Platform Money*

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Abstract

This paper examines a digital platform issuing proprietary money to compete against a legacy fiat marketplace. Platform money kickstarts two-sided network effects, complementing matching services and strengthening market power. The platform's inflation markup over the Friedman rule is disciplined internally by demand elasticity and externally by legacy inflation. Platform money improves welfare when the issuer possesses superior matching technology. Competitive public currencies (e.g., CBDCs) structurally constrain platform monopoly power. Finally, platforms may optimally reject outside money to strategically segment the user base, softening competition to extract higher seigniorage.

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1 Introduction

The rise of the platform economy is evidenced by the growing dominance of digital super-apps that seamlessly connect vast networks of buyers and sellers. Empowered by advanced data processing, platforms such as Alibaba, Tencent, Mercado Libre, Safaricom, and Gojek deliver superior matching capabilities that traditional brick-and-mortar marketplaces cannot replicate. Increasingly, these platforms are embedding proprietary payment systems and digital currencies, such as Alipay, WeChat Pay, Mercado Pago, M-Pesa, and GoPay, directly into their commercial ecosystems. This development raises a fundamental question: How does the ability to issue private money affect the competitive dynamics of digital platforms? Furthermore, do these platforms face the same irresistible temptation to overissue currency that Friedman (1960) warned against or does competition discipline their issuance decisions as argued by Hayek (1976)?

To address these questions, we build on the New Monetarist framework of Lagos and Wright (2005). A two-sided platform issues its own currency and competes against a legacy marketplace that uses government-issued fiat money. Both marketplaces face search frictions with cross-group network externalities: more buyers attract sellers and more sellers attract buyers. We show that private money gives the platform a unique strategic instrument: by subsidizing buyer entry through monetary policy, it assembles a critical mass of buyers, which in turn allows it to extract high explicit fees from sellers in equilibrium.

This framework formalizes Hayek's argument of competitive monetary discipline based on the industrial organization of two-sided marketplace competition. A platform treats money not as its sole product, but as a complement to its core matchmaking business. Its monetary policy is designed to gain a competitive advantage over a legacy market that does not have control over its own inflation. This policy is disciplined by competition from outside inflation and anchored at an interior optimum: an inflation markup over the socially optimal Friedman rule that follows a monetary Lerner index, inversely proportional to the semi-elasticity of buyer demand.

In contrast, in standard macroeconomic models, a standalone private issuer operates without a complementary service to anchor money demand. The value of pure fiat money is either indeterminate or must be hardwired, as when money enters the utility function directly.

In our framework, money demand is derived from the gains from trade in the matching market. This provides a structural foundation for monetary discipline: the matching service anchors the value of private money, competition from the legacy marketplace constrains the inflation markup, and the congestion effects inherent in two-sided matching further dampen the platform's inflation response.

Moreover, we uncover several striking equilibrium properties regarding how platforms compete with central bank policy. We find that the platform's monetary policy exhibits a form of *strategic dampening*. As central bank inflation rises, the legacy market is forced to cut explicit fees to prevent user exit. In response, the platform raises its own inflation by *less* than one-for-one, strategically moderating its rate to compete against the legacy market's lower fees and maintain an "inflation shelter" for its buyers. Furthermore, if consumers are "inattentive" and perceive inflation costs as less salient than explicit fees, the platform exploits this behavioral friction by implementing obfuscated pricing, that is, replacing salient transaction fees entirely with less salient inflation charges.

We also evaluate the welfare implications of platform money. In the planner's optimum, private money is socially inefficient in our baseline setup because the platform uses the inflation subsidy to attract an excessive ratio of buyers relative to sellers, creating congestion. However, if the platform possesses a sufficiently superior matching technology, this result is overturned. By subsidizing buyer entry, the platform draws agents to the more efficient marketplace, and this reallocation benefit outweighs the congestion cost. In this case, private money yields a strictly higher social welfare than a standard fee-only platform.

Our framework highlights the vital role of macroeconomic policy in regulating these digital ecosystems. A well-managed public currency, or the introduction of a retail Central Bank Digital Currency (CBDC), allows the central bank to fine-tune the competitive advantage of platform money. When the platform's matching technology offers no efficiency gain, low central bank inflation strengthens the legacy market as a competitive outside option, limiting the platform's ability to draw excessive buyers and the resulting congestion costs. When the platform possesses superior matching technology, the central bank optimally raises legacy inflation to accelerate adoption of the more efficient marketplace. In both cases, macroeconomic policy acts as a structural instrument for regulating platform market power.

We also demonstrate that our core mechanisms are robust when the platform explicitly internalizes its impact on the equilibrium seller-to-buyer ratio. By internalizing market tightness, the platform utilizes its monetary policy not merely for seigniorage, but as an active lever to manage two-sided network externalities. This introduces a monetary analog to the classic cross-subsidy pricing in platform economics (Rochet and Tirole, 2003). The platform optimally loosens or tightens its inflation markup depending on which cross-group externality dominates. Ultimately, whether the platform uses low inflation to subsidize buyers or extracts rent directly, the fundamental structural motive to issue and strategically price private money remains intact.

Finally, we rationalize why platforms often enforce a "Closed Policy" (rejecting outside fiat) rather than an "Open Policy" (interoperability). We introduce buyer heterogeneity, where a fraction of users strictly prefer outside money. We show that the platform may optimally reject outside money as a mechanism for *strategic segmentation*. By ceding biased buyers to the legacy market, the platform effectively turns the legacy market into a monopoly over that captive segment. Secure in its monopoly, the legacy market raises its fees. This high-fee environment relaxes the competitive constraint on the platform, allowing it to extract a higher inflation tax from its remaining flexible users without losing them. Thus, "walled gardens" are sustained not just by technological lock-in, but as an endogenous monetary strategy to soften price competition.

While our framework abstracts from specific institutional details to highlight core economic mechanisms, the industrial organization of platform money we describe closely mirrors the evolution of digital payment ecosystems worldwide. Prominent examples of these dynamics include the historical use of Tencent's Q-Coin as unregulated private fiat, Gojek's GoPay overcoming legacy frictions through asymmetric pricing, and the cross-group subsidies inherent in airline frequent flyer programs. Furthermore, the strategic inflation dampening and segmentation (the "Closed Policy") predicted by our model are actively observable in platforms like Mercado Pago, Alipay, and WeChat Pay. We provide a comprehensive discussion of how these specific platform monies operate in practice to Appendix A.

Literature Our work bridges three distinct strands of literature: two-sided markets with search frictions, the emerging theory of digital tokens, and classical monetary competition.

Our model builds on the foundational literature of two-sided markets. Early work focused on platforms with exogenous network effects, analyzing pricing structures where one side is subsidized by the other (Rochet and Tirole, 2003; Caillaud and Jullien, 2003; Armstrong, 2006; Weyl, 2010). A second stream of literature endogenizes these network effects using search and matching frictions (Chen and Huang, 2012; Goos et al., 2014; Gautier et al., 2023). We contribute to this tradition by introducing a new instrument: *private money*. Unlike standard models where platforms compete solely via fees, our platform manages buyer participation through its monetary policy (inflation and seigniorage), effectively bundling the matching service with a medium of exchange.

There is limited study of private money for two-sided marketplaces. However, there is a growing tokenomics literature. For instance, Rogoff and You (2023) model tokens as non-tradable loyalty rewards, while Sockin and Xiong (2023), Goldstein et al. (2024), Li and Mann (2025), and Lyandres et al. (2025) analyze tokens as commitment instruments, coordination devices or financing vehicles. We diverge from this literature by modelling platform money explicitly as a circulating medium of exchange for two-sided search and matching marketplaces that generates gains from trade and potentially seigniorage, and studying its impact on competition dynamics. Our finding that platform money can be welfare-enhancing in the competitive equilibrium is a unique insight.[1]

We relate to the literature on currency competition (Schilling and Uhlig, 2019; Fernández-Villaverde and Sanches, 2019; Skeie, 2019; Benigno et al., 2022; Guennewig, 2025; Brunnermeier and Payne, 2024). While this literature primarily analyzes strategic Nash equilibria between seigniorage-maximizing entities, we approach the viability of private money from a structural angle: the industrial organization of a private platform competing against a legacy fiat marketplace. Rather than modeling the central bank as a strategic player, we depart from the "money-as-product" view by treating money as a *complement* to the platform's core matching service. Consequently, we generate imperfect substitutability between

[1]Furthermore, we enrich this analysis by incorporating behavioral insights motivated by evidence that consumers underweight non-salient costs (Bordalo et al., 2022; Blake et al., 2021) and exhibit money illusion (Shafir et al., 1997; Brunnermeier and Julliard, 2008).

currencies endogenously via the users' discrete choice of marketplace, rather than embedding an elasticity parameter directly into the utility function over real balances (e.g., Benigno et al., 2022).

Our result also offers a resolution to the classic indeterminacy puzzle of Kareken and Wallace (1981). They show that if private currencies are perfect substitutes, the exchange rate between them is indeterminate and volatile. In contrast, our model generates a determinate exchange rate because we treat platform money and outside money as *imperfect substitutes*. The demand for platform money is anchored by the platform's superior matching technology. By bundling the currency with a unique matching service, the platform creates a specific transactional demand that pins down the price level and the exchange rate, thereby eliminating the indeterminacy found in standard currency competition models.[2]

The rest of the paper is organized as follows. Section 2 sets up the model and provides the equilibrium definition. Section 3 analyses equilibrium properties. Section 4 solves the planner's problem. Section 5 conducts a robustness check. Section 6 characterizes when a platform should accept outside money. Section 7 concludes.

2 The Model

2.1 The Environment

Time is discrete, lasts forever, and is indexed by $t \in \{0, 1, ...\}$. There are two marketplaces: a private platform P and a legacy market L. There are three types of agents: a measure $\bar{N}_b$ of buyers, a measure $\bar{N}_s$ of sellers, and the owners of the two trading marketplaces. Decentralized search and matching between buyers and sellers occurs in the two marketplaces.

The discount factor between periods is $\beta \in (0, 1)$ and each period is divided into two stages. In the first stage, the decentralized marketplaces (DM) are open to trade a perishable

[2]Recent literature provides varying results on the welfare implications of currency competition. While Guennewig (2025) finds that private money disciplines the central bank toward the Friedman Rule, others highlight the limits of this competition. For instance, Brunnermeier and Payne (2024) emphasize that while competition limits rent extraction, it may also impair the platform's ability to enforce contracts. Our model, instead, demonstrates that such competition can be socially beneficial by incentivizing platforms to offer private money that maximizes gains from trade via two-sided matching.

consumption good y, paid for with money and produced only by sellers at zero marginal cost. Buyers obtain u from consuming one unit and do not value more units. In each period, buyers and sellers are able to participate in one and only one of the trading marketplaces. When a buyer and a seller match, the payment is made in the marketplace's money: platform money on platform P and outside money on legacy market L. We also refer to this DM stage as the consumption goods market.

In the second stage, a centralized, frictionless settlement market (CM) is established. In this market, the owners of the trading platforms set buyer and seller fees, and agents decide which future DM consumption good marketplace to join, pay the corresponding fee, and rebalance their portfolios of platform and outside money. Consequently, the platform money as a medium of exchange is priced. We assume that all types of agents consume the perishable CM good, x, and can supply labor, h, to produce the good x via a linear production technology with a 1:1 ratio. All agents obtain utility $U(x, h)$ by consuming x of the CM good but incur dis-utility from labor. To simplify the exposition, we assume that $U(x, h) = x - h$. We also refer to this CM stage as money market.

Both market owners impose fees denoted by k_t^j to sellers and f_t^j to buyers, where $j \in \{P, L\}$ (with the fees measured in units of x) while the owner of the platform (P) also chooses the amount of additional platform money to issue. Sellers and buyers observe these fees, pay the fee associated with the market they choose to enter and adjust their money portfolios accordingly.

Under the assumption that consumption goods x and y are perishable, the only forms of money in this economy are platform money and outside money. We denote the money price in each market by ϕ_t^j, where $j \in \{P, L\}$, and the platform money supply by M_t^P. In the legacy market, the central bank sets the inflation rate to achieve macroeconomic objectives. In contrast, the platform determines the holding cost of its own money to maximize profit. In practice, platforms extract this monetary revenue (seigniorage) by retaining the yield earned on users' idle balances rather than passing it back to them, or by gradually devaluing the purchasing power of native platform tokens (e.g., airlines adjusting award charts for frequent flyer miles). By controlling these parameters, the platform sets the inflation rate borne by buyers.

The law of motion for the platform money supply is:

$$M_{t+1}^P = \mu_t^P M_t^P. \tag{1}$$

For outside money used in the legacy market, we assume its purchasing power declines at a constant, exogenous gross inflation rate $\tilde{\mu}^L$, such that $\phi_t^L/\phi_{t+1}^L = \tilde{\mu}^L$ for all t. This reflects the standard premise that central bank policy is driven by broader macroeconomic mandates rather than the internal dynamics of any individual platform or legacy market. We assume $\tilde{\mu}^L \geq 1$ (the central bank does not run a deflationary policy). Therefore, $\tilde{\mu}^L > \beta$, which means that agents' demand for outside money is finite. Hence, a seller doesn't carry any money across periods. A buyer doesn't carry the money of the market where she doesn't trade and carries the optimal amount of money necessary to trade in DM. Figure 2.1 summarizes the aforementioned events in this economic environment.

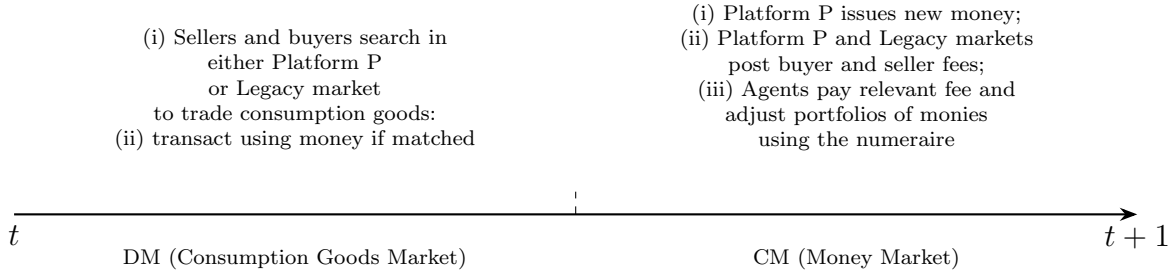


Figure 1: Timeline

2.2 Centralized Market

We denote an agent's value function in the CM by W_t and in the DM by V_t. The CM value function of a buyer denoted by subscript b is given by:

$$W_{b,t}(m_{b,t}^P, m_{b,t}^L) = \hat{\Pi}_{b,t}^P W_{b,t}^P(m_{b,t}^P, m_{b,t}^L) + \hat{\Pi}_{b,t}^L W_{b,t}^L(m_{b,t}^P, m_{b,t}^L), \tag{2}$$

where $\hat{\Pi}_{b,t}^j$ is the buyer's endogenously determined probability of choosing market j in the CM. This formulation allows buyers to bring both types of money from CM to DM, but in fact buyers will bring at most one type of money (or none), since holding money is costly

if not for transaction purposes. That is, buyers pay only the entry fee and purchase the relevant money of the chosen marketplace j. The value of going to market j is:

$$W^j_{b,t}(m^P_{b,t}, m^L_{b,t}) = \max_{x_t, h_t, m^j_{b,t+1} \geq 0} x_t - h_t + \beta V^j_{b,t+1}(m^j_{b,t+1}, 0) \tag{3}$$

$$\text{s.t.} \quad x_t + f_j + \phi^j_t m^j_{b,t+1} \leq h_t + \phi^P_t m^P_{b,t} + \phi^L_t m^L_{b,t}, \tag{4}$$

where buyers choose consumption, labour, and money holding amount optimally. By substituting for $x_t - h_t$, we obtain

$$W^j_{b,t}(m^P_{b,t}, m^L_{b,t}) = \phi^P_t m^P_{b,t} + \phi^L_t m^L_{b,t} + W^j_{b,t}(0,0),$$

where

$$W^j_{b,t}(0,0) = \max_{m^j_{b,t+1} \geq 0} -\phi^j_t m^j_{b,t+1} - f_j + \beta V^j_{b,t+1}(m^j_{b,t+1}, 0) \tag{5}$$

Thus, a buyer's optimization problem in CM is not history dependent and her value function in CM can be written as

$$W_{b,t}(m^P_{b,t}, m^L_{b,t}) = \phi^P_t m^P_{b,t} + \phi^L_t m^L_{b,t} + \left[\hat{\Pi}^P_{b,t} W^P_{b,t}(0,0) + \hat{\Pi}^L_{b,t} W^L_{b,t}(0,0)\right]. \tag{6}$$

Similarly, a seller's value function denoted by subscript s can be written as:

$$W_{s,t}(m^P_{s,t}, m^L_{s,t}) = \phi^P_t m^P_{s,t} + \phi^L_t m^L_{s,t} + \left[\hat{\Pi}^P_{s,t} W^P_{s,t}(0,0) + \hat{\Pi}^L_{s,t} W^L_{s,t}(0,0)\right]. \tag{7}$$

where k is the seller entry fee and

$$W^j_{s,t}(0,0) = \max_{m^j_{s,t+1} \geq 0} -\phi^j_t m^j_{s,t+1} - k_j + \beta V^j_{s,t+1}(m^j_{s,t+1}, 0). \tag{8}$$

2.3 Decentralized Market

Trading in a decentralized marketplace is subject to search frictions that we capture with a matching function. For any market j with N_s sellers and N_b buyers, the matching function $Q^j(N_s, N_b)$ represents the total number of successful matches in j. We assume that

Q^j exhibits constant returns to scale, is strictly increasing in both arguments, and is concave in both arguments. It is also useful to denote the ratio of sellers to buyers in market j, by $n_j \equiv N_s/N_b$. Using this definition, the probability that a *buyer* successfully finds a match in market j is related to the seller-to-buyer ratio in the following manner:

$$a_{jb}(n_j) \equiv \frac{Q^j(N_s, N_b)}{N_b} = Q^j(n_j, 1) \tag{9}$$

Similarly, the probability that a *seller* finds a match in market j is:

$$a_{js}(n_j) \equiv \frac{Q^j(N_s, N_b)}{N_s} = \frac{1}{n_j} Q^j(n_j, 1) \tag{10}$$

It follows that $a_{jb}(n)$ is strictly increasing and concave in n, while $a_{js}(n)$ is strictly decreasing in n. We assume that the platform has weakly superior matching technology: $Q^P(N_s, N_b) = A \cdot Q^L(N_s, N_b)$ for some $A \geq 1$. This implies $a_{Pj}(n) = A \cdot a_{Lj}(n)$ for $j \in \{b, s\}$: the platform is uniformly better at matching by a factor A on both sides. All derivative conditions follow automatically: $a'_{Pb}(n) \geq a'_{Lb}(n)$ and $|a'_{Ps}(n)| \geq |a'_{Ls}(n)|$.

For either marketplace $j \in \{P, L\}$ define

$$h(n) \equiv \frac{a'_{jb}(n)}{|a'_{js}(n)|} = \frac{a'_{Lb}(n)}{|a'_{Ls}(n)|},$$

which measures how much buyers gain relative to how much sellers lose from a marginal increase in the seller-to-buyer ratio n. The ratio does not depend on j: the factor A multiplies both slopes on the platform and cancels, so h is a property of the common matching technology rather than of either marketplace.

Assumption 1 (Matching regularity) *The ratio $h(n) = a'_{jb}(n)/|a'_{js}(n)|$, which is the same for $j = P$ and $j = L$, is non-decreasing in n.*

Assumption 1 says that, as sellers become more abundant relative to buyers, the response of buyers' matching rate to tightness rises relative to the response of sellers' matching rate: relative sensitivity shifts toward the side that is becoming scarce. It restricts how the ratio changes with tightness, not its level, and it restricts the shape of the matching technology, not the strength of network effects: it is satisfied by Cobb-Douglas matching ($Q(N_s, N_b) =$

$N_s^{\alpha} N_b^{1-\alpha}$) for every elasticity α and every technology gap A; there $h(n) = \frac{\alpha}{1-\alpha}\, n$, which is below one for $n < (1-\alpha)/\alpha$ and above one beyond it. Its role is to ensure that own-side congestion dominates cross-side thickness effects, so that the feedback loop in which buyers attract sellers who attract buyers is damped; this is what delivers strategic dampening (Proposition 6) and the single-instrument result (Proposition 9).

Conditional on a successful match, the buyer and seller bargain over the terms of trade using proportional bargaining (Kalai (1977)). With proportional bargaining under liquidity constraints the buyer brings real balances just sufficient to purchase the optimal quantity given the surplus split (Hu and Rocheteau (2020)), so the real price of good y is $p^j \phi^j = u(1-\gamma)$. We adopt Kalai's proportional bargaining because it is more empirically relevant under liquidity constraints (Duffy et al. (2021)), delivers well-defined monetary trade even with zero seller costs, and provides expositional clarity. Online Appendix C provides the details of bargaining with liquidity constraints under both Kalai and Nash bargaining protocols (Nash (1950, 1953)) and shows our results are robust to the choice of bargaining protocol.

2.4 Buyers in the DM

Given the matching probabilities, we obtain the DM value function for each individual buyer who chooses to trade on market $j \in \{P, L\}$ as

$$V_{b,t}^j(m_{b,t}^j, 0) = a_{jb}(n_{jt})[u + W_{b,t}(m_{b,t}^j - p_t^j, 0)] + (1 - a_{jb}(n_{jt}))W_{b,t}(m_{b,t}^j, 0). \tag{11}$$

The first term states that, conditional on being matched, the buyer gains utility u and carries the after-trade money balance $m_{b,t}^j - p_t^j$ into the CM. The second term captures the case where the buyer simply carries over their money to the CM if not matched. By plugging for $W_{b,t}(\cdot)$, we simplify the value function as follows:

$$V_{b,t}^j(m_{b,t}^j, 0) = a_{jb}(n_{jt})[u - \phi_t^j p_t^j] + \phi_t^j m_{b,t}^j + W_{b,t}(0, 0). \tag{12}$$

The real price of the DM good is set by the bargaining rule:

$$\phi_t^j m_{b,t}^j = \phi_t^j p_t^j = u(1-\gamma) \tag{13}$$

since buyers bring the exact amount of money to pay for the DM good.

The buyer acquires $m^j_{b,t+1}$ tokens at CM_t price ϕ^j_t for use in DM_{t+1}, and expects that each token will be worth $\phi^{j,e}_{t+1}$ at CM_{t+1}. The real cost of the required balances is therefore $\frac{\phi^j_t}{\phi^{j,e}_{t+1}}\, u(1-\gamma)$. We write $\tilde{\mu}^{j,e}_t \equiv \phi^j_t/\phi^{j,e}_{t+1}$ for the expected inflation rate on market j. Note that for the legacy market, $\tilde{\mu}^{L,e}_t = \tilde{\mu}^L$ for all t by assumption.

