Cooperative Fiscal and Monetary Policy at the Zero Lower Bound *

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Abstract

We construct a model of the international transmission of shocks to the savings rate which may push optimal policy interest rates down to their zero bound. We explore the optimal policy response to these shocks, when governments cooperate on both fiscal and monetary responses. The key aspect of the model is to emphasize how both the transmission of shocks, and the optimal policy responses are affected by the degree of trade openness of each economy. When the savings shock emanates from one economy, the degree to which the partner economy is pushed towards a liquidity trap is determined by the degree of trade openness. When trade openness is full, both economies enter a liquidity trap simultaneously, and the optimal policy response is to have identical, expansionary fiscal packages. When trade is less than fully open, the economy suffering the shock is the worst hit. Then the optimal policy response is to have fiscal expansion in that economy, a much smaller expansion in the foreign economy, combined with monetary tightening in the foreign economy. Strikingly, the foreign economy may choose to raise interest rates even though its optimal policy rate should be zero, from the perspective of a closed economy.

Keywords:Liquidity Trap, Monetary Policy, Fiscal Policy, International Spillovers

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1 Introduction

During the great moderation, an orthodox monetary policy developed which focused on setting short-term interest rates to stabilize demand conditions, focusing on price stability. At the same time, the use of fiscal policy as an instrument for stabilization policy fell out of favor; perhaps, in part, because of the perceived success of monetary tools in providing short-run stability and low inflation. The experience of Japan over the last decade, and the United States more recently, has rekindled interest in using fiscal policy for macroeconomic stabilization. Both countries have been beset by disinflationary demand slumps that have lead central banks to cut policy rates to a level of zero. The infeasibility of reducing interest rates beneath the zero lower bound has lead to an interest in less orthodox policies (see Krugman, 1998).

Expansions in government spending may be especially powerful at the zero lower bound. An expansion in government spending will tend to be inflationary. A persistent rise in government spending will increase inflationary expectations. When the interest rate is immobile, as at the zero lower bound, an increase in inflationary expectations reduces real interest rates, further stimulating aggregate demand under sticky prices. In a canonical closed economy New Keynesian model, Eggertson (2010) shows that a persistent government spending expansion will have a multiplier exceeding one when the economy is operating at the zero lower bound. Indeed, Woodford (2010) shows that optimal discretionary fiscal policy during a persistent slump in a closed economy is expansionary, even if that fiscal policy is wasteful in the sense that such spending would be at a level in which the direct marginal utility would be lower than the equilibrium marginal cost.

Though much of the focus on the zero lower bound inspired by Japan's experience with the liquidity trap has focused on the closed economy case (exceptions include .....), the global Great Recession of 2008 has created new impetus to examine the international aspect of macroeconomic management. This paper will focus on optimal fiscal and monetary policy at the lower bound in an open economy in which two trading partners are well integrated through financial markets but less than perfectly integrated in goods markets. Introducing international trade generates some novel complications. For one, the impact of fiscal policy on real interest rates will also impact the real terms of trade through real interest parity. Cook and Devereux (2010 EER forthcoming) show that open economy multipliers on domestic fiscal policy will exceed the closed economy multiplier at the zero lower bound, regardless of the degree of trade integration. Expansionary fiscal policy which reduces the real interest rate will also depreciate the real exchange rate, stimulating external trade. At the same

time, fiscal policy will have negative spillovers on demand for goods produced by trading partners, so the impact on

Despite this, optimal cooperative fiscal policy in the wake of a demand shock that creates a liquidity trap is hardly more complicated in the case of a perfectly integrated trading environment than in the closed economy. The reason is that when international trade is so perfectly integrated that there is no tendency of consumers to concentrate demand on home produced goods, firms in all countries will feel equally the effects of any slump in demand. This means that output and inflation in all countries will respond symmetrically to demand shocks regardless of the source of the shock. The optimal fiscal policy will also be symmetric and will be precisely equivalent to optimal policy in a closed economy. Because both countries are equally affected by the 'zero lower bound', monetary policy is not effective[1].

However, some economies which have been pushed into a liquidity trap, most prominently Japan and the United States display substantial home bias in their international trade with exports making up less than 20% of overall demand. Theoretically, even in the case of strong home bias, a sufficiently large demand shock in one country can push the equilibrium interest rate in a trading partner below the zero bound. However, the introduction of home bias complicates the analysis of optimal policy, since the international effects of a demand shock are not shared equally amongst trading partners, but are concentrated on the country that is the primary source of the shock. We are able to show that the average level of the output gap in the world economy declines and the average optimal fiscal policy response should be expansionary. Moreover, the decline in the world output gap and the expansionary world fiscal policy are invariant to home bias and precisely equal to that observed in a closed economy.

However, the distribution of the effects of the shock are less clear when home bias is important. It is clear that the response of fiscal policy to the shock should be larger and more expansionary in the country that is the primary source of the shocks. If home bias is small and international trade is well integrated, a foreign trading partner might be strongly affected by a loss of demand in the source country and need expansionary fiscal policy to offset the deflationary effects. But if the economies are less open to international trade, both the international tranmission of shocks, and the optimal policy response, may be very different. With substantially restricted trade, the decline in global interest rates that occur during the slump may actually be expansionary for global trading partners. In such a case, the optimal policy response will be to combine very slight fiscal expansion with monetary

[1] As discussed below, in the paper we are looking at discretionary macroeconomic policies only, so we ignore the potential use of announcement effects of future monetary expansion.

contraction. That is, the foreign economy should only minimally engage in a cooperative fiscal expansion, but should raise its policy rates. Strikingly, we find that the best policy (from a global cooperative perspective), is for the foreign country to tighten monetary policy, even though using the standard criterion from the closed economy logic, it should still be in a liquidity trap (where its 'natural' real interest rate is below zero). The logic behind this policy is that a foreign interest rate increase acts so as to dampen the (positive) response of the foreign economy to the home country savings shock, while at the same time facilitating a weakening of the home currency, and generating an expenditure switching of demand towards the worst hit (home) economy.

The paper builds on a substantial recent literature on monetary and fiscal policy in a 'liquidity trap'. In particular, Krugman (1999), Eggertson and Woodford (2003, 2005), Jung et al. (2005), Svensson (2003), Auerbach and Obstfeld (2004) and many other writers explored how monetary and fiscal policy could be usefully employed even when the authorities have no further room to reduce short term nominal interest rates. Recently, a number of authors have revived this literature in light of the very similar problems now encountered by the economies of Western Europe and North America. Papers by Christiano et al (2009), Eggertson (2009), Taylor et al. (2008) have explored the possibility for using government spending expansions, tax cuts, and monetary policy when the economy is in a 'liquidity trap'. For the most part, these papers did not focus on the international dimension of liquidity traps. Some recent expections are Fujiwara et al. (2009, 2010), Erceg et al. (2009) and Jeanne (2009). Jeanne (2009) examines a 'global liquidity trap' in a model of one-period ahead pricing similar to that of Krugman (2009). Erceg. et al (2009) use a fully specific two country DSGE model to examine the international transmission of shocks when one country is in a liquidity trap, but do not focus on optimal monetary policy or fiscal policy choices. In particular, our model is essentially a convex combination of the models of Beetsma and Jensen (2005), who incorporated government spending into a two country open economy environment, and Engel (2010), who looked at an optimal policy environment with home bias in consumer preferences[2]. In both cases, the assumption is that there is a complete set of markets for consumption insurance across countries. . Fujiwara et al. (2009) examine the optimal monetary problem with commitment in a multi country situation, but do not examine the determination of fiscal policy, or the transmission of demand shocks across countries. Fujiwara et al. (2010) look at the impact of the international effects of fiscal policy in a liquidity trap.

