Buy Box and Price Parity: Competitive Effects and Consumer Welfare in Online Marketplaces

Problem definition: Amazon's Buy Box (now the Featured Offer) grants selected sellers prominent placement that substantially increases visibility and sales. Buy Box eligibility partly depends on whether a seller's Amazon price is no higher than the product's rival-platform price, effectively imposing an implicit price parity rule. This practice has drawn regulatory scrutiny: the FTC argues that it stifles competition and harms consumers by compelling sellers to raise prices on rival platforms to maintain eligibility. ***Methodology/results***: We develop a game-theoretic model of a dominant platform enforcing the eligibility-linked price parity rule. Unlike contractual Most-Favored-Nation clauses, implicit parity lets sellers opt out when compliance is costly, generating an endogenous response absent from standard parity models. Analyzing both single-seller and two-seller market structures yields the following findings. First, where parity binds, its price effects depend on market structure: under a single seller, it lowers the dominant platform's price but raises the rival's, whereas under competing sellers, it lowers both prices. Second, under multi-homing, welfare effects hinge on seller compliance rather than on the direction of prices: parity raises consumer welfare when Buy Box eligibility is retained, but lowers it when compliance costs induce opt-out, even as both prices fall; welfare also falls when the seller single-homes on the dominant platform. Third, parity never raises the dominant platform's commission revenue under competing sellers. ***Managerial implications***: Our findings challenge the view that parity rules uniformly harm consumers. Eligibility-linked price parity can improve welfare when the seller complies while multi-homing, but backfires when the seller opts out or single-homes. Blanket prohibitions therefore risk suppressing the rule where it is pro-competitive.

Key words: Buy Box, Price Parity, Online Marketplaces, Platform Operations, Antitrust

1. Introduction

Online marketplaces have transformed how consumers shop and how sellers compete, with Amazon emerging as the dominant intermediary in global e-commerce. Roughly 40% of U.S. online purchases occur on Amazon, compared to about 7% on its closest competitor, Walmart. Amazon hosts more than one million active third-party sellers and has ranked as the lowest-priced online retailer for eight consecutive years (Black 2024). An important element of this scale and price leadership is the Buy Box (now, the Featured Offer; Figure 1), which prominently displays a selected seller and captures approximately 83% of platform sales for featured products (Team (2023)). When no seller qualifies for

Buy Box eligibility, consumers see "No featured offers available" and must actively search among alternative sellers, substantially increasing search costs.[1]

Buy Box placement confers a substantial visibility and demand boost; however, eligibility is not unconditional. To qualify, a seller's Amazon price of a product must be no higher than its price on rival platforms (offered by the same seller or any other sellers). Amazon has been alleged to continuously monitor rival-platform prices, and sellers risk demotion from the Buy Box if lower prices are offered elsewhere, even just by a small margin (Weise 2023). As a result, Buy Box eligibility effectively imposes an implicit price parity constraint that links pricing decisions across platforms. Sellers respond strategically by adjusting prices across channels in order to preserve Buy Box access and the associated demand advantage.

This practice has drawn significant regulatory scrutiny. The U.S. Federal Trade Commission (FTC) alleges that Amazon uses the Buy Box to penalize sellers who discount on competing platforms, thereby discouraging cross-platform price competition and harming consumers (Harrington 2023). From this perspective, implicit price parity raises rival platform prices and softens competition. However, Buy Box eligibility-linked price parity could also promote consumer welfare by ensuring prices on Amazon match rival platforms, putting a price pressure on Amazon sellers. Whether this practice ultimately raises or lowers consumer welfare therefore remains an open question.

Despite extensive discussion of Amazon's marketplace power and growing empirical evidence on Buy Box effects (Hunold et al. 2022, Gallino et al. 2025), existing theoretical work on price parity has primarily focused on explicit Most Favored Nation (MFN) clauses rather than Buy Box eligibility-linked mechanisms (Boik and Corts 2016, Johnson 2017). Unlike contractual MFNs, implicit parity operates through threatened loss of visibility rather than formal penalties, and sellers can optimally violate eligibility by forgoing Buy Box eligibility altogether. How such an eligibility-linked price parity rule reshapes cross-platform pricing incentives and welfare therefore remains theoretically underexplored. Moreover, prior work has not systematically examined how the cross-platform market structure, e.g., single seller vs. competing sellers, mediates the effects of parity. We

[1] See Figure 1 (in section 3.1) and Figure A1 (in the Online Appendix) for examples of Buy Box eligibility and ineligibility, respectively.

address these gaps by developing a game-theoretic model of Buy Box eligibility that links seller incentives to equilibrium prices and welfare outcomes.

We model Buy Box eligibility as a conditional demand boost and compare two eligibility policies enforced by a dominant platform. Under *Buy Box with implicit price parity* (**BP**), sellers receive a demand boost only when their price on the dominant platform does not exceed that on a rival platform. Under *Buy Box without parity* (**B**), the demand boost is granted unconditionally. Under BP, sellers may intentionally violate the eligibility condition when compliance is too costly, thereby forgoing Buy Box access. We refer to this equilibrium outcome as the *opt-out* regime: while the Buy Box feature remains institutionally available, sellers receive no demand boost in equilibrium.

The analysis considers two market structures. Under the *single seller* structure, one seller may offer the product on both the dominant platform and a rival platform, choosing participation as well as prices to maximize total profit across platforms. Although stylized, this environment captures important real-world cases: brand manufacturers selling through both their direct channels and a dominant platform (for example, Nike, Apple) or large third-party sellers maintaining presence across multiple marketplaces.

Under the *two-seller* structure, two independent sellers compete across platforms, with one seller operating exclusively on the dominant platform and another operating exclusively on the rival platform. It captures a common form of marketplace competition in which sellers list on a single platform and compete with sellers on rival platforms. Fundamentally, the key distinction between the two market structures is not the number of sellers per se, but whether cross-platform pricing decisions are internalized by a single decision-maker or arise from competitive interaction between independent sellers.

Our analysis yields three main insights into the impact of eligibility-linked price parity. First, price effects depend on market structure. Under the single-seller structure, when Buy Box eligibility binds, the seller lowers the dominant platform's price and raises the rival platform's price to satisfy the price parity condition. Notably, this finding—that binding parity raises rival platform prices—aligns with regulatory concerns about implicit parity softening cross-platform competition. In contrast, when prices are set competitively across the two platforms under the two-seller structure, implicit parity's price effects change fundamentally— it no longer operates through within-seller price

coordination across platforms but instead through strategic competition between independent sellers. In the eligibility-binding regime, equilibrium prices fall on both platforms relative to Buy Box without parity, reflecting heightened competition rather than the price compression observed under the single-seller structure.

Second, welfare effects hinge on seller compliance—retaining Buy Box eligibility or opting out. When parity compliance can be achieved with reasonable price adjustments, the seller on the dominant platform gains Buy Box eligibility and benefits from the associated demand boost. Under the single-seller structure, when eligibility binds, parity lowers the dominant platform's price and raises the rival's by equal amounts. The resulting shift of demand toward the dominant platform, at unchanged aggregate quantity, raises consumer welfare relative to Buy Box without parity. Under the two-seller structure, when eligibility binds, implicit parity lowers prices on both platforms and shifts demand toward the dominant platform, and increases consumer welfare despite lower seller profits.

By contrast, when parity compliance is too costly and the seller on the dominant platform optimally forgoes Buy Box eligibility, prices fall on both platforms relative to Buy Box without parity, yet consumer welfare declines due to the loss of the Buy Box-induced demand boost. Thus, welfare effects depend critically on whether parity binds or induces opt-out. *Importantly, eligibility-linked parity cannot be evaluated based on observed price changes alone: welfare consequences depend on sellers' endogenous compliance decisions and the resulting preservation or loss of Buy Box eligibility*.

Third, the dominant platform does not reliably gain from the implicit price parity rule. Under the single-seller structure, its commission revenue rises when the rival platform's market size is small enough to induce single-homing, and also when the seller complies and cross-platform substitutability is high, so that the gain in demand more than offsets the lower price. It falls when the seller opts out of eligibility, and when that price reduction outweighs the demand it attracts. Under the competing-seller structure, this benefit disappears: compliance intensifies price competition and opt-out removes the Buy Box demand advantage, so the dominant platform never gains from its own price parity rule.

Together, these findings challenge the presumption that price parity rules uniformly harm consumers and stifle competition. When the two platforms are comparable in size, the eligibility constraint binds and the seller complies, retaining the Buy Box's demand

boost while parity disciplines prices. In this regime, implicit parity improves consumer welfare in both market structures: by reallocating demand toward the dominant platform under a single seller, and by strengthening price competition under competing sellers. More broadly, the analysis demonstrates that implicit price parity mechanism can strengthen rather than suppress competition—but only when the market structure and sellers' compliance incentives are properly accounted for.

This paper makes three contributions to the literature. First, we formalize Buy Box eligibility-linked price parity as distinct from contractual MFN clauses. Unlike explicit parity, eligibility-linked parity allows sellers to optimally forgo eligibility and abandon the Buy Box's demand boosting benefit, creating an endogenous participation option absent from standard parity models. Second, we show that market structure, specifically whether cross-platform pricing decisions are internalized by a single seller or decentralized across competing sellers, fundamentally mediates parity's price effects. When pricing decisions are internalized (as under the single-seller structure), binding parity raises the rival platform's price but lowers the dominant platform's price. In contrast, when prices are controlled by independent sellers (as under the two-seller structure), parity intensifies competition and lowers prices across platforms. Third, we characterize the welfare implications of eligibility-linked price parity and show that they hinge on seller compliance: parity can enhance welfare when it compresses prices without deterring seller participation, but reduce welfare when it induces sellers to opt out.

The remainder of the paper proceeds as follows. Section 2 reviews the related literature. Section 3 presents the institutional background and modeling framework. Sections 4 and 5 analyze equilibrium outcomes under the single-seller and two-seller structures, respectively. Section 6 discusses policy implications and concludes.

2. Related Literature

Digital platforms have become an important subject of operations and economics research, with Chen et al. (2020) classifying emerging platform types and outlining an operations-management research agenda. This paper contributes to and integrates two streams of this literature: (1) platform operations and marketplace design, and (2) price parity and Most Favored Nation clauses. Each stream illuminates important aspects of platform design and competition, but typically studies them in isolation. Our analysis connects these literatures by examining how Buy Box eligibility, when conditioned on price parity, reshapes pricing incentives and welfare across platforms.

2.1. Platform Operations and Marketplace Design

A large literature studies how digital platforms shape the behavior and performance of market participants through incentive design, contracting, information provision, ranking rules, and operational policies. In gig markets, Chen et al. (2026) analyze how competing platforms use bonuses alongside commissions to attract workers, showing that the resulting competitive and welfare effects depend on the availability of gig labor. Complementing this strategic perspective, Dai et al. (2022) examine how workers learn from different types of delivery experience and how this learning affects productivity and service quality. In social commerce, Qi et al. (2022) study how manufacturers choose between regular and top influencers and design compensation arrangements under asymmetric demand information. Within e-commerce marketplaces, Hagiu and Wright (2015) examine the trade-offs platforms face when choosing between operating as a marketplace and as a reseller, highlighting how this choice affects seller incentives. Zhu and Liu (2018) empirically show that Amazon's decisions to compete with third-party sellers depend on category-level demand potential. Research on Fulfillment by Amazon studies how platform-provided logistics services affect seller participation and competition (Lai et al. 2022, Li et al. 2025). Beyond platforms' own design choices, regulation of online retailers can produce effects opposite to those intended once firms respond strategically: Xu et al. (2025) show that data-privacy regulation such as the GDPR can paradoxically lower consumer surplus, illustrating that the welfare effects of well-intentioned regulation depend on the equilibrium responses it induces.

A growing strand of this literature focuses on discoverability, ranking, and prominence. Dinerstein et al. (2018) show that improved search and ranking can intensify price competition while steering consumers toward preferred products. Hagiu and Wright (2024) develop a theory of optimal discoverability, highlighting a central trade-off: increasing visibility attracts transactions but can erode seller participation if competition becomes too intense. Ranking rules also shape sellers' incentives to manipulate their standing: Jin et al. (2023) show that sales-based rankings can induce sellers to place fake orders, so that consumer welfare is non-monotonic in enforcement: stricter measures can shift distortion toward lower-quality sellers, and placing greater weight on sales volume need not improve consumer welfare. Related studies of prominence competition show that sellers

strategically adjust prices in response to algorithmic ordering (Ciotti and Madio 2022, Eisfeld 2023, Garcia et al. 2024). Empirical evidence indicates that algorithmic nudges and featured placements influence pricing and demand allocation (Dash et al. 2024, Raval 2022). What distinguishes visibility from other design levers is that it is allocated rather than posted: the platform decides which seller receives it and under what conditions.

Several empirical studies examine the Buy Box specifically. Hunold et al. (2022) document that Buy Box eligibility depends not only on prices posted on Amazon but also on prices offered on rival platforms, providing direct evidence of cross-platform price monitoring. Sikdar et al. (2022) also find that Buy Box selection penalizes high prices from both first- and third-party sellers. Other work identifies non-price determinants such as seller ratings and fulfillment choices (Chen et al. 2016, Gómez-Losada et al. 2022). Gallino et al. (2025) show that the Buy Box reduces search frictions but can increase market concentration, echoing the efficiency–competition trade-off identified in the broader search literature. Together, these studies establish that the Buy Box is a price-sensitive and economically consequential mechanism and that its eligibility rules can transmit pricing incentives across platform boundaries.

2.2. Price Parity and Most Favored Nation Clauses

A separate literature studies price parity and Most Favored Nation (MFN) clauses, which restrict sellers from offering lower prices on competing channels. Boik and Corts (2016) show that platform MFNs can raise platform fees and retail prices, while Johnson (2017) analyze parity clauses in agency settings and identify conditions under which they soften competition and increase prices. This work provides important benchmarks for understanding the competitive effects of explicit, contractually enforced parity.

Our framework differs in how parity is enforced. Rather than a contractual restriction, we study parity sustained through eligibility for a demand-enhancing placement. Under eligibility-based parity, sellers are not bound to comply: they can violate eligibility and forgo the placement whenever compliance cost exceeds its value. This distinction introduces an endogenous compliance margin absent from standard MFN models.

2.3. Our Contributions

Existing research thus explains how platforms influence seller conduct through access and service design, how algorithmic visibility reshapes competition, and how price parity clauses affect prices. What remains missing is an integrated framework that combines

these elements. Our paper provides such a framework by studying how a platform's visibility mechanism, when conditioned on cross-platform price parity, alters pricing incentives and welfare under different market structures. In doing so, we show how eligibility-linked parity can discipline prices, induce strategic opt-out, and generate welfare effects that differ sharply from those predicted by models of explicit parity or within platform competition alone.

3. Model

This section introduces the institutional background and modeling framework.

3.1. Institutional Background and Modeling Motivation

Amazon Marketplace is one of the world's largest e-commerce platforms and hosts a vast number of third-party sellers. A defining feature of Amazon's product pages is the Buy Box, which allows consumers to purchase a product directly from a selected seller without searching among alternative offers (Figure 1). Sellers listed in the Buy Box receive a substantial increase in visibility and sales, while sellers not listed experience sharply reduced demand. When no seller qualifies for Buy Box eligibility (Figure A1), consumers must actively search among competing sellers, increasing search costs and reducing conversion rates. Empirical evidence suggests that a large majority of consumers transact through the Buy Box rather than exploring alternative sellers.

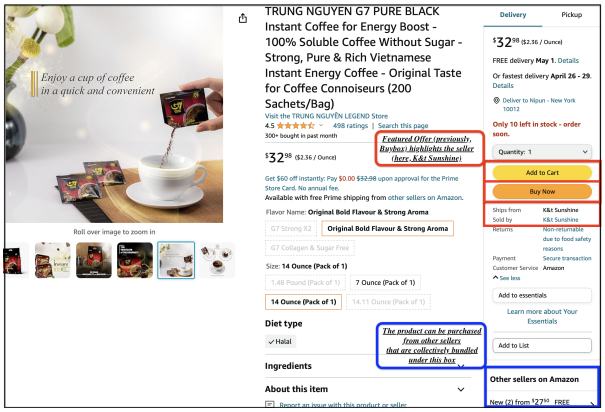


Figure 1 **Amazon's product page with Buy Box. The featured seller receives algorithmic placement with prominent "Add to Cart" and "Buy Now" buttons, capturing the majority of platform sales.**

Buy Box eligibility is determined by an algorithm that considers several factors, including price, seller reputation, and fulfillment characteristics. While the precise algorithm

is proprietary, one salient and widely documented feature is that Buy Box eligibility is sensitive to a seller's prices on competing platforms. In practice, Amazon monitors rival-platform prices and issues alerts when it detects lower prices elsewhere, warning sellers that their Buy Box eligibility may be at risk. We abstract from the details of the Buy Box algorithm and model its effect parsimoniously as a demand boost that is conditional on relative prices across platforms.

To capture this environment, we consider competition between two platforms. Platform 1 (subsequently referred to as the dominant platform) features the Buy Box and enforces implicit price parity through eligibility conditions. Platform 2 represents a rival marketplace that does not offer a Buy Box and does not impose parity. This asymmetric setup reflects a common industry configuration in which Amazon combines enhanced visibility with implicit price parity, while rival platforms compete primarily through prices and baseline demand.[2]

We study two structures that differ in the seller market. In the single seller model, one seller sets prices on both platforms and internalizes cross-platform pricing trade-offs. In the two-seller model, each seller controls only its own price, and parity operates through strategic interaction.

Both single-seller and two-seller market structures are relevant for studying parity-linked Buy Box eligibility. When determining Featured Offer eligibility, Amazon defines the competitive price as the lowest price recently found at another reputable retailer for the item, or for an item customers may regard as equivalent, regardless of whether it is sold by the same seller on Amazon.[3]

3.2. Single Seller

In the single seller setting, a seller can list an identical product on two platforms. The seller chooses prices to maximize total profit and may optimally single-home (sell on only one platform) or multi-home (sell on both). The seller's marginal cost is normalized to zero, so the seller's profit equals its (sales) revenue.

Timing. The seller simultaneously chooses whether to single-home or multi-home and sets the corresponding price(s) (p_1, p_2). When multi-homing, Buy Box eligibility requires that the seller's price on Platform 1 not exceed its price on Platform 2, i.e., $p_1 \leq p_2$. Since eligibility depends on relative pricing, the seller's optimization problem is piecewise.

[2] In our framework, Buy Box–only environments can be captured through an exogenous demand expansion.

[3] For details, see https://sell.amazon.com/blog/buy-box-featured-offer

Multi-homing demand. When the seller operates on both platforms, demand is given by

$$\begin{aligned} q_1^{BP} &= \alpha_1 - p_1 + \beta p_2 + \theta \cdot \mathbf{1}\{p_1 \leq p_2\}, \\ q_2^{BP} &= \alpha_2 - p_2 + \beta p_1. \end{aligned} \tag{1}$$

Single-homing demand. If the seller chooses to operate only on Platform 1 (so that $q_2^{BP} = 0$), the effective demand on Platform 1 is

$$q_1^{BP} = \alpha_1 + \theta + \beta\alpha_2 - (1 - \beta^2)p_1. \tag{2}$$

Similarly, if the seller operates only on Platform 2 (so that $q_1^{BP} = 0$), the demand on Platform 2 is

$$q_2^{BP} = \alpha_2 + \beta\alpha_1 - (1 - \beta^2)p_2. \tag{3}$$

The parameters $\alpha_i > 0$ capture baseline demand and $\beta \in [0, 1)$ measures substitutability across platforms. The demand boost $\theta > 0$ captures the expected demand gain from Buy Box eligibility, a status exclusive to Platform 1, and θ is relevant only when $q_1^{BP} > 0$. The superscript BP denotes this Buy Box regime with implicit parity (Model BP). The linear demand systems above are consistent with the quadratic utility framework of Singh and Vives (1984) and Zanchettin (2006); the full micro-foundation is in Online Appendix B.

