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# Portfolio Allocation for Public Pension Funds*

by

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July 22, 2010

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## Abstract

This paper presents a dynamic model of a public pension fund's choice of portfolio risk. Optimal portfolio allocation is derived when the pension fund manager maximizes the utility of wealth of a representative taxpayer. Alternatively, portfolio decisions are derived when the pension fund manager maximizes his own utility of compensation. The model's implications are examined using annual data on the portfolio allocations and plan characteristics of 125 state pension funds over the 2000 to 2009 period. Consistent with agency behavior by public pension fund management, we find evidence that funds chose greater overall asset – liability portfolio risk following periods of relatively poor investment performance. In addition, pension plans that select a relatively high rate with which to discount their liabilities tend to choose riskier portfolios. Moreover, consistent with a desire to gamble for higher benefits, pension plans take more risk when they have greater proportions of plan participants on their Boards of Trustees.

## I. Introduction

This paper examines the portfolio allocation policies of state and local government pension funds. It presents a model of public pension fund investment choice that can be interpreted either a normative guide to portfolio allocation or a positive theory of pension fund portfolio allocation and risk-taking behavior. The model's implications are examined using annual data on 125 state pension funds over the period from 2000 to 2009.

How a public pension fund allocates its investment portfolio across broad asset classes is critical to understanding the problem of public pension under-funding. A public pension fund's annual investment return is typically much larger in magnitude than its annual employer and employee contributions (Munnell and Soto (2007)). Furthermore, the fund's portfolio allocation across broad asset classes is the major determinant of its investment return (Brinson, Hood, and Beebower (1986) and Brinson, Singer, and Beebower (1991)). Hence, a pension fund's portfolio allocation policy will have first-order consequences for its funding status.

Public pension fund asset allocation also is of interest because, in aggregate, it has changed drastically over time. Based on Federal Reserve Flow of Funds data, Figure 1 shows that state and local government pension funds invested almost entirely in cash and fixed income instruments during the 1950s, but gradually increased their portfolio allocations in equities and, more recently, in other investments (including real estate, private equity, and hedge fund investments).[1] This trend slowed over the last decade, a period when public pension portfolio allocations to equities, fixed income, other investments, and cash have averaged 59%, 30%, 9%, and 2%, respectively.

A benchmark policy for assessing a pension fund's investment choice is a portfolio allocation that best hedges or "immunizes" the risk of its liabilities. The value of a pension fund's liabilities equals the value of the retirement annuities that it is obligated to pay its employees and retirees. These retirement annuities typically are linked to a worker's wages and years of service, and most often payments are partially indexed to inflation. Hence, the value of pension liabilities is exposed to risks of not only real or nominal interest rate changes but also changes wage rates. Thus, a satisfactory analysis of portfolio choice with a goal of liability immunization should account for uncertainty in both interest rates and wage growth.

Portfolio allocations that deviate from the benchmark portfolio that best immunizes liabilities introduce what we referred to as "tracking error." In this paper, we consider how different pension fund objective functions influence the choice of tracking error volatility and

[1] Munnell and Soto (2007) also document this trend in public pension allocation. Similar behavior has occurred with corporate defined-benefit pension plans. See, for example, in Ruthen (2005).

how this volatility may be influenced by the pension plan's funding ratio, past returns, and other characteristics. Our paper is perhaps the first to examine the overall portfolio risks of a time series – cross section sample of pension funds' asset – liability portfolios. We use data on state and local government wages and various investment categories to create a single measure of tracking error volatility for each pension fund during each year. We estimate the risks of wage changes and changes in the returns on long-term real and nominal liability payments as well as the risks of returns on seven different investment categories: U.S. equities, foreign equities, U.S. fixed income, foreign fixed income, real estate, private equity, and hedge funds. Using the covariance matrix of these assets' returns and an individual pension fund's portfolio allocation and liability structure, we are able to develop a measure of the fund's overall tracking error volatility which then is related to the fund's characteristics.

The plan of the paper is as follows. Section II briefly discusses related theoretical and empirical work on the portfolio allocations of state and local pension funds. Our model is presented in Section III. Section IV describes our data and variable construction, while Section V presents the empirical results. Concluding comments are in Section VI.

**II. Related Literature on Public Pension Fund Portfolio Allocation**

The focus of this paper is a public pension fund's portfolio allocation relative to a benchmark portfolio that best hedges (immunizes) the fund's liability risks. To evaluate liability risk, one must first determine a method for valuing pension liabilities since they are not marketable securities. There is disagreement on how this should be done, with the major conflict between the actuarial approach specified by the Government Accounting Standards Board (GASB) and the market value approach based on finance theory.[2] The GASB actuarial approach discounts a pension plan's future retirement payments using the expected rate of return on the pension plan's assets, rather than a discount rate appropriate to the actual risk of the pension plan's retirement payments.

As pointed out in many papers, most recently by Brown and Wilcox (2009), Lucas and Zeldes (2009), and Novy-Marx and Rauh (2009), there is no logical basis for valuation using the GASB actuarial standard. Moreover, its use creates moral hazard incentives in the form of "accounting arbitrage": a pension plan has the incentive to invest in assets with high systematic

[2] GASB is a private organization established in the early 1980s. While not obligated to do so, in practice state and local pension plans report the value of their pension liabilities using GASB standards. GASB also sets standards for contributions that public pension funds should make to prevent significant underfunding. Munnell, Haverstick, Sass, and Aubrey (2008) credit the creation of GASB with improved funding by public pension plans.

risk in order to justify a higher discount rate that will reduce the actuarial valuation of its liabilities. The GASB standard also can lead to accounting gimmicks that reach beyond the public pension fund by reducing the accounting value of a municipality's overall budget deficit via the issuance of pension obligation bonds.[3]

A financial valuation approach that prohibits such arbitrage opportunities eliminates this moral hazard. We take it as a settled question that such financial theory-based market valuation should be used to evaluate the risk of pension liabilities. A more subtle issue is whether the most relevant market value measure should be the pension fund's Accumulated Benefit Obligation (ABO) or Projected Benefit Obligation (PBO). Typically, the annual annuity payment paid by a public pension plan to participant is the product of the participant's final average salary over the last one to five years, years of credited service, and a benefit multiplier that is typically 1% to 2.5%. The ABO measures the present value of this payment based on current years of service and the current average salary, whereas the PBO measures the present value based on current years of service and an estimated future average salary just prior to retirement.

Arguments for using ABO include Bulow (1982), Bodie (1990), Gold and Hudson (2003), and Brown and Wilcox (2009). In contrast, Lucas and Zeldes (2006), Peng (2009), and Munnell, Kopcke, Aubry, and Quinby (2010) provide support for using PBO. In part, the difference depends on the purpose for which the liability measure is to be used. For hedging liability risk when the public pension plan is likely to be a continuing concern, the PBO seems more appropriate to us, and it will be the measure we consider for our benchmark immunizing portfolio.[4] The primary effect is to include future wage uncertainty in overall liability risk.

Black (1989) recommends that if a pension fund manager takes a narrow view by hedging the ABO measure of pension liabilities, the pension portfolio should invest almost exclusively in duration-matching bonds. If a broader PBO view is taken, then he recommends some allocation to stocks assuming stock returns are positively correlated with wage growth. Peskin (2001) supports this view and finds that a 20% to 90% allocation to equities could be optimal depending on the characteristics of a particular public pension fund.

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[3] Munnell, Calabrese, Monk, and Aubry (2010) show that issuance of pension obligation bonds became a losing investment strategy as a result of the recent financial crisis.

[4] Bodie (1990) argues that a corporate pension fund's relevant obligation to be hedged is its ABO because its PBO is not guaranteed by the corporation or by the Pension Benefit Guaranty Corporation (PBGC) should the corporation fail. This reasoning is less relevant for public pension plans. Peng (2009) argues that public plan benefits are relatively more secure because, unlike corporate plans, municipalities typically cannot extinguish their obligations to pay pension benefits, even following bankruptcy. As a consequence, a public-sector worker who continues to be employed is likely to receive her PBO at retirement.

Lucas and Zeldes (2009) come to a similar conclusion from a model where a municipality wishes to minimize tax distortions and pension liabilities are positively correlated with stocks. Their model predicts that pension funds should invest more in stocks if their liabilities are more wage-sensitive, which should be the case if a pension fund has a relatively high ratio of currently-employed pension participants to pension plan retirees. However, they find no empirical evidence for this prediction based on year 2006 Boston College Center for Retirement Research (CRR) data on equity allocations for state and local pension funds. They did find that pension plans with a higher funding ratio allocated more to stocks.

## III. A Public Pension Fund Model

A public pension fund's portfolio choice will derive from its objective function. Therefore, we begin by considering an appropriate normative or positive objective of a public pension fund's investment manager.

### III.A The Public Pension Fund's Investment Objective

As will be discussed, academics and practitioners often do not agree on the proper objective of a public pension plan. Partly, the divergence of views might be due to the difficulty of separating a public pension fund's objective from the overall objective of the municipal government of which it is a part. As one component of the government, a public pension fund's portfolio choices might be made in conjunction with the municipality's other asset and liability decisions. For example, pension portfolio decisions might account for a desire to hedge liability risks that include municipal bonds and other benefit obligations, in addition to pension liabilities.

Taking an even broader Ricardian (1820) / Modigliani-Miller (1958) perspective would imply that even the municipal government's overall objective is irrelevant. As discussed in Bader and Gold (2007), arguments along the lines of Barro (1974) imply that any balance sheet (including pension fund) decision made by a municipal government would be offset by the savings and portfolio decisions of rational private agents. If private individuals and firms recognize the future tax consequences of a government's (dis-) savings and portfolio decisions, those public decisions could be over-turned by private portfolio decisions.[5]

However, as summarized in Elmendorf and Mankiw (1999), the conditions that would enable private individuals to fully neutralize government savings and portfolio decisions are unlikely to hold in practice. Heterogeneity amongst individuals, borrowing constraints, tax

[5] For example, McDonald (1983) argues for a Miller Theorem result where increased municipal bond issuance will be offset by a reduction in equity issuance by private corporations. Lucas and Zeldes (2009)

distortions, and imperfect information regarding government policies implies that public pension policies very likely effect the net tax burdens by individuals and, therefore, have real welfare consequences.

Due to such frictions, Peskin (2001) argues that the risk that pension fund returns fail to match pension fund liabilities imposes intergenerational transfers because future generations are not compensated for the taxes they pay to cover current pension obligations resulting from pension underfunding. Intergenerational equity is likely to be improved if unfunded pension costs are not borne by later generations of taxpayers. Moreover, relative to the federal government, state and local governments have more limited means with which to cope with underfunding: they cannot inflate-away the value of their liabilities via money creation. Thus, unlike the federal social security program that operates on a pay-as-you-do basis, state and local governments may want to maintain a funded pension plan in order to minimize the possibility of unsustainable fiscal imbalances. Peng (2009) believes this is the reason why almost all state and local pension plans are pre-funded.

