Losing grip? The Quantity Theory of Money under Currency Competition*

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Abstract

This study examines monetary policy under competition between a centrally managed currency (the "Dollar") and a fixed-supply alternative (the "Peso"). Theory and laboratory evidence reveal two equilibrium types. In the pass-through equilibrium, Dollar inflation is high and trade effects are limited. In the substitution equilibrium, Dollar dominance rises and velocity falls, muting Dollar inflation while generating inflation spillovers to Peso despite its fixed supply. Which equilibrium prevails depends on self-fulfilling inflation beliefs, not policy alone, and Dollar trade shares reveal these beliefs. The central bank therefore faces a trade-off between Dollar dominance and inflation control.

Key words: currency competition, cryptocurrencies, Quantity Theory of Money, Gresham's law, Thier's law

JEL codes: E5, E4, C92

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1 Introduction

The rise of alternative currencies has turned currency competition from a theoretical possibility into a practical monetary-policy problem. As of June 2026, stablecoins have a market capitalization above $300 billion and adjusted annual transaction volume of about $10 trillion, the global cryptocurrency market exceeds $2 trillion, and central banks around the world are studying or developing central bank digital currencies (CBDCs). The U.S. Dollar already competes directly with national currencies in a number of economies. In Cambodia, both the U.S. Dollar and the Khmer Riel are widely used for payments; in Zimbabwe, the U.S. Dollar circulates alongside the ZiG; in Lebanon, extensive dollarization has left the U.S. Dollar circulating alongside the Lebanese Pound; and in El Salvador, the U.S. Dollar has circulated alongside Bitcoin as legal tender since 2021 (Alvarez, Argente, and Van Patten, 2023).[1]

At the same time, monetary balances can earn interest. As of October 2026, the Federal Reserve pays 3.90 percent on U.S. Dollar reserve balances, while the Cambodian central bank remunerates required foreign-currency reserves, predominantly held in U.S. Dollars, but not required Riel reserves. Private digital currencies provide similar examples. PayPal pays 4 percent annual rewards on PYUSD, a Dollar stablecoin issued by Paxos with a market capitalization of approximately $2.8 billion and usable for payments and transfers. Ether provides a protocol-based counterpart: ETH is used for transactions, while staked ETH currently earns about 2.6 percent annually, with rewards paid in ETH and partly financed through new issuance.[2]

This paper examines, both theoretically and in a laboratory experiment, how monetary policy implemented through interest payments on currency balances affects currency competition. We study two currencies circulating side by side, one of which pays interest on balances while the other does not. The interest rate is also the money-growth factor: raising it increases the nominal return to holding the interest-bearing currency while simultaneously expanding its supply.[3] This creates a central tension for monetary policy. A higher interest rate makes the currency more attractive relative to its competitor, but the associated monetary expansion also puts upward pressure on its inflation rate. Under currency competition, does monetary expansion still pass through one-for-one into the price level of the interest-bearing currency, as predicted by the quantity theory of money?

[1] A large literature studies the design, adoption, and market structure of digital currencies; see, Duffie (2019), Huberman, Leshno, and Moallemi (2021), Prat and Walter (2021), Saleh (2021), Andolfatto and Martin (2022), Auer et al. (2022), Chiu et al. (2023), and Sockin and Xiong (2023).

[2] Further examples include USDO, a Bermuda-issued U.S. Dollar stablecoin that distributes reserve income through increases in token balances. Although many CBDC designs do not currently provide for interest payments, a CBDC could similarly pay interest, for example to encourage adoption when initial demand is low.

[3] Our framework naturally extends to competing currencies paying distinct interest rates, as shown in Appendix A.4.

Our answer is: not necessarily. We show theoretically and experimentally that the same monetary-policy parameter can support different rational-expectations equilibria with sharply different inflation outcomes. Monetary expansion may pass primarily into the price of the expanding currency, as in the conventional quantity-theory benchmark. But it may instead increase the currency's use in trade and reduce its velocity, absorbing the monetary expansion through quantities rather than prices.

We formalize this mechanism in an endowment economy based on Kareken and Wallace (1981) and Schilling and Uhlig (2019) with a policy currency, which we call "Dollar," and a competing currency, which we call "Peso." These labels are placeholders. The Dollar can represent a currency that remunerates central-bank reserves, an interest-bearing CBDC, an interest-paying stablecoin, or a protocol token whose supply expands through balance-contingent issuance, such as staking rewards. Peso represents a competing currency whose supply is fixed, at least over the relevant horizon. Infinitely lived agents alternate between buying and selling consumption goods. As sellers, they choose how much output to offer for Dollars rather than Peso. As buyers, they choose how much of each currency to spend and how much to save. We hold aggregate output and individual seller endowments constant and symmetric to isolate the effects of an exogenous increase in the money supply. Trade shares, currency velocities, and price levels are endogenous equilibrium objects. Prices are fully flexible, and both currency markets clear in every period.

In a single-currency economy, standard quantity-theory logic links monetary expansion to higher inflation when velocity is unchanged. With a competing currency, our framework makes both trade shares and currency velocities endogenous margins of adjustment, alongside prices, through which monetary expansion can be absorbed. In Kareken and Wallace (1981) and Schilling and Uhlig (2019), equilibrium velocities are pinned down at one. Moreover, whereas goods are simultaneously available for trade in either currency in these models, sellers in our setting preallocate their exogenous endowments across currency markets before trade and commit to that allocation, paralleling the timing in Angeletos and La'O (2013), where production precedes trade. This timing allows us to disentangle sellers' trade-share decisions from buyers' currency-velocity decisions in response to changes in inflation expectations.

The paper makes two main theoretical contributions. First, we establish a new equilibrium response to monetary expansion under currency competition. Monetary expansion need not primarily raise Dollar prices. It can instead increase the Dollar trade share and reduce Dollar velocity through greater Dollar saving, generating a novel "currency-substitution equilibrium." Equilibrium multiplicity itself is not new in this literature (Kareken and Wallace, 1981; Schilling and Uhlig, 2019); what is new is that monetary expansion can be absorbed through en-

dogenous adjustments in trade shares and velocities, thereby muting price pass-through despite fully flexible prices. Dollar prices can thus appear sticky, as in Head, Liu, Menzio, and Wright (2012), even without nominal rigidity.[4] Relatedly, Alvarez, Atkeson, and Edmond (2009) show in a single-currency economy that an endogenous decline in velocity can generate a sluggish price-level response to monetary expansion even when prices are fully flexible. Currency competition introduces an additional adjustment margin in our model: monetary expansion can be absorbed through both lower Dollar velocity and a higher Dollar trade share. In a single-currency economy with menu costs, Golosov and Lucas (2007) show that substantial individual price stickiness need not imply muted aggregate price adjustment. Our result establishes the converse: muted aggregate price pass-through despite fully flexible prices, as monetary expansion is absorbed through endogenous adjustments in currency trade shares and velocities.

For the mechanism, monetary policy via interest paid on Dollar balances has two opposing effects. It raises the nominal return to accepting and saving Dollars, attracting both sellers and buyers toward the Dollar. At the same time, because the interest is financed through an expansion of Dollar balances, it creates inflationary pressure and makes Dollars less attractive. The strength of this second effect depends on how much inflation agents expect the monetary expansion to generate. Crucially, agents' inflation expectations are not uniquely pinned down by the money-growth factor. Different inflation beliefs therefore lead agents to resolve this trade-off differently, so the policy parameter alone does not determine which force dominates.

Our second theoretical contribution is to develop a belief-based mechanism of equilibrium selection. Monetary policy alone does not select among the multiple equilibria. We show that inflation beliefs provide the missing selection criterion.

Lower expected Dollar inflation relative to Peso selects the *currency-substitution equilibrium*: sellers shift trade toward Dollars and buyers save more Dollars at given prices, hoping to take advantage of the nominal interest rate. Through market clearing, Dollar inflation is muted, monetary expansion spills over into higher Peso inflation, or both. This equilibrium appears paradoxical: agents increasingly accept and hoard the currency whose supply expands. Yet their behavior is rational: by raising the Dollar trade share and saving more Dollars, their actions mute realized Dollar inflation, validating their initial beliefs in a self-fulfilling manner.[5] The cross-currency inflation spillover arises because, with output fixed,

[4]Other frameworks also analyze the transmission of monetary shocks, but emphasize different adjustment margins: Chari, Kehoe, and McGrattan (2000) study real output responses under nominal price rigidity, whereas we hold output fixed and prices fully flexible, examining how monetary expansion is absorbed through endogenous adjustments in currency trade shares and velocities, thereby muting price pass-through.

[5]This outcome resembles a form of "rational money illusion": agents increasingly favor a currency undergoing monetary expansion, seemingly disregarding its inflationary consequences. Unlike conventional money illusion, however, agents do not misperceive nominal and real values. Their optimal

sellers shift goods toward the Dollar market, leaving fewer goods available for Peso transactions. A fixed Peso supply therefore chases fewer goods, raising the Peso price level. Thus, inflation "missing" from Dollar prices can reappear in Peso prices even without any expansion of the Peso supply. More generally, as already noted by Friedman, inflation can arise from a contraction in goods traded against a currency, not only from growth in that currency's supply. Higher expected Dollar inflation instead selects the traditional *pass-through equilibrium*, in which trade shifts toward Peso, buyers spend more Dollars at given prices, and monetary expansion passes more strongly into Dollar prices.

Moreover, pass-through into prices is inversely related to adjustment through trade shares and velocities. Monetary policy is therefore non-neutral for the real allocation of trade across currencies, even though aggregate output remains unchanged and prices are flexible.

While our results do not require us to impose a particular updating rule linking realized inflation to future expectations, any updating process in which lower realized inflation prevents a substantial upward revision of subsequent inflation expectations can sustain low-inflation beliefs over time. Conversely, persistently high realized inflation can sustain high inflation expectations.

Because the equilibrium response depends on inflation beliefs rather than on monetary policy alone, the monetary authority controls the policy instrument but not the equilibrium that is selected.[6] It therefore cannot uniquely determine the price level of its own currency, a loss of monetary control. At the same time, however, monetary policy acquires a powerful cross-currency dimension: it can shift trade toward or away from both currencies and affect inflation in the competing foreign currency even without changing that currency's supply.

To make the equilibrium-selection criterion precise, we combine monotone comparative statics with stochastic orders. We construct these orders so that moving upward corresponds to lower expected Dollar inflation relative to Peso and shifts agents' indifference conditions monotonically. Thus, if an agent is initially indifferent at a given action and belief, a change in beliefs along the order breaks that indifference and induces her to reoptimize, for example by changing how much to save or how much output to offer for Dollars. Because the indifference condition is also monotone in the agent's action, the direction of this adjustment is uniquely determined, yielding a monotone belief-to-action mapping. In the aggregate, sellers reveal relative inflation beliefs through the allocation of trade across currencies, while buyers respond through their saving decisions at given prices. For a given policy and state of the economy, a belief profile therefore pins down the trade allocation, and all equilibria it supports are ordered by prices and ve-

decisions endogenously sustain the low inflation they expect.

[6]This limitation applies under the assumption where the monetary authority cannot perfectly coordinate agents' inflation beliefs. If it could fully anchor beliefs on a particular inflation path, this source of equilibrium multiplicity would disappear.

locities, while different belief profiles select and rationalize different equilibrium paths in otherwise identical economies.

We use our selection result to reveal underlying inflation expectations and make equilibrium selection empirically observable. A higher Dollar trade share k_t reveals lower expected Dollar inflation relative to Peso among sellers, while an index J_t constructed from trade shares and velocities measures how much Dollar money growth is absorbed rather than passed through to prices. Unlike direct elicitation of inflation beliefs in the laboratory, the trade share allows us to infer otherwise unobserved inflation beliefs from equilibrium behavior using the theory, and, together with J_t, to track equilibrium selection within and across experimental economies.

Our equilibrium-selection argument allows for heterogeneity in inflation beliefs across agents, a natural feature of both experimental economies and real-world markets. Even without such heterogeneity, strategic uncertainty introduces a separate channel through which changes in expectations about future trade shares and velocities can affect realized inflation, see Section 3.5.

As another benchmark, perfect foresight clarifies why strategic uncertainty is essential for inflation multiplicity in Section 3.4. Perfect foresight does not eliminate equilibrium multiplicity: trade shares and velocities remain indeterminate under interest-bearing monetary policy, allowing for Dollar crowding-in or crowding-out. Unlike in Kareken and Wallace (1981), however, Dollar and Peso inflation are uniquely pinned down by sequential market clearing. Dollar inflation exhibits a unique tempered pass-through, while Peso inflation is independent of Dollar monetary policy. Strong and time-varying deviations from the conventional QTM pass-through benchmark, as well as monetary-policy spillovers into Peso, therefore require strategic uncertainty about equilibrium play.

We test these predictions in the laboratory. Subjects trade in two currency-specific markets and alternate between buyer and seller roles, reproducing the central intertemporal incentives of the model. In the benchmark treatment, monetary policy is muted, $m = 1$. Under active monetary policy, Dollar balances grow by 30 percent per period, $m = 1.3$. The laboratory is particularly well suited to studying this question because the economic fundamentals and policy rule are held fixed across independent economies. Differences in prices, trade shares, and velocities therefore reveal equilibrium selection rather than differences in fundamentals.

The experimental evidence supports the main mechanisms of the theory. Under muted monetary policy, the currencies are perfect substitutes, the economy resembles that of Kareken and Wallace (1981), and equilibrium outcomes are comparatively stable. Under active policy, otherwise identical economies display substantial heterogeneity in Dollar inflation, Dollar dominance, velocity, and Peso inflation. Dollar inflation remains well below the QTM pass-through benchmark on average, while Peso inflation is positive despite its fixed supply. Dollar trade

shares rise substantially, showing that monetary expansion can attract rather than repel trade from the expanding currency, while agents simultaneously hoard Dollars. Across individual economies, we observe outcomes ranging from strong currency substitution to substantially stronger price pass-through, as well as reversals between these regions over repeated sequences. In some individual economies, Peso inflation is as high as Dollar inflation, although the two arise through distinct mechanisms. Dollar trade shares reveal the corresponding movements in sellers' inflation beliefs, while the absorption index J_t tracks how much of the monetary expansion passes through to Dollar prices.

Literature Theoretical (Kareken and Wallace, 1981; Manuelli and Peck, 1990; Schilling and Uhlig, 2019) and experimental (Cardozo et al., 2024) work on currency competition has largely studied monetary expansion through lump-sum transfers. In Kareken and Wallace (1981), the exchange rate is constant but indeterminate, so the same monetary policy supports multiple inflation paths; Manuelli and Peck (1990) extend this indeterminacy to a stochastic setting without studying monetary policy, and Schilling and Uhlig (2019) show that a Dollar expansion can raise either the Dollar price level or that of the competing currency. In our perfect-foresight benchmark, by contrast, sequential market clearing uniquely pins down both inflation rates, and the exchange rate grows at the Dollar growth factor, restoring a unique quantity-theory mapping while trade shares and velocities remain indeterminate. Unlike these papers, we study pass-through into trade shares and velocities and provide a mechanism for selecting among equilibria; Section 6.2 discusses these differences in detail. In the experiment of Cardozo et al. (2024), the quantity theory largely holds and monetary expansion shifts trade toward the competing currency; under interest-bearing transfers, our results point in the opposite direction.

Work on interest-bearing money has focused on single-currency environments, where proportional transfers preserve one-for-one pass-through: theoretically in Lucas (1972), empirically at low frequencies in Lucas (1980), and experimentally in Jiang, Puzzello, and Zhang (2023). Under currency competition, this pass-through need not obtain, because monetary expansion can be absorbed through trade shares and velocities.

Other currency-competition models explain currency use through transaction costs, risk, adoption costs, or issuance technologies. Berentsen (1998) models digital-money adoption as a coordination problem with network externalities, Camera, Craig, and Waller (2004) study competition between safe and risky fiat monies, and related work studies public versus private money and competition among privately issued monies (Monnet, 2006; Marimon et al., 2003, 2012; Fernández-Villaverde and Sanches, 2019; Rahman, 2018; Garratt and Wallace, 2018). In Choi and Rocheteau (2022), government money grows through lump-sum transfers,

while policy directly controls its real return to keep real balances constant. In our model, the monetary authority controls only the nominal return, which is mechanically tied to supply growth, while the real return is determined endogenously by inflation, trade shares, and velocities. Biais, Rochet, and Villeneuve (2025) impose pass-through, whereas it is an endogenous equilibrium object here.

Our selection mechanism also relates to global games (Carlsson and van Damme, 1993; Morris and Shin, 1998, 2001), tested experimentally by Heinemann et al. (2004), and to sentiment-driven fluctuations in Angeletos and La'O (2013). Unlike in these settings, fundamentals are perfectly observed, beliefs concern endogenous future inflation, and ordered beliefs select among multiple equilibria rather than eliminating multiplicity; see Section 6.2.

Experimental work on money mostly studies how intrinsically worthless objects emerge as media of exchange or how competing monies are selected in decentralized trade.[7] Marimon and Sunder (1993, 1994) study equilibrium selection in single-currency OLG economies with seigniorage-financed deficits and find inflation dynamics consistent with adaptive expectations. Closer to us, several experiments study currency choice without monetary policy: Camera, Noussair, and Tucker (2003) show that history and hoarding incentives can sustain a return-dominated fiat money, Rietz (2019) and Jiang and Zhang (2018) study the acceptance of competing currencies in search models, Ding and Puzzello (2020) study legal restrictions and acceptance costs, and Baeriswyl, My, and Cornand (2025) study competition between a risky and a risk-free currency. In none of these does monetary policy change both the return on and the supply of a currency, as it does here.

Finally, expectations affect inflation through sticky prices in Werning (2022), and sticky prices generate the real effects of dominant currencies in Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Møller (2020); in our setting, expectations instead shift currency choice and saving under fully flexible prices. The continuum of inflation paths also relates to Lorenzoni and Werning (2023), although the coordination object here is beliefs about the relative future purchasing power of competing currencies.

2 The model

We study competition between two fiat currencies, an interest-bearing currency termed "Dollar" and a fixed-supply currency termed "Peso," in a Samuelson (1958) monetary economy, following the two-currency tradition of Kareken and Wallace

[7] See Kiyotaki and Wright (1989); Camera and Corbae (1999); Lagos and Wright (2005); Duffy and Ochs (1999, 2002) and Camera and Casari (2014) experimentally study the emergence of money, while related experiments study price formation in monetary exchange (Duffy and Puzzello, 2014a,b; Jiang, Norman, Puzzello, Sultanum, and Wright, 2024).

(1981) and Schilling and Uhlig (2019). Unlike these contributions, which implement monetary policy through lump-sum transfers, we assume that the Dollar pays interest on outstanding balances. Appendix A.4 extends the analysis to the case in which both currencies pay interest, at distinct rates.

2.1 Environment

We consider an infinite-horizon endowment economy in discrete time, indexed by $t = 1, 2, \ldots$. Figure 1 illustrates the structure of the economy.

There is a single, perishable consumption good (chips packages) that can be traded using either of the two competing currencies, named "Dollars" and "Pesos." The Dollar supply $D_t > 0$ is set at the discretion of a monetary authority, such as a central bank, a blockchain protocol, or a smart contract responding to market conditions. Peso is in fixed supply $B_t \equiv B > 0$ and represents a currency that lacks such an authority, or for which such policy is inactive.[8]

Agents, endowments, and preferences The economy consists of two equally sized and infinitely lived groups of agents: N green agents and N red agents, for a total of $2N$ agents. Agent i obtains utility $u(c_{i,t})$ from consumption $c_{i,t} \geq 0$, where $u(\cdot)$ is strictly increasing, strictly concave, twice continuously differentiable, and satisfies $u(0) = 0$ and $u'(0) = \infty$.[9] Money does not enter utility.

Aggregate output is constant at $Y_t = Y$, and also individual endowments are constant over time at $y_t = \bar{y}$, $Y = N\bar{y}$, for all $t > 0$. Hence, there is no idiosyncratic or aggregate risk. The roles of green and red agents alternate over time. In even periods, red agents are sellers, each endowed with the exogenous amount of $\bar{y}$ packages, and green agents are buyers who consume and have no goods endowment. In odd periods, green agents become sellers endowed with $\bar{y}$, and red agents are buyers.

In period $t = 1$, each buyer is endowed with $d_1 > 0$ Dollars and $b_1 > 0$ Pesos, while sellers hold no currency. The initial aggregate money supplies are $D_1 = Nd_1$ and $B_1 = Nb_1 \equiv B$. Agents derive utility only when they are buyers and choose consumption and currency savings to maximize expected discounted lifetime utility,

$$U = \mathbb{E}\left[\sum_{t=1}^{\infty} \xi_{i,t}\, \beta^t\, u(c_{i,t})\right], \qquad \beta \in (0,1) \tag{1}$$

where $\xi_{i,t} = 1$ when agent i is a buyer and zero otherwise. Unlike in an overlapping-generations economy, agents do not exit after buying. Instead, they carry money

[8] The Bitcoin protocol fits this interpretation: a predetermined supply path capped at 21 million coins, with no discretionary adjustment. Ether instead fits the "Dollar," with staking yields and no hard cap.

[9] The CRRA utility function $u(c) = \sqrt{c}$ applied in the experiment or any $u(c) = c^{\alpha}, \alpha \in (0,1)$, $u(c) = \log(1 + \sqrt{c})$ satisfy those assumptions.

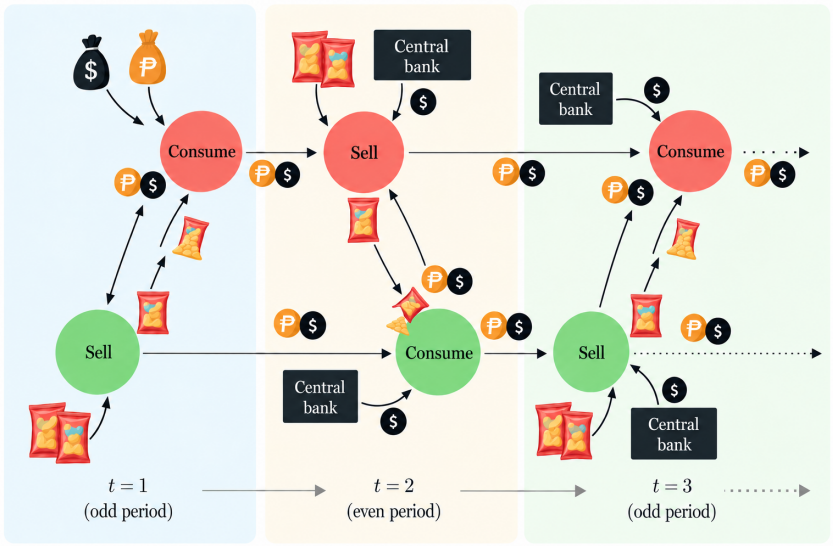


Figure 1: Schematic representation of the structure of the economy

balances forward and subsequently become sellers again. A seller thus plays the role of a young agent in an OLG model: she sells her endowment for money that finances consumption in the following period, when she returns as a buyer.[10]

Market segmentation and trade shares We assume semi-segmented markets: in each period, each seller i preallocates her endowment across two currency-denominated goods markets before trade opens and cannot reallocate during trading. Markets are otherwise centralized and frictionless, with free and instantaneous trade.

At every t, seller i chooses the number of packages offered in the Peso market, $x_{i,t} \geq 0$, and in the Dollar market, $z_{i,t} \geq 0$, subject to $x_{i,t} + z_{i,t} \leq \bar{y}$. Buyers make no allocation choice, and we adopt the convention $x_{i,t} = z_{i,t} = 0$ for buyers. Aggregate allocations satisfy $Y_t^D = \sum_{i=1}^{2N} z_{i,t}$, $Y_t^B = \sum_{i=1}^{2N} x_{i,t}$, and $Y_t^D + Y_t^B \leq Y$. The Dollar trade share $k_t \equiv Y_t^D / Y \in [0,1]$ summarizes sellers' relative currency preferences. We refer to $Y_t^D > Y_t^B$ as "Dollar dominance."

Sellers allocate goods across currencies according to their expected future purchasing power. Unlike in Kareken and Wallace (1981) and Schilling and Uhlig (2019), where goods are simultaneously available for purchase with either currency, our preallocation timing ensures that trade shares reflect sellers' preferences rather than buyers' spending decisions. This distinction is important for our equilibrium-selection result and matches the experimental design in Section 4.

Prices adjust flexibly to clear the two centralized goods markets. Let P_t denote the market-clearing Dollar price of a package and $\hat{P}_t$ the market-clearing

[10] The relevant expectation horizon for a seller compares prices in t and $t+1$, whereas a buyer compares prices in t and $t+2$, when she buys again.

Peso price of a package. We define Dollar inflation as $\pi_t = P_t/P_{t-1}$ and Peso inflation similarly as $\hat{\pi}_t$. While excluding direct exchange between currencies, define Q_t as the implicit exchange rate given by the law of one price, $Q_t = \frac{P_t}{\hat{P}_t}$, i.e. the price of one Peso denominated in Dollars. Buyer i's consumption decomposes as $c_{i,t} = q^D_{i,t} + q^B_{i,t}$, where $q^D_{i,t}$ and $q^B_{i,t}$ are consumption units purchased in the Dollar and Peso markets. With beginning-of-period currency holdings $(d_{i,t}, b_{i,t})$, the buyer faces budget constraints $P_t q^D_{i,t} \le d_{i,t}$, $\hat{P}_t q^B_{i,t} \le b_{i,t}$. For t-sellers, we adopt the convention $c_{i,t} = q^D_{i,t} = 0 = q^B_{i,t}$.

Monetary policy, money balances, and velocity The Peso supply is constant, $B_t \equiv B$. The Dollar supply grows through interest payments on outstanding Dollar balances at gross rate $m \ge 1$, financed by additional Dollar issuance.[11] Thus, $D_t = mD_{t-1} = m^{t-1}D_1$, $t > 1$, $\quad D_1 = N \times d_1$, where D_t is the Dollar supply at the beginning of period t, after interest payments and before trading. For $m = 1$, both currency supplies are constant, and the currencies are symmetric, as in Kareken and Wallace (1981).

If agent i enters period t as a buyer with balances $(d_{i,t}, b_{i,t})$, her balances after purchases and interest payments are $d_{i,t+1} = m\left(d_{i,t} - q^D_{i,t}P_t\right)$, $\; b_{i,t+1} = b_{i,t} - q^B_{i,t}\hat{P}_t$. If she enters as a seller, her balances after sales and interest payments are $d_{i,t+1} = m\left(d_{i,t} + z_{i,t}P_t\right)$, $\quad b_{i,t+1} = b_{i,t} + x_{i,t}\hat{P}_t$. Individual balances sum to aggregate supplies: $D_t = \sum_{i=1}^{2N} d_{i,t}$ and $B = \sum_{i=1}^{2N} b_{i,t}$. Currency velocities measure the share of each currency's total supply spent within a period:

$$V^B_t = \frac{\sum_{i \in N_{B,t}} (b_{i,t} - b_{i,t+1})}{B} \in [0,1], \qquad V^D_t = \frac{\sum_{i \in N_{B,t}} (d_{i,t} - d_{i,t+1}/m)}{D_t} \in [0,1].$$

The corresponding savings rates are $s^B_t = 1 - V^B_t$ and $s^D_t = 1 - V^D_t$, measuring the shares of currency balances carried forward before interest payments. Market clearing implies the quantity-theory identities $P_t Y^D_t = D_t V^D_t$ and $\hat{P}_t Y^B_t = BV^B_t$.

Unlike in standard OLG models, where agents spend their remaining balances before exiting, infinitely lived agents may save currency across periods. Velocities are therefore endogenous and bounded above by one, since each currency unit can change hands at most once per period. Together with trade shares, endogenous velocities can attenuate monetary-policy pass-through.

History, timing, and beliefs Although the economy has no fundamental uncertainty, agents face strategic uncertainty about future equilibrium outcomes. Let $H_t \equiv (Y^D_t, Y^B_t, V^D_t, V^B_t, P_t, \hat{P}_t)$ denote the aggregate state at the end of period t. Let $\mathcal{F}_{i,t}$ denote all information available to agent i at that time, including the history

[11] If interest were financed in real terms, for example through taxes or an external return, the associated inflationary force would be weaker. Such financing could introduce direct redistributive effects, as taxing all agents while remunerating only Dollar holders would disadvantage agents with relatively large Peso positions.

of aggregate states and her own past actions, balances, and consumption. When the agent index is suppressed, we write $\mathscr{F}_t$.

Let $\phi_{i,t}$ denote agent i's belief kernel (density) over future states. Because agents alternate between seller and buyer roles, the same underlying beliefs enter different decision problems depending on the agent's role at t. We therefore write $\phi^S_{i,t}$ and $\phi^B_{i,t}$ for the role-specific marginals relevant to the seller and buyer problems, respectively; the corresponding payoff-relevant expectations are characterized below.

At $t = 1$, agents have no prior history on which to condition their beliefs, so initial beliefs are drawn exogenously. Thereafter, agents update their belief kernels in light of observed history. We do not impose a specific belief updating rule. This follows the rational-expectations tradition, in which beliefs are disciplined by equilibrium consistency rather than by a separately specified updating process (Lucas, 1972; Sargent and Wallace, 1975; Lucas and Stokey, 1983).

Each period t consists of two stages. In the pretrade stage t^-, sellers move before period-t prices are known. Seller i chooses her allocation $(z_{i,t}, x_{i,t})$ given $\mathscr{F}_{i,t-1}$ and belief $\phi^S_{i,t}$. Sellers commit to these allocations before trade opens.

Buyers move second, after seller allocations (Y^D_t, Y^B_t) have been fixed. During trade, each buyer observes the contemporaneous market prices $(P_t, \hat{P}_t)$ and takes them as given when choosing consumption and saving. Let $\mathscr{F}^P_{i,t} \equiv \mathscr{F}_{i,t-1} \vee \sigma(Y^D_t, Y^B_t, P_t, \hat{P}_t)$ denote the information available to buyer i at this stage. Given $\mathscr{F}^P_{i,t}$ and her buyer-relevant beliefs, she chooses consumption and saving. In the aggregate, these choices determine currency velocities, while prices adjust to clear both markets. Thus, prices and aggregate buyer spending are jointly determined in equilibrium, although each individual buyer is a price taker. The resulting aggregate state H_t completes the period.

2.2 Individual decision rules and equilibrium

A seller decision rule at date t specifies an $\mathscr{F}_{i,t-1}$-measurable allocation $(z_{i,t}, x_{i,t})$ that, given belief $\phi^S_{i,t}$, maximizes

$$\mathbb{E}^{\phi^S_{i,t}}_{t^-}\left[\sum_{k\geq 0}\beta^{2k+1}u(c_{i,t+2k+1}) \mid \mathscr{F}_{i,t-1}\right]$$

subject to $z_{i,t} + x_{i,t} \leq \bar{y}$ and the seller's money-balance laws of motion defined above. A buyer decision rule at date t specifies an $\mathscr{F}^P_{i,t}$-measurable choice $(q^D_{i,t}, q^B_{i,t}, d_{i,t+1}, b_{i,t+1})$ that, given belief $\phi^B_{i,t}$, maximizes

$$u(q^D_{i,t} + q^B_{i,t}) + \mathbb{E}^{\phi^B_{i,t}}_{t}\left[\sum_{k\geq 1}\beta^{2k}u(c_{i,t+2k}) \mid \mathscr{F}^P_{i,t}\right]$$

subject to the buyer's budget constraints and money-balance laws of motion defined above. The policy parameter m is common knowledge and is suppressed from the information notation.

We allow beliefs to differ across agents. Belief kernels have densities with full support over feasible future states. For later use, define the cross-sectional average role-specific beliefs by $\bar{\phi}_t^S \equiv \frac{1}{|N_{S,t}|}\sum_{i\in N_{S,t}} \phi_{i,t}^S$, $\bar{\phi}_t^B \equiv \frac{1}{|N_{B,t}|}\sum_{i\in N_{B,t}} \phi_{i,t}^B$.

Definition 2.1 (Rational expectations equilibrium (REE))**.** *Given initial money holdings $\{d_{i,1}, b_{i,1}\}_{i=1}^{2N}$ and policy $m \geq 1$, a rational expectations equilibrium consists of sequences of prices $\{P_t, \hat{P}_t\}_{t\geq 1}$, allocations $\{Y_t^D, Y_t^B\}_{t\geq 1}$, velocities $\{V_t^D, V_t^B\}_{t\geq 1}$, individual seller and buyer decision rules, and belief kernels $\{\phi_{i,t}\}_{i=1,\ldots,2N;\, t\geq 1}$, with role-specific marginals $\phi_{i,t}^S$ and $\phi_{i,t}^B$, such that, for every period t:*

1. ***Optimality:*** *each seller and buyer decision rule solves the corresponding optimization problem above, conditional on the agent's information and relevant belief.*
2. ***Market clearing:*** $Y_t^D = \sum_{i\in N_{S,t}} z_{i,t} = \sum_{i\in N_{B,t}} q_{i,t}^D$, $Y_t^B = \sum_{i\in N_{S,t}} x_{i,t} = \sum_{i\in N_{B,t}} q_{i,t}^B$.
3. ***Nominal accounting:*** $P_t Y_t^D = V_t^D D_t$, $\hat{P}_t Y_t^B = V_t^B B$.
4. ***Belief consistency:***
   (a) Individual dynamic consistency. *For each agent, successive belief kernels are conditional distributions induced by a common subjective probability measure over future states. Hence, the agent's conditional expectations satisfy the law of iterated expectations.*
   (b) Aggregate rational-expectations consistency. *Conditional on the information available when agents move, the cross-sectional average role-specific beliefs $\bar{\phi}_t^S$ and $\bar{\phi}_t^B$ correctly forecast the equilibrium distributions of the corresponding payoff-relevant future states generated by equilibrium decision rules and market clearing. Individual beliefs may differ across agents, but their cross-sectional averages must be correct in equilibrium.*

The distribution from which initial beliefs are drawn is exogenous, but the realized belief profile is part of the equilibrium. Thus, for given primitives (Y, m), different admissible belief profiles can support different REE. Below we show that all equilibria supported by a given belief profile share the same trade allocation and are ordered by prices and velocities.

Our rational-expectations restriction differs slightly from the standard representative-agent formulation. In the usual rational-expectations benchmark, agents' expectations are model-consistent with the equilibrium law of motion (Lucas, 1972; Sargent and Wallace, 1975; Lucas and Stokey, 1983). Here, we allow individual beliefs to differ across agents and require model-consistency only in the cross-sectional average within each role. Thus, individual forecast errors need not vanish, but they must wash out in the aggregate: the average seller and average buyer beliefs must coincide with the corresponding equilibrium distributions. This relaxation preserves rational-expectations consistency at the aggregate level while allowing for belief heterogeneity across agents.

Definition 2.2 (Strong monetary equilibrium)**.** *A* strong monetary equilibrium *is an REE in which both currencies retain value and are used in every period:* $P_t \sum_i q^D_{i,t} > 0$ *and* $\hat{P}_t \sum_i q^B_{i,t} > 0$ *for all* $t \geq 1$.

The environment also admits non-monetary equilibria, in which both currencies are valueless, and single-currency equilibria. We focus on strong monetary equilibria because currency choice becomes degenerate when one currency ceases to circulate.[12] Co-circulation restricts the support of the belief kernels. A currency has value today only if it is expected to have value tomorrow.