The seller chooses prices and platform participation to maximize total profit, summed across the two platforms,

$$\pi_I^{BP} = p_1 q_1^{BP} + p_2 q_2^{BP}.$$

To isolate the effect of price parity, we consider a Buy Box regime without price parity (denoted by superscript B), where the seller receives an unconditional demand boost.

$$\begin{aligned} q_{1,I}^{B} &= \alpha_1 + \theta - p_{1,I}^{B} + \beta p_{2,I}^{B}, \\ q_{2,I}^{B} &= \alpha_2 - p_{2,I}^{B} + \beta p_{1,I}^{B}. \end{aligned} \tag{4}$$

3.3. Two Sellers

The two-seller setting retains the same demand structure and Buy Box rules but replaces the single seller with two competing sellers. Seller 1 operates exclusively on Platform 1 and Seller 2 operates exclusively on Platform 2, so neither faces a participation decision. Sellers offer the same product but compete through Bertrand pricing across differentiated platforms. As in the single-seller model, each seller's marginal cost is normalized to zero, so that their profits equal their (sales) revenues. Here, however, each seller earns profit Π_k on its own platform, unlike the single seller, whose profit π_I is earned across both platforms.

Timing. The two sellers simultaneously set prices $p_{1,II}^{BP}$ and $p_{2,II}^{BP}$ to maximize their individual profits. Buy Box eligibility requires $p_{1,II}^{BP} \leq p_{2,II}^{BP}$, making Seller 1's optimization problem piecewise.

Demand. Under Buy Box with implicit price parity, the demands are as follows:

$$\begin{aligned} q_{1,II}^{BP} &= \alpha_1 - p_{1,II}^{BP} + \beta p_{2,II}^{BP} + \theta \cdot \mathbf{1}\{p_{1,II}^{BP} \leq p_{2,II}^{BP}\}, \\ q_{2,II}^{BP} &= \alpha_2 - p_{2,II}^{BP} + \beta p_{1,II}^{BP}. \end{aligned} \tag{5}$$

As in the single seller case, Buy Box without parity (Model B) serves as a benchmark:

$$\begin{aligned} q_{1,II}^{B} &= \alpha_1 + \theta - p_{1,II}^{B} + \beta p_{2,II}^{B}, \\ q_{2,II}^{B} &= \alpha_2 - p_{2,II}^{B} + \beta p_{1,II}^{B}. \end{aligned} \tag{6}$$

We use consumer surplus as our measure of consumer welfare. Section 4.5 calculates consumer surplus using the quadratic utility representation underlying the demand system. Platform i's commission revenue is proportional to seller revenue $p_i q_i$; we therefore use $p_i q_i$ to compare platform revenue across Models B and BP. Table 1 summarizes the notations used throughout the analysis.

Variables	Description
α_i	Baseline demand on Platform $i \in \{1,2\}$
$\beta \in [0,1)$	Cross-platform substitutability
θ	Demand boost from Buy Box eligibility
$q_{i,k}^{j}$	Demand on Platform i under Model $j \in \{BP, B\}$ with $k \in \{I, II\}$ sellers
$p_{i,k}^{j}$	Price on Platform i under Model $j \in \{BP, B\}$ with $k \in \{I, II\}$ sellers
π_I^j	Single seller's total profit across both platforms under Model $j \in \{BP, B\}$
$\Pi_{i,k}^{j}$	Seller profit on Platform i under Model $j \in \{BP, B\}$ with $k \in \{I, II\}$ sellers
$R_{i,k}^{j}$	Platform i's commission revenue under Model $j \in \{BP, B\}$ with $k \in \{I, II\}$ sellers
CS_k^j	Consumer surplus under Model $j \in \{BP, B\}$ with $k \in \{I, II\}$ sellers
TS_k^j	Total surplus under Model $j \in \{BP, B\}$ with $k \in \{I, II\}$ sellers

Table 1 Key Notations

Our analysis will focus on the empirically relevant case $\theta > 0$, $\beta \in (0,1)$, and $0 < \alpha_2 < \alpha_1 + \theta$: a positive Buy Box boost, genuine cross-platform substitution, and a rival platform whose demand does not exceed the boosted demand of the dominant platform. When

$\alpha_2 \geq \alpha_1 + \theta$, the eligibility condition is slack and Models B and BP coincide, so that region is of no relevant interest.

4. Analysis: Single Seller

We now characterize the single seller's optimal pricing and platform participation decisions, i.e., whether to single-home or multi-home and how to set prices accordingly. We start with a benchmark (Model B) in which the Buy Box demand boost is received unconditionally, and then analyze the main model (Model BP), where the boost is contingent on cross-platform price parity. Because Buy Box access under BP is conditional on relative prices, a multi-homing seller faces a fundamental trade-off: preserving eligibility requires cross-platform price consistency, while forgoing eligibility restores pricing flexibility at the cost of losing the Buy Box demand advantage. Platform participation itself is also endogenous — concentrating on Platform 1 alone resolves the trade-off entirely by making the eligibility condition vacuous.

4.1. Benchmark: Buy Box without Parity (Model B)

Under Model B, the seller receives the demand boost θ unconditionally and simultaneously chooses prices $p^B_{1,I}$ and $p^B_{2,I}$ to maximize total profit across both platforms:

$$\pi^B_I \;=\; p^B_{1,I}\, q^B_{1,I} \;+\; p^B_{2,I}\, q^B_{2,I}, \tag{7}$$

where $q^B_{1,I}$ and $q^B_{2,I}$ are given by (4). Because θ enters demand on Platform 1 unconditionally, the seller faces an effective demand intercept of $\alpha_1 + \theta$ on that platform; the seller's problem is equivalent to a standard two-product monopoly. The first-order conditions yield a unique interior solution, characterized in the following proposition.

PROPOSITION 1 **(Model B: Equilibrium Structure)**. *Under Model B, the seller* ***multi-homes*** *and the unique equilibrium prices are*

$$\left(p^{B*}_{1,I}, p^{B*}_{2,I}\right) = \left(\frac{\alpha_1 + \theta + \beta\alpha_2}{2(1-\beta^2)},\; \frac{\alpha_2 + \beta\alpha_1 + \beta\theta}{2(1-\beta^2)}\right). \tag{8}$$

Notably, the seller multi-homes for every $\alpha_2 > 0$: the incremental profit from listing on Platform 2 is $\alpha_2^2/4 > 0$ regardless of the boost. Under the unconditional Buy Box benchmark, therefore, the rival platform is never foreclosed — a property that fails once eligibility is conditioned on relative prices.

Since θ accrues to the seller without any price constraint, it functions as a pure upward shift in the effective demand intercept on Platform 1. Accordingly, $p_{1,I}^{B*}$ responds to θ exactly as it responds to an equal increase in α_1, at rate $1/[2(1-\beta^2)]$. The price $p_{2,I}^{B*}$ inherits the boost indirectly: since the seller jointly maximizes profit across both platforms, the higher Platform 1 price raises the optimal Platform 2 price through the cross-price demand effect, at β times that rate.

Model B therefore isolates the pure effect of the demand boost, absent any eligibility requirement. Model BP adds the eligibility condition, so any difference between the two models is attributable to parity alone. Section 4.3 measures these differences.

4.2. Buy Box with Parity (Model BP)

The defining feature of BP is that access to the Buy Box, and the associated demand boost, is conditional on the seller setting a *competitive price on Platform 1*, i.e., satisfying $p_1 \leq p_2$. Because Buy Box eligibility depends endogenously on the seller's pricing choices, parity affects incentives even when it does not bind prices in equilibrium—as in the single-homing regime, where the seller avoids Platform 2 entirely, making the condition vacuously satisfied. Understanding how this conditionality shapes equilibrium behavior is therefore central to the analysis.

We refer to *opt-out* as the limiting outcome in which the seller forgoes Buy Box eligibility entirely, receiving no demand boost (Figure A1). This emerges endogenously under BP whenever the cost of satisfying price parity exceeds the benefit of eligibility.

Under Model BP, the eligibility requirement introduces a fundamental tension: the seller can earn the demand boost θ only by satisfying the parity constraint, yet doing so may require costly price distortions. As we show below, the resolution of this trade-off depends critically on the baseline demand of the rival platform, α_2. The seller's problem can be viewed as two nested decisions: first, a *participation decision* — whether to single-home on one platform or multi-home across both platforms; and second, conditional on multi-homing, a *compliance decision* — whether to satisfy the eligibility condition $p_1 \leq p_2$ and retain the Buy Box demand advantage, or to forfeit it entirely.

Intuitively, when α_2 is sufficiently low, Platform 2 generates negligible profit and the seller optimally concentrates on Platform 1, retaining the Buy Box demand advantage without any compliance cost. As α_2 rises and Platform 2 becomes a meaningful revenue source, the seller lists on both platforms and faces the compliance decision directly. In the

seller's unconstrained optimum, the higher effective demand on Platform 1 — reflecting the demand asymmetry so long as $\alpha_2 < \alpha_1 + \theta$ — leads the seller to charge a higher price on Platform 1 than on Platform 2. This cross-platform price difference is precisely what the eligibility condition $p_1 \leq p_2$ prohibits: satisfying it requires the seller to compress its prices toward a uniform price (as established in the equilibrium characterization below), forgoing the profit from this price difference.

How the seller resolves this trade-off depends on α_2, generating three multi-homing regimes. For low-to-intermediate α_2, the seller forfeits eligibility: the *eligibility-violated* regime, also referred to as *opt-out*. For higher α_2, the seller complies and sets identical prices across both platforms: the *eligibility-binding* regime. When α_2 is large enough that the unconstrained optimum already satisfies $p_1 \leq p_2$, the condition imposes no distortion: the *eligibility-slack* regime. These regimes are formally stated in the following proposition.

PROPOSITION 2 **(Model BP: Equilibrium Regimes)**. *The seller's unique equilibrium strategy falls into one of the following four regimes, depending on the rival-platform demand α_2:*

*i) When $\alpha_2 \in [0, \tilde{\alpha})$, the seller **single-homes** on Platform 1, with $q_{2,I}^{BP*} = 0$ and*

$$p_{1,I}^{BP*} = \frac{\alpha_1 + \beta\alpha_2 + \theta}{2(1-\beta^2)}. \tag{9}$$

*ii) When $\alpha_2 \geq \tilde{\alpha}$, the seller **multi-homes**, with strictly positive demand on both platforms:*

*ii-a) The **eligibility-violated** regime arises when $\alpha_2 \in [\tilde{\alpha}, \max\{\tilde{\alpha}, \bar{\alpha}\})$;*

*ii-b) The **eligibility-binding** regime arises when $\alpha_2 \in [\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)$;*

*ii-c) The **eligibility-slack** regime arises when $\alpha_2 \in [\alpha_1 + \theta, \infty)$.*

$\tilde{\alpha}$ and $\bar{\alpha}$ denote the single-homing and compliance thresholds, respectively. The eligibility-violated regime is non-empty if and only if $\tilde{\alpha} < \bar{\alpha}$. Equilibrium prices and demands are given in Table 2.

The single-homing threshold $\tilde{\alpha}$ is defined as the point of indifference at which the seller's profit from single-homing on Platform 1 equals the profit from operating on both platforms. When α_2 is below $\tilde{\alpha}$, the incremental profit from listing on Platform 2 is insufficient to offset the value of the Buy Box demand advantage on Platform 1 at zero compliance cost. The seller therefore concentrates entirely on Platform 1. Notably, α_2 still enters the optimal price in (9) through the cross-platform substitutability term $\beta\alpha_2$, reflecting the underlying consumer utility framework that ties the two platforms' demand systems together even when Platform 2 attracts no purchases in equilibrium. The price in (9) is

strictly increasing in θ: since the eligibility constraint is vacuous when the seller is absent from Platform 2, the full boost θ is captured without any compliance cost.

The compliance threshold $\bar{\alpha}$ is defined as the point of indifference at which the seller's profit from complying with the eligibility condition equals the profit from forfeiting the Buy Box boost. The three multi-homing regimes reflect distinct resolutions of the compliance trade-off, each arising for a different range of Platform 2's baseline demand.

In the eligibility-violated regime $\big(\alpha_2 \in [\tilde{\alpha}, \max\{\tilde{\alpha}, \bar{\alpha}\}]\big)$, Platform 2's baseline demand α_2 is large enough to make multi-homing profitable, but the profit loss from distorting prices to satisfy $p_1 \leq p_2$ exceeds the value of retaining Buy Box placement. The seller therefore optimally forfeits Buy Box eligibility. Consequently, the equilibrium prices are the standard two-product monopoly prices at intercepts (α_1, α_2), and the eligibility condition is violated in this regime.

In the eligibility-binding regime $\big(\alpha_2 \in [\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)\big)$, the demand boost θ becomes sufficiently valuable relative to the distortion cost that the seller finds compliance worthwhile. Because the seller's unconstrained optimum has $p_1 > p_2$ throughout this regime, the eligibility condition is actively binding at the optimum. Hence, the seller sets identical prices across both platforms. Parity thus compresses prices by raising Platform 2's price and suppressing Platform 1's price simultaneously, relative to Model B.

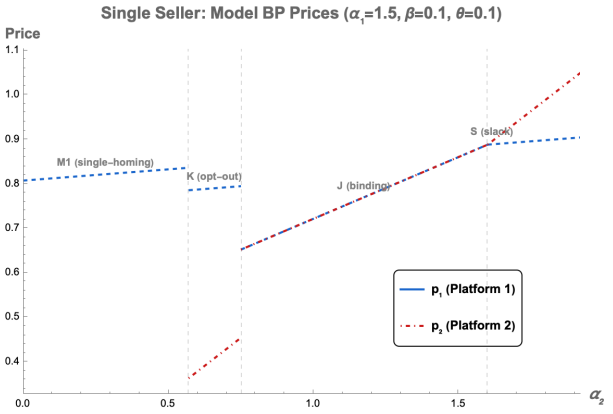
In the eligibility-slack regime $\big(\alpha_2 \in [\alpha_1 + \theta, \infty)\big)$, Platform 2's baseline demand is sufficiently strong that the seller's unconstrained optimum naturally satisfies $p_{1,I}^{BP} \leq p_{2,I}^{BP}$ without any distortion. The equilibrium prices are therefore identical to the benchmark prices $p_{1,I}^{B*}$ and $p_{2,I}^{B*}$ from Proposition 1: in this regime, the parity constraint is costless and the two models are observationally equivalent.

Taken together, these regimes illustrate how eligibility-linked price parity and the associated demand boost reshape pricing incentives. When the rival platform's market size is small, the seller single-homes, foreclosing the rival platform entirely while retaining Buy Box eligibility at no compliance cost. When it is moderately large, the seller multi-homes, either by forfeiting Buy Box eligibility or by distorting prices to satisfy the parity condition. When it is large, the seller's original price gap satisfies the eligibility condition automatically.

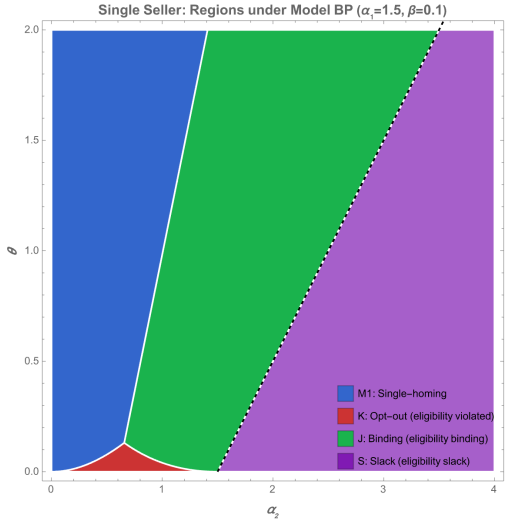
PROPOSITION 3 **(Single-Homing Comparative Statics)**. *The seller **single-homes** on Platform 1 if and only if $\alpha_2 < \tilde{\alpha}$, and **multi-homes** otherwise. $\tilde{\alpha}$ satisfies the following properties:*

	Single-Homing	Multi-Homing		
	(Platform 1 only)	**Eligibility-violated** *(Opt-out)*	**Eligibility-binding** *(Binding)*	**Eligibility-slack** *(Slack)*
Price on Platform 1	$\frac{\alpha_1+\theta+\beta\alpha_2}{2(1-\beta^2)}$	$\frac{\alpha_1+\beta\alpha_2}{2(1-\beta^2)}$	$\frac{\alpha_1+\theta+\alpha_2}{4(1-\beta)}$	$\frac{\alpha_1+\theta+\beta\alpha_2}{2(1-\beta^2)}$
Price on Platform 2	—	$\frac{\beta\alpha_1+\alpha_2}{2(1-\beta^2)}$	$\frac{\alpha_1+\theta+\alpha_2}{4(1-\beta)}$	$\frac{\beta(\alpha_1+\theta)+\alpha_2}{2(1-\beta^2)}$
Demand on Platform 1	$\frac{\alpha_1+\theta+\beta\alpha_2}{2}$	$\frac{\alpha_1}{2}$	$\frac{3(\alpha_1+\theta)-\alpha_2}{4}$	$\frac{\alpha_1+\theta}{2}$
Demand on Platform 2	0	$\frac{\alpha_2}{2}$	$\frac{3\alpha_2-(\alpha_1+\theta)}{4}$	$\frac{\alpha_2}{2}$
Buy Box eligibility status	Satisfied	Not satisfied	Satisfied under binding price parity	Satisfied without price adjustment

Table 2 Equilibrium outcomes under Buy Box with parity (BP) in the single-seller setting



(a) Equilibrium prices across regimes. Note the binding equality $p_1 = p_2$ in regime J.



(b) Equilibrium regimes in the (α_2, θ) parameter space.

Figure 2 Equilibrium characterization under Model BP with a single seller ($\alpha_1 = 1.5$, $\beta = 0.1$, $\theta = 0.1$).

i) ***(Buy Box Boost)*** *A larger demand boost expands the single-homing regime:* $\dfrac{\partial \tilde{\alpha}}{\partial \theta} > 0$.

ii) ***(Platform 1 Base Demand)*** *A larger Platform 1's base demand expands the single-homing regime:* $\dfrac{\partial \tilde{\alpha}}{\partial \alpha_1} > 0$.

iii) ***(Cross-Platform Substitutability)*** *The effect of β is regime-dependent:*

$$\frac{\partial \tilde{\alpha}}{\partial \beta} \begin{cases} > 0, & \text{if } \tilde{\alpha} < \bar{\alpha}, \\ < 0, & \text{if } \tilde{\alpha} \geq \bar{\alpha}. \end{cases}$$

Proposition 3 characterizes how the single-homing threshold responds to key parameters of the model. The comparative static in part (1) reflects the opportunity cost of cross-platform entry: a larger demand boost θ raises the value of retaining Buy Box eligibility on Platform 1 alone, making single-homing relatively more attractive and therefore requiring a stronger rival platform to justify entry. The logic in part (2) is analogous — a larger

α_1 raises the profit from single-homing on Platform 1, raising the bar that cross-platform profits must clear to induce entry. Together, parts (1) and (2) imply that eligibility-based parity endogenously forecloses the rival platform over a wider parameter range when either the Buy Box boost or the dominant platform's baseline demand is large. Under Model B the seller multi-homes for every $\alpha_2 > 0$, so this foreclosure is induced by the eligibility rule rather than by the demand asymmetry alone — a finding with direct relevance to the regulatory concerns about platform foreclosure.