Even if pension fund deficits are covered by an immediate rise in taxes paid by the current generation of taxpayers, risk-aversion and intra-generational equity may provide an additional rationale for hedging. Not only might a pension fund's objective be to fund the present value of pension obligations as they accrue, but also reduce the uncertainty that returns on invested funds fail to match the change in the present value of obligations due to changing market conditions. A fully-funded pension fund whose investments are chosen to best hedge the change in value of its liabilities fits this ideal of minimizing the uncertainty of taxes needed to fund pension obligations.[6]

We find this argument compelling, with one caveat. It is unlikely that a municipal government's other expenditures in the form of non-pension benefits are fully funded at each point in time by current tax revenues. In particular, during economic downturns, tax revenues (excluding pension contributions) are likely to fail to cover non-pension expenditures. If one wished to hedge the net tax surplus of the aggregate municipal balance sheet, pension investments might be chosen such that their return will tend to exceed the return on pension liabilities during poor economic conditions and trail the return on pension liabilities during good economic times.

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also discuss how a Modigliani-Miller (1958) Theorem could hold where private portfolio decisions undo public ones.

[6] The Lucas and Zeldes (2009) model with costly tax distortions provides these insights.

An implication is that portfolio choice should be biased toward investments that outperform pension liabilities during poor market conditions.[7]

Thus, from a normative viewpoint based on minimizing inter- and intra-generational inequities, a strong argument may be made that pension liabilities should be funded as they accrue and investment policy should be that which best hedges liabilities and, possibly, the municipality's other budget deficits.

In practice, state and local governments do not appear to combine pension fund investment decisions within a single, aggregate framework for managing all of their assets and liabilities. Rather, state or local governments delegate the administration of a public pension plan to a Board of Trustees who may be appointed or may be elected by plan participants. The Board's scope of responsibility tends to focus narrowly on the pension plan, without regard to other municipal assets and liabilities. Typically, the Board sets policy objectives regarding investment policies and asset allocation that are carried out by a supporting staff headed by an executive director. The Board and staff often further delegate investment decisions to professional pension plan consultants and money managers.

The typical procedure is for the Board to establish broad asset allocations in various investment classes, such as fixed income, equities, and alternative investments. Within these classes, internal and/or external money managers are given responsibility for choosing particular securities and investments. Most frequently, the performance of money managers are measured against market benchmarks or their peers that have similar investment styles.

Evidence suggests that the most important aspect of a pension fund's overall investment performance is its decisions regarding allocations into various asset classes, rather than the choice of money managers for each given asset class. Empirical studies of large corporate pension funds by Brinson, Hood, and Beebower (1986) and Brinson, Singer, and Beebower (1991) find that over 90% of investment returns are explained by a fund's asset allocation policy. Hence, it would appear that the pension Board's asset-class allocation decisions are critical.

Presumably, the objectives set by a public pension fund's Board determine the chosen asset-class allocations. Peng (2009) shows that typically these objectives state numerical goals for ex-post investment returns, rather than a desire to hedge the risk of liabilities. In addition, asset-class portfolio weights often are chosen based on mean-variance portfolio efficiency that ignores the risk characteristics of liabilities.

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[7] One indication of such a situation is when a municipal government finds it difficult to make pension contributions to cover accrued liabilities during poor economic conditions.

In practice, there appears to be little connection between a pension fund's investment objectives and the risk characteristics of its pension liabilities.[8] A potential explanation for this behavior is that archaic GASB pension fund accounting clouds the true market valuation of pension liabilities. Thus, rather than be judged on how the pension fund's investments hedge the market risk of its liabilities, the pension Board and staff's performance is gauged against the investment performance of similar public pension funds. Peskin (1991) describes such a peer group benchmark as belonging to the "traditional" approach to public pension investment management. Park (2009) argues that such a peer group benchmark is a result of career concerns by the public pension plan's Board of Trustees and staff and is reinforced by "prudent person" fiduciary standards.

III.B Public Pension Fund Portfolio Choice

Given the previous discussion, we begin with a model that assumes a non-Ricardian environment where a pension fund's objective recognizes that its future funding status has tax consequences for a representative resident of the municipality. Given the narrow focus of most public pension plans' Boards of Trustees, we also assume that the pension fund's objective is separable from the other assets and liabilities composing the municipality's overall balance sheet. Because future funding status depends on the risk of pension liabilities, in addition to pension investments, our analysis will first recognize the role of liability risk and consider a liability immunizing portfolio as a benchmark investment strategy. We will later consider the consequences of an investment objective that is divorced from liability risk, and the pension fund's benchmark becomes the average performance of its peers.

The model is similar to Chen and Pennacchi (2009), and details regarding its derivation can be found in that paper. Let the initial date be 0 and let the end of the pension fund's performance horizon be date $T$. The interval from date 0 to $T$ might be interpreted as the state or local government's fiscal year or some other longer horizon over which pension over- or under-funding will have tax consequences and thereby affect the wealth of the municipality's residents. Since the paper's focus is on portfolio allocation given an initial level of funding, we assume that contributions by the pension fund's sponsoring government and its employees are made just prior to date 0, as are any cash outflows to pay retirement benefits. Thus, during the interval from date 0 to date $T$, the only changes in the values of pension assets and liabilities are assumed to be due to their market rates of return.

[8] Ruthen (2005) argues that a pension fund's typically large allocation to equities while its liabilities are fixed income in nature exposes them to rising underfunding when interest rates unexpectedly decline.

During the interval from dates 0 to $T$, the pension fund's benchmark, which for now is assumed to be its liabilities, $L_t$, follows the process

$$\frac{dL_t}{L_t} = \alpha_L dt + \sigma_L dz_L \tag{1}$$

where $dz_L$ is a Brownian motion process. The Appendix shows how a rate of return process of this form can be derived from the value of individual employees' projected benefits and retirees' annuities. In particular, the process depends on risks from changes in wages and changes in the value of nominal or, in the case of Cost of Living Adjustments (COLAs), inflation-indexed (real) bonds.

The pension fund can choose investments that are assumed to perfectly match (immunize) the above rate of return on liabilities. It can also choose another portfolio of "alternative" securities. These alternative securities are defined as the portion of the fund's total assets that accounts for the difference between the fund's portfolio and one that is invested solely in the liability immunizing benchmark portfolio. Let $A_t$ be the date $t$ value of this alternative securities portfolio. Its rate of return follows the process[9]

$$\frac{dA_t}{A_t} = \alpha_A dt + \sigma_A dz_A \tag{2}$$

where $dz_A$ is another Brownian motion such that $\sigma_L dz_L \sigma_A dz_A = \sigma_{AL} dt$. For simplicity, $\sigma_A$, $\sigma_L$, and $\sigma_{AL}$ are assumed to be constants. $\alpha_A$ and $\alpha_L$ may be time varying, as would be the case when market interest rates are stochastic, but it is assumed that their spread, $\alpha_A - \alpha_L$, is constant.

If at date $t$ the pension fund allocates a portfolio proportion of $1-\omega_t$ to the immunizing portfolio and a proportion $\omega_t$ to the alternative investments, then the value of the pension fund's asset portfolio, $V_t$, satisfies

$$\begin{aligned} \frac{dV_t}{V_t} &= (1-\omega_t)\frac{dL_t}{L_t} + \omega_t \frac{dA_t}{A_t} \\ &= \left[(1-\omega_t)\alpha_L + \omega_t \alpha_A\right] dt + (1-\omega_t)\sigma_L dz_L + \omega_t \sigma_A dz_A \end{aligned} \tag{3}$$

[9] The appendix of Chen and Pennacchi (2009) shows that the pension fund's optimal choice of individual alternative securities is one where their relative portfolio proportions do not vary over time. This implies that the intertemporal portfolio choice problem can be transformed to one of allocating a portion of the fund's portfolio to the liability immunizing portfolio and the remaining portion to a single alternative investment portfolio.

Whenever $\omega_t \neq 0$, the fund's return in equation (3) deviates from the liability immunizing portfolio's return. Now define $G_t \equiv V_t/L_t$ to be the pension fund's date $t$ funding ratio; that is, value of the fund's assets relative to that of its liabilities. At date 0, $G_0 = V_0 / L_0$ and then by Itô's lemma evolves over the interval from date 0 to date $T$ according to

$$\frac{dG_t}{G_t} = \omega_t \left( \alpha_A - \alpha_L + \sigma_L^2 - \sigma_{AL} \right) dt + \omega_t \left( \sigma_A dz_A - \sigma_L dz_L \right) \tag{4}$$

Given that a public pension fund must pay its promised retirement benefits, future over- or under-funding at date $T$ is assumed to be the obligation of the municipality's taxpaying citizens. Thus, an aggregate change of wealth equal to $V_T - L_T$ accrues to taxpayers at date $T$. For analytical convenience, the population of representative taxpaying individuals at date $T$ is assumed to be proportional to the value of date $T$ liabilities.[10] Thus, the representative taxpayer's date $T$ wealth might then be written as

$$\begin{aligned} W_T &= \hat{w} + \lambda \frac{V_T - L_T}{L_T} = \hat{w} - 1 + \lambda G_T \\ &= w + \lambda G_T \end{aligned} \tag{5}$$

where $w \equiv \hat{w} - 1$ is assumed to be a constant that includes wealth unrelated to pension funding and $\lambda > 0$ is the ratio of pension liabilities per representative taxpayer. The larger is $\lambda$, the more sensitive is the representative taxpayer's wealth to the pension plan's funding status. If taxpayers' utilities display constant relative risk aversion with coefficient $(1-\gamma) > 0$, and pension investment policy has the objective of maximizing their utility, then the pension fund's asset allocation problem is

$$\underset{\omega(t) \forall t \in [0,T]}{\text{Max}} \; E_0 \left[ \left( \frac{w + \lambda G_T}{\gamma} \right)^{\gamma} \right] \tag{6}$$

subject to the pension funding ratio dynamics in equation (4). The solution for the optimal portfolio proportion invested in the alternative securities for any date $t \in [0,T]$ is[11]

$$\omega_t^* = \frac{\alpha_G}{(1-\gamma)\sigma_G^2} \left( 1 + \frac{w}{\lambda G_t} \right) \tag{7}$$

[10] This would be approximately the case if there is a steady-state ratio of pension benefits to population, $\lambda$.
[11] See Chen and Pennacchi (2009) for the derivation.

where $\alpha_G \equiv \alpha_A - \alpha_L + \sigma_L^2 - \sigma_{AL}$ and $\sigma_G^2 \equiv \sigma_A^2 - 2\sigma_{AL} + \sigma_L^2$. Equation (7) says that when $\alpha_G$ is positive (*negative*), the pension fund takes a long (*short*) position in the alternative securities.[12] This deviating position is tempered by the relative volatility of the alternative securities, $\sigma_G$, and also the representative taxpayer's coefficient of relative risk aversion, (1- $\gamma$).

