1.382
Median-firm value per unit of capital V/K, rational	1.424	1.466
Aggregate capital stock K, biased / rational	1.820	1.135
Median-firm capital stock K, biased / rational	1.518	1.097

Table 6: Analysis of the Bias in Firms' Marginal Investment Rates

Note: Bias in Marginal Investment Rate = Biased Manager's Marginal Inv. Rate – Rational Manager's Marginal Inv. Rate

Panel A: Summary Statistics for the Bias in Marginal Investment Rates

Estimation	Variable	Equal-Weighted Mean	Median	Std Dev	Aggregate
13-Moment	Bias in Marginal Investment Rate	0.0138	0.0153	0.0692	0.0313
15-Moment	Bias in Marginal Investment Rate	0.0040	0.0040	0.0381	0.0074

Panel B: The Cross Section of the Bias in Marginal Investment Using Simulated Data based on the 13-Moment Estimates

Regression	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Q	0.0154					-0.0119	-0.0155	0.0021
Investment		0.0460				0.0523	0.0911	0.0864
Cash Flow			0.0083				-0.0515	-0.0636
Size				0.0243				0.0328
Past One-year Return					0.0103			0.0162
R^2	0.0542	0.4426	0.0152	0.1246	0.0237	0.4650	0.7351	0.9291

Panel C: The Cross Section of the Bias in Marginal Investment Using Simulated Data based on the 15-Moment Estimates

Regression	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Q	0.0022					0.0001	-0.0048	0.0010
Investment		0.0109				0.0109	0.0477	0.0618
Cash Flow			-0.0064				-0.0443	-0.0679
Size				0.0040				0.0243
Past One-year Return					0.0018			0.0106
R^2	0.0066	0.0831	0.0286	0.0122	0.0032	0.0841	0.5430	0.8426

Table 7: Simulated Moments in 13-Moment Models with Different Biases

Bias in the Model	Extrapolation	Overconfidence	No Biases	Both Biases	Empirical
Mean Tobin's Q	1.5965	1.4510	1.445	1.6317	1.5801
Mean Cash Flow	0.1125	0.1185	0.1198	0.1102	0.1317
Std Dev of Q	0.5087	0.3041	0.2979	0.9649	0.9370
Std Dev of Abnormal Return	0.2026	0.1560	0.1644	0.1997	0.2108
Std Dev of Cash Flow	0.0868	0.0907	0.0907	0.1040	0.0940
Std Dev of Investment	0.0866	0.0653	0.0643	0.0968	0.1686
Persistence of Cash Flow	0.7959	0.8032	0.7975	0.8066	0.7901
Abn Ret Sensitivity to Cash Flow	3.0180	1.4084	2.4307	1.7593	1.7569
Investment Sensitivity to Q	0.1344	0.1832	0.1854	0.0635	0.0666
Investment Sensitivity to Cash Flow	0.7077	0.2292	0.4443	0.6551	0.6603
Cash Flow – Year 1	-0.0826	0.0648	-	0.0408	0.0139
Tobin's Q – Year 1	-0.0540	-0.0480	-	-0.0265	-0.0638
Tobin's Q – Years 2 and 3	-0.0409	-0.0301	-	-0.0169	-0.0456
Investment – Year 1	-0.0747	-0.0518	-	-0.0353	-0.0671
Investment – Years 2 and 3	-0.0552	-0.0327	-	-0.0225	-0.0242

Table 8: True Mispricing and One-Year Expected Returns in 13-Moment Models with Different Biases

Panel A: True Mispricing in Portfolios Sorted by Firm Characteristics

Sorting Variable	Mispricing		Q		Investment		Cash Flow	
Bias in the Model	Extrapolat.	Overconf.	Extrapolat.	Overconf.	Extrapolat.	Overconf.	Extrapolat.	Overconf.
Lowest Decile	0.7123	0.7112	0.9649	1.0343	0.9246	1.0266	0.8998	0.8145
2	0.7606	0.8043	0.9075	0.9893	0.8920	0.9912	0.8840	0.8818
3	0.7856	0.8529	0.8814	0.9684	0.8723	0.9728	0.8690	0.9074
4	0.8062	0.8925	0.8606	0.9634	0.8564	0.9632	0.8556	0.9318
5	0.8248	0.9283	0.8425	0.9504	0.8419	0.9499	0.8432	0.9471
6	0.8425	0.9637	0.8250	0.9419	0.8293	0.9415	0.8323	0.9645
7	0.8606	1.0009	0.8064	0.9339	0.8140	0.9299	0.8191	0.9823
8	0.8815	1.0443	0.7857	0.9209	0.7977	0.9201	0.8056	0.9959
9	0.9079	1.1009	0.7610	0.9032	0.7781	0.9076	0.7869	1.0137
Highest Decile	0.9657	1.2196	0.7128	0.8827	0.7405	0.8867	0.7511	1.0504