Part (3) is the most subtle result. The effect of cross-platform substitutability β on the single-homing threshold reverses sign depending on whether the eligibility-violated regime is non-empty. When $\tilde{\alpha} < \bar{\alpha}$, the marginal comparison at $\tilde{\alpha}$ is between single-homing and opt-out. The incremental profit from listing on Platform 2 under opt-out is exactly $\alpha_2^2/4$, independent of β; what higher substitutability changes is the opportunity cost of multi-homing, because the boost retained under single-homing is worth $\theta(2\alpha_1 + 2\beta\alpha_2 + \theta)/[4(1-\beta^2)]$, which is increasing in β. The single-homing regime therefore expands. When $\tilde{\alpha} \geq \bar{\alpha}$, the comparison is instead between single-homing and multi-homing with uniform pricing: higher substitutability raises the value of uniform pricing faster than the gains from demand consolidation on Platform 1, so the single-homing regime shrinks.

In the following subsections, we compare these equilibrium outcomes to those of Model B, characterizing the price parity's implications for prices, demand, profitability, and welfare.

4.3. Parity's Effects on Prices and Demands

This subsection examines how implicit price parity affects equilibrium prices and demands by comparing Model BP with Model B. A common concern about parity-based Buy Box policies is that they raise prices on rival platforms. However, we show that this is not always the case. The following proposition formalizes this comparison.

PROPOSITION 4 **(Price Effects: Single Seller)**. *In the single-seller setting, the following price comparisons hold between Model BP and Model B (the benchmark):*

i) ***(Platform 1)*** *Under single-homing, $p_{1,I}^{BP*} = p_{1,I}^{B*}$. Under multi-homing, $p_{1,I}^{BP*} < p_{1,I}^{B*}$.*

ii) ***(Platform 2)*** *Under multi-homing, $p_{2,I}^{BP*} < p_{2,I}^{B*}$ in the eligibility-violated regime, and $p_{2,I}^{BP*} > p_{2,I}^{B*}$ in the eligibility-binding regime.*

iii) ***(Price Compression)*** *Under multi-homing, the price movements on the two platforms in the eligibility-binding regime are equal and opposite:*

$$p_{1,I}^{B*} - p_{1,I}^{BP*} = p_{2,I}^{BP*} - p_{2,I}^{B*} = \delta \equiv \frac{\alpha_1 + \theta - \alpha_2}{4(1+\beta)} > 0.$$

When the seller single-homes, Platform 1's equilibrium price is identical across the two models. This is because the seller captures the full boost θ in both cases — unconditionally under Model B and without any compliance cost under Model BP because the parity rule is vacuous when the seller does not operate on Platform 2.

In the multi-homing eligibility-violated regime, where $\alpha_2 \in [\tilde{\alpha}, \max\{\tilde{\alpha}, \bar{\alpha}\})$, the profit loss from complying with $p_{1,I}^{BP*} \leq p_{2,I}^{BP*}$ exceeds the value of the boost θ. The seller therefore optimally forfeits the Buy Box demand advantage, pricing as if $\theta = 0$. The economic mechanism is straightforward: under Model B, the boost θ expands effective demand on Platform 1, which raises both prices through the cross-price demand linkage — directly on Platform 1 and indirectly on Platform 2 via substitutability β. Forfeiting θ removes this demand support entirely, lowering prices on *both* platforms simultaneously: $p_{1,I}^{BP*} < p_{1,I}^{B*}$ and $p_{2,I}^{BP*} < p_{2,I}^{B*}$. The price reduction is therefore not a distortion imposed by the parity constraint but a consequence of the seller's optimal decision to operate without the demand boost.

In the multi-homing eligibility-binding regime, where $\alpha_2 \in [\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)$, the boost θ is sufficiently valuable that the seller satisfies the eligibility condition, with identical prices across both platforms. This uniform pricing eliminates the cross-platform price differences present in Model B, compressing prices in opposite directions relative to the benchmark: $p_{1,I}^{BP*} < p_{1,I}^{B*}$ on Platform 1 and $p_{2,I}^{BP*} > p_{2,I}^{B*}$ on Platform 2. The two movements are of equal magnitude, $\delta = (\alpha_1 + \theta - \alpha_2)/[4(1+\beta)]$, since the common price is the midpoint of the two benchmark prices; δ is strictly decreasing in α_2, so the compliance distortion weakens as the rival platform strengthens.

REMARK 1 (PLATFORM 2 PRICE IN THE SINGLE-HOMING REGIME). In the single-homing regime, any $p_2 \geq \alpha_2 + \beta p_{1,I}^{BP*}$ sustains $q_{2,I}^{BP*} = 0$, and the seller is indifferent among them. Selecting one with $p_{2,I}^{BP} \geq p_{1,I}^{BP*}$ is therefore costless and secures eligibility, so the boost is retained without any compliance cost.

To summarize, Proposition 4 establishes that the parity constraint never raises the Platform 1 price, and that its effect on Platform 2's price is regime-dependent: as α_2 increases

from $\tilde{\alpha}$ toward $\alpha_1 + \theta$, the constraint first suppresses the Platform 2 price below the benchmark in the eligibility-violated regime and then inflates it above the benchmark in the eligibility-binding regime. Three mechanisms generate this pattern: (i) a participation mechanism that sustains single-homing when α_2 is small, (ii) an opt-out mechanism in which the seller forgoes the Buy Box demand boost to avoid compliance costs, and (iii) a compliance mechanism that eliminates cross-platform price differences when the boost is sufficiently valuable. These mechanisms operate over distinct ranges of α_2.

We now examine how price parity reshapes equilibrium demand across platforms. Unlike the price comparison, where Platform 1's price is weakly lower under Model BP in every regime, the demand effects do not follow the price effects monotonically and depend critically on whether the seller retains Buy Box eligibility.

COROLLARY 1 **(Demand Effects: Single Seller)**. *In the single-seller setting, the following demand comparisons hold between Model BP and Model B (the benchmark):*

i) ***(Platform 1)*** *Under single-homing, $q_{1,I}^{BP*} > q_{1,I}^{B*}$. Under multi-homing, $q_{1,I}^{BP*} < q_{1,I}^{B*}$ in the eligibility-violated regime, and $q_{1,I}^{BP*} > q_{1,I}^{B*}$ in the eligibility-binding regime.*

ii) ***(Platform 2)*** *Under single-homing, $q_{2,I}^{BP*} < q_{2,I}^{B*}$. Under multi-homing, $q_{2,I}^{BP*} = q_{2,I}^{B*}$ in the eligibility-violated regime, and $q_{2,I}^{BP*} < q_{2,I}^{B*}$ in the eligibility-binding regime.*

Under Model B, whenever $\alpha_1 + \theta > \alpha_2$, the seller exploits the demand asymmetry across platforms (the effective demand intercept is higher on Platform 1) by charging more on Platform 1 and less on the rival platform. The eligibility requirement removes this cross-platform price gap, and whether the result is a concentration of demand on Platform 1 depends on which regime prevails.

In the single-homing regime, Platform 1's demand is strictly higher under Model BP despite prices being equal across the two models. The source of this demand gain is demand shifting, which is reflected directly in the demand function, where the term $\beta\alpha_2$ captures the additional demand redirected to Platform 1 when Platform 2 has zero demand in equilibrium due to the seller's decision to single home.

In the multi-homing eligibility-violated regime, the mechanism produces the opposite outcome on Platform 1. The seller optimally forfeits Buy Box eligibility, and θ drops out of equilibrium entirely. Although both prices are lower relative to Model B, the loss of the demand boost dominates the price effect, and Platform 1 demand falls below the benchmark.

In the multi-homing eligibility-binding regime, the demand gain on Platform 1 operates through price differences alone. The Buy Box boost is not a differential channel here, since both models retain it. Both price movements push demand the same way: the lower Platform 1 price raises its own demand, and the higher Platform 2 price diverts demand toward Platform 1. Platform 1 demand therefore rises above the benchmark. The offsetting change on Platform 2 is exactly equal in magnitude, $q_{2,I}^{BP*} - q_{2,I}^{B*} = -(\alpha_1 + \theta - \alpha_2)/4$, so compliance leaves aggregate consumption unchanged and operates purely as a shift of demand across platforms. This property drives the consumer surplus comparison in Section 4.5.

Platform 2's demand comparison reveals a consistent pattern across regimes — Platform 2's demand is weakly lower under Model BP than under Model B in all regimes, and exactly equal only in the eligibility-violated regime.

We examine the implications for seller and platforms profits in the next subsection.

4.4. Parity's Effects on the Profitability of the Seller and the Platforms

The following proposition establishes that the eligibility-linked price parity requirement unambiguously harms the seller, regardless of the equilibrium regime.

PROPOSITION 5 **(Effects on the Seller's Profit: Single Seller)**. *In the single-seller setting, the seller's total profit is strictly lower under Model BP than under Model B (the benchmark):* $\pi_I^{BP*} < \pi_I^{B*}$.

The result follows from a simple feasibility argument: conditioning the demand boost on a pricing requirement cannot expand the seller's feasible payoff set relative to receiving it unconditionally. The result holds across all equilirium regimes.

In the multi-homing eligibility-binding regime, the seller successfully retains θ yet total profit remains strictly below the benchmark. The reason is that satisfying the eligibility condition requires the seller to abandon its preferred platform-specific pricing, compressing prices toward a uniform level by suppressing the Platform 1 price and inflating the Platform 2 price. The foregone profit from platform-specific pricing is the compliance cost: it equals $(\alpha_1 + \theta - \alpha_2)^2/[8(1+\beta)] = 2(1+\beta)\delta^2$, quadratic in the price distortion δ and vanishing only as $\alpha_2 \to \alpha_1 + \theta$. It is strictly positive even when the eligibility condition is met successfully, since Model B delivers the same boost θ without any such restriction.

We now examine price parity's effects on the platforms. Notice that platform commission revenue is proportional to $p_{i,I}^{m*} q_{i,I}^{m*}$, therefore we use them directly to compare platform commission revenue across Models B and BP, denoted by $R_{i,I}^{m}$, where $i \in \{1,2\}$ and $m \in \{B, BP\}$. Because eligibility influences both price and demand on Platform 1, revenue effects arise through two channels: a *price effect* and a *demand effect*. The two channels may reinforce or oppose one another, depending on how the seller responds to the eligibility condition. Inducing compliance therefore does not automatically imply higher commission revenue for Platform 1, as we characterize next.

PROPOSITION 6 **(Effects on Platform 1's Commission Revenue: Single Seller)**. *Let* $z \equiv \frac{1-3\beta}{1+\beta}(\alpha_1 + \theta)$. *In the single-seller setting, the following comparisons of Platform 1's commission revenue hold between Model BP and Model B (the benchmark):*

i) Under single-homing, $R_{1,I}^{BP} > R_{1,I}^{B}$.

ii) Under multi-homing, in the eligibility-violated regime, $R_{1,I}^{BP} < R_{1,I}^{B}$.

iii) Under multi-homing, in the eligibility-binding regime, $R_{1,I}^{BP} < R_{1,I}^{B}$ *for* $\alpha_2 < z$, *and* $R_{1,I}^{BP} \geq R_{1,I}^{B}$ *for* $\alpha_2 \geq z$.

Under single-homing, prices on Platform 1 coincide across models, so no price suppression arises. Demand is strictly higher under Model BP due to demand shift from Platform 2 (captured by the $\beta\alpha_2$ term in the demand function), and the quantity effect operates without any offsetting price distortion. Platform 1 revenue therefore rises.

Under multi-homing, in the eligibility-violated regime, the seller forfeits the Buy Box demand advantage and sets prices as if $\theta = 0$. Both price and demand on Platform 1 fall strictly below the benchmark, so the two channels reinforce each other and Platform 1 revenue falls.

Under multi-homing, in the eligibility-binding regime, the revenue comparison is driven entirely by the price compression that compliance imposes, since both models retain the Platform 1 demand boost θ. This compression works in two opposing directions: the lower Platform 1 price reduces revenue per unit, while the lower p_1 and higher p_2 shifts demand toward Platform 1, raising its quantity. The threshold z is where these two effects exactly offset. For $\alpha_2 < z$ the price effect dominates and Platform 1's revenue falls below the benchmark; for $\alpha_2 \geq z$ the quantity effect dominates and revenue rises above it. When cross-platform substitutability is high ($\beta \geq 1/3$), $z \leq 0$ lies below the entire

regime, so the quantity effect dominates throughout and revenue stays at or above the benchmark.

Proposition 6 underscores that the implicit price-parity rule can reduce, rather than increase, Platform 1's revenue. Building on the three channels identified earlier, the participation channel raises revenue under single-homing, whereas the opt-out channel lowers revenue when the seller forfeits eligibility. The compliance channel has an ambiguous effect: the required reduction in Platform 1's price lowers revenue per unit but increases demand. Revenue rises when the demand gain outweighs the lower price and falls otherwise.

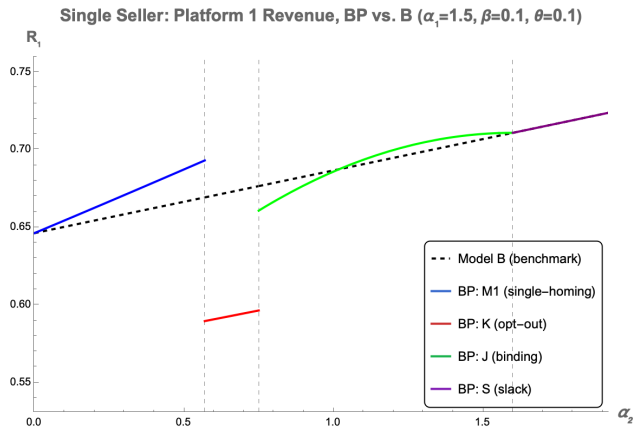


Figure 3 **Platform 1 revenue ($R_{1,I}$) under Model BP versus Model B (dashed). BP revenue exceeds the benchmark in the single-homing regime, falls below it in the eligibility-violated regime, rises above it again in the binding regime, and converges to the benchmark under slack eligibility.**

Platform 2's revenue, by contrast, is weakly lower under Model BP in every regime (see Online Appendix C.1). Because seller profit under Model BP is uniformly lower than under Model B, any welfare gains from parity must arise through consumer surplus rather than seller rents.

4.5. Parity's Effects on Welfare

Consumer surplus (welfare) is calculated using the quadratic utility representation underlying the demand system, which aggregates surplus across both platforms. Writing $S = q_{1,I}^{m*} + q_{2,I}^{m*}$ for aggregate consumption and $d = q_{1,I}^{m*} - q_{2,I}^{m*}$ for the quantity gap between platforms,

$$CS^m = \frac{\left(q_{1,I}^{m*}\right)^2 + \left(q_{2,I}^{m*}\right)^2 + 2\beta\, q_{1,I}^{m*}\, q_{2,I}^{m*}}{2} = \frac{(1+\beta)S^2 + (1-\beta)d^2}{4}, \qquad m \in \{B, BP\}. \quad (10)$$

Consumer surplus therefore rises with aggregate consumption and, at fixed aggregate consumption, with the concentration of demand on the higher-demand platform. In the single-homing regime the two coincide because $q_{2,I}^{BP*} = 0$ leads to $S = d = q_{1,I}^{BP*}$.

PROPOSITION 7 **(Consumer Surplus Effects: Single Seller)**. *In the single-seller setting, consumer surplus is strictly greater under Model BP than under Model B (the benchmark) if and only if the multi-homing eligibility-binding regime arises.*

In the single-homing regime, the seller concentrates entirely on Platform 1 under Model BP, so $q_{2,I}^{BP*} = 0$: aggregate consumption S falls, while the quantity gap d widens. By the decomposition above, the two move consumer surplus in opposite directions, but the decrease in S dominates the increase in d. Hence, the consumer surplus decreases. In the eligibility-violated regime, Platform 1 demand is strictly lower under Model BP while Platform 2 demand is unchanged. Since CS^m is strictly increasing in each quantity, the consumer surplus falls unambiguously relative to the benchmark.

The multi-homing eligibility-binding regime delivers the central consumer surplus result. Compliance induces a symmetric price redistribution of magnitude $\delta > 0$, lowering Platform 1's price and raising Platform 2's price by equal amounts. Because the demand system is symmetric in the price coefficients, these equal and opposite price shifts generate equal and opposite quantity shifts, concentrating consumption on Platform 1 while leaving aggregate quantity unchanged. In 10, S is unchanged while d exactly doubles, from $(\alpha_1 + \theta - \alpha_2)/2$ under Model B to $\alpha_1 + \theta - \alpha_2$ under Model BP, yielding $CS^{BP} > CS^B$. The gain thus comes entirely from the concentration of consumption on Platform 1 rather than from any expansion in consumption or from the boost θ (since both models retain it in this regime).

Taken together, Proposition 7 shows that the welfare effect of the parity constraint for consumers is non-monotone in α_2. Consumers are worse off when the seller single-homes or forfeits eligibility, but strictly better off when the eligibility condition binds and the seller absorbs the associated distortion. This generates a sharp tension between seller surplus and consumer surplus in the eligibility-binding regime: it is the only regime in which the parity constraint harms the seller while benefiting consumers.

These results establish the baseline: in the single-seller setting, the welfare effects of parity depend entirely on whether the eligibility condition binds. Total surplus comparisons are provided in Online Appendix C.1. Section 5 examines how strategic interaction between competing sellers changes this calculus.

5. Analysis: Two Sellers

Section 4 analyzed implicit price parity when a single seller controls prices on both platforms. We now examine how parity operates when two independent sellers each operate exclusively on their own platform. In this setting, the compliance calculus changes fundamentally: Buy Box eligibility is no longer part of a joint profit-maximization problem but a unilateral decision by Seller 1. As we show below, this strategic friction alters both the nature of equilibrium and the impact of price parity relative to the single-seller case.

To save space, equilibrium is characterized in Online Appendix C.2. Under Model B, Seller 1's demand boost is unconditional and the equilibrium is standard Bertrand competition with effective demand intercept $\alpha_1 + \theta$. Under Model BP, three regimes arise as α_2 rises: Seller 1 forfeits the boost and prices without constraint (*eligibility violated*); Seller 1 complies by matching Seller 2 at the common price $\alpha_2/(2-\beta)$ (*eligibility binding*); and Seller 1 is eligible without adjustment, so outcomes coincide with Model B (*eligibility slack*). An intermediate interval separating the violated and binding regimes admits no pure-strategy equilibrium; all comparisons below are restricted to the regimes in which one exists. This section compares equilibrium outcomes between model BP and model B.

Because price parity affects outcomes only in the eligibility-violated and eligibility-binding regimes, our analysis is restricted to them; when eligibility is slack, outcomes coincide across the two models by Proposition 13 in the Online Appendix. No single-homing regime arises here because each seller operates on one platform and participation is exogenous.

5.1. Parity's Effects on Prices and Demands

PROPOSITION 8 **(Price Effects: Two Sellers)**. *In the two-seller setting, equilibrium prices on both platforms are strictly lower under Model BP than under Model B (the benchmark):* $p_{k,II}^{BP*} < p_{k,II}^{B*}, \quad k \in \{1,2\}$.

In the *eligibility-violated regime*, Seller 1 forfeits the boost and prices as if $\theta = 0$. Under Model B, θ expands effective demand on Platform 1, raising its price directly and Platform 2's indirectly through strategic complementarity. Forfeiting θ removes this demand

support entirely, so both prices fall. The mechanism is not active price distortion but the *absence* of θ from equilibrium: prices fall because Seller 1 optimally relinquishes the boost, not because parity forces any price down.

In the *eligibility-binding regime*, the boost θ is sufficiently valuable that Seller 1 lowers its price to match Seller 2's and satisfy the eligibility condition. Because prices are strategic complements, Seller 1's lower price makes *undercutting* less profitable for Seller 2, which best-responds by lowering its own price. Compliance thus propagates through competition rather than being internalized by a single seller: a constraint imposed on one seller drags down the price of the other, so both platforms' prices fall below their benchmarks throughout the regime. This mechanism is absent in the single-seller model, where a single seller sets both prices and the parity constraint compresses them in opposite directions rather than pushing both down.

These price effects translate directly into the equilibrium demand for the two sellers.