Perhaps the most interesting insight from equation (7) is how allocations away from the perfectly immunizing portfolio vary with the current pension funding level, $G_t$:

$$\frac{\partial \left|\omega_t^*\right|}{\partial G_t} = -w \frac{\left|\alpha_G\right|}{(1-\gamma)\lambda G_t^2 \sigma_G^2} \tag{8}$$

This derivative is negative whenever $w$ is positive, so that when the representative taxpayer's wealth unrelated to pension funding is sufficiently large, declines in $G_t$ raise $\left|\omega_t^*\right|$. In other words, a fall in the pension plan's funding ratio leads to a larger deviating allocation and greater "tracking error" risk, which might be interpreted as "gambling" behavior. The reverse holds when $w$ is negative, so that when wealth unrelated to pension funding is small, the pension fund decreases its deviating allocation (tracking error) with pension underfunding; that is, it better hedges its liabilities as its funding ratio declines. The intuition for this behavior stems from the desire to avoid negative wealth when $w < 0$ makes it a possibility.

Based on our previous discussion, these model results come with several caveats. First, the representative taxpayer faces taxation risk not just from the municipality's pension under-funding but also from other deficits/surpluses that may arise from the government's other activities. Recognizing these other sources of tax uncertainty in the individual's wealth in equation (5) could motivate the pension fund to hedge those risks. For example, if budget surpluses were procyclical, the pension fund might wish to choose counter-cyclical alternative securities that produce high (*low*) returns relative to the benchmark during economic recessions (*expansions*).

Second, the model takes the individual's portion of wealth unrelated to pension returns, $w$, as non-stochastic, as might be the case if the individual invested entirely in a default-free bond maturing at date $T$. If this other wealth were invested in risky assets, then again there would be a hedging motive when the pension fund chose its portfolio weight in the alternative securities. Third, if the representative taxpayer's choice of assets that composed her personal wealth were endogenous, then the pension fund's portfolio choice and the uncertainty of $G_T$ could be offset by

[12] It can be shown that the optimal portfolio behavior ensures that the quantity $[1 + w/(\lambda G_t)]$ is always

personal portfolio decisions. Fourth, if personal portfolio decisions were endogenous and the municipality wished to maximize the representative taxpayer's utility of wealth after paying federal income taxes, then Bader and Gold (2007) show that the pension fund should be invested in bonds, rather than equities. Because equities are taxed at a lower federal personal tax rate relative to bonds, but all pension fund investments are federal tax-exempt, individuals would pay lower federal taxes if their exposure to (relatively high-taxed) bonds were held by the pension fund and they held equities in their personal portfolio.

Finally, it may be unrealistic to assume that when the pension fund ends with a surplus ($G_T > 1$) that the representative taxpayer's wealth is incremented by that amount. Political pressure leads to a sharing of the surplus with employees in the form of a reduction in employee contributions or an increase in pension benefits.[13]

These numerous caveats cast doubt on whether a public pension fund's investment strategy should attempt to provide an optimal tradeoff of expected return for risk on behalf of taxpayers as outlined in equation (7). Rather, these qualifications would tend to favor an investment policy that passively follow the liability immunizing strategy where $\omega_t = 0 \; \forall \; t$. Such a strategy would be transparent to taxpayers, allowing them to focus on their individual portfolios. It would also avoid generating surpluses that taxpayers would be forced to share with employees. In addition, since it entails primarily fixed-income investments, it provides federal tax savings.

Shifting from a normative to a positive theory of public pension fund investment behavior, we note that the practice of delegating pension fund management could lead to agency problems where the Board of Trustees and staff maximize their own utility of wealth rather than that of a representative taxpayer. In addition, since stated objectives guiding pension plan investments often ignore the risk of pension liabilities, the Board and staff may be judged against an alternative benchmark based on the investment performance of peer pension plans. In this light, the wealth in equation (5) can be re-interpreted as that of the pension Board and staff where the process followed by $L_t$ in (1) is now the average rate of return earned by other public pension funds. Due to explicit or implicit (career concern) compensation tied to the fund's performance relative to its peers, the pension Board and staff's wealth is arguably a function of its future relative performance, $G_T = V_T/L_T$.

The solution for optimal portfolio choice continues to satisfy equations (7) and (8), so that if the Board and staff's wealth unrelated their pension performance is sufficiently large ($w >$

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positive.

[13] This point is made by Peskin (2001), Bader and Gold (2007), Peng (2009), among others.

0), they will increase the fund's tracking error risk as their relative performance declines. If their wealth that is unrelated to performance is low ($w < 0$), the pension fund's management will decrease its tracking error risk as its performance declines.

The next sections of the paper examine the empirical evidence related to our model based on a time series and cross section of state pension plans. We investigate how a state pension plan's choice of tracking error volatility, which from equation (4) is given by $|\omega_t|\sigma_G$, relates to its characteristics, such as the plan's funding ratio and its performance relative to its peers. We also consider other characteristics related to plan design, governance, and participant demographics that could determine tracking error risk.

**IV. Data and Variable Construction**

Our data on state pension funds comes from two sources. The first source is Wilshire Associates who generously provided us with an annual time series of investment information on 125 state pension funds over the 2000 to 2009 period.[14] This data gives each fund's actuarial values of liabilities and actuarial and market values of assets for each of the ten years. Also for each year, it gives every fund's proportion of assets allocated to eight different categories: U.S. equities, non-U.S. equities, U.S. fixed income, non-U.S. fixed income, real estate, private equity, hedge funds, and other. Also included is each fund's assumed rate for discounting liabilities and the total payroll for active participants in the pension fund.

The second source of pension fund information comes from the Boston College Center for Retirement Research (CRR). This is publicly-available data on 112 state pension funds for the year 2006.[15] It includes individual pension fund characteristics pertaining to governance, the type of plan participants (general employees, teachers, or police and firefighters), the design of plan benefits (e.g., the type of COLA), and numbers of active members and annuitants.

Comparing the state pension funds in the Wilshire data to those in the CRR data led to 97 matches. Based on these 97 state plans, we selected from the CRR the following variables: the ratio of a pension fund's Board members who are plan participants to the total Board members of the pension fund; a dummy variable equaling 1 if the pension fund had a separate investment council (zero otherwise); and a dummy variable equaling 1 if the contribution rate of the pension fund sponsor was statutorily set (zero otherwise).

As previously mentioned, our measure of pension fund risk-taking is its tracking error volatility. To translate a given pension fund's asset-class allocations for a given year into this

[14] Due to late reporting, information on only 50 state funds is available for the year 2009.
[15] This data is at http://crr.bc.edu/frequently_requested_data/state_and_local_pension_data_4.html.

risk measure, we collected for the period January 1997 to April 2010 monthly time series of asset returns in order to estimate a covariance matrix of returns for seven different asset classes.[16] The following asset return series were chosen: U.S. equities – Vanguard Total Stock Market Index Institutional Fund; Non-U.S. equities – Vanguard Total International Stock Index Fund; U.S. fixed income – Vanguard Total Bond Market Index Institutional Fund; Non-U.S. fixed income – Barclays Capital Global Majors, Ex. U.S., Fixed-Income Index; Real estate – Wilshire U.S. REIT Index; Hedge funds – Morningstar MSCI Composite Hedge Fund Returns; and Private equity – Cambridge Associates U.S. Private Equity Returns.[17]

Time series were also collected on wage growth and returns on nominal and real (inflation-indexed) bonds. This data was used to estimate the variances of market returns on pension fund liabilities as well as these returns' covariances with the different asset classes. For wages, we used the Bureau of Labor Statistics quarterly Employment Cost Index for State and Local Government Workers. For bonds, a monthly time series of 15-year maturity, zero-coupon bond returns were constructed for nominal Treasuries and Treasury Inflation-Protected Securities (TIPS) using the data of Gürkaynak, Sack, and Wright (2007, 2008).[18] We choose return series for nominal and real bonds having a maturity of 15 years because several sources indicate that the typical pension fund's liabilities have a duration of that length.[19]

In our model, tracking error volatility was represented by the quantity $|\omega_t|\sigma_G$, which is mathematically equivalent to the square root of the variance of the difference in the rates of return on pension assets ($V_t$) and pension liabilities ($L_t$): $\sigma_V^2 + \sigma_L^2 - 2\rho_{VL}\sigma_V\sigma_L$.[20] We estimated the variance of a pension fund's assets in a given year as $\sigma_V^2 = \omega'\Omega\omega$, where Ω is the estimated

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[16] Recall that the Wilshire data reports eight different asset allocation classes, but one of them is "Other." Because the typical fund had a small allocation to this category (the average "other" allocation was 1.9%), we ignored it when computing tracking error volatility and proportionally increased the other allocation weights so that they summed to one.

[17] Each of these asset return series is at a monthly frequency except for the Cambridge Associates U.S. Private Equity Returns, which is quarterly. We used the monthly series to estimate the covariance matrix of returns, except for those matrix elements relating to the private equity returns, which were estimated based on a quarter time series of returns.

[18] This nominal Treasury and TIPS yield data is available at http://www.federalreserve.gov/econresdata/researchdata.htm. We converted the monthly real returns on a 15-year TIPS to nominal ones using the Consumer Price Index (CPI) for the month.

[19] Ryan and Fabozzi (2002) state "an average 15.5 duration should be close to the median or average duration of the pension industry." Also, consulting firm Mercer LLC (2010) uses a 15-year duration for the average pension plan and states that a pension plan with a typical mix of active members and retirees has a duration between 13 and 16 years.

[20] Note $\sigma_V^2 = (1-\omega_t)^2\sigma_L^2 + \omega_t^2\sigma_A^2 + 2\omega_t(1-\omega_t)\sigma_A\sigma_L\rho$ and $\rho_{LV}\sigma_V\sigma_L = (1-\omega_t)\sigma_L^2 + \omega_t\sigma_A\sigma_L\rho$.

covariance matrix of returns for the seven asset classes and $\omega$ is the 7×1 vector of the fund's portfolio weights (allocations) in these seven asset classes.

As shown in the Appendix, pension liabilities under a PBO measure are composed of wage and bond risk, where the bond risk is either nominal or real (in the case of COLA benefits). Thus, the standard deviation of pension liabilities $\sigma_L$ can be computed from the estimated standard deviations of wage growth ($\sigma_{Lw}$) and nominal or real bond returns $\sigma_{Lp}$ as well as their correlation $\rho_{wp}$: $\sigma_L^2 = \sigma_{Lw}^2 + \sigma_{Lp}^2 + 2\rho_{wp}\sigma_{Lw}\sigma_{Lp}$. We calculated this pension liability standard deviation, $\sigma_L$, for four different assumptions: 1) nominal liabilities; 2) real (COLA) liabilities; 3) nominal liabilities adjusted for the ratio of active to total plan participants; 4) real (COLA) liabilities adjusted for the ratio of active to total plan participants. The Appendix details how, in the latter two cases, the standard deviations of wage growth and bond risk (duration) were adjusted using the fund's ratio of active (employed) to total plan participants (including retirees). These adjustments permit a fund's liabilities to have a duration between 6 and 20 years.

As discussed in the previous section, in practice pension funds may not view their liabilities as a benchmark but rather may benchmark their performance based on peer public pension funds. Thus, we created one additional tracking error volatility measure where, instead of the liability volatilities just discussed, we set $\sigma_L^2 = \omega_a' \Omega \omega_a$ where $\omega_a$ is a 7×1 vector of portfolio weights (allocations) that are the averages across the 125 state funds for a particular year.