Panel B: One-Year Expected Returns in Portfolios Sorted by Firm Characteristics

Sorting Variable	1-Yr Expected Return		Q		Investment		Cash Flow	
Bias in the Model	Extrapolat.	Overconf.	Extrapolat.	Overconf.	Extrapolat.	Overconf.	Extrapolat.	Overconf.
Lowest Decile	-0.0372	-0.0740	0.0382	0.0330	0.0458	0.0341	0.0491	-0.0343
2	-0.0232	-0.0439	0.0172	0.0157	0.0239	0.0189	0.0266	-0.0150
3	-0.0155	-0.0282	0.0098	0.0083	0.0144	0.0115	0.0163	-0.0080
4	-0.0091	-0.0157	0.0043	0.0067	0.0075	0.0075	0.0090	-0.0011
5	-0.0029	-0.0043	-0.0002	0.0025	0.0024	0.0028	0.0026	0.0029
6	0.0035	0.0075	-0.0034	-0.0002	-0.0027	-0.0006	-0.0030	0.0079
7	0.0103	0.0198	-0.0062	-0.0024	-0.0077	-0.0045	-0.0084	0.0126
8	0.0183	0.0339	-0.0092	-0.0062	-0.0126	-0.0081	-0.0141	0.0161
9	0.0296	0.0527	-0.0110	-0.0108	-0.0183	-0.0121	-0.0208	0.0206
Highest Decile	0.0554	0.0938	-0.0147	-0.0153	-0.0279	-0.0181	-0.0325	0.0299

Table 9: Real Effects in 13-Moment Models with Different Biases

Model	Extrapolation Only	Overconfidence Only
Aggregate value added V – K, biased / rational	0.907	0.819
Capital-weighted value per unit of capital V/K, biased	1.221	1.311
Capital-weighted value per unit of capital V/K, rational	1.385	1.385
Median-firm value per unit of capital V/K, biased	1.269	1.344
Median-firm value per unit of capital V/K, rational	1.424	1.424
Aggregate capital stock K, biased / rational	1.567	1.007
Median-firm capital stock K, biased / rational	1.460	1.015
Median marginal investment rate, based – rational	0.014	0.000
Std dev marginal investment rate, based – rational	0.027	0.051