COROLLARY 2 **(Demand Effects: Two Sellers)**. *In the two-seller setting, the following demand comparisons hold between Model BP and Model B:*

i) In the eligibility-violated regime, $q_{1,II}^{BP*} < q_{1,II}^{B*}$ *and* $q_{2,II}^{BP*} < q_{2,II}^{B*}$.

ii) In the eligibility-binding regime, $q_{1,II}^{BP*} > q_{1,II}^{B*}$ *and* $q_{2,II}^{BP*} < q_{2,II}^{B*}$.

In the *eligibility-violated regime*, demand falls for both sellers relative to the benchmark. The absence of the demand boost θ from equilibrium contracts Seller 1's effective demand intercept, and this effect dominates the stimulus from lower prices. Seller 2's demand falls through the direct cross-price effect of Seller 1's lower price — since the two sellers' products are substitutes, Seller 1's lower price shifts demand away from Seller 2, reducing Seller 2's demand below the benchmark despite Seller 2's own price also falling.

In the *eligibility-binding regime*, both sellers lower their prices. The gain to Seller 1's demand from its own lower price outweighs the loss from Seller 2's lower price, so Seller 1's demand rises on net. Conversely, Seller 2's demand falls. Demand therefore shifts toward Platform 1. Moreover, because both prices fall rather than moving in opposite directions, this shift is not zero-sum: the lower overall price level draws in additional demand, so Seller 1's gain exceeds Seller 2's loss and aggregate consumption rises. The single-seller binding regime involves only this demand shift: parity there compresses prices in opposite directions, moving demand at unchanged aggregate quantity.

Taken together, Corollary 2 reveals that the direction of the demand shift under BP depends entirely on whether Seller 1 retains eligibility. Seller 1's demand rises above the benchmark when eligibility binds but falls below it when eligibility is violated. Seller 2's demand, by contrast, falls in both regimes, since Seller 1's lower price draws demand away from it whether or not the boost is retained. Notably, prices fall in both regimes, so the direction of prices alone does not reveal how demand is reallocated. The next subsection examines how these price and demand effects translate into seller profits.

5.2. Parity's Effects on the Profitability of the Sellers and the Platforms

We next compare seller profits between Model BP and Model B.

PROPOSITION 9 **(Effects on Seller Profits: Two Sellers)**. *In the two-seller setting, both sellers earn strictly lower profits under Model BP than under Model B (the benchmark), that is,* $\Pi_{k,II}^{BP*} \leq \Pi_{k,II}^{B*}$ *where* $k \in \{1, 2\}$.

In the *eligibility-binding regime*, Seller 1 gains the Buy Box boost by pricing down to Seller 2's level, eliminating the competitive price differential that would otherwise prevail under Model B. As established in Proposition 8, equilibrium prices fall on both platforms relative to Model B. Although Seller 1's demand rises, the price reduction dominates the demand gain, and Seller 1's profit falls strictly below the benchmark. Seller 2's price and demand both fall, so its profit falls as well. Seller 1 cannot unilaterally sustain a higher price while retaining eligibility, so the value of the boost is more than offset by the price compression that compliance requires.

In the *eligibility-violated regime*, Seller 1 forfeits the boost rather than distort its price to satisfy the eligibility condition. The absence of θ from equilibrium eliminates the upward demand shift that Model B provides to Seller 1, while both prices fall relative to the benchmark. Lower prices combined with lower demand decrease Seller 1's profit unambiguously. Seller 2's profit falls for the same reason: its own price falls, and its demand falls as well, since Seller 1's lower price diverts demand through the cross-price effect. The profit loss in this regime is not the cost of compliance but the cost of opt-out: by forfeiting the boost, Seller 1 removes the demand support that Model B extends to both platforms.

COROLLARY 3 **(Effects on Platform 1's Commission Revenue: Two Sellers)**. *In the two-seller setting, the dominant platform never gains from its own price parity rule.*

In the single-seller environment, the seller maximized joint profit across both platforms, so its incentives did not align with either platform's individual revenue interest, and a separate revenue analysis was necessary (Proposition 6; Proposition 11 in Online Appendix C.1). In the two-seller environment, by contrast, each seller operates exclusively on one platform and maximizes profit there. Since platform commissions are proportional to seller revenue, each platform's interest is aligned with its own seller's by construction. The profit comparisons in Proposition 9 therefore apply directly to platform revenues: both platforms earn strictly lower revenue under Model BP than under Model B in the eligibility-binding and eligibility-violated regimes.

These profit effects, combined with the price and demand responses characterized above, determine the welfare consequences of parity in the two-seller environment. The welfare results mirror the single-seller case in direction but not in mechanism. The difference is that compliance here lowers both prices and expands aggregate consumption, whereas under single-seller pricing the welfare gain arises from demand shifting at unchanged aggregate quantity.

5.3. Parity's Effects on Welfare

We examine the welfare implications of Model BP relative to Model B in the two-seller environment. Welfare is decomposed into consumer surplus and seller profit. Consumer surplus (welfare) is measured using the quadratic utility representation in equation (18) (See Appendix B).

PROPOSITION 10 **(Consumer Surplus Effects: Two Sellers)**. *In the two-seller setting, the consumer surplus is strictly higher under Model BP than under Model B (the benchmark) if and only if the eligibility-binding regime arises.*

In the *eligibility-violated regime*, Seller 1 finds compliance too costly and forgoes the Buy Box entirely. Without the demand boost θ, Seller 1's effective demand contracts sharply — and despite both prices falling below the Model B benchmark, this demand contraction dominates the price stimulus, reducing aggregate consumption and pulling consumer surplus below the benchmark.

In the *eligibility-binding regime*, Seller 1's compliance decision pulls both prices below their unconstrained Bertrand levels through strategic interaction. Since $\alpha_1 + \theta > \alpha_2$

throughout this regime, these price reductions expand aggregate consumption: consumers pay less on both platforms and buy more in total. At the same time, demand shifts toward Seller 1, whose higher demand base means this shift generates additional surplus gains. The eligibility mechanism thus delivers a double dividend for consumers: lower prices and demand concentration reinforce each other. Total surplus comparisons are provided in Corollary 5 (Online Appendix C.2).

6. Conclusion

This paper examines how conditioning Buy Box eligibility on cross-platform price comparison, i.e., implicit price parity, reshapes seller choices, equilibrium prices, and welfare. The central feature of our framework is endogenous compliance—absent from standard MFN models—a seller may comply with the price parity condition and retain the Buy Box demand boost, forfeit it when compliance becomes too costly. These responses are governed by a trade-off between the cost of satisfying the parity rule and the value of retaining Buy Box eligibility.

Our analysis reveals three key findings. First, where parity binds, seller market structure determines whether implicit price parity raises or lowers the rival platform's price. With a single seller operating across platforms, compliance requires the seller to lower its price on the dominant platform and raise it on the rival platform, whereas opting out lowers prices on both platforms. With two competing sellers each operating exclusively on one platform, prices fall on both platforms whether the seller complies or opts out.

Second, welfare effects of parity hinge on seller compliance, and the pattern is consistent across market structures: consumer surplus rises whenever eligibility binds, and falls when the seller violates the constraint or escapes it by single-homing.

Third, parity does not uniformly serve the dominant platform's revenue interest. With a single seller operating across platforms, commission revenue rises when the rival platform's baseline demand is low enough to induce single-homing, or when eligibility binds and high cross-platform substitutability lets the demand gain outweigh the price cut; otherwise it falls. Under seller competition, revenue never rises: compliance intensifies price competition, while opting out depresses both price and demand on the dominant platform because of the forgone Buy Box demand boost.

These findings have direct implications for regulatory assessments of Buy Box parity practices. Whether rival-platform prices rise or fall and whether consumers gain or lose

depend not on the eligibility rule alone, but on whether one seller or competing sellers set prices and on the rival platform's baseline demand, which together shape the seller's compliance decision. Price observations are therefore an unreliable guide to welfare: under a single seller, the rival-price rise under compliance accompanies a welfare gain while the price declines under opt-out accompany a loss; under seller competition, prices fall in either regime, yet welfare rises under compliance and falls under opt-out.

Several extensions arise from the framework developed here. First, the model captures Buy Box eligibility as a stark binary — the seller receives either the full demand boost or none — which isolates the compliance trade-off cleanly. A natural generalization would allow graduated enforcement, where noncompliance reduces rather than eliminates the demand boost, and examine how the strength of parity-linked eligiblity shapes the compliance threshold and welfare. Second, a seller entry stage would connect parity-linked eligibility to marketplace dynamics and the evolution of cross-platform competition — in particular, whether eligibility rules deter sellers from establishing a presence on rival platforms. Third, allowing each seller in the two-seller setting to list on both platforms would introduce seller-side multi-homing, permitting a richer analysis of how parity enforcement shapes competition within and across platforms.

References

Black M (2024) Amazon Still Leads as Lowest-priced Retailer; Target Gains Ground Ahead of Holidays. URL `https://www.profitero.com/press-releases/amazon-still-leads-target-gains-ground-ahead-of-holidays-0`.

Boik A, Corts KS (2016) The Effects of Platform Most-Favored-Nation Clauses on Competition and Entry. *The Journal of Law and Economics* 59(1):105–134, URL `http://dx.doi.org/10.1086/686971`.

Chen L, Cui Y, Liu J, Liu X (2026) Bonus competition in the gig economy. *Production and Operations Management* 35(5):1729–1746.

Chen L, Mislove A, Wilson C (2016) An Empirical Analysis of Algorithmic Pricing on Amazon Marketplace. *Proceedings of the 25th International Conference on World Wide Web*, 1339–1349 (Montréal Québec Canada: International World Wide Web Conferences Steering Committee), ISBN 978-1-4503-4143-1.

Chen YJ, Dai T, Korpeoglu CG, Körpeoğlu E, Sahin O, Tang CS, Xiao S (2020) Om forum—innovative online platforms: Research opportunities. *Manufacturing & Service Operations Management* 22(3):430–445.

Ciotti F, Madio L (2022) Competition for prominence. Technical report, Mimeo.

Dai H, Swaminathan JM, Xu Y (2022) Gig worker learning: An empirical study on on-demand delivery platform. *Available at SSRN 4106978* .

Dash A, Chakraborty A, Ghosh S, Mukherjee A, Gummadi KP (2024) Investigating Nudges toward Related Sellers on E-commerce Marketplaces: A Case Study on Amazon. *Proceedings of the ACM on Human-Computer Interaction* 8(CSCW2):1–31, URL http://dx.doi.org/10.1145/3686994.

Dinerstein M, Einav L, Levin J, Sundaresan N (2018) Consumer Price Search and Platform Design in Internet Commerce. *American Economic Review* 108(7):1820–1859.

Eisfeld L (2023) How do online product rankings influence sellers' pricing behavior? .

Gallino S, Karacaoglu N, Moreno A (2025) Algorithmic assortment curation: An empirical study of buybox in online marketplaces. *Manufacturing & Service Operations Management* 27(3):917–934.

Garcia D, Tolvanen J, Wagner AK (2024) Strategic Responses to Algorithmic Recommendations: Evidence from Hotel Pricing. *Management Science* mnsc.2022.03740.

Gómez-Losada Asencio-Cortés G, Duch-Brown N (2022) Automatic Eligibility of Sellers in an Online Marketplace: A Case Study of Amazon Algorithm. *Information* 13(2):44.

Hagiu A, Wright J (2015) Marketplace or Reseller? *Management Science* 61(1):184–203.

Hagiu A, Wright J (2024) Optimal Discoverability on Platforms. *Management Science* 70(11):7770–7790.

Harrington C (2023) Amazon's All-Powerful 'Buy Box' Is at the Heart of Its New Antitrust Troubles. *Wired* URL https://www.wired.com/story/amazons-all-powerful-buy-box-is-at-the-heart-of-its-new-antitrust-troubles/.

Hunold M, Laitenberger U, Thébaudin G (2022) Bye-box: An analysis of non-promotion on the amazon marketplace URL https://ideas.repec.org/p/hal/wpaper/hal-04104183.html.

Jin C, Yang L, Hosanagar K (2023) To brush or not to brush: Product rankings, consumer search, and fake orders. *Information Systems Research* 34(2):532–552.

Johnson JP (2017) The Agency Model and MFN Clauses. *The Review of Economic Studies* 84(3):1151–1185.

Lai G, Liu H, Xiao W, Zhao X (2022) "Fulfilled by Amazon": A Strategic Perspective of Competition at the e-Commerce Platform. *Manufacturing & Service Operations Management* 24(3):1406–1420.

Li Y, Lu LX, Van Mieghem JA, Xiao W (2025) Fulfillment by Amazon as a strategic lever: Anticompetitive or welfare enhancing? *Available at SSRN 5376166* .

Qi A, Sethi S, Wei L, Zhang J (2022) Top or regular influencer? contracting in live-streaming platform selling. *Contracting in live-streaming platform selling (August 28, 2022)* .

Raval D (2022) Steering in one click: Platform self-preferencing in the amazon buy box. *Unpublished manuscript* .

Sikdar S, Kadiyali V, Hooker G (2022) Does amazon's dual role weaken marketplace competition? *Journal of Marketing* 00222429261417677.

Singh N, Vives X (1984) Price and Quantity Competition in a Differentiated Duopoly. *The RAND Journal of Economics* 15(4):546, URL http://dx.doi.org/10.2307/2555525.

Team T (2023) Winning the Amazon Buy Box [Algorithm Tips for 2024]. URL https://tinuiti.com/blog/amazon/win-amazon-buy-box/.

Weise K (2023) Here Are the 2 Tactics Amazon Used to Undermine Competition, the F.T.C. Says. *The New York Times* URL https://www.nytimes.com/2023/09/26/technology/amazon-ftc-lawsuit-antitrust.html.

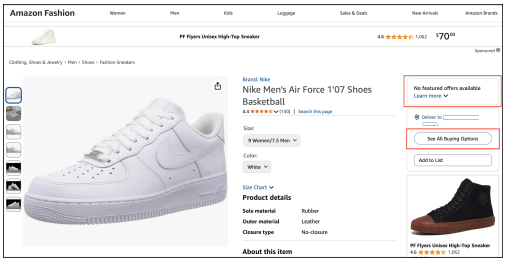
Xu F, Wang X, Zhang F (2025) Consumer privacy in online retail supply chains. *Management Science* 71(10):8371–8389.

Zanchettin P (2006) Differentiated duopoly with asymmetric costs. *Journal of Economics & Management Strategy* 15(4):999–1015.

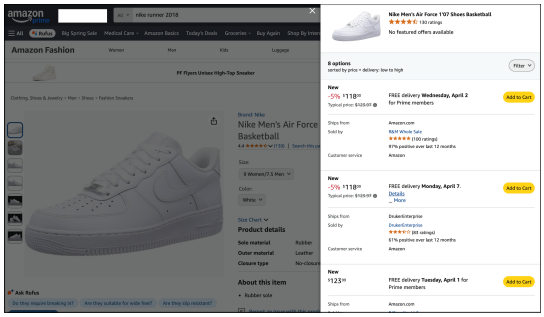
Zhu F, Liu Q (2018) Competing with complementors: An empirical look at Amazon.com. *Strategic Management Journal* 39(10):2618–2642, URL http://dx.doi.org/10.1002/smj.2932.

Online Appendix of "Buy Box and Price Parity: Competitive Effects and Consumer Welfare in Online Marketplaces"

Appendix A. Figures



(a) No Buy Box Offer available



(b) Alternative seller list

Figure A1 **Amazon product page without Buy Box offer. Panel (a) displays the message "No featured offers available" when no seller qualifies for Buy Box. Panel (b) displays a list of competing sellers when consumers click 'See All Buying Options', which requires active search and price comparison rather than a one-click purchase.**

Appendix B. Demand System: Micro-Foundations

Step 1: Duopoly demand from quadratic utility. A representative consumer chooses quantities (q_1, q_2) to maximize

$$\max_{q_1,q_2} \left\{ a_1 q_1 + a_2 q_2 - \frac{1}{2}\left(q_1^2 + 2\gamma q_1 q_2 + q_2^2\right) - p_1 q_1 - p_2 q_2 \right\}, \tag{11}$$

where $a_1, a_2 > 0$ are taste parameters and $\gamma \in [0,1)$ governs substitutability. This is the standard Singh–Vives (1984) quadratic utility. The first-order conditions, solved under $|\gamma| < 1$, yield reduced-form demands

$$q_1 = \alpha_1 - p_1 + \beta p_2, \qquad q_2 = \alpha_2 - p_2 + \beta p_1, \tag{12}$$

where $\beta \equiv \gamma$, prices are normalised as $p_i \equiv p_i^{orig}/(1-\beta^2)$, and the reduced-form intercepts are

$$\alpha_1 \equiv \frac{a_1 - \beta a_2}{1-\beta^2} > 0, \qquad \alpha_2 \equiv \frac{a_2 - \beta a_1}{1-\beta^2} > 0. \tag{13}$$

The *interior duopoly regime* is defined by $q_1 > 0$ and $q_2 > 0$; outside it, at least one platform is inactive.

Step 2: Demand boost in the duopoly interior. We extend (11) by allowing platform 1 to receive an *expected demand boost* $\theta \geq 0$ whenever the seller prices no higher on platform 1 than on platform 2, i.e. $p_1 \leq p_2$. Formally, this is implemented by shifting $a_1 \mapsto a_1 + \theta$ and $a_2 \mapsto a_2 + \beta\theta$ in (11) contingent on $p_1 \leq p_2$, which under (13) shifts only $\alpha_1 \mapsto \alpha_1 + \theta$. The resulting piecewise demand system is

$$(q_1, q_2) = \begin{cases} (\alpha_1 + \theta - p_1 + \beta p_2,\ \alpha_2 - p_2 + \beta p_1), & p_1 \leq p_2, \\ (\alpha_1 - p_1 + \beta p_2,\ \alpha_2 - p_2 + \beta p_1), & p_1 > p_2, \end{cases} \tag{14}$$

with both components positive throughout the duopoly interior.

Step 3: Monopoly regimes and asymmetric boost eligibility. A key feature of the model is that θ is tied to platform 1 being active: the boost condition $p_1 \leq p_2$ is economically relevant only when $q_1 > 0$. This generates an asymmetry across the two monopoly corners.

Monopoly on platform 1 ($q_2 = 0$, $q_1 > 0$). Setting $q_2 = 0$ corresponds to the seller choosing p_2 high enough to drive platform-2 demand to zero. Inverting (13) gives $a_1 = \alpha_1 + \beta\alpha_2$, so the corner case of (11) yields

$$q_1 = \alpha_1 + \beta\alpha_2 + \theta - (1 - \beta^2)p_1. \tag{15}$$

The boost θ applies here because $q_1 > 0$ and the condition $p_1 \leq p_2$ is trivially satisfied for any sufficiently large p_2.

Monopoly on platform 2 ($q_1 = 0$, $q_2 > 0$). When platform 1 is inactive, $q_1 = 0$ and the boost condition $p_1 \leq p_2$ has no economic content: there is no platform-1 demand to boost. Accordingly, θ does *not* apply, and the monopoly demand is simply

$$q_2 = \alpha_2 + \beta\alpha_1 - (1 - \beta^2)p_2. \tag{16}$$

Step 4: Consumer surplus in each regime. Consumer surplus is defined as

$$CS \;=\; U(q_1, q_2) - p_1^{orig} q_1 - p_2^{orig} q_2, \tag{17}$$

where U is the primitive utility (11). Using the first-order conditions to substitute $(a_i - p_i^{orig})$ and collecting terms, one obtains a unified formula:

$$CS \;=\; \tfrac{1}{2}\left(q_1^2 + 2\beta\, q_1 q_2 + q_2^2\right). \tag{18}$$

Note that (18) holds regardless of whether the boost θ is active, because θ shifts equilibrium quantities but leaves the algebraic relationship between CS and quantities unchanged. The three equilibrium regimes then give:

Duopoly ($q_1 > 0$, $q_2 > 0$). Both platforms are active, so

$$CS^D \;=\; \tfrac{1}{2}\left(q_1^2 + 2\beta\, q_1 q_2 + q_2^2\right), \tag{19}$$

where (q_1, q_2) are evaluated at the duopoly equilibrium prices from (14). The cross-term $2\beta q_1 q_2 > 0$ reflects the substitutability between platforms: consumer surplus is higher when both platforms are active and serve as competitive alternatives.