[2]Engel's paper is focused more on the role of local currency pricing for the monetary policy problem in open economies. We ignore this additional complication in our analysis - assuming the simpler framework of producer currency pricing.

The rest of the paper is organized as follows. The next section develops the basic model. Section 3 examines the solution under sticky prices. Then in section 4 we analyze the impact of fiscal policies at the zero lower bound, and the role of international spillovers of policies. Section 5 examines the optimal policy making problem in a global cooperative agreement. Some conclusions are then offered.

2 A two country model of interacting monetary and fiscal policy

There are two countries in the world economy here. In each country, households consume both private and government goods, and supply labour. Denote the countries as 'home' and 'foreign' with foreign variables denoted with an asterisk superscript. The population of each country is normalized to unity. Each country produces a range of differentiated goods. Complete asset markets allow full insurance of consumption risk across countries. Households also hold their own country's nominal government bonds. Firms produce private goods, while governments produce government goods which are distributed uniformly across households. Firms production and supply is constrained by sticky prices. Governments have access to lump sum taxation.

2.1 Households

Utility of a representative infinitely lived home household evaluated from date 0 is:

$$U_t = E_0 \sum_{t=0}^{\infty} (\beta)^t (U(C_t, \xi_t) - V(N_t) + J(G_t)) \tag{1}$$

where U, V, and J represent the utility of the composite home consumption bundle C_t, disutility of labour supply N_t, and utility of the government supplied public good G_t,respectively. The variable ξ_t represents a shock to preferences or 'demand' . We assume that $U_{12} > 0$..

Composite consumption is defined as

$$C_t = \Phi C_{Ht}^{v/2} C_{Ft}^{1-v/2}, \quad v \geq 1$$

where $\Phi = \left(\frac{v}{2}\right)^{\frac{v}{2}} \left(1 - \left(\frac{v}{2}\right)\right)^{\frac{v}{2}}$, C_H is the consumption of the home country composite good by the home household, and C_F is consumption of the foreign composite good. If $v > 1$ then there is a home preference bias for domestic goods. We will focus on the home bias case,

where v is strictly greater than 1. This case will be most realistic for thinking about policy in large open economies.

Consumption aggregates, C_H and C_F are composites, defined over a range of home and foreign differentiated goods, with elasticity of substitution θ between goods, so that:

$$C_H = \left[\int_0^1 C_H(i)^{1-\frac{1}{\theta}} di\right]^{\frac{1}{1-\frac{1}{\theta}}}, \qquad C_F = \left[\int_0^1 C_F(i)^{1-\frac{1}{\theta}} di\right]^{\frac{1}{1-\frac{1}{\theta}}}, \quad \theta > 1.$$

Price indices for home and foreign consumption are:

$$P_H = \left[\int_0^1 P_H(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}, \qquad P_F = \left[\int_0^1 P_F(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}},$$

while the aggregate (CPI) price index for the home country is $P = P_H^{\frac{v}{2}} P_F^{1-\frac{v}{2}}$ and for the foreign is $P_t^* = P_F^{*v/2} P_H^{*1-v/2}$

Demand for each differentiated good ($j = H, F$) is

$$\frac{C_j(i)}{C_j} = \left(\frac{P_j(i)}{P_j}\right)^{-\theta}$$

The law of one price holds for each good so $P_j(i) = SP_j^*(i)$.where S_t is the nominal exchange rate (home price of foreign currency). Relative demand for the composites is:

$$\frac{C_H}{C_F} = \frac{P_F}{P_H} = \frac{SP_F^*}{P_H}$$

Home government spending falls on the home composite good and foreign government spending on the foreign composite good. Thus, government spending is assumed to have full 'home bias'. I addition, we assume that government spending demand for each variety of home goods has price elasticity θ, the same as that for private spending.

The household's implicit labour supply at nominal wage W_t is:

$$U_C(C_t, \xi_t) W_t = P_t V'(N_t). \tag{2}$$

Optimal risk sharing implies

$$U_C(C_t, \xi_t) = U_C(C_t^*, \xi_t^*) \frac{S_t P_t^*}{P_t} = U_C(C_t^*, \xi_t^*) T_t^{v-1}, \tag{3}$$

.Nominal bonds pay interest, R_t. The Euler equation is:

$$\frac{U_C(C_t,\xi_t)}{P_t} = \beta R_t E_t \frac{U_C(C_{t+1},\xi_{t+1})}{P_{t+1}}. \tag{4}$$

Foreign household preferences and choices can be defined exactly symmetrically. The foreign representative household has weight $v/2$, $(1-v/2)$ on the foreign (home) compositive good in preferences.

2.2 Firms

Each firm i employs labor to produce a differentiated good.

$$Y_t(i) = N_t(i),$$

Profits are $\Pi_t(i) = P_{Ht}(i)Y_t(i) - W_t H_t(i)\frac{\theta-1}{\theta}$ indicating a subsidy financed by lump-sum taxation to eliminate steady state first order inefficiencies. Each firm re-sets its price according to Calvo pricing with probability of adjusting prices equal to $1-\kappa$. Firms that adjust their price set new price given by $\widetilde{P}_{Ht}(i)$:

$$\widetilde{P}_{Ht}(i) = \frac{E_t \sum_{j=0} m_{t+j}\kappa^j \frac{W_{t+j}}{A_{t+j}} Y_{t+j}(i)}{E_t \sum_{j=0} m_{t+j}\kappa^j Y_{t+j}(i)}. \tag{5}$$

where stochastic discount factor $m_{t+j} = \frac{P_t}{U_C(C_t,\varepsilon_t)}\frac{U_C(C_{t+j},\xi_{t+j})}{P_{t+j}}$. In the aggregate, the price index for the home good then follows the process given by:

$$P_{Ht} = [(1-\kappa)\widetilde{P}_{Ht}^{1-\theta} + \kappa P_{Ht-1}^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{6}$$

The behaviour of foreign firms and the foreign good price index may be described analogously.

2.3 Market Clearing

Equilibrium in the market for good i as

$$Y_{Ht}(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta}\left[\frac{v}{2}\frac{P_t}{P_{Ht}}C_t + (1-\frac{v}{2})\frac{S_t P_t^*}{P_{Ht}}C_t^* + G_t\right],$$

where G_t represents total home government spending. Aggregate market clearing in the home good is:

$$Y_{Ht} = \frac{v}{2}\frac{P_t}{P_{Ht}}C_t + (1-\frac{v}{2})\frac{S_t P_t^*}{P_{Ht}}C_t^* + G_t. \tag{7}$$

Here $Y_{Ht} = V_t^{-1} \int_0^1 Y_{Ht}(i)di$ is aggregate home country output, where we have defined $V_t = \int_0^1 \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} di$. It follows that home country employment (employment for the representative home household) is given by $N_t = \int_0^1 N(i)di = A^{-1}Y_{Ht}V_t$.

The aggregate market clearing condition for the foreign good is

$$Y_{Ft} = \frac{v}{2}\frac{P_t^*}{P_{Ft}^*}C_t^* + (1-\frac{v}{2})\frac{P_t}{S_t P_{Ft}^*}C_t^* + G_t^*, \tag{8}$$

where: $N_t^* = \int_0^1 N_t^*(i)di = A^{*-1}Y_{Ft}V_t^*$, where $V_t^* = \int_0^1 \left(\frac{P_{Ft}^*(i)}{P_{Ft}^*}\right)^{-\theta} di$.

