The Great Diversification and its Undoing*

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Abstract

We investigate the hypothesis that macroeconomic fluctuations are primitively the results of many microeconomic shocks, and show that it has significant explanatory power for the evolution of macroeconomic volatility. We define "fundamental" volatility as the volatility that would arise from an economy made entirely of idiosyncratic microeconomic shocks, occurring primitively at the level of sectors or firms. In its empirical construction, motivated by a simple model, the sales share of different sectors vary over time (in a way we directly measure), while the volatility of those sectors remains constant. We find that fundamental volatility accounts for the swings in macroeconomic volatility in the US and the other major world economies in the past half century. It accounts for the "great moderation" and its undoing. Controlling for our measure of fundamental volatility, there is no break in output volatility. The initial great moderation is due to a decreasing share of manufacturing between 1975 and 1985. The recent rise of macroeconomic volatility is due to the increase of the size of the financial sector. As the origin of aggregate shocks can be traced to identifiable microeconomic shocks, we may better understand the origins of aggregate fluctuations. (JEL: E32, E37)

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1 Introduction

This paper explores the hypothesis that changes in the microeconomic composition of the economy during the post-war period can account for the "great moderation" and its unraveling, both in the US and in the other major world economies. We call "fundamental volatility" the volatility that would come only from microeconomic shocks. If aggregate shocks come in large part from microeconomic shocks (augmented by amplification mechanisms), then aggregate volatility should track fundamental volatility. To operationalize this idea, the key quantity we consider (which constitutes one departure from other studies) is the following definition of "fundamental volatility":

$$\sigma_{Ft} = \sqrt{\sum_{i=1}^{n} \left(\frac{S_{it}}{\text{GDP}_t} \right)^2 \sigma_i^2} \tag{1}$$

where S_{it} is the gross output (not just value added) of sector i, and σ_i is the standard deviation of the total factor productivity (TFP) in the sector. Note that in this measure the weights do not add up to one. These are the "Domar weights" that research in productivity studies (Domar 1961, Hulten 1978) has identified as the proper weights to study the impact of microeconomic shocks.

Figure 1 plots σ_{Ft} for the US. We see a local peak around 1975, then a fall (due to the decline of manufacturing), followed by a new rise (which we will relate to the rise of finance). This looks tantalizingly like the evolution of the volatility of US GDP growth. Indeed, we show statistically that the volatility of the innovations to GDP is well explained by the fundamental volatility σ_{Ft}. In particular, our measure explains the great moderation: the existence of a break in the volatility of US GDP around 1984. After controlling for fundamental volatility, there is no break in GDP volatility. Our measure also accounts for the rise of GDP volatility, as finance became large from the mid-1990s onward, creating an increase in fundamental volatility.

In Figure 2 we present a similar analysis for the major economies for which we could get disaggregated data about shares and TFP movements: Japan, Germany, France, and the United Kingdom. The results also indicate that fundamental volatility tracks GDP volatility.

Our conclusion is that fundamental volatility appears to be a quite useful explanatory construct. It provides an operational way to understand the evolution of volatility, and shed more light on its origins.

Hence, our paper may bring us closer to a concrete understanding of the origin of macro-

economic shocks. What causes aggregate fluctuations? It has proven convenient to think about aggregate productivity shocks, but their origin is mysterious: What is the common high-frequency productivity shock that affects Wal-Mart and Boeing? This is why various economists have progressively developed the hypothesis that macroeconomic fluctuations can be traced back to microeconomic fluctuations. This literature includes Long and Plosser (1983), who proposed a baseline model where sectors have constant sizes. Its implementation is relatively complex, as it requires sector sizes that are constant over time (unlike the evidence we rely on), and the use of input-output matrices. Horvath (1998, 2000) perhaps made the greatest strides toward developing these ideas empirically, in the context of a rich model with dynamic linkages. The richness of the model might make it difficult to see what drives its empirical features, and certainly prevents the use of a simple concept like the concept of fundamental volatility. Carvalho (2009) shows how viewing the links between narrowly defined sectors as a network gives extra flesh to the way microeconomic fluctuations affect aggregate volatility.

Dupor (1999) disputes that the origins of shocks can be microeconomic, on the grounds of the central limit theorem: if there is a large number of sectors, aggregate volatility should vanish, proportionally to the square root of the number of sectors. Hence Horvath's result would stem from poorly disaggregated data.[1] Gabaix (2010) points out that "sectors" may be arbitrary constructs, and the fat-tailed Zipf distribution of firms (or perhaps very disaggregated sectors) necessarily leads to a high amount of aggregate volatility coming from microeconomic shocks, something he dubs the "granular" hypothesis. In that view, microeconomic fluctuations create a fat-tailed distribution of firms or sectors (Simon 1955, Gabaix 1999, Luttmer 2007). In turn, the large firms or sectors coming from that fat-tailed distribution create GDP fluctuations. Gabaix also highlights the conceptual usefulness of the notion of fundamental volatility. Di Giovanni and Levchenko (2009) how that perspective helps explain the volatility of trade fluctuations across countries.

Against this backdrop, we use a simple way to cut through the complexity of the situation, and rely on a simple, transparent microeconomic construct, fundamental volatility, to predict an important macroeconomic quantity, GDP volatility.

By bringing fundamental volatility into the picture, we contribute to the literature on the origins of the "great moderation," a term coined by Stock and Watson (2002): the decline in

[1] Interesting other conceptual contributions for the micro origins of macro shocks include Bak *et al.* (1993), Jovanovic (1987), Durlauf (1993), and Nirei (2006).

the volatility of US output growth around 1984, up until about 2007 and the financial crisis. The initial contributions (McConnell and Perez-Quiros 2000, Blanchard and Simon 2001) diagnosed the decline in volatility, and conjectured that some basic explanations (including sectoral shifts – to which we will come back) did not seem promising. Perhaps better inventory management (Irvine and Schuh 2005) or better monetary policy were prime candidates. However, given the difficulty of relating those notions to data, much of the discussion was conjectural. Later, more full-fledged theories of the great moderation have been advanced. Arias, Hansen, and Ohanian (2007) attribute the changes in volatility to changes in TFP volatility within a one-sector model. Our work sheds light on the observable microeconomic origins of this change in TFP volatility. Justiniano and Primiceri (2008) document that much of the great moderation could be traced back to a change in the volatility of the investment demand function. Gali and Gambetti (2009) document a change in both the volatility of initial impulses and the impulse-response mechanism. Compared to those studies, we use much more disaggregated data, which allows us to calculate the fundamental volatility of the economy. Because we use richer disaggregated data, we can obviate some of the more heavy artillery of dynamic stochastic general equilibrium (DSGE) models, and have a parsimonious toolkit to think about volatility. We defer the discussion of Jaimovich and Siu (2009) to Section 6.2. We view our proposal as complementary to those other mechanisms presented in the literature.

Finally, we relate to the literature on technological diversification and its effects on aggregate volatility. Imbs and Wacziarg (2003) as well as Koren and Tenreyro (2007) have shown that cross-country variation in the degree of diversification in the menu of technologies being used helps explain cross-country variation in GDP volatility and its relation with the level of development of an economy. Here we concentrate on the time series dimension of this mechanism and how it can generate the observed long swings in volatility for a given economy. We also differ in that we build our measures of diversification and sectoral level volatility from microeconomic TFP accounting rather than from unspecified sectoral shocks. Koren and Tenreyro (2009) and Moro (2009) on the other hand provide models of, respectively, technological diversification (through an expanding number of input varieties) and structural change in a two-sector economy. Their respective calibrations show that it is possible to build models that generate a decline of volatility with the level of development/structural change that is consistent with empirical evidence. However, they do not contemplate the possibility of a reversal of these patterns. We instead show, through our fundamental volatility construct, that such a reversal is present and is key to explaining the recent period of higher volatility.

The methodological principle of this paper is to use as simple and transparent an approach as possible. In particular, we find a way to avoid the use of the input-output matrix, which has no claim to be stable over time, and is not necessary in our framework. We examine the economics through a very simple two-period model, rather than an infinite-horizon DSGE model. Useful as they are for a host of macroeconomic questions, those models have many free parameters, and we find it instructive to simply focus our attention on a zero-free parameter construct, the fundamental volatility of the economy. This said, a potentially fruitful next step is to build a DSGE model on the many sectors in the economy.

Section 2 presents a very simple framework that motivates our concept of fundamental volatility, and its implementation. Section 3 presents the basic empirical results. Section 4 presents a brief history of fundamental volatility. Section 5 how the role of correlation in the business cycle, and why previous analyses were pessimistic about the role of microeconomic shocks. Section 6 expands on a variety of points, including the use of value added vs. sales, and firms vs. sectors. Section 7 concludes on the role of policy and the use of fundamental volatility as an early warning system. The Appendices provide an account of the data and procedures we employ, as well as the proofs.

2 Framework and Motivation

In this section, we present a very simple model that exposes the basic ideas and motivates our empirical work. Section 5 presents a fuller model.

There are n sectors that produce intermediate and final goods, and two primitive factors, capital and labor. Sector i uses inputs capital and labor inputs K_i and L_i, and a vector of intermediary inputs $\boldsymbol{X}_i = (X_{ij})_{j=1...n} \in \mathbb{R}^n$: it uses a quantity X_{ij} from sector j. It produces a gross output $Y_i = A_i F_i \left(K_i^{1-\alpha} L_i^{\alpha}, \mathbf{X}_i\right)$, where F_i is homogenous of degree 1. We call $\boldsymbol{C} = (C_1, \ldots, C_n)$ and $\boldsymbol{A} = (A_1, \ldots, A_n)$ the vectors of consumption and productivity.

The utility function is $\mathcal{C}(\boldsymbol{C}) - L^{1+1/\phi}$, and where $\mathcal{C}$ is homogenous of degree 1, so that $\mathcal{C}$ is like an aggregate good. Hence, given inputs K and L, the aggregate production function is defined as:

$$\begin{aligned} F(K, L; \boldsymbol{A}) &= \max_{\substack{C_i, L_i, K_i, X_{ij} \\ 1 \leq i,j \leq n}} \mathcal{C}(\boldsymbol{C}) \text{ subject to} \\ \sum_i K_i &\leq K; \qquad \sum_i L_i \leq L; \qquad \forall i, C_i + \sum_j X_{ji} \leq A_i F_i \left(K_i^{1-\alpha} L_i^{\alpha}, \boldsymbol{X}_i\right). \end{aligned}$$

Note that this economic structure admits quite general linkages between sectors via the production functions F_i and the factor markets. The next Lemma, whose proof is in the Appendix, describes basic aggregation results in this economy.

Lemma 1 *The aggregate production function can be written:*

$$F\left(K, L; \boldsymbol{A}\right) = \Lambda\left(\boldsymbol{A}\right) K^{1-\alpha} L^{\alpha}$$

After sectoral-level TFP shocks dA_i, the shock to aggregate TFP Λ is:

$$\frac{d\Lambda}{\Lambda} = \sum_{i=1}^{N} \frac{S_i}{Y} \frac{dA_i}{A_i} \tag{2}$$

where S_{it} is the dollar value of the gross output of sector i.

Formula (2) is nothing Hulten (1978)'s result. We rewrite it slightly, using the "hat" notation for fractional changes, $\widehat{Z} \equiv dZ/Z$.[2], and time subscript. Shock $\widehat{A}_{it}$ to the productivity in sector i result in a aggregate TFP growth $\widehat{\Lambda}_t$ expressed as:

$$\widehat{\Lambda}_t = \sum_{i=1}^{N} \frac{S_{it}}{Y_t} \widehat{A}_{it} \tag{3}$$

where S_{it} is the dollar value of sales (gross output) in sector i, and Y_t is GDP. S_{it}/Y_t is called the "Domar" weight.

Note that the sum of the weights $\sum_{i=1}^{N} S_{it}/Y_t$ can be greater than 1. This is a well-known and important feature of models of complementarity. If on average the value added to sales ratio is 2, and each sector has a TFP increase of 1%, the aggregate TFP increase is 2%. This effect comes from the fact that a Hicks-neutral productivity shock increases gross output (sales), not just value added, and has been analyzed by Domar (1961), Hulten (1978), and Jones (2009).[3]

[2] The rules are well known and come from taking the logarithm and differentiating. For instance, $\widehat{X^{\alpha} Y^{\beta} Z^{\gamma}} = \alpha \widehat{X} + \beta \widehat{Y} + \gamma \widehat{Z}$.

[3] The intuition for (3) is the following: Suppose there are just two sectors, say cars (a final good, sector 1) and plastics (both a final and intermediary good, sector 2). Cars use plastics as an intermediary input. Suppose there is productivity growth of $\widehat{A}_{1t} = 1\%$ in cars and $\widehat{A}_{2t} = 3\%$ in plastics. Suppose that, after the shock, there is no reallocation of factors. We then have 1% more cars in the economy and 3% more plastics. Those goods have not yet been reallocated to production, but still, they have a "social value,"

Consider the baseline case where productivity shocks $\widehat{A}_{it}$ are uncorrelated across i's, and unit i has a variance of shocks $\sigma_i^2 = var\left(\widehat{A}_{it}\right)$. Then, we have $\sigma_\Lambda = \sigma_F$, where we define:

$$\sigma_{Ft} = \sqrt{\sum_{i=1}^{N} \left(\frac{S_{it}}{Y_t}\right)^2 \sigma_i^2} \tag{4}$$

This defines the "fundamental" volatility, which comes from microeconomic shocks. Gabaix (2010) calls this the "granular" volatility.

To see the changes in GDP, we assume that capital can be rented at a price r. The agent's consumption is $Y - rK$ with $Y = \Lambda K^{1-\alpha} L^{\alpha}$. The competitive outcomes implement the planner's problem, which is to maximize the agent's utility subject to the resource constraint:

$$\max_{K,L} C - L^{1+1/\varphi} \text{ subject to } C = \Lambda K^{1-\alpha} L^{\alpha} - rK.$$

The solution is obtained by standard methods detailed in the proof of Proposition 2.

$$Y = k\Lambda^{\frac{1+\varphi}{\alpha}},$$

for an unimportant constant k. Taking logs, $\ln Y = \frac{1+\varphi}{\alpha} \ln \Lambda + \ln k$, and a change in TFP $\widehat{\Lambda}$ creates a change in GDP equal to

$$\widehat{Y} = \frac{1+\varphi}{\alpha} \widehat{\Lambda}.$$

Given that the volatility of TFP is the fundamental volatility σ_{Ft}, the volatility of GDP is $\sigma_{GDPt} = \frac{1+\varphi}{\alpha} \sigma_{Ft}$. We summarize the situation in the next proposition.

Proposition 2 *The volatility of GDP growth is*

$$\sigma_{Yt} = \mu \cdot \sigma_{Ft}, \tag{5}$$

where the fundamental volatility σ_{Ft} is given by (4), and the productivity multiplier μ is equal to

$$\mu = \frac{1+\varphi}{\alpha} \tag{6}$$

Here α is the labor share and φ is the Frisch elasticity of labor supply.

captured by their price. Hence, if the economy uses the same quantity of factors, GDP has increased by dY_t =1%×initial value of cars + 3%×initial value of plastic, i.e. $dY_t = S_{1t} \times \widehat{A}_{1t} + S_{2t} \times \widehat{A}_{2t}$. Dividing by Y_t, we get $\frac{dY_t}{Y_t} = \sum_{i=1}^{2} \frac{S_{it}}{Y_t} \cdot \widehat{A}_{it}$. However, what has increased is the productive capacity of the economy. So, it is really TFP that has increased by $\widehat{\Lambda}_t = \sum_{i=1}^{2} \frac{S_{it}}{Y_t} \widehat{A}_{it}$. GDP might increase more or less once we take into account the response of labor supply, something we shall consider very soon.

