When does the algorithm beat data? Gaining momentum in dynamic platform competition*

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Abstract

We consider a dynamic model with two horizontally differentiated platforms that serve heterogeneous users. In each period, platforms collect data from users that improve their services and remain useful in later periods, subject to depreciation. The incumbent platform starts with a larger initial data base, while the entrant platform has a higher standalone quality. Holding platform policies fixed, we derive an explicit law of motion based on a single condition on parameters that determines which platform has *momentum*, i.e., gains market share in the short-run. The entrant can gain share while it remains smaller and collects less new data, because the incumbent's relative data advantage depreciates. Weak data feedback leads to a unique long-run steady state, which may involve stable coexistence or entrant monopoly. With stronger feedback, three steady states are possible: incumbent monopoly, entrant monopoly and unstable coexistence. The same condition that determines short-run momentum also selects unique long-run equilibrium. Greater product differentiation makes it easier for incumbent to gain market share, but can also sustain coexistence. A policy that mandates reciprocal data sharing between platforms leads to long-run coexistence, with both platforms being active in the long run.

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1 Introduction

In today's digital economy, platforms increasingly rely on user data accumulation to enhance their offerings---through personalization, algorithmic refinement, and predictive services. Unlike traditional network effects, where value is created by interactions among users, data accumulation can be leveraged across users and over time, even after users leave. This turns past user participation into an advantage. potentially improving its service and attracting more users in future periods. For example, search engines can learn from past queries and users' clicks to improve results for subsequent users. Similarly, recommendation services can use viewing or listening behavior to refine suggestions for other users, while generative AI services can use feedback on earlier responses to improve their answers to future queries.

An inherited data advantage, however, need not be self-sustaining. As older data lose relevance, maintaining that advantage requires a continuing flow of new data. A platform can therefore be the market leader while its relative data advantage erodes. This raises the questions: When do large platforms enjoy momentum where data accumulation enables them to continuously grow their market share? Under what conditions do small platforms gain momentum? Moreover, how does the evolution of the platforms' market share shape long-run market outcomes? In particular, do platforms with an early lead in market share always end up dominating in the long run?

This paper explores the dynamics of platform market share momentum in data-enabled environments. We present a dynamic model with heterogeneous users in which the market shares of two horizontally differentiated platforms evolve over time. Each platform improves its service by accumulating user data, but this data depreciates over time. Thus, the value of a platform's service at any point in time reflects not only its base quality but also the stock of user data it has acquired---even if those users are no longer with the platform. Base quality captures the appeal of a platform's service independently of its accumulated user data. It may come, for example, from the quality of its underlying algorithm. The incumbent enters the analysis with an accumulated data stock, while the entrant offers higher base quality.

We hold platform policies fixed and set user prices to zero. By abstracting from user pricing, our model isolates the role data accumulation and product-quality differences play in determining the evolution of market shares. Users choose between services based on their own preferences and platforms' current quality, while their participation generates data that shapes future service quality. We derive an explicit law of motion for the data stocks and use it to determine which steady state the market approaches from given initial conditions. Although multiple steady states may exist, the dynamics select a unique long-run outcome, which can involve either coexistence or monopoly. This provides a tractable framework for assessing how initial data advantages, and base quality advantage affect market evolution.

We show that even when a platform has a larger market share than its competitor and can exploit it to collect more data, it does not necessarily gain momentum over time. To preserve its data advantage, the incumbent must collect enough additional data to offset the depreciation of its lead over the entrant. Collecting more new data than the entrant is insufficient if the difference does not compensate for this depreciation. The entrant can consequently gain share even while it remains smaller and collects less new data. Its persistent quality advantage and the erosion of the incumbent's data advantage jointly support its momentum.

Whether the incumbent gains or loses ground market share depends on how much users benefit from each additional unit of data. We identify a single threshold that separates these two possibilities: when the data benefit exceeds the threshold, the incumbent gains share; below it, the entrant gains share; exactly at the threshold, market shares remain constant. This threshold depends only on the primitives: the platforms' quality difference, the initial data stock, how quickly data depreciates, and the strength of horizontal preferences and contemporaneous network effects. For example, stronger horizontal differentiation raises the threshold, to the entrant's favor. Users who strongly prefer the entrant's service are less easily drawn away by the incumbent's data advantage. Their continued participation helps the entrant gain momentum over a wider range of values of data.

Knowing which platform gains share does not yet tell us how the market will look in the long run. The entrant's gains may diminish as the platforms approach stable coexistence, or they may lead to monopoly. Our law of motion distinguishes these outcomes. When users benefit little from additional data, the market converges to a unique steady state, where the entrant's base-quality advantage secures it a positive market

share and possibly monopoly. The incumbent's initial data advantage cannot change this eventual outcome, although a larger initial stock can delay the entrant's growth. When users benefit more from data, either the entrant's base-quality advantage or the incumbent's data advantage can sustain dominance, and multiple steady states may exist. In this case, the same threshold that determines short-run momentum also selects the long-run outcome: a data benefit below the threshold leads to entrant dominance; above it, to incumbent dominance; at equality, market shares remain constant. This threshold depends on the incumbent's initial data stock. Thus, even as the original data depreciate away, they can leave a lasting effect through the users they attract and the new data those users generate.

Our model offers important policy insights. In particular, while mandating a one-time data transfer to entrants, at the time of entry, eliminates the incumbent's informational head start and ensures that the highest-quality platform dominates, this outcome is not efficient because users with stronger preferences for one of the platforms are served by the other one. In contrast, continuous reciprocal data-sharing requirement implements the welfare maximizing outcome where both platforms maintain constant market shares, and the entrant–being the superior quality platform–holds a larger market share.

2 Related Literature

Our model integrates persistent yet depreciating data, heterogeneous user preferences, and horizontal product differentiation into a unified dynamic framework. We contribute to a growing body of work that examines how accumulated user data improves platform quality over time and drives learning. Hagiu and Wright (2023) develop a model of data-enabled learning with homogeneous users and all-or-nothing competition. They find that an increase in the strength of network effects reduces the incumbent advantage because entrants can catch up more quickly. Our model departs from theirs by incorporating heterogeneous users and horizontal differentiation, allowing for interior equilibria in which both platforms survive or in which either the entrant or the incumbent can gain momentum. Moreover, we find that in contrast to the case where competition is all-or-nothing, when users are heterogeneous and data persists beyond user participation, an increase in the strength of network effects strengthens the incumbent's grip.

Our analysis connects to the broader literature on platform competition with network effects (Katz and Shapiro, 1985; Farrell and Saloner, 1985; see Jullien et al.,

2021, for a review of the literature). Hałaburda and Yehezkel (2013; 2016; 2019), and Markovich and Yehezkel (2022; 2024) consider platform competition and coordination in the context of a static game. Hałaburda et al. (2020), and Biglaiser and Crémer (2020) consider dynamic competition. This literature shows that because of network effects, there are multiple equilibria, depending on users beliefs regarding the decisions of other users to which platform to join. A key difference between this literature and our paper is that in our setting with data accumulation, users' value from the platform depends not only on its current user base but also on data from past users even if they are no longer on the platform. We contribute to this literature by showing how dynamics and the platforms' momentum over time serve as an equilibrium selection. In particular, while we find that in the long-run there are multiple steady-state equilibria, our analysis of the short-run dynamics and platforms' momentum shows that the market converges to a unique long-run steady state.

Our momentum results contribute to the literature on dynamic competition with switching and learning (Cabral, 2011) and connect to prior work on market tipping and path dependence. Markovich (2008) examines dynamic competition in differentiated markets and shows how market structure can evolve toward monopoly or coexistence depending on switching costs and network effects. Similar to that work, we focus on dynamic trajectories and thresholds that determine whether an entrant can overcome an incumbent's advantage; however, our model replaces switching costs with persistent data-driven advantages.

3 The Model

Environment and timing.

Two horizontally differentiated platforms, E (entrant) and I (incumbent), compete in infinite discrete time $t = 0, 1, 2, \ldots$.

Platform I is active from $t = 0$, and E enters at $t = 1$. In each period $t \geq 1$, users choose between the two platforms, and each platform collects data from its current users. These data improve its service in the same period; yet, the value of data depreciates over time. Platforms provide their services for free and earn payoffs that increase with their number of users (e.g., through advertising or data revenues). Hence, each platform's objective is market-share maximization.

Users, Hotelling preferences, and market shares.

A unit mass of users is uniformly distributed on $[0,1]$. Platform E is located at 0 and I at 1. If a user of type $x \in [0,1]$ joins in period t, their utilities from joining each platform are

$$U_{E,t}(x) = Q + q + \beta n_{E,t} + \lambda D_{E,t} - \theta x, \qquad U_{I,t}(x) = Q + \beta n_{I,t} + \lambda D_{I,t} - \theta(1-x).$$

Here Q is large enough to ensure full coverage; $q \in (0,1)$ is E's persistent quality advantage; $\theta > 0$ governs horizontal differentiation; $\beta \geq 0$ measures the same-period network effects; and $n_{i,t}$ is the number of users on platform i in period t. The data the platform has available in period t is represented by $D_{i,t}$, and $\lambda > 0$ represents the platform's learning from this data. Users choose a platform in each period to maximize that period's utility; each user is small and therefore does not take into account how their choice affects future data stocks. Throughout the main analysis we assume $\theta - \lambda > \beta$ to ensure the demand system is stable (i.e., network effects do not dominate horizontal differentiation).

Let $x_t \in [0,1]$ denote E's share in period t. Because users are uniformly distributed and the market is fully covered, $n_{E,t} = x_t$ and $n_{I,t} = 1 - x_t$. When both platforms are active, x_t is the location of the user who is indifferent between them: users to the left of x_t join E, and users to its right join I.

Data as a persistent (depreciating) asset.

Let $D_{i,t}$ denote platform i's data stock in t. Each period, platforms collect data from their current users and retain a fraction $\gamma \in (0,1)$ of their previous stock, so that data depreciates at rate $1-\gamma$:

$$D_{i,t} = n_{i,t} + \gamma D_{i,t-1}, \qquad i \in \{E, I\}.$$

At entry, I has an initial stock $D_{I,0} = D_0 > 0$ and E has $D_{E,0} = 0$. Therefore, D_0 measures the incumbent's initial data advantage.