Against this background, we now derive the main theoretical results.

3 Theory results

3.1 Primer: QTM under Currency Competition

In a single-currency economy, the QTM equates the nominal value of output to total money expenditure, $P_t Y = D_t V^D_t = m D_{t-1} V^D_t$. With constant output, sequential market clearing implies

$$\frac{P_t}{P_{t-1}} = m \frac{V^D_t}{V^D_{t-1}}.$$

Thus, with constant velocity, monetary expansion passes one-for-one into prices, and hence inflation. Under currency competition, sellers can reallocate goods between the Dollar and Peso markets. Market clearing requires $P_t Y^D_t = D_t V^D_t$ and $\hat{P}_t Y^B_t = B V^B_t$, where $Y^D_t + Y^B_t = Y$. Importantly, the aggregate QTM remains an accounting identity when both currencies are expressed in Dollar terms: $P_t Y = D_t V^D_t + Q_t B V^B_t$, $Q_t = \frac{P_t}{\hat{P}_t}$. However, monetary expansion need not pass one-for-one into Dollar prices. Sequential market clearing in each currency market yields

$$\frac{P_t}{P_{t-1}} = m \frac{V^D_t}{V^D_{t-1}} \frac{Y^D_{t-1}}{Y^D_t}, \qquad \frac{\hat{P}_t}{\hat{P}_{t-1}} = \frac{V^B_t}{V^B_{t-1}} \frac{Y^B_{t-1}}{Y^B_t}. \tag{2}$$

Currency competition introduces an additional margin of adjustment that can mute the effect of Dollar money growth on Dollar prices. Dollar money growth can be absorbed through higher Dollar prices, lower Dollar velocity, or a larger Dollar trade share. Moreover, because aggregate output is fixed, reallocating goods toward the Dollar market reduces the supply of goods available for Peso transactions, potentially raising Peso prices.

[12] We rule out barter for simplicity, since search-theoretic models already show that under random matching money arises as a medium of exchange that resolves the double coincidence of wants. A seller's only route to future utility is therefore to accept money today, anticipating that future sellers will do the same when roles reverse. In the experiment, at least one currency has to be valued for subjects to accrue utility but nothing imposes that both currencies need to be.

Proposition 3.1 (Monetary policy pass-through under currency competition)**.** *Let $\Delta_t \equiv Y_t^D - Y_{t-1}^D$ denote the change in Dollar trade, with the corresponding change in Peso trade $\hat{\Delta}_t = -\Delta_t$. Then, all else equal,*

$$P_t = m\frac{V_t^D}{V_{t-1}^D}\delta_P P_{t-1}, \qquad \hat{P}_t = \frac{V_t^B}{V_{t-1}^B}\hat{\delta}\,\hat{P}_{t-1}, \tag{3}$$

where $\delta_P \equiv Y_{t-1}^D / Y_t^D$ and $\hat{\delta} \equiv Y_{t-1}^B / Y_t^B$. For $\Delta_t > 0$, we have $\delta_P < 1$ and $\hat{\delta} > 1$: holding velocity changes fixed, reallocating trade toward Dollars dampens Dollar price pass-through and amplifies Peso inflation. The effects reverse for $\Delta_t < 0$.

The result follows from sequential market clearing. Since $Y_t^D = Y_{t-1}^D + \Delta_t$ and $Y_t^B = Y_{t-1}^B - \Delta_t$, market clearing in consecutive periods implies $\frac{D_t V_t^D}{P_t} = \frac{D_{t-1}V_{t-1}^D}{P_{t-1}} + \Delta_t$, $\frac{BV_t^B}{\hat{P}_t} = \frac{BV_{t-1}^B}{\hat{P}_{t-1}} - \Delta_t$. Solving for prices, using $D_t = mD_{t-1}$, yields (3), with $\delta_P = Y_{t-1}^D / Y_t^D$ and $\hat{\delta} = Y_{t-1}^B / Y_t^B$. With unchanged trade allocations, $\Delta_t = 0$, both factors equal one, $\delta_P = \hat{\delta} = 1$, reproducing the single-currency price recursion for Dollars. When Dollar trade expands, $\Delta_t > 0$, $\delta_P < 1$ and $\hat{\delta} > 1$: holding velocity changes fixed, a larger quantity of goods sold in the Dollar market dampens Dollar price pass-through, while the contraction of the Peso market amplifies Peso price growth. The effects reverse for $\Delta_t < 0$.

Proposition 3.1 holds velocities fixed and therefore provides only a partial-equilibrium benchmark. The following analysis determines how monetary policy affects trade shares and velocities jointly in general equilibrium.

3.2 Efficiency and Equilibria

Lemma 3.1 (Efficiency)**.** *For any $m \geq 1$, all goods are offered for sale, $z_{it} = \bar{y} - x_{it}$, and $Y_t^D = Y - Y_t^B$. All offered goods are purchased, $\sum_i^N x_{it} = Y_t^B = \sum_i^N q_i^B$, $\sum_i^N z_{it} = Y_t^D = \sum_i^N q_i^D$. Aggregate consumption is constant over time, $\sum_{i=1}^N c_{i,t} = \sum_{i=1}^N c_{i,t+1}$ for all $t \geq 1$.*

Although aggregate consumption remains constant, the allocation of goods across the Dollar and Peso markets can vary with monetary policy. We next characterize individual optimality and equilibrium behavior for any $m \geq 1$, where monetary policy is active for $m > 1$ and muted for $m = 1$. Throughout, we suppress the agent index i. Consumption c_t denotes individual consumption, and expectations are conditional on the agent's beliefs and information: $(\phi_{i,t}^S, \mathscr{F}_{i,t-1})$ for sellers and $(\phi_{i,t}^B, \mathscr{F}_{i,t}^P)$ for buyers.

Proposition 3.2 (Rational expectations monetary equilibrium)**.** *For every $m \geq 1$:*

1. ***Sellers' indifference (inflation and exchange rates):***
   (a) *t-Sellers strictly prefer selling for Dollars over Peso if they expect*
   $\mathbb{E}_{t^-}\left[\frac{\hat{P}_t}{\hat{P}_{t+1}} u'(c_{t+1}) \middle| m, \mathscr{F}_{t-1}\right] < m \cdot \mathbb{E}_{t^-}\left[\frac{P_t}{P_{t+1}} u'(c_{t+1}) \middle| m, \mathscr{F}_{t-1}\right]$.

(b) *t-Sellers are indifferent between currencies if*

$$\mathbb{E}_{t^-}\left[\frac{\hat{P}_t}{\hat{P}_{t+1}}u'(c_{t+1})\middle| m, \mathscr{F}_{t-1}\right] = m \cdot \mathbb{E}_{t^-}\left[\frac{P_t}{P_{t+1}}u'(c_{t+1})\middle| m, \mathscr{F}_{t-1}\right]. \tag{4}$$

2. ***Buyers' optimization (pricing conditions):***

(a) *t-Buyers spend all Dollars if* $\beta^2 m^2 \mathbb{E}_t\left[\frac{u'(c_{t+2})}{P_{t+2}}\middle| m, \mathscr{F}_t^P\right] < \frac{u'(c_t)}{P_t}$, *and spend all Pesos if* $\beta^2 \mathbb{E}_t\left[\frac{u'(c_{t+2})}{\hat{P}_{t+2}}\middle| m, \mathscr{F}_t^P\right] < \frac{u'(c_t)}{\hat{P}_t}$.

(b) *t-Buyers are indifferent between consuming and saving if*

$$\frac{u'(c_t)}{P_t} = \beta^2 m^2 \mathbb{E}_t\left[\frac{u'(c_{t+2})}{P_{t+2}}\middle| m, \mathscr{F}_t^P\right], \qquad \textit{(Dollar goods market)} \tag{5}$$

$$\frac{u'(c_t)}{\hat{P}_t} = \beta^2 \mathbb{E}_t\left[\frac{u'(c_{t+2})}{\hat{P}_{t+2}}\middle| m, \mathscr{F}_t^P\right], \qquad \textit{(Peso goods market).} \tag{6}$$

3. ***Market clearing:*** *the Dollar and Peso-denominated markets clear:* $Y_t^D = Y - Y_t^B$, $Y_t^D = \frac{D_t V_t^D}{P_t}$, $Y_t^B = \frac{BV_t^B}{\hat{P}_t}$.

The Inada condition $u'(0) = \infty$ rules out zero consumption in any buying period, so $c_t > 0$. The notation $\mathbb{E}_{t^-}$ denotes sellers' expectations formed before observing period-t prices, whereas $\mathbb{E}_t$ denotes buyers' expectations conditional on contemporaneous prices and their pretrade information.

For $m = 1$, the equilibrium conditions coincide with those of Schilling and Uhlig (2019): both currencies offer the same nominal return, and currency choice depends on expected relative inflation. Under perfect foresight, the conditions reduce further to Kareken and Wallace (1981), with an indeterminate constant exchange rate. For $m > 1$, our equilibrium conditions differ from the lump-sum transfer setting of Schilling and Uhlig (2019), also studied experimentally by Cardozo, Rosokha, and Zhang (2024). Monetary policy operates through two opposing forces. Higher expected Dollar inflation encourages sellers to shift trade toward Pesos and buyers to spend Dollars sooner. At the same time, the interest payment m encourages sellers to accept Dollars and buyers to save them. Which force dominates depends on inflation expectations, allowing different equilibrium trade shares and velocities under the same monetary policy. Agents' indifference conditions can therefore hold at distinct allocations and savings levels, generating the equilibrium multiplicity that we study next.

Equilibrium types Proposition 3.2 characterizes the expected returns that make sellers and buyers indifferent but does not uniquely pin down equilibrium outcomes. The six aggregate variables $(Y_t^D, Y_t^B, V_t^D, V_t^B, P_t, \hat{P}_t)$ are linked through market clearing and $Y = Y_t^D + Y_t^B$, while future indifference conditions depend on the evolution of beliefs. We distinguish two classes of equilibria, constructed below:

Definition 3.1 (Currency-Substitution and Pass-through Equilibria)**.** *Fix* $m > 1$ *and*

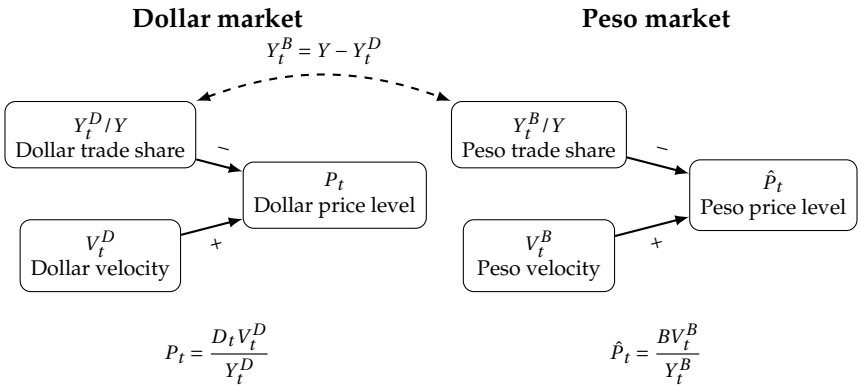


Figure 2: Market clearing implies that each currency's price level rises with its velocity and falls with its trade share. The two markets are interconnected because total output is sold against either the first or the second currency, so that $Y_t^B = Y - Y_t^D$.

Y. We distinguish two types of rational-expectations equilibria according to their observable outcomes:

1. *Currency-substitution equilibrium (money illusion, Gresham's law). Trade shifts toward Dollars, accompanied by Dollar hoarding. Dollar inflation remains muted, while Peso prices rise through monetary-policy spillovers.*
2. *Pass-through equilibrium (Thiers' law). Trade shifts toward Pesos, while agents spend Dollars more rapidly. Dollar prices rise sharply, while Peso prices remain stable or fall.*

These are two extremes of a continuum of equilibria, distinguished by observable trade shares, velocities, and prices. Stronger Dollar pass-through is associated with a lower Dollar trade share and/or higher Dollar velocity, together with weaker Peso spillovers. The belief profiles that support and select among these equilibria are characterized below. Both patterns are consistent with market clearing and individual indifference. A higher Dollar trade share combined with lower Dollar velocity dampens Dollar inflation and, by reducing goods available for Peso transactions, raises Peso prices, generating a Gresham-like pattern.[13] Conversely, a lower Dollar trade share and higher Dollar velocity amplify Dollar pass-through, generating a Thiers-like pattern.

Equilibrium multiplicity is well established in the currency competition literature (Kareken and Wallace, 1981; Schilling and Uhlig, 2019). In Section 6.2, we discuss how our belief-based equilibrium-selection mechanism extends this literature and contrasts with the single-currency rational-expectations benchmark of Lucas (1972).

[13]The classical statement of Gresham's law refers to "bad" (overvalued) money driving out "good" (undervalued) money when the relative value of the two currencies is fixed. Following the broader usage in currency-competition models such as Camera et al. (2004), we use Gresham's law to describe the allocation of currencies between trade and hoarding under flexible exchange rates.

3.3 Equilibrium Selection and Belief Classification

The previous section established that currency competition admits multiple equilibrium responses to Dollar supply expansion: pass-through raises Dollar prices, whereas currency substitution increases the Dollar trade share and lowers Dollar velocity, muting Dollar inflation. While equilibrium multiplicity is well established in the literature, a mechanism explaining which equilibrium response agents coordinate on is missing. Our second main contribution is to characterize how inflation beliefs select among equilibria through agents' currency allocations and saving decisions.

Because sellers allocate goods across currency markets before trade, whereas buyers move after observing allocations and prices, the two selection margins can be studied separately within each period. Sellers determine trade shares; conditional on these allocations, buyers determine spending, saving, and hence velocities. Across periods, agents alternate between roles, linking the two margins dynamically.

Seller beliefs and the relative-return order. Let Ω denote the state space of relevant future histories. For a feasible seller allocation $(x_t, z_t) \in [0, \bar{y}]^2$ and seller belief kernel ϕ^S on Ω, define the seller payoff differential

$$I^S_{i,t}(x_t, z_t \mid m, \mathscr{F}_{i,t-1}, \phi^S) \equiv \mathbb{E}^{\phi^S}_{t^-}\left[\left(m\frac{P_t}{P_{t+1}} - \frac{\hat{P}_t}{\hat{P}_{t+1}}\right)u'(c_{t+1}) \,\middle|\, m, \mathscr{F}_{i,t-1}\right]. \tag{7}$$

We suppress the agent subscript below. At a fixed seller allocation (x_t, z_t), $I^S_t > 0$ means that selling for Dollars yields a higher expected marginal return than selling for Peso, so the seller strictly prefers selling for Dollars. Conversely, $I^S_t < 0$ implies a strict preference for selling for Peso. In an interior equilibrium in which both currencies are accepted, seller indifference requires $I^S_t = 0$.

The seller differential is the difference between two expectations taken over different random variables: the interest-adjusted expected real return on Dollars and the expected real return on Peso. A stochastic order on the distribution of either return alone determines the direction of change in only one of these terms, and therefore does not in general sign the change in the differential.

We therefore order seller beliefs directly by the value of this difference.

Definition 3.2 (Relative-return (RR) order on seller belief kernels)**.** *Let ϕ^S and $\tilde{\phi}^S$ be seller belief kernels on Ω. We say that ϕ^S relative-return dominates $\tilde{\phi}^S$, written $\phi^S \succeq_{\mathscr{R}} \tilde{\phi}^S$, if*

$$I^S_t(x_t, z_t \mid m, \mathscr{F}_{t-1}, \phi^S) \geq I^S_t(x_t, z_t \mid m, \mathscr{F}_{t-1}, \tilde{\phi}^S)$$

for every feasible $(x_t, z_t) \in [0, \bar{y}]^2$. Dominance is strict, $\phi^S \succ_{\mathscr{R}} \tilde{\phi}^S$, if the inequality is strict at every feasible allocation.

Thus the RR order is constructed exactly so that the seller payoff differential

is monotone in beliefs when the allocation is held fixed. Economically, an RR-dominant kernel implies a larger expected interest-adjusted real return on Dollars relative to the expected real return on Peso.

Equivalently, it embodies lower expected Dollar inflation relative to Peso inflation. Distinct seller beliefs need not change this expected return difference, so not all belief kernels are strictly RR-ordered.

Buyer beliefs and first-order stochastic dominance. Buyers face a different problem. Conditional on the seller-determined trade allocation, a buyer chooses how much of each currency to spend on consumption or save. Her Dollar and Peso payoff differentials are

$$I_t^{B,D}(q_t^D \mid m, \mathscr{F}_t, \phi^{B,D}) \equiv \frac{u'(c_t)}{P_t} - \beta^2 m^2 \mathbb{E}_{\phi^{B,D}}\left[\frac{u'(c_{t+2})}{P_{t+2}} \,\middle|\, m, \mathscr{F}_t\right], \tag{8}$$

$$I_t^{B,B}(q_t^B \mid m, \mathscr{F}_t, \phi^{B,B}) \equiv \frac{u'(c_t)}{\hat{P}_t} - \beta^2 \mathbb{E}_{\phi^{B,B}}\left[\frac{u'(c_{t+2})}{\hat{P}_{t+2}} \,\middle|\, m, \mathscr{F}_t\right]. \tag{9}$$

Each buyer belief kernel ϕ^B over future histories induces marginal belief distributions $\phi_t^{B,D}$ over a Dollar random variable $u'(c_{t+2})/P_{t+2}$ and $\phi_t^{B,B}$ over the Peso random variable $u'(c_{t+2})/\hat{P}_{t+2}$ by integrating out all other future variables. Unlike the seller differential, each buyer differential depends on the expectation of a single random variable. First-order stochastic dominance is therefore the natural ordering: it ranks buyer beliefs so that, at every fixed feasible action, the corresponding payoff differential moves monotonically with the belief order. For Dollar beliefs, define the cdfs $F_D(x) \equiv \int_{-\infty}^{x} \phi_t^{B,D}(s)\, ds$, $\quad \tilde{F}_D(x) \equiv \int_{-\infty}^{x} \tilde{\phi}_t^{B,D}(s)\, ds$.

Definition 3.3 (First-order stochastic dominance on buyer belief kernels)**.** *We write $\phi_t^{B,D} \succeq_{\text{FOSD}} \tilde{\phi}_t^{B,D}$ if $F_D(x) \le \tilde{F}_D(x)$ for all x and every savings pair (d_{t+1}, b_{t+1}), where both cdfs are evaluated at the same savings pair. Dominance is strict, $\phi_t^{B,D} \succ_{\text{FOSD}} \tilde{\phi}_t^{B,D}$, if the inequality is strict on a set of positive Lebesgue measure.*

The Peso order is defined analogously from the cdfs of $u'(c_{t+2})/\hat{P}_{t+2}$.

Savings must be held fixed in this belief comparison. For a given belief, the expected future marginal consumption depends on how much the buyer saves today: higher savings raise expected future consumption c_{t+2} and lower expected future marginal utility. We denote the expected marginal continuation values of a saved Dollar and a saved Peso by

$$f_i^D(d_{t+1}, b_{t+1}) \equiv \beta^2 m^2 \mathbb{E}_{\phi_t^{B,D}}\left[\frac{u'(c_{t+2})}{P_{t+2}} \,\middle|\, d_{t+1}, b_{t+1}\right], \tag{10}$$

$$f_i^B(d_{t+1}, b_{t+1}) \equiv \beta^2 \mathbb{E}_{\phi_t^{B,B}}\left[\frac{u'(c_{t+2})}{\hat{P}_{t+2}} \,\middle|\, d_{t+1}, b_{t+1}\right]. \tag{11}$$

For a fixed belief, we assume that f_i^D and f_i^B are nonincreasing in both d_{t+1} and b_{t+1}, which we refer to as the wealth effect of savings: saving more of either

currency raises expected future consumption and therefore lowers the expected marginal value of an additional unit of either currency.[14]

In this notation, the buyer payoff differentials (8) simplify to $I_t^{B,D} = \frac{u'(c_t)}{P_t} - f_i^D(d_{t+1}, b_{t+1})$, $\quad I_t^{B,B} = \frac{u'(c_t)}{\hat{P}_t} - f_i^B(d_{t+1}, b_{t+1})$.

The standard implication of FOSD is a corresponding ordering of expectations (Shaked and Shanthikumar, 2007). In the present setting this directly orders the buyer payoff differentials.

Corollary 3.1 (FOSD and buyer payoff differentials). *If $\phi_t^{B,D} \succeq_{\text{FOSD}} \tilde{\phi}_t^{B,D}$, then at every savings pair: $f_i^D(d_{t+1}, b_{t+1} \mid \phi_t^{B,D}) \geq f_i^D(d_{t+1}, b_{t+1} \mid \tilde{\phi}_t^{B,D})$, and therefore, for every fixed feasible consumption choice (q_t^D, q_t^B), that is, at fixed savings,*

$$I_t^{B,D}(q_t^D \mid m, \mathscr{F}_t, \phi^{B,D}) \leq I_t^{B,D}(q_t^D \mid m, \mathscr{F}_t, \tilde{\phi}^{B,D}).$$

Both inequalities are strict under strict FOSD. The same statements hold for Peso under the corresponding FOSD ordering.

A FOSD-dominant Dollar belief therefore places more weight on states in which the future marginal value of Dollars, $u'(c_{t+2})/P_{t+2}$, is high at every level of savings, and thus embodies lower Dollar inflation expectations. Analogously, FOSD dominance for Peso embodies lower Peso inflation expectations.

3.3.1 Belief-based Equilibrium Selection

Definition 3.4 (Currency-substitution and pass-through belief profiles). *Fix a date t. Let $\Phi^{\text{abs}} \equiv \big(\{\phi_{i,t}^S\}_{i\in N_{S,t}}, \{\phi_{i,t}^{B,D}\}_{i\in N_{B,t}}, \{\phi_{i,t}^{B,B}\}_{i\in N_{B,t}}\big)$ and $\Phi^{\text{pt}} \equiv \big(\{\tilde{\phi}_{i,t}^S\}_{i\in N_{S,t}}, \{\tilde{\phi}_{i,t}^{B,D}\}_{i\in N_{B,t}}, \{\tilde{\phi}_{i,t}^{B,B}\}_{i\in N_{B,t}}\big)$. We call Φ^{abs} a* date-t currency-substitution belief profile *relative to Φ^{pt}, and Φ^{pt} a* date-t pass-through belief profile *relative to Φ^{abs}, if for every active agent i*

$$\phi_{i,t}^S \succ_{\mathscr{R}} \tilde{\phi}_{i,t}^S, \qquad \phi_{i,t}^{B,D} \succeq_{\text{FOSD}} \tilde{\phi}_{i,t}^{B,D}, \qquad \text{and} \qquad \tilde{\phi}_{i,t}^{B,B} \succeq_{\text{FOSD}} \phi_{i,t}^{B,B}.$$

In that case we also write $\Phi^{\text{abs}} \succ \Phi^{\text{pt}}$. The dominance is imposed agent by agent, not merely on the cross-sectional average. Under these orderings, every seller under Φ^{abs} expects a strictly higher real return on Dollars relative to Peso than under Φ^{pt}, and every buyer expects weakly lower Dollar inflation and weakly higher Peso inflation.

Weak dominance suffices for buyers.[15]

[14] $f_i^D/(\beta m)$ and f_i^B/β are the derivatives of the buyer's expected continuation value with respect to Dollar and Peso savings; see Appendix A.1. That f_i^D is nonincreasing in d_{t+1} and f_i^B in b_{t+1} follows from concavity of the buyer's continuation value. The substantive part of the assumption is the cross effect: additional Peso savings lower the marginal value of a saved Dollar, and additional Dollar savings lower the marginal value of a saved Peso. The two cross effects are equivalent, since f_i^D/m and f_i^B are partial derivatives of the same function, $\partial f_i^D/\partial b_{t+1} = m\partial f_i^B/\partial d_{t+1}$, while the own effects may differ across currencies.

[15] This is because the strictness of the selection result in Proposition 3.3 comes from the sellers' strict shift in the trade share; the buyer side enters the argument only through weak inequalities between

Assumption 3.1 (Beliefs, preferences, and continuation values). *(A1) Every belief profile considered supports a strong monetary REE, in which both currencies are valued and used at every date. In particular, beliefs assign positive probability to both currencies retaining value in all future periods, i.e., to prices $(P_{t+\tau}, \hat{P}_{t+\tau})$ remaining finite for all $\tau \geq 0$.*

(A2) Relative risk aversion is at most one: $-c\,u''(c)/u'(c) \leq 1$ for all $c > 0$.

(A3) For every buyer i, $\mathbb{E}_t^{\phi_{i,t}^B}[S_i]$ is twice continuously differentiable and strictly concave in $(d_{i,t+1}, b_{i,t+1})$, and f_i^D, f_i^B are nonincreasing in both arguments.

Assumption (A1) restricts the set of belief profiles we consider. Some belief profiles support nonmonetary equilibria, in which agents expect neither currency to retain value, so that neither is accepted and both are valueless. Other belief profiles support single-currency monetary equilibria, in which agents expect only one currency to retain value and sellers allocate all goods to that currency's market. We exclude both classes. In a nonmonetary equilibrium no trade takes place, and in a single-currency equilibrium currency choice is degenerate, so neither class speaks to currency competition. Assumption (A2) includes log utility, the standard benchmark, as its boundary case, as well as the power utility used in our experiment.[16]

The strict concavity in (A3) is needed only along directions that shift savings from one currency to the other while holding current consumption fixed. We impose it as an assumption. It reflects a diversification motive and holds whenever the buyer's beliefs about the relative future real return on the two currencies are nondegenerate, as discussed in the proof. It fails only if she believes that the future real values of the two currencies always move together, in which case Dollars and Pesos are perfect substitutes as stores of value.

We obtain the second main result of the theory part:

Proposition 3.3 (Equilibrium Characterization and Selection). *Let $m \geq 1$, fix a date t and the history $\mathscr{F}_{t-1}$, and suppose Assumption 3.1 holds.*

1. ***Characterization at a given belief profile** Φ.*
   - *(a) All date-t equilibrium outcomes under Φ share the same trade allocation (Y_t^D, Y_t^B).*
   - *(b) Any two strong monetary equilibrium outcomes under Φ, the second denoted by primes, are strictly ordered: $P_t' > P_t \iff \hat{P}_t' > \hat{P}_t \iff V_t^{D\prime} > V_t^D \iff V_t^{B\prime} > V_t^B$.*
2. ***Selection.** Let $\Phi^{\text{abs}} \succ \Phi^{\text{pt}}$ as in Definition 3.4. Compare any strong monetary equilibrium outcome under Φ^{abs}, denoted by primes, with any strong monetary equilibrium outcome under Φ^{pt}, and call a seller interior if her allocation under Φ^{pt} is interior.*
   - *(a) Trade shares satisfy $Y_t^{D\prime} \geq Y_t^D$ and $Y_t^{B\prime} \leq Y_t^B$, strictly if at least one seller is interior.*

velocity schedules. By Corollary 3.1, the buyer orderings imply $f_i^D(\cdot \mid \Phi^{\text{abs}}) \geq f_i^D(\cdot \mid \Phi^{\text{pt}})$ and $f_i^B(\cdot \mid \Phi^{\text{abs}}) \leq f_i^B(\cdot \mid \Phi^{\text{pt}})$ at every given savings pair.

[16] Examples are $u(c) = \log c$, with relative risk aversion equal to one; $u(c) = c^\alpha$ with $\alpha \in (0,1)$, with relative risk aversion $1-\alpha$, including $u(c) = \sqrt{c}$ used in the experiment; and $u(c) = \log(1+\sqrt{c})$, with relative risk aversion $\frac{1}{2} + \frac{\sqrt{c}}{2(1+\sqrt{c})} \in (\frac{1}{2}, 1)$. Of these, c^α and $\log(1+\sqrt{c})$ also satisfy $u(0) = 0$ and $u'(0) = \infty$.

(b) *If* $Y_t^{D\prime} > Y_t^D$, *then* $P_t' < P_t$ *or* $\hat{P}_t' > \hat{P}_t$, *or both. If* $Y_t^{D\prime} = Y_t^D$, *either* $P_t' < P_t$ *or* $\hat{P}_t' > \hat{P}_t$, *or both, holds or* $P_t' = P_t$ *and* $\hat{P}_t' = \hat{P}_t$.

(c) *If at least one seller is interior and* $P_t' \geq P_t$, *then* $\hat{P}_t' > \hat{P}_t$ *and* $V_t^{D\prime} > V_t^D$.

(d) *If at least one seller is interior and* $\hat{P}_t' \leq \hat{P}_t$, *then* $P_t' < P_t$ *and* $V_t^{B\prime} < V_t^B$.

In particular, lower expected Dollar inflation relative to Peso is self-fulfilling in a sense that is robust to equilibrium multiplicity: whichever equilibria are selected, if at least one seller is interior, the economy under Φ^{abs} *trades more in Dollars and has a lower Dollar price, a higher Peso price, or both.*

Proposition 3.3 compares belief profiles while holding the economy and history fixed. The comparison is between rational-expectations equilibria, not out-of-equilibrium deviations: different inflation beliefs, arising from different initial belief draws or updating processes, can support different self-fulfilling outcomes in the same economy. The mapping from belief profiles to outcomes is not one-to-one in either direction. A given profile Φ can support multiple equilibria, including nonmonetary ones, and distinct profiles may imply the same expected values, and hence the same equilibrium choices and observable outcomes, so not all profiles can be ranked. Across all strong monetary equilibria under a fixed Φ, however, the trade share is unique, since sellers' pre-trade allocations depend only on their beliefs and not on prices. Equilibria can therefore differ only in buyers' spending and saving and the resulting prices, and part 1 shows that they form a chain: an outcome with a higher Dollar price also has a higher Peso price, strictly higher velocities in both currencies, and strictly lower aggregate buyer savings. The chain thus ranges from high saving and low prices to low saving and high prices.

3.3.2 Economic Mechanism of Equilibrium Selection

Our selection argument applies the logic of implicit-function comparative statics in belief space. Let $I(a \mid \phi)$ denote an agent's payoff differential at action a under belief kernel ϕ, so that an interior optimum satisfies $I(a^*(\phi) \mid \phi) = 0$. For a scalar parameter, one signs how a change in ϕ shifts I at a fixed action and combines this with the monotonicity of I in a. Here, however, the parameter is an infinite-dimensional belief kernel. We therefore define stochastic orders over beliefs *precisely so that, at every fixed feasible action, the payoff differential is monotone as beliefs increase along the order* (strictly so for sellers).[17] For sellers, this logic applies directly. For buyers, who choose savings in both currencies jointly and whose continuation values depend on both, it is implemented through monotone comparative statics on a lattice, as shown in the proof of Proposition 3.3. The second

[17] Stochastic orders compare beliefs without imposing a parametric form on their distributions. They reduce high-dimensional belief kernels to a one-dimensional ranking that isolates precisely the variation in inflation beliefs that shifts agents' optimality conditions in a systematic direction.

ingredient is the monotonicity of each differential in the agent's own action.[18]

Lemma 3.2 (Seller Differential Monotonicity)**.** *Suppose both currencies have value in period t. Then $I_t^S(x_t^i, z_t^i \mid m, \mathscr{F}_{t-1}, \phi^S)$ is strictly decreasing in the Dollar allocation z_t^i and therefore strictly increasing in $x_t^i = \bar{y} - z_t^i$, for $z_t^i, x_t^i \in [0, \bar{y}]$.*

The proof is in Appendix Section A.2.1. Together with Definition 3.2, the lemma yields the seller-side comparative static illustrated in Figure 3. Starting from indifference at z_1 under $\tilde{\phi}^S$, an RR-dominant belief ϕ^S raises the differential at z_1; since a larger Dollar allocation lowers it, indifference is restored at some $z_2 > z_1$. Aggregating, $Y_t^D = \sum_{i \in N_{S,t}} z_{i,t}^*(\phi_{i,t}^S)$ is weakly increasing in the seller belief profile, and strictly so if at least one seller with an interior allocation experiences a strict upward shift. The restriction to strong monetary equilibria is substantive: the seller differential compares Dollar and Peso returns and is undefined if one currency is valueless.

Lemma 3.3 (Buyer Differential Monotonicity)**.** *Holding beliefs and q_t^B fixed, $I_t^{B,D}(q_t^D \mid m, \mathscr{F}_t, \phi^{B,D})$ is strictly decreasing in Dollar-financed consumption q_t^D and hence strictly increasing in Dollar saving $d_{t+1}^i = m(d_t^i - P_t q_t^D)$, $q_t^D \in [0, d_t^i / P_t]$. Holding beliefs and q_t^D fixed, $I_t^{B,B}(q_t^B \mid m, \mathscr{F}_t, \phi^{B,B})$ is strictly decreasing in Peso-financed consumption q_t^B and hence strictly increasing in Peso saving $b_{t+1}^i = b_t^i - \hat{P}_t q_t^B$, $q_t^B \in [0, b_t^i / \hat{P}_t]$.*

The proof is in Appendix Section A.2.1. Analogously, a strictly FOSD-dominant Dollar belief lowers the buyer's Dollar differential at every fixed level of spending, so indifference is restored at lower Dollar spending: the buyer saves more Dollars. This one-dimensional argument holds Peso spending and prices fixed; the proof of Proposition 3.3 establishes the joint statement that, at every given price pair, a buyer saves weakly more Dollars and weakly fewer Pesos under the dominant belief. In the aggregate, the Dollar velocity schedule is therefore weakly lower and the Peso velocity schedule weakly higher at every price pair.

These individual mappings determine the trade share and the velocity schedules; market clearing then determines prices and velocities jointly. Consider a move from Φ^{pt} to Φ^{abs}. Sellers allocate weakly more goods to the Dollar market, strictly so if at least one seller's allocation is interior.[19] At given prices, buyers spend weakly fewer Dollars and weakly more Pesos. Through market clearing,

[18] Sellers and buyers require different orders not because they hold different beliefs: an agent has one belief kernel, but the two roles depend on different functionals of it. A seller compares the expected marginal returns of the two currencies, which gives rise to the RR order; a buyer compares the marginal value of spending today with the expected marginal value of each currency's future balances, which gives rise to the FOSD orders. Both restrict beliefs about marginal utility per unit of currency: lower expected Dollar inflation and higher expected Peso inflation raise the value of Dollar balances relative to Peso balances, inducing both greater Dollar acceptance by sellers and greater Dollar saving by buyers.

[19] If all sellers choose corner allocations, the trade share may remain unchanged: a seller who already allocates all goods to the Dollar market cannot allocate more, and a seller who allocates all goods to the Peso market changes her allocation only if the belief shift makes selling for Dollars strictly profitable at the margin.

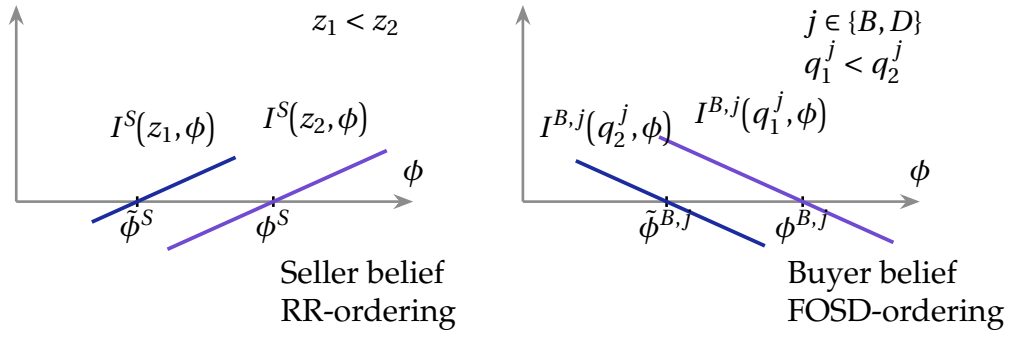


Figure 3: Beliefs shift seller and buyer payoff differentials; choices adjust until the relevant differential returns to zero.