Our hypothesis is that, indeed, σ_{Ft} explains a substantial part of GDP volatility, as motivated by (5). We construct σ_{Ft} as in equation (4), keeping σ_i time-independent. We do this for two reasons. First, as we will see later, the volatility of TFP at the micro level does not exhibit any marked trend. It is very similar in the first and second parts of the sample. Second, by using a constant σ_i, we highlight that the changes in fundamental volatility come only from changes in the shares of the largest sectors in the economies, rather than their volatility (which would make the explanation run the risk of being circular). We explain time-varying GDP volatility solely with time-varying shares of economic activity within the economy.

To interpret the results, it is useful to comment on the calibration. We interpret the elasticity of labor supply broadly, including not only changes in hours worked per employed worker, but also changes in employment, and changes in effort. Using this notion, recent research (e.g. summarized in Hall 2009a,b) is consistent with a value of $\varphi = 2$, in part because of the large reaction of employment and effort (as opposed to simply hours worked per employed worker) to business cycle conditions. Using these values and the labor share of $\alpha = 2/3$, we obtain a multiplier of $\mu = 4.5$.

Figure 1 shows the fundamental volatility graphs from 1960 to 2008. We see that fundamental volatility and GDP volatility track each other rather well. By 2008 we're already at mid-80s levels. This suggests a good correlation between fundamental volatility and GDP volatility. The next section studies this systematically.

3 Fundamental Volatility and Low Frequency Movements in GDP Volatility

3.1 US Evidence

3.1.1 Basic Facts

As a baseline measure of cyclical volatility, we first obtain deviations from the HP trend of log quarterly real GDP (smoothing parameter 1600; sample 1947:Q1 to 2009:Q4, source FRED database). We then compute the standard deviation at quarter t using a rolling window of 10 years (41 quarters, centered around quarter t) In order to extend the period to the latest recession, from 2005:Q1 till 2009:Q4 we use uncentered (i.e. progressively more one-sided) windows for this period. We refer to this measure as $\sigma_{Yt}^{\text{Roll}}$.

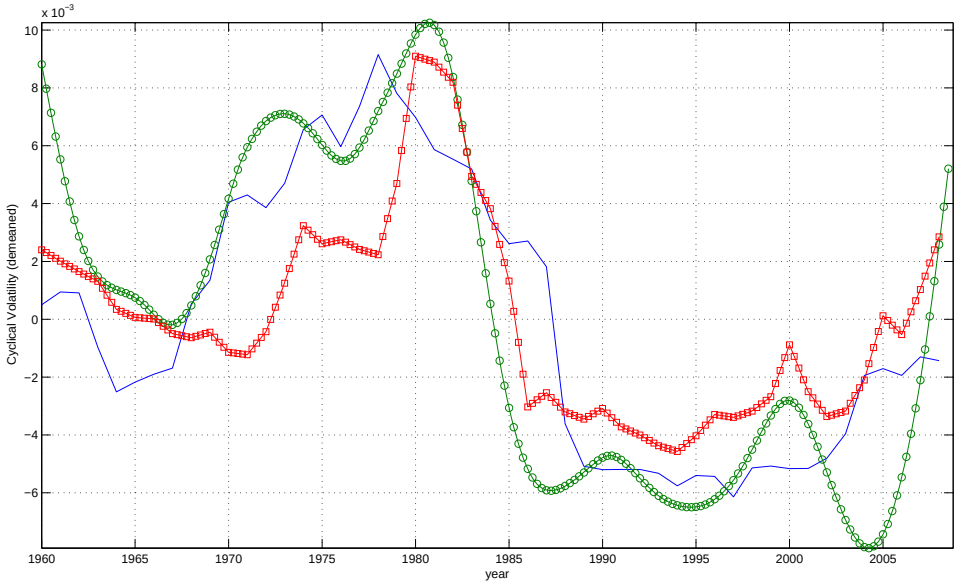


Figure 1: Fundamental Volatility and GDP Volatility. The squared line gives the fundamental volatility ($4.5\sigma_{Ft}$, also demeaned). The solid and circle lines are annualized (and demeaned) estimates of GDP volatility. The solid line depicts rolling window estimates of the standard deviation of GDP volatility. The solid-circle line depicts the HP trend of instantaneous standard deviation.

As a robustness check, we also consider a different measure of cyclical volatility, namely the instantaneous quarterly standard deviation as computed by McConnell and Quiros (2000). For this measure, we start by fitting an AR(1) model to real GDP growth rates (1960:Q1 until 2008:Q4):

$$\Delta y_t = \psi + \phi \Delta y_{t-1} + \epsilon_t \tag{7}$$

where y_t is log GDP. We obtain as estimates $\psi = 0.006$ ($t = 6.78$) and $\phi = 0.292$ ($t = 4.20$). As is well known, an unbiased estimator of the annualized standard deviation is given by $2\sqrt{\frac{\pi}{2}}|\widehat{\epsilon}_t|$, where the factor 2 converts quarterly volatility into annualized volatility, and the $\sqrt{\frac{\pi}{2}}$ comes from the fact that, if $\varepsilon \sim N(0, \sigma^2)$, then $\sigma = E\left[\sqrt{\frac{\pi}{2}}\,|\varepsilon|\right]$. We refer to

$$\sigma_{Yt}^{\text{Inst}} \equiv 2\sqrt{\frac{\pi}{2}}|\widehat{\epsilon}_t| \tag{8}$$

as the "instantaneous" measure of GDP volatility. We shall also use σ_{Yt}^{HP}, the Hodrick-Prescott smoothing of the instantaneous volatility $\sigma_{Yt}^{\text{Inst}}$.

Figure 1 plots the familiar great moderation graphs depicting the halving of volatility in the mid-80s. Interestingly, both measures also point to a significant increase in volatility from the early 2000s on, mostly as a result of the recent crisis. It also depicts the sample fit of our fundamental volatility measure, σ_{Ft}, for the annual case given a baseline value of $\mu = 4.5$. In particular, it shows $\sigma_{Yt}^{\text{Roll}}$ and σ_{Yt}^{HP} (annualized and demeaned), together with $4.5\sigma_{Ft}$ (demeaned).

We run least squares regressions of the type:

$$\sigma_{Yt} = a + b\sigma_{Ft} + \eta_t \tag{9}$$

where σ_{Ft} is our measure of fundamental volatility and σ_{Yt} is one of the measures of volatility described above: $\sigma_{Yt}^{\text{Roll}}$ for the rolling window estimate, $\sigma_{Yt}^{\text{Inst}}$ for the instantaneous standard deviation measure. Note that σ_{Ft} is only available on an annual basis. As such we pursue two different strategies: i) annualizing the standard deviation measure by averaging the left-hand side over four quarters ("annual" below), or ii) linearly interpolating our measures of fundamental volatility in order to obtain quarterly frequency data.

Table 1: GDP Volatility and Fundamental Volatility

	Annual Data-$\sigma_{Yt}^{\text{Roll}}$	Annual Data- $\sigma_{Yt}^{\text{Inst}}$	Quarterly Data- $\sigma_{Yt}^{\text{Roll}}$	Quarterly Data- $\sigma_{Yt}^{\text{Inst}}$
$\widehat{a}$	-0.029 (-5.53;0.005)	-0.0459 (-3.53;0.013)	-0.031 (-12.26;0.003)	-0.0478 (-4.10;0.012)
$\widehat{b}$	4.614 (8.39;0.574)	6.741 (4.70;1.434)	5.056 (18.18;0.281)	6.955 (5.40;1.288)
R^2	0.60	0.33	0.63	0.13

Notes: Regression of GDP volatility on fundamental volatility: $\sigma_{Yt} = a + b\sigma_{Ft} + \eta_t$. In parentheses are t-statistics and standard errors.

Table 1 summarizes the results, at respectively annual and quarterly frequency. We find good statistical and economic significance of σ_F. [4] It is the sole regressor, and its R^2 is around

[4] Our model predicts an intercept $a = 0$. Simple variants could predict a positive a, or a negative a, as we find here empirically. A positive a is generated by adding other shocks to GDP. A negative a is generated if the multiplier μ is increasing in σ_F rather than constant, i.e. if the economy's technologies are more flexible when the environment is more volatile.

60% for the rolling estimate of volatility.[5] This shows that σ_{Ft} explains a good fraction of the historical evolution on GDP volatility. Of course, the R^2 of $\sigma_{Yt}^{\text{Inst}}$ is lower than for $\sigma_{Yt}^{\text{Roll}}$, as $\sigma_{Yt}^{\text{Inst}}$ is a much more volatile measure of GDP volatility.

Note that, in our regressions, all the movements come from the sizes of sectors: their volatilities are fixed in our construction of σ_F. We do it for parsimony's sake, and also because it is warranted by the evidence: the average volatility of sectoral-level microeconomic volatility did not have noticeable trends in the sample. Indeed, the average sectoral-level volatility is 3.4% in the 1960-2005 period, 3.5% in the 1960-1984 period, and 3.2% in the 1984-2005 period. We cannot reject the null of equal mean volatility across the two sample periods (the $p-$value is 0.18). Hence, our construction of σ_F allows to isolate the impact of the changes in microeconomic composition of the economy.

3.1.2 Accounting for the Break in US GDP Volatility

A common way of encoding the great moderation is to test the null hypothesis of a constant level in GDP volatility

$$\sigma_{Yt} = a + \eta_t$$

against an alternative representation featuring a break in the level

$$\sigma_{Yt} = a + cD_t + \eta_t$$

where D_t is a dummy variable assuming a value of 1 from period for $t \geq T$ for an estimated break date T. Following McConnell and Quiros (2000), we take σ_{Yt} to be given by the instantaneous volatility measure $\sigma_{Yt}^{\text{Inst}}$ and test for the presence of a break in level using Andrews' (1993) F-test statistic (the SupF statistic).[6] In what follows, we look for a single break date T, where we assume T lies in a range $[T_1, T_2]$ with $T_1 = 0.2n$ and $T_2 = 0.8n$, and n is the total number of observations (i.e. the trimming percentage is set at 20% of the sample).[7]

To assure comparability and since our sample period does differ, we start by reconfirming the findings in McConnell and Quiros (2000). We do find strong support for a level break with

[5] As a two-step OLS can be slightly inefficient econometrically, we have also performed an ARCH-type maximum-likelihood estimation, based on the joint system (7) and $\sigma_t = \alpha + \beta\sigma_{Ft} + \eta_t$. Its results are very similar to those in Table 1.

[6] We use code made available by Qu and Perron (2007).

[7] Bai and Perron (2006) find that serial correlation can induce significant size distortions when low values of the trimming percentage are used, and recommend values of 15% or higher.

SupF statistic of 32.326 (the 5% critical value 8.752). The estimated break date T is 1984:1 and is estimated with a 90% confidence interval given by 1981:2–1986:4. The estimated value of c is -0.0104 ($t = -5.50$), implying a permanent decrease in aggregate volatility after this date.

Table 2: Break Tests with Fundamental Volatility

	Break Test With or Without Fundamental Volatility on the Right-Hand Side			
	Without	With		
	H_0: No break in a	H_0: No Break in a	H_0: No Break in b	H_0: No Break in a, b
	(i)	(ii)	(iii)	(iv)
SupF stat.	32.33	8.60	8.96	9.33
Null of no break	Reject	Accept	Accept	Accept
Est. break date	1984:1	None	None	None

Notes: We perform a break test for equation $\sigma_{Yt}^{Inst} = a + \eta_t$ (column i) and $\sigma_{Yt}^{Inst} = a + b\sigma_{Ft} + \eta_t$, the regression of instantaneous GDP volatility on fundamental volatility (columns ii-iv). Column (i) confirms that, without conditioning on fundamental volatility, there is a break in GDP volatility (the great moderation). Next, column (ii) performs a test on the a coefficient. We cannot reject the null hypothesis of no break. This means that, once we control for fundamental volatility, there is no break in GDP volatility. The subsequent tests for breaks in b and (a, b) are extra robustness checks (columns iii-iv); they confirm the conclusion that, after controlling for fundamental volatility, there is no break in GDP volatility. From Andrews (1993) the 10% asymptotic critical values for $SupF$ statistic are 11.35 for (ii) and (iii) and 9.54 for (iv).

We next test the hypothesis that, once our fundamental volatility measure is accounted for in the dynamics of $\sigma_{Yt}^{\text{Inst}}$, there is no such level break in aggregate volatility. That is, we test for the null of no break in the intercept of the following equation:

$$\sigma_{Yt}^{\text{Inst}} = a + b\sigma_{Ft} + \eta_t$$

To rule out the additional possibility that the break in aggregate volatility is the result of a break in its link with fundamental volatility, we also consider testing the null of no break on

the slope parameter b and the joint null of no break in both a and b. To maximize the total number of observations, we opt to use a quarterly interpolation of our fundamental volatility measure.

The results are in Table 2. We cannot reject the null of no break in any of these settings. We conclude that, after controlling for the time series behavior of fundamental volatility, there is no break in GDP volatility. This is the sense in which fundamental volatility explains the great moderation (and its undoing): after controlling for the changes in fundamental volatility, there is no statistical evidence of a residual great moderation.

3.2 International Evidence

We now extend the previous analysis to the four other major economies: France, Japan, Germany, and the UK. As is well known (see Stock and Watson 2005), these countries have exhibited quite different low-frequency dynamics of GDP volatility throughout the last half-century. Under the hypothesis of this paper, it should be the case that the evolution of our measure of fundamental volatility is also heterogeneous across these economies.

Relative to the US, we face greater data limitations, both on the time series and cross-sectional dimensions. We are able to construct the Domar weight measures from 1970 to 2005 (from 1973 for Japan). Though we are able to get considerable sectoral details for nominal measures, sector-specific price indexes are only available for half or less of the sectors in each country.[8] This renders impossible an accurate weighting by sectoral TFP volatility. Therefore we choose to consider a special case of our fundamental volatility measure given by

$$\overline{\sigma}_{Ft} = \overline{\sigma}\sqrt{\sum_{i=1}^{N}\left(\frac{S_{it}}{Y_t}\right)^2} \tag{10}$$

where $\overline{\sigma}$ is the average standard deviation of sectoral TFP over the entire sample period in the country under consideration. The values of $\overline{\sigma}$ are 2.3% for France, 2.0% for Germany, 3.2% for Japan, 2.2% for the UK, and 3.4% for the US. Finally, motivated by our discussion above, we consider a multiplier $\mu = 4.5$ to obtain the volatility of GDP implied by our fundamental volatility measure, i.e. $\sigma_{Yt} = 4.5\overline{\sigma}_{Ft}$. As in the US case, we take as a baseline measure of cyclical volatility the 10-year rolling window standard deviation of HP filtered quarterly real GDP (smoothing parameter 6.5). Figure 2 compares the evolution of these measures (where, again, we demean both measures).

[8] See Appendix A for more details on the sources, description, and construction of these measures.