An equilibrium in the world economy with positive nominal interest rates may be described by the equations (3), and (2), (4), (5) (6) and (**??**) for the home and foreign economy, as well as (7) and (8). For given values of V_t and V_t^*, and given government spending policies, these equations determine an equilibrium sequence for the variables $C_{t,}C_t^*, W_t, W_t^*, S_t, P_{Ht}$, $P_{Ft}^*, \widetilde{P}_{Ht}, \widetilde{P}_{Ft}^*, R_t, R_t^*$, and N_t, N_t^*.

3 New Keynesian Open Economy Model

3.1 Demand Shocks and Natural Interest Rates

Define $\sigma \equiv -\frac{U_{CC}\overline{C}}{U_C}$ as the inverse of the elasticity of intertemporal substitution in consumption, $\phi \equiv -\frac{V''\overline{H}}{V'}$ as the elasticity of the marginal disutility of hours worked and $\sigma_g \equiv -\frac{J''\overline{G}}{J'}$ as the elasticity of marginal utility of public goods. We assume that $\sigma_g = \sigma > 1$. Finally, $\varepsilon_t = \frac{U_{C\xi}}{U_C}\ln(\xi_t)$ is the measure of a positive demand shock in the home country, with an equivalent definition for the foreign country. Define $c_y = \frac{C}{Y}$ is the steady state share of consumption in output.We examine the effects of once and for all demand shocks which have a probability $1-\mu$ of reverting to mean.

For any variable x_t, define the world average and world relative level, $x_t^W = \frac{x_t + x_t^*}{2}$ and $x_t^R = \frac{x_t - x_t^*}{2}$. Solving a first order apprixmation of the flexible price, zero inflation version of the above economy, we can derive the Wicksellian natural interest rate of the home and foreign economy, $\widetilde{r}_t$ as a function of the demand shock.

$$\widetilde{r}_t = \overline{r} + \left(\frac{\phi c_y}{\phi+\sigma}\varepsilon_t^W + \frac{\phi c_y(v-1)}{\Delta}\varepsilon_t^R\right)(1-\mu) \tag{9}$$

The foreign efficient nominal interest rate is:

$$\widetilde{r}_t^* = \overline{r} + \left(\frac{\phi c_y}{\phi + \sigma} \varepsilon_t^W - \frac{\phi c_y (v-1)}{\Delta} \varepsilon_t^R \right) (1-\mu) \tag{10}$$

where $\Delta \equiv \phi c_y D + \phi(1-c_y) + \sigma$ and $\sigma > D \equiv (\sigma v(2-v) + (1-v)^2) > 1$. The way to interpret (9) and (10) is as the Fisherian, consumption based real real interest rates that would obtain in an economy with fully flexible prices and no other distortions. These are critical variables for our analysis, since they govern the degree to which monetary policy can be efficiently employed to stabilize the economy. In particular, our model has the characteristic that when (9) and (10) are both positive, then monetary policy can perfectly achieve the joint target of zero inflation and zero output gaps.

Note that in the no home bias case, when $v = 1$, then the natural interest rate for both economies should be the same. $\widetilde{r}_t = \widetilde{r}_t^* = \overline{r} + \left(\frac{\phi c_y}{\phi + \sigma} \varepsilon_t^W \right) (1-\mu)$. This is an example where the real exchange rate is constant, and given integrated financial markets, consumption-based real interest rates are equated across countries. But in fact, the case $v = 1$ is not particularly realistic. For most economies, and particlarly for large open economies, the lion's share of demand will come from the domestic economy, making the home bias case most relevant. We will therefore focus on the more general case where $v > 1$.

We also focus on the case where the home country is the source of the shocks. In particular, we will assume that home consumers are affected by preference shocks which affect their propensity to save, whereas consumers in the foreign economy is not directly affected by these shocks. Of course foreign consumers will be indirectly affected by the shock, however, since integrated financial markets lead to linkages between interest rates. Thus, a saving shock that emanates from the home economy, pushing the monetary authority into a liquidity trap situation, may have similar effects on the foreign economy, even though the foreign consumers are not directly affected by the shock.

Making this assumption, we have in this case, $\varepsilon_t^* = 0$ and $\varepsilon_t^W = \varepsilon_t^R = \frac{\varepsilon_t}{2}$ and we can write the home country natural rate as

$$\widetilde{r}_t = \overline{r} + \left(\frac{\Delta + (\phi + \sigma)(v-1)}{(\phi + \sigma)\Delta} \right) (1-\mu) \phi c_y \frac{\varepsilon_t}{2} \tag{11}$$

Then the foreign natural rate is:

$$\widetilde{r}_t^* = \overline{r} + \left(\frac{\Delta - (\phi + \sigma)(v-1)}{(\phi + \sigma)\Delta} \right) (1-\mu) \phi c_y \frac{\varepsilon_t}{2} \tag{12}$$

It is clear, that when there is home bias, $(v > 1)$, and then $\Delta + (\phi + \sigma)(v-1) >$

$\Delta - (\phi + \sigma)(v-1) > 0$[3]. A home preference shock that is sufficiently large can push the natural interest rate of both countries below zero, regardless of the degree of home bias.[4]

3.2 First Order Approximation

We derive a sticky price log-linear approximation of the model in terms of inflation and output gaps in a similar manner to Clarida et al. (2002) and Engel (2010). Let $\widehat{x}_t$ be the percentage deviation of a given variable x_t from the efficient zero flexible price equilibrium. As defined before, $D \equiv \sigma v(2-v)+(1-v)^2 > 1$. In addition, let $s \equiv \frac{\sigma}{c_y D}$, and $s > s_D \equiv \frac{s}{D} > 1$. In order to explore the implications of the zero lower bound constraint, we begin with the standard forward looking inflation equations and open economy IS relationships for the home and foreign economy.

The home and foreign forward looking inflation equations are

$$\pi_{Ht} = k(\phi \widehat{n}_t + \frac{s_D}{2}\left[\widehat{ng}_t(1+D) + \widehat{ng}^*_t(D-1)\right]) + \beta E_t \pi_{Ht+1} \tag{13}$$

.

$$\pi^*_{Ft} = k(\phi \widehat{n}^*_t + \frac{s_D}{2}\left[\widehat{ng}^*_t(1+D) + \widehat{ng}_t(D-1)\right]) + \beta E_t \pi^*_{Ft+1} \tag{14}$$

Likewise, the home and foreign 'dynamic IS equations' are

$$\begin{aligned} & s_D E_t(\widehat{ng}_{t+1} - \widehat{ng}_t)(D+1) + s_D E_t(\widehat{ng}^*_{t+1} - \widehat{ng}^*_t)(D-1) \\ = \; & 2E_t\left(r_t - \widetilde{r}_t - \pi_{Ht+1}\right) \end{aligned} \tag{15}$$

$$\begin{aligned} & s_D E_t(\widehat{ng}^*_{t+1} - \widehat{ng}^*_t)(D+1) + s_D E_t(\widehat{ng}_{t+1} - \widehat{ng}_t)(D-1) \\ = \; & 2E_t\left(r^*_t - \widetilde{r}^*_t - \pi_{Ft+1}\right) \end{aligned} \tag{16}$$

where π_{Ht} and π^*_{Ft} are the inflation rates of the domestic and foreign composite goods, $\widehat{ng}_t = (\widehat{n}_t - (1-c_y)\widehat{g}_t)$ and $\widehat{ng}^*_t = (\widehat{n}^*_t - (1-c_y)\widehat{g}^*_t)$, and the coefficient k depends on the

[3]Note that $\Delta - (\phi+\sigma)(v-1) = \phi c_y(D-1) + (\phi+\sigma)(2-\nu)$

[4]However, it is also possible, in the case of strong home bias or in the case of a sufficiently small preference shock, that the natural interest rate of the home economy will be pushed below zero, while the foreign economy will retain a Wicksellian rate that will be positive.

degree of price rigidity. Note that, approximated around the steady state, $\widehat{n}_t \approx \widehat{y}_t$, $\widehat{n}^*_t \approx \widehat{y}^*_t$, so the labor gap will stand in for the output gap.