Plugging the real price, the money holding, and (12) into (5), we obtain:

$$W^j_{b,t}(0,0) = -\underbrace{f_{j,t}}_{\text{Fee}} + \underbrace{\beta a_{jb}(n_{j,t+1})\gamma u}_{\text{Utility from trade}} + \underbrace{(\beta - \tilde{\mu}^{j,e}_t)(1-\gamma)u}_{\text{Cost of holding money}} + \underbrace{\beta W_{b,t+1}(0,0)}_{\text{Continuation value}}. \tag{14}$$

2.5 Buyers' marketplace choice

Next, we formalize the buyer's marketplace selection as a random discrete choice problem. In this framework, the buyer's decision is influenced not only by the anticipated value each marketplace offers but also by an idiosyncratic choice shock and a behavioral bias toward inflation. In our interpretation, a buyer's actual experienced payoff is $W^j_{b,t}(0,0)$ but at the choice stage each buyer l uses their perceived payoffs $\hat{W}^j_{l,b,t}$ plus a white noise η_{jl} to choose between the two marketplaces. The idiosyncratic noise term η_{jl} captures the randomness of the choice stage. Formally, the perceived payoff of buyer l is given by:

$$\hat{W}^j_{l,b,t}(\xi) = -f_{j,t} + \beta a_{jb}(n_{j,t+1})\gamma u + (\beta - \xi\tilde{\mu}^{j,e}_t)(1-\gamma)u + \beta W_{b,t+1}(0,0) \tag{15}$$

where the parameter $\xi \in [0,1]$ captures the salience of inflation (from either outside money or platform money) to the buyer. That is, the buyer does not fully account for inflation costs when choosing between marketplaces. Thus, the perceived advantage of platform (P) over legacy (L) marketplace is

$$\Delta_{b,t+1} \equiv \underbrace{\beta\left(a_{Pb}(n_{P,t+1}) - a_{Lb}(n_{L,t+1})\right)\gamma u}_{\text{Utility from trade difference}} + \underbrace{\xi(\tilde{\mu}^L - \tilde{\mu}^{P,e}_t)(1-\gamma)u}_{\text{Inflation cost difference}} + \underbrace{(f_{L,t} - f_{P,t})}_{\text{Fee difference}}. \tag{16}$$

It is important to note that the salience of inflation can either help or hinder platform P's ability to attract buyers. When the legacy marketplace faces higher inflation set by the central bank, any inflation cost savings offered by platform P become less compelling to buyers, who tend to discount these savings. Conversely, if the legacy marketplace experiences

lower inflation, platform P can afford to impose a higher inflation rate, and buyers may not fully account for this increased inflation in their decision-making.

Each buyer l draws a pair of independent idiosyncratic shocks (η_{Pl}, η_{Ll}), one for each marketplace, capturing taste heterogeneity at the choice stage. Buyer l chooses platform P whenever the perceived advantage plus the net shock favors it, $\Delta_{b,t+1} + \eta_{Pl} - \eta_{Ll} \geq 0$, so the fraction of buyers choosing P is given by the *attraction function*:

$$\Pi_b(\Delta_{b,t+1}) = \Pr\left\{l : \Delta_{b,t+1} \geq \eta_{Ll} - \eta_{Pl}\right\}, \tag{17}$$

which is the CDF of the noise difference $\eta_{Ll} - \eta_{Pl}$ evaluated at $\Delta_{b,t+1}$. Since the noise terms are i.i.d. across buyers, $\Pi_b(\Delta_{b,t+1})$ also gives the measure of buyers entering the platform.[3]

2.6 Sellers

Sellers do not want to hold any additional money in the CM and would convert all the money they have received (if matched) to CM goods immediately to avoid facing inflation cost in the next period. A seller's value function if he chooses to enter market j to trade is then:

$$W^j_{s,t}(0,0) = -\underbrace{k_{j,t}}_{\text{Fee}} + \underbrace{\beta a_{js}(n_{j,t+1})(1-\gamma)u}_{\text{Utility from trade}} + \underbrace{\beta W_{s,t+1}(0,0)}_{\text{Continuation value}} \tag{18}$$

Each seller similarly draws a pair of independent idiosyncratic choice shocks (η_{Ps}, η_{Ls}), one for each marketplace. The attraction function of a seller for platform P over legacy L is then $\Pi_s(\Delta_{s,t+1})$, where

$$\Delta_{s,t+1} = \beta\left(a_{Ps}(n_{P,t+1}) - a_{Ls}(n_{L,t+1})\right)(1-\gamma)u + (k_{L,t} - k_{P,t}). \tag{19}$$

The attraction function $\Pi_s(\Delta_{s,t+1})$ yields the fraction of sellers who choose to trade on P.

We assume that the attraction functions Π_b and Π_s are twice continuously differentiable, strictly increasing, and satisfy $\Pi_i(0) = \frac{1}{2}$ for $i \in \{b, s\}$. Moreover, we assume the density of

[3]We assume idiosyncratic shocks affect only the *perceived* payoffs at the choice stage to generate smooth attraction functions. They do not enter the agent's actual *experienced* utility. Therefore, the ex-ante value is the linear probability-weighted sum as in equation (2).

the noise difference $\eta_{Li} - \eta_{Pi}$ is log-concave, which ensures that both the attraction functions Π_i and the survival functions $1 - \Pi_i$ are log-concave for $i \in \{b, s\}$.[4]

2.7 Equilibrium

In our baseline model, we assume that marketplace owners take matching probabilities (or equivalently equilibrium market tightness) as given and maximize their revenue by optimally setting the entry fees for sellers and buyers ($k_j \geq 0$ and $f_j \geq 0$ where $j \in \{P, L\}$) and choosing the rate of money growth in the case of platform P. This serves as a tractable benchmark that clearly isolates the platform's direct rent-extraction motives. In Section 5, we fully relax this assumption, allowing the platform to explicitly internalize its effect on the seller-to-buyer ratio, and show that our core economic mechanisms are robust.

The within-period timing is as follows. At each CM_t, the platform sets its fees $(f_{P,t}, k_{P,t})$ and expands the money supply from M_t^P to M_{t+1}^P; the legacy market sets fees $(f_{L,t}, k_{L,t})$. After observing fees and the platform's monetary policy, buyers and sellers simultaneously decide which marketplace to enter. Buyers who choose the platform acquire platform money at the prevailing CM_t price for use in DM_{t+1}, where agents are matched and trade within their chosen marketplace.

The platform commits at time 0 to a monetary policy for the entire horizon, and all agents observe this policy when forming expectations about the future value of platform money. Because the platform's problem is recursive, with the inherited buyer base $N_{b,t}^P$ as its only payoff-relevant state variable (Section 2.8), this commitment amounts to announcing a one-period-ahead policy rule that the platform finds optimal to follow each period. The legacy market, by contrast, has no intertemporal linkages and sets fees to maximize per-period profit.

Each buyer entering the platform acquires $\frac{u(1-\gamma)}{\phi_{t+1}^{P,e}}$ units of platform money, where $\phi_{t+1}^{P,e}$ is

[4]The symmetry condition $\Pi_i(0) = \frac{1}{2}$ reflects that when the two marketplaces offer identical terms, each attracts half the population. By Bagnoli and Bergstrom (2005), a log-concave density implies both the CDF and the survival function are log-concave. In the standard discrete-choice specification the individual shocks are i.i.d. Type I extreme value (Gumbel), so their difference is logistic. More generally, the log-concave class includes the logistic, normal, uniform, Laplace, and extreme value distributions.

the expected price at $t+1$. The market clearing condition for platform money at CM_t is

$$M_{t+1}^P \;=\; \frac{N_{b,t+1}^P\, u(1-\gamma)}{\phi_{t+1}^{P,e}}, \tag{20}$$

where $N_{b,t+1}^P = \bar{N}_b \Pi_b(\Delta_{b,t+1})$ is the number of buyers choosing the platform at CM_t. We impose perfect foresight as an equilibrium condition below, hence the expected price equals the actual price at $t+1$.

Definition 1 *An equilibrium with active platform money consists of sequences of seller-to-buyer ratio $(n_{P,t+1}, n_{L,t+1})$, platform policies $\left(f_{P,t}, k_{P,t}, \mu_t^P\right)$, legacy market fees $(f_{L,t}, k_{L,t})$ and users' participation strategies such that at every t:*

1. *Buyers and sellers optimally choose which market to enter at CM_t, given current fees, inflation rates, matching probabilities, and the platform's committed monetary policy.*

2. *Given $(n_{P,t+1}, n_{L,t+1}, f_{L,t}, k_{L,t})$ and users' entry strategies, the platform's fees $(f_{P,t}, k_{P,t})$ and money supply growth rate μ_t^P maximize its discounted present value.*

3. *Given $\left(n_{P,t+1}, n_{L,t+1}, f_{P,t}, k_{P,t}, \mu_t^P\right)$ and buyers' and sellers' entry decisions, legacy market's fees $f_{L,t}$ and $k_{L,t}$ maximize the legacy market's per-period profit.*

4. *The seller-to-buyer ratio is consistent with entry decisions:*

$$\begin{aligned} \frac{\overline{N}_s \Pi_s\left(\Delta_{s,t+1}\right)}{\overline{N}_b \Pi_b\left(\Delta_{b,t+1}\right)} &= n_{P,t+1}, \\ \frac{\overline{N}_s\left(1-\Pi_s\left(\Delta_{s,t+1}\right)\right)}{\overline{N}_b\left(1-\Pi_b\left(\Delta_{b,t+1}\right)\right)} &= n_{L,t+1}. \end{aligned}$$

5. *Expectations are rational: $\phi_{t+1}^{P,e} = \phi_{t+1}^P$ for all t.*

From part 5 we immediately obtain $\tilde{\mu}_t^{P,e} = \tilde{\mu}_t^P$, so henceforth we drop the e superscript for the platform. Writing (20) at dates t and $t+1$ and dividing gives

$$\frac{\phi_t^P}{\phi_{t+1}^P} \;\cdot\; \frac{M_t^P}{M_{t+1}^P} \;=\; \frac{N_{b,t}^P}{N_{b,t+1}^P}.$$

Recognising the left-hand side as $\tilde{\mu}_t^P/\mu_t^P$, where $\mu_t^P \equiv M_{t+1}^P/M_t^P$ is the money supply growth rate, we obtain

$$\mu_t^P \;=\; \tilde{\mu}_t^P \;\cdot\; \frac{N_{b,t+1}^P}{N_{b,t}^P}\,. \tag{21}$$

Money supply growth and inflation coincide in stationarity, when $N_{b,t+1}^P = N_{b,t}^P$ (Proposition 1).

The platform's commitment to $\{\mu_t^P\}$ fixes the money supply path, so agents can compute the future price of platform money from (20). Part 5 requires that agents' price expectations are self-consistent with the prices implied by market clearing. Together, they determine the inflation rate $\tilde{\mu}_t^P$ as a function of the committed μ_t^P and the endogenous buyer base through (21). In the platform's optimization below, we equivalently use $\tilde{\mu}_t^P$ as the control variable, since (21) provides a one-to-one mapping between μ_t^P and $\tilde{\mu}_t^P$ for given participation.

Definition 1 takes the platform's commitment as given. In Online Appendix E, we show that the commitment equilibrium can also be supported as a subgame-perfect equilibrium: if the platform ever deviates from its announced policy, agents lose trust in the platform's money, destroying the monetary matching ecosystem. The platform adheres to its committed policy whenever the continuation value of the monetary equilibrium exceeds the one-shot deviation payoff.

2.8 The Platform's Dynamic Problem

The platform chooses fees (f_P, k_P) each period and commits to the money supply growth rate μ_t^P. Unlike the legacy market, the platform's problem is inherently dynamic: buyers attracted at CM_t carry real balances $u(1-\gamma)$ into DM_{t+1}, and the inherited buyer base $N_{b,t}^P$ determines the platform's outstanding monetary obligations. We formulate the platform's optimization as a Bellman equation with $N_{b,t}^P$ as the state variable.

Seigniorage. Using the market clearing condition for platform money and (13), the money stock satisfies $M_t^P = \bar{N}_b \Pi_b(\Delta_{b,t}) \cdot u(1-\gamma)/\phi_t^P$. The seigniorage revenue from issuing new

money at CM_t is:

$$\text{Seigniorage}_t = (\mu_t^P - 1) M_t^P \phi_t^P = (\mu_t^P - 1) N_{b,t}^P \, u(1-\gamma), \tag{22}$$

where $N_{b,t}^P = \bar{N}_b \Pi_b(\Delta_{b,t})$ is the number of buyers on the platform in period t.

Per-period buyer-side revenue. The total buyer-side revenue at CM_t consists of seigniorage from the inherited base $N_{b,t}^P$ and entry fees from the new base $N_{b,t+1}^P$. Using the identity $\mu_t^P = \tilde{\mu}_t^P \cdot N_{b,t+1}^P / N_{b,t}^P$ from (21), this revenue can be written as:

$$R_t^{\text{buyer}} = N_{b,t+1}^P \big(\tilde{\mu}_t^P \, u(1-\gamma) + f_P \big) - N_{b,t}^P \, u(1-\gamma). \tag{23}$$

The key feature is that R_t^{buyer} involves *two* buyer counts: the old base $N_{b,t}^P$ (an obligation) and the new base $N_{b,t+1}^P$ (a choice). It is this intertemporal structure, which is not present in the legacy market's problem, that distinguishes the platform's optimization.

The Bellman equation. Recall the inflation rate $\tilde{\mu}_t^P \equiv \phi_t^P / \phi_{t+1}^P$ and the money supply growth rate $\mu_t^P \equiv M_{t+1}^P / M_t^P$, linked by (21). Buyers' participation decisions depend on $\tilde{\mu}_t^P$, and the platform can implement any desired $\tilde{\mu}_t^P$ by setting the appropriate μ_t^P given the inherited base. Within each period, the legacy market's fees and the seller-to-buyer ratio are determined simultaneously as part of the within-period Nash equilibrium; accordingly, the buyer and seller advantages $\Delta_{b,t+1}$ and $\Delta_{s,t+1}$ are functions of the platform's current instruments only. We therefore write the Bellman with $\tilde{\mu}_t^P$ as the control variable:

$$\mathcal{V}(N_{b,t}^P) = \max_{\tilde{\mu}_t^P,\, f_P \geq 0,\, k_P} \Big\{ \bar{N}_s \Pi_s(\Delta_s) k_P + N_{b,t+1}^P \big(\tilde{\mu}_t^P \, u(1-\gamma) + f_P \big) - N_{b,t}^P \, u(1-\gamma) + \beta \, \mathcal{V}(N_{b,t+1}^P) \Big\} \tag{24}$$

where $N_{b,t+1}^P = \bar{N}_b \, \Pi_b(\Delta_{b,t+1})$ and $\Delta_{b,t+1}$ depends on $(\tilde{\mu}_t^P, f_P)$ but *not* on the inherited state $N_{b,t}^P$.

Proposition 1 (Existence, Stationarity, and Uniqueness) *An equilibrium with active platform money exists for all $\beta \in (0,1)$, and it is unique.*[5] *Any such equilibrium has the*

[5] As is standard in models of fiat money (Lagos and Wright, 2005), a non-monetary equilibrium ($\phi^P = 0$) also always exists; we focus on the active monetary equilibrium throughout.

following properties:

1. *The optimal inflation rate $\tilde{\mu}^P$ and fees (f_P^*, k_P^*) are state-independent: they do not depend on the inherited buyer base $N_{b,t}^P$.*

2. *The buyer base is stationary: $N_{b,t}^P = N_b^* \equiv \bar{N}_b \, \Pi_b(\Delta_b^*)$ for all $t \geq 1$, regardless of the initial condition. The legacy market fees (f_L^*, k_L^*) and seller-to-buyer ratios (n_P^*, n_L^*) are also stationary.*

3. *In stationarity ($t \geq 1$), money supply growth and inflation coincide: $\mu^P = \tilde{\mu}^P$.*

Proof. See Appendix B.1. □

The key structural observation is that $N_{b,t+1}^P$ does not depend on $N_{b,t}^P$: the state enters the Bellman equation (24) only through the additive term $-N_{b,t}^P \, u(1-\gamma)$, which drops out of the first-order conditions. This *additive separability* causes the dynamic problem to decompose period by period and makes the value function affine in the inherited base.

Why the inherited base is a liability, not an asset. The economics behind the separability is that platform money is a one-period claim. A buyer acquires platform money at CM_t in order to spend it in DM_{t+1}; when she returns to the CM at $t+1$ she rebalances from scratch and chooses a marketplace anew. The inherited buyer base is therefore not a stock of customers but a stock of liabilities. The only trace the past leaves on the platform's problem is the redemption bill $N_{b,t}^P \, u(1-\gamma)$, the real value the platform has promised to deliver to last period's holders. Everything of value to the platform, its seigniorage and its seller fees, comes from the flow of buyers and sellers it attracts anew this period, and that flow depends only on current terms.

Four features of the environment deliver this result. First, marketplace choice is memoryless: buyers and sellers draw fresh idiosyncratic shocks every period, face no switching costs, and under quasi-linear preferences carry the same continuation value whichever marketplace they used last, so there is no installed base for the platform to defend or exploit. Second, quasi-linearity in the CM makes the distribution of money holdings degenerate (Lagos and Wright, 2005): no cohort or wealth heterogeneity is carried over, and the inherited base is summarized by a single number. Third, proportional bargaining fixes real balances per buyer

at $u(1-\gamma)$, so each inherited buyer is a liability of exactly this amount and each new buyer a source of exactly $\tilde{\mu}_t^P\, u(1-\gamma)$ in revenue; the effective inflation rate $\tilde{\mu}_t^P$ fully controls the terms faced by new entrants, while the headline growth rate μ_t^P absorbs the inherited base through (21). Fourth, the platform's monetary policy is committed and observed at the time buyers acquire money, so the real value delivered to old holders is $u(1-\gamma)$ regardless of what the platform does today: the redemption bill is predetermined. Without commitment, the value delivered to old holders would depend on the platform's current choice relative to what they expected, the redemption bill would no longer be predetermined, and the separability would fail; we return to the role of commitment in Online Appendix E.

The separability has a sharp implication: $\partial \mathcal{V}/\partial N = -u(1-\gamma) < 0$. An additional inherited buyer lowers the platform's value by exactly the real balances she is owed, and all of the platform's franchise value is the present value of the flow of new entrants, which does not depend on the inherited state. This is also why the platform has no incentive to inflate away its outstanding obligations: with the redemption bill predetermined, surprise dilution gains nothing, and the optimal policy is the same in every state. Conversely, any feature that turned the inherited base into an asset would break the separability and make monetary policy genuinely state-dependent: switching costs or habit, installed-base network effects, carry-over of unmatched agents across periods, binding liquidity constraints on the acquisition of platform money, or a non-degenerate distribution of money holdings as in Araujo and Camargo (2008). In each case the platform would trade off exploiting its current base against building a future one.

Substituting the affine value function into (24) and cancelling terms, the platform's reduced per-period problem becomes:

$$\max_{k_P,\, \tilde{\mu}^P,\, f_P \geq 0} \quad \underbrace{\bar{N}_s \Pi_s\left(\Delta_s\right) k_P}_{\text{Fee revenue from sellers}} + \underbrace{\bar{N}_b \Pi_b\left(\Delta_b\right)\left[u\left(1-\gamma\right)\left(\tilde{\mu}^P - \beta\right) + f_P\right]}_{\text{Per-period extraction from buyers}}. \tag{25}$$

The factor $(\tilde{\mu}^P - \beta)$ captures the dynamic nature of the platform's monetary extraction. Each buyer attracted today contributes $\tilde{\mu}^P\, u(1-\gamma)$ in current revenue, but creates a future obligation of $u(1-\gamma)$. Discounting this future cost by β gives a net present value of $(\tilde{\mu}^P - \beta)\, u(1-\gamma)$ per buyer.

Proposition 1 establishes that the platform's buyer base and monetary policy reach their stationary equilibrium values in a single period. Because quasi-linearity absorbs all wealth effects, buyers face no binding liquidity constraints when acquiring platform money. Consequently, they can instantaneously adjust their portfolios to the optimal real balances required for the active monetary equilibrium, collapsing the transition phase into a single period.

2.9 Equilibrium Characterization

Legacy market. The legacy market (L) owner maximizes per-period profit, yielding the standard pricing conditions:

$$k_L = \frac{1 - \Pi_s}{\Pi'_s}, \tag{26}$$

$$f_L = \frac{1 - \Pi_b}{\Pi'_b}. \tag{27}$$

Platform Policy. The first order condition with regard to the seller fee k_P is separable from the buyer side and gives:

$$k_P = \frac{\Pi_s}{\Pi'_s}. \tag{28}$$

The fee is set so that the marginal revenue from sellers equals the marginal loss from those who exit. For the buyer side, the FOC with respect to f_P requires $-\Pi'_b[u(1-\gamma)(\tilde{\mu}^P - \beta) + f_P] + \Pi_b = 0$, while the FOC with respect to $\tilde{\mu}^P$ requires $-\xi\,\Pi'_b[u(1-\gamma)(\tilde{\mu}^P - \beta) + f_P] + \Pi_b = 0$. When $\xi < 1$, these two conditions cannot hold simultaneously, so the fee constraint $f_P \geq 0$ binds and the optimal solution is:

$$f_P = 0 \tag{29}$$

$$\tilde{\mu}^P = \beta + \frac{1}{\xi\,(1-\gamma)\,u}\frac{\Pi_b\,(\Delta_b)}{\Pi'_b\,(\Delta_b)} \tag{30}$$

When $\xi = 1$, the two FOCs coincide and the platform is indifferent between extracting via fees or inflation: any pair $(f_P, \tilde{\mu}^P)$ satisfying $u(1-\gamma)(\tilde{\mu}^P - \beta) + f_P = \Pi_b/\Pi'_b$ is optimal. Henceforth, we set $f_P = 0$ throughout the paper, including the case $\xi = 1$. This pins down

the platform's inflation rate $\tilde{\mu}^P$ uniquely via Equation (30) for all $\xi \in (0, 1]$, and is without loss of generality when $\xi = 1$ since total buyer extraction $u(1-\gamma)(\tilde{\mu}^P - \beta) + f_P$ is the same along the indifference locus. This indifference concerns the choice of *instrument*, not the choice of *money*: both f_P and $\tilde{\mu}^P$ are charges levied by a platform that already issues its own currency. As we show below, even when $\xi = 1$ the platform strictly prefers to charge buyers in its own money rather than to accept outside money.

Equation (30) shows that the platform sets inflation as a markup above the Friedman rule ($\tilde{\mu}^P = \beta$). Because the platform takes the seller-to-buyer ratio as given, its buyer-side and seller-side pricing problems separate. The markup takes the form of a monetary Lerner index: it is proportional to the inverse semi-elasticity of buyer demand (Π_b/Π_b'), and inversely proportional to both the real money balances per buyer, $(1-\gamma)u$, and the salience of inflation, ξ.

This reveals a unique form of obfuscated pricing. When buyers do not fully account for inflation costs ($\xi < 1$), the platform sets the salient entry fee to zero ($f_P = 0$) and replaces it with less salient inflation charges to attract buyers while maintaining real revenue. When buyers are fully attentive ($\xi = 1$), the salience motive disappears and the platform is indifferent between the two instruments; we select the $f_P = 0$ equilibrium by convention. The indifference does not, however, make private money redundant.

The Issuer's Advantage over legacy. Even if buyers are fully attentive ($\xi = 1$), the platform strictly prefers to issue its own money rather than accept outside money, because every point on the indifference locus contains a rent that only the issuer of the money can collect. The per-buyer extraction decomposes as:

$$(\tilde{\mu}^P - \beta)\, u(1-\gamma) = \underbrace{(\tilde{\mu}^P - 1)\, u(1-\gamma)}_{\text{inflation tax}} + \underbrace{(1-\beta)\, u(1-\gamma)}_{\text{issuer's discounting benefit}} . \tag{31}$$

While the first term requires inflation, the second term is a fundamental structural advantage. On any marketplace, buyers must hold real balances for one period before spending them, incurring an opportunity cost of $(1-\beta)u(1-\gamma)$, an "interest-free" loan to the money issuer. Because this cost is identical on both the platform and the legacy market, it washes out of

the buyer's participation decision.[6] However, by issuing its own money, the platform acts as the counterparty and captures this $(1-\beta)$ discounting cost as pure rent *without losing a single buyer*. A platform that accepts outside money instead collects only its fee, and the holding cost that its buyers bear accrues to the issuer of the outside money.

Proposition 2 *Platform P strictly prefers to adopt its own money.*

3 Equilibrium Properties

We first record an accounting identity linking the seller-to-buyer ratios of the two marketplaces to the attraction shares on each side of the market.