$P_t = D_t V_t^D / Y_t^D$ and $\hat{P}_t = B V_t^B / Y_t^B$, these changes move prices, and the price response works against the initial adjustment: a lower Dollar price, for instance, raises the real value of a Dollar spent today and encourages Dollar spending. Because a buyer's saving in one currency also affects her current and future consumption, and thus her marginal values in both currencies, realized velocities cannot be ranked in general. Nevertheless, the combined shifts must lower the Dollar price, raise the Peso price, or both, strictly so whenever the trade share rises strictly. A currency's velocity can be ranked only when its price and its trade share move in the same direction: if the Dollar price does not fall, Dollar velocity is strictly higher under Φ^{abs}, and if the Peso price does not rise, Peso velocity is strictly lower.

In this sense, Φ^{abs} selects outcomes closer to the *currency-substitution equilibrium (Gresham's law)* of Definition 3.1, and Φ^{pt} outcomes closer to the *pass-through equilibrium (Thiers' law)*: the Dollar trade share is weakly higher, and the Dollar price is lower or the Peso price higher. These equilibria are the two poles of a spectrum, whose degree we quantify in Section 3.3.3.

3.3.3 Equilibrium Indexing: Revealed Beliefs and Price Absorption

Proposition 3.3 shows that equilibrium outcomes reveal inflation beliefs through sellers' currency allocations. Because sellers allocate goods before prices are observed, the Dollar trade share is uniquely determined by their beliefs and the history, even when a belief profile supports multiple equilibria, and it increases with the ordering of seller beliefs. Buyers' spending and saving, by contrast, respond monotonically to beliefs only at given prices, so realized velocities cannot generally be ranked across belief profiles. We therefore use the Dollar trade share as our measure of revealed beliefs:

$$k_t \equiv \frac{Y_t^D}{Y}. \tag{12}$$

Among ordered belief profiles, a higher k_t reflects lower expected Dollar inflation relative to Peso, that is, a higher expected real return on Dollars, which induces

sellers to allocate more trade to Dollars. Unlike velocities and prices, k_t is unaffected by equilibrium multiplicity in buyers' spending decisions.

Corollary 3.2 (Revealed Beliefs through Trade Shares)**.** *Suppose Assumption 3.1 holds, and compare belief profiles supporting strong monetary equilibria at the same date and history. If* $\Phi \succ \tilde{\Phi}$*, then*

$$k_t(\Phi) \geq k_t(\tilde{\Phi}), \tag{13}$$

with strict inequality if at least one seller's allocation under $\tilde{\Phi}$ *is interior. Conversely, if the two profiles are known to be strictly ordered in one direction or the other, then* $k_t(\Phi) > k_t(\tilde{\Phi})$ *implies* $\Phi \succ \tilde{\Phi}$*.*

Proof. The first statement follows from part 2(a) of Proposition 3.3, dividing by Y. For the converse, $\tilde{\Phi} \succ \Phi$ would imply $k_t(\tilde{\Phi}) \geq k_t(\Phi)$ by part 2(a), a contradiction. □

The direction of the belief ordering can thus be inferred from observable currency allocations, without eliciting numerical inflation forecasts. This interpretation has three limitations. First, the ordering is partial: trade shares cannot rank profiles that are incomparable under Definition 3.4. Second, k_t does not reveal individual sellers' numerical inflation expectations or their average: different beliefs may induce the same individual allocation, and different individual allocations may sum to the same trade share.[20] Third, Proposition 3.3 compares outcomes at a fixed date and history. In the experiment, sellers enter successive periods with different inherited portfolios, which may affect the continuation value of accepting either currency, so changes in k_t over time need not reflect changes in beliefs alone. Nevertheless, because agents alternate roles, today's buyers are tomorrow's sellers, so the trade-share path reflects the beliefs of all agents over time, not only those of current sellers.

Realized Dollar price absorption. The trade share does not by itself measure how much Dollar money growth passes through to prices. For this purpose, we define the index

$$J_t \equiv \frac{Y_t^D / Y}{V_t^D}, \tag{14}$$

which increases with the Dollar trade share and decreases with Dollar velocity.[21] Dollar market clearing implies $J_t = D_t/(YP_t)$, the real Dollar money stock per unit of output: within a period, a higher J_t corresponds exactly to a lower Dollar price, and the normalization by D_t removes the mechanical money-supply trend. Be-

[20] For example, with two equally endowed sellers, Dollar allocations of 20% and 80% yield the same trade share as 50% and 50%. Because allocations depend nonlinearly on beliefs and individual states, equal trade shares need not imply equal average expectations.

[21] Both components are dimensionless, with $k_t \in [0,1]$ and $V_t^D \in (0,1]$, and $\ln J_t = \ln k_t - \ln V_t^D$, so equal proportional changes in the two have equal and opposite effects on the index.

cause $D_t = mD_{t-1}$ and output is constant,

$$m = \frac{P_t}{P_{t-1}} \frac{J_t}{J_{t-1}}, \tag{15}$$

which decomposes Dollar money growth into Dollar inflation and changes in J_t. A constant J_t implies full pass-through, while a rising J_t implies that part of the monetary expansion is absorbed through trade shares and velocity, muting Dollar inflation. Unlike k_t, J_t is not monotone in beliefs: a given belief profile can support several equilibria with the same trade share but different velocities and prices. Movements in J_t may thus reflect changes in beliefs, differences across equilibria under the same beliefs, or both. We therefore use k_t to reveal beliefs and J_t to measure realized absorption.

Both measures allow outcomes to be compared across economies and over time. Their evolution need not represent convergence to a unique equilibrium, since equilibrium is not unique, nor learning about a fixed state, since there is none.[22]

3.3.4 Discussion and Implications of Equilibrium Selection

Substitution equilibria generate a rational form of money illusion. Even as the Dollar supply expands, agents may increase Dollar trade and hoard Dollars, seemingly confusing nominal and real currency values. They do not. Agents are fully rational: first, they maximize expected utility given their beliefs; second, these beliefs are self-fulfilling. Lower expected Dollar inflation relative to Peso induces sellers to allocate more trade to Dollars and buyers to save more Dollars and fewer Pesos at given prices. These choices lower the realized Dollar price, raise the realized Peso price, or both, in line with the initial expectations. Although we impose no specific updating rule, this validation may help sustain low Dollar inflation expectations over time.

The selection result is strict under a single condition: if at least one seller's allocation under Φ^{pt} is interior, the Dollar trade share rises strictly, and the realized Dollar price is strictly lower, the realized Peso price strictly higher, or both. Realized velocities, by contrast, can be ranked only when the price and the trade share of a currency move in the same direction, as in parts 2(c) and 2(d) of Proposition 3.3.

The selection mechanism requires strategic uncertainty about the relative real return on the two currencies. An agent's payoff differentials respond to her own choices because a change in her allocation or saving changes her own future consumption by different amounts across future price states. Under perfect foresight,

[22] Differences between two economies e and e' observed over T common periods can be measured by the Euclidean distances $d_k(e, e') = [\sum_{t=1}^{T}(k_{e,t} - k_{e',t})^2]^{1/2}$ and $d_J(e, e') = [\sum_{t=1}^{T}(J_{e,t} - J_{e',t})^2]^{1/2}$. Both are dimensionless and require no rescaling; d_k measures differences in revealed seller beliefs, d_J differences in realized price absorption.

this channel disappears: the return gap is deterministic, a price-taking seller cannot move her payoff differential through her allocation, and the selection mechanism becomes inoperative, as Section 3.4 demonstrates.

Heterogeneous beliefs play a different role. With a representative agent, consumption equals $\bar{y}$ in every state in equilibrium, so marginal utility cancels from the equilibrium indifference conditions, which then restrict beliefs about future prices directly: sellers must expect the interest-adjusted real return on Dollars to equal the real return on Pesos, $m\mathbb{E}_{t^-}\left[\frac{P_t}{P_{t+1}}\right] = \mathbb{E}_{t^-}\left[\frac{\hat{P}_t}{\hat{P}_{t+1}}\right]$. Together with the requirement that each agent's beliefs be correct, this severely restricts the set of admissible belief profiles; with heterogeneous beliefs, only the cross-sectional average must be correct. We therefore work with heterogeneous beliefs, for two reasons: it matches the experimental setting, in which subjects form their own expectations, and it provides a richer space of belief profiles consistent with rational expectations.

The agent-by-agent belief dominance imposed in Definition 3.4 is sufficient, but not necessary, for the rankings of Proposition 3.3. Ordering only cross-sectional average beliefs is, however, generally insufficient to rank aggregate outcomes.[23]

3.4 Perfect foresight equilibria

To build intuition for how beliefs generate multiple inflation paths in the stochastic economy, we now impose perfect foresight. Without strategic uncertainty, agents correctly anticipate all future prices, trade shares, and velocities. Proposition 3.2 becomes:

Proposition 3.4 (Symmetric perfect foresight monetary (SPFM) equilibrium)**.** *For every $m \geq 1$, a symmetric perfect foresight monetary equilibrium is characterized by $c_t = \bar{y} = c_{t+2}$ and*

1. ***Sellers' indifference:*** $\frac{P_{t+1}}{P_t} = m\frac{\hat{P}_{t+1}}{\hat{P}_t}$, *implying* $Q_{t+1} = mQ_t = m^t Q_1$.
2. ***Buyers' optimality:*** $\frac{P_{t+2}}{P_t} = \beta^2 m^2, \qquad \frac{\hat{P}_{t+2}}{\hat{P}_t} = \beta^2$.
3. ***Market clearing:*** $Y_t^D = Y - Y_t^B = \frac{D_t V_t^D}{P_t}, \qquad Y_t^B = \frac{BV_t^B}{\hat{P}_t}$.

For $j \in \{D, B\}$, define $\Delta_t^{Y,j} \equiv Y_t^j / Y_{t-1}^j$ as the change in currency-j trade share and $\Delta_t^{V,j} \equiv V_t^j / V_{t-1}^j$ as the change in its velocity.

Proposition 3.5 (Properties of SPFM equilibria)**.** *For every $m \geq 1$, active or mute, every SPFM equilibrium satisfies:*

(i) Tempered Dollar pass-through $\frac{P_{t+2}}{P_t} = \beta^2 m^2$, and
no Peso spillover: $\frac{\hat{P}_{t+2}}{\hat{P}_t} = \beta^2$.

[23] Individual optimal seller allocations $z_{i,t}^*$ and buyer saving decisions $(d_{i,t+1}^*, b_{i,t+1}^*)$ depend nonlinearly on beliefs and individual states, including heterogeneous currency balances, utility curvature, and corner solutions. Consequently, evaluating optimal choices at average beliefs need not yield the same aggregate outcome as aggregating individual choices under heterogeneous beliefs.

(ii) Master relation: *velocity and trade share of each currency move together, via*

$$\Delta_t^{V,j} = \beta \Delta_t^{Y,j}, \text{ for every } t \tag{16}$$

subject to feasibility $V_t^j \in (0,1]$ *and* $Y_t^j \in (0, Y)$.

(iii) Constructing trade-share shifts: *the equilibrium Dollar trade share can be designed to rise* ($\Delta_t^{Y,D} > 1$) *or fall* ($\Delta_t^{Y,D} \in (0,1)$)*, as long as the desired growth factor lies in the nonempty interval* $\max\left\{0,\ \frac{Y}{Y_{t-1}^D} - \frac{Y_{t-1}^B}{\beta Y_{t-1}^D V_{t-1}^B}\right\} \leq \Delta_t^{Y,D} \leq \min\left\{\frac{1}{\beta V_{t-1}^D},\ \frac{Y}{Y_{t-1}^D}\right\}$*, which contains* 1.

(iv) Constructing velocity shifts: *the Dollar velocity can be designed to rise or fall, for any desired velocity growth factor in* $\max\left\{0,\ \frac{\beta Y - Y_{t-1}^B / V_{t-1}^B}{Y_{t-1}^D}\right\} < \Delta_t^{V,D} \leq \min\left\{\frac{1}{V_{t-1}^D},\ \beta \frac{Y}{Y_{t-1}^D}\right\}$*, which contains* β*, and analogously for the Peso velocity. By (ii) with* $\beta \leq 1$*, a falling trade share forces a falling velocity and a rising velocity forces a rising trade share, whereas a rising trade share admits either falling or growing velocity.*

(v) Knife-edge cases: *a constant-trade-share equilibrium exists for every* $\beta \in (0,1]$*, with velocities declining at* $V_t^j = \beta V_{t-1}^j$*; a constant-velocity equilibrium exists if and only if* $\beta = 1$*, in which case trade shares are constant as well.*

Proposition 3.5 shows that perfect foresight uniquely pins down two-period inflation even though equilibrium allocations remain indeterminate.[24] The Dollar exhibits tempered pass-through, $\frac{P_{t+2}}{P_t} = \beta^2 m^2 < m^2$, while Peso inflation is independent of monetary policy, $\frac{\hat{P}_{t+2}}{\hat{P}_t} = \beta^2 < 1$. The Dollar-Peso exchange rate therefore grows at rate m, $Q_{t+1} = mQ_t$. For muted policy, $m = 1$, we recover the Kareken and Wallace (1981) result $Q_{t+1} = Q_t$.

Corollary 3.3. *Strong and time-varying deviations from the quantity theory of money beyond the tempered perfect-foresight benchmark, i.e.,* $\frac{P_{t+2}}{P_t} \ll \beta^2 m^2 < m^2$*, and Peso inflation spillovers, i.e., cases in which* $\frac{\hat{P}_{t+2}}{\hat{P}_t}$ *varies with* m*, require strategic uncertainty about equilibrium play, since neither can arise under perfect foresight.*

Perfect foresight nevertheless leaves trade shares and velocities indeterminate within the bounds of Proposition 3.5. Equilibrium may therefore feature either Dollar crowding-in or crowding-out and either rising or falling Dollar velocity. Moreover, these allocation dynamics do not require active monetary policy: the admissible adjustments in trade shares and velocities are independent of m, while only the inflation path depends on policy. Figure 4 illustrates this multiplicity. These results distinguish us from the overlapping generations model of Kareken

[24] One-period inflation need not be constant. For example, one can construct alternating inflation rates satisfying $P_{t+1}/P_t = \pi_1$ and $P_{t+2}/P_{t+1} = \beta^2 m^2/\pi_1$, while preserving $P_{t+2}/P_t = \beta^2 m^2$. Thus, our model also permits cyclical inflation under perfect foresight, but restricts its two-period growth factor. In contrast, the overlapping-generations model of Grandmont (1985), studied experimentally by Arifovic, Hommes, and Salle (2019), admits perfect-foresight equilibria with cycles of arbitrary periodicity and chaotic dynamics.

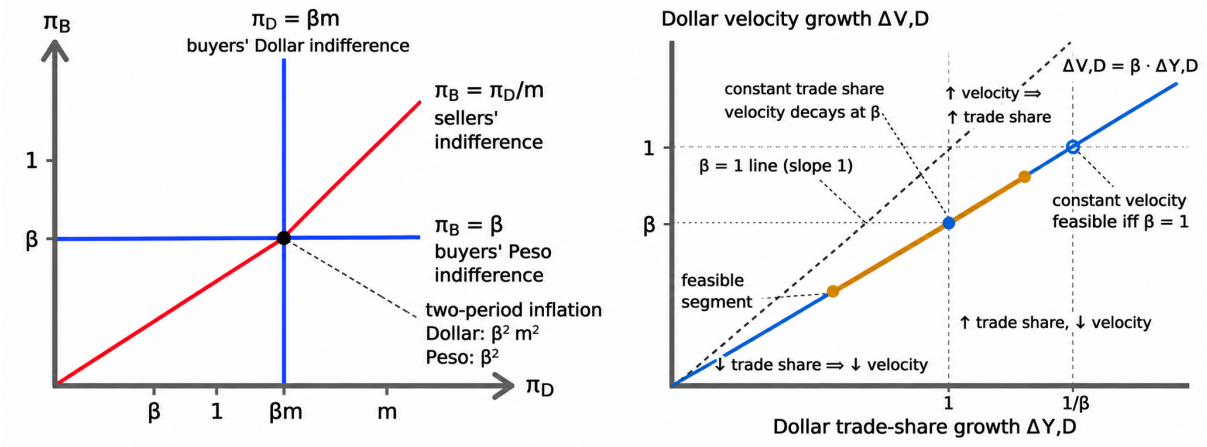


Figure 4: Perfect foresight equilibria: inflation and the relation between velocity growth and trade-share growth. The left panel shows the buyers' and sellers' indifference conditions, which pin down Dollar inflation at $\pi_D = \beta m$ and Peso inflation at $\pi_B = \beta$. The right panel illustrates the relation between velocity growth and trade-share growth. For each currency j, admissible one-period moves lie on the ray $\Delta_t^{V,j} = \beta \Delta_t^{Y,j}$ of slope $\beta \leq 1$, with feasibility restricting them to the highlighted segment around the constant-trade-share point. Constant velocity is attainable only at $\beta = 1$. The same relation holds for both currencies, but their adjustments are interdependent: since $Y_t^D + Y_t^B = Y$, an increase in the Dollar trade share requires a decrease in the Peso trade share and hence a decrease in Peso velocity.

and Wallace (1981) and the infinitely-lived agents model in Schilling and Uhlig (2019) where agents spend all currency every period, putting equilibrium velocity at one.

The master relation (16) further restricts this multiplicity. Within each currency, velocity and trade-share growth are positively linked, with $\Delta_t^{V,j} = \beta \Delta_t^{Y,j}$. Hence, a falling trade share implies a falling velocity, and a rising velocity implies a rising trade share, while a rising trade share can coexist with either rising or falling velocity. Across currencies, movements are negatively linked because $Y_t^D + Y_t^B = Y$: a rise in the Dollar trade share requires a fall in the Peso trade share and, through (16), a decline in Peso velocity. Figure 5 illustrates this cross-currency coupling.

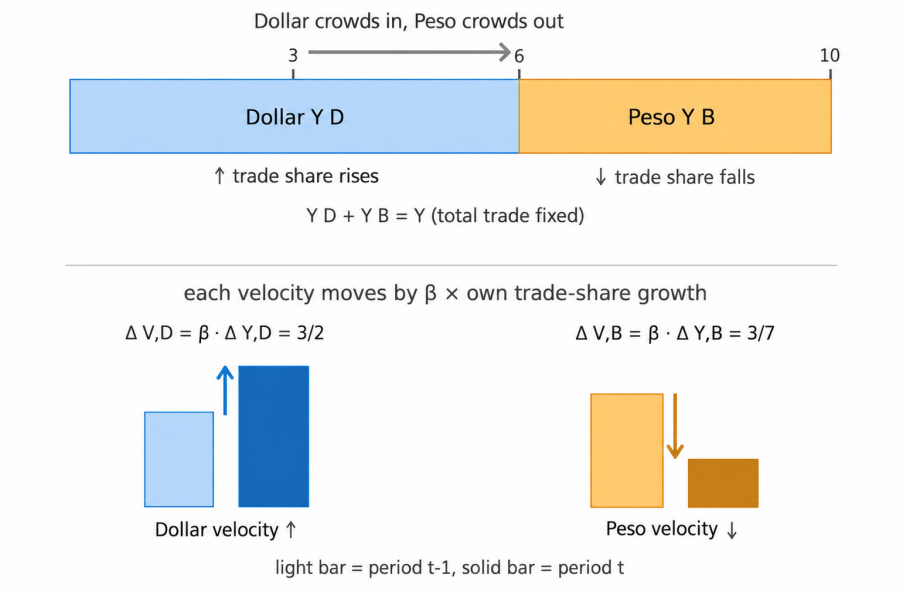


Figure 5: Example: cross-currency coupling under symmetric perfect foresight. A rising Dollar trade share ($Y^D : 3 \to 6$, $\Delta^{Y,D} = 2$) forces an offsetting fall in the Peso trade share ($Y^B : 7 \to 4$, $\Delta^{Y,B} = 4/7$) through the fixed total $Y^D + Y^B = Y$; by (16) each velocity then moves by β times its own trade-share growth ($\Delta^{V,D} = \beta\Delta^{Y,D} = \frac{3}{2}$, $\Delta^{V,B} = \beta\Delta^{Y,B} = \frac{3}{7}$, at $\beta = \frac{3}{4}$), so the Dollar velocity rises while the Peso velocity falls.

Next, we show that when agents cannot *ex ante* predict which of the many monetary equilibria will prevail, strategic uncertainty makes the realized path stochastic and enlarges the equilibrium set beyond this perfect-foresight benchmark. In particular, in the stochastic economy, even the tempered QTM equilibrium that prevails under perfect foresight can fail.

3.5 Representative-Agent Equilibria: Strategic Uncertainty

The selection result above is cross-sectional: it compares belief profiles at a single date. We now turn to a complementary time-series question: along a single realized equilibrium path, what does observed inflation reveal about how buyers' expectations evolve? To isolate this channel, we consider a representative agent under strategic uncertainty, an intermediate case between the two environments studied so far. Perfect foresight (Section 3.4) shuts down all belief channels and pins inflation at tempered pass-through, with no spillover. Heterogeneous beliefs (Proposition 3.3) add a cross-sectional selection channel, because individual consumption, and hence marginal utility, differs across agents and states. A representative agent under strategic uncertainty activates only the time variation of expectations, so any deviation from the tempered benchmark is attributable to that channel alone.

The representative-agent economy differs from the heterogeneous one in two ways. First, each agent's beliefs must themselves be correct, not only on average, which severely restricts the set of admissible belief profiles. Second, consump-

tion is pinned at $c_t = \bar{y} = c_{t+2}$ along every equilibrium path. Agents nevertheless remain price takers, with market clearing imposed only after optimization (Lucas and Prescott, 1971; Ljungqvist and Sargent, 2018): an agent's own future consumption responds to her own choices and to expected prices, and $c = \bar{y}$ results from market clearing and symmetry rather than being a constraint she imposes. Strategic uncertainty also persists, since sellers choose a currency before trade opens and must form beliefs about buyers' spending.[25]

We extend the single-currency decomposition of Sargent and Wallace (1973) to two currencies. Because every agent consumes $\bar{y}$ in a symmetric equilibrium, marginal utility cancels from the equilibrium Euler and seller indifference conditions. The buyer's Dollar Euler condition then expresses the realized price through expected prices, trade, and velocity,

$$P_t = \frac{1}{\beta^2 m^2}\,\mathbb{E}_t\big[1/P_{t+2}\big]^{-1} = \frac{1}{\beta^2}\,D_t\,\mathbb{E}_t\big[Y^D_{t+2}/V^D_{t+2}\big]^{-1}, \tag{17}$$

and seller indifference requires

$$\mathbb{E}_{t^-}\left[\frac{\hat{P}_t}{\hat{P}_{t+1}}\right] = m \cdot \mathbb{E}_{t^-}\left[\frac{P_t}{P_{t+1}}\right]. \tag{18}$$

In a strong monetary equilibrium, these conditions restrict equilibrium beliefs rather than quantities, and they do not uniquely determine the underlying belief distributions, since many distributions share the same expectations. Because prices must clear goods markets, expectations about future prices are equivalently expectations about future trade shares and velocities.

Lemma 3.4 (Realized inflation reflects expectations of trade and velocity)**.** *Under strategic uncertainty, realized Dollar and Peso inflation between t and $t+2$ equal the change over time in expectations about trade quantities and velocities:*

$$\frac{P_{t+2}}{P_t} = \mathbb{E}_{t+2}\big[1/P_{t+4}\big]^{-1}\,\mathbb{E}_t\big[1/P_{t+2}\big] = m^2\,\mathbb{E}_{t+2}\big[Y^D_{t+4}/V^D_{t+4}\big]^{-1}\,\mathbb{E}_t\big[Y^D_{t+2}/V^D_{t+2}\big], \tag{19}$$

$$\frac{\hat{P}_{t+2}}{\hat{P}_t} = \mathbb{E}_{t+2}\big[1/\hat{P}_{t+4}\big]^{-1}\,\mathbb{E}_t\big[1/\hat{P}_{t+2}\big] = \mathbb{E}_{t+2}\big[(Y - Y^D_{t+4})/V^B_{t+4}\big]^{-1}\,\mathbb{E}_t\big[(Y - Y^D_{t+2})/V^B_{t+2}\big]. \tag{20}$$

The second equalities use $D_{t+2} = m^2 D_t$. These characterizations yield conditions on expectations under which the economy departs from the tempered QTM:

Proposition 3.6 (Equilibrium deviations from the tempered QTM)**.** *Under strategic uncertainty, deviations from the tempered QTM pass-through depend on how price (or trade) expectations evolve over time.*

1. ***Dampened pass-through.*** *If buyers' expected Dollar prices fall sufficiently over time relative to the perfect-foresight benchmark, equivalently their expected Dollar*

[25] Knowledge of the model and of market clearing does not reveal which path obtains (Guesnerie, 1992; Van Huyck et al., 1990). Prices therefore still enter through an expectations operator.

trade share rises or their expected Dollar velocity falls sufficiently,

$$\mathbb{E}_t\big[Y^D_{t+2}/V^D_{t+2}\big] \le \beta^2\,\mathbb{E}_{t+2}\big[Y^D_{t+4}/V^D_{t+4}\big] \;\Leftrightarrow\; \mathbb{E}_t\big[1/P_{t+2}\big] \le \beta^2 m^2\,\mathbb{E}_{t+2}\big[1/P_{t+4}\big], \quad (21)$$

then inflation undershoots and pass-through is weaker than the perfect-foresight benchmark, $P_{t+2}/P_t \le \beta^2 m^2$.

2. ***Amplified pass-through.*** *If those expectations move the opposite way,*

$$\mathbb{E}_t\big[Y^D_{t+2}/V^D_{t+2}\big] > \beta^2\,\mathbb{E}_{t+2}\big[Y^D_{t+4}/V^D_{t+4}\big] \;\Leftrightarrow\; \mathbb{E}_t\big[1/P_{t+2}\big] > \beta^2 m^2\,\mathbb{E}_{t+2}\big[1/P_{t+4}\big], \quad (22)$$

then inflation overshoots, $P_{t+2}/P_t > \beta^2 m^2$.

3. ***Peso spillover.*** *If and only if buyers' expected Peso prices rise sufficiently over time relative to the perfect-foresight benchmark, equivalently their expected Peso trade share falls (Dollar trade share rises) or their expected Peso velocity rises sufficiently,*

$$\mathbb{E}_t\big[(Y - Y^D_{t+2})/V^B_{t+2}\big] > \beta^2\,\mathbb{E}_{t+2}\big[(Y - Y^D_{t+4})/V^B_{t+4}\big] \;\Leftrightarrow\; \mathbb{E}_t\big[1/\hat{P}_{t+2}\big] > \beta^2\,\mathbb{E}_{t+2}\big[1/\hat{P}_{t+4}\big], \quad (23)$$

then Peso inflation overshoots the perfect foresight benchmark, $\hat{P}_{t+2}/\hat{P}_t > \beta^2$.

Under perfect foresight, the trade-to-velocity expectation ratios in Lemma 3.4 equal β^2, and the inverse-price ratios equal $\beta^2 m^2$ for the Dollar and β^2 for the Peso, so all velocity and trade-share paths deliver the same tempered Dollar pass-through and the same Peso deflation, with no spillover, as in Proposition 3.5. Stronger deviations from the QTM and spillovers into Peso prices therefore require that expectations over prices, trade shares, or velocities evolve across periods, as Proposition 3.6 formalizes. Schilling and Uhlig (2019) first show that a monetary expansion can raise the price of the competing currency, but do not characterize the role of expectations in generating this spillover. Under interest-bearing policy, such spillovers arise even for a representative agent, purely through the time variation of expectations.

3.6 Persistence of Muted Inflation

Can muted Dollar inflation persist despite permanent monetary expansion? A natural question is whether a low-pass-through equilibrium can persist when the Dollar supply grows by the factor $m > 1$ every period, or whether Dollar inflation must eventually "catch up" with money growth. It need not. Combining Dollar market clearing in (2) with $D_t = mD_{t-1}$ gives $\frac{P_t}{P_{t-1}} = m\frac{V^D_t}{V^D_{t-1}}\frac{Y^D_{t-1}}{Y^D_t}$. Hence continued Dollar money growth can be absorbed by a rising Dollar trade share, a falling Dollar velocity, or both. The trade-share margin is necessarily bounded: in a strong monetary equilibrium $Y^D_t < Y$, and if the Dollar trade share converges, for example to $0.999Y$, then $Y^D_{t-1}/Y^D_t \to 1$, so changes in the trade share eventually cease to

absorb money growth. Velocity is different. It can remain strictly positive in every period while converging arbitrarily close to zero. In particular, once the trade share is approximately constant, any constant gross Dollar inflation rate $\pi^D < m$ can be sustained by setting

$$V_t^D := \frac{\pi^D}{m} V_{t-1}^D$$

so that V_t^D falls but never reaches zero at any finite date. Thus, continued Dollar money growth does not mechanically require an eventual transition to high Dollar inflation.

The natural question is why agents would coordinate on the low-inflation equilibrium rather than on the seemingly more natural high-inflation equilibrium when the Dollar supply is expanding permanently. The answer is that money growth by itself does not determine the relative return on the two currencies. What matters for an individual is the equilibrium behavior she expects from others. If she expects other agents to continue coordinating on the substitution equilibrium, selling predominantly for Dollars and to retain a large fraction of their Dollar balances, she expects a high Dollar trade share and low Dollar velocity, and hence only muted Dollar inflation. Dollars then offer a relatively high real return compared with Peso. Given this anticipated aggregate play, switching unilaterally toward Peso is not attractive: as a seller, she prefers to continue accepting Dollars, and as a buyer, she prefers to continue saving Dollars. These individual best responses sustain the aggregate behavior that keeps Dollar inflation low, and validates expectations ex post in a self-fulfilling manner. Because each individual is a price taker, a unilateral deviation does not overturn this aggregate price path. The high-inflation equilibrium is equally possible if agents instead expect rapid Dollar spending and a shift of trade toward Peso, but permanent Dollar money growth alone gives no individual agent a reason to abandon the low-inflation equilibrium once others are expected to continue playing it.

The logic is analogous to other coordination equilibria. Just as a bank run can be self-fulfilling when agents optimally respond to the expectation that others will run, muted Dollar inflation can be self-fulfilling when agents optimally respond to the expectation that others will continue to accept and retain Dollars.

The theory therefore allows transitions between the two equilibrium regions, but neither the timing nor the occurrence of such a transition is implied by continued growth of the Dollar supply alone.

Moreover, the muted realized inflation generated by the substitution equilibrium may reinforce the beliefs supporting it. An updating process that places weight on recent realized inflation may revise low Dollar inflation expectations only weakly when those expectations are repeatedly validated by realized outcomes. Such updating is not required for the existence of the substitution equilibrium, but provides one mechanism through which coordination on it can persist over time.

3.7 Summary and discussion of the main model mechanisms

The central implication of the model is that monetary policy does not uniquely determine inflation under currency competition. Because the policy parameter m raises both the nominal return on Dollar balances and the Dollar supply, its equilibrium effects depend on inflation beliefs. Under strategic uncertainty, the same m supports a continuum of REE. Lower expected Dollar inflation relative to Peso induces a higher Dollar trade share and greater Dollar saving at given prices, lowering the Dollar price, raising the Peso price, or both. Higher expected Dollar inflation instead selects equilibria with stronger pass-through. Unlike under perfect foresight, inflation itself therefore varies across equilibria. Under perfect foresight, multiple equilibria exist, but all share the same inflation path, with only trade shares and velocities remaining indeterminate. These contrasting outcomes motivate the following testable hypotheses.

3.8 Testable Hypotheses

Lemma 3.1 yields the first testable prediction for the experiment:

Hypothesis 1 (Market efficiency)**.** *All sellers sell their entire package endowment in every period* $t \geq 1$.

With policy muted, our setting resembles the deterministic framework of Kareken and Wallace (1981) and its stochastic counterpart in Schilling and Uhlig (2019). In both, monetary policy operates through lump-sum transfers that do not scale with money balances and therefore do not create a relative-return wedge between currencies. We obtain the comparable benchmark by setting $m = 1$. In that case, neither currency pays interest, the relative-return wedge disappears, and the two currencies become perfect substitutes, yielding a special case of Schilling and Uhlig (2019). Kareken and Wallace (1981) famously show that, under perfect foresight, exchange rates are constant but indeterminate.

Hypothesis 2 (Benchmark: currency competition without monetary policy ($m = 1$))**.** *Sellers hold trade shares constant; buyers hold currency velocities constant or decreasing; the exchange rate is constant across time, with possibly different Dollar and Peso prices that stay constant or decline over time.*

Under perfect foresight with $\beta < 1$, velocities and prices decline at rate β (Proposition 3.5). In the experiment, however, agents face strategic uncertainty and hold heterogeneous beliefs, so deviations from this benchmark are to be expected even under static policy. Without monetary expansion, nothing in the environment shifts relative returns over time, which suggests constant velocities once agents have coordinated on an equilibrium.

Based on Proposition 3.3, under active monetary policy, we conjecture one of two possible dynamics:

Hypothesis 3 (Currency competition with monetary policy ($m > 1$))**.**

1. ***Substitution equilibria (Gresham's Law).***
   - *Sellers reallocate trade toward Dollars away from Peso (higher Y_t^D, lower Y_t^B)*
   - *Buyers hoard Dollars, lowering velocity V_t^D.*
   - *Dampened pass-through: Dollar prices P_t grow slowly or decline.*
   - *Spillover into Peso: Peso prices $\hat{P}_t$ rise.*
   - *Exchange rate $Q_t = P_t/\hat{P}_t$ rises slowly below rate m or stays flat.*
2. ***Pass-through equilibria (Thiers' Law).***
   - *falling Dollar dominance in trade (lower Y_t^D, higher Y_t^B), more Peso trade*
   - *Buyers reduce Dollar savings, raising velocity V_t^D.*
   - *Strong pass-through: Dollar price P_t rises fast in m.*
   - *No or reversed spillover: Peso prices $\hat{P}_t$ fall or stay constant.*
   - *Exchange rate Q_t rises quickly at rate close to m.*

4 The laboratory experiment

We design a group experiment with a between-subject design based on our theoretical framework to test Hypotheses 1-3.

4.1 The experimental environment

In each session, we run an experimental economy composed of 12 subjects, six 'red agents' and six 'green agents' who alternate in the role of buyers and sellers in every period.[26] We consider two treatments: the benchmark treatment with $m = 1$, and a treatment denoted 'MP' with active monetary policy and use $m = 1.3$. Each treatment involves six independent economies.