The final step in constructing tracking error volatility is to calculate the covariance between a fund's chosen assets and the selected liability/peer performance for a particular year: $\rho_{VL}\sigma_V\sigma_L$. Then the final measure of tracking error volatility is the square root of $\sigma_V^2 + \sigma_L^2 - 2\rho_{VL}\sigma_V\sigma_L$.

Table 1 gives summary statistics for the seven different asset class rates of return, as well as the state and local employee wage growth and returns on 15-year zero coupon nominal and real bonds. It is the annualized standard deviations from this table, which were calculated over the 1997 to 2010 period, that we use to construct tracking error volatilities. In addition, construction of this variable requires correlations between these returns and wage growth, which are presented in Table 2.

Of particular interest is the estimated correlation of -0.27 between state and local wage growth and U.S. equities. Prior research, including Black (1989), Cardinale (2003), and Lucas and Zeldes (2006, 2009) advocate pension fund investments in equities in order to hedge wage

uncertainty. Such a recommendation assumes equities and wages are positively correlated. Our negative correlation estimate appears to conflict with this presumption.

A potential criticism of our negative wage-equity correlation estimate is that it is a short-run correlation calculated over a quarterly holding period. Recommendations for using stocks to hedge wage growth assume that their correlation is positive over a longer holding period. Thus, as a check, we calculated correlations between wages and equity returns over longer holding periods. Because the BLS state and local worker wage index extends back to only 1981, we computed correlations using the BLS national wage index, which extends back to 1952. For this comparison, the return on equities was taken to be the S&P 500 return and the return on bonds was a 10-year and 20-year maturity Treasury bond return obtained from the Center for Research in Security Prices (CRSP).

Table 3 presents estimated wage – asset return correlations for holding periods from one year to nine years. From Panel A of the table, one sees that there is almost no evidence of a positive correlation between wages and stock returns. If anything, the correlation point estimates appear to trend more negative as the holding period increases. In contrast, Panels B and C find a correlation between wages and bond returns whose point estimates becomes more positive as the holding period increases. Though none of these correlation estimates in Table 3 are statistically significant, the evidence raises doubt that there is a long-run positive correlation between U.S. equity returns and wages.[21]

Given our estimated asset return and wage growth standard deviations and correlations, the final input in constructing tracking error volatilities is each pension fund's asset allocations for each year. Summary statistics of these allocations and the resulting tracking error volatilities are given in Table 4. Figure 2 graphs the average of the 125 state pension funds' allocations during each year that are reported in Table 4. Over the 2000-2009 period, there was a moderate decline in the allocation to U.S. equities that was offset by a rise in Non-U.S. equities and private equity. The allocation to U.S. fixed-income peaked at 33% in 2002 following the decline in technology stocks, but then trended downward to its minimum of 25% in 2009.

The average tracking error volatilities in Table 4 show the average overall risk taken by the pension funds. The volatilities tend to be lower when we adjust for fund-specific demographic differences as measured by the ratio of active (employed) to total participants. It may be unsurprising that when we use the average of funds' allocations as the benchmark, rather than a fund's liabilities, then tracking error volatilities typically are the least.

[21] This evidence is consistent with the theory and empirical work in Lustig and Van Nieuwerburgh (2008). They find a negative correlation between innovations in human capital and financial asset returns.

Table 4 also includes the yearly averages for two variables that our model predicts may influence tracking error volatility: the plan's funding ratio and its investment return. The funding ratio is measured as the market value of assets divided by the actuarial value of liabilities. The plan's investment return in a given year is estimated as the product of its asset-class allocation weights times the returns earned by each of the asset classes.[22] As might be expected, the average funding ratio of these 125 state pension plans peaked at 109 % during 2000 prior to the technology stock decline and was at a minimum funding ratio of 58 % during 2009. The significant underfunding in 2009 followed what we estimate to be a greater than 30% investment loss in 2008.

## V. Empirical Tests

### V.A Immunizing Allocations

As a prelude to examining how state pension plans' tracking error volatilities vary with their plan characteristics, we first ask what would be the asset portfolio allocations that minimize the typical pension plan's tracking error volatility. This was done using our estimated covariance matrix for the returns on the seven asset classes as well as these returns' covariances with wages and the 15-year duration liabilities of a typical pension plan.[23] We demographically adjusted this typical pension fund's liabilities assuming a ratio of active to total participants of 66.6%, which was the average for our sample. The portfolio allocations that best hedge (immunize) this pension fund's liabilities are given in Table 5.

Columns 1 and 2 of Table 5 assume that the pension fund's liabilities are purely nominal, having no COLAs. As shown in column 1, many of the asset allocations are small, but there is a 19% allocation to U.S. equities and an 86% allocation to U.S. fixed income. Surprisingly, the allocation calls for a -47% (short) position in private equity and a 55% allocation to hedge fund investments. Since these large positions in alternative investments are not typical of most pension funds, in column 2 we consider what would be the volatility minimizing allocation if no private equity or hedge fund investments were permitted. There we see that the allocation to fixed income becomes 106%, with a 9% short position in Non-U.S. equities and a near-zero

[22] While we have no data on each fund's actual investment return but only the fund's portfolio allocation for different asset classes and the asset classes' returns, the evidence Brinson, Hood, and Beebower (1986) and Brinson, Singer, and Beebower (1991) suggests that our proxy should be a close approximation.

[23] The tracking error minimizing weights are given by $\Omega^{-1}\Psi$ where $\Omega$ is the 7×7 covariance matrix of asset returns and $\Psi$ is a 7×1 vector of covariances between the asset returns and pension liabilities. See Pennacchi (2008) pages 50-54 for a discussion.

allocation to U.S. equities. Apparently, the previous long position in U.S. equities was, in part, offsetting the large short position in private equity.

Columns 3 and 4 of Table 5 assume that the pension fund's liabilities are fully inflation-indexed, real liabilities. With this change, column 3 shows that the allocation to U.S. fixed income becomes 61% and the allocation to Non-U.S. fixed income is a large 23%. This makes sense since foreign fixed income is likely to be a good hedge against U.S. inflation, assuming that exchange rates adjust with purchasing power parity. When private equity and hedge fund investments are not permitted, column 4 indicates that the allocation to U.S. fixed income becomes 78% and Non-U.S. fixed income becomes 25%. Short positions in U.S. and Non-U.S. equities total -7%. In summary, these allocations seem reasonable.

V.B Tracking Error Volatility and Plan Characteristics

Let us now investigate the relationship between a fund's choice of tracking error volatility and its characteristics. Our examination is based on the following linear regression model:

$$\begin{aligned}\sigma_{TE,i,t} = {} & \beta_1 \times \left(\text{Funding ratio}\right)_{t-1} + \beta_2 \times \left(\text{Return relative to peers}\right)_{t-1} \\ & + \beta_3 \times \left(\text{Governance variables}\right) + \beta_4 \times \left(\text{Other Control variables}\right) + \varepsilon_{it}\end{aligned} \tag{9}$$

where $\sigma_{TE,i,t}$ is the tracking error volatility of fund *i* in year *t*. Taking the normative view that the pension fund should maximize the utility of a representative taxpayer, our model predicts that tracking error should be negatively related to the plan's funding ratio ($\beta_1 < 0$) if the taxpayer's wealth is sufficiently large ($w > 0$).

Instead, if the pension fund is managed to maximize the utility of wealth of the pension Board and staff, and their benchmark for compensation is the "traditional" one of return performance relative to the fund's peers, then tracking error volatility should vary with the fund's past investment return. In particular, if the wealth of the pension Board and staff is sufficiently large ($w > 0$), then tracking error risk should increase following a poor return relative to the fund's peers. Consequently, this interpretation of the model predicts $\beta_2 < 0$.

The regression includes additional explanatory variables listed in Table 6. These include governance-related variables as well as other controls related to the type of participants and other characteristics such as a proxy for fund size (the natural log of the market value of pension assets), the ratio of payroll to pension liabilities, and the rate chosen by the fund to discount its liabilities.

The regression results are given in Table 7. In each regression specification, we control for time (year) fixed-effects. In columns 1, 3, 5, 7, and 9, the regressions include time-invariant fund characteristics from the CRR data, so our observations drop to 97 pension plans per year. In columns 2, 4, 6, 8, and 10 we exclude the CRR data but control for fund fixed-effects, in which case our observations equal 125 pension plans per year.

The results in Table 7 are qualitatively the same no matter which construction of tracking error volatility is used as the dependent variable. There are only three explanatory variables that are statistically significant: the prior year's investment return relative to peers, the fund's chosen rate used to discount liabilities, and the proportion of members of the Board of Trustees who are participants.

The negative coefficient on the fund's prior-year return relative to its peers in consistent with gambling behavior on the part of the Board and its staff. If compensation depends on the fund's investment performance relative to its peers, then our model predicts that when Board wealth is sufficiently large ($w > 0$), the pension fund chooses greater tracking error volatility following poor investment performance.

The positive relationship between a fund's chosen rate to discount liabilities and its tracking error volatility also seems consistent with risk-taking behavior, though the direction of causality is unclear.[24] Funds with higher tracking error volatility may be able to justify a higher discount rate because they are investing in assets with higher systematic (priced) risks. Or, it might be the case that a fund is motivated to select a higher discount rate in order to understate its liabilities and fictitiously increase its net worth in order to justify greater tracking error risk.

The finding that a fund takes more tracking error risk when it has greater participant representation on its Board of Trustees might be explained in a couple of ways. If pension participants are less financially literate than typical Board members, they may have less ability to select asset class allocations that provide a good hedge of the pension plan's liabilities. Alternatively, participants may knowingly wish to take more tracking error risk. If such a gamble results in a significant pension surplus, Peskin (2001), Bader and Gold (2007), and Peng (2009) point out that this surplus is typically shared with participants in the form of increased benefits or lower employee contribution rates. In contrast, if the gamble ends in a greater deficit, benefits for current participants typically cannot be cut and greater contributions by the state's taxpayers often result.

[24] That funds choosing higher discount rates also choose higher tracking error volatility is consistent with Park (2009) who finds that public plans selecting higher discount rates are more likely to invest in real estate and alternative investments (including private equity and hedge funds).

## VII. Conclusion

This paper presented a model of public pension fund asset allocation that can be a normative guide to a municipality's pension portfolio choice. However, the model's insights also extend to a setting where career concerns dictate the behavior of public pension fund managers. Indeed, our empirical results appear most consistent with such agency behavior. A pension fund's Board of Trustees and staff appear to act in their own self-interest when selecting pension investments.

Consistent with this positive, rather than normative, interpretation of the model, our empirical analysis of 125 state pension plans over the 2000-2009 period finds that a fund tends to take more overall asset – liability "tracking error" risk following declines in its relative investment performance. Tracking error volatility also is higher for pension funds that select a high rate with which to discount their liabilities and those pension funds that have a greater proportion of participants on their Boards of Trustees.

Our analysis suggests that portfolio allocation of public pension plans is far from the liability immunizing strategies advocated by many academics. Rather, public plans appear to be following a "traditional" investment strategy where their investments are chosen independently from their funds' liability risks. The excessive risks resulting from these strategies will be primarily borne by taxpayers.