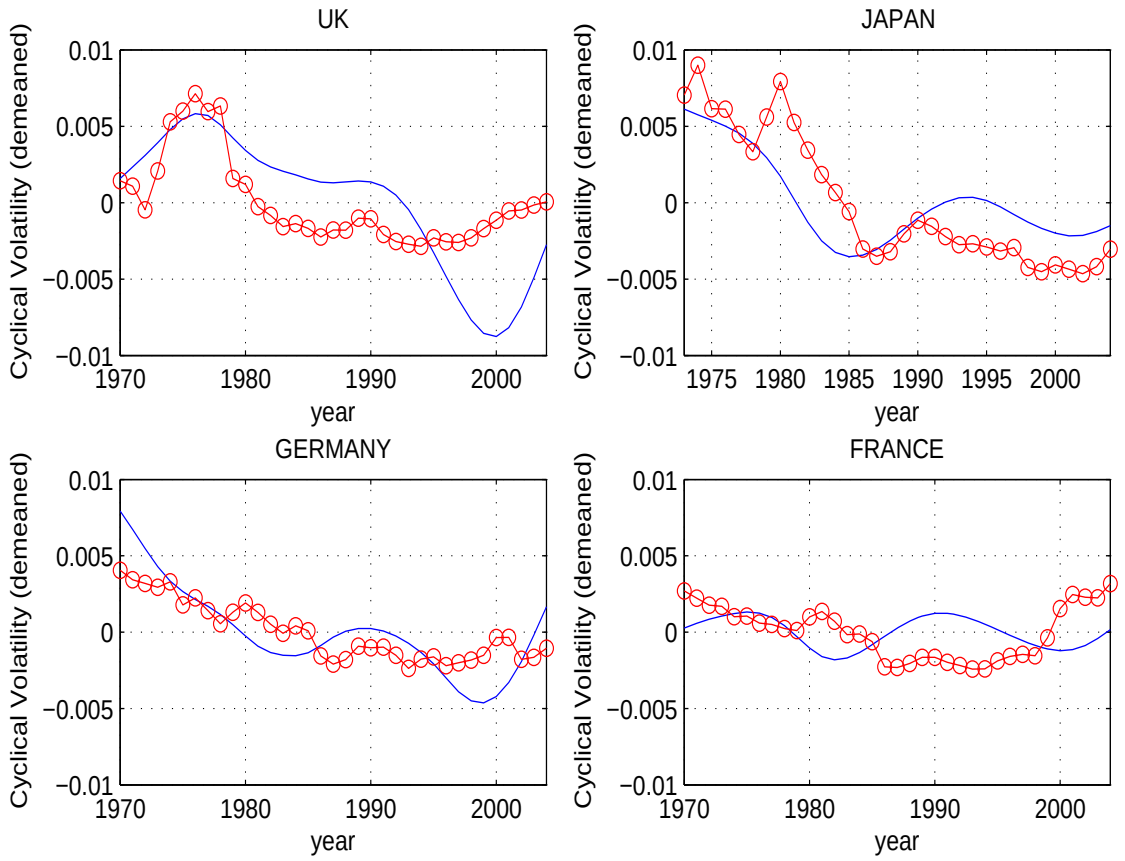


Figure 2: GDP Volatility and Fundamental Volatility in Four OECD countries. Solid line: Smoothed rolling window standard deviation of deviations from HP-trend of quarterly real GDP. Circle line: Fundamental volatility measure, $\sigma_{Yt} = 4.5\overline{\sigma}_{Ft}$. Both measures are demeaned. We report results for the four large countries for which we have enough disaggregated data.

As in the US case, our proposed measure seems to account well for the (different) low-frequency movements in GDP volatility in this set of countries. In the UK it captures the strong reduction in volatility in the late 1970s and its leveling off until the mid 1990s. We cannot account for the short lived drop in UK volatility around 2000 (but notice that by 2005 the levels of our measure and the data are again very close). As Stock and Watson (2005) noticed, Germany provides a different picture, that of a large but gradual decline. Again, our measure also displays a much smoother negative trend. For Japan, fundamental volatility tracks well the fall in GDP volatility in the late 1970s and early 1980s as well as the often noted reversal occurring in the mid 1980s. For France, our measure displays no discernible trend, hovering around its mean throughout the sample period. This is in line with the muted low frequency dynamics of French GDP volatility.

Table 3: GDP Volatility and Fundamental Volatility: International Evidence

	σ_{ct}	σ_{ct}
$\widehat{\beta}$	3.253 (7.91;0.411)	1.941 (3.63;0.534)
χt	No	Yes
Observations	172	172

Notes: We run the regression $\sigma_{Yct} = \alpha_c + \chi t + \beta \overline{\sigma}_{Fct} + \varepsilon_{ct}$, where σ_{Yct} is the country volatility using a rolling window measure, $\overline{\sigma}_{Fct}$ is the fundamental volatility of the country defined in (10), α_c a country fixed effect and χt a linear time trend. t-statistics and standard errors in parentheses.

To complement this we consider running panel regressions

$$\sigma_{Yct} = \alpha_c + \chi t + \beta \overline{\sigma}_{Fct} + \varepsilon_{ct}$$

where σ_{Yct} is our rolling window measure of cyclical volatility for country c in year t and $\overline{\sigma}_{Fct}$ are the country-specific fundamental volatility measures. We include the US along with the four other economies mentioned above. To preserve comparability, we construct and use the $\overline{\sigma}_{Fct}$ for the US (rather than σ_{Ft} described in the previous section). We use country fixed effects α_c and consider running the above panel with and without a common linear time trend χt. We view the specification without a time trend as the cross-country analog

of the regressions run above for the US alone. The specification with a time trend allows us to control for potential common factors affecting volatility in all countries and therefore identifies β through cross-country timing differences in the evolution of fundamental volatility. While this specification renders the value of β not comparable to the values obtained for the simple US regression, it strengthens our results by minimizing possible spurious regression type problems in our baseline specification.[9]

Table 3 reports the results. All results are significant at the 1% level.[10] Again, we confirm the existence of a tight link between aggregate volatility and our fundamental volatility measure. Notice that, for the no linear trend case, our measure is quantitatively similar in the cross-country case (a value of 4.5 is well within the 95% confidence multiplier). Its significance survives when we allow for a linear time trend and when instrumented by its own lag.

4 A Brief History of Fundamental Volatility

The previous section has shown that fundamental volatility correlates well with GDP volatility. In this section, we present a brief account of the evolution of our fundamental volatility measure in the last half-century.[11] We will sometimes use:

$$H_i(t_1, t_2) = \frac{\left(\left(\frac{S_{it_2}}{Y_{t_2}}\right)^2 - \left(\frac{S_{it_1}}{Y_{t_1}}\right)^2\right)\sigma_i^2}{\sigma_{Ft_2}^2 - \sigma_{Ft_1}^2} \tag{11}$$

That is, $H_i(t_1, t_2)$ indicates how much of the change in squared fundamental volatility between t_2 and t_1 can be explained by the corresponding change in the squared Domar weight of industry i.

4.1 USA

We now dig deeper and provide an account of the evolution of our fundamental volatility measure in the last half-century. We find it useful to break this account into three questions: i) What accounts for the "long and large decline" of fundamental volatility from the 1960s to the early 1990s? ii) What accounts for the interruption of this trend from the mid-70s to

[9] We report heteroskedasticity-autocorrelation robust standard errors by using a Newey-West estimator with 2 lags.

[10] The results are also significant without the US.

[11] See Jorgenson and Timmer (forth.) for another analysis of structural change.

the early 1980s? iii) What is behind the reversal of fundamental volatility dynamics observed around the mid 1990s and its subsequent increase until 2008?

Our answers are the following: i) The long and large decline of fundamental volatility from the 1960s to the early 1990s is due to the smaller size of a handful of heavy manufacturing sectors. ii) The growth of the oil sector (which itself can be traced to the rise of the oil price) accounts for the burst of volatility in the mid 1970s. iii) The rise the size of the financial sector is an important determinant of the increase in fundamental volatility. We now detail our answers.

We find that the low-frequency decline in fundamental volatility observed from 1960 to 1990 can be accounted for almost entirely by the demise of a handful of heavy manufacturing sectors: Construction, Primary Metals, Fabricated Metal Products, Machinery (excluding Computers) and Motor Vehicles.[12] These sectors, while only moderately large in the value-added sense in 1960 – they accounted for 18% of total value added in 1960 – are both relatively more intensive intermediate inputs users and relatively more volatile, thus accounting for a disproportionately large fraction of aggregate fundamental volatility in 1960 (30% of σ_F^2).[13] In this sense, the relatively high aggregate volatility in the early 1960s was the result of an undiversified technological portfolio, loading heavily on a few heavy manufacturing industries Their demise, starting around the early 1970s and accelerating around 1980, meant that by 1990 they accounted for only 10% of aggregate volatility.

Another way to see this is to compute a counterfactual fundamental volatility measure where we fix the Domar weights of these sectors to their sample average, while using the actual, time-varying, Domar weights for the other sectors. This enables us to ask what would have happened to fundamental volatility had these sectors not declined during the period of analysis. We find that in this counterfactual economy the level of fundamental volatility would have barely changed from the early 1960s to the early 1990s. At the same time, it is also clear that the dynamics of these sectors do not account either for the spike in fundamental volatility around 1980 nor do they play a role in its continued rise from the mid 1990s onwards.

[12] A simple way to assess the sources of the movements in σ_{Ft} is to decompose aggregate changes according to the (volatility-weighted) contribution of each individual technology.

$$\sigma_{Ft}^2 - \sigma_{F,t-j}^2 = \sum_{i=1}^{N} \left[\left(\frac{S_{it}}{Y_t} \right)^2 - \left(\frac{S_{it-j}}{Y_{t-j}} \right)^2 \right] \sigma_i^2$$

[13] The share of σ_F^2 due to sector i is defined as $\left(\frac{S_{it}}{Y_t} \right)^2 \sigma_i^2 / \sigma_F^2$. Those shares add up to 1.

Instead, we find that the spectacular rise and precipitous decline in fundamental volatility from the early 1970s to the mid 1980s is largely accounted for by the dynamics of two energy-related sectors: "Oil and gas extraction" and "Petroleum and coal products." By 1981, these two sectors accounted for 41% of fundamental volatility, a fourfold increase from the average over the remainder of the sample. The rise and fall of these two sectors account for 94% of the increase in aggregate volatility from 1971 to 1980 and 86% of its decline during the 1980-1986 period.

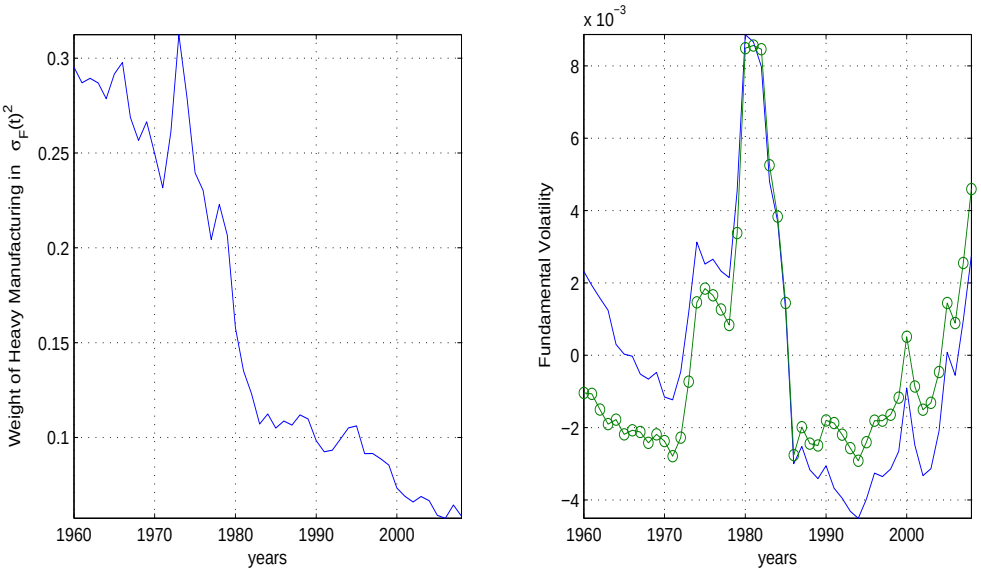


Figure 3: **Left**: Weight of heavy manufacturing sectors in σ_{Ft}^2. **Right**: The continuous line is the baseline fundamental volatility measure ($4.5\sigma_{Ft}$ demeaned). The circled line gives counterfactual volatility measure (also demeaned) where weights of heavy manufacturing sectors are fixed at their sample average.

The importance of these two energy sectors for the volatility spike around 1980 can also be gauged by resorting to the same type of counterfactual exercise: What would have happened to fundamental volatility had the weight of these sectors remained constant throughout the sample? The answer, not surprisingly, is that there would have been no heightened volatility in the late 1970s. Rather, what emerges is a steady decline in fundamental volatility from the early 1970s to the early 1990s, being led by the dynamics of the manufacturing sectors described above.

Also apparent from the graphs above is that, while energy-related sectors have contributed

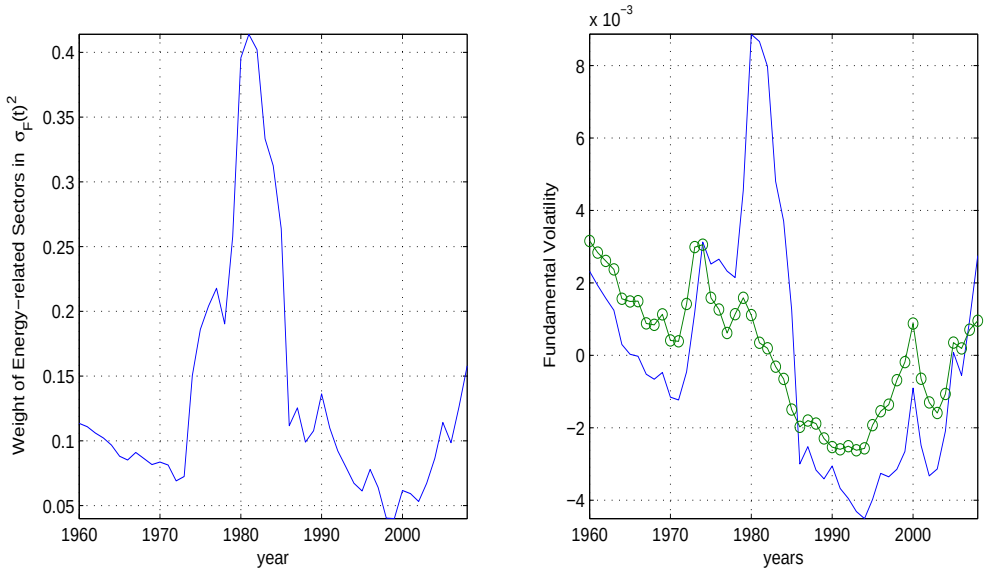


Figure 4: **Left**: Weight of energy-related sectors in σ_{Ft}^2. **Right**: The continuous line is the baseline fundamental volatility measure ($4.5\sigma_{Ft}$ demeaned). The circled line gives counterfactual volatility measure (also demeaned) where weights of energy-related sectors are fixed at their sample average.

