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What fiscal policy is effective at zero interest rates?[1]

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Abstract

Tax cuts can deepen a recession if the short-term nominal interest rate is zero, according to a standard New Keynesian business cycle model. An example of a contractionary tax cut is a reduction in taxes on wages. This tax cut deepens a recession because it increases deflationary pressures. Another example is a cut in capital taxes. This tax cut deepens a recession because it encourages people to save instead of spend when more spending is needed. Fiscal policies aimed directly at stimulating aggregate demand work better. These policies include 1) a temporary increase in government spending; 2) temporary tax cuts directly aimed at stimulating aggregate demand rather than aggregate supply, such as an investment tax credit or a cut in sales taxes. The results are special to an environment in which the interest rate is close to zero, as observed in large parts of the world today.

Key words: tax and spending multipliers, zero interest rates, deflation
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Table 1

	Labor taxcut multiplier	**Capital taxcut multiplier**	**Gov. sp. multiplier**
Pos. interest rate	0.16	0.0	0.48
Zero interest rate	-1	-0.1	2.3

1 Introduction

The economic crisis of 2008 started one of the most heated debates about U.S. fiscal policy in the past half a century. With the federal funds rate close to zero – and output, inflation, and employment at the edge of a collapse – U.S. based economists argued over alternatives to interest rate cuts to spur a recovery. Meanwhile, several other central banks slashed interest rates close to zero, including the European Central Bank, the Bank of Japan, the Bank of Canada, the Bank of England, the Riksbank of Sweden, and the Swiss National Bank, igniting similar debates in all corners of the world. Some argued for tax cuts, mainly a reduction in taxes on labor income (see, e.g., Hall and Woodward (2008), Bils and Klenow (2008), and Mankiw (2008)) or tax cuts on capital (see, e.g., Feldstein (2009) and Barro (2009)). Others emphasized an increase in government spending (see, e.g., Krugman (2009) and De Long (2008)). Yet another group of economists argued that the best response would be to reduce the government, i.e., reduce *both* taxes *and* spending.[2] Even if there was no professional consensus about the correct fiscal policy, the recovery bill passed by Congress in 2009 marks the largest fiscal expansion in U.S. economic history since the New Deal, with projected deficits (as a fraction of GDP) in double digits. Many governments followed the U.S. example. Much of this debate was, explicitly or implicitly, within the context of old-fashioned Keynesian models or the frictionless neoclassical growth model.

This paper takes a standard New Keynesian dynamic stochastic general equilibrium (DSGE) model, which by now is widely used in the academic literature and utilized in policy institutions, and asks a basic question: What is the effect of tax cuts and government spending under the economic circumstances that characterized the crisis of 2008? A key assumption is that the model is subject to shocks so that the short-term nominal interest rate is zero. This means that, in the absence of policy interventions, the economy experiences excess deflation and an output contraction. The analysis thus builds on a large recent literature on policy at the zero bound on the short-term nominal interest rates, which is briefly surveyed at the end of the introduction. The results are perhaps somewhat surprising in the light of recent public discussion. Cutting taxes on labor or capital is *contractionary* under the special circumstances the U.S. is experiencing today. Meanwhile, the effect of temporarily increasing government spending is large, much larger than under normal circumstances. Similarly, some other forms of tax cuts, such as a reduction in sales taxes and investment tax credits, as suggested for example by Feldstein (2002) in the context of Japan's "Great Recession," are extremely effective.[3]

[2] This group consisted of 200 leading economists, including several Nobel Prize winners, who signed a letter prepared by the Cato Institute.

[3] For an early proposal for temporary sales tax cuts as an effective stabilization tool, see for example Modigliani

The contractionary effects of labor and capital tax cuts, and the strong expansionary effect of government spending, are special to the peculiar environment created by zero interest rates. This point is illustrated by a numerical example in Table 1. It shows the "multipliers" of cuts in labor taxes, capital taxes and of increasing government spending; several other multipliers are also discussed in the paper. The multipliers summarize by how much output decreases/increases if the government cuts tax rates by 1 percent or increases government spending by 1 percent (as a fraction of GDP). At positive interest rates, a labor tax cut is expansionary, as the literature has emphasized in the past. But at zero interest rates, it flips signs and tax cuts become contractionary. Similarly while capital tax cuts are almost irrelevant in the model at positive interest rate (up to second decimal point) they become strongly negative at zero. Meanwhile, the multiplier of government spending not only stays positive at zero interest rates, but becomes almost five times larger. This illustrates that empirical work on the effect of fiscal policy based on data from the post-WWII period, such as the much cited and important work of Romer and Romer (2008), may not be directly applicable for assessing the effect of fiscal policy on output today. Interest rates are always positive in their sample, as in most other empirical research on this topic. To infer the effects of fiscal policy at zero interest rates, then, we can rely on experience only to a limited extent . Reasonably grounded theory may be a better benchmark with all the obvious weaknesses such inference entails, since the inference will never be any more reliable than the model assumed.

The starting point of this paper is the negative effect of labor income tax cuts, i.e., a cut in the tax on wages. These tax cuts cause deflationary pressures in the model by reducing marginal costs of firms, thereby increasing the real interest rate. The Federal Reserve can't accommodate this by cutting the federal funds rate, since it is already close to zero. Higher real interest rates are contractionary. I use labor tax cuts as a starting point, not only because of their prominence in the policy discussion but to highlight a general principle for policy in this class of models. *The principal goal of policy at zero interest rates should not be to increase aggregate supply by manipulating aggregate supply incentives. Instead, the goal of policy should be to increase aggregate demand – the overall level of spending in the economy.* This diagnosis is fundamental for a successful economic stimulus once interest rates hit zero in the model. At zero interest rates, output is demand-determined. Accordingly, aggregate supply is mostly relevant in the model because it pins down expectations about future inflation. The result derived here is that policies aimed at increasing aggregate supply are counterproductive because they can create deflationary expectations at zero interest rates. At a loose and intuitive level, therefore, policy should not be aimed at increasing the supply of goods when the problem is that there are not enough buyers.

Once the general principle is established, it is straightforward to consider a host of other fiscal policy instruments, whose effect at first blush may seem puzzling. Consider first the idea of cutting taxes on capital, another popular policy proposal in response to the crisis of 2008. A permanent reduction in capital taxes increases investment and the capital stock under normal circumstances, which increases the production capacities of the economy. More shovels and trac-

and Steindel (1977).

tors, for example, mean that people can dig more and bigger holes, which increases steady-state output. But at zero interest rate, the problem is not that the production capacity of the economy is inadequate. Instead, the problem is insufficient aggregate spending. Cutting capital taxes gives people the incentive to save instead of spend, when precisely the opposite is needed. A cut in capital taxes will reduce output because it reduces consumption spending. One might think that the increase in people's incentive to save would in turn increase aggregate savings and investment. But everyone starts saving more, which leads to lower demand, which in turns leads to lower income for households, thus reducing their *ability* to save. Paradoxically, a consequence of cutting capital taxes is therefore a collapse in *aggregate saving* in general equilibrium because everyone tries to save more!

From the same general principle – that the problem of insufficient demand leads to below-capacity production – it is easy to point out some effective tax cuts and spending programs, and the list of examples provided in the paper is surely not exhaustive. Temporarily cutting sales taxes and implementing an investment tax credit are both examples of effective fiscal policy. These tax cuts are helpful not because of their effect on aggregate supply, but because they directly stimulate aggregate spending. Similarly, a temporary increase in government spending is effective because it directly increases overall spending in the economy. For government spending to be effective in increasing demand, however, it has to be directed at goods that are imperfect substitutes with private consumption (such as infrastructure or military spending). Otherwise, government spending will be offset by cuts in private spending, leaving aggregate spending unchanged.

A natural proposal for a stimulus plan, at least in the context of the model, is therefore a combination of temporary government spending increases, temporary investment tax credits, and a temporary elimination of sales taxes, all of which can be financed by a temporary increase in labor and/or capital taxes. There may, however, be important reasons outside the model that suggest that an increase in labor and capital taxes may be unwise and/or impractical. For these reasons I am not ready to suggest, based on this analysis alone, that raising capital and labor taxes is a good idea at zero interest rates. Indeed, my conjecture is that a reasonable case can be made for a temporary budget deficit to finance a stimulus plan as further discussed in the paper and the footnote.[4]

The contractionary effect of labor and capital taxes are applications of two paradoxes, one

[4]The contractionary labor tax cuts studied, although entirely standard in the literature, are very special in many respects. They correspond to variations in linear tax rates on labor income, while some tax cuts on labor income in practice resemble more lump-sum transfers to workers and may even, in some cases, imply an effective *increase* in marginal taxes (Cochrane (2008)). Similarly, this form of taxes does not take into account the "direct" spending effect tax cuts have in some old-fashioned Keynesian models and as modeled more recently in a New Keynesian model by Gali, Lopez–Salido, and Valles (2007). A similar comment applies to taxes on capital. There could be a "direct" negative demand effect of increasing this tax through households' budget constraints. Another problem is that an increase in taxes on capital would lead to a decline in stock prices. An important channel not being modeled is that a reduction in equity prices can have a negative effect on the ability of firms to borrow, through collateral constraints as in Kiyotaki and Moore (1995), and thus contract investment spending. This channel is not included in the model and is one of the main mechanisms emphasized by Feldstein (2009) in favor of reducing taxes on capital.

new and one old. The result on the contractionary effect of capital tax cuts is an application of the old Keynesian *paradox of thrift* (see, e.g., Christiano (2004) for discussion of this paradox)[5]. The contractionary effect of labor tax cuts, however, is a new paradox illustrated in Eggertsson (2010) who coins it the *paradox of toil*. While the paradox of thrift is that if everybody tries to save, there will be less savings in the aggregate, the paradox of toil is that if everybody tries to work more, there will be less work in the aggregate once the nominal interest rate hits zero.

The main focus of the paper is temporary tax and spending policies in response to an economic crisis. The results can be very different if instead we assume a permanent change in policy. I illustrate this by some numerical examples. While a temporary increase in government spending, for example, can be very effective at zero interest rate, an expansion in the government can instead even be *contractionary* if it is expected to be permanent under some specifications of policy. This explains the different of the results reported in Table 1 from some recent studies on the effect of government spending in DSGE models as further discussed below.

2 Relation to other literature

The first paper to study the effect of government spending at zero interest rate in a New Keynesian DSGE model is Eggertsson (2001). That paper characterize the optimal policy under commitment and discretion, where the government has as policy instruments the short-term nominal interest rate and real government spending and assumes taxes are lump sum. Relative to that paper, this paper studies much more general menu of fiscal instruments, such as the effect various distortionary taxes, and gives more attention to the quantitative effect of fiscal policy. Moreover, the current paper does not take a direct stance on the optimality of fiscal policy but instead focuses on "policy multipliers", i.e. the effect of policy at the margin as in Christiano (2004). This allows me to obtain clean closed form solutions and illuminate the general forces at work.

This paper also builds upon a large literature on optimal monetary policy at the zero bound, such as Summers (1991), Fuhrer and Madigan (1997), Krugman (1998), Reifschneider and Williams (2000), Svensson (2001, 2003), Eggertsson and Woodford (2003 and 2004), Christiano (2004), Wolman (2005), Eggertsson (2006a), Adam and Billi (2006), and Jung et al. (2005).[6] The analysis of the variations in labor taxes builds on Eggertsson and Woodford (2004), who study value added taxes (VAT) that show up in a similar manner. One difference is that while they focus mostly on commitment equilibrium (in which fiscal policy plays a small role because optimal monetary commitment does away with most of the problems) the assumption here is that the central bank

[5] The connection to the paradox of thrift was first pointed out to me by Larry Christiano in an insightful dicussion of Eggertsson and Woodford (2003). See Christiano (2004). Krugman (1998) also draws a comparison to the paradox of thrift in a similar context.

[6] This list is not nearly complete. See Svensson (2003) for an excellent survey of this work. All these papers treat the problem of the zero bound as a consequence of real shocks that make the interest rate bound binding. Another branch of the literature has studied the conseqence of binding zero bound in the context of self-fulfilling expectations. See, e.g., Benhabib, Schmitt-Grohe, and Uribe (2002), who considered fiscal rules that eliminate those equilibria.

is unable to commit to future inflation, an extreme assumption, but an useful benchmark. This assumption can also be defended because the optimal monetary policy suffers from a commitment problem, while fiscal policy does not to the same extent, as first formalized in Eggertsson (2001).[7] The contractionary effect of cutting payroll taxes is closely related to Eggertsson (2008b), who studies the expansionary effect of the National Industrial Recovery Act (NIRA) during the Great Depression. In reduced form, the NIRA is equivalent to an increase in labor taxes in this model. The analysis of real government spending also builds on Eggertsson (2004, 2006b) and Christiano (2004), who find that increasing real government spending is very effective at zero interest rates if the monetary authority cannot commit to future inflation and Eggertsson (2008a), who argues based on those insights that the increase in real government spending during the Great Depression contributed more to the recovery than is often suggested.[8] Christiano, Eichenbaum and Rebelo (2009, CCR), building on Christiano (2004), calculate the size of the multiplier of government spending in a much more sophisticated empirically estimated model than previous studies, taking the zero bound explicitly into account, and find relatively similar quantitative conclusions as reported here, see Denes and Eggertsson (2009) for further discussion (that paper describes the estimation strategy I follow in this paper and compares it to other recent work in the field). CCR also study various other issues related to the government spending multiplier, such as timing, sensitivity, and other shocks, not addressed directly here. Cogan, Cwik, Taylor, and Wieland (2009) study the effect on increasing government spending in a DSGE model which is very similar to the CCR model and report small multipliers. The reason for the different finding is that they assume that the increase in spending is *permanent*, while this paper and CCR assume that the fiscal spending is a *temporary* stimulus in response to temporary deflationary shocks. This is made explicit in the analysis in section 12 that shows that an increase in government spending which is expected to be permanent can even be contractionary.

3 A Microfounded Model

This section summarizes a standard New Keynesian DSGE model.[9] (Impatient readers can skip directly to the next section.) At its core, this is a standard stochastic growth model (real business cycle model) but with two added frictions: a monopolistic competition among firms, and frictions in the firms' price setting through fixed nominal contracts that have a stochastic duration as in

[7] Committing to future inflation may not be so trivial in practice. As shown by Eggertsson (2001,2006a), the central bank has an incentive to promise future inflation and then renege on this promise; this is the deflation bias of discretionary policy. In any event, optimal monetary policy is relatively well known in the literature, and it is of most interest in order to understand the properties of fiscal policy in the "worst case" scenario if monetary authorities are unable and/or unwilling to inflate.

[8] Other papers that studied the importance of real government spending and found a substantial fiscal policy multiplier effect at zero interest rate include Williams (2006). That paper assumes that expectations are formed according to learning, which provides a large role for fiscal policy.

[9] See, .e.g., Clarida, Gali, and Gertler (1999), Benigno and Woodford (2003), Smets and Wouters (2007), and Christiano, Eichenbaum, and Evans (2005). Several details are omitted here, but see, e.g., Woodford (2003) for a textbook treatment.

Calvo (1983). Relative to standard treatments, this model has a more detailed description of taxes and government spending. This section summarizes a simplified version of the model that will serve as the baseline illustration. The baseline model abstracts from capital, but Section 14 extends the model to include it.

There is a continuum of households of measure 1. The representative household maximizes

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} \xi_T \left[u(C_T + G_T^S) + g(G_T^N) - \int_0^1 v(l_T(j)) dj \right], \tag{1}$$

where β is a discount factor, C_t is a Dixit-Stiglitz aggregate of consumption of each of a continuum of differentiated goods, $C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$, with an elasticity of substitution equal to $\theta > 1$, P_t is the Dixit-Stiglitz price index, $P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}$, and $l_t(j)$ is the quantity supplied of labor of type j. Each industry j employs an industry-specific type of labor, with its own wage $W_t(j)$. The disturbance ξ_t is a preference shock, and $u(.)$ and $g(.)$ are increasing concave functions while $v(.)$ is an increasing convex function. G_T^S and G_T^N are government spending that differ only in how they enter utility and are also defined as Dixit-Stiglitz aggregates analogous to private consumption. G_t^S is perfectly substitutable for private consumption, while G_t^N is not. For simplicity, we assume that the only assets traded are one-period riskless bonds, B_t. The period budget constraint can then be written as

$$\begin{aligned} & (1+\tau_t^s) P_t C_t + B_t \\ = \; & (1-\tau_{t-1}^A)(1+i_{t-1}) B_{t-1} + (1-\tau_t^P) \int_0^1 Z_t(i) di + (1-\tau_t^w) \int_0^1 W_t(j) l_t(j) dj - T_t, \end{aligned} \tag{2}$$

where $Z_t(i)$ is profits that are distributed lump sum to the households. I do not model optimal stock holdings (i.e., the optimal portfolio allocation) of the households, which could be done without changing the results.[10] There are five types of taxes in the baseline model: a sales tax τ_t^s on consumption purchases, a payroll tax τ_t^w, a tax on financial assets τ_t^A, a tax on profits τ_t^p, and finally a lump-sum tax T_t, all represented in the budget constraint. Observe that I allow for different tax treatments of the risk-free bond returns and dividend payments, while in principle we could write the model so that these two underlying assets are taxed in the same way. I do this to clarify the role of taxes on capital. The profit tax has no effect on the household consumption/saving decision (it would only change how stocks are priced in a more complete description of the model) while taxes on the risk-free debt have a direct effect on households' saving and consumption decisions. This distinction is helpful to analyze the effect of capital taxes on households' spending and savings (τ_t^A) on the one hand, and the firms' investment, hiring, and pricing decisions on the other (τ_t^P), because we assume that the firms maximize profits net of taxes. Households take prices and wages as given and maximize utility subject to the budget constraint by their choices of $c_t(i)$, $l_t(j), B_t$ and $Z_t(i)$ for all j and i at all times t.

[10] It would simply add asset-pricing equations to the model that would pin down stock prices.