Virtual system and clipping.

To characterize the dynamics, we first study a virtual system in which the law of motion that determines the indifferent user when both platforms are active applies in

every period, even when it implies a share outside $[0,1]$. We denote the virtual share by $\tilde{x}_t$. The virtual system is an analytical device: starting from an interior allocation, the actual and virtual shares coincide as long as $\tilde{x}_t \in (0,1)$. Once the virtual share leaves $[0,1]$, the actual share is set at the corresponding boundary, and data accumulate from this actual allocation. We refer to this adjustment as *clipping*. Section 6 shows that a boundary reached from an interior allocation is absorbing, so the actual path is the virtual path clipped at the boundaries, $x_t = \min\{1, \max\{0, \tilde{x}_t\}\}$.

We start the analysis with two definitions. Our central state variable is *momentum*:

Definition 1. ***Momentum*** *is the period-to-period change in E's market share:*

$$\mu_t \equiv x_{t+1} - x_t.$$

We say that E (I) has momentum when $\mu_t > 0$ (< 0). In the virtual system, momentum is $\tilde{\mu}_t \equiv \tilde{x}_{t+1} - \tilde{x}_t$.

A platform that has momentum increases its market share over time. Because the market is fully covered, in each period at most one of the platforms can have momentum. Momentum determines the market short-run trajectory: which platform increases its market share over time. As we show in Section 6, this statistic will tie short-run trajectories to long-run equilibrium selection.

Definition 2. *We say that momentum* ***accelerates*** *in t if $|\mu_{t+1}| > |\mu_t|$ and* ***decelerates*** *in t if $|\mu_{t+1}| < |\mu_t|$.*

The magnitude of momentum may change over time, in which case momentum accelerates or decelerates. As we show below, this statistic determines the speed at which short-run dynamics converge to the long-run equilibrium.

4 Short-run dynamics: the direction and persistence of momentum

This section characterizes momentum in the virtual system. We study how the platforms' market shares evolve over time in the short-run, starting from the first period onward (the next section considers the long-run steady states) We show that a single threshold in the value of data determines which platform has momentum, that momen-

tum never changes direction, and that its magnitude changes at a constant rate over time. Together, these results pin down the entire short-run trajectory of market shares.

Consider any period $t \geq 1$ in which both platforms are active. The indifferent user, $\widetilde{x}_t$, solves

$$Q + q - (\theta - \beta)\widetilde{x}_t + \lambda(\widetilde{x}_t + \gamma D_{E,t-1}) = Q - (\theta - \beta)(1 - \widetilde{x}_t) + \lambda(1 - \widetilde{x}_t + \gamma D_{I,t-1}), \quad (1)$$

hence:

$$\widetilde{x}_t = \frac{1}{2} + \frac{q}{2(\theta - \beta - \lambda)} - \frac{\lambda\gamma(D_{I,t-1} - D_{E,t-1})}{2(\theta - \beta - \lambda)}. \quad (2)$$

Intuitively, platform E gains a higher market share than platform I ($\widetilde{x}_t > 1/2$), if its relative quality advantage, q, is large and platform I's relative data-advantage, $D_{I,t-1} - D_{E,t-1}$, is low. Equation (2) defines the virtual share $\tilde{x}_t$ in every period and shows why same-period network effects, β, alone do not generate momentum. Specifically, when $\lambda = 0$, $\widetilde{x}_t = \frac{1}{2} + \frac{q}{2(\theta-\beta)}$ in every period, regardless of β. Momentum requires data to remain valuable after the period in which they are collected. Same-period network effects then amplify users' response to such data, through the term $\theta - \beta - \lambda$.

Using (2) recursively, we can express $\widetilde{x}_{t+1}$ as a function of the platforms' relative data advantage, $D_{I,t-1} - D_{E,t-1}$:

$$\widetilde{x}_{t+1} = \frac{1}{2} + \frac{q(\theta - \beta - \lambda(1 - \gamma))}{2(\theta - \beta - \lambda)^2} - \frac{\lambda\gamma^2(\theta - \beta)(D_{I,t-1} - D_{E,t-1})}{2(\theta - \beta - \lambda)^2}. \quad (3)$$

We can now state the following result:

Theorem 1. ***(Momentum-direction threshold)*** *In the virtual system, the entrant has momentum if learning from data, λ, is low, while the incumbent has momentum otherwise. That is, there exists a momentum-direction threshold $\overline{\lambda}$*

$$\overline{\lambda} \equiv (\theta - \beta)(1 - \gamma) + \frac{q}{D_0},$$

such that $\tilde{\mu}_t$'s sign is time-invariant

$$\text{sign}(\tilde{\mu}_t) = \text{sign}(\bar{\lambda} - \lambda) \qquad \textit{for all } t \geq 1$$

Thus, $\lambda < \bar{\lambda}$ implies $\tilde{\mu}_t > 0$ (entrant has momentum), $\lambda > \bar{\lambda}$ implies $\tilde{\mu}_t < 0$ (incumbent has momentum), and $\lambda = \bar{\lambda}$ implies $\tilde{\mu}_t = 0$ (market shares are stationary).

Importantly, Theorem 1 implies that the question of which platform has momentum

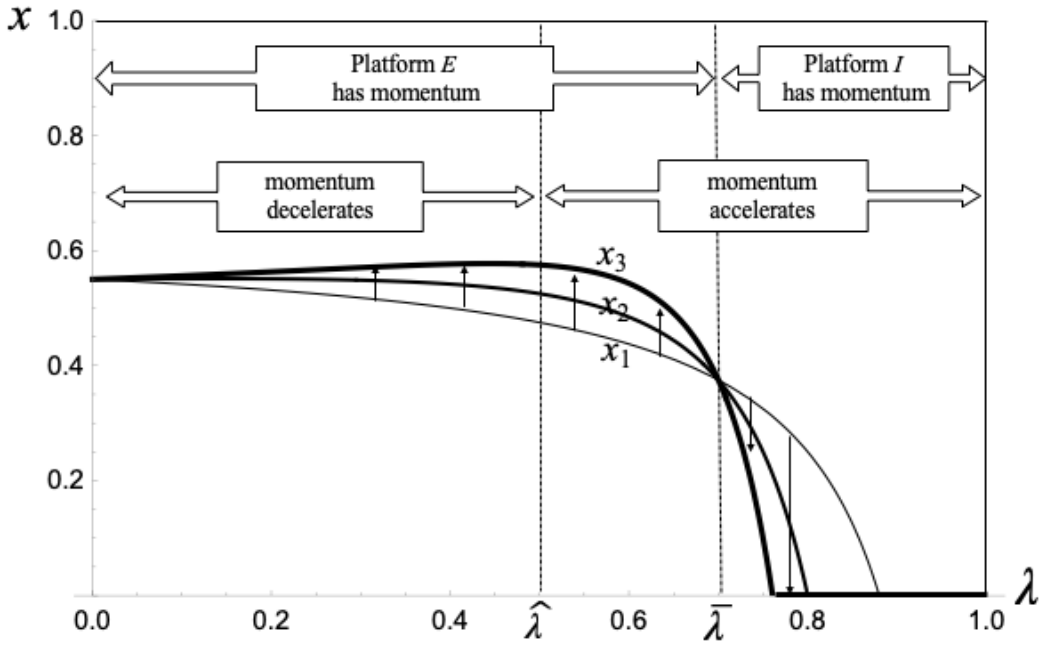


Figure 1: The platforms' market shares in $t = 1, 2, 3$ ($q = 0.1$, $\gamma = 0.5$, $D_0 = 0.5$ and $\theta - \beta = 1$)

does not depend on t. Moreover, if a platform has momentum between periods t and $t+1$, it will continue to have momentum in all future periods in the virtual system.

Figure 1 illustrates the market shares of the two platforms as a function of λ, for $t = 1, 2, 3$. While we only show the market shares in the first three periods after entry, by Theorem 1, the results follow to all t (until the market reaches corner solutions, on which we elaborate in section 6). Starting with the case of $\lambda = 0$, the figure shows that the market remains constant over time, because accumulating data does not increase the users' utility. When λ is lower than the momentum threshold $\overline{\lambda}$, platform E's market share increases over time. In this case, even when platform E's market share in period $t = 1$ is below $1/2$, platform E enjoys momentum where its market share increases with t until it eventually captures most or all of the market. The opposite result holds when $\lambda > \overline{\lambda}$, in which case platform I gains momentum as t increases (until eventually covering the entire market). When $\lambda = \overline{\lambda}$, the market shares of the two platforms remain stationary over time.

The intuition for this result is that when λ is higher than the momentum threshold (i.e., $\lambda > \overline{\lambda}$), platform I's data-driven incumbency advantage is stronger than platform E's quality advantage, resulting in an increase in platform I's market share, i.e., $\widetilde{x}_2 <$

$\widetilde{x}_1$. This increases platform I's data advantage even further and enables platform I to increase its market share even further in the next period: $\widetilde{x}_3 < \widetilde{x}_2$, and so on. The opposite case occurs if $\lambda < \overline{\lambda}$. Here, platform E's quality advantage makes it sufficiently attractive to users despite platform I's data-driven advantage, that the incumbent does not obtain enough new data to offset depreciation of its stock. This in turn increases relative attractiveness and market share of platform E in the next period. When $\lambda = \overline{\lambda}$, the two effects of initial data collection and initial quality advantage balance each other. Therefore, $\widetilde{x}_2 = \widetilde{x}_1$. As market share in the second period did not change, the two effects continue to balance each other such that $\widetilde{x}_3 = \widetilde{x}_2$ and so on.