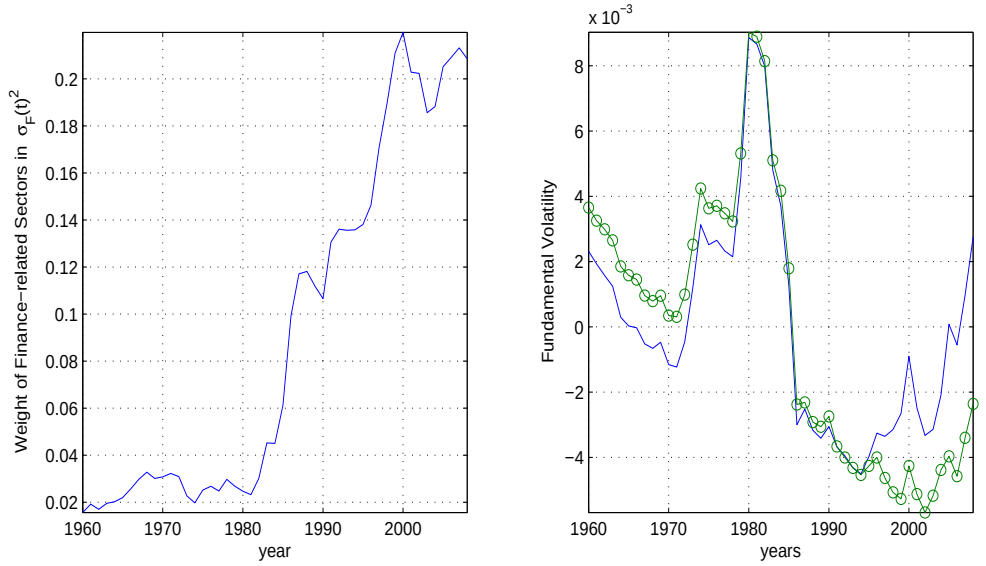


Figure 5: **Left**: Weight of finance-related sectors in σ_{Ft}^2. **Right**: The continuous line is the baseline fundamental volatility measure ($4.5\sigma_{Ft}$ demeaned). The circled line gives counterfactual volatility measure (also demeaned) where weights of finance-related sectors are fixed at their sample average.

somewhat to the increase in fundamental volatility from 2000 onwards, they cannot account for the mid-90s turning point in fundamental volatility nor for its heightened level by 2008. That is, even when we fix the weights of these sectors to their sample mean, our counterfactual volatility measure still points to a marked increase in fundamental volatility over the last decade and a half, undoing most of the (low frequency) volatility declines of the previous three decades.[14] We find that the answer for this lies elsewhere, and is given by the rise of finance.

To show this, we build on Philippon's (2008) analysis of the evolution of the GDP share of the financial sector, but revisit it through the metric of fundamental volatility. We find that the combined contribution of three finance-related sectors – Depository Institutions, Non-Depository Financial Institutions (including Brokerage Services and Investment Banks), and Insurance – to fundamental volatility increased tenfold from the early 1980s to the 2000s, with the latest of these sharp movements occurring in the mid 1990s and coinciding with the rise of our fundamental volatility measure. From the late 1990s onward, these three sectors have accounted for roughly 20% of fundamental volatility.

In a counterfactual economy where the weights of these sectors are held fixed, fundamental volatility would have prolonged its trend decline until the early 2000s. As discussed above, while the renewed exposure to energy-related sectors would have reversed this trend somewhat, the implied level of fundamental volatility at the end of the sample would have been lower, in line with that observed in the early 1990s (and not that of the early 1960s and 1970s as our baseline measure implies). The rise of finance is thus the key to explaining the undoing of the great moderation from the mid 1990s onwards: As the US economy loaded more and more on these sectors, fundamental volatility rose, reflecting a return to a relatively undiversified portfolio of sectoral technologies.

We next turn to our four other major economies.

[14] The left panel tells why: While on the rise from the 2000s onwards, the structural transformation of the US economy has rendered it less dependent on energy sectors. As such, the weight of these energy sectors in aggregate fundamental volatility is about a third of what it was in 1980. This is reminiscent of the analysis in Blanchard and Gali (2008) who show that, despite the unprecedented price increases in this period (larger than those observed in the 1970s), the impact of oil shocks on aggregate activity has been reduced threefold.

4.2 United Kingdom

The UK time series starts off with a short-term spike in fundamental volatility between 1972 and 1976 which is almost single-handedly explained by the high-frequency developments for Petroleum and coal products, yielding $H_i(1972, 1976) = 0.595$. Thereafter, we see a big decline from 1978 until the mid/late 1980s. Petroleum and coal products also explain this "great moderation" ($H_i(1978, 1988) = 0.607$), which is much more emphasized than in Germany and France.

Until the mid 1980s, the UK was labeled as the "sick man of Europe" due to weak GDP per capita compared to the major economies in Continental Europe. Among the industries that led to the UK's catching up in terms of productivity growth in the 1980s is Construction where TFP levels were even higher than in the U.S. – that is, if it were not for the construction sector, the "great moderation" in the UK would have been even greater. As can be seen in Figure 2, fundamental volatility stabilizes in 1985 and then gradually increases until 2005. A large portion of that surge in fundamental volatility can be attributed to Construction, as demonstrated by the low-frequency $H_i(1985, 2005)$ of 0.410.

What is more, the rise of the finance sector also contributes to the modest but continuous rise in fundamental volatility since 2000: The combined $H_i(2000, 2005)$ for Financial Intermediation and Insurance equals 0.456.

4.3 Japan

The most salient characteristic of the Japanese time series is the steep decline in fundamental volatility from 1970 to 1987. Two interrelated factors play into this: First, the long-term decline of the steel industry following the oil crisis explains the development in fundamental volatility very well with $H_i(1970, 1987) = 0.572$ and 0.328 for Basic Metals and Construction, respectively. This process was simultaneously accompanied by a slow rise of the low-volatility service sector. Second, energy-intensive industries, such as steel, sought ways to increase their productivity and thus weaken their dependence on natural resources.[15] As a consequence, technological advances led to a decline in the standard deviation of TFP in the respective sectors.

[15] The importance of technology in the Japanese steel industry during the 1980s is epitomized by the performance of two distinct groups in the industry – integrated mills and minimills (Nair and Kotha, 2001) – which are largely characterized by differences in technology.

Despite the decline from 1970 to 1987, fundamental volatility reaches very high levels in the 1970s, especially compared to Germany and France, as can be witnessed in Figure 2 and the short-term spikes between 1973 and 1974, as well as between 1978 and 1980. The prominent role of the steel industry also explains this pattern with $H_i(1973, 1974) = 0.639$ and $H_i(1978, 1980) = 0.234$ for Basic Metals, and $H_i(1978, 1980) = 0.235$ for Construction. While the time series only begins in 1973, the preceding developments in the Japanese steel industry rationalize these high-frequency movements. The rise of the Japanese steel industry took place rapidly in the mid 1960s, documenting exponential growth in the Japanese market share of global steel production (0.3% in 1913 compared to 2.5% after World War II and almost 25% in 1973). Ever since, Japan has been one of the top three producers of steel. In the early 1970s, Japan became an important exporter of steel.

After the above-described episode, we observe a sharp increase in fundamental volatility from 1987 to 1990. This rise can in large part be attributed to Construction – as evidenced by the corresponding $H_i(1987, 1990)$ which is 1.022.

4.4 Germany

The major trend in the German time series of fundamental volatility is the latter's downturn from 1970 to 1987 which is very well explained by the drop in GDP shares for Basic Metals and Construction: The respective values for $H_i(1970, 1987)$ are 0.728 and 0.569. Prior to the rise of the Japanese steel industry, Germany used to be among the top three steel producers in the world, establishing it as a traditional player in the respective market. Basic Metals explain the rise in fundamental volatility in Japan during the early 1970s so well because Japan was not an immediate victim of the steel crisis (which followed the 1973 oil crisis). On the other hand, Germany and other western economies did face the consequences of the steel crisis as their steel mills were driven out of business, e.g. by very low international steel prices. In this manner, Basic Metals and Construction explain large portions of the downward trend of fundamental volatility in Germany from 1970 until the end of the 1980s.

Furthermore, the downturn of fundamental volatility during the 1980s can in part be explained by Petroleum and coal products, with $H_i(1981, 1988) = 0.189$.

4.5 France

The French time series can be split as follows – a decline in fundamental volatility from 1970 to 1987/8, followed by a ten-year sequence of hardly any movement, and a steep increase from 1998 to 2005.

Similarly to Germany, France showcases a downward trend of fundamental volatility starting in 1970. Just like in Germany, Construction heavily contributes to this drop ($H_i(1970, 1987) = 0.501$), and is accompanied by Petroleum and coal products in the 1980s ($H_i(1981, 1988) = 0.233$). Lastly, the steep increase in fundamental volatility since 1998 is by "other business activities", which contains the bulk of business services. This category comprises very heterogeneous activities, ranging from operative services, such as security activities to intensive services requiring highly qualified human capital, including advanced consultancy services.

5 Idiosyncratic Shocks and Comovement

5.1 Motivation and Summary

We next discuss how our results are consistent with comovement in the economy. We need some notations. Calling Y_i the value added of sector i, GDP growth is $\widehat{Y} = \sum_i \frac{Y_i}{Y}\widehat{Y}_i$, and its variance can be decomposed as:

$$\sigma_{Yt}^2 = D_t + N_t \tag{12}$$

$$D_t = \sum_{i=1}^{n} \left(\frac{Y_{it}}{Y_t}\right)^2 var\left(\widehat{Y}_{it}\right), \qquad N_t = 2 \sum_{1 \leq i < j \leq n} \frac{Y_{it}}{Y_t}\frac{Y_{jt}}{Y_t} cov\left(\widehat{Y}_{it}, \widehat{Y}_{jt}\right)$$

The term D_t represents the diagonal terms in GDP growth, while the term N_t represents the non-diagonal terms, i.e., in an accounting sense, the terms that come from common shocks or from linkages in the economy.

In this section, we address why previous research was pessimistic about the importance of microeconomic shocks (Blanchard and Simon 2001, McConnell and Perez-Quiros 2000, Stock and Watson 2002), and answer the following questions. (i) Previous research showed that comovement (the N_t term) accounts for the bulk of GDP volatility, so why focus on the diagonal terms? (ii) Previous literature showed that the off-diagonal N_t terms fell in volatility, doesn't that mean that the common shock they reflect is the main story (see, for instance, Stiroh 2009)? (iii) Don't we also detect comovement of TFP across sectors, which must mean that there is an extra common factor to take into account?

Before laying out the model that helps think about those questions, we present a summary of our answers. (i) We present a model with comovement in which all the primitive shocks are idiosyncratic (at the sector level). However, because of linkages, there is comovement. Indeed, to a good approximation, in this model

$$D_t \simeq c_D \sigma_{Ft}^2, \qquad N_t \simeq c_N \sigma_{Ft}^2 \tag{13}$$

for two coefficients c_D and c_N. In our calibration, like in data, about 90% of the variance of output is indeed due to comovement ($N_t/\sigma_Y^2 \simeq 0.9$). That comovement itself comes entirely from the primitive diagonal shocks whose variance is measured by σ_{Ft}^2. Hence, the off-diagonal terms are just the shadow of the primitive shocks, which are the diagonal terms captured by σ_{Ft}^2.

(ii) In terms of time-series evolution, in our hypothesis the prime mover is the change in σ_{Ft}^2, which comes from sectoral-level shares. By linkages, the off-diagonal terms N_t will reflect those changes. The fact that the magnitude of off-diagonal terms changes simply reflects the fact that the primitive shocks captured by σ_{Ft}^2 change.[16]

(iii) In our model, all primitive shocks are idiosyncratic. Hence, TFP movements across sectors are uncorrelated. However, even small measurement errors will create a comovement in measured TFP. Suppose that, when times are good (i.e. when the average idiosyncratic shock is positive), people work more, but this partly comes as higher effort, so that measured employment underestimates the true increase in labor supply. Each firm will look more productive than it really is. There will be a measured common productivity increase, but it's only due to mismeasurement. Quantitatively, our model shows that a small measurement error can account for the observed comovement in measured TFP.

We next proceed to the model that allows us to reach the above conclusions.

5.2 Model Setup

We now extend the model of section 2 in a way that allows us to quantify comovement. The cost is that it is more specific – it relies on a CES production function for one final

[16] However, in the model of Section 5, the fall in fundamental volatility will make all sector's volatilities fall, and will indeed make a simulated shock $\widehat{Y}_t^* = \sum_{i=1}^N (Y_i/Y)^* \cdot \widehat{Y}_{it}$ fall in variance, keeping constant output shares $(Y_i/Y)^*$ but using the observed shocks $\widehat{Y}_{it}$. (A more analytical note on this point is available upon request.)

good.[17] The benefit is that we can solve in closed form the structure of this economy, and quantify its comovement. Our model differs in a series of small ways from Long and Plosser (1983), Horvath (1998, 2000), Shea (2002), Foerster, Sarte, and Watson (2008) and Carvalho (2009).[18] Its main virtue is that it is solvable in closed form, so that the mechanisms are fairly transparent.

There is an aggregate good and n intermediary goods. Unit i uses L_i, K_i, X_i of labor, capital, and the aggregate good, to produce Q_i goods i:

$$Q_i = \kappa A_i \left(L_i^{\alpha} K_i^{1-\alpha}\right)^b X_i^{1-b} \tag{14}$$

with $\kappa = 1/\left(b^b (1-b)^{1-b}\right)$, and b is the share of intermediate inputs, and will also be the ratio of value added to sales, both at the level of the unit and of the economy. GDP is production net of the intermediate inputs, the X_i's:

$$Y = \left(\sum_i Q_i^{1/\psi}\right)^{\psi} - \sum_i X_i \tag{15}$$

with $\psi > 1$. The transformation from the goods Q_i to the final good $\left(\sum_i Q_i^{1/\psi}\right)^{\psi}$ and the intermediary inputs is made by a competitive fringe of firms.

The representative agent's utility function is $U = Y - L^{1+1/\varphi}$. Capital can be rented at a rate r. Thus, the social planner's problem is: $\max_{\{K_i, L_i, X_i\}} Y - L^{1+1/\varphi} - rK$ subject to $\sum K_i = K; \sum L_i = L$. We assume that the prices equal marginal cost. This could be caused by competition, or by an input subsidy equal to ψ for the intermediary firms.

[17] In the CES world that we parameterize (with positive elasticity of substitution), a positive TFP shock increases a sector's size. This is not necessarily a good thing. When a sector is very new (say electronic gadgets), the size of that sector grows as the sector becomes more productive (that is, as more products are invented). However, perhaps in a long run sense, some sectors shrink as they become very productive (e.g. agriculture). Following the macro tradition, we eschew here a calibrated modeling of this heterogeneity in the link between productivity and size.

[18] Long and Plosser (1983) impose a Cobb-Douglas structure, with zero idiosyncratic movement in the sales per employee and dollar sales.

The model gives:

$$\text{GDP} : Y = \Lambda L^{\alpha} K^{1-\alpha} \tag{16}$$

$$\text{TFP} : \Lambda = \left(\sum_i A_i^{1/(\psi-1)} \right)^{(\psi-1)/b} \tag{17}$$

$$\text{Sum of sales} : H = \sum p_i Q_i = \frac{Y}{b} \tag{18}$$

$$\frac{\text{Sales}_i}{\text{GDP}} : \frac{p_i Q_i}{Y} = \frac{1}{b} \left(\frac{A_i}{\Lambda^b} \right)^{1/(\psi-1)} \tag{19}$$

5.3 Comovement in Output

To study comovement, we assume that we start from a steady state equilibrium, and study the one-shot response of our economy to shocks.

Models such as (14) always deliver a Sales / Employees ratio that is independent of the unit's productivity. The reason for this almost surely counterfactual prediction is that labor is assumed to be costlessly adjustable. To capture the realistic case of labor adjustment costs, we assume that a fraction $1-\nu$ of labor is a quasi-fixed factor, in the sense of Oi (1962). Technically, we represent $L_i = L_{V,i}^{\nu} L_{F,i}^{1-\nu}$, where $L_{V,i}$ and $L_{F,i}$ are respectively the variable part of labor and the quasi-fixed part of labor. After a small shock, only $L_{V,i}$ adjusts. The disutility of labor remains $L^{1+1/\varphi}$, where $L = L_V^{\nu} L_F^{1-\nu}$ is aggregate labor. We assume that capital and intermediary inputs are flexible. The Appendix relaxes that assumption.