The household optimal plan needs to satisfy the following first order conditions

$$u_c(C_t + G_t^S) = (1 + i_t)(1 - \tau_t^A)\beta E_t u_c(C_{t+1} + G_{t+1}^S)\frac{\xi_{t+1}}{\xi_t}\frac{P_t}{P_{t+1}}\frac{1 + \tau_t^s}{1 + \tau_{t+1}^s}, \tag{3}$$

$$\frac{1 - \tau_t^W}{1 + \tau_t^S}\frac{W_t(j)}{P_t} = \frac{v_l(l_t(j))}{u_c(C_t + G_t^S)}.$$

$$\lim_{T\to\infty} E_t \frac{B_T}{P_T(1 + \tau_T^s)} u_c(C_T + G_T^S) = 0. \tag{4}$$

There is a continuum of firms in measure 1. Firm i sets its price and then hires the labor inputs necessary to meet any demand that may be realized. A unit of labor produces one unit of output. The preferences of households and the assumption that the government distributes its spending on varieties in the same way as households imply a demand for good i of the form $y_t(i) = Y_t(\frac{p_t(i)}{P_t})^{-\theta}$, where $Y_t \equiv C_t + G_t^N + G_t^S$ is aggregate output. We assume that all profits are paid out as dividends and that the firm seeks to maximize post-tax profits. Profits can be written as $Z_t(i) = p_t(i)Y_t(p_t(i)/P_t)^{-\theta} - W_t(j)Y_t(p_t(i)/P_t)^{-\theta}$, where i indexes the firm and j the industry in which the firm operates. Following Calvo (1983), let us suppose that each industry has an equal probability of reconsidering its price in each period. Let $0 < \alpha < 1$ be the fraction of industries with prices that remain unchanged in each period. In any industry that revises its prices in period t, the new price p_t^* will be the same. The maximization problem that each firm faces at the time it revises its price is then to choose a price p_t^* to maximize

$$\max_{p_t^*} E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T (1 - \tau_T^P)[p_t^* Y_T(p_t^*/P_T)^{-\theta} - W_T(j)Y_T(p_t^*/P_T)^{-\theta}] \right\}.$$

where λ_T is the marginal utility of nominal income for the representative household. An important assumption is that the price the firm sets is *exclusive* of the sales tax. This means that if the government cuts sales taxes, then consumers face a lower store price of exactly the amount of the tax cuts for firms that have not reset their prices. This maximization problem yields the following first order condition.

$$E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{u_c(C_T + G_T^S)\xi_T}{P_T} \frac{(1 - \tau_T^P)}{(1 + \tau_T^s)} (\frac{p_t^*}{P_T})^{-\theta-1} Y_T [\frac{p_t^*}{P_T} - \frac{\theta}{\theta - 1}\frac{1 + \tau_t^s}{1 - \tau_t^W}\frac{v_l(Y_T(p_t^*/P_T)^{-\theta})}{u_c(C_T + G_T^S)}] \right\} = 0 \tag{5}$$

Using the assumption of that a fraction of α keep their prices fixed, while $1 - \alpha$ set them at p_t^* we can express the price index as

$$P_t = [(1 - \alpha)(p_t^*)^{1-\theta} + \alpha P_{t-1}^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{6}$$

All output is either consumed by the government or the private sector

$$Y_t = C_t + G_t^S + G_t^N \tag{7}$$

Without going into details about how the central bank implements a desired path for nominal interest rates, we assume that it cannot be negative so that[11]

$$i_t \geq 0. \tag{8}$$

We assume that the central bank follows a Taylor rule, whose parameters are detailed further below, in accordance to

$$i_t = \max(0, \phi(\frac{P_t}{P_{t-1}}, Y_t, \xi_t, \psi_t)). \tag{9}$$

Fiscal policy, on the other hand, is a set of functions for each of the policy variables, $\tau_t^w, \tau_t^s, \tau_t^A, \tau_t^P, T_t$ and G_t^N, G_t^S. I will be more specific about the rules that governs fiscal policy in coming sections.

An equilibrium of the model can now defined as a collection of stochastic processes for the endogenous variables $\{p_t^*, Y_t, P_t, C_t\}$ for $t \geq t_0$ given an initial condition P_{t_0-1}, an exogenous sequence $\{\xi_t\}$, a stochastic process for $\{i_t\}$ that satisfies (8)-(9) and a collection of stochastic processes for $\{\tau_t^w, \tau_t^s, \tau_t^A, \tau_t^P, T_t, G_t^N, G_t^S\}$ given the fiscal policy rules to be specified. We now describe how we approximate to model by a log-linear approximation of the equilibrium conditions around a zero-inflation steady state (summarized in Proposition 1).[12]

4 An approximated equilibrium

This section summarizes a log-linearized version of the model. It is convenient to summarize the model by "aggregate demand" and "aggregate supply." By the aggregate demand, I mean the equilibrium condition derived from the optimal consumption decisions (3) of the household where the aggregate resource constraint (7) is used to substitute out for consumption. By aggregate supply, I mean the equilibrium condition derived by the optimal production and pricing decisions of the firms (5) and (6). Aggregate demand (AD) is

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(i_t - E_t\pi_{t+1} - r_t^e) + (\hat{G}_t^N - E_t\hat{G}_{t+1}^N) + \sigma\chi^s E_t(\hat{\tau}_{t+1}^s - \hat{\tau}_t^s) + \sigma\chi^A\hat{\tau}_t^A, \tag{10}$$

where i_t is the one-period risk-free nominal interest rate, π_t is inflation, and E_t is an expectation operator and the coefficients $\sigma, \chi^A, \chi^s > 0$, $\hat{Y}_t \equiv \log Y_t/\bar{Y}$, $\hat{G}_t^N \equiv \log G_t^N/\bar{Y}$, while $\hat{\tau}_t^s \equiv \tau_t^s - \bar{\tau}^s$, $\hat{\tau}_t^A \equiv (1-\beta)^{-1}(\tau_t^A - \bar{\tau}^A)$ and r_t^e is an exogenous disturbance that is only a function of the shock ξ_t (for details see footnote on the rationale for this notation).[13] The aggregate supply (AS) is

$$\pi_t = \kappa\hat{Y}_t + \kappa\psi(\chi^w\hat{\tau}_t^w + \chi^s\hat{\tau}_t^s - \sigma^{-1}\hat{G}_t^N) + \beta E_t\pi_{t+1}, \tag{11}$$

[11] See e.g. Eggertsson and Woodford (2003) for further discussion.

[12] For details on the log-linearization and steady state of this class of models see for example Woodford (2003) and for special consideration that arize due to the zero bound see Eggertsson and Woodford (2003).

[13] The coefficients of the model are defined as follows $\sigma \equiv -\frac{\bar{u}_{cc}}{\bar{u}_c\bar{Y}}$, $\omega \equiv \frac{\bar{v}_l\bar{L}}{\bar{v}_{ll}}$, $\psi \equiv \frac{1}{\sigma^{-1}+\omega}$, $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\sigma^{-1}+\omega}{1+\omega\theta}$, where bar denotes that the variable is defined in steady state. The shock is defined as $r_t^e \equiv \log\beta^{-1} + E_t(\hat{\xi}_t - \hat{\xi}_{t+1})$, where $\hat{\xi}_t \equiv \log\xi_t/\bar{\xi}$. Finally we define $\chi^A \equiv \frac{1-\beta}{(1-\bar{\tau}^A)}$, $\chi^w \equiv \frac{1}{1-\bar{\tau}^w}$, $\chi^s \equiv \frac{1}{1+\bar{\tau}^s}$.

In terms of our previous notation, i_t now actually refers to $log(1+i_t)$ in the log-linear model. Observe also that this variable, unlike the others, is not defined in deviations from steady state. I do this so that we can still express the zero bound simply as the requirement that i_t is nonnegative.

where the coefficients $\kappa, \psi > 0$ and $0 < \beta < 1$ and the zero bound is

$$i_t \geq 0. \tag{12}$$

The policy rule (9) is approximated to yield:

$$i_t = \max(0, r_t^e + \phi_\pi \pi_t + \phi_y \hat{Y}_t), \tag{13}$$

where the coefficients $\phi_\pi > 1$ and $\phi_y > 0$. For a given policy rule for taxes and spending equations (10)-(13) close the model. An approximate equilibrium can now be defined as a collection of stochastic processes for $\{\hat{Y}_t, \pi_t\}$, given an exogenous path for $\{r_t^e\}$, a monetary policy specifying the process $\{i\}$ that satisfies (12) and (13) and fiscal policy rules which determines the path for $\{\hat{\tau}_t^w, \hat{\tau}_t^s, \hat{\tau}_t^A, \hat{G}_t^N\}$ which I will be specific about in the coming policy experiments. To summarize:

Proposition 1 The steady of the model is a set of constants shown in the Appendix . An approximate equilibrium, which is accurate up to a first order, is a collection of stochastic processes for $\{\hat{Y}_t, \pi_t, r_t^e, i_t, \}$ that solve equations (10)-(13) given a path for $\{\hat{\tau}_t^w, \hat{\tau}_t^s, \hat{\tau}_t^A, \hat{G}_t^N\}$ determined by fiscal policy, where we have linearized about the steady state stated in this proposition.

Proof: See Appendix.

Observe that this list of equations for the equilibrium determination does not include the government budget constraint. I assume that Ricardian equivalence holds, so that temporary variations in either $\hat{\tau}_t^w, \hat{\tau}_t^s$ or $\hat{G}_t^N$are offset either by lump-sum transfers (i.e. changes in $\hat{T}_t$) in period t or in future periods $t+j$ (the exact date is irrelevant because of Ricardian equivalence).[14] Observe also that I do not need to determine the path for $\hat{G}_t^S$ and $\hat{C}_t$ for output and inflation

For the purposes of interpretation I have scaled the fiscal policy variables in a particular way in order to have a more meaningful comparison of "multipliers" in coming sections. $\hat{G}_t^N$ is the percentage changes of government spending from its steady state as fraction of steady state output. We want the other tax instrument to be of similar order to facilitate comparison of alternative fiscal actions. Because all production is consumed then, in the absence of shocks, a one percent increase in the sales tax, local to steady state, will increase revenues by exactly the same amount as the increase in government spending, which is why we define $\hat{\tau}_t^s \equiv \tau_t^s - \bar{\tau}^s$, i.e., it has the interpretation of a percentage increase in the sales tax rate (which would exactly finance an increase in $\hat{G}_t^N$ one to one). Similarly $\hat{\tau}_t^w \equiv \tau_t^w - \bar{\tau}^w$ measures the percentage increase the tax rate on labor. Given these definitions I scale each of the tax instruments with χ^s and χ^w. Finally, I define the variable $\hat{\tau}_t^A$ so that a one percent increase in this variable correponds to a one percent increase in the tax on *capital income* per year, to be comparable with the tax on labor income.

[14] This assumption simplifies that analysis quite a bit, since otherwise, when considering the effects of particular tax cuts, I would need to take a stance on what combination of taxes would need to be raised to offset the effect of the tax cut on the government budget constaint and at what time horizon. Moreover, I would need to take a stance on what type of debt the government could issue. While all those issues are surely of some interest in future extensions, this approach seems like the most natural first step since it allows us to analyze the effect of each fiscal policy instrument in isolation (abstracting from their effect on the government budget). Note that by making this critical assumption, I do not need to talk in detail about several issues related to fiscal policy in the fully non-linear model, see e.g. Eggertsson and Woodford (2003) for discussion.

determination. The reason for this is that any variation in $\hat{G}_t^S$ will be offset by a corresponding reduction in $\hat{C}_t$ and hence it is irrelevant for the determination of inflation and output (more on this in section 8). Finally observe that the tax instrument $\hat{\tau}_t^P$ does not appear in any of the equations. This suggests that taxes on profits, under our assumption, do not have a first order effect on inflation, output and interest rates.[15]

5 Long and short run equilibrium allocations

This section summarizes the model's equilibrium allocation. The rest of the paper is devoted to explaining and evaluating this allocation under a variety of additional assumptions. Accordingly, the discussion of the propositions here is short and terse, the proofs relegated to the Appendix.

The *long run* in the model is defined as the time at which the shock, r_t^e, has gone to steady state. The short run is the period in which the economy is subject to temporary disturbance. More precisely, the short run is defined by $r_t^e = r_S^e$. This shock reverts back to steady state, $\bar{r}$, with probability μ in each period. Let us call the stochastic period in which the shocks are back to steady state T^e. Then $t \geq T^e$ is defined as the *long run*, while $t < T^e$ is the *short run*. Monetary policy follows (13) and (for now) fiscal policy is perfectly correlated with the shock, that is, we consider tax cuts/increases and government spending increases/cuts that are a direct reaction to the shock, so that, $(\hat{\tau}_t^w, \hat{\tau}_t^s, \hat{\tau}_t^A, \hat{G}_t^N) = (\hat{\tau}_S^w, \hat{\tau}_S^s, \hat{\tau}_S^A, \hat{G}_S^N)$ in the short run, $t < \tau$, and $(\hat{\tau}_t^w, \hat{\tau}_t^s, \hat{\tau}_t^A, \hat{G}_t^N) = (0, 0, 0, 0)$ in the long run $t \geq T^e$. We are focusing, in other words, on fiscal policy that is aimed at short-run stabilization.

The assumption about a short and long-run is very convenient because of the structure of the model. As we will see, it allows us to boil the model down to just a few static "short run" equations, even if in principle we are dealing with an infinite-horizon model. This allows me to show everything in closed form, and even illustrate the key results by the aid of simple short-run diagrams.

Proposition 2 In the long run, $t \geq T^e$, there is a unique bounded solution such that $\pi_t = \hat{Y}_t = 0$ and $i_t = \bar{r}$.

Proof: See Appendix.

We now state two conditions that are helpful to analyze the short run:

$$C1 \qquad r_S^e < -\frac{\kappa\phi_\pi + (1-\beta\mu)\phi_y}{(1-\mu+\sigma\phi_y)(1-\beta\mu)+\kappa\sigma(\phi_\pi-\mu)}\sigma\chi^A\hat{\tau}_S^A - \frac{(1-\mu)\kappa\psi\phi_\pi + \sigma\mu\kappa\psi\phi_y}{(1-\mu+\sigma\phi_y)(1-\beta\mu)+\kappa\sigma(\phi_\pi-\mu)}\chi^w\hat{\tau}_S^w - \frac{\kappa\sigma^{-1}(1-\mu)(\sigma-\psi)\phi_\pi + [(1-\mu)(1-\beta\mu)-\kappa\psi\mu)]\phi_y}{(1-\mu+\sigma\phi_y)(1-\beta\mu)+\kappa\sigma(\phi_\pi-\mu)}[\hat{G}_S^N - \sigma\chi^s\hat{\tau}_t^s]$$

[15] In the model with capital $\hat{\tau}_t^P$ will matter.

$$C2 \qquad L(\mu) \equiv (1-\mu)(1-\mu\beta) - \kappa\mu\sigma > 0,$$

Observe that C1 imposes a condition on the fundamental shock, r_S^e and a limit on the fiscal expansion. If there is no countercyclical short-run policy this condition simply states that $r_S^e < 0$. This condition needs to hold to ensure that the zero bound is binding.

Proposition 3 In the short run, $t < \tau$, then we consider two cases:

1.(*Positive interest rates in the short run*) If C1 does *not* hold, then there is a unique bounded equilibrium at positive interest rates such that

$$\begin{aligned}\pi_S &= -\frac{\kappa\sigma^{-1}[(1-\mu+\sigma\phi_y)\psi - \sigma(1-\mu)]}{[(1-\mu+\sigma\phi_y)(1-\beta\mu)+\kappa\sigma(\phi_\pi-\mu)]}[\hat{G}_S^N - \sigma\chi^s\tau_S^s] \\ &+\frac{(1-\mu+\sigma\phi_y)\kappa\psi}{[(1-\mu+\sigma\phi_y)(1-\beta\mu)+\kappa\sigma(\phi_\pi-\mu)]}\chi^w\hat{\tau}_S^w + \frac{\kappa\sigma}{[(1-\mu+\sigma\phi_y)(1-\beta\mu)+\kappa\sigma(\phi_\pi-\mu)]}\chi^A\hat{\tau}_S^A\end{aligned}$$

$$\begin{aligned}\hat{Y}_S &= \frac{[(1-\mu)(1-\beta\mu)+\kappa\psi(\phi_\pi-\mu)]}{(1-\beta\mu)(1-\mu+\sigma\phi_y)+\kappa\sigma(\phi_\pi-\mu)}[\hat{G}_S^N - \sigma\chi^s\hat{\tau}_S^s] \\ &-\frac{\sigma(\phi_\pi-\mu)\kappa\psi}{(1-\beta\mu)(1-\mu+\sigma\phi_y)+\kappa\sigma(\phi_\pi-\mu)}\chi^w\hat{\tau}_S^w + \frac{\sigma(1-\beta\mu)}{(1-\beta\mu)(1-\mu+\sigma\phi_y)+\kappa\sigma(\phi_\pi-\mu)}\chi^A\hat{\tau}_S^A,\end{aligned}$$

$$i_t = i_S = r_S^e + \phi_\pi\pi_S + \phi_y\hat{Y}_S$$

2. (*Zero interest rates in the short run*) If C1 *and C2 hold,* there is a unique bounded equilibrium at zero interest rates such that

$$\begin{aligned}\pi_S &= \frac{\kappa\sigma}{(1-\mu)(1-\mu\beta)-\kappa\mu\sigma}r_S^e + \frac{\kappa(1-\mu)[(1-\psi\sigma^{-1}]}{(1-\mu)(1-\mu\beta)-\kappa\mu\sigma}[\hat{G}_S^N - \sigma\chi^s\hat{\tau}_S^s] \\ &+\frac{\kappa\sigma}{(1-\mu)(1-\mu\beta)-\kappa\mu\sigma}\chi^A\hat{\tau}_S^A + \frac{(1-\mu)\kappa\psi}{(1-\mu)(1-\mu\beta)-\kappa\mu\sigma}\chi^w\hat{\tau}_S^w\end{aligned}$$

$$\begin{aligned}\hat{Y}_S &= \frac{\sigma(1-\mu\beta)}{(1-\mu)(1-\mu\beta)-\mu\sigma\kappa}r_S^e + \frac{(1-\mu)(1-\mu\beta)-\mu\kappa\psi}{(1-\mu)(1-\mu\beta)-\mu\sigma\kappa}[\hat{G}_S^N - \sigma\chi^s\hat{\tau}_S^s] \\ &+\frac{\sigma(1-\mu\beta)}{(1-\mu)(1-\mu\beta)-\mu\sigma\kappa}\chi^A\hat{\tau}_S^A + \frac{\mu\sigma\kappa\psi}{(1-\mu)(1-\mu\beta)-\mu\sigma\kappa}\chi^w\hat{\tau}_S^w\end{aligned}$$

$$i_S = 0.$$

Proof See Appendix.