Sequences of play To emulate an infinite-horizon economy and discounting in the lab, we implement independent repetitions, called sequences, of the experimental game with a stochastic ending, following the related literature (Fréchette and Yuksel, 2017). The stochastic ending prevents backward induction and induces impatience among subjects. Our subjects play repeatedly sequences com-

[26]Six market participants are considered sufficient to induce competition and hinder collusion, see e.g. Dufwenberg and Gneezy (2000), while keeping the scale of the experiment small enough to collect 12 independent observations where no subject participated more than once to the experiment, see below. Six buyers and six sellers do not compromise price-taking behavior in the laboratory: the continuous double auction competes away individual price impact even in thin markets, and Smith (1982) documents convergence to the competitive, price-taking-consistent equilibrium with as few as two sellers at near-full efficiency, so our design is comfortably within range. The price-taking assumption therefore rests on the trading institution rather than on a large-numbers limit, which our design does not invoke. This is consistent with the high market efficiency observed in our data (Section 5): strategic price impact would manifest as unexhausted gains from trade, whereas the double auction drives allocations toward full efficiency regardless of which equilibrium the economy selects.

posed of an unknown number of rounds so as to fill a two-hour lab session. Sequence lengths were randomly drawn from a geometric distribution with success parameter 1/6. In other words, at the beginning of each round, a sequence continues with probability $\beta = 5/6$, which acts as a proxy for the discount factor of the theoretical model. This probability is common knowledge among participants. With probability 1/6, a sequence ends and a new one begins among the same participants if enough time remains: participants were recruited for 2 hours and 15 minutes, with sequences designed to not exceed 1 hour and 50 minutes. This timing ensures that the end of the final sequence was kept sufficiently distant to remain unpredictable, maintaining a constant perceived ending probability throughout the experiment. To ensure comparability across sessions, the sequence lengths drawn for the first session are implemented for all sessions. Each round within a sequence represents one time period of the model.

The sequence lengths balance the trade off between emulating impatience through shorter sequences and providing sufficient scope for experience accumulation that is inherent to longer laboratory sessions. After a two-period training sequence to familiarize subjects with the trading interface and tasks without affecting their earnings, four sequences of play follow, consisting of seven, eleven, six, and eight periods, totaling 32 periods of actual play per economy.

Trading In the experiment, we use two separate, currency-specific auction houses with a continuous double auction (CDA) mechanism, see the trading interface in Appendix D.2 for illustration. CDA experiments are well-known in the experimental literature to quickly and accurately converge to the predictions of the competitive market model (90 to 100 percent allocative efficiency) and induce market clearing even with few traders and limited information (Smith, 1982; Plott, 2008). They show that using a CDA, where price discovery happens via a sequence of accepted bilateral bids and asks, is without loss of generality relative to the centralized market clearing of the theory. We therefore treat the within-period market-clearing price as the object our theory pins down and the CDA discovers, while cross-period equilibrium selection is governed by agents' beliefs rather than by the trading institution.

We use the labels "Currency A" for Dollars and "Currency B" for Peso to avoid framing. The sellers submit asks, while buyers submit bids, with each ask and bid consisting of a quantity and a unit price. The experiment enforces budget and capacity constraints. Buyers may submit multiple bids but once accepted, their outstanding bids are deleted if they exceed their remaining budget. Participants can retract bids or asks as long as they are not matched. Trade occurs whenever a seller's ask price is less than or equal to a buyer's bid price and the transaction price is the seller's ask price. The number of packages exchanged is always the minimum of the buyer's demand and the seller's supply. Additionally, each

market has a shortcut for participants to instantly accept the best standing trade. Sellers cannot reallocate packages across markets during trade.

Sequence of events Each round of a given sequence proceeds as follows:

1. In the first round of each sequence ($t = 1$), six of the 12 subjects are randomly assigned to start as buyers and six as sellers. Each buyer receives $d_{i,1} = 100$ Dollar and $b_{i,1} = 100$ Peso, and each seller is endowed with $y_{i,1} = 5$ packages. In subsequent rounds ($t > 1$), sellers again receive $y_{i,t} = 5$ packages, while buyers' balances reflect the proceeds from previous sales, possibly adjusted by monetary policy. Hence $B = 600$, $D_1 = 600$, and $Y = 30$.
2. Sellers have 30 seconds to allocate their packages between the two currency-specific auction houses. Any allocation is feasible, the only departure from the theory being that sellers allocate, and buyers purchase, only full (integer) packages.
3. Subjects trade for up to 90 seconds, with an auction house closing earlier if one side is exhausted: buyers accumulate packages toward their real-world earnings (see hereafter), and sellers accrue Dollar and Peso balances from sales.
4. After trade closes, monetary policy multiplies each Dollar (currency A) by $m \geq 1$ for buyers and sellers, $D_t = m^t 600$; the Peso quantity is unchanged.
5. Participants are then told whether the sequence continues. If so, roles flip, balances carry over, and unsold packages perish; if it ends, a new sequence begins with balances reinitialized as in Step 1.

Payoff Subjects are paid based solely on the number of packages they purchase, with payoffs converted into euros using the CRRA function $\sqrt{\#\text{packages}}$ to mimic consumption smoothing. Balances in experimental currencies (Dollars and Peso) are not paid out, preventing hoarding and reinforcing their role as fiat money (i.e., intrinsically worthless). Since roles alternate, subjects accrue payoff only every other period, namely when they are buyers. As sellers, they accumulate currency to spend in future rounds. Final earnings are based on two randomly chosen sequences to limit wealth effects while ensuring reasonable payoffs. Average earnings are €13 (€2 standard deviation), excluding a €9 show-up fee.

4.2 Experimental implementation

The experiment was conducted using the oTree platform (Chen et al., 2016) and was run in-person at the CREED lab of the University of Amsterdam. Six sessions took place in Spring 2022 and six in Spring 2023 (with both treatments run each time), with a total of $12 \times 2 \times 6 = 144$ unique student participants. On average, par-

ticipants were 22 years old; 52% identified as female,[27] around 60% studied Economics or Business, and most others Social Sciences or Law. Approximately 60% were from Europe and 30% from Asia; 91% reported being proficient in English, the experiment's language. Fewer than half had prior lab experiment experience. Treatment assignment was randomized and balanced across demographics.[28]

Participants first read the instructions privately at their own pace, followed by a verbal reading by the experimenter. All experimental material is provided in Appendix D. Participants then completed a four-question comprehension quiz before starting the game. At the end of the last sequence, participants completed a questionnaire on demographics, strategies, and comments.

5 Experimental results

5.1 Efficiency and Welfare

We begin by examining the dynamics of unsold packages and subjects' utility (payoff) as measures of market efficiency, pooling data per treatment; see Figure 6.

Experimental results 1 (Market efficiency and welfare)**.** *Across both treatments, the number of unsold packages declines significantly over rounds, yielding a significant increase in subjects' payoffs across rounds and sequences.*

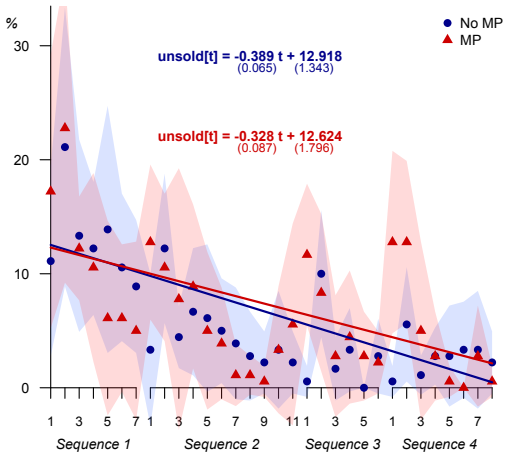
We observe a steady increase in efficiency: the number of unsold packages declines across rounds, consistent with Hypothesis 1. This indicates that repeated trading rounds and independent sequences fostered coordination and a reduction of market frictions. Interest payments under $m = 1.3$ significantly increase heterogeneity across buyers in prices and quantities purchased on the Dollar-denominated market (see Table C.1 in Appendix). However, across agents and periods, these distributional effects do not affect welfare (13.7 versus 13.5 in the benchmark versus the MP treatment, $p = 0.589$) or welfare dispersion across agents (0.320 vs. 0.45 with $p = 0.394$).[29] The absence of a treatment effect on welfare is consistent with the theory. Equilibrium selection changes prices, currency trade shares, and velocities, but with aggregate output fixed, aggregate consumption is constant across equilibrium types. Nor does our equilibrium classification predict how consumption is distributed across individual agents. We therefore do not expect active monetary policy to systematically raise or lower welfare relative to the benchmark.

[27] Response options were male, female, and other.

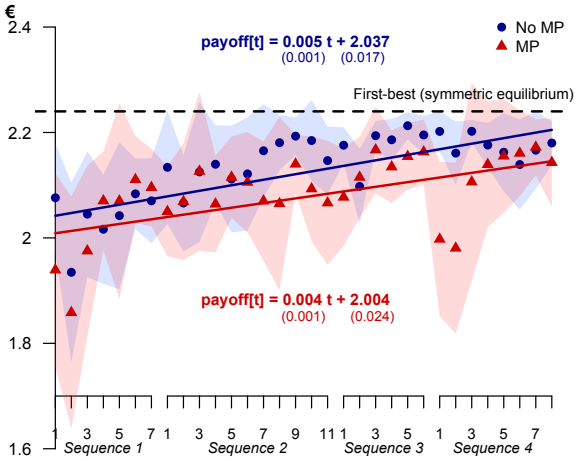
[28] The F-statistic of joint significance of the demographics on treatment assignments are 2.00 and 1.22 for the benchmark and the MP treatment respectively.

[29] All tests are exact Wilcoxon rank-sum tests at the matching group level with six observations in each group.

(a) Average unsold packages per round



(b) Welfare per round



Notes: Dots represent averages across economies per period, the shaded area indicates ± one standard deviation, lines are trends, represented by the linear regression line. Robust standard errors are reported between brackets underneath estimated coefficients.

Figure 6: Efficiency and welfare per treatment over time.

With constant output and utilitarian welfare, the first best is attained at the symmetric allocation, where each of the six buyers consumes the same number of packages, i.e., $c_{i,t} = 5$.

5.2 Overview of equilibrium selection

Figure 7 displays median values and interquartile ranges of the main variables across rounds and sequences in both treatments; individual economies are shown in Appendix B. We summarize the main outcomes before analyzing them in detail.

Experimental results 2 (Economies with muted monetary policy ($m = 1$))**.** *The experimental data do not reject Hypothesis 2, that states that the Dollar sales share and the exchange rate are constant over time, and that prices in both currency markets are constant or declining. Savings in both currencies persist over time.*

As for the monetary policy treatment, we make the following observations:

Experimental results 3 (Economies with active monetary policy ($m = 1.3$))**.**

- *The Dollar trade share rises over time to stabilize above 50%, before decreasing.*
- *Dollar velocity drops over time then flattens out, i.e. savings rates increase then stabilize.*

- *There is increasing Dollar inflation, i.e. the pass-through strengthens across rounds, but falls short of the rate suggested by the QTM (i.e. $\pi < 1.3$), approaching rather the tempered QTM equilibrium where $\pi = \frac{5}{6} \cdot 1.3$.*
- *As the Dollar trade share reaches a high enough level, there is Peso inflation due to scarcity (monetary policy spill-over)*
- *The Peso price in Dollars (exchange rate) strictly increases over time but at a rate below the QTM benchmark m.*
- *These aggregate patterns mask substantial heterogeneity across economies and over time, consistent with heterogeneous beliefs underlying distinct equilibria selected along the continuum from full pass-through to full substitution.*

Appendix Table C.3 complements Figure 7 with regressions on time variation. Under muted policy, subjects use both currencies roughly equally, with a Dollar trade share between 50 and 60% that trends slightly toward 50% (blue lines in Figure 7b; Column I). Transaction prices in the two currencies are similar, so the exchange rate is stable around one (Figures 7f, 7g, and 7c; Columns IV–VI), in line with Kareken and Wallace (1981). Unlike in Kareken and Wallace (1981), however, velocities are below one. Saving rates oscillate with role alternation but are stable and strictly positive across every other round, averaging about 18% in Dollars and 25% in Pesos, that is, velocities of about 82% and 75% (Figures 7d–7e; Columns II–III).

Positive saving persists although discounting ($\beta = 5/6$), the absence of any terminal value of currency, and $u(0) = 0$ all favor current spending. It is nonetheless consistent with our theory: infinitely lived agents carry balances across periods, and strategic uncertainty about future purchasing opportunities can rationally induce precautionary saving even when neither currency pays interest. Consistent with this motive, consumption differs markedly across buyers within the same economy (Table C.1 in Appendix C). The trading institution can add execution frictions: within the finite trading window, a buyer may miss a purchase, lose it to a faster buyer, or wait for a better price, leaving balances unspent (see also the word cloud in Appendix Figure D.4). These frictions apply equally under $m = 1$ and $m > 1$.

Under active monetary policy (orange lines), the Dollar trade share starts at a similar level but rises significantly and remains well above 50% in all sequences. Dollar saving rates are higher and rise over time, averaging about 29% (a velocity of about 71%), while Peso saving rates remain comparatively stable at about 33% (a velocity of about 67%). Both currencies exhibit inflation. Dollar inflation falls short of the 30% QTM benchmark (dotted grey line) and eventually approaches the tempered benchmark (dotted black line), while Peso prices rise despite the fixed Peso supply. The exchange rate accordingly rises by less than a factor of 1.3 per



Median values per round across six independent experimental economies. The dashed colored lines represent Q1 and Q3. The shaded areas depict the interquartile range (50% of observations). The black dotted lines represent theoretical reference values: a stable 30% growth path for the exchange rate in panel b) and a 30% growth path (grey), respectively $\beta*$ 30% growth path (black), for the Dollar price in Panel e) given the median initial values in each sequence observed in the experiment. Prices are computed as an average weighted by sold quantities per round.

Figure 7: Dynamics with and without monetary policy

round (Columns IV–VI): part of the "missing" Dollar inflation reappears in Peso prices.

5.3 Equilibrium selection and Evolution over time

We now turn to how equilibrium selection evolves across experimental sequences. Although budgets reset at the start of each sequence, subjects carry forward memory of past outcomes, which may shift their inflation expectations and hence the

equilibrium selected in subsequent sequences. Because m does not uniquely determine inflation, these changes should not be interpreted as learning about an unobserved state or convergence toward a uniquely correct expectation, since there is neither a latent state nor a unique equilibrium. Rather, changes in observed outcomes across sequences can reflect changes in beliefs and, consequently, in equilibrium selection.

Figure 8 reports the average inflation (grey) in each sequence of the six economies ($4 \times 6 = 24$ sequences) in each treatment using the same representation in the (π^D, π^B) plane as in Section 3.4. The figure also compares average inflation in the first (blue) and the last sequence (black), each averaged across six economies. Combined with the scatterplots in Figure 9 that illustrate how subjects adjust behavior over repeated sequences, we can describe the experimental outcomes through the lens of belief changes.

Benchmark economies ($m = 1$) In Figure 8a, we observe a strong concentration around the Dollar-Peso inflation couple $(1, 1)$, indicating that across all sequences, prices in both currencies remain stable and approximately equal over time. This suggests that under muted policy, subjects do not substantially adjust their beliefs and their play over time. The share of goods offered for Dollars remains largely stable across sequences, which is consistent with inflation outcomes remaining on the locus that represents sellers' indifference (brown diagonal line). The stability of trade shares, velocities, and prices under $m = 1$, despite subjects' repeated experience with the same trading institution, also provides a direct control for generic familiarity effects: repeated experience alone does not generate systematic drift in equilibrium play.

MP economies ($m = 1.3$) Figure 8b shows that under active monetary policy, realized Dollar-Peso inflation pairs are more dispersed than under muted policy, suggesting more heterogeneous expectations, and thus more diverse equilibria across experimental economies than in the $m = 1$ benchmark. Moreover, Dollar inflation tends to be lower in the early sequences and higher in the later ones, remaining just below 1.1 initially and rising toward 1.1 thereafter. In all cases, Dollar inflation remains strictly below the QTM benchmark of 1.3. Peso inflation exhibits no comparable time trend but remains positive on average across all sequences, i.e., above the $y = 1$ line. These patterns are compatible with changes in subjects' expectations about future Dollar inflation across sequences.

Comparing the early and late experimental sequences in Figure 9, average Dollar trade shares are initially lower but rise over time, indicating a shift in trade away from Peso. This pattern is consistent with sellers developing lower expected Dollar inflation relative to Peso across sequences, which raises the Dollar trade share; see Corollary 3.2. The resulting expansion in Dollar trade, in turn, tem-

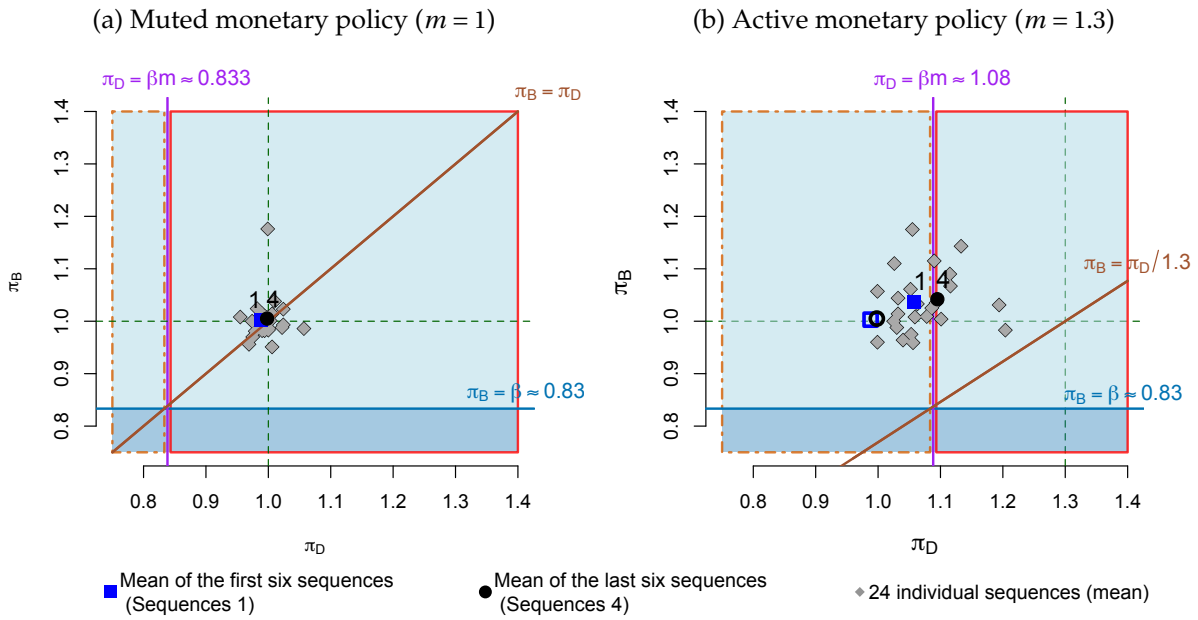
pers realized Dollar inflation, corresponding to outcomes above the brown line in Figure 8b.

Figure 9b reports average currency velocities in the (V_D, V_B) plane. Subjects increase both Peso and, more markedly, Dollar velocity from the first to the last experimental sequence. This pattern is consistent with rising Dollar inflation expectations among buyers, although realized velocities also depend on equilibrium prices and therefore do not by themselves reveal buyer beliefs. As a robustness check, we use the number of unsold packages reported in the efficiency analysis to identify rounds with particularly severe aggregate non-execution. Under active monetary policy, unsold packages are concentrated in the first two rounds of each sequence. Excluding these rounds leaves the main dynamics unchanged: Dollar saving continues to increase across rounds, and the corresponding decline in Dollar velocity persists. Likewise, the Dollar price level remains approximately flat in Sequences 1-3 and rises more strongly in Sequence 4. Hence, the velocity and price dynamics are not driven by the rounds with unusually high numbers of unsold packages.

The increase in Dollar velocity pushes the economy toward stronger Dollar price pass-through, whereas the increase in the Dollar trade share works in the opposite direction, toward the currency-substitution equilibrium. This divergence between buyer and seller behavior is noteworthy because subjects alternate between the two roles across periods. One possible explanation is the different waiting time before Dollar balances can finance consumption. Sellers can spend the Dollars they receive in the next period, when they become buyers, whereas buyers who save Dollars must wait two periods before they can use those balances for consumption. The expected duration of the experiment and the possibility of early termination may therefore affect buyers and sellers differently.

Overall, the buyer-velocity effect dominates the seller trade-share effect. From the first to the last sequence of play, average Dollar inflation rises, moving the economy toward the tempered pass-through equilibrium. We revisit this effect below when visualizing the absorption index in Section 5.4.

The theory shows that rising Dollar dominance or falling Dollar velocity does not require active monetary policy. Why, then, do trade shares and velocities shift in the laboratory only under active policy? Without interest payments, nothing alters agents' incentives to trade or save in a particular currency, and inflation expectations appear to remain stable. Under active policy, by contrast, growing Dollar balances continuously shape subjects' beliefs and play. The resulting pattern matches the rational money illusion predicted by the theory: to benefit from interest payments, sellers increasingly accept Dollars and buyers hoard them, as if the monetary expansion were costless, and the resulting muted pass-through validates the low inflation expectations ex post. Both this money illusion and the scarcity-driven Peso inflation are absent in Schilling and Uhlig (2019).



Notes: Average inflation rates per sequence are shown for all 24 sequences under treatment $m = 1$ and all 24 under $m = 1.3$, represented by gray diamonds. Solid blue squares indicate the average inflation rates of the first sequences ('1') across the six economies, while solid black circles represent the averages of the fourth sequences ('4'). RHS: The blue square and black circle contours correspond to the averages under mute monetary policy, included for direct comparison.

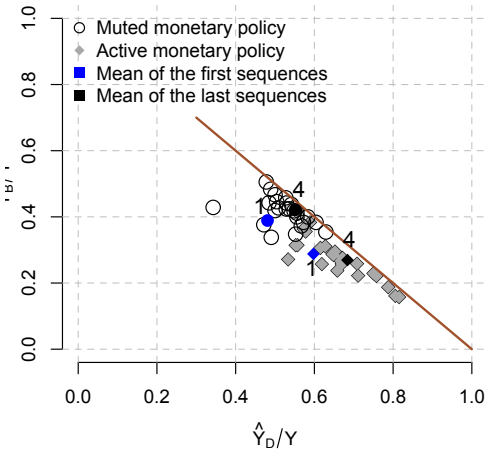
Figure 8: Average realized inflation rates in the (π^D, π^B) space

We did not elicit numerical inflation forecasts. Instead, an open-ended question asked what drove subjects' decisions, without mentioning inflation, money growth, or interest. LLM-assisted coding classifies whether responses refer to saving and returns or to inflation and prices (Appendix Table C.2). Under $m = 1$, references to either mechanism are naturally rare. In the MP treatment, 15% of subjects spontaneously mention saving or returns, whereas only 3% mention inflation or rising prices: unprompted, subjects perceive the 30% adjustment predominantly through its return channel rather than through its effect on prices.

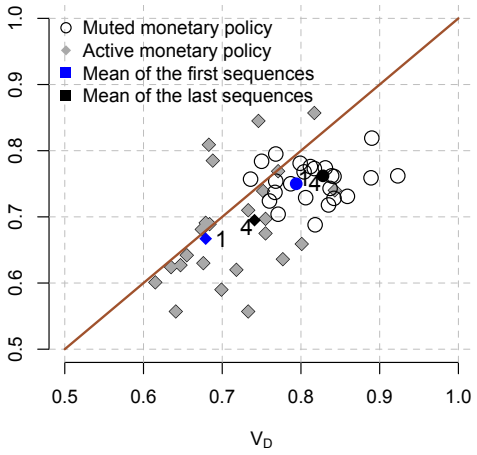
In contrast to the lump-sum transfer experiment of Cardozo, Rosokha, and Zhang (2024), where monetary expansion shifts trade toward Peso with strong Dollar pass-through, interest payments on Dollar balances shift trade toward Dollars and mute pass-through, showing that the two are fundamentally different monetary-policy instruments.

Individual economies The aggregate patterns discussed above mask variation across individual economies, some of which show a reversal or even a collapse in the Dollar trade share toward the end of the sequences. Figure 11 illustrates notable heterogeneity across experimental economies. In the MP treatment, Economy 3 (purple) maintains Dollar trade shares mostly between 80 and 90% from

(a) Market size by currency: trade share



(b) Average dynamics of currency velocities



Notes: each symbol represents the average over a sequence, with 24 sequences per treatment (white circles for $m = 1$ and gray diamonds for $m = 1.3$). The corresponding blue and black symbols represent the average over the first six and last six sequences in each treatment.

Figure 9: Left: Average Dollar (x-axis) versus Peso (y-axis) trade shares. Right: Average Dollar (x-axis) versus Peso (y-axis) velocities across experimental sequences and economies.

Sequence 2 onward. In contrast, the Dollar trade share of Economy 1 peaks at around 70% in Sequence 2, falls below 50% at its end, and remains mostly around 60% thereafter. Consequently, across all sequences, Economy 1 consistently trades fewer goods in Dollars compared to Economy 3. Economy 5 experiences large Dollar inflation by the end of Sequence 2 (see Figure B.3). In the subsequent sequences 3 and 4, possibly in response to this inflation episode, subjects in Economy 5 significantly increase Dollar velocity (i.e., reduced savings), as shown in Figure B.7, while maintaining a high Dollar trade share (Figure 11). This indicates that while buyers adapt their behavior by spending Dollars more quickly, sellers continue to prefer transacting in Dollars. Furthermore, Figure B.5 reveals that several economies under active monetary policy, particularly Economies 2 and 5, experience clear Peso inflation. In Economy 2, the scarcity effect on Peso prices is so pronounced that Peso inflation nearly matched Dollar inflation, with Peso-denominated goods selling for 40–50 Peso, comparable to Dollar goods prices ranging between 30–50 and occasionally reaching 70.

5.4 Revealed Beliefs and Monetary Absorption

We measure the absorption of Dollar money growth with the index J_t of Section 3.3.3. By (15), Dollar inflation falls short of money growth m when J_t rises, equals it when J_t is constant, and exceeds it when J_t falls. A rising index thus indicates that

part of the monetary expansion is absorbed through a higher Dollar trade share or lower Dollar velocity, as in the substitution type, while a constant or falling index indicates full pass-through to the Dollar price, as in the pass-through type. We report the dynamics of this index in Figure 10 for the MP treatment. We defer the benchmark $m = 1$ to Figure B.10. With no Dollar monetary expansion in these economies, the index is empirically comparatively stable across time and economies, averaging about 0.7.[30]

We infer beliefs from the Dollar trade share k_t, shown by economy in Figure 11. By Corollary 3.2, a higher trade share reveals lower expected Dollar inflation relative to Peso among sellers. Changes over time and differences across economies may additionally reflect inherited currency portfolios. Because some offered packages remain unsold in the laboratory, the share of packages sold for Dollars can differ slightly from the share sellers offered for Dollars, which is the choice that reveals their beliefs; in theory, all offered packages are sold (Lemma 3.1), and the two coincide.

Focusing on Sequence 2, which is the longest of the four, is particularly revealing for assessing transition between different equilibrium types over time and diverging equilibrium selection across economies. For instance, under active MP, Economy 5 exhibits relatively stronger price pass-through across all sequences. Its low J-index is driven by high Dollar velocity: its Dollar trade share remains comparatively high, mostly between 70 and 85 percent, so pass-through operates through buyers' spending rather than through sellers' currency allocations.

Economy 1 also exhibits a low index, but here the trade share is the lowest in the treatment, mostly around 60 percent and falling below 50 percent at the end of Sequence 2, revealing comparatively Peso-favorable seller beliefs.

By contrast, in Economy 3, the J- index rises steadily into the substitution region, so that Dollar money growth is increasingly absorbed rather than passed through to prices. Its trade share remains the highest in the treatment, mostly between 80 and 90 percent from Sequence 2 onward, revealing persistently Dollar-favorable seller beliefs; with a roughly constant trade share, the rising index reflects falling Dollar velocity, that is, increasing Dollar savings.

Economies 2 and 6 are particularly interesting because they exhibit strong reversals. Economy 2 initially moves clearly toward the substitution-equilibrium region, with the J-index rising sharply in Sequence 2, but then collapses in Se-

[30]Because the laboratory trading institution can generate temporary coordination and execution imperfections, the observed trade shares and index need not coincide exactly with their frictionless theoretical counterparts in every round. Such imperfections can affect realized trade shares and velocities under both treatments and therefore add noise to k_t and J_t. We consequently interpret systematic movements in k_t and J_t jointly with Dollar velocity and with the associated price dynamics, rather than attributing every round-to-round fluctuation in k_t to changes in beliefs or every fluctuation in J_t to changes in absorption. Although we did not separately elicit subjects' price forecasts, the open-ended strategy responses provide independent qualitative evidence on how subjects perceived the monetary-policy treatment. They do not, however, provide a quantitative measure of beliefs against which the theoretical belief-to- trade-share mapping can be tested directly.

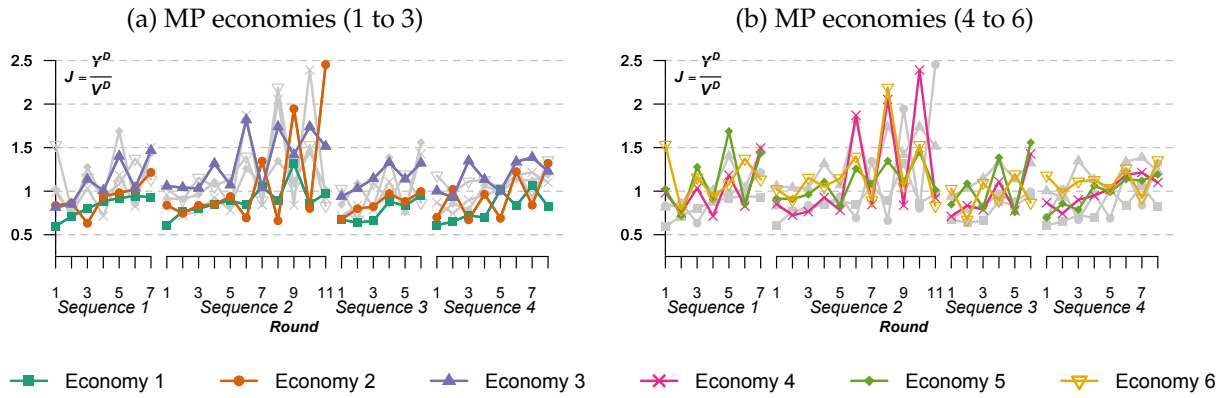


Figure 10: Dollar price absorption J_t in the MP treatment

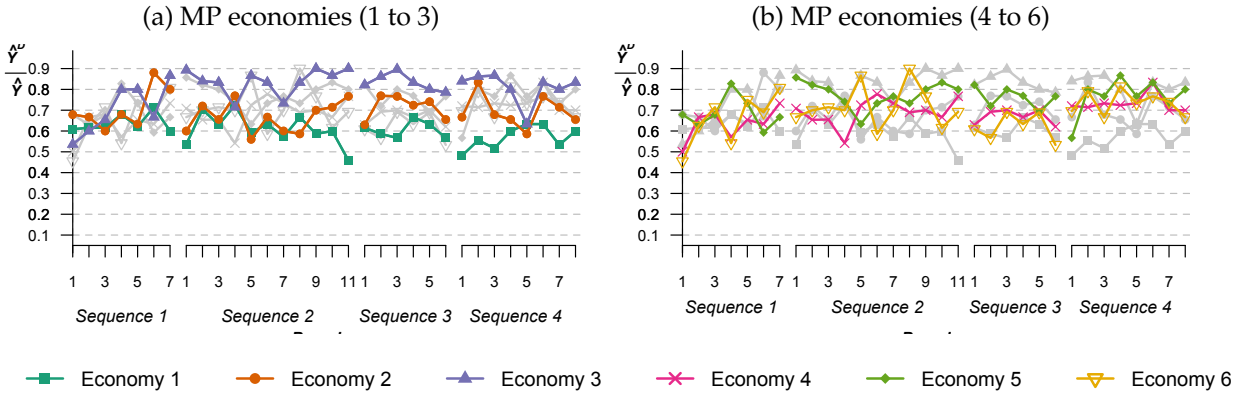


Figure 11: Dollar trade share $k_t = Y_t^D / Y$ in the MP treatment. By Corollary 3.2, a higher trade share reveals lower expected Dollar inflation relative to Peso among sellers.

quence 3. The trade share, however, does not reverse: it rises toward the end of Sequence 2 and stays above 70 percent for most of Sequence 3. The collapse in absorption therefore runs through higher Dollar velocity rather than through a shift in revealed seller beliefs. Economy 6 exhibits an even sharper reversal within Sequence 2, first moving strongly into the substitution-equilibrium region before reversing toward stronger pass-through. Here the trade share reverses as well, peaking at about 90 percent in round 8 of Sequence 2, falling to about 60 percent two rounds later, and remaining lower throughout Sequence 3. This reveals a shift in seller beliefs toward higher expected Dollar inflation relative to Peso.

6 Discussion

6.1 Implications for central banking

A conventional view of monetary policy is that the central bank chooses the money-growth factor m to determine Dollar inflation. Proposition 3.3 shows that this logic breaks down under currency competition. For a given m, inflation beliefs are not uniquely determined. The same policy can therefore support different Dollar and

Peso inflation expectations, which in turn generate different trade shares, velocities, and price paths. We observe this multiplicity in the experiment as well. Thus, by choosing m, the central bank does not uniquely determine the price level of its own currency and cannot directly steer Dollar inflation or intentionally crowd out Peso through m alone.

The reason is that, by choosing m, the central bank changes the incentives associated with Dollar balances but does not determine how agents respond to those incentives. When agents expect low Dollar inflation, a higher interest factor can attract trade toward Dollars, increase Dollar dominance, and temper Dollar price pass-through. When agents instead raise their Dollar inflation expectations, the same policy can induce faster Dollar spending, reduce Dollar dominance, and crowd in Peso. The central bank therefore cannot reliably select among these equilibria through m alone. In this sense, currency competition weakens its control over its own money and its ability to govern the Dollar price level.

Our theory and experiment instead show that belief evolution and coordination, rather than the central bank policy rate alone, determine whether monetary expansion translates primarily into higher prices or greater Dollar dominance. This ambiguity provides an additional rationale for central-bank communication, forward guidance, and other expectation-management tools.

More broadly, the term "Dollar" is only a placeholder for an interest-bearing medium of exchange. The same mechanism therefore applies to an interest-paying CBDC, a stablecoin, or a cryptocurrency such as Ether when balances earn staking rewards. In particular, the Ether staking yield alone does not determine ETH prices, because investors' inflation expectations and the return on competing currencies also shape equilibrium demand and prices. The same logic applies to an interest-bearing CBDC. Much of the economic literature studies interest-paying CBDC without simultaneously tracing the consequences of the associated expansion in the CBDC supply required to pay that interest.

The mechanism can also be viewed from the perspective of the passive currency rather than the actively expanding one. An interest-paying foreign CBDC can generate inflation spillovers into a more supply-rigid currency, such as the Canadian Dollar or the British pound, or alternatively attract trade away from that currency. Our results therefore have direct implications for competition among currencies that may themselves pay interest; see Appendix A.4 for an extension of the model in which the "Peso" in our model also earns interest $n > 1$.