Lemma 1 describes the features of the entrants' stationary market share when $\lambda = \overline{\lambda}$, defined as $\widetilde{x}^s$. Notice that in this case, platform I has a higher market share than platform E in all periods (i.e., $\widetilde{x}^s < 1/2$), as the following lemma shows:

Lemma 1. *At the momentum threshold $\overline{\lambda}$, $\widetilde{x}_t = \widetilde{x}^s$ for all t, where*

$$\widetilde{x}^s \equiv \frac{1}{2} - \frac{(1-\gamma)D_0}{2}.$$

(i) *The entrant has momentum if and only if its first-period share exceeds $\widetilde{x}^s$.*

(ii) *When market shares remain constant over time, platform E holds a smaller market share than platform I. That is, $\widetilde{x}^s < \frac{1}{2}$ and is independent of θ. Moreover, $\widetilde{x}^s > 0$ if and only if $D_0 < \frac{1}{1-\gamma}$.*

Lemma 1 is also helpful in describing the intuition for Theorem 1. Recall that in the first period after entry ($t = 1$) platform I holds the data advantage carried over from $t = 0$ of D_0, while platform E has a quality advantage, q, that does not depend on previous data collection. Unlike E's quality advantage, however, I's data advantage depreciates over time: in the first period, a fraction $1-\gamma$ of its initial stock, $(1-\gamma)D_0$, is lost. To maintain its data advantage, I must therefore collect more new data than E by exactly this amount, $(1 - x_1) - \widetilde{x}_1 = (1-\gamma)D_0$, that is, E's first-period share must equal $\widetilde{x}^s$. Lemma 1 shows that E has momentum if and only if its first-period share exceeds this level. Because I's data advantage depreciates, maintaining it requires I to serve more than half of the users, so $\widetilde{x}^s < \frac{1}{2}$.

Theorem 1 and Lemma 1 describe the virtual system. The actual share path $\{x_t\}$, however, must satisfy $x_t \in [0, 1]$, so the interior motion may at some point cross a boundary. Section 6 shows that clipping can stop the motion of actual shares but never

reverses its direction, so the results of this section determine the direction of the actual path.

Next, we move to identify the speed of momentum:

Theorem 2. ***(Acceleration and deceleration of momentum)*** *In the virtual system, momentum $\tilde{\mu}_t = \tilde{x}_{t+1} - \tilde{x}_t$ satisfies the linear law*

$$\tilde{\mu}_{t+1} = \kappa\, \tilde{\mu}_t, \qquad \kappa = \frac{\gamma(\theta - \beta)}{\theta - \beta - \lambda} > 0.$$

Hence, there is a threshold, $\widehat{\lambda} = (1-\gamma)(\theta-\beta)$, such that nonzero virtual momentum accelerates (i.e., $\kappa > 1$) when $\lambda > \widehat{\lambda}$, decelerates (i.e., $\kappa < 1$) when $\lambda < \widehat{\lambda}$ and remains constant over time (i.e., $\kappa = 1$) when $\lambda = \widehat{\lambda}$. Zero momentum remains zero at every date.

Intuitively, if λ is sufficiently high, the market shares change at an increasing rate, i.e., momentum accelerates, because learning from previous data has a strong effect on users' choices in the following period. Notice that nonzero momentum accelerates whether its positive or negative. Momentum also accelerates as γ increases, because then a larger proportion of each platform's data stock remains useful in the next period. Figure 1 shows when momentum accelerates or decelerates. Notice that because $\widehat{\lambda} < \overline{\lambda}$, momentum can only decelerate when platform E has momentum. When momentum accelerates, it is either the case that platform I or E has momentum.

Next we move to how market conditions affect $\overline{\lambda}$. Corollary 1 follows directly from Theorem 1:

Corollary 1. ***(Comparative statics of the momentum-direction threshold)*** *The momentum threshold $\overline{\lambda}$ defined by $\overline{\lambda} = (\theta - \beta)(1 - \gamma) + \frac{q}{D_0}$ satisfies*

$$\frac{\partial \overline{\lambda}}{\partial q} = \frac{1}{D_0} > 0, \qquad \frac{\partial \overline{\lambda}}{\partial \theta} = -\frac{\partial \overline{\lambda}}{\partial \beta} = 1 - \gamma > 0,$$

$$\frac{\partial \overline{\lambda}}{\partial D_0} = -\frac{q}{D_0^2} < 0, \qquad \frac{\partial \overline{\lambda}}{\partial \gamma} = -(\theta - \beta) < 0.$$

Hence:

(i) *Greater base-quality advantage q or stronger horizontal differentiation θ raises the threshold (favoring the entrant), while*

(ii) *a larger initial incumbent data stock D_0, or more persistent data (higher γ), or stronger same-period network effects β lower it (favoring the incumbent).*

Intuitively, as platform E's quality advantage, q, increases, it is easier for platform E to gain momentum, which enables it to gain momentum for higher values of λ. Likewise, as platform I's data-advantage, D_0 (or the persistence of accumulated data, γ), increases, it is easier for platform I to gain momentum, which enables it to gain momentum for lower values of λ. A less straightforward result is that as differentiation (measured by θ) increases, it is easier for the entrant to gain momentum. To see the intuition for this result, recall from Lemma 1 that in order to balance the data-driven advantage and the quality advantage (which occurs when $\lambda = \overline{\lambda}$), platform I needs to have a superior market share, because I's data-driven advantage can be sustained over time only if it continues to collect more data than platform E in all periods. Hence, at $\lambda = \overline{\lambda}$, the indifferent user, $\widetilde{x}^s < \frac{1}{2}$, is closer in preferences to platform E. As θ increases and users' preferences become stronger, it becomes more costly for this user to join the incumbent, which offers a more distant service than platform E. Hence, this user will no longer be indifferent and now *strictly prefers* to join platform E. As a result, a higher λ is needed to make this user indifferent between the two platforms, resulting in an increase in $\overline{\lambda}$.

Note also that κ depends on λ, γ, and $\theta - \beta$, but not on q or D_0. The quality advantage and the initial data stock therefore affect the direction and initial magnitude of momentum, whereas learning and differentiation determine how momentum subsequently grows or shrinks.

We conclude this section by noting that it is possible that the whole feasible range of λ is below the momentum threshold $\bar{\lambda}$ when the incumbent's initial data stock is sufficiently small.

Remark. (Small D_0 case: universal entrant momentum) If $D_0 \leq \frac{q}{\gamma(\theta-\beta)}$, then the momentum threshold satisfies

$$\overline{\lambda} = (\theta - \beta)(1 - \gamma) + \frac{q}{D_0} \;\geq\; \theta - \beta.$$

Consequently, for all $0 < \lambda < \theta - \beta$, $\lambda < \overline{\lambda}$, and thus in the virtual system the entrant has strictly positive momentum,

$$\widetilde{\mu}_t > 0 \qquad \textit{for all } t \geq 1.$$

Yet, it is always the case that $\widehat{\lambda} < \theta - \beta$.

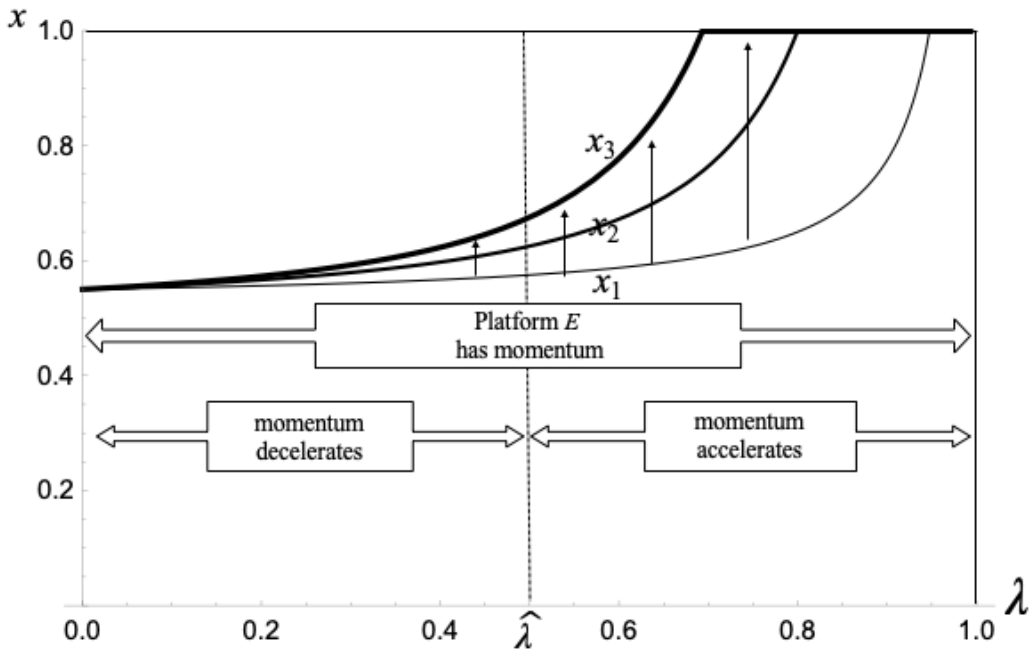


Figure 2: Market share of the two platforms ($q = 0.1$, $\gamma = 0.5$, $D_0 = 0.1$ and $\theta - \beta = 1$)

Figure 2 illustrates the case where $D_0 \leq \frac{q}{\gamma(\theta-\beta)}$. In this case, platform E always has momentum (i.e., for all values of $\lambda \in (0, \theta - \beta]$), though it is always the case that $\widehat{\lambda} < \theta - \beta$ such that platform E's momentum can accelerate or decelerate. This result may appear counterintuitive, because although $D_0 > 0$ (the incumbent has an initial data advantage), and yet it can be seen from Figure 2 that as λ increases, platform E's market share increases, as well as the speed of momentum. The intuition for this result is that although $D_0 > 0$, platform E benefits from a quality advantage, which enables it to attract more users, collect their data, and use this data to further attract more users. This mechanism becomes stronger as λ increases, increasing platform E's competitive advantage over platform I.