Lemma 1 *The platform has a lower seller-to-buyer ratio than the legacy market if and only if its attraction share is lower for sellers than for buyers:*

$$n_P \leq n_L \Leftrightarrow \Pi_s(\Delta_s) \leq \Pi_b(\Delta_b). \tag{32}$$

Moreover, the economy-wide ratio always lies between the two: if $\Pi_s(\Delta_s) \leq \Pi_b(\Delta_b)$ *then* $n_P \leq \overline{N}_s/\overline{N}_b \leq n_L$*, and if* $\Pi_s(\Delta_s) \geq \Pi_b(\Delta_b)$ *the ordering is reversed, with strict inequalities throughout whenever* $\Pi_s(\Delta_s) \neq \Pi_b(\Delta_b)$*.*

Proof. Market clearing gives $n_P = \overline{N}_s\Pi_s/(\overline{N}_b\Pi_b)$ and $n_L = \overline{N}_s(1-\Pi_s)/(\overline{N}_b(1-\Pi_b))$, so $n_P \leq \overline{N}_s/\overline{N}_b \Leftrightarrow \Pi_s \leq \Pi_b \Leftrightarrow n_L \geq \overline{N}_s/\overline{N}_b$, and $n_P/n_L = \big[\Pi_s/(1-\Pi_s)\big]\big/\big[\Pi_b/(1-\Pi_b)\big] \leq 1 \Leftrightarrow \Pi_s \leq \Pi_b$. □

The lemma uses nothing about matching technology or pricing. Since $\overline{N}_s/\overline{N}_b = \Pi_b n_P + (1-\Pi_b)n_L$ is a buyer-weighted average of the two ratios, one marketplace is always on each side of the economy-wide ratio. We call a marketplace *seller-thin* when its seller-to-buyer ratio is below $\overline{N}_s/\overline{N}_b$ and *seller-rich* when it is above. The lemma says that the platform is seller-thin exactly when it attracts a smaller share of sellers than of buyers, and the legacy market is then seller-rich. (We reserve "thick" and "thin" without a qualifier for the overall size of a marketplace.)

[6] This equivalence holds because both markets impose the same one-period holding structure.

Building on this, we can demonstrate how the platform leverages its monetary capabilities. By controlling its own money supply, the platform possesses a distinct tool to subsidize buyers. This buyer influx ensures that sellers on the platform face a thicker pool of potential trading partners, strictly improving their chances of transacting.

Proposition 3 *In equilibrium, sellers are more likely to be matched on the platform than on the legacy market, i.e.,* $a_{Ps}(n_P) > a_{Ls}(n_L)$.

If the platform and legacy market possess identical or highly similar matching technologies, the only way a seller can achieve a higher matching probability on the platform is if the platform maintains a strictly lower seller-to-buyer ratio.

Corollary 1 *If the platform's technological advantage is sufficiently small (*A *close to* 1*), then* $n_P < n_L$.

Proof. With $A = 1$, Proposition 3 gives $a_{Ls}(n_P) > a_{Ls}(n_L)$, and a_{Ls} is strictly decreasing, so $n_P < n_L$. The equilibrium is unique (Proposition 1) and the equilibrium conditions are continuously differentiable in A, so by the implicit function theorem $n_P - n_L$ varies continuously with A and remains strictly negative for A in a neighbourhood of 1. □

The qualifier in the corollary matters. When the platform's matching technology is substantially better, the inflow of sellers can be strong enough that the platform ends up seller-rich: $\Pi_s > \Pi_b$ and, by Lemma 1, $n_L < \overline{N}_s/\overline{N}_b < n_P$. Proposition 3 then holds because of the technology gap itself rather than because the platform is seller-thin.

By drawing a disproportionately large buyer base from the legacy market, the platform's monetary policy initiates a strong positive cross-group externality for sellers. By offering sellers a faster, more reliable matching process, the platform generates a higher expected trade surplus. This superior liquidity translates directly into pricing power.

Crucially, because the value of the elevated matching probability outweighs the cost of the higher fee, the platform can charge a higher explicit price while simultaneously drawing a larger overall volume of sellers.

Proposition 4 *Platform* P *charges a higher seller fee than the legacy market (*$k_P > k_L$*) and attracts more sellers (*$\Delta_s > 0$*).*

Does this influx of fee-paying sellers eventually crowd out the initial advantage? Owing to the self-reinforcing nature of two-sided network externalities, it does not. The arrival of additional sellers increases the matching probability for the buyers on the platform. Thus, buyers are drawn not merely by the monetary subsidy, but by the thick presence of sellers.

Proposition 5 *There are more buyers on the platform ($\Delta_b > 0$).*

Taken together, Propositions 3 through 5 illustrate a powerful reinforcing mechanism. The platform leverages its monetary policy to subsidize buyer entry. This elevated buyer volume fundamentally improves market liquidity for sellers, allowing the platform to extract high seller fees. Finally, this deep pool of sellers feeds back into the value proposition for buyers, guaranteeing that the platform permanently sustains a larger buyer base. Private money thus serves a dual role in equilibrium: it generates seigniorage revenue directly, and by drawing buyers into the platform it triggers cross-group externalities that thicken the market and sustain the platform's pricing power on the seller side.

Strategic Dampening of Macroeconomic Policy. Having established the structural advantages of the platform's matching ecosystem, we turn to how its monetary policy reacts to the macroeconomic environment. The following proposition establishes that the platform does not simply mirror outside monetary policy, but instead engages in a form of strategic dampening.

Proposition 6 *Under Assumption 1, for any $\xi \in (0,1]$, $0 < \frac{\partial \mu^P}{\partial \tilde{\mu}^L} < 1$. Consequently, there is a threshold value $\hat{\mu}^L \geq 1$ such that if $\tilde{\mu}^L \lessgtr \hat{\mu}^L$ then $\mu^P \gtrless \tilde{\mu}^L$.*

The proposition says that the platform raises its inflation less than one-for-one with legacy inflation. To understand why, consider what happens when central bank inflation $\tilde{\mu}^L$ rises by a small amount. This makes the legacy market less attractive to buyers, who begin shifting toward the platform. As Δ_b rises, the platform's optimal inflation markup Π_b/Π_b' increases (by log-concavity of Π_b), so the platform raises its own inflation.

Three forces explain why the response is less than one-for-one. First, the legacy market cuts its buyer fee to retain buyers in response to higher inflation, which puts competitive pressure on the amount of inflation the platform can impose. Second, when more buyers

shift to the platform to escape high legacy inflation, the seller-to-buyer ratio on the platform falls, reducing each buyer's matching probability and making the platform less attractive at the margin. Third, sellers who follow buyers to the platform face higher platform fees, limiting the seller shift. The overall effect of these three forces overwhelms the cross-side feedback loop (that is, more buyers attract more sellers, which attract more buyers) induced by the platform matching technology under Assumption 1, resulting in strategic dampening. The argument is agnostic about which marketplace is seller-rich: the proof uses only that the sign of $h(n_P) - h(n_L)$ agrees with that of $\Pi_s - \Pi_b$ (Lemma 1), so dampening holds in both regimes identified after Corollary 1.

Since $\mu^P > 1$ when $\tilde{\mu}^L = 1$ (the platform inflates to generate seigniorage) while $\partial \mu^P / \partial \tilde{\mu}^L < 1$, there exists a unique crossing point $\hat{\mu}^L > 1$. Below this threshold the platform inflates more than the central bank; above it, the platform moderates its rate below legacy inflation, offering buyers an "inflation shelter."

The Role of Inflation Salience. The result holds for all $\xi \in (0, 1]$: strategic dampening is driven by competitive dynamics, not buyer inattention. At $\xi = 1$, full rationality compresses the platform's optimal markup but leaves the qualitative mechanism unchanged. While we model ξ in reduced form, it naturally proxies for the outcome of a rational inattention problem (Sims, 2003)—acquiring precise information about high-frequency token depreciation is cognitively costly relative to evaluating a posted fee, so a rationally inattentive agent endogenously underweights inflation ($\xi < 1$).

4 The Planner's Problem

In this section, we analyze social welfare by studying the planner's solution. The planner's objective is to maximize the total utility for all buyers, sellers, and the owners of marketplaces in this economy. Since all the transactions in the CM as well as the payments from buyers to sellers are transfers between agents, maximization of total utility is equivalent to maximization of the total surplus from trade in DM.[7] By Proposition 1, the equilibrium is stationary, so we state the planner's problem directly in steady state. The planner's problem

[7]Detailed derivation of the planner's objective is in the online appendix D.

can be stated as:

$$\begin{aligned}\max_{\Pi_b,\Pi_s} \quad & \overline{N}_b\gamma u\left[\Pi_b a_{Pb}(n_P)+(1-\Pi_b)a_{Lb}(n_L)\right] \\ & +\overline{N}_s\left(1-\gamma\right)u\left[\Pi_s a_{Ps}(n_P)+(1-\Pi_s)a_{Ls}(n_L)\right]\end{aligned} \tag{33}$$

subject to the market clearing conditions:

$$n_P=\frac{\overline{N}_s\Pi_s}{\overline{N}_b\Pi_b} \text{ and } n_L=\frac{\overline{N}_s\left(1-\Pi_s\right)}{\overline{N}_b\left(1-\Pi_b\right)}. \tag{34}$$

Note that we allow the planner to allocate the shares of buyers (Π_b) and of sellers (Π_s) to each marketplace directly. Clearly, any allocation that the planner can achieve by choosing fees and money growth rates, she can also achieve by directly allocating buyers and sellers. In fact, the opposite is also true. The planner can achieve any allocation of buyers and sellers by choosing the fees to buyers and sellers appropriately.

We can simplify the above objective function using $a_{js}(n_j)=a_{jb}(n_j)/n_j$ and plugging in for the expressions of n_P and n_L. The objective becomes:

$$\max_{\Pi_b,\Pi_s}\Pi_b a_{Pb}(n_P)+(1-\Pi_b)a_{Lb}(n_L). \tag{35}$$

That is, maximizing the trading surplus is equivalent to maximizing the combined matching probabilities for buyers on the two marketplaces.

Proposition 7 *When the matching technology is symmetric across marketplaces, the planner's solution is* $n_P=n_L=\overline{N}_s/\overline{N}_b$. *When platform* P *has superior matching technology, the planner's solution is* $\Pi_s=\Pi_b=1$.

Due to the concavity of the matching function, in any marketplace where there is trade, it is optimal to set seller-to-buyer ratio equal to $\overline{N}_s/\overline{N}_b$. With symmetric technology any allocation of buyers and sellers to the two marketplaces that preserves the optimal seller-to-buyer ratio is socially optimal. When platform P has superior matching technology, it is optimal to have all sellers and buyers on the platform which automatically preserves the optimal seller-to-buyer ratio.

Recall from Corollary 1 that with symmetric (or nearly symmetric) technology $n_P < n_L$. This leads to the next corollary.

Corollary 2 *When the matching technology is the same across two marketplaces, competitive equilibrium with private money does not achieve the social optimum because the seller-buyer ratio on platform P is too low.*

The inefficiency arises because private money acts as a subsidy for buyers. This subsidy leads to excessive buyer entry, creating a congestion effect where the seller-to-buyer ratio (n_P) drops below the socially optimal level ($\overline{N}_s/\overline{N}_b$). The platform's private incentive to maximize seigniorage thus misaligns with the social goal of maximizing matching efficiency, unless the platform's matching technology is sufficiently superior to compensate for this distortion. Thus, the welfare loss stems not just from search frictions, but from the platform's strategic use of money as a tool for rent extraction rather than efficiency.

However, this monetary wedge between private and social incentives can act as a *second-best instrument* to correct search frictions when technologies are asymmetric. If the platform possesses a superior matching technology, the social optimum requires shifting a large mass of buyers to the platform. By issuing private money, the platform effectively subsidizes buyer entry to capture seigniorage. While this instrument is profit driven, it coincidentally aligns with the social goal of relocating agents to the more efficient marketplace. Thus, platform money can improve welfare not by eliminating the congestion externality, but by counteracting the friction that prevents agents from adopting superior technology. The following proposition states this result formally.

Proposition 8 *When platform P has strictly better matching technology, allowing private money in the decentralized equilibrium can achieve a better outcome from the planner's perspective than the case where private money is not allowed.*

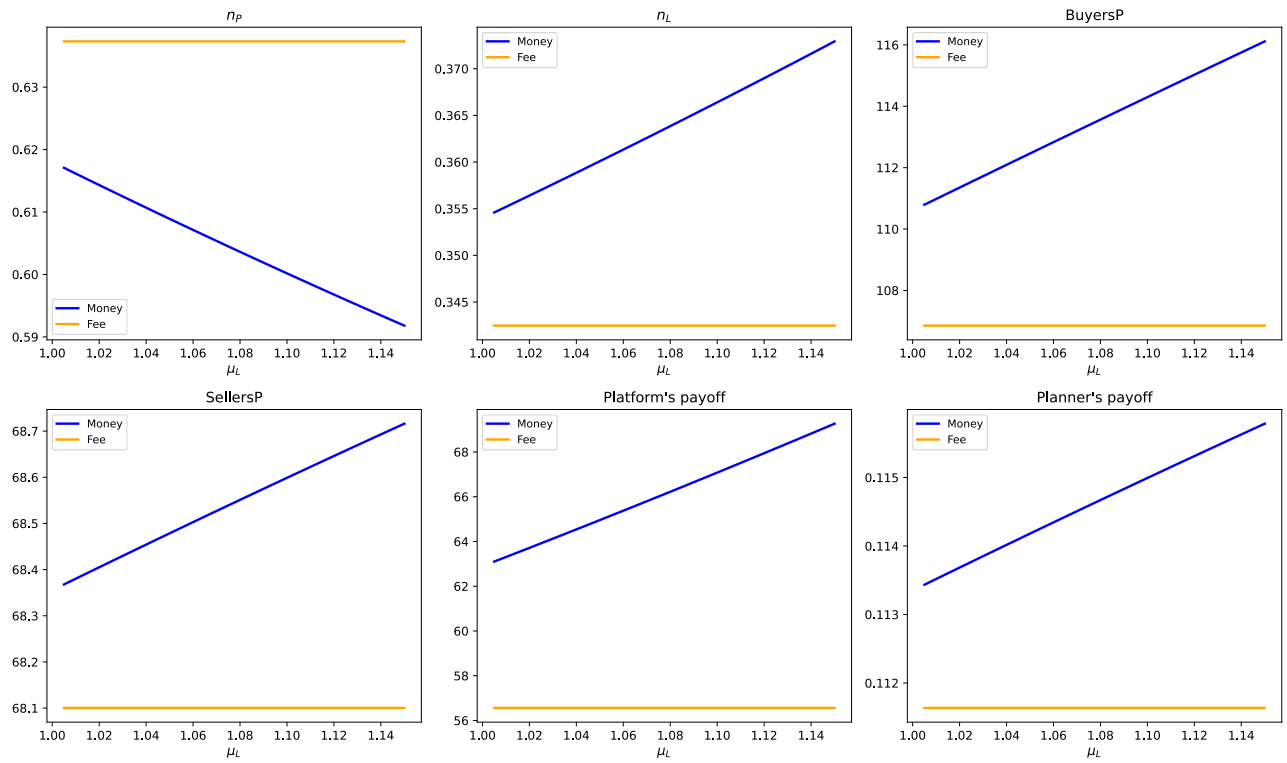
The logic of Proposition 8 is as follows. When platform P has superior matching technology, the social optimum is for all buyers and sellers to be on the platform. There are two important outcome differences between the competitive equilibrium and the social optimum: the seller-to-buyer ratio is generically not equal to $\overline{N}_s/\overline{N}_b$ and the number of buyers or sellers on the platform is too low. Controlling platform money helps platform P to attract buyers,

moving the competitive equilibrium towards the social optimum. However, it also has a cost: as more buyers flock to the platform, seller entry fails to keep pace due to the high entry fees set by the platform owner. Because of the concavity of the matching function, the total number of matches does not grow proportionally with this influx of buyers. Consequently, the per-buyer matching probability declines as seller-to-buyer ratio falls, causing the competitive outcome to still deviate from the first-best social optimum.

We illustrate the result in Proposition 8 in a numerical example shown in Figure 2. In this example, we compare how the planner's payoff and each of the other equilibrium outcomes (n_P, n_L, number of buyers (BuyerP), number of sellers (SellerP), and platform's payoff) varies with the legacy inflation for the following two cases: the case when the platform is allowed to use private money (labeled as money in the blue line) and the case where the private platform money is not allowed (labeled as fee in the orange line). In these graphs, the legacy inflation is plotted on the x-axis while the y-axis represents a specific outcome variable. We observe that when the platform is not allowed to use private money and charges a fee denominated in outside money, the equilibrium outcomes do not vary with the legacy inflation. This is because in this case legacy inflation does not affect how the platform competes with the legacy marketplace as both experience the same inflation rate. The second graph on the bottom row shows that allowing private money in the equilibrium the planner achieves a higher payoff than the fee only outcome. In the equilibrium where the platform money is allowed, the seller-to-buyer ratio on the platform is lower (further away from the social optimum), but the number of buyers on the platform is larger (closer to the social optimum), than the case where the platform is only allowed to charge a fixed fee.

Implementation via Macroeconomic Policy. Having established the socially optimal allocations, we now consider the implementation of this optimum by a benevolent central bank. Because the monetary authority cannot directly command user participation, it must utilize the legacy fiat inflation rate, $\tilde{\mu}^L$, as an instrument to influence the buyers' decentralized marketplace choice. The efficacy of this policy relies fundamentally on the platform's strategic dampening behavior established in Proposition 6. Because the platform raises its inflation less than one-for-one with legacy inflation ($0 < \partial\mu^P/\partial\tilde{\mu}^L < 1$), the central bank recognizes that an increase in $\tilde{\mu}^L$ strictly widens the effective cost differential between the

Figure 2: Money vs Fiat-Fee



The graphs have $\tilde{\mu}^L$ on x-axis and an outcome variable (n_p, n_L, buyer number on platform (BuyersP), seller number on platform (SellersP), platform owner's payoff, planner's payoff) on y-axis. Platform money in the blue line and platform fee (denominated in outside money) only in the orange line. Matching technology: CES of den Haan et al. (2000), $a_{Pb}(n) = (n^{-\rho}+1)^{-1/\rho}$, $a_{Lb}(n) = a_{Pb}(n)/A$, $a_{js}(n) = a_{jb}(n)/n$, with $\rho = 0.5$ and $A = 10$, so that all matching probabilities lie in $(0, 1)$. Other parameters: $\beta = 0.9$; $u = 1$; $\sigma_b = 0.2$; $\sigma_s = 0.05$; $\gamma = 0.5$; $\bar{N}_s = 100$; $\bar{N}_b = 200$; $\xi = 0.95$. Replication: `draft/figures_v12.py`.

two markets, thereby driving buyers toward the platform.

Consequently, the central bank's optimal policy is strictly contingent upon the platform's relative technological advantage. When the platform and the legacy market possess identical matching technologies, Corollary 2 demonstrates that platform money generates inefficient congestion by excessively subsidizing buyer entry. To maximize social welfare, the central bank must minimize the platform's market share and correct this distortion. It achieves this by setting the legacy inflation rate to its lower bound, adhering to the Friedman rule ($\tilde{\mu}^L \to \beta$). Under this symmetric technology regime, the threat of private platform money strictly disciplines the central bank, forcing it to maintain a highly competitive and non-

distortionary public currency.

Conversely, if the platform possesses a strictly superior matching technology, Proposition 7 establishes that the social optimum necessitates shifting a large mass of buyers to the platform. To induce this efficient reallocation, the central bank optimally implements a strictly positive inflation tax ($\tilde{\mu}^L > \beta$). By intentionally degrading the payoff of the legacy market, the central bank utilizes legacy inflation as a Pigouvian tax to overcome search frictions, actively migrating users toward the platform's superior matching infrastructure. In this asymmetric regime, rather than being disciplined, the central bank effectively free-rides on the platform's private ecosystem to maximize the total trade surplus. Thus, endogenizing central bank policy reveals that private platform money does not universally exert downward pressure on public inflation; rather, depending on the underlying industrial organization, it can actively enable the monetary authority to deploy inflation as an instrument for structural reallocation.

Furthermore, this implementation highlights the role of public money as a fundamental constraint on private platform market power. As established in the platform's first-order conditions, the optimal inflation markup is inversely proportional to the semi-elasticity of buyer demand, which is directly governed by the relative attractiveness of the legacy market. By maintaining a highly credible, low-inflation public currency ($\tilde{\mu}^L \to \beta$), or by upgrading the legacy infrastructure to reduce search frictions, such as through the introduction of a retail Central Bank Digital Currency (CBDC), the monetary authority significantly enhances the outside option available to users. This improvement in the public baseline compresses the platform's perceived buyer advantage, thereby increasing the elasticity of demand for the platform's services. Stripped of its local monopoly insulation, the platform is forced to compress its own inflation markup toward the socially optimal Friedman rule simply to prevent user attrition. Consequently, a well-managed public currency does not merely serve as a macroeconomic medium of exchange; it acts as a vital structural check that strictly limits the capacity of digital platforms to extract monopolistic rents from their enclosed ecosystems.

5 Robustness to Internalizing Seller-Buyer Ratio

In this section we discuss how the platform's monetary policy changes when it internalizes its effect on the seller-to-buyer ratio and show that our main message based on the baseline model is robust to this change.

Internalizing the seller-to-buyer ratio allows the platform to actively manage the composition of the market. In the baseline model, the platform takes seller to buyer ratio as given and its monetary policy affects the seller side of the market only through its equilibrium effect. Under internalization, the platform recognizes that monetary policy shapes the mix of buyers and sellers. Depending on which cross-side channel dominates, the platform may tighten monetary policy to retain buyers whose presence improves seller matching, or loosen it to finance seller subsidies that improve buyer matching. This gives the platform an additional reason to control its own monetary policy: beyond seigniorage extraction, own money serves as a lever for managing the two-sided externality.

The cross-subsidy logic that drives internalization is the monetary analog of the Rochet and Tirole (2003) optimal pricing formula for two-sided platforms, where network externalities on each side create incentives to subsidize one side of the market using revenue from the other. The distinctively monetary feature is that the platform can adjust the *tightness* of its monetary policy to manage market composition, rather than relying solely on fees.

Formulation. The only change relative to the baseline is that the platform now accounts for the effect of its instruments on the seller-to-buyer ratios. As in Definition 1, it continues to take the legacy market's fees (f_L, k_L) as given, so the equilibrium concept is unchanged; fees remain non-negative, $k_P \geq 0$ and $f_P = 0$, and we characterize the interior solution of the internalized problem. We keep $f_P = 0$ and let the platform choose the shares (Π_b, Π_s) directly, writing the seller fee and the per-buyer seigniorage as the inverse demands that implement them given (f_L, k_L), as in Armstrong (2006) and Weyl (2010). This change of variables is without loss because, for given platform and legacy instruments, the equilibrium shares are unique (Proposition 1). A remark after Proposition 9 covers the alternative in which the platform also anticipates the legacy market's fee response.

Relative to the baseline, internalizing market tightness changes each pricing rule through

two opposing forces.

Residual-demand elasticity (pushes both $\tilde{\mu}^P$ and k_P up). The baseline platform computes its Lerner markup from the partial-equilibrium slope of buyer demand, $1/\Pi_b'$. Once it internalizes tightness, the inverse demand it faces is steeper by a factor

$$F_b \;=\; 1 \;+\; \theta_b \Pi_b' \;>\; 1.$$

The first term is the baseline slope. The second is congestion rebalancing: more buyers on the platform lower n_P and raise n_L, which erodes the matching gain that attracted them ($\theta_b > 0$ is the fall in buyers' matching advantage per unit of buyer share, defined in the appendix). Attracting share therefore costs the platform more than the baseline assumes, because each arrival congests its own side. Buyer demand is therefore less elastic than the baseline assumes, and the markup scales up by F_b. The seller side has the analogous factor $F_s = 1 + \theta_s \Pi_s' > 1$.

Cross-side externality (pushes both $\tilde{\mu}^P$ and k_P down). The baseline platform ignores the indirect cost of each instrument: a buyer driven away by inflation worsens seller matching and reduces fee revenue, and a seller driven away by the fee worsens buyer matching and reduces seigniorage. The internalizing platform recognizes both linkages. This is the Pigouvian correction: each buyer generates not only direct seigniorage but also indirect fee revenue, and each seller generates not only fee revenue but also indirect seigniorage.