## Appendix

This appendix derives the value and risk characteristics of a pension fund's projected benefit obligation (PBO) when an employee's retirement benefit takes the form of annuity payments proportional to the employee's wages over the last *N* years of work. It is assumed that both wage rates and interest rates are stochastic. One source of uncertainty that we neglect is the worker's probability of separation, though it would be relatively straightforward to incorporate this risk.[25]

Let the date *t* wage rate be $w_t$, which is assumed to grow at a rate that is a constant $\delta$ less than the rate of return on a tradeable security that has the same priced risk as the wage rate.[26] Then the risk-neutral process followed by the wage rate can be modeled as

$$dw_t / w_t = \left(r_t - \delta\right) dt + \sigma_w dz_w \tag{A.1}$$

where $\sigma_w$ is the standard deviation of wage growth and $r_t$ is the instantaneous-maturity default-free interest rate.

It is assumed that the term structure of default-free interest rates follows the model of Vasicek (1977), which assumes that the risk-neutral process for $r_t$ is

$$dr_t = \alpha\left(\overline{r}^Q - r_t\right) dt + \sigma_r dz_r \tag{A.2}$$

where $\alpha$ measures the strength of mean reversion of $r_t$ to its unconditional mean, $\overline{r}^Q$, and the volatility $\sigma_r$ is a constant. The risk-neutral unconditional mean equals $\overline{r}^Q = \overline{r} + \theta\sigma_r / \alpha$, where $\overline{r}$ is the interest rate's physical unconditional mean and $\theta$ is a market price of interest rate risk. The wage process and interest rate process may be correlated, such that $dz_w dz_r = \rho dt$.

This model of the instantaneous-maturity interest rates implies that the date *t* price of a zero-coupon bond that pays \$1 at date $t+\tau$ is given by

$$P\left(r_t, \tau\right) = A\left(\tau\right) e^{-r_t B(\tau)} \tag{A.3}$$

where $A\left(\tau\right) \equiv \exp\left[\left(B\left(\tau\right) - \tau\right)\left(\overline{r}^Q - \tfrac{1}{2}\sigma_r^2 / \alpha^2\right)\right]$ and $B\left(\tau\right) \equiv \left(1 - e^{-\alpha\tau}\right) / \alpha$. An application of Itô's lemma to equation (A.3) implies that the risk-neutral process for the bond satisfies

$$\frac{dP\left(r_t, \tau\right)}{P\left(r_t, \tau\right)} = r_t dt + \sigma_p\left(\tau\right) dz_p \tag{A.4}$$

where $\sigma_p\left(\tau\right) \equiv \sigma_r B\left(\tau\right)$ and $dz_p = - dz_r$.

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[25] It would be straightforward to introduce a termination date that differs from the date when the annuity begins, since our framework already values individual cashflows that occur following retirement. Demographic data could be used to estimate a worker's probability of termination (or survival) during a maximum working horizon, so that valuation would assign a probability of different horizon annuities for each worker. Valuation would be particularly easy if termination probabilities were uncorrelated with interest rates and wages.

[26] See Pennacchi (1999) for a similar modeling of wages. His footnote 18 discusses how the parameter $\delta$ can be estimated.

The employee is assumed to retire at date $T$ and receive an annual annuity payment for $m$ years. Each annual payment is set to $c\sum_{n=0}^{N-1} w_{T-n}$ ; that is, a proportion of the last $N$ years of wages.[27] To value this annuity, we can divide it into $N$ annuities, each starting at date $T$ (with the first payment at date $T$+1) but set to a different year's prior wage. The value of the annuity linked to the wage $w_{T-n}$ will be denoted $V_E^n$. Its value at date $t < T-n$ is

$$\begin{aligned}
V_E^n &= E_t^Q\left[e^{-\int_t^T r_s ds} c w_{T-n}\sum_{i=1}^m P\left(r_T,i\right)\right] \\
&= c w_t E_t^Q\left[e^{-\int_t^T r_s ds+\int_t^{T-n}\left(r_s-\alpha_w-\frac{1}{2}\sigma_w^2\right)ds+\sigma_w\int_t^{T-n}dz_w}\sum_{i=1}^m P\left(r_T,i\right)\right] \\
&= c w_t e^{-\left(\alpha_w+\frac{1}{2}\sigma_w^2\right)(T-n-t)} E_t^Q\left[e^{-\int_{T-n}^T r_s ds+\sigma_w\left(z_{wT-n}-z_{wt}\right)}\sum_{i=1}^m E_T^Q\left[e^{-\int_T^{T+i} r_s ds}\right]\right] \\
&= c w_t e^{-\left(\alpha_w+\frac{1}{2}\sigma_w^2\right)(T-n-t)} E_t^Q\left[e^{\sigma_w\left(z_{wT-n}-z_{wt}\right)}\sum_{i=1}^m E_{T-n}^Q\left[e^{-\int_{T-n}^{T+i} r_s ds}\right]\right] \\
&= c w_t e^{-\left(\alpha_w+\frac{1}{2}\sigma_w^2\right)(T-n-t)} E_t^Q\left[e^{\sigma_w\left(z_{wT-n}-z_{wt}\right)}\sum_{i=1}^m P\left(r_{T-n},i+n\right)\right] \\
&= c w_t e^{-\left(\alpha_w+\frac{1}{2}\sigma_w^2\right)(T-n-t)}\sum_{i=1}^m A\left(i+n\right) E_t^Q\left[e^{\sigma_w\left(z_{wT-n}-z_{wt}\right)-r_{T-n}B(i+n)}\right]
\end{aligned} \tag{A.5}$$

Given that the $r_t$ follows the process in equation (A.2), its discrete-time distribution is normal with mean $E_t^Q\left[r_T\right] = r_t e^{-\alpha(T-t)} + \bar{r}^Q\left(1-e^{-\alpha(T-t)}\right)$ and variance $Var_t\left[r_T\right] = \sigma_r{}^2\left(1-e^{-2\alpha(T-t)}\right)/\left(2\alpha\right)$. This implies that the risk-neutral expectation in (A.5) can be evaluated as

$$\begin{aligned}
&E_t^Q\left[e^{\sigma_w\left(z_{wT-n}-z_{wt}\right)-r_{T-n}B(i+n)}\right] \\
&= e^{-\left[r_t e^{-\alpha(T-n-t)}+\bar{r}^Q\left(1-e^{-\alpha(T-n-t)}\right)\right]B(i+n)+\frac{1}{2}\sigma_w^2(T-n-t)+\frac{1}{2}B(i+n)^2\frac{\sigma_r^2}{2\alpha}\left(1-e^{-2\alpha(T-n-t)}\right)} \\
&\quad\times e^{\rho\sigma_w\sigma_r B(i+n)\sqrt{(T-n-t)\left(1-e^{-2\alpha(T-n-t)}\right)/(2\alpha)}} \\
&= e^{-E_t^Q[r_{T-n}]B(i+n)+\frac{1}{2}\sigma_w^2(T-n-t)+\psi(i+n,T-n-t)}
\end{aligned} \tag{A.6}$$

where

$$\begin{aligned}
\Psi\left(i+n,T-n-t\right) \equiv{}& \tfrac{1}{2}B\left(i+n\right)^2\frac{\sigma_r^2}{2\alpha}\left(1-e^{-2\alpha(T-n-t)}\right) \\
&+\rho\sigma_w\sigma_r B\left(i+n\right)\sqrt{\left(T-n-t\right)\left(1-e^{-2\alpha(T-n-t)}\right)/\left(2\alpha\right)}
\end{aligned} \tag{A.7}$$

Substituting this result into (A.5), we obtain

$$\begin{aligned}
V_E^n &= c w_t e^{-\alpha_w(T-n-t)}\sum_{i=1}^m A\left(i+n\right)e^{-E_t^Q[r_{T-n}]B(i+n)+\psi(i+n,T-n-t)} \\
&= c w_t e^{-\alpha_w(T-t-n)}\sum_{i=1}^m P\left(E_t^Q\left[r_{T-n}\right],i+n\right)e^{\psi(i+n,T-n-t)}
\end{aligned} \tag{A.8}$$

[27] Typically, annuity payments are computed as the product of years of service, average wages over the last $N$ years of service, and a conversion ratio. To value the plan's accrued liabilities adjusted for likely wage growth, the parameter $c$ could equal the employee's current years of service times the conversion ratio.

The value of the entire annuity composed of those linked to each wage is then the sum:

$$\begin{aligned} V_E &= \sum_{n=0}^{N-1} V_E^n \\ &= cw_t e^{-\alpha_w(T-t)} \sum_{n=0}^{N-1} e^{\alpha_w n} \sum_{i=1}^{m} A(i+n) e^{-E_t^Q[r_{T-n}]B(i+n)+\psi(i+n,T-n-t)} \\ &= cw_t e^{-\alpha_w(T-t)} \sum_{n=0}^{N-1} \sum_{i=1}^{m} P\left(E_t^Q\left[r_{T-n}\right], i+n\right) e^{\alpha_w n+\psi(i+n,T-n-t)} \end{aligned} \tag{A.9}$$

Equation (A.9) shows that the present value of the PBO obligation is a function of the current wage rate, $w_t$, various times until maturity ($T$-$t$ + $i$ where $i$ = 0, 1,…,$m$), and the current interest rate, $r_t$, through the expectation term $E_t^Q\left[r_{T-n}\right] = r_t e^{-\alpha(T-n-t)} + \overline{r}^Q\left(1-e^{-\alpha(T-n-t)}\right)$.

