Trade and the Global Recession*

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PRELIMINARY AND INCOMPLETE

First Draft: July 2009

This Version: July 2010

Abstract

The ratio of global trade to GDP declined by nearly 30 percent during the global recession of 2008-2009. This large drop in international trade has generated significant attention and concern. Did the decline simply reflect the severity of the recession for traded goods industries? Or alternatively, did international trade shrink due to factors unique to cross border transactions? This paper merges an input-output framework with a gravity trade model and solves numerically several general equilibrium counterfactual scenarios which quantify the relative importance for the decline in trade of the changing composition of global GDP and changes in trade frictions. Our results suggest that the relative decline in demand for manufactures was the most important driver of the decline in manufacturing trade. Changes in demand for durable manufactures alone accounted for 65 percent of the cross-country variation in changes in manufacturing trade/GDP. The decline in total manufacturing demand (durables and non-durables) accounted for more than 80 percent of the global decline in trade/GDP. Trade frictions increased and played an important role in reducing trade in some countries, notably China and Japan, but decreased or remained relatively flat in others. Globally, the impact of these changes in trade frictions largely cancel each other out.

*We thank Costas Arkolakis, Christian Broda, Lorenzo Caliendo, Marty Eichenbaum, Chang-Tai Hsieh, Anil Kashyap, and Ralph Ossa as well as participants at numerous seminars for helpful comments. Tim Kehoe, Kanda Naknoi, Denis Novy, Andy Rose, Jonathan Vogel, and Kei-Mu Yi gave excellent discussions. Fernando Parro and Kelsey Moser provided outstanding research assistance. This research was funded in part by the Neubauer Family Foundation and the Charles E. Merrill Faculty Research Fund at the University of Chicago Booth School of Business. Eaton and Kortum gratefully acknowledge the support of the National Science Foundation under grant number SES-0820338.

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1 Introduction

Peak to trough, estimates suggest that the ratio of global trade to GDP declined by nearly 30 percent.[1] The four panels of Figure 1 plot the average of imports and exports relative to GDP for the four largest countries in the world: U.S. Japan, China, and Germany. Trade to GDP ratios sharply declined in the recent recession in each of these economies.

This large drop in international trade has generated significant attention and concern, even against a backdrop of plunging final demand and collapsed asset prices. For example, Eichengreen (2009) writes, "The collapse of trade since the summer of 2008 has been absolutely terrifying, more so insofar as we lack an adequate understanding of its causes." *International Economy* (2009) asks in its symposium on the collapse, "World trade has been falling faster than global GDP – indeed, faster than at any time since the Great Depression. How is this possible?" Dozens of researchers posed hypotheses in Baldwin (2009), a timely and insightful collection of short essays aimed at the policy community and titled, "The Great Trade Collapse: Causes, Consequences and Prospects."

Given traded goods sectors such as durable manufactures are procyclical, trade may have fallen relative to GDP due to the changing composition of global output. Alternatively, increasing trade frictions at the international border, broadly defined, might be the culprit. This distinction is important because if trade has fallen faster than GDP purely due to compositional effects, then international trade patterns can only contribute to our understanding of the cross-country transmission of the recession. Imagine instead that the decline in trade reflects increases in international trade frictions, such as the reduced availability of trade credit, protectionist measures, or the home-bias implicit in stimulus measures. In such a case, in addition to the initial shock that led to a decline in final demand, there would be an independent contribution from trade amplifying the shock and worsening the recession.

This paper aims to quantify the relative contributions of these explanations, both globally and at the country level. Our conclusion is that the bulk of the decline in international trade is attributable

[1] The global trade index was obtained by multiplying the world trade volume index by the world trade price index available from the Netherlands Bureau for Economic Policy Analysis. This index was divided by our own estimations of world GDP.

to the decline in the share of demand for tradables. Changes in demand for durable manufactures alone accounted for about 65 percent of the cross-country variation in changes in manufacturing trade/GDP from the first quarter of 2008 to the first quarter of 2009, a period encompassing the steep decline in trade. The decline in total manufacturing demand (durables and non-durables) accounted for more than 80 percent of the global decline in trade/GDP in 2008 and 2009.

The decline in trade for some countries (and between some country pairs) did exceed what one would expect simply from the changing patterns of demand. Hence, increasing trade frictions reflected an independent contribution to the troubles facing the global economy and played an important role in some countries, particularly China and Japan. Our calculations suggest, however, that other countries saw reductions in trade frictions over this period. Globally, these effects largely cancel out. This result need not be the case in our framework, and is driven by the data over this period, not the model. When we perform related calculations on data from the Great Depression, we find evidence suggesting a dramatic increase in trade frictions for the United States in the early 1930s.

Our analytic tool is a multi-sector model of production and trade, calibrated to global data from recent quarters. We run counterfactuals to determine what the path of trade would have been without the shift in demand away from the manufacturing sectors and without the increase in trade frictions. The spirit of our exercise is similar to that of growth accounting as well as the "wedges" approach for business cycle accounting in Chari, Kehoe, and McGratten (2007). Just as growth accounting uses a theoretical framework to decompose output growth into the growth of labor and capital inputs as well as a Solow residual term, we use our model to decompose changes in trade flows into factors such as changes in trade frictions and the composition of demand. Closer to Chari, Kehoe, and McGratten (2007), however, our decomposition relies on model-based general equilibrium counterfactual responses to various shock scenarios.

Our basic exercise is simple: we wish to tie the decline in final demand for tradable goods to the decline in trade flows in the recent global recession. The practical implementation of this exercise requires overcoming three empirical difficulties: (1) countries have different input-output

structures tying trade and production flows to final demand; (2) the country-level accounting must be consistent with changing patterns in bilateral trade flows; and (3) data must be at a high enough frequency to capture the contours of the recession.

We solve the first problem by building a multi-sector model with a global input-output structure incorporating country differences. Guided by results such as Engel and Wang (2009) and Lewis, Levchenko, and Tesar (2009) that stress the different cyclical properties of durables and non-durables (generally as well as during the recent recession), we define our sectors as durable manufacturing, non-durable manufacturing, and non-manufacturing. We solve the second problem by merging our global input-output structure with a gravity model of trade. Thus we account for bilateral trade flows between each of 22 countries and the rest of the world, separately for durables and non-durables. Third, we base our measures on monthly data. The decline in trade steepened in the summer of 2008, and reversed sometime in mid-to-late 2009. Annual data would likely miss the key dynamics of the episode (and complete data for 2009 are just now starting to become available). Using a procedure called "temporal disaggregation," we infer monthly production values from annual totals using information contained in monthly industrial production (IP) and producer price (PPI) indices, both widely available for many countries. We first convert these monthly data into dollars and then aggregate to form totals for each quarter, which is our basic unit of time throughout the analysis. This is preferable to starting with quarterly data because converting with average monthly exchange rates is more accurate than converting at average quarterly exchange rates.

We calibrate our multi-country general equilibrium model to fully account for changes in macroeconomic and trade variables over 4-quarter periods to eliminate seasonal effects. Some global results are shown for rolling four-quarter periods starting in 2006 and through the end of 2009. Other results focus on the period from the first quarter of 2008 to the first quarter of 2009. We focus on trade in the durable and non-durable manufacturing sectors. To quantify the impact of global or country-specific shocks on trade flows in our model, we run counterfactual scenarios and relate the outcomes with what we observe in the data.

2 Trade Decline: Hypotheses

The shorter pieces mentioned above and other academic papers have generated several potential explanations for the decline in trade flows relative to overall economic activity. Levchenko, Lewis, and Tesar (2009), for example, use U.S. data to show that the recent decline in trade is large relative to previous recessions. They present evidence of a relative decline in demand for tradables, particularly durable goods. Their paper, as well as the input-output analysis by Bems, Johnson, and Yi (2010), suggest that the changing composition of GDP can largely account for the decline in trade relative to GDP.

Other work suggests that trade frictions, or phenomena that increase home-bias and resemble increasing trade frictions, are of first-order importance. For instance, given that many economies' banking systems have been in crisis, one leading hypothesis is that a collapse in trade credit has contributed to the breakdown in trade. Amiti and Weinstein (2009) demonstrate, with earlier data, that the health of Japanese firms' banks significantly affected the firms' trading volumes, presumably through their role in issuing trade credit. Using U.S. trade data during the recent episode, Chor and Manova (2009) show that sectors requiring greater financing saw a greater decline in trade volume. McKinnon (2009) and Bhagwati (2009) also focus on the role of reduced trade credit availability in explaining the recent trade collapse.

Others note that protectionist measures have exerted an extra drag on trade.[2] Brock (2009) writes, "...many political leaders find the old habits of protectionism irresistible ... This, then, is a large part of the answer to the question as to why world trade has been collapsing faster than world GDP." Another hypothesis is that, since trade flows are measured in gross rather than value added terms, a disintegration of international vertical supply chains may be driving the decline.[3] In addition, dynamics associated with the inventory cycle may be generating disproportionately severe contractions in trade, as in Alessandria, Kaboski, and Midrigan (2009, 2010). Finally, fiscal stimulus measures implemented worldwide may be implicitly home-biased due to political

[2] See www.globaltradealert.org for real-time tracking of protectionist measures implemented during the recent global downturn.

[3] Eichengreen (2009) writes, "The most important factor is probably the growth of global supply chains, which has magnified the impact of declining final demand on trade," and a similar hypothesis is found in Yi (2009).

pressures on government purchases. All of these potential disruptions can be broadly construed as reflecting international trade frictions, where some factor is directly affecting goods which cross the international border per se.

Results such as Levchenko et al. and Chor and Manova only analyze U.S. data in partial equilibrium, but are able to use highly disaggregated data which allow for clean identification of various effects. We view our work as complementary to these U.S.-based empirical studies. Our framework has the benefit of being able to evaluate hypotheses for the trade decline in a multi-country quantitative general equilibrium model.

3 A Brief Look at the Data

Given that the share of spending on tradables typically drops during recessions, it is not surprising that the ratio of trade to GDP also typically declines in recessions. Figures 2 and 3 show proxies for growth in manufacturing spending relative to GDP in the left columns, and growth in non-oil imports relative to GDP in the right columns, also for the four largest economies.[4] Some of these ratios are highly procyclical, others are not. The data points from the recent recession are plotted with red squares, distinguishing them from historical data plotted with blue circles. The squares in the plots of imports/GDP are often right along the regression line, such as for the United States and Germany. For some countries, particularly for Japan and China, they are below the line, indicating the drop in trade is unusual relative to the historical relationship. These simple summary relationships suggest cyclical factors are crucial in understanding some of the decline in global trade flows. They cannot on their own indicate whether composition or trade frictions are most important, however, since trade frictions might themselves be highly cyclical. Hence, a richer general equilibrium framework will be required to determine whether the unprecedentedly large decline in trade is simply a reflection of the unprecedentedly large drop in tradable sector activity, or whether additional forces are driving down global trade flows.

[4] Due to data limitations, China's plots are of manufacturing production / GDP and manufacturing imports / GDP.

4 A Framework to Analyze the Global Recession

We now turn to our general equilibrium framework, which builds upon the models of Eaton and Kortum (2002), Lucas and Alvarez (2008), and Dekle, Eaton, and Kortum (2008). Our setup is most closely related to recent work by Caliendo and Parro (2009), which uses a multi-sector generalization of these models to study the impact of NAFTA.[5] Our paper is also related to Bems, Johnson, and Yi (2010), which uses the input-output framework of Johnson and Noguera (2009) to link changes in final demand across many countries during the recent global recession to changes in trade flows throughout the global system. One crucial difference is that we endogenize changes in bilateral trade shares, an important feature to match the recent experience.

We start by describing the input-output structure. Next, we merge this structure with trade share equations from gravity models.

4.1 Demand and Input-Output Structure

Consider a world of $i = 1, \ldots, I$ countries with constant return to scale production and perfectly competitive markets. There are three sectors indexed by j: durable manufacturing ($j = D$), non-durable manufacturing ($j = N$), and non-manufacturing ($j = S$). The label S was chosen because "services" are a large share of non-manufacturing, although our category also includes agriculture, petroleum and other raw materials. We let $\Omega = \{D, N, S\}$ denote all sectors and $\Omega_M = \{D, N\}$ the manufacturing sectors.

We model international trade explicitly only for the manufacturing sectors. Net trade in the S sector is exogenous in our framework. Within manufactures, we distinguish between durables and non-durables because these two groups have been characterized by shocks of different sizes, as documented in Levchenko, Lewis, and Tesar (2009).

Let Y_i^j denote country i's gross production in sector $j \in \Omega$. Country i's gross absorption of j is

[5] Their model contains significantly more sectors and input-output linkages, but unlike our work, does not seek to "account" for changes in trade patterns with various shocks.

X_i^j and $D_i^j = X_i^j - Y_i^j$ is its deficit in sector j. Country i's overall deficit is:

$$D_i = \sum_{j \in \Omega} D_i^j,$$

while, for each $j \in \Omega$,

$$\sum_{i=1}^{I} D_i^j = 0.$$

Denoting GDP by Y_i, aggregate spending is $X_i = Y_i + D_i$. The relationship between GDP and sectoral gross outputs depends on the input-output structure, to which we now turn.

Sectoral outputs are used both as inputs into production and to satisfy final demand. We assume a Cobb-Douglas aggregator of sectoral inputs. [6] Value-added is a share β_i^j of gross production in sector j of country i, while γ_i^{jl} denotes the share of sector l in among intermediates used by sector j, with $\sum_l \gamma_i^{jl} = 1$ for each $j \in \Omega$. We assume that the coefficients β_i^j and γ_i^{jl} are fixed over time but vary across countries. Figure 4 plots examples of the input-output coefficients for several large economies for both 2000 and 2005 and offers empirical support for this assumption. For example, while Korea's value added share in durable manufacturing is significantly lower than the U.K.'s (i.e. $\beta_{Korea}^D < \beta_{U.K.}^D$), neither of these technological parameters changes much over time.

We can now express GDP as the sum of sectoral value added:

$$Y_i = \sum_{j \in \Omega} \beta_i^j Y_i^j. \tag{1}$$

We ignore capital and treat labor as perfectly mobile across sectors so that:

$$Y_i = \sum_{j \in \Omega} w_i L_i^j = w_i L_i.$$

[6] To avoid uninteresting constants in the cost functions that follow, we specify this Cobb-Douglas aggregator as:

$$B_i^j = \left(\frac{l_i^j}{\beta_i^j}\right)^{\beta_i^j} \prod_{k \in \Omega} \left(\frac{y_i^{jk}}{\gamma_i^{jk}(1-\beta_i^j)}\right)^{\gamma_i^{jk}(1-\beta_i^j)},$$

where B_i^j are input bundles used to produce sector j output. Here l_i^j is labor input in sector j, and y_i^{jk} is sector-k intermediate input used in sector-j production.

Finally, we denote by α_i^j the share of sector j consumption in country i's aggregate final demand, so that the total demand for sector j in country i is:

$$X_i^j = \alpha_i^j X_i + \sum_{l \in \Omega} \gamma_i^{lj} (1 - \beta_i^l) Y_i^l. \tag{2}$$

To interpret (2), consider the case of durables manufacturing, $j = D$. The first term represents the final demand for durables manufacturing as a share of total final absorption X_i. A disproportionate drop in final spending on automobiles, trucks, and tractors in country i can be captured by a decline in α_i^D. Some autos, trucks, and tractors, however, are used as inputs to make additional durable manufactures, non-durable manufactures, and even services. The demand for durable manufactures as intermediate inputs for those sectors is represented by the second term of (2). The sum of these two terms – demand for durable manufactures used as final consumption and demand for durables manufactures used as intermediates – generates the total demand for durable manufactures in country i, X_i^D.

It is helpful to define the 3-by-3 matrix $\mathbf{\Gamma}_i$ of input-output coefficients, with $\gamma_i^{lj}(1 - \beta_i^l)$ in the l'th row and j'th column, where we've ordered the sectors as D, N, and S. We can now stack equations (2) for each value of j and write the linear system:

$$\mathbf{X}_i = \mathbf{Y}_i + \mathbf{D}_i = \boldsymbol{\alpha}_i X_i + \mathbf{\Gamma}_i^T \mathbf{Y}_i, \tag{3}$$

where $\mathbf{\Gamma}_i^T$ is the transpose of $\mathbf{\Gamma}_i$ and the boldface variables $\mathbf{X}_i$, $\mathbf{Y}_i$, $\mathbf{D}_i$, and $\boldsymbol{\alpha}_i$ are 3-by-1 vectors, with each element containing the corresponding variable for sectors D, N, and S. We can thus express production in each sector as:

$$\mathbf{Y}_i = (\mathbf{I} - \mathbf{\Gamma}_i^T)^{-1} (\boldsymbol{\alpha}_i X_i - \mathbf{D}_i). \tag{4}$$

Through the input-output structure, production in each sector depends on the entire vector of final demands across sectors, net of the vector of sectoral trade deficits.

The input-output structure has implications for the cost of production in different sectors. We

first consider the cost of inputs for each sector and then introduce a model of sectoral productivity, that, in turn, determines sectoral price levels and trade patterns for durable and non-durable manufactures.

For now we take wages w_i and sectoral prices, p_i^l for $l \in \Omega$, as given. The Cobb-Douglas aggregator implies that the minimized cost of a bundle of inputs used by sector $j \in \Omega$ producers is:

$$c_i^j = w_i^{\beta_i^j} \prod_{l \in \Omega} \left(p_i^l\right)^{\gamma_i^{jl}(1-\beta_i^j)}. \tag{5}$$

As noted above, we do not explicitly model trade in sector S. Instead we simply specify productivity for that sector as A_i^S so that $p_i^S = c_i^S / A_i^S$. Taking into account round-about production we get:

$$p_i^S = \left(\frac{1}{A_i^S} w_i^{\beta_i^S} \prod_{l \in \Omega_M} \left(p_i^l\right)^{\gamma_i^{Sl}(1-\beta_i^S)} \right)^{\frac{1}{1-\gamma_i^{SS}(1-\beta_i^S)}}.$$

We can substitute this expression for the price of services back into the cost functions expressions (5) for $j \in \Omega_M$. We are treating the manufacturing sectors as if they had integrated the production of all service-sector intermediates into their operations. After some algebra we can write the resulting expression for the cost of an input bundle in a way that brings out the parallels to (5):

$$c_i^j = \frac{1}{A_i^{jS}} w_i^{\widetilde{\beta}_i^j} \prod_{l \in \Omega_M} \left(p_i^l\right)^{\widetilde{\gamma}_i^{jl}(1-\widetilde{\beta}_i^j)}, \tag{6}$$

for $j \in \Omega_M$. Here, the productivity term is

$$A_i^{jS} = \left(A_i^S\right)^{\gamma_i^{jS}(1-\beta_i^j)/[1-\gamma_i^{SS}(1-\beta_i^S)]},$$

while the input-output parameters become

$$\widetilde{\beta}_i^j = \beta_i^j + \frac{\gamma_i^{jS}(1-\beta_i^j)\beta_i^S}{1-\gamma_i^{SS}(1-\beta_i^S)},$$

and

$$\widetilde{\gamma}_i^{jl} = \gamma_i^{jl} + \gamma_i^{jS} \frac{\gamma_i^{Sl}(1-\beta_i^S) + \gamma_i^{jl}\beta_i^S}{1 - \gamma_i^{SS}(1-\beta_i^S) - \gamma_i^{jS}\beta_i^S}.$$

The term A_i^{jS} captures the pecuniary spillover from service-sector productivity to sector j costs. The parameter $\widetilde{\beta}_i^j$ is the share of value added used directly in sector j as well as the value added embodied in service-sector intermediates used by sector j. The share of manufacturing intermediates is $1-\widetilde{\beta}_i^j$, with $\widetilde{\gamma}_i^{jl}$ representing the share of manufacturing sector l intermediates among those used by sector j, with:

$$\sum_{l \in \Omega_M} \widetilde{\gamma}_i^{jl} = 1.$$

Substituting out the service sector leaves two sectoral demand equations for each country, in place of (2):

$$X_i^j = \widetilde{\alpha}_i^j (w_i L_i + D_i) - \delta_i^j D_i^S + \sum_{l \in \Omega_M} \widetilde{\gamma}_i^{lj} (1 - \widetilde{\beta}_i^l) Y_i^l, \tag{7}$$

for $j \in \Omega_M$, where

$$\delta_i^j = \frac{\gamma_i^{Sj}(1-\beta_i^S)}{1 - \gamma_i^{SS}(1-\beta_i^S)},$$

and

$$\widetilde{\alpha}_i^j = \alpha_i^j + \delta_i^j \alpha_i^S.$$

All that remains of the service sector is its trade deficit, if any, which we treat as exogenous.

4.2 International Trade

Any country's production in each sector $j \in \Omega_M$ must be absorbed by demand from other countries or from itself. Define π_{ni}^j as the share of country n's expenditures on goods in sector j purchased from country i. Thus, we require:

$$Y_i^j = \sum_{n=1}^{I} \pi_{ni}^j X_n^j. \tag{8}$$

To complete the picture, we next detail the production technology across countries, which leads to an expression for trade shares.

Durable and non-durable manufactures consist of disjoint unit measures of differentiated goods,

indexed by z.[7] We denote country i's efficiency making good z in sector j as $a_i^j(z)$. The cost of producing good z in sector j in country i is thus $c_i^j/a_i^j(z)$, where c_i^j is the cost of an input bundle, given by (6).