One can now study the effect of a productivity shock $\widehat{A}_i$ to each unit i. We call $S_i = p_i Q_i$ the dollar sales of unit i. The next proposition, whose proof is in the Online Appendix (which contains a generalization to the case where all factors have finite elasticity), describes how the economy reacts to microeconomic shocks.

Proposition 3 *(Aggregate factor emerging from microeconomic shocks) Suppose that each unit i receives a productivity shock $\widehat{A}_i$. Macroeconomic variables change according to*

$$TFP: \ \widehat{\Lambda} = \sum \frac{S_i}{Y} \widehat{A}_i = \sum \frac{Sales_i}{GDP} \widehat{A}_i \tag{20}$$

$$GDP: \ \widehat{Y} = \frac{1+\varphi}{\alpha} \widehat{\Lambda} \tag{21}$$

$$Employment: \widehat{L} = \frac{\varphi}{1+\varphi} \widehat{Y} \tag{22}$$

$$Wage: \widehat{w} = \frac{1}{1+\varphi} \widehat{Y} \tag{23}$$

and microeconomic-level variables change according to

$$\textit{Dollar sales and dollar value added}: \widehat{S}_i = \widehat{Y}_i = \beta \widehat{A}_i + \overline{\Phi}\widehat{Y} \tag{24}$$

$$\textit{Production}: \widehat{Q}_i = \psi\beta \widehat{A}_i + (1 - \psi\Phi)\, \widehat{Y} \tag{25}$$

$$\textit{Price}: \widehat{p}_i = -(\psi - 1)\, \beta \widehat{A}_i + (\psi - 1)\, \beta\Phi\widehat{Y} \tag{26}$$

$$\textit{Employment}: \widehat{L}_i = \nu\beta \widehat{A}_i + \left(\frac{\varphi}{1+\varphi} - \nu\Phi\right) \widehat{Y} \tag{27}$$

$$\textit{Use of intermediary inputs and capital}: \widehat{X}_i = \widehat{K}_i = \widehat{S}_i \tag{28}$$

$$\beta = \frac{1}{\psi - 1 + b\alpha\,(1 - \nu)} \tag{29}$$

$$\Phi = \frac{\beta b\alpha}{1+\varphi}, \qquad \overline{\Phi} = 1 - \Phi \tag{30}$$

In other terms, this economy exhibits a common factor $\widehat{Y}$ (equations 24-28) which is itself nothing but a sum of idiosyncratic shocks (equations 20-21).

The new results are the sector-level changes, in equations (24)-(28) where (20) is Hulten's (1978) equation (TFP is entirely the sum of idiosyncratic unit-level shocks) and (21)-(23) are standard. The economy behaves like a one-factor model with an "aggregate shock," the GDP factor $\widehat{Y}$. Again, this factor stems from a multitude of idiosyncratic shocks. It causes all microeconomic-level quantities to comove. Economically, when sector i has a positive shock, it makes the aggregate economy more productive (equations 17 and 20) and affects the other sectors in three different ways. First, other sectors can use more intermediary inputs produced by sector i, hence increasing their production. Second, sector i demands more inputs from the other firms (equation 24), which leads their production to increase. Third, given sector i commands a large share of output, it will use more of the inputs of the economy, which tends to reduce the other sectors' output. The net effect depends on the magnitudes of the elasticities.

We calibrate the model using conventional parameters to the extent possible, with parameters summarized in Table 4.

Using the decomposition (12), the ratio $f^{GDP} \equiv N/\sigma_Y^2$ captures how much of GDP variance is due to comovement.

Proposition 4 *(Magnitude of the comovement in output and employment) Call f^{GDP} (resp. f^{Labor}) the fraction of GDP (resp. employment) variance attributed to comovement in a*

Table 4: Calibration

Calibrated Values	
Labor share	$\alpha = 2/3$
One minus share of intermediate inputs	$b = 1/2$
Elasticity of labor supply	$\varphi = 2$
Product differentiation parameter	$\psi = 1.2$
Share of labor that is variable in the short run	$\nu = 1/2$
Fraction of mismeasured temporary labor utilization	$\theta = 0.3$
Resulting Values	
Micro-level productivity multiplier	$\beta = 2.7$
Aggregate productivity multiplier	$\mu = 4.5$
Elasticitity Φ	$\Phi = 0.3$
Fraction of GDP variance attributed to comovement	$f^{GDP} = 0.90$
Fraction of employment variance attributed to comovement	$f^{Labor} = 0.95$
Fraction of measured TFP variance attributed to comovement	$f^{TFP,m} = 0.5$

Notes: The first part of the table shows the postulated values. The second part shows the resulting values for a few quantities. Because of linkages, the fraction of variances attributed to comovement is non-zero although all primitive shocks are assumed to be idiosyncratic.

variance-accounting sense. An exact value is given in equation (38) of the Appendix. If most shocks are idiosyncratic at the micro level ($\beta^2\sigma_A^2 \gg \overline{\Phi}^2\sigma_Y^2$), we have

$$f^{GDP} = 1 - \left(\frac{b\beta}{\mu}\right)^2 \tag{31}$$

$$f^{Labor} = 1 - \left(\nu\left(1+\frac{1}{\phi}\right)\frac{b\beta}{\mu}\right)^2 \tag{32}$$

and we have (13) with $c_D \simeq b^2\beta^2$ and $c_N \simeq \mu^2 - b^2\beta^2$. However, economically, all the shocks are primitively idiosyncratic.

Of course, as μ increases, so does the fraction attributed to comovement. Using our calibration, we find $f^{GDP} = 91\%$ and $f^{Labor} = 95\%$. This is to say that, even though primitive shocks are purely idiosyncratic in our model, linkages create such a large comovement that, in

a volatility accounting sense, 93% are mechanically attributed to comovement. This measure is congruent with the empirical findings of Shea (2002), who finds estimates of f^{GDP} in the range 80%-85% and $f^{Labor} = 95\%$.

5.4 Comovement in Measured TFP

The data show some positive correlation in measured TFP innovations across sectors. The previous model generates positive comovement from independent TFP shocks. Hence, if there is perfect measurement of TFP, it will generate no comovement of TFP. Against this background, we interpret the data in the following way. We say that a fraction θ of the change in the effective number of hours is not measured. For instance, a secretary will work harder when there's lots of work to do, and less intensely when there's less work. Still, the total number of hours that are counted is the same, say 40 hours per week. In that case, $\theta = 1$. If she does some overtime, so that some of her extra efforts appear in the labor supply statistics, then $\theta < 1$. For simplicity, we assume that only labor is mismeasured (the same argument would go through if more factors were mismeasured). The measured number of hours is:

$$\widehat{L}_i^m = (1-\theta)\,\widehat{L}_i$$

where the superscript m denotes the $m$easured quantities. Measured TFP growth will be:

$$\begin{aligned}\widehat{A}_i^m &= \widehat{Q}_i - b\alpha\widehat{L}_i^m - b\,(1-\alpha)\,\widehat{K}_i - (1-b)\,\widehat{X}_i \\ &= \left[\widehat{Q}_i - b\alpha\widehat{L}_i - b\,(1-\alpha)\,\widehat{K}_i - (1-b)\,\widehat{X}\right] + \theta b\alpha\left(\widehat{L}_i - \widehat{L}_i^m\right)\end{aligned}$$

Hence:

$$\widehat{A}_i^m = \widehat{A}_i + \theta b\alpha\widehat{L}_i \tag{33}$$

In other terms, the measured TFP is the true TFP plus the increase in effective labor ($\widehat{L}_i$) times labor share in output-cum-intermediary-inputs ($b\alpha$) times the mismeasurement factor θ.

In this benchmark economy, the comovement in true productivity growth $\widehat{A}_i$ is 0. However, there will be some comovement in measured productivity growth, as all sectors increase factor utilization (in a partially unmeasured way) during books. The following proposition quantifies this.

Proposition 5 *(Magnitude of the comovement in measured TFP) Measured TFP follows the factor structure:*

$$\widehat{A}_i^m = c_A \widehat{A}_i + c_Y \widehat{Y} \tag{34}$$

where $c_A \equiv 1 + \theta b \alpha \nu \beta$ *and* $c_Y \equiv \theta b \alpha \left(\frac{\varphi}{1+\varphi} - \nu \Phi \right)$*. Hence, if there is mismeasurement, measured TFP covaries. Call* $f^{TFP,m}$ *the fraction of measured TFP variance attributed to comovement in a variance-accounting sense. If most shocks are idiosyncratic at the micro level, we have*

$$f^{TFP,m} = 1 - \left(1 + \frac{c_Y \mu}{c_A b} \right)^{-2}. \tag{35}$$

Note that, if there is no mismeasurement ($\theta = 0$*),* $c_Y = 0$ *and* $f^{TFP,m} = 0$*: there is no comovement in TFP. In addition,*

Empirically, we measure $f^{TFP,m} \simeq 0.5$. Solving for θ in equation (35) this corresponds to $\theta = 30\%$ of the variable labor input being undermeasured. That seems reasonable to us. The corresponding value of σ_F^m / σ_F is simply $c_A = 1.14$. So, mismeasurement of inputs affects a lot the apparent comovement between sectors (as it is the cause of comovement, and the productivity multiplier is large), but only relatively little the measurement of sectoral-level productivity (idiosyncratic factors generally dominate aggregate factors at the microeconomic level). In our model, all primitive shocks come from idiosyncratic microeconomic shocks, but there is comovement in output because of production linkages. In other terms, there is positive comovement in measured TFP because statistical agencies do not control well for unmeasured increases in labor inputs, i.e. "effort" or "utilization."

We wish to conclude that our model simply illustrates important quantitative features of an economy with comovement. We suspect that the features highlighted will survive with other sources of comovement, e.g. a financial accelerator (Bernanke and Gertler 1989) or expectations (Lorenzoni 2009).

6 Robustness Checks and Extensions

6.1 Variants in the Empirical Constructs

6.1.1 Gross vs. Net output

In this paper, we use the concept of gross output, rather than net output (i.e. value added). In that we follow the common best practice of the productivity literature (e.g. Basu, Fernald

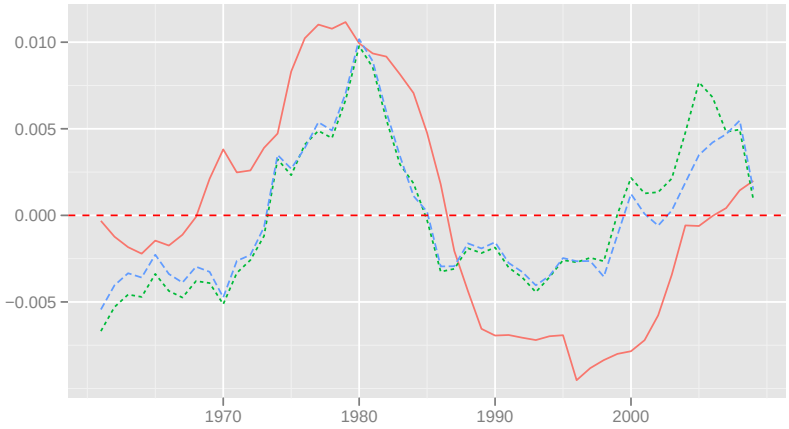


Figure 6: Firm-based Fundamental Volatility and GDP Volatility. The solid line gives the GDP volatility, $\sigma_{Yt}^{\text{Roll}}$ (demeaned). The dotted and dashed lines give the fundamental volatility based on firms (rather than sectors), $4.5\sigma_{F}^{\text{Firms}}$ (demeaned). The dashed line is based on the largest 100 firms by sales in each year. The dotted line is based on all firms in Compustat.

and Kimball 2006). Part of the reason is data availability, part is conceptual: most models use inputs such as labor, capital, and other goods (the intermediary inputs), and for productivity there is no good reason to subtract the intermediary inputs.[19] Nonetheless, we did examine our results with a value-added productivity notion. We found them to be quite similar.

6.1.2 Sectors vs. Firms

In this paper, we primarily use sectors, because we have measures of gross output and value added for sectors for several countries. The firm-level data is spottier, though it is encouraging, as we shall see. We start with firms in the USA, using the Compustat data set. We defined the firm-based fundamental volatility as for sectors, (1). To implement this formula, Compustat

[19] To think about the problem, take a production gross output function $F(L,X)$: The inputs are labor and an intermediary input X, the net output is $V(L,X) = F(L,X)$. A Hicks-neutral increase gross output productivity by a ratio of A means that the production function becomes $AF(L,X)$, so that the net output changes by $AF(L,X) - X$. In constrast, a neutral increase in producivity of net output would change the production function to $AF(L,X) - AX$. The interpretation of the latter is rather odd: With $A = 1.1$, the firm can produce 10% more gross output, but also has become 10% less efficient at handling the inputs. This is a bit odd. This is one reason why it is easier conceptually to think about productivity in the gross output function.

has several important limitations, which are rather important when studying long-run trends. It is not quite consistent over time, as it covers more and more firms. Additionally, the data is on worldwide sales, rather than domestic output. There is some data on domestic sales, but it is unfortunately too spotty to be used.

For firm-level volatility, a firm-by-firm estimation gives quite volatile numbers, so we proceed as in the international section of this paper, using constant values of σ_i across firms. We use $\sigma_i = 12\%$, the typical volatility of sales/employees, sales and employee growth found in Gabaix (2010).

Figure 6 plots the detrended firm-level fundamental volatility for the top 100 firms and for all firms in the data set. As the top 100 firms are very large, most of the variation in σ_F^{Firms} is driven by them. We see that they track GDP volatility quite well. We also see that firm-based fundamental volatility and GDP volatility have had a similar evolution.

To be more quantitative, we proceed as in Table 1 and regress $\sigma_{Yt} = a + b\sigma_{Ft}^{\text{Firms}}$. We find a coefficient $b = 4.8$ (s.e. 0.8) for $\sigma_{Yt}^{\text{Roll}}$ and yearly data, with a R^2 equal to 0.44. We also replicate the break test of Table 2. Controlling for $\sigma_{Ft}^{\text{Firms}}$, there is no break in GDP volatility at the conventional confidence level. We conclude that the firm-based fundamental volatility does account for the great moderation and its undoing, just like the sector-based fundamental volatility we use in the rest of the paper.

The chief difficulty is for non-US data. For European countries the main dataset is Amadeus. Unfortunately, it starts only in 1996, which is too short a time period to detect the long run trends that are the key object of this paper. We can only hope that a researcher will compile a historical database of the actions of the top firms in the major economies. We conjecture that going to more disaggregated data would enrich the economic understanding of the microeconomic development (e.g., the big productivity growth of the retail sector was due to Wal-Mart, rather than a mysterious shock affecting a whole sector), but data availability prevents us from pursuing that idea in this paper.

We conclude that the firm-level analysis is hard to do because of data availability, but the extant US evidence is nonetheless favorable, and confirms with firms what we saw with sectoral data.

6.2 Time-Varying Elasticity of Labor Supply

Jaimovich and Siu (JS, 2009) find that changes in the composition of the labor force account for part of the movement in GDP volatility across the G7 countries. As the young have a more elastic labor supply, the aggregate labor supply elasticity should be increasing in the fraction of the labor force who is between 15 and 29 years old – which is the JS_t variable.