Recognizing that the risk-neutral expected rate of return on the employee's benefit must equal $r_t$ and its risk-neutral and physical volatilites are the same and given by Itô's lemma, we can write this risk-neutral process as

$$\begin{aligned} \frac{dV_E}{V_E} &= r_t dt + \frac{1}{V_E}\frac{\partial V_E}{\partial w_t}\sigma_w w_t dz_w + \frac{1}{V_E}\frac{\partial V_E}{\partial r_t}\sigma_r dz_r \\ &= r_t dt + \sigma_w dz_w + \frac{1}{V_E}\frac{\partial V_E}{\partial r_t}\sigma_r dz_r \end{aligned} \tag{A.10}$$

The partial derivatives with respect to the interest rate can be rewritten as

$$\begin{aligned} \frac{\partial V_E}{\partial r_t} &= cw_t e^{-\alpha_w(T-t)} \sum_{n=0}^{N-1} \sum_{i=1}^{m} \frac{\partial P\left(E_t^Q\left[r_{T-n}\right], i+n\right)}{\partial E_t^Q\left[r_{T-n}\right]} e^{-\alpha(T-n-t)} e^{\alpha_w n+\psi(i+n,T-n-t)} \\ &= -cw_t e^{-\alpha_w(T-t)} \sum_{n=0}^{N-1} \sum_{i=1}^{m} P\left(E_t^Q\left[r_{T-n}\right], i+n\right) e^{\alpha_w n+\psi(i+n,T-n-t)} B(i+n) e^{-\alpha(T-n-t)} \end{aligned} \tag{A.11}$$

Dividing this derivative by $V_E$,

$$\begin{aligned} \frac{1}{V_E}\frac{\partial V_E}{\partial r_t} &= \frac{-\sum_{n=0}^{N-1}\sum_{i=1}^{m} P\left(E_t^Q\left[r_{T-n}\right], i+n\right) e^{\alpha_w n+\psi(i+n,T-n-t)} B(i+n) e^{-\alpha(T-n-t)}}{\sum_{n=0}^{N-1}\sum_{i=1}^{m} P\left(E_t^Q\left[r_{T-n}\right], i+n\right) e^{\alpha_w n+\psi(i+n,T-n-t)}} \\ &= -\sum_{n=0}^{N-1}\sum_{i=1}^{m} \omega_i^n B(i+n) e^{-\alpha(T-n-t)} \end{aligned} \tag{A.12}$$

where

$$\omega_i^n \equiv \frac{P\left(E_t^Q\left[r_{T-n}\right], i+n\right) e^{\alpha_w n+\psi(i+n,T-n-t)}}{\sum_{n=0}^{N-1}\sum_{i=1}^{m} P\left(E_t^Q\left[r_{T-n}\right], i+n\right) e^{\alpha_w n+\psi(i+n,T-n-t)}} \tag{A.13}$$

is a weight for the annuity payment having the horizon of $i$ years linked to the wage $w_{T-n}$. Using (A.12) allows us to re-write (A.10) as

$$\begin{aligned}\frac{dV_E}{V_E} &= r_t dt + \sigma_w dz_w - \left[\sum_{n=0}^{N-1}\sum_{i=1}^{m}\omega_i^n \sigma_r B(i+n) e^{-\alpha(T-n-t)}\right] dz_r \\ &= r_t dt + \sigma_w dz_w + \left[\sum_{n=0}^{N-1}\sum_{i=1}^{m}\omega_i^n e^{-\alpha(T-n-t)} \sigma_p (i+n)\right] dz_p\end{aligned} \tag{A.14}$$

From (A.14) we see that the stochastic process for the PBO liability of a given worker includes the same stochastic component as wage growth and a weighted average of discounted stochastic components of zero-coupon bonds of various maturities.

Equations (9) and (14) give the value and rate of return process, respectively, for the PBO liability for a given worker. Next, consider the similar quantities for a retired participant in the pension fund. The retiree's annuity payment is known (in nominal terms if there is no COLA and in real terms if there is a COLA). Let $c_r$ be the retiree's annual benefit that is to be paid for $m_r$ remaining years. Then the value of the annuity is

$$V_A = c_r \sum_{i=1}^{m_r} P(r_t, i) \tag{A.15}$$

and its risk-neutral process satisfies

$$\frac{dV_A}{V_A} = r_t dt + \frac{1}{V_A}\frac{\partial V_A}{\partial r_t}\sigma_r dz_r \tag{A.16}$$

where

$$\frac{1}{V_A}\frac{\partial V_A}{\partial r_t} = -\frac{\sum_{i=1}^{m_r} P(r_t, i) B(i)}{\sum_{i=1}^{m_r} P(r_t, i)} \tag{A.17}$$

so that

$$\frac{dV_A}{V_A} = r_t dt + \sum_{i=1}^{m_r} \omega_i^p \sigma_p (i)\, dz_p \tag{A.18}$$

where $\omega_i^p \equiv P(r_t, i) / \sum_{i=1}^{m_r} P(r_t, i)$ is the value of annuity payment *i* relative to the value of all remaining annuity payments.

To determine the value and the rate of return process for the pension fund's aggregate liabilities, we next define $V_{j,E}(t)$ to be the date *t* value of current employee *j*'s annuity, which reflects her particular retirement date, *T*, and wage level, *c*. Then if the pension fund has $N_E$ eligible employees, the value of liabilities for all current employees is

$$L_t^E = \sum_{j=1}^{N_E} V_{j,E}(t) \tag{A.19}$$

Let $\gamma_{j,E} \equiv V_{j,E}(t) / L_t^E$ be the value weight of employee *j*'s annuity to the value of all employee annuities. Then the rate of return on the pension fund's total employee liabilities is

$$\frac{dL_t^E}{L_t^E} = \sum_{j=1}^{N_E} \gamma_{j,E} \frac{dV_{j,E}}{V_{j,E}} \tag{A.20}$$

where $dV_{j,E}/V_{j,E}$ follows the process in (A.14). Since $\sum_{j=1}^{N_E}\gamma_{j,E}=1$, this implies that total employee liabilities follow a risk-neutral process of the form

$$\frac{dL_t^E}{L_t^E}=r_t dt+\sigma_w dz_w+\sum_{j=1}^{N_E}\gamma_{i,E}\Omega_{i,E}dz_p \tag{A.21}$$

where $\Omega_{j,E}$ is the term in square brackets in the last line of equation (A.14) that pertains to employee $j$.

Similar logic implies that the pension fund's total liabilities for its $N_A$ retirees satisfies:

$$\frac{dL_t^A}{L_t^A}=r_t dt+\sum_{j=1}^{N_A}\gamma_{i,A}\Omega_{i,A}dz_p \tag{A.22}$$

where $L_t^A=\sum_{j=1}^{N_A}V_{j,A}(t)$, $\gamma_{j,A}\equiv V_{j,A}(t)/L_t^A$, and $\Omega_{j,A}$ is the term multiplying $dz_p$ in equation (A.18) that pertains to retiree $j$.

Finally, let $L_t=L_t^E+L_t^A$ be the pension fund's total liabilities. The previous results implies that its risk-neutral process satisfies

$$\frac{dL_t}{L_t}=r_t dt+\frac{L_t^E}{L_t}\sigma_w dz_w+\left(\frac{L_t^E}{L_t}\sum_{j=1}^{N_E}\gamma_{i,E}\Omega_{i,E}+\frac{L_t^A}{L_t}\sum_{j=1}^{N_A}\gamma_{i,A}\Omega_{i,A}\right)dz_p \tag{A.23}$$

Equation (A.23) shows that the pension fund's liabilities has a standard deviation associated with wage risk equal to the volatility of wage growth times the weight of employee liabilities to total liabilities. The standard deviation associated with bond risk is a linear combination of different maturity zero coupon bond standard deviations that correspond to the times when employees' and retirees' benefit payments are to be made. If these standard deviations are approximately constant over time, so that the last two terms in (A.23) can be written as $\sigma_{Lw}dz_w+\sigma_{Lp}dz_p\equiv\sigma_L dz_L$ then the physical process corresponding to the risk-neutral process in (A.23) will equal equation (1) in the text where $\sigma_L^2=\sigma_{Lw}^2+\sigma_{Lp}^2-2\rho\sigma_{Lw}\sigma_{Lp}$ and $\alpha_L=r_t+\theta_L\sigma_L$ where $\theta_L$ is the market price of risk associated with the Brownian motion $dz_L$.

Our empirical work approximates $L_t^E/L_t=1-L_t^A/L_t$ by the pension fund's number of active participants to the total number of active participants and annuitants; that is, $N^E/(N^E+N^A)$. Also, we approximate the bond standard deviation term in parentheses in equation (A.23) as $\varphi\sigma_p(15)$, where

$$\varphi=0.4+0.558\frac{N^E}{N^E+N^A}+0.0425\left(\frac{N^E}{N^E+N^A}\right)^2 \tag{A.24}$$

and $\sigma_p(15)$ is the standard deviation of a nominal or real zero-coupon Treasury bond having a maturity of 15 years. Note that when $N^E/(N^E+N^A)=0$, $\varphi=0.4$ and the pension fund's bond standard deviation will approximately equal that of a bond with maturity 6 years, which Mercer (2010) states is the duration of a pension fund with all retires. When $N^E/(N^E+N^A)=1$, $\varphi=1.33$ and the pension fund's bond standard deviation will approximately equal that of a bond with maturity 20 years, which Mercer (2010) states is the duration of a pension fund with all active participants. Finally, when $N^E/(N^E+N^A)=0.6657$, which is the average for our sample of public

pension funds, $\varphi = 1$ and the pension fund's bond standard deviation will approximately equal that of a bond with maturity 15 years, which Mercer (2010) states is the duration of a "typical" pension fund.

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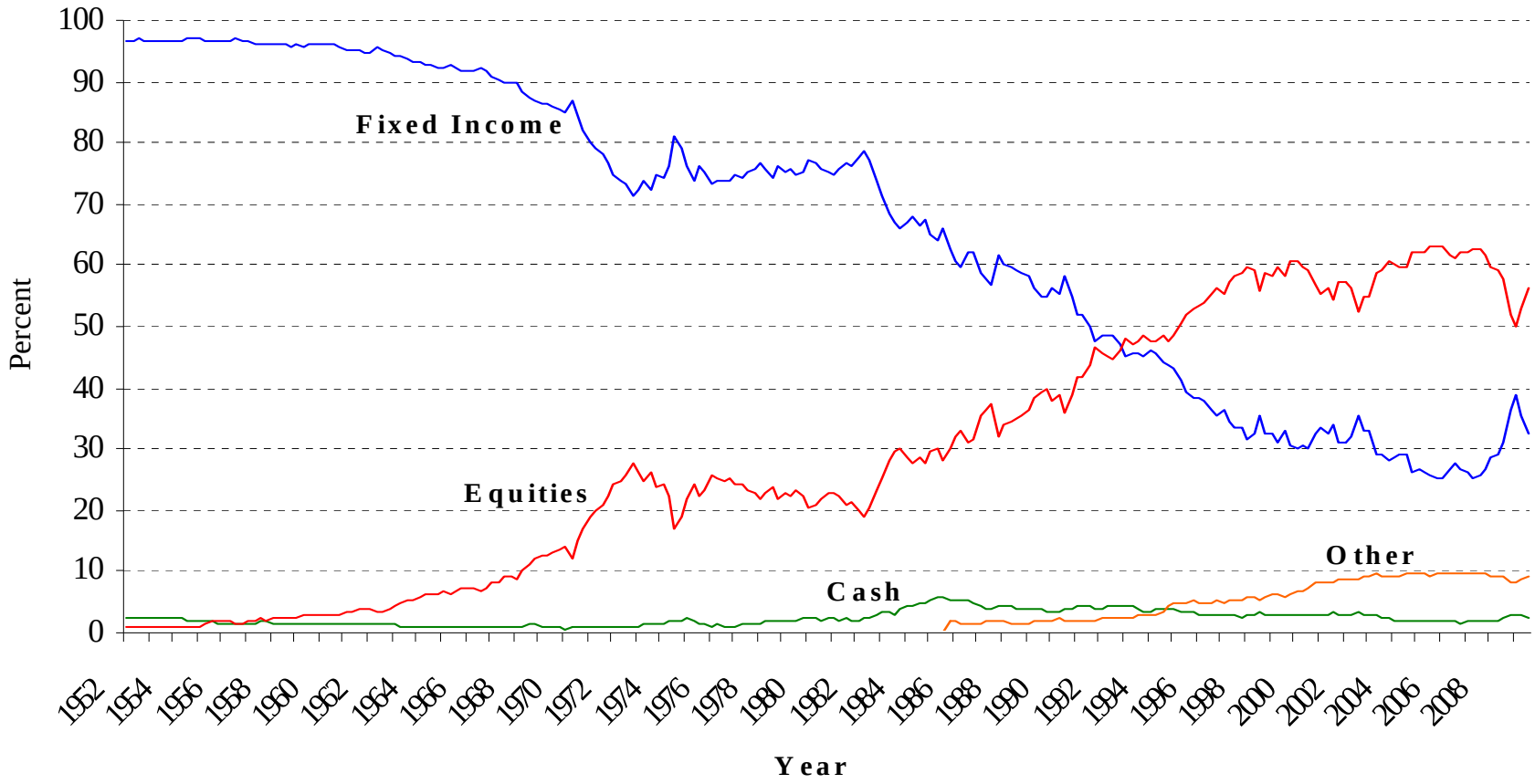
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**Figure 1**