If the natural interest rates of both economies are always above zero, then the monetary and fiscal authorities can retain perfect price and output stability by setting the nominal interest rate equal to the natural real interest rate and keeping the fiscal gaps, $\widehat{g}_t$ and $\widehat{g}^*_t$ closed. However, a world wide liquidity trap, where desired real interest rates are both negative, will prevent the monetary authorities from achieving this outcome. As a result, there is a role for fiscal policy in stabilizing inflation and output gaps.

3.3 The World and Relative Economy

We can simplify the equations by writing them in terms of world average and relative levels. The world average for global inflation is written as:

$$\pi^W_t = k(\phi + s)\widehat{n}^W_t - ks \cdot \widehat{cg}^W_t + \beta E_t \pi^W_{t+1} \tag{17}$$

$$sE_t(\widehat{n}^W_{t+1} - \widehat{n}^W_t) - sE_t(\widehat{cg}^W_{t+1} - \widehat{cg}^W_t) = E_t\left(r^W_t - \widetilde{r}^W_t - \pi^W_{t+1}\right) \tag{18}$$

The world 'relative' variables are written as:

$$\pi^R_t = k(\phi + s_D)\widehat{n}^R_t - ks_D\widehat{cg}^R_t + \beta E_t \pi^R_{t+1} \tag{19}$$

$$s_D E_t(\widehat{n}^R_{t+1} - \widehat{n}^R_t) - s_D E_t(\widehat{cg}^R_{t+1} - \widehat{cg}^R_t) = E_t\left(r^R_t - \widetilde{r}^R_t - \pi^R_{t+1}\right) \tag{20}$$

where $\widehat{cg}^W_t \equiv (1-c_y)\widehat{g}^W_t$ and $\widehat{cg}^R_t \equiv (1-c_y)\widehat{g}^R_t$. A requirement for stability and determinacy of this set of equations is that $\Delta^D_3 \equiv s_D(1-\beta\mu)(1-\mu) - \mu k\,(\phi + s_D) > 0$, an assumption that we will maintain throughout.

If we abstract from fiscal gaps, (i..e $\widehat{cg}^W_t = \widehat{cg}^R_t = 0$)we can rewrite these equations so as to most clearly understand them:

$$\pi^W_t = k(\phi + s)\widehat{n}^W_t + \beta E_t \pi^W_{t+1} \tag{21}$$

$$sE_t(\widehat{n}^W_{t+1} - \widehat{n}^W_t) = E_t\left(r^W_t - \widetilde{r}^W_t - \pi^W_{t+1}\right) \tag{22}$$

$$\pi_t^R = k(\phi + s_D)\widehat{n}_t^R + \beta E_t \pi_{t+1}^R \tag{23}$$

$$s_D E_t(\widehat{n}_{t+1}^R - \widehat{n}_t^R) = E_t\left(r_t^R - \widetilde{r}_t^R - \pi_{t+1}^R\right) \tag{24}$$

We can clearly see that both systems of equations (for the world average and the world relative economies) are in the canonical form of the New Keynesian closed economy equations. The only difference comes in the parameterization of the inverse elasticity of consumption: s , in the case of the average economy; and, s_D, in the case of the relative world economy. Note that $s_D < s$, so the world average level of demand is less sensitive to the average interest rate than the relative level of demand is sensitive to the relative interest rate. This reflects the expenditure switching effect of terms of trade changes. When worldwide interest rates are relatively low, then for intertemporal reasons, world demand will be relatively high. But when there is a large gap in interest rates across country, demand will be relatively high in the low interest rate country for the same intertemporal reasons, but on top of that, a relatively low interest rate country will be characterized by a current deterioration in its terms of trade (or a real exchange rate depreciation), implying relatively high current demand for their goods, through the expenditure switching channel.

4 Global Liquidity Traps

We now focus on the policy dilemma when there is a global liquidity trap. We assume that the liquidity trap is created by a negative shock to home country preferences, pushing up home country savings, and reducing both home and foreign natural real interest rates below zero. As a result countries are pushed to the zero lower bound of the policy rate, so that $r_t^R = r_t^W = 0$. As we noted before, it is assumed that the preference shock reverts back to zero with probability $1 - \mu$ per period. As a result of this, all other variables in the world economy will inherit the same persistence, in expectation, so that for any variable x_t is $E_t[x_{t+1}] = \mu x_t$ within the liquidity trap[5]. Consider first the response of output and inflation to a negative demand shock from the home economy when the govenment spending

[5]Note that the implicit assumption is that, when the shock disappears, monetary policy will immediately close all gaps thereafter, and any fiscal gaps that are created by policy during the liquidity trap are also closed off.

gap is equal to zero. Using (17)-(20), we obtain:

$$\begin{aligned}\Delta_3 \widehat{n}_t^W &= (1-\beta\mu)\widetilde{r}_t^W \\ \Delta_3^D \widehat{n}_t^R &= (1-\beta\mu)\widetilde{r}_t^R\end{aligned}$$

where $\Delta_3 \equiv s(1-\beta\mu)(1-\mu)-\mu k\left(\phi+s\right) > \Delta_3^D > 0$. When both economies are in a liquidity trap, driven by a home preference shock, it must be that $\widetilde{r}_t^W < 0$ and $\widetilde{r}_t^R < 0$. So therefore, both $\widehat{n}_t^W < 0$, and $\widehat{n}_t^R < 0$. Since $\widehat{n}_t = \widehat{n}_t^W + \widehat{n}_t^R$, it must be that home country output falls. However, foreign output, defined as $\widehat{n}_t = \widehat{n}_t^W - \widehat{n}_t^R$, may or may not fall. As discussed in Cook and Devereux (2010c), the international transmission of a 'liquidity trap shock' may be positive or negative. In particular, if the preference shock is not too large, so that the foreign economy is only 'slightly' in the liquidity trap, then the fall in foreign policy rates to zero stimulates foreign spending, offseting the fall in demand coming from the home country, so that the foreign output gap may rise.

4.1 Fiscal Multiplier

In the absence of further monetary stimulus when both countries are in a liquidity trap, fiscal spending policies can be used to reduce output gaps and eliminate deflationary pressures. Consider the world fiscal multiplier in a global liquidity trap. For convenience define $\Delta_4 = s(1-\beta\mu)(1-\mu)-\mu k\left(s\right) > \Delta_4^D = s_D(1-\beta\mu)(1-\mu)-\mu k\left(s_D\right)$. Also note that $\Delta_4 > \Delta_3$, $\Delta_4^D > \Delta_3^D$ and $\Delta_4 = D\Delta_4^D$.