Table 5: Fundamental Volatility and Jaimovic-Siu Variable

	(1)	(2)	(3)	(4)
$\widehat{\beta}$	1.580 (3.38;0.468)	2.179 (3.75;0.580)	1.508 (3.16;0.477)	2.123 (3.38;0.627)
$\widehat{\gamma}$	0.041 (3.76;0.011)	0.0569 (4.99;0.011)	0.044 (3.64;0.012)	0.058 (4.96;0.012)
$\widehat{\delta}$	-	-	2.788 (0.91;3.05)	1.168 (0.71;3.164)
χt	No	Yes	No	Yes
Observations	134	134	134	134

Notes: We run the regression $\sigma_{Yct} = \alpha_c + \beta\overline{\sigma}_{Fct} + \gamma JS_{ct} + \delta\left(\overline{\sigma}_{Fct} - \overline{\sigma_F}\right) \times \left(JS_{ct} - \overline{JS}\right) + \chi t + \varepsilon_{ct}$, where σ_{Yct} is the country volatility using a rolling window measure, $\overline{\sigma}_{Fct}$ is the fundamental volatility of the country defined in (10), $\overline{\sigma_F}$ is the mean of the $\overline{\sigma}_{Fct}$'s, JS_{ct} is the Jaimovich-Siu (2009) measure of the labor force share of the "volatile age group," $\overline{JS}$ is the mean of the JS_{ct}'s, α_c a country fixed effect and χt a linear time trend. t-statistics and Newey-West standard errors (2 lags) in parentheses.

In terms of our model in (5)-(6), this corresponds to having a time-varying Frisch elasticity of labor supply φ_t, i.e.

$$\sigma_{Yt} = \frac{1+\varphi_t}{\alpha}\sigma_{Ft}. \tag{36}$$

The JS composition of the workforce effect is in the term $\varphi(t) = A + B \cdot JS_t$, while the effect we focus on in this paper is the σ_{Ft} term. In other terms, the JS variable is about the amplification of primitive shocks, while our variable is about the primitive shocks themselves. To investigate (36), we run the panel regression:

$$\sigma_{Yct} = \alpha_c + \beta\overline{\sigma}_{Fct} + \gamma JS_{ct} + \delta\left(\overline{\sigma}_{Fct} - \overline{\sigma_F}\right) \times \left(JS_{ct} - \overline{JS}\right) + \chi t + \varepsilon_{ct}$$

That is, we return to the cross-country exercise above, this time including the JS measure of the labor force share of the "volatile age group" in each of our economies, JS_{ct}. The inclusion

of this measure shortens our sample somewhat as it extends only to 1999 (and only begins in 1979 for the case of the UK). We also include an interaction term; to lessen orthogonality, we interact the demeaned versions of the variables, $(\overline{\sigma}_{Fct} - \overline{\sigma_F}) \times \left(JS_{ct} - \overline{JS}\right)$. Finally, we consider running this regression with and without a linear time trend χt. Table 5 reports the results.

Both coefficients β and γ are positive and significant in specifications. The interaction term has the expected sign but is insignificant, perhaps due to the lack of power to detect second-order terms. We conclude that the JS labor supply elasticity and the fundamental volatility are both relevant to explain the cross-country evolution of business cycle volatility.

6.3 Time-Varying Tail Risk in the Aggregate Economy

Recent events have forced economists and policy makers to reassess the mid-2000s belief of an ever more stable economy, and to update the probability of large fluctuations, or tail events, in aggregate GDP growth. For example, Blanchard, Dell'Ariccia, and Mauro (2010) state that "the great moderation led too many (including policy makers and regulators) to understate macroeconomic risk, ignore, in particular, tail risk and take positions [...] which turned out to be much riskier after the fact." In this section we ask whether, from the vantage point of our fundamental volatility construct, this understatement of tail risk was warranted.

We use our equation $\widehat{Y} = \mu \sum_{i=1}^{N} \frac{S_{it}}{Y_t} \cdot \widehat{A_{it}}$, with $\mu = 4.5$. For each time period, we feed the corresponding Domar weight vector and perform 10,000 draws of independent sectoral TFP shocks, normally distributed with mean zero and where the variance of each sector's TFP growth is fixed at its full-sample empirical estimate. We then compute the probability of having that year's GDP growth below $-1.64\sigma_{GDP}$, where σ_{GDP} is the model implied average volatility of aggregate GDP for the full-sample period. The value 1.64 is chosen so that, if the volatility is constant, the tail probability is 5% ($P(X \leq -1.64) = 0.05$ if X is a standard normal). We then repeat this for all time periods and obtain "tail risk probabilities" from 1960-2008.

Figure 7 reports the result. The overall sample mean of tail risk is 0.051, very close to a prior of 0.05. Not surprisingly, tail risk follows the dynamics of our fundamental volatility measure.

Tail risk was reduced fourfold during the great moderation, from the high water mark of 0.11 in 1980 (implying a large fluctuation every 9 years) to a low of 0.028 in 1994 (a large

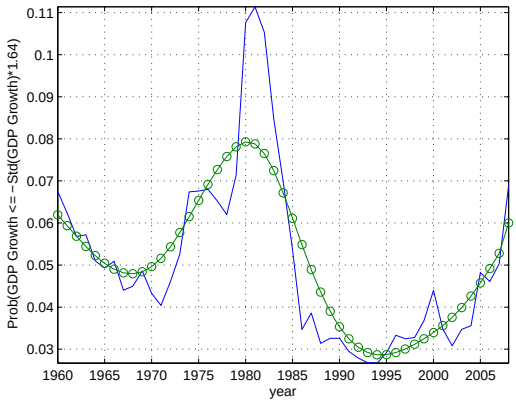


Figure 7: Tail Risk Probability. This figure plots for each year the probability of negative GDP growth in excess of 1.64 standard deviations, given the level of fundamental volatility.

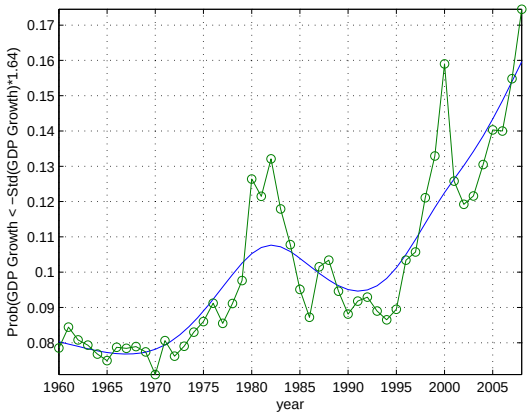


Figure 8: Tail Risk Probability and Fragility to the Non-Depository Financial Sector. This figure plots for each year the probability of negative GDP growth in excess of 1.64 standard deviations, given two-standard-devation shocks to the financial sector.

fluctuation every 36 years). Taking the early 1960s as a benchmark implies a halving of tail risk. However, tail risk has been on the rise since the mid 1990s, reaching 0.07 in 2008 (a large fluctuation every 14 years), roughly the same probability observed in the late 1970s. We conclude that, while the great moderation did imply a marked decrease of tail risk in GDP growth, larger (negative and positive) events were to be expected by the mid 2000s.

Notice also that, by virtue of averaging over many shocks for each sector-year, this tail-risk measure holds true for *typical* sectoral TFP disturbances. However, it does not inform us of the fragility of aggregate growth to *tail events in particular sectors* of the economy. To explore this alternative notion of fragility, or *conditional* tail risk, we consider what would have happened if a given sector had suffered a two-standard-deviation shock to its TFP growth rate. To be precise, we implement the following procedure: Define Σ to be a diagonal matrix with non-zero entries given by each sector's (full-sample) variance of TFP growth. Let $B = \Sigma^{1/2}$, and define $G_t = BX_t$, i.e. $G_{it} = \sum_j B_{ij} X_{jt}$. Let j^* be the index of the particular sector we are interested in exploring. Then, the procedure is: Simulate X_{jt} i.i.d. with mean zero and unit standard deviation for $j \neq j^*$, and set $X_{j^*t} = -2$ (for a shock equal to two standard deviations). Then, compute $G_{it} = BX_t$, and finally TFP. We repeat this for every t from 1960-2008, feeding in the actual Domar Weights.

Given the recent focus on risks posed by the "shadow banking system," we focus on a particular finance sector in our sample: non-depository financial institutions (including security and commodity brokerage services and investment banks). Figure 8 shows a dramatic rise in the exposure of the aggregate economy to tail events in this sector. By 2008, the probability of a large negative event in GDP conditional on a negative tail event in this sector reaches a peak of 17.3% (or one every six years). While, by definition, the conditioning event is a rare one, we note that the fragility of the aggregate economy to any such event in this sector had been unprecedentedly high since the late 1990s: the 1998-2008 average is 14%.

7 Conclusion

We have investigated the explanatory power of "fundamental volatility" to understand the swings in macroeconomic volatility, and found it to be quite good. Fundamental volatility explains the great moderation and its undoing. It is directly observable, with no parameter adjustment.

Our findings do support the view that the key to macroeconomic volatility might be

found in microeconomic shocks. Of course, they need to be enriched by some "propagation mechanisms." Their identification might be simplified if we think that microeconomic shocks are the primary factors that are propagated. For instance, we do not deny that monetary shocks may be important. However, they may largely be part of the response to other shocks (e.g. real shocks caused by oil or finance), rather than really primitive shocks.

Our findings pose the welfare consequences of the microeconomic composition of an economy. In models with financial frictions, a rise in volatility is typically welfare-reducing. Perhaps finance was too big and created too much volatility in the 2000s? Perhaps the oil-dependent industries were too big and created too much volatility in the 1970s?

In addition, fundamental volatility can serve as an "early warning system" to measure future volatility. In retrospect, the surge in the size of finance in the 2000s could have been used to detect a great source of new macroeconomic volatility.

In any case, we think that fundamental microeconomic volatility is a useful tool to consider when thinking about the causes and consequences of aggregate fluctuations.

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A Data Appendix

USA data. The main data source for this paper was constructed by Dale Jorgenson and associates[20], and gives a detailed breakdown of the entire US economy into 88 sectors. The data is annual and covers the period between 1960 and 2005. The original sources are input-output tables and industry gross output data compiled by the Bureau of Labor Statistics and the Bureau of Economic Analysis The data is organized according to the KLEM methodology reviewed in Jorgenson, Gallop, and Fraumeni (1987) and Jorgenson, Ho, and Stiroh (2005). In particular, the input data incorporate adjustments for quality and composition of capital and labor. To the best of our knowledge, this is the most detailed (balanced) panel coverage of US sectors available, offering a unified dataset for the study of sectoral productivity.[21]

From this dataset, and for each year-industry pair, we observe nominal values of sectoral gross output, capital, labor, and material inputs supplied by the 88 sectors (plus non-competing imports) as well as the corresponding price deflators. Following Jorgenson, Ho, and Stiroh (2008) and Basu et al. (2009), we concentrate on private sector output, thus excluding services produced by the government, but including purchases of private-sector goods and services by the government (a robustness check shows that this does not materially affect our result). We also exclude from the analysis the imputed service flow from owner-occupied housing.[22] This yields a panel of 77 sectors which forms the basis of all computations.

International data. For UK, Japan, Germany, and France we resort to the EU KLEMS database (see Timmer et al. 2007 for a full account of the dataset). As the name indicates, this database is again organized according the KLEM methodology proposed by Jorgenson and associates. To preserve comparability, we focus on private sector accounts, thus excluding publicly provided goods and services. We thus exclude sectors under the heading 'non-market services' in the dataset. These also include real estate services as the database does not make a distinction between real estate market services and the service flow from owner occupied residential buildings (see Timmer et al. 2007, Appendix Table 3 for definitions and discussion).

[20] The dataset is available for download at Dale Jorgenson's Dataverse website, `http://dvn.iq.harvard.edu/dvn/dv/jorgenson`.

[21] The NBER-CES manufacturing database provides further detail but only includes manufacturing industries. As it will be made clear in the paper, it is crucial to account for the growth of service sectors when looking at cross-sectoral diversification and the great moderation.

[22] Finally we drop two sectors for which there is no data in the original dataset, "Uranium Ore" and "Renting of Machinery."

For each country we obtain a panel of nominal sectoral gross output and value added at the highest level of disaggregation possible. For the UK we end up with 66 sectors, Japan 58, Germany 50, and France 46. For roughly half of these sectors we can compute TFP growth from the gross output perspective. We then compute the average (across sectors) standard deviation of TFP growth during the entire sample period. For UK, Germany, and France the resulting panel runs from 1970-2005. For Japan it starts in 1973, due to the unavailability of earlier data.

B Proof Appendix

Proof of Lemma 1: The Lagrangean is

$$\mathcal{L} = \mathcal{C}(\boldsymbol{C}) + \sum_i p_i \left(A_i F_i \left(K_i^{1-\alpha} L_i^{\alpha}, \mathbf{X}_i \right) - C_i + \sum_j X_{ji} \right) + r \left(K - \sum_i K_i \right) + w \left(\sum_i L_i - L \right)$$

The Lemma makes two claims: the Cobb-Douglas form, and the Hulten-formula. We start with the the Cobb-Douglas form. We have

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial K_i} &= \frac{1-\alpha}{K_i} p_i A_i \partial_1 F_i - r = 0 \\ \frac{\partial \mathcal{L}}{\partial L_i} &= \frac{\alpha}{L_i} p_i A_i \partial_1 F_i - w = 0 \end{aligned}$$

hence

$$\frac{K_i}{L_i} = \frac{1-\alpha}{\alpha} \frac{w}{r} = \frac{\sum K_j}{\sum L_j} = \frac{K}{L}$$

Calling $I_i = K_i^{-\alpha} L_i^{\alpha}$ the aggregate factor input in sector i, and $I = \sum_i I_i$ the total aggregate input, we have

$$\frac{K_i}{K} = \frac{L_i}{L} = \left(\frac{K_i}{K} \right)^{1-\alpha} \left(\frac{L_i}{L} \right)^{\alpha} = \frac{I_i}{K^{1-\alpha} L^{\alpha}}$$

hence

$$1 = \sum_i \frac{K_i}{K} = \sum_i \frac{I_i}{K^{\alpha} L^{\alpha}} = \frac{I}{K^{1-\alpha} L^{\alpha}}$$

so that $I = K^{1-\alpha} L^{\alpha}$.

The production function is homogenous of degree 1 in (I_i, X_{ij}), if a plan (I_i, X_{ij}) is doable, a plan $(\lambda I_i, \lambda X_{ij})$ is also. Hence, the production function has a form

$$Y = I \cdot \Lambda(A_1, .., A_n) = K^{1-\alpha} L^{\alpha} \Lambda(A_1, ..., A_n)$$

Second, we handle the Hulten formula. Shocks dA_i creates a change in welfare:

$$d\mathcal{L} = \sum_i p_i F_i \left(K_i^{1-\alpha} L_i^{\alpha}, \mathbf{X}_i\right) dA_i = \sum_i A p_i F_i \cdot \frac{dA_i}{A_i} = \sum_i S_i \frac{dA_i}{A_i}$$

i.e. $\frac{dY}{Y} = \sum_i \frac{S_i}{Y} dA_i$, or $\frac{d\Lambda}{\Lambda} = \sum_i \frac{S_i}{Y} dA_i$.