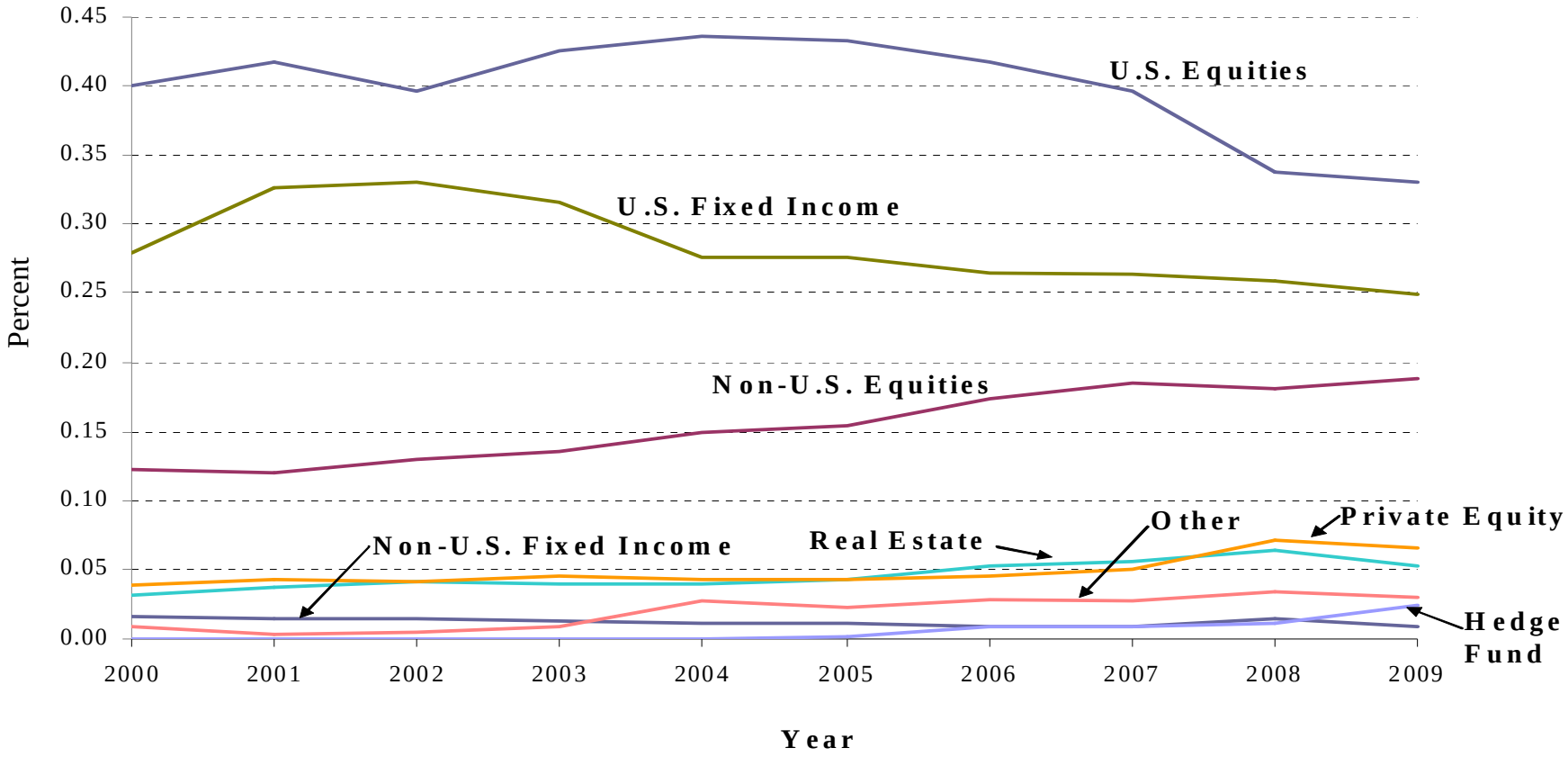
**Portfolio Allocations of State and Local Government Pension Funds**




Source: Federal Reserve Flow of Funds Accounts Table L.119

**Figure 2**

**Portfolio Allocations of Sample State Pension Funds**




Source: Wilshire Associates

## Table 1

## Summary Statistics of Asset Returns and Wage Growth

| Variable | Observations | Mean | Std. Dev. | Minimum | Maximum |
|---|---|---|---|---|---|
| **Asset Rates of Return:** | | | | | |
| US Equities | 159 | 0.0533 | 0.1714 | -0.1939 | 0.1007 |
| Non-US Equities | 159 | 0.0488 | 0.1891 | -0.2496 | 0.1341 |
| US Fixed Income | 159 | 0.0727 | 0.0885 | -0.0908 | 0.1072 |
| Non-US Fixed Income | 159 | 0.0529 | 0.0838 | -0.0609 | 0.0768 |
| US-Real Estate | 159 | 0.0858 | 0.2447 | -0.3913 | 0.2841 |
| Private Equity | 52 | 0.1295 | 0.1232 | -0.1567 | 0.1466 |
| Hedge Fund | 159 | 0.0815 | 0.0512 | -0.0537 | 0.0501 |
| **Liability Component Rates of Return:** | | | | | |
| State and Local Employee Wage Growth | 52 | 0.0293 | 0.0107 | 0.0010 | 0.0196 |
| 15-Year Nominal Bond | 159 | 0.0795 | 0.1376 | -0.1466 | 0.1760 |
| 15-Year Real Bond | 159 | 0.0795 | 0.0971 | -0.1285 | 0.0976 |

Note: Summary statistics for rates of return on seven different asset categories: U.S. Equities; Non-U.S. Equities; U.S. Fixed Income; Non-U.S. Fixed Income; U.S. Real Estate; Private Equity; and Hedge Funds, as well as State and Local Employee Cost Index wage growth and 15-Year Nominal and Real (TIPS) Bond returns. State and local employee wage growth and private equity returns are calculated based on quarterly data from the second quarter of 1997 to the first quarter of 2010. All other statistics are calculated from monthly return data between February 1997 and April 2010. Means and standard deviations are annualized. Minimums and maximums for wage growth and private equity are one quarter rates while minimums and maximums for the other series are one month rates.

## Table 2

## Correlations of Asset Returns and State and Local Wage Growth

| | US Equities | Non-US Equities | US Fixed Income | Non-US Fixed Income | US-Real Estate | Private Equity | Hedge Funds | S&L Wage Growth | 15-Year Nominal Bond | 15-Year Real Bond |
|---|---|---|---|---|---|---|---|---|---|---|
| US Equities | 1 | | | | | | | | | |
| Non-US Equities | 0.87091 | 1 | | | | | | | | |
| US Fixed Income | 0.06412 | 0.11068 | 1 | | | | | | | |
| Non-US Fixed Income | 0.04328 | 0.22146 | 0.49819 | 1 | | | | | | |
| US-Real Estate | 0.57669 | 0.57607 | 0.17921 | 0.16747 | 1 | | | | | |
| Private Equity | 0.79986 | 0.73599 | -0.20965 | -0.17387 | 0.46705 | 1 | | | | |
| Hedge Funds | 0.63777 | 0.69356 | 0.12148 | 0.02080 | 0.33082 | 0.74276 | 1 | | | |
| S&L Wage Growth | -0.27389 | -0.29040 | 0.20711 | 0.14081 | -0.00980 | -0.25375 | -0.21839 | 1 | | |
| 15-Year Nominal Bond | -0.07907 | -0.04836 | 0.94010 | 0.46061 | 0.02485 | -0.29953 | -0.01632 | 0.25426 | 1 | |
| 15-Year Real Bond | 0.04059 | 0.11604 | 0.72275 | 0.49579 | 0.16940 | -0.18294 | 0.14551 | 0.18128 | 0.68908 | 1 |

Note: Correlation table for rates of return on seven different asset categories: U.S. Equities; Non-U.S. Equities; U.S. Fixed Income; Non-U.S. Fixed Income; U.S. Real Estate; Private Equity; and Hedge Funds, as well as State and Local Employee Cost Index wage growth and 15-Year Nominal and Real (TIPS) Bond returns. S&L wage growth and private equity returns are calculated based on quarterly data from the second quarter of 1997 to the first quarter of 2010. All other correlations are calculated based on monthly return data between February 1997 and April 2010.

## Table 3

## Correlations of Equity and Bond Returns with National Wage Growth, 1952- 2008

Panel A: Correlation between National Wage Index Growth and S&P500 Return

| Holding Period | Correlation | Observations | t-statistics | p-value |
|---|---|---|---|---|
| one-year | -0.0389 | 57 | -0.2890 | 0.7737 |
| three-year | 0.0246 | 19 | 0.1014 | 0.9205 |
| five-year | -0.4348 | 11 | -1.4483 | 0.1815 |
| seven-year | -0.2860 | 8 | -0.7312 | 0.4922 |
| nine-year | -0.4406 | 6 | -0.9816 | 0.3819 |

Panel B: Correlation between National Wage Index Growth and 20-year Treasury bond Return

| Holding Period | Correlation | Observations | t-statistics | p-value |
|---|---|---|---|---|
| one-year | -0.2163 | 57 | -1.6427 | 0.1061 |
| three-year | -0.3201 | 19 | -1.3932 | 0.1815 |
| five-year | -0.2645 | 11 | -0.8228 | 0.4319 |
| seven-year | 0.0859 | 8 | 0.2113 | 0.8397 |
| nine-year | 0.2449 | 6 | 0.5053 | 0.6399 |

Panel C: Correlation between National Wage Index Growth and 10-year Treasury note Return

| Holding Period | Correlation | Observations | t-statistics | p-value |
|---|---|---|---|---|
| one-year | -0.1095 | 57 | -0.8171 | 0.4174 |
| three-year | -0.0961 | 19 | -0.3979 | 0.6957 |
| five-year | -0.0746 | 11 | -0.2245 | 0.8274 |
| seven-year | 0.2753 | 8 | 0.7013 | 0.5094 |
| nine-year | 0.4752 | 6 | 1.0802 | 0.3408 |

Note: National Wage Index is taken from the U.S. Bureau of Labor Statistics. The returns on the S&P 500 Index, the 10-year Treasury note and the 20-year Treasury bond were obtained from the Center for Research in Security Prices (CRSP).