It is useful to note the asymmetry implied by the formula $\overline{\lambda} = (\theta - \beta)(1 - \gamma) + q/D_0$. Since $\theta - \beta > 0$, $0 < \gamma < 1$, and $q, D_0 > 0$, we always have $\overline{\lambda} > 0$. Consequently, there is no parameter region in which $\bar{\lambda}$ lies to the left of the entire feasible range of λ. Thus, while small D_0 can shift $\overline{\lambda}$ above $\theta - \beta$ and thus give the entrant positive momentum for every feasible λ, there is no analogous regime in which the incumbent enjoys universal momentum dominance across all $0 < \lambda < \theta - \beta$.

5 Long-run Steady States

The previous section analyzed market-share dynamics using the virtual system, which allows shares to move outside the feasible range. We now restrict actual shares to lie between zero and one and characterize three steady-state market structures: coexistence, the entrant captures the whole market (E-wins), and the incumbent captures the whole market (I-wins). In a steady state, market shares and data stocks remain constant, so the allocation reproduces itself each period under user choice and data accumulation. Characterizing these steady states identifies which outcomes can persist, but not which one the market reaches from a given initial condition. In particular, the existence of a monopoly steady state does not imply that the market will tip toward that platform. The next section uses the short-run dynamics to determine which steady state is selected.

Starting with the steady-state in which both platforms have positive market shares, define the interior indifferent user in the coexistence steady state by x^{co}. In each period, platform E collects x^{co} data and therefore accumulates $D_{t,E} = \frac{x^{co}}{1-\gamma}$. Likewise, platform I accumulates $D_{t,I} = \frac{1-x^{co}}{1-\gamma}$. The indifferent user solves:

$$q - (\theta - \beta)x^{co} + \lambda \frac{x^{co}}{1-\gamma} = -(\theta - \beta)(1 - x^{co}) + \lambda \frac{1 - x^{co}}{1-\gamma}, \tag{4}$$

or

$$x^{co} = \frac{1}{2} - \frac{(1-\gamma)q}{2\left(\lambda - (\theta - \beta)(1-\gamma)\right)}, \quad \text{for } \lambda \neq (\theta - \beta)(1-\gamma). \tag{5}$$

There is an equilibrium with positive market shares if $x^{co} \in (0, 1)$.

In a steady state in which platform E wins the market in all periods, user $x = 1$ prefers joining platform E over platform I, given that in all periods, all users join platform E:

$$q - (\theta - \beta) + \frac{\lambda}{1-\gamma} > 0 \iff \lambda > (\theta - \beta - q)(1-\gamma). \tag{6}$$

Likewise, in a steady state in which platform I wins, user $x = 0$ prefers joining platform I over platform E, given that in all periods, all users join platform B:

$$-(\theta - \beta) + \frac{\lambda}{1-\gamma} > 0 \iff \lambda > (\theta - \beta + q)(1-\gamma). \tag{7}$$

Notice that a main feature of all three steady states is that they do not depend on D_0, because D_0 entirely depreciates in the long-run. Define $\lambda^{multi} \equiv (\theta - \beta + q)(1 - \gamma)$ as the critical value of λ below which there is a single steady state and above which there are multiple steady states. Lemma 2 below summarizes the steady states:

Lemma 2. ***(Multiplicity of steady states)*** *Number of possible distinctive steady states depend on λ:*

(i) *For $\lambda \leq (\theta - \beta - q)(1 - \gamma)$, single interior steady state exists with $x^{co} \in (\frac{1}{2}, 1]$, where x^{co} is increasing with λ and reaches 1 as λ reaches $\lambda = (\theta - \beta - q)(1 - \gamma)$,*

(ii) *For $(\theta - \beta - q)(1 - \gamma) < \lambda < \lambda^{multi}$, only the E-wins steady state exists,*

(iii) *For $\lambda = \lambda^{multi}$, two distinctive steady state exist: E-wins and I-wins,*

(iv) *For $\lambda > \lambda^{multi}$, three distinctive steady state exist: E-wins, I-wins and an interior with $x^{co} \in (0, \frac{1}{2})$, where $x^{co} = 0$ at $\lambda = \lambda^{multi}$, and x^{co} is increasing with λ.*

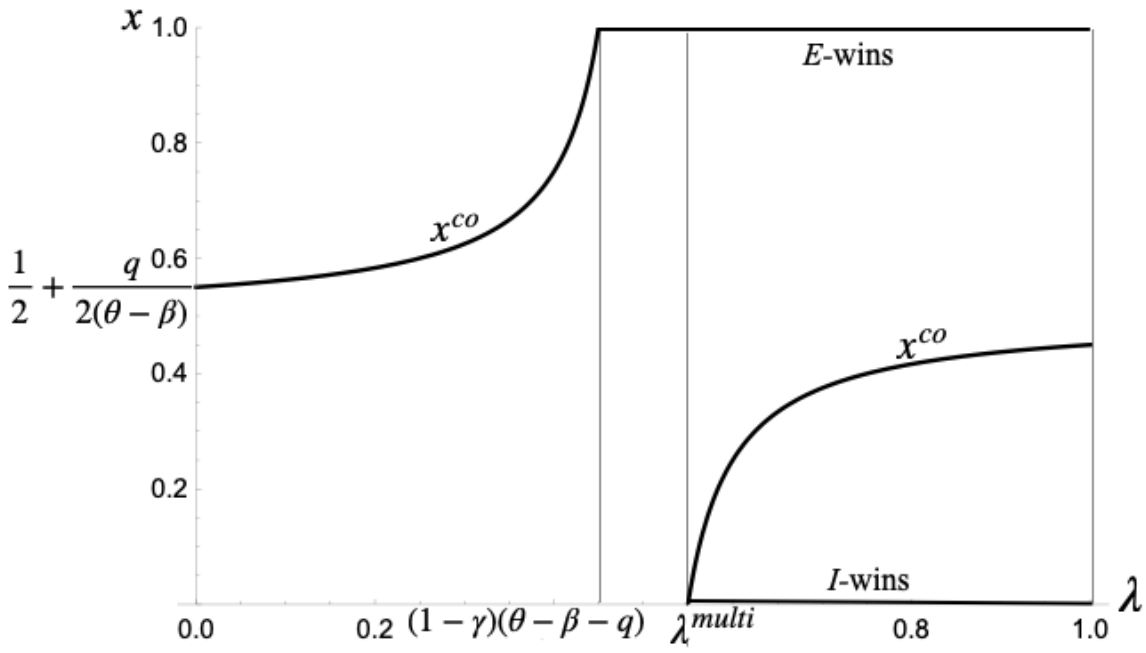


Figure 3: Steady states depending on λ ($q = 0.1$, $\gamma = 0.5$, and $\theta - \beta = 1$). The set of steady states does not depend on D_0.

Figure 3 illustrates the steady states as a function of λ. At $\lambda = 0$, the two platforms share the market and platform E gains a higher market share due to its quality advantage. This market share increases as λ increases because, as explained above, platform

E's quality advantage enables it to attract users, collect their data and use this data to collect more users. This effect becomes stronger as λ increases, increasing platform $E's$ market share until eventually, at $\lambda = (1-\gamma)(\theta-\beta-q)$, platform E takes over the market. Notice that if $q > \theta - \beta$ such that $\frac{1}{2} + \frac{q}{2(\theta-\beta)} > 1$, the steady state starts at $\lambda = 0$ with platform E dominating the market.

The figure further shows that when $\lambda > \lambda^{multi}$, there are three steady states, in which either platform E or I dominates the market, or the two platforms share the market where platform I gains a higher market share. Intuitively, as the value of data is high, if a platform accumulated sufficient data, it can dominate the market, regardless of the base quality advantage. Yet, there is also a steady-state in which the platforms share the market. As in the short-run case, in this equilibrium the higher-quality platform E has lower market share than the lower-quality platform I. Specifically, in order to obtain a steady-state equilibrium with positive market shares, platform I needs to balance the quality advantage of platform E by consistently accumulating more data, which requires it to capture a larger market share. In this equilibrium, platform E's market share increases with λ, because as λ increases a smaller amount of data accumulation is needed for platform I to balance the quality advantage of platform E.

6 Equilibrium Selection

We now determine which steady state the market approaches from the initial conditions. The virtual system describes the trajectory when the allocation is interior. Once a boundary constraint binds, however, subsequent data accumulation reflects the actual monopoly allocation. Establishing equilibrium selection therefore requires showing whether the platform that captures the entire market retains it as both platforms' data stocks continue to adjust.

Lemma 3 below establishes that a boundary reached from an interior allocation is absorbing. That is, once the market reaches the boundary, the market stays at this boundary forever. Then, we will move to Proposition 1 that connects this boundary result to the virtual dynamics: for an interior initial allocation, clipping can bring share movements to an end, but cannot reverse their direction. This proposition allows us to use the momentum results to characterize the actual market trajectory.

We first establish that when the first period has an interior market share, $x_1 \in (0,1)$, and momentum drives the market share to one of the boundaries, either $x_t = 1$ or $x_t = 0$,

then the market share stays at the absorbing boundary ($x_t = 1$ or $x_t = 0$) in all future periods.

Lemma 3. *(Absorbing boundaries) Suppose the market starts with any share that is not* 0*, i.e.,* $x_1 \in (0, 1]$*, then the boundaries are absorbing. That is:*

(i) *If* $x_t = 1$ *for some* t*, then* $x_{t+k} = 1$ *for all* $k \geq 0$*.*

(ii) *If* $x_t = 0$ *for some* t*, then* $x_{t+k} = 0$ *for all* $k \geq 0$*.*

The actual trajectory of the market shares follows the virtual dynamics in Theorem 1 until a boundary constraint binds, and the absorbing-boundary result determines its continuation thereafter. The following proposition formally shows that clipping can truncate momentum but cannot reverse its direction. At the same time, boundary need not be reached, since the trajectory may instead converge to an interior steady state.