We can then combine equations (17) and (19) to get:

$$\Delta_3 \widehat{n}_t^W - \Delta_4 \widehat{cg}_t^W = (1-\beta\mu)\widetilde{r}_t^W,$$

so that:

$$\widehat{n}_t^W = \frac{\Delta_4}{\Delta_3}\widehat{cg}_t^W + \frac{(1-\beta\mu)}{\Delta_3}\widetilde{r}_t^W \tag{25}$$

From (25) , we have (noting that output and employment gaps are equivalent in this model) $\frac{d\widehat{y}^W}{d\widehat{g}^W} \approx \frac{\Delta_4}{\Delta_3}(1-c_y)$. But we can define $\frac{d\widehat{y}^W}{d\widehat{g}^W} = \frac{\frac{dY^W}{Y}}{\frac{dG^W}{G}} = \frac{dY^W}{dG^W}\frac{G^W}{Y^W} = \frac{dY^W}{dG^W}(1-c_y)$. Thus, the world government spending multiplier is given by $\frac{dY^W}{dG^W} = \frac{\Delta_4}{\Delta_3}$. Note that by the definition of $\Delta_3$and Δ_4, the world multiplier is independent of the degree of home bias, ν. In the extreme of $\nu = 2$, with complete home bias we would be dealing with two closed economies. However,

equation (25) indicates that we can also view $\frac{\Delta_4}{\Delta_3}$ as equal to the multiplier in the closed economy . By definition $\Delta_4 > \Delta_3$ so the world, or closed economy multiplier is greater than one. From the forward looking inflation equation, we see that an increase in government spending will have a positive effect on inflation. Thus, a persistent increase in government spending will have a positive effect on expected inflation. At the zero lower bound, this will reduce the real interest rate. If an expansion in world government spending reduces world real interest rates, this will stimulate demand beyond the direct spending of government.

We can also construct a parallel equation for the relative levels of the economies by combining (18) and (20), using

$$\Delta_3^D \widehat{n}_t^R - \Delta_4^D \widehat{cg}_t^R = (1-\beta\mu)\widetilde{r}_t^R,$$

so that:

$$\widehat{n}_t^R = \frac{\Delta_4^D}{\Delta_3^D}\widehat{cg}_t^R + \frac{(1-\beta\mu)}{\Delta_3^R}\widetilde{r}_t^R \tag{26}$$

Note that the *relative* government spending multiplier is also greater than one, $\frac{\Delta_4^D}{\Delta_3^D} > 1$, by the definition of $\Delta_4^D > \Delta_3^D$. That is, a rise in home relative to foreign government spending increases home relative to the foreign output gap more than one for one. The relative multiplier may be worth further examination. Noting that $Ds_D = s$, we have:

$$\begin{aligned}\frac{\Delta_4^D}{\Delta_3^D} &= \frac{D\Delta_4^D}{D\Delta_3^D} = \frac{Ds_D(1-\beta\mu)(1-\mu) - \mu k\,(Ds_D)}{Ds_D(1-\beta\mu)(1-\mu) - \mu k\,(D\phi + Ds_D)} \\ &= \frac{\Delta_4}{\Delta_3 - (D-1)\mu k\phi}\end{aligned}$$

Notice that the home bias term only enters through D. Remember $D = (\sigma v(2-v) + (1-v)^2) > 1$ so $\frac{dD}{dv} = -2(\sigma-1)(v-1) < 0$. Therefore, the greater is home bias, the smaller will be the relative multiplier. We can also see that the relative multiplier is larger than the world multiplier. A country that is doing a relatively large amount of government spending will have a relatively low interest rate, stimulating demand intertemporally. But the relatively low interest rate will also translate into a terms of trade depreciation, concentrating demand on the output of the country with high government spending. Then the high spending country will receive even higher demand through the expenditure switching effect, thus explaining why the relative multiplier is stronger than the world multiplier.

Now, we can examine how government spending in one country will affect both its own own output gap, and the output gap of its trading partner. Consider the effect of spending

by the home economy alone, so that $\widehat{cg}_t^W = \widehat{cg}_t^R = \frac{\widehat{cg}_t}{2}$.

Proposition 1 *In a persistent global liquidity trap ($\mu > 0$), the multiplier of government spending on domestic output will be greater than one. The domestic output multiplier is a decreasing function of home bias; for $v < 2$, the domestic output multiplier is larger than the closed economy multiplier. The multiplier of government spending on foreign output is negative. This foreign multiplier is approaches zero as home bias increases.*

Proof. . $\widehat{n}_t = \widehat{n}_t^R + \widehat{n}_t^W$. Add both sides of (25) and (26) to get

$$\begin{aligned} \widehat{n}_t^R + \widehat{n}_t^W &= \widehat{n}_t = \frac{\Delta_4}{\Delta_3}\frac{\widehat{cg}_t}{2} + \frac{\Delta_4^D}{\Delta_3^D}\frac{\widehat{cg}_t}{2} + ... \\ &= \left[\frac{1}{2}\frac{\Delta_4}{\Delta_3} + \frac{1}{2}\frac{\Delta_4^D}{\Delta_3^D}\right]\widehat{cg}_t \end{aligned}$$

The multiplier on domestic government spending is the average of the closed economy multipier and the relative multiplier. As the relative multiplier is larger than the closed economy multiplier and both are greater than 1, the multiplier is also greater than one. Also, since the relative multiplier is a declining function of home bias, so is the multiplier on domestic government spending.

Also the foreign output gap is defined as: $\widehat{n}_t^* = \widehat{n}_t^W - \widehat{n}_t^R$. Subtract (26) from (25) to get

$$\begin{aligned} \widehat{n}_t^W - \widehat{n}_t^R &= \widehat{n}_t^* = \frac{\Delta_4}{\Delta_3}\frac{\widehat{cg}_t}{2} - \frac{\Delta_4^D}{\Delta_3^D}\frac{\widehat{cg}_t}{2} + ... \\ &= \frac{1}{2}\left[\frac{\Delta_4}{\Delta_3} - \frac{\Delta_4^D}{\Delta_3^D}\right]\widehat{cg}_t \end{aligned}$$

The multiplier on foreign goverment spending is half the difference between the closed economy multiplier and the relative multiplier. Since the relative multiplier is larger, the multiplier on foreign government spending is negative. Also, since the relative multiplier gets smaller as v gets larger, the multiplier on foreign government spending gets closer to zero as home bias increases. ■

In addition to the standard intertemporal effects, in a liquidity trap, expansionary government spending will weaken the domestic currency, leading to enhanced demand beyond the closed economy multiplier. By the same token, this tends to reduce demand for foreign output through the expenditure switching effect. But the greater is home bias, the less important is the expenditure switching effect. As home bias increases, the domestic multiplier goes to the closed economy multiplier and the foreign multiplier will go to zero.

One important factor in the calculation of the size of government spending multipliers is the degree of persistence of the liquidity trap shock itself. This is made clear in the following proposition.

Proposition 2 *In a transitory global liquidity trap ($\mu = 0$), the multiplier of government spending on domestic output will be equal to one regardless of home bias. The multiplier of government spending on foreign output is zero regardless of home bias.*

Proof. If $\mu = 0$ then $\Delta_3^D = \Delta_4^D = s_D$ and $\Delta_3 = \Delta_4 = s$ so the closed economy multiplier equals the relative multiplier $\frac{\Delta_4}{\Delta_3} = \frac{\Delta_4^D}{\Delta_3^D} = 1$. ■

If the initial shock is purely transitory, so that the government spending response is also transitory, there will be no impact of government spending on *expected* inflation. Thus, there will be no effects on real interest rates, and therefore no intertemporal or expenditure switching effects. An increase in government demand in the home economy will translate one-for-one into an increase in domestic output but will have no impact on the foreign economy. More generally, the size of the government spending multiplier will be much smaller, the less persistent is the initial savings shock.