Figure 1: Percentage Modeling Errors = (Simulated Moment / Empirical Moment) – 1

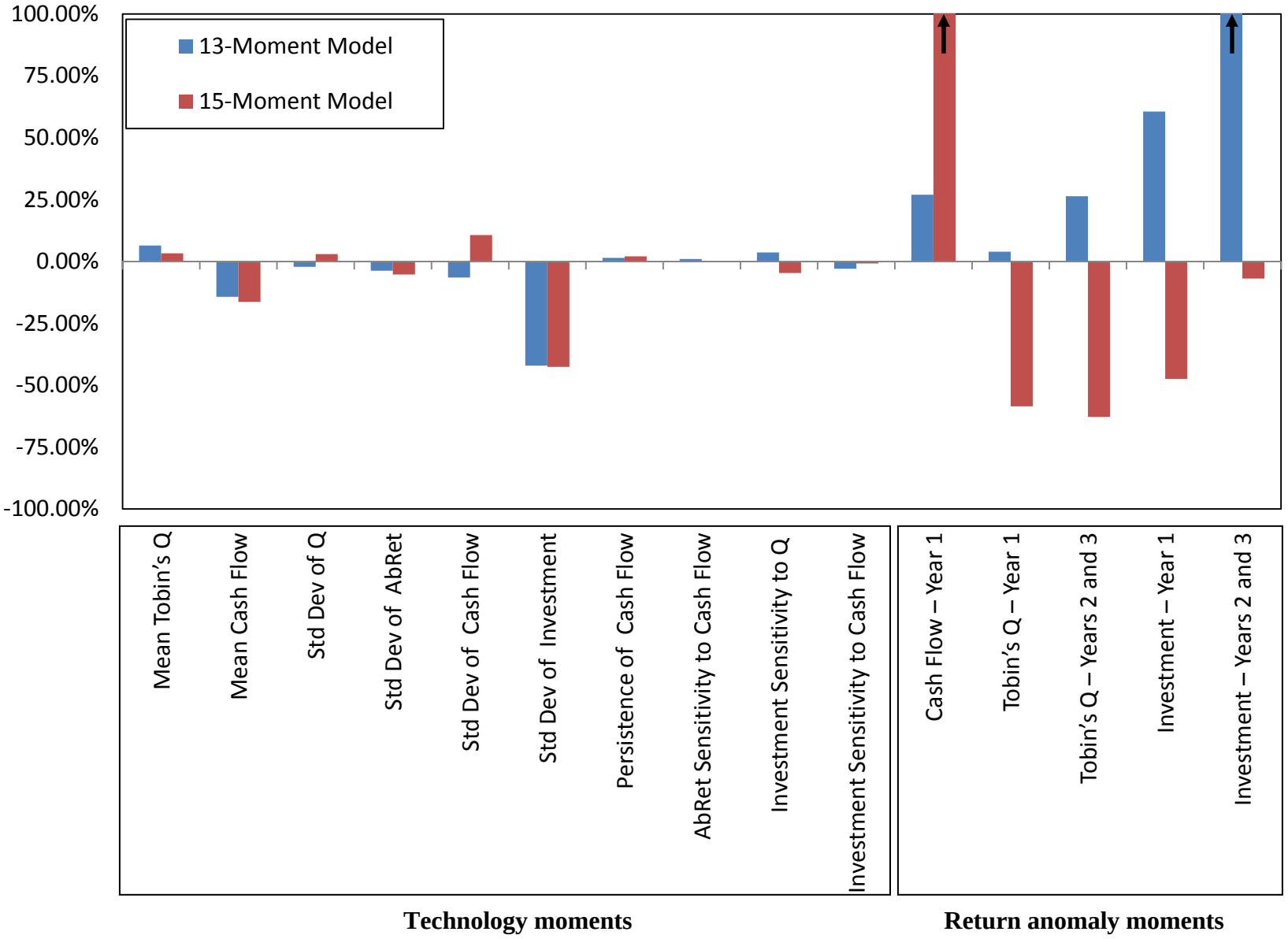


Figure 2: Expected abnormal returns of top and bottom decile portfolios

(Using 13-moment estimates)

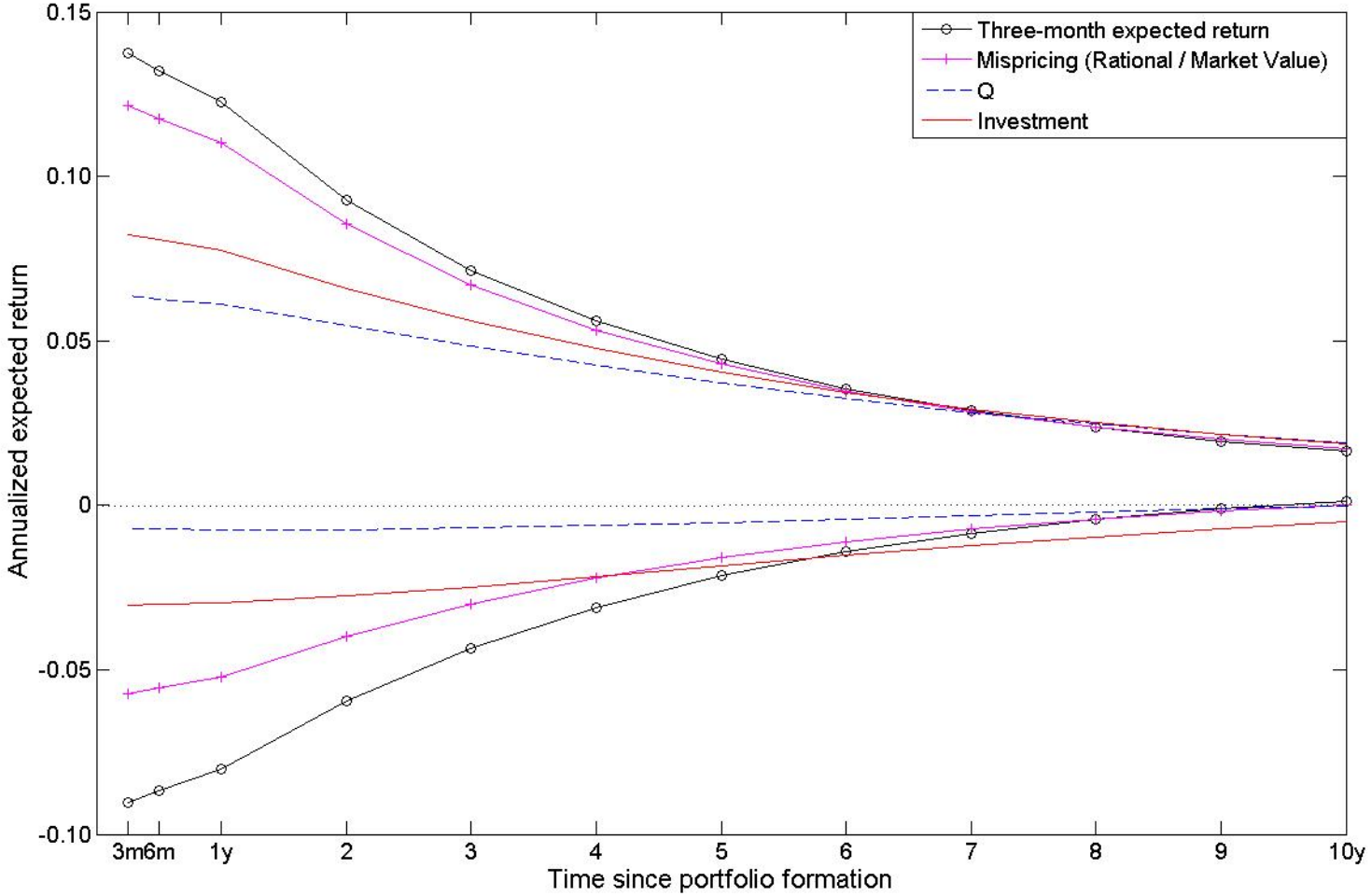


Figure 3: Expected abnormal returns of top and bottom decile portfolios

(Using 15-moment estimates)