Proposition 1. ***(Clipped dynamics and direction of motion)*** *Let* $\tilde{x}_t$ *denote the virtual path from Theorem 1. Then, the actual shares satisfy* $x_t := \min\{1, \max\{0, \tilde{x}_t\}\}$*. Let* $\mu_t := x_{t+1} - x_t$*. For any initial share* $x_1 \in (0, 1)$*, the dynamics are characterized by the relationship between* λ *and the threshold* $\bar{\lambda}$*. In particular,*

$$\mu_t \cdot (\overline{\lambda} - \lambda) \geq 0 \quad \textit{for all } t \geq 0,$$

with strict inequality whenever $x_t \in (0, 1)$*. And specifically:*

(i) *If* $\lambda < \overline{\lambda}$*, then* $\tilde{\mu}_t > 0$ *for all* t *and the actual path* $\{x_t\}$ *is weakly increasing:* $\mu_t > 0$ *whenever* $x_t \in (0, 1)$*, and if for some* t*,* x_t *hits* 1 *we have* $\mu_t = 0$ *forever.*

(ii) *If* $\lambda > \overline{\lambda}$*, then* $\tilde{\mu}_t < 0$ *for all* t *and the actual path* $\{x_t\}$ *is weakly decreasing:* $\mu_t < 0$ *whenever* $x_t \in (0, 1)$*, and if for some* t*,* x_t *hits* 0 *we have* $\mu_t = 0$ *forever.*

(iii) *If* $\lambda = \overline{\lambda}$*, then* $\tilde{\mu}_t = 0$ *and hence shares remain constant at the initial level, i.e., for all* t*,* $x_t = x_1 \equiv \widetilde{x}^s \in (0, \frac{1}{2})$*. Moreover, this stationary share is always strictly less than* 1/2*, and interior stationary share exists if and only if* $D_0 < \frac{1}{1-\gamma}$*.*

This is how short-run momentum operates once the system begins in the interior region. Theorem 1 and Proposition 1 show that the momentum threshold $\bar{\lambda}$ plays a central role in short-run direction of motion: it selects whether the entrant or the incumbent gains

share each period, and --- through clipping --- whether this motion results in rapid absorption at a boundary or gradual adjustment within the interior.

Proposition 1 and Figure 2 show that gaining a higher market share in the first period does not guarantee momentum. Specifically, the incumbent gains more than a 50% market share in the first period, when the entrant's quality advantage is not large enough, compared with the incumbent's data advantage. In this case, the incumbent's data advantage should increase in the second period even further because with more than 50% in the first period, the incumbent collected more data than the entrant. Yet, it is still possible the incumbent to lose momentum as the data the entrant collected in that period may be large enough to attract in $t+1$ more users than in the entry period t. This is because, the incumbent's larger flow of new data must be compared with depreciation of its data stock. The entrant can therefore increase its share while the incumbent still serves more than half of the market. The following corollary highlights this result, which follows directly because $\widetilde{x}^s < 1/2$:

Corollary 2. ***(Incumbent majority with entrant momentum)*** *Suppose that $\overline{\lambda} \in (0, \beta - \theta)$. Then, at least when λ is slightly lower than $\bar{\lambda}$, the entrant has momentum even though the incumbent's market share in the first period is higher than 50%.*

The intuition for this result is that when λ is slightly lower than $\overline{\lambda}$, in the first period the incumbent collects more data than the entrant due to the high D_0. Yet, the difference in their data inflows does not offset depreciation of the incumbent's lead, D_0 depreciates in the second period, causing the incumbent to lose momentum.

The result in Lemma 3 that when $x_1 \in (0, 1]$, the boundaries are absorbing may appear trivial at first. Yet, note that Lemma 3, as most of our discussion so far, holds only when the entrant's initial market share is strictly positive; i.e., $x_1 \in (0, 1]$. In contrast, in Proposition 2 we show that when $x_1 = 0$, the boundary may not be absorbing and instead $x_{1+k} > 0$ for $k > 0$.

Proposition 2. ***(Delayed entry)*** *If the incumbent has the entire market in the first period, the entrant gains momentum after some periods if data-learning is sufficiently small. That is, suppose that the market starts at the boundary, $x_1 = 0$.*

(i) *If $\lambda < \lambda^{multi}$, then there exists a finite $t > 1$ such that for all $k < t$, $x_k = 0$ and for all $k \geq t$, $x_k > 0$. Moreover, $\mu_k > 0$ for all $k \geq t$.*

(ii) *Otherwise, the boundary is absorbing.*

Delayed entry requires the incumbent's initial data stock to exceed the stock it can sustain by serving the entire market under the accumulation process $D_0 > 1/(1-\gamma)$. While the entrant remains excluded, the incumbent continues to collect data from all users, but depreciation reduces its stock toward this sustainable level of $1/(1-\gamma)$. When data learning is sufficiently weak, the declining data advantage eventually allows the entrant to attract users. Such a large initial stock of the incumbent could reflect additional historical data sources or temporarily higher data collection before entry.

The incumbent's initial data stock determines the ordering of the momentum-direction threshold and the multiplicity threshold, because λ^{multi} is independent of D_0 while the momentum-direction threshold $\bar{\lambda}$ decreases with D_0. The ordering of these thresholds also clarifies the role of clipping. Recall that in the virtual system, entrant has momentum for all $\lambda < \bar{\lambda}$ and incumbent has momentum for $\lambda > \bar{\lambda}$. When $D_0 < 1/(1-\gamma)$, we have $\lambda^{multi} < \bar{\lambda}$. For $\lambda^{multi} \leq \lambda < \bar{\lambda}$, incumbent monopoly is one of the three steady states, but the virtual dynamics favor the entrant. The actual market follows this direction toward entrant monopoly: the incumbent's initial data advantage is insufficient to sustain its position along the equilibrium path. When $D_0 > 1/(1-\gamma)$, the ordering reverses. For $\bar{\lambda} < \lambda < \lambda^{multi}$, the virtual system predicts incumbent momentum, yet Proposition 2 implies eventual entrant dominance, after delayed entry. Here, the virtual entrant share is negative at entry, so the virtual trajectory of incumbent's momentum is infeasible from the outset for the actual market shares. Clipping limits the incumbent to the entire market and therefore caps its inflow of new data. Even collecting data from the entire market cannot maintain incumbent's oversized initial stock, which declines over time toward $1/(1-\gamma)$. With entrant's quality advantage sufficient to put $\lambda < \bar{\lambda}$, this decline in incumbent's data stock eventually allows entrant to enter and capture the market.

Define x^* as the entrant's long-run market share. We can now fully describe the short-run trajectory and the equilibrium selection as a function of λ:

Proposition 3. ***(Long-run market structure)*** *Suppose $q > 0$. The equilibrium path is uniquely determined by the initial parameters, and its long-run outcome is as follows:*

(i) *Stable coexistence. If $\lambda < (\theta - \beta - q)(1-\gamma)$, in the equilibrium steady state the entrant share is $x^* = x^{co} \in (1/2, 1)$. Any initial exclusion of the entrant ends in finite time.*

(ii) *Entrant dominance. If $(\theta-\beta-q)(1-\gamma) \leq \lambda < \max\{\overline{\lambda}, \lambda^{multi}\}$, in the equilibrium*

steady state $x^* = 1$.

(iii) *Incumbent dominance. If* $\lambda > \max\{\overline{\lambda}, \lambda^{multi}\}$, *or if* $\lambda = \lambda^{multi} \geq \overline{\lambda}$, *in the equilibrium steady state* $x^* = 0$.

(iv) *Fragile coexistence. If* $\lambda = \overline{\lambda} > \lambda^{multi}$, *then in the equilibrium steady state* $x^* = x^{co} = \widetilde{x}^s \in (0, 1/2)$. *This interior steady-state equilibrium is unstable.*

In each case the data stocks converge to $D_E^* = x^*/(1-\gamma)$ *and* $D_I^* = (1-x^*)/(1-\gamma)$.

We complete this section by highlighting the role of D_0 on the short-run trajectory. Notice that for $\lambda < \lambda^{multi}$, D_0 does not affect the long-term outcome: the entrant has momentum and eventually captures most of the market (if $\lambda < (1-\gamma)(\theta - \beta - q)$, or all the the market (when $(1-\gamma)(\theta - \beta - q) < \lambda < \lambda^{multi}$). Yet, when $\lambda > \lambda^{multi}$, the short-run trajectory is determined by the value of λ relative to $\overline{\lambda}$, which in turn depends on D_0. Recall from Corollary 1 that $\overline{\lambda}$ is decreasing with D_0. While for $\lambda > \lambda^{multi}$, an increase in D_0 makes it easier for the incumbent to dominate, i.e, the incumbent can dominate for lower values of λ. For $\lambda < \lambda^{multi}$, D_0 does not provide the incumbent with a competitive advantage. The following corollary summarizes these results:

Corollary 3. *When* $\lambda < \lambda^{multi}$, D_0 *has no effect on the long-run equilibrium outcome: the entrant is always going to have majority of the market. Otherwise,* D_0 *selects the trajectory toward the long-run equilibrium,*

7 Implications

Policy implications

In this subsection we study the policy implications of our results. We focus on the case of $\lambda > \lambda^{multi}$, such that there are three steady state outcomes (the incumbent dominates, the entrant dominates, and the two platforms share the market), because in this range public policy can affect the outcome selected in the long-run equilibrium.

Notice that in this model, due to market dominance and product differentiation, there are two sources for loss of welfare. First, the equilibrium in which the incumbent dominates the market provides lower total welfare than the equilibrium in which the entrant dominates the market. This is because the incumbent offers a lower base quality than the entrant. Yet, because of product differentiation, the equilibrium in which the

entrant dominates the market does not always maximize welfare. Users with strong preferences towards the incumbent (i.e., with x close to 1) may prefer to join the incumbent over the entrant. Hence, a second loss of welfare occurs when a platform dominates the market, depriving users of the choice of a competing platform that is closer to their preferences. Whether entrant monopoly or coexistence maximize welfare depends on particular environment parameters.