Single-homing on platform 1 ($q_1 > 0$, $q_2 = 0$). Setting $q_2 = 0$ in (18),

$$CS^{M_1} \;=\; \tfrac{1}{2}\, q_1^2, \tag{20}$$

where q_1 is evaluated at the platform-1 monopoly equilibrium (15), which includes the boost θ.

Single-homing on platform 2 ($q_1 = 0$, $q_2 > 0$). Setting $q_1 = 0$ in (18),

$$CS^{M_2} \;=\; \tfrac{1}{2}\, q_2^2, \tag{21}$$

where q_2 is evaluated at the platform-2 monopoly equilibrium (16), which does *not* include θ.

Appendix C. Additional Analysis

C.1. Single Seller

Platform 2 Revenue. We now examine how eligibility affects Platform 2 revenue relative to the unconditional benchmark Model B.

PROPOSITION 11 **(Platform 2 Revenue: Single Seller)**. *Platform 2's revenue is weakly lower under Model BP than under Model B:*

$$R_{2,I}^{BP} \leq R_{2,I}^{B},$$

with strict inequality for all $\alpha_2 \in (0, \alpha_1 + \theta)$ whenever $\beta > 0$.

In the single-homing regime, $q_{2,I}^{BP*} = 0$ under Model BP, so Platform 2 earns no revenue, whereas it earns strictly positive revenue under Model B. The revenue loss operates entirely through the quantity channel.

In the eligibility-violated regime, the seller forfeits the Buy Box demand advantage and sets prices as if $\theta = 0$, yielding a strictly lower price on Platform 2 than under Model B. Since demand coincides across models in this regime, the revenue loss operates purely through the price channel.

In the eligibility-binding regime, compliance raises the Platform 2 price by $\delta = (\alpha_1 + \theta - \alpha_2)/(4(1+\beta))$ - the same magnitude by which the Platform 1 price is suppressed - while contracting Platform 2 demand. Throughout the binding regime, the demand contraction outweighs the positive price effect on revenue.

Total Surplus. Total surplus equals the sum of consumer surplus and seller profit:

$$TS^m = CS^m + (1-\beta^2)\pi_I^{m*}, \qquad m \in \{B, BP\}.$$

COROLLARY 4 **(Total Surplus Comparison)**. *Total surplus is strictly higher under Model BP than under Model B in the eligibility-binding regime and strictly lower otherwise.*

Since seller profit is strictly lower under Model BP in all regimes, changes in total surplus are driven entirely by changes in consumer surplus. In the single-homing and eligibility-violated regimes, consumer surplus also falls, so total surplus falls immediately. In the eligibility-binding regime, the consumer surplus gain from eligibility-driven demand exceeds the weighted profit loss from compliance distortion. Parity is therefore welfare-improving if and only if the eligibility condition binds in equilibrium. These results suggest that welfare assessments of parity-based platform practices should account for sellers' endogenous compliance and participation responses, not merely observed price changes.

C.2. Two Sellers

Benchmark: Buy Box without Parity, Model B.

Under Model B, Seller 1 receives the demand boost θ unconditionally and both sellers simultaneously choose prices to maximize individual profits:

$$\Pi_{1,II}^{B} = p_{1,II}^{B}(\alpha_1 + \theta - p_{1,II}^{B} + \beta p_{2,II}^{B}), \quad \Pi_{2,II}^{B} = p_{2,II}^{B}(\alpha_2 - p_{2,II}^{B} + \beta p_{1,II}^{B}). \tag{22}$$

Because θ enters Seller 1's demand unconditionally, Seller 1's problem is equivalent to standard Bertrand competition with an effective demand intercept of $\alpha_1 + \theta$. The first-order conditions yield a unique interior equilibrium, characterized in the following proposition.

PROPOSITION 12 **(Model B: Equilibrium with Two Sellers)**. *Under Model B, the unique equilibrium prices are*

$$\left(p_{1,II}^{B*},\, p_{2,II}^{B*}\right) = \left(\frac{2(\alpha_1+\theta)+\beta\alpha_2}{4-\beta^2},\, \frac{2\alpha_2+\beta(\alpha_1+\theta)}{4-\beta^2}\right). \tag{23}$$

As in the single-seller benchmark, θ enters as a pure upward shift to Seller 1's effective demand intercept. Seller 1's equilibrium price therefore responds to θ in exactly the same way it responds to an equal increase in α_1. Seller 2's price inherits the boost indirectly: the higher Platform 1 price induces Seller 2 to raise its price through strategic complementarity — since prices are strategic complements, Seller 2's best response is increasing in p_1. The Model B equilibrium therefore serves as the natural baseline against which the pricing distortions induced by parity-based eligibility are measured in Section 5.1.

Equilibrium Characterization under Buy Box with Parity, Model BP.

Under Model BP, the demand boost θ is contingent: Seller 1 earns it only if $p_{1,II}^{BP} \leq p_{2,II}^{BP}$. Seller profits are:

$$\Pi_{1,II}^{BP} = p_{1,II}^{BP}(\alpha_1 - p_{1,II}^{BP} + \beta p_{2,II}^{BP} + \theta\,\mathbf{1}\{p_{1,II}^{BP} \leq p_{2,II}^{BP}\}), \quad \Pi_{2,II}^{BP} = p_{2,II}^{BP}(\alpha_2 - p_{2,II}^{BP} + \beta p_{1,II}^{BP}). \tag{24}$$

The eligibility requirement introduces a discontinuity in Seller 1's profit at the threshold $p_{1,II}^{BP} = p_{2,II}^{BP}$ - the point at which Buy Box eligibility is exactly satisfied - rendering the standard first-order approach insufficient; the full derivation is provided in the Appendix. Unlike the single-seller case, where the compliance decision involved a joint revenue trade-off internalized by one seller, here each seller responds to the other's price strategically. Seller 1 chooses between adjusting its price to satisfy the eligibility condition and gain θ, or pricing without the eligibility constraint and forgoing the boost entirely, anticipating Seller 2's best response in either case. Seller 2, facing no eligibility requirement, simply best-responds to Seller 1's price. The resolution of this strategic interaction depends critically on the rival platform's market size, α_2, and the demand boost, θ.

PROPOSITION 13 **(Model BP: Equilibrium with Two Sellers)**. *Under Buy Box with implicit price parity (BP), equilibrium pricing partitions into three regimes:*

1. ***Eligibility-violated*** $\left(\alpha_2 \leq \dfrac{(4-\beta^2)\Delta - \beta\alpha_1}{2}\right)$*: Seller 1 prices strictly above Seller 2 and forgoes Buy Box eligibility, where* Δ *is defined in the Appendix.*
2. ***Eligibility-binding*** $\left(\alpha_2 \in \left[(2-\beta)\Delta,\, \alpha_1+\theta\right)\right)$*: Sellers set identical prices,* $p_{1,II}^{BP*} = p_{2,II}^{BP*}$*, and Seller 1 retains Buy Box eligibility exactly.*
3. ***Eligibility-slack*** $(\alpha_2 \geq \alpha_1+\theta)$*: Seller 1 remains eligible without adjusting its price; equilibrium outcomes coincide with those of Model B.*

The gap between these thresholds corresponds to a regime of non-existence, characterized in Remark 2. Equilibrium prices and demands in each regime are summarized in Table A1.

REMARK 2 (NON-EXISTENCE OF PURE-STRATEGY EQUILIBRIUM). There exists an intermediate parameter regime, lying between the eligibility-binding and eligibility-violated regimes, in which no pure-strategy Nash equilibrium exists. In this regime, the payoff discontinuity at $p_1 = p_2$ generates undercutting incentives that preclude any stable price vector: Seller 1 can always gain by marginally adjusting its price relative

to the eligibility threshold, yet no price pair satisfies mutual best-response consistency. This non-existence arises endogenously from the indicator-function structure of the eligibility condition and is characterized formally in the Appendix.

The three regimes reflect distinct resolutions of the compliance trade-off under strategic pricing, each arising for a different configuration of (α_2, θ).

In the *eligibility-violated regime*, the rival platform's market size is sufficiently small that Seller 2's unconstrained best-response price falls below Seller 1's unconstrained best-response price. Satisfying the eligibility condition $p_{1,II}^{BP} \leq p_{2,II}^{BP}$ would therefore require Seller 1 to price below its profit-maximizing level given Seller 2's response - a distortion whose cost exceeds the profit value of gaining θ. Seller 1 consequently forfeits the boost entirely and prices without the eligibility constraint. Equilibrium prices in this regime are identical to the standard Bertrand outcome with $\theta = 0$, and the parity constraint imposes no price distortion.

In the *eligibility-binding regime*, the profit value of gaining θ is sufficiently large relative to the compliance cost that Seller 1 finds it worthwhile to satisfy the eligibility condition. Under Model B, the demand asymmetry $\alpha_1 + \theta > \alpha_2$ generates a competitive price differential — Seller 1's equilibrium price exceeds Seller 2's price — and the eligibility condition requires this differential to be eliminated. The mechanism operates through strategic interaction: Seller 1 prices down to Seller 2's level, and both sellers set a common equilibrium price $p_{1,II}^{BP*} = p_{2,II}^{BP*} = \alpha_2/(2-\beta)$, which is independent of θ. Seller 1's compliance decision depends on the demand boost θ, but the resulting equilibrium price does not.

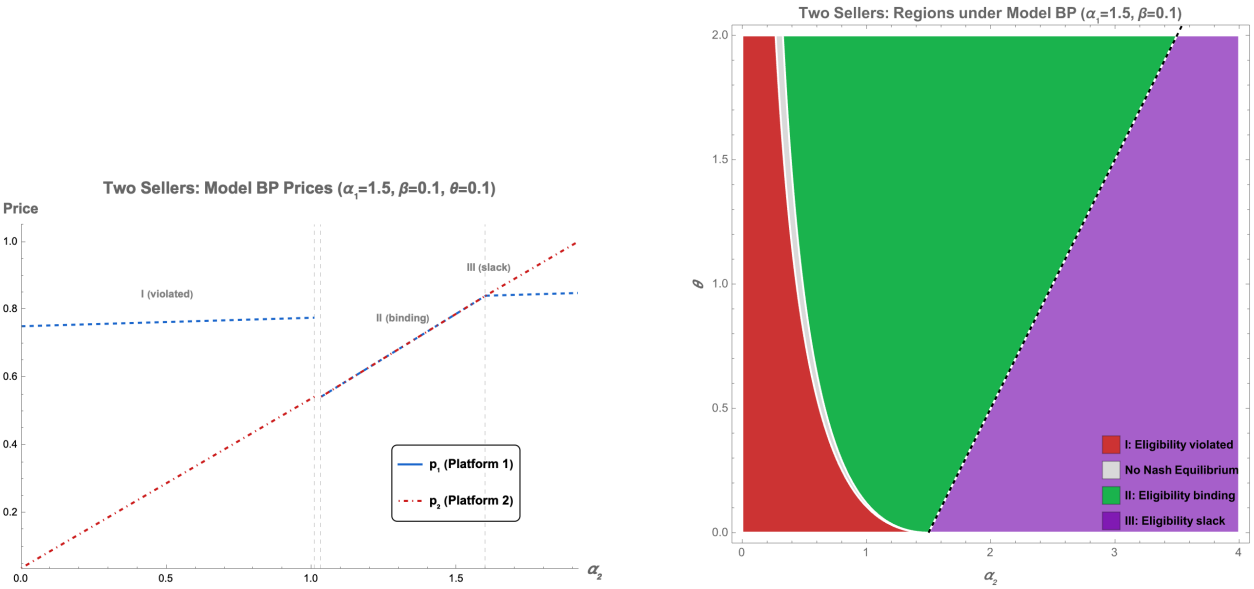
In the *eligibility-slack regime*, Seller 2's unconstrained best-response price is already weakly above Seller 1's unconstrained best-response price. The eligibility condition is therefore satisfied without any price adjustment by either seller, and equilibrium outcomes coincide exactly with Model B. In this regime, parity-based eligibility is costless and the two models are observationally equivalent.

When the rival platform is weak, Seller 1 forfeits the boost rather than distort its price to satisfy the eligibility condition. When the rival platform is moderately strong, Seller 1 complies by matching Seller 2's price, with the equilibrium price anchored by Seller 2's market size rather than the boost itself. When the rival platform is sufficiently strong, eligibility is satisfied passively and imposes no competitive distortion.

The structural comparison with the single-seller analysis is instructive. Both environments generate the same three qualitative regimes - violated, binding, and slack - and the slack regime is observationally equivalent across the two settings by construction. Notably, the single-homing regime that arises in the single-seller analysis has no analog here: since each seller prices exclusively on its own platform, platform participation is exogenous and the participation decision is absent from the two-seller model. The key difference lies in the binding regime: under single-seller pricing, the common price is increasing in θ because the seller jointly optimizes both prices with the boost in the objective. Under independent pricing, the binding price is determined solely by Seller 2's market size and substitutability β, because Seller 2 faces no demand boost and its optimal price is therefore independent of θ. Seller 1's compliance decision depends on θ, but the resulting equilibrium price does not. This distinction, as we show in Section 5.1, has direct consequences for how prices and profits compare across the two environments.

	Eligibility-violated	Eligibility-binding	Eligibility-slack
Price on Platform 1	$\frac{2\alpha_1 + \beta\alpha_2}{4-\beta^2}$	$\frac{\alpha_2}{2-\beta}$	$\frac{2(\alpha_1+\theta) + \beta\alpha_2}{4-\beta^2}$
Price on Platform 2	$\frac{2\alpha_2 + \beta\alpha_1}{4-\beta^2}$	$\frac{\alpha_2}{2-\beta}$	$\frac{2\alpha_2 + \beta(\alpha_1+\theta)}{4-\beta^2}$
Demand on Platform 1	$\frac{2\alpha_1 + \beta\alpha_2}{4-\beta^2}$	$\alpha_1 + \theta - \frac{\alpha_2(1-\beta)}{2-\beta}$	$\frac{2(\alpha_1+\theta) + \beta\alpha_2}{4-\beta^2}$
Demand on Platform 2	$\frac{2\alpha_2 + \beta\alpha_1}{4-\beta^2}$	$\frac{\alpha_2}{2-\beta}$	$\frac{2\alpha_2 + \beta(\alpha_1+\theta)}{4-\beta^2}$
Buy Box eligibility status	Not satisfied	Satisfied under binding price parity	Satisfied without price adjustment

Table A1 Equilibrium outcomes under Buy Box with implicit price parity (BP) in the two-seller environment.



(a) Equilibrium prices across regimes. Note the binding equality $p_1 = p_2$ in the binding regime.

(b) Equilibrium regimes in the (α_2, θ) space.

Figure C1 Equilibrium characterization under Model BP with two sellers ($\alpha_1 = 1.5$, $\beta = 0.1$, $\theta = 0.1$).

COROLLARY 5 **(Total Surplus: Two Sellers)**. *Total surplus is strictly higher under Model BP than under Model B in the eligibility-binding regime, and strictly lower in the eligibility-violated regime.*

Corollary 5 follows from combining Proposition 10 with the seller profit results in Proposition 9. In the eligibility-violated regime, consumer surplus and seller profits both fall under Model BP, so total surplus falls immediately. In the eligibility-binding regime, seller profits are strictly lower under Model BP while consumer surplus is strictly higher. As confirmed by direct comparison in closed form (see proof of Corollary 5), the consumer surplus gain dominates throughout the binding regime. The price compression generated by compliance benefits consumers by more than it costs sellers in foregone profit: Seller 1 is constrained by the eligibility condition, while Seller 2's competitive best response prevents it from restoring benchmark prices given Seller 1's lower price. The consumer surplus gain exceeds the seller profit loss because compliance not only redistributes surplus from sellers to consumers but also expands aggregate consumption through lower prices — creating new surplus that pushes total surplus above the benchmark.

Parity therefore improves total surplus in the two-seller environment if and only if the eligibility condition binds. Prices fall in both regimes, so the direction of prices does not identify the welfare effect.

Appendix D. Proofs

***Proof of Proposition** 1:* The seller maximizes total profit over both platforms:

$$\pi^D(p_1,p_2) = p_1(\alpha_1+\theta-p_1+\beta p_2) + p_2(\alpha_2-p_2+\beta p_1). \tag{25}$$

First-order conditions. $\frac{\partial \pi^D}{\partial p_1} = \alpha_1+\theta-2p_1+2\beta p_2 = 0$, $\frac{\partial \pi^D}{\partial p_2} = \alpha_2-2p_2+2\beta p_1 = 0$, which can be rewritten as $p_1-\beta p_2 = \frac{\alpha_1+\theta}{2}$, $\quad p_2-\beta p_1 = \frac{\alpha_2}{2}$.

Solving this linear system under $\beta \in [0,1)$ yields the unique candidate: $p_{1,I}^{B*} = \frac{\alpha_1+\beta\alpha_2+\theta}{2(1-\beta^2)}$, $p_{2,I}^{B*} = \frac{\alpha_2+\beta(\alpha_1+\theta)}{2(1-\beta^2)}$. Since the Hessian of π^D has diagonal entries -2 and off-diagonal entries 2β, its determinant is $4(1-\beta^2) > 0$ for $\beta \in [0,1)$, confirming strict concavity and uniqueness of the interior optimum.

Equilibrium quantities and interior duopoly. Substituting back, $q_1^{B*} = \frac{\alpha_1+\theta}{2} > 0$, $\quad q_2^{B*} = \frac{\alpha_2}{2} > 0$, so the duopoly demand system applies and the interior regime is valid under $\alpha_1+\theta > 0$ and $\alpha_2 > 0$.

Equilibrium profit. $\pi^{B*} = p_{1,I}^{B*}q_1^{B*} + p_{2,I}^{B*}q_2^{B*} = \frac{(\alpha_1+\theta)^2+\alpha_2^2+2\beta\alpha_2(\alpha_1+\theta)}{4(1-\beta^2)}$.

Multi-homing dominates single-platform monopoly. Under monopoly on platform 1, the consistent demand is $q_1^{M1} = \alpha_1+\beta\alpha_2+\theta-(1-\beta^2)p_1$, yielding $\pi^{M1} = \frac{(\alpha_1+\beta\alpha_2+\theta)^2}{4(1-\beta^2)}$.

Under monopoly on platform 2, $q_2^{M2} = \alpha_2+\beta\alpha_1-(1-\beta^2)p_2$, yielding $\pi^{M2} = \frac{(\alpha_2+\beta\alpha_1)^2}{4(1-\beta^2)}$.

Computing the gaps: $\pi^{B*}-\pi^{M1} = \frac{\alpha_2^2(1-\beta^2)}{4(1-\beta^2)} = \frac{\alpha_2^2}{4} \geq 0$, $\quad \pi^{B*}-\pi^{M2} = \frac{\alpha_1^2(1-\beta^2)+2\alpha_1\theta+2\beta\alpha_2\theta+\theta^2}{4(1-\beta^2)} \geq 0$, with both inequalities strict whenever $\alpha_2 > 0$ and $\alpha_1+\theta > 0$ respectively. Therefore multi-homing strictly dominates both single-platform alternatives, confirming that $(p_{1,I}^{B*}, p_{2,I}^{B*})$ is the unique global optimum.