I do not consider the case when C1 holds but C2 does not. In this case either the local approximation is no longer valid (the model explodes) or there is an indeterminacy of equilibria. Proposition 2 is the heart of the paper. In fact, by manipulating it, we can compute all the relevant fiscal multipliers. The rest of the paper is to a large extent about interpreting this proposition, both qualitatively and quantitatively.

6 An output collapse in the absence of fiscal interventions

Output can collapse under the special circumstances when interest rates are zero. This environment is the focus of the paper. We consider the model first when there is no fiscal intervention, i.e. each of the fiscal variables are at steady state. The propositions and condition C1 suggest that when $r_t^e < 0$ then the zero bound is binding, so that $i_t = 0$. A negative r_t^e generates a recession in the model and plays a key role.

Before going further, it is natural to ask: Where does this shock come from? In the simplest version of the model, a negative r_t^e is equivalent to a preference shock and so corresponds to a lower ξ_t in period t in the utility function (1) that reverts back to steady state with probability $1-\mu$. Everyone suddenly wants to save more so the real interest rate must decline for output to stay constant. More sophisticated interpretations are possible, however. Curdia and Eggertsson (2010), building on Curdia and Woodford (2008), show that a model with financial frictions can also be reduced to equations (10)-(11). In this more sophisticated model, the shock r_t^e corresponds to an exogenous increase in the probability of default by borrowers. What is nice about this interpretation is that r_t^e can now be mapped into the wedge between a risk-free interest rate and an interest rate paid on risky loans. Both rates are observed in the data. The wedge implied by these interest rates exploded in the U.S. economy during the crisis of 2008, providing empirical evidence for a large negative shock to r_t^e. A banking crisis – characterized by an increase in probability of default by borrowers– is my story for the model's recession.

Let us now go through the equilibrium determination informally (the formal proof is in Propositions 1 and 2). Panel (a) in Figure 1 illustrates assumption about the shock graphically. Under this assumption, the shock r_t^e remains negative in the short run denoted S, until some stochastic date T^e, when it returns to steady state. For starters, let us assume that $\hat{\tau}_t^w = \hat{\tau}_t^A = \hat{\tau}_t^s = \hat{G}_t^N = 0$. Given the shock, monetary policy takes the following form:

$$i_t = r_S^e = \bar{r} \text{ for } t \geq T^e \tag{14}$$

$$i_t = 0 \text{ for } 0 < t < T^e, \tag{15}$$

In the periods $t \geq T^e$, the solution is $\pi_t = \hat{Y}_t = 0$. In periods $t < T^e$, the assumption about the shock implies that inflation in the next period is either zero (with probability $1-\mu$) or the same as at time t, i.e., $\pi_t = \pi_S$ (with probability μ).[16] Hence the solution in $t < T^e$ satisfies the AD and the AS equations:

$$AD \qquad \hat{Y}_S = \mu \hat{Y}_S + \sigma \mu \pi_S + \sigma r_S^e \tag{16}$$

$$AS \qquad \pi_S = \kappa \hat{Y}_S + \beta \mu \pi_S \tag{17}$$

It is helpful to graph the two equations in $(\hat{Y}_S, \pi_S)$ space. Consider first the special case in which $\mu = 0$, i.e., the shock r_S^e reverts back to steady state in period 1 with probability 1. This case is shown in Figure 2. It applies only to the equilibrium determination in period 0.

[16] This conjecture is formally proved in Proposition 2 and 3.

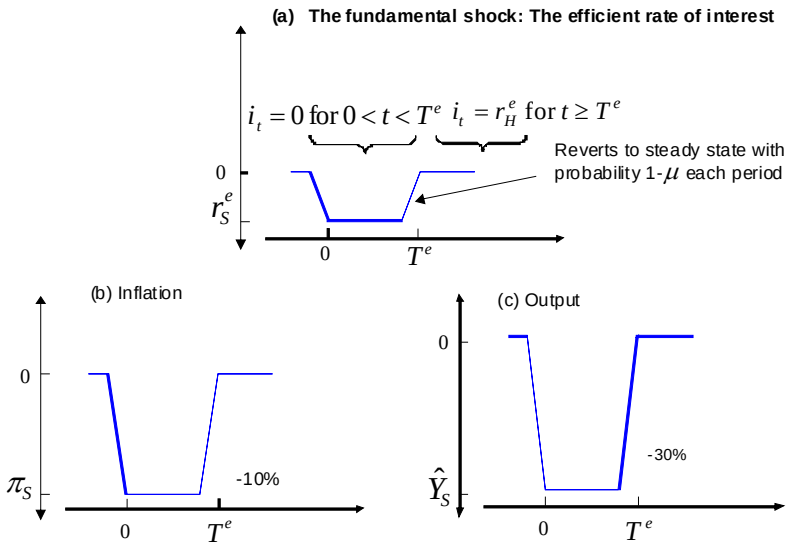


Figure 1: The effect of negative r_t^e on output and inflation in the absence of any fiscal policy.

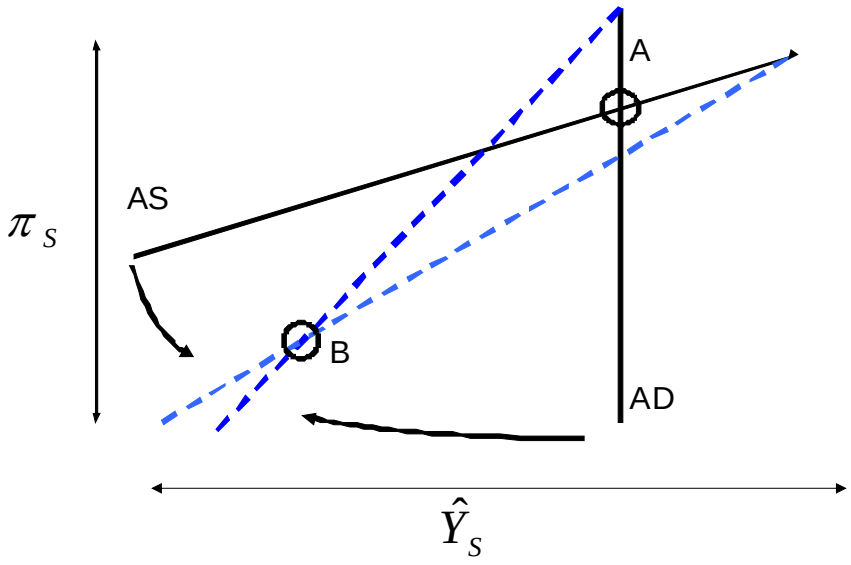


Figure 2: The effect of multiperiod recession.

The equilibrium is shown where the two solid lines intersect at point A. At point A, output is completely demand-determined by the vertical AD curve and pinned down by the shock r_t^e.[17] For a given level of output, then, inflation is determined by where the AD curve intersects the AS curve. It is worth emphasizing again: *Output is completely demand-determined, i.e., it is completely determined by the AD equation.*

Consider now the effect of increasing $\mu > 0$. In this case, the contraction is expected to last for longer than one period. Because of the simple structure of the model, and the two-state Markov process for the shock, the equilibrium displayed in the figure corresponds to all periods $0 \leq t < T^e$. The expectation of a possible future contraction results in movements in both the AD and the AS curves, and the equilibrium is determined at the intersection of the two dashed curves, at point B. Observe that the AD equation is no longer vertical but upward sloping in inflation, i.e., higher inflation *expectations* $\mu\pi_S$ increase output. The reason is that, for a given nominal interest rate ($i_S = 0$ in this equilibrium), any increase in expected inflation reduces the real interest rate, making current spending relatively cheaper and thus increasing demand. Conversely, expected deflation, a negative $\mu\pi_S$, causes current consumption to be relatively more expensive than future consumption, thus suppressing spending. Observe, furthermore, the presence of the expectation of future contraction, $\mu\hat{Y}_S$, on the right-hand side of the AD equation. The expectation of future contraction makes the effect of both the shock and the expected deflation even stronger.

Let us now turn to the AS equation (17). Its slope is now steeper than before because the expectation of future deflation will lead the firms to cut prices by more for a given demand slack, as shown by the dashed line. The net effect of the shift in both curves is a more severe contraction and deflation shown by the intersection of the two dashed curves at point B in Figure 2 (see the footnote on why the AD curve has to be steeper than the AS curve).[18]

The more severe depression at point B is triggered by several contractionary forces. First,

[17] A higher efficient rate of interest, r_S^e, corresponds to an autonomous increase in the willingness of the household to spend at a given nominal interest rate and expected inflation and thus shifts the AD curve. Note that the key feature of the shock assumption is that we are considering a shock that results in a negative efficient interest rate, which in turn causes the nominal interest rate to decline to zero. Another way of stating this is that it corresponds to an "autonomous" decline in spending for given prices and a nominal interest rate. This shock thus corresponds to what the old Keynesian literature referred to as "demand" shocks, and one can interpret it as a stand-in for any exogenous reason for a decline in spending. Observe that in the model all output is consumed. If we introduce other sources of spending, such as investment, a more natural interpretation of a decline in the efficient interest rate is an autonomous shock to the cost of investment in addition to the preference shock (see further discussion in the section of the paper with endogenous capital).

[18] The contraction is amplified without a bound as μ increases. As μ increases, the AD curve becomes flatter and the AS curve steeper, and the cutoff point moves further down in the $(\hat{Y}_S, \pi_S)$ plane in Figure 2. At a critical value $1 > \bar{\mu} > 0$ when $L(\bar{\mu}) = 0$ in C2, the two curves are parallel, and no solution exists. The point $\bar{\mu}$ is called a *deflationary black hole.* In the remainder of the paper we assume that μ is small enough so that the deflationary black hole is avoided and the solution is well defined and bounded (this is guaranteed by the inequality in assumption C2). A deflationary solution always exists as long as the shock μ is close enough to zero because $L(0) > 0$ (at $\mu = 0$, the shock reverts back to steady state with probability 1 in the next period). Observe, furthermore, that $L(1) < 0$ and that in the region $0 < \mu < 1$ the function $L(\mu)$ is strictly decreasing, so there is some critical value $\bar{\mu} = \mu(\kappa, \sigma, \beta) < 1$ in which $L(\mu)$ is zero and the model has no solution.

because the contraction is now expected to last more than one period, output is falling in the price level because there is expected deflation, captured by $\mu\pi_S$ on the right-hand side of the AD equation. This increases the real interest rate and suppresses demand. Second, the expectation of future output contraction, captured by the $\mu\hat{Y}_S$ term on the right-hand side of the AD equation, creates an even further decline in output. Third, the strong contraction, and the expectation of it persisting in the future, implies an even stronger deflation for given output slack, according to the AS equation.[19] Note the role of the aggregate supply, or the AS equation. It is still really important to determine the expected inflation in the AD equation. This is the sense in which output is demand-determined in the model even when the shock lasts for many periods. That is what makes tax policy so tricky, as we soon will see. It is also the reason why government spending and cuts in sales taxes can have a big effect.

The two-state Markov process for the shock allows us to collapse the model into two equations with two unknown variables, as shown in Figure 2 and illustrated by Proposition 3. It is important to keep in mind, however, the stochastic nature of the solution. The output contraction and the deflation last only as long as the stochastic duration of the shock, i.e., until the stochastic date T^e, and the equilibrium depicted in Figure 2 applies only to the "recession" state, i.e. the short run. This is illustrated in Figure 1, which shows the solution for an arbitrary contingency in which the shock lasts for T^e periods. I have added for illustration numerical values in this figure, using the parameters from Table 2. The values assumed for the structural parameters are relatively standard. (The choice of parameters and shocks in Table 2 is described in more detail in Appendix B and in Eggertsson and Denes (2009)). The values are obtained by maximizing the posterior distribution of the model to match a 30 percent decline in output and a 10 percent deflation in the short run. Both these numbers correspond to the trough of the Great Depression in the first quarter of 1933 before President Franklin D. Roosevelt assumed power, when the nominal interest rate was close to zero. I ask the model to match the data from the Great Depression, because people have often claimed that the goal of fiscal stimulus in 2008 was to avoid a dire scenario of that kind.[20]

[19] Observe the vicious interaction between the contractionary forces in the AD and AS equations. Consider the pair $\hat{Y}_S^A, \pi_S^A$ at point A as a candidate for the new equilibrium. For a given $\hat{Y}_S^A$, the strong deflationary force in the AS equation reduces expected inflation so that we need to have $\pi_S < \pi_S^A$. Owing to the expected deflation term in the AD equation, this again causes further contraction in output, so that $\hat{Y}_S < \hat{Y}_S^A$. The lower $\hat{Y}_S$ then feeds again into the AS equation, triggering even further deflation and thus a further drop in output according to the AD equation, and so on and on, leading to a vicious deflation-output contractionary spiral that converges to point B in panel (a), where the dashed curves intersect.

[20] Note that the value of τ^s, τ^w and τ^A, only matters for determining χ^A, χ^w and χ^s which simply scales the multipliers. Hence changing the value of these parameters will just move the multipliers I report by exactly the same amount as they change the $\chi's$. The value of ϕ_π and ϕ_y are those suggested by Taylor (1993). Note that in his paper he expressed the rule in terms of annualized interest rates and inflation so that his interest rate referred to $4i_t$ and his inflation measure to $4\pi_t$ in my notation. Hence the coefficient on $\hat{Y}_t$ is 0.5/4 according to my notation.

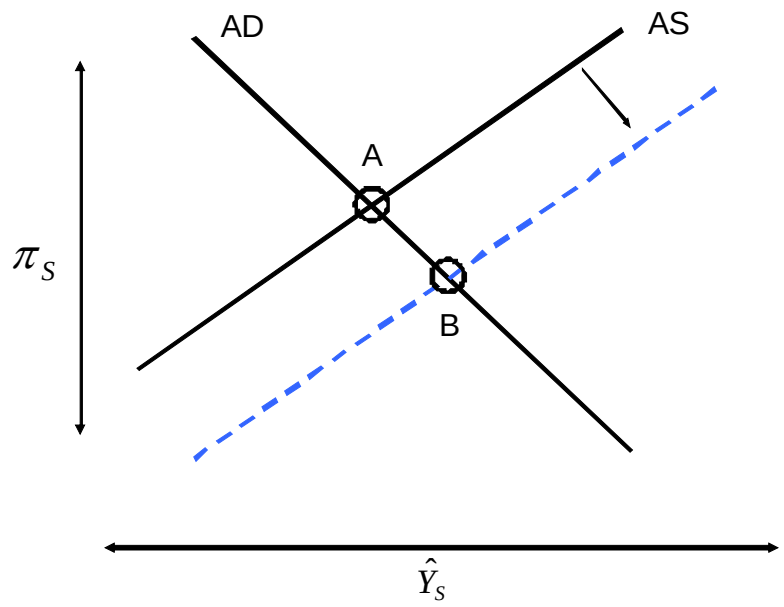


Figure 3: The effect of cutting taxes at a positive interest rate.

Table 2: parameters, mode

	σ^{-1}	β	ω	α	θ
Parameters (mode)	1.1599	0.9970	1.5692	0.7747	12.7721

	ϕ_π	ϕ_y	$\bar{\tau}^s$	$\bar{\tau}^w$	$\bar{\tau}^A$
Parameters (calibrated)	1.5	0.5/4	0.05	0.2	0

	r_S^e	μ
Shocks (mode)	-0.0104	0.9030

7 Why labor tax cuts are contractionary

Can fiscal policy reverse the output collapse shown in the last section? We start by considering tax cuts on labor. Before going further, it is helpful to study tax cuts under regular circumstance, i.e., *in the absence of the shock*. Under normal circumstances, a payroll-tax cut is *expansionary* in the baseline model. This is presumably why this policy proposal has gained much currency in recent policy discussions. Consider a temporary tax cut $\hat{\tau}_t^w = \hat{\tau}_S^w < 0$ in period t that is reversed with probability $1-\rho$ in each period to steady state $\hat{\tau}_t^w = 0$. Let us call the date on which the tax cut reverses to steady state T^τ. Let $\hat{G}_t^N = \hat{\tau}_t^s = \hat{\tau}_t^A = 0$. Because the model is perfectly forward-looking, this allows us again to collapse the model into only two states, the short run

when $\hat{\tau}_S^w < 0$ and the long run when $\hat{\tau}_t^w = \hat{\tau}_L^w = 0$. Observe that in the steady state $t > T^\tau$ then $\hat{Y}_t = \pi_t = 0$. Substituting 13 into the AD equation, we can write the AD and AS equation in the short run as

$$\hat{Y}_S = -\sigma \frac{\phi_\pi - \rho}{1 - \rho + \sigma\phi_y} \pi_S \tag{18}$$

$$(1 - \beta\rho)\pi_S = \kappa \hat{Y}_S + \kappa\psi\chi^w \tau_S^w. \tag{19}$$

Figure 3 shows the AS and AD curves (18) and (19). This figure looks like any undergraduate textbook AS-AD diagram! A tax cut shifts down the AS curve. Why? Now people want to work more since they get more money in their pocket for each hour worked. This reduces real wages, so that firms are ready to supply more goods for less money, creating some deflationary pressure. In response, the central bank accommodates this shift by cutting interest rates in order to curb deflation, which is why the AD equation is downward sloping.[21] A new equilibrium is found at point B. We can compute the multiplier of tax cuts by using the method of undetermined coefficients.[22] The tax cut multiplier is

$$\frac{\Delta \hat{Y}_S}{-\Delta \hat{\tau}_S^w} = \chi^w \frac{\sigma\phi_\pi\kappa\psi}{(1 - \rho + \sigma\phi_y)(1 - \rho\beta) + \sigma\phi_\pi\kappa} > 0. \tag{20}$$

Here, Δ denotes change relative to the benchmark of no variations in taxes. To illustrate the multiplier numerically, I use the values reported in Table 2 and assume $\rho = \mu$. The multiplier is 0.16. If the government cut the tax rate $\hat{\tau}_L^w$ by 1 percent in a given period, then output increases by 0.16 percent. Table 2 also reports 5 percent and 95 percent posterior bands for the multiplier, giving the reader a sense of the sensitivity of the result, given the priors distributions described in more detail in Appendix B. We can also translate this into dollars. Think of the tax cuts in terms of dollar cuts in tax collections in the absence of shocks, i.e., tax collection in a "steady state." Then the meaning of the multiplier is that each dollar of tax cuts buys you a 16 cent increase in output.