5 Optimal Discretionary Policy

We now turn to the key part of the paper, which is the analysis of the optimal policy response to a liquidity trap shock. We explore optimal *cooperative* fiscal policy responses, where governments in each country cooperate on policies. In addition, we will show that in some cases, even when natural real interest rates in both countries are pushed below zero, monetary policy may be effectively used in the foreign country (which is not directly affected by the preference shock).

While a complete analysis of the determination of fiscal and monetary policy in a global liquidity trap would also require an exploration of the strategic interaction between non-cooperative policy authorities, this raises difficult technical issues (see Benigno and Benigno 2005), and so is left as a topic for future research. Focusing on the cooperative problem is a desirable first approach. since it sets out a benchmark for choosing a policy so as to maximize world welfare in response to a negative demand shock which undermines the normal mechanism of monetary policy[6].

[6]The cooperative approach to fiscal policy in a global liquidity trap is not necessarily unrealistic. In the immediate aftermath of the financial crash of 2008, the G20 group agreed on a joint policy response to the crisis which assigned target levels of fiscal stimulus to each member country.

In order to analyze optimal policy, we first need to define an objective function. As shown in Cook and Devereux (2010a), a second order approximation to an equally weighted world social welfare can also be constructed in world averages and world differences.

$$\begin{aligned} V_t &= -(\widehat{n}_t^R)^2 \cdot \frac{A}{2} - (\widehat{n}_t^W)^2 \frac{B}{2} - (\widehat{cg}_t^R)^2 \cdot \frac{F}{2} - (\widehat{cg}_t^W)^2 \cdot \frac{H}{2} - J(\widehat{n}_t^R)(\widehat{cg}_t^R) \\ &\quad -L(\widehat{n}_t^W)(\widehat{cg}_t^W) - \frac{\theta}{4k}(\pi_t^W + \pi_t^R)^2 - \frac{\theta}{4k}(\pi_t^W - \pi_t^R)^2 \end{aligned} \tag{27}$$

where

$$\begin{aligned} A &\equiv \left\{ \frac{(1+\phi c_y)}{c_y^2} + \frac{(\sigma - D)}{D}(1 + \frac{(1-c_y^2)}{c_y^2 D}) \right\} \\ B &\equiv \frac{(\sigma + \phi c_y)}{c_y^2} = \frac{(s+\phi)}{c_y}, \\ H &\equiv \frac{1}{(1-c_y)} \frac{\sigma}{c_y^2} = \frac{1}{(1-c_y)} \frac{s}{c_y} \quad L \equiv \frac{-\sigma}{c_y^2} = \frac{s}{c_y} \end{aligned}$$

$$\begin{aligned} J &\equiv \left[-\frac{1}{c_y^2} - \frac{(\sigma - D)}{c_y^2 D^2}(1 + (v-1)(D-1)c_y^2) \right] \\ F &\equiv \frac{((1-c_y) + c_y \sigma)}{(1-c_y)c_y^2} + \frac{(\sigma - D)}{c_y^2 D^2}(1 + (v-1)(D-1)c_y^2) \end{aligned}$$

In the analysis here, we will concentrate exclusively on optimal discretionary fiscal policy responses. This implies that governments cannot commit themselves to future policy actions, so all policies become history independent. As a result, when both countries are in a liquidity trap, with desired real interest rates below zero, monetary policy becomes entirely ineffective, since policy makers cannot commit to expansionary monetary policy in the future, after natural real interest rates have risen again. In some cases, however, the foreign policy-maker may not be in a liquidity trap, because it can choose to set its policy rate higher than the natural real interest rate in the foreign country. Thus, we allow for the possibility that the foreign policy interest rate is not constrained by the zero lower bound.

In the case of policy discretion, the cooperative fiscal policy response to a liquidity trap involves maximizing (27) in each period, taking expectations of all future variables as given, subject to the inflation equations for world averages and differences, given by (17) and (19), and subject to the non-negativity constraints on nominal interest rates in each country. Since the non-negatively constraint on the home country policy rate will also bind for the

duration of the shock, we only impose the non-negativity condition on the foreign interest rate.

Given this, we have the Lagrangean expression:

$$\begin{aligned}
\max_{\widehat{n}_t^R,\widehat{n}_t^W,cg_t^R,cg_t^W,\pi_t^W,\pi_t^R} L_t \;=\; & -(\widehat{n}_t^R)^2\cdot\frac{A}{2}-(\widehat{n}_t^W)^2\frac{B}{2}-(c\widehat{g}_t^R)^2\cdot\frac{F}{2}-(\widehat{cg}_t^W)^2\cdot\frac{H}{2} \\
& -J(\widehat{n}_t^R)(\widehat{cg}_t^R)-L(\widehat{n}_t^W)(\widehat{cg}_t^W)-\frac{\theta}{4k}(\pi_t^W+\pi_t^R)^2-\frac{\theta}{4k}(\pi_t^W-\pi_t^R)^2 \\
& +\lambda_{1t}\left[\pi_t^W-k(\phi+s)\widehat{n}_t^W+ks\cdot\widehat{cg}_t^W-\beta E_t\pi_{t+1}^W\right] \\
& +\lambda_{2t}\left[\pi_t^R-k(\phi+s_D)\widehat{n}_t^R+ks_D\widehat{cg}_t^R-\beta E_t\pi_{t+1}^R\right] \\
& +\psi_{1t}\left[sE_t(\widehat{n}_{t+1}^W-\widehat{n}_t^W)-sE_t(\widehat{cg}_{t+1}^W-\widehat{cg}_t^W)-E_t\left(\frac{r_t^*}{2}-\widetilde{r}_t^W-\pi_{t+1}^W\right)\right] \\
& +\psi_{2t}\left[s_DE_t(\widehat{n}_{t+1}^R-\widehat{n}_t^R)-s_DE_t(\widehat{cg}_{t+1}^R-\widehat{cg}_t^R)-E_t\left(-\frac{r_t^*}{2}-\frac{\widetilde{r}_t^R}{2}-\pi_{t+1}^R\right)\right] \\
& +\gamma_t\left[r_t^*\right]
\end{aligned}$$

The first two constraints are the inflation equations in average and relative terms. The second two constraints are the average and relative 'IS' equations. The final constraint is the non-negativity constraint on the foreign policy interest rate. The policy optimum involves the choice of the output gaps, the government spending 'gaps', and the inflation rates to maximize this Lagrangean. The first order conditions of the maximization are:

$$-A\widehat{n}_t^R-J(\widehat{cg}_t^R)=\lambda_2k(\phi+s_D)+s_D\psi_2\equiv\Upsilon_2 \tag{28}$$

$$-B\widehat{n}_t^W-L(\widehat{cg}_t^W)=\lambda_1k(\phi+s)+s\psi_1\equiv\Upsilon_1 \tag{29}$$

$$F\widehat{cg}_t^R+J(\widehat{n}_t^R)=ks_D\lambda_2+s_D\psi_2=\Upsilon_2-\phi(k\lambda_1) \tag{30}$$

$$H\widehat{cg}_t^W+L(\widehat{n}_t^W)=ks\lambda_1+s\psi_1=\Upsilon_2-\phi(k\lambda_1) \tag{31}$$

$$k\lambda_1=\theta\pi_t^W \tag{32}$$

$$k\lambda_2=\theta\pi_t^R \tag{33}$$

$$\psi_{2t}-\psi_{1t}+\gamma_t=0 \tag{34}$$

We analyze these first order conditions in the next three propositions.