Stablecoins such as Tether or USDC attempt to maintain a fixed peg through mint-and-burn arbitrage but typically do not remunerate balances. Much of the stablecoin literature adopts partial-equilibrium models that treat arbitrage as the principal peg-stabilizing mechanism. Our general-equilibrium perspective shows that persistent trade reallocation can itself place substantial pressure on the peg. Even with effective arbitrage, persistent trade imbalances, much like those that

ultimately led the Swiss National Bank to abandon its Euro peg, can overwhelm stabilization efforts and trigger depegging.

Relation to existing international finance theory. This result distinguishes our framework from standard international finance theory in two respects. First, even under fully flexible prices, a monetary expansion can increase the Dollar trade share. This contrasts with standard two-country models, in which a permanent increase in the money supply raises the price level and the nominal exchange rate proportionally, leaving the real exchange rate and net exports unchanged, so that money is neutral; see Obstfeld and Rogoff (1995, Section II) and the long-run neutrality result in Dornbusch (1976). In our economy, monetary policy is instead non-neutral for the real allocation of trade: it changes the quantities of goods sellers allocate across the Dollar and Peso markets even though aggregate output and welfare remain unchanged.

Second, when Dollar inflation beliefs coordinate on a low value, our equilibrium resembles the classic currency-war result typically derived under sticky prices: with free capital flows, an increase in the money supply can expand Dollar trade through a depreciation of the home currency (Fleming, 1962; Mundell, 1963; Obstfeld and Rogoff, 1995; Rodrik, 2008; Freund and Pierola, 2012; McLeay and Tenreyro, 2025). The key difference is that prices are fully flexible in our setting and all goods can be traded simultaneously in both currencies. The expansion in Dollar-denominated trade therefore reflects a reallocation of expenditure across currencies rather than across countries.

Unlike the traditional predictions under either sticky or flexible prices, our economy can alternate between regimes depending on how inflation beliefs coordinate, and the central bank cannot determine this coordination through monetary policy alone. Moreover, price pass-through and trade reallocation are inversely related: a larger increase in the Dollar trade share necessarily comes at the expense of Dollar price pass-through.

Interpretation of the Dollar and its growth factor Our Dollar is an interest-bearing liquid claim that agents hold directly and use in payment, with counterparts in remunerated reserves and interest-paying digital currencies (see the Introduction). Its nominal return is paid in newly issued Dollars, so the growth factor m is both the gross nominal return to Dollar holders and the gross growth rate of the Dollar supply. Although the interest rate on reserves and their aggregate supply can be controlled independently (Goodfriend, 2002; Keister et al., 2008), our model links the two through the direct issuance of interest payments. Households and firms cannot hold reserves, but government money market fund shares and tokenized Treasury funds provide close substitutes: they pay a return that tracks the policy rate, and the interest can be reinvested in additional shares.

Two features of actual monetary transmission lie outside this interpretation. First, most payment money consists of bank deposits, whose quantity is determined by bank lending and deposit demand rather than by the interest the central bank pays, and no stable money multiplier restores a mechanical link: U.S. reserve requirements have been zero since March 2020, and with ample reserves the ratio of deposits to reserves is an equilibrium outcome. Second, banks pass policy-rate changes through to deposit rates only partially, and depositors respond by shifting funds toward other assets (Drechsler et al., 2017). Since our mechanism works through agents who reallocate balances in response to anticipated real returns, we model Dollar holders as facing full pass-through, consistent with those who move out of deposits when the policy rate rises. Our Dollar should therefore be interpreted as an interest-bearing liquid claim whose nominal return equals the growth rate of its supply, rather than as conventional bank deposits.

Finally, following Kareken and Wallace (1981), we model money as outside money supplied according to a preannounced rule, with no banking sector, but with transfers proportional to Dollar holdings rather than lump sum, which provides a direct incentive to hold Dollars. This abstraction also serves the experiment: subjects can observe and verify the policy instrument, and the supply law $D_t = m^{t-1}D_1$ implements a known nominal return without adding bank intermediation as a separate source of strategic uncertainty.

6.2 Relation to Existing Literature and Contribution

The possibility that Dollar money growth need not pass one-for-one into Dollar inflation under currency competition is not new. In Kareken and Wallace (1981), perfect-foresight monetary equilibria feature a constant but indeterminate exchange rate, while the common inflation rate is not pinned down by the growth rate of an individual currency supply. Schilling and Uhlig (2019) extend this indeterminacy to a stochastic environment and explicitly construct multiple responses to monetary policy: an increase in the Dollar supply can raise either Dollar prices or prices in the competing currency. Thus, neither muted Dollar inflation nor positive Peso inflation spillovers are, by themselves, novel to our theory. Our contribution is to introduce endogenous currency-specific trade shares and velocities as distinct equilibrium adjustment margins and to characterize how inflation beliefs select among the resulting equilibrium responses. In Kareken and Wallace (1981); Schilling and Uhlig (2019), monetary expansion occurs through lump-sum transfers, goods are simultaneously available for purchase with either currency, and equilibrium velocity is one. In our model, sellers preallocate goods across currency markets, while buyers choose how much of each currency to spend and save. Moreover, interest payments on Dollar balances simultaneously increase their nominal return and expand their supply. Consequently, monetary expansion can attract rather than repel trade from the expanding currency, while in-

creased Dollar saving lowers its velocity. Both adjustments mute Dollar price pass-through despite fully flexible prices. Our equilibrium-selection mechanism establishes how inflation beliefs determine these adjustments, while the experiment demonstrates that monetary expansion can increase Dollar use and generate inflation spillovers into Peso through trade reallocation.

Our analysis also relates to the single-currency rational-expectations benchmark of Lucas (1972), where exogenous fundamentals determine a unique equilibrium price function and monetary non-neutrality arises through signal extraction rather than equilibrium indeterminacy.

Our equilibrium-selection result is also related to Angeletos and La'O (2013), where extrinsic sentiment shocks generate fluctuations in economic outcomes without changes in fundamentals. The distinction concerns equilibrium uniqueness. In Angeletos and La'O (2013), the information structure, including the distribution of sentiment shocks, is a primitive of the economy, and their Theorem 1 establishes a unique rational-expectations equilibrium for any given information structure. Beliefs about the equilibrium terms of trade follow from this structure and equilibrium strategies through Bayesian updating. Consequently, variation in sentiments generates different realizations of a unique equilibrium rather than selecting among multiple equilibria. In our setting, by contrast, fixing beliefs selects among equilibria rather than specifying the economy. For the same preferences, output, money supplies, and monetary policy, different admissible inflation-belief profiles support distinct rational-expectations equilibria, while all equilibria supported by a given profile share the same trade allocation and are ordered by prices and velocities. The difference also reflects the intertemporal structure of the two models. In Angeletos and La'O (2013), islands exchange goods through barter without carrying assets across periods, and equilibrium is determined by a static fixed-point problem. In our economy, agents repeatedly trade and carry money balances forward, so sellers' currency allocations and buyers' saving decisions depend on anticipated future currency returns. Because these returns are endogenous and not uniquely determined by fundamentals, different self-fulfilling inflation beliefs select equilibria with different trade shares, velocities, and inflation rates.

Finally, our belief-based equilibrium-selection mechanism relates to the global-games literature of Carlsson and van Damme (1993); Morris and Shin (1998), experimentally studied by Heinemann, Nagel, and Ockenfels (2004). In global games, agents receive noisy private information about an uncertain fundamental, and heterogeneity in signals can select a unique equilibrium. Our environment has no uncertain fundamental: monetary policy and aggregate output are perfectly observed, and different equilibrium selections need not differ in aggregate output or efficiency. Rather than eliminating multiplicity, we use stochastic orderings of heterogeneous inflation beliefs to rank agents' optimal currency allocations and saving decisions. Aggregation and market clearing then map these decisions into

equilibrium trade shares, velocities, and prices. Conditional on an admissible belief profile, all equilibria share the same trade allocation and are ordered by prices and velocities, while distinct belief profiles can support different equilibria. The Dollar trade share k_t ranks the equilibrium-relevant implications of strictly ordered inflation beliefs without requiring numerical inflation forecasts, while the index J_t measures how much Dollar money growth is absorbed rather than passed through to prices. In the experiment, we do not directly observe subjects' inflation beliefs as quantitative forecasts. Instead, we observe their equilibrium consequences in prices, trade shares, and velocities, complemented by open-ended strategy responses that provide qualitative evidence on how subjects interpret monetary policy. This allows us to study belief-based equilibrium selection experimentally even when the underlying beliefs are not directly observed.

7 Conclusions

This paper studies monetary policy under currency competition when monetary expansion takes the form of interest payments on money balances. Our first contribution is to establish a new form of equilibrium multiplicity. For the same policy parameter m, the economy can support a pass-through equilibrium with high Dollar inflation, rising Dollar velocity, and declining Dollar trade, or a currency-substitution equilibrium in which Dollar trade rises, Dollar velocity falls, and price pass-through is muted. In the latter case, trade is drawn away from Peso, generating Peso inflation despite its fixed supply.

Our second contribution is to develop a belief-driven mechanism of equilibrium selection. Given equilibrium multiplicity, monetary policy alone does not select which equilibrium is realized. A given belief profile may still support several equilibria, but these share the same trade allocation and are ordered by prices and velocities. Lower expected Dollar inflation relative to Peso raises the Dollar trade share and lowers the Dollar price or raises the Peso price, while higher expected Dollar inflation generates stronger pass-through. The Dollar trade share therefore reveals sellers' inflation beliefs, while an absorption index based on trade shares and velocities measures how much Dollar money growth is absorbed rather than passed through to prices. The laboratory evidence supports both equilibrium types, the inverse relation between Dollar dominance and Dollar pass-through, and inflation spillovers into Peso.

The results imply a new constraint on monetary sovereignty. A central bank can choose the interest paid on its currency but cannot thereby uniquely determine the resulting inflation rate. Strengthening the currency's role in trade may come at the cost of weaker price pass-through, while stronger pass-through may coincide with declining currency dominance. Under currency competition, monetary policy therefore determines incentives, but inflation beliefs help determine how those

incentives translate into prices, trade, and velocity.

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Internet Appendix

A Theoretical appendix

Proof. [Proof Lemma 3.1] Sellers offer their entire endowment for sale because packages are perishable and cannot be stored. An unsold package yields zero continuation value, whereas selling it at any positive price in a valued currency yields positive income that can be carried into the next period, when the seller becomes a buyer and can use it to finance consumption. Hence, no seller optimally lets a package decay, and $x_{i,t} + z_{i,t} = \bar{y}$, so that $x_{i,t} = \bar{y} - z_{i,t}$.

All goods offered for sale are purchased. Since buyers derive strictly positive marginal utility from consumption, they are willing to purchase additional packages at a sufficiently low positive price whenever they hold the corresponding currency. Sellers, in turn, prefer selling at any positive price to letting a package decay. Thus, for every market with positive supply, there exists a positive price at which demand equals supply. Since prices adjust flexibly to clear markets, $\sum_i^N x_{i,t} = Y_t^B = \sum_i^N q_{i,t}^B$ and $\sum_i^N z_{i,t} = Y_t^D = \sum_i^N q_{i,t}^D$.

Because all packages are offered and purchased, aggregate consumption equals aggregate output in every period, $\sum_{i=1}^N c_{i,t} = Y_t^D + Y_t^B = Y$. Therefore, $\sum_{i=1}^N c_{i,t} = \sum_{i=1}^N c_{i,t+1}$ for all $t \geq 1$.

□

A.1 Equilibrium characterization Bellman Equation

We consider the optimization problem of a green agent, who is a buyer and consumes in even periods and a seller in odd periods; let $\xi_t = 1$ when t is even and zero when odd. Take the state entering a buyer period $2t$ to be the money balance pair (d_{2t}, b_{2t}), and let W be the value of entering that period optimally. One cycle spans a buy at $2t$ and a sale at $2t+1$. Because z_{2t+1} is chosen only at the subsequent seller node and current utility does not depend on it, the cycle separates into a buyer stage and a seller stage,

$$W(d_{2t}, b_{2t}) = \max_{q_{2t}^D,\, q_{2t}^B} u\big(q_{2t}^D + q_{2t}^B\big) + \beta \mathbb{E}_{2t}[S(d_{2t+1}, b_{2t+1})],$$

$$S(d_{2t+1}, b_{2t+1}) = \max_{0 \leq z_{2t+1} \leq \bar{y}} \beta \mathbb{E}_{(2t+1)^-}[W(d_{2t+2}, b_{2t+2})],$$

with the buyer choosing under $\mathscr{F}_{2t}$ and the seller under $\mathscr{F}_{(2t+1)^-}$. The buyer stage is subject to the money-balance transitions (prices $P_{2t}, \hat{P}_{2t}$ taken as given)

$$d_{2t+1} = m\big(d_{2t} - q_{2t}^D P_{2t}\big), \qquad b_{2t+1} = b_{2t} - q_{2t}^B \hat{P}_{2t}.$$

At the seller stage, by Lemma 3.1, the seller offers her entire endowment, so that choosing z_{2t+1} for the Dollar market implies the Peso allocation $\bar{y} - z_{2t+1}$. Her money balances therefore evolve as (prices $P_{2t+1}, \hat{P}_{2t+1}$ not yet realized)

$$d_{2t+2} = m(d_{2t+1} + z_{2t+1}P_{2t+1}), \qquad b_{2t+2} = b_{2t+1} + \left(\bar{y} - z_{2t+1}\right)\hat{P}_{2t+1},$$

The buyer choices satisfy the budget constraints $q^D_{2t}P_{2t} \le d_{2t}$, $q^B_{2t}\hat{P}_{2t} \le b_{2t}$ and $q^D_{2t}, q^B_{2t} \ge 0$, while the seller choice satisfies $0 \le z_{2t+1} \le \bar{y}$.

Over the two stages, the agent's controls are the buyer's currency-specific consumption (q^D_{2t}, q^B_{2t}), equivalently his savings (d_{2t+1}, b_{2t+1}), and the seller's Dollar allocation z_{2t+1}, which implies the Peso allocation $x_{2t+1} = \bar{y} - z_{2t+1}$. The continuation balances d_{2t+2}, b_{2t+2} depend on $P_{2t+1}, \hat{P}_{2t+1}$, which lie outside $\mathscr{F}_{2t}$, so they are the induced continuation state, not date-$2t$ decision variables. The only choice at the seller node is z_{2t+1}. Because the seller moves under the richer information set $\mathscr{F}_{(2t+1)^-}$, the seller value $S(d_{2t+1}, b_{2t+1})$ is random from the date-$2t$ buyer's perspective.

Expectations are taken under the subjective kernels, ϕ^B_{2t} at the buyer stage and ϕ^S_{2t+1} at the seller stage. The belief updates $\phi^B_{2t} \to \phi^S_{2t+1}$ and $\phi^S_{2t+1} \to \phi^B_{2t+2}$ may, but need not, depend on the history realized at the end of the preceding period, namely $H_{2t} = (Y^D_{2t}, Y^B_{2t}, V^D_{2t}, V^B_{2t}, P_{2t}, \hat{P}_{2t})$ for the former and $H_{2t+1} = (Y^D_{2t+1}, Y^B_{2t+1}, V^D_{2t+1}, V^B_{2t+1}, P_{2t+1}, \hat{P}_{2t+1})$ for the latter.

The buyer's saving choice. Each buyer currency saving, d_{2t+1} and b_{2t+1}, is either interior at the optimum, so that its first-order condition holds with equality, or at the spend-all bound with zero savings. Writing the saving choice as the carried balances $d_{2t+1} = m(d_{2t} - q^D_{2t}P_{2t}) \ge 0$ and $b_{2t+1} = b_{2t} - q^B_{2t}\hat{P}_{2t} \ge 0$, optimality requires

$$u'(c_{2t}) \;\ge\; \beta^2 m^2 P_{2t}\,\mathbb{E}_{2t}[W_d(d_{2t+2}, b_{2t+2})], \qquad \text{with equality if } d_{2t+1} > 0, \tag{24}$$

$$u'(c_{2t}) \;\ge\; \beta^2 \hat{P}_{2t}\,\mathbb{E}_{2t}[W_b(d_{2t+2}, b_{2t+2})], \qquad \text{with equality if } b_{2t+1} > 0. \tag{25}$$

In the savings region ($d_{2t+1}, b_{2t+1} > 0$) the marginal utility of current consumption equals the marginal continuation value of carrying the balance forward, so (24) and (25) bind. At a spend-all corner the current margin strictly dominates, the inequality is strict, and the corresponding balance is exhausted, $d_{2t+1} = 0$ or $b_{2t+1} = 0$. The seller's allocation first order condition,

$$\mathbb{E}_{(2t+1)^-}\left[mP_{2t+1}\,W_d(d_{2t+2}, b_{2t+2}) - \hat{P}_{2t+1}\,W_b(d_{2t+2}, b_{2t+2})\right] = 0, \tag{26}$$

is treated in the reallocation step below.

If the buyer uses both markets, $q_{2t}^D, q_{2t}^B > 0$, we claim that[31]

$$W_d(d_{2t}, b_{2t}) = \frac{u'(c_{2t})}{P_{2t}}, \qquad W_b(d_{2t}, b_{2t}) = \frac{u'(c_{2t})}{\hat{P}_{2t}}, \tag{27}$$

in the savings region and at the spend-all corner alike.

In the savings region, differentiate $W(d_{2t}, b_{2t}) = \max_{q_{2t}^D, q_{2t}^B} u(q_{2t}^D + q_{2t}^B) + \beta S(d_{2t+1}, b_{2t+1})$ in d_{2t} at fixed optimal controls. If a rise in the budget does not affect consumption today, a marginal rise in d_{2t} raises only savings, so the utility term $u(q_{2t}^D + q_{2t}^B)$ is flat in d_{2t} and only continuation value βS contributes through a change in d_{2t+1}, with $\partial d_{2t+1}/\partial d_{2t} = m$,

$$W_d(d_{2t}, b_{2t}) = \beta\, S_d(d_{2t+1}, b_{2t+1})\, m. \tag{28}$$

The interior first-order condition of W for q_{2t}^D, using $\partial d_{2t+1}/\partial q_{2t}^D = -mP_{2t}$, gives

$$u'(c_{2t}) - \beta\, S_d(d_{2t+1}, b_{2t+1})\, mP_{2t} = 0 \implies \beta\, S_d\, m = \frac{u'(c_{2t})}{P_{2t}}, \tag{29}$$

and substituting (29) into (28) delivers (27).

At the spend-all corner the Dollar budget binds, $q_{2t}^D P_{2t} = d_{2t}$, so the carried balance is pinned at zero, $d_{2t+1} = m(d_{2t} - q_{2t}^D P_{2t}) = 0$, and stays there under a marginal change in entering wealth,

$$\frac{\partial c_{2t}}{\partial d_{2t}} = \frac{\partial q_{2t}^D}{\partial d_{2t}} = \frac{1}{P_{2t}}, \qquad \frac{\partial d_{2t+1}}{\partial d_{2t}} = 0. \tag{30}$$

The entering balance now reaches the value through consumption rather than through the continuation, and differentiating W along the binding constraint, a marginal budget increase now raises consumption today but leaves the continuation value unaffected,

$$W_d(d_{2t}, b_{2t}) = u'(c_{2t})\,\frac{\partial c_{2t}}{\partial d_{2t}} + \beta\, S_d(d_{2t+1}, b_{2t+1})\,\frac{\partial d_{2t+1}}{\partial d_{2t}} = \frac{u'(c_{2t})}{P_{2t}}, \tag{31}$$

the same expression as in the savings region.

In the mixed case the buyer neither exhausts the Dollar balance nor saves it in full. Define the marginal propensity to consume out of real balances at $2t$,

$$\kappa_{2t} \;\equiv\; P_{2t}\,\frac{\partial c_{2t}}{\partial d_{2t}} \;\in (0, 1), \tag{32}$$

[31] A strong monetary equilibrium guarantees positive spending in both currencies only in the aggregate. If an individual buyer spends no Dollars at $2t$, (27) holds with inequality, $W_d = \beta m S_d \geq u'(c_{2t})/P_{2t}$, and the marginal value of a Dollar equals the discounted expected marginal utility per Dollar at her next date with positive Dollar spending; analogously for Pesos. The proof of Proposition 3.3 uses f_i^D and f_i^B only as derivatives of the expected continuation value (Step 2) and is unaffected; the expressions (10) and (11), and the FOSD order, then refer to that later date.

Because the two currency budgets are separate, a marginal change in Dollar balances affects that period's Dollar-financed consumption but leaves Peso-financed consumption unchanged, $\partial q^B_{2t}/\partial d_{2t} = 0$. Hence, $\partial c_{2t}/\partial d_{2t} = \partial q^D_{2t}/\partial d_{2t}$.

Then a marginal increase in Dollar wealth d_{2t} raises current optimal consumption by $\partial c_{2t}/\partial d_{2t} = \kappa_{2t}/P_{2t}$ and, through the budget $d_{2t+1} = m(d_{2t} - q^D_{2t}P_{2t})$, raises the carried balance by $\partial d_{2t+1}/\partial d_{2t} = m(1-\kappa_{2t})$. In the mixed case both channels are active, $\kappa_{2t} \in (0,1)$, and differentiating W in d_{2t} at fixed optimal controls picks up both,

$$W_d(d_{2t}, b_{2t}) = u'(c_{2t})\,\frac{\partial c_{2t}}{\partial d_{2t}} + \beta\, S_d(d_{2t+1}, b_{2t+1})\,\frac{\partial d_{2t+1}}{\partial d_{2t}} = u'(c_{2t})\,\frac{\kappa_{2t}}{P_{2t}} + \beta\, S_d\, m\,(1-\kappa_{2t}). \tag{33}$$

Since the Dollar saving is interior in the mixed case, its first-order condition (29) holds with equality, $\beta S_d m = u'(c_{2t})/P_{2t}$. Substituting into (33), the two channels carry the same marginal value and the split κ_{2t} cancels,

$$W_d(d_{2t}, b_{2t}) = \kappa_{2t}\,\frac{u'(c_{2t})}{P_{2t}} + (1-\kappa_{2t})\,\frac{u'(c_{2t})}{P_{2t}} = \frac{u'(c_{2t})}{P_{2t}}, \tag{34}$$

the same expression as in the pure savings and pure spend-all regimes. The consumed fraction contributes $\kappa_{2t}\,u'(c_{2t})/P_{2t}$ through the utility channel and the carried fraction contributes $(1-\kappa_{2t})\,u'(c_{2t})/P_{2t}$ through the continuation channel; because interior saving equates the marginal value of a Dollar spent now to the marginal value of one carried forward, the two coincide and W_d is independent of κ_{2t}, $\partial W_d/\partial \kappa_{2t} = 0$.

The Peso identity follows symmetrically, so (27) requires only $q^D_{2t}, q^B_{2t} > 0$, not a regime.

The buyer's Euler conditions. Substituting (27) one cycle ahead into (24) and (25) and dividing by P_{2t} and $\hat{P}_{2t}$ yields the buyer's Euler conditions in complementary-slackness form,

$$\frac{u'(c_{2t})}{P_{2t}} \;\geq\; \beta^2 m^2\, \mathbb{E}_{2t}\left[\frac{u'(c_{2t+2})}{P_{2t+2}}\right], \qquad \text{with equality if } d_{2t+1} > 0, \tag{35}$$

$$\frac{u'(c_{2t})}{\hat{P}_{2t}} \;\geq\; \beta^2\, \mathbb{E}_{2t}\left[\frac{u'(c_{2t+2})}{\hat{P}_{2t+2}}\right], \qquad \text{with equality if } b_{2t+1} > 0. \tag{36}$$

In the savings region the relevant condition binds; at a spend-all corner it holds strictly and the balance is exhausted. If the buyer uses both markets, $q^D_{2t}, q^B_{2t} > 0$, each of (35), (36) binds or slackens on its own, giving the four configurations save/spend in each currency stated in Proposition 3.2.

The seller's reallocation gain. At $(2t+1)^-$ the seller enters with balances (d_{2t+1}, b_{2t+1}) fixed by her buyer-stage choice and picks the Dollar allocation $z_{2t+1} \in [0, \bar{y}]$ to solve

(37),

$$S(d_{2t+1}, b_{2t+1}) = \max_{0 \le z_{2t+1} \le \bar{y}} \beta \mathbb{E}_{(2t+1)^-}[W(d_{2t+2}, b_{2t+2})], \tag{37}$$

with continuation money balances

$$d_{2t+2} = m(d_{2t+1} + z_{2t+1} P_{2t+1}), \qquad b_{2t+2} = b_{2t+1} + (\bar{y} - z_{2t+1})\hat{P}_{2t+1}, \tag{38}$$

so that $\partial d_{2t+2}/\partial z_{2t+1} = mP_{2t+1}$ and $\partial b_{2t+2}/\partial z_{2t+1} = -\hat{P}_{2t+1}$. The allocation is chosen before prices realize, so z_{2t+1} is common to every price realization and, by dominated convergence, the derivative passes inside the expectation. Writing S_z for the partial of (37) in z_{2t+1}, the interior first-order condition is $S_z = 0$ with

$$S_z = \beta \mathbb{E}_{(2t+1)^-}\left[mP_{2t+1} W_d(d_{2t+2}, b_{2t+2}) - \hat{P}_{2t+1} W_b(d_{2t+2}, b_{2t+2}) \right], \tag{39}$$

the marginal value of shifting one more package into the Dollar market. This is the seller allocation condition (26). Using (27) at date $2t+2$, it reads in primitives

$$S_z = \beta \mathbb{E}_{(2t+1)^-}\left[u'(c_{2t+2}) \left(m \frac{P_{2t+1}}{P_{2t+2}} - \frac{\hat{P}_{2t+1}}{\hat{P}_{2t+2}} \right) \right]. \tag{40}$$

A.2 Equilibrium Selection

First of all, to see how changes in beliefs affect expected values, recall that a belief kernel distributes probability mass over future states, which shapes the expected return. Writing the Dollar return explicitly (Peso analogously),

$$\mathbb{E}_{t^-}^{\phi^S}\left[\mathscr{R}^D(\omega_t) \mid m, \mathscr{F}_{t-1} \right] \equiv \int_\Omega \frac{p}{p'} u'(c') \, \phi^S(p, p', c' \mid m, \mathscr{F}_{t-1}) \, dp \, dp' \, dc', \tag{41}$$

the expectation is high when ϕ^S concentrates probability mass on states with high Dollar real return $\frac{P(h)}{P(h')} u'(c')$, that is, states of low Dollar inflation and less mass on states with lower Dollar real return. If $\phi^S \succeq_{\mathscr{R}} \tilde{\phi}^S$, then relative to $\tilde{\phi}^S$ the dominant kernel ϕ^S places more mass on high-Dollar-return states and less on low-Dollar-return ones, or more weight on low-Peso-return states and less on high-Peso-return ones.

The following corollary follows directly from Definition 3.2 and Lemma 3.2:

Corollary A.1 (Beliefs and Equilibrium Seller Allocations)**.** *The more dominant the seller belief kernel ϕ^S in the relative-return ordering (the lower expected Dollar inflation relative to Peso inflation), the weakly larger the seller's Dollar allocation $z_{i,t}$ and the weakly smaller her Peso allocation $x_{i,t} = \bar{y} - z_{i,t}$. If the belief ordering is strict, $\phi^S \succ_{\mathscr{R}} \tilde{\phi}^S$, and the allocation under $\tilde{\phi}^S$ is interior, the ordering is strict: $z_{i,t}(\phi^S) > z_{i,t}(\tilde{\phi}^S)$.*

Given a strict belief ordering, $\phi^S \succ_{\mathscr{R}} \tilde{\phi}^S$, the indices are also strictly ordered, $I_t^S(x_t, z_t \mid m, \mathscr{F}_{t-1}, \phi^S) > I_t^S(x_t, z_t \mid m, \mathscr{F}_{t-1}, \tilde{\phi}^S)$. Moreover, then, $z_t(\phi^S) \ge z_t(\tilde{\phi}^S)$. In the aggregate, this causes the Dollar trade share to be weakly larger when Dollar

inflation beliefs are lower. Whether this allocation ordering is strict depends on the allocation under the dominated belief. If $z_t(\tilde{\phi}^S) \in (0, \bar{y})$, then $z_t(\phi^S) > z_t(\tilde{\phi}^S)$. If instead $z_t(\tilde{\phi}^S) = \bar{y}$, then $z_t(\phi^S) = \bar{y} = z_t(\tilde{\phi}^S)$, and if $z_t(\tilde{\phi}^S) = 0$, the allocation may remain at zero. A strong monetary equilibrium does not exclude these corners for individual sellers, since it requires only that both currencies circulate in the aggregate; see Step 6 of the proof of Proposition 3.3.

The following corollary follows directly from Corollary 3.1 and Lemma 3.3: FOSD dominance shifts the buyer payoff differential downward at every fixed purchase, while lower current purchases raise the differential back toward zero.

Corollary A.2 (Beliefs and Equilibrium Buyer Consumption-Savings)**.** *At given prices and given spending in the other currency, the more dominant the buyer Dollar belief kernel $\phi^{B,D}$ in the FOSD ordering (the lower expected Dollar inflation), the lower Dollar-financed consumption q_t^D. Likewise for Peso: the more dominant $\phi^{B,B}$ (the lower expected Peso inflation), the lower Peso-financed consumption q_t^B. If the belief ordering is strict and the buyer does not spend all of currency j, the consumption orderings are strict.*

The joint statement for both currencies is established in Step 7 of the proof of Proposition 3.3.

Given a strict belief ordering, $\tilde{\phi}^{B,j} \succ_{\text{FOSD}} \phi^{B,j}$, the indices are also strictly ordered, $I_t^{B,j}(q_t^j \mid m, \mathscr{F}_t^P, \phi^B) > I_t^{B,j}(q_t^j \mid m, \mathscr{F}_t^P, \tilde{\phi}^B)$. Moreover, $q_t^j(\phi^B) \geq q_t^j(\tilde{\phi}^B)$. Whether this consumption ordering is strict depends on consumption under the dominant belief. If $q_t^j(\tilde{\phi}^B) < d_t/P_t$, i.e. if the budget constraint is slack, then $q_t^j(\phi^B) > q_t^j(\tilde{\phi}^B)$. If instead the buyer spends all currency j, $q_t^j(\tilde{\phi}^B) = d_t/P_t$, then $q_t^j(\phi^B) = q_t^j(\tilde{\phi}^B)$.

A.2.1 Proof: Seller payoff differential Monotonicities

Proof. [Lemma 3.2]

Suppose a t-seller with inflation belief ϕ_t^S is not indifferent between allocating packages to the Dollar and Peso markets at $(z_t, \bar{y} - z_t)$, with

$$I^S(z_t, \bar{y} - z_t) = \mathbb{E}_{t^-}\left[\left(m\frac{P_t}{P_{t+1}} - \frac{\hat{P}_t}{\hat{P}_{t+1}}\right)u'(c_{t+1}) \;\middle|\; m, \mathscr{F}_{t-1}, \phi_t^S\right] > 0. \tag{42}$$

By Proposition 3.2 the seller then strictly prefers selling for Dollars at $(z_t, \bar{y} - z_t)$, because the expected real return in the Dollar market corrected for the interest rate is larger. Assume $z_t < \bar{y}$, so that increasing the Dollar allocation is feasible.

We show that the differential expression (42) is strictly decreasing in z_t, so that raising z_t moves the seller strictly toward indifference, $I^S = 0$. Since $x_t = \bar{y} - z_t$, the payoff differential is equivalently strictly increasing in x_t, which is the claim of the lemma.

Once I_t^S is strictly decreasing in z_t, the indifference condition $I_t^S(z_t) = 0$ has at most one solution. Hence, whenever an interior strong monetary equilibrium exists, the corresponding seller allocation is unique.

The seller does not consume in t but anticipates consumption in $t+1$, when she becomes a buyer, financed by Dollar and Peso purchases, $c_{t+1} = q^D_{t+1} + q^B_{t+1}$, subject to the $t+1$ budget constraints that depend on her period-t allocation through $d_{t+1} = m(d_t + z_t P_t)$ and $b_{t+1} = b_t + (\bar{y} - z_t)\hat{P}_t$.

The derivative of (42) in z_t is taken holding the seller at her (generally suboptimal) date-t allocation z_t, while her date-$(t+1)$ consumption $c_{t+1} = c^*_{t+1}(z_t)$ is the buyer's optimum conditional on the balances that z_t delivers.

Although z_t is generally suboptimal under the premise $I^S(z_t) > 0$, the date-$(t+1)$ buyer problem is optimized conditional on every z_t, so the envelope identities established in Section A.1 apply pointwise in z_t:

$$\frac{\partial W_{t+1}}{\partial d_{t+1}} = \frac{u'(c^*_{t+1})}{P_{t+1}}, \qquad \frac{\partial W_{t+1}}{\partial b_{t+1}} = \frac{u'(c^*_{t+1})}{\hat{P}_{t+1}}. \tag{43}$$

They link the differential expression to the seller's expected continuation value $\mathbb{E}_{t^-}[W_{t+1}(d_{t+1}, b_{t+1})]$, evaluated at the z_t-dependent balances $d_{t+1}(z_t), b_{t+1}(z_t)$. Passing the derivative inside the expectation and through the two balances, with prices taken as given, and applying (43),

$$\frac{\partial}{\partial z_t}\mathbb{E}_{t^-}[W_{t+1}(d_{t+1}, b_{t+1})] = \mathbb{E}_{t^-}\left[mP_t \frac{\partial W_{t+1}}{\partial d_{t+1}} - \hat{P}_t \frac{\partial W_{t+1}}{\partial b_{t+1}}\right] = \mathbb{E}_{t^-}\left[\left(m\frac{P_t}{P_{t+1}} - \frac{\hat{P}_t}{\hat{P}_{t+1}}\right) u'(c^*_{t+1})\right], \tag{44}$$

using $\partial d_{t+1}/\partial z_t = mP_t$ and $\partial b_{t+1}/\partial z_t = -\hat{P}_t$ in the first equality and (43) in the second. The right-hand side is the payoff differential expression (42). At an interior optimum $\partial_{z_t}\mathbb{E}_{t^-}[W_{t+1}] = 0$ if and only if $I^S = 0$; by the premise $I^S > 0$ the current z_t does not maximize the expected continuation value and the seller strictly prefers to raise z_t. We want to show that the expected continuation value is strictly concave in z_t,

$$\frac{\partial^2}{\partial z_t^2}\mathbb{E}_{t^-}[W_{t+1}(d_{t+1}, b_{t+1})] < 0. \tag{45}$$

Equivalently, the differential expression (42) is strictly decreasing in z_t. As the seller raises z_t, the marginal expected continuation value declines, moving the seller toward indifference, $I^S_t = 0$. Hence the indifference condition can have at most one solution in z_t.