With the standard "iceberg" assumption about trade, delivering one unit of a good in sector j from country i to country n requires shipping $d_{ni}^j \geq 1$ units, with $d_{ii}^j = 1$ for all $j \in \Omega_M$. Thus, a unit of good z in sector j in country n from country i costs:

$$p_{ni}^j(z) = c_i^j d_{ni}^j / a_i^j(z).$$

The price actually paid in country n for this good is:

$$p_n^j(z) = \min_k \left\{ p_{nk}^j(z) \right\}.$$

Country i's efficiency $a_i^j(z)$ in making good z in sector j can be treated as a random variable with distribution: $F_i^j(a) = \Pr[a_i^j(z) \leq a] = e^{-T_i^j a^{-\theta^j}}$, which is drawn independently across i and j. Here $T_i^j > 0$ is a parameter that reflects country i's overall efficiency in producing any good in sector j. In particular, average efficiency in sector j of country i scales with $\left(T_i^j\right)^{1/\theta^j}$. The parameter θ^j is an inverse measure of the dispersion of efficiencies. (We derive the model allowing for differences in each sector's θ^j but the results that follow are all simulated with a homogenous θ.)

We assume that the individual manufacturing goods, whether used as intermediates or in final demand, are combined in a constant-elasticity-of-substitution aggregator, with elasticity $\sigma^j > 0$. As detailed in Eaton and Kortum (2002), we can then derive the price index by integrating over the prices of individual goods to get:

$$p_n^j = \varphi^j \left[\sum_{i=1}^{I} T_i^j \left(c_i^j d_{ni}^j \right)^{-\theta^j} \right]^{-1/\theta^j}, \tag{9}$$

[7] Goods from different sectors with the same index z have no connection to one another. On the other hand, goods from different countries in the same sector with the same index are perfect substitutes.

where φ^j is a function of θ^j and σ^j, requiring $\theta^j > (\sigma^j - 1)$. Substituting (6) into (9), we get:

$$p_n^j = \varphi^j \left[\sum_{i=1}^{I} \left(w_i^{\widetilde{\beta}_i^j} \left(p_i^j \right)^{\widetilde{\gamma}_i^{jj}\left(1-\widetilde{\beta}_i^j\right)} \left(p_i^l \right)^{\widetilde{\gamma}_i^{jl}\left(1-\widetilde{\beta}_i^j\right)} \frac{d_{ni}^j}{A_i^j} \right)^{-\theta^j} \right]^{-1/\theta^j}, \tag{10}$$

where $l \neq j$ is the other manufacturing sector and

$$A_i^j = A_i^{jS} \left(T_i^j \right)^{1/\theta^j},$$

captures the combined effect on costs of better technology in manufacturing sector j and cost reductions brought about by productivity gains in the services sector. Expression (10) links sector-j prices in country n to the prices of labor and intermediates around the world.

Imposing that each destination purchases each differentiated good z from the lowest cost source, and invoking the law of large numbers, leads to an expression for sector-j trade shares:

$$\pi_{ni}^j = \frac{T_i^j \left[c_i^j d_{ni}^j \right]^{-\theta^j}}{\sum_{k=1}^{I} T_k^j \left[c_k^j d_{nk}^j \right]^{-\theta^j}}.$$

We can use (9) and (6) to rewrite the trade-share expression as:

$$\pi_{ni}^j = \left[w_i^{\widetilde{\beta}_i^j} \left(p_i^j \right)^{\widetilde{\gamma}_i^{jj}\left(1-\widetilde{\beta}_i^j\right)} \left(p_i^l \right)^{\widetilde{\gamma}_i^{jl}\left(1-\widetilde{\beta}_i^j\right)} \frac{\varphi^j d_{ni}^j}{A_i^j p_n^j} \right]^{-\theta^j}. \tag{11}$$

4.3 Global Equilibrium

We can now express the conditions for global equilibrium. Substituting (8) into (7) we obtain "global input-output" equations linking spending in each sector $j \in \Omega_M$ around the world:

$$X_i^j = \widetilde{\alpha}_i^j (w_i L_i + D_i) - \delta_i^j D_i^S + \sum_{l \in \Omega_M} \widetilde{\gamma}_i^{lj} (1 - \widetilde{\beta}_i^l) \left(\sum_{n=1}^{I} \pi_{ni}^l X_n^l \right). \tag{12}$$

Summing (8) across the two manufacturing sectors gives "global market clearing" equations for each country:

$$X_i^D + X_i^N - (D_i - D_i^S) = \sum_{l \in \Omega_M} \sum_{n=1}^{I} \pi_{ni}^l X_n^l. \tag{13}$$

Following Alvarez and Lucas (2007), we take world GDP as the numeraire and hence, we cannot in this paper account for the global decline in real GDP. Rather, ours is a model of the movements of country-level variables such as GDP and trade relative to the global totals. Equilibrium is a set of wages w_i for each country $i = 1, ..., I$ and, for sectors $j \in \Omega_M$, spending levels X_i^j, price levels p_i^j, and trade shares π_{ni}^j that solve equations (12), (13), (10), and (11) given labor endowments L_i and deficits D_i and D_i^S. Production, deficits, and employment by country for sectors $j \in \Omega_M$ are then implied by (8).

4.4 Interpretation of Shocks

Trade flows for each sector in our model are driven entirely by four categories of shocks: (i) demand shocks (or more precisely, shocks to a sector's share in final demand), (ii) deficit shocks, (iii) productivity shocks, and (iv) trade-friction shocks. We emphasize, however, that while we derived our system from a particular model, these shocks are consistent with a variety of different structural interpretations.

The first category of shocks in our model is the country-specific share α_i^j of final demand that is spent on sector-j goods. Fluctuations in α_i^j are consistent with any changes in the domestic absorption of good j that are not attributable to the current demand for intermediate inputs. For example, non-homothetic preferences over consumption may imply a relative decline in final consumption demand for durables during recessions. In our model, this type of effect – such as a reduction in the purchase of automobiles – would manifest as a decline in α_i^D. Similarly, any shocks which reduce final investment activity would map to a change in α_i^D because that term reflects the purchase of machinery or capital goods that are not used up in the production of intermediates. A reduction in durable inventories, since inventories have not yet been used up in the production of intermediates, will also produce a decline in α_i^D.

The second category of shocks in our model is deficits. In particular, equilibrium is a function of each country's overall deficit D_i and its non-manufacturing deficit D_i^S. Since our tool of investigation is a static trade model, we have none of the features required to evaluate the intertemporal trade-offs that determine deficits – endogenizing deficits would require us to embed our framework in a dynamic model. Of our four categories of shocks, this is the only one without a flexible interpretation.

The third and fourth categories of shocks are productivity and trade friction shocks and are isomorphic to many different structural representations. We derived the price index (10) and trade share expression (11) from a particular Ricardian model, but emphasize that any model generating these two aggregate equations would be equally valid in our analysis. For instance, Appendix A shows that these expressions emerge in, among others, the Armington (1969) model elaborated in Anderson and Van Wincoop (2003), the Krugman (1980) model implemented in Redding and Venables (2004), the Ricardian model of Eaton and Kortum (2002), and the Melitz (2003) model expanded in Chaney (2008).[8] In the Armington setup, for example, one would simply re-interpret shocks to A_i^j as preference shocks for that country's goods. For instance, a world-wide decline in demand for cars produced in Japan would map to a reduction in Japan's durable-good productivity in our framework.

Finally, the shocks d_{ni}^j can be interpreted as trade frictions in a broad sense. Anything causing an increase in home-bias, or a reduction in absorption of imports relative to absorption of domestic production, will map in our framework to a change in d_{ni}^j. The simplest examples of such shocks would be changes in shipping costs (relative to domestic shipping costs), changes in tariffs, and changes in non-tariff trade barriers, such as the so-called "Buy America" provision in the U.S. fiscal stimulus package. Difficulties in obtaining trade finance relative to other types of credit, as in Amiti and Weinstein (2009), would also influence the d_{ni}^j term in our model. The scenario where the inventories of importers adjust more due to fixed costs of trade, as detailed in the model of Alessandria, Kaboski, and Midrigan (2010), would also map to a change in d_{ni}^j.[9]

[8] The deep similarity in the predicted trade patterns from such seemingly disparate models is striking and is the subject of Arkolakis, Costinot, and Rodriguez-Clare (2009).

[9] To reiterate, a uniform reduction in inventories – whether the goods are imported or not – will appear in our

5 Measuring the Shocks in the Data

An empirical implementation of the above framework requires data on bilateral trade, production, and input-output structure at the sector-level for many countries, as well as standard macro data such those on output and exchange rates. Further, the model requires information on monthly nominal production levels, though typically only indices of real production are available at monthly frequency. Appendix B details our sources for these data and the procedures required to generate separate monthly nominal production values for durable and non-durable manufacturing sectors. With these data, we can examine various measures of the shocks that drive the model.

5.1 Demand Shocks

The demand shocks can be calculated through a manipulation of (4):

$$\boldsymbol{\alpha}_i = \frac{1}{X_i}\left(\mathbf{X}_i - \boldsymbol{\Gamma}_i^T \mathbf{Y}_i\right),$$

where data for all the right hand side terms have been described above.[10] Figure 5 plots the paths of α_i^D and α_i^N for four large countries since 2000. The dashed vertical lines on the right of the plot correspond to the period starting in the first quarter of 2008 and ending in the first quarter of 2009. We highlight this window because it will be the period we use for several of our counterfactual analyses. The recent recession has led to a steep decline in the share of final demand for manufactures in all these countries, with a particularly steep decline in durables. This share begins to increase again in most countries toward the end of 2009.

model as a decline in demand for that sector. A disproportionately large decline in imported good inventories, however, would appear in our framework as an increase in trade frictions.

[10] Service sector production is imputed as: $Y_i^S = (Y_i - \beta_i^D Y_i^D - \beta_i^N Y_i^N)/\beta_i^S$, as implied by (1). For the rest of the world ($i = ROW$) we first need to construct sectoral production for $j \in \Omega_M$. We start by averaging sectoral value added as a fraction of GDP $\beta_i^j Y_i^j / Y_i$ across the countries in our sample. We then multiply the result by Y_{ROW} to estimate value added by sector for rest of world. We divide by β_{ROW}^j to estimate Y_{ROW}^j, where β_{ROW}^j is estimated as the median value of β_i^j across the countries in our sample.

5.2 Trade Deficits

Trade deficits are treated as exogenous in our framework, and are one of the shocks in the model. This shock can be measured directly. Trade deficits changed dramatically over the current recession. Figure 6 shows overall and non-manufacturing trade deficits for several key countries. The sharp reduction in the overall U.S. trade deficit during the recession is balanced by reduced surpluses for Japan, Germany, and China.

5.3 Trade Frictions: Head-Ries Index

Trade frictions cannot be directly measured in the data, unlike the macro aggregates above. Hence, in this section, we derive the Head-Ries index, an inverse measure of trade frictions implied by our trade share equation (11), or any gravity model. The index is an easily measurable object that reflects changes in trade frictions and is invariant to the scale of tradable good demand or the relative size and productivity of trading partners. Head and Ries (2001) use this expression – equation (8) in their paper – to measure the border effect on trade between the U.S. and Canada for several manufacturing industries. Jacks, Meissner, and Novy (2009) studies a very similar object for a span of over 100 years to analyze long-term changes in trade frictions.

Denote country n's spending on manufactures of type j from country i by X_{ni}^j, measured in U.S. Dollars. All variables are indexed by time (other than the elasticity θ^j), though we generally omit this from our notation. We have:

$$\frac{X_{ni}^j}{X_{nn}^j} = \frac{\pi_{ni}^j}{\pi_{nn}^j} = \frac{T_i^j \left[c_i^j d_{ni}^j\right]^{-\theta^j}}{T_n^j \left[c_n^j\right]^{-\theta^j}}, \tag{14}$$

where we normalize $d_{nn}^j = 1$. Domestic absorption of goods of type j, X_{nn}^j, is equal to gross production less exports: $X_{nn}^j = Y_n^j - \sum_{i=1}^I X_{in}^j$.[11]

[11] Grouping together country-level terms as $S_i^j = T_i^j \left(c_i^j\right)^{-\theta^j}$ and taking logs of both sides of (14), we could run a regression at date t on country fixed effects. We might do this hoping to sweep out the components S_i^j so that we would be left with $\left(d_{ni}^j\right)^{-\theta^j}$, which is the object we would like to input into our analysis. Such a procedure would be misleading, however, due to a fundamental identification problem. For any set of parameters $\left\{S_i^j, d_{ni}^j\right\}$ we can fit

Multiplying (14) by the parallel expression for what i buys from n in sector j and taking the square root, we generate:

$$\Theta_{ni}^{j} = \left(\frac{X_{ni}^{j}}{X_{nn}^{j}} \frac{X_{in}^{j}}{X_{ii}^{j}} \right)^{1/2} = \left[d_{ni}^{j} d_{in}^{j} \right]^{-\theta^{j}/2}. \tag{15}$$

This index implies that, for given trade costs, the product of bilateral trade flows in both directions should be a fixed share of the product of the countries' domestic absorption of tradable goods.

This index will change only in response to movements in trade frictions. Other measures which might have been used to capture these movements include "openness" indices, similar to the left-hand side of (14), or the summation of bilateral trade flows relative to the summation of any pair of countries' final demands. These other measures, however, have the disadvantage of being unable to isolate trade frictions.

It is difficult to extract information from the I^2 different bilateral Head-Ries indices, so as a way of summarizing historical trends in trade frictions at the country level, we apply a regression framework to these bilateral indices.[12] We start with the assumption that each directional transport cost reflects aggregate, exporter, and importer components that change over time, as well as a bilateral term that is fixed, and finally an idiosyncratic shock. Given importers and exporters enter symmetrically in (15), we cannot learn about distinct importer and exporter frictions, but we can

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the same data with another set of parameters $\left\{ \widetilde{S}_{i}^{j}, \widetilde{d}_{ni}^{j} \right\}$ where:

$$\widetilde{S}_{i}^{j} = \phi_{i}^{j} S_{i}^{j},$$

and

$$\widetilde{d}_{ni}^{j} = \left[\frac{\phi_{i}^{j}}{\phi_{n}^{j}} \right]^{1/\theta^{j}} \widetilde{d}_{ni}^{j}.$$

The problem is that there are no restrictions on ϕ_{i}^{j}, so this procedure would be unable to determine whether the d_{ni}^{j} changed or the S_{i}^{j} changed. Going back to the primitives of the model, any change in trade shares can be explained by an infinite number of combinations of changes in $\left\{ T_{i}^{j} \right\}$ and $\left\{ d_{ni}^{j} \right\}$. There is hope, however. Notice that if we multiply d_{ni}^{j} by d_{in}^{j}, the ambiguity goes away. This fact is the key motivation for our use of the Head-Ries index. In our counterfactual analysis below, we obtain additional restrictions by confronting the model's implications for prices.

[12] See Anderson and Yotov (2009) for a related exercise which estimates bilateral trade frictions from observable proxies for bilateral trade costs. They then use a theory-based aggregation of these bilateral terms to determine the buyer's and seller's incidence of trade frictions and to study the evolution of this incidence over time.

extract their combined effects by estimating the pooled regression for all i, n, and t:

$$\ln \Theta_{ni}^{j}(t) = \beta_{n}^{j}(t) + \beta_{i}^{j}(t) + \gamma_{ni}^{j} + \varepsilon_{ni}^{j}(t). \tag{16}$$

We do this separately for each manufacturing industry, $j = D, N$. Note that each regression contains only N country dummy variables each period, any given observation will be influenced by two of these country dummies. Again, each dummy represents the sum of the trade frictions experienced by that country's exporters and importers.

Figures 7 and 8 plot the four-quarter moving average of the country-time effects β_i^j from a weighted estimation of (**??**) for selected countries. We use a moving average due to the strong seasonal effects in the data. The coefficients are normalized to zero in the first quarter of 2000 and extend through the fourth quarter of 2009. The country-time effects act proportionately on the Head-Ries indices for all bilateral pairs involving any given country. For instance, if the series for country i increases from 0 to 0.1, it implies that the index would increase 10 percent for all pairs in which i is an exporter or an importer.

Looking at Figure 7, we see examples of countries where the recession did not bring with it marked increases in trade frictions. Only a small share (or a negative share) of any declines in trade flows for Germany, the U.S., Mexico, and Italy should, according to this measure, be attributed to declining trade frictions. Figure 8, by contrast, includes only countries for which there is a steep increase in trade frictions (a decline in the index) during the recession. These countries include Japan, China, Austria, and Finland, among others not shown. One important conclusion is that, while there is evidence of increasing trade frictions in some countries, this shock appears to be quite heterogenous across countries and is generally relatively muted. For some countries, in fact, reduced trade frictions could have ameliorated the trade collapse.

5.4 Trade Frictions During the Great Depression

These results suggest there was not a large universal trade friction shock associated with the recent recession. This should be interpreted as resulting from the data and not from any predisposition of

the calculation to attenuate an underlying increase in trade frictions. To confirm that our measure can pick up changes in trade frictions, we compare calculations made with data from the recent recession to those using data from the Great Depression, a period with a major collapse in trade that is widely believed to have reflected, in part, increased trade barriers (see, for example, Irwin, 1998)..

We have sufficient Depression-era data to construct Head-Ries Indices (15) for the bilateral trade between the United States and eight partners: Austria, Canada, Finland, Germany, Japan, Spain, Sweden, and the United Kingdom. Appendix B includes the details on the data required for this exercise. Figures 9 and 10 compare these Head-Ries indices from the Great Depression (seen in the solid blue lines and corresponding to the lower x-axis) with the equivalent Head-Ries indices from the recent recession (seen in the dotted red lines and corresponding to the upper x-axis). Unlike our earlier analyses, these indices are calculated at the annual level and pool data from both manufacturing sectors, to make an appropriate comparison.

Of the eight bilateral pairs, it just so happens that three of the countries, Austria, Finland, and Japan, are among those displaying the largest declines of the Head-Ries indices in the recent period. The declines for those three pairs for the recent recession are similar to the declines in the first few years of the Great Depression. The other five countries show markedly larger Depression-era drops, however, with average peak-to-trough declines in the Head-Ries index exceeding 50 percent, compared to flat or increasing indices in the recent recession. In sum, the data we have on trade and production in the Great Depression suggest that global Head-Ries indices dropped far more broadly and more dramatically in the early 1930s than in the late 2000s.

6 Calibration

Having set up the model, discussed the four categories of shocks that can change trade flows, and given historical context on the path of these shocks, we now calibrate the model to perfectly match the period from the first quarter of 2008 to the first quarter of 2009. The calibration exercise only includes a balanced panel of countries for which we have good data on input-output structure,

production, and imports from and exports to all other included countries. After constructing trade, production, GDP, deficit, and input-output information for each country, and balancing this panel, we are left with a dataset containing complete data for 22 countries responsible for about 75 percent of global manufacturing trade and global GDP.[13] Table 1 lists the included countries, shares in trade, and shares in global GDP, before and after the crisis, as well as a residual category "rest of world."[14]

First, we re-formulate the model to facilitate computing its implications for changes in endogenous variables. Next we describe how we parameterize the model for calculating changes.

6.1 Change Formulation

For any time-varying variable x in the model we denote its beginning-of-period or baseline value as x and its end-of-period or counterfactual value as x', with the "change" over the period (or counterfactual change) denoted $\hat{x} = x'/x$. We will take labor supply as fixed so that $Y_i' = \hat{w}_i Y_i$.

In terms of counterfactual levels and changes, the global input-output equations (12), for sectors $j \in \Omega_M$ and countries $i = 1, 2, ..., I$, become

$$\left(X_i^j\right)' = \left(\widetilde{\alpha}_i^j\right)' \left(\hat{w}_i Y_i + D_i'\right) - \delta_i^j \left(D_i^S\right)' + \sum_{l \in \Omega_M} \widetilde{\gamma}_i^{lj} (1 - \widetilde{\beta}_i^l) \left[\sum_{n=1}^{I} \left(\pi_{ni}^l\right)' \left(X_n^l\right)'\right]. \tag{17}$$

The global market clearing conditions (13) become:

$$\left(X_i^D\right)' + \left(X_i^N\right)' - \left[D_i' - \left(D_i^S\right)'\right] = \sum_{n=1}^{I} \left(\pi_{ni}^D\right)' \left(X_n^D\right)' + \sum_{n=1}^{I} \left(\pi_{ni}^N\right)' \left(X_n^N\right)'. \tag{18}$$

The price equations (10) become:

$$\hat{p}_n^j = \left(\sum_{i=1}^{I} \pi_{ni}^j \hat{w}_i^{-\theta^j \widetilde{\beta}_i^j} \left(\hat{p}_i^j\right)^{-\theta^j \widetilde{\gamma}_i^{jj} (1 - \widetilde{\beta}_i^j)} \left(\hat{p}_i^l\right)^{-\theta^j \widetilde{\gamma}_i^{jl} (1 - \widetilde{\beta}_i^j)} \left(\frac{\hat{d}_{ni}^j}{\hat{A}_i^j}\right)^{-\theta^j}\right)^{-1/\theta^j}, \tag{19}$$

[13] These shares are highly similar before and after the crisis, suggesting we have a representative sample in terms of the declines in trade and output.