To begin with, assume that the zero bound constraint on r_t^* is binding. In this case, $\gamma_t>0$, and we may solve (28)-(33) for output gaps, government spending gaps, and inflation

rates. First insert (28) and (33) into (30) to get:

$$-(J+F)\widehat{cg}_t^R - (J+A)(\widehat{n}_t^R) = \phi\theta\pi_t^R \tag{35}$$

Then insert (29) and (32) into (31) to get:

$$-(H+L)\widehat{cg}_t^W - (B+L)\widehat{n}_t^W = \phi\theta\pi_t^W \tag{36}$$

Using these conditions, we establish the following:

Proposition 3 *In a global liquidity trap, the optimal discretionary average level of the fiscal gap, $\widehat{cg}_t^W$, will be positive, independent of the degree of home bias, and equal to the optimal closed economy fiscal gap*

Proof. See Appendix ■

The full proof of proposition () is contained in the Appendix. To see the intuition, we first take equation (36). Then, using the condition that all expectations terms of variable x_t will satisfy $E_t(x_{t+1}) = \mu x_t$, we can solve (17) and (18) for a second relationship between $\widehat{cg}_t^W$ and $\widehat{n}_t^W$, given by:

$$-\Delta_3\widehat{n}_t^W + \Delta_4\widehat{cg}_t^W = -(1-\beta\mu)\widehat{r}_t^W \tag{37}$$

Then, combining (36) and (37), we arrive at

$$[(\Delta_3 HL + \Delta_4 BL) + \phi f(1-\mu)s]\widehat{cg}_t^W = -[f(\phi+s) + (1-\beta\mu)BL]\,\widehat{r}_t^W \tag{38}$$

where the expressions $HL>0$ and $BL > 0$ and $f > 0$ are defined in the Appendix. From this, it is clear that when the world average natural rate falls, the world average fiscal gap must increase.

Note that, outside a liquidity trap, it would never be desirable to have a non-zero fiscal gap. But when the policy rate is constrained by the zero lower bound, fiscal spending, by creating anticipated inflation, can reduce real interest rates, stimulate private demand, and reduce the current world output gap. Moreover, since all terms in (38) are independent of the degree of home bias v, the optimal fiscal gap for the world average is the same as that in a closed economy, despite the fact that the initial shock in this exercise emanates from the home country alone.

The key focus of interest however, is the breakdown of the fiscal spending policies between the home and foreign economy. On the surface, because opening up fiscal gaps is costly

in itself, in welfare terms, it would seem that the optimal cooperative policy should involve sharing the burden of fiscal adjustment across the two countries. But the appropriate response for fiscal policy depends upon how the policy itself can impact upon expected inflation, and the output gap in each country, which in turn depends on the degree of home bias, and other features of the model, as discussed in section () above. Nevertheless, it is easy to establish that the home fiscal gap should always be positive, as stated in the following proposition.

Proposition 4 *In a global liquidity trap, both the world relative fiscal gap and the optimal home fiscal gap will be positive,* $\widehat{cg}_t^R > 0$ *and* $\widehat{cg}_t > 0$

Proof. *See Appendix.* ■

The first part of the proposition is established along the same lines as Proposition (). Putting together the first order conditions for the world relative variables, and the conditions from the model equations (19) and (20), we may derive a relationship between the world relative shock to the natural interest rate and the optimal response of the world relative fiscal spending gap as follows:

$$\left[\Delta_3^D HL + \Delta_4^D JA + f\phi s_D(1-\mu)\right]\widehat{cg}_t^R = -\left[f(\phi + s_D) + (1-\beta\mu)JA\right]\widetilde{r}_t^R \tag{39}$$

where the terms HL, and JA are positive, and defined in the Appendix. Since $\widetilde{r}_t^R > 0$ for a shock that emanates from the home economy, it must be that the relative fiscal gap $\widehat{cg}_t^R$ is positive also. But since from the definition of $\widehat{cg}_t^W$ and $\widehat{cg}_t^R$, we have $\widehat{cg}_t = \widehat{cg}_t^W + \widehat{cg}_t^R$, it then follows that the home fiscal spending gap is positive. Thus, an optimal response to the liquidity trap for the home country is to follow an expansionary fiscal policy.

What is not so clear, however, is the optimal fiscal response for the foreign economy, when both economies are in a liquidity trap. Note, that $\widehat{cg}_t^* = \widehat{cg}_t^W - \widehat{cg}_t^R$. Since both $\widehat{cg}_t^W$ and $\widehat{cg}_t^R$ are positive, we cannot obviously sign this expression. Using the solutions from the Appendix, we can write

$$\begin{aligned} & \widehat{cg}_t^W - \widehat{cg}_t^R \\ = & -\frac{[f(\phi+s)+(1-\beta\mu)BL]}{[(\Delta_3 HL + \Delta_4 BL) + \phi f(1-\mu)s]}\widetilde{r}_t^W + \frac{[f(\phi+s_D)+(1-\beta\mu)JA]}{[\Delta_3^D HL + \Delta_4^D JA + f\phi s_D(1-\mu)]}\widetilde{r}_t^R \end{aligned} \tag{40}$$

The first term on the right hand side is positive, while the second term is negative, when the (negative) shock hits the home economy. So in principle, it would seem as if the foreign fiscal expansion (when both countries are in a liquidity trap) could go either way. But we

must also consider the condition (34), since for some parameters or shock values, the optimal foreign policy rate may be above zero.

We tackle this problem by solving the model numerically. To evaluate the economy quantitatively, we adopt some parameters from Cook and Devereux (2010). Let $\beta = 0.99$, so each period is a quarter, and this translates to a value of the steady state interest rate $\overline{r} = 0.01$. The Frisch labor supply elasticity is $\phi = 1$. Price stickiness is $\kappa = 0.85$, so that $k = 0.027$, as in Christiano et al. (2009). Let the share of government in output be 20 percent, $c_y = 0.8$. We assume the inverse of intertemporal demand elasticity σ, is equal to 2. The persistence of the demand shock is set at 0.8 ($\mu = 0.8$) implying an expected length of the slump to be 5 quarters. We set the elasticity of substitution between good varieties within a country, θ, equal to 5. Finally, we set the preference shock in the home country ε_t so that at $v = 1$ (the case without any home bias), the natural real interest rate at the quarterly frequency would fall from 1 percent to -1.7 percent, with persistence μ.

Figure 1 illustrates the response of home and foreign output gaps, home and foreign government spending gaps, home and foreign inflation, and the foreign country optimal policy rate, as well as the foreign natural real interest rate, for different values of v, when the optimal fiscal and monetary policy response is chosen. The figure takes account of condition (34), so that, at each value of v, the non-negativity constraint on r_t^* is tested, and if it is not binding, the optimal foreign policy rate is chosen. The first thing to note is that at $v = 1$, then clearly the liquidity trap is binding in both countries, and all variables respond in the same way in the two countries. The output gap falls by over 7 percent in both countries, and this is coupled with a fall in the rate of inflation by equal amounts. Since both countries are affected equally, it must be that the zero lower bound is binding in both countries, and the foreign policy rate is set at zero. The response of policy is illustrated by the positive response of the fiscal gap in each country. Thus, fiscal policy should behave countercyclically, and equally so in each country for a world without home bias in preferences.