We now show that this very same tax cut has the opposite effect under the special circumstances when the zero bound is binding. Again, consider a temporary tax cut, but now one that is explicitly aimed at "ending the recession" created by the negative shock that caused all the trouble in the last section. Assume the tax cut takes the following form:

$$\hat{\tau}_S^w = \phi_\tau r_S^e < 0 \text{ when } 0 < t < T^e \tag{21}$$

with $\phi_\tau > 0$ and

$$\hat{\tau}_t^w = 0 \text{ when } t \geq T^e. \tag{22}$$

[21] A case where the central bank targets a particular inflation rate, say zero, corresponds to $\phi_\pi - > \infty$. In this case, the AD curve is horizonal and the effect of the tax cut is very large because the central bank will accomodate it with aggressive interest rates cuts.

[22] Note that the two-state Markov process we assumed gives the same result as if we had assumed the stochastic process $\hat{\tau}_t = \mu_\tau \hat{\tau}_{t-1} + \epsilon_t$ where ϵ_t is normally distributed iid. In that case, the multiplier applies to output in period 0.

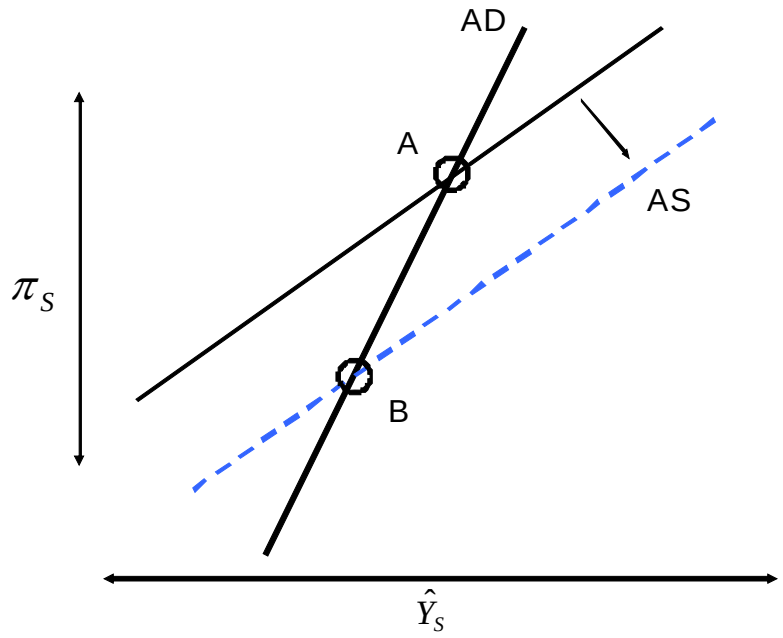


Figure 4: The effect of cutting labor taxes at a zero interest rate.

Consider now the solution in the periods when the zero bound is binding but the government follows this policy. The AS curve is exactly the same as under the "normal circumstance" shown in equation 19, but now we have replaced ρ with the probability of the duration of the shock, i.e., $\rho = \mu$. The big difference is the AD curve, because of the shock r_S^e and because the zero bound is binding. Hence we replace equation (18) with equation (16) from the last section. These two curves are plotted in Figure 4, and it should now be clear that the effect of the tax cut is the opposite from what we had before. Just as before, the increase in $\hat{\tau}_L^w$ shifts the AS curve outwards as denoted by a dashed line in Figure 4. As before, this is just a traditional shift in "aggregate supply" outwards; the firms are now in a position to charge lower prices on their products than before. But now the slope of the AD curve is different from before, so that a new equilibrium is formed at the intersection of the dashed AS curve and the AD curve at lower output and prices, i.e., at point B in Figure 4. The general equilibrium effect of the tax cut is therefore an output contraction!

The intuition for this result (as clarified in the following paragraphs) is that the expectation of lower taxes in the recession creates deflationary expectations *in all states of the world in which the shock* r_t^e *is negative*. This makes the real interest rate higher, which reduces spending according to the AD equation. We can solve the AD and AS equations together to show analytically that

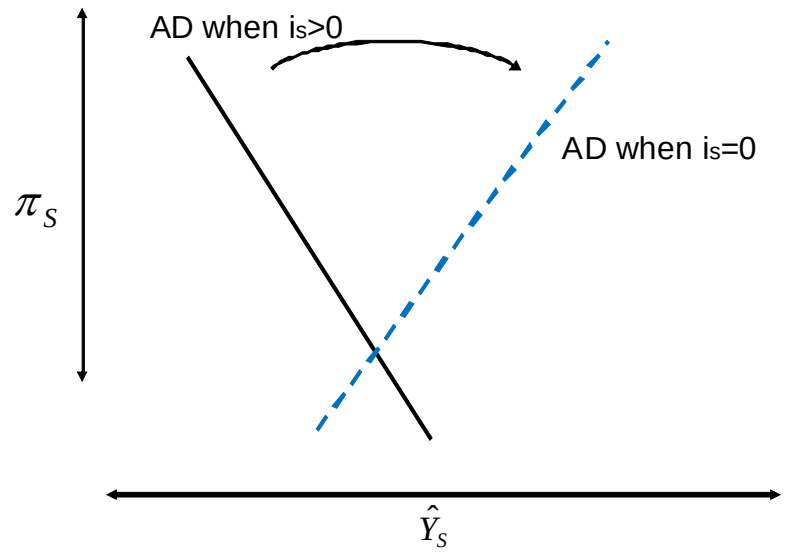


Figure 5: How aggregate demand changes once the short-term interest rate hits zero.

output and inflation are reduced by these tax cuts:

$$\begin{aligned} \hat{Y}_S^{taxcut} &= \frac{1}{(1-\mu)(1-\beta\mu)-\mu\sigma\kappa}[(1-\beta\mu)\sigma r_S^e + \mu\kappa\sigma\psi\chi^w\hat{\tau}_S^w] < \hat{Y}_t^{notax} \text{ if } t < T^e \\ \text{and } \hat{Y}_S^{taxcut} &= 0 \text{ if } t \geq T^e \end{aligned}$$

$$\pi_t^{taxcut} = \frac{\kappa}{1-\beta\mu}(\hat{Y}_t^{tax} + \psi\chi^w\hat{\tau}_S^w) < \pi_t^{notax} \text{ if } t < T^e \text{ and } \hat{\pi}_t^{tax} = 0 \text{ if } t \geq T^e.$$

Figure 5 clarifies the intuition for why labor tax cuts become contractionary at zero interest rates while being expansionary under normal circumstances. *The key is aggregate demand.* At positive interest rates the AD curve is downward-sloping in inflation. The reason is that as inflation decreases, the central bank will cut the nominal interest rate more than 1 to 1 with inflation (i.e., $\phi_\pi > 1$, which is the Taylor principle; see equation 13). Similarly, if inflation increases, the central bank will increase the nominal interest rate more than 1 to 1 with inflation, thus causing an output contraction with higher inflation. As a consequence, the real interest rate *will decrease with deflationary pressures* and expanding output, because any reduction in inflation will be met by a more than proportional change in the nominal interest rate. This, however, is no longer the case at zero interest rates, because interest rates can no longer be cut. This means that the central bank will no longer be able to offset deflationary pressures with aggressive interest rate cuts, shifting the AD curve from downward-sloping to upward-sloping in $(\hat{Y}_S, \pi_S)$ space, as shown in Figure 5. The reason is that lower inflation will now mean *a higher real rate*, because

the reduction in inflation can no longer be offset by interest rate cuts. Similarly, an increase in inflation is now expansionary because the increase in inflation will no longer be offset by an increase in the nominal interest rate; hence, higher inflation implies lower real interest rates and thus higher demand.

We can now compute the multiplier of tax cuts at zero interest rates. It is negative and given by

$$\frac{\Delta \hat{Y}_S}{-\Delta \hat{\tau}_S^w} = -\chi^w \frac{\mu\kappa\sigma\varphi}{(1-\mu)(1-\beta\mu)-\mu\sigma\kappa} < 0. \tag{23}$$

Using the numerical values in Table 2, this corresponds to a multiplier of -1.02 (with the 5 percent and 95 percent posterior bands reported in the table. This means that if the government reduces taxes rate $\hat{\tau}_S^w$ by 1 percent at zero interest rates, then aggregate output declines by 1 percent. To keep the multipliers (20) and (23) comparable, I assume that the expected persistence of the tax cuts is the same across the two experiments, i.e., $\mu = \rho$.

Table 3: Multipliers of temporary policy changes

(First line denotes mode while the second line denotes 5-95 percent posterior bands.)

	Multiplier (mode) $i_t > 0$ (5%,95%)	Multiplier (mode) $i_t = 0$ (5%,95%)
τ_t^w(Payroll Tax Cut)	0.1612 (0.0768, 0.2207)	-1.0191 (-1.7076, -0.2604)
G_t^S(Government Spending 1 Increase)	**0** (0)	**0** (0)
G_t^N (Government Spending 2 Increase)	0.4652 (0.4211, 0.5560)	2.2793 (1.4275, 3.2528)
τ_t^S (Sales Tax Cut)	0.5139 (0.3549, 0.8557)	2.5179 (1.5040, 3.7898)
τ_t^A (Capital Tax Cut)	-0.0013 -0.0024, -7.2439e-04)	-0.1012 (-0.2029, -0.0353)

8 Expansionary government spending: Timely, targeted and temporary

Let us now consider the effect of government spending. Consider first the effect of increasing $\hat{G}_t^S$. It is immediate from our derivation of the model in Section 4 that increasing government spending, which is a perfect substitute for private spending, has no effect on output or inflation. The reason is that the private sector will reduce its own consumption by exactly the same amount. The formal way to verify this is to observe that the path for $\{\pi_t, \hat{Y}_t\}$ is fully determined by equations (10)-(13), along with a policy rule for the tax instruments and $\hat{G}_t^N$, which makes no reference to the policy choice of $\hat{G}_t^S$. Let us now turn to government spending, which is not a perfect substitute for private consumption, $\hat{G}_t^N$.

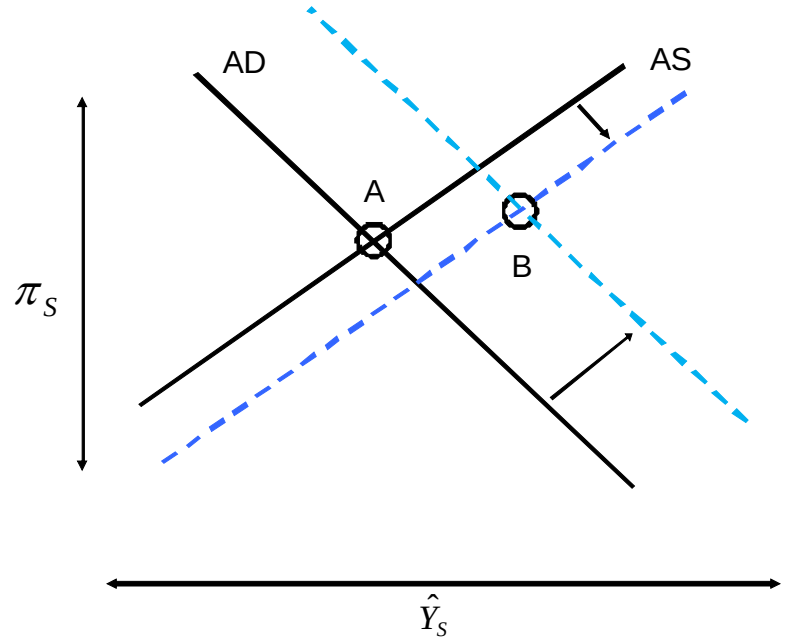


Figure 6: Increasing government spending at positive interest rates.

Consider the effect of increasing government spending, $\hat{G}_t^N$, in the absence of the deflationary shock so that the short-term nominal interest rate is positive. In particular, consider an increase $\hat{G}_S^N > 0$ that is reversed with probability $1-\rho$ in each period to steady state. Substituting the Taylor rule into the AD equation we can write the AD and AS equations as

$$(1-\rho+\sigma\phi_y)\hat{Y}_S = -\sigma(\phi_\pi-\rho)\pi_S + (1-\rho)\hat{G}_S^N \tag{24}$$

$$(1-\beta\rho)\pi_S = \kappa\hat{Y}_S - \kappa\psi\sigma^{-1}\hat{G}_S^N. \tag{25}$$

The experiment is shown in Figure 6. It looks identical to a standard undergraduate textbook AD-AS diagram. An increase in $\hat{G}_S^N$ shifts out demand for all the usual reasons, i.e., it is an "autonomous" increase in spending. In the standard New Keynesian model, there is an additional kick, however, akin to the effect of reducing labor taxes. Government spending also shifts out aggregate supply. Because government spending takes away resources from private consumption, people want to work more in order to make up for lost consumption, shifting out labor supply and reducing real wages. This effect is shown in the figure by the outward shift in the AS curve. The new equilibrium is at point B. Using the method of undetermined coefficients, we can compute the multiplier of government spending at positive interest rates as

$$\frac{\Delta\hat{Y}_S}{\Delta\hat{G}_S^N} = \frac{(1-\rho)(1-\rho\beta)+(\phi_\pi-\rho)\kappa\psi}{(1-\rho+\sigma\phi_y)(1-\rho\beta)+(\phi_\pi-\rho)\sigma\kappa} > 0.$$

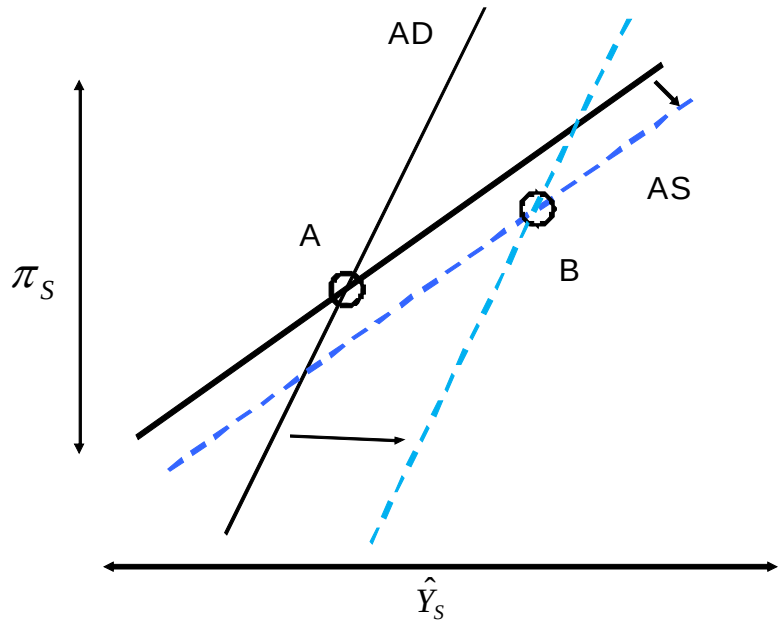


Figure 7: The effect of increasing government spending at zero interest rates.

Using the parameter values in Table 1, we find that one dollar in government spending increases output by 0.46, which is more than three times the multiplier of tax cuts at positive interest rates.

Consider now the effect of government spending at zero interest rates. In contrast to tax cuts, increasing government spending is very effective at zero interest rates. Consider the following fiscal policy:

$$\hat{G}_t^N = \hat{G}_S^N > 0 \quad \text{for} \quad 0 < t < T^e \tag{26}$$

$$\hat{G}_t^N = 0 \quad \text{for} \quad t \geq T^e. \tag{27}$$

Under this specification, the government increases spending in response to the deflationary shock and then reverts back to steady state once the shock is over.[23] The AD and AS equations can be written as

$$\hat{Y}_S = \mu \hat{Y}_S + \sigma \mu \pi_S + \sigma r_S^e + (1-\mu)\hat{G}_S^N \tag{28}$$

$$\pi_S = \kappa \hat{Y}_S + \beta \mu \pi_S - \kappa \psi \sigma^{-1} \hat{G}_S^N. \tag{29}$$

Figure 7 shows the effect of increasing government spending. Increasing $\hat{G}_S^N$ shifts out the AD equation, stimulating both output and prices. At the same time, however, it shifts out the AS equation as we discussed before, so there is some deflationary effect of the policy, which arises from an increase in the labor supply of workers. This effect, however, is too small to overcome

[23] This equilibrium form of policy is derived from microfoundations in Eggertsson (2008a) assuming a Markov perfect equilibrium.

the stimulative effect of government expenditures. In fact, solving these two equations together, we can show that the effect of government spending is always positive and always greater than 1. Solving (28) and (29) together yields the following multiplier:[24]

$$\frac{\Delta \hat{Y}_S}{\Delta \hat{G}_S^N} = \frac{(1-\mu)(1-\beta\mu) - \mu\kappa\psi}{(1-\mu)(1-\beta\mu) - \sigma\mu\kappa} > 1, \tag{30}$$

i.e., one dollar of government spending, according to the model, has to increase output by more than 1. In our numerical example, the multiplier is 2.3, i.e., each dollar of government spending increases aggregate output by 2.3 dollars. Why is the multiplier so large? The main cause of the decline in output and prices was the expectation of a future *slump* and *deflation*. If the private sector expects an increase in future government spending in all states of the world in which the zero bound is binding, contractionary expectations are changed in all periods in which the zero bound is binding, thus having a large effect on spending in a given period. Thus, expectations about future policy play a key role in explaining the power of government spending, and a key element of making it work *is to commit to sustain the spending spree until the recession is over*. One of the consequences of expectations driving the effectiveness of government spending is that it is not of crucial importance if there is an implementation lag of a few quarters. It is the announcement of the fiscal stimulus that matters more than the exact timing of its implementation. This is in sharp contrast to old-fashioned Keynesian models (see Christiano et al (2009) for further discussion of this point).