Requiring the incumbent to share its initial data accumulation

Consider a policy that requires the incumbent to share with the entrant the data it collected prior to the entrant's entry. Sharing means copying the data – the incumbent retains the stock. Only the initial data is shared: no platform is obligated to share data with its competitor in future periods. In our model, this implies that in period $t = 1$, the incumbent has to share D_0 with the entrant, and then the two platforms continue to accumulate their own data, given their market shares. The following proposition shows that this policy solves the first source of loss of welfare:

Proposition 4. *Suppose that $D_{E,t} = D_{I,t}$ and $\lambda > \lambda^{multi}$. Then, after period t, the entrant has momentum (i.e., $x_{t+1} > x_t$) if and only if $q > 0$.*

Proposition 4, along with our previous results, show that if the incumbent shares its initial data with the entrant, then, the highest quality platform has momentum. And since we are considering region with three steady states, the platform with momentum will therefore dominate the market in the long-run. This result has three implications for welfare. First, it has been argued that if a high-quality incumbent platform share its data with a new platform, the incumbent will lose a welfare enhancing competitive advantage to a lower-quality platform. For example, against antitrust claims of monopolization, Google argued that its high market share is due to its superior quality.[1] However, if we extend the logic of Proposition 4 to the case where the incumbent offers a superior quality than the entrant (which corresponds to $q < 0$), we can see that sharing the initial D_0 with the entrant will not prevent the incumbent from gaining momentum and dominating the market. Second, notice that in the long-run, this policy is relevant only if $\lambda > \overline{\lambda}$. Recall that if $\lambda < \overline{\lambda}$, the entrant gains momentum and dominates the market without this policy, though in the short-run, this policy may expedite the convergence to the steady-state in which the entrant wins. Finally, notice that while under

[1] See, for example: https://www.reuters.com/legal/transactional/us-takes-google-much-anticipated-antitrust-trial-2023-09-12/.

this policy, the best platform wins the market, the market is not necessarily efficient. As explained above, welfare can be improved if both platforms are active in the market.

Requiring both platforms to continuously share their data

To show how welfare can be maximized, consider a rather extreme policy in which both platforms need to share their data with each other in all periods, starting the data from period $t = \bar{t} \geq 1$. That is, in some period $\bar{t}$, platforms need to share all their data prior to period $\bar{t}$, and continue to share each period's data starting from $\bar{t}$. In this case, in each period, each platform collects data equals to 1 regardless of its market share. Therefore, in all future periods, $D_{E,t} = D_{I,t}$, consequently the indifferent user is characterized by:

$$Q + q - (\theta - \beta)x_t + \lambda D_{E,t-1} = Q - (\theta - \beta)(1 - x_t) + \lambda D_{I,t-1} \tag{8}$$

$$\Updownarrow$$

$$x_t = \frac{1}{2} + \frac{q}{2(\theta - \beta)}.$$

Because both platforms have access to the same pool of data, λ does not affect x_t and therefore x_t is constant over time. Moreover, $\frac{1}{2} < x_t < 1$ when $0 < q < (\theta - \beta)$. We therefore have the following result:

Proposition 5. *Consider a policy that requires both platforms to share their data in all periods and suppose that* $\lambda > \lambda^{multi}$ *and* $0 < q < (\theta - \beta)$. *Then, in all periods, users with* $x < \frac{1}{2} + \frac{q}{2(\theta-\beta)}$ *join the entrant and users with* $x > \frac{1}{2} + \frac{q}{2(\theta-\beta)}$ *join the incumbent.*

Proposition 5 shows that this policy results in a different market outcome. Both platforms are active, both have the same set of data and the higher-quality the entrant gains a higher market share. Yet, we acknowledge that such a regulation can raise two problems. First, from a legal perspective, this regulation requires platforms to give up on their intellectual property – the data that they collect – in all periods. This is different from a one-time sharing of data from the incumbent to the entrant as in the previous subsection. Second, sharing data in a continuous manner may raise problems of data security and privacy that our paper does not model. Third, such policy may also weaken the incentive for the platforms to collect the data, which would result in lower overall quality of the products offered to the users. Consequently, this policy

should be interpreted as a benchmark case of what policy can enhance welfare in an ideal, unconstrained, market.

Managerial implications

Short-run market share and long-run dominance

It might be expected that if a platform has a short-run market share above 50%, and data from current users accumulates over time, then this platform is bound to gain momentum, increase its market share further and end up dominating the market. Yet, the results of this paper show that this is not always the case. Consider the incumbent, whose competitive advantage is in its initial data accumulation. Suppose that λ is slightly below $\overline{\lambda}$. Because $x^s < \frac{1}{2}$, the incumbent has a market share of above 50%. However, because $\lambda < \overline{\lambda}$, the incumbent does not have momentum: it will gradually lose market share until eventually the entrant will dominate the market. Therefore, the incumbent cannot rely on its current superior market share: it will need to take measures that reduce $\overline{\lambda}$ (such as increasing its base quality) to prevent the entrant from dominating the market.

Notice that the same argument does not apply to the entrant, whose competitive advantage is in its superior quality. If the entrant benefits from a short-run market share above 50%, it will always (taking other market conditions as given) have momentum and dominate the market. To see why, notice that x_t can be higher than $\frac{1}{2}$ only if $\lambda < \overline{\lambda}$, in which case the entrant has momentum.

Commercializing data and the threat of losing market dominance

Related to the previous managerial implications, suppose that a platform (say, the entrant) decides to commercialize data (or charge an access price), in order to generate positive revenues. Suppose further that commercializing data inflicts a disutility of users and therefore decreases the entrant's quality advantage. This reduces q, which, in turn, reduces $\overline{\lambda}$. In the short-run, commercializing data result in losing a certain market share, due to the decrease in q. Yet, in the long-run, if, after data commercialization, it is still the case that $\lambda < \overline{\lambda}$, the entrant still has momentum. Whatever market share lost due to data commercialization, the entrant will gain it back in future periods and eventually dominate the market. If, however, the decrease in $\overline{\lambda}$ results in $\overline{\lambda} < \lambda$, then the entrant will start losing momentum and eventually loose the entire market.

It is possible to apply this argument to the incumbent, though recall from the previous policy implication the difference between the two platforms. Suppose, for example, that the entrant has, say, 70% of the market, and due to data commercialization, in the short-run its market share drops to 60%. Then, it is always the case that the entrant still has momentum and will gain its high market share back. This is not necessarily the case for the incumbent. Suppose, for example, that the incumbent has 70% and has momentum, with an increase in market share in each period. Suppose further that once the incumbent starts commercializing data, in the short-run its market share drops to 60%. Although its market share is still above 50%, it is not necessarily the case that the incumbent still has momentum. It could be that the decrease in its base quality (equivalent to an increase in q in our model) increases $\overline{\lambda}$ from values under which the incumbent has momentum ($\overline{\lambda} < \lambda$) to values under which it does not ($\lambda < \overline{\lambda}$). In other words, for the incumbent, a market share above 50% does not necessarily place it in a position in which it can start commercializing data.

Increased product differentiation result in entrant's market dominance

It might be expected that as differentiation increases, it is more likely that competing platforms share the market. As a result, incumbents might be less fearful from entry threats as products are more substitutes. Yet, the results of this paper show that as differentiation increases (an increase in θ), $\overline{\lambda}$ increases and it is more likely that the entrant has momentum and will eventually dominate the market. Consequently, differentiation creates a threat of entry by providing a stronger competitive advantage to entrants. For example, when asking both ChatGPT and Gemini what are the differences between them, the two AI tools highlight differences, stating that each has unique advantages in serving specific needs. According to our model, such differentiation is more likely to benefit the entrant platform.

8 Conclusion

In the competitive landscape of digital platforms, does a data-rich incumbent hold an indisputable advantage against a higher-quality entrant? This paper demonstrates that this is not the case. The evolution of market dominance hinges on the interaction between the quality-data tradeoff and the dynamics of the new data inflow. At the entry, the users face the trade-off between the economic value coming from incumbent's

accumulated data versus the value coming from higher intrinsic quality of the service that the entrant provides without accumulated data stock. Over time, both platforms collect additional data further improving overall quality of their products. Yet, because the usefulness of the data declines with time, the incumbent may not be able to collect enough new data to fully counter the deterioration of its initial data stock. In this case, the total value provided to the users by the entrant grows faster than incumbents, resulting in entrant's growing market share.

Our contribution is a tractable framework connecting short-run platform momentum to long-run market structure. Holding platform policies fixed, we derive an explicit law of motion that determines the market's trajectory from given initial conditions. A single condition, derived from the model's primitives, governs which platform gains share and, when multiple steady states exist, also selects the long-run outcome. This connects the direction of competition upon entry to its eventual long-run equilibrium and reveals when historical advantages have lasting consequences.

In some markets, the incumbent's initial data stock affects how quickly the market adjusts but not where it ends up. In others, it determines which platform ultimately dominates. Thus, the consequences of an initial data advantage can outlast the data themselves, persisting through the users they attract and the subsequent data those users generate. Moreover, perhaps more surprisingly, greater product differentiation does not insulate the incumbent. Instead, it amplifies the entrant's quality advantage among its core user base, making an eventual market takeover more, not less, likely.

These results show that current market leadership can give a misleading picture of future competitive strength. A large incumbent may be losing the advantage that sustains its position, while a small entrant may already be on a path toward expansion. Distinguishing these cases helps assess whether an inherited data advantage constitutes a lasting barrier to competition. It also matters for data-access policies. A one-time data transfer can permanently change the eventual winner if it puts the market on a path toward a different steady state. Where the long-run outcome is independent of initial stocks, the same intervention affects only the transition. Continuing data sharing, in turn, changes how participation creates future advantages. Evaluating these policies therefore requires understanding both the market's starting position and how competition evolves from it.

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Appendix A

Below are the proofs for all Theorems, propositions and Lemmas in the text.

Proof of Theorem 1:
Let $\Delta D_t = D_{E,t} - D_{I,t}$. For an interior allocation,

$$x_t = \frac{1}{2} + \frac{q + \lambda \Delta D_t}{2(\theta - \beta)} = \frac{1}{2} + \frac{q + \lambda\gamma \Delta D_{t-1}}{2(\theta - \beta - \lambda)}. \tag{9}$$

The virtual system uses (9) without restricting shares to $[0, 1]$ and accumulates data from those virtual shares. Its shares and momentum are denoted $\widetilde{x}_t$ and $\widetilde{\mu}_t = \widetilde{x}_{t+1} - \widetilde{x}_t$.