## Table 4

### Time Series of Average Asset Allocations, Tracking Error Volatilities, and Funding Ratio

| | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Asset Class Allocations:** | | | | | | | | | | |
| US Equities | 0.3996 | 0.4174 | 0.3966 | 0.4261 | 0.4363 | 0.4322 | 0.4172 | 0.3957 | 0.3378 | 0.3299 |
| Non-US Equities | 0.1231 | 0.1197 | 0.1303 | 0.1348 | 0.1497 | 0.1534 | 0.1743 | 0.1848 | 0.1802 | 0.1880 |
| US Fixed Income | 0.2796 | 0.3258 | 0.3306 | 0.3162 | 0.2769 | 0.2757 | 0.2648 | 0.2629 | 0.2588 | 0.2486 |
| Non-US Fixed Income | 0.0156 | 0.0144 | 0.0141 | 0.0122 | 0.0116 | 0.0114 | 0.0087 | 0.0085 | 0.0143 | 0.0090 |
| US-Real Estate | 0.0314 | 0.0372 | 0.0418 | 0.0405 | 0.0396 | 0.0434 | 0.0520 | 0.0553 | 0.0647 | 0.0524 |
| Private Equity | 0.0378 | 0.0427 | 0.0419 | 0.0456 | 0.0429 | 0.0432 | 0.0454 | 0.0499 | 0.0715 | 0.0659 |
| Hedge Fund | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0002 | 0.0013 | 0.0089 | 0.0082 | 0.0115 | 0.0238 |
| Other | 0.0091 | 0.0027 | 0.0037 | 0.0087 | 0.0269 | 0.0235 | 0.0286 | 0.0267 | 0.0342 | 0.0303 |
| **Tracking Error Volatilities for Benchmark:** | | | | | | | | | | |
| Nominal Liabilities | 0.2292 | 0.2257 | 0.2253 | 0.2284 | 0.2337 | 0.2343 | 0.2370 | 0.2376 | 0.2356 | 0.2354 |
| Real Liabilities | 0.2036 | 0.2000 | 0.1993 | 0.2027 | 0.2083 | 0.2087 | 0.2112 | 0.2116 | 0.2093 | 0.2088 |
| Nominal Liabilities Demographically Adjusted | 0.2107 | 0.2072 | 0.2060 | 0.2091 | 0.2138 | 0.2146 | 0.2163 | 0.2172 | 0.2146 | 0.2155 |
| Real Liabilities Demographically Adjusted | 0.1833 | 0.1798 | 0.1784 | 0.1817 | 0.1867 | 0.1873 | 0.1889 | 0.1896 | 0.1868 | 0.1871 |
| Average Asset Allocation of Peers | 0.1684 | 0.1639 | 0.1629 | 0.1669 | 0.1735 | 0.1739 | 0.1762 | 0.1762 | 0.1717 | 0.1706 |
| **Funding Ratio** | 1.0852 | 0.9226 | 0.7912 | 0.7775 | 0.8250 | 0.8454 | 0.8618 | 0.9336 | 0.7831 | 0.5832 |
| **Return on Investments** | -0.0129 | -0.0524 | -0.0723 | 0.2093 | 0.1304 | 0.0807 | 0.1429 | 0.0678 | -0.3067 | 0.1881 |

Note: Sample average over 125 state pension plans for eight different asset allocations, various tracking error volatilities, funding ratios (market value of assets divided by actuarial value of liabilities), and return on investments. The return on investments were estimated as the product of their asset-class allocation weights and the returns earned by each of the asset classes.

**Figure 5**

**Tracking Error Minimizing Allocations**

| | Nominal Liabilities | | Real Liabilities | |
|---|---|---|---|---|
| | All Asset Classes | No Private Equity and Hedge Funds | All Asset Classes | No Private Equity and Hedge Funds |
| US Equities | 0.1883 | 0.0075 | -0.0458 | -0.0527 |
| Non-US Equities | -0.0649 | -0.0927 | -0.0586 | -0.0166 |
| US Fixed Income | 0.8565 | 1.0556 | 0.6098 | 0.7846 |
| Non-US Fixed Income | -0.0478 | 0.0535 | 0.2273 | 0.2479 |
| US-Real Estate | -0.0101 | -0.0240 | 0.0359 | 0.0368 |
| Private Equity | -0.4739 | - | -0.0049 | - |
| Hedge Fund | 0.5520 | - | 0.2364 | - |

Note: The entries are the pension asset portfolio weights that minimize a typical pension fund's tracking error volatility. This typical fund's liabilities have a 15-year duration and a ratio of active to total participants of 66.6%.

**Table 6**

**Regression Variable Summary Statistics**

| Variable | Observations | Mean | Std. Dev. | Minimum | Maximum |
|---|---|---|---|---|---|
| **Dependent Variable; Tracking Error Volatility:** | | | | | |
| Nominal Liabilities | 1122 | 0.2321 | 0.0135 | 0.1853 | 0.2707 |
| Real Liabilities | 1122 | 0.2062 | 0.0141 | 0.1563 | 0.2378 |
| Nominal Liabilities Demographically Adjusted | 904 | 0.2123 | 0.0208 | 0.1425 | 0.2757 |
| Real Liabilities Demographically Adjusted | 904 | 0.1848 | 0.0183 | 0.1339 | 0.2389 |
| Average Asset Allocation of Peers | 1122 | 0.1704 | 0.0083 | 0.1500 | 0.1996 |
| **Primary Explanatory Variables:** | | | | | |
| Funding Ratio | 1122 | 0.8533 | 0.1976 | 0.1908 | 1.7520 |
| Return on Investments | 1122 | 0.0529 | 0.1370 | -0.3053 | 0.2927 |
| **Governance Variables:** | | | | | |
| Participants to Total Board Members | 922 | 0.5647 | 0.2226 | 0 | 1 |
| Dummy for Separate Investment Council | 931 | 0.3845 | 0.4867 | 0 | 1 |
| Dummy for Legal Restrictions | 931 | 0.3083 | 0.4620 | 0 | 1 |
| **Other Control Variables:** | | | | | |
| Natural log of Market Value of Assets | 1122 | 22.6895 | 1.3556 | 19.21909 | 26.1743 |
| Payroll to Actuarial Liabilities | 1122 | 0.2360 | 0.2028 | 0.0085 | 2.6248 |
| Discount Rate | 1122 | 0.0801 | 0.0039 | 0.07 | 0.09 |
| Dummy for Teachers Fund | 931 | 0.5070 | 0.5002 | 0 | 1 |
| Dummy for General State Fund | 931 | 0.6230 | 0.4849 | 0 | 1 |
| Dummy for Police and Fire Fighters Fund | 931 | 0.4447 | 0.4972 | 0 | 1 |

**Figure 7 Tracking Error Regression Results**

| Tracking Error Volatility Benchmark | Dependent Variable: Tracking Error Volatility | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Liabilities Unadjusted | | | | Liabilities Demographically Adjusted | | | | Average Asset Allocation of Peers | |
| | Nominal Liabilities | | Real Liabilities | | Nominal Liabilities | | Real Liabilities | | | |
| Lag Funding Ratio | -0.0009 | -0.0047 | -0.0016 | -0.0063 | -0.0018 | -0.0034 | -0.0027 | -0.0049 | -0.0028 | -0.0036 |
| | (.79) | (.26) | (.63) | (.15) | (.59) | (.42) | (.42) | (.26) | (.17) | (.17) |
| Lag Return Relative to Peer Average | -0.0545 | -0.0525 | -0.0627 | -0.0613 | -0.0404 | -0.0382 | -0.0481 | -0.0461 | -0.0359 | -0.0291 |
| | (.00) | (.00) | (.00) | (.00) | (.00) | (.00) | (.00) | (.00) | (.00) | (.00) |
| Lag of Ln Market Value of Assets | 0.0006 | 0.0038 | 0.0008 | 0.0045 | -0.0004 | 0.0016 | 0.0001 | 0.0023 | 0.0007 | 0.0009 |
| | (.49) | (.36) | (.38) | (.30) | (.82) | (.68) | (.94) | (.58) | (.18) | (.75) |
| Lag Discount Rate | 1.0034 | 0.7027 | 1.1512 | 0.8819 | 0.646 | 0.6968 | 0.8071 | 0.8726 | 0.4246 | 0.4348 |
| | (.00) | (.00) | (.00) | (.00) | (.00) | (.00) | (.00) | (.00) | (.00) | (.00) |
| Lag Payroll to Actuarial Liabilities | -0.0049 | 0.0017 | -0.0054 | 0.0015 | 0.0104 | 0.0001 | 0.0103 | -0.0006 | -0.0032 | -0.0003 |
| | (.22) | (.78) | (.19) | (.82) | (.04) | (.98) | (.03) | (.92) | (.18) | (.95) |
| Participants to Total Board Members | 0.0109 | | 0.0117 | | 0.0198 | | 0.0196 | | 0.0055 | |
| | (.01) | | (.01) | | (.02) | | (.01) | | (.03) | |
| Dummy for Teachers Fund | 0.0009 | | 0.0008 | | 0.0028 | | 0.0021 | | 0.0002 | |
| | (.69) | | (.75) | | (.53) | | (.58) | | (.86) | |
| Dummy for General State Fund | -0.0003 | | -0.0004 | | 0.0022 | | 0.0013 | | 0.0003 | |
| | (.90) | | (.86) | | (.61) | | (.72) | | (.82) | |
| Dummy for Police and Fire Fighters Fund | -0.0028 | | -0.0032 | | -0.0019 | | -0.0025 | | -0.0007 | |
| | (.21) | | (.17) | | (.67) | | (.50) | | (.59) | |
| Dummy for Separate Investment Council | 0.0011 | | 0.0013 | | -0.0014 | | -0.0007 | | 0.0013 | |
| | (.58) | | (.52) | | (.72) | | (.83) | | (.28) | |
| Dummy for Legal Restrictions | 0.0007 | | 0.0004 | | 0.0024 | | 0.0016 | | 0.0002 | |
| | (.75) | | (.86) | | (.57) | | (.64) | | (.88) | |
| Fund Fixed Effects | No | Yes | No | Yes | No | Yes | No | Yes | No | Yes |
| N | 811 | 986 | 811 | 986 | 787 | 795 | 787 | 795 | 811 | 986 |
| $R^2$ | 0.36 | 0.16 | 0.37 | 0.16 | 0.18 | 0.039 | 0.22 | 0.057 | 0.42 | 0.39 |

Note: Each regression includes time (year) fixed effects. Numbers in parenthesis are p-values. The dependent variable in all regressions is tracking error volatility. In columns (1) and (2) tracking error volatility is constructed assuming 15-year duration nominal liabilities with no demographic adjustments. In columns (3) and (4) tracking error volatility is constructed assuming 15-year duration real liabilities with no demographic adjustment. In columns (5) and (6) tracking error volatility is constructed assuming 15-year duration nominal liabilities with demographic adjustment for the ratio of active to total participants. In columns (7) and (8) tracking error volatility is constructed assuming 15-year duration real liabilities with demographic adjustment for the ratio of active to total participants. In columns (9) and (10) tracking error volatility is constructed assuming a benchmark equal to the average sample asset allocation of peer public pension funds.