The allocation z_t moves W_{t+1} solely through the two money balances, with $\partial d_{t+1}/\partial z_t = mP_t$ and $\partial b_{t+1}/\partial z_t = -\hat{P}_t$. To characterize the induced consumption response, differentiate the envelope identities (43). Since the buyer is a price taker, P_{t+1} and $\hat{P}_{t+1}$ are fixed in these derivatives, so

$$W_{dd} = \frac{u''(c^*_{t+1})}{P_{t+1}}\frac{\partial c^*_{t+1}}{\partial d_{t+1}}, \qquad W_{db} = \frac{u''(c^*_{t+1})}{P_{t+1}}\frac{\partial c^*_{t+1}}{\partial b_{t+1}}, \tag{46}$$

and

$$W_{bb} = \frac{u''(c^*_{t+1})}{\hat{P}_{t+1}}\frac{\partial c^*_{t+1}}{\partial b_{t+1}}, \qquad W_{bd} = \frac{u''(c^*_{t+1})}{\hat{P}_{t+1}}\frac{\partial c^*_{t+1}}{\partial d_{t+1}}. \tag{47}$$

Because the Hessian of W_{t+1} is symmetric, $W_{db} = W_{bd}$. Hence, $\frac{u''(c^*_{t+1})}{P_{t+1}} \frac{\partial c^*_{t+1}}{\partial b_{t+1}} = \frac{u''(c^*_{t+1})}{\hat{P}_{t+1}} \frac{\partial c^*_{t+1}}{\partial d_{t+1}}$, and therefore

$$P_{t+1} \frac{\partial c^*_{t+1}}{\partial d_{t+1}} = \hat{P}_{t+1} \frac{\partial c^*_{t+1}}{\partial b_{t+1}}.$$

We can thus define the buyer's marginal propensity to consume out of real balances at $t+1$ by

$$\kappa_{t+1} \equiv P_{t+1} \frac{\partial c^*_{t+1}}{\partial d_{t+1}} = \hat{P}_{t+1} \frac{\partial c^*_{t+1}}{\partial b_{t+1}} \in [0,1], \tag{48}$$

where $\kappa_{t+1} \geq 0$ because prices are positive and utility is strictly increasing, so equilibrium consumption is weakly increasing in the buyer's budget. Moreover, $\kappa_{t+1} \leq 1$ because an additional unit of currency can increase current consumption by at most its real value, $1/P_{t+1}$ for Dollars and $1/\hat{P}_{t+1}$ for Peso, attained when the entire additional balance is spent. Multiplying by the corresponding price level therefore yields $\kappa_{t+1} \leq 1$. Hence, $\kappa_{t+1} \in [0,1]$. By concavity of the utility function we can hence follow, $W_{dd} \leq 0$, $W_{bb} \leq 0$.

Differentiating c^*_{t+1} along the buyer's optimal plan with respect to the seller's allocation z_t gives

$$\frac{\partial c^*_{t+1}}{\partial z_t} = \frac{\partial c^*_{t+1}}{\partial d_{t+1}} \frac{\partial d_{t+1}}{\partial z_t} + \frac{\partial c^*_{t+1}}{\partial b_{t+1}} \frac{\partial b_{t+1}}{\partial z_t}. \tag{49}$$

Using $\partial d_{t+1}/\partial z_t = mP_t$, $\partial b_{t+1}/\partial z_t = -\hat{P}_t$, and (48), this becomes

$$\frac{\partial c^*_{t+1}}{\partial z_t} = \kappa_{t+1} \left(m \frac{P_t}{P_{t+1}} - \frac{\hat{P}_t}{\hat{P}_{t+1}} \right). \tag{50}$$

Show: The equilibrium consumption response is strictly positive, $\kappa_{t+1} > 0$. Two cases remain relevant. If the Dollar budget binds at $t+1$, $q^D_{t+1} P_{t+1} = d_{t+1}$, then a marginal rise in d_{t+1} is consumed in full through the Dollar market. Since, as established above, $\partial q^B_{t+1}/\partial d_{t+1} = 0$, $\frac{\partial c^*_{t+1}}{\partial d_{t+1}} = \frac{\partial q^D_{t+1}}{\partial d_{t+1}} = \frac{1}{P_{t+1}} > 0$, and hence $\kappa_{t+1} = 1$.

Now consider the case with interior Dollar savings, where $q^D_{t+1} P_{t+1} < d_{t+1}$ and the Dollar Euler equation binds. We show $\kappa_{t+1} > 0$ by contradiction. Suppose $\kappa_{t+1} = 0$. By (48), $\frac{\partial c^*_{t+1}}{\partial d_{t+1}} = \frac{\partial q^D_{t+1}}{\partial d_{t+1}} = 0$, meaning the additional Dollar is entirely saved, leaving equilibrium consumption unchanged $\frac{\partial d_{t+2}}{\partial d_{t+1}} = m \left(1 - P_{t+1} \frac{\partial q^D_{t+1}}{\partial d_{t+1}} \right) = m > 0$.

The marginal balance must eventually raise consumption at some future buyer date. Otherwise the additional balance would be carried forever without ever affecting utility, while consuming an arbitrarily small part of it at $t+1$ would be feasible and, since $u' > 0$, would strictly increase utility. Let $t+1+2k$, $k \geq 1$, denote the first future buyer date at which the marginal increase in d_{t+1} raises consumption.

By strong monetary equilibrium, the Dollar market is active, $q^D_{t+1} > 0$. Moreover, because Dollar saving is interior, $q^D_{t+1} \in (0, d_{t+1}/P_{t+1})$, the Dollar Euler equation holds with equality between dates $t+1$ and $t+3$, before the marginal increase in d_{t+1} is introduced.

Under the hypothesis $\kappa_{t+1} = 0$, the marginal increase in d_{t+1} leaves consump-

tion at $t+1$ unchanged. Hence, to preserve the Euler equality as the budget rises, consumption at $t+3$ must also remain unchanged; otherwise marginal utility at $t+3$ would drop due to increased consumption c_{t+3} such that

$$\frac{u'(c^*_{t+1})}{P_{t+1}} > \beta^2 m^2 \mathbb{E}_{t+1}\left[\frac{u'(c_{t+3})}{P_{t+3}}\middle| m, \mathscr{F}_{t+1}\right]$$

and the buyer would strictly prefer to consume more today, contradicting $\frac{\partial c^*_{t+1}}{\partial d_{t+1}} = 0$. The marginal Dollar must therefore be carried beyond $t+3$, so $k > 1$. Repeating the argument, if the Euler condition holds with equality between $t+1$ and $t+1+2j$ before the budge rise and the marginal Dollar has not yet raised consumption by date $t+1+2j$, then the rise in d_{t+1} leaves consumption at both endpoints unchanged and the marginal Dollar must again be carried forward. Hence $k > j$. By induction, the Dollar Euler condition holds with equality between $t+1$ and every intervening buyer date $t+1+2j$, $j < k$. Dynamic consistency of beliefs and the law of iterated expectations therefore imply for $j < k$:

$$\begin{aligned}\frac{u'(c^*_{t+1})}{P_{t+1}} &= \beta^2 m^2 \mathbb{E}_{t+1}\left[\frac{u'(c_{t+3})}{P_{t+3}}\middle| m, \mathscr{F}_{t+1}\right] \\ &= \beta^{2j} m^{2j} \mathbb{E}_{t+1}\left[\frac{u'(c_{t+1+2j})}{P_{t+1+2j}}\middle| m, \mathscr{F}_{t+1}\right], \qquad j < k. \end{aligned} \tag{51}$$

For the marginal increase in d_{t+1} to leave consumption unchanged at all dates $t+1+2j$, $j < k$, and to be consumed first at $t+1+2k$, the marginal value of consumption at $t+1+2k$ must strictly exceed that of consumption at $t+1$ *before* the increase in the Dollar budget.

$$\frac{u'(c^*_{t+1})}{P_{t+1}} < \beta^{2k} m^{2k} \mathbb{E}_{t+1}\left[\frac{u'(c_{t+1+2k})}{P_{t+1+2k}}\middle| m, \mathscr{F}_{t+1}\right]. \tag{52}$$

Equation (52), however, cannot hold if the buyer spends Dollars at $t+1$. If it did, the buyer would strictly prefer to reduce Dollar-financed consumption at $t+1$ and carry the released balance forward to raise consumption at $t+1+2k$. The only case in which such a reduction is infeasible is when the buyer is already at the lower bound, $q^D_{t+1} = 0$. A strong monetary equilibrium rules out this case only in the aggregate, so we consider a buyer who spends in both currencies at $t+1$ (see the footnote in Appendix A.1). For her, (52) cannot hold.

Since complete deferral of the marginal increase in d_{t+1} requires (52), the marginal increase must raise current consumption to some degree, and hence $\kappa_{t+1} > 0$.

If instead Dollars are exhausted while Peso saving is interior, the same argument applies symmetrically to the Peso balance. Hence $\kappa_{t+1} > 0$ when-

ever the buyer spends in both currencies at $t+1$.

Finally, since sellers take prices as given, the return gap $mP_t/P_{t+1} - \hat{P}_t/\hat{P}_{t+1}$ does not depend on z_t, so differentiating the payoff differential (42) in z_t acts only on $u'(c^*_{t+1})$, and by (49), $\partial_{z_t} u'(c^*_{t+1}) = u''(c^*_{t+1})\,\kappa_{t+1}\,(mP_t/P_{t+1} - \hat{P}_t/\hat{P}_{t+1})$. Hence

$$\frac{\partial}{\partial z_t} I^S = \frac{\partial}{\partial z_t} \mathbb{E}_{t^-}\left[\left(m\frac{P_t}{P_{t+1}} - \frac{\hat{P}_t}{\hat{P}_{t+1}}\right) u'(c_{t+1})\right] = \mathbb{E}_{t^-}\left[\kappa_{t+1}\, u''(c^*_{t+1})\left(m\frac{P_t}{P_{t+1}} - \frac{\hat{P}_t}{\hat{P}_{t+1}}\right)^2\right] < 0 \tag{53}$$

since $\kappa_{t+1} > 0$, $u'' < 0$, and because the premise $I^S > 0$ implies that the return gap $m\frac{P_t}{P_{t+1}} - \frac{\hat{P}_t}{\hat{P}_{t+1}}$ is nonzero on a set of positive probability.

The strict payoff differential monotonicity relies on marginal utility $u'(c^*_{t+1})$ varying across states. The seller takes prices as given, so her only lever on the payoff differential is the marginal-utility $u'(c^*_{t+1})$ attached to each currency's real return; reallocating z_t moves this weight through the consumption response (53) only because $u'(c^*_{t+1})$ differs across states. Under perfect foresight $u'(c^*_{t+1})$ is a common deterministic factor and cancels from both terms of the payoff differential, which reduces to the inflation equalization $P_{t+1}/P_t = m\hat{P}_{t+1}/\hat{P}_t$. With prices taken as given, z_t does not enter this condition, so a price-taking seller with $I^S \neq 0$ cannot reallocate herself toward indifference: the sign of the payoff differential does not depend on z_t. The strict result therefore holds only away from perfect foresight. □

Representative-agent benchmark. In a symmetric representative-agent equilibrium, $c_t = c_{t+1} = \bar{y}$, so marginal utility cancels from the seller's indifference condition at the equilibrium allocation, and indifference restricts beliefs rather than allocations; see Section 3.5. Away from the equilibrium allocation, however, the price-taking seller's own future consumption still responds to her own allocation, so under strategic uncertainty her payoff differential remains strictly decreasing in z^i_t.[32] Thus, the payoff-differential monotonicity requires strategic uncertainty, but not cross-sectional heterogeneity.

A.2.2 Buyer payoff differential monotonicity

Proof. [Lemma 3.3]

[32] Under perfect foresight, the seller payoff differential becomes $I^S_t(z^i_t) = u'(c^*_{t+1}(z^i_t))\left(m\frac{P_t}{P_{t+1}} - \frac{\hat{P}_t}{\hat{P}_{t+1}}\right)$ and remains sensitive to the seller's allocation through its effect on next-period consumption $c^*_{t+1}(z^i_t)$ whenever, off equilibrium, $m\frac{P_t}{P_{t+1}} \neq \frac{\hat{P}_t}{\hat{P}_{t+1}}$. However, because the return gap is independent of the price-taking seller's allocation and $u' > 0$, she cannot attain indifference by varying z^i_t: $I^S_t = 0$ if and only if $m\frac{P_t}{P_{t+1}} = \frac{\hat{P}_t}{\hat{P}_{t+1}}$.

We prove the lemma using the buyer payoff differential for the Dollar. Suppose at the buyer belief ϕ^B and Dollar-financed consumption q_t^D it holds $I_t^{B,D}(q_t^D) > 0$: the buyer is not indifferent and strictly prefers to raise Dollar-financed consumption today, increasing q_t^D. We show that doing so lowers the payoff differential strictly toward zero, so raising q_t^D moves the buyer toward indifference $I_t^{B,D} = 0$. That is, we show

$$\frac{\partial}{\partial q_t^D} I_t^{B,D}(q_t^D | m, \mathscr{F}_t, \phi^B) \equiv \frac{\partial}{\partial q_t^D} \left(u'(c_t)/P_t - \beta^2 m^2 \mathbb{E}_{\phi_t^B}[\rho^{B,D}(\omega_{t+2})] \right) < 0 \tag{54}$$

Two remarks on how the payoff differential in (8) is evaluated, since we assess it before the date-t optimization rather than at its optimum. First, the term $u'(c_t)/P_t$ is written directly in terms of current consumption, not as $W_d(d_t, b_t)$: the envelope identity $W_d(d_t, b_t) = u'(c_t)/P_t$ holds only once q_t^D satisfies its own first-order condition, which under the premise $I_t^{B,D} > 0$ it does not. The first term of the payoff differential is therefore the marginal value of spending a Dollar at the current, generally suboptimal, q_t^D and $u'(c_t)/P_t = W_d$ is reached only in the limit $I_t^{B,D} \to 0$.

Second, the expected future marginal-value term is an envelope derivative and is legitimate at every q_t^D: the continuation consumption $c_{t+2} = c_{t+2}^*$ is the buyer's future optimum conditional on the balances d_{t+2}, b_{t+2} that q_t^D delivers, so $W_d(d_{t+2}, b_{t+2}) = u'(c_{t+2}^*)/P_{t+2}$ holds throughout. The payoff differential thus contrasts a not-yet-optimized current margin with an already-optimized continuation margin. At an interior optimum, the two marginal values are equal, so $I_t^{B,D} = 0$.

We study the first term of the payoff differential first. An extra unit of Dollar-financed consumption raises today's consumption one-for-one, $\partial c_t / \partial q_t^D = 1$, so

$$\frac{\partial}{\partial q_t^D} \frac{u'(c_t)}{P_t} = \frac{u''(c_t)}{P_t} \frac{\partial c_t}{\partial q_t^D} = \frac{u''(c_t)}{P_t} < 0 \tag{55}$$

by concavity, $u'' < 0$. The first term in the buyer payoff differential declines strictly.

For the second term of the payoff differential, recall that the buyer payoff differential is connected to the first order condition of his optimization problem the following way. The t- buyer chooses q_t^D to maximize

$$W_t(d_t, b_t) = \max_{q_t^D, q_t^B} u(q_t^D + q_t^B) + \beta \mathbb{E}_t [S(d_{t+1}, b_{t+1})], \qquad d_{t+1} = m(d_t - q_t^D P_t). \tag{56}$$

whereas the seller maximizes

$$S(d_{t+1}, b_{t+1}) = \max_{z_{t+1} \in [0, \bar{y}]} \beta \mathbb{E}_{(t+1)^-}[W_{t+2}(d_{t+2}, b_{t+2})], \tag{57}$$

given the carried balance

$$d_{t+1} = m(d_t - q_t^D P_t), \qquad d_{t+2} = m(d_{t+1} + z_{t+1} P_{t+1}) = m\big(m(d_t - q_t^D P_t) + z_{t+1} P_{t+1}\big), \tag{58}$$

By the envelope identity established in Section A.1, $W_d(d_t, b_t) = u'(c_t)/P_t$, evaluated at the buyer's optimum and holding in the savings and spend-all regimes alike. The buyer Dollar payoff differential compares the marginal value of a Dollar spent now with its marginal value carried one cycle forward, so $I_t^{B,D} = 0$ is exactly the buyer's Dollar Euler equation at an interior consumption-savings choice in (56). The premise $I_t^{B,D} > 0$ says this first-order condition is not satisfied at the current q_t^D. The balances $d_{t+2}(q_t^D, z_{t+1}^*), b_{t+2}(q_t^D, z_{t+1}^*)$ entering the second term of the buyer payoff differential are determined by q_t^D and the interim seller allocation z_{t+1}^* through the transitions, the latter chosen optimally, while the date-$(t+2)$ consumption is the buyer's future optimum given those balances.

Economically, raising q_t^D depletes the Dollar balance carried forward, which lowers future Dollar-financed consumption and hence raises its expected marginal value $\mathbb{E}_t[u'(c_{t+2}^*)/P_{t+2}]$; since this term enters the payoff differential negatively, it too pushes $I_t^{B,D}$ down. Establishing this rigorously requires care, because the balance carried into $t+2$ also depends on the interim seller allocation z_{t+1}^*, which itself responds to q_t^D. We therefore route the future term through the seller value S_d, where the envelope theorem disposes of that response, rather than differentiating c_{t+2}^* directly. By the envelope theorem, at the optimal allocation z_{t+1}^*,

$$S_d(d_{t+1}, b_{t+1}) = \beta m \mathbb{E}_{(t+1)^-}[W_d(d_{t+2}, b_{t+2})]\,. \tag{59}$$

because $\frac{\partial d_{t+2}}{\partial d_{t+1}} = m$.

If we can show that $S_{dd} \le 0$, then the second term of the buyer payoff differential rises in q_t^D by:

$$\begin{aligned}
\frac{\partial}{\partial q_t^D} \beta^2 m^2 \mathbb{E}_t\left[\frac{u'(c_{t+2}^*)}{P_{t+2}}\right] &= \frac{\partial}{\partial q_t^D} \beta^2 m^2 \mathbb{E}_t[W_d(d_{t+2}, b_{t+2})] && (60)\\
&= \frac{\partial}{\partial q_t^D} \beta^2 m^2 \mathbb{E}_t\big[\mathbb{E}_{(t+1)^-}[W_d(d_{t+2}, b_{t+2})]\big] && (61)\\
&= \beta m \frac{\partial}{\partial q_t^D} \mathbb{E}_t[S_d(d_{t+1}, b_{t+1})] && (62)\\
&= \beta m \mathbb{E}_t\left[S_{dd} \frac{\partial d_{t+1}}{\partial q_t^D}\right] && (63)\\
&= -\beta m^2 P_t \mathbb{E}_t[S_{dd}] \ \ge 0, && (64)
\end{aligned}$$

using $\partial d_{t+1}/\partial q_t^D = -mP_t$ and the law of iterated expectations and the envelope identity (59). We next show $S_{dd} \le 0$:

Differentiating (59) again in d_{t+1}, the balances respond both directly and through

the allocation $z^*_{t+1}(d_{t+1})$. Using $\partial d_{t+2}/\partial d_{t+1} = m$ (direct) together with $\partial d_{t+2}/\partial z_{t+1} = mP_{t+1}$, $\partial b_{t+2}/\partial z_{t+1} = -\hat{P}_{t+1}$ (allocation),

$$S_{dd} = \underbrace{\beta m^2 \mathbb{E}_{(t+1)^-}[W_{dd}]}_{\text{direct curvature}} + \underbrace{\beta m \mathbb{E}_{(t+1)^-}\left[\left(mP_{t+1} W_{dd} - \hat{P}_{t+1} W_{db}\right)\right] \frac{\mathrm{d}z^*_{t+1}}{\mathrm{d}d_{t+1}}}_{\text{allocation response}}. \tag{65}$$

The sign of the allocation response is determined by the seller's optimality condition. If the optimal allocation is at a bound, it does not respond to a marginal change in d_{t+1}, the allocation-response term vanishes, and $S_{dd} \le 0$ follows from the first term alone. If it is interior, $z^*_{t+1} \in (0, \bar{y})$, it satisfies $S_z = 0$; differentiating this identity in d_{t+1} gives the comparative static (implicit function theorem)

$$S_{zd} + S_{zz} \frac{\mathrm{d}z^*_{t+1}}{\mathrm{d}d_{t+1}} = 0 \qquad \Longrightarrow \qquad \frac{\mathrm{d}z^*_{t+1}}{\mathrm{d}d_{t+1}} = -\frac{S_{zd}}{S_{zz}}, \tag{66}$$

which is well defined because

$$S_{zz} = \beta \frac{\partial^2}{\partial z^2_{t+1}} \mathbb{E}_{(t+1)^-}[W_{t+2}(d_{t+2}, b_{t+2})] < 0, \tag{67}$$

the strict concavity of the seller's continuation value in her allocation established in the proof of Lemma 3.2, inequality (45), applied one cycle later at $t+1$.

From $S_z = \beta \mathbb{E}_{(t+1)^-}[mP_{t+1} W_d - \hat{P}_{t+1} W_b]$ in (39),

$$S_{zd} = \beta \mathbb{E}_{(t+1)^-}\left[mP_{t+1} W_{dd} - \hat{P}_{t+1} W_{db}\right] \tag{68}$$

is exactly the coefficient multiplying $\mathrm{d}z^*_{t+1}/\mathrm{d}d_{t+1}$ in (65), up to the factor m. Substituting (66) and (68) into the allocation-response term,

$$S_{dd} = \beta m^2 \mathbb{E}_{(t+1)^-}[W_{dd}] + m S_{zd}\left(-\frac{S_{zd}}{S_{zz}}\right) = \beta m^2 \mathbb{E}_{(t+1)^-}[W_{dd}] - \frac{m S^2_{zd}}{S_{zz}}. \tag{69}$$

The first term is nonpositive because $W_{dd} \le 0$ by (46), and the second is nonnegative because $S^2_{zd} \ge 0$ while $S_{zz} < 0$ by (67), so $-m S^2_{zd}/S_{zz} \underline{\le} 0 \ge 0$. Their sum is nonetheless nonpositive because S is concave in d_{t+1}: it is the maximum over z_{t+1} of the seller's objective, which is jointly concave in (d_{t+1}, z_{t+1}), since W is concave and the balances (d_{t+2}, b_{t+2}) are affine in (d_{t+1}, z_{t+1}).

Hence $S_{dd} \le 0$. By (60) the expected future marginal-value term therefore rises weakly in q^D_t. The first term falls strictly by (55) and the second rises weakly, so their difference $I^{B,D}_t$ falls strictly in q^D_t, establishing $\partial I^{B,D}_t/\partial q^D_t < 0$.

Because q^D_t and d_{t+1} are inversely related through the budget, the comparative static in d_{t+1} is reversed. The same argument applied to (56) in q^B_t, using $W_b(d_t, b_t) = u'(c_t)/\hat{P}_t$ and the Peso Euler equation with no interest compounding, gives the corresponding monotonicity of the Peso payoff differential $I^{B,B}_t$ in q^B_t and b_{t+1}.

Unlike the seller payoff differential, the buyer payoff differential does not degenerate under perfect foresight: its two terms weight consumption at different dates, $u'(c_t)$ and $u'(c^*_{t+2})$, so no common marginal-utility factor cancels. Raising q^D_t still lowers $u'(c_t)/P_t$ strictly by (55) and, through the depleted carried balance, weakly raises the future term, so $I^{B,D}_t$ is strictly decreasing in q^D_t whether or not the path is deterministic. □

A.2.3 Monotonicity in General Equilibrium

The proofs above establish monotonicity of each agent's indifference condition with respect to her own action. Characterizing aggregate equilibrium outcomes, however, requires accounting for the feedback between individual decisions and market prices. For sellers, this feedback is absent because they make their decisions before observing prices: a seller's allocation depends only on her belief and her history. For buyers, by contrast, consumption decisions affect not only individual marginal utilities but also, in the aggregate, currency velocities and hence market prices. These price adjustments, in turn, feed back into buyers' indifference conditions, even though each buyer takes prices as given. The buyer problem is further complicated by two cross effects between the currency markets. First, spending in one currency raises current consumption $c_{i,t}$ and thereby lowers the marginal value of spending the other currency. Second, saving in one currency raises expected future consumption and thereby lowers the continuation value of the other currency, $\partial f^D_i/\partial b_{i,t+1} \le 0$. Characterizing buyer outcomes therefore requires analyzing both buyer indifference conditions jointly with the market-clearing equations that determine prices, a fixed-point problem. We show that, although this problem can have several solutions, its solutions are ordered and respond monotonically to beliefs.

Proof. [Proposition 3.3] *Step 1: Sellers and trade shares.* Fix Φ. Seller i chooses $z_{i,t}$ before period-t prices are observed, using only her belief $\phi^S_{i,t}$ and her history $\mathscr{F}_{i,t-1}$. Her expected continuation value as a function of $z_{i,t}$ is $\beta\mathbb{E}_{t^-}[W(d_{i,t+1}, b_{i,t+1})]$ with $d_{i,t+1} = m(d_{i,t} + z_{i,t}P_t)$ and $b_{i,t+1} = b_{i,t} + (\bar{y} - z_{i,t})\hat{P}_t$, where $W(d_{i,t+1}, b_{i,t+1})$ is her expected continuation value of entering period $t+1$ as a buyer with balances $(d_{i,t+1}, b_{i,t+1})$,

$$W(d_{i,t+1}, b_{i,t+1}) = \max_{q^D_{i,t+1},\, q^B_{i,t+1}} u\big(q^D_{i,t+1} + q^B_{i,t+1}\big) + \beta\mathbb{E}_{t+1}\big[S(d_{i,t+2}, b_{i,t+2})\big],$$

subject to $P_{t+1}q^D_{i,t+1} \le d_{i,t+1}$ and $\hat{P}_{t+1}q^B_{i,t+1} \le b_{i,t+1}$, as defined in Appendix A.1. By (44), the seller's expected continuation value $\beta\mathbb{E}_{t^-}[W(d_{i,t+1}, b_{i,t+1})]$ is differentiable in $z_{i,t}$ with derivative $\beta I^S_i(z_{i,t} \mid \phi^S_{i,t})$. By (A1), the seller's beliefs assign value to both currencies, so Lemma 3.2 applies and I^S_i is strictly decreasing in $z_{i,t}$. Hence the seller's objective is strictly concave on $[0, \bar{y}]$ and has a unique maximizer $z^*_{i,t}$.

If $I_i^S > 0$ on $[0, \bar{y}]$, the objective is strictly increasing and $z_{i,t}^* = \bar{y}$; if $I_i^S < 0$ on $[0, \bar{y}]$, it is strictly decreasing and $z_{i,t}^* = 0$. Otherwise I_i^S changes sign on $[0, \bar{y}]$, so the maximizer satisfies $I_i^S(z_{i,t}^* \mid \phi_{i,t}^S) = 0$, and this root is unique because I_i^S is strictly decreasing:

$$z_{i,t}^* = \begin{cases} \bar{y} & \text{if } I_i^S(z \mid \phi_{i,t}^S) > 0 \text{ for all } z \in [0, \bar{y}], \\ 0 & \text{if } I_i^S(z \mid \phi_{i,t}^S) < 0 \text{ for all } z \in [0, \bar{y}], \\ \text{the unique root of } I_i^S(\cdot \mid \phi_{i,t}^S) & \text{otherwise.} \end{cases}$$

Equivalently, $z_{i,t}^*$ is the feasible allocation that minimizes $|I_i^S(z_{i,t} \mid \phi_{i,t}^S)|$ over $[0, \bar{y}]$. Because $z_{i,t}^*$ depends only on $\phi_{i,t}^S$ and $\mathscr{F}_{i,t-1}$, it does not depend on buyers' date-t choices or on realized date-t prices. By Lemma 3.1, sellers offer their entire endowment. Hence every date-t equilibrium outcome under Φ has the same trade allocation $Y_t^D = \sum_{i \in N_{S,t}} z_{i,t}^*$ and $Y_t^B = Y - Y_t^D$. This proves part 1(a).

If $z_{i,t}^* \in (0, \bar{y})$ for at least one seller, or if all sellers choose boundary allocations but at least one seller chooses $z_{i,t}^* = 0$ and another chooses $z_{j,t}^* = \bar{y}$, goods are supplied in both currency markets, so $Y_t^D \in (0, Y)$ and $Y_t^B \in (0, Y)$, as required for a strong monetary equilibrium. If instead $z_{i,t}^* = \bar{y}$ for all i, then $Y_t^D = Y$ and $Y_t^B = 0$; if $z_{i,t}^* = 0$ for all i, then $Y_t^D = 0$ and $Y_t^B = Y$. In these cases one currency does not circulate at date t, but the allocation is again unique given Φ.

Step 2: The buyer problem in terms of savings. Fix Φ and the seller allocation, and consider buyer i with balances $(d_{i,t}, b_{i,t})$ facing prices $(P_t, \hat{P}_t)$. Choosing purchases $(q_{i,t}^D, q_{i,t}^B)$ is equivalent to choosing savings $(d_{i,t+1}, b_{i,t+1}) \in [0, m d_{i,t}] \times [0, b_{i,t}]$, with

$$q_{i,t}^D = \frac{d_{i,t} - d_{i,t+1}/m}{P_t}, \qquad q_{i,t}^B = \frac{b_{i,t} - b_{i,t+1}}{\hat{P}_t}, \qquad c_{i,t} = q_{i,t}^D + q_{i,t}^B.$$

The buyer maximizes

$$F_i(d_{i,t+1}, b_{i,t+1}; P_t, \hat{P}_t) \equiv u\left(\frac{d_{i,t} - d_{i,t+1}/m}{P_t} + \frac{b_{i,t} - b_{i,t+1}}{\hat{P}_t}\right) + \beta \mathbb{E}_t^{\phi_{i,t}^B}\left[S_i(d_{i,t+1}, b_{i,t+1})\right].$$

Four properties follow. First, the feasible set $X_i \equiv [0, m d_{i,t}] \times [0, b_{i,t}]$ depends neither on current prices nor on beliefs, and current prices $(P_t, \hat{P}_t)$ enter the buyer's objective only through current consumption $c_{i,t}$. Second, F_i is strictly concave in $(d_{i,t+1}, b_{i,t+1})$, because $\mathbb{E}_t^{\phi_{i,t}^B}[S_i]$ is strictly concave by (A3) and $u(c_{i,t})$ is concave, as u is concave and $c_{i,t}$ is affine in savings. Concavity of $\mathbb{E}_t^{\phi_{i,t}^B}[S_i]$ holds without (A3), since u is concave and the budget constraints are linear in balances; (A3) adds strictness.[33] Since F_i is continuous and X_i is a compact set, F_i at-

[33] Strictness is needed because $u(c_{i,t})$ is constant along shifts of savings across currencies that leave $c_{i,t}$ unchanged, so along such shifts the strict concavity of F_i comes from $\mathbb{E}_t^{\phi_{i,t}^B}[S_i]$ alone. Such a shift leaves current consumption unchanged and alters only the mix of currencies the buyer carries forward. Strict concavity along it reflects a diversification motive. If the buyer believes that the future real values of Dollars and Pesos can differ across states, tilting savings toward one currency raises future consumption

tains a maximum on X_i. Since F_i is strictly concave and X_i is convex, this maximizer is unique: if $x \neq x'$ were both maximizers, strict concavity would give $F_i\big(\frac{1}{2}x + \frac{1}{2}x'\big) > \frac{1}{2}F_i(x) + \frac{1}{2}F_i(x') = \max_{X_i} F_i$, a contradiction, because $\frac{1}{2}x + \frac{1}{2}x' \in X_i$. We denote this unique maximizer by $\big(d^*_{i,t+1}(P_t, \hat{P}_t \mid \Phi),\, b^*_{i,t+1}(P_t, \hat{P}_t \mid \Phi)\big)$. Third, if $d_{i,t} + b_{i,t} > 0$, the maximizer satisfies $c_{i,t} > 0$ by $u'(0) = \infty$.

Fourth, the derivatives of the expected continuation value $\beta \mathbb{E}_t^{\phi^B_{i,t}}[S_i]$ with respect to Dollar and Peso savings equal f_i^D/m and f_i^B, as defined in (10) and (11). Recall that

$$S_i(d_{i,t+1}, b_{i,t+1}) = \max_{0 \le z_{i,t+1} \le \bar{y}} \beta \mathbb{E}_{(t+1)^-}\Big[W\big(m(d_{i,t+1} + z_{i,t+1}P_{t+1}),\, b_{i,t+1} + (\bar{y} - z_{i,t+1})\hat{P}_{t+1}\big)\Big].$$

Here $S_i(d_{i,t+1}, b_{i,t+1})$ is agent i's value of entering period $t+1$ as a seller with balances $(d_{i,t+1}, b_{i,t+1})$, with her allocation $z_{i,t+1}$ chosen optimally, so that $\beta \mathbb{E}_t^{\phi^B_{i,t}}[S_i(d_{i,t+1}, b_{i,t+1})]$ is her expected continuation value of the savings she carries forward.

A marginal change in $d_{i,t+1}$ or $b_{i,t+1}$ affects S_i directly through the balances $(d_{i,t+2}, b_{i,t+2})$, and indirectly through the optimal allocation $z^*_{i,t+1}$. The indirect effect does not contribute to the derivative of S_i. The bounds $0 \le z_{i,t+1} \le \bar{y}$ do not depend on balances, so either $z^*_{i,t+1}$ is interior, where the derivative of the seller's objective with respect to $z_{i,t+1}$ is zero, or $z^*_{i,t+1}$ remains at the same bound, so that the allocation does not change. Hence, using $\partial d_{i,t+2}/\partial d_{i,t+1} = m$ and $\partial b_{i,t+2}/\partial b_{i,t+1} = 1$,

$$\frac{\partial S_i}{\partial d_{i,t+1}} = \beta m \mathbb{E}_{(t+1)^-}\big[W_d(d_{i,t+2}, b_{i,t+2})\big], \qquad \frac{\partial S_i}{\partial b_{i,t+1}} = \beta \mathbb{E}_{(t+1)^-}\big[W_b(d_{i,t+2}, b_{i,t+2})\big].$$

By (27), $W_d = u'(c_{i,t+2})/P_{t+2}$ and $W_b = u'(c_{i,t+2})/\hat{P}_{t+2}$, where $c_{i,t+2}$ is the consumption the buyer attains at $t+2$ given the savings $(d_{i,t+1}, b_{i,t+1})$. Taking expectations at date t and comparing with (10) and (11),

$$\beta \frac{\partial \mathbb{E}_t^{\phi^B_{i,t}}[S_i]}{\partial d_{i,t+1}} = \beta^2 m \mathbb{E}_t\left[\frac{u'(c_{i,t+2})}{P_{t+2}}\right] = \frac{f_i^D}{m}, \qquad \beta \frac{\partial \mathbb{E}_t^{\phi^B_{i,t}}[S_i]}{\partial b_{i,t+1}} = \beta^2 \mathbb{E}_t\left[\frac{u'(c_{i,t+2})}{\hat{P}_{t+2}}\right] = f_i^B.$$

These identities hold at every savings pair $(d_{i,t+1}, b_{i,t+1})$, not only at the buyer's date-t maximizer. The maximization in S_i is over future choices only, namely the allocation $z_{i,t+1}$ and all later decisions, while the savings $(d_{i,t+1}, b_{i,t+1})$ are its arguments, determined at t. S_i is therefore a function of savings, defined at every

in states where that currency retains its value and lowers it in states where it loses value. Since u is strictly concave, each further tilt costs more in the bad states than it gains in the good ones, so the expected continuation value is strictly concave along the shift. This requires two conditions. First, future consumption must respond to the balances carried forward, $\kappa > 0$, as in the proof of Lemma 3.2. Second, under the buyer's belief, the shift must change future consumption in some states with positive probability, even after she reoptimizes her allocation as a seller in period $t+1$; that is, no reallocation can fully undo the shift in every state. Full-support beliefs over future prices satisfy the second condition. Strictness fails only if the buyer believes that the future real values of the two currencies always move together, as under perfect foresight, in which case Dollars and Pesos are perfect substitutes as stores of value and the currency composition of her savings is indeterminate.

feasible savings pair, and its derivatives are evaluated at the future optimal choices given that pair.