The 5 percent and 95 percent posterior bands for the government spending correspond to 1.41 and 3.3. Thus, while the government spending multiplier cannot be smaller than 1, it can be much larger, and there is even 5 percent of the posterior for the multiplier larger than 3.3, given the prior distribution for the parameters we assume (and that are explained in the Appendix). Eggertsson and Denes (2009) explain in more detail the parameter configurations that give rise to such large multipliers. As can be seen in expression (30), this occurs when the denominator is close to zero, i.e., when the AD and AS curves are close to parallel in Figure 2. As $(1-\mu)(1-\beta\mu) - \sigma\mu\kappa$ approaches zero, the multiplier approaches infinity in the limit.

9 The case for a temporary sale tax holiday

Not all temporary tax cuts are contractionary in the model. Perhaps the most straightforward expansionary one is a cut in sales taxes.[25] Observe that, according to the AD and AS equations (10) and (11), the sales tax enters these two equations in exactly the same form as the negative of government spending, except that it is multiplied by the coefficient $\sigma\chi^s$. Hence, the analysis from the last section about the expansionary effect of increases in government spending goes through

[24] Note that the denominator is always positive according to C2. See the discussion in footnote 18.

[25] This is essentially Feldstein's (2002) idea in the context of Japan, although he suggested that Japan should *commit to raising future VAT*. As documented below, there are some subtle reasons for why VAT may not be as well suited for this proposal because of how they typically interact with price frictions.

unchanged by replacing $\hat{G}_t^N$ with $-\sigma\chi^s\hat{\tau}_t^s$, and we can use both the graphical analysis and the analytical derivation of the multiplier from the last section.

Why do sales tax cuts increase demand? A temporary cut in sales taxes makes consumption today relative to the future cheaper and thus stimulates spending. Observe also that it increases the labor supply because people want to work more because their marginal utility of income is higher. The relative impact of a 1 percent decrease in the sale tax versus a 1 percent increase in spending depends on $\sigma\chi^s$ and, in the baseline calibration, because $\sigma\chi^s < 1$, sales tax cuts have a bigger effect in the numerical example.

One question is of practical importance: Is reducing the sales tax temporarily enough to stimulate the economy out of the recession in the numerical example? In the baseline calibration, it is not, because it would imply a cut in the sales tax rate over 10 percent. Since sales takes in the U.S. are typically in the range of 3-8 percent, this would imply a large *sales subsidy in the model*. A subsidy for consumption is impractical, because it would give people the incentive to sell each other the same good ad infinitum and collect subsidies (although there may of course be ways of getting around this). However, the case for a temporary sales tax holiday appears relatively strong in the model and could go a long way toward eliminating the recession in the model. Another complication with sales taxes in the U.S. is that they are collected by each individual state, so it might be politically complicated to use them as a stimulative device.

It is worth pointing out that the model may not support the policy of cutting value added taxes (VAT). As emphasized by Eggertsson and Woodford (2004), VAT of the kind common in Europe enter the model differently from American sales taxes, because of how VAT typically interact with price frictions. We assumed in the case of sale taxes that firms set their price exclusively of the tax, so that a 1 percent reduction in the tax will mean that the customer faces a 1 percent lower purchasing price for the goods he/she purchases even if the firms themselves have not revised their own pricing decisions. This assumption is roughly in line with empirical estimates of the effect of variations in sales taxes in the United States; see, e.g., Poterba (1996). This assumption is maybe less plausible for VAT, however, because posted prices usually include the tax (often by law). Let us then suppose the other extreme, as in Eggertsson and Woodford (2004), that the prices the firms post are inclusive of the tax. In this case, if there is a 1 percent decrease in the VAT, this will only lead to a decrease in the price the consumer face if the firms whose goods they are purchasing have revisited their pricing decision (which only happens with stochastic intervals in the model). As a consequence, as shown in Eggertsson and Woodford (2004), the VAT shows up in the AS and AD equations exactly in the same way as the *payroll tax,* so that the analysis in Section 7 goes through unchanged. The implication is that while I have argued that cutting sales taxes is expansionary, cutting VAT works in exactly the opposite way, at least if we assume that the pricing decisions of firms are made inclusive of the tax. The intuition for this difference is straightforward. Sales tax cuts stimulate spending because a cut implies an immediate drop in the prices of goods, and consumers expect them to be relatively more expensive as soon as the recession is over. In contrast, because VAT are included in the posted price, eliminating them will show up in prices only once the firm revisits its price (which happens with a stochastic

probability). This could take a some time. As a consequence, people may hold off their purchases to take advantage of lower prices in the future. In any event, the sensitivity of the effect of tax policy to the nature of price rigidities (that is if taxes are included or not in the posted price), and the absence of much empirical or theoretical literature on it, suggests a relatively big unexplored research area that may be of practical policy importance.

10 Why capital tax cuts can be contractionary

So far, we have only studied variations in taxes on labor and consumption expenditures. A third class of taxes are taxes on capital, i.e., a tax on the financial wealth held by households. In the baseline specification, I included a tax that is proportional to aggregate savings, i.e., the amount people hold of the one-period riskless bond, through τ_t^A, and then I assumed there was tax τ_t^P on dividends. Observe that even if the firm maximizes profits net of taxes, τ_t^p, it drops out of the first-order approximation of the firm Euler equation (AS). Capital taxes thus appear only in the AD equation through τ_t^A.

Consider, at positive interest rates, a tax cut in period t that is reversed with a probability $1-\rho$ in each period. A cut in this tax will reduce demand, according to the AD equation. Why? Because saving today is now relatively more attractive than before and this will encourage households to save instead of consume. This means that the AD curve shifts backward in Figure 3, leading to a contraction in output and a decline in the price level. The multiplier of cutting this tax is given by

$$\frac{\Delta \hat{Y}_S}{-\Delta \hat{\tau}_S^A} = -\chi^A \frac{\sigma\left(1-\rho\beta\right)}{1-\rho+\sigma\phi_y+\sigma(\phi_\pi-\rho)\kappa} < 0$$

and is equal to -0.0013 in our numerical example, a small number. Recall that, in reporting this number, I have scaled $\hat{\tau}_S^A$ so that a 1 percent change in this variable corresponds to a tax cut that is equivalent to a cut in the tax on real *capital income* of 1 percent per year in steady state (see footnote 13).

This effect is much stronger at zero interest rates. As shown in Figure 8, a cut in the tax on capital shifts the AD curve backward and thus again reduces both output and inflation. The multiplier is again negative and given by

$$\frac{\Delta \hat{Y}_S}{-\Delta \hat{\tau}_S^A} = -\chi^A \frac{1-\beta\mu}{(1-\mu)(1-\beta\mu)-\mu\sigma\kappa} < 0.$$

In this case, however, the quantitative effect is much bigger and corresponds to -0.1 in our numerical example. This means that a tax cut that is equivalent to a 1 percent reduction in the tax rate on capital income (in steady state) reduces output by -0.1 percent.

Observe that the contractionary effect of capital tax cuts is prevalent at either positive or zero interest rates (although it is virtually absent up to two decimal point at positive interest rates). It is worth pointing out, however, that in principle the central bank can fully offset this effect at positive interest rates by cutting the nominal interest rates further, so the degree to which

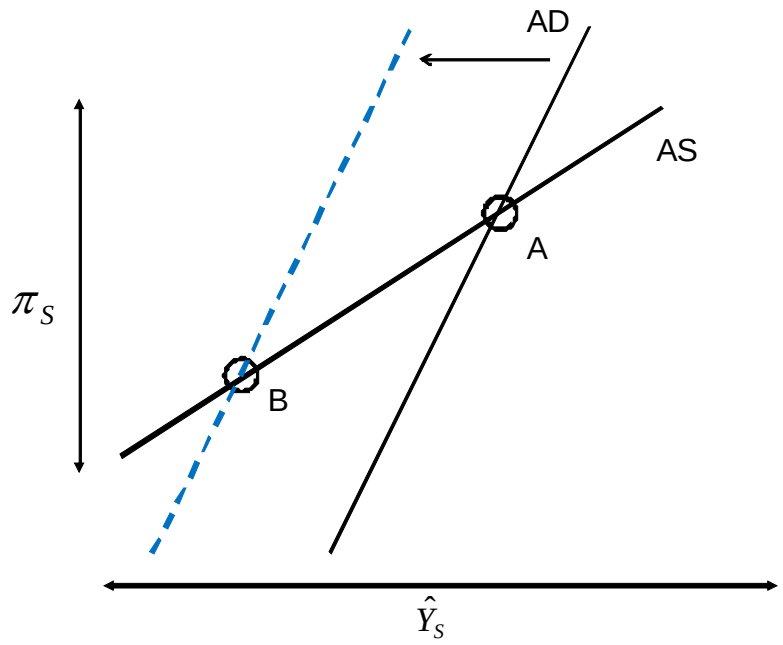


Figure 8: The effect of cutting capital taxes.

this is contractionary at positive interest rates depends on the reaction function of the central bank.[26] Accommodating this tax cut, however, is not feasible at zero interest rates. This tax cut is therefore always contractionary at zero short-term interest rates.

There is an important institutional difference between the capital tax in the model and capital taxes in the U.S. today. The tax in the model is a tax on the *stock* of *savings.* The way in which capital taxes work in practice, however, is that they are a tax on *nominal capital income.* Let us call a tax on nominal capital income τ_t^{AI}. In the case of a one-period riskless bond, therefore, the tax on nominal capital income τ_t^{AI} is equivalent to the tax on financial assets in the budget constraint (2) if we specify that tax as

$$\tau_t^A = \frac{i_t}{1+i_t}\tau_t^{AI}.$$

We can then use our previous equations to study the impact of changing taxes on capital income. Observe, however, *that at zero interest rates this tax has to be zero by definition, because at that point the nominal income of owning a one-period risk-free bond is zero.* The relevant tax rate τ_t^A on one-period bonds – which is the pricing equation that matters for policy – is therefore constrained to be zero under the current institutional framework in the U.S. Hence this tax instrument cannot be used absent institutional changes. It follows that the government would need to rewrite the

[26] In the rule we assume once $\phi_\pi \to \infty$ there is no effect. If there is a time-varying coefficient in the Taylor rule that depends on taxes there can also be no effect.

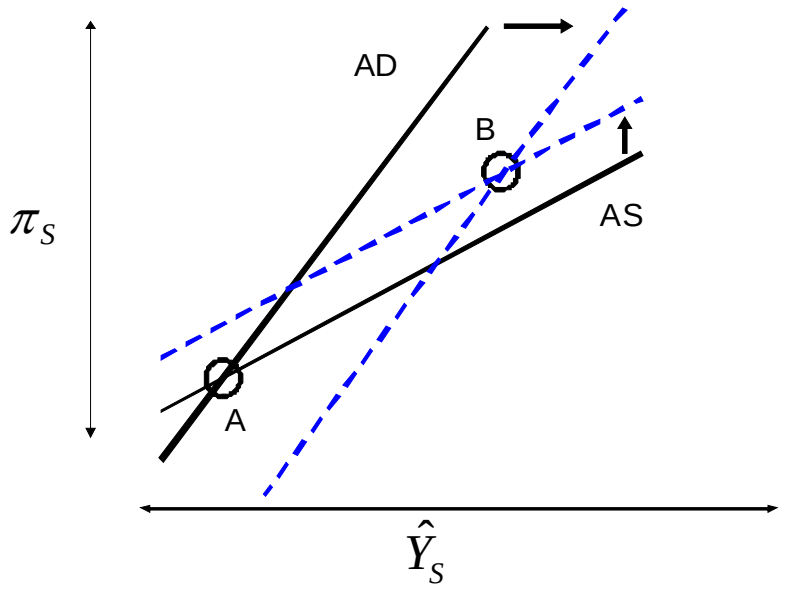


Figure 9: Commitment to inflate at zero nominal interest rates.

tax code and directly tax savings if it wants to stimulate spending by capital tax increases, a proposal that may be harder to implement than other alternatives outlined in this paper.

One argument in favor of cutting taxes on capital is that, in equilibrium, savings is equal to investment, so that higher savings will equal higher investment spending and thus can stimulate demand. Furthermore, higher capital increases the capital stock and thus the production capacities of the economy. In the baseline specification, we have abstracted from capital accumulation. Hence a cut in capital taxes reduced the willingness of consumers to consume at given prices without affecting investment spending or the production capacity of the economy.

Section 14 considers how our results change by explicitly modeling investment spending. This enriched model, however, precludes closed-form solutions, which is why I abstract from capital accumulation in the baseline model. To preview the result, I find that capital accumulation does not affect the results in a substantive way. It does, however, allow us to consider investment tax credits and also how taxes on savings affect aggregate savings, which will fall in response to tax cuts. It thus also puts a nice structure on the old Keynesian idea of the paradox of thrift.

11 The scope for monetary policy: A commitment to inflate and credibility problems

Here, I consider another policy to increase demand: a commitment to inflate the currency. Expansionary monetary policy is modeled as a commitment to a higher growth rate of the money supply in the future, i.e., at $t \geq T^e$. As shown by several authors, such as Eggertsson and Woodford (2003) and Auerbach and Obstfeld (2005), it is only the expectation about future money supply (once the zero bound is no longer binding) that matters at $t < T^e$ when the interest rate is zero. Consider the following monetary policy rule:

$$i_t = \max\{0, r_t^e + \pi^* + \phi_\pi(\pi_t - \pi^*) + \phi_y(\hat{Y}_t - \hat{Y}^*)\}, \tag{31}$$

where π^* denotes the implicit inflation target of the government and $\hat{Y}^* = (1-\beta)\kappa^{-1}\pi^*$ is the implied long-run output target. Under this policy rule, a higher π^* corresponds to a credible inflation commitment. Consider a simple money constraint as in Eggertsson (2008a), $M_t/P_t \geq \chi Y_t$, where M_t is the money supply and $\chi > 0$. A higher π^* corresponds to a commitment to a higher growth rate of the money supply in $t \geq T^e$ at the rate of π^*. The assumption about policy in (13) is a special case of this policy rule with $\pi^* = 0$.

What is the effect of an increase in the inflation target? It is helpful to write out the AD and AS equations in periods $0 < t < \tau$ when the zero bound is binding:

$$AD \qquad \hat{Y}_S = \mu\hat{Y}_S + (1-\mu)\hat{Y}^* + \sigma\mu\pi_S + \sigma(1-\mu)\pi^* + \sigma r_S^e \tag{32}$$

$$AS \qquad \pi_S = \kappa\hat{Y}_S + \beta\mu\pi_S + \beta(1-\mu)\pi^*. \tag{33}$$

Consider the effect of increasing $\pi^* = 0$ to a positive number $\pi^* > 0$. As shown in Figure 9, this shifts the AD curve to the right and the AS curve to the left, increasing both inflation and output. The logic is straightforward: A higher inflation target in period $t \geq T^e$ reduces the real rate of interest in period $t < T^e$, thus stimulating spending in the depression state. This effect can be quite large, owing to a similar effect as described in the case of fiscal policy. The effect of π^* not only increases inflation expectations at dates $t \geq T^e$, but also increases inflation in all states of the world in which the zero bound is binding. In general equilibrium, the effect of inflating the currency is very large for this reason.

Expansionary monetary policy can be difficult if the central bank cannot commit to future policy. The problem is that an inflation promise is not credible for a discretionary policy maker. The welfare function in the model economy is given by the utility of the representative household, which to a second order can be approximated as[27]

$$E_t \sum_{t=0}^{\infty} \beta^t \{\pi_t^2 + \lambda_Y \hat{Y}_t^2 + \lambda_G (G_t^N)^2\}.$$

[27] See, e.g., Eggertsson and Woodford (2004). Our assumption about the shocks is such that $\hat{Y}_t^* = 0$ in their notation. See the discussion in Section 1.2 of that paper and also Eggertsson (2008a), who discusses this assumption in some detail.

The central bank has an incentive to promise future inflation at date $t < T^e$, but then to renege on this promise at data $t \geq T^e$ since at that time the bank can achieve both zero inflation and set output at trend, which is the ideal state of affairs according to this welfare function. This credibility problem was first shown formally in Eggertsson (2006) that coins it the "deflation bias" of discretionary monetary policy at zero interest rates. Government spending does not have this problem. In fact, the policy under full discretion will take exactly the same form as the spending analyzed in Section 8 (see, e.g., Eggertsson (2001,2004, 2006a), who analyzes the Markov perfect equilibrium). The intuition is that fiscal policy not only requires promises about what the government will do in the future, but also involves direct actions today. And those actions are fully consistent with those the government promises in the future (namely, increasing government spending throughout the recession period).

It seems quite likely that, in practice, a central bank with a high degree of credibility, can make credible announcements about its future policy and thereby have considerable effect on expectations. Moreover, many authors have analyzed explicit steps, such as expanding the central bank balance sheet through purchases of various assets such as foreign exchange, mortgage-backed securities, or equities, that can help make an inflationary pledge more credible (see, e.g., Eggertsson (2006), who shows this in the context of an optimizing government, and Jeanne and Svensson (2004), who show that an independent central bank that cares about its balance sheet can also use real asset purchases as a commitment device). Finally, if the government accumulates large amounts of nominal debt, this, too, can be helpful in making an inflation pledge credible. However, the assumption of no credible commitment by the central bank, as implied by the benchmark policy rule here, is a useful benchmark for studying the usefulness of fiscal policy in the worst case scenario in which monetary policy looses its bite.