These two forces appear directly in the internalized pricing rules. We show in the appendix that setting the internalized first-order conditions to zero yields

$$(\tilde{\mu}^{P*} - \beta)(1-\gamma)u \;=\; \underbrace{\frac{\Pi_b}{\xi\,\Pi_b'}\,F_b}_{\text{elasticity correction}} \;-\; \underbrace{\frac{\bar{N}_s\,\Pi_s}{\bar{N}_b}\,\phi_{sb}}_{\text{cross-subsidy}}, \qquad k_P^* \;=\; \underbrace{\frac{\Pi_s}{\Pi_s'}\,F_s}_{\text{elasticity correction}} \;-\; \underbrace{\frac{\bar{N}_b\,\Pi_b}{\xi\,\bar{N}_s}\,\phi_{bs}}_{\text{cross-subsidy}},$$

where $\phi_{sb} > 0$ is the buyers-help-sellers externality, the rise in sellers' matching advantage per unit of buyer share (each buyer lets the platform charge every seller ϕ_{sb} more), and $\phi_{bs} > 0$ the sellers-help-buyers externality, the rise in buyers' matching advantage per unit of seller share (each seller raises the seigniorage the platform can collect per buyer by ϕ_{bs}/ξ). Each cross-subsidy term is the value of a marginal user on one side to the other side, as in the

two-sided pricing formulas of Rochet and Tirole (2003) and Armstrong (2006). Compared to the baseline rules $(\tilde{\mu}^{P,0} - \beta)(1-\gamma)u = \Pi_b/(\xi\Pi_b')$ and $k_P^0 = \Pi_s/\Pi_s'$, the elasticity corrections scale both markups up, and the cross-subsidy terms pull both down. Each instrument falls when its cross-subsidy term dominates.

Lowering either instrument below its baseline level is a cross-subsidy: the platform gives up margin on one side to thicken the other. Whether a cross-subsidy pays is a ratio test. Since $F_i - 1 = \theta_i \Pi_i'$, the pricing rules give: the seller fee falls when $\phi_{bs}/\theta_s > \xi n_P$, and inflation falls when $\phi_{sb}/\theta_b > 1/(\xi n_P)$, where $n_P = \bar{N}_s\Pi_s/(\bar{N}_b\Pi_b)$ is the platform's seller-to-buyer ratio. Each cut pays when the subsidized side's help to the other side, relative to its own congestion, exceeds a threshold set by the platform's composition: the platform subsidizes the side that is relatively scarce. The two thresholds are reciprocals, so multiplying the two conditions cancels ξ and the composition: both instruments can fall only if cross-side help exceeds own-side congestion in product, $\phi_{bs}\phi_{sb} > \theta_b\theta_s$. The appendix shows that this never happens, for a reason that lies in the matching externalities themselves. Under Assumption 1 the externality matrix

$$\begin{pmatrix} \theta_b & -\phi_{bs} \\ -\phi_{sb} & \theta_s \end{pmatrix}$$

has a non-negative determinant: own-side congestion dominates cross-side thickness *jointly*, though not necessarily side by side. This is the analogue of the stability condition in two-sided-market models, that the product of the cross-group externalities not exceed the product of the own-group effects (Armstrong, 2006; Weyl, 2010), and it is the same condition that delivers strategic dampening in Proposition 6. The intuition is the one behind Lemma 1: on each marketplace it is the scarce side whose arrivals move tightness, and Assumption 1 says that where a side is scarce its own matching rate is relatively more responsive to tightness than on the other marketplace. A scarce-side arrival therefore congests its own side more than it helps the other, which damps the loop in which buyers attract sellers who attract buyers. Since $\theta_b\theta_s \geq \phi_{bs}\phi_{sb}$, the platform can afford *at most one* cross-subsidy.

Proposition 9 (At most one instrument falls) *Under Assumption 1, the seller fee and the inflation rate cannot both fall when the platform internalizes the seller-to-buyer ratio.*

That is, for any $\xi > 0$, at most one of k_P and $\tilde{\mu}^P$ decreases relative to the baseline.

Three regimes therefore arise, depending on which cross-side channel dominates: (a) tight money with a higher seller fee ($\tilde{\mu}^P$ falls, k_P rises) when the "buyers help sellers" channel dominates; (b) loose money with a lower seller fee (k_P falls, $\tilde{\mu}^P$ rises) when the "sellers help buyers" channel dominates; or (c) both instruments rise when cross-side externalities are weak relative to the elasticity corrections on both sides.

Remark 1 (Anticipating the legacy market's fee response) *If the platform also anticipated the legacy market's equilibrium fee responses, $f_L = (1 - \Pi_b)/\Pi_b'$ and $k_L = (1 - \Pi_s)/\Pi_s'$, acting as a Stackelberg leader in shares, each elasticity factor would gain a term, $F_i = 1 + (1 - R_i) + \theta_i \Pi_i'$, where $R_i \leq 1$ is a curvature term defined in the appendix and $(1 - R_i) \geq 0$ reflects that the legacy cuts its fee when it loses participants, making share harder to gain. Then $(F_s - 1)(F_b - 1) \geq \theta_b \theta_s \Pi_b' \Pi_s' \geq \phi_{bs}\phi_{sb}\Pi_b'\Pi_s'$, so Proposition 9 holds with more slack: the legacy's fee response only reinforces the result.*

Buyer inattention ($\xi < 1$). Inattentive buyers do not fully perceive inflation, so per-buyer seigniorage scales as $1/\xi$, which exceeds the buyer's perceived cost. This amplifies the "sellers help buyers" channel: each seller retained on the platform attracts buyers who generate amplified seigniorage revenue. Consequently, buyer inattention tilts the platform toward regime (b), which is loose monetary policy combined with a lower seller fee. The parameter ξ governs which regime obtains: high ξ favors tight money (regime a), low ξ favors loose money (regime b).

Strategic dampening under internalization. The strategic dampening result of Proposition 6 ($0 < \partial \mu^P / \partial \tilde{\mu}^L < 1$) extends to the internalized setting. To see why, write the platform's optimal inflation in terms of the shares as $\tilde{\mu}^P = \tilde{\mu}^L + \Gamma(\Pi_b, \Pi_s)/[\xi(1-\gamma)u]$, where $\Gamma(\Pi_b, \Pi_s) \equiv M_b(\Pi_b, \Pi_s) + (1 - \Pi_b)/\Pi_b' - \Delta_b(\Pi_b)$ captures the buyer surplus gap extracted by the platform and does not depend directly on $\tilde{\mu}^L$. Strategic dampening ($\partial \tilde{\mu}^P / \partial \tilde{\mu}^L < 1$) therefore requires $d\Gamma/d\tilde{\mu}^L < 0$. Decompose:

$$\frac{d\Gamma}{d\tilde{\mu}^L} = \underbrace{\frac{\partial \Gamma}{\partial \Pi_b}}_{-\tilde{F}_b/\Pi_b' < 0} \frac{d\Pi_b}{d\tilde{\mu}^L} + \underbrace{\frac{\partial \Gamma}{\partial \Pi_s}}_{\phi_{bs} > 0} \frac{d\Pi_s}{d\tilde{\mu}^L}.$$

Here $\tilde{F}_b \equiv F_b + (1 - R_b) \geq F_b$ also includes the legacy market's equilibrium fee response, because Γ is evaluated along the equilibrium path on which $f_L = (1-\Pi_b)/\Pi_b'$ adjusts. Higher legacy inflation drives buyers to the platform ($d\Pi_b/d\tilde{\mu}^L > 0$), which worsens congestion and tightens the buyer surplus gap, so the first term is unambiguously negative. Under internalization, two channels reinforce this dampening relative to the baseline. First, the cross-side externality gives the platform an additional reason to moderate inflation: each buyer lost to higher $\tilde{\mu}^P$ degrades seller matching and reduces fee revenue, so the internalized platform responds more cautiously to a given increase in $\tilde{\mu}^L$. Second, the elasticity factor ($\tilde{F}_b > 1$) amplifies the magnitude of $\partial\Gamma/\partial\Pi_b$, strengthening the congestion channel. These forces ensure that the first (negative) term dominates the second, preserving $d\Gamma/d\tilde{\mu}^L < 0$ and hence strategic dampening.[8]

6 Should the Platform Accept Outside Money?

In the baseline model, the platform strictly prefers to use platform money and has no incentive to accept outside fiat money. In practice, however, many platforms accept both platform money and outside payment options. To capture this, we extend the model by distinguishing between flexible buyers, who are indifferent to the payment medium, and biased buyers, who prefer outside money. We show that as long as flexible buyers exist, the platform always optimally issues its own currency to capture seigniorage. Consequently, the strategic decision reduces to whether to additionally accept outside money (Open Policy) or to enforce exclusivity (Closed Policy).

We uncover a trade-off between market share and strategic segmentation. By accepting outside money (Open policy) the platform competes directly for all buyers, maximizing its potential user base. However, by rejecting outside money (Closed Policy), the platform effectively cedes the biased buyers to the legacy market. This strategic segmentation can

[8] A formal proof proceeds via the implicit function theorem applied to the Hessian of the internalized revenue function $R(\Pi_b, \Pi_s; \tilde{\mu}^L)$. Since $\tilde{\mu}^L$ enters R only through the per-buyer seigniorage $E \equiv (1-\gamma)u(\tilde{\mu}^P - \beta)$ and $\partial^2 R/\partial\Pi_s\partial\tilde{\mu}^L = 0$, the seller-side second-order condition $R_{ss} < 0$ ensures $d\Pi_b/d\tilde{\mu}^L = -R_{ss}\bar{N}_b(1-\gamma)u/D > 0$ where $D = R_{bb}R_{ss} - R_{bs}^2 > 0$ is the Hessian determinant. The sign of $d\Gamma/d\tilde{\mu}^L$ then reduces to $\tilde{F}_b|R_{ss}|/\Pi_b' > \phi_{bs}R_{sb}$, which holds whenever the own-side second-order effect dominates the cross-side revenue complementarity, a condition implied by the second-order conditions of the platform's problem.

soften competition: it turns the legacy market into a monopoly over the biased segment, encouraging it to raise fees. This high-fee environment, in turn, allows the platform to extract higher seigniorage from its own flexible users.

To simplify the analysis, we introduce two modifications to the baseline model. First, we assume the attraction functions take the logistic form: $\Pi_b(\Delta_b) = [1 + \exp(-\Delta_b/\sigma_b)]^{-1}$ and $\Pi_s(\Delta_s) = [1 + \exp(-\Delta_s/\sigma_s)]^{-1}$. These standard functional forms arise when idiosyncratic payoff shocks follow a Gumbel distribution with scale parameters σ_b and σ_s.[9] Second, we fix the number of sellers on each market and focus only on the buyer side of the market. Specifically, we assume that the number of sellers on the platform is $\bar{N}_s\Pi_s$ and on the legacy market is $\bar{N}_s(1 - \Pi_s)$. We also assume that $\tilde{\mu}^L > 1$.

Buyers are heterogeneous in their payment preferences. A fraction ψ are flexible (indifferent to the medium of exchange), while $1 - \psi$ are biased: they incur a utility cost κ when using platform money. In this environment, we compare the following two policies available to the platform:

(i) Closed policy: accept only platform money, as in the baseline model; or

(ii) Open policy: accept both platform and outside money.

We denote the variables under the closed policy with the superscript c and write the platform's and the legacy market's objective functions respectively as:

$$\bar{N}_b \left(\psi\Pi_b\left(\Delta_b^c\right) + \left(1 - \psi\right)\Pi_b\left(\Delta_b^c - \kappa\right)\right)\left[u\left(1 - \gamma\right)\left(\mu^{P,c} - \beta\right)\right] \tag{36}$$

and

$$\bar{N}_b \left(1 - \psi\Pi_b\left(\Delta_b^c\right) - \left(1 - \psi\right)\Pi_b\left(\Delta_b^c - \kappa\right)\right) f_L^c \tag{37}$$

where the perceived advantage of the platform over the legacy market for the flexible buyers,

[9] As the scale parameter approaches ∞, agents choose venues purely randomly (shares approach 0.5). As it approaches 0, the choice becomes perfectly deterministic. Under this specification, the inverse semi-elasticity of demand shaping the platform's optimal markup (Equation 30) takes the form $\Pi_b/\Pi_b' = \sigma_b/(1 - \Pi_b)$: the platform's pricing power scales with the variance of user preferences (σ_b) and convexly with its market share (Π_b).

Δ_b^c, is defined analogously to (16):

$$\Delta_b^c = \beta \left(a_{Pb}(n_P^c) - a_{Lb}(n_L^c)\right) \gamma u + \xi \left(\tilde{\mu}^L - \mu^{P,c}\right) (1-\gamma)\, u + f_L^c, \tag{38}$$

and seller-to-buyer ratios are given by:

$$n_P^c = \frac{\bar{N}_s \Pi_s}{\bar{N}_b \left(\psi \Pi_b \left(\Delta_b^c\right) + (1-\psi)\, \Pi_b \left(\Delta_b^c - \kappa\right)\right)}, \tag{39}$$

$$n_L^c = \frac{\bar{N}_s \left(1 - \Pi_s\right)}{\bar{N}_b \left(1 - \psi \Pi_b \left(\Delta_b^c\right) - (1-\psi)\, \Pi_b \left(\Delta_b^c - \kappa\right)\right)}. \tag{40}$$

Note that under the closed policy, the perceived advantage of the platform for the biased buyers is reduced by κ, which is the additional cost that these buyers experience when they hold platform money.[10]

Under the open policy we denote the perceived advantage of the platform over the legacy market for the flexible (the biased) buyers by Δ_b^{of} (Δ_b^{ob}) and the other variables with the superscript o. We characterize the equilibrium where flexible buyers use platform money and biased buyers use outside money. We write the platform's and the legacy market's objective functions respectively as:

$$\bar{N}_b \left(\psi \Pi_b \left(\Delta_b^{of}\right) \left[u \left(1-\gamma\right) \left(\mu^{P,o} - \beta\right)\right] + (1-\psi)\, \Pi_b \left(\Delta_b^{ob}\right) f_P^o\right) \tag{41}$$

and

$$\bar{N}_b \left(1 - \psi \Pi_b \left(\Delta_b^{of}\right) - (1-\psi)\, \Pi_b \left(\Delta_b^{ob}\right)\right) f_L^o \tag{42}$$

where

$$\Delta_b^{of} = \beta \left(a_{Pb}(n_P^o) - a_{Lb}(n_L^o)\right) \gamma u + \xi \left(\tilde{\mu}^L - \mu^{P,o}\right) (1-\gamma)\, u + f_L^o, \tag{43}$$

$$\Delta_b^{ob} = \beta \left(a_{Pb}(n_P^o) - a_{Lb}(n_L^o)\right) \gamma u + \left(f_L^o - f_P^o\right), \tag{44}$$

[10] We assume that this cost is fixed and does not depend on the amount of money that the biased buyers hold. It is easy to show that all the results in this section go through even if the cost is proportional to the amount of money that the biased buyers hold.

$$n_P^o = \frac{\bar{N}_s \Pi_s}{\bar{N}_b \left(\psi \Pi_b \left(\Delta_b^{of} \right) + (1-\psi) \Pi_b \left(\Delta_b^{ob} \right) \right)}, \tag{45}$$

$$n_L^o = \frac{\bar{N}_s (1-\Pi_s)}{\bar{N}_b \left(1 - \psi \Pi_b \left(\Delta_b^{of} \right) - (1-\psi) \Pi_b \left(\Delta_b^{ob} \right) \right)}. \tag{46}$$

Under the open policy, flexible buyers use platform money and the biased buyers use outside money which introduces two incentive compatibility constraints. The first constraint, $\Delta_b^{of} \geq \Delta_b^{ob}$, requires that the flexible buyers use platform money and the second constraint, $\Delta_b^{of} - \kappa \leq \Delta_b^{ob}$, requires that the biased buyers use outside money. We next show that if the open policy is optimal then the first constraint cannot be binding and flexible buyers strictly prefer using platform money.

Lemma 2 *If the platform prefers the open policy and accepts both types of money then the flexible consumers strictly prefer to use platform money, i.e.,* $\Delta_b^{of} > \Delta_b^{ob}$.

Intuitively, the proof is based on the platform's unique competitive advantage to attract flexible consumers by controlling its own money supply, that is, setting a lower inflation while collecting the seigniorage income.

So far, we have not considered the possibility that the platform accepts only outside money. Next, we show that as long as there are some flexible buyers, accepting only outside money cannot be optimal for the platform.[11] This result holds even if most buyers are biased and they have strong preference for outside money.

Proposition 10 *As long as* $\psi > 0$*, platform either uses only platform money or accepts both types of money but never uses only outside money.*

The proof requires a detailed comparison of the platform's payoff under two cases: the case where the platform only allows the use of outside money and charges buyers a fee, versus the case where it allows both types of money, charging flexible buyers platform money inflation and biased buyers a fee. There are two effects that go in the same direction.

[11] This result is different from Proposition 2 which shows that when *all* buyers are flexible, platform strictly prefers using platform money over outside money.

First, we show that, in absence of the strategic response from the legacy, that is, when the legacy marketplace fee is fixed, platform can always achieve a higher payoff by allowing both types of money. This is again due to the unique advantage of the platform to control its own money supply and collect seigniorage.

Second, we show that, using both platform and outside money as opposed to only outside money elicits strategic response from the legacy marketplace leading to a higher platform payoff. When the platform allows both types of money, since flexible buyers prefer to use platform money (as shown in Lemma 2) and are attracted to the platform, legacy mainly compete with the platform over biased buyers rather than a mixed group of biased and flexible buyers. This means that legacy faces a reduced competition since they only need to attract biased buyers now, and hence can raise its fees. In response to the higher fee in legacy, the platform can raise its charge to its buyers via both inflation and fee and obtain higher payoffs consequently. It is then easy to show that the combined effects always favour accepting both types of money as opposed to only outside money.

Next, we consider two edge cases to characterize the platform's choice of allowing the use of outside money to highlight the underlying economic trade-off. In the first edge case, the disutility of using platform money for biased buyers, κ, is small. In the second edge case, $\kappa = \infty$ so that biased buyers never hold platform money.

In the first edge case, if the platform follows the closed policy and accepts only platform money, it loses a very small fraction of biased buyers to the legacy market. When $\tilde{\mu}^L > 1$, the platform's profit from the biased buyers that it captures is higher when it charges them through inflating the platform money as opposed to a fee that is paid in outside money. Hence, when the bias is small enough, the platform strictly prefers the closed policy. The result in the following proposition is immediate.

Proposition 11 *There exists $\bar{\kappa} > 0$ such that when $\kappa < \bar{\kappa}$, the platform adopts the closed policy (accepts only platform money).*

In the second edge case, suppose $\kappa = \infty$ so that biased buyers never hold platform money. In this case there are thresholds $\underline{\psi} \leq \overline{\psi}$ such that if $\psi < \underline{\psi}$ then the platform follows the open policy and if $\psi > \overline{\psi}$ then the platform follows the closed policy. Underlying this result

again are two effects. The first is a direct effect: with the closed policy the platform loses the biased buyers to the legacy market. The second is a strategic effect: with the closed policy the legacy market effectively becomes a monopolist for the biased buyers, raises its buyer fee, in turn allowing the platform to raise its seigniorage income. When ψ is low, the direct effect dominates and the open policy is optimal. When ψ is high, the strategic effect dominates and the closed policy is optimal.

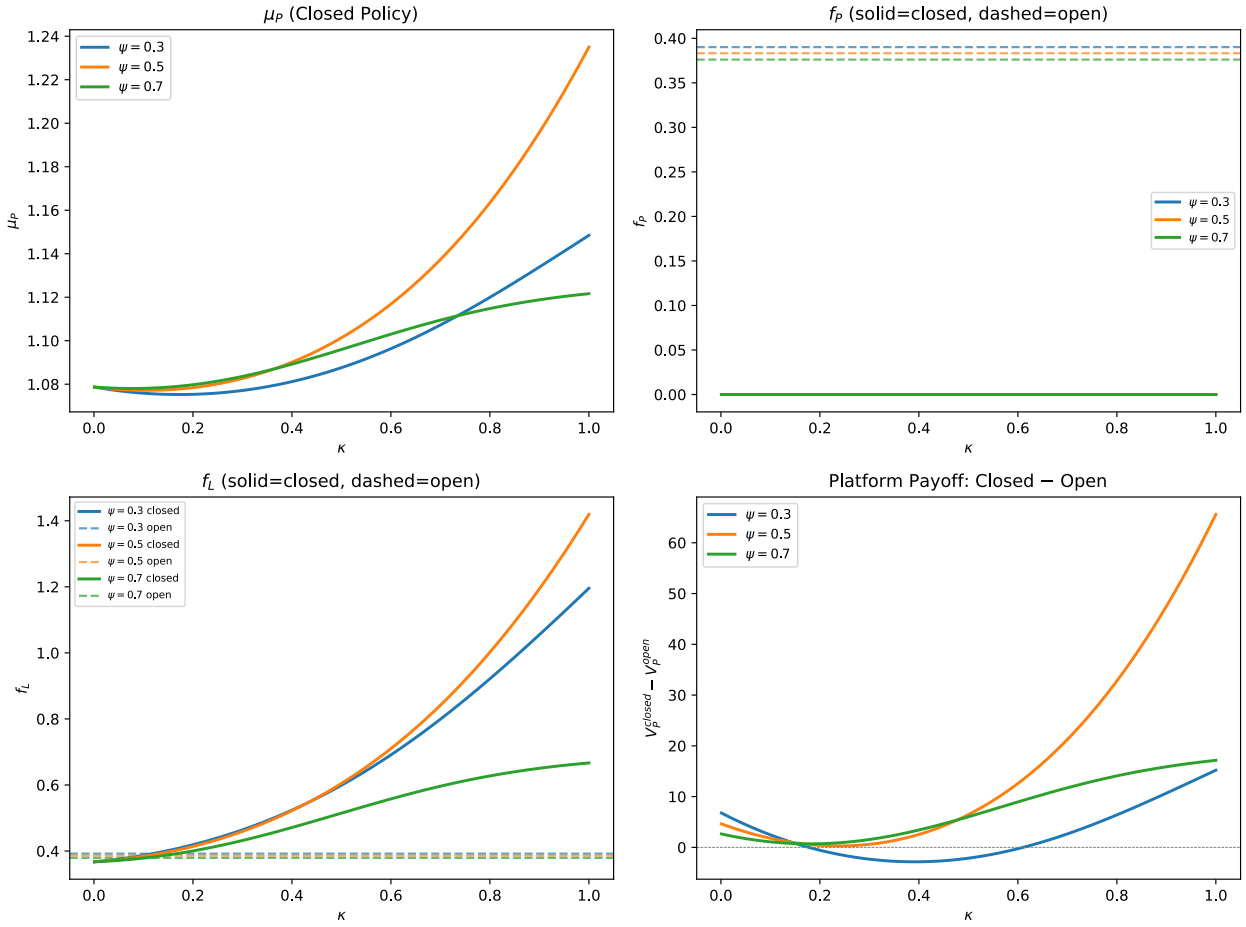
Proposition 12 *Suppose $\kappa = \infty$. There exist thresholds $\underline{\psi}$ and $\overline{\psi}$ such that if $\psi < \underline{\psi}$ then the platform follows the open policy (accepts both outside money and platform money) and if $\psi > \overline{\psi}$ then the platform follows the closed policy (accepts only platform money).*

While it may seem intuitive that under high buyer bias κ the platform would accept outside money, Proposition 12 states that this is not necessarily the case. The deciding factor is the fraction of flexible buyers ψ. If the fraction of flexible buyers is small, this intuition is correct and, the platform has to allow outside money in order to attract the large population of extremely biased buyers. However, if the fraction of flexible buyers is large enough, this intuition does not apply and platform chooses not to accept outside money at all.

This result is driven by strategic complementarity. As κ increases, the Closed Policy effectively segments the market: the legacy market captures the biased buyers, while the platform serves the flexible ones. Secure in its monopoly over the biased 'captive' segment, the legacy market optimally raises its buyer fees (f_L). Crucially, this higher legacy fee reduces the attractiveness of the outside option for flexible buyers. This relaxes the competitive constraint on the platform, allowing it to charge a higher inflation (μ^P) to its own flexible users without losing them. Thus, by refusing to compete for biased buyers, the platform induces a high-fee equilibrium that maximises its seigniorage revenue from the flexible majority.