Step 3: Buyer savings respond monotonically to prices. Current consumption is

$$c_{i,t} = \frac{d_{i,t} - d_{i,t+1}/m}{P_t} + \frac{b_{i,t} - b_{i,t+1}}{\hat{P}_t}.$$

Holding savings fixed and using $q^D_{i,t} = (d_{i,t} - d_{i,t+1}/m)/P_t$ and $q^B_{i,t} = (b_{i,t} - b_{i,t+1})/\hat{P}_t$,

$$\frac{\partial c_{i,t}}{\partial P_t} = -\frac{d_{i,t} - d_{i,t+1}/m}{P_t^2} = -\frac{q^D_{i,t}}{P_t}, \qquad \frac{\partial c_{i,t}}{\partial \hat{P}_t} = -\frac{b_{i,t} - b_{i,t+1}}{\hat{P}_t^2} = -\frac{q^B_{i,t}}{\hat{P}_t},$$

and holding prices fixed,

$$\frac{\partial c_{i,t}}{\partial d_{i,t+1}} = -\frac{1}{mP_t}, \qquad \frac{\partial c_{i,t}}{\partial b_{i,t+1}} = -\frac{1}{\hat{P}_t}.$$

By Step 2, $\beta \partial \mathbb{E}_t^{\phi^B_{i,t}}[S_i]/\partial d_{i,t+1} = f_i^D/m$ and $\beta \partial \mathbb{E}_t^{\phi^B_{i,t}}[S_i]/\partial b_{i,t+1} = f_i^B$. Further, f_i^D, f_i^B do not depend on current prices. The first-order derivatives of F_i are therefore

$$\frac{\partial F_i}{\partial d_{i,t+1}} = \frac{1}{m}\left(f_i^D - \frac{u'(c_{i,t})}{P_t}\right), \qquad \frac{\partial F_i}{\partial b_{i,t+1}} = f_i^B - \frac{u'(c_{i,t})}{\hat{P}_t}.$$

Differentiating these once more yields the cross-partial derivatives of F_i

$$\frac{\partial^2 F_i}{\partial d_{i,t+1}\partial b_{i,t+1}} = \frac{u''(c_{i,t})}{mP_t\hat{P}_t} + \frac{1}{m}\frac{\partial f_i^D}{\partial b_{i,t+1}} < 0,$$

$$\frac{\partial^2 F_i}{\partial d_{i,t+1}\partial P_t} = \frac{u'(c_{i,t}) + u''(c_{i,t})\, q^D_{i,t}}{mP_t^2} \geq 0, \qquad \frac{\partial^2 F_i}{\partial b_{i,t+1}\partial P_t} = \frac{u''(c_{i,t})\, q^D_{i,t}}{P_t\hat{P}_t} \leq 0,$$

$$\frac{\partial^2 F_i}{\partial d_{i,t+1}\partial \hat{P}_t} = \frac{u''(c_{i,t})\, q^B_{i,t}}{mP_t\hat{P}_t} \leq 0, \qquad \frac{\partial^2 F_i}{\partial b_{i,t+1}\partial \hat{P}_t} = \frac{u'(c_{i,t}) + u''(c_{i,t})\, q^B_{i,t}}{\hat{P}_t^2} \geq 0.$$

The first inequality uses $u'' < 0$ and the wealth effect $\partial f_i^D/\partial b_{i,t+1} \leq 0$ from (A3). The inequalities involving $u' + u'' q^j_{i,t}$ use (A2): since $u'' < 0$ and $0 \leq q^j_{i,t} \leq c_{i,t}$, we have $u'(c_{i,t}) + u''(c_{i,t}) q^j_{i,t} \geq u'(c_{i,t}) + u''(c_{i,t}) c_{i,t} = u'(c_{i,t})\big(1 + c_{i,t}u''(c_{i,t})/u'(c_{i,t})\big) \geq 0$, by relative risk-aversion at most one. [34] To show that buyers' optimal savings move monotonically with prices and beliefs, we define a lattice structure on the set of feasible savings pairs. For two savings pairs $x = (d_{i,t+1}, b_{i,t+1})$ and $x' = (d'_{i,t+1}, b'_{i,t+1})$, write $x \succeq_s x'$ if $d_{i,t+1} \geq d'_{i,t+1}$ and $b_{i,t+1} \leq b'_{i,t+1}$, that is, if x holds weakly more Dollars and weakly fewer Pesos. Under this order, the join and meet of x and x'

[34] The cross-partial derivatives are defined for $c_{i,t} > 0$. We can ignore $c_{i,t} = 0$, since by $u'(0) = \infty$ it is never optimal.

are

$$x \vee x' = \big(\max\{d_{i,t+1}, d'_{i,t+1}\}, \min\{b_{i,t+1}, b'_{i,t+1}\}\big), \qquad x \wedge x' = \big(\min\{d_{i,t+1}, d'_{i,t+1}\}, \max\{b_{i,t+1}, b'_{i,t+1}\}\big).$$

The set of feasible savings pairs $X_i \equiv [0, m d_{i,t}] \times [0, b_{i,t}]$ contains $x \vee x'$ and $x \wedge x'$ whenever it contains x and x', so X_i is a lattice under $\succeq_s$, for the definition see (Milgrom and Shannon, 1994; Topkis, 1998). Moreover, X_i depends neither on current prices nor on beliefs. A function F_i on the lattice $(X_i, \succeq_s)$ is supermodular if

$$F_i(x \vee x') + F_i(x \wedge x') \;\geq\; F_i(x) + F_i(x') \qquad \text{for all } x, x' \in X_i.$$

For a twice differentiable F_i, this holds if and only if $\partial^2 F_i / \partial d_{i,t+1} \partial b_{i,t+1} \leq 0$, since under $\succeq_s$ a higher element holds more Dollars and fewer Pesos. Since $\partial^2 F_i / \partial d_{i,t+1} \partial b_{i,t+1} < 0$, F_i is supermodular on $(X_i, \succeq_s)$.

Under $\succeq_s$, F_i has increasing differences in $\big((d_{i,t+1}, b_{i,t+1}), P_t\big)$ and in $\big((d_{i,t+1}, b_{i,t+1}), -\hat{P}_t\big)$. Increasing differences in $\big((d_{i,t+1}, b_{i,t+1}), P_t\big)$ means that for every $P'_t > P_t$, the difference $F_i(x; P'_t, \hat{P}_t) - F_i(x; P_t, \hat{P}_t)$ is nondecreasing in x under $\succeq_s$, that is, it weakly rises when Dollar savings rise and when Peso savings fall. Since F_i is twice differentiable for $c_{i,t} > 0$, this holds if and only if

$$\frac{\partial^2 F_i}{\partial d_{i,t+1} \partial P_t} \geq 0 \qquad \text{and} \qquad \frac{\partial^2 F_i}{\partial b_{i,t+1} \partial P_t} \leq 0,$$

which are the signs derived above. Economically, a higher Dollar price lowers the value of spending Dollars today, which raises the marginal value of saving Dollars, and lowers the value of the Dollar-financed part of current consumption, which raises current marginal utility and hence lowers the marginal value of saving Pesos. Analogously, increasing differences in $\big((d_{i,t+1}, b_{i,t+1}), -\hat{P}_t\big)$ holds if and only if

$$\frac{\partial^2 F_i}{\partial d_{i,t+1} \partial(-\hat{P}_t)} = -\frac{\partial^2 F_i}{\partial d_{i,t+1} \partial \hat{P}_t} \geq 0 \qquad \text{and} \qquad \frac{\partial^2 F_i}{\partial b_{i,t+1} \partial(-\hat{P}_t)} = -\frac{\partial^2 F_i}{\partial b_{i,t+1} \partial \hat{P}_t} \leq 0,$$

which again follow from the signs derived above: a lower Peso price raises the marginal value of saving Dollars and lowers the marginal value of saving Pesos.

The three properties established above, namely that X_i is a lattice under $\succeq_s$ that does not depend on prices, that F_i is supermodular, and that F_i has increasing differences in $\big((d_{i,t+1}, b_{i,t+1}), P_t\big)$ and in $\big((d_{i,t+1}, b_{i,t+1}), -\hat{P}_t\big)$, are exactly the conditions of the monotone comparative statics theorem of Topkis (1978) and Milgrom and Shannon (1994), which implies that the buyer's optimal savings are monotone in prices. For completeness, we reproduce their argument for the case of a unique maximizer. Fix $\hat{P}_t$, let $P'_t > P_t$, and let x and x' denote the maximizers of F_i at P_t and P'_t. Since X_i is a lattice, $x \vee x'$ and $x \wedge x'$ are feasible. Supermodularity and the

optimality of x at P_t give

$$F_i(x \vee x'; P_t) - F_i(x'; P_t) \;\geq\; F_i(x; P_t) - F_i(x \wedge x'; P_t) \;\geq\; 0.$$

Since $x \vee x' \succeq_s x'$, increasing differences give $F_i(x \vee x'; P_t') - F_i(x \vee x'; P_t) \geq F_i(x'; P_t') - F_i(x'; P_t)$. Rearranging and using the previous inequality,

$$F_i(x \vee x'; P_t') - F_i(x'; P_t') \;\geq\; F_i(x \vee x'; P_t) - F_i(x'; P_t) \;\geq\; 0.$$

Hence $x \vee x'$ is also a maximizer of $F_i(\cdot; P_t')$ on the lattice of feasible savings pairs X_i. Since by Step 2 this maximizer is unique and equals x', we have $x \vee x' = x'$, that is, $x' \succeq_s x$. Since $x = \big(d^*_{i,t+1}(P_t, \hat{P}_t \mid \Phi), b^*_{i,t+1}(P_t, \hat{P}_t \mid \Phi)\big)$ and $x' = \big(d^*_{i,t+1}(P_t', \hat{P}_t \mid \Phi), b^*_{i,t+1}(P_t', \hat{P}_t \mid \Phi)\big)$, the relation $x' \succeq_s x$ means $d^*_{i,t+1}(P_t', \hat{P}_t \mid \Phi) \geq d^*_{i,t+1}(P_t, \hat{P}_t \mid \Phi)$ and $b^*_{i,t+1}(P_t', \hat{P}_t \mid \Phi) \leq b^*_{i,t+1}(P_t, \hat{P}_t \mid \Phi)$.

The same argument with $-\hat{P}_t$ in place of P_t applies to changes in the Peso price. Therefore $d^*_{i,t+1}$ is nondecreasing in P_t and nonincreasing in $\hat{P}_t$, while $b^*_{i,t+1}$ is nonincreasing in P_t and nondecreasing in $\hat{P}_t$. A higher Dollar price thus induces the buyer to save more Dollars and spend more Pesos, and a higher Peso price induces the reverse.

Step 4: Velocity schedules and market clearing. Define the buyers' price-contingent velocity schedules under Φ by

$$V_t^D(P_t, \hat{P}_t \mid \Phi) \equiv \frac{1}{D_t} \sum_{i \in N_{B,t}} \Big(d_{i,t} - \frac{d^*_{i,t+1}(P_t, \hat{P}_t \mid \Phi)}{m}\Big), \qquad V_t^B(P_t, \hat{P}_t \mid \Phi) \equiv \frac{1}{B} \sum_{i \in N_{B,t}} \Big(b_{i,t} - b^*_{i,t+1}(P_t, \hat{P}_t \mid \Phi)\Big).$$

By Step 3, V_t^D is nonincreasing in P_t and nondecreasing in $\hat{P}_t$, and V_t^B is nondecreasing in P_t and nonincreasing in $\hat{P}_t$.

Market clearing in each currency is equivalent to an equation in the velocity schedules. Multiplying Dollar market clearing $\sum_{i \in N_{B,t}} q^D_{i,t} = Y_t^D$ by $P_t > 0$ and using $P_t q^D_{i,t} = d_{i,t} - d^*_{i,t+1}/m$ gives $\sum_{i \in N_{B,t}} \big(d_{i,t} - d^*_{i,t+1}/m\big) = P_t Y_t^D$, whose left-hand side equals $D_t V_t^D$ by definition. The same steps apply to the Peso market. Hence market clearing is equivalent to

$$P_t Y_t^D = D_t\, V_t^D(P_t, \hat{P}_t \mid \Phi), \qquad \hat{P}_t Y_t^B = B\, V_t^B(P_t, \hat{P}_t \mid \Phi). \tag{70}$$

A date-t equilibrium outcome under Φ is a price pair $(P_t, \hat{P}_t)$ solving (70) given the trade allocation of Step 1, together with the induced buyer savings. In each equation the left-hand side is strictly increasing in the own price level whenever the trade share is positive, while the right-hand side is weakly decreasing in the own price level and weakly increasing in the other currency's price level.

Step 5: Ordering of outcomes at a given belief profile (part 1(b)). Consider two strong monetary equilibrium outcomes under Φ with prices $(P_t, \hat{P}_t)$ and $(P_t', \hat{P}_t')$. By Step

1, both share the same $Y_t^D, Y_t^B > 0$. Suppose $P_t' > P_t$ and $\hat{P}_t' \le \hat{P}_t$. Then, by Step 4,

$$D_t V_t^D(P_t', \hat{P}_t' \mid \Phi) \le D_t V_t^D(P_t', \hat{P}_t \mid \Phi) \le D_t V_t^D(P_t, \hat{P}_t \mid \Phi) = P_t Y_t^D < P_t' Y_t^D,$$

contradicting Dollar market clearing at $(P_t', \hat{P}_t')$. Hence $P_t' > P_t$ implies $\hat{P}_t' > \hat{P}_t$. Symmetrically, suppose $\hat{P}_t' > \hat{P}_t$ and $P_t' \le P_t$. Then

$$B\, V_t^B(P_t', \hat{P}_t' \mid \Phi) \le B\, V_t^B(P_t, \hat{P}_t' \mid \Phi) \le B\, V_t^B(P_t, \hat{P}_t \mid \Phi) = \hat{P}_t Y_t^B < \hat{P}_t' Y_t^B,$$

contradicting Peso market clearing. Hence $\hat{P}_t' > \hat{P}_t$ implies $P_t' > P_t$, and therefore $P_t' = P_t$ if and only if $\hat{P}_t' = \hat{P}_t$. If prices coincide, each buyer's unique optimum from Step 2 coincides, so the two outcomes are identical.

Velocities are strictly higher in the equilibrium with higher prices: By (70), $V_t^D = P_t Y_t^D / D_t$ and $V_t^B = \hat{P}_t Y_t^B / B$ are strictly increasing in the respective price, since $Y_t^D, Y_t^B > 0$. This establishes the chain of equivalences in part 1(b). Aggregate buyer savings are $\sum_{i \in N_{B,t}} d_{i,t+1} = m\big(\sum_{i \in N_{B,t}} d_{i,t} - P_t Y_t^D\big)$ and $\sum_{i \in N_{B,t}} b_{i,t+1} = \sum_{i \in N_{B,t}} b_{i,t} - \hat{P}_t Y_t^B$. Since buyers' date-$t$ balances $d_{i,t}, b_{i,t}$ are predetermined, both $\sum_{i \in N_{B,t}} d_{i,t+1}$ and $\sum_{i \in N_{B,t}} b_{i,t+1}$ are strictly lower in the outcome with higher prices.

Step 6: Seller selection (part 2(a)). We show that each seller allocates weakly more goods to the Dollar market under Φ^{abs} than under Φ^{pt}, and strictly more if her allocation under Φ^{pt} is interior.[35] Fix a seller i, and let $\tilde{z} \equiv z_{i,t}^*(\tilde{\phi}_{i,t}^S)$ and $z \equiv z_{i,t}^*(\phi_{i,t}^S)$ denote her unique optimal allocations under Φ^{pt} and Φ^{abs}, respectively, where we suppress the seller index i and the date t in $\tilde{z}$ and z for readability.

By Step 1, under either belief her expected continuation value is strictly concave in $z_{i,t}$, with derivative $\beta I_i^S(z_{i,t} \mid \cdot)$, and I_i^S is strictly decreasing in $z_{i,t}$. By Definition 3.2, $\phi_{i,t}^S \succ_{\mathscr{R}} \tilde{\phi}_{i,t}^S$ means

$$I_i^S(z_{i,t} \mid \phi_{i,t}^S) > I_i^S(z_{i,t} \mid \tilde{\phi}_{i,t}^S) \qquad \text{for every } z_{i,t} \in [0, \bar{y}].$$

If $\tilde{z} = 0$, then $z \ge \tilde{z}$ holds trivially. Suppose $\tilde{z} > 0$. By the characterization of $z_{i,t}^*$ in Step 1, either $I_i^S(\cdot \mid \tilde{\phi}_{i,t}^S) > 0$ on $[0, \bar{y}]$ and $\tilde{z} = \bar{y}$, or $\tilde{z}$ is the root of $I_i^S(\cdot \mid \tilde{\phi}_{i,t}^S)$. In both cases $I_i^S(\tilde{z} \mid \tilde{\phi}_{i,t}^S) \ge 0$, and therefore $I_i^S(\tilde{z} \mid \phi_{i,t}^S) > 0$. Since $I_i^S(\cdot \mid \phi_{i,t}^S)$ is strictly decreasing and $I_i^S(\tilde{z} \mid \phi_{i,t}^S) > 0$, either it has a root, which then lies strictly above $\tilde{z}$, so that $z > \tilde{z}$, or it is strictly positive on all $[0, \bar{y}]$, so that $z = \bar{y} \ge \tilde{z}$. Hence $z \ge \tilde{z}$, with strict inequality whenever $\tilde{z} \in (0, \bar{y})$.

[35] A strong monetary equilibrium does not by itself guarantee that any individual seller's allocation is interior. It requires only that goods are offered in both markets, $Y_t^D \in (0, Y)$, which also holds if every seller chooses a corner allocation, some allocating all goods to the Dollar market and the others all goods to the Peso market. In that case the Dollar trade share need not rise under Φ^{abs}: a seller at $z_{i,t}^* = \bar{y}$ cannot allocate more goods to the Dollar market, and a seller at $z_{i,t}^* = 0$ whose payoff differential at zero remains negative under Φ^{abs} does not change her allocation. Strictness of the trade-share ranking therefore requires at least one seller with an interior allocation under Φ^{pt}. Thus, the assumption of a strong monetary equilibrium guarantees only the weak ranking $Y_t^D(\Phi^{\text{abs}}) \ge Y_t^D(\Phi^{\text{pt}})$.

Summing over sellers gives

$$Y_t^D(\Phi^{\text{abs}}) = \sum_{i \in N_{S,t}} z_{i,t}^*(\phi_{i,t}^S) \geq \sum_{i \in N_{S,t}} z_{i,t}^*(\tilde{\phi}_{i,t}^S) = Y_t^D(\Phi^{\text{pt}}),$$

with strict inequality if at least one seller's allocation under Φ^{pt} is interior. Since $Y_t^B = Y - Y_t^D$ under both profiles, $Y_t^B(\Phi^{\text{abs}}) \leq Y_t^B(\Phi^{\text{pt}})$, with the same strictness.

Step 7: Buyer savings respond monotonically to beliefs. We show that, at every price pair, each buyer saves weakly more Dollars and weakly fewer Pesos under Φ^{abs} than under Φ^{pt}. As a consequence, at every price pair, the Dollar velocity schedule is weakly lower and the Peso velocity schedule weakly higher under Φ^{abs}.

Fix prices $(P_t, \hat{P}_t)$ and a buyer i. Both profiles share the history $\mathscr{F}_{t-1}$, and hence the same set of buyers $N_{B,t}$, the same balances $(d_{i,t}, b_{i,t})$, and the same supplies D_t and B. Write $F_i(\cdot \mid \Phi)$ for the buyer's objective from Step 2 under belief profile Φ. By Step 2, applied under each profile, $F_i(\cdot \mid \Phi^{\text{pt}})$ and $F_i(\cdot \mid \Phi^{\text{abs}})$ have unique maximizers on the set of feasible savings pairs X_i, which we denote $\tilde{x}$ and x, respectively. The two objectives share the current-utility term $u(c_{i,t})$, which depends only on savings and prices. Their difference is therefore

$$\Delta_i(y) \equiv F_i(y \mid \Phi^{\text{abs}}) - F_i(y \mid \Phi^{\text{pt}}) = \beta\Big(\mathbb{E}_t^{\phi_{i,t}^B}\big[S_i(y)\big] - \mathbb{E}_t^{\tilde{\phi}_{i,t}^B}\big[S_i(y)\big]\Big), \qquad y \in X_i.$$

By Step 2, its partial derivatives at every savings pair y are

$$\frac{\partial \Delta_i}{\partial d_{i,t+1}} = \frac{1}{m}\Big(f_i^D(y \mid \Phi^{\text{abs}}) - f_i^D(y \mid \Phi^{\text{pt}})\Big) \geq 0, \qquad \frac{\partial \Delta_i}{\partial b_{i,t+1}} = f_i^B(y \mid \Phi^{\text{abs}}) - f_i^B(y \mid \Phi^{\text{pt}}) \leq 0,$$

where both inequalities hold at every savings pair because, by Definition 3.4, the buyer's Dollar belief under Φ^{abs} first-order stochastically dominates her Dollar belief under Φ^{pt}, while her Peso belief under Φ^{pt} first-order stochastically dominates her Peso belief under Φ^{abs}. By Corollary 3.1, these orderings imply $f_i^D(\cdot \mid \Phi^{\text{abs}}) \geq f_i^D(\cdot \mid \Phi^{\text{pt}})$ and $f_i^B(\cdot \mid \Phi^{\text{abs}}) \leq f_i^B(\cdot \mid \Phi^{\text{pt}})$.

Hence Δ_i is nondecreasing on $(X_i, \succeq_s)$. To see this, let $y = (d, b)$ and $y' = (d', b')$ be two feasible savings pairs in X_i with $y \succeq_s y'$, so that $d \geq d'$ and $b \leq b'$, and note that $(d, b') \in X_i$ because X_i is a rectangle. Then, adding zero,

$$\Delta_i(y) - \Delta_i(y') = \big[\Delta_i(d, b) - \Delta_i(d, b')\big] + \big[\Delta_i(d, b') - \Delta_i(d', b')\big] \geq 0,$$

since the first bracket lowers Peso savings, along which Δ_i weakly rises, and the second raises Dollar savings, along which Δ_i also weakly rises. In words, F_i has increasing differences in savings and beliefs: the gain from moving to a savings pair with more Dollars and fewer Pesos is weakly larger under Φ^{abs}.

We now show that $x \succeq_s \tilde{x}$, using the argument of Step 3 with the belief profile in place of the price. Since X_i is a lattice, $\tilde{x} \vee x$ and $\tilde{x} \wedge x$ are feasible. By Step 3,

$F_i(\cdot \mid \Phi^{\text{pt}})$ is supermodular, since (A3) holds under every belief profile considered. Supermodularity and the optimality of $\tilde{x}$ under Φ^{pt} give

$$F_i(\tilde{x} \vee x \mid \Phi^{\text{pt}}) - F_i(x \mid \Phi^{\text{pt}}) \;\geq\; F_i(\tilde{x} \mid \Phi^{\text{pt}}) - F_i(\tilde{x} \wedge x \mid \Phi^{\text{pt}}) \;\geq\; 0. \tag{71}$$

We now apply the monotonicity of Δ_i with $y = \tilde{x} \vee x$ and $y' = x$. By the definition of the join, $\tilde{x} \vee x \succeq_s x$, so that $\Delta_i(\tilde{x} \vee x) \geq \Delta_i(x)$, that is, $F_i(\tilde{x} \vee x \mid \Phi^{\text{abs}}) - F_i(\tilde{x} \vee x \mid \Phi^{\text{pt}}) \geq F_i(x \mid \Phi^{\text{abs}}) - F_i(x \mid \Phi^{\text{pt}})$. Rearranging this inequality, and using positivity derived in (71),

$$F_i(\tilde{x} \vee x \mid \Phi^{\text{abs}}) - F_i(x \mid \Phi^{\text{abs}}) \;\geq\; F_i(\tilde{x} \vee x \mid \Phi^{\text{pt}}) - F_i(x \mid \Phi^{\text{pt}}) \;\geq\; 0. \tag{72}$$

Hence $\tilde{x} \vee x$ also maximizes $F_i(\cdot \mid \Phi^{\text{abs}})$, and uniqueness of the maximizer implies $\tilde{x} \vee x = x$, that is, $x \succeq_s \tilde{x}$. By definition of the maximizers, $x = (d_x, b_x) = \big(d^*_{i,t+1}(P_t, \hat{P}_t \mid \Phi^{\text{abs}}), b^*_{i,t+1}(P_t, \hat{P}_t \mid \Phi^{\text{abs}})\big)$ and $\tilde{x} = (d_{\tilde{x}}, b_{\tilde{x}}) = \big(d^*_{i,t+1}(P_t, \hat{P}_t \mid \Phi^{\text{pt}}), b^*_{i,t+1}(P_t, \hat{P}_t \mid \Phi^{\text{pt}})\big)$. By the definition of the join, $\tilde{x} \vee x = x$ means $(d_x, b_x) = \big(\max\{d_{\tilde{x}}, d_x\}, \min\{b_{\tilde{x}}, b_x\}\big)$, so that $d_x \geq d_{\tilde{x}}$ and $b_x \leq b_{\tilde{x}}$. The relation $x \succeq_s \tilde{x}$, therefore reads

$$d^*_{i,t+1}(P_t, \hat{P}_t \mid \Phi^{\text{abs}}) \geq d^*_{i,t+1}(P_t, \hat{P}_t \mid \Phi^{\text{pt}}), \qquad b^*_{i,t+1}(P_t, \hat{P}_t \mid \Phi^{\text{abs}}) \leq b^*_{i,t+1}(P_t, \hat{P}_t \mid \Phi^{\text{pt}}). \tag{73}$$

Substituting these inequalities into the definitions of the velocity schedules in Step 4, and using that balances and supplies coincide across the two profiles, we obtain at every price pair

$$V^D_t(P_t, \hat{P}_t \mid \Phi^{\text{abs}}) \leq V^D_t(P_t, \hat{P}_t \mid \Phi^{\text{pt}}), \qquad V^B_t(P_t, \hat{P}_t \mid \Phi^{\text{abs}}) \geq V^B_t(P_t, \hat{P}_t \mid \Phi^{\text{pt}}). \tag{74}$$

Step 8: Price selection (parts 2(b), 2(c) and 2(d)). We show that every strong monetary equilibrium under Φ^{abs} has a strictly lower Dollar price or a strictly higher Peso price than every strong monetary equilibrium under Φ^{pt}, unless the two equilibria have identical prices and trade allocations, and that the latter case is excluded if at least one seller's allocation under Φ^{pt} is interior.

Take any strong monetary equilibrium under Φ^{pt}, with prices $(P_t, \hat{P}_t)$ and trade allocation (Y^D_t, Y^B_t), and any strong monetary equilibrium under Φ^{abs}, with prices $(P'_t, \hat{P}'_t)$ and trade allocation $(Y^{D\prime}_t, Y^{B\prime}_t)$. In a strong monetary equilibrium, both trade allocations are strictly positive. Suppose, contrary to the claim, that $P'_t \geq P_t$ and $\hat{P}'_t \leq \hat{P}_t$. Then

$$\begin{aligned} D_t V^D_t(P'_t, \hat{P}'_t \mid \Phi^{\text{abs}}) \leq D_t V^D_t(P'_t, \hat{P}'_t \mid \Phi^{\text{pt}}) \leq D_t V^D_t(P_t, \hat{P}'_t \mid \Phi^{\text{pt}}) \leq D_t V^D_t(P_t, \hat{P}_t \mid \Phi^{\text{pt}}) \\ = P_t Y^D_t \leq P_t Y^{D\prime}_t \leq P'_t Y^{D\prime}_t = D_t V^D_t(P'_t, \hat{P}'_t \mid \Phi^{\text{abs}}). \end{aligned}$$

The first inequality follows from Step 7, equation (74). The second follows from $P'_t \geq P_t$, since V^D_t is nonincreasing in P_t by Step 4. The third follows from $\hat{P}'_t \leq \hat{P}_t$, since V^D_t is nondecreasing in $\hat{P}_t$ by Step 4. The equality that follows is Dollar market clearing (70) under Φ^{pt}. The fourth inequality follows from $Y^{D\prime}_t \geq Y^D_t$ by

Step 6, and the fifth from $P'_t \geq P_t$. The final equality is Dollar market clearing under Φ^{abs}. Since the chain begins and ends with the same expression, all inequalities must hold with equality. In particular, $P_t Y_t^{D\prime} = P'_t Y_t^{D\prime}$ with $Y_t^{D\prime} > 0$ gives $P'_t = P_t$, and $P_t Y_t^D = P_t Y_t^{D\prime}$ with $P_t > 0$ gives $Y_t^{D\prime} = Y_t^D$, hence $Y_t^{B\prime} = Y_t^B$.

Turning to the Peso market,

$$\begin{aligned} B\, V_t^B(P'_t, \hat{P}'_t \mid \Phi^{\text{abs}}) \geq B\, V_t^B(P'_t, \hat{P}'_t \mid \Phi^{\text{pt}}) = B\, V_t^B(P_t, \hat{P}'_t \mid \Phi^{\text{pt}}) \geq B\, V_t^B(P_t, \hat{P}_t \mid \Phi^{\text{pt}}) \\ = \hat{P}_t Y_t^B = \hat{P}_t Y_t^{B\prime} \geq \hat{P}'_t Y_t^{B\prime} = B\, V_t^B(P'_t, \hat{P}'_t \mid \Phi^{\text{abs}}). \end{aligned}$$

The first inequality follows from Step 7 with equation (74), and the subsequent equality from $P'_t = P_t$. The second inequality follows from $\hat{P}'_t \leq \hat{P}_t$, since V_t^B is nonincreasing in $\hat{P}_t$ by Step 4. The next equality is Peso market clearing under Φ^{pt}, followed by $Y_t^{B\prime} = Y_t^B$. The last inequality follows from $\hat{P}'_t \leq \hat{P}_t$, and the final equality is Peso market clearing under Φ^{abs}. Again all inequalities must hold with equality, and $\hat{P}_t Y_t^{B\prime} = \hat{P}'_t Y_t^{B\prime}$ with $Y_t^{B\prime} > 0$ gives $\hat{P}'_t = \hat{P}_t$.

Hence $P'_t \geq P_t$ and $\hat{P}'_t \leq \hat{P}_t$ imply that the two equilibria have identical prices and trade allocations, and therefore, by (70), identical velocities, since D_t and B are common to both profiles.

Contrapositively, either $P'_t < P_t$ or $\hat{P}'_t > \hat{P}_t$ or both, unless prices and trade allocations coincide. If $Y_t^{D\prime} > Y_t^D$, the last case is excluded, so that $P'_t < P_t$ or $\hat{P}'_t > \hat{P}_t$. If $Y_t^{D\prime} = Y_t^D$, either $P'_t < P_t$, or $\hat{P}'_t > \hat{P}_t$, or both prices coincide. This proves part 2(b).

For part 2(c), suppose that at least one seller's allocation under Φ^{pt} is interior, so that $Y_t^{D\prime} > Y_t^D$ by Step 6, and assume that $P'_t \geq P_t$. By part 2(b), it follows $\hat{P}'_t > \hat{P}_t$. By (70),

$$V_t^{D\prime} = \frac{P'_t Y_t^{D\prime}}{D_t} > \frac{P_t Y_t^D}{D_t} = V_t^D,$$

since $P'_t \geq P_t > 0$ and $Y_t^{D\prime} > Y_t^D > 0$. The Peso velocity cannot be ranked in general: by (70), $V_t^{B\prime} = \hat{P}'_t Y_t^{B\prime}/B$ and $V_t^B = \hat{P}_t Y_t^B/B$, where the Peso price is higher, $\hat{P}'_t > \hat{P}_t$, but the Peso trade share is lower, $Y_t^{B\prime} < Y_t^B$.

For part 2(d), suppose that at least one seller's allocation under Φ^{pt} is interior, so that $Y_t^{D\prime} > Y_t^D$ and hence $Y_t^{B\prime} < Y_t^B$ by Step 6, and assume that $\hat{P}'_t \leq \hat{P}_t$. By part 2(b), it must hold $P'_t < P_t$. By (70),

$$V_t^{B\prime} = \frac{\hat{P}'_t Y_t^{B\prime}}{B} < \frac{\hat{P}_t Y_t^B}{B} = V_t^B,$$

since $0 < \hat{P}'_t \leq \hat{P}_t$ and $0 < Y_t^{B\prime} < Y_t^B$.

The velocity of a currency can be ranked across the two profiles only when its price and its trade share move in the same direction. Parts 2(c) and 2(d) cover the two cases in which this holds for one of the currencies. In the remaining case, $P'_t < P_t$ and $\hat{P}'_t > \hat{P}_t$, neither velocity can be ranked: by (70), $V_t^{D\prime}/V_t^D = (P'_t/P_t)(Y_t^{D\prime}/Y_t^D)$, where the Dollar price is lower but the Dollar trade share is higher, and $V_t^{B\prime}/V_t^B = (\hat{P}'_t/\hat{P}_t)(Y_t^{B\prime}/Y_t^B)$, where the Peso price is higher but the Peso trade share is lower.

□

A.3 Symmetric perfect foresight monetary equilibria

Although inflation is pinned down, the trade shares and velocities are not free either. By market clearing $P_t = D_t V_t^D / Y_t^D$ and $\hat{P}_t = B V_t^B / Y_t^B$ together with the inflation rates above, their growth factors satisfy the master relation of part (ii) (Appendix A.3.1),

$$\Delta_t^{V,j} = \beta \Delta_t^{Y,j}, \qquad j \in \{D, B\}. \tag{75}$$

Across periods, a currency's velocity and trade share move in the same direction, velocity growth being the fraction β of trade-share growth. This offsets two price pressures: selling more goods for a currency, a higher trade share, is deflationary because more goods chase the same money, while spending it faster, a higher velocity, is inflationary, and holding inflation at the rate required by indifference forces the two to move together in the fixed proportion (75). It is exactly this offsetting that keeps Dollar pass-through at $P_{t+2}/P_t = \beta^2 m^2$ across all SPFM equilibria even as trade shares and velocities vary: the variation is confined to quantities and cancels in the price.

Two restrictions complete the picture. The two trade shares are linked by goods feasibility, $Y_t^D + Y_t^B = Y$, so a rising Dollar share is a falling Peso share, whereas the velocities, each bounded in $(0, 1]$, are not linked across currencies. The open dimension is therefore the direction and size of the trade-share and velocity shifts, not the inflation rate, which the policy factor m alone pins down.

Three features stand out. First, the Dollar pass-through is unique for every m. Unlike under strategic uncertainty, where the same m can support different Dollar and Peso inflation rates, perfect foresight delivers a single tempered pass-through, $P_{t+2}/P_t = \beta^2 m^2$, with Peso prices unaffected. We call this weaker form of the Quantity Theory of Money the tempered Quantity Theory of Money (TQTM). Because monetary shocks are absorbed through the Dollar price level, velocities, and trade shares but never through Peso prices, stronger deviations from the Quantity Theory of Money, and any spillover into Peso prices, as observed experimentally, require the strategic uncertainty of Section 3.5.