[14] We use most countries for which we have high quality data, with the exceptions of Belgium and the Netherlands. Belgium and the Netherlands are omitted because their manufacturing exports often exceed their manufacturing production (due to re-exports), and our framework is not capable of handling this situation.

where $l \neq j$ is the other manufacturing sector. The trade share equations (11) become:

$$\left(\pi_{ni}^{j}\right)' = \pi_{ni}^{j}\hat{w}_i^{-\theta^j\widetilde{\beta}_i^j}\left(\hat{p}_i^j\right)^{-\theta^j\widetilde{\gamma}_i^{jj}(1-\widetilde{\beta}_i^j)}\left(\hat{p}_i^l\right)^{-\theta^j\widetilde{\gamma}_i^{jl}(1-\widetilde{\beta}_i^j)}\left(\frac{\hat{d}_{ni}^j}{\hat{A}_i^j\hat{p}_n^j}\right)^{-\theta^j}. \tag{20}$$

Equations (17), (18), (19), and (20) determine the changes in endogenous variables implied by a given set of shocks. We solve this set of equations for: (i) changes in wages $\hat{w}_i$, (ii) counterfactual levels of spending $(X_i^j)'$, (iii) changes in prices $\hat{p}_i^j$, and (iv) counterfactual trade shares $\left(\pi_{ni}^j\right)'$ for countries $i = 1, ..., I$ and sectors $j \in \Omega_M$. Baseline trade shares and GDPs are used to calibrate the model. The forcing variables are the end-of-period or counterfactual demand shocks $\left(\widetilde{\alpha}_i^j\right)'$ and deficits $\left(D_i^S\right)'$ and D_i', changes in trade frictions $\hat{d}_{ni}^j$, and changes in productivity $\hat{A}_i^j$.[15]

The system can be solved as follows. Given a vector of possible wage changes, (19) is solved for price changes. Wage and price changes then imply counterfactual trade shares via (20). Given counterfactual trade shares and wage changes, (17) can be solved as a linear system for counterfactual levels of spending. If these levels of spending satisfy (18), then we have an equilibrium. If not, we adjust wage changes according to where there is excess demand (with world GDP fixed) and return to (19). Details are described in Appendix D.

Given the solution described above, we can use equation (8), as it applies to the counterfactual levels:

$$\left(Y_i^j\right)' = \sum_{n=1}^{I}\left(\pi_{ni}^j\right)'\left(X_n^j\right)', \tag{21}$$

to obtain counterfactual levels of sectoral production and deficits.

6.2 Parameter Values and Shocks

We start by setting $\theta^D = \theta^N = 2$. This value is between the smaller values typically used in the open-economy macro literature and the larger values used in Eaton and Kortum (2002).

We have described above our procedure for backing out end-of period demand shocks $(\alpha_i^j)'$. We

[15] As described in Appendix C, equilibrium outcomes for everything but price changes are invariant to productivity shocks of a labor-augmenting form, i.e. $\hat{A}_i^j = \lambda^{\widetilde{\beta}_i^j}$ for some $\lambda > 0$. Such shocks lead to price changes equal to $1/\lambda$. Furthermore, shocks to service-sector productivity, given $\hat{A}_i^j$, do not perturb the equilibrium outcomes. Either type of productivity shock will likely alter welfare, but is irrelevant to the model's implications for international trade.

use them to construct the demand shocks as they enter the model through equation (17):

$$\left(\widetilde{\alpha}_i^j\right)' = \left(\alpha_i^j\right)' + \frac{\gamma_i^{Sj}(1-\beta_i^S)}{1-\gamma_i^{SS}(1-\beta_i^S)}\left(\alpha_i^S\right)',$$

for $j \in \Omega_M$. End of period deficits D_i^j and $\left(D_i^S\right)'$ can be read directly from the data. They enter the model via equations (17) and (18). Our residual country, "Rest of World," has its deficit defined such that the global deficit equals zero.

We have described the Head-Ries index above. Calculating squared changes of it yields:

$$\left(\hat{\Theta}_{ni}^j\right)^2 = \frac{\hat{\pi}_{ni}^j \hat{\pi}_{in}^j}{\hat{\pi}_{nn}^j \hat{\pi}_{ii}^j} = \left(\hat{d}_{ni}^j\right)^{-\theta^j}\left(\hat{d}_{in}^j\right)^{-\theta^j}.$$

Here we need to decompose this measure to isolate $\left(\hat{d}_{ni}^j\right)^{-\theta^j}$. Dividing both sides of (20) by π_{ni}^j we get an expression for $\hat{\pi}_{ni}^j$. Dividing by the corresponding expression for $\hat{\pi}_{ii}^j$ and rearranging yields:

$$\left(\hat{d}_{ni}^j\right)^{-\theta^j} = \frac{\hat{\pi}_{ni}^j}{\hat{\pi}_{ii}^j}\left(\frac{\hat{p}_i^j}{\hat{p}_n^j}\right)^{\theta^j}. \tag{22}$$

We implement this equation using the changes in sectoral PPI's we constructed earlier.[16] We can also retrieve productivity changes by rearranging (20) as it applies to $n = i$:

$$\hat{A}_i^j = \left(\hat{\pi}_{ii}^j\right)^{1/\theta^j} \hat{w}_i^{\widetilde{\beta}_i^j}\left(\hat{p}_i^j\right)^{\widetilde{\gamma}_i^{jj}(1-\widetilde{\beta}_i^j)-1}\left(\hat{p}_i^l\right)^{\widetilde{\gamma}_i^{jl}(1-\widetilde{\beta}_i^j)}. \tag{23}$$

The trade-friction and productivity shocks both enter the model through equation (19) and (20).[17]

We present cross-country evidence on these shocks, together with some of the underlying variables used to construct them, from the first quarter of 2008 to the first quarter of 2009. We begin with the demand and deficit shocks shown above as they varied over time within countries. The

[16] We estimate $\hat{p}_{ROW}^j$ for $j \in \Omega_M$ by simply averaging $\hat{p}_i^j$ across the countries in our sample.

[17] Price data, such as the PPI data we use here, are required to disentengle changes in productivity from changes in trade frictions. As discussed in Appendix C, however, the combined impact of these two shocks on all non-price variables is robust to any procedure that separates them in an internally consistent way. For example, we can choose an arbitrary vector for the productivity shocks, back out the implied trade friction shocks, and other than prices, nothing in the model will change.

four panels in Figure 11 plot on the y-axis the changes in the durables and non-durables demand shocks and the overall and non-manufacturing deficits. The change in trade to GDP ratios during the crisis are plotted along the x-axis. Figure 12 plots the corresponding changes in the durable and non-durable productivity shocks, calculated according to equation (23), and changes in prices in the two sectors (measured in U.S. dollars and relative to our numeraire of world GDP).

The trade-friction shocks, constructed according to equation (22), have both an importer and exporter dimension. The two panels of Figure 13 contain histograms of the durable and non-durable trade friction changes, $\left(\widehat{d}_{ni}^{j}\right)^{-\theta^j}$.[18] The histograms exclude the largest and smallest 5 percentile values (generally small country-pair outliers).

Table 2 lists the combined impact (on imports and exports relative to GDP) of all the shocks associated with the recession, across all the countries used in our counterfactual exercises. By construction, the combined effect of our shocks fully accounts for the actual decline in trade from the first quarter of 2008 to the first quarter of 2009. The top row of data in the table, in boldface and labeled "World," shows that in global trade declined by 19 percent relative to GDP, with durables dropping by 22 percent and non-durables dropping by 11 percent.

7 Counterfactuals

We now discuss our counterfactual exercises. Given values for the changes in the forcing variables we solve (17), (18), (19), and (20), using an algorithm adapted from Dekle, Eaton, and Kortum (2008). In the results that follow we treat all end-of-period deficits (as a share of world GDP) as exogenous, so that wage changes are endogenous. In future drafts we will consider a case of exogenous wage changes and endogenous end-of-period manufacturing deficits.

It will be convenient to define the set of all shocks:

$$\Xi' = \left\{ \left\{ \widehat{\alpha}_i^D \right\}, \left\{ \widehat{\alpha}_i^N \right\}, \left\{ \widehat{D}_i \right\}, \left\{ \widehat{D}_i^S \right\}, \left\{ \widehat{d}_{ni}^D \right\}, \left\{ \widehat{d}_{ni}^N \right\}, \left\{ \widehat{A}_i^D \right\}, \left\{ \widehat{A}_i^S \right\} \right\},$$

[18] To back out the implied change in the trade friction itself, these changes should be divided by $\theta^j = 2$.

for all countries $i, n \in I$.[19] For any given four-quarter period and any given set of shocks Ξ, we can solve our model to generate changes in all values and flows in the global system relative to the base period. As an example, consider the case where we choose the first quarter of 2008 as the base period. If we solve the model with all shocks in Ξ' equal to one, implying the shocks did not change at all relative to this base period, the model would generate outcome variables (such as production, trade, GDP, etc.) precisely equal to those seen in the first quarter of 2008, as if the recession and the shocks that generated it never occurred. If, on the other hand, we solve the model with the set of shocks $\Xi' = data$, where "$data$" means that the shock values are those for 2008:Q1 to 2009:Q1 as given in the previous tables and plots, the model would generate values precisely equal to those seen in the first quarter of 2009. It will be convenient to define these two special cases of the shock matrices as Ξ^{08Q1} and Ξ^{09Q1}, respectively.

7.1 Accounting the Trade Decline from 2006-2009

We start by considering a series of four-quarter changes, beginning with the period from 2006:Q1 to 2007:Q1 all the way through to the period from 2008:Q4 to 2009:Q4. We run the model for each of these 12 four-quarter periods under various counterfactual assumptions and consider the implications at the global level. For instance, when we run the simulation after inputting all observed shocks, the counterfactual in each period shows the actual gross percentage change in world trade/GDP over the previous four quarters, as would be found in the raw data. Figure 14 plots these results as the boldfaced black line labeled "Data." (The 12 overlapping four-quarter changes are plotted as a continuous line, but it should be remembered that each calculation is done independently of those that came before. The changes are not cumulative.)

One sees that after mild rates of growth in the periods ending in 2007, global manufacturing trade/GDP was essentially unchanged until the fourth quarter of 2008, when it dropped nearly 10 percent relative to its value four quarters earlier. The drop continued and world trade/GDP in the first and second quarters of 2009 were about 20 percent below its respective levels in the first

[19] We note that while we write $\widehat{D}_i^S$ and $\widehat{D}_i$, we really only need information on $\left(D_i^S\right)'$and D_i', and so do not run into problems if $\widehat{D}_i^S$ and $\widehat{D}_i$ are undefined because initial deficits are zero.

and second quarters of 2008. By the end of the dataset, annual growth in global trade/GDP had flattened, as represented by the black line approaching the value 1.0. We expect the line to exceed one in future quarters as trade levels recover.

Next, we consider the question of what might have happened to global trade/GDP if we did the identical exercise, but instead of introducing all shocks, we only introduce the shocks to the share of durable and non-durable manufacturing. Formally, for each of the 12 simulations, we input the shock matrix $\Xi' = \left\{\left\{\widehat{\alpha}_i^D\right\}, \left\{\widehat{\alpha}_i^N\right\}, 1, 1, 1_{Ix1}, 1_{Ix1}, 1, 1\right\}$ for all countries I and generate the counterfactual change in the global trade/GDP ratio. These counterfactual results are plotted in the red line and demonstrate that the model with demand shocks alone performs quite well in capturing the magnitude of the decline across all of the four-quarter windows. When we consider the same exercise inputting only productivity shocks, only trade friction shocks, or only deficit shocks, the implied paths of global trade/GDP are essentially flat. None of the other shocks, on their own, come close to matching the actual pattern of declines. It is this result that leads us to conclude that demand shocks are the most significant driver of the decline in global trade/GDP. The red line dips down more than 80 percent of the way toward the black line during the recession.

Heterogeneity in the Head-Ries indices found earlier, suggest that trade friction shocks may be more successful in explaining the experiences of some countries. In Figure 15, we examine the profiles for some large countries that display different qualitative patterns. The United States and Germany largely mirror the World, with the set of pure demand shocks explaining most changes in trade to GDP. For Japan, the actual declines are larger in the depths of the recession, and no single shock type can on its own account for the majority of these declines. In China, the decline started earlier and, like Japan, no single shock type captures it. For both Japan and China, the trade friction shocks are arguably the largest factors.

7.2 Focusing on 2008:Q1 to 2009:Q1

To get a better sense for the experiences of all 23 countries (including "rest of world"), we now focus on the period from the first quarter of 2008 to the first quarter of 2009. We saw in Table 3

that world trade dropped 19 percent relative to economic activity over this period. Compared to this 19 percent, Table 3 shows that a 15 percent decline is generated from a counterfactual recession in which manufacturing demand dropped as it did but with no other shocks. Table 4 shows that a counterfactual recession in which the only change is the shock to trade frictions produces only a 1 percent decline in global trade. In addition to these aggregate results, Tables 2 through 4 list separately the experiences of each country in the data, in the counterfactual with only demand shocks, and in the counterfactual with only trade friction shocks.

We now introduce a measure to summarize the ability of our counterfactuals to match the cross-country pattern. We write the gross change in any particular outcome variable ξ for country i as $\widehat{\xi}_i(\Xi') = \xi_i'/\xi_i^{08Q1}$ to represent its value when the system is solved using the set of shocks Ξ' relative to the value that was observed in the first quarter of 2008. For example, if ξ_i is country i's overall trade to GDP ratio, then $\widehat{\xi}_i(\Xi^{09Q1})$ is the gross percentage change in trade to GDP observed from 2008:Q1 to 2009:Q1 in that country. (Note that with this base period, $\widehat{\xi}_i(\Xi^{08}) = 1$ for any variable ξ_i, by definition.)

We construct the following measure:

$$\upsilon\left(\Xi'\right) = \sum_{i}^{I} w_i \left(\widehat{\xi}_i\left(\Xi'\right) - \widehat{\xi}_i\left(\Xi^{09Q1}\right)\right)^2.$$

It is a weighted sum of squared deviations of the vector $\widehat{\boldsymbol{\xi}}\left(\Xi'\right)$ from the vector $\widehat{\boldsymbol{\xi}}\left(\Xi^{09Q1}\right)$, with each element's deviation weighted by w_i, with $\sum_i w_i = 1$. An important feature of this measure is that it does not net out the mean value of the deviation. For instance, if ξ is the trade to GDP ratio, then $\upsilon\left(\Xi^{08Q1}\right)$ measures total squared changes across countries in trade to GDP ratios during the recession. To measure the share of these total changes in ξ over the recession that are captured by a set of shocks Ξ', we define:

$$\mathbb{V}\left(\Xi'\right) = 1 - \frac{\upsilon\left(\Xi'\right)}{\upsilon\left(\Xi^{08Q1}\right)}.$$

Imagine running a counterfactual scenario with all shocks equal to 1, except for changes in countries' non-manufacturing trade deficits, which are set equal to what was observed in the data.

The scenario would generate a counterfactual vector of changes in trade to GDP ratios. The x-axis in the top-left plot of Figure 16 plots the vector $\widehat{\boldsymbol{\xi}}\left(\Xi^{09Q1}\right)$, while the y-axis plots the vector $\widehat{\boldsymbol{\xi}}\left(\Xi'\right)$.[20] If all the points were on the 45 degree line, it would indicate that the observed changes in the non-manufacturing deficits alone can fully explain the cross-country changes in trade to GDP during that four-quarter period. In such a case, $\mathbb{V}\left(\Xi'\right)$ would equal 1. As is easy to see, however, this counterfactual was far from aligning the points along the 45 degree line. Using shares of pre-recession global trade as our weights (w_i), we calculate $\mathbb{V}\left(\Xi'\right) = 0.05$. Thus, the subtitle of the top-left plot of Figure 16 says "Share of Trade-Weighted Variance Explained: 5%," and we conclude that the non-manufacturing deficit shocks can explain very little of the pattern of trade changes in the recession.[21]

The other three plots in Figure 16 show results from counterfactual scenarios simulated with both trade friction shocks, with durable manufacturing shocks, and with all shocks included. The top-right plot, capturing trade frictions, explains a bit more of the cross-country pattern than did non-manufacturing deficits. The share of trade-weighted variance explained is listed as 9 to 17 percent, where the upper bound of this range is generated from adding all trade frictions except those estimated with the "Rest of World," since those measurements involve more assumptions and less raw data than the others. The most notable result is the durable demand shock on the bottom-left panel of Figure 16. Durable demand shocks, on their own, explain 64 percent of the trade-weighted variance. Finally, as shown in the bottom-right panel of Figure 16, when all shocks are implemented, they perfectly explain changes in the economic system. This result is, of course, true by construction.

Table 5 lists the shares of trade-weighted variance explained by various shock combinations, including those displayed in Figure 16. Those combinations that include trade frictions list a range, where the larger value assumes no change in trade frictions with the "rest of the world." The contributions of each shock are not orthogonal to the others and hence, the shares of variance explained by each shock will not sum to one. Given these shocks may not be introduced independently in

[20] The plots actually show the net rates of change, that is, $\widehat{\boldsymbol{\xi}}\left(\Xi\right) - 1$.

[21] Note that this calculation can very well be negative. We would expect this with any shock that pushes the vector of outcome variables even further away from the post-recession data.

some recessions, it is useful to observe that the only combinations the generate very large contributions involve durable demand shocks. Further, adding productivity or deficit shocks to the demand shocks increases their explanatory power only slightly. Hence, though one might prefer to consider counterfactuals using combinations of these shocks, the demand shocks remain the most important proximate driver of changes in the pattern of trade/GDP.

7.3 Other Counterfactuals

Given the heterogeneity in the shocks affecting countries in the recent recession, we also consider counterfactuals run at the country- or region-level. As an example, imagine one wants to know the global impact of the decline in durables demand just in the U.S. The top panel of Table 6 shows simulated trade flows at the country and global level (for selected countries) when the only shock we introduce into the system is $\widehat{\alpha}^D_{US}$. The impact of this single shock on the world is large – it reduces global durables trade by about 3 percent relative to GDP. One also notes the impact of geography. Mexico and Canada are affected very significantly, while Germany, for example, is relatively insulated.

The bottom panel of Table 6 shows an alternative exercise where the only shocks introduced are the changes in trade frictions observed in China and Japan. These reduce total global trade by about 3 percent relative to GDP, but also have interesting cross-country implications. For example, the counterfactual produces trade diversion as manifest in the increase in South Korea's trade to GDP ratio.

8 Conclusion

A prominent characteristic of the recent global recession was a large and rapid drop in trade relative to GDP. Motivated by these dramatic changes in the cross-country pattern of trade, production, and GDP, we build an accounting framework relating them to shocks to demand, trade frictions, deficits, and productivities across several sectors. Applying our framework to the recent recession, we find that the bulk of the decline in trade/GDP can be explained by the shocks to manufacturing

demand, with a particularly important role for the shocks to durable manufacturing demand. The trade declines in China and Japan, however, reflect a moderate contribution from increased trade frictions.

We developed this approach with the recent recession in mind. We anticipate, however, that the framework can be applied quite generally to study the geography of global booms and busts.

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Tables

Country	Share of Global GDP in USD (Percent)		Share of Global Exports (Percent)		Share of Global Imports (Percent)	
	2008:Q1	2009:Q1	2008:Q1	2009:Q1	2008:Q1	2009:Q1
Austria	0.7	0.7	1.7	1.5	1.5	1.5
Canada	2.6	2.3	2.7	2.5	3.1	3.3
China	6.6	8.1	12.2	13.6	7.0	7.1
Czech Republic	0.4	0.3	1.3	1.2	1.2	1.1
Denmark	0.6	0.5	0.9	0.9	0.9	0.9
Finland	0.5	0.4	0.9	0.7	0.7	0.6
France	4.9	4.7	5.2	5.1	5.3	5.6
Germany	6.3	5.8	13.1	12.7	8.9	9.4
Greece	0.6	0.6	0.2	0.2	0.6	0.7
Hungary	0.3	0.2	1.0	0.9	0.9	0.8
India	2.2	2.1	1.1	1.3	1.4	1.7
Italy	3.9	3.7	4.7	4.4	3.9	3.7
Japan	8.2	9.5	7.0	5.7	3.9	4.2
Mexico	1.9	1.5	2.1	2.3	2.4	2.5
Poland	0.9	0.7	1.5	1.4	1.6	1.4
Romania	0.3	0.3	0.4	0.4	0.7	0.6
Slovakia	0.1	0.2	0.6	0.6	0.6	0.6
South Korea	1.8	1.3	3.5	3.7	2.5	2.4
Spain	2.7	2.6	2.3	2.1	3.2	2.6
Sweden	0.8	0.7	1.6	1.4	1.3	1.2
United Kingdom	4.8	3.8	3.7	3.5	5.0	4.7
United States	23.9	26.6	9.9	10.9	14.0	14.7
Rest of World	25.1	23.3	22.6	22.9	29.5	28.7

Table 1: Country Coverage in Data

Notes:

	No Recession	Recession (2008:Q1 to 2009:Q1)					
Shocks:	None	All Shocks Introduced					
		Exports / GDP			Imports / GDP		
	All Vars	All	Durables	Non-Durables	All	Durables	Non-Durables
World	**1.00**	**0.81**	**0.78**	**0.89**	**0.81**	**0.78**	**0.89**
Austria	1.00	0.78	0.73	0.89	0.84	0.80	0.92
Canada	1.00	0.89	0.83	1.03	0.99	0.93	1.14
China	1.00	0.74	0.72	0.79	0.67	0.67	0.68
Czech Republic	1.00	0.80	0.79	0.87	0.80	0.76	0.90
Denmark	1.00	0.90	0.84	0.97	0.89	0.86	0.94
Finland	1.00	0.72	0.69	0.79	0.77	0.72	0.91
France	1.00	0.83	0.79	0.89	0.88	0.85	0.94
Germany	1.00	0.85	0.81	0.95	0.92	0.89	0.99
Greece	1.00	0.84	0.75	0.91	0.91	0.88	0.94
Hungary	1.00	0.86	0.83	0.99	0.84	0.80	0.96
India	1.00	1.02	1.13	0.90	1.05	1.05	1.04
Italy	1.00	0.81	0.78	0.88	0.82	0.76	0.93
Japan	1.00	0.57	0.56	0.66	0.76	0.68	0.91
Mexico	1.00	1.05	1.03	1.19	1.02	1.01	1.03
Poland	1.00	0.93	0.90	1.00	0.87	0.82	0.96
Romania	1.00	0.92	0.93	0.90	0.78	0.67	0.99
Slovakia	1.00	0.75	0.74	0.80	0.77	0.72	0.91
South Korea	1.00	1.15	1.16	1.11	1.03	1.02	1.05
Spain	1.00	0.76	0.70	0.86	0.70	0.63	0.83
Sweden	1.00	0.86	0.79	1.04	0.89	0.82	1.05
United Kingdom	1.00	0.98	0.90	1.14	0.98	0.90	1.14
United States	1.00	0.81	0.79	0.84	0.77	0.73	0.87
Rest of World	1.00	0.88	0.82	0.98	0.85	0.83	0.92

Table 2: Imports/GDP and Exports/GDP over Recession

Notes: All variables expressed relative to global GDP. Calculations using $\beta = 1$ restricted interpolations and extrapolations.