Now, as v rises above unity, we know that the impact of the shock on the foreign natural interest rate becomes muted, while the opposite occurs for the home natural interest rate. The negative response of the foreign output gap is then reduced, while that of the home output gap is increased. As v rises more and more, we find, as discussed above, that the foreign fiscal gap may actually increase. A similar dynamic occurs in the response of the inflation rates in the two countries - home inflation becomes more and more negative as v rises, while the negative response of foreign inflation becomes less and less. The optimal response of fiscal policy gaps is illustrated in panel b of the Figure. As v rises, home fiscal policy becomes more aggressive, while the foreign fiscal policy becomes more muted.

Panel d illustrates the optimal response of the foreign country policy rate, alongside the foreign country natural real interest rate. Note that at $v = 1$, the foreign policy rate is stuck at zero, while the natural real interest rate is at -0.017. As v rises, the response of the foreign natural interest rate becomes less and less, as is obvious from the formula (10). Eventually, as v rises to 2, the foreign country would be entirely unaffected by the shock, and the foreign natural interest rate would rise to -0.01, the steady state natural interest rate. But the key feature of panel d is the the foreign country will raise its policy rate above zero for values of $\widetilde{r}_t^* < 0$. That is, the foreign country will choose positive interest rates, as part of an optimal cooperative policy package, even though, by the usual closed economy logic, it should be still in a liquidity trap, since its natural rate of interest is below zero. Equivalently, the foreign country will not follow a policy of offsetting the movement in the foreign natural interest rate, to the greatest extent that it can, so long as the policy rate is above the zero bound. Rather, it chooses to raise policy rates, even though $\widetilde{r}_t^* < 0$. In fact, panel d makes clear that as v rises, it may be optimal for the foreign country to raise its policy rate *above the steady state natural rate of interest.*

Thus, surprisingly, an optimal cooperative policy response to a liquidity trap can be characterized by expansionary fiscal policy in all countries, but contractionary monetary policy in the least affected country. This seemingly paradoxical result is related to the results of section (4) above. As v rises, the home economy is significantly more affected by the negative demand shock. An optimal policy response is to raise world demand, and to re-orient world demand towards the home country. The raising of the foreign policy rate is associated with an appreciation of the foreign currency, which generates an additional expenditure switching of demand towards the home country. Since the impact of the home country shock on foreign output is positive in any case, when v is sufficiently greater than unity, the rise in the foreign policy rate has the additional benefit that it helps to minimize the response of the foreign output gap to the home country shock.

5.1 Optimal Policy with Semi-commitment

To be added..

6

7 Conclusions

To be added

References

[29] Auerbach, Alan, and Maurice Obstfeld (2006) "The Role for Open Market Operations in a Liquidity Trap", American Economic Review.

[29] Beetsma Roel and Henrik Jensen, (2005), "Monetary and Fiscal Policy Interactions in a Micro-Founded Model of a Monetary Union", Journal of International Economics, 67, 320-352.

[3] Benhabib, Jess, Stephanie Schmitt Grohe, and Martin Uribe, (2002) "Avoiding Liquidity Traps", Journal of Political Economy.

[29] Benigno, Gianluca and Pierpaolo Benigno (2006), "Designing Targeting Rules for International Monetary Policy Cooperation", Journal of Monetary Economics, 53, 473-506.

[29] Blanchard, Olivier and Roberto Perotti, (2002), "An Empirical Characterization of the Dynamic Effects of changes in Government Spending and Taxes on Output", Quarterly Journal of Economics, 117, 1329-1368.

[29] Bodenstein, Martin, Christopher J. Erceg, and Luca Guerrieri (2009) "The Effect of Foreign Shocks When Interest Rates are at Zero" IFdp 983, Board of Governors of the Federal Reserve System.

[7] Christiano, Larry, Martin Eichenbaum, and Sergio Rebelo, (2009) "When is the Government Spending Multiplier Large?", Northwestern University.

[29] Cook, David, and Michael B. Devereux, (2010a) "International Monetary and Fiscal Policy Coordination in a Liquidity Trap", European Economic Review, forthcoming.

[29] Cook, David and Michael B. Devereux (2010b) "Global vs Local Liquidity Traps", mimeo

[10] Cogan, John, Tobias Cwik, John Taylor, and Volker Wieland, (2009) "New Keynesian Versus Old Keynesian Government Spending Multipliers", mimeo

[29] Davig, Troy and Eric M. Leeper (2009) "Monetary Fiscal Interactions and Fiscal Stimulus", NBER d.p. 15133.

[29] Devereux, Michael B. (2010) "Fiscal Deficits, Debt, and Monetary Policy in a Liquidity Trap", Central Bank of Chile, Working Paper 581.

[29] Engel, Charles (2010), "Currency Misalignments and Optimal Monetary Policy: A Reinvestigation", mimeo

[29] Eggerston, Gauti, (2010) "What Fiscal Policy is Effective at Zero Interest Rates?", NBER Macroeconomics Annual, forthcoming.

[15] Eggertson Gauti and Michael Woodford, (2003) "The Zero Interest Bound and Optimal Monetary Policy", Brookings Papers on Economic Activity.

[16] Eggertsson, Gauti and Michael Woodford, (2005) "Optimal Monetary and Fiscal Policy in a Liquidity Trap", ISOM Annual.

[29] Faia, Ester and Tommaso Monacelli, (2008) "Optimal Monetary Policy in a Small Open Economy with Home Bias," Journal of Money, Credit and Banking, 40, 721-750.

[29] Fujiwara, Ippei, Nao Sudo, and Yuki Teranishi, (2009) "The Zero Lower Bound and Monetary Policy in a Global Economy: A Simple Analytical Investigation", International Journal of Central Banking.

[29] Fujiwara, Ippei, Tomoyuki Nakajimaz, Nao Sudox, and Yuki Teranishi, (2010) "Global Liquidity Trap", mimeo, Bank of Japan

[29] Fujiwara, Ippei, and Kozu Ueda (2010) "The Fiscal Multiplier and Spillover in a Global Liquidity Trap", IMES d.p. 2010-E-3. Bank of Japan.

[29] Jeanne, Olivier (2009) "The Global Liquidity Trap", mimeo

[22] Krugman, Paul, (1998), "It's baaack: Japan's Slump and the Return of the Liquidity Trap", Brookings Papers on Economic Activity", 2, 137-87.

[23] Jung, Taehun, Yuki Terinishi, and Tsutomo Watanabe, (2005) "The Zero Bound on Nominal Interest Rates and Optimal Monetary Policy", Journal of Money Credit and Banking.

[29] Monacelli, Tomasso and Roberto Perotti, (2008), "Fiscal Policy, Wealth Effects and Markups", NBER d.p. 14584.

[29] Perotti, Roberto, (2007) "In Search of the Transmission Mechanism of Fiscal Policy", NBER 13143.

[26] Rotemberg, J. and M. Woodford (1998) "An Optimization Based Econometric Framework for the Evaluation of Monetary Policy", NBER Technical Working Paper 233.

[29] Svensson, Lars E. (2003) "Escaping from a Liquidity Trap and Deflation: The Foolproof Way and Others" Journal of Economic Perspectives 17, 145-166.

[28] Woodford, Michael (2003) *Interest and Prices,* MIT Press

[29] Woodford, Michael (2010) "Simple Analytics of the Government Expenditure Multiplier", mimeo, Columbia University.

Figure 1: Liquidity Trap Shock in the Home Economy