Second, while the inflation rate is unique, the trade and velocity paths are not. The same m, and hence the same anticipated Dollar and Peso inflation, supports a range of equilibria in which the Dollar trade share rises or falls and Dollar velocity rises or falls. Uncertainty about inflation is therefore not necessary to generate either direction of Dollar dominance or either direction of velocity. This is striking against the construction under strategic uncertainty, where such movements are driven by differences in inflation expectations: lower seller Dollar inflation expectations raise the Dollar trade share, and lower buyer Dollar inflation expectations raise Dollar saving at given prices (Proposition 3.3). The starkest case is a high

interest factor m, and hence high anticipated Dollar inflation $\beta^2 m^2$: even then the Dollar trade share can rise and Dollar velocity can fall, responses that run counter to what high Dollar inflation would suggest and that constitute a further form of rational money illusion.

Third, the central bank's reach stops at the inflation rate. Under perfect foresight it steers Dollar inflation directly through m, yet it cannot steer the direction of trade shares or velocities, exactly as under strategic uncertainty. The point is sharper still: the feasible range of trade-share and velocity growth is itself independent of m (Proposition 3.5(iii) and (iv)), since m enters the Dollar price path but neither the master relation (75) nor the feasibility constraints $V_t^j \in (0,1]$ and $Y_t^j \in (0,Y)$. The central bank thus governs the Dollar inflation rate and nothing else in the real allocation, and the set of sustainable trade and velocity paths is identical under active and mute policy, the two differing only in the equilibrium Dollar inflation rate, βm against β.[36] It follows that the central bank cannot reliably attract users into the Dollar by paying interest to defend its dominance, nor can agents consistently resist money illusion: Dollar hoarding, that is lower velocity, is only one possible outcome, faster Dollar spending is equally consistent, and belief-driven miscoordination is not necessary for either currency to dominate or be dominated.

These are monetary equilibria. If sellers refuse Peso, $Y_t^B = 0$, it loses all value and its price is indeterminate, so both currencies retain value only if $Y_t^B, Y_t^D \in (0,Y)$ for all t. Zero velocity with a positive goods supply is never an equilibrium: goods are perishable, so by Lemma 3.1 all packages sell, and zero velocity is consistent only with a zero goods allocation, that is, a non-monetary equilibrium.

A.3.1 Proof: perfect foresight equilibrium construction

Proposition 3.5. All three constructions share the same machinery, which we record once. We work in the symmetric representative economy, $c_t = \bar{y} = c_{t+2}$, write $\Delta_t^{Y,j} \equiv Y_t^j / Y_{t-1}^j$ and $\Delta_t^{V,j} \equiv V_t^j / V_{t-1}^j$ for $j \in \{D,B\}$, and use sequential market clearing

$$P_t Y_t^D = D_t V_t^D, \qquad \hat{P}_t Y_t^B = B V_t^B, \qquad D_t = m D_{t-1}, \qquad Y_t^D + Y_t^B = Y, \text{ for all } t \geq 1,$$

taking as given past realizations $\{((Y_s^D, Y_s^B), (V_s^D, V_s^B), P_s, \hat{P}_s)\}_{0 \leq s \leq t-1}$.

Proposition 3.5 (i) Under perfect foresight and symmetry, buyer indifference over the two-period horizon requires $P_{t+2}/P_t = \beta^2 m^2$ and $\hat{P}_{t+2}/\hat{P}_t = \beta^2$, and seller indifference requires $P_{t+1}/P_t = m\hat{P}_{t+1}/\hat{P}_t$. This yields Proposition 3.5(i). These conditions pin down only two-period inflation and admit a family of per-period paths;

[36] Under mute policy, perfect foresight requires both Dollar and Peso inflation to equal the discount factor (deflation), so the shifts cannot be read as money illusion. The rate βm is constant across equilibria within a treatment and differs only across treatments.

it suffices to construct equilibria with constant per-period inflation

$$\frac{P_t}{P_{t-1}} = \beta m, \qquad \frac{\hat{P}_t}{\hat{P}_{t-1}} = \beta,$$

which satisfy all three (the seller condition because $\beta m = m \cdot \beta$, the buyer conditions because the rates compound to $\beta^2 m^2$ and β^2). The constant per-period inflation is only a selection: seller indifference fixes relative inflation each period while buyer indifference fixes only the two-period rates, so any path with $\hat{P}_t/\hat{P}_{t-1} = a_t$ and $a_t a_{t-1} = \beta^2$ (equivalently $P_t/P_{t-1} = m a_t$) is also a symmetric perfect foresight equilibrium. Inflation may therefore oscillate from period to period, for instance in a two-cycle $a, \beta^2/a, \ldots$, and remains an equilibrium as long as two-period inflation stays at $\beta^2 m^2$ for the Dollar and β^2 for Peso. The oscillating paths are one such possibility; for the construction we restrict attention to the monotone case of constant per-period inflation, $P_t/P_{t-1} = \beta m$ and $\hat{P}_t/\hat{P}_{t-1} = \beta$.

Proposition 3.5 (ii) Dividing the one-period clearing identities across t and $t-1$ gives $P_t/P_{t-1} = m\Delta_t^{V,D}/\Delta_t^{Y,D}$ and $\hat{P}_t/\hat{P}_{t-1} = \Delta_t^{V,B}/\Delta_t^{Y,B}$. Imposing the constant per-period inflation yields the master relation

$$\Delta_t^{V,j} = \beta \Delta_t^{Y,j}, \qquad j \in \{D, B\}, \tag{76}$$

together with the feasibility condition $Y_t^D + Y_t^B = Y$, we can connect the change in the Dollar trade share to the change in the Peso trade share,

$$\Delta_t^{Y,B} = \frac{Y - \Delta_t^{Y,D} Y_{t-1}^D}{Y_{t-1}^B}. \tag{77}$$

Equation (76) is the key identity of the construction: per period, velocity and trade share of each currency move in the same direction, velocity growth being the fraction β of trade-share growth. Feasibility requires $V_t^j \in (0, 1]$ and $Y_t^j \in (0, Y)$ at every t. This establishes Proposition 3.5(ii). The parts (iii)-(v) differ only in which growth factor is taken as free; we postpone part (v) as a special case.

Proposition 3.5 (iii), directed shifts in trade share Fix any target $\Delta_t^{Y,D} > 0$. To implement it in a symmetric perfect foresight equilibrium, $\Delta_t^{Y,B}$ is given by (77) and the velocities follow from (76), $V_t^D = \beta \Delta_t^{Y,D} V_{t-1}^D$, $\quad V_t^B = \beta V_{t-1}^B \frac{Y - \Delta_t^{Y,D} Y_{t-1}^D}{Y_{t-1}^B}$. Three feasibility constraints bound the target. The constraint $Y_t^B > 0$, so that Peso keeps value, gives $\Delta_t^{Y,D} < Y/Y_{t-1}^D$; the constraint $V_t^D \le 1$ gives $\Delta_t^{Y,D} \le 1/(\beta V_{t-1}^D)$; and the constraint $V_t^B \le 1$ gives the lower bound $\Delta_t^{Y,D} \ge \frac{Y}{Y_{t-1}^D} - \frac{Y_{t-1}^B}{\beta Y_{t-1}^D V_{t-1}^B}$. The Dollar trade-share growth is therefore feasible on

$$\max\left\{0, \frac{Y}{Y_{t-1}^D} - \frac{Y_{t-1}^B}{\beta Y_{t-1}^D V_{t-1}^B}\right\} \le \Delta_t^{Y,D} \le \min\left\{\frac{1}{\beta V_{t-1}^D}, \frac{Y}{Y_{t-1}^D}\right\}, \tag{78}$$

At a constant trade share $\Delta_t^{Y,D} = 1$ both bounds hold with slack, since the velocities then fall by the factor β, $V_t^j = \beta V_{t-1}^j \leq 1$, so a nonempty interval around 1 is sustainable, covering both rising ($\Delta_t^{Y,D} > 1$) and falling ($\Delta_t^{Y,D} \in (0,1)$) Dollar trade shares. Sellers are indifferent since $P_t/P_{t-1} = \beta m = m\hat{P}_t/\hat{P}_{t-1}$.

Proposition 3.5 (iv), directed shifts in velocity Fix any target $\Delta_t^{V,D} > 0$. Inverting (76) gives $\Delta_t^{Y,D} = \Delta_t^{V,D}/\beta$, hence $Y_t^D = \frac{1}{\beta}\Delta_t^{V,D} Y_{t-1}^D$, after which (77) fixes $\Delta_t^{Y,B}$ and thus $\Delta_t^{V,B} = \beta\Delta_t^{Y,B}$. Three feasibility constraints bound the target. The constraint $V_t^D \leq 1$ gives $\Delta_t^{V,D} \leq 1/V_{t-1}^D$; the constraint $Y_t^D < Y$, so that Peso keeps value ($Y_t^B > 0$), gives $\Delta_t^{V,D} < \beta Y/Y_{t-1}^D$; and the constraint $V_t^B \leq 1$ gives, by multiplying the trade-share bound by β,

$$\Delta_t^{V,D} \geq \frac{\beta Y - Y_{t-1}^B/V_{t-1}^B}{Y_{t-1}^D}.$$

The Dollar velocity growth is therefore feasible on

$$\max\left\{0,\ \frac{\beta Y - Y_{t-1}^B/V_{t-1}^B}{Y_{t-1}^D}\right\} < \Delta_t^{V,D} \leq \min\left\{\frac{1}{V_{t-1}^D},\ \beta\frac{Y}{Y_{t-1}^D}\right\}. \tag{79}$$

This interval is nonempty: it always contains $\Delta_t^{V,D} = \beta$, i.e. the constant-trade-share equilibrium ($\Delta_t^{Y,D} = 1$), where $V_t^D = \beta V_{t-1}^D \leq 1$ and $V_t^B = \beta V_{t-1}^B \leq 1$ hold with slack. Choosing $\Delta_t^{V,D} < 1$ yields a falling Dollar velocity, and choosing $\Delta_t^{V,D} > 1$, which is feasible whenever $\min\{1/V_{t-1}^D,\ \beta Y/Y_{t-1}^D\} > 1$, yields a rising one, so both occur in equilibrium. The Peso side is determined analogously.

Proposition 3.5 (v), constant trade share or velocity *Constant trade share.* Setting $\Delta_t^{Y,D} = 1$ in Proposition 3.5(iii) is feasible for every $\beta \in (0,1]$ and gives, by the master relation, declining velocities $V_t^j = \beta V_{t-1}^j$.

Constant velocity. Setting $\Delta_t^{V,D} = \Delta_t^{V,B} = 1$ forces, by the master relation, $\Delta_t^{Y,D} = \Delta_t^{Y,B} = 1/\beta$, so $Y_t^D + Y_t^B = \frac{1}{\beta}(Y_{t-1}^D + Y_{t-1}^B) = Y/\beta$, which equals Y if and only if $\beta = 1$. Hence a constant-velocity equilibrium exists if and only if $\beta = 1$, in which case trade shares are constant as well; for $\beta < 1$ none exists. □

A.4 Peso as a foreign CBDC with active monetary policy

So far "Peso" has stood for a foreign currency with passive monetary policy, for instance a foreign central bank digital currency (CBDC) competing with the U.S. Dollar. Suppose instead it pays gross interest n on Peso balances, so that supply grows as $B_t = n^{t-1}B_1$. The seller's indifference condition then becomes

$$\mathbb{E}_{t^-}\left[\frac{\hat{P}_t}{\hat{P}_{t+1}}\,u'(c_{t+1})\right] = \frac{m}{n}\,\mathbb{E}_{t^-}\left[\frac{P_t}{P_{t+1}}\,u'(c_{t+1})\right]. \tag{80}$$

The factor m/n shows that n partially offsets, or overcompensates, the Dollar rate m, and a rise in n leads agents to revise Peso inflation expectations, rapidly or gradually as in the Dollar case. For $n > m$, depending on the speed of updating and prevailing Dollar beliefs, trade may shift toward Peso or pass-through into Peso prices may strengthen; for $n < m$, the Peso rate dampens the Dollar bank's influence, limiting the shift to Dollar trade or weakening Dollar inflation pass-through. The buyers' consumption-savings conditions become

$$\frac{u'(c_t)}{P_t} = \beta^2 m^2\,\mathbb{E}_t\left[\frac{u'(c_{t+2})}{P_{t+2}}\right], \qquad \text{(Dollar goods market)}, \tag{81}$$

$$\frac{u'(c_t)}{\hat{P}_t} = \beta^2 n^2\,\mathbb{E}_t\left[\frac{u'(c_{t+2})}{\hat{P}_{t+2}}\right], \qquad \text{(Peso goods market)}. \tag{82}$$

If Peso's active policy used symmetric lump-sum transfers instead of interest, all indifference conditions would be unchanged from our Propositions.

This logic in our paper has direct implications for other competing currencies. In our model Peso is a rigid-supply currency with no active monetary authority. Stablecoins such as Tether or USDC attempt to hold a fixed peg through mint-and-burn arbitrage but typically do not remunerate balances, and much of the stablecoin literature adopts partial-equilibrium models that treat arbitrage as the sole peg-stabilizing mechanism. Our general-equilibrium perspective shows instead that persistent trade reallocations create strong pressure on the peg: without effective supply adjustments, Dollar monetary expansion would lead the stablecoin to appreciate at rate $Q_{t+1} = mQ_t$. Equivalently, the interest-paying Dollar can be interpreted as staked Ether; our results show that the staking yield alone does not determine ETH prices, since investors' inflation expectations and the interest paid on a competing currency play an equal role. Proportional Dollar monetary policy thus places strong pressure on any peg mechanism that seeks to hold $Q_{t+1} = Q_t$ through supply adjustments alone. Even with effective arbitrage, persistent trade imbalances, much like those that forced the Swiss National Bank to abandon its Euro peg, can ultimately overwhelm stabilization efforts and trigger depegging.

B Single economies

The following plots display the behavior of individual economies across treatments. It is important to emphasize that, although we label economies as "1–6" across treatments, the participants differ across these economies and treatments. Specifically, the subjects in economies 1–3 of the benchmark treatment are not the same individuals as those in economies 1–3 of the MP treatment. Similarly, economies 4–6 in the benchmark treatment consist of different participants from economies 4–6 in the MP treatment.

B.1 Dollar sales share

Muted Monetary Policy Treatment

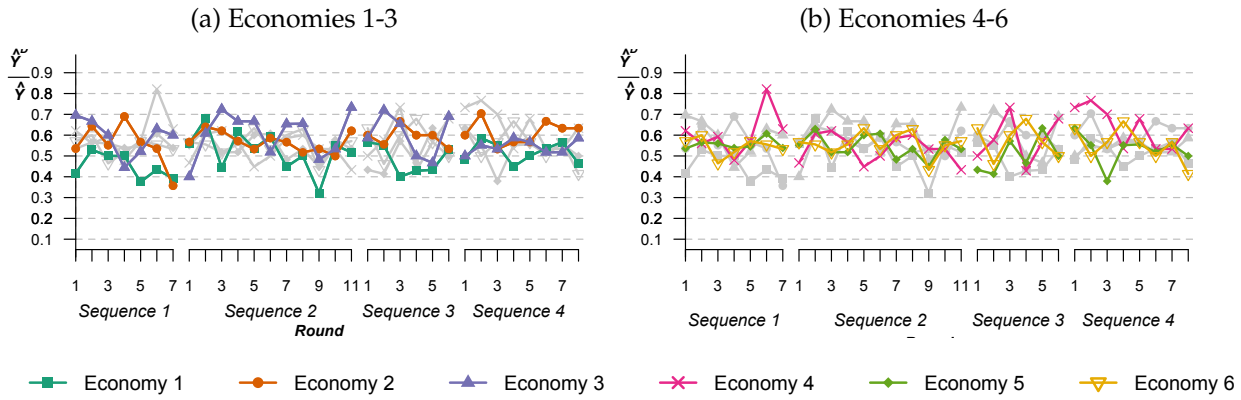


Figure B.1: Dollar sales share across single experimental economies in the benchmark setting

Active Monetary Policy Treatment

The Dollar trade share for the individual economies in the MP treatment is reported in Figure 11 in Section 5.4.

B.2 Dollar goods price ("currency A")

Muted Monetary Policy Treatment

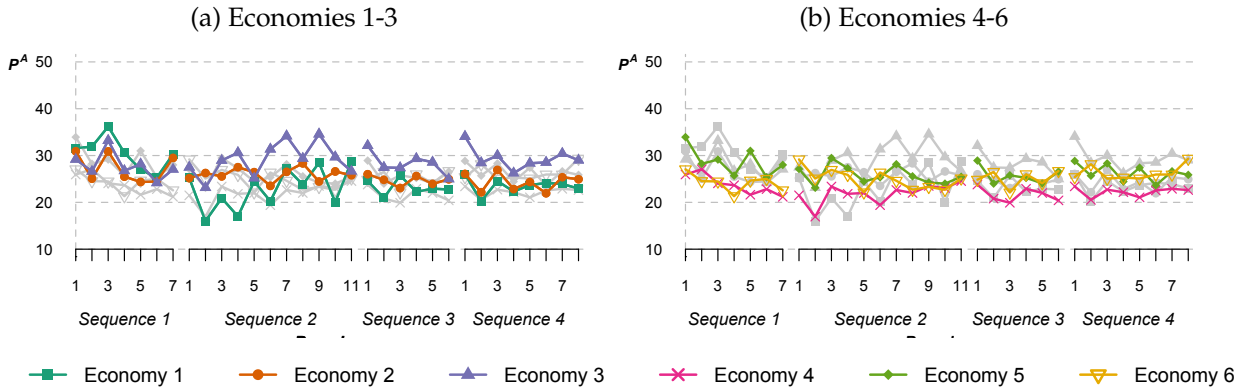


Figure B.2: Realized Dollar goods prices across single experimental economies in the benchmark setting

Active Monetary Policy Treatment

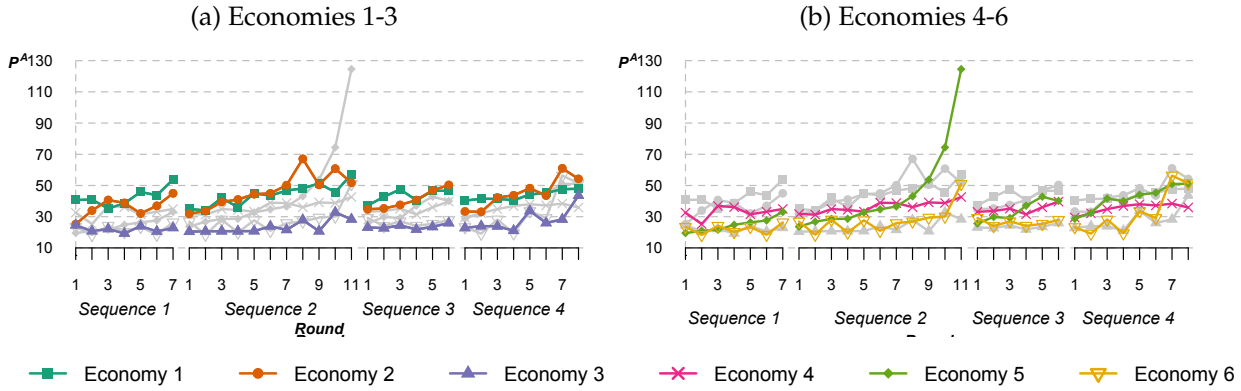


Figure B.3: Realized Dollar goods prices across single experimental economies in the monetary policy setting

B.3 Peso goods price ("currency B")

Muted Monetary Policy Treatment

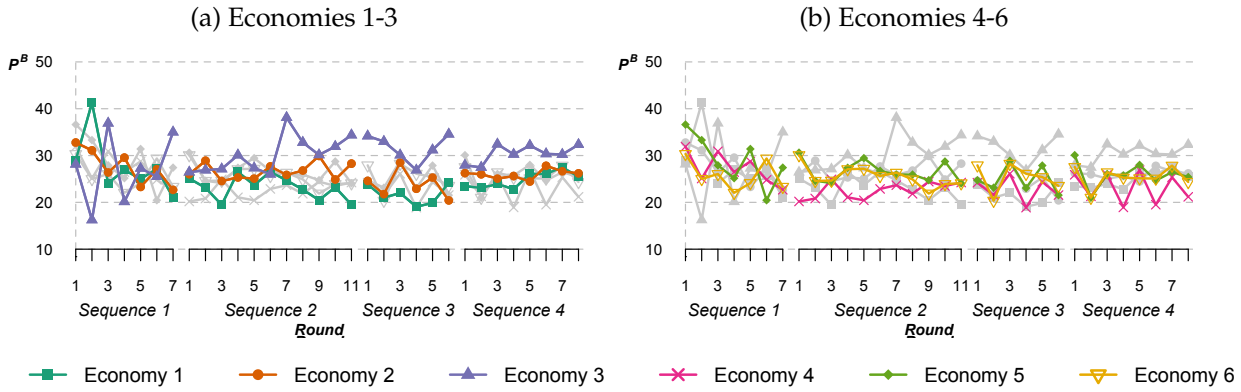


Figure B.4: Realized Peso goods prices across single experimental economies in the benchmark setting

Active Monetary Policy Treatment

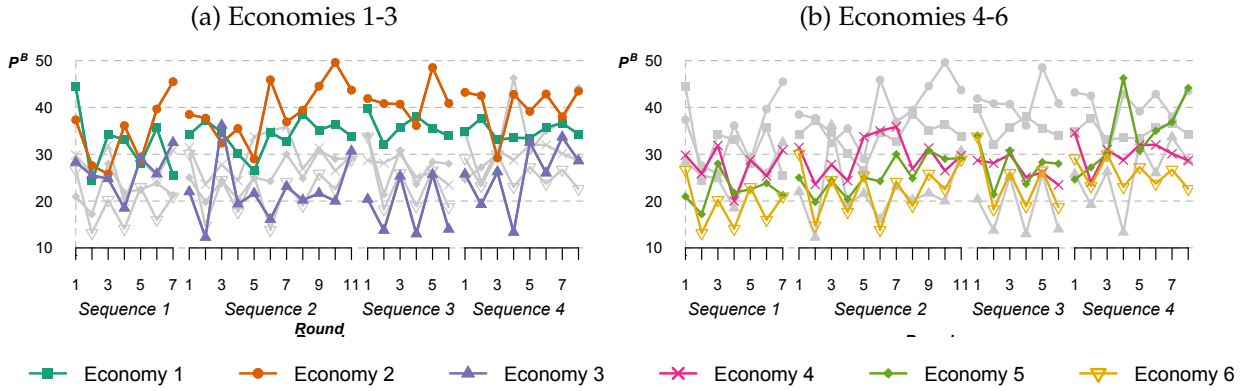


Figure B.5: Realized Peso goods prices across single experimental economies in the active monetary policy setting

B.4 Dollar savings rate

Muted Monetary Policy Treatment

(a) Economies 1-3 (b) Economies 4-6

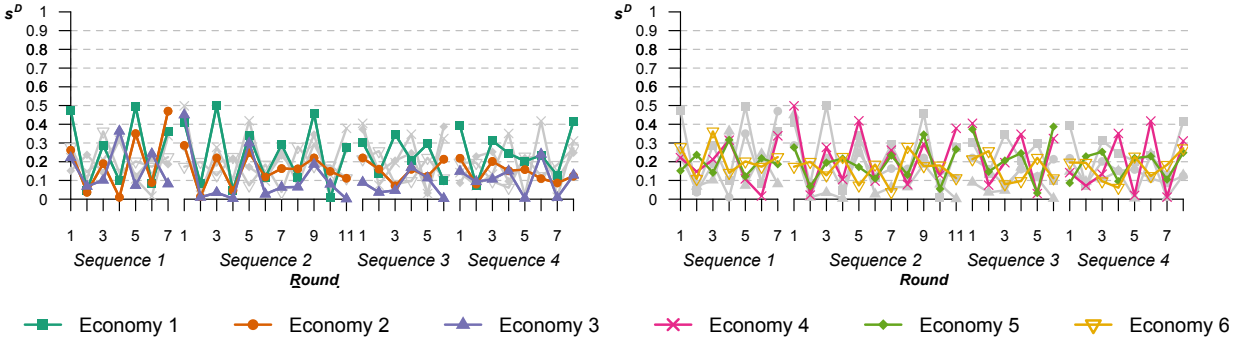


Figure B.6: Realized Dollar savings rate ($s^D = 1 - V^D$) across single experimental economies in the benchmark setting

Active Monetary Policy Treatment

(a) Economies 1-3 (b) Economies 4-6

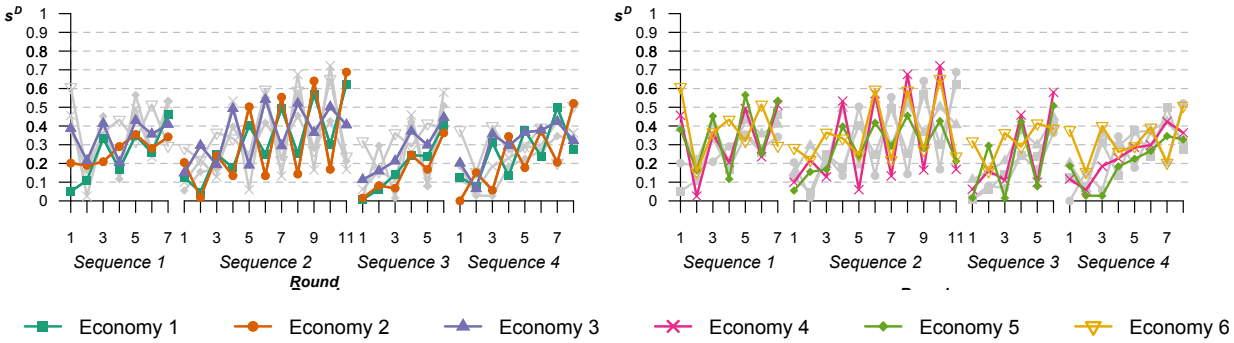
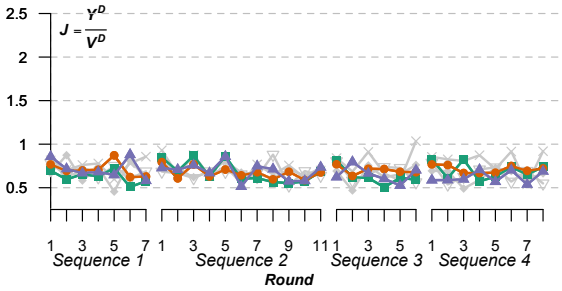


Figure B.7: Realized Dollar savings rate (the share of unspent currency) ($s^D = 1 - V^D$) across single experimental economies in the monetary policy setting

(a) Benchmark economies (1 to 3)



(b) Benchmark economies (4 to 6)

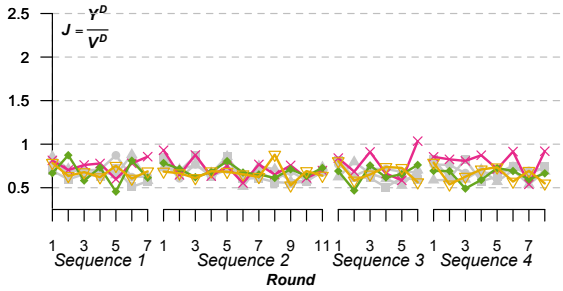


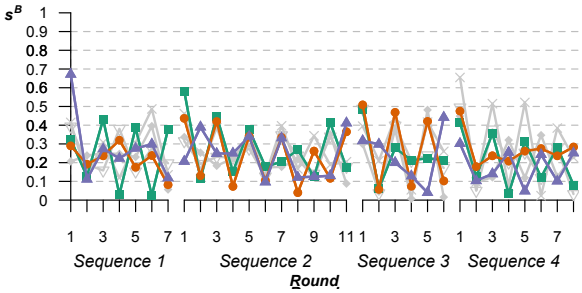


Figure B.10: Dollar price absorption J_t in the benchmark treatment ($m = 1$)

B.5 Peso savings rate

Muted Monetary Policy Treatment

(a) Economies 1-3



(b) Economies 4-6

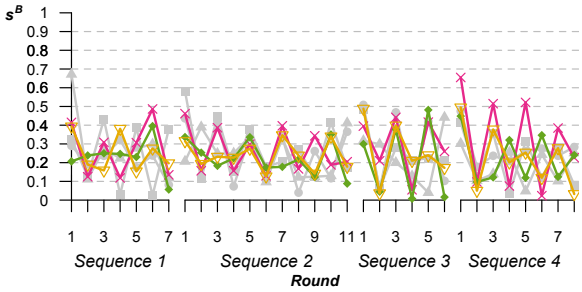
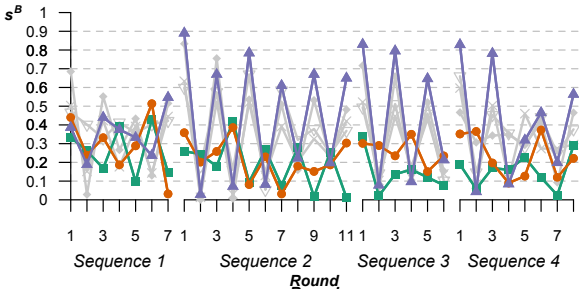




Figure B.8: Realized Peso savings rate ($s^B = 1 - V^B$) across single experimental economies in the benchmark setting

Active Monetary Policy Treatment

(a) Economies 1-3



(b) Economies 4-6

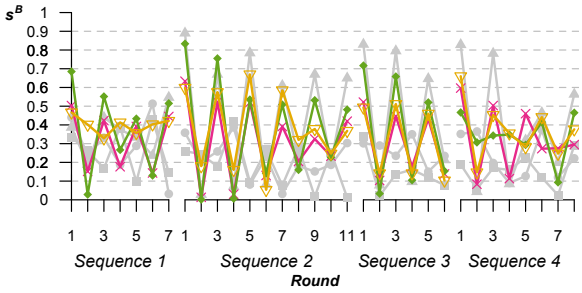




Figure B.9: Realized Peso savings rate (the share of unspent currency) ($s^D = 1 - V^D$) across single experimental economies in the monetary policy setting

C Additional experimental results

	Panel A. Price paid in Dollar	*Panel B. Price paid in Peso*
(I) $m = 1$	0.829 (0.162)	1.040 (0.377)
(II) $m = 1.3$	2.153 (2.133)	1.951 (0.453)
p-value (I) vs. (II)	0.009***	0.026**
	Panel C. Packages bought in Dollar	*Panel D. Packages bought in Peso*
(I) $m = 1$	0.640 (0.030)	0.984 (0.046)
(II) $m = 1.3$	1.395 (0.373)	1.056 (0.049)
p-value (I) vs. (II)	0.004***	0.590
	Panel E. Total consumption	
(I) $m = 1$	56.040 (135.881)	
(II) $m = 1.3$	113.288 (4667.42)	
p-value (I) vs. (II)	0.041**	

Notes: Each cell reports the relative dispersion of a given variable across the six buyers for each treatment, averaged over the 32 paid periods and the six independent sessions; standard deviations are reported next in parentheses. Each p-value corresponds to the exact two-sided Mann-Whitney rank-sum test of cross-treatment comparison, performed on matching-group level with six observations per treatment variable.

Table C.1: Relative dispersion of buyers' prices and consumption levels

Keyword	No MP	MP	p (MP vs. no MP)
Saving / return (i)	2 (3%)	10 (15%)	0.013
Inflation (π)	1 (1%)	2 (3%)	0.727
N answered	67	68	

Table C.2: LLM-assisted classification of open-ended strategy responses. Entries report the number (share) of subjects whose responses mention saving or returns, or inflation, by treatment. The final column reports exact Wilcoxon rank-sum tests based on session-level shares, comparing the MP treatment ($m = 1.3$) with the no-MP treatment ($m = 1$), with six sessions per treatment.

> One of the clearest references to the inflationary implication of money growth comes from a subject who explains the decision to sell in the fixed-supply market by noting that "the money supply remains constant there."

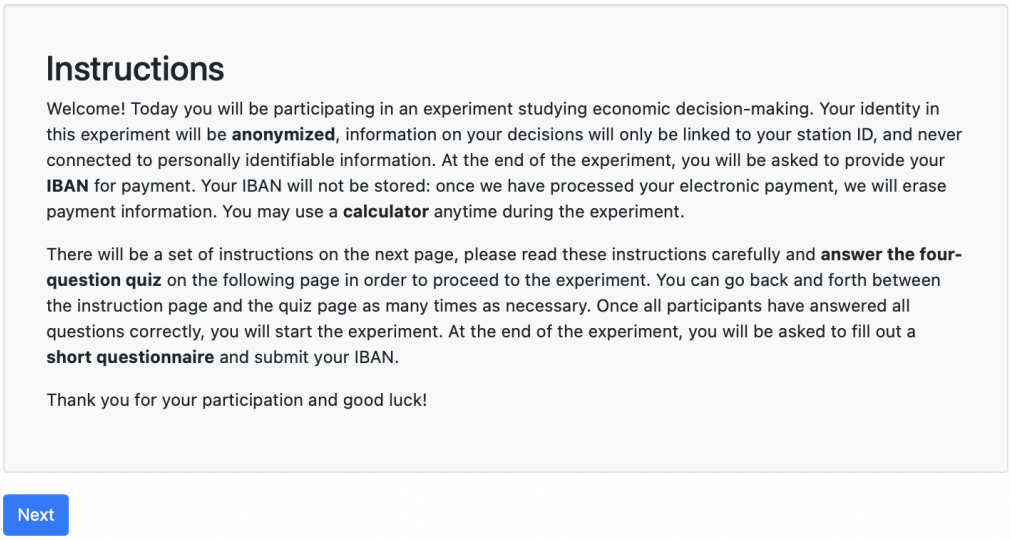
Dependent variables:	Dollar trade share	Dollar saving rate	Peso saving rate	Dollar market price	Peso market price	Exchange rate
	(I)	(II)	(III)	(IV)	(V)	(VI)
$\mathbf{1}_{\{m=1.3\}}$ (omitted: $m=1$)	0.102^{***}	-0.056	0.067^{**}	-1.373	1.254	-0.047
	(0.027)	(0.041)	(0.033)	(3.629)	(2.375)	(0.071)
round	-0.004^{***}	-0.002	-0.013^{***}	-0.053	-0.078	0.002
	(0.001)	(0.002)	(0.001)	(0.070)	(0.147)	(0.006)
$\mathbf{1}_{\{m=1.3\}}\times$ round	0.009^{***}	0.034^{***}	0.001	2.341^{***}	0.415^{*}	0.060^{***}
	(0.002)	(0.005)	(0.002)	(0.689)	(0.222)	(0.021)
Constant	0.577^{***}	0.230^{***}	0.310^{***}	24.805^{***}	26.024^{***}	0.975^{***}
	(0.015)	(0.017)	(0.012)	(1.287)	(0.942)	(0.041)
F-stat	4.57^{**}	79.7^{***}	7.00^{***}	103.7^{***}	3.78^{*}	72.3^{***}
R2	0.405	0.295	0.074	0.402	0.086	0.293
Obs.	384	384	384	384	384	384

Notes: OLS estimates with cluster-robust standard errors at the session level reported between brackets, sequence fixed effects included. The F-statistic and p-value correspond to the linear restriction test H_0: round$+\mathbf{1}_{\{m=1.3\}}\times$round$=0$, i.e. the variable