From the indifferent-user condition,

$$\widetilde{\mu}_t = \widetilde{x}_t - \widetilde{x}_{t-1} = \frac{\lambda}{2(\theta - \beta)}(\Delta D_{t+1} - \Delta D_t).$$

Data evolve as $D_{i,t} = n_{i,t} + \gamma D_{i,t-1}$. In the virtual system, $n_{E,t} = \widetilde{x}_t$ and $n_{I,t} = 1 - \widetilde{x}_t$, so

$$\Delta D_{t+1} = 2\widetilde{x}_{t+1} - 1 + \gamma \Delta D_t.$$

Substituting for $\widetilde{x}_{t+1}$ and rearranging gives

$$\Delta D_{t+1} = \frac{q}{\theta - \beta - \lambda} + \frac{\gamma(\theta - \beta)}{\theta - \beta - \lambda}\Delta D_t, \qquad \kappa := \frac{\gamma(\theta - \beta)}{\theta - \beta - \lambda} > 0. \tag{10}$$

Consequently,

$$\Delta D_{t+2} - \Delta D_{t+1} = \kappa(\Delta D_{t+1} - \Delta D_t), \qquad \widetilde{\mu}_{t+1} = \kappa \widetilde{\mu}_t. \tag{11}$$

To compute the sign, use $\Delta D_0 = -D_0$:

$$\begin{aligned} \Delta D_1 - \Delta D_0 &= \frac{q - D_0[\lambda - (\theta - \beta)(1 - \gamma)]}{\theta - \beta - \lambda} \\ &= \frac{D_0(\overline{\lambda} - \lambda)}{\theta - \beta - \lambda}. \end{aligned}$$

The first post-entry momentum is therefore

$$\widetilde{\mu}_1 = \frac{\lambda}{2(\theta-\beta)}(\Delta D_2 - \Delta D_1)$$
$$= \frac{\lambda\kappa}{2(\theta-\beta)}(\Delta D_1 - \Delta D_0) = \frac{\lambda\gamma D_0(\overline{\lambda}-\lambda)}{2(\theta-\beta-\lambda)^2}. \tag{12}$$

All factors multiplying $\overline{\lambda} - \lambda$ are positive. The recurrence in (12), with $\kappa > 0$, preserves that sign at every date.

Proof of Theorem 2:
The recurrence was obtained in (11). The denominator in formula for κ is positive, as $\theta - \lambda > \beta$. And numerator is positive, as $\lambda > 0$. For nonzero momentum, the three cases are obtained by directly comparing κ to 1. Zero momentum remains zero and cannot strictly accelerate.

When $q > 0$, $\overline{\lambda} > \widehat{\lambda}$, so negative virtual momentum always accelerates, while deceleration is possible only with positive momentum.

Proof Corollary 1:
Differentiate the threshold formula, holding the other parameters fixed. The signs follow from $D_0 > 0$, $0 < \gamma < 1$, $\theta > \beta$, and $q > 0$, with the stated exception at $q = 0$.

Proof Lemma 1:
Theorem (1) gives zero virtual momentum at $\lambda = \overline{\lambda}$. Substitute $\overline{\lambda} = (\theta-\beta)(1-\gamma)+q/D_0$ into

$$\widetilde{x}_1 = \frac{1}{2} + \frac{q - \lambda\gamma D_0}{2(\theta-\beta-\lambda)}.$$

The numerator becomes

$$q - \overline{\lambda}\gamma D_0 = -(1-\gamma)D_0(\theta-\beta-\overline{\lambda}),$$

giving the stated share. Since $D_0 > 0$, the share is below $1/2$; it is above zero exactly when $D_0 < 1/(1-\gamma)$. In that case the virtual path is feasible at every date and coincides with the actual path.

Proof of Lemma 2:
In a steady state, the stock equations give $D_{E,t} = x^{co}/(1-\gamma)$ and $D_{I,t} = (1-x^{co})/(1-\gamma)$. The interior indifferent-user condition is

$$q-(\theta-\beta)x^{co}+\frac{\lambda x^{co}}{1-\gamma}=-(\theta-\beta)(1-x^{co})+\frac{\lambda(1-x^{co})}{1-\gamma},$$

which gives (5) when $\lambda \neq (\theta-\beta)(1-\gamma)$. At equality, the equation would require $q=0$, so there is no interior candidate when $q>0$.

If $\lambda < (\theta-\beta)(1-\gamma)$, the candidate is above 1/2, and

$$x^{co}\leq 1 \quad \Longleftrightarrow \quad \lambda \leq (\theta-\beta-q)(1-\gamma).$$

If $\lambda > (\theta-\beta)(1-\gamma)$, the candidate is below 1/2, and

$$x^{co}\geq 0 \quad \Longleftrightarrow \quad \lambda \geq (\theta-\beta+q)(1-\gamma)=\lambda^{multi}.$$

At entrant monopoly, the least favorable consumer, $x=1$, weakly prefers E if and only if

$$q-(\theta-\beta)+\frac{\lambda}{1-\gamma}\geq 0 \quad \Longleftrightarrow \quad \lambda \geq (\theta-\beta-q)(1-\gamma).$$

At incumbent monopoly, the consumer most inclined toward E, $x=0$, weakly prefers I if and only if

$$-q-(\theta-\beta)+\frac{\lambda}{1-\gamma}\geq 0 \quad \Longleftrightarrow \quad \lambda \geq \lambda^{multi}.$$

Combining these conditions and counting a candidate at an endpoint only once gives all four cases. Weak inequalities are appropriate because a single indifferent endpoint consumer has zero mass. Finally,

$$\frac{\mathrm{d}x^{co}}{\mathrm{d}\lambda}=\frac{(1-\gamma)q}{2[(\theta-\beta)(1-\gamma)-\lambda]^2}>0.$$

The expressions for shares and stationary stocks contain no D_0.

Proof of Lemma 3:

Part (i): entrant monopoly. Suppose $x_t=1$ but $x_{t+1}<1$. Using (9), we have:

$$x_t=\min\left\{1,\max\left\{0,\frac{1}{2}+\frac{q+\lambda\gamma\Delta D_{t-1}}{2(\theta-\beta-\lambda)}\right\}\right\}. \tag{13}$$

By (13),

$$\Delta D_{t-1} \geq \frac{\theta - \beta - \lambda - q}{\lambda\gamma}, \qquad \Delta D_t < \frac{\theta - \beta - \lambda - q}{\lambda\gamma}.$$

Thus $\Delta D_t < \Delta D_{t-1}$. But $x_t = 1$ gives $\Delta D_t = 1 + \gamma \Delta D_{t-1}$, so the strict decrease requires

$$\Delta D_{t-1} > \frac{1}{1-\gamma}.$$

This is impossible: since $D_{E,0} = 0$ and $x_k \leq 1$,

$$\Delta D_{t-1} \leq D_{E,t-1} \leq \sum_{k=0}^{t-2} \gamma^k = \frac{1-\gamma^{t-1}}{1-\gamma} < \frac{1}{1-\gamma},$$

with the same strict upper bound at $t = 1$. Hence an entrant-monopoly period cannot be followed by a share below one.

Part (ii): incumbent monopoly reached after positive entry. Suppose $x_t = 0$ but $x_{t+1} > 0$. Since $x_1 > 0$, necessarily $t > 1$. The two demand conditions give

$$\Delta D_{t-1} \leq -\frac{q + \theta - \beta - \lambda}{\lambda\gamma}, \qquad \Delta D_t > -\frac{q + \theta - \beta - \lambda}{\lambda\gamma}. \tag{14}$$

Thus $\Delta D_t > \Delta D_{t-1}$. Because $x_t = 0$ implies $\Delta D_t = -1 + \gamma \Delta D_{t-1}$, this requires $\Delta D_{t-1} < -1/(1-\gamma)$. For every feasible history,

$$\Delta D_{t-1} \geq -\gamma^{t-1} D_0 - \frac{1-\gamma^{t-1}}{1-\gamma}. \tag{15}$$

If $D_0 \leq 1/(1-\gamma)$, the lower bound is at least $-1/(1-\gamma)$, a contradiction. Hence exit from zero would require $D_0 > 1/(1-\gamma)$. In that case, since $t > 1$, (15) instead implies

$$\Delta D_{t-1} > -D_0 = \Delta D_0.$$

Initial positive share gives

$$\Delta D_0 > -\frac{q + \theta - \beta - \lambda}{\lambda\gamma}.$$

Combining these last two inequalities contradicts the first inequality in (14). Thus incumbent monopoly reached after positive entry also cannot be left. Applying these one-period conclusions repeatedly proves absorption for every subsequent date.

Proof of Proposition 1:

Theorem 1 gives the virtual momentum's constant sign. The actual and virtual shares and stocks coincide until the first boundary because they solve the same interior equations with the same initial stocks.

If $\lambda < \overline{\lambda}$, the virtual shares are strictly increasing. Either they remain in $(0, 1)$ forever, in which case the actual and virtual paths coincide forever, or there is a first T with $\widetilde{x}_T < 1$ and $\widetilde{x}_{T+1} \geq 1$. For $t < T$, $\mu_t = \widetilde{\mu}_t > 0$. At T, the actual increment is $\mu_T = 1 - \widetilde{x}_T > 0$. Lemma 3 gives $x_t = 1$ and $\mu_t = 0$ for $t \geq T + 1$. Since the virtual path keeps increasing, clipping it gives the same actual shares thereafter.

If $\lambda > \overline{\lambda}$, the virtual shares are strictly decreasing. They either remain interior, or there is a first T with $\widetilde{x}_T > 0$ and $\widetilde{x}_{T+1} \leq 0$. At that date $\mu_T = -\widetilde{x}_T < 0$, and shares remain zero thereafter by Lemma 3. Clipping again gives the actual share path.