These theoretical findings can also be demonstrated numerically for the intermediate cases where the disutility of using platform money for biased buyers, κ, is moderate, and the fraction of flexible buyers, $\psi \in (0, 1)$. We next conduct a numerical exercise, and in Figure 3 graph the equilibrium outcome variables: platform inflation, platform buyer fee, legacy buyer fee, and difference in platform payoff between the closed versus the open policy when κ varies from 0 to 1 and ψ takes values 0.3, 0.5, and 0.7.

Figure 3: Comparison of the Closed and Open Policies



The graphs have κ on the x-axis and μ^P, f_P, f_L, and the platform's payoff under the closed (accept only platform money) versus the open policy (accept both forms of money) on the y-axis. Matching technology as in Figure 2 with $\rho = 0.5$ and symmetric technology, $A = 1$. Other parameters: $\beta = 0.99$; $u = 10$; $\gamma = 0.5$; $\sigma_b = 0.2$; $\sigma_s = 0.2$; $\bar{N}_s = 100$; $\bar{N}_b = 200$; $\xi = 0.99$; $\tilde{\mu}^L = 1.02$. Replication: `draft/figures_v12.py`.

The bottom right panel of Figure 3 shows that platform follows the closed policy when κ is small or large, echoing the theoretical findings in Proposition 11 and Proposition 12. The numerical exercise illustrates that for the intermediate range of κ, open policy can become optimal. This result follows from our earlier discussion. As κ increases the direct effect can dominate the strategic effect favoring the open policy, but eventually the strategic effect overtakes the direct effect and the platform optimally follows the closed policy.

While inflation salience (ξ) appears in the perceived-advantage expressions and our numerical parameterization, our qualitative results regarding strategic segmentation do not rely

on this behavioral bias. The mechanism is fundamentally driven by the platform's structural market power rather than consumer inattention. Even if buyers are fully attentive ($\xi = 1$), the platform still optimally utilizes the closed policy to segment the market and soften competition. The salience parameter merely shifts the magnitude of the seigniorage benefit enjoyed by the platform, potentially amplifying or dampening the baseline incentive depending on the level of outside inflation.

7 Conclusion

Our analysis demonstrates that when a platform possesses the ability to issue its own money, it can strategically control its money supply to subsidize buyer entry. This influx of buyers subsequently draws more sellers, launching and amplifying cross-group network externalities, especially when combined with superior matching technologies. By commanding a thicker, more efficient market, the platform exercises considerable market power, enabling it to extract high seller fees. While the platform's monetary policy creates inefficient congestion when competing technologies are similar, it becomes socially desirable when the platform possesses superior matching technology, as the inflation subsidy effectively corrects search frictions that a fee-only model cannot.

Furthermore, our analysis explains why platforms often resist interoperability. We show that a "Closed Policy" (i.e., rejecting outside money) is not merely about forcing adoption, but about strategic segmentation. By ceding biased buyers to the legacy market, the platform induces its competitor to raise fees, creating a high-cost environment that paradoxically allows the platform to extract higher seigniorage from its own flexible users. This suggests that "walled gardens" may be sustained not just by technological lock-in, but by an endogenous monetary strategy that softens price competition.

Beyond strategic segmentation, our analysis emphasizes the critical disciplinary role of public macroeconomic policy. We demonstrate that a well-managed public currency, or the deployment of a retail CBDC, acts as a structural antitrust instrument against platform market power. By upgrading the legacy infrastructure and improving the public outside option, the central bank directly compresses the platform's perceived competitive advantage.

Stripped of its local monopoly insulation, the platform is forced to compress its own inflation markup toward the socially optimal Friedman rule, effectively preventing the extraction of monopolistic rents.

Ultimately, our findings provide a formal foundation for the competitive discipline of private money argued by Hayek (1976). By embedding private money within the industrial organization of a two-sided platform, we show that competition from a legacy marketplace anchors the platform's monetary policy at an inflation markup over the Friedman rule. The equilibrium relies on commitment, but the platform's incentive to honor it extends beyond preserving future seigniorage: overissuance would also destroy the cross-group network externalities and seller fee revenue that underpin the matchmaking business. This provides a more economically grounded basis for monetary discipline than standard models where commitment must be sustained by seigniorage alone.

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Appendix

A Discussion: Platform Money in Practice

While our framework abstracts from specific institutional details to highlight core economic mechanisms, the industrial organization of platform money described in our model closely mirrors the evolution of digital payment ecosystems worldwide. The mechanisms of cross-group subsidies, strategic segmentation, and inflation dampening are particularly visible in developing economies and tech-forward nations where historical frictions in legacy banking allowed digital platforms to leverage their superior matching technologies.

Unregulated Private Fiat and Endogenous Discipline: Tencent's Q-Coin. Perhaps the purest historical example of our baseline model is Tencent's Q-Coin in the mid-2000s. Originally issued merely as a digital token to purchase virtual avatars within the QQ messaging platform, Q-Coin became so widely held that third-party vendors began accepting it for real-world goods. It effectively evolved into a circulating private fiat currency. Crucially, as our model predicts, Tencent did not succumb to the temptation to hyperinflate Q-Coin to extract infinite seigniorage, as doing so would have destroyed user trust and unraveled the network effects of its core messaging and gaming ecosystem. Ultimately, Q-Coin became such a viable parallel currency that the People's Bank of China had to intervene and explicitly ban its use for physical goods to protect the central bank's monopoly.

Overcoming Legacy Frictions and Asymmetric Pricing: GoPay. Gojek's GoPay in Indonesia provides a prime example of a platform using asymmetric pricing to overcome severe legacy market frictions. In an economy characterized by a large unbanked population and heavy reliance on physical cash, the legacy fiat infrastructure suffered from significant transaction frictions. Gojek leveraged its ride-hailing network to digitize payments, providing a vastly superior matching technology. Following our model's predictions ($f_P^* = 0$, $k_P > 0$), GoPay charged users zero entry fees and heavily subsidized the buyer side with cashback promotions to build a captive digital liquidity pool. Once consumers were locked into the

digital wallet, millions of traditional micro-merchants (*warungs*) and restaurants eagerly joined the ecosystem to accept offline QR and delivery payments. These merchants willingly absorb the platform's transaction fees (seller fees) to access this concentrated buyer demand, perfectly illustrating how private money serves as a subsidized instrument to launch highly profitable cross-group network externalities, drastically improving the overall trade surplus and driving widespread financial inclusion.

Loyalty Points and Cross-Group Subsidies: Airline Frequent Flyer Miles. While digital wallets are a recent phenomenon, airline frequent flyer programs represent one of the oldest and most successful implementations of platform money. In this ecosystem, the airline acts as the two-sided platform and issues mileage points as its private currency. To build a large captive buyer base, airlines heavily subsidize consumers by awarding miles for flights and everyday credit card spending. This locked-in liquidity generates powerful cross-group network externalities, attracting a vast network of "sellers" such as affiliated hotels, car rental agencies, retail merchants, and crucially, co-branded credit card banks. As predicted by our model's asymmetric pricing result, the platform extracts its primary monopoly rents from this seller side, famously selling miles to partner banks at a significant premium. Furthermore, while airlines frequently adjust award charts (effectively causing inflation in the mileage currency), they are structurally constrained from hyperinflating miles to worthlessness. Doing so would destroy the consumer trade surplus, unraveling the highly profitable seller-side network effects that sustain the airline's most lucrative revenue streams.

Strategic Dampening in High-Inflation Environments: Mercado Pago. Finally, Proposition 6 predicts that platforms will engage in strategic dampening when legacy fiat experiences high inflation ($\tilde{\mu}^L$), positioning their ecosystem as an "inflation shelter" to preserve market share. This dynamic is clearly observable in Latin America through Mercado Libre's proprietary payment system, Mercado Pago. In economies like Argentina, which suffer from severe legacy hyperinflation, Mercado Pago allows users to frictionlessly sweep their digital balances into algorithmic, yield-bearing accounts. By offering a structural shelter against the macroeconomic depreciation of the legacy fiat, the platform protects its buyers' purchasing

power, ensuring they remain on the platform to generate the transaction volume required to sustain the marketplace's core e-commerce revenues.

Regulatory Pegs, Yield Sweeps, and Walled Gardens: Alipay and WeChat Pay. While modern regulatory constraints force platforms like Alibaba (Alipay) and Tencent (WeChat Pay) to maintain a strict 1:1 peg with the sovereign Chinese RMB, they still actively manage the effective cost of holding their platform money. Because they are legally prohibited from paying direct interest on standard wallet balances, both platforms created seamless integrations with algorithmic money market funds (such as Alipay's Yu'ebao and WeChat's Lingqiantong). By automatically sweeping idle balances into these yield-bearing accounts, the platforms pass market yield back to their users. This achieves the exact same economic outcome as the strategic inflation dampening predicted by our model, creating an "inflation shelter" that keeps user liquidity locked within the ecosystem.

Once this captive buyer base is secured, their behavior perfectly illustrates the "Closed Policy" and strategic segmentation analyzed in Section 6. For years, these platforms famously blocked each other's payment links. Users could not use WeChat Pay on Alibaba's Taobao marketplace, and Taobao links were blocked on WeChat. By maintaining these "walled gardens," the platforms successfully segment the market. As our model demonstrates, refusing interoperability is not merely technological foreclosure; it is a strategic pricing mechanism that operates directly on the cost of holding money. By ceding biased buyers to legacy alternatives, the platform induces its competitor to raise legacy buyer fees. This softens overall price competition, allowing the platform to safely extract higher monetary revenue (seigniorage) from its remaining flexible users without triggering an exodus.

Cross-Group Externalities and Macroeconomic Limits: GrabPay. The asymmetric pricing strategy derived from our first-order conditions ($f_P^* = 0$, combined with high seller fees k_P) is the standard playbook for Southeast Asian "super-apps" like Grab (GrabPay). The platform heavily subsidizes consumers with zero transaction fees and rewards to lock liquidity inside its proprietary digital wallet. Forced to chase this captive liquidity, offline merchants eagerly join the ecosystem and willingly absorb the platform's merchant discount rates (seller fees).

Furthermore, GrabPay's recent evolution provides a fascinating illustration of how external macroeconomic environments ($\tilde{\mu}^L$) dictate platform monetary policy. To offset the opportunity cost of holding idle balances, Grab initially experimented with algorithmic money-market sweeps (AutoInvest and Earn+), attempting the same "strategic dampening" seen in other digital wallets. However, unlike Mercado Pago which operates in hyperinflationary Argentina, Grab operates primarily in low-inflation environments like Singapore. Because legacy fiat inflation remained relatively stable, and Grab's near-monopoly on ride-hailing and delivery provided an overwhelmingly superior matching surplus, the platform did not strictly need to provide an "inflation shelter" to satisfy the buyers' participation constraint. Recognizing this structural advantage, Grab officially shut down its yield-bearing products in 2023. By returning users to standard non-yielding wallets, the platform demonstrated that when legacy inflation is low and the matching surplus is large, a platform can safely rely on the value of its core business rather than costly monetary subsidies, reflecting the impact of a low and stable inflationary central bank monetary policy.

B Proofs

B.1 Proof of Proposition 1: Existence, Stationarity, and Uniqueness

We prove each claim in turn. Recall that the Bellman (24) is written with the inflation rate $\tilde{\mu}_t^P$ as the control variable, and that $N_{b,t+1}^P = \bar{N}_b\,\Pi_b(\Delta_{b,t+1})$ where $\Delta_{b,t+1}$ depends on $(\tilde{\mu}_t^P, f_P)$ but not on the state $N_{b,t}^P$.

Preliminary: Affine value function. The state $N_{b,t}^P$ enters the right-hand side of (24) *only* through the additive term $-N_{b,t}^P\,u(1{-}\gamma)$. The new buyer base $N_{b,t+1}^P$, seller revenue $\bar{N}_s\Pi_s(\Delta_s)k_P$, and the continuation value $\beta\,\mathcal{V}(N_{b,t+1}^P)$ all depend on $(\tilde{\mu}_t^P, f_P, k_P)$ but not on $N_{b,t}^P$. Therefore the Bellman can be separated:

$$\mathcal{V}(N_{b,t}^P) = -N_{b,t}^P\,u(1{-}\gamma) + \underbrace{\max_{\tilde{\mu}_t^P,\,f_P,\,k_P} \Big\{ \bar{N}_s\Pi_s(\Delta_s)k_P + N_{b,t+1}^P\big(\tilde{\mu}_t^P\,u(1-\gamma) + f_P\big) + \beta\,\mathcal{V}(N_{b,t+1}^P) \Big\}}_{\text{does not depend on } N_{b,t}^P}.$$

(B.1)

This establishes that $\mathcal{V}(N) = -N\,u(1-\gamma) + K$ where K is the value of the maximization on the right. Substituting back into (24), cancelling $-N^P_{b,t}\,u(1-\gamma)$ from both sides, and collecting terms on $N^P_{b,t+1}$:

$$(1-\beta)\,K = \max_{\tilde{\mu}^P,\, f_P \geq 0,\, k_P} \bar{N}_s\,\Pi_s(\Delta_s)\,k_P + \bar{N}_b\,\Pi_b(\Delta_b)\,\big[(\tilde{\mu}^P - \beta)\,u(1-\gamma) + f_P\big]. \tag{B.2}$$

The right-hand side involves only the controls $(\tilde{\mu}^P, f_P, k_P)$ and not the state $N^P_{b,t}$, confirming the affine form. The platform's reduced objective is (25).

Item 1: State-independent policy. The first-order conditions (28)–(30) are derived from (B.2), which does not involve $N^P_{b,t}$. Therefore $(\tilde{\mu}^P, f^*_P, k^*_P)$ are constants, independent of the state.

Item 2: Stationarity. Since platform policies $(\tilde{\mu}^P, f^*_P, k^*_P)$ are constants and the legacy inflation $\tilde{\mu}^L$ is fixed, the within-period equilibrium is the same at every CM_t. In particular, the buyer's perceived advantage is constant:

$$\Delta_{b,t+1} = \beta(a_{Pb} - a_{Lb})\gamma u + \xi(\tilde{\mu}^L - \tilde{\mu}^P)(1-\gamma)u + f_L \equiv \Delta^*_b \quad \text{for all } t \geq 0.$$

Therefore $N^P_{b,t+1} = \bar{N}_b\,\Pi_b(\Delta^*_b) \equiv N^*_b$ for all $t \geq 0$, regardless of the initial condition $N^P_{b,0}$. The buyer base reaches its stationary level in one period because $N^P_{b,t+1}$ depends only on the (constant) policy and not on the inherited state $N^P_{b,t}$. By the same logic, Δ^*_s, $\Pi_s(\Delta^*_s)$, the seller-to-buyer ratios $n^*_P = \bar{N}_s\Pi^*_s/(\bar{N}_b\Pi^*_b)$ and $n^*_L = \bar{N}_s(1-\Pi^*_s)/(\bar{N}_b(1-\Pi^*_b))$, and the legacy market fees $f^*_L = (1-\Pi^*_b)/\Pi^{*\prime}_b$, $k^*_L = (1-\Pi^*_s)/\Pi^{*\prime}_s$ are all stationary.

Item 3: Money supply growth equals inflation. The money supply growth rate is $\mu^P_t = \tilde{\mu}^P_t \cdot N^P_{b,t+1}/N^P_{b,t}$. For $t \geq 1$, $N^P_{b,t} = N^P_{b,t+1} = N^*_b$ by Item 2, so $\mu^P_t = \tilde{\mu}^P$. During the transition at $t = 0$, the growth rate $\mu^P_0 = \tilde{\mu}^P \cdot N^*_b/N^P_{b,0}$ may differ from $\tilde{\mu}^P$ if $N^P_{b,0} \neq N^*_b$, but this discrepancy vanishes after one period.

Step 6: Uniqueness. By the assumption on the attraction functions (Π_b and Π_s) specified in Section 2.6, $\log \Pi_b$ is concave. Define $G(\tilde{\mu}^P) \equiv \Pi_b(\Delta_b(\tilde{\mu}^P))\,(\tilde{\mu}^P - \beta)$ on (β, ∞), where Δ_b is linear in $\tilde{\mu}^P$ with slope $\partial\Delta_b/\partial\tilde{\mu}^P = -\xi(1-\gamma)u \equiv -c < 0$. We have $G(\beta) = 0$, $G > 0$ on

(β, ∞), and $G(\tilde{\mu}^P) \to 0$ as $\tilde{\mu}^P \to \infty$. To show G has a unique interior maximum, consider $g(\tilde{\mu}^P) \equiv \log G(\tilde{\mu}^P) = \log \Pi_b(\Delta_b) + \log(\tilde{\mu}^P - \beta)$. Then:

$$g'(\tilde{\mu}^P) = -\frac{\Pi_b'(\Delta_b)}{\Pi_b(\Delta_b)}\, c + \frac{1}{\tilde{\mu}^P - \beta},$$

$$g''(\tilde{\mu}^P) = \underbrace{\frac{d}{d\Delta_b}\left(\frac{\Pi_b'}{\Pi_b}\right) c^2}_{\leq 0 \text{ (log-concavity of } \Pi_b)} - \underbrace{\frac{1}{(\tilde{\mu}^P - \beta)^2}}_{>0} < 0.$$

The first term is non-positive because concavity of $\log \Pi_b$ is equivalent to $\frac{d}{d\Delta_b}(\Pi_b'/\Pi_b) \leq 0$. The second term is strictly negative. Hence g is strictly concave. Since $g'(\beta^+) = +\infty$ and $g'(\tilde{\mu}^P) \to -(\Pi_b'/\Pi_b)\, c < 0$ as $\tilde{\mu}^P \to \infty$, g' crosses zero exactly once. Because $G > 0$ on (β, ∞), the zeros of G' coincide with those of g', so the FOC has a unique solution $\tilde{\mu}^P$, and the equilibrium is unique. □

B.2 Proof of Proposition 2

Consider a platform that does not issue its own money and instead accepts only outside money (with inflation rate $\tilde{\mu}^L$). Such a platform collects revenue only through fees (f_P, k_P). Now suppose the platform issues its own money and sets $\tilde{\mu}^P = \tilde{\mu}^L$ and $f_P = 0$. From the buyer's perspective, the inflation cost is identical to outside money and the entry fee has been removed, so Δ_b weakly increases. The platform's per-buyer extraction is $(\tilde{\mu}^L - \beta)\, u(1-\gamma) = \underbrace{(\tilde{\mu}^L - 1)\, u(1-\gamma)}_{\text{inflation tax}} + \underbrace{(1-\beta)\, u(1-\gamma)}_{>0}$. The first component replicates the purchasing-power loss that buyers already bear under outside money (and hence does not affect participation), while the second component—the issuer's discounting benefit—is strictly positive for $\beta < 1$. Because seller-side pricing is separable, seller revenue is unchanged. The platform therefore earns strictly more than under outside money alone. □

B.3 Proof of Proposition 3

Suppose toward a contradiction that $a_{Ps}(n_P) \leq a_{Ls}(n_L)$. Since $a_{Ps}(n) \geq a_{Ls}(n)$ for all n and a_{Ls} is strictly decreasing, this implies $n_P \geq n_L$. By Lemma 1, $\Pi_s(\Delta_s) \geq \Pi_b(\Delta_b)$.

Step 1: $\Pi_s(\Delta_s) \leq 1/2$**.** Substituting the FOCs into the seller advantage expression gives

$\Delta_s = \beta(a_{Ps}(n_P) - a_{Ls}(n_L))(1-\gamma)u + (1 - 2\Pi_s(\Delta_s))/\Pi_s'(\Delta_s)$. Under the hypothesis, the matching term is ≤ 0. If $\Pi_s > 1/2$, the fee term $(1 - 2\Pi_s)/\Pi_s'$ is also < 0, so $\Delta_s < 0$, giving $\Pi_s(\Delta_s) < 1/2$—a contradiction. Hence $\Pi_s(\Delta_s) \leq 1/2$.

Step 2: $\Pi_b(\Delta_b) > 1/2$**.** Similarly, $\Delta_b = \beta(a_{Pb}(n_P) - a_{Lb}(n_L))\gamma u + \xi(\tilde{\mu}^L - \beta)(1-\gamma)u + (1 - 2\Pi_b(\Delta_b))/\Pi_b'(\Delta_b)$. The buyer matching term is ≥ 0 (since $n_P \geq n_L$ and $a_{Pb}(n) \geq a_{Lb}(n)$) and the monetary term $\xi(\tilde{\mu}^L - \beta)(1-\gamma)u > 0$. If $\Pi_b < 1/2$, the fee term $(1 - 2\Pi_b)/\Pi_b'$ is also > 0, so all three terms are positive and $\Delta_b > 0$, giving $\Pi_b(\Delta_b) > 1/2$—a contradiction. Hence $\Pi_b(\Delta_b) \geq 1/2$. In fact, if $\Pi_b = 1/2$ then $\Delta_b = 0$, but then the positive matching and monetary terms sum to zero, which is impossible. So $\Pi_b(\Delta_b) > 1/2$.

Contradiction. Steps 1–2 give $\Pi_s(\Delta_s) \leq 1/2 < \Pi_b(\Delta_b)$, contradicting $\Pi_s(\Delta_s) \geq \Pi_b(\Delta_b)$. □

B.4 Proof of Proposition 4

Using FOCs for k_P and k_L, we have $k_P > k_L$ if and only if $\frac{\Pi_s(\Delta_s)}{\Pi_s'(\Delta_s)} > \frac{1-\Pi_s(\Delta_s)}{\Pi_s'(\Delta_s)}$. This is true if and only if $\Pi_s(\Delta_s) > \frac{1}{2}$ if and only if $\Delta_s > 0$. Note $\Delta_s = \beta\left(a_{Ps}(n_P) - a_{Ls}(n_L)\right)(1-\gamma)u + (k_L - k_P)$. The first term is strictly positive. Suppose that $k_P \leq k_L$, the second term is also positive and hence, $\Delta_s > 0$, which implies $k_P > k_L$. Thus, we must have $k_P > k_L$. We must also have $\Delta_s > 0$.

B.5 Proof of Proposition 5

More buyers on platform P if

$$\Delta_b = \beta\left(a_{Pb}(n_P) - a_{Lb}(n_L)\right)\gamma u + \xi(1-\gamma)u\left(\tilde{\mu}^L - \beta\right) + \left(\frac{1 - 2\Pi_b(\Delta_b)}{\Pi_b'(\Delta_b)}\right) > 0. \quad \text{(B.3)}$$

Towards a contradiction, let us suppose that $\Delta_b < 0$ or equivalently $\Pi_b(\Delta_b) < 0.5$. In this case, it must be that $a_{Pb}(n_P) < a_{Lb}(n_L)$ since otherwise all the terms on the right side of the above equation are positive (note that $\tilde{\mu}^L \geq 1 > \beta$ ensures the second term is positive).

However, recall that the platform possesses a weak technological advantage, meaning $a_{Pb}(n) \geq a_{Lb}(n)$ for any given seller-to-buyer ratio. Therefore, if $a_{Pb}(n_P) < a_{Lb}(n_L)$, it

strictly requires that $n_P < n_L$ because the buyer matching probability is strictly increasing in n. By Lemma 1, $n_P < n_L$ implies $\Pi_s(\Delta_s) < \Pi_b(\Delta_b) < 1/2$, so $\Delta_s < 0$.

But $\Delta_s < 0$ is a contradiction since we already established $\Delta_s > 0$ in Proposition 4. Then, we must have $\Delta_b > 0$.