12 Equilibrium allocations when policy changes are expected to be permanent

A key assumption in our policy exercises has been that the tax cuts, or government expansion, are temporary short-run policies. As soon as the fundamental shock, r_S^e, reverts back to its long-run value, we have assumed that policy does so as well. This is natural, since much of the recent discussion has been about the appropriate "stimulus packages" in response to the economic crisis of 2008. One obvious question, however, is how do our result change, if we assume that the policy change is permanent? Not only is this helpful to clarify through what mechanism short-run stabilization policy works in the model, it also helps illustrating the difference between the result in this paper and some recent work by Cogan et al (2009) that argue that the multiplier of government spending is small in a model that bears close resemblance to this one. They assume that the expansion in government spending is permanent. As we will see this distinction is crucial. It is so important, in fact, that we can even get a negative multiplier for government spending at zero interest rate, if it is expected to be permanent, flipping our previous result on its head.

Let me stress right up-front, however, that I do not think that the numerical examples reported here are a realistic description of what would happen in response to tax or spending changes, for reason I clarify at the end of the section. They are, however, quite helpful in order to unwrap the mechanism of fiscal policy in this class of models and illustrates how important the persistence of policy can be.

The next two propositions and Table 4 summarize the answer to the following question: How does a permanent change in taxes and spending affect output? The propositions show both the short and long run effect of the policy. In our previous analysis the long-run effect of policy was zero since we assumed that all instruments would revert back to steady state in the long run. In contrast, permanent effects are now important, since the policy change is permanent. The proposition and the table also shows short-run multipliers which are comparable to the statistics we reported earlier in the paper.

Proposition 1 In the long run, $t \geq T^e$, there is a unique bounded solution such that $\pi_t = \pi_L$, $\hat{Y}_t = \hat{Y}_L$ and $i_t = i_L$ where

$$\hat{Y}_L = \frac{(1-\beta)\hat{\tau}_L^A - \kappa\psi(\phi_\pi - 1)\chi^w\hat{\tau}_L^w + \kappa\psi\sigma^{-1}(\phi_\pi - 1)[\hat{G}_L^N - \sigma\chi^s\hat{\tau}_L^s]}{\kappa(\phi_\pi - 1) + (1-\beta)\phi_y}$$

$$\pi_L = \frac{\kappa\chi^A\hat{\tau}_L^A + \kappa\psi\phi_y\chi^w\hat{\tau}_L^w - \kappa\psi\sigma^{-1}\phi_y[\hat{G}_L^N - \sigma\chi^s\tau_L^s]}{\kappa(\phi_\pi - 1) + (1-\beta)\phi_y}$$

$$i_L = \bar{r} + \phi_\pi\pi_L + \phi_y\hat{Y}_L$$

Proposition 2 In the short run, $t < T^e$, then we consider two cases:

1.(*Positive interest rates in the short run*) If C1 does *not* hold, then there is a unique bounded equilibrium at positive interest rates such that

$$\hat{Y}_S = \frac{(1-\mu)(1-\mu\beta)\hat{Y}_L + (1-\mu)\pi_L + (\phi_\pi - \mu)\kappa\psi[\hat{G}_L^N - \sigma\chi^s\hat{\tau}_L^s - \sigma\chi^w\hat{\tau}_L^w] + (1-\mu\beta)\chi^A\hat{\tau}_L^A}{(1-\mu+\sigma\phi_y)(1-\mu\beta) + \sigma\kappa(\phi_\pi - \mu)}$$

$$\begin{aligned}\pi_S &= \frac{(1-\mu)\kappa\hat{Y}_L + (1-\mu)[\kappa\sigma + \beta(1-\mu+\sigma\phi_y)]\pi_L}{(1-\mu\beta)(1-\mu+\sigma\phi_y) + \sigma\kappa(\phi_\pi - \mu)} \\ &\quad \frac{-\kappa\psi\sigma^{-1}(1-\mu+\sigma\phi_y)[\hat{G}_L^N - \sigma\chi_L\hat{\tau}_L^s] + \sigma\kappa\chi^A\hat{\tau}_S^A + \kappa\psi(1-\mu+\sigma\phi_y)\chi^w\hat{\tau}_L^w}{(1-\mu\beta)(1-\mu+\sigma\phi_y) + \sigma\kappa(\phi_\pi - \mu)}\end{aligned}$$

$$i_S = r_S^e + \phi_\pi\pi_S + \phi_y\hat{Y}_S$$

2. (*Zero interest rates in the short run*) If C1 *and C2 hold,* there is a unique bounded

equilibrium at zero interest rates such that

$$\begin{aligned}\hat{Y}_S &= \frac{(1-\mu\beta)(1-\mu)}{(1-\mu\beta)(1-\mu)-\sigma\mu\kappa}\hat{Y}_L + \frac{\sigma(1-\mu\beta)}{(1-\mu\beta)(1-\mu)-\sigma\mu\kappa}\pi_L + \sigma(1-\mu\beta)r_S^e] \\ &+\frac{\sigma\mu\kappa\psi}{(1-\mu\beta)(1-\mu)-\sigma\mu\kappa}\chi^w\hat{\tau}_L^w - \frac{\mu\kappa\psi}{(1-\mu\beta)(1-\mu)-\sigma\mu\kappa}[\hat{G}_L^N + \sigma\chi^s\hat{\tau}_L^s] \\ &+\frac{\sigma(1-\mu\beta)}{(1-\mu\beta)(1-\mu)-\sigma\mu\kappa}\chi^A\hat{\tau}_L^A\end{aligned}$$

$$\begin{aligned}\pi_S &= \frac{\kappa(1-\mu)}{(1-\mu\beta)(1-\mu)-\mu\sigma\kappa}\hat{Y}_L + \frac{(1-\mu)[\sigma\kappa+(1-\mu)\beta]}{(1-\mu\beta)(1-\mu)-\mu\sigma\kappa}\pi_L + \frac{\sigma\kappa}{(1-\mu\beta)(1-\mu)-\mu\sigma\kappa}r_S^e \\ &+\frac{\sigma\kappa}{(1-\mu\beta)(1-\mu)-\mu\sigma\kappa}\chi^A\hat{\tau}_L^A + \frac{\kappa\psi(1-\mu)}{(1-\mu\beta)(1-\mu)-\mu\sigma\kappa}\chi^w\hat{\tau}_L^w \\ &-\frac{\kappa\psi\sigma^{-1}(1-\mu)}{(1-\mu\beta)(1-\mu)-\mu\sigma\kappa}[\hat{G}_L^N - \sigma\chi^s\hat{\tau}_L^s]\end{aligned}$$

$$i_S = 0.$$

Proof See Appendix.

Table 4: Multipliers of permanent policy changes[28]

	Short-run	Short-run	Long-run	Long-run
	$\hat{Y}_S$ multiplier $i_t > 0$ (5%,95%)	$\hat{Y}_S$ multiplier $i_t = 0$ (5%,95%)	$\hat{Y}_L$ multiplier (5%,95%)	π_L multiplier (5%,95%)
$\tau_t^w \downarrow$	0.0195 (0.0092,0.0384)	−2.5924 (-3.8469,0.5443)	0.4207 (0.2116, 0.6079)	−0.1052 (-0.1520,-0.0529)
$G_t^S \uparrow$	**0** (0)	**0** (0)	0 (0)	0 (0)
$G_t^N \uparrow$	0.0181 (0.0099,0.0323)	−2.4055 (-3.5461,-0.5926)	0.3904 (0.2116,0.5524)	−0.0976 (-0.1381,-0.0529)
$\tau_t^S \downarrow$	0.0149 (0.0070,0.0292)	−1.9751 (-2.9310,-0.4147)	0.3205 (0.1612,0.4632)	−0.0801 (-0.1158,-0.0403)
$\tau_t^A \downarrow$	0 (0,0)	−0.2495 (-0.4348,-0.0842)	0.4207 (0.2116,0.6079)	−0.0056 (-0.0091,-0.0035)

To focus the discussion, I mainly discuss the effect of a permanent expansion in government spending below. I then briefly comment on permanent variations in the other policy instruments. Consider first the long run effect of an increase in government spending. According to Proposition 3 it is

$$\hat{Y}_L = \frac{\kappa\psi\sigma^{-1}(\phi_\pi - 1)}{\kappa(\phi_\pi - 1) + (1-\beta)\phi_y}\hat{G}_L^N \tag{34}$$

[28] Posterior bands to be added in a revised version of the paper.

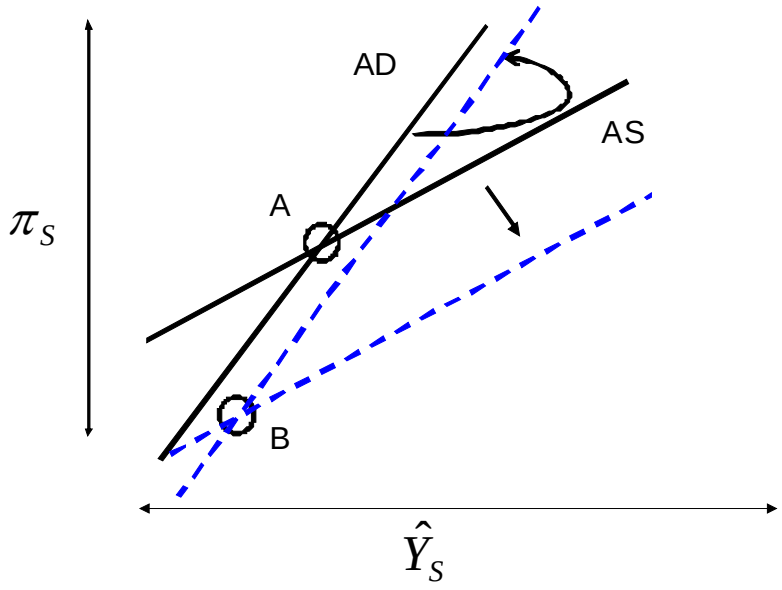


Figure 10: Permanent increase in government spending that is contractionary.

$$\pi_L = -\frac{\kappa\psi\sigma^{-1}\phi_y}{\kappa(\phi_\pi - 1) + (1-\beta)\phi_y}\hat{G}_L^N \tag{35}$$

that is a permanent increase in government spending raises permanent output (because people will work more to make up for lost private consumption) and it lowers inflation permanently (because output is now above steady state). This latter effect is highly dependent on the monetary policy commitment, e.g., it disappears if either $\phi_\pi \to \infty$ or $\phi_y \to 0$ in the policy rule. The long run multipliers in our baseline calibration are shown in Table 3. Of even greater interest, however, is how does this change our short-run analysis?

Consider the AD and the AS equations, where short run spending is $\hat{G}_S$, long run spending $\hat{G}_L$, and the long run effect of policy on output and inflation is captured by $\hat{Y}_L, \pi_L$ computed from the formulas (34) and (35)

$$AD \qquad \hat{Y}_S = \mu\hat{Y}_S + (1-\mu)\hat{Y}_L - \sigma\mu\pi_S + \sigma(1-\mu)\pi_L + \sigma r_S^e + \{\hat{G}_S^N - \mu G_S^N + (1-\mu)G_L^N\} \tag{36}$$

$$\pi_S = \kappa\hat{Y}_S - \kappa\psi\sigma^{-1}\hat{G}_S^N + \beta\mu\pi_L + \beta(1-\mu)\pi_S \tag{37}$$

Again we can plot these two curves up in $(\hat{Y}_S, \pi_S)$ space and their slope is identical to our earlier analysis as shown in Figure 10. What differs is how fiscal spending shifts these two curves. The first thing to observe is that if the increase in government spending is permanent then $\hat{G}_S^N = G_L^N$ and the last term in the AD equation disappears. Hence the spending effect we considered in section 8 on aggregate demand is simply not there. The reason for this clarifies why government

spending had an effect in our previous exercise. The main problem at the zero bound is insufficient spending, i.e. the economy "needs" negative real interest rate, which calls for higher spending today. However, this is not "insufficient spending" in the abstract. It refers to spending in the short run *relative* to the long run. If the government increase spending both in the long and the short run there is no effect on spending in the short run relative to the long run and thus no effect on demand.

For aggregate demand, however, the story does not end here. In our previous example we assumed that in the long run things would go back to steady state, hence $\hat{Y}_L = \pi_L = 0$. With a permanent expansion in the government this is no longer the case, and this very fact has an influence on expectations, captured by $(1-\mu)\hat{Y}_L$ and $(1-\mu)\pi_L$, and thus *aggregate demand in the short run.* On the one hand, the permanent increase in government spending will increase labor supply because it takes output away from consumption to government use (thus increasing marginal utility of private consumption) and thereby increasing $\hat{Y}_L$ and shifting out demand as shown in figure 10. On the other hand there may be an effect on inflation in the long run, depending on how monetary policy reacts to this increase in long run output. Our monetary commitment in equation 13 implies that monetary policy will offset the output increase to some extent thus leading to a lower π_L thus moving back demand in figure 10. What is the net effect on demand? This depends on the parameters, as seen by the formulas in Proposition 3. The total effect can be either positive or negative. Table 3 suggests that for the mode of our baseline calibration the net effect is $\frac{\Delta Y_L}{\Delta G_L}+\sigma\frac{\Delta \pi_L}{\Delta G_L} = 0.3063$, a small but nontrivial number. We can compare this to the demand effect we saw in section 8, where demand moved one to one with government spending increase (in that case $\frac{\Delta Y_L}{\Delta G_L}+\sigma\frac{\Delta \pi_L}{\Delta G_L} = 0$ and the effect was completely driven by change in spending in the short run vs the long run). The demand effect of temporary government spending in response to the shock, in other words, *is more than three times greater than if the change is expected to be permanent!*

This is only half of the story. Let us now turn to the AS equation. As before the increase in government spending shifts the AS curve out. Given the upward sloping AD curve, however, this is contractionary in the short run as we have already discovered. On top of this we have the effect of lower long-run inflation expectations, $\mu\pi_L$, which shifts the AS curve out even further. The question now becomes which force is stronger in the short run, the contractionary effect of government spending due to its negative effect on short-run inflation expectations due the increase in aggregate supply it causes, or the positive effect of permanent spending on long run output expectations. The answer is in Table 3. In our numerical example the short-run effect of a permanent increase in government spending is contractionary according to the mode of our baseline calibration. Thus while temporary government spending is extremely effective to battle a recession, a permanent spending increase is contractionary! Observe that the contractionary effect of a permanent increase in government spending hinges very much on the fact that it leads to a decline in long-run inflation expectations, i.e. $\frac{\Delta \pi_L}{\Delta G_L} < 0$. This effect was driven by the monetary policy commitment (13). We can abstract away from this channel by assuming that $\phi_\pi \to \infty$ or $\phi_y \to 0$. In this case a permanent increase in government spending is no longer contractionary

but has a small positive effect. The multiplier is 0.4250. The small, or even negative, effect of a permanent increase in government spending explains why Cogan et al (2009) report fiscal multipliers that are much smaller than I reported in section 8. They assume that the expansion in fiscal spending is permanent. Note that permanent sales tax cuts have exactly the same effect as government spending, but weighted by $\sigma\chi^s$.

Moving to other tax cuts, similar forces are at work for labor taxes as before, i.e., in the short run a labor tax cut increases aggregate supply and creates short-run deflationary expectations. Through that mechanism a cut in labor taxes is contractionary as before. Now, however, there is an effect of these tax cuts on long run expectations, and this can have an effect on both supply and demand. Lets focus on the demand side since this is what is new in this case. The effect comes about, as with government spending, through the term $\frac{\Delta\hat{Y}_L}{\Delta\tau_L^w} + \sigma\frac{\Delta\pi_L}{\Delta\tau_L^w} = 0.2997$ so that on net there is an increase in demand due to higher expectations of future income (because labor taxes are lower in the long run output in the long run is also higher). As before, however, the total effect of a permanent decrease in labor taxes *in the short run* is negative, as can be seen in Table 3. Finally we see that the multiplier of capital tax cuts is again negative but small. This is of less interest, since if one wants to take the long-run effects of capital taxes seriously, we need to take endogenous capital accumulations into account, which we do in section 14.

Let me conclude this section by emphasizing that I do not consider the results reported Table 4 as a reliable estimator of a "stimulus" for at least two reasons. The first, and perhaps less important, is that it seems to me to be unrealistic to assume that, for example, an increase in government spending is permanent. The fiscal stimulus in 2008 (and 1933), for example, involved a fair number of infrastructure projects which clearly seemed to be aimed at temporarily creating employment (one could argue that some entitlements spending, e.g. due to health care reforms, have a stronger persistence to them. Those type of spending, however, would seem more accurately described as an increase in $\hat{G}_t^S$ which is substitutable with private consumption and has no effect on inflation and output). The second reason to view Table 4 with some caution is that a large part of the effect is coming from the long term monetary commitment, i.e., the effect of the permanent increase in spending on long term inflation π_L. One might argue, instead, that π_L should be some constant number that does not depend on policy (e.g. because monetary policy only responds to the deviation of output from its "natural/flexible price" level). In any event, the general point is that once we start thinking about permanent policy changes, their effect depends very much on how those policy changes affect the long-term stance of monetary policy. That assumption need to be explicit and a key part of the analysis.

13 The consequence of deficit spending

So far I have not modelled the dynamics of the government budget constraint. The reason I could get away with this in the context of the model, is because I assumed lump-sum taxes and that any variation in government spending or one of the tax instrument would be offset by current or future lump-sum taxes. The dynamics of debt, therefore, play no role since the timing of

lump-sum taxes is irrelevant due to Ricardian equivalence.

More realistically, variation in taxes and spending are not counteracted with current and/or future lump sum taxes. Instead, it is likely, that an increase in debt today will be offset by future distortionary taxes or a reduction in future government spending. How might this change the results? Will it lead us to favor or disfavor deficit spending?