Proof of Proposition 2 The planner's problem is $\max_{K,L} \Lambda K^{1-\alpha} L^{\alpha} - L^{1+1/\varphi} - rK$. The first order conditions with respect to K and L give: $(1-\alpha)\frac{Y}{K} = r$, $\alpha \frac{Y}{L} = (1+1/\varphi) L^{1/\varphi}$, so that $K = (1-\alpha) Y/r$, $L = \left(\frac{\alpha}{1+1/\varphi} Y\right)^{\frac{\varphi}{1+\varphi}}$, and

$$Y = \Lambda K^{1-\alpha} L^{\alpha} = \Lambda \left((1-\alpha)\frac{Y}{r}\right)^{1-\alpha} \left(\frac{\alpha}{1+1/\varphi} Y\right)^{\frac{\alpha\varphi}{1+\varphi}} = \left(\frac{1-\alpha}{r}\right)^{1-\alpha} \left(\frac{\alpha}{1+1/\varphi}\right)^{\frac{\alpha}{1+\varphi}} \Lambda Y^{1-\frac{\alpha}{1+\varphi}}$$

Finally $Y = k\Lambda^{\frac{1+\varphi}{\alpha}}$ with $k = \left[\left(\frac{1-\alpha}{r}\right)^{1-\alpha} \left(\frac{\alpha}{1+1/\varphi}\right)^{\frac{\alpha}{1+\varphi}}\right]^{\frac{1+\varphi}{\alpha}}$.

Proof of Proposition 4 In this model, value added is proportional to sales, $Y_i = bS_i$ (this comes from the first order condition with respect to X_i). So

$$\begin{aligned} D &\equiv \sum_i \left(\frac{Y_i}{Y}\right)^2 var\left(\widehat{Y}_i\right) = \sum_i b^2 \left(\frac{S_i}{Y}\right)^2 var\left(\widehat{S}_i\right) \\ &= b^2 \sum_i \left(\frac{S_i}{Y}\right)^2 var\left(\beta \widehat{A}_i + \overline{\Phi}\widehat{Y}\right) \end{aligned} \tag{37}$$

and $f = N/\sigma_Y^2 = 1 - D/\sigma_Y^2$. Consider first the case where at the microeconomic level most shocks are idiosyncratic, i.e. $\beta^2\sigma_A^2 \gg \overline{\Phi}^2\sigma_Y^2$. Then,

$$f^{GDP} \simeq 1 - \frac{b^2 \sum_i \left(\frac{S_i}{Y}\right)^2 \beta^2 \sigma_A^2}{\sigma_Y^2} = 1 - \frac{b^2\beta^2\sigma_F^2}{\mu^2\sigma_F^2} = 1 - \frac{b^2\beta^2}{\mu^2}.$$

and $c_D \simeq b^2\beta^2$ and $c_N \simeq \mu^2 - b^2\beta^2$.

In the general case, give (20) implies $cov\left(\widehat{A}_i, \widehat{Y}\right) = \frac{S_i}{Y}\mu\sigma_A^2$, (37) gives:

$$\begin{aligned} f^{GDP} &= 1 - \frac{b^2 \sum_i \left(\frac{S_i}{Y}\right)^2 \left[\beta^2\sigma_A^2 + \overline{\Phi}^2\mu^2\sigma_F^2 + 2\beta\overline{\Phi}\left(\frac{S_i}{Y}\right)\mu\sigma_A^2\right]}{\mu^2\sigma_F^2} \\ &= 1 - \frac{b^2\beta^2}{\mu^2} - \overline{\Phi}^2 b^2 \left(\sum_i \left(\frac{S_i}{Y}\right)^2\right) - \frac{2b^2\beta\overline{\Phi}\sigma_A^2}{\mu\sigma_F^2} \sum_i \left(\frac{S_i}{Y}\right)^3 \end{aligned}$$

so

$$f^{GDP} = 1 - \frac{b^2\beta^2}{\mu^2} - \overline{\Phi}^2 b^2 \sum_i \left(\frac{S_i}{Y}\right)^2 - \frac{2b^2\beta\overline{\Phi}}{\mu}\frac{\sum_i \left(\frac{S_i}{Y}\right)^3}{\sum_i \left(\frac{S_i}{Y}\right)^2}. \tag{38}$$

Likewise, the comovement in labor follows, using (22), (27) and $L_i/L = bS_i/Y$

$$\begin{aligned} 1 - f^{Labor} &= \frac{\sum_i var\left(\widehat{L}_i\right)(L_i/L)^2}{var\left(\widehat{L}\right)} \simeq \frac{\sum_i (\nu\beta)^2 \sigma_A^2 (bS_i/Y)^2}{\left(\frac{\varphi}{1+\varphi}\right)^2 \sigma_Y^2} \\ &= \left(\frac{\nu\beta(1+\phi)}{\phi}\right)^2 \frac{b^2\sigma_F^2}{\mu^2\sigma_F^2} = \left(\frac{\nu(1+\phi)}{\phi}\frac{b\beta}{\mu}\right)^2 \end{aligned}$$

Proof of Proposition 5 Equation (34) comes from (27) and (33). The measured change in productivity is (using $\sum_i A_i/Y = 1/b$)

$$\widehat{\Lambda}^m = \sum_i \frac{A_i}{Y}\widehat{A}_i^m = \sum_i \frac{A_i}{Y}\left(c_A\widehat{A}_i + c_Y\widehat{Y}\right) = c_A\widehat{\Lambda} + \frac{c_Y}{b}\widehat{Y} = \left(c_A + \frac{c_Y}{b}\mu\right)\widehat{\Lambda}$$

so the volatility of measured TFP, $\sigma_F^{Full,m} = Var\left(\widehat{\Lambda}^m\right)^{1/2}$, is $\sigma_F^{Full,m} = \left(c_A + \frac{c_Y}{b}\mu\right)\sigma_F$. On the other hand, the measured productivity using only diagonal term is $\sigma_F^m = c_A\sigma_F$ by (34). Hence, we obtain

$$\frac{\sigma_F^{Full,m}}{\sigma_F^m} = 1 + \frac{c_Y\mu}{c_A b} \tag{39}$$

Finally, we get $f^{TFP,m} = 1 - \left(\frac{\sigma_F^{Full,m}}{\sigma_F^m}\right)^{-2}$.

Online Appendix (not for publication) to
"The Great Diversification and Its Undoing"

C Additional Derivations

Proof of Proposition 3 We found it useful to state a general proposition with an arbitrary number of fixed and variable factors. We call $\mathcal{F}$ the primitive factors (e.g. labor and capital), and $\mathcal{F} \cup X$ the set of all factors – the primitive factor and the intermediary inputs. Consider the microeconomic production function:

$$Q_i = \kappa A_i \left(\prod_f F_{if}^{\alpha_f} \right)^b X_i^{1-b}$$

with $\kappa = 1/\left(b^b (1-b)^{1-b}\right)$, F_{if} is the amount of of factor $f \in \mathcal{F}$ in sector i, and X_i is the amount of intermediary inputs. This can be rewritten

$$Q_i = \kappa A_i \prod_{f \in \mathcal{F} \cup X} F_{if}^{\gamma_f} \tag{40}$$

where we define $\gamma_f = b\alpha_f$ for f a primitive factor, and $\gamma_f = (1-b)$ for f the intermediary input. Using this notation, the intermediary inputs in sector i are $X_i = F_{iX}$.

For instance, in the economy studied in section 5 there are three factors:

$$(F_f)_{f=1...3} = (\text{Labor,Capital,Intermediary inputs}) = (L, K, X)$$

and their weights are: $\left(\gamma_f\right)_{f=1...3} = (\alpha b, (1-\alpha)\, b, 1-b)$. We will call it the "3-factor economy."

Each factor F_f has a cost $C_f F_f^{1/\xi_f}$ for a constant f. In the the 3-factor economy, $1/\xi_1 = 1 + 1/\varphi$, i.e. $\xi_1 = \varphi/(1+\varphi)$. On the other hand, as the cost of the intermediary good X is linear in X, $\xi_3 = C_3 = 1$. If capital is elastic in the short run, $\xi_2 = 1$, and $C_2 = r$; while if it is completely inelastic, $\xi_2 = 0$.

We define the output gross of intermediary inputs by

$$H = \left(\sum_i Q_i^{1/\psi} \right)^{\psi} \tag{41}$$

GDP is output net of intermediary inputs as: $Y = H - X = H - \sum_i F_{iX}$. GDP is the solution of the planner's problem:

$$\max_{F_{if}} H - \sum_f C_f F_f^{1/\xi_f} \text{ s.t. for all } f, \sum_i F_{if} = F_f$$

We start with a general Proposition.

Proposition 6 *(General case) The static equilibrium is described by* $Y = \Lambda \prod_{f \in \mathcal{F}} F_f^{\alpha_f}$ *with (17)-(19). Furthermore, suppose that each unit* i *receives a productivity shock* $\widehat{A}_i$*. Macroeconomic variables change according to:*

$$TFP: \ \widehat{\Lambda} = \sum_i \frac{S_i}{Y} \widehat{A}_i = \sum_i \frac{Sales_i}{GDP} \widehat{A}_i \tag{42}$$

$$GDP: \ \widehat{Y} = \frac{1}{1 - \sum_f \alpha_f \xi_f} \widehat{\Lambda} \tag{43}$$

$$Employment \text{ of factor } f: \ \widehat{F}_f = \xi_f \widehat{Y} \tag{44}$$

$$Wage \text{ of factor } f: \widehat{w}_f = \left(1 - \xi_f\right) \widehat{Y} \tag{45}$$

Microeconomic-level variables change according to:

$$Dollar \text{ sales}: \ \widehat{S}_i = \beta \widehat{A}_i + \overline{\Phi} \widehat{Y} \tag{46}$$

$$Production: \widehat{Q}_i = \psi \beta \widehat{A}_i + (1 - \psi \Phi) \widehat{Y} \tag{47}$$

$$Price: \widehat{p}_i = -(\psi - 1) \beta \widehat{A}_i + (\psi - 1) \beta \Phi \widehat{Y} \tag{48}$$

$$Employment \text{ of factor } f: \widehat{F}_{if} = \nu_f \beta \widehat{A}_i + \left(\xi_f - \nu_f \Phi\right) \widehat{Y} \tag{49}$$

$$Use \text{ of intermediary input}: \widehat{X}_i = \widehat{S}_i \tag{50}$$

where

$$\beta = \frac{1}{\psi - \sum_f \gamma_f \nu_f} = \frac{1}{\psi - (1 - b) - b \sum_f \alpha_f \nu_f}$$
$$= \frac{1}{\psi - \sum_{f \in \mathcal{F} \cup X} Share \text{ of factor } f \times Flexibility \text{ ratio of factor } f}$$

and

$$\Phi = \beta \left(1 - \sum_f \gamma_f \xi_f\right) = \beta b \left(1 - \sum_f \alpha_f \xi_f\right)$$
$$= \beta \left(1 - \sum_{f \in \mathcal{F} \cup X} Share \text{ of factor } f \times Adjusted \text{ supply elasticity of factor } f\right)$$

where $f \in \mathcal{F} \cup X$ denotes the primitive factors (labor, capital) and also the intermediary input X.

Proof of Proposition 6 *Step 1. Frictionless equilibrium.* The price of unit i is: $p_i = \frac{\partial H}{\partial Q_i}$, hence by (41): $S_i/H = p_i Q_i/H = Q_i \frac{\partial H}{\partial Q_i}/H$, and

$$\frac{S_i}{H} = \left(\frac{Q_i}{H}\right)^{1/\psi} \tag{51}$$

Because H is homogenous of degree 1, $H = \sum \frac{\partial H}{\partial Q_i} Q_i = \sum S_i$. H is the sum of sales in the economy.

Unit i solves: $\max_{F_{if}} p_i Q_i - \sum_f w_f F_{if}$, which gives: $F_{if} = S_i \gamma_f / w_f \propto S_i$. We use $\propto$ to mean that the variables are proportional, up to a factor that does not depend on i. So, $S_i^{\psi} \propto Q_i \propto A_i S_i$ by (14), so $S_i \propto A_i^{1/(\psi-1)}$. Calling $B = \sum_i A_i^{1/(\psi-1)}$, and using the adding up constraint $\sum F_{if} = F_f$, we find the constant of proportionality: $F_{if} = F_f A_i^{1/(\psi-1)}/B$. Plugging this in (41), we obtain $H = \kappa B^{\psi-1} \prod_{f \in \mathcal{F} \cup X} F_f^{\gamma_f}$. Now, we solve for X, i.e. solve $\max_X Y = H - X$, i.e. $\max_X \kappa B^{\psi-1} \left(\prod_{f \in \mathcal{F}} F_f^{\alpha_f}\right)^b X^{1-b} - X$. The solution yields $X = (1-b)\, H$, $Y = H - X = B^{(\psi-1)/b} \prod_{f \in \mathcal{F}} F_f^{\alpha_f}$, i.e.

$$Y = \Lambda \prod_{f \in \mathcal{F}} F_f^{\alpha_f}, \qquad \Lambda = B^{(\psi-1)/b} \tag{52}$$

as announced in the statement of Proposition 6. In the 3-factor economy we obtain (16). Also, $Y = H - X = bH$.

Step 2. Changes, assuming $\nu_f = 1$. To keep the proof streamlined, we first consider the case $\nu_f = 1$, i.e. the case with no frictions in the adjustment of labor, and with the possibility that $\sum_f \gamma_f$ is not 1. TFP growth comes from (17), and is also Hulten's formula. $Y = bH$ gives $\widehat{Y} = \widehat{H}$. The optimal use of factor f maximizes $Y - \sum C_f F_f^{1/\xi}$, for constants C_f. Hence $\gamma_f Y/F_f = C_f F_f^{1/\xi_f - 1}$, and $F_f = Y^{\xi_f}$ times a constant, and

$$\widehat{F_f} = \xi_f \widehat{Y} \tag{53}$$

Equation (52) implies that:

$$\widehat{Y} = \widehat{\Lambda} + \sum_f \alpha_f \widehat{F}_f = \widehat{\Lambda} + \left(\sum_f \alpha_f \xi_f\right) \widehat{Y}$$

and $\widehat{Y} = \frac{\widehat{\Lambda}}{1-\sum_f \alpha_f \xi_f}$. The wage is $w_f = \frac{1}{\xi_f} L^{1/\xi_f - 1}$, so: $\widehat{w}_f = \left(\frac{1}{\xi_f} - 1\right) \widehat{F}_f$, hence

$$\widehat{w}_f = \left(1 - \xi_f\right) \widehat{Y}. \tag{54}$$

It is convenient that one can solve for changes in the macroeconomic variables without revisiting the sectors' decision problems.

We now turn to the unit-level changes. Optimization of the demand for labor gives $w_f F_{if} = \gamma_f S_i$, so

$$\widehat{F}_{if} = \widehat{S}_i - \widehat{w_f} = \widehat{S}_i - \left(1 - \xi_f\right) \widehat{Y}$$

We have, from (14),

$$\begin{aligned} \widehat{Q}_i &= \widehat{A}_i + \sum_f \gamma_f \widehat{F}_{if} = \widehat{A}_i + \sum_f \gamma_f \left(\widehat{S}_i - \left(1 - \xi_f\right) \widehat{Y}\right) \\ &= \widehat{A}_i + \left(\sum_f \gamma_f\right) \widehat{S}_i - \sum_f \gamma_f \left(1 - \xi_f\right) \widehat{Y} \end{aligned}$$

Eq. (51) gives $\widehat{Q}_i = \psi \widehat{S}_i + (1 - \psi) \widehat{H}$, and using $\widehat{Y} = \widehat{H}$,

$$\psi \widehat{S}_i + (1 - \psi) \widehat{Y} = \widehat{Q}_i = \widehat{A}_i + \left(\sum_f \gamma_f\right) \widehat{S}_i - \sum_f \gamma_f \left(1 - \xi_f\right) \widehat{Y}$$

which gives

$$\begin{aligned} \widehat{S}_i &= \frac{\widehat{A}_i + \left[\psi - 1 - \sum_f \gamma_f \left(1 - \xi_f\right)\right] \widehat{Y}}{\psi - \sum_f \gamma_f} = \frac{\widehat{A}_i}{\psi - \sum_f \gamma_f} + \left(1 - \frac{1 - \sum_f \gamma_f \xi_f}{\psi - \sum_f \gamma_f}\right) \widehat{Y} \\ &= \beta \widehat{A}_i + \overline{\Phi} \widehat{Y} \end{aligned}$$

where

$$\beta = \frac{1}{\psi - \sum_f \gamma_f}, \qquad \Phi = \beta \left(1 - \sum_f \gamma_f \xi_f\right)$$

and as $\widehat{Q}_i = \psi \widehat{S}_i + (1 - \psi) \widehat{Y}$, we obtain the announced expressions for $\widehat{S}_i$ and $\widehat{Q}_i$. $\widehat{F}_{if}$ comes from $\widehat{F}_{if} = \widehat{S}_i - \widehat{w}_f$. S_i was defined as $S_i = p_i Q_i$, which gives $\widehat{p}_i = \widehat{S}_i - \widehat{Q}_i$.