	No Recession	Demand Shocks Only					
Shocks:	None	Durable and Non-Durable Demand Shocks Introduced					
		Exports / GDP			Imports / GDP		
	All Vars	All	Durables	Non-Durables	All	Durables	Non-Durables
World	**1.00**	**0.85**	**0.82**	**0.92**	**0.85**	**0.82**	**0.92**
Austria	1.00	0.96	0.92	1.08	0.95	0.95	0.96
Canada	1.00	0.90	0.87	0.97	0.92	0.90	0.97
China	1.00	0.98	0.94	1.08	0.91	0.88	1.01
Czech Republic	1.00	0.80	0.78	0.89	0.78	0.72	0.93
Denmark	1.00	0.87	0.81	0.94	0.87	0.85	0.89
Finland	1.00	0.83	0.79	0.91	0.78	0.74	0.88
France	1.00	0.82	0.77	0.89	0.82	0.80	0.86
Germany	1.00	0.91	0.88	1.01	0.86	0.83	0.91
Greece	1.00	0.74	0.68	0.79	0.89	0.86	0.92
Hungary	1.00	0.83	0.81	0.93	0.82	0.77	0.96
India	1.00	0.93	0.87	0.99	0.95	0.94	0.99
Italy	1.00	0.81	0.77	0.88	0.77	0.71	0.87
Japan	1.00	0.90	0.88	1.01	0.79	0.75	0.86
Mexico	1.00	0.88	0.87	0.97	0.90	0.87	0.98
Poland	1.00	0.82	0.78	0.91	0.83	0.80	0.90
Romania	1.00	0.75	0.72	0.82	0.84	0.74	1.05
Slovakia	1.00	0.74	0.72	0.82	0.73	0.70	0.83
South Korea	1.00	0.87	0.85	0.97	0.81	0.76	0.94
Spain	1.00	0.70	0.67	0.76	0.77	0.69	0.91
Sweden	1.00	0.87	0.84	0.95	0.84	0.79	0.96
United Kingdom	1.00	0.87	0.83	0.95	0.91	0.86	1.01
United States	1.00	0.80	0.77	0.87	0.85	0.84	0.90
Rest of World	1.00	0.80	0.76	0.87	0.85	0.82	0.93

Table 3: Counterfactual Results with Demand Shocks Only

Notes: $\theta^D = \theta^N = 2$. All variables expressed relative to global GDP. Calculations using $\beta = 1$ restricted interpolations and extrapolations.

	No Recession	Trade Frictions Only					
Shocks:	None	Durable and Non-Durable Trade Friction Shocks Introduced					
		Exports / GDP			Imports / GDP		
	All Vars	All	Durables	Non-Durables	All	Durables	Non-Durables
World	**1.00**	**0.99**	**0.99**	**1.01**	**0.99**	**0.99**	**1.01**
Austria	1.00	0.95	0.82	1.27	0.92	0.89	0.98
Canada	1.00	1.02	0.95	1.18	1.02	1.00	1.08
China	1.00	0.89	0.82	1.07	0.84	0.91	0.62
Czech Republic	1.00	1.06	1.11	0.85	1.06	1.08	1.02
Denmark	1.00	1.02	0.96	1.08	1.02	1.00	1.04
Finland	1.00	1.00	0.98	1.07	0.97	0.97	0.98
France	1.00	1.08	1.11	1.01	1.07	1.05	1.11
Germany	1.00	1.04	1.04	1.05	1.05	1.06	1.05
Greece	1.00	1.11	1.07	1.14	1.05	1.04	1.06
Hungary	1.00	1.10	1.24	0.57	1.13	1.15	1.06
India	1.00	1.16	1.30	1.01	1.14	1.17	1.05
Italy	1.00	1.05	1.05	1.05	1.05	1.06	1.05
Japan	1.00	0.85	0.83	0.93	0.81	0.78	0.86
Mexico	1.00	1.22	1.31	0.72	1.18	1.15	1.24
Poland	1.00	1.08	1.07	1.10	1.09	1.07	1.12
Romania	1.00	0.95	0.95	0.95	1.00	1.01	0.99
Slovakia	1.00	1.04	1.03	1.06	1.04	1.04	1.07
South Korea	1.00	1.24	1.34	0.83	1.27	1.27	1.27
Spain	1.00	0.93	0.88	1.03	0.97	0.97	0.96
Sweden	1.00	1.07	1.01	1.20	1.04	1.03	1.06
United Kingdom	1.00	1.04	0.97	1.17	1.06	1.04	1.10
United States	1.00	1.00	1.04	0.91	0.98	0.93	1.12
Rest of World	1.00	1.02	1.00	1.05	1.03	1.03	1.03

Table 4: Counterfactual Results with Trade Friction Shocks Only

Notes: $\theta^D = \theta^N = 2$.All variables expressed relative to global GDP. Calculations using $\beta = 1$ restricted interpolations and extrapolations.

Shock:	Demand		Trade Frictions		Deficits		Productivities		Share of Trade-Weighted Variance Explained (2008:Q1 to 2009:Q1)
Sector:	D	N	D	N	S	All	D	N	
	("X" Indicates Shocks are Included)								
	X								0.64
		X							0.17
			X						0.07 to 0.13
				X					0.08 to 0.12
					X				0.05
						X			-0.01
							X		0.11
								X	0.01
	X	X							0.65
			X	X					0.09 to 0.17
					X	X			0.10
							X	X	0.12
	X	X			X	X			0.71
	X	X					X	X	0.74
			X	X			X	X	0.18 to 0.26
	X	X	X	X	X	X	X	X	1.00

Table 5: Cross-Sectional Explanatory Power of Combinations of Shocks, 2008:Q1 to 2009:Q1

Notes: $\theta^D = \theta^N = 2$.

	No Recession	Only U.S. Durables Demand Shock					
Shocks:	None	Durable Demand Shock for U.S. Introduced					
		Exports / GDP			Imports / GDP		
	All Vars	All	Durables	Non-Durables	All	Durables	Non-Durables
World	**1.00**	**0.98**	**0.97**	**1.00**	**0.98**	**0.97**	**1.00**
Canada	1.00	0.97	0.92	1.07	0.98	0.97	0.98
China	1.00	1.00	0.99	1.03	0.99	0.99	0.99
Germany	1.00	1.00	1.00	1.01	1.00	1.00	1.00
Japan	1.00	1.00	0.99	1.03	0.98	0.98	0.99
Mexico	1.00	0.96	0.94	1.09	0.97	0.97	0.96
South Korea	1.00	1.00	0.99	1.02	0.99	0.99	0.99
United Kingdom	1.00	0.99	0.99	1.01	1.00	1.00	1.00
United States	1.00	0.89	0.89	0.88	0.91	0.85	1.05
Rest of World	1.00	0.99	0.98	1.02	1.00	0.99	1.00

	No Recession	Only Trade Friction Shocks in China and Japan					
Shocks:	None	Both Trade Frictions Shocks in China and Japan					
		Exports / GDP			Imports / GDP		
	All Vars	All	Durables	Non-Durables	All	Durables	Non-Durables
World	**1.00**	**0.97**	**0.97**	**0.97**	**0.97**	**0.97**	**0.97**
Canada	1.00	1.01	1.03	0.97	1.01	1.01	1.02
China	1.00	0.87	0.81	1.04	0.83	0.90	0.61
Germany	1.00	1.01	1.02	0.99	1.01	1.01	1.01
Japan	1.00	0.84	0.83	0.93	0.82	0.79	0.88
Mexico	1.00	1.03	1.04	0.95	1.02	1.02	1.03
South Korea	1.00	1.12	1.15	0.96	1.13	1.13	1.15
United Kingdom	1.00	1.01	1.02	1.00	1.01	1.01	1.02
United States	1.00	0.96	0.98	0.93	0.98	0.95	1.03
Rest of World	1.00	0.97	0.98	0.97	0.99	0.98	1.00

Table 6: Country/Region-specific Counterfactuals

Notes: $\theta^D = \theta^N = 2$.All variables expressed relative to global GDP. Calculations using $\beta = 1$ restricted interpolations and extrapolations.

Durable Manufacturing

(1) Wood and products of wood and cork
(2) Other non-metallic mineral products
(3) Iron & steel
(4) Non-ferrous metals
(5) Fabricated metal products, except machinery & equipment
(6) Machinery & equipment, nec
(7) Office, accounting & computing machinery
(8) Electrical machinery & apparatus, nec
(9) Radio, television & communication equipment
(10) Medical, precision & optical instruments
(11) Motor vehicles, trailers & semi-trailers
(12) Building & repairing of ships & boats
(13) Aircraft & spacecraft
(14) Railroad equipment & transport equip n.e.c.
(15) 50 percent of: Manufacturing nec; recycling (include Furniture)

Non-Durable Manufacturing

(1) Food products, beverages and tobacco
(2) Textiles, textile products, leather and footwear
(3) Pulp, paper, paper products, printing and publishing
(4) Chemicals excluding pharmaceuticals
(5) Pharmaceuticals
(6) Rubber & plastics products
(7) 50 percent of: Manufacturing nec; recycling (include Furniture)

Non-Manufacturing

(1) Agriculture, hunting, forestry and fishing
(2) Mining and quarrying (energy)
(3) Mining and quarrying (non-energy)
(4) Coke, refined petroleum products and nuclear fuel
(5) Production, collection and distribution of electricity
(6) Manufacture of gas; distribution of gaseous fuels through mains
(7) Steam and hot water supply
(8) Collection, purification and distribution of water
(9) Construction
(10) Wholesale & retail trade; repairs
(11) Hotels & restaurants
(12) Land transport; transport via pipelines
(13) Water transport
(14) Air transport
(15) Supporting and auxiliary transport activities; activities of travel agencies
(16) Post & telecommunications
(17) Finance & insurance
(18) Real estate activities
(19) Renting of machinery & equipment
(20) Computer & related activities
(21) Research & development
(22) Other Business Activities
(23) Public admin. & defence; compulsory social security
(24) Education
(25) Health & social work
(26) Other community, social & personal services
(27) Private households with employed persons & extra-territorial organisations & bodies

Table B1: Sector definitions in the OECD Input-Output tables

Notes:

Figures

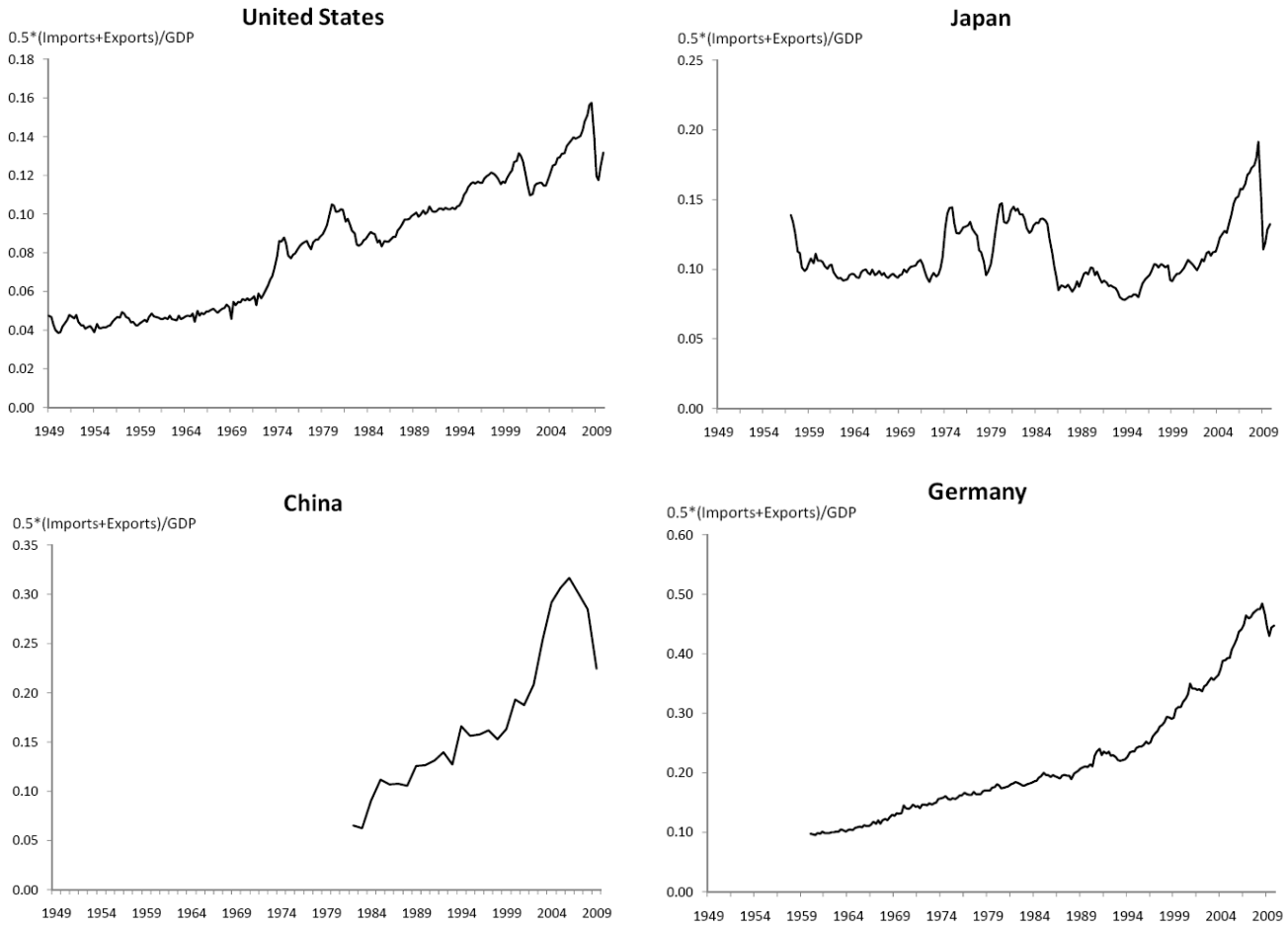


Figure 1: Trade as a Share of Output in the Four Largest Economies

Notes: All data through the end of 2009. United States quarterly data taken from BEA national accounts. Japan quarterly data taken from IMF's IFS database. Germany's quarterly data taken from Source.OECD database. China data only available annually. China's data taken from IFS through 2008. 2009 Trade data for taken from WTO database and GDP estimate from IMF's WEO for China. Trade for United States, Germany, and Japan is goods and services, China is just goods.

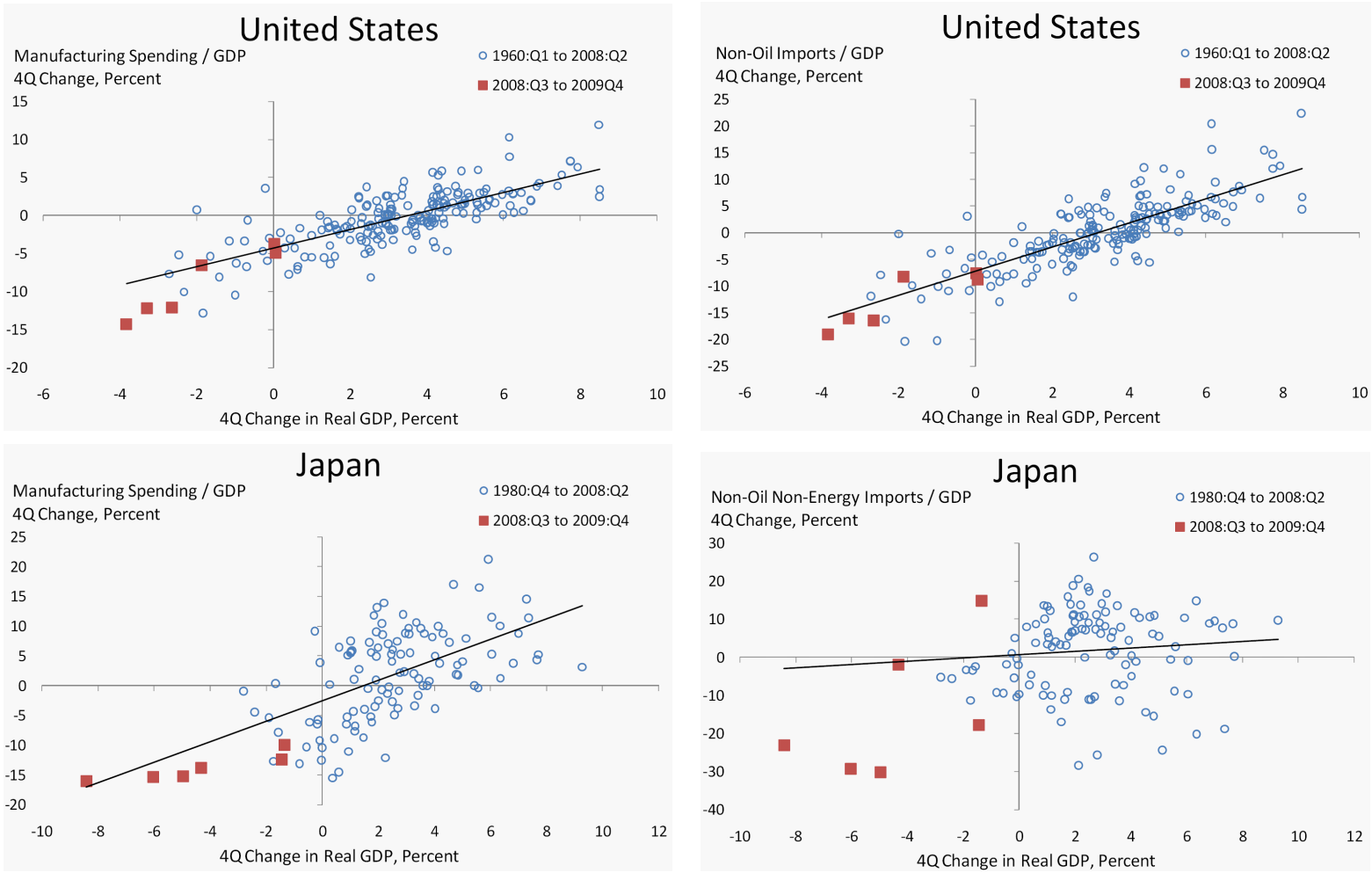


Figure 2: The Cyclical Properties of Tradable-Sector Activity and Trade in U.S. and Japan

Notes: For the U.S., manufacturing spending is the sum of non-farm non-financial business capital spending, personal consumption expenditures on durables, and half of personal consumption expenditures on non-durables (only half because it includes some non-manufactures and services). Non-oil imports are total imports less imports of petroleum and petroleum products. For Japan, manufacturing spending is the sum of business expenditure on new plant and equipment, household expenditure on durable goods, and household expenditure on semi-durables. Non-oil imports are total imports less imports of crude and partly refined petroleum. All series are from Datastream.