***Proof of Proposition** 2:* Let $x \equiv \alpha_1+\theta$. Under the BP demand system, the seller's profit is $\Pi^{BP}(p_1,p_2) = p_1(\alpha_1-p_1+\beta p_2+\theta\mathbf{1}\{p_1 \leq p_2\}) + p_2(\alpha_2-p_2+\beta p_1)$, with prices restricted to $p_1,p_2 \geq 0$. Define the smooth quadratic $\Phi(p_1,p_2) \equiv p_1(\alpha_1+\theta-p_1+\beta p_2)+p_2(\alpha_2-p_2+\beta p_1)$, so that

$$\Pi(p_1,p_2) = \begin{cases} \Phi(p_1,p_2), & p_1 \leq p_2 \quad \text{(boost active)}, \\ \Phi(p_1,p_2)-p_1\theta, & p_1 > p_2 \quad \text{(opt-out)}. \end{cases}$$

Hence the global optimum is $\max_{p_1,p_2\geq 0}\Pi(p_1,p_2) = \max\left\{ \max_{\substack{p_1,p_2\geq 0 \\ p_1\leq p_2}} \Phi(p_1,p_2),\ \max_{\substack{p_1,p_2\geq 0 \\ p_1>p_2}} \big(\Phi(p_1,p_2)-p_1\theta\big) \right\}$.

Step A (Slack regime). Φ is strictly concave with unique maximiser (p_1^0,p_2^0) satisfying $p_2^0-p_1^0 = (1-\beta)(\alpha_2-x)/[2(1-\beta^2)] \geq 0$ iff $\alpha_2 \geq x$. For $\alpha_2 \geq x$ the maximiser lies in the eligible region $\{p_1 \leq p_2\}$ where $\Pi^{BP} = \Phi$, and is therefore the global maximiser of Π^{BP}, coinciding with Model B. This is the eligibility-slack regime. Henceforth $\alpha_2 \in [0,x)$.

Step B (Single-homing Candidates; elimination of M2). Under M1 (single-homing on Platform 1), the seller prices Platform 2 demand to zero and $p_2 > p_1$, so the boost is earned vacuously. Substituting into Φ yields the maximization problem: $\max_{p_1}\ p_1\big[(x+\beta\alpha_2)-p_1(1-\beta^2)\big]$,

with solution $p_1^{M1} = (x+\beta\alpha_2)/[2(1-\beta^2)]$ and $\Pi^{M1} = (x+\beta\alpha_2)^2/[4(1-\beta^2)]$.

Under M2 (single-homing on Platform 2), the seller prices Platform 1 demand to zero and the maximization problem is as follows: Substituting yields $\max_{p_2}\ p_2\big[(\alpha_2+\beta\alpha_1)-p_2(1-\beta^2)\big]$,

with solution $p_2^{M2} = (\alpha_2 + \beta\alpha_1)/[2(1-\beta^2)]$ and $\Pi^{M2} = (\alpha_2 + \beta\alpha_1)^2/[4(1-\beta^2)]$.

Since $\Pi^{M1} - \Pi^{M2} = [(1+\beta)(\alpha_1+\alpha_2)+\theta][(1-\beta)(\alpha_1-\alpha_2)+\theta]/[4(1-\beta^2)] > 0$ for all $\alpha_2 \in [0, x)$, M2 is strictly dominated and discarded. The remainder of the proof determines which of M1, K, J is globally optimal at each $\alpha_2 \in [0, x)$ where K and J are given as follows:

1. Under K, the seller maximizes $\Phi - \theta p_1$ with $q_1, q_2 > 0$ and forfeits the Buy Box advantage since $p_1 > p_2$.
2. Under J, the seller maximizes Φ with $q_1, q_2 > 0$ and setting identical prices on the two platforms ($p_1 = p_2$) winning the Buy Box advantage θ.

Case 1: $\theta \geq 2\alpha_1$. The feasibility regions of K and J are disjoint, so we compare M1 against each separately.

M1 vs K. On $[0, \alpha_1)$, $\Pi^{M1} - \Pi^K = N_K/[4(1-\beta^2)]$ where $N_K(\alpha_2) \equiv -(1-\beta^2)\alpha_2^2 + 2\beta\theta\alpha_2 + \theta(\theta + 2\alpha_1)$ is a downward-opening parabola with $N_K(0) > 0$. Since $N_K(\alpha_1) = \theta^2 + 2(1+\beta)\alpha_1\theta - (1-\beta^2)\alpha_1^2$ is strictly increasing in θ with $N_K(\alpha_1)|_{\theta=2\alpha_1} = \alpha_1^2(7 + 4\beta + \beta^2) > 0$, we have $N_K > 0$ throughout $[0, \alpha_1)$ for all $\theta \geq 2\alpha_1$: M1 strictly dominates K.

M1 vs J. On $(x/3, x)$, $\Pi^{M1} - \Pi^J = N_J/[8(1-\beta^2)]$ where $N_J(\alpha_2) \equiv 2(x + \beta\alpha_2)^2 - (1+\beta)(x+\alpha_2)^2$ is strictly concave with $N_J(x/3) = 2x^2(1-\beta)^2/9 > 0$ and $N_J(x) = 2x^2(1+\beta)(\beta - 1) < 0$. By strict concavity and the Intermediate Value Theorem there is a unique root $\tilde{\alpha} = (k-1)x/(\beta - k)$, $k \equiv \sqrt{(1+\beta)/2}$, in $(x/3, x)$; since $\bar{\alpha} = x/3$ and $\tilde{\alpha} > \bar{\alpha}$, the opt-out interval is empty and the equilibrium has M1 optimal on $[0, \tilde{\alpha})$ and J on $[\tilde{\alpha}, x)$.

Case 2: $\theta < 2\alpha_1$. Since $x/3 < \alpha_1$, K and J are simultaneously feasible on $(x/3, \alpha_1)$. We first construct the J–K envelope, then compare M1 against it.

Step D (J–K envelope). On the overlap $(x/3, \alpha_1)$, $\frac{d}{d\alpha_2}(\Pi^J - \Pi^K) = \frac{(1-\beta)(\alpha_1-\alpha_2)+(1+\beta)\theta}{4(1-\beta^2)} > 0$,
so $\Pi^J - \Pi^K$ is strictly increasing, ending at the positive value $\theta(4\alpha_1 + \theta)/[8(1-\beta)]$ at $\alpha_2 = \alpha_1$. Define

$$\bar{\alpha} \equiv \begin{cases} x/3 & \text{if } \Pi^J(x/3) \geq \Pi^K(x/3), \\ \alpha^* \in (x/3, \alpha_1) & \text{if } \Pi^J(x/3) < \Pi^K(x/3), \end{cases}$$

where α^* is the unique zero of $\Pi^J - \Pi^K$ on $(x/3, \alpha_1)$, guaranteed by the IVT and strict monotonicity. Taking the envelope right-continuously at $x/3$: K is the best multi-homing strategy on $[0, \bar{\alpha})$ and J on $[\bar{\alpha}, x)$.

Step E (M1 vs envelope). At $\alpha_2 = 0$: $\Pi^{M1}(0) - \Pi^K(0) = \theta(\theta + 2\alpha_1)/[4(1-\beta^2)] > 0$. Near $\alpha_2 = x$: $\Pi^J(x) - \Pi^{M1}(x) = x^2/4 > 0$. By continuity and right-continuity of the envelope at $\bar{\alpha}$, there exists at least one crossing $\tilde{\alpha} \in (0, x)$. Uniqueness follows because the difference $D \equiv \Pi^{M1} -$ envelope is globally concave on $[0, x)$: each piece (N_K, N_J) has a negative leading coefficient; D is continuous at $\bar{\alpha}$ (since $\Pi^K(\bar{\alpha}) = \Pi^J(\bar{\alpha})$); and the slope of D decreases at $\bar{\alpha}$ by the amount $d(\Pi^J - \Pi^K)/d\alpha_2|_{\bar{\alpha}} > 0$, a downward kink that preserves global concavity. With $D(0) > 0$ and $D(x^-) < 0$, the standard chord argument gives exactly one zero.

The global optimum follows from $\tilde{\alpha}$ and $\bar{\alpha}$: $\underbrace{[0, \tilde{\alpha})}_{\text{M1}}$, $\underbrace{[\tilde{\alpha}, \max\{\tilde{\alpha}, \bar{\alpha}\})}_{\text{K (opt-out)}}$, $\underbrace{[\max\{\tilde{\alpha}, \bar{\alpha}\}, x)}_{\text{J (binding)}}$, $\underbrace{[x, \infty)}_{\text{slack}}$.

The opt-out interval is non-empty iff $\tilde{\alpha} < \bar{\alpha}$; when $\tilde{\alpha} \geq \bar{\alpha}$ it degenerates to the empty set and the equilibrium has three regimes. Equilibrium prices in each regime are given by:

$$\left(p_{1,I}^{BP*},\, p_{2,I}^{BP*}\right) = \begin{cases} \left(\dfrac{\alpha_1+\beta\alpha_2+\theta}{2(1-\beta^2)},\, —\right) & \text{if } \alpha_2 \in [0, \tilde{\alpha}) \\ \left(\dfrac{\alpha_1+\beta\alpha_2}{2(1-\beta^2)},\, \dfrac{\alpha_2+\beta\alpha_1}{2(1-\beta^2)}\right) & \text{if } \alpha_2 \in [\tilde{\alpha},\, \max\{\tilde{\alpha}, \bar{\alpha}\}) \\ \left(\dfrac{\alpha_1+\alpha_2+\theta}{4(1-\beta)},\, \dfrac{\alpha_1+\alpha_2+\theta}{4(1-\beta)}\right) & \text{if } \alpha_2 \in [\max\{\tilde{\alpha}, \bar{\alpha}\},\, \alpha_1+\theta) \\ \left(\dfrac{\alpha_1+\theta+\beta\alpha_2}{2(1-\beta^2)},\, \dfrac{\alpha_2+\beta(\alpha_1+\theta)}{2(1-\beta^2)}\right) & \text{if } \alpha_2 \in [\alpha_1+\theta, \infty). \end{cases} \tag{26}$$

Q.E.D.

***Proof of Proposition** 3:* We previously calculated seller's optimal profits under single-homing on Platform 1 (M1) as: $\Pi^{M1} = \frac{(x+\beta\alpha_2)^2}{4(1-\beta^2)}$.

Similarly, seller's profit when multi-homing on two platforms while forfeiting Buy Box (K) and winning Buy Box with equal prices (J) equals: $\Pi^K = \frac{(\alpha_1+\alpha_2)^2-2(1-\beta)\alpha_1\alpha_2}{4(1-\beta^2)}$, $\Pi^J = \frac{(x+\alpha_2)^2}{8(1-\beta)}$.

Define the comparison functions

$$N_K(\alpha_2) \equiv 4(1-\beta^2)\left[\Pi^{M1} - \Pi^K\right] = -(1-\beta^2)\alpha_2^2 + 2\beta\theta\alpha_2 + \theta(\theta+2\alpha_1),$$
$$N_J(\alpha_2) \equiv 8(1-\beta^2)\left[\Pi^{M1} - \Pi^J\right] = 2(x+\beta\alpha_2)^2 - (1+\beta)(x+\alpha_2)^2.$$

From the proof of Proposition 2, we have the J–K envelope threshold $\bar{\alpha} \in [x/3, \alpha_1)$: K yields higher profit than J for $\alpha_2 < \bar{\alpha}$ and J for $\alpha_2 \geq \bar{\alpha}$. The threshold $\tilde{\alpha}$ is therefore the unique zero of N_K when $\tilde{\alpha} < \bar{\alpha}$ (K sub-regime, opt-out non-empty) and of N_J when $\tilde{\alpha} \geq \bar{\alpha}$ (J sub-regime, opt-out empty).

In both cases N crosses zero from above at $\tilde{\alpha}$. This follows from the global concavity of $D \equiv \Pi^{M1} -$ envelope on $[0, x)$: each piece of D has a negative leading coefficient in α_2, the pieces join continuously at $\bar{\alpha}$, and the slope of D falls at $\bar{\alpha}$ by $d(\Pi^J - \Pi^K)/d\alpha_2|_{\bar{\alpha}} > 0$, preserving global concavity.

Since $D(0) > 0$ and $D(\alpha_2) < 0$ for α_2 sufficiently close to x (as $\lim_{\alpha_2 \to x^-} D(\alpha_2) = -x^2/4 < 0$), the IVT gives at least one zero in $(0, x)$; and since D is concave with $D(0) > 0$, any two zeros $c_1 < c_2$ would satisfy $D(c_1) \geq D(0)(c_2 - c_1)/c_2 > 0$ by the chord inequality, contradicting $D(c_1) = 0$, so the zero is unique.

So $\left.\frac{\partial N}{\partial \alpha_2}\right|_{\tilde{\alpha}} < 0$. The Implicit Function Theorem then gives $\operatorname{sgn}\left(\frac{\partial \tilde{\alpha}}{\partial \lambda}\right) = \operatorname{sgn}\left(\left.\frac{\partial N}{\partial \lambda}\right|_{\alpha_2=\tilde{\alpha}}\right)$ for each parameter $\lambda \in \{\theta, \alpha_1, \beta\}$.

Parts (1) and (2). K sub-regime: $\partial N_K/\partial\theta = 2(\beta\tilde{\alpha} + \theta + \alpha_1) > 0$ and $\partial N_K/\partial\alpha_1 = 2\theta > 0$.

J sub-regime: Since $\partial x/\partial\theta = \partial x/\partial\alpha_1 = 1$, $\left.\frac{\partial N_J}{\partial\theta}\right|_{\tilde{\alpha}} = \left.\frac{\partial N_J}{\partial\alpha_1}\right|_{\tilde{\alpha}} = 2(1-\beta)(x-\tilde{\alpha}) > 0$,
using $\beta < 1$ and $\tilde{\alpha} \in (x/3, x)$. Hence $\partial\tilde{\alpha}/\partial\theta > 0$ and $\partial\tilde{\alpha}/\partial\alpha_1 > 0$ in both sub-regimes.

Part (3). K sub-regime: $\partial N_K/\partial\beta = 2\tilde{\alpha}(\beta\tilde{\alpha} + \theta) > 0$, so $\partial\tilde{\alpha}/\partial\beta > 0$.

J sub-regime: Using $N_J(\tilde{\alpha}) = 0$ to write $(x+\tilde{\alpha})^2 = 2(x+\beta\tilde{\alpha})^2/(1+\beta)$:

$$\left.\frac{\partial N_J}{\partial\beta}\right|_{\tilde{\alpha}} = 4\tilde{\alpha}(x+\beta\tilde{\alpha}) - (x+\tilde{\alpha})^2 = \frac{2(x+\beta\tilde{\alpha})}{1+\beta}\left[\tilde{\alpha}(2+\beta) - x\right].$$

The prefactor is positive. Substituting $\tilde{\alpha} = (k-1)x/(\beta-k)$, $k \equiv \sqrt{(1+\beta)/2}$, and $\beta = 2k^2 - 1$ into the bracket: $\tilde{\alpha}(2+\beta) - x = x \cdot \frac{(k-1)(2+\beta)-(\beta-k)}{\beta-k} = x \cdot \frac{2k(k-1)^2}{\beta-k}$.

Since $k > 0$ and $(k-1)^2 > 0$, the numerator $2k(k-1)^2 > 0$. For the denominator, $2\beta^2 - \beta - 1 = (2\beta + 1)(\beta - 1) < 0$ for $\beta \in [0,1)$, giving $\beta^2 < (1+\beta)/2 = k^2$, and since $k, \beta \geq 0$ this gives $k > \beta$, hence $\beta - k < 0$. Therefore $\tilde{\alpha}(2+\beta) - x < 0$, giving $\partial N_J/\partial\beta < 0$ and $\partial\tilde{\alpha}/\partial\beta < 0$. The sign of $\partial\tilde{\alpha}/\partial\beta$ is thus positive when $\tilde{\alpha} < \bar{\alpha}$ (opt-out non-empty) and negative when $\tilde{\alpha} \geq \bar{\alpha}$ (opt-out empty).

Proof of Proposition 4: We compare prices regime by regime for $\alpha_2 < \alpha_1 + \theta$, where Models BP and B differ. For $\alpha_2 \geq \alpha_1 + \theta$, both models yield identical prices and there is nothing to compare.

Regime $M1$ **(single-homing,** $\alpha_2 \in [0, \tilde{\alpha})$**).** Under Model BP the seller single-homes on Platform 1 at price $p_{1,I}^{BP*} = \frac{\alpha_1 + \beta\alpha_2 + \theta}{2(1-\beta^2)}$, which coincides exactly with the Model B Platform 1 price $p_{1,I}^{B*}$. Hence $p_{1,I}^{BP*} - p_{1,I}^{B*} = 0$, establishing the equality case in part (1). For Platform 2, the seller sets a prohibitively high price under Model BP so that $q_{2,I}^{BP*} = 0$, while under Model B the seller always multi-homes at the finite price $p_{2,I}^{B*} = (\alpha_2 + \beta(\alpha_1 + \theta))/[2(1-\beta^2)] < \infty$. Hence $p_{2,I}^{BP*} > p_{2,I}^{B*}$ throughout the single-homing regime, consistent with part (2) (see Remark 1).

Regime K **(opt-out,** $\alpha_2 \in [\tilde{\alpha}, \max\{\tilde{\alpha}, \bar{\alpha}\})$**).** Under Model BP the equilibrium prices are $p_{1,I}^{BP*} = \frac{\alpha_1 + \beta\alpha_2}{2(1-\beta^2)}$, $\quad p_{2,I}^{BP*} = \frac{\alpha_2 + \beta\alpha_1}{2(1-\beta^2)}$.

Comparing with Model B prices $p_{1,I}^{B*} = (\alpha_1 + \beta\alpha_2 + \theta)/[2(1-\beta^2)]$ and $p_{2,I}^{B*} = (\alpha_2 + \beta(\alpha_1 + \theta))/[2(1-\beta^2)]$:

$$p_{1,I}^{BP*} - p_{1,I}^{B*} = -\frac{\theta}{2(1-\beta^2)} < 0, \qquad p_{2,I}^{BP*} - p_{2,I}^{B*} = -\frac{\beta\theta}{2(1-\beta^2)} \leq 0.$$

Hence Platform 1 price is strictly lower under BP, establishing part (1) for this regime. Platform 2 price is also weakly lower under BP (strictly so when $\beta > 0$), which is the exception stated in part (2).

Regime J **(eligibility-binding,** $\alpha_2 \in [\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)$**).** Under Model BP the seller sets equal prices $p_{1,I}^{BP*} = p_{2,I}^{BP*} = p^J$ where $p^J = \frac{\alpha_1 + \alpha_2 + \theta}{4(1-\beta)}$.

Comparing with Model B prices: $p_{1,I}^{BP*} - p_{1,I}^{B*} = \frac{\alpha_1+\alpha_2+\theta}{4(1-\beta)} - \frac{\alpha_1+\beta\alpha_2+\theta}{2(1-\beta^2)} = \frac{(1-\beta)(\alpha_2 - (\alpha_1+\theta))}{4(1-\beta^2)} < 0$, since $\alpha_2 < \alpha_1 + \theta$, establishing part (1) for this regime. For Platform 2:

$p_{2,I}^{BP*} - p_{2,I}^{B*} = \frac{\alpha_1+\alpha_2+\theta}{4(1-\beta)} - \frac{\alpha_2+\beta(\alpha_1+\theta)}{2(1-\beta^2)} = \frac{(1-\beta)(\alpha_1+\theta-\alpha_2)}{4(1-\beta^2)} > 0$, since $\alpha_2 < \alpha_1 + \theta$, establishing part (2) for this regime.

Combining all three regimes completes the proof of both parts of Proposition 4.