The answer depends on what sort of theory we have of how the future debt will be paid off, i.e., with what combinations of taxes increases and/or spending cuts. Eggertsson (2006), for example, points out that if the debt is nominal then future governments have an incentive to inflate. He analyzes a Markov Perfect Equilibrium of the game between the government and the private sector. Since one of the main goal of policy in the short run in the model is to counteract deflationary expectations, increasing future inflation expectations through higher nominal debt can have a large positive effect on demand in the short-run. Eggertsson (2008a) argues that this mechanism may in fact have played a role in the turnaround in 1933 when the US started recovering from the Great Depression, moving from two digit deflation into modest inflation. It is worth stressing, however, that deficit spending can also have some contractionary effects. To the extent that future debt will be paid off by taxes that reduce output in the long run, this may have a negative effect on demand in the short run. Sorting out various different channels and mechanism through which the dynamics of the governments budget constraint feed into expectations of future taxation, inflation and output is an important topic for future research and may have non-trivial effect on the estimates of fiscal multipliers in general equilibrium. Finally it is worth stressing that our analysis suggested that within the model it is actually possible to compute a balance budget stimulus package which is even more stimulatory than tax cuts or spending increases that are financed by lump-sum taxes. An example of this is a temporary cut in sales taxes (which is expansionary) that is financed by a simultaneous increase in labor taxes or capital taxes (both of which are also expansionary in the model). As noted in the introduction, however, there may be some reasons out of the model which makes such a policy counterproductive (e.g. liquidity constraints on both firms and consumers).

14 The paradox of thrift

I conclude the paper by illustrating the paradox of thrift in the extended model that allows for endogenous capital. Besides confirming our previous conclusion and illustrating the paradox of thrift, this model is also interesting because it allows us to consider investment tax credits and also gives a more important role for varying taxes on profits. Consider now an economy in which each firm uses both capital and labor as inputs in production, i.e., $y_t(i) = K_t(i)^{\gamma} l_t(i)^{1-\gamma}$, and $K_t(i)$ is firm-specific capital. Following Christiano (2004) and Woodford (2003), let us assume that, in order to increase the capital stock to $K_{t+1}(i)$ from $K_t(i)$, the firm invests at time t

$$I_t(i) = \phi(\frac{K_{t+1}(i)}{K_t(i)}, \xi_t) K_t(i),$$

where the function ϕ satisfies $\phi(1,\bar{\xi}) = \zeta$, $\phi^I(1,\bar{\xi}) = 1$, $\phi^{II} \geq 0$, $\phi^{\xi}(1,\bar{\xi}) = 0$ and $\phi^{I\xi}(1,\bar{\xi}) \neq 0$. The variable λ corresponds to the depreciation rate of capital. At time t, the capital stock is predetermined. I allow for the shock to appear in the cost-of-adjustment function. The shock to the cost of adjustment, in addition to taxes, is the only difference relative to Christiano (2004) and Woodford (2003). Accordingly, the description of the model below is brief (readers can refer to these authors for details).

Here, $I_t(i)$ represents purchases of firm i of the composite good, defined over all the Dixit-Stiglitz good varieties, so that we can write

$$y_t(i) = Y_t(\frac{p_t(i)}{P_t})^{-\theta}.$$

Firm i in industry j maximizes present discounted value of profits. The pre-capital-tax profit is

$$Z_t(i)^{pretax} = p_t(i)y_t(i) - W_t(j)l_t(i) - (1+\tau_t^s)P_tI_t(i).$$

However, we assume that profits are taxed at a rate τ_t^P, owing to the tax on dividends. Furthermore, we assume that there is an investment tax credit given by τ_t^I. The tax bill is

$$\tau_t^P[p_t(i)y_t(i) - P_tn_t(i)h_t(i) - P_td(\frac{p_t(i)}{p_{t-1}(i)}) - (1+\tau_t^I)(1+\tau_t^s)P_tI_t(i)].$$

The firm maximizes after-tax profits by its choice of investment and its price. Let us denote $I_t^N(i) \equiv \frac{K_{t+1}(i)}{K_t(i)}$ as the net increase in the capital stock in each period. Endogenous capital accumulation gives rise to the following first-order condition:

$$\begin{aligned} &-\phi^I(I_t^N(i),\xi_t)(1-\tau_t^P(1+\tau_t^I))(1+\tau_t^s) \qquad (38)\\ &+E_tQ_{t+1}\Pi_{t+1}[\rho_{t+1}(i) + \phi I(I_{t+1}^N(i),\xi_{t+1})I_{t+1}^N(1-\tau_{t+1}^P(1+\tau_{t+1}^I))(1+\tau_{t+1}^s) - \phi(I_{t+1}^A(i),\xi_{t+1})], \end{aligned}$$

where

$$\rho_t(i) \equiv \frac{\gamma}{1-\gamma}\frac{l_t(i)}{K_t(i)}W_t(j)\frac{1-\tau_t^w}{1+\tau_t^s}. \qquad (39)$$

Below, I summarize the equations of the model that define an equilibrium once that model has been approximated around steady state:[29]

$$\hat{C}_t = E_t\hat{C}_{t+1} - \sigma(i_t - E_t\pi_{t+1} - r_t^e - \chi^A\hat{\tau}_t^A) + \sigma\chi^sE_t(\hat{\tau}_{t+1}^s - \hat{\tau}_t^s)$$

$$\begin{aligned} \hat{I}_t^N &= \beta E_t\hat{I}_{t+1}^N - \sigma_I(i_t - E_t\pi_{t+1} - r_t^e - \chi^A\hat{\tau}_t^A) + \chi E_t\hat{\rho}_{t+1}\\ &\quad + \frac{\bar{\tau}^P}{1-\tau^P}[\hat{\tau}_t^I - \beta(1-\lambda)E_t\hat{\tau}_{t+1}^I] + [\chi^P\hat{\tau}_t^P - \beta(1-\lambda)\chi^PE_t\hat{\tau}_{t+1}^P] - \chi^s[\hat{\tau}_t^s - \beta(1-\lambda)E_t\hat{\tau}_{t+1}^s] \end{aligned}$$

$$\hat{\rho}_t = (1+\nu)\hat{L}_t + \sigma^{-1}\hat{C}_t - \hat{K}_t + \chi^s\hat{\tau}_t^s + \chi^w\hat{\tau}_t^w - \chi^p\hat{\tau}_t^P$$

$$\hat{Y}_t = \hat{C}_t + \hat{G}_t + \delta_K\hat{I}_t^N + \lambda\delta_K\hat{K}_t$$

[29] In steady state, we have $\rho = (\beta^{-1} - 1 + \zeta)(1-\tau^P)(1+\tau^s)$, $\frac{K}{Y} = \frac{\alpha}{\rho}\frac{\theta-1}{\theta}(1-\tau^p)$. Note that in these equations I have not kept track of G_t^S, this could be done without changing the results.

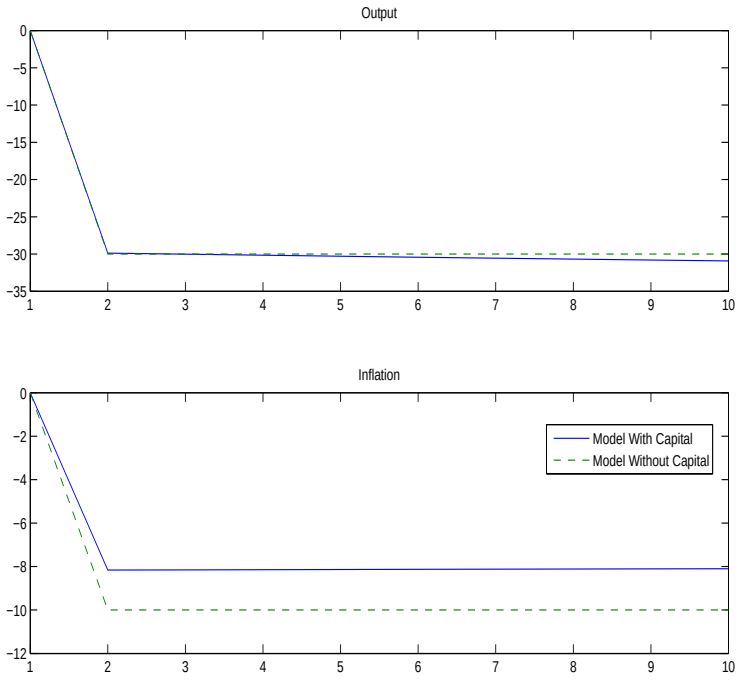


Figure 11: Comparing the model with and without capital

$$\hat{I}_t^N = \hat{K}_{t+1} - \hat{K}_t$$

$$\hat{Y}_t = \gamma \hat{K}_t + (1-\gamma)\hat{L}_t = 0$$

$$\pi_t = \kappa \hat{Y}_t - \kappa\psi\sigma^{-1}[\hat{G}_t + \delta_K \hat{I}_t^N] - \kappa_K \hat{K}_t + \beta E_t \pi_{t+1} + \kappa\psi(\chi^s \hat{\tau}_t^s + \chi^w \hat{\tau}_t^w)$$

where κ_K, ν, σ_I, and the $\chi' s$ are coefficients greater than zero.[30] Observe that, instead of one aggregate demand equation as in previous sections, there are now two Euler equations that determine aggregate demand: the investment Euler equation and the consumption Euler equation. The basic form of the two equations is the same, however; both investment spending and consumption spending depend on the current and expected path of the short-term real interest rate. The firm-pricing Euler equation is the same as in the model without capital, but with an additional term involving the capital stock. An important assumption is that we assume that the shock enters the cost of adjustment of investment, which is a key difference from Christiano (2004). This assumption is consistent with the interpretation that this disturbance is due to banking troubles that raise the cost of loans, which should affect investment spending and consumption spending in the same way.

I do not attempt to estimate the model, but instead do a preliminary calibration, leaving the estimation to future research. I assume the same coefficients as in the model without capital,

[30] σ and ψ are defined as before. Other parameters are defined as follows: $\sigma^I = \frac{(1-\lambda+\rho)}{\phi^{II}} \frac{\beta\rho}{(1-\tau^P)(1+\tau^s)}, \chi \equiv \frac{\beta\rho}{\phi^{II}(1-\tau^P)(1+\tau^s)}$, $\delta_K \equiv \frac{K}{Y}$, $\nu \equiv \frac{v_h h}{v_{hh}}$ (Note that ν and ω are related as follows: $\omega = \frac{\nu}{1-\gamma} + \frac{\gamma}{1-\gamma}$). $\kappa_K \equiv \kappa\psi\frac{\gamma}{1-\gamma}\nu$. The parameter κ is defined in Woodford (2003), and it solves a polynomial defined in that paper.

i.e., I choose parameters so that ω, σ β, and κ correspond to one another in the two models and assume exactly the same value for shocks. I then need to choose values for τ^P, γ, λ, and $\phi_{II.}$. The values are summarized in Table 4. The parameters λ and γ are taken from the literature, but the value for ϕ_{II} is chosen so that the output in the fourth quarter of the "contraction" is -30 percent. (It is assumed that investment declines in the same proportion as consumption). Figure 11 compares the dynamics of output and inflation in the model with and without endogenous capital stock. They are almost identical, although the deflation is slightly less, reflecting the extra terms in the AS equation with endogenous capital tend to increase marginal costs (and thus limit the deflation). To achieve this fit, the degree of capital adjustment is $\phi_{II} = 71.9$. Future work should include a more systematic analysis of the model, taking investment data more explicitly into account, and explicitly pick the parameters to maximize the posterior of the model, as we did in previous sections. As the figure shows, capital dynamics do not add much to the analysis, at least in terms of inflation and output dynamics. This result is somewhat at odds with the findings of Christiano (2004), who finds that adding capital gives somewhat different quantitative conclusions. The main reason for this may be that I have added similar shocks to the investment Euler equation as to the consumption Euler equation (by adding the shock to the investment adjustment cost), together with the strategy I follow in calibrating the model. Table 5 shows how the multipliers change quantitatively with this extension given the calibration strategy just described. As the table reveals, they do not change much. The difference might even be smaller if we followed the same estimation strategy for the model with capital, as we used with the model with fixed capital stock.

Several things are interesting about this extension apart from confirming the robustness of the previous analysis. Endogenous investment allows us to consider one alternative instrument, i.e., the investment tax credit. Table 5 shows the multiplier of a tax credit: A tax credit that allows firms to deduct one additional percent on top of the purchasing price of their investment from taxable profits would lead to a 0.31 percent increase in output. This expansionary effect occurs because an investment tax credit gives firms an incentive to invest today relative to in the future, thus stimulating spending.

Another interesting statistic is the effect of cutting the tax on savings, τ_t^A. Cutting this capital tax will give consumers an incentive to save more. Since, in equilibrium, savings must be equal to investment, one might expect that this would stimulate investment. The calibrated model, however, gives the opposite conclusion. A 1 percent decrease in $\hat{\tau}_t^A$ at zero interest rates will instead lower investment by 0.2384. The main reason is that even if a lower tax on capital gives the household more incentive to save, it reduces aggregate income at the same time. In equilibrium, this effect is strong enough so that even if each household saves more for a given income, aggregate saving declines. This is the classic paradox of thrift, first suggested by Keynes.

As before, a decrease in $\hat{\tau}_t^A$ results in a reduction in output, of similar order as in the model without capital, and the logic of the result is the same. The effect of cutting the tax on profits now is no longer neutral. If the tax on profit is reduced, then given the way I model this tax, the firm has an incentive to delay investment in order to pay out as much profits as possible at the

lower tax rate now. Hence, to stimulate investment, the government should increase the tax on current profits, with a promise to reduce them in the future. This will give firms an incentive to spend their cash on investment today rather than on dividends payments.

Table 4

	γ	ϕ_{II}	λ	$\bar{\tau}^P$
parameters	0.25	71.935	0.025	0.3

Table 5: Comparing multipliers of temporary policy changes in the model with and without capital

	Without capital $i_t = 0$	With capital $i_t = 0$
τ_t^w(Payroll Tax Cut)	-1.0191	-1.2706
G_t^S(Government Spending 1 Increase)	0	0
G_t^N (Government Spending 2 Increase)	2.2793	2.69
τ_t^S (Sales Tax Cut)	2.5179	-2.73
τ_t^A (Capital Tax Cut)	-0.1012	- 0.0752
τ_t^P (Capital Tax Cut)	—	-0.4670
τ_t^I	—	0.3116

15 Conclusions

The main problem facing the model economy I have studied in this paper is insufficient demand. In this light, the emphasis should be on policies that stimulate spending. Payroll tax cuts may not be the best way to get there. The model shows that they can even be contractionary. What should be done, according to the model? Traditional change in government spending is one approach, although it needs to be targeted and temporary, or sales tax cuts and investment tax credits. Another is a commitment to inflate. Ideally, all should go together. Government spending, sale tax cuts and investment credits have an advantage over inflation policy in that no credibility problems are associated with them. Inflation policy, however, has the advantage of not requiring any public spending or variations in the tax code which may be at its "first best level" in the steady state of the model studied here. Any fiddling around with the tax code should take into account that deflation might be a problem. In that case, shifting out aggregate supply can make things worse.

It is worth stressing that the way taxes are modeled here, although standard, is special in a number of respects. In particular, tax cuts do not have any "direct" effect on spending. The labor tax cut, for example, has an effect only through the incentive it creates for employment and thus "shifts aggregate supply," lowering real wages. One can envision various environments in which tax cuts stimulate spending, such as old-fashioned Keynesian models or models where people have limited access to financial markets. In those models, there will be a positive spending effect of tax cuts, even payroll tax cuts like the ones in the standard New Keynesian model.

It is also worth raising another channel through which tax cuts can stimulate the economy.

Tax cuts would tend to increase budget deficits and thus increase government debt. That gives the government a higher incentive to inflate the economy. As we have just seen in Section 11, higher inflation expectations have a strong positive impact on demand at zero interest rates. Eggertsson (2006) models this channel explicitly. In his model, taxes have no effect on labor supply, but instead generate tax collection costs. In that environment, tax cuts are expansionary because they increase debt and, through that, inflation expectations.

What should we take out of all this? There are two general lessons to be drawn from this paper in my view. The first is that insufficient demand is the main problem once the zero bound is binding, and policy should first and foremost focus on ways in which the government can increase spending. Policies that expand supply, such as some (but not all) tax cuts and also a variety of other policies, can have subtle counterproductive effects at zero interest rates by increasing deflationary pressures. This should – and can – be avoided by suitably designed policy.

The second lesson is that policy makers today should view with some skepticism empirical evidence on the effect of tax cuts or government spending based on post-WWII U.S. data. The number of these studies is high, and they are frequently cited in the current debate. The model presented here, which has by now become a workhorse model in modern macroeconomics, predicts that the effect of tax cuts and government spending is fundamentally different at zero nominal interest rates than under normal circumstances.

16 References

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17 Appendix A: Proofs

17.1 Proof of Proposition 1

For steady state we assume that each of the fiscal instruments are equal to some constant value, that is $\tau_t^s = \bar{\tau}^s$, $\tau_t^w = \bar{\tau}^w$, $\tau_t^P = \bar{\tau}^P$, $G_t = \bar{G}^N$, $G_t^S = \bar{G}^S$. The value of $T_t = \bar{T}$ is then backed out as residual to solve the budget constraint (2) in steady state. Given these values it is easy to verify that the following constants, $\Pi_t = \frac{P_t}{P_{t-1}} = \frac{p_t^\star}{P_t} = \bar{\Pi} = 1$, $Y_t = \bar{Y}$, $l_t(j) = y_t(i) = Y_t = \bar{Y}$, $i_t = \bar{\imath} = \beta^{-1} - 1$, $C_t = C_t = \bar{Y} - G^N - G^S$ solve equations (2)-(9) and the rest of the model where $\bar{Y}$ is the solution to

$$\frac{\theta}{\theta - 1} \frac{1 + \bar{\tau}^s}{1 - \bar{\tau}^W} \frac{v_l(\bar{Y})}{u_c(\bar{Y} - \bar{G}^N)} = 1$$

Observe that there is no need to assume that the steady state is efficient.