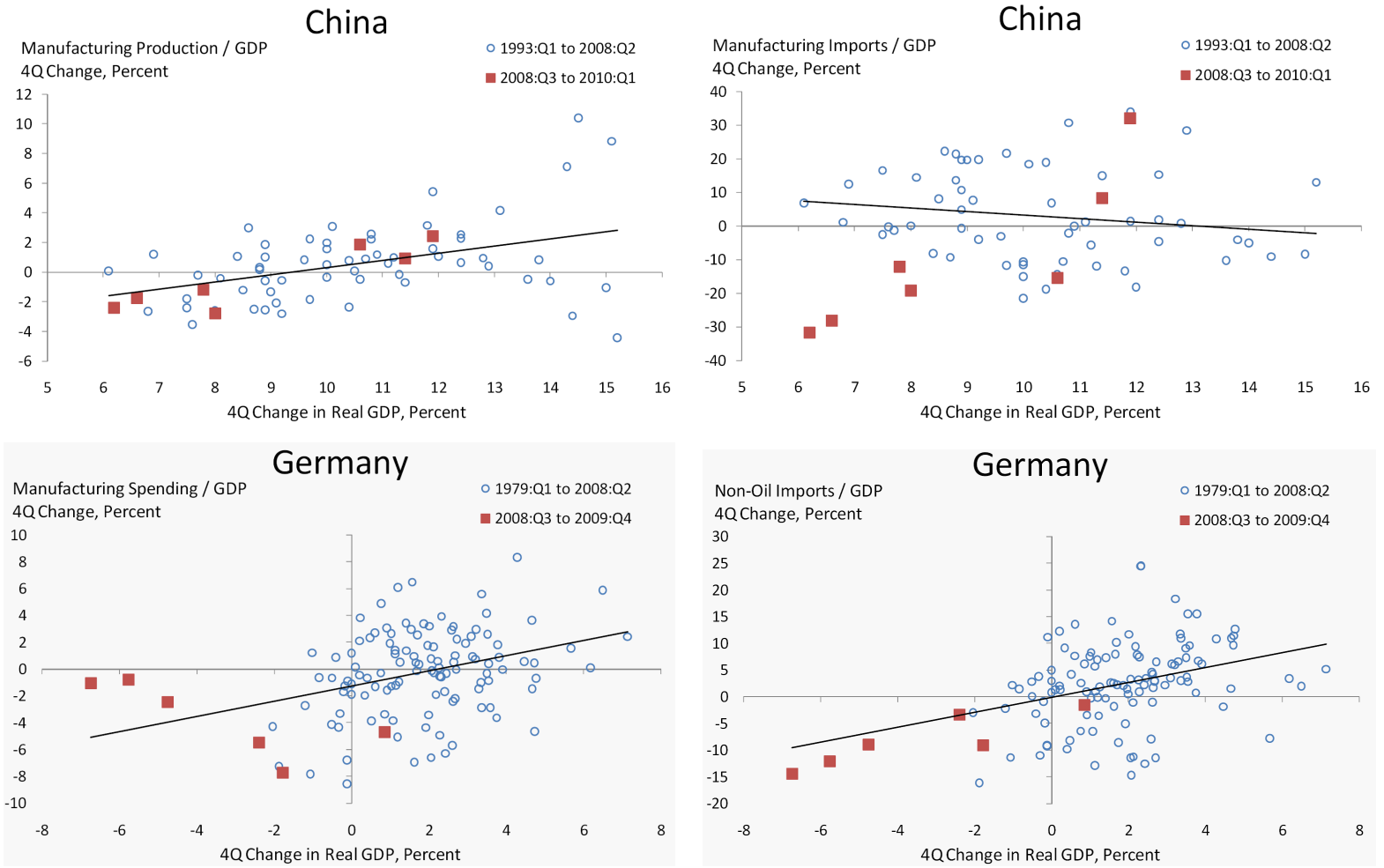


Figure 3: The Cyclical Properties of Tradable-Sector Activity and Trade in China and Germany

Notes: For China, manufacturing production is the GDP of secondary industry. Manufacturing imports are imports of manufactured goods. For Germany, growth in manufacturing spending to GDP is one third of the growth rate of machinery and equipment investment to GDP plus two-thirds of the growth rate of retail sales including motor vehicles and petrol to GDP. All series are from Datastream.

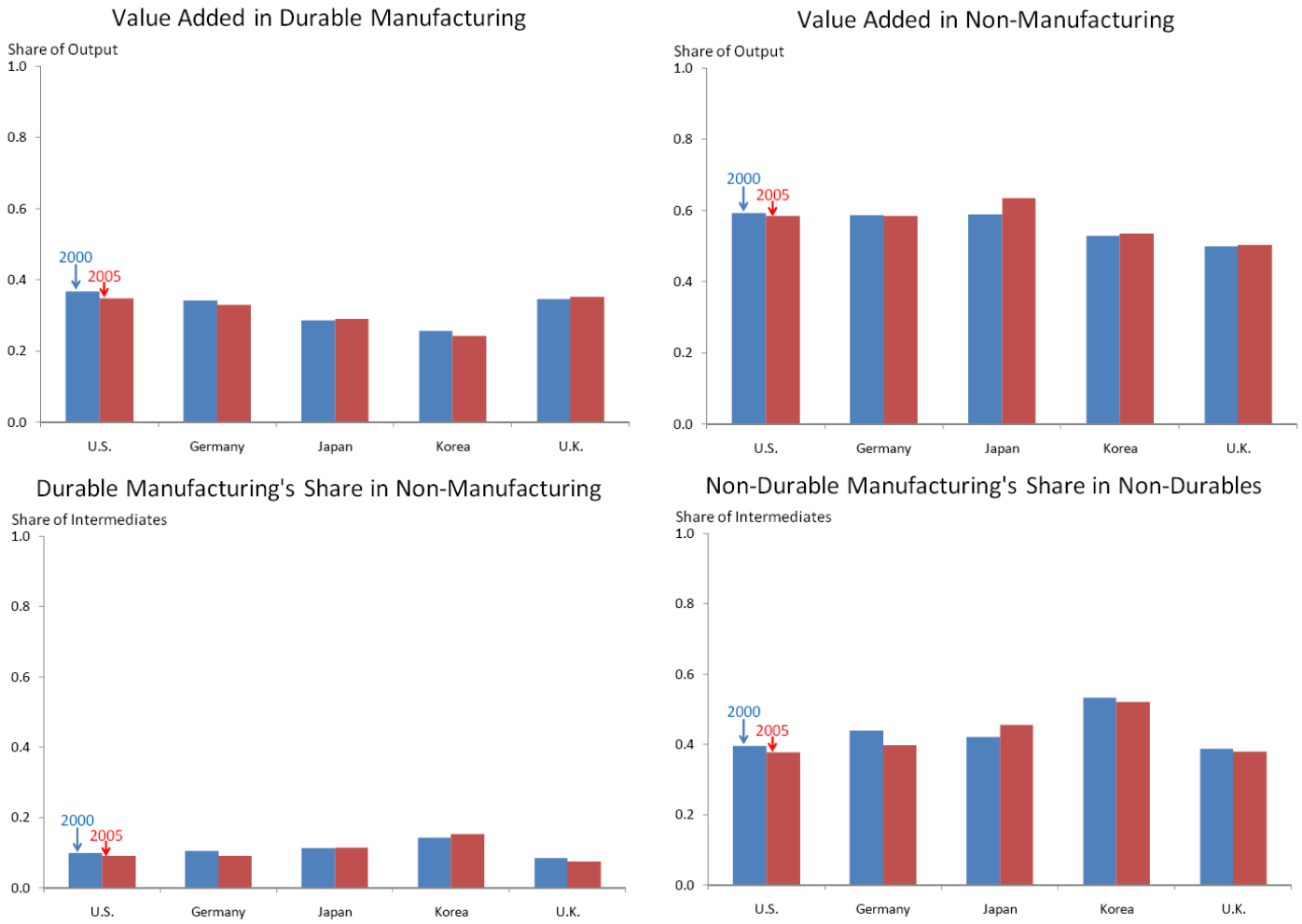


Figure 4: Sample Input-Output Coefficients (β_i^D, β_i^N, γ_i^{ND}, and γ_i^{NN})

Notes: Input-Output coefficients taken from OECD input-Output database, version 2009. See Table B1 for sectoral definitions.

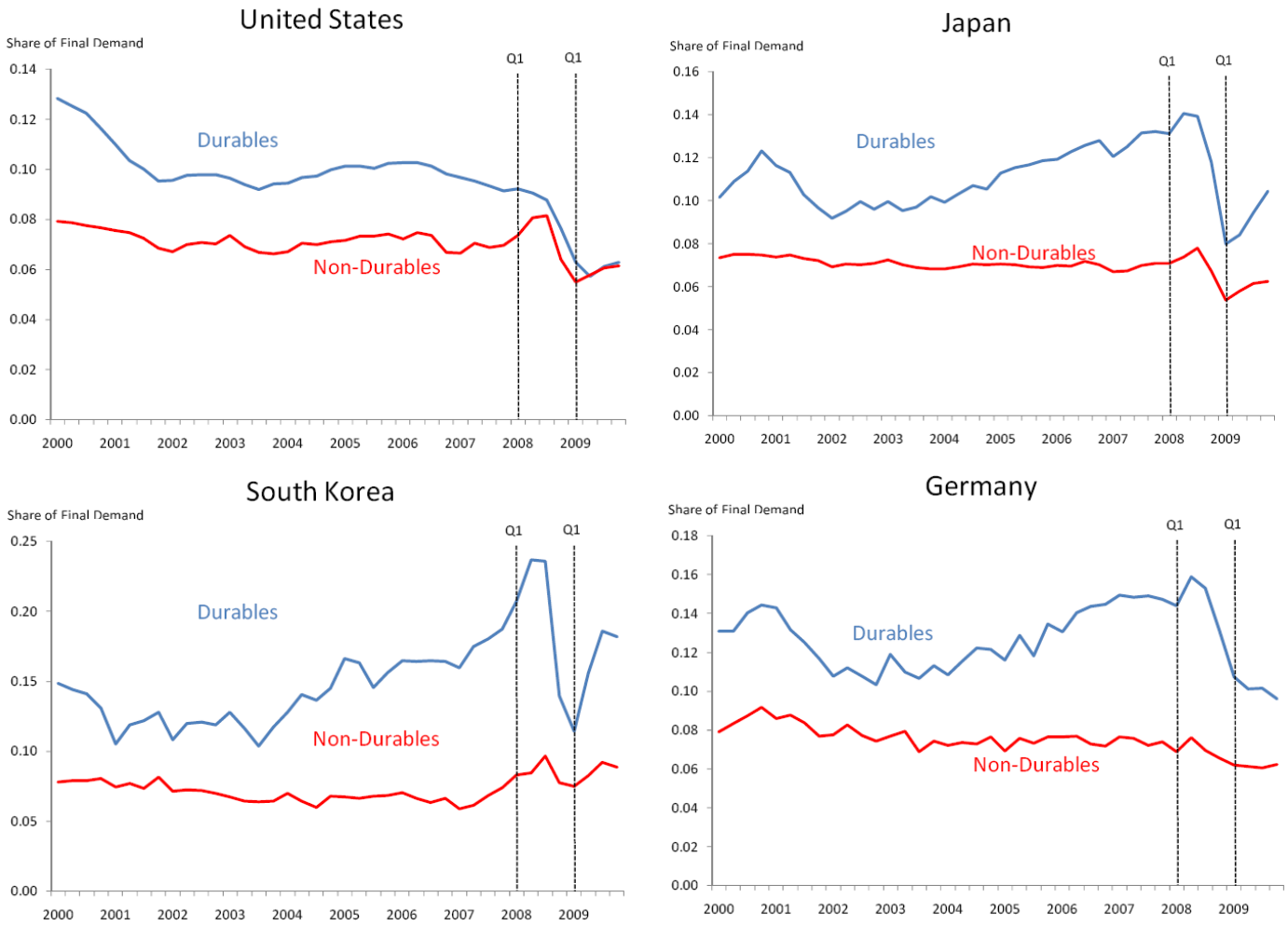


Figure 5: Shares of Manufacturing in Final Demand

Notes: Generated using interpolation procedure with elasticities set to equal one.

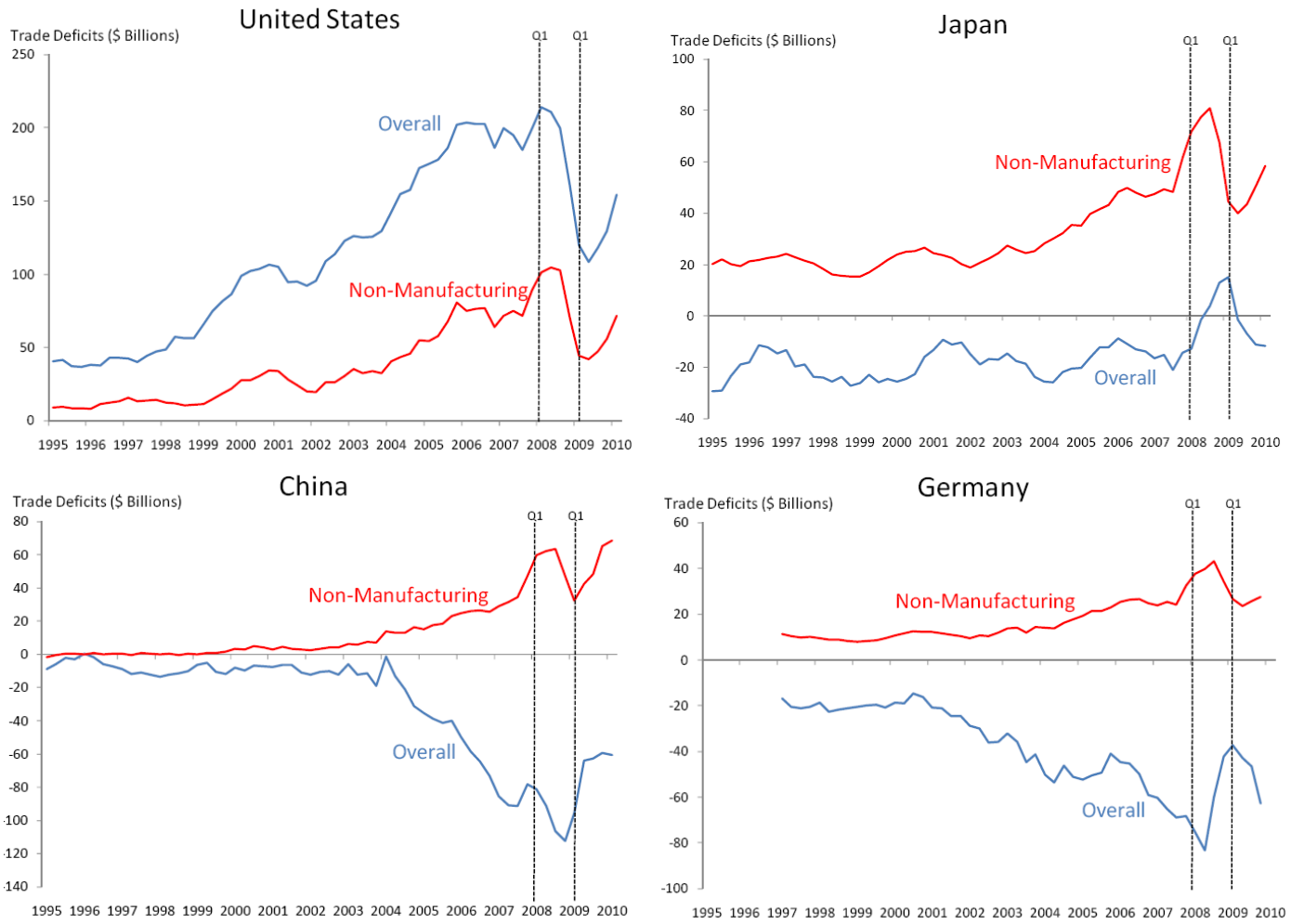


Figure 6: Overall and Non-Manufacturing Trade Deficits

Notes:

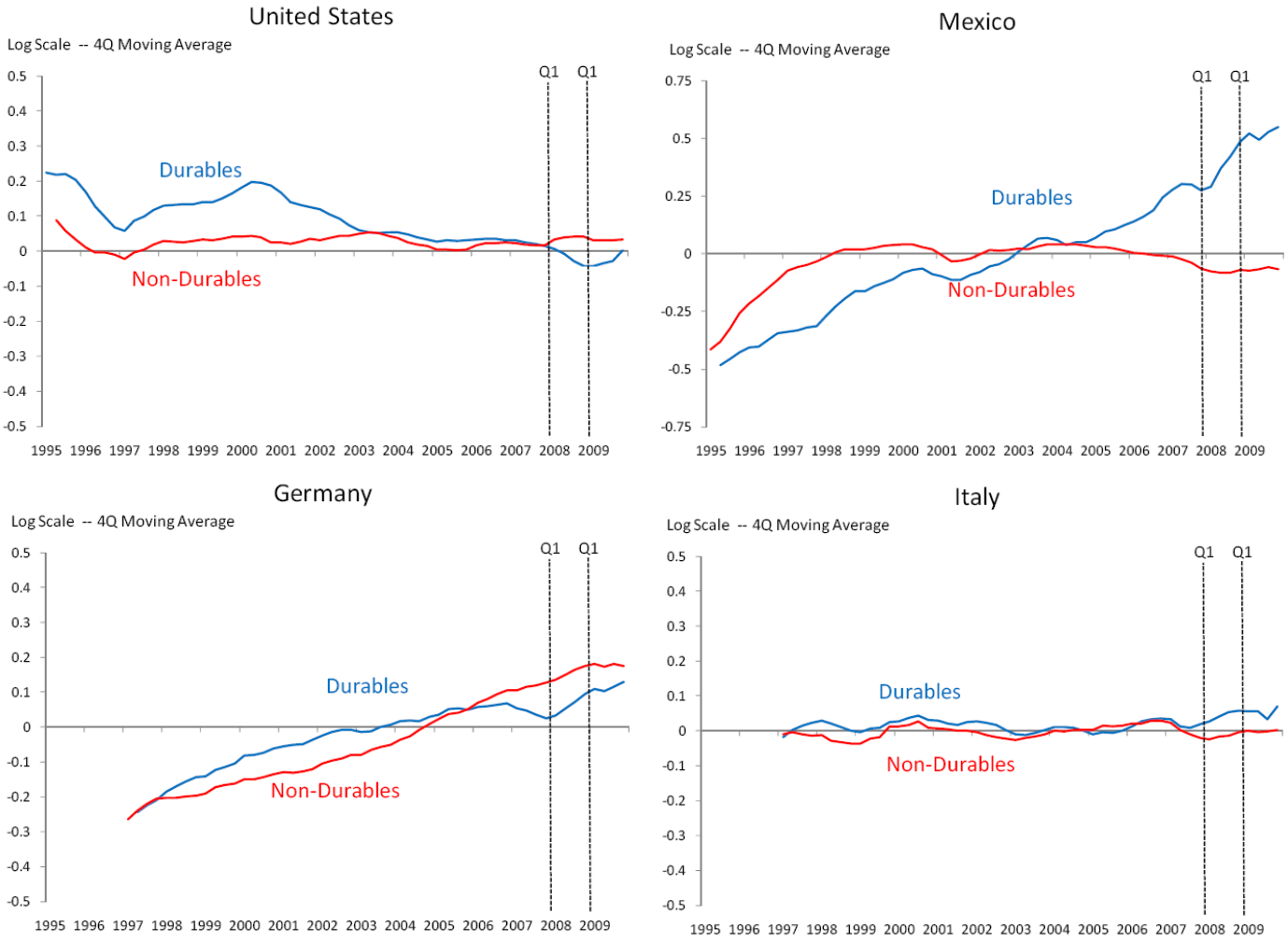


Figure 7: Countries without Large Negative Shock to Trade Frictions

Notes: Generated using interpolation procedure with endogenous elasticites.



Figure 8: Countries with Large Negative Shock to Trade Frictions

Notes: Generated using interpolation procedure with endogenous elasticites.