B.6 Proof of Proposition 6

Differentiating the equilibrium system with respect to $\tilde{\mu}^L$, substituting for $\partial n_P/\partial\tilde{\mu}^L$ and $\partial n_L/\partial\tilde{\mu}^L$, and collecting terms yields a 2×2 linear system. To state it compactly, define the following shorthand (all evaluated at equilibrium):

$$H' \equiv 1 - \frac{\Pi_b''\,\Pi_b}{(\Pi_b')^2}, \qquad R \equiv \frac{-(1-\Pi_b)\Pi_b''}{(\Pi_b')^2}, \qquad Q \equiv \frac{(1-2\Pi_s)\Pi_s''}{(\Pi_s')^2},$$

$$\theta_b \equiv \beta\gamma u\left(\frac{a'_{Pb}(n_P)\,n_P}{\Pi_b} + \frac{a'_{Lb}(n_L)\,n_L}{1-\Pi_b}\right), \qquad \phi_{bs} \equiv \beta\gamma u\left(\frac{a'_{Pb}(n_P)\,n_P}{\Pi_s} + \frac{a'_{Lb}(n_L)\,n_L}{1-\Pi_s}\right),$$

$$\theta_s \equiv \beta(1-\gamma)u\left(\frac{|a'_{Ps}(n_P)|\,n_P}{\Pi_s} + \frac{|a'_{Ls}(n_L)|\,n_L}{1-\Pi_s}\right), \qquad \phi_{sb} \equiv \beta(1-\gamma)u\left(\frac{|a'_{Ps}(n_P)|\,n_P}{\Pi_b} + \frac{|a'_{Ls}(n_L)|\,n_L}{1-\Pi_b}\right),$$

and let $F_b \equiv H' + (2-R) + \theta_b\Pi_b'$ and $F_s \equiv (3+Q) + \theta_s\Pi_s'$. Here θ_b and θ_s are the own-side congestion effects, the fall in buyers' (respectively sellers') matching advantage per unit of own-side share, and ϕ_{bs} and ϕ_{sb} the cross-side thickness effects, the rise in buyers' matching advantage per unit of seller share and in sellers' per unit of buyer share; multiplying by Π_i' converts them to share units. The same four objects appear in the internalized pricing rules of Section 5. The system is:

$$F_b\,\frac{\partial\Delta_b}{\partial\tilde{\mu}^L} = \phi_{bs}\Pi_s'\,\frac{\partial\Delta_s}{\partial\tilde{\mu}^L} + \xi(1-\gamma)u, \tag{B}$$

$$F_s\,\frac{\partial\Delta_s}{\partial\tilde{\mu}^L} = \phi_{sb}\Pi_b'\,\frac{\partial\Delta_b}{\partial\tilde{\mu}^L}. \tag{S}$$

(The platform's FOC response H' has already been absorbed into F_b.)

Substituting (S) into (B): $(F_b - \phi_{bs}\phi_{sb}\Pi_b'\Pi_s'/F_s)\,\partial\Delta_b/\partial\tilde{\mu}^L = \xi(1-\gamma)u$. Since the plat-

form's FOC gives $\frac{\partial \mu^P}{\partial \tilde{\mu}^L} = \frac{H'}{\xi(1-\gamma)u} \frac{\partial \Delta_b}{\partial \tilde{\mu}^L}$, the dampening ratio is:

$$\frac{\partial \mu^P}{\partial \tilde{\mu}^L} = \frac{H'}{F_b - \phi_{bs}\phi_{sb}\Pi_b'\Pi_s'/F_s}. \tag{B.4}$$

Note that ξ cancels: the result holds for any $\xi \in (0,1]$. We now show the denominator exceeds H', establishing $0 < \partial\mu^P/\partial\tilde{\mu}^L < 1$. Since $F_b - H' = (2-R) + \theta_b\Pi_b'$, we need $(2 - R + \theta_b\Pi_b')\, F_s > \phi_{bs}\phi_{sb}\Pi_b'\Pi_s'$. Expanding:

$$\begin{aligned}(2-R+\theta_b\Pi_b')\, F_s - \phi_{bs}\phi_{sb}\Pi_b'\Pi_s' = \underbrace{(2-R)(3+Q)}_{\text{(i)}} + \underbrace{(2-R)\,\theta_s\Pi_s'}_{\text{(ii)}} \\ + \underbrace{\theta_b\Pi_b'(3+Q)}_{\text{(iii)}} + \underbrace{(\theta_b\theta_s - \phi_{bs}\phi_{sb})\,\Pi_b'\Pi_s'}_{\text{(iv)}}.\end{aligned} \tag{B.5}$$

Terms (i)–(iii) are positive. Log-concavity of Π_b implies $\Pi_b\Pi_b'' \le (\Pi_b')^2$, so $H' > 0$. Log-concavity of $1-\Pi_b$ implies $R \le 1$, hence $2 - R > 1$. Log-concavity of both Π_s and $1-\Pi_s$ implies $3 + Q > 1$.[12] The congestion terms $\theta_b\Pi_b', \theta_s\Pi_s'$ are manifestly positive.

Term (iv) is non-negative. Writing $A_P = a_{Pb}'(n_P)n_P$, $B_L = a_{Lb}'(n_L)n_L$, $C_P = |a_{Ps}'(n_P)|n_P$, $D_L = |a_{Ls}'(n_L)|n_L$, a direct calculation shows:

$$\begin{aligned}&(\theta_b\theta_s - \phi_{bs}\phi_{sb})\,\Pi_b'\Pi_s' \\ &\qquad = \frac{\beta^2\gamma(1-\gamma)u^2\,\Pi_b'\Pi_s'}{\Pi_b\Pi_s(1-\Pi_b)(1-\Pi_s)}\,\big(A_PD_L - B_LC_P\big)\,\big(\Pi_s - \Pi_b\big).\end{aligned}$$

Under proportional superiority, $h_P(n) = h_L(n) \equiv h(n)$ (the factor A cancels), so $A_PD_L - B_LC_P = C_PD_L\big[h(n_P) - h(n_L)\big]$. By Lemma 1, $n_P \gtrless n_L$ if and only if $\Pi_s \gtrless \Pi_b$. Since h is non-decreasing (Assumption 1), the two factors $\big[h(n_P) - h(n_L)\big]$ and $(\Pi_s - \Pi_b)$ always have the same sign, so their product is non-negative.

Since all four terms in (B.5) are positive, the denominator of (B.4) strictly exceeds H', giving $0 < \partial\mu^P/\partial\tilde{\mu}^L < 1$.

Since $\mu^P > 1$ at $\tilde{\mu}^L = 1$ (the platform inflates to generate seigniorage) while $\partial\mu^P/\partial\tilde{\mu}^L < 1$,

[12] When $\Pi_s > 1/2$: log-concavity of Π_s gives $\Pi_s'' \le (\Pi_s')^2/\Pi_s$, so $Q \ge (1-2\Pi_s)/\Pi_s > -1$. When $\Pi_s \le 1/2$: log-concavity of $1-\Pi_s$ gives $\Pi_s'' \ge -(\Pi_s')^2/(1-\Pi_s)$, so $Q \ge -(1-2\Pi_s)/(1-\Pi_s) > -1$. In either case, $3 + Q > 2$.

there exists a unique crossing point $\hat{\mu}^L > 1$ where $\mu^P = \tilde{\mu}^L$.

B.7 Proof of Proposition 7

The first order condition with respect to $\Pi_b < 1$ is:

$$a_{Pb}(n_P) - a'_{Pb}(n_P)n_P \;\; \geq \;\; a_{Lb}(n_L) - a'_{Lb}(n_L)n_L, \tag{B.6}$$

where the above condition holds with equality if $\Pi_b < 1$.

The first order condition with respect to $\Pi_s < 1$ is:

$$a'_{Pb}(n_P) \;\; \geq \;\; a'_{Lb}(n_L), \tag{B.7}$$

where the above condition holds with equality if $\Pi_s < 1$.

We observe $a_{Pb}(0) = a_{Lb}(0) = 0$. We assume that $a'_{Pb}(n) > a'_{Lb}(n), \forall n > 0$. That is, the concave matching function for the platform has a steeper slope for the same seller-to-buyer ratio than the legacy marketplace.

Step 1: $\Pi_s = 1$ *(by contradiction).* Suppose for contradiction that $\Pi_s < 1$. Then the seller FOC holds with equality: $a'_{Pb}(n_P) = a'_{Lb}(n_L)$. By concavity of a_{Pb} and a_{Lb}, $n_P > n_L$. However, we find that

$$\begin{aligned}
a_{Pb}(n_P) - a'_{Pb}(n_P)n_P &= \\
a_{Pb}(n_L) - a'_{Pb}(n_P)n_L + \int_{n_L}^{n_P} a'_{Pb}(n)dn - a'_{Pb}(n_P)\left(n_P - n_L\right) &> \\
a_{Pb}(n_L) - a'_{Pb}(n_P)n_L + \int_{n_L}^{n_P} a'_{Pb}(n_P)dn - a'_{Pb}(n_P)\left(n_P - n_L\right) &= \\
a_{Pb}(n_L) - a'_{Pb}(n_P)n_L &> \\
a_{Lb}(n_L) - a'_{Lb}(n_L)n_L.
\end{aligned}$$

The second inequality holds because $a'_{Pb}(n) > a'_{Pb}(n_P)$ for all $n \in (n_L, n_P)$. Since the first order condition with respect to Π_b holds in strict inequality, we obtain $\Pi_b = 1$ (conditional on $\Pi_s < 1$). Plugging $\Pi_b = 1$ into the objective function, we get:

$$\max_{\Pi_s, \Pi_b} \Pi_b a_{Pb}\left(\frac{\overline{N}_s \Pi_s}{\overline{N}_b \Pi_b}\right) + (1 - \Pi_b) a_{Lb}\left(\frac{\overline{N}_s\left(1 - \Pi_s\right)}{\overline{N}_b\left(1 - \Pi_b\right)}\right) = \max_{\Pi_s, \Pi_b} a_{Pb}\left(\frac{\overline{N}_s \Pi_s}{\overline{N}_b}\right).$$

This maximization problem implies that $\Pi_s = 1$ since a_{Pb} is increasing, contradicting $\Pi_s < 1$. Hence $\Pi_s = 1$.

Step 2: $\Pi_b = 1$. Plugging $\Pi_s = 1$ into the planner's objective function, we obtain

$$\begin{aligned}\max_{\Pi_s,\Pi_b} \Pi_b a_{Pb}\left(\frac{\overline{N}_s\Pi_s}{\overline{N}_b\Pi_b}\right) + (1-\Pi_b)a_{Lb}\left(\frac{\overline{N}_s\left(1-\Pi_s\right)}{\overline{N}_b\left(1-\Pi_b\right)}\right) &= \\ \max_{\Pi_s,\Pi_b} \Pi_b a_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right) + (1-\Pi_b)a_{Lb}(0) &= \max_{\Pi_b} \Pi_b a_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right).\end{aligned}$$

The first order condition with respect to Π_b becomes:

$$a_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right) - a'_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right)\frac{\overline{N}_s}{\overline{N}_b\Pi_b} \geq a_{Lb}(0) - a'_{Lb}(0)0 = 0. \tag{B.8}$$

Or

$$\frac{a_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right)}{\frac{\overline{N}_s}{\overline{N}_b\Pi_b}} \geq a'_{Pb}\left(\frac{\overline{N}_s}{\overline{N}_b\Pi_b}\right). \tag{B.9}$$

Since a_{Pb} is concave, the above cannot hold with equality. So we must have $\Pi_b = 1$.

B.8 Proofs of the Results in Section 6

In this section we prove Lemma 2 and Propositions 10 and 12.

B.8.1 Proof of Lemma 2

Suppose the open policy is optimal. Suppose also that flexible buyers who are on the platform are indifferent between platform money and outside money and biased buyers strictly prefer outside money. That is, $\Delta_b^{of} = \Delta_b^{ob}$ and $\Delta_b^{of} - \kappa < \Delta_b^{ob}$. From the binding constraint we obtain:

$$\mu^{P,o} = \frac{f_P^o}{\xi\left(1-\gamma\right)u} + \tilde{\mu}^L. \tag{B.10}$$

Plugging $\mu^{P,o}$ into the platform's objective function we obtain:

$$\Pi_b\left(\Delta_b^{ob}\right)\left(\left(\frac{\psi}{\xi}+(1-\psi)\right)f_P^o+\psi\left(\tilde{\mu}^L-\beta\right)u\left(1-\gamma\right)\right) \tag{B.11}$$

and platform's fee for biased buyers is:

$$f_P^o=\frac{\Pi_b\left(\Delta_b^{ob}\right)}{\Pi_b'\left(\Delta_b^{ob}\right)}-\frac{\psi\left(\tilde{\mu}^L-\beta\right)u\left(1-\gamma\right)}{\left(\frac{\psi}{\xi}+(1-\psi)\right)} \tag{B.12}$$

and, under the binding constraint $\Delta_b^{of}=\Delta_b^{ob}$, the implied platform money inflation is:

$$\mu^{P,o}=\beta+\frac{1}{\xi\left(1-\gamma\right)u}\frac{\Pi_b\left(\Delta_b^{ob}\right)}{\Pi_b'\left(\Delta_b^{ob}\right)}+\frac{\xi\left(1-\psi\right)\left(\tilde{\mu}^L-\beta\right)}{\psi+\xi\left(1-\psi\right)}. \tag{B.13}$$

Substituting back into platform's objective we obtain:

$$\left(\frac{\psi}{\xi}+(1-\psi)\right)\frac{\left(\Pi_b\left(\Delta_b^{ob}\right)\right)^2}{\Pi_b'\left(\Delta_b^{ob}\right)}$$

where $\Delta_b^{ob}=\beta\left(a_{Pb}(n_P^o)-a_{Lb}(n_L^o)\right)\gamma u+(f_L^o-f_P^o)$. If the platform is not constrained it would choose $\mu^P=\beta+\frac{1}{\xi(1-\gamma)u}\frac{\Pi_b\left(\Delta_b^{ob}\right)}{\Pi_b'\left(\Delta_b^{ob}\right)}$, which is strictly lower than the above value. But then if the platform lowers $\mu^{P,o}$ slightly, its profit from flexible buyers increases. Moreover, Δ_b^{of} goes up and both constraints hold. Hence at an optimum flexible buyers must strictly prefer platform money.

B.8.2 Proof of Proposition 10

Suppose $\psi>0$. We consider two cases:

Case i: The platform uses only outside money. In this case platform chooses f_P to maximize its payoff $\Pi_b(\Delta_b^{out})f_P$ and the legacy chooses f_L to maximize its payoff $(1-\Pi_b(\Delta_b^{out}))f_L$ where

$$\Delta_b^{out}=\beta(a_{Pb}(n_P^{out})-a_{Lb}(n_L^{out}))\gamma u+(f_L-f_P)$$

and n_P^{out}, n_L^{out} are the seller to buyer ratios.

Case ii: The platform uses both platform and outside money. In this case platform chooses μ^P and f_P to maximize its payoff $\psi\Pi_b(\Delta_b^{of})u(1-\gamma)(\mu^P-\beta)+(1-\psi)\Pi_b(\Delta_b^{ob})f_P$ and the legacy chooses f_L to maximize its payoff $(1-\psi\Pi_b(\Delta_b^{of})-(1-\psi)\Pi_b(\Delta_b^{ob}))f_L$ where

$$\Delta_b^{of}=\beta(a_{Pb}(n_P^o)-a_{Lb}(n_L^o))\gamma u+\xi(\tilde{\mu}^L-\mu^P)(1-\gamma)u+f_L$$

$$\Delta_b^{ob}=\beta(a_{Pb}(n_P^o)-a_{Lb}(n_L^o))\gamma u+(f_L-f_P)$$

and n_P^o, n_L^o are the seller to buyer ratios. We also impose the constraints $\Delta_b^{of}\geq\Delta_b^{ob}$ and $\Delta_b^{of}-\kappa\leq\Delta_b^{ob}$ although these constraints do not play a role in the proof.[13]

We begin by proving three lemmas.

Lemma 3 *For a fixed f_L the platform can guarantee itself a higher payoff in case ii.*

Proof. Suppose that for a given f_L, platform's optimal fee in case i is f_P^{out}. Now suppose the platform accepts both types of money, and sets $f_P^o=f_P^{out}$, and sets $\mu^P=\tilde{\mu}^L+\frac{f_P^{out}}{\xi u(1-\gamma)}$. By construction $\Delta_b^{of}=\Delta_b^{ob}=\Delta_b^{out}$ and the constraints are automatically satisfied. And, the platform's payoff goes up since:

$$\psi\Pi_b(\Delta_b^{out})((\tilde{\mu}^L-\beta)(1-\gamma)u+\frac{f_P}{\xi})+(1-\psi)\Pi_b(\Delta_b^{out})f_P>\Pi_b(\Delta_b^{out})f_P.$$

□

Lemma 4 *Let $\bar{\Pi}=\psi\Pi_b(\Delta_b^{of})+(1-\psi)\Pi_b(\Delta_b^{ob})$. In case ii, legacy's optimal fee satisfies $f_L^o\geq\frac{\sigma}{\bar{\Pi}}$.*

Proof. We can write legacy's optimal fee in case ii as:

$$f_L^o=\sigma\frac{1-\bar{\Pi}}{\psi\Pi_b(\Delta_b^{of})(1-\Pi_b(\Delta_b^{of}))+(1-\psi)\Pi_b(\Delta_b^{ob})(1-\Pi_b(\Delta_b^{ob}))}$$

Since $x(1-x)$ is strictly concave, by Jensen's Inequality,

$$\psi\Pi_b(\Delta_b^{of})(1-\Pi_b(\Delta_b^{of}))+(1-\psi)\Pi_b(\Delta_b^{ob})(1-\Pi_b(\Delta_b^{ob}))\leq\bar{\Pi}(1-\bar{\Pi})$$

[13] In the proof we use logistic functional forms for $\Pi_b(\cdot)$. The proof can be extended to the case where $\Pi_b'(\cdot)$ is concave.

which implies $f_L^o \geq \frac{\sigma}{\Pi}$. $\square$

Lemma 5 *In case i, the platform's payoff is increasing in f_L.*

Proof. Let $B(\Pi_b) = \beta(a_{Pb}(n_P^{out}(\Pi_b)) - a_{Lb}(n_L^{out}(\Pi_b)))\gamma u$ and $\theta \equiv -B'(\Pi_b) > 0$ (since n_P^{out} is decreasing and n_L^{out} is increasing in Π_b, while a_{Pb} and a_{Lb} are increasing). The buyer indifference is $\Delta_b^{out} = B(\Pi_b(\Delta_b^{out})) + f_L - f_P(\Delta_b^{out})$, and the FOC gives $\Pi_b' f_P = \Pi_b$. Differentiating the on-path payoff $V_P^{out} = \Pi_b f_P$ with respect to Δ_b, using $f_P' = \Pi_b/(1-\Pi_b)$ (logistic):

$$\frac{dV_P^{out}}{d\Delta_b} = \Pi_b' f_P + \Pi_b f_P' = \Pi_b + \frac{\Pi_b^2}{1-\Pi_b} = \frac{\Pi_b}{1-\Pi_b} > 0.$$

By the implicit function theorem, $\frac{d\Delta_b^{out}}{df_L} = \frac{1}{\frac{1}{1-\Pi_b}+\theta\Pi_b'} > 0$. Therefore $\frac{dV_P^{out}}{df_L} = \frac{\Pi_b}{(1-\Pi_b)\left[\frac{1}{1-\Pi_b}+\theta\Pi_b'\right]} > 0$. $\square$

To finish the proof suppose towards a contradiction that $V_p^o < V_p^{out}$. Using the logistic $\Pi_b' = \Pi_b(1-\Pi_b)/\sigma_b$, the FOCs give $f_P^{out} = \sigma_b/(1-\Pi_b(\Delta_b^{out}))$, $f_P^o = \sigma_b/(1-\Pi_b(\Delta_b^{ob}))$, and $u(1-\gamma)(\mu^P-\beta) = \sigma_b/(\xi(1-\Pi_b(\Delta_b^{of})))$. Substituting into the payoffs:

$$V_P^{out} = \sigma_b \frac{\Pi_b(\Delta_b^{out})}{1-\Pi_b(\Delta_b^{out})}, \qquad V_P^o = \sigma_b\left[\frac{\psi}{\xi}\frac{\Pi_b(\Delta_b^{of})}{1-\Pi_b(\Delta_b^{of})} + (1-\psi)\frac{\Pi_b(\Delta_b^{ob})}{1-\Pi_b(\Delta_b^{ob})}\right].$$

Dividing $V_P^o < V_P^{out}$ by σ_b and using $\psi/\xi \geq \psi$ (since $\xi \leq 1$) to weaken the left side:

$$\left[\psi\left(\frac{\Pi_b(\Delta_b^{of})}{1-\Pi_b(\Delta_b^{of})}\right) + (1-\psi)\left(\frac{\Pi_b(\Delta_b^{ob})}{1-\Pi_b(\Delta_b^{ob})}\right)\right] < \frac{\Pi_b(\Delta_b^{out})}{1-\Pi_b(\Delta_b^{out})}.$$

Since $\frac{x}{1-x}$ is an increasing and convex function Jensen's inequality gives:

$$\frac{\bar{\Pi}}{1-\bar{\Pi}} < \left[\psi\left(\frac{\Pi_b(\Delta_b^{of})}{1-\Pi_b(\Delta_b^{of})}\right) + (1-\psi)\left(\frac{\Pi_b(\Delta_b^{ob})}{1-\Pi_b(\Delta_b^{ob})}\right)\right] < \frac{\Pi_b(\Delta_b^{out})}{1-\Pi_b(\Delta_b^{out})}.$$

which implies $\bar{\Pi} < \Pi_b(\Delta_b^{out})$. From Lemma 4 we obtain:

$$f_L^o \geq \frac{\sigma}{\bar{\Pi}} > \frac{\sigma}{\Pi_b(\Delta_b^{out})} = f_L^{out}.$$

Combining Lemma 3 and Lemma 5 we obtain $V_P^o(f_L^o) \geq V_P^{out}(f_L^o) > V_P^{out}(f_L^{out})$ which is a contradiction.

B.8.3 Proof of Proposition 12

Let $\psi \in [0,1]$ be the fraction of flexible buyers and let $\epsilon = 1 - \psi$ be the fraction of biased buyers. Let $V_j^c(\psi)$ and $V_j^o(\psi)$ be the payoff of market $j \in \{P, L\}$ under closed and open policies respectively as functions of ψ. We can write these functions explicitly as:

$$
\begin{aligned}
V_P^c(\epsilon) &= \bar{N}_b(1-\epsilon)\Pi_b\big(\Delta_b^c\big)\Big[u(1-\gamma)\big(\mu^{P,c} - \beta\big)\Big], \\
V_L^c(\epsilon) &= \bar{N}_b\Big(1 - (1-\epsilon)\Pi_b\big(\Delta_b^c\big)\Big) f_L^c, \\
V_P^o(\epsilon) &= \bar{N}_b\Big((1-\epsilon)\Pi_b\big(\Delta_b^{of}\big)\Big[u(1-\gamma)\big(\mu^{P,o} - \beta\big)\Big] + \epsilon\Pi_b\big(\Delta_b^{ob}\big) f_P^o\Big), \\
V_L^o(\epsilon) &= \bar{N}_b\Big(1 - (1-\epsilon)\Pi_b\big(\Delta_b^{of}\big) - \epsilon\Pi_b\big(\Delta_b^{ob}\big)\Big) f_L^o.
\end{aligned}
$$

W.l.o.g we set $\bar{N}_b = 1$.

We define the profit difference of the platform under closed and open policies as $D(\psi) = V_P^c(\psi) - V_P^o(\psi)$.

Observe that:

1. At $\psi = 0$ ($\epsilon = 1$): Biased buyers cannot join the closed platform ($\kappa = \infty$), so $V_P^c(0) = 0$. Under the open policy, they generate fee revenue, so $V_P^o(0) > 0$. Thus $D(0) < 0$.

2. At $\psi = 1$ ($\epsilon = 0$): There are no biased buyers. The optimization problem for the flexible majority is identical under both policies. Thus $D(1) = 0$.

To prove the platform strictly prefers the closed policy for a large flexible share ($\epsilon \to 0$), we show that $D(1-\epsilon)$ is strictly increasing in ϵ at the limit $\epsilon = 0$. Equivalently, we show $\frac{dD}{d\epsilon} > 0$ at $\epsilon = 0$.