The AD equation is a straight forward first order approximation of the two equations (3) and (7_), for details about the accuracy of this approach, which is an application of the implicit function theorem, see Woodford (2003), Appendix A, and for special consideration arizing due to the zero bound see Eggertsson and Woodford (2003). The derivation of the AS equation is slightly more involved, and the main steps are outlined briefly below (see proof of Proposition 3.5 in Woodford (2003) Appendix B.4 for the standard case and more details for each step of the derivation.)

The price index (6) can be approximated to yield

$$\log P_t = \alpha \log P_{t-1} + (1-\alpha)\log p_t^*$$

while an approximation of 5 yields

$$E_t\left\{\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}[(1+\omega\theta)(\log p_t^* - \log P_T) - \chi^s\hat{\tau}_T^s - \chi^w\hat{\tau}_T^s - (\omega+\sigma^{-1})\hat{Y}_T + \sigma^{-1}\hat{G}_T^N]\right\} = 0$$

Solving this euqation for $\log p_t^*$ and subtracting $\log P_t$ from both sides, and expressing the expected change in the price index over various future horizons in terms of expected inflation rate at various future dates, we obtain

$$\begin{aligned}\hat{p}_t^* &= (1-\alpha\beta)E_t\left\{\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}[\sum_{s=t+1}^{T}\pi_s + \frac{1}{1+\omega\theta}\chi^s\hat{\tau}_T^s + \frac{1}{1+\omega\theta}\chi^w\hat{\tau}_T^s + \frac{\omega+\sigma^{-1}}{1+\omega\theta}\hat{Y}_T - \frac{\sigma^{-1}}{1+\omega\theta}\hat{G}_T^N]\right\} \\ &= E_t\left\{\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}[\alpha\beta\pi_{T+1} + \frac{1}{1+\omega\theta}\chi^s\hat{\tau}_T^s + \frac{1}{1+\omega\theta}\chi^w\hat{\tau}_T^s + \frac{\omega+\sigma^{-1}}{1+\omega\theta}\hat{Y}_T - \frac{\sigma^{-1}}{1+\omega\theta}\hat{G}_T^N]\right\}\end{aligned}$$

where $\hat{p}_t^* = \log(p_t^*/P_t)$. This implies that

$$\hat{p}_t^* = \alpha\beta E_t\pi_{t+1} + \frac{(1-\alpha\beta)}{(1+\omega\theta)}\chi^s\hat{\tau}_t^s + \frac{(1-\alpha\beta)}{(1+\omega\theta)}\chi^w\hat{\tau}_t^s + \frac{(\omega+\sigma^{-1})(1-\alpha\beta)}{(1+\omega\theta)}\hat{Y}_t - \frac{\sigma^{-1}(1-\alpha\beta)}{(1+\omega\theta)}\hat{G}_t^N + \alpha\beta E_t\hat{p}_{t+1}^*$$

The linearization of the price index implies (subtracting $\log P_t$ from both sides) suggests that the inflation rate is

$$\pi_t = \frac{1-\alpha}{\alpha}\hat{p}_t^*$$

Substitute this into the equation above, and one obtains the AS equation in the text. QED.

17.2 Proposition 2 and 3

While the steps in the proofs for propositions 2 and 3 should be familiar to most readers., they are provided here for completeness. For the reader's convenience, I first reproduce a result in linear rational-expectations models that is shown in Woodford (2003). I will use his results in my proofs of Propositions 2 and 3.

17.3 Proposition A.1

Consider a linear rational-expectations model of the form

$$E_t z_{t+1} = Az_t + ae_t,$$

where z_t is a two-by-two vector of nonpredetermined endogenous state variables, e_t is a vector of exogenous disturbances, A is a two-by-two matrix of coefficients, and a is two times the number of exogenous disturbances. Rational-expectations equilibrium is determinate if and only if the

matrix A has both eigenvalues outside the unit circle (i.e., with modulus $|\lambda| > 1$). Denote the determinant of the matrix A as $det(A)$ and its trace as $tr(A)$.

The condition for a unique bounded solution is satisfied if and only if

$$\text{Case 1} \quad (a)\ det(A) > 1 \qquad (b)\ det(A) - tr(A) > -1 \qquad (c)\ det(A) + tr(A) > -1$$

$$\text{Case 2} \quad (d)\ det(A) - tr(A) < -1 \quad (e)\ det(A) + tr(A) < -1$$

Proof: See proof to Proposition C.1 in Woodford 2003, p. 670-71.

17.4 Proof of Proposition 2

The system can be written in the form

$$E_t z_{t+1} = A z_t,$$

where $z_t \equiv \begin{bmatrix} \pi_t \\ \hat{Y}_t \end{bmatrix}$ and $A \equiv \begin{bmatrix} \beta^{-1} & -\beta^{-1}\kappa \\ \sigma(\phi_\pi - \beta^{-1}) & 1 + \sigma(\phi_x + \kappa\beta^{-1}) \end{bmatrix}$.

Observe that $tr(A) = 1 + \beta^{-1} + \sigma\kappa\beta^{-1} + \sigma\phi_x$ and $det(A) = \beta^{-1}(1 + \sigma(\phi_x + \kappa\phi_\pi))$. Case 2 of Proposition A.1 clearly does not apply, see, e.g., condition e is violated. Now consider Case 1. As seen above, condition a is satisfied because the determinant is positive. Condition c is also clearly satisfied since both det(A) and tr(A) are positive, which leaves condition b, which we show applies if $\phi_\pi + \frac{(1-\beta)}{\kappa}\phi_x > 1$.

$\beta^{-1}(1 + \sigma(\phi_x + \kappa\phi_\pi)) - 1 - \beta^{-1} - \sigma\kappa\beta^{-1} - \sigma\phi_x > -1 \Leftrightarrow \phi_\pi + \frac{(1-\beta)}{\kappa}\phi_x > 1$. It is now easy to confirm that the analytic solution in the proposition (which, from the discussion above, we know is unique) is the one given in the proposition, i.e. $\pi_t = \hat{Y}_t = 0$ and $i_t = \bar{r}$. QED

17.5 Proof of Proposition 3

Consider the solution at time $t < \tau$. Given the result from Proposition 1, we can write expectations of inflation and output as

$$E(Y_{t+1} \backslash t < \tau) = (1-\mu)E_t Y^S_{t+1} + \mu * 0 = (1-\mu)E^S_t Y^S_{t+1},$$

where the notation E^S_t is used as the expectation of the variable Y_{t+1} conditional on the shock being in the S state, i.e., $t+1 < \tau$. Similarly, the notation Y^S_{t+1} is used to signify that this is the value of Y_{t+1} conditional on $t+1 < \tau$. We can similarly write inflation as

$$E(\pi_{t+1} \backslash t < \tau) = (1-\mu)E_t \pi^S_{t+1} + \mu * 0 = (1-\mu)E^S_t \pi^S_{t+1}.$$

Hence

$$Y^S_t = \mu E_t Y^S_{t+1} - \sigma i^S_t + \sigma\mu E_t \pi^S_{t+1} + \sigma r^e_S + (1-\mu)\hat{G}^N_S - \sigma(1-\mu)\hat{\tau}^s_S + \sigma\hat{\tau}^A_S, \tag{40}$$

$$\pi^S_t = \kappa\hat{Y}^S_t + \kappa\psi(\hat{\tau}^w_S + \hat{\tau}^s_S) - \kappa\psi\sigma^{-1}\hat{G}^N_S + \beta\mu E_t \pi^S_{t+1}, \tag{41}$$

$$i^S_t = \max(0, r^e_S + \phi_\pi \pi^S_t).$$

17.5.1 Part i

Consider first part i, and let us conjecture that the zero bound is not binding. Then we can write

$$E_t z_{t+1}^S = A z_t^S + a e_t^S$$

where $z_t^S \equiv \begin{bmatrix} \hat{Y}_t^S \\ \pi_t^S \end{bmatrix}$, and $A \equiv \begin{bmatrix} [\frac{1+\sigma(\phi_y+\kappa\beta^{-1})}{\mu}] & \frac{\sigma(\phi_\pi-\beta^{-1})}{\mu} \\ -\frac{\kappa\beta^{-1}}{\mu} & \frac{\beta^{-1}}{\mu} \end{bmatrix}$

$$e_t \equiv \begin{bmatrix} \hat{G}_S^N \\ \hat{\tau}_S^w \\ \hat{\tau}_S^s \\ \hat{\tau}_S^A \end{bmatrix} a \equiv \begin{bmatrix} -\frac{\kappa\psi\beta^{-1}+1-\mu}{\mu} & \frac{\sigma\kappa\psi\beta^{-1}}{\mu} & \sigma\frac{\kappa\psi\beta^{-1}+1-\mu}{\mu} & -\frac{\sigma}{\mu} \\ \frac{\kappa\psi\sigma^{-1}\beta^{-1}}{\mu} & -\frac{\kappa\psi\beta^{-1}}{\mu} & -\frac{\kappa\psi\beta^{-1}}{\mu} & 0 \end{bmatrix}.$$

Observe that $tr(A) = \frac{1+\beta^{-1}+\sigma\kappa\beta^{-1}+\sigma\phi_y}{\mu}$ and $det(A) = \frac{\beta^{-1}+\sigma\kappa\beta^{-1}\phi_\pi+\sigma\beta^{-1}\phi_y}{\mu^2}$. Case 2 of Proposition A.1 clearly does not apply, e.g., condition e is violated. Hence, consider Case 1. As seen above, condition a is satisfied because the determinant is positive since $\mu > 0$. Condition c is also clearly satisfied since both $det(A)$ and $tr(A)$ are positive. Which leaves condition b, which is shown to apply below:

$$\begin{aligned} & \frac{\beta^{-1}+\sigma\kappa\beta^{-1}\phi_\pi+\sigma\beta^{-1}\phi_y}{\mu^2} - \frac{1+\beta^{-1}+\sigma\kappa\beta^{-1}+\sigma\phi_y}{\mu} \\ > \quad & -1 \\ \Leftrightarrow \quad & \mu^2 - (1+\beta^{-1}+\sigma\kappa\beta^{-1}+\sigma\phi_y)\mu + \beta^{-1}+\sigma\kappa\beta^{-1}\phi_\pi+\sigma\beta^{-1}\phi_y > 0, \end{aligned}$$

which holds for any $\mu \in [0,1]$ as long as $\phi_\pi + \frac{(1-\beta)}{\kappa}\phi_x > 1$. This proves that there is a unique bounded solution in the short run of the form stated in the proposition. The algebraic form of the solution in the proposition can be found using standard methods, e.g., method of undetermined coefficients. Note in particular that the analytical solution for the nominal interest rate suggests that it is only satisfied as long as C1 does not apply. QED.

17.5.2 Part ii

We now consider the case in which both C1 and C2 apply. From the solution for the nominal interest rate in part i of the proposition we see that this implies that the zero bound is binding. Then we can write

$$E_t z_{t+1}^S = A z_t^S + a e_t,$$

where $z_t^S \equiv \begin{bmatrix} \hat{Y}_t^S \\ \pi_t^S \end{bmatrix}$ $A \equiv \begin{bmatrix} \frac{1+\frac{\sigma\kappa}{\beta}}{\mu} & -\frac{\sigma\beta^{-1}}{\mu} \\ -\frac{\kappa}{\beta\mu} & \frac{1}{\beta\mu} \end{bmatrix}$,

$$e_t \equiv \begin{bmatrix} r_S^e \\ \hat{G}_S^N \\ \hat{\tau}_S^w \\ \hat{\tau}_S^s \\ \hat{\tau}_S^A \end{bmatrix} a \equiv \begin{bmatrix} \sigma & -\frac{\kappa\psi\beta^{-1}+1-\mu}{\mu} & \frac{\sigma\kappa\psi\beta^{-1}}{\mu} & \sigma\frac{\kappa\psi\beta^{-1}+1-\mu}{\mu} & -\frac{\sigma}{\mu} \\ 0 & \frac{\kappa\psi\sigma^{-1}\beta^{-1}}{\mu} & -\frac{\kappa\psi\beta^{-1}}{\mu} & -\frac{\kappa\psi\beta^{-1}}{\mu} & 0 \end{bmatrix}.$$

Observe that $tr(A) = \frac{1+\beta^{-1}+\frac{\sigma\kappa}{\beta}}{\mu} = \frac{(1+\beta^{-1}+\frac{\sigma\kappa}{\beta})\mu}{\mu^2}$ and $det(A) = \frac{\beta^{-1}}{\mu^2}$. Case 2 of Proposition A.1 clearly does not apply, because, for example, condition e is violated. Consider Case 1. As seen above, condition a is satisfied because the determinant is positive since $1 > \mu > 0$. Condition c is also satisfied since both $det(A)$ and $tr(A)$ are positive. Which leaves condition b which is shown to apply below:

$$\begin{aligned} & \frac{\beta^{-1}}{(1-\mu)^2} - \frac{(1+\beta^{-1}+\frac{\sigma\kappa}{\beta})(1-\mu)}{(1-\mu)^2} \\ > \quad & -1 \Leftrightarrow \beta\mu^2 + \mu(1-\beta+\sigma\kappa) - \sigma\kappa = \mu(1-\beta(1-\mu)) - (1-\mu)\sigma\kappa > 0 \end{aligned}$$

$$\frac{\beta^{-1}}{\mu^2} - \frac{(1+\beta^{-1}+\frac{\sigma\kappa}{\beta})\mu}{\mu^2} > -1 \Leftrightarrow \beta\mu^2 - (1+\beta^{-1}+\frac{\sigma\kappa}{\beta})\beta\mu + 1 = (1-\mu)(1-\mu\beta) - \kappa\mu\sigma > 0$$

which is condition C2. This proves that there is a unique bounded solution in the short run of the form stated in the proposition, provided C1 and C2 apply. The algebraic form of the solution in the proposition can be found using standard methods, e.g., method of undetermined coefficients. QED.

17.6 Proofs of Propositions 4 and 5

These proofs of these propositions follow exactly the same steps as proofs 2 and 3 and again one can find the algebraic solution by using method of undetermined coefficients.

18 Appendix B: The numerical simulation

	Prior 5%	Prior 50%	Prior 95%	Posterior 5%	Posterior 50%	Posterior 95%	Mode
alpha	0.5757	0.6612	0.7402	0.7026	0.7633	0.8164	0.7747
beta	0.9949	0.9968	0.9981	0.9948	0.9967	0.9981	0.9970
1-mu	0.0198	0.0740	0.1788	0.0705	0.1045	0.1523	0.0971
omega	0.1519	0.8200	2.4631	0.7756	1.9033	3.8332	1.5692
reL	-0.0196	-0.0094	-0.0036	-0.0266	-0.0146	-0.0066	-0.0104
sigma^-1	1.2545	1.9585	2.8871	0.8107	1.2161	1.7779	1.1599
theta	3.7817	7.6283	13.4871	8.3394	13.2496	19.7410	12.7721

Priors and posteriors and mode of parameters.

Assuming the normally distributed random discrepancy between the model and the data specified in the text, the log of the posterior likelihood of the model is

$$\log L = -\frac{(\pi_S - (-0.1/4))^2}{2\sigma_\pi^2} - \frac{(\hat{Y}_S - (-0.3))^2}{2\sigma_Y^2} + \sum_{\psi_s \in \Omega} f(\psi_s), \tag{42}$$

where Y_L and π_L are given by Proposition 2 for output and inflation in the absence of policy intervention. I write the likelihood conditional on the hypothesis that the shock r_S^e is in the "low state." The only data I match are that output is -30 percent and inflation is -10 percent. The functions $f(\psi_s)$ measure the distance of the variables in Ω from the priors imposed where the parameters and shocks are denoted $\psi_s \in \Omega$. The distance functions $f(\psi_s)$ are given by the statistical distribution of the priors listed in Table 6. I use gamma distribution for parameters that are constrained to be positive and beta distribution for parameters that have to be between 0 and 1.

The priors for the parameters are relatively standard. The priors for the shocks, however, are chosen as follows. It is assumed that the mean of the shock r_S^e in the low state is equivalent to a 2-standard-deviation shock to a process fitted to ex ante real interest rates in post-WWII data. While ex ante real rates would be an accurate measure of the efficient rate of interest only in the event output is at its efficient rate at all times, this gives at least some sense of a reasonably "large" shock as a source of the Great Depression. I'm working on forming priors mapping the model into spreads. The prior on the persistence of the shock is that it is expected to reach steady state in ten quarters, which is consistent with the stochastic process of estimated ex ante real rates. It also seems reasonable to suppose that in the midst of the Great Depression people expected it to last for several years. All these priors are specified as distributions, and Table 1 provides information on this. Observe that the values of $\sigma_{\pi,t}^2$ and $\sigma_{Y,t}^2$ measure how much we want to match the data against the priors. The measurement error is there only for computational reasons. I assume that it is extremely small such that the estimation hits the data very accurately

I use a Metropolis algorithm to simulate the posterior distribution (42) . Let y^Tdenote the set of available data and Ω the vector of coefficients and shocks. Moreover, let Ω^j denote the jth draw from the posterior of Ω. The subsequent draw is obtained by drawing a candidate value, $\tilde{\Omega}$, from a Gaussian proposal distribution with mean Ω^j and variance sV. We then set $\Omega^{(j+1)} = \tilde{\Omega}$ with probability equal to

$$\min\{1, \frac{p(\Omega/y^T)}{p(\Omega^j/y^T)}\}.$$

If the proposal is not accepted, we set $\Omega^{(j+1)} = \Omega^j$.

The algorithm is initialized around the posterior mode, found using a standard Matlab maximization algorithm. We set V to the inverse Hessian of the posterior evaluated at the mode, while s is chosen in order to achieve an acceptance rate approximately equal to 25 percent. We run two chains of 1000,000 draws and discard the first 200,000 to allow convergence to the ergodic distribution.