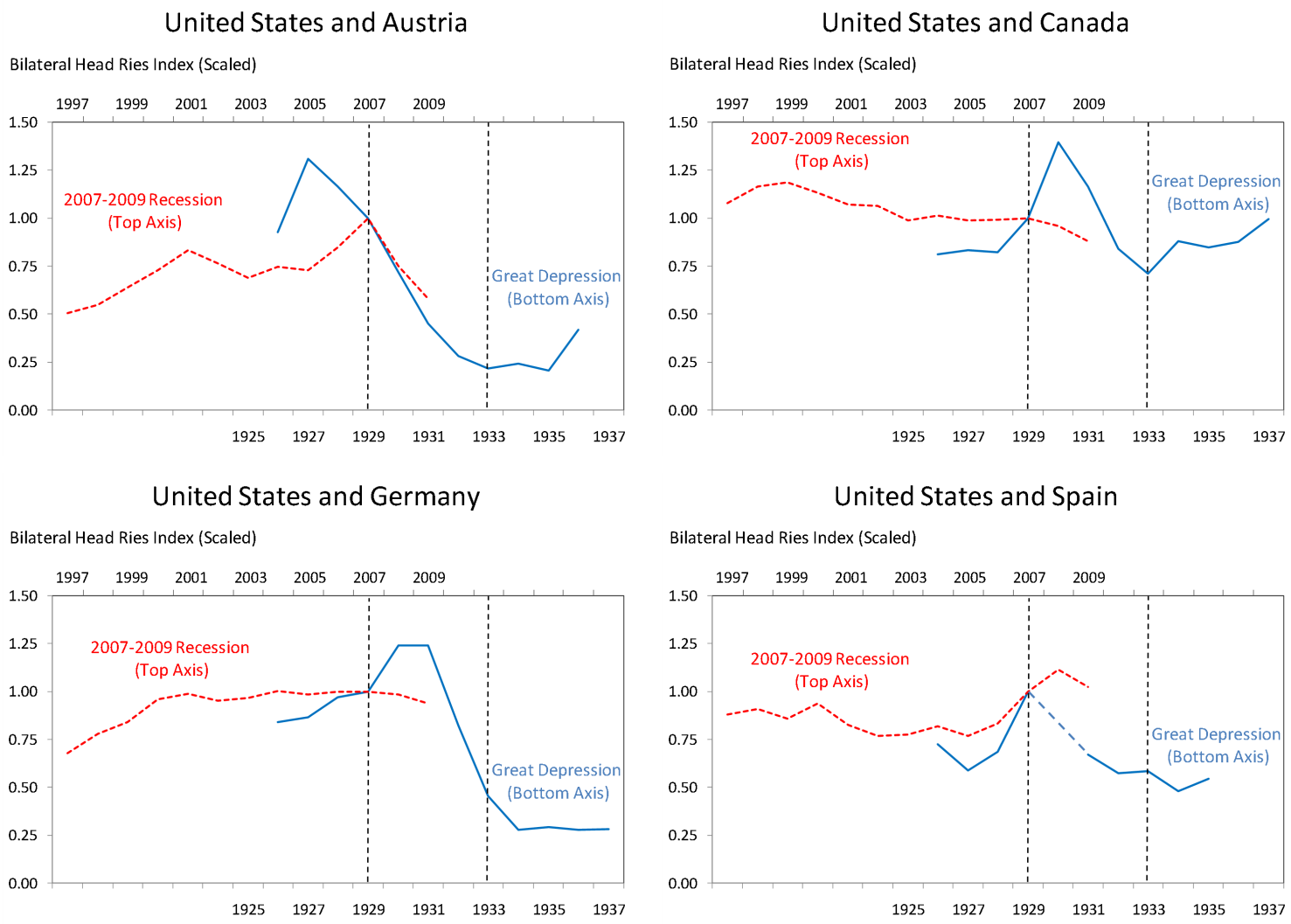


Figure 9: Head-Ries Indices in the Great Depression and Recent Recession

Notes: Red dotted lines (top axis) plot scaled Head-Ries indices with recent annual data on total manufactures (durables and non-durables). Blue solid lines (bottom axis) plot scaled Head-Ries indices with annual data on total manufactures from the Great Depression. For most countries, the index decline during the Great Depression is far more severe than what was witnessed in the 2007-2009 recession.

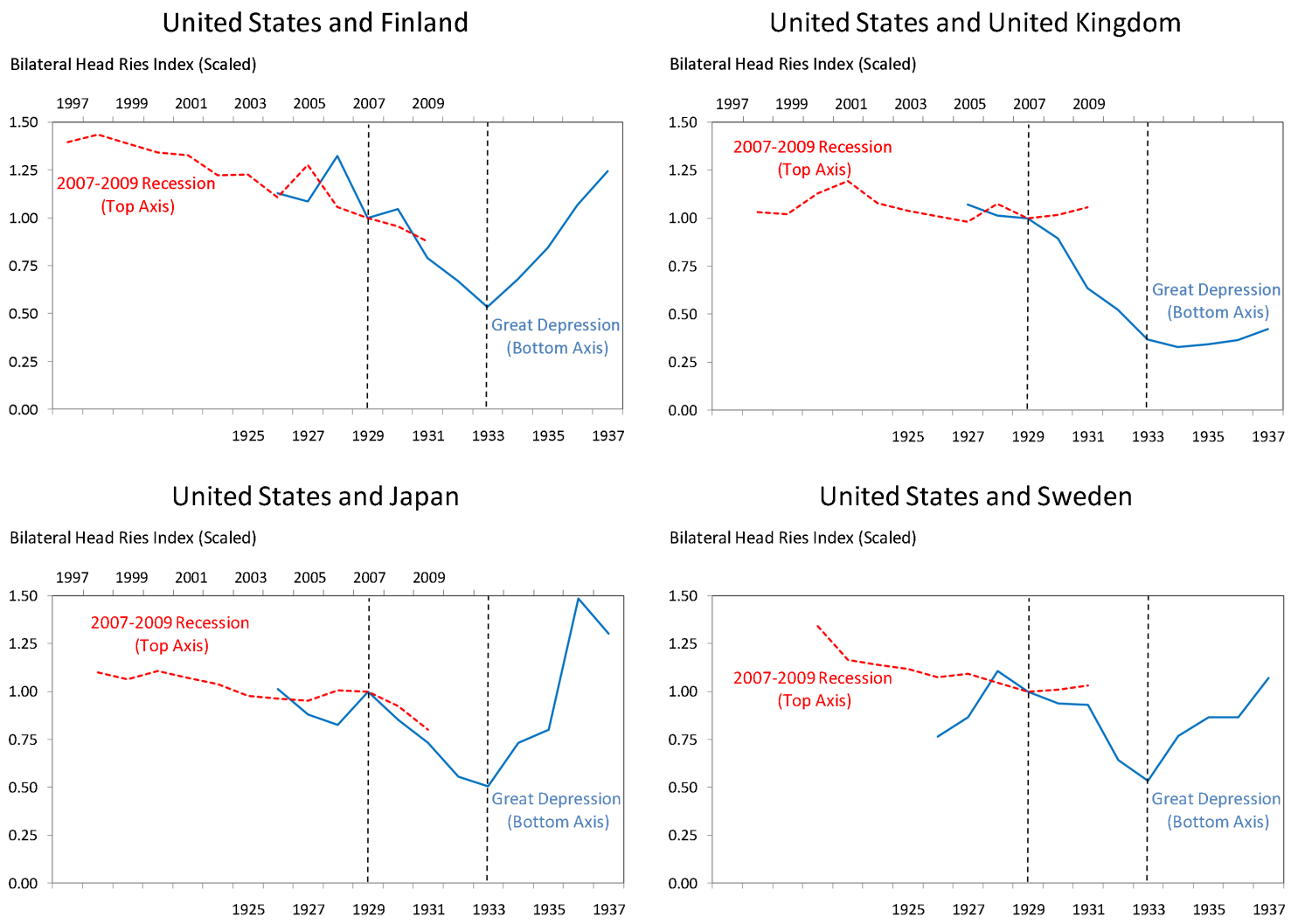


Figure 10: Head-Ries Indices in the Great Depression and Recent Recession

Notes: Red dotted lines (top axis) plot scaled Head-Ries indices with recent annual data on total manufactures (durables and non-durables). Blue solid lines (bottom axis) plot scaled Head-Ries indices with annual data on total manufactures from the Great Depression. For most countries, the index decline during the Great Depression is far more severe than what was witnessed in the 2007-2009 recession.



Figure 11: Sector Shares, Deficit Shocks, and Trade from 2008:Q1 to 2009:Q1

Notes: Changes in deficit levels are relative to global GDP.



Figure 12: Shocks to Productivity, Prices, and Trade from 2008:Q1 to 2009:Q1

Notes:

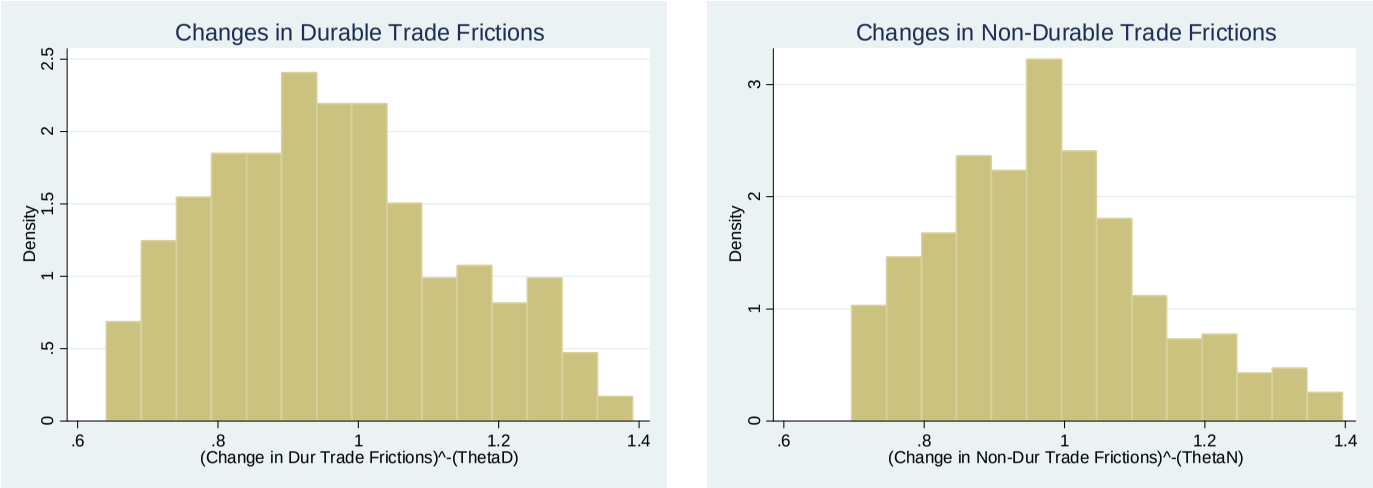


Figure 13: Shocks to Bilateral Trade Frictions from 2008:Q1 to 2009:Q1

Notes: Histograms exclude largest and smallest 5 percentile shocks

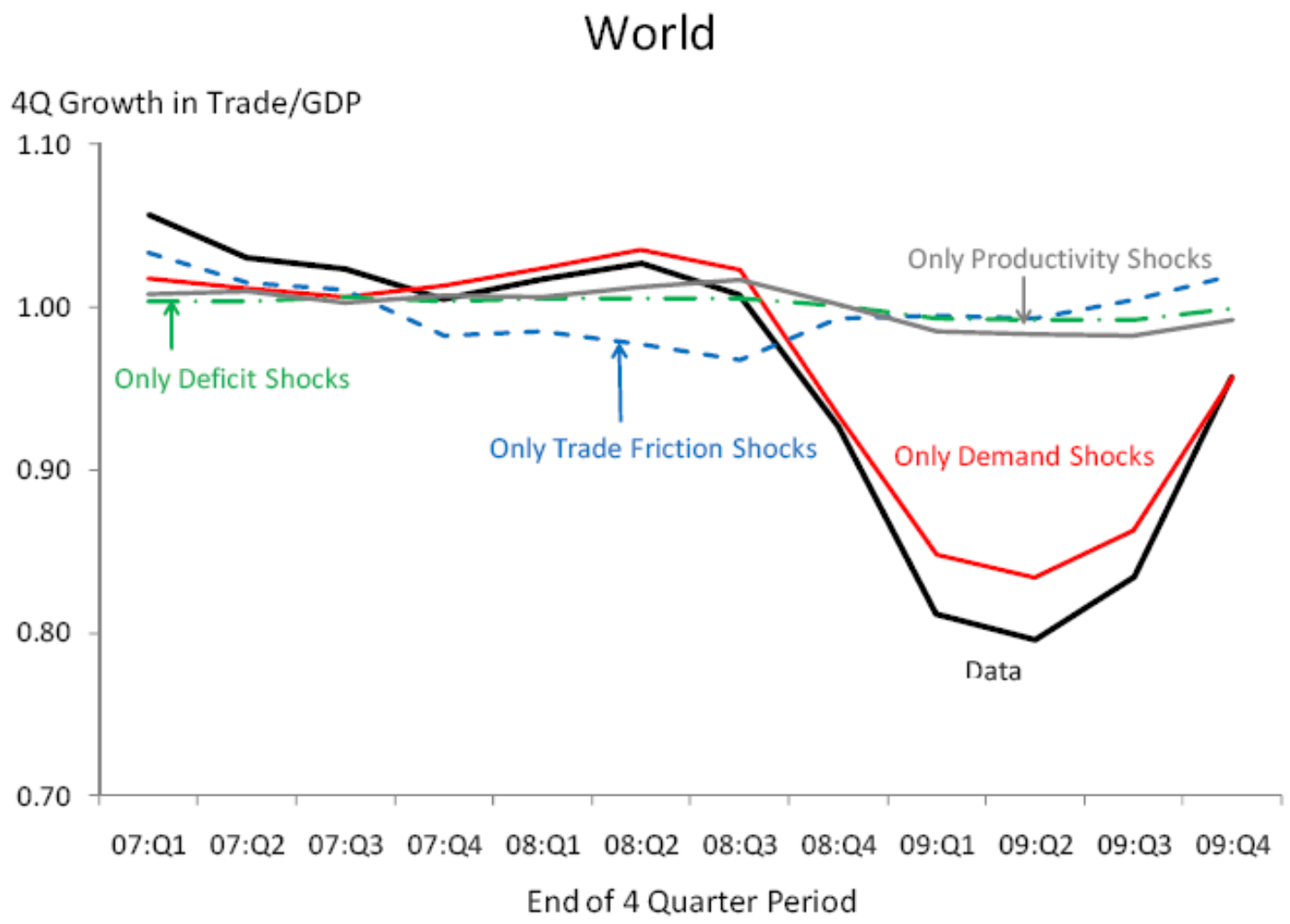


Figure 14: Global Trade/GDP Across Many Four-Quarter Periods in Data and Counterfactuals

Notes:

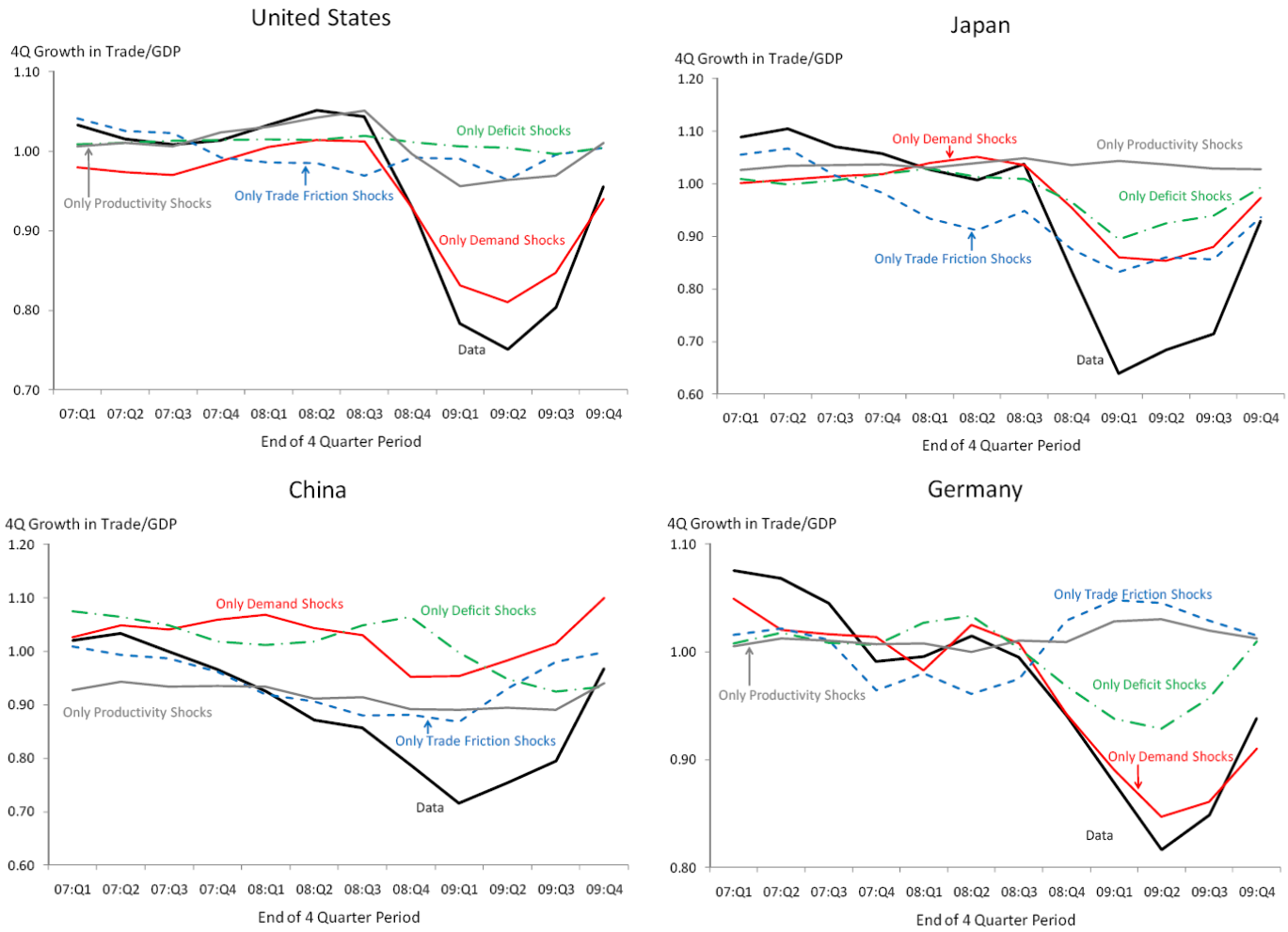


Figure 15: Country Trade/GDP Across Many Four-Quarter Periods in Data and Counterfactuals

Notes: $\theta^D = \theta^N = 2$

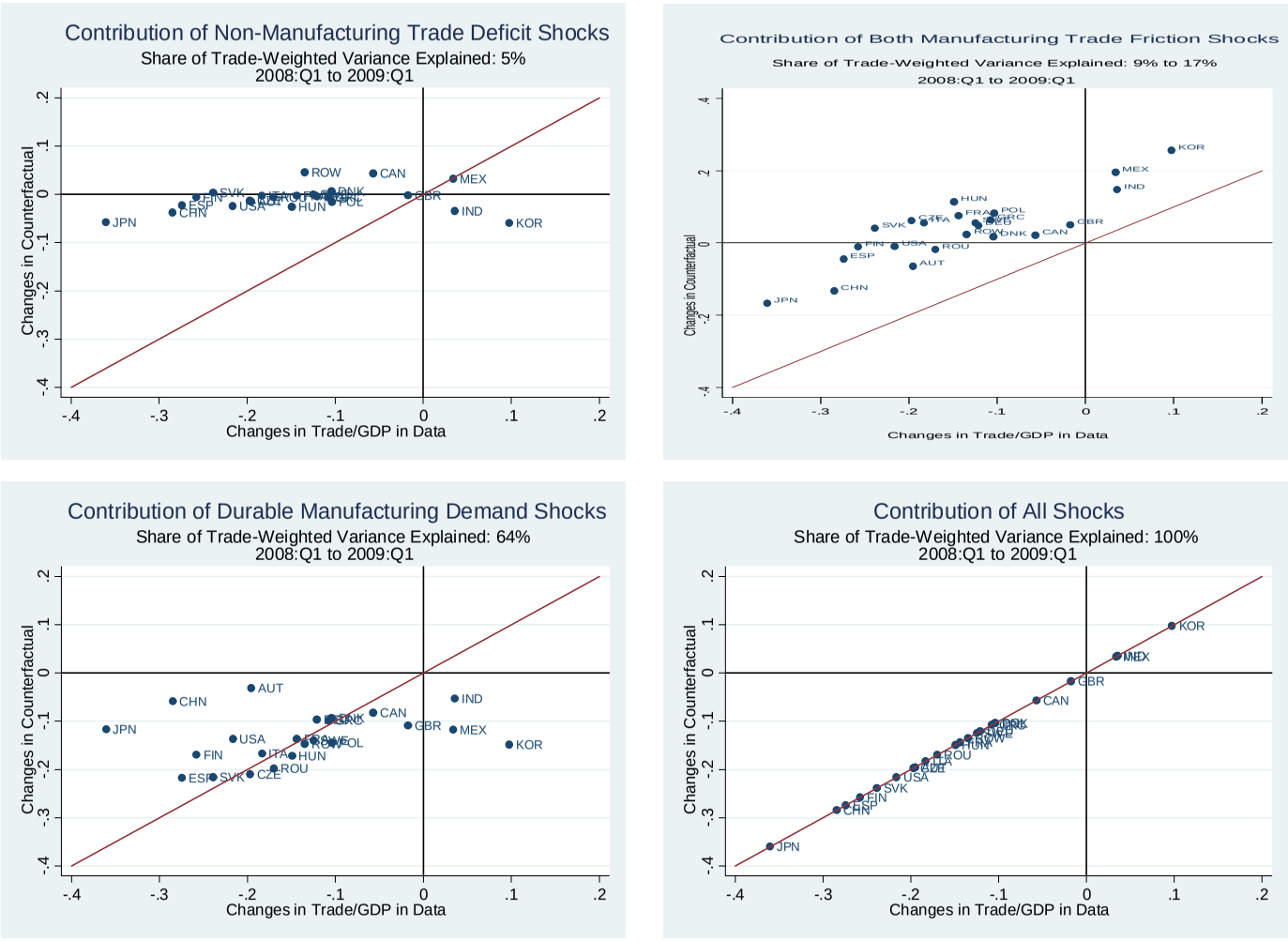


Figure 16: Cross-Sectional Explanatory Power of Various Shocks

Notes: $\theta^D = \theta^N = 2$

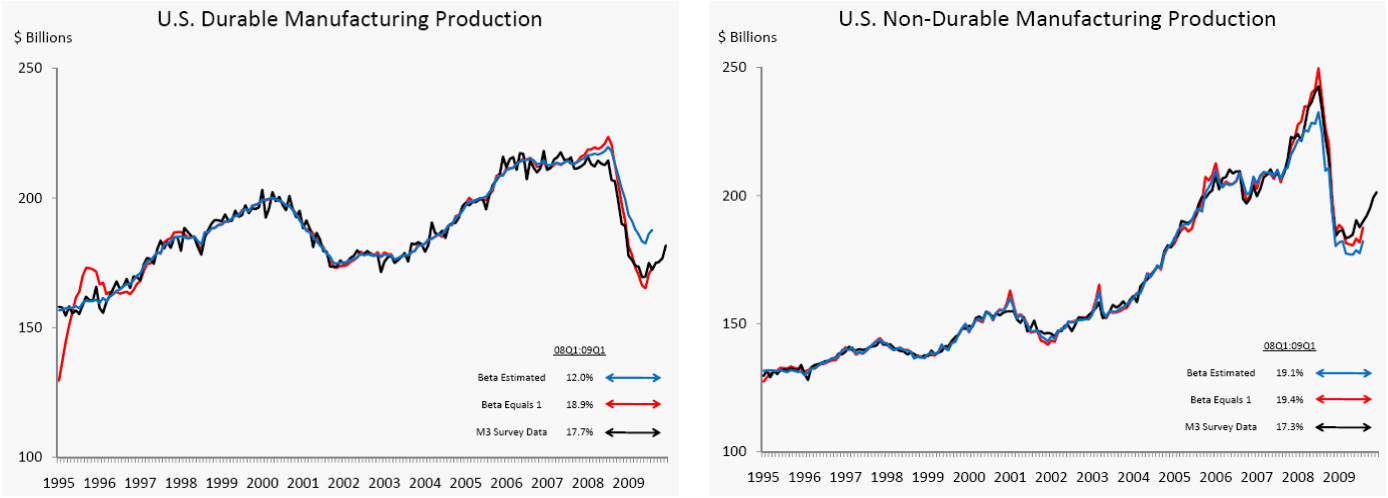


Figure B1: Checking Accuracy of Temporal Disaggregation Procedure for U.S.