Proof of Corollary 1: Let $x := \alpha_1 + \theta$. Under Model B, equilibrium demands are $q_{1,I}^{B*} = x/2$ and $q_{2,I}^{B*} = \alpha_2/2$ for all $\alpha_2 \geq 0$ (Proposition 1). For $\alpha_2 \geq x$, Models BP and B coincide and there is nothing to compare. For $\alpha_2 < x$, direct comparison of the Model BP equilibrium demands reported in Table 2 with the Model B benchmarks yields:

- *Regime* $M1$ ($\alpha_2 \in [0, \tilde{\alpha})$): $q_{1,I}^{BP*} - q_{1,I}^{B*} = \beta\alpha_2/2 \geq 0$ and $q_{2,I}^{BP*} - q_{2,I}^{B*} = -\alpha_2/2 \leq 0$.
- *Regime* K ($\alpha_2 \in [\tilde{\alpha}, \max\{\tilde{\alpha}, \bar{\alpha}\})$): $q_{1,I}^{BP*} - q_{1,I}^{B*} = -\theta/2 < 0$ and $q_{2,I}^{BP*} - q_{2,I}^{B*} = 0$.
- *Regime* J ($\alpha_2 \in [\max\{\tilde{\alpha}, \bar{\alpha}\}, x)$): $q_{1,I}^{BP*} - q_{1,I}^{B*} = (x - \alpha_2)/4 > 0$ and $q_{2,I}^{BP*} - q_{2,I}^{B*} = (\alpha_2 - x)/4 < 0$.

Both parts of the corollary follow immediately.

Proof of Proposition 5 For any price pair (p_1, p_2), the BP and B objectives satisfy

$$\begin{aligned} \Pi^{BP}(p_1, p_2) &= p_1(\alpha_1 - p_1 + \beta p_2 + \theta \cdot \mathbf{1}\{p_1 \leq p_2\}) + p_2(\alpha_2 - p_2 + \beta p_1) \\ &\leq p_1(\alpha_1 - p_1 + \beta p_2 + \theta) + p_2(\alpha_2 - p_2 + \beta p_1) = \Pi^{B}(p_1, p_2), \end{aligned} \tag{27}$$

since $\theta \cdot \mathbf{1}\{p_1 \le p_2\} \le \theta$ for all (p_1, p_2). Taking the maximum over all prices gives $\pi_I^{BP*} = \max_{p_1,p_2} \Pi^{BP}(p_1, p_2) \le \max_{p_1,p_2} \Pi^B(p_1, p_2) = \pi_I^{B*}$. It remains to show the inequality is strict for $\alpha_2 < \alpha_1 + \theta$. Under Model B the unique unconstrained maximizer satisfies $p_{1,I}^{B*} - p_{2,I}^{B*} = \frac{\alpha_1+\theta-\alpha_2}{2(1+\beta)} > 0 \quad$ for all $\alpha_2 < \alpha_1 + \theta$, so $p_1^{B*} > p_2^{B*}$. Let (p_1^{BP*}, p_2^{BP*}) denote the BP maximizer.

We consider two cases: **Case 1:** $p_1^{BP*} \le p_2^{BP*}$**.** Since Π^B is strictly concave and (p_1^{B*}, p_2^{B*}) is its unique maximizer with $p_1^{B*} > p_2^{B*}$, any price pair satisfying $p_1 \le p_2$ is strictly suboptimal under Model B: $\Pi^B(p_1^{BP*}, p_2^{BP*}) < \Pi^B(p_1^{B*}, p_2^{B*}) = \pi_I^{B*}$. Therefore, $\pi_I^{BP*} = \Pi^{BP}(p_1^{BP*}, p_2^{BP*}) \le \Pi^B(p_1^{BP*}, p_2^{BP*}) < \pi_I^{B*}$.
Case 2: $p_1^{BP*} > p_2^{BP*}$**.** Then $\mathbf{1}\{p_1^{BP*} \le p_2^{BP*}\} = 0$, so $\Pi^{BP}(p_1^{BP*}, p_2^{BP*}) < \Pi^B(p_1^{BP*}, p_2^{BP*}) \le \pi_I^{B*}$.

In both cases $\pi_I^{BP*} < \pi_I^{B*}$ for all $\alpha_2 < \alpha_1 + \theta$, completing the proof.

***Proof of Proposition** 6* Throughout, let $x = \alpha_1 + \theta$ and recall from Proposition 5 that $R_1^{B*}(\alpha_2) = \frac{(\alpha_1+\theta)(\alpha_1+\theta+\beta\alpha_2)}{4(1-\beta^2)}$.

Part 1: Single-homing regime $\alpha_2 \in [0, \tilde{\alpha})$**.** From Proposition 5, $R_1^{M1*} - R_1^{B*} = \frac{\beta\alpha_2(\alpha_1+\theta+\beta\alpha_2)}{4(1-\beta^2)} \ge 0$, with strict inequality whenever $\alpha_2 > 0$ and $\beta > 0$. Hence $R_{1,I}^{BP} \ge R_{1,I}^{B}$ on $[0, \tilde{\alpha})$.

Part 2: Eligibility-violated regime $\alpha_2 \in [\tilde{\alpha}, \max\{\tilde{\alpha}, \bar{\alpha}\})$**.** From Proposition 5, $R_1^{B*} - R_1^{K} = \frac{\theta(2\alpha_1+\beta\alpha_2+\theta)}{4(1-\beta^2)} > 0$, since $\theta > 0$ and $\alpha_1 + \beta\alpha_2 + \theta > 0$. Hence $R_{1,I}^{BP} < R_{1,I}^{B}$ strictly throughout this regime.

Part 3: Eligibility-binding regime $\alpha_2 \in [\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)$**.** From Proposition 5,

$$R_1^J - R_1^{B*} = \frac{(\alpha_1 + \theta - \alpha_2)\big[(3\beta - 1)(\alpha_1 + \theta) + (1 + \beta)\alpha_2\big]}{16(1 - \beta^2)}.$$

Since $\alpha_2 < \alpha_1 + \theta$, the factor $(\alpha_1 + \theta - \alpha_2) > 0$ on $[\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)$, so the sign of $R_1^J - R_1^{B*}$ is determined by the linear factor $L(\alpha_2) \equiv (3\beta - 1)(\alpha_1 + \theta) + (1 + \beta)\alpha_2$, which is strictly increasing in α_2 and has a unique zero $z = \frac{1-3\beta}{1+\beta}(\alpha_1 + \theta)$.

(Also note that $\bar{\alpha} \ge (\alpha_1 + \theta)/3$ by definition, hence $\max\{\tilde{\alpha}, \bar{\alpha}\} \ge (\alpha_1 + \theta)/3 > 0$ whenever $\alpha_1 + \theta > 0$, so $\alpha_2 = 0$ is not contained in $[\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)$.) Moreover, at $\alpha_2 = \alpha_1 + \theta$ the factor $(\alpha_1 + \theta - \alpha_2)$ vanishes, so $R_1^J = R_1^{B*}$ at $\alpha_2 = \alpha_1 + \theta$. *Case (a): $\beta \ge 1/3$ or $z < \max\{\tilde{\alpha}, \bar{\alpha}\}$.* If $\beta \ge 1/3$, then $(3\beta - 1) \ge 0$ and thus $L(\alpha_2) \ge 0$ for all $\alpha_2 \ge 0$. If instead $\beta < 1/3$ but $z < \max\{\tilde{\alpha}, \bar{\alpha}\}$, then for all $\alpha_2 \in [\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)$, $L(\alpha_2) \ge L(\max\{\tilde{\alpha}, \bar{\alpha}\}) > L(z) = 0$.

In both subcases $L(\alpha_2) > 0$ for all $\alpha_2 \in [\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)$, hence $R_1^J > R_1^{B*}$ throughout $[\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)$, with equality only at the boundary $\alpha_2 = \alpha_1 + \theta$ (where $\alpha_1 + \theta - \alpha_2 = 0$).

Case (b): $\beta < 1/3$ and $z = \max\{\tilde{\alpha}, \bar{\alpha}\}$. Then $L(\max\{\tilde{\alpha}, \bar{\alpha}\}) = L(z) = 0$ and $L(\alpha_2) > 0$ for all $\alpha_2 \in (\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)$. Hence $R_1^J \ge R_1^{B*}$ throughout $[\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)$, with equality at $\alpha_2 = \max\{\tilde{\alpha}, \bar{\alpha}\}$ and at $\alpha_2 = \alpha_1 + \theta$.

Case (c): $\beta < 1/3$ and $z \in (\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)$. Since $L(\max\{\tilde{\alpha}, \bar{\alpha}\}) < L(z) = 0$ and L is strictly increasing, we have $L(\alpha_2) < 0$ for $\alpha_2 \in [\max\{\tilde{\alpha}, \bar{\alpha}\}, z)$ and $L(\alpha_2) > 0$ for $\alpha_2 \in (z, \alpha_1 + \theta)$. Hence $R_1^J < R_1^{B*}$ on $[\max\{\tilde{\alpha}, \bar{\alpha}\}, z)$ and $R_1^J \ge R_1^{B*}$ on $[z, \alpha_1 + \theta)$, with equality at $\alpha_2 = z$ and at $\alpha_2 = \alpha_1 + \theta$.

Combining Parts 1–3 establishes the stated revenue comparisons across regimes.

***Proof of Proposition** 7* Under Model B, equilibrium quantities are $q_1^{B*} = (\alpha_1 + \theta)/2$ and $q_2^{B*} = \alpha_2/2$, giving $CS^B = \frac{(\alpha_1+\theta)^2+\alpha_2^2+2\beta(\alpha_1+\theta)\alpha_2}{8}$. We compare regime by regime for $\alpha_2 \in (0, \alpha_1 + \theta)$.

Regime $M1$ **(**$\alpha_2 \in [0, \tilde{\alpha})$**).** Under BP, $q_1^{M1*} = (\alpha_1 + \theta + \beta\alpha_2)/2$ and $q_2^{M1*} = 0$, so $CS^{BP} - CS^B = \frac{(\alpha_1+\theta+\beta\alpha_2)^2}{8} - \frac{(\alpha_1+\theta)^2+\alpha_2^2+2\beta(\alpha_1+\theta)\alpha_2}{8} = \frac{(\beta^2-1)\alpha_2^2}{8} \leq 0$, with strict inequality whenever $\alpha_2 > 0$ and $\beta < 1$. Hence $CS^{BP} < CS^B$ throughout this regime.

Regime K **(**$\alpha_2 \in [\tilde{\alpha}, \max\{\tilde{\alpha}, \bar{\alpha}\}$**).** Under BP, $q_1^K = \alpha_1/2$ and $q_2^K = \alpha_2/2$, so

$$CS^{BP} - CS^B = \frac{\alpha_1^2 + \alpha_2^2 + 2\beta\alpha_1\alpha_2}{8} - \frac{(\alpha_1+\theta)^2 + \alpha_2^2 + 2\beta(\alpha_1+\theta)\alpha_2}{8} = \frac{-\theta(2\alpha_1 + \theta + 2\beta\alpha_2)}{8} < 0,$$

since $\theta > 0$ and $2\alpha_1 + \theta + 2\beta\alpha_2 > 0$. Hence $CS^{BP} < CS^B$ throughout this regime.

Regime J **(**$\alpha_2 \in [\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)$**).** Under BP, $q_1^J = (3(\alpha_1+\theta) - \alpha_2)/4$ and $q_2^J = (3\alpha_2 - (\alpha_1+\theta))/4$, so

$$CS^{BP} = \frac{(3(\alpha_1+\theta) - \alpha_2)^2 + (3\alpha_2 - (\alpha_1+\theta))^2 + 2\beta(3(\alpha_1+\theta) - \alpha_2)(3\alpha_2 - (\alpha_1+\theta))}{32}.$$

$CS^{BP} - CS^B = \frac{(3(\alpha_1+\theta)-\alpha_2)^2+(3\alpha_2-(\alpha_1+\theta))^2+2\beta(3(\alpha_1+\theta)-\alpha_2)(3\alpha_2-(\alpha_1+\theta))}{32} - \frac{(\alpha_1+\theta)^2+\alpha_2^2+2\beta(\alpha_1+\theta)\alpha_2}{8}$.

Expanding the numerator of the combined expression yields $CS^{BP} - CS^B = \frac{3(1-\beta)(\alpha_1+\theta-\alpha_2)^2}{16} > 0$, since $\beta < 1$ and $\alpha_2 < \alpha_1 + \theta$. Hence $CS^{BP} > CS^B$ throughout this regime.

Combining all three regimes, $CS^{BP} > CS^B$ iff $\alpha_2 \in [\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)$, establishing the proposition.

Proof for Proposition 8 We can compare the sellers' prices under Model BP and Model B from their pricing strategy obtained in Proposition 13.

1. When *eligibility is binding:* For seller 1, we need to compare his prices under BP, $p_{1,II}^{BP}$, with that under Model B, $p_{1,II}^B$. Therefore, $p_{1,II}^{BP} - p_{1,II}^B = \frac{2[\alpha_2-(\alpha_1+\theta)]}{(4-\beta^2)} < 0$, since $\alpha_2 < \alpha_1 + \theta$.

   Similarly for seller 2, we need to compare his prices under BP with that under Model B. $p_{2,II}^{BP} - p_{2,II}^B = \frac{\beta[\alpha_2-(\alpha_1+\theta)]}{(4-\beta^2)} < 0$, since $\alpha_2 \leq \alpha_1 + \theta$.

2. When *eligibility is violated:* $p_{1,II}^B - p_{1,II}^{BP} = \frac{2\theta}{4-\beta^2}$, and $p_2^B - p_2^{BP} = \frac{\beta\theta}{4-\beta^2}$. Therefore, since $\theta \geq 0$, $p_i^B \geq p_i^{BP}$ for $i = 1, 2$ with equality if and only if $\theta = 0$.

   Therefore, under Buy Box with price parity, equilibrium prices on both platforms are lower than that under Buy Box without price parity model. Q.E.D.

Proof for Corollary 2 Let $D := 4 - \beta^2 = (2-\beta)(2+\beta)$.

1. **Case:** ***Eligibility-violated.*** For seller 1 (Platform 1), $q_{1,II}^B - q_{1,II}^{BP} = \frac{2(\alpha_1+\theta)+\beta\alpha_2-(2\alpha_1+\beta\alpha_2)}{D} = \frac{2\theta}{D} > 0$, so $q_1^{BP} < q_1^B$. For seller 2 (Platform 2), $q_{2,II}^B - q_{2,II}^{BP} = \frac{2\alpha_2+\beta(\alpha_1+\theta)-(2\alpha_2+\beta\alpha_1)}{D} = \frac{\beta\theta}{D} > 0$, so $q_2^{BP} < q_2^B$.

   Hence, relative to B, both sellers have lower quantities under BP *when eligibility is violated*.

2. **Case:** ***Eligibility-binding.*** For seller 2 (Platform 2), $q_{2,II}^B - q_{2,II}^{BP} = \frac{2\alpha_2+\beta(\alpha_1+\theta)}{D} - \frac{\alpha_2}{2-\beta} = \frac{\beta(\alpha_1+\theta-\alpha_2)}{(2-\beta)(2+\beta)}$. Since $(2-\beta)(2+\beta) > 0$, and $\alpha_1 + \theta > \alpha_2$, we obtain $q_{2,II}^{BP} < q_{2,II}^B$.

   For seller 1 (Platform 1), $q_{1,II}^B - q_{1,II}^{BP} = \frac{2(\alpha_1+\theta)+\beta\alpha_2}{D} - \left(\alpha_1 + \theta - \frac{(1-\beta)\alpha_2}{2-\beta}\right) = \frac{(2-\beta^2)(\alpha_2-(\alpha_1+\theta))}{(2-\beta)(2+\beta)}$, $\implies q_{1,II}^{BP} > q_{1,II}^B$ since $(2-\beta^2) > 0$ and $\alpha_1 + \theta > \alpha_2$. Q.E.D.

Proof for Proposition 9 Under Buy Box with price parity (BP), both sellers earn weakly lower profits than under B. Let $x := \alpha_1 + \theta$ and $D := 4 - \beta^2 > 0$. Under Buy Box only (B), equilibrium quantities satisfy $q_i^B = p_i^B$, hence $\Pi_{1,II}^B = \frac{(2x+\beta\alpha_2)^2}{D^2}$, $\Pi_{2,II}^B = \frac{(2\alpha_2+\beta x)^2}{D^2}$.

1. *Eligibility-violated.* Here, equilibrium profits under Buy Box with price parity are given by: $\Pi_{1,II}^{BP} = \frac{(2\alpha_1+\beta\alpha_2)^2}{D^2}$, $\Pi_{2,II}^{BP} = \frac{(2\alpha_2+\beta\alpha_1)^2}{D^2}$.

   For seller 1 (Platform 1), $\Pi_{1,II}^{B} - \Pi_{1,II}^{BP} = \frac{(2x+\beta\alpha_2)^2-(2\alpha_1+\beta\alpha_2)^2}{D^2} = \frac{4\theta\left(2\alpha_1+\beta\alpha_2+\theta\right)}{D^2} \geq 0$, strictly if $\theta > 0$.

   For seller 2 (Platform 2), $\Pi_{2,II}^{B} - \Pi_{2,II}^{BP} = \frac{(2\alpha_2+\beta x)^2-(2\alpha_2+\beta\alpha_1)^2}{D^2} = \frac{\beta\theta\left(4\alpha_2+2\beta\alpha_1+\beta\theta\right)}{D^2} \geq 0$, strictly if $\theta > 0$.

   Hence, $\Pi_{i,II}^{B} \geq \Pi_{i,II}^{BP}$ for $i = 1, 2$ when *eligbility is violated*.

2. *Eligibility-binding.* In this case, profits under Buy Box with Price Parity (BP) equals: $p_{1,II}^{BP} = p_{2,II}^{BP} = \frac{\alpha_2}{2-\beta}$, $q_{2,II}^{BP} = \frac{\alpha_2}{2-\beta}$, $q_{1,II}^{BP} = x - \frac{(1-\beta)\alpha_2}{2-\beta}$. Thus $\Pi_{2,II}^{BP} = \frac{\alpha_2^2}{(2-\beta)^2}$, $\Pi_{1,II}^{BP} = \frac{\alpha_2}{2-\beta}\left(x - \frac{(1-\beta)\alpha_2}{2-\beta}\right)$.

   For seller 1 (Platform 1), $\Pi_{1,II}^{BP} - \Pi_{1,II}^{B} = -\frac{(x-\alpha_2)\left(4x+\alpha_2(\beta^3+2\beta^2-4)\right)}{(2-\beta)^2(2+\beta)^2}$. Since $x \geq \alpha_2$ and $4x + \alpha_2(\beta^3 + 2\beta^2 - 4) = 4(x - \alpha_2) + \alpha_2(\beta^3 + 2\beta^2) \geq 0$, we obtain $\Pi_{1,II}^{BP} \leq \Pi_{1,II}^{B}$, with strict inequality if $x > \alpha_2$.

   For seller 2 (Platform 2), $\Pi_{2,II}^{BP} - \Pi_{2,II}^{B} = -\frac{\beta(x-\alpha_2)\left(\beta x+(4+\beta)\alpha_2\right)}{(2-\beta)^2(2+\beta)^2} \leq 0$, with strict inequality if $x > \alpha_2$.

   Thus, $\Pi_{i,II}^{BP} \leq \Pi_{i,II}^{B}$ for $i = 1, 2$.

All inequalities are strict whenever $\theta > 0$ and $\alpha_1 + \theta > \alpha_2$. Q.E.D.

***Proof of Proposition** 11* Under Model B, Platform 2 revenue is $R_2^{B*} = \frac{\alpha_2(\alpha_2+\beta(\alpha_1+\theta))}{4(1-\beta^2)}$ for all $\alpha_2 > 0$. We compare regime by regime for $\alpha_2 \in (0, \alpha_1 + \theta)$.

Regime $M1$ **(**$\alpha_2 \in [0, \tilde{\alpha})$**).** The seller does not participate on Platform 2 under BP, so $R_{2,I}^{BP} = 0 < R_2^{B*}$ for all $\alpha_2 > 0$.