At the limit $\epsilon = 0$ define Δ^* and Π^* so that $\Delta_b^c = \Delta_b^{of} = \Delta^*$, $\Pi_b = \Pi^*$, and $\Pi_b' = \Pi^{*\prime}$. Write $\Pi^o = \Pi_b(\Delta_b^{ob})\big|_{\epsilon=0}$ and $\Pi^{o\prime} = \Pi_b'(\Delta_b^{ob})\big|_{\epsilon=0}$. From Lemma 7 below, $\Delta_b^{of} > \Delta_b^{ob}$ at $\epsilon = 0$, so $\Pi^* > \Pi^o$.

Lemma 6 *The difference in the marginal response of the legacy fee to an increase in biased buyers between the closed (c) and open (o) policies satisfies:*

$$\frac{df_L^c}{d\epsilon} - \frac{df_L^o}{d\epsilon} > 0. \tag{B.14}$$

Proof:

Equilibrium system. The buyer advantage Δ_b is determined jointly with the legacy fee f_L by two equations. The first is the legacy market's FOC for f_L, which we denote $G(\epsilon, f_L, \Delta_b) = 0$ (with G^c and G^o defined explicitly below for the closed and open policies).

The second is the seller-to-buyer ratio consistency condition. Recall that

$$\Delta_b = \underbrace{\beta\big(a_{Pb}(n_P) - a_{Lb}(n_L)\big)\gamma u}_{\equiv M(\Pi_b)} + \xi(\tilde{\mu}^L - \mu^P)(1-\gamma)u + f_L,$$

where seller-to-buyer ratio depends on buyer participation:

$$n_P = \frac{\bar{N}_s \Pi_s}{\bar{N}_b \Pi_b(\Delta_b)}, \qquad n_L = \frac{\bar{N}_s(1-\Pi_s)}{\bar{N}_b(1-\Pi_b(\Delta_b))}.$$

Since Δ_b appears on both sides (through n_P, n_L), it is pinned down by the fixed-point equation

$$F(\epsilon, f_L, \Delta_b) \;\equiv\; \Delta_b - M\big(\Pi_b(\Delta_b, \epsilon)\big) - \xi(\tilde{\mu}^L - \mu^P)(1-\gamma)u - f_L \;=\; 0, \tag{B.15}$$

where we write $M(\Pi_b) \equiv \beta(a_{Pb}(n_P(\Pi_b)) - a_{Lb}(n_L(\Pi_b)))\gamma u$ for the matching-advantage component. Define the seller-to-buyer ratio sensitivity parameter

$$\theta \equiv -M'(\Pi^*) = \beta\gamma u\left[\frac{a'_{Pb}(n_P^*)\, n_P^*}{\Pi^*} + \frac{a'_{Lb}(n_L^*)\, n_L^*}{1-\Pi^*}\right] > 0, \tag{B.16}$$

which measures the congestion feedback: when more buyers join P, matching worsens on P and improves on L, reducing Δ_b.

Solving $F(\epsilon, f_L, \Delta_b) = 0$ for $\Delta_b(\epsilon, f_L)$ by the implicit function theorem:

$$\frac{\partial \Delta_b}{\partial \epsilon} = -\frac{F_\epsilon}{F_{\Delta_b}}, \qquad \frac{\partial \Delta_b}{\partial f_L} = \frac{1}{F_{\Delta_b}}, \tag{B.17}$$

where $F_{f_L} = -1$ since f_L enters Δ_b directly with coefficient 1. Substituting into the legacy FOC gives the reduced-form $\tilde{G}(\epsilon, f_L) \equiv G(\epsilon, f_L, \Delta_b(\epsilon, f_L))$, and the total response is $\frac{df_L}{d\epsilon} = -\frac{\tilde{G}_\epsilon}{\tilde{G}_{f_L}}$. By the chain rule:

$$\tilde{G}_\epsilon = G_\epsilon - G_{\Delta_b}\frac{F_\epsilon}{F_{\Delta_b}}, \tag{B.18}$$

$$\tilde{G}_{f_L} = G_{f_L} + \frac{G_{\Delta_b}}{F_{\Delta_b}}. \tag{B.19}$$

Closed policy. Define $G^c(\epsilon, f_L, \Delta_b) = (1 - (1-\epsilon)\Pi_b(\Delta_b^c)) - (1-\epsilon)\Pi_b'(\Delta_b^c) f_L$. Since $G^c(0, f_L^*, \Delta^*) = (1-\Pi^*) - \Pi^{*\prime} f_L^* = 0$ and $f_L^* = (1-\Pi^*)/\Pi^{*\prime}$: $G_\epsilon^c\big|_{\epsilon=0} = \Pi^* + \Pi^{*\prime} f_L^* = 1$. Under the closed policy, biased buyers are expelled entirely from P, so the aggregate buyer fraction on P is $(1-\epsilon)\Pi_b(\Delta_b^c)$. From (B.15), $F_\epsilon = -M'(\Pi_b^{\text{agg}}) \cdot \partial \Pi_b^{\text{agg}}/\partial \epsilon$. At $\epsilon = 0$, $\partial \Pi_b^{\text{agg}}/\partial \epsilon = -\Pi^*$, so $F_\epsilon^c = -M'(\Pi^*)(-\Pi^*) = -\Pi^* \theta$.

Open policy. Define $G^o(\epsilon, f_L, \Delta_b) = \epsilon\, A(f_L, \Delta_b) + (1-\epsilon)\, B(f_L, \Delta_b)$ where

$$A = (1 - \Pi_b(\Delta_b^{ob})) - f_L \Pi_b'(\Delta_b^{ob}), \qquad B = (1 - \Pi_b(\Delta_b^{of})) - f_L \Pi_b'(\Delta_b^{of}).$$

At $\epsilon = 0$, the FOC sets $B = 0$. The explicit numerator is $G_\epsilon^o|_{\epsilon=0} = A_0 \equiv A\big|_{\epsilon=0}$. Using $f_L^* = \sigma_b/\Pi^*$:

$$A_0 = (1 - \Pi^o) - \frac{\sigma_b}{\Pi^*} \cdot \frac{\Pi^o(1-\Pi^o)}{\sigma_b} = \frac{(1-\Pi^o)(\Pi^* - \Pi^o)}{\Pi^*}\,.$$

Under the open policy, the aggregate buyer fraction on P is $(1-\epsilon)\Pi_b(\Delta_b^{of}) + \epsilon \Pi_b(\Delta_b^{ob})$, so $\partial \Pi_b^{\text{agg}}/\partial \epsilon\big|_{\epsilon=0} = -\Pi^* + \Pi^o = -(\Pi^* - \Pi^o)$. Hence $F_\epsilon^{of} = -(\Pi^* - \Pi^o)\theta$.

Common quantities at $\epsilon = 0$. Using $\Pi_b' = \frac{1}{\sigma_b}\Pi_b(1-\Pi_b)$, $\Pi_b'' = \frac{1}{\sigma_b}\Pi_b'(1-2\Pi_b)$, and $f_L^* = (1-\Pi^*)/\Pi^{*\prime}$:

$$G_{f_L} + G_{\Delta_b} = -2\Pi^{*\prime} - f_L^* \Pi^{*\prime\prime} = -\frac{\Pi^{*\prime}}{\Pi^*}\,, \qquad G_{\Delta_b} = -\frac{\Pi^{*\prime}(1-\Pi^*)}{\Pi^*}\,,$$

Differentiating (B.15) with respect to Δ_b, noting that μ^P is also endogenous (set by the platform's FOC, hence a function of Δ_b):

$$F_{\Delta_b} = \underbrace{1}_{\text{direct}} \underbrace{-M'(\Pi^*)\Pi^{*\prime}}_{\theta\Pi^{*\prime}\text{ (seller-to-buyer ratio)}} + \underbrace{\xi\frac{\partial \mu^P}{\partial \Delta_b}(1-\gamma)u}_{\text{platform pricing}}.$$

The last term equals $H'(\Delta^*) = \Pi^*/(1-\Pi^*)$, where $H(\Delta_b) \equiv \Pi_b/\Pi_b' = \sigma_b/(1-\Pi_b)$ and we used the platform's FOC $\xi u(1-\gamma)(\mu^P-\beta) = H(\Delta_b)$. Hence

$$F_{\Delta_b} = 1 + \theta\Pi^{*\prime} + \frac{\Pi^*}{1-\Pi^*} = \frac{1}{1-\Pi^*} + \theta\Pi^{*\prime} > 1. \tag{B.20}$$

Assembling. From (B.18), the reduced-form numerators are $\tilde{G}_\epsilon^c = 1 + G_{\Delta_b}\Pi^*\theta/F_{\Delta_b}$ and $\tilde{G}_\epsilon^o = A_0 + G_{\Delta_b}(\Pi^* - \Pi^o)\theta/F_{\Delta_b}$. Their difference is

$$\tilde{G}_\epsilon^c - \tilde{G}_\epsilon^o = (1 - A_0) + G_{\Delta_b}\frac{\Pi^o\theta}{F_{\Delta_b}}. \tag{B.21}$$

Since $1 - A_0 = \Pi^o(1 + \Pi^* - \Pi^o)/\Pi^*$ and $G_{\Delta_b} < 0$, the second term reduces the difference. We show it remains positive. We need

$$\frac{\Pi^o(1 + \Pi^* - \Pi^o)}{\Pi^*} > \frac{\Pi^{*\prime}(1 - \Pi^*)\Pi^o\theta}{\Pi^* F_{\Delta_b}}.$$

Cancel Π^o/Π^* and multiply both sides by $F_{\Delta_b} = 1/(1 - \Pi^*) + \theta\Pi^{*\prime}$:

$$(1 + \Pi^* - \Pi^o)\left[\frac{1}{1 - \Pi^*} + \theta\Pi^{*\prime}\right] > (1 - \Pi^*)\Pi^{*\prime}\theta.$$

The left side equals $\frac{1+\Pi^*-\Pi^o}{1-\Pi^*}+(1+\Pi^*-\Pi^o)\theta\Pi^{*\prime}$. Since $\frac{1+\Pi^*-\Pi^o}{1-\Pi^*} > 1$ and $(1+\Pi^*-\Pi^o) > (1-\Pi^*)$ (because $2\Pi^* > \Pi^o$), the left side exceeds $(1 - \Pi^*)\Pi^{*\prime}\theta$. Hence $\tilde{G}_\epsilon^c - \tilde{G}_\epsilon^o > 0$.

From (B.19), the common reduced-form denominator is

$$\tilde{G}_{f_L} = G_{f_L} + \frac{G_{\Delta_b}}{F_{\Delta_b}} = -\frac{\Pi^{*\prime}}{\Pi^*} + \frac{\Pi^{*\prime}(1 - \Pi^*)}{\Pi^*}\left(1 - \frac{1}{F_{\Delta_b}}\right) < 0.$$

Since $\tilde{G}^c_\epsilon - \tilde{G}^o_\epsilon > 0$ and $\tilde{G}_{f_L} < 0$:

$$\frac{df^c_L}{d\epsilon} - \frac{df^o_L}{d\epsilon} = \frac{-(\tilde{G}^c_\epsilon - \tilde{G}^o_\epsilon)}{\tilde{G}_{f_L}} > 0. \quad \text{Q.E.D.}$$

Lemma 7 *Assume $\xi \leq 1$ and legacy inflation $\tilde{\mu}^L > 1$. At the limit $\epsilon \to 0$:*

$$u(1-\gamma)(\mu^P - \beta) > \frac{f_P}{\xi} \tag{B.22}$$

Proof: Consider the open policy at the limit $\epsilon = 0$. The platform chooses inflation μ^P and fee f_P to maximize profit from the flexible majority while retaining optimal conditions for the vanishing biased segment. The first-order conditions (FOCs) for μ^P and f_P are:

$$\xi u(1-\gamma)(\mu^P - \beta) = \frac{\Pi_b(\Delta^{of}_b)}{\Pi'_b(\Delta^{of}_b)} \quad \text{and} \quad f_P = \frac{\Pi_b(\Delta^{ob}_b)}{\Pi'_b(\Delta^{ob}_b)} \tag{B.23}$$

Let $H(\Delta) \equiv \frac{\Pi_b(\Delta)}{\Pi'_b(\Delta)}$. For the logistic Π_b, $H'(\Delta) > 0$.

The perceived advantages for flexible buyers (Δ^{of}_b) and biased buyers (Δ^{ob}_b) are defined as:

$$\Delta^{of}_b = \beta\left(a_{Pb}(n^o_P) - a_{Lb}(n^o_L)\right)\gamma u + f_L + \xi(1-\gamma)u(\tilde{\mu}^L - \mu^P) \tag{B.24}$$

$$\Delta^{ob}_b = \beta\left(a_{Pb}(n^o_P) - a_{Lb}(n^o_L)\right)\gamma u + f_L - f_P \tag{B.25}$$

Subtracting these yields:

$$\Delta^{of}_b - \Delta^{ob}_b = \xi u(1-\gamma)(\tilde{\mu}^L - \beta) - (\xi u(1-\gamma)(\mu^P - \beta) - f_P) \tag{B.26}$$

Let $Y = \xi u(1-\gamma)(\mu^P - \beta) - f_P$ and $K = \xi u(1-\gamma)(\tilde{\mu}^L - \beta)$. Since $\tilde{\mu}^L > 1 > \beta$, $K > 0$. The difference in advantages is $\Delta^{of}_b - \Delta^{ob}_b = K - Y$.

Substituting the FOCs back into the definition of Y:

$$Y = H(\Delta^{of}_b) - H(\Delta^{ob}_b) = H(\Delta^{ob}_b + K - Y) - H(\Delta^{ob}_b) \tag{B.27}$$

Define the function $\Phi(Y) = H(\Delta^{ob}_b + K - Y) - H(\Delta^{ob}_b) - Y$. We seek Y such that $\Phi(Y) = 0$.

- At $Y = 0$: $\Phi(0) = H(\Delta_b^{ob} + K) - H(\Delta_b^{ob})$. Since H is strictly increasing and $K > 0$, $\Phi(0) > 0$.

- Slope: $\Phi'(Y) = -H'(\cdot) - 1 < 0$.

Since $\Phi(0) > 0$ and $\Phi'(Y) < 0$, the unique solution Y^* must be strictly positive ($Y^* > 0$). Therefore, $\xi u(1-\gamma)(\mu^P - \beta) - f_P > 0$, which implies $u(1-\gamma)(\mu^P - \beta) > f_P/\xi$. Q.E.D.

To finish the proof, recall that the platform's FOC gives $\xi u(1-\gamma)(\mu^P-\beta) = H(\Delta_b)$ where $H(\Delta_b) = \Pi_b/\Pi_b'$. Since μ^P is determined by Δ_b through this FOC, both Π^* and μ^P move when Δ_b changes. By the product rule:

$$\frac{d}{d\Delta_b}\big[\Pi^*\, u(1-\gamma)(\mu^P-\beta)\big] = \Pi^{*\prime}\, u(1-\gamma)(\mu^P-\beta) \;+\; \Pi^*\, u(1-\gamma)\frac{d\mu^P}{d\Delta_b}\,.$$

Differentiating the FOC gives $\xi\, u(1-\gamma)\frac{d\mu^P}{d\Delta_b} = H'(\Delta_b) = \frac{\Pi^*}{1-\Pi^*}$ (computed above). The FOC itself gives $\Pi^{*\prime}\, u(1-\gamma)(\mu^P-\beta) = \Pi^{*\prime}\, H/\xi = \Pi^*/\xi$. Substituting:

$$= \frac{\Pi^*}{\xi} + \frac{\Pi^{*2}}{\xi(1-\Pi^*)} = \frac{\Pi^*}{\xi(1-\Pi^*)} = \frac{u(1-\gamma)(\mu^P-\beta)\,\Pi^{*\prime}}{1-\Pi^*}\,,$$

where the last equality uses the FOC $\Pi^*/\xi = \Pi^{*\prime}\, u(1-\gamma)(\mu^P-\beta)$ once more.

At $\epsilon = 0$, using $d\Delta_b/d\epsilon = (-F_\epsilon + df_L/d\epsilon)/F_{\Delta_b}$ from (B.17):

$$\frac{d\bar{V}_P^c}{d\epsilon} = -\Pi^*\, u(1-\gamma)(\mu^P-\beta) + \frac{u(1-\gamma)(\mu^P-\beta)\,\Pi^{*\prime}}{(1-\Pi^*)F_{\Delta_b}}\left[\Pi^*\theta + \frac{df_L^c}{d\epsilon}\right],$$
$$\frac{d\bar{V}_P^o}{d\epsilon} = -\Pi^*\, u(1-\gamma)(\mu^P-\beta) + \Pi^o f_P + \frac{u(1-\gamma)(\mu^P-\beta)\,\Pi^{*\prime}}{(1-\Pi^*)F_{\Delta_b}}\left[(\Pi^*-\Pi^o)\theta + \frac{df_L^o}{d\epsilon}\right],$$

where the $\Pi^*\theta$ and $(\Pi^*-\Pi^o)\theta$ terms capture the seller-to-buyer ratio-induced shift in Δ_b when ϵ increases.

Taking the difference:

$$\left.\frac{dD}{d\epsilon}\right|_{\epsilon=0} = \frac{u(1-\gamma)(\mu^P-\beta)\,\Pi^{*\prime}}{(1-\Pi^*)F_{\Delta_b}}\left[\Pi^o\theta + \frac{df_L^c}{d\epsilon} - \frac{df_L^o}{d\epsilon}\right] - \Pi^o f_P. \tag{B.28}$$

Combining the seller-to-buyer ratio benefit and strategic gap. From (B.21), the strategic gap from Lemma 6 is $\frac{df_L^c}{d\epsilon} - \frac{df_L^o}{d\epsilon} = \big[(1-A_0) - |G_{\Delta_b}|\Pi^o\theta/F_{\Delta_b}\big]/|\tilde{G}_{f_L}|$. Adding $\Pi^o\theta$ and using

$|\tilde{G}_{f_L}| - |G_{\Delta_b}|/F_{\Delta_b} = |G_{f_L}| = \Pi^{*\prime}$ from (B.19):

$$\Pi^o\theta + \frac{df_L^c}{d\epsilon} - \frac{df_L^o}{d\epsilon} = \frac{(1-A_0) + \Pi^o\theta\,\Pi^{*\prime}}{|\tilde{G}_{f_L}|} = \frac{\Pi^o\left[(1+\Pi^*-\Pi^o) + \Pi^*\theta\Pi^{*\prime}\right]}{\Pi^*\,|\tilde{G}_{f_L}|}\,, \tag{B.29}$$

where we used $1-A_0 = \Pi^o(1+\Pi^*-\Pi^o)/\Pi^*$. Substituting $|\tilde{G}_{f_L}| = \Pi^{*\prime}(\Pi^* F_{\Delta_b}+1-\Pi^*)/(\Pi^* F_{\Delta_b})$ from (B.19) into (B.28):

$$\left.\frac{dD}{d\epsilon}\right|_{\epsilon=0} = \Pi^o\Big[u(1-\gamma)(\mu^P-\beta)\,\tilde{R} - f_P\Big]\,, \tag{B.30}$$

where

$$\tilde{R} \;\equiv\; \frac{(1+\Pi^*-\Pi^o) + \Pi^*\theta\Pi^{*\prime}}{(1-\Pi^*)\,(\Pi^* F_{\Delta_b} + 1 - \Pi^*)}\,. \tag{B.31}$$

Claim: $\tilde{R} > 1$. Set $a = \theta\Pi^{*\prime} \geq 0$, $p = \Pi^*$, $q = \Pi^o$. Using $F_{\Delta_b} = 1/(1-p) + a$, the denominator expands to $(1-p)(pF_{\Delta_b} + 1-p) = 1-p+p^2+pa(1-p)$. Cross-multiplying, $\tilde{R} > 1$ iff

$$(1+p-q) + pa \;>\; 1-p+p^2+pa(1-p),$$

which simplifies to

$$\underbrace{p(2-p) - q}_{>0} \;+\; \underbrace{p^2 a}_{\geq 0} \;>\; 0\,,$$

which is strictly positive since $p > q$ (from $\Pi^* > \Pi^o$) and $p(2-p) > p > q$.

Since $\tilde{R} > 1$, applying Lemma 7 and $\xi \leq 1$:

$$u(1-\gamma)(\mu^P-\beta)\,\tilde{R} > \frac{f_P}{\xi}\cdot 1 \geq f_P\,.$$

Hence $dD/d\epsilon > 0$ strictly, which proves that for a sufficiently small mass of biased buyers, the platform earns strictly higher profits under the closed policy, implying $D(1-\epsilon) > 0$. This completes the proof of the proposition.

B.9 Proof of Proposition 9

All notation follows the paper. We work in stationarity with $f_P = 0$.

Setup. The buyer and seller advantages satisfy:

$$\Delta_b = \underbrace{\beta\big(a_{Pb}(n_P) - a_{Lb}(n_L)\big)\gamma u}_{M_b(\Pi_b,\Pi_s)} + \xi(\tilde{\mu}^L - \tilde{\mu}^P)(1-\gamma)u + \frac{1-\Pi_b}{\Pi_b'}, \tag{B.32}$$

$$\Delta_s = \underbrace{\beta\big(a_{Ps}(n_P) - a_{Ls}(n_L)\big)(1-\gamma)u}_{M_s(\Pi_b,\Pi_s)} + \frac{1-\Pi_s}{\Pi_s'} - k_P, \tag{B.33}$$

with $n_P = \bar{n}\,\Pi_s/\Pi_b$, $n_L = \bar{n}\,(1-\Pi_s)/(1-\Pi_b)$, and $\bar{n} = \bar{N}_s/\bar{N}_b$. Per-period revenue with $E \equiv u(1-\gamma)(\tilde{\mu}^P - \beta)$:

$$R = \underbrace{\bar{N}_s\,\Pi_s\,k_P}_{R_s} + \underbrace{\bar{N}_b\,\Pi_b\,E}_{R_b}. \tag{B.34}$$

Choice of shares. As in Definition 1, the platform takes the legacy market's fees (f_L, k_L) as given; in equilibrium they equal $(1-\Pi_b)/\Pi_b'$ and $(1-\Pi_s)/\Pi_s'$, but the platform does not internalize their response. Given (f_L, k_L), the platform's two instruments $(k_P, \tilde{\mu}^P)$ implement a pair (Π_b, Π_s), uniquely by Proposition 1. Following Armstrong (2006) and Weyl (2010), we let the platform choose the shares directly and back out the instruments as inverse demands, writing $\Delta_i(\Pi_i)$ for the inverse of the attraction function:

$$k_P(\Pi_b, \Pi_s) = M_s(\Pi_b, \Pi_s) + k_L - \Delta_s(\Pi_s), \tag{B.35}$$

$$\xi E(\Pi_b, \Pi_s) = M_b(\Pi_b, \Pi_s) + f_L - \Delta_b(\Pi_b) + \xi(\tilde{\mu}^L - \beta)(1-\gamma)u. \tag{B.36}$$

The proof proceeds in three steps: (A) derive the partial derivatives of the inverse demand functions, (B) characterize the direction each instrument moves, and (C) show the two conditions cannot hold simultaneously.

Step A: Partial derivatives of inverse demands.

Define the matching externality parameters, the same four objects as in the proof of

Proposition 6. When Π_b changes (holding Π_s fixed), n_P and n_L shift, affecting both sides:

$$\theta_b \equiv \beta\gamma u\left[\frac{A_P}{\Pi_b}+\frac{B_L}{1-\Pi_b}\right], \qquad \phi_{sb} \equiv \beta(1-\gamma)u\left[\frac{C_P}{\Pi_b}+\frac{D_L}{1-\Pi_b}\right], \tag{B.37}$$

where $A_P = a'_{Pb}(n_P)\, n_P$, $B_L = a'_{Lb}(n_L)\, n_L$, $C_P = |a'_{Ps}(n_P)|\, n_P$, $D_L = |a'_{Ls}(n_L)|\, n_L$. Similarly, when Π_s changes:

$$\theta_s \equiv \beta(1-\gamma)u\left[\frac{C_P}{\Pi_s}+\frac{D_L}{1-\Pi_s}\right], \qquad \phi_{bs} \equiv \beta\gamma u\left[\frac{A_P}{\Pi_s}+\frac{B_L}{1-\Pi_s}\right]. \tag{B.38}$$

Define the **residual-demand elasticity factors**:

$$F_b \equiv 1+\theta_b\Pi'_b > 1, \qquad F_s \equiv 1+\theta_s\Pi'_s > 1. \tag{B.39}$$

Each factor is the ratio of the slope of the inverse demand the platform faces when it accounts for tightness to the partial-equilibrium slope $1/\Pi'_i$ it uses in the baseline: the leading 1 is the platform's own slope and $\theta_i\Pi'_i$ is congestion rebalancing. (If the platform also anticipated the legacy market's fee response, substituting $f_L = (1-\Pi_b)/\Pi'_b$ and $k_L = (1-\Pi_s)/\Pi'_s$ into the inverse demands, each factor would gain the term $1-R_i \geq 0$, where $R_i \equiv -(1-\Pi_i)\Pi''_i/(\Pi'_i)^2 \leq 1$ by log-concavity, since $df_L/d\Pi_b = -(1-R_b)/\Pi'_b$; see the remark at the end of this proof.)

Lemma 8 (Partial derivatives of inverse demands)

$$\frac{\partial k_P}{\partial \Pi_s} = -\frac{F_s}{\Pi'_s} < 0, \tag{B.40}$$

$$\frac{\partial k_P}{\partial \Pi_b} = \phi_{sb} \quad > 0, \tag{B.41}$$

$$\frac{\partial(\xi E)}{\partial \Pi_b} = -\frac{F_b}{\Pi'_b} < 0, \tag{B.42}$$

$$\frac{\partial(\xi E)}{\partial \Pi_s} = \phi_{bs} \quad > 0. \tag{B.43}$$

Proof. Differentiate (B.35) with respect to Π_s, holding k_L fixed. Two contributions: (i) $\partial M_s/\partial \Pi_s = -\theta_s$ (more sellers worsen seller matching on both markets); (ii) $-d\Delta_s/d\Pi_s =$

$-1/\Pi_s'$ (a larger share requires a larger seller advantage). Hence

$$\frac{\partial k_P}{\partial \Pi_s} = -\theta_s - \frac{1}{\Pi_s'} = -\frac{1 + \theta_s \Pi_s'}{\Pi_s'} = -\frac{F_s}{\Pi_s'}.$$

For the cross-derivative (B.41): only M_s depends on Π_b, and $\partial M_s/\partial \Pi_b = \phi_{sb} > 0$ (more buyers improve seller matching on both markets).

The buyer-side derivatives (B.42)–(B.43) follow by the same logic with (b, s) exchanged. □

Step B: Direction conditions.

The platform maximizes $R(\Pi_b, \Pi_s) = \bar{N}_s \Pi_s \, k_P(\Pi_b, \Pi_s) + \bar{N}_b \Pi_b \, E(\Pi_b, \Pi_s)$.

Seller FOC ($\partial R/\partial \Pi_s$). The product rule gives

$$\frac{\partial R}{\partial \Pi_s} = \bar{N}_s \left[k_P + \Pi_s \frac{\partial k_P}{\partial \Pi_s} \right] + \frac{\bar{N}_b \, \Pi_b}{\xi} \frac{\partial (\xi E)}{\partial \Pi_s}.$$

Substituting the partial derivatives from Lemma 8 ($\partial k_P/\partial \Pi_s = -F_s/\Pi_s'$ and $\partial(\xi E)/\partial \Pi_s = \phi_{bs}$):

$$\frac{\partial R}{\partial \Pi_s} = \bar{N}_s \left[k_P - \frac{\Pi_s \, F_s}{\Pi_s'} \right] + \frac{\bar{N}_b \, \Pi_b}{\xi} \, \phi_{bs}.$$

Setting $\partial R/\partial \Pi_s = 0$ and solving for k_P:

$$k_P^* = \frac{\Pi_s}{\Pi_s'} \, F_s - \frac{\bar{N}_b \, \Pi_b}{\xi \, \bar{N}_s} \, \phi_{bs}. \tag{B.44}$$

The first term is the baseline Lerner markup $k_P^0 = \Pi_s/\Pi_s'$ scaled up by the elasticity factor $F_s > 1$; the second is the cross-subsidy correction. Subtracting the baseline:

$$k_P^* - k_P^0 = \frac{\Pi_s}{\Pi_s'} \, (F_s - 1) - \frac{\bar{N}_b \, \Pi_b}{\xi \, \bar{N}_s} \, \phi_{bs}.$$

The seller fee falls ($k_P^* < k_P^0$) when the second term exceeds the first. Multiplying both sides

by $\bar{N}_s \, \Pi'_s > 0$:

$$k_P \text{ falls} \quad \Longleftrightarrow \quad \bar{N}_s \, \Pi_s \, (F_s - 1) \; < \; \frac{\bar{N}_b \, \Pi_b}{\xi} \, \phi_{bs} \Pi'_s. \tag{B.45}$$

The left side is the elasticity-correction gain from raising the fee; the right side is the cross-subsidy benefit of lowering it (each seller contributes ϕ_{bs} to buyer matching, scaled by Π'_s to convert to fee units).

Buyer FOC ($\partial R / \partial \Pi_b$). By the same steps with (b, s) exchanged:

$$\frac{\partial R}{\partial \Pi_b} = \bar{N}_s \, \Pi_s \, \phi_{sb} + \frac{\bar{N}_b}{\xi} \left[\xi E - \frac{\Pi_b \, F_b}{\Pi'_b} \right].$$

Setting to zero: $E^* = (\Pi_b / (\xi \Pi'_b)) F_b - (\bar{N}_s \Pi_s / \bar{N}_b) \phi_{sb}$. Subtracting the baseline $E_0 = \Pi_b / (\xi \Pi'_b)$ and multiplying by $\bar{N}_b \Pi'_b / \xi > 0$:

$$E \text{ falls} \quad \Longleftrightarrow \quad \frac{\bar{N}_b \, \Pi_b}{\xi} \, (F_b - 1) \; < \; \bar{N}_s \, \Pi_s \, \phi_{sb} \Pi'_b. \tag{B.46}$$

Step C: The two conditions cannot both hold.

Suppose for contradiction that both conditions hold. The seller condition (B.45) says

$$(F_s - 1) \; < \; \frac{\bar{N}_b \, \Pi_b}{\xi \, \bar{N}_s \, \Pi_s} \, \phi_{bs} \Pi'_s,$$

and the buyer condition (B.46) says

$$(F_b - 1) \; < \; \frac{\xi \, \bar{N}_s \, \Pi_s}{\bar{N}_b \, \Pi_b} \, \phi_{sb} \Pi'_b.$$

Multiplying these two inequalities, the factors $\bar{N}_i \Pi_i / \xi$ cancel, leaving:

$$(F_s - 1)(F_b - 1) \; < \; \phi_{bs} \phi_{sb} \, \Pi'_b \Pi'_s. \tag{B.47}$$

We show this is impossible. By (B.39), $F_i - 1 = \theta_i \Pi'_i$, so (B.47) reads $\theta_b \theta_s < \phi_{bs} \phi_{sb}$. The following lemma shows the reverse inequality always holds.

Lemma 9 (Strategic dampening) *Under Assumption 1, $\theta_b\theta_s \geq \phi_{bs}\phi_{sb}$, with equality only if $\Pi_b = \Pi_s$.*

Proof. Compute directly. Using the definitions (B.37)–(B.38):

$$\begin{aligned}
\theta_b &= \frac{\beta\gamma u}{\Pi_b(1-\Pi_b)}\big[A_P(1-\Pi_b) + B_L\Pi_b\big],\\
\theta_s &= \frac{\beta(1-\gamma) u}{\Pi_s(1-\Pi_s)}\big[C_P(1-\Pi_s) + D_L\Pi_s\big],\\
\phi_{bs} &= \frac{\beta\gamma u}{\Pi_s(1-\Pi_s)}\big[A_P(1-\Pi_s) + B_L\Pi_s\big],\\
\phi_{sb} &= \frac{\beta(1-\gamma) u}{\Pi_b(1-\Pi_b)}\big[C_P(1-\Pi_b) + D_L\Pi_b\big].
\end{aligned}$$

Collecting the common factor $\beta^2\gamma(1-\gamma)u^2/[\Pi_b(1-\Pi_b)\Pi_s(1-\Pi_s)]$:

$$\begin{aligned}
\theta_b\theta_s &\propto \big[A_P(1-\Pi_b) + B_L\Pi_b\big]\big[C_P(1-\Pi_s) + D_L\Pi_s\big],\\
\phi_{bs}\phi_{sb} &\propto \big[A_P(1-\Pi_s) + B_L\Pi_s\big]\big[C_P(1-\Pi_b) + D_L\Pi_b\big].
\end{aligned}$$

Expanding both products and subtracting:

$$\theta_b\theta_s - \phi_{bs}\phi_{sb} \;\propto\; (A_P D_L - B_L C_P)(\Pi_s - \Pi_b).$$

Write $h(n) \equiv a'_{Lb}(n)/|a'_{Ls}(n)|$. Since $A_P = h(n_P)\,C_P$ and $B_L = h(n_L)\,D_L$ (the factor A cancels from buyer-to-seller derivative ratios), $A_P D_L - B_L C_P = C_P D_L\,[h(n_P) - h(n_L)]$. Since $n_P \gtrless n_L$ if and only if $\Pi_s \gtrless \Pi_b$, and h is non-decreasing (Assumption 1), the two factors $[h(n_P) - h(n_L)]$ and $(\Pi_s - \Pi_b)$ always have the same sign. Their product is non-negative. □

By Lemma 9, $(F_s - 1)(F_b - 1) = \theta_b\theta_s\Pi'_b\Pi'_s \geq \phi_{bs}\phi_{sb}\Pi'_b\Pi'_s$, contradicting (B.47). Therefore k_P and $\tilde{\mu}^P$ cannot both fall. □

Remark (anticipating the legacy market's fee response). If the platform instead substitutes the legacy's best-response fees into the inverse demands, $k_L = (1-\Pi_s)/\Pi'_s$ and $f_L = (1-\Pi_b)/\Pi'_b$, then $dk_L/d\Pi_s = -(1-R_s)/\Pi'_s$ and $df_L/d\Pi_b = -(1-R_b)/\Pi'_b$ add to the derivatives in Lemma 8, so that $F_i = 1 + (1-R_i) + \theta_i\Pi'_i$. Steps A–C go through verbatim,

and

$$(F_s-1)(F_b-1) = \underbrace{(1-R_s)(1-R_b)}_{\geq 0} + \underbrace{(1-R_s)\theta_b\Pi'_b}_{\geq 0} + \underbrace{(1-R_b)\theta_s\Pi'_s}_{\geq 0} + \theta_b\theta_s\Pi'_b\Pi'_s \;\geq\; \phi_{bs}\phi_{sb}\Pi'_b\Pi'_s,$$

so the proposition holds with more slack.

C Online Appendix: Bargaining Protocol

C.1 Nash Bargaining with Liquidity Constraints

We assume that the seller's cost of producing x units of the good is xc where $0 \leq c < u$. We also assume that a buyer obtains utility xu if the seller produces $0 \leq x \leq 1$ and does not value more than one unit. (Alternatively we can assume the seller has the production capacity of one unit instead.) To begin, suppose the buyer is not liquidity constrained. The (generalized) Nash bargaining problem is :

$$\max_{x,q} (xu - q)^{\gamma} (q - xc)^{1-\gamma} \tag{C.1}$$

where q is the terms of trade for exchanging the consumption goods. The solution is: $x^* = 1$ and $q^* = (1-\gamma)\, u + \gamma c$. Hence the buyer's surplus is: $\gamma\,(u-c)$ and the seller's surplus is $(1-\gamma)\,(u-c)$.

Now suppose the buyer is liquidity constrained so that they will not be able to bring enough money to pay for the consumption good price from the above Nash solution. That is, $q \leq q^*$. The liquidity constraint is binding so the Nash bargaining problem becomes:

$$\max_{x,q} (xu - q)^{\gamma} (y - xc)^{1-\gamma} \tag{C.2}$$

subject to $q \leq q^*$. The first order condition is

$$\frac{q - xc}{xu - q} = \frac{1-\gamma}{\gamma}\frac{c}{u} \Rightarrow x = \frac{(\gamma u + (1-\gamma)\, c)}{cu} q.$$

Therefore, the solution is

$$x^* = \begin{cases} 1 & \text{if } \frac{(\gamma u+(1-\gamma)c)}{cu} q \geq 1 \\ \frac{(\gamma u+(1-\gamma)c)}{cu} q & \text{otherwise} \end{cases}$$

Hence, the term of trade is:

$$q^{**} = \frac{cu}{\gamma u + (1-\gamma)\, c}. \tag{C.3}$$

. The seller's surplus is:

$$\frac{cu}{\gamma u + (1-\gamma)\, c} - c = \frac{(1-\gamma)\, c\, (u-c)}{\gamma u + (1-\gamma)\, c}. \tag{C.4}$$

The buyer's surplus is:

$$u - \frac{cu}{\gamma u + (1-\gamma)\, c} = \frac{\gamma u\, (u-c)}{\gamma u + (1-\gamma)\, c}. \tag{C.5}$$

The Nash bargaining solution indicates that the buyer when liquidity constrained chooses to bring q^{**} specified in eq. (C.3) from the CM to the DM in order to trade the consumption goods. That is, Thus, the resulting real price for the good y is: $p^j \phi^j = cu/(\gamma u + (1-\gamma)\, c)$. The welfare analysis will be based on the split of the total surplus $u - c$ specified in eq. (C.4) and eq. (C.5). The qualitative results on the platform use of platform money to gain competitive advantage remain the same but the quantitative implications are different. This is because the liquidity constrained buyer potentially would bring less money, platform money in this case give the platform owner less advantages than the Kalai bargaining solution.

However, note that there is one peculiarity in this Nash bargaining solution that we consider undesirable. The derivative of the seller's surplus with respect to c has same sign as: $\gamma u^2 - 2\gamma cu - (1-\gamma)\, c^2$. It is negative quadratic in c and at $c = 0$ this is $\gamma u^2 > 0$ and at $c = u$ this is $-u^2 < 0$. Hence, if the seller has a low cost, he has incentive to exaggerate cost or engage in wasteful expenditure.

C.2 Kalai Bargaining with and without Liquidity Constraints

Suppose that the seller's cost of producing x units of the good is xc where $0 \leq c < u$. The buyer obtains utility xu if the seller produces $0 \leq x \leq 1$ and does not value more than one unit.[14] We can also interpret x as probability of trade.

First, suppose the buyer is not liquidity constrained. Under Kalai's proportional bargaining with bargaining parameter γ, the optimization problem is to choose the terms of trade q and the amount to produce x to maximize:

$$\begin{aligned} \max_{x,q} \quad & xu - q \\ \text{subject to} \quad & \tfrac{xu-q}{q-xc} = \tfrac{\gamma}{1-\gamma} \text{ and } x \leq 1. \end{aligned}$$

Solving for q from the constraint and plugging into the objective we see that objective is increasing in x. Hence, the solution without liquidity constraint is $x^* = 1$ and $q^* = (1-\gamma)\,u + \gamma c$.[15]

Now, suppose the real value of the money that the buyer brings to the DM is $\bar{q}$ and the buyer is liquidity constrained so that $\bar{q} \leq q^*$. Now the problem becomes:

$$\max_{x,q} \quad xu - q \tag{C.6}$$

$$\text{subject to} \quad \tfrac{xu-q}{q-xc} = \tfrac{\gamma}{1-\gamma} \text{ and } q \leq \bar{q}. \tag{C.7}$$

Solving for q and plugging into the objective, we can rewrite this problem as:

$$\max_{x} \quad x\,(u-c) \tag{C.8}$$

$$\text{subject to} \quad x((1-\gamma)u + \gamma c) \leq \bar{q}. \tag{C.9}$$

The solution is

$$x^{**} = \frac{\bar{q}}{(1-\gamma)\,u + \gamma c} \text{ and } q^{**} = \bar{q} \tag{C.10}$$

The buyer's utility under this solution is $\left(\gamma\,(u-c)\,/((1-\gamma)\,u + \gamma c)\right)\bar{q}$. Now, let's step

[14] Alternatively, we can assume that the seller has the production capacity of one unit.

[15] When buyers are not liquidity constrained, the Kalai and the Nash bargaining solutions are the same.

back and ask how much liquidity the buyer brings to the DM. Buyer's utility is increasing in $\bar{q}$ (up to q^*). Hence, the buyer's optimal liquidity, or the price of the DM good, is $q^* = (1-\gamma)\,u + \gamma c$. Consequently, the buyer's utility is $\gamma\,(u-c)$ and the seller's utility is $(1-\gamma)\,(u-c)$. When $c = 0$, the resulting real price for the consumption good y is: $p^j \phi^j = u(1-\gamma)$.

D Online Appendix: Planner's Objective Derivation

Claim 1 *The social planner's objective function can be written as:*

$$\max \overline{N}_b \gamma u \left[\Pi_b a_{Pb}(n_P) + (1-\Pi_b) a_{Lb}(n_L)\right] + \overline{N}_s\,(1-\gamma)\,u\left[\Pi_s a_{Ps}(n_P) + (1-\Pi_s) a_{Ls}(n_L)\right] \quad \text{(D.1)}$$

By summing all agents' utility:

$$\begin{aligned}
\max & \underbrace{\overline{N}_b[\Pi_b\,(\Delta_b)\,W_b^P(0,0) + [1-\Pi_b\,(\Delta_b)]W_b^L(0,0)]}_{\text{Buyers' Utility}} \\
& + \underbrace{\overline{N}_s[\Pi_s\,(\Delta_s)\,W_s^P(0,0) + (1-\Pi_s\,(\Delta_s))\,W_s^L(0,0)]}_{\text{Sellers' Utility}} \\
& + \underbrace{\overline{N}_s \Pi_s\,(\Delta_s)\,k_P + \overline{N}_b \Pi_b\,(\Delta_b)\,[f_P + (\mu_P - 1)u(1-\gamma)]}_{P \text{ Owner's Profit}} \\
& + \underbrace{\overline{N}_s\,(1-\Pi_s\,(\Delta_s))\,k_L + \overline{N}_b\,(1-\Pi_b\,(\Delta_b))\,f_L}_{L \text{ Owner's Profit}} \\
& + \underbrace{\overline{N}_b\,(1-\Pi_b\,(\Delta_b))\,(\mu_L - 1)u(1-\gamma)}_{\text{Tax rebate for outside money seigniorage}},
\end{aligned}$$

which is subject to

$$\begin{aligned}
n_P[\bar{N}_b \Pi_b\,(\Delta_b)] &= \bar{N}_s \Pi_s\,(\Delta_s) \\
n_L[\bar{N}_b(1-\Pi_b\,(\Delta_b))] &= \bar{N}_s(1-\Pi_s\,(\Delta_s)).
\end{aligned}$$

Next, we plug in (5) and (18) and after some algebra we get to equation (D.1).

E Online Appendix: Self-Enforceability and Punishment Strategies

Under commitment, the monetary equilibrium exists for all $\beta \in (0,1)$ (Proposition 1). Here we characterize when the commitment is self-enforcing, i.e., when the platform would not wish to deviate from its announced policy even if it could.

We isolate the buyer-side monetary value $\hat{\mathcal{V}}$ by netting out the separable seller present value: $\mathcal{V}(N_b) = \bar{N}_s \Pi_s k_P/(1-\beta) + \hat{\mathcal{V}}(N_b)$. Writing $z \equiv u(1-\gamma)$ and $N_b' \equiv \bar{N}_b\, \Pi_b(\Delta_b(\tilde{\mu}^P))$:

$$\hat{\mathcal{V}}(N_b) = \max\Big\{0,\ \max_{\tilde{\mu}^P} \left[\tilde{\mu}^P\, N_b'\, z - N_b\, z + \beta\, \hat{\mathcal{V}}(N_b')\right]\Big\}, \tag{E.1}$$

where the outer max captures the option to abandon money issuance. By the separability of the seller fee (equation (28)), the buyer-side component inherits the affine structure: $\hat{\mathcal{V}}(N) = -Nz + \hat{K}$, where $\hat{K} = N_b^*(\tilde{\mu}^P - \beta)z/(1-\beta)$.

Proposition 13 (Self-enforcing monetary equilibrium) *Define*

$$\beta_{\text{crit}} \ \equiv\ 1 - \frac{\Pi_b(\Delta_0)}{\xi\, z\, \Pi_b'(\Delta_0)}\,, \tag{E.2}$$

where $\Delta_0 \equiv \Delta_b(1)$ is the buyer advantage when $\tilde{\mu}^P = 1$. The commitment equilibrium is self-enforcing if and only if $\beta > \beta_{\text{crit}}$. Whenever this holds, the stationary policy satisfies $\tilde{\mu}^P > 1$: the platform inflates the money supply.

Non-negative buyer-side value on path. The candidate buyer-side value at the stationary state is $\hat{\mathcal{V}}(N_b^*) = \hat{K} - N_b^* z$. Using $\hat{K} = N_b^*(\tilde{\mu}^P - \beta)z/(1-\beta)$:

$$\hat{\mathcal{V}}(N_b^*) \ =\ N_b^*\, z\, \frac{\tilde{\mu}^P - 1}{1-\beta}\,.$$

Hence $\hat{\mathcal{V}}(N_b^*) > 0$ if and only if $\tilde{\mu}^P > 1$, i.e., $\beta > \beta_{\text{crit}}$ (from (E.2)).

On-path optimality. On the equilibrium path, $N_b = N_b^*$ and the platform plays $\tilde{\mu}^P$. Since $\tilde{\mu}^P$ maximizes $N_b'(\tilde{\mu}^P)(\tilde{\mu}^P - \beta)z$ by the first-order condition (30), no deviation to another $\tilde{\mu}^{P\prime}$

that keeps money issuance going is profitable: the platform's choice is the unconstrained argmax of the buyer-side per-period objective.

Deviations to abandoning money are self-defeating. Suppose the platform contemplates a deviation to some $\tilde{\mu}^{P\prime}$ after which it would abandon money issuance. If the platform stops honouring its currency at CM $t{+}1$, money at $t{+}1$ is worthless ($\phi^P_{t+1} = 0$). Sellers anticipate this and refuse to accept platform tokens in DM $t{+}1$. But then buyers have no reason to acquire tokens at CM t: the deviation generates zero buyer-side revenue. Any deviation that leads to abandoning money is therefore self-defeating. It produces no seigniorage, not a large one-time payoff.

Proof. [Proof of Proposition 13] *Sufficiency ($\beta > \beta_{\text{crit}}$): self-enforcing equilibrium exists.* When $\beta > \beta_{\text{crit}}$, we have $\tilde{\mu}^P > 1$ and $\hat{\mathcal{V}}(N_b^*) > 0$. On path, the stationary policy is optimal by the FOC. Off path, any deviation either (i) keeps money issuance going, in which case the FOC ensures $\tilde{\mu}^P$ dominates, or (ii) leads to abandoning money, which is self-defeating and generates zero buyer-side revenue. Hence $\hat{\mathcal{V}}(N) = -Nz + \hat{K}$ satisfies the Bellman equation (E.1).

Necessity ($\beta \leq \beta_{\text{crit}}$): no self-enforcing equilibrium. When $\beta \leq \beta_{\text{crit}}$, $\tilde{\mu}^P \leq 1$ and $\hat{\mathcal{V}}(N_b^*) \leq 0$. The buyer-side monetary value is non-positive, so the platform prefers fee-only operations and would abandon money. By backward induction, if the platform will abandon its currency, money is worthless: sellers refuse to accept it, buyers refuse to acquire it, and no monetary equilibrium can be sustained. Since the affine value function from Proposition 1 is the unique candidate in the operating region, no self-enforcing monetary equilibrium exists. □