Notes: Checking procedure with durable (AMDMVS) and non-durable (AMNMVS) series from Federal Reserve M3 survey (note this is different source from analysis in paper). Annual totals included from 1995-2007 only, even though data starts earlier and is available through 2009, to mirror extent of data used for other countries.

Appendix A: Derivations of Expression (11)

In this appendix, we demonstrate that one can derive the Head-Ries index from many classes of trade models, such as a structure with Armington preferences, as in Anderson and van Wincoop (2003), monopolistic competition as in Redding and Venables (2004), the Ricardian structure in Eaton and Kortum (2002), or monopolistic competition with heterogeneous producers, as in Melitz (2003) and Chaney (2008). To do so, we need only show that each theory of international trade lead to a bilateral import share equation with the same form as equation (11). From there, the derivation of (15) follows exactly as in Section 2. This implies that for the first sections of the paper, we need not specify a particular trade structure, so long as it is in this larger set of models.

1. Consider the model of Armington (1969), as implemented in Anderson and van Wincoop (2003). Consumers in country n maximize:

$$\left(\sum_i \beta_i^{(1-\sigma)/\sigma} c_{ni}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)},$$

subject to the budget constraint $\sum_i p_{ni} c_{ni} = y_n$, where σ is a preference parameter representing the elasticity of substitution across goods produced in different countries, $\beta_i > 0$ is a parameter capturing the desirability of of country i's goods, y_n is the nominal income of country n, and p_{ni} and c_{ni} are the price and quantity of the traded good supplied by country i to country n. In their setup, prices reflect a producer-specific cost and a bilateral-specific trade cost: $p_{ni} = p_i t_{ni}$. Solving for the nominal demand of country i for goods from country j then yields their equation (6):

$$x_{ni} = \left(\frac{\beta_i p_i t_{ni}}{P_n} \right)^{1-\sigma} y_n,$$

where $P_n = \left[\sum_k \left(\beta_k p_k t_{nk} \right)^{1-\sigma} \right]^{1/(1-\sigma)}$ is the price index of country n. Substituting this definition and with goods markets clearing, $y_n = \sum_j x_{nj}$, we obtain:

$$\pi_{ni} = \frac{x_{ni}}{\sum_j x_{nj}} = \frac{\left(\beta_i p_i t_{ni} \right)^{1-\sigma}}{\sum_k \left(\beta_k p_k t_{nk} \right)^{1-\sigma}}.$$

Relabeling $\theta = \sigma - 1$ and $T_i = \beta_i^{-\theta}$, we recover an expression equivalent to (11).

2. Consider the model of Krugman (1980), as implemented in Redding and Venables (2004). Like Anderson and van Wincoop, they use a constant elasticity formulation, but they include a fixed cost for firms operating in each country. They express, in their equation (9), the total value of imports to country n from i:

$$x_{ni} = \left(n_i p_i^{1-\sigma} \right) t_{ni}^{1-\sigma} \left(E_n G_n^{\sigma-1} \right),$$

where they refer to $\left(E_n G_n^{\sigma-1} \right)$ as the "market capacity" of the importing country n because it refers to the size of n's market, the number of competing firms that can cover the fixed cost of operation, and the level of competition as summarized by the price index G. They refer to the term $\left(n_i p_i^{1-\sigma} \right)$ as the "supply capacity" of the exporting country i, because fixing the market capacity, the volume of sales is linearly homogeneous in that term. Finally, $T_{ni}^{1-\sigma}$ is the iceberg trade cost for shipping from i to n. Hence, this model too leads to an expression:

$$\pi_{ni} = \frac{x_{ni}}{\sum_j x_{nj}} = \frac{\left(n_i p_i^{1-\sigma} \right) T_{ni}^{1-\sigma}}{\sum_k \left(n_k p_k^{1-\sigma} \right) T_{nk}^{1-\sigma}}.$$

Again, this expression can be relabeled and made equivalent to (11).

3. Consider the competitive model of Eaton and Kortum (2002), where θ and T_i are parameters of a Fréchet distribution of producer efficiency capturing, respectively, heterogeneity across producers (inversely) and country i's absolute advantage. The property of this distribution is such that the probability that country i is the lowest price (production plus transport costs) provider of a good to

country n is an expression identical to (11), their equation (8). Given that average expenditure per good in their model does not vary by source and invoking the low of large numbers, it follows that this probability is equivalent to the trade share.

4. Consider Chaney (2008), which builds on Melitz (2003). Firm productivities are distributed Pareto with shape parameter γ and in addition to iceberg costs τ_{ni}, to sell in market n also requires employing f_{ni} units of local labor. This leads to an expression for total imports by country n from country i, his equation (10) (where we've dropped sectoral terms indexed by h):

$$x_{ni} = \frac{Y_i Y_n}{Y} \theta_n^{\gamma} w_i^{-\gamma} \tau_{ni}^{-\gamma} f_{ni}^{-[\gamma/(\sigma-1)-1]},$$

where notation is similar to the examples above, and θ_n measures what he refers to as country n's "remoteness" from the rest of the world. Summing this over all bilaterals implies:

$$\pi_{ni} = \frac{x_{ni}}{\sum_j x_{nj}} = \frac{Y_i w_i^{-\gamma} \left(\tau_{ni} f_{ni}^{[1/(\sigma-1)-1/\gamma]}\right)^{-\gamma}}{\sum_k Y_k w_k^{-\gamma} \left(\tau_{nk} f_{nk}^{[1/(\sigma-1)-1/\gamma]}\right)^{-\gamma}},$$

which, again, is clearly in the same form as (11).

Appendix B: Data Sources and Related Procedures

In this appendix, we first detail our sources for trade, production, input-output, and macroeconomic data. We next describe the construction of sectoral industrial production and producer price indices and review the temporal disaggregation procedure that uses these indices along with annual data to generate monthly production values for each manufacturing sector. Finally, we list the data sources and procedures required to calculate the Head-Ries index on Great Depression-era data.

Trade Data

Trade data are readily available at a monthly frequency – we use monthly bilateral trade data from the Global Trade Atlas Database. These data are not seasonally adjusted and are provided in dollars. We aggregate appropriate 2-digit HS categories to generate the total bilateral and multilateral trade flows in each manufacturing sector.

Concordances Linking Trade and Production

A many-to-many concordance was constructed to link the 2-digit harmonized system (HS) trade data to the International Standard Industrial Classification (ISIC) codes used in the production data. We start by downloading the mapping of 6-digit HS codes (including all revisions) to ISIC codes from the WITS website. This concordance was then merged with COMTRADE data on the volume of world trade at the 6-digit level for 2007-2008 (also accessed through WITS). We estimate the proportion of each HS 2-digit code that belongs in each ISIC category using these detailed worldwide trade weights. Then we can use the same concordance in the last step to map production and trade to our sectors $j \in \Omega_M$.

Input-Output Coefficients

The input-output coefficients – β_i^j and γ_i^{jl} – were calculated from the 2009 edition of the OECD's country tables.[22] We concord and combine the 48 sectors used in these tables to form input-output tables for the three sectors $j \in \Omega$. Table B1 shows how we classified these 48 sectors into durables, non-durables, and non-manufactures. To determine β_i^j, we divide the total value added in sector j of country i by that sector's total output. To determine the values for γ_i^{jl}, we divide total spending in country i by sector j on inputs from sector l and divide this by that sector's total intermediate use at basic prices (i.e. net of taxes on products). The OECD input-output tables are often available for the same countries for multiple years. In such cases, we use the most recent year of data available.

Additional Macro Data

Exchange rates to translate local currency production values into dollars (to match the dollar-denominated trade flows) are from the OECD.Stat database and from the International Financial Statistics database from the IMF. Other standard data used in the paper, such as quarterly GDP and trade deficits, are taken from the Economist Intelligence Unit (EUI). Trade and production data are translated using exchange rates at the monthly frequency before being aggregated to the quarterly frequency that we use in our regressions and counterfactuals.

IP and PPI Indices by Sector

To generate monthly production levels for durables and non-durables, we first need IP and PPI indices at the sector level. The exact methodology used to construct the series depended on what series were available on DataStream, as this is not consistent across countries. Essentially, three different methodologies were used.

We applied a first methodology when Datastream contains IP or PPI series on durable manufacturing and nondurable manufacturing for the country. Included in this category for IP are Canada, China, and the United States. Included in this category for PPI are China and the United States. For China the series are actually "Heavy Industry" and "Light Industry". The key difference appears to be that one group of nondurable manufactures - chemicals - is included in heavy industry.

[22] The only exception is China's input-output table, which was obtained from Robert Feenstra and is analyzed in Feenstra and Hong (2007).

Next, there are several countries for which Datastream contains IP or PPI series for capital goods, durable consumer goods, nondurable consumer goods, and intermediate goods. We classify capital goods as durable, but need to be able to decompose the intermediate goods into durable goods (such as metals) and nondurable goods (such as paper). The presence of more detailed manufacturing industry data allows us to do this using regression analysis. We regress monthly log-changes in intermediate goods IP or PPI series on underlying detailed manufacturing industry series to reveal the composition of the intermediate series and exclude countries for which there is not a good fit. The regression results give us estimates of the industry composition of intermediate goods manufacturing, and we combine this with our industry concordances to durable and non-durable to generate durable and nondurable intermediate goods IP or PPI series. We then combine all the more aggregated categories, using their weights in production from the annual data, to generate indices for overall durables and non-durables. This methodology applies to the construction of our IP and PPI series for Austria, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Italy, Romania, Slovakia, South Korea, Spain, Sweden, and the United Kingdom.

There are some countries with IP or PPI data for multiple manufacturing industries together with aggregate manufacturing (or occasionally, total industry IP or PPI).We use similar regression analysis to ascertain the industry composition of the broad measure, and then use our data on the durable and non-durable composition of each of these industries to construct durable and nondurable series. When the regression analysis does not yield a good fit (judged by a high R-squared and coefficients that sum close to 1), we do not include that country. This procedure was used for India, Japan, Mexico and Poland.

Finally, we also require IP and PPI indices for overall manufacturing. These monthly data for China were found at chinadataonline.com, and for the other countries were downloaded from the OECD Main Economic Indicators Database (MEI) and the EIU Database.

Annual Production by Sector

In addition to monthly IP and PPI indices, we need annual production levels for each sector and for overall manufacturing. These annual data are taken from the OECD Structural Analysis Database (STAN) and the United Nations National Accounts and Industrial Statistics Database (UNIDO). For China, Chang-Tai Hsieh provided us with cross-tabs from 4-digit manufacturing production data from the census of manufacturing production. We used these data to determine the durables/non-durables split and multipled these shares by the manufacturing total from http://chinadataonline.org.

We concord International Standard Industrial Classification (ISIC Rev. 3) 2-digit manufacturing production data to the appropriate sector definition (whatever is required to match the IP/PPI indices) to get annual totals for each of these categories.[23] Our definition of manufacturing comprises ISIC industries 15 through 36 excluding 23 (petroleum). We further divide goods into capital goods, durable consumer goods, nondurable consumer goods, durable intermediate goods, and non-durable intermediate goods using the U.S. import end use classification. Harmonized System (HS) trade data are simultaneously mapped into the end use classification using a concordance from the U.S. Census Bureau and into the ISIC classification using the concordances from the World Bank's World Integrated Trade Solution (WITS) website. World trade volumes at the 6-digit level for 2007-2008 are again used to estimate what proportion of each ISIC classification belongs in each of the categories.

Temporal Disaggregation

The United States reports monthly estimates of the level of manufacturing production, but such data are not available for most of the countries we study. However, annual production data in levels are available for all of these countries. We now describe how we use the monthly indicators of production constructed as described above to disaggregate annual production levels into internally consistent monthly values, as well as to generate out-of-sample predictions that reflect all up-to-date information for the months subsequent to the previous year's end. This problem, referred to in the econometrics and forecasting literature as temporal disaggregation, was studied as early as the 1950s by, among others, Milton Friedman (see Friedman, 1962).

We disaggregate and extrapolating the annual production data in country i and sector j using the estimated relationship with the industrial production and producer price indices in that same country and sector.[24] The details follow below, but for intuition, think of a linear regression of the annual gross production of manufacturers on the annual sum of the monthly totals of the high frequency variables. Chow and Lin (1971) starts by using the coefficient estimates from such a regression to generate predicted monthly values.

[23] Occasionally, a 2-digit sector will be dropped for one year, so we impute an alternative series where production levels are "grown" backward from the more recent and most complete data, only using the growth rates from categories reported in both years.

[24] The procedure was adapted from the code in Quilis, Enrique. "A Matlab Library of Temporal Disaggregation and Interpolation Methods: Summary," 2006.

Next, the Chow-Lin procedure distributes the regression residuals equally to each of these monthly predicted values for any given year. This procedure creates an internally consistent monthly series that sums up to the actual annual data. However, it generally creates artificial jumps from December to January since the corrections for residuals are different only from year to year. Our procedure makes two additional changes to this basic structure.

First, we follow Fernandez (1981) and allow for serial correlation in the monthly residuals, which eliminates spurious jumps between the last period of one year and the first period of the subsequent year. Second, we follow Di Fonzi (2002) in adjusting the data so the procedure works for a log-linear, rather than linear, relationship. The monthly indicators used are the index of industrial production (IP) and the producer price index (PPI), so a relationship in logs is clearly most sensible.

Again, we wish to generate an estimate of the monthly series for gross manufacturing production $Y^M(t)$, but we only have the annual totals for this series:

$$\overline{Y}^M(\tau) = \sum_{t=12(\tau-1)+1}^{12\tau} Y^M(t), \tag{B1}$$

where $\tau = 1..T$ denotes the year and $t = 1..12T$ denotes the month. We use two related series that contain information on the underlying gross production series – industrial production and the producer price index – plus a third series of ones to capture a constant. We write each of the three related series in vector form and join the vectors in a $(12T)$-by-3 matrix Z.

We also write the annual data in vector form as $\overline{Y}^M = [\overline{Y}^M_1, .., \overline{Y}^M_T]'$ and the estimates for $Y^M(t)$ in vector form as $\widehat{Y^M} = [\widehat{Y^M_1}, .., \widehat{Y^M_{12T}}]'$. Assume a linear relationship between the related series and monthly production:

$$Y^M = Z\beta + \varepsilon, \tag{B2}$$

where $\beta = [\beta_1, \beta_2, \beta_3]'$ and ε is a random vector with mean 0 and covariance matrix $E[\varepsilon\varepsilon'] = \Omega$. We can write (B2) as:

$$\overline{Y}^M = B'Y^M = B'Z\beta + B'\varepsilon,$$

where

$$B = I_T \otimes \Psi,$$

and I_T is the T-by-T identity matrix and Ψ is a 12-by-1 column vector of ones. Hence, $\widehat{\beta}$ and $\widehat{Y^M}$ can be obtained using GLS as:

$$\begin{aligned} \widehat{\beta} &= [Z'B(B'\Omega B)^{-1}B'Z]^{-1}Z'B\,(B'\Omega B)^{-1}\,\overline{Y}^M \\ \widehat{Y^M} &= Z\widehat{\beta} + \Omega B(B'\Omega B)^{-1}[\overline{Y}^M - B'Z\widehat{\beta}]. \end{aligned} \tag{B3}$$

Consider the simplest assumption that there is no serial correlation and equal variance in the monthly residuals, or $\Omega = \sigma^2 I_{12T}$. Then, equation (B3) simplifies to:

$$\widehat{Y^M} = Z\widehat{\beta} + B[\overline{Y}^M - B'Z\widehat{\beta}]\frac{1}{12}$$

because $(B'B)^{-1} = 1/12$. This implies that the annual discrepancy $B'\varepsilon$ be distributed evenly across each month of that year. Given the failure of the zero serial correlation assumption in the data, this would create obvious and spurious discontinuities near the beginning and end of each year.

We now follow Fernandez (1981) and consider a similar procedure, but with a transformation designed to transform a model with serially correlated residuals into one with classical properties, and then to apply a procedure similar to the one above, to deal with the disaggregation of annual values. Consider the case where the error term from equation (B2) followed a random walk:

$$\varepsilon_t = \varepsilon_{t-1} + \mu_t,$$

where μ_t has no serial correlation, zero mean, and constant variance σ^2. Consider the first difference

transformation D:

$$D_{12T\text{-by-}12T} = \begin{bmatrix} 1 & 0 & 0 & . & 0 & 0 \\ -1 & 1 & 0 & . & 0 & 0 \\ 0 & -1 & 1 & . & 0 & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & -1 & 1 \end{bmatrix}.$$

One can premultiply the error in equation (B2) by this matrix to generate: $DY^M - DZ\beta$, which converts the both left and right hand sides of the model into first-difference form, with the exception being the first terms given the upper left hand element equals one. With these first-differenced series, we can re-write the model as:

$$D\overline{Y}^M = DZ\beta + D\varepsilon.$$

Note that $\Omega = E[D\varepsilon\varepsilon' D'] = E[\mu\mu'] = \sigma^2 I_{12T}$, so errors in this reformulated model have classical properties. Fernandez shows that the expression for the best linear estimator in this context is the same as (B3), but with $\Omega = (D'D)^{-1}$:

$$\begin{aligned} \widehat{\beta} &= [Z'B(B'(D'D)^{-1}B)^{-1}B'Z]^{-1}Z'B\left(B'(D'D)^{-1}B\right)^{-1}\overline{Y}^M \\ \widehat{Y^M} &= Z\widehat{\beta} + (D'D)^{-1}B(B'(D'D)^{-1}B)^{-1}[\overline{Y}^M - B'Z\widehat{\beta}]. \end{aligned} \tag{B4}$$

The relationship (B2) is written in levels, but it is clearly more appropriate for our purposes to write the relationship between production and production indicators in log-levels, such that a given percentage change in one variable leads to a percentage change in the other:

$$\ln Y^M = (\ln Z)\,\beta + \varepsilon. \tag{B5}$$

This can be somewhat difficult to handle in the above framework because the sum of the log of monthly totals will not equal the log of the annual total when the adding-up constrain does hold in levels. We deal with this by running the algorithm on annual data that has been converted such that the sum of fitted monthly data will approximate the original annual levels. This cannot be achieved exactly, so a second-stage procedure is then implemented to distribute the residuals across the months and ensure the aggregation constraints bind exactly.

Following Di Fonzi (2002), we consider the first order Taylor series approximation of $\ln Y^M$ around the log of the arithmetic average for the monthly totals, $\ln(\overline{Y}^M/12)$. We write:

$$\ln Y^M = \widetilde{Y^M} \approx \ln\frac{\overline{Y}^M}{12} + \frac{12}{\overline{Y}^M}\left(Y^M - \frac{\overline{Y}^M}{12}\right) = \ln\overline{Y}^M - \ln 12 + 12\frac{Y^M}{\overline{Y}^M} - 1.$$

Summing this expression up over the twelve months, we get:

$$\sum_{j=1}^{12}\widetilde{Y_j^M} = 12\ln\overline{Y}^M - 12\ln 12.$$

Hence, we can follow the above procedure, except we replace the left hand size of (B5) with $\widetilde{Y^M} = 12\ln\overline{Y}^M - 12\ln 12$ and the right hand size with $\sum_{j=1}^{12}\ln Z_j$.