Step 3. With a general $\lambda \in [0, 1]$. After the changes $\widehat{A}_i$, only $L_{V,i}$ can adjust. The planner optimizes the variable part of labor supply: $\max_{L_{Vf}} A \prod_f \left(F_{Vf}^{\nu_f} F_{Ff}^{1-\nu_f}\right)^{\gamma_f} - \left(F_{Vf}^{\nu_f} F_{Ff}^{1-\nu_f}\right)^{1/\xi_f}$. Note that this is isomorphic to optimizing the total labor supply, defining $F_f = F_{Vf}^{\nu_f} F_{Ff}^{1-\nu_f}$. Hence, we have (53) and (54).

For the unit-level variables, one replaces $(\alpha_f, \gamma_f, \xi_f)$ by $(\alpha'_f, \gamma'_f, \xi'_f) = (\alpha_f \nu_f, \gamma_f \nu_f, \xi_f/\nu_f)$, which delivers

$$\beta = \frac{1}{\psi - \sum_f \gamma_f \nu_f}, \qquad \Phi = \beta \left(1 - \sum_f \gamma_f \xi_f\right).$$

Remember that (24). Then, the expression for employment stemming from the optimization of labor demand becomes:

$$\begin{aligned} \widehat{F}_{V,i} &= \nu_f \left[\widehat{S}_i - \left(1 - \frac{\xi_f}{\nu_f}\right)\widehat{Y}\right] = \nu_f \left[\beta \widehat{A}_i + (1-\Phi)\widehat{Y} - \left(1 - \frac{\xi_f}{\nu_f}\right)\widehat{Y}\right] \\ &= \nu_f \left[\beta \widehat{A}_i + \left(\frac{\xi_f}{\nu_f} - \Phi\right)\widehat{Y}\right] = \nu_f \beta \widehat{A}_i + \left(\xi_f - \nu_f \Phi\right)\widehat{Y} \end{aligned}$$

This concludes the proof of Proposition 6.

Proof of Proposition 3 We apply the results from Proposition 6. We particularize it to the case of flexible capital ($\xi_2 = 1, \nu_2 = 1$), and flexible intermediary inputs ($\xi_3 = 1, \nu_3 = 1$), while labor is less flexible ($\xi_1 = \xi, \nu_1 = \nu$ can be less than 1). Then, using $\xi = \varphi/(1+\varphi)$,

$$\widehat{Y} = \frac{1}{1 - (1-\alpha) - \alpha\xi_1}\widehat{\Lambda} = \frac{1}{\alpha(1-\xi)}\widehat{\Lambda} = \frac{1+\varphi}{\alpha}\widehat{\Lambda}$$

$$\widehat{L} = \xi \widehat{Y} = \frac{\varphi}{1+\varphi}\widehat{Y}, \qquad \widehat{w} = (1-\xi)\widehat{Y} = \frac{\widehat{Y}}{1+\varphi}.$$

Also,

$$\beta^{-1} = \psi - \sum_f \gamma_f \nu_f = \psi - b\alpha\nu - b(1-\alpha) - (1-b) = \psi - 1 + b\alpha(1-\nu)$$

$$\Phi = \beta\left(1 - \sum_f \gamma_f \xi_f\right) = \beta\left(1 - b\alpha\xi - b(1-\alpha) - (1-b)\right) = \beta b\alpha(1-\xi) = \beta b\alpha \frac{1}{1+\varphi}.$$

D Some Additional Empirical Results

D.1 Including the covariance terms

There is an extension we will sometimes consider. Call $\rho_{ij} = corr\left(\widehat{A}_i, \widehat{A}_j\right)$ the cross-correlation between sector i and j. We can also define:

$$\sigma_{Ft}^{Full} = \sqrt{\sum_{i,j=1...N} \left(\frac{S_{it}}{Y_t}\right)\left(\frac{S_{jt}}{Y_t}\right)\rho_{ij}\sigma_i\sigma_j} \tag{55}$$

Note that σ_{Ft}^{Full} should be, essentially by construction, the volatility of TFP. The advantage of this construct, though, will be to do the following thought experiment. Suppose that the shares S_{it}/Y_t change, and the variance-covariance matrix $\left(\rho_{ij}\sigma_i\sigma_j\right)$ doesn't change, how much should GDP volatility change? Figure 9 shows the "fundamental volatility" graph including the full covariance matrix, i.e. accounting for cross terms. Cross terms help to get the level right but at the cost of decreasing any dramatic increases in the 2000s.

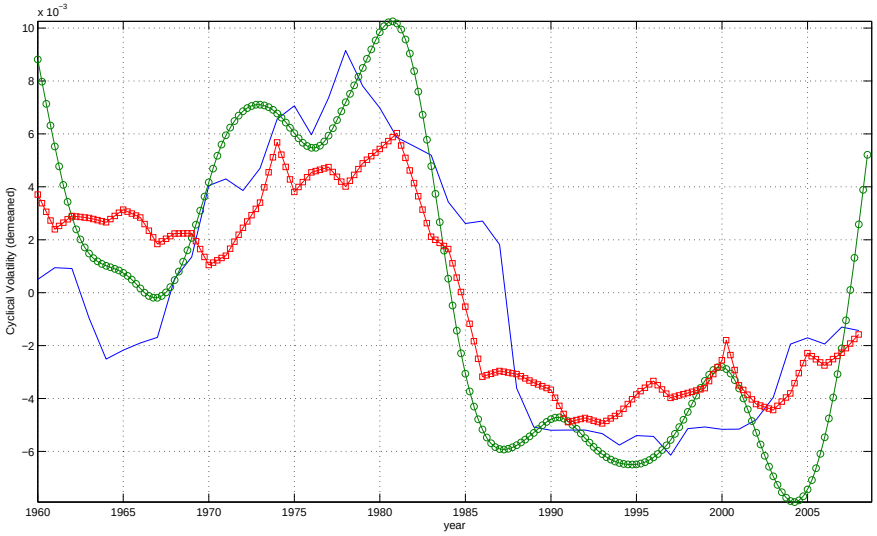


Figure 9: Fundamental Volatility (full matrix) and GDP volatility. The squared line gives the fundamental volatility drawn from full variance-covariance matrix of TFP ($4.5\sigma_{Ft}^{Full}$, also demeaned). The solid and circle lines are annualized (and demeaned) estimations of GDP volatility. The solid line depicts rolling window estimates of standard deviation of GDP volatility. The solid-circle line depicts the HP trend of instantaneous standard deviation.

Table 6 show that the results of Table 1 hold up also with σ_{Ft}^{Full}, the R^2 are actually a bit higher. The advantage of σ_F, with only the diagonal terms, compared to σ_F^{Full}, which has both diagonal and non-diagonal term, is that σ_F is easier to interpret (volatility coming from independent shocks) and requires less data, so it is easier to apply to non-USA countries (where the data is sparser, and we replace all variances by a constant and all covariances by 0).

Table 6: GDP Volatility and Fundamental Volatility

	Annual Data- $\sigma_{Yt}^{\text{Roll}}$	Annual Data- $\sigma_{Yt}^{\text{Inst}}$	Quarterly Data- $\sigma_{Yt}^{\text{Roll}}$	Quarterly Data- $\sigma_{Yt}^{\text{Inst}}$
$\widehat{a}$	-0.041 (-7.13;0.006)	-0.0459 (-4.27;0.013)	-0.041 (-14.46;0.003)	-0.067 (-4.88;0.014)
$\widehat{b}$	3.981 (9.74;0.409)	5.727 (4.70;1.085)	4.046 (19.66;0.206)	5.919 (5.98;0.989)
R^2	0.67	0.38	0.67	0.16

Notes: Regression of GDP volatility on fundamental volatility. We regress $\sigma_{Yt} = a + b\sigma_{Ft}^{Full} + \eta_t$ on fundamental volatility. In parenthesis are t-statistics and standard errors.

D.2 Technological Diversification Patterns

Recall that in the construction of our fundamental volatility measure, the only time varying element that we allow for is the sum of squared Domar weights, $H_t^D = \sum_{i=1}^{N} (S_{it}/Y_t)^2$, where S_{it} is sector i nominal gross output in year t and Y_t gives the total (nominal) value added for the private sector economy in year t. While Domar weights do not sum to one - as gross output at the sector level exceeds sectoral value added by the amount of intermediate input consumed by that sector - this measure is akin to the more usual Herfindahl indexes of concentration. In particular, looking at the cross-sectional (uncentered) second moment to characterize dispersion/concentration in technology loadings is still valid. The graph below shows the evolution of this measure for the US.

From the peak in 1960 to the through in 1997, there is a 33% drop in the H_t^D measure. These dynamics are key for explaining the evolution of our fundamental volatility measure. As such, it is important to perform a number of robustness checks and confirm that the same pattern obtains: i) in more disaggregated data, and across different classification systems; ii) for different dispersion/concentration measures and iii) for value added shares.

D.2.1 More Disaggregated Data

First we look at the raw BLS data underlying much of the construction of Jorgenson's dataset. This data is both defined at a more disaggregated level and according to different classification systems. Namely, we source two different vintages of BLS "interindustry relationships" data(i.e. Input-Output data). The first is based on SIC classification (a mixture of two and

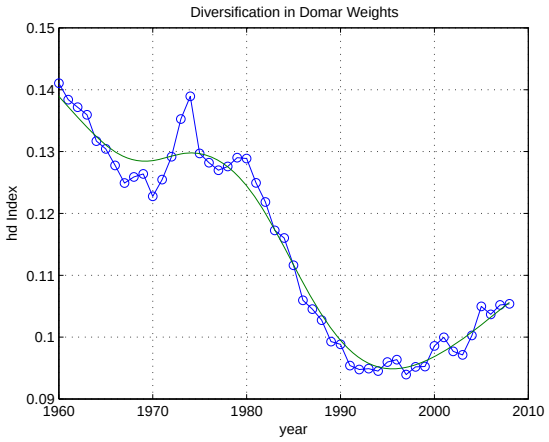


Figure 10: Evolution of H_t^D, 1960-2008

three digit SIC sectors) running from 1977 to 1995 for a total of 185 sectors. The second is the latest vintage produced by the BLS, based on the newer NAICS classification system and runs from 1993-2008 (for a total of 200 sectors).

Based on this data we calculate the implied herfindahl-like measure. Notice that given the difference in underlying classification systems the levels are not comparable, neither among themselves nor with the ones reported above. Nevertheless the dynamics seem to be in broad agreement with those above: fall in technological concentration in late 1970s and early 1980s and if anything a stronger reversal of this pattern from late 1990s onwards.

D.2.2 Checking against other dispersion measures

Though directly relevant to the story of this paper, herfindahl are not the only dispersion measure. However, looking at alternative measures of concentration, such as the Gini Index, the broad story is unchanged: decline in concentration for both measures up until the early 1990s followed by re-concentration from mid to late 1990s, where the latter now shows up much more strongly.

The patterns above are robust when we consider other measures of dispersion: cross sectional standard deviation; coefficient of variation; max-min spread and max-median spread.

An alternative is to look into diversification patters in sectoral value added shares, where the corresponding herfindahl at year t, H_t^{VA}, is defined as $H_t^{VA} = \sum_{i=1}^{N} (Y_{it}/Y_t)^2$, where Y_{it} is nominal value added in sector i at year t. Figure 13 depicts the evolution of this index and

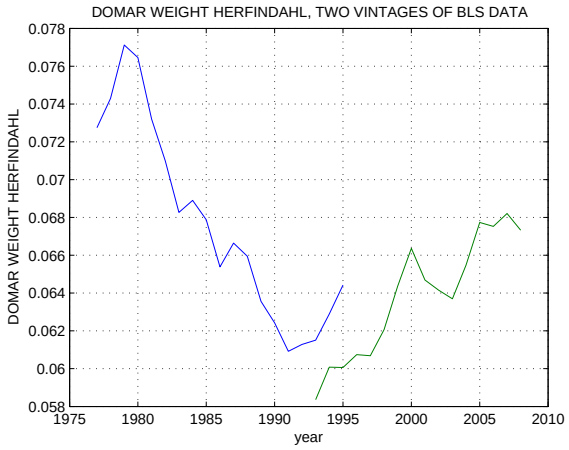


Figure 11: Herfindahl measure for Domar weights computed from two vintages of raw BLS data (1977-1995 SIC data; 1993-2008 NAICS data)

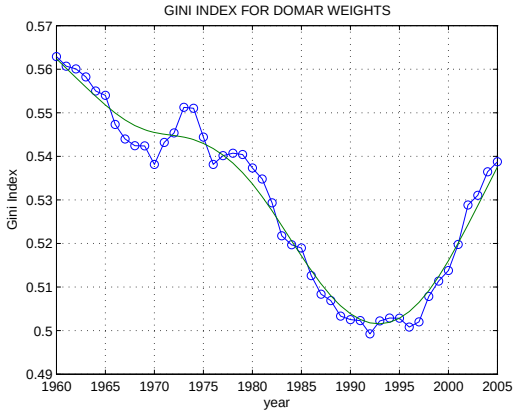


Figure 12: Gini Index for Sectoral Domar Weights.

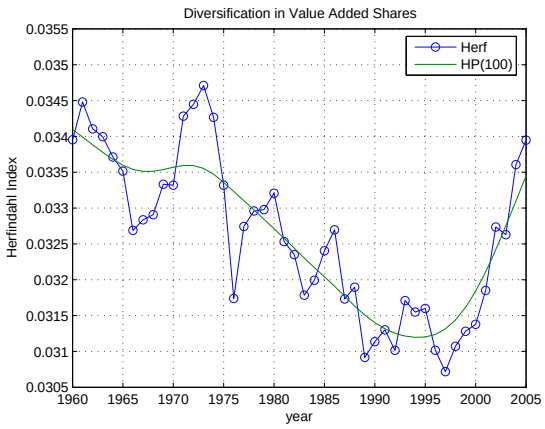


Figure 13: Herfindahl index of sectoral concentration from value added shares data from 1960-2005.

the corresponding HP-filtered series.

Again, looking at value-added shares leads to the same U-shaped pattern. We also see that, quantitatively, the peak to through movement in the value added Herfindahl is smaller. This is as it should be: manufacturing technologies are relatively more intermediate input intensive. As such the gradual move away from manufacturing and into services implies that the Domar weights-based measure of concentration fell relatively more than the value added one: not only has economic activity relied more on what were initially small - in a value added sense - sectors but these latter sectors have relatively lower gross output to value added ratios.[23]

[23] A way to confirm this is to regress the average growth rate of each sector's value added share on the average weight of intermediate input purchases in sectoral gross output (average over the entire sample period 1960-2005). We obtain a negative and significant slope coefficient.