If $\lambda = \overline{\lambda}$, Theorem 1 gives zero momentum at every date. Lemma 1 gives the share's value and shows it is interior exactly when $D_0 < 1/(1-\gamma)$. These three cases prove all the claims. **Proof of Corollary 2:**

At $\lambda = \overline{\lambda}$, Lemma 1 gives $0 < x_1 = x^s < 1/2$. The initial interior share

$$\frac{1}{2} + \frac{q - \lambda\gamma D_0}{2(\theta - \beta - \lambda)}$$

is continuous in λ near $\overline{\lambda} < \theta - \beta$. Therefore it remains strictly between zero and $1/2$ when λ is sufficiently close below $\overline{\lambda}$. Proposition 1 then gives positive entrant momentum while the share is interior. **Proof of Proposition 2:**

We start by examining the incumbent's stock while entry is blocked. The initial condition $x_1 = 0$ is equivalent to

$$D_0 \geq \frac{q + \theta - \beta - \lambda}{\lambda\gamma}. \tag{16}$$

While $x_k = 0$, the entrant collects no data and the incumbent's stock obeys $D_{I,k} = 1 + \gamma D_{I,k-1}$. Thus

$$D_{I,k} = \frac{1}{1-\gamma} + \gamma^k \left(D_0 - \frac{1}{1-\gamma} \right). \tag{17}$$

Positive share in period k occurs exactly when

$$D_{I,k-1} < \frac{q + \theta - \beta - \lambda}{\lambda\gamma}. \tag{18}$$

Part (i): eventual entry and positive momentum. If $\lambda < \lambda^{multi}$, then

$$\frac{1}{1-\gamma} < \frac{q+\theta-\beta-\lambda}{\lambda\gamma},$$

because the difference between the right and left sides is $(\lambda^{multi}-\lambda)/[\lambda\gamma(1-\gamma)] > 0$. Together with (16), this gives $D_0 > 1/(1-\gamma)$. The stock in (17) therefore falls strictly toward $1/(1-\gamma)$ and crosses below the entry cutoff in finite time. Let T be the first date with positive share.

If $x_T = 1$, the argument in Lemma 3(i), starting with $D_{E,T-1} = 0$, gives immediate absorption. Otherwise the new starting share is interior, with initial incumbent stock $D_{I,T-1}$ and zero entrant stock. Applying the momentum calculation in Theorem 1 at that date, its sign is the sign of

$$q - D_{I,T-1}[\lambda - (\theta-\beta)(1-\gamma)]. \tag{19}$$

If $\lambda \leq \widehat{\lambda}$, this is positive. If $\widehat{\lambda} < \lambda < \lambda^{multi}$, (18) gives

$$\begin{aligned} q - D_{I,T-1}(\lambda - \widehat{\lambda}) &> q - \frac{q+\theta-\beta-\lambda}{\lambda\gamma}(\lambda - \widehat{\lambda}) \\ &= \frac{(\theta-\beta-\lambda)(\lambda^{multi}-\lambda)}{\lambda\gamma} > 0. \end{aligned}$$

Thus momentum is positive upon interior entry. Proposition 1, applied from date T, shows that shares subsequently increase while interior and remain at one if they reach it.

Part (ii): absorbing exclusion. If $\lambda \geq \lambda^{multi}$, then

$$\frac{1}{1-\gamma} \geq \frac{q+\theta-\beta-\lambda}{\lambda\gamma}.$$

Both D_0 and $1/(1-\gamma)$ are now weakly above the blocking cutoff. Every stock in (17) is a convex combination of these two values and is therefore also weakly above the cutoff. Consequently, (18) never holds, and induction verifies $x_k = 0$ at every date, including the equality case $\lambda = \lambda^{multi}$.

Proof of Proposition 3:

(i) Stable coexistence. If $\lambda < (\theta-\beta-q)(1-\gamma)$, then $x^{co} \in (\frac{1}{2}, 1)$ by Lemma 2 and (6), and $\kappa < 1$ by Theorem 2. Moreover, $\lambda < \lambda^{multi}$, so Proposition 3(i) implies that

any initial exclusion ends in finite time and that momentum is strictly positive after entry while shares remain interior.

The first positive share is strictly below one: in this parameter region, $q < \theta - \beta - \lambda$, and the entrant has no inherited data at its first positive-share date. For $\lambda \neq \widehat{\lambda}$, equations (9), (13), and (6) imply that, whenever $0 < x_t < 1$,

$$x_{t+1} = \min\left\{1, \max\left\{0,\, x^{co} + \kappa\left[x_t - x^{co}\right]\right\}\right\}.$$

Since $\kappa < 1$, positive momentum implies that the first positive share lies below $x^{co}(\lambda)$. In the recursion, the distance from the steady stage shrinks by the factor of κ, staying strictly between its preceding value and x^{co}. Thus the path remains interior after entry, with $\mu_t > 0$ and x_t^{co}.

(ii) Entrant dominance. Suppose

$$(\theta - \beta - q)(1 - \gamma) \leq \lambda < \max\{\overline{\lambda}, \lambda^{multi}\}.$$

If $\lambda < \lambda^{multi}$, Proposition 3(i) gives finite entry and positive momentum while interior, or immediate absorbing entrant monopoly.

If $\lambda \geq \lambda^{multi}$, the condition upper bound requires $\lambda < \overline{\lambda}$. Initial exclusion is then impossible: the initial-share cutoff in Proposition 3 would otherwise imply

$$D_0 \geq \frac{q + \theta - \beta - \lambda}{\lambda\gamma} \geq \frac{q}{\lambda - (\theta - \beta)(1 - \gamma)},$$

contradicting $\lambda < \overline{\lambda}$. Hence Proposition 3(ii) again gives positive momentum while interior, or immediate absorbing entrant monopoly.

To determine the limit, first suppose $\kappa < 1$. If the path remained interior, equations (9) and (10) would imply

$$x_{t+1} - x_t = (1 - \kappa)(1 - x_t) + \frac{\lambda - (\theta - \beta - q)(1 - \gamma)}{2(\theta - \beta - \lambda)}.$$

For $\lambda > (\theta - \beta - q)(1 - \gamma)$, these increments are bounded below by a strictly positive constant. Hence the share must reach one in finite time, after which Lemma 3 gives absorption.

At $\lambda = (\theta - \beta - q)(1 - \gamma)$, the first positive share is strictly below one by the

argument in part (i). The update (13) then gives

$$1 - x_{t+1} = \kappa(1 - x_t) > 0$$

from every interior share. Since $0 < \kappa < 1$, the share remains below one at every finite date and converges to one asymptotically.

If instead $\kappa \geq 1$, equation (11) implies that, as long as the path remains interior, its strictly positive increments cannot decrease. Consequently, it must reach one in finite time. Lemma 3 gives absorption upon reaching one. In all cases, $\mu_t \geq 0$ for every t.

(iii) Incumbent dominance. Suppose first that $\lambda > \max\{\overline{\lambda}, \lambda^{multi}\}$. An initial share of one is impossible: its demand condition gives

$$q - D_0(\lambda - \widehat{\lambda}) \geq (\theta - \beta - \lambda)\big[1 + (1 - \gamma)D_0\big] > 0,$$

which would imply $\lambda < \overline{\lambda}$. If $x_1 = 0$, Proposition 3(ii) gives permanent exclusion. If $0 < x_1 < 1$, the same result gives strictly decreasing shares until zero is reached in finite time, followed by absorption.
Now suppose $\lambda = \lambda^{multi} \geq \overline{\lambda}$. Since

$$\overline{\lambda} - \lambda^{multi} = q\left(\frac{1}{D_0} - (1 - \gamma)\right),$$

we have $D_0 \geq 1/(1 - \gamma)$. At $\lambda = \lambda^{multi}$, the initial-exclusion cutoff in (16) equals $1/(1 - \gamma)$. Hence $x_1 = 0$, and Proposition 2(ii) gives permanent exclusion. Thus, throughout this case, x_t reaches zero in finite time, possibly at date 1, and $\mu_t \leq 0$ for every t.

(iv) Fragile coexistence. If $\lambda = \overline{\lambda} > \lambda^{multi}$, then

$$\overline{\lambda} - \lambda^{multi} = q\left(\frac{1}{D_0} - (1 - \gamma)\right) > 0,$$

so $D_0 < 1/(1 - \gamma)$. Lemma 1 and (6) therefore give

$$x_t = x^* = x^{co} = \frac{1}{2} - \frac{(1 - \gamma)D_0}{2} \in (0, \tfrac{1}{2}), \qquad \mu_t \equiv 0.$$

Since $\overline{\lambda} > \lambda^{multi} > \widehat{\lambda}$, Theorem 2 gives $\kappa > 1$. The interior recursion above multiplies

deviations from this steady-state share by κ, so the steady state is unstable.

Finally, uniqueness follows recursively from (13) and the stock equations. Since shares converge and $0 < \gamma < 1$, iterating the stock equations gives

$$D_{E,t} \longrightarrow \frac{x^*}{1-\gamma}, \qquad D_{I,t} \longrightarrow \frac{1-x^*}{1-\gamma}.$$

Indeed, the initial stocks and any fixed finite history receive vanishing geometric weights, while the remaining shares converge to their respective limits. This completes the proof.

Proof of Proposition 4**:**
Recall that

$$x_{t+1} - x_t = \frac{\lambda\gamma}{2(\theta-\beta-\lambda)^2}\left[q - (D_{I,t-1} - D_{E,t-1})(\lambda - (\theta-\beta)(1-\gamma))\right],$$

Substituting $D_{E,t-1} = D_{I,t-1}$ into the gap $x_{t+1} - x_t$ yields $x_{t+1} - x_t = \frac{\beta\gamma}{2(\theta-\beta-\lambda)^2}[q]$, which is positive if and only if $q > 0$.

Proof of Proposition 5:
The result that under this policy, users with $x < \frac{1}{2} + \frac{q}{2(\theta-\beta)}$ join the entrant and users with $x > \frac{1}{2} + \frac{q}{2(\theta-\beta)}$ join the incumbent follows directly from the text.