This approximation should come close to satisfying the temporal aggregation constraints, but will fail to do so exactly. Hence, the final step is to adjust the estimates following Denton (1971). Denoting the initial fitted values as $\widehat{Y^M}$ and the residuals $\overline{Y}^M - \sum_{t=1}^{12}\widehat{Y_t^M} = R$ (in vector form), we make the final adjustment:

$$\widehat{Y^{M*}} = \widehat{Y^M} + (D'D)^{-1}B'(B(D'D)^{-1}B')^{-1}R.$$

To check the quality of the procedure, we compared the monthly fitted series produced using this algorithm and the actual monthly data released by the U.S. Census Bureau on the value of shipments in durable and non-durable manufacturing. The U.S. monthly data are collected as part of the M3 manufacturing

survey.[25] In all other parts of the paper, our monthly series sum to the annual production totals found in the UN and OECD data, but for this test of the algorithm we re-run the procedure using annual totals from the M3 survey. Though M3 data are available through 2009, we only use annual totals for 1995-2007 to ensure the procedure uses the same amount of data as we would have for other countries in our sample. We test both a procedure which estimates the beta coefficients from (B2) for the relationship between annual production and the monthly indicators as well as one in which we set the coefficients in the relationship equal to one for all countries and sectors.

Appendix Figure B1 demonstrates that both procedures do an excellent job of matching movements in the time series for non-durables, including the out-of-sample decline in production during the recent recession. Our "Beta equals 1" procedure does an excellent job both in-sample and out-of sample. The procedure with estimated coefficients underestimates the decline in production of durables during the recent recession. Given this, the results in the text reflect the results of this procedure when the beta coefficients from (B2) for the relationship between production and IP/PPI are automatically set equal to one, implying that the product of IP and PPI will generally move perfectly with nominal production. We have checked all our results with the endogenous estimation of these beta coefficients and none of the global results changes meaningfully.

When applied at the sector level, this procedure generates monthly series of these disaggregated categories from which we obtain a monthly series of the share of durables in manufacturing. Given the highest quality production data from these databases are for the total manufacturing sector, we then multiply these shares by total manufacturing production, which is interpolated in exactly the same way but with IP/PPI indices for the whole of manufacturing. We then have monthly series for durable and non-durable manufacturing production which are consistent with published annual (and implied monthly) levels of total manufacturing production.

Calculating Head-Ries Indices During the Great Depression

We obtained data on bilateral and multilateral manufacturing trade as well as exchange rates for 1926-1937 from the annual *Foreign Commerce Yearbooks*, published by the U.S. Department of Commerce. Total U.S. multilateral manufacturing imports and exports were taken from Carter et al. (2006). The gross value of manufacturing, required for the denominator of (15), were obtained from a variety of country-specific sources.[26] The U.S. ratio of gross output to value added in manufacturing, found in Carter (2006), was applied to foreign manufacturing value added when output data were unavailable.

The bilateral trade and the manufacturing totals often reflect changing availability of data for disaggregated categories. For example, one year's total growth may reflect both 20% growth in Paper Products as well as the initial measurement (relative to previous missing values) of Transportation Equipment. Since inspection suggests that such missing values do not simply reflect zero values, we calculate year-to-year growth rates using only the common set of recorded goods. For manufacturing production, we not only need the growth rate, but the level also matters because we subtract the level of exports to measure absorption. We apply the growth rate backwards from the most complete, typically also the most recent, series value.

[25] The monthly totals are extrapolated from a sampling procedure that covers a majority of manufacturers with $500 million or more in annual shipments as well as selected smaller companies in certain industries. See http://www.census.gov/indicator/www/m3/m3desc.pdf for additional details.

[26] Where needed, U.S. Department of Commerce (1968) was used to convert currency or physical units into U.S. dollars. Austria: Bundesamt fur Statistik (1927-1936) was used to obtain product-specific production data, either in hundreds of Austrian schilling or in kilograms. Canada: Value of manufacturing data were available in U.S. dollars from Urquhart (1983). Germany: Data were obtained from Statistishen Reichsamt (1931, 1935, 1940). Finland, Japan, Spain, and Sweden: Value added in manufacturing, in local currency units, were taken from Smits (2009). Peru: Output data in Peruvian pounds and soles obtained from Ministerio de Hacienda y Comercio (1939). United Kingdom: Data were obtained from United Kingdom Board of Trade (1938). These annual numbers combined less frequent results from the censuses in 1924, 1930, and 1935, with industrial production data, taken yearly, from 1927-1937.

Appendix C: Trade Frictions and Productivity

In this appendix, we describe how changes in trade frictions and changes in productivity are intimately connected. We can bring out this connection, and get some insights into the logic of the model, by combining trade friction shocks and productivity shocks in the term:

$$\hat{\delta}_{ni}^{j} = \frac{\hat{\Phi}_{n}^{j}}{\hat{\Phi}_{i}^{j}} \hat{d}_{ni}^{j}, \tag{C1}$$

where the $\hat{\Phi}_{i}^{j}$ represent productivity changes through:

$$\hat{A}_{i}^{j} = \left(\hat{\Phi}_{i}^{j}\right)^{1-\widetilde{\gamma}_{i}^{jj}(1-\widetilde{\beta}_{i}^{j})} \left(\hat{\Phi}_{i}^{l}\right)^{-\widetilde{\gamma}_{i}^{jl}(1-\widetilde{\beta}_{i}^{j})}. \tag{C2}$$

In addition, we define a productivity-adjusted price change by

$$\hat{q}_{i}^{j} = \hat{p}_{i}^{j} \hat{\Phi}_{i}^{j}. \tag{C3}$$

Using this reparameterization, (17) and (18) remain unchanged while (19) becomes:

$$\hat{q}_{n}^{j} = \left(\sum_{i=1}^{I} \pi_{ni}^{j} \hat{w}_{i}^{-\theta^{j}\widetilde{\beta}_{i}^{j}} \left(\hat{q}_{i}^{j}\right)^{-\theta^{j}\widetilde{\gamma}_{i}^{jj}(1-\widetilde{\beta}_{i}^{j})} \left(\hat{q}_{i}^{l}\right)^{-\theta^{j}\widetilde{\gamma}_{i}^{jl}(1-\widetilde{\beta}_{i}^{j})} \left(\hat{\delta}_{ni}^{j}\right)^{-\theta^{j}} \right)^{-1/\theta^{j}}, \tag{C4}$$

and (20) becomes:

$$\left(\pi_{ni}^{j}\right)' = \pi_{ni}^{j} \hat{w}_{i}^{-\theta^{j}\widetilde{\beta}_{i}^{j}} \left(\hat{q}_{i}^{j}\right)^{-\theta^{j}\widetilde{\gamma}_{i}^{jj}(1-\widetilde{\beta}_{i}^{j})} \left(\hat{q}_{i}^{l}\right)^{-\theta^{j}\widetilde{\gamma}_{i}^{jl}(1-\widetilde{\beta}_{i}^{j})} \left(\frac{\hat{\delta}_{ni}^{j}}{\hat{q}_{n}^{j}}\right)^{-\theta^{j}}. \tag{C5}$$

Note that productivity changes do not enter directly into (C4) or (C5) as they are embedded in the $\hat{\delta}_{ni}^{j}$ and $\hat{q}_{i}^{j}$.

The solution to (17), (18), (C4), and (C5) is also the solution to (17), (18), (19), and (20). To see why, substitute (C1) into (C4) to get:

$$\hat{q}_{n}^{j} = \left(\sum_{i=1}^{I} \pi_{ni}^{j} \hat{w}_{i}^{-\theta^{j}\widetilde{\beta}_{i}^{j}} \left(\hat{q}_{i}^{j}\right)^{-\theta^{j}\widetilde{\gamma}_{i}^{jj}(1-\widetilde{\beta}_{i}^{j})} \left(\hat{q}_{i}^{l}\right)^{-\theta^{j}\widetilde{\gamma}_{i}^{jl}(1-\widetilde{\beta}_{i}^{j})} \left(\frac{\hat{\Phi}_{n}^{j}}{\hat{\Phi}_{i}^{j}} \hat{d}_{ni}^{j}\right)^{-\theta^{j}} \right)^{-1/\theta^{j}}.$$

Grouping terms, the left hand side becomes $\hat{q}_{n}^{j}/\hat{\Phi}_{n}^{j}$ and the price terms on the right hand side become $\hat{q}_{i}^{j}/\hat{\Phi}_{i}^{j}$ and $\hat{q}_{i}^{l}/\hat{\Phi}_{i}^{l}$, leaving a term on the right hand side equal to $\left(\hat{\Phi}_{i}^{j}\right)^{\theta^{j}[1-\widetilde{\gamma}_{i}^{jj}(1-\widetilde{\beta}_{i}^{j})]} \left(\hat{\Phi}_{i}^{l}\right)^{-\theta^{j}\widetilde{\gamma}_{i}^{jl}(1-\widetilde{\beta}_{i}^{j})}$. This expression then replicates (19) after substituting in (C3) and (C2). Similarly, substituting (C1) into (C5), applying (C3) and (C2), yields (20).

One implication of this result is that productivity shocks of the form

$$\hat{A}_{i}^{j} = \lambda^{\widetilde{\beta}_{i}^{j}},$$

for any $\lambda > 0$, leave equilibrium wages, spending, and trade shares unaffected. The resulting price changes are $\hat{p}_{i}^{j} = 1/\lambda$ for $j \in \Omega_{M}$. Furthermore, changes in service-sector productivity do not change equilibrium outcomes given the $\hat{A}_{i}^{j}$. Such changes would have welfare consequences, but are irrelevant to the equilibrium considered here.

Another implication of this result is that to solve the model for changes in wages and trade shares, all we need is $\hat{\delta}_{ni}^{j}$ rather than $\hat{d}_{ni}^{j}$ and $\hat{A}_{i}^{j}$ separately. We can decompose the contribution of trade friction and productivity shocks using additional restrictions or data, as we do above with data on sectoral price changes.

If we do not wish to impose further restrictions, we can calibrateion the $\hat{\delta}_{ni}^{j}$ directly. Start by dividing

both sides of equation (C5) by π_{ni}^j to get an expression for $\hat{\pi}_{ni}^j$. Dividing by the corresponding expression for $\hat{\pi}_{ii}^j$ gives:

$$\left(\hat{\delta}_{ni}^j\right)^{-\theta^j} = \frac{\hat{\pi}_{ni}^j}{\hat{\pi}_{ii}^j}\left(\frac{\hat{q}_i^j}{\hat{q}_n^j}\right)^{\theta^j}. \tag{C6}$$

We can then use (C5) (for $n = i$) and (C4) to get:

$$\hat{\pi}_{ii}^j = \widehat{w}_i^{-\theta^j\widetilde{\beta}_i^j}\left(\hat{q}_i^j\right)^{\theta^j\left[1-\widetilde{\gamma}_i^{jj}(1-\widetilde{\beta}_i^j)\right]}\left(\hat{q}_i^l\right)^{-\theta^j\widetilde{\gamma}_i^{jl}(1-\widetilde{\beta}_i^j)}, \tag{C7}$$

where $l \neq j$ is the other manufacturing sector. Combining these equations for the two manufacturing sectors, and rearranging yields:

$$\left(\hat{q}_i^j\right)^{\theta^j} = \left(\hat{\pi}_{ii}^j\hat{w}_i^{\theta^j\widetilde{\beta}_i^j}\right)^{\frac{1-\widetilde{\gamma}_i^{ll}(1-\widetilde{\beta}_i^l)}{\Delta_i}}\left(\hat{\pi}_{ii}^l\hat{w}_i^{\theta^l\widetilde{\beta}_i^l}\right)^{\frac{\theta^j\widetilde{\gamma}_i^{jl}(1-\widetilde{\beta}_i^j)}{\theta^l\Delta_i}},$$

where

$$\Delta_i = \prod_{l\in\Omega_M}\left(1-\widetilde{\gamma}_i^{ll}(1-\widetilde{\beta}_i^l)\right) - \prod_{l,j\in\Omega_M, l\neq j}\widetilde{\gamma}_i^{lj}(1-\widetilde{\beta}_i^l).$$

These expressions for price changes can be plugged into (C6) to get:

$$\left(\hat{\delta}_{ni}^j\right)^{-\theta^j} = \frac{\hat{\pi}_{ni}^j}{\hat{\pi}_{ii}^j}\frac{\left(\hat{\pi}_{ii}^j\hat{w}_i^{\theta^j\widetilde{\beta}_i^j}\right)^{\frac{1-\widetilde{\gamma}_i^{ll}(1-\widetilde{\beta}_i^l)}{\Delta_i}}\left(\hat{\pi}_{ii}^l\hat{w}_i^{\theta^l\widetilde{\beta}_i^l}\right)^{\frac{\theta^j\widetilde{\gamma}_i^{jl}(1-\widetilde{\beta}_i^j)}{\theta^l\Delta_i}}}{\left(\hat{\pi}_{nn}^j\hat{w}_n^{\theta^j\widetilde{\beta}_n^j}\right)^{\frac{1-\widetilde{\gamma}_n^{ll}(1-\widetilde{\beta}_n^l)}{\Delta_n}}\left(\hat{\pi}_{nn}^l\hat{w}_n^{\theta^l\widetilde{\beta}_n^l}\right)^{\frac{\theta^j\widetilde{\gamma}_n^{jl}(1-\widetilde{\beta}_n^j)}{\theta^l\Delta_n}}}. \tag{C8}$$

This way of calibrating the model is consistent with how we proceed in the paper, except that it does not allow for calculation of the contribution of productivity shocks separate from trade-friction shocks.

One method of separating the contribution of productivity and trade-friction shocks is to use price data. This is our approach in the main text. We now consider a different method the separates the contribution of productivity and trade-friction shocks by imposing symmetry on the two changes in trade-frictions between any given pair of countries. Again, this will change the relative contributions of the productivity and trade friction shocks, but will not in any way impact their joint contribution or the contributions from the other shocks. Since the Head-Ries index is $\Theta_{ni}^j = \left[d_{ni}^j d_{in}^j\right]^{-\theta^j/2}$, imposing $d_{ni}^j = d_{in}^j$ implies, in changes, $\hat{\Theta}_{ni}^j = \left(\hat{d}_{ni}^j\right)^{-\theta^j} = \left(\hat{d}_{in}^j\right)^{-\theta^j}$. Combining with (C1), and allowing for deviations μ_{ni}^j around symmetry, we get:

$$\frac{\hat{\Theta}_{ni}^j}{\left(\hat{\delta}_{ni}^j\right)^{-\theta^j}} = \left(\frac{\hat{\Phi}_n^j}{\hat{\Phi}_i^j}\right)^{\theta^j} e^{\mu_{ni}^j}.$$

Taking logs gives an estimating equation:

$$\ln\hat{\Theta}_{ni}^j + \theta^j\ln\delta_{ni}^j = \theta^j\ln(\hat{\Phi}_n^j) - \theta^j\ln(\hat{\Phi}_i^j) + \mu_{ni}^j. \tag{C9}$$

The left-hand side can be calculated from our data (employing (15) and (C8)), while for the right-hand side we estimate the coefficients on a set of N dummy variables, one for each country. For each (n, i) observation, there are two non-zero dummy values. The first, corresponding to country n, takes a value of $(+1)$, while the second, corresponding to country i, takes a value of (-1). We estimate $\hat{\Phi}_i^j$ by dividing the corresponding coefficients (on the dummy variables for country i) by θ^j (for each sector j) and exponentiating the result. We drop "Rest of World" since a common scalar won't change anything. Finally, to recover changes in sectoral productivity, we substitute these estimates into (C2).

Appendix D: Solving for the Equilibrium

In this appendix, we explain in more detail how we solve for the system's equilibrium. Given a vector of wage changes $\widehat{\boldsymbol{w}}$, we solve (19) and (20) jointly for changes in trade shares and prices. Denote the solution for changes in trade shares by $\pi_{ni}^{j}(\widehat{w}) = \left(\pi_{ni}^{j}\right)'$.

Second, we can substitute the service sector out of equation (3) to get

$$\begin{bmatrix} \left(X_i^D\right)' \\ \left(X_i^N\right)' \end{bmatrix} = \widetilde{\boldsymbol{\alpha}}_i' \left(Y_i' + D_i'\right) - \boldsymbol{\delta}_i \left(D_i^S\right)' + \widetilde{\boldsymbol{\Gamma}}_i^T \begin{bmatrix} \left(Y_i^D\right)' \\ \left(Y_i^N\right)' \end{bmatrix}, \tag{D1}$$

where the 2 by 1 vector $\widetilde{\boldsymbol{\alpha}}_i$ has elements

$$\left(\widetilde{\alpha}_i^j\right)' = \left(\alpha_i^j\right)' + \left(\alpha_i^S\right)' \delta_i^j,$$

the 2 by 1 vector $\boldsymbol{\delta}_i$ has elements

$$\delta_i^j = \frac{\gamma_i^{Sj}(1-\beta_i^S)}{1-\gamma_i^{SS}(1-\beta_i^S)},$$

and the 2 by 2 matrix $\widetilde{\boldsymbol{\Gamma}}_i$ contains

$$\widetilde{\gamma}_i^{jl}(1-\widetilde{\beta}_i^j)$$

in its j'th row and l'th column for all $j, l \in \Omega_M$.

Third, we follow the approach of Caliendo and Parro (2009) and substitute (21) into the right hand side of (D1). Given wage changes, we obtain a linear system in the $\left(X_i^j\right)'$'s by stacking (D1) across all countries:

$$\mathbb{X}' = \left(\widetilde{\boldsymbol{\alpha}}\boldsymbol{X}\right)' - \left(\delta\mathbb{D}^S\right)' + \widetilde{\boldsymbol{\Gamma}}^T \left[\boldsymbol{\Pi}(\widehat{w})\right]^T \mathbb{X}'.$$

Here

$$\mathbb{X}' = \left[\left(X_1^D\right)', \left(X_2^D\right)', ..., \left(X_I^D\right)', \left(X_1^N\right)', \left(X_2^N\right)', ..., \left(X_I^N\right)'\right]^T,$$

$$\left(\widetilde{\boldsymbol{\alpha}}\boldsymbol{X}\right)' = \left[\left(\widetilde{\alpha}_1^D X_1\right)', \left(\widetilde{\alpha}_2^D X_2\right)', ..., \left(\widetilde{\alpha}_I^D X_I\right)', \left(\widetilde{\alpha}_1^N X_1\right)', \left(\widetilde{\alpha}_2^N X_2\right)', ..., \left(\widetilde{\alpha}_I^N X_I\right)'\right]^T,$$

with

$$\left(\widetilde{\alpha}_i^j X_i\right)' = \left(\widetilde{\alpha}_i^j\right)' \left(\hat{w}_i Y_i + D_i'\right),$$

$$\left(\delta\mathbb{D}^S\right)' = \left[\delta_1^D \left(D_1^S\right)', \delta_2^D \left(D_2^S\right)', ..., \delta_I^D \left(D_I^S\right)', \delta_1^N \left(D_1^S\right)', \delta_2^N \left(D_2^S\right)', ..., \delta_I^N \left(D_I^S\right)'\right]^T,$$

$$\widetilde{\boldsymbol{\Gamma}} = \begin{bmatrix} \widetilde{\gamma}_1^{DD}(1-\widetilde{\beta}_1^D) & 0 & 0 & \widetilde{\gamma}_1^{DN}(1-\widetilde{\beta}_1^D) & 0 & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \\ 0 & 0 & \widetilde{\gamma}_I^{DD}(1-\widetilde{\beta}_I^D) & 0 & 0 & \widetilde{\gamma}_I^{DN}(1-\widetilde{\beta}_I^D) \\ \widetilde{\gamma}_1^{ND}(1-\widetilde{\beta}_1^N) & 0 & 0 & \widetilde{\gamma}_1^{NN}(1-\widetilde{\beta}_1^N) & 0 & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \\ 0 & 0 & \widetilde{\gamma}_I^{ND}(1-\widetilde{\beta}_I^N) & 0 & 0 & \widetilde{\gamma}_I^{NN}(1-\widetilde{\beta}_I^N) \end{bmatrix},$$

and

$$\boldsymbol{\Pi}(\widehat{w}) = \begin{bmatrix} \Pi^D(\widehat{w}) & 0 \\ 0 & \Pi^N(\widehat{w}) \end{bmatrix},$$

where $\left(\Pi^j\right)'(\widehat{w})$ has $\pi^j_{ni}(\widehat{w})$ in its n'th row and i'th column. We can denote the solution by

$$\mathbb{X}(\widehat{w}) = \left[\boldsymbol{I} - \widetilde{\boldsymbol{\Gamma}}^T \left[\boldsymbol{\Pi}(\widehat{w})\right]^T\right]^{-1} \left[(\widetilde{\boldsymbol{\alpha}}\boldsymbol{X})' - \left(\delta \mathbb{D}^S\right)'\right],$$

where the elements of $\mathbb{X}(\widehat{w})$ are $X^j_i(\widehat{w}) = \left(X^j_i\right)'$.

Finally, summing up (21) over $j \in \Omega_M$ yields

$$X^D_i(\widehat{w}) + X^N_i(\widehat{w}) - \left(D'_i - \left(D^S_i\right)'\right) = \sum_{n=1}^{I} \pi^D_{ni}(\widehat{w}) X^D_n(\widehat{w}) + \sum_{n=1}^{I} \pi^N_{ni}(\widehat{w}) X^N_n(\widehat{w}). \tag{D2}$$

This non-linear system of equations can be solved for the $I-1$ changes in wages.