Regime K **(**$\alpha_2 \in [\tilde{\alpha}, \max\{\tilde{\alpha}, \bar{\alpha}\})$**).** Platform 2 revenue under BP is $R_2^K = \frac{\alpha_2(\alpha_2+\beta\alpha_1)}{4(1-\beta^2)}$, so $R_2^{B*} - R_2^K = \frac{\alpha_2\beta\theta}{4(1-\beta^2)} \geq 0$, with strict inequality whenever $\beta > 0$ and $\alpha_2 > 0$.

Regime J **(**$\alpha_2 \in [\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)$**).** Platform 2 revenue under BP is $R_2^J = \frac{(\alpha_1+\theta+\alpha_2)(3\alpha_2-(\alpha_1+\theta))}{16(1-\beta)}$, so $R_2^{B*} - R_2^J = \frac{Q(\alpha_2)}{16(1-\beta^2)}$, where $Q(\alpha_2) = (1-3\beta)\alpha_2^2 + 2(\beta-1)(\alpha_1+\theta)\alpha_2 + (1+\beta)(\alpha_1+\theta)^2$.

We establish $Q(\alpha_2) > 0$ for all $\alpha_2 \in (0, \alpha_1 + \theta)$ and $\beta \in [0, 1)$ via the identity $Q(\alpha_2) = (\alpha_1 + \theta - \alpha_2)\big((3\beta - 1)\alpha_2 + (1+\beta)(\alpha_1+\theta)\big)$. For $\alpha_2 \in (0, \alpha_1 + \theta)$, $Q(\alpha_2) > 0$ throughout, implying $R_2^{B*} > R_2^J$ strictly.

Since for each $\alpha_2 \in (0, \alpha_1 + \theta)$ the equilibrium falls into one of the three regimes, and in each regime we have $R_2^{B*} \geq R_{2,I}^{BP}$ (strictly if $\beta > 0$), we conclude $R_{2,I}^{BP} \leq R_{2,I}^{B}$ for all $\alpha_2 \in (0, \alpha_1 + \theta)$, strictly when $\beta > 0$.

***Proof of Corollary** 4:* Total surplus satisfies $TS^m = CS^m + (1-\beta^2)\pi_I^{m*}$ for $m \in \{B, BP\}$. We use the regime-by-regime profit and CS comparisons established in Propositions 5 and 7.

Regime $M1$ **(**$\alpha_2 \in [0, \tilde{\alpha})$**).** From Proposition 5, $\pi^{BP} < \pi^B$, and from Proposition 7, $CS^{BP} < CS^B$. Hence $TS^{BP} < TS^B$ strictly.

Regime K **(**$\alpha_2 \in [\tilde{\alpha}, \max\{\tilde{\alpha}, \bar{\alpha}\})$**).** From Proposition 5, $\pi^{BP} < \pi^B$, and from Proposition 7, $CS^{BP} < CS^B$. Hence $TS^{BP} < TS^B$ strictly.

Regime J **(**$\alpha_2 \in [\max\{\tilde{\alpha}, \bar{\alpha}\}, \alpha_1 + \theta)$**).** From the profit comparison, $\pi^B - \pi^J = \frac{(\alpha_1+\theta-\alpha_2)^2}{8(1+\beta)}$, and from the CS comparison, $CS^{BP} - CS^B = \frac{3(1-\beta)(\alpha_1+\theta-\alpha_2)^2}{16}$.

Therefore, $TS^{BP} - TS^B = (CS^{BP} - CS^B) - (1-\beta^2)(\pi^B - \pi^J) = \frac{(1-\beta)(\alpha_1+\theta-\alpha_2)^2}{16}$, which is strictly positive since $\alpha_2 < \alpha_1 + \theta$.

Combining all three regimes establishes the corollary.

***Proof of Proposition** 12:* Under Model B, seller 1 and seller 2 have profit functions

$$\Pi_1(p_1,p_2) = p_1\big(\alpha_1 - p_1 + \beta p_2 + \theta\big), \qquad \Pi_2(p_1,p_2) = p_2\big(\alpha_2 - p_2 + \beta p_1\big).$$

Seller 1. Taking the derivative with respect to p_1, $\frac{\partial \Pi_1}{\partial p_1} = \alpha_1 - 2p_1 + \beta p_2 + \theta$. The first-order condition (FOC) implies $p_1(p_2) = \frac{\alpha_1+\theta+\beta p_2}{2}$.

Since $\frac{\partial^2 \Pi_1}{\partial p_1^2} = -2 < 0$, Π_1 is strictly concave in p_1, so the best response is unique.

Seller 2. Taking the derivative with respect to p_2, $\frac{\partial \Pi_2}{\partial p_2} = \alpha_2 - 2p_2 + \beta p_1$.

The FOC implies $p_2(p_1) = \frac{\alpha_2+\beta p_1}{2}$. Since $\frac{\partial^2 \Pi_2}{\partial p_2^2} = -2 < 0$, Π_2 is strictly concave in p_2, so the best response is unique.

Equilibrium. Solving the two best-response functions simultaneously, $p_1 = \frac{\alpha_1+\theta+\beta\left(\frac{\alpha_2+\beta p_1}{2}\right)}{2}$. Rearranging, $4p_1 = 2(\alpha_1+\theta) + \beta\alpha_2 + \beta^2 p_1$, which implies $p_1^* = \frac{2(\alpha_1+\theta)+\beta\alpha_2}{4-\beta^2}$.

Substituting into seller 2's best response yields $p_2^* = \frac{2\alpha_2+\beta(\alpha_1+\theta)}{4-\beta^2}$. Since $4-\beta^2 > 0$ under $0 \le \beta < 1$, the linear best-response system has a unique solution. Hence the equilibrium prices are $\left(p_{1,II}^{B*}, p_{2,II}^{B*}\right) = \left(\frac{2(\alpha_1+\theta)+\beta\alpha_2}{4-\beta^2}, \frac{2\alpha_2+\beta(\alpha_1+\theta)}{4-\beta^2}\right)$.

Proof for Proposition 13 We can write the profits for seller 1 and 2 as:

$$\Pi_1(p_1,p_2) = p_1(\alpha_1 - p_1 + \beta p_2 + \theta \mathbf{1}_{p_1 \le p_2}),\ \Pi_2(p_1,p_2) = p_2(\alpha_2 - p_2 + \beta p_1)$$

Moreover, for any price p_1, we can find seller 2's best response as: $\frac{\partial \Pi_2}{\partial p_2} = 0 \implies p_2(p_1) = \frac{\alpha_2+\beta p_1}{2}$.

Then, for any price p_2 set by the seller, Seller 1's best response can be characterized as:

$$p_1(p_2) = \arg\max \left\{ \max_{p_1 \le p_2} p_1(\alpha_1 - p_1 + \beta p_2 + \theta), \max_{p_1 > p_2} p_1(\alpha_1 - p_1 + \beta p_2) \right\}.$$

Let p_1^0 denote the unconstrained maximizer of $p_1(\alpha_1 - p_1 + \beta p_2 + \theta)$ then using the first order condition, we have $p_1^0 = \frac{\alpha_1+\beta p_2+\theta}{2}$.

1. If $p_1^0 \le p_2$, (*i.e.*, $p_2 \ge \frac{\alpha_1+\theta}{2-\beta}$), then $p_1(p_2) = p_1^0 = \frac{\alpha_1+\beta p_2+\theta}{2}$.
2. Otherwise, when $p_2 < \frac{\alpha_1+\theta}{2-\beta}$. It suffices to compare $R(p_2,\alpha_1,\beta,\theta) := p_2\left(\alpha_1 - p_2 + \beta p_2 + \theta\right)$ vs. $S(p_2,\alpha_1,\beta) := \max_{p_1 > p_2} p_1(\alpha_1 - p_1 + \beta p_2)$.

   Let $p_1^\Delta := \operatorname{argmax}_{p_1} p_1(\alpha_1 - p_1 + \beta p_2)$ be the unconstrained maximizer then $p_1^\Delta = \frac{\alpha_1+\beta p_2}{2}$.

   So, if $p_1^\Delta \le p_2$, (*i.e.*, $\frac{\alpha_1}{2-\beta} \le p_2$), then $p_1(p_2) = p_2$ and we have the best response strategy for player 1, since $R(p_2,\alpha_1,\beta,\theta) > S(p_2,\alpha_1,\beta)$. Otherwise, for $p_2 < \frac{\alpha_1}{2-\beta}$, we need to compare Seller 1 profits under the two strategies: pricing equal to seller 2's price, or pricing above Seller 2. In the former case, his profit equals $R(p_2,\alpha_1,\beta,\theta) = p_2(\alpha_1 - p_2 + \beta p_2 + \theta)$, and under latter his profit equals $S(p_2,\alpha_1,\beta) = \frac{(\alpha_1+\beta p_2)^2}{4}$.

The difference between the two profit functions, $R(p_2,\alpha_1,\beta,\theta) - S(p_2,\alpha_1,\beta) = -\frac{1}{4}(2-\beta)^2 p_2^2 + \frac{1}{2}(2-\beta)\alpha_1 p_2 + \theta p_2 - \frac{\alpha_1^2}{4}$, increases over $p_2 \in \left[0, \frac{\alpha_1}{2-\beta}\right]$, and changes sign from negative to positive only once.

Let $\Delta \in [0, \frac{\alpha_1}{2-\beta}]$ be the solution to $R - S = 0$, that is, when Seller 2 sets price as Δ, Seller 1 is indifferent between setting the same price as seller 2 or adopting the strategy of pricing more than seller

2. $R(p_2,\alpha_1,\beta,\theta)\Big|_{p_2=\Delta} - S(p_2,\alpha_1,\beta)\Big|_{p_2=\Delta} = 0 \implies \alpha_1 = (2-\beta)\Delta + 2\sqrt{\theta\Delta}$. Also, $R(p_2,\alpha_1,\beta,\theta)\Big|_{p_2=0} - S(p_2,\alpha_1,\beta)\Big|_{p_2=0} = -\frac{\alpha_1^2}{4} < 0$.

Therefore, we can characterize Seller 1's best response as:

$$p_1(p_2) = \begin{cases} p_1^{\Delta} = \frac{\alpha_1+\beta p_2}{2} & \text{for } p_2 \in [0,\Delta), \\ p_2 & \text{for } p_2 \in \left[\Delta, \frac{\alpha_1+\theta}{2-\beta}\right), \\ p_1^{0} = \frac{\alpha_1+\beta p_2+\theta}{2} & \text{for } p_2 \geq \frac{\alpha_1+\theta}{2-\beta}. \end{cases}$$

And, we had previously found Seller 2's best response as $p_2(p_1) = \frac{\alpha_2+\beta p_1}{2}$. There are only three pure-strategy candidates for Nash Equilibrium, $\left(p_1^I, p_2^I\right)$, $\left(p_1^{II}, p_2^{II}\right)$, or $\left(p_1^{III}, p_2^{III}\right)$ where each of them is given by:
$\left(p_1^I, p_2^I\right) = \left(\frac{\alpha_1+\beta p_2^I}{2}, \frac{\alpha_2+\beta p_1^I}{2}\right) \Rightarrow \left(p_1^I, p_2^I\right) = \left(\frac{2\alpha_1+\beta\alpha_2}{4-\beta^2}, \frac{2\alpha_2+\beta\alpha_1}{4-\beta^2}\right)$.
$\left(p_1^{II}, p_2^{II}\right) = \left(p_2^{II}, \frac{\alpha_2+\beta p_1^{II}}{2}\right) \Rightarrow \left(p_1^{II}, p_2^{II}\right) = \left(\frac{\alpha_2}{2-\beta}, \frac{\alpha_2}{2-\beta}\right)$.
$\left(p_1^{III}, p_2^{III}\right) = \left(\frac{\alpha_1+\beta p_2^{III}+\theta}{2}, \frac{\alpha_2+\beta p_1^{III}}{2}\right) \Rightarrow \left(p_1^{III}, p_2^{III}\right) = \left(\frac{2(\alpha_1+\theta)+\beta\alpha_2}{4-\beta^2}, \frac{2\alpha_2+\beta(\alpha_1+\theta)}{4-\beta^2}\right)$.
However,

$$\left(p_1^I, p_2^I\right) \text{ is the only NE } \Leftrightarrow p_2^I < \Delta, \textit{ i.e.,} \frac{2\alpha_2+\beta\alpha_1}{4-\beta^2} < \Delta. \quad (1)$$

$$\left(p_1^{II}, p_2^{II}\right) \text{ is the only NE } \Leftrightarrow \Delta < p_2^{II} < \frac{\alpha_1+\theta}{2-\beta}, \textit{ i.e.,} \Delta < \frac{\alpha_2}{2-\beta} < \frac{\alpha_1+\theta}{2-\beta}. \quad (2)$$

$$\left(p_1^{III}, p_2^{III}\right) \text{ is the only NE } \Leftrightarrow p_2^{III} > \frac{\alpha_1+\theta}{2-\beta}, \textit{ i.e.,} \alpha_2 > \alpha_1+\theta. \quad (3)$$

We do not have multiple equilibrium since none of the conditions overlap, moreover, we can restate conditions 1 and 2 as: Condition 1 $\Leftrightarrow \alpha_2 < \frac{(4-\beta^2)\Delta-\beta\alpha_1}{2}$, & Condition 2 $\Leftrightarrow (2-\beta)\Delta \leq \alpha_2 \leq \alpha_1+\theta$.

Since $(2-\beta)\Delta - \frac{(4-\beta^2)\Delta-\beta\alpha_1}{2} = \frac{\beta(\alpha_1-(2-\beta)\Delta)}{2} > 0$, there is no Nash Equilibrium if $\alpha_2 \in \left[\frac{(4-\beta^2)\Delta-\beta\alpha_1}{2}, (2-\beta)\Delta\right)$. Q.E.D.

Proof for Proposition 10 Let $x := \alpha_1+\theta$, $D := 4-\beta^2$ and we have $x \geq \alpha_2$. We will compare consumer surplus under Buy Box with price parity relative to the Buy Box only benchmark (B) under two regimes:

1. *Eligbility Binding:* $CS_{II}^{BP} \geq CS_{II}^{B}$, with strict inequality if $\alpha_1+\theta > \alpha_2$.

   Consumer surplus for any price vector (p_1,p_2) admits the representation given in equation (18) and since $CS(p_1,p_2)$ is the value function of the consumer's maximization problem, the envelope theorem implies $\frac{\partial CS}{\partial p_1}(p_1,p_2) = -(1-\beta^2)q_1(p_1,p_2)$, $\frac{\partial CS}{\partial p_2}(p_1,p_2) = -(1-\beta^2)q_2(p_1,p_2)$.

   In the interior regime ($q_1,q_2>0$), hence CS is strictly decreasing in each price.

   Equilibrium prices under Buy Box only (B) and Buy Box with Price Parity (BP) are $p_{1,II}^{B} = \frac{2x+\beta\alpha_2}{D}$, $p_{2,II}^{B} = \frac{2\alpha_2+\beta x}{D}$, $p_{1,II}^{BP} = p_{2,II}^{BP} = \frac{\alpha_2}{2-\beta} = \frac{\alpha_2(2+\beta)}{D}$. Hence, $p_{1,II}^{B} - p_{1,II}^{BP} = \frac{2(x-\alpha_2)}{D} \geq 0$, $p_{2,II}^{B} - p_{2,II}^{BP} = \frac{\beta(x-\alpha_2)}{D} \geq 0$, with at least one strict inequality whenever $x > \alpha_2$.

   Since CS is decreasing in each price coordinate, it follows that $CS_{II}^{BP} \geq CS_{II}^{B}$, strictly if $\alpha_1+\theta > \alpha_2$.

2. *Eligbility Violated:* $CS_{II}^{B} \geq CS_{II}^{BP}$, with strict inequality if $\theta > 0$.

   Substituting the equilibrium prices for BP (when eligibility is violated) and B into the expression for consumer surplus (equation 19) and simplifying yields $CS_{II}^{B} - CS_{II}^{BP} =$

$\frac{\theta\left(10\alpha_1\beta^2+8\alpha_1+2\alpha_2\beta^3+16\alpha_2\beta+5\beta^2\theta+4\theta\right)}{2(2-\beta)^2(2+\beta)^2}$. For $0 \leq \beta < 1$, denominator is strictly positive. Also, the term in parentheses in the numerator is strictly positive for $\theta > 0$, $\alpha_1 > 0$, and $\alpha_2 \geq 0$. Therefore, $CS_{II}^{B} - CS_{II}^{BP} > 0$ for $\theta > 0$, with equality only when $\theta = 0$. Q.E.D.

Proof for Corollary 5 Let $x := \alpha_1 + \theta$, $D := 4 - \beta^2 = (2-\beta)(2+\beta)$, and define total surplus as $TS = CS + (1-\beta^2)(\Pi_1 + \Pi_2)$, so that $TS^{BP} - TS^{B} = (CS^{BP} - CS^{B}) + (1-\beta^2)(\Pi_1^{BP} - \Pi_1^{B}) + (1-\beta^2)(\Pi_2^{BP} - \Pi_2^{B})$.

1. *Eligibility-violated.* By Proposition 10, $CS^{BP} - CS^{B} < 0$ whenever $\theta > 0$. By Proposition 9, $\Pi_1^{BP} - \Pi_1^{B} < 0$, and $\Pi_2^{BP} - \Pi_2^{B} \leq 0$ whenever $\theta > 0$. Therefore, $TS^{BP} - TS^{B} = (CS^{BP} - CS^{B}) + (1-\beta^2)(\Pi_1^{BP} - \Pi_1^{B}) + (1-\beta^2)(\Pi_2^{BP} - \Pi_2^{B}) < 0$, since at least two terms are strictly negative. Hence $TS^{B} > TS^{BP}$ strictly whenever $\theta > 0$.

2. *Eligibility-binding.* Here the CS and profit effects oppose each other -CS rises under BP while both profits fall - so additivity alone does not determine the sign. Instead we work directly with the quantity representations. Let $x := \alpha_1 + \theta$, $p^* := \frac{\alpha_2}{2-\beta}$. By equation 19, consumer surplus admits the quadratic quantity representation $CS = \frac{1}{2}\left(q_1^2 + 2\beta\, q_1 q_2 + q_2^2\right)$.

   Using this representation and $\Pi_k = p_k q_k$: $TS^{BP} = \frac{1}{2}\left[(q_1^{BP})^2 + 2\beta\, q_1^{BP} q_2^{BP} + (q_2^{BP})^2\right] + (1-\beta^2)p^*(q_1^{BP} + q_2^{BP})$, where $q_1^{BP} = x - (1-\beta)p^*$, $q_2^{BP} = p^*$, and $q_1^{BP} + q_2^{BP} = x + \beta p^*$.

   Under Model B, $p_k^{B} = q_k^{B}$, so $\Pi_k^{B} = (q_k^{B})^2$ and: $TS^{B} = \frac{1}{2}\left[(q_1^{B})^2 + 2\beta\, q_1^{B} q_2^{B} + (q_2^{B})^2\right] + (1-\beta^2)(q_1^{B})^2 + (1-\beta^2)(q_2^{B})^2$, where $q_1^{B} = \frac{2x+\beta\alpha_2}{D}$ and $q_2^{B} = \frac{2\alpha_2+\beta x}{D}$.

   Substituting and simplifying, we get $TS^{BP} \geq TS^{B}$, and the equality holds iff $(x - \alpha_2) = 0$. Q.E.D.

***Proof of Corollary** 3:* Platform 1's commission revenue equals $p_{1,II}^{m*} q_{1,II}^{m*} = \Pi_{1,II}^{m*}$ for $m \in \{B, BP\}$, since Seller 1 has no marginal cost. Platform 1's revenue is strictly lower under Model BP than under Model B in both the eligibility-binding and eligibility-violated regimes. The claim follows from Proposition 9 and the coincidence of Models B and BP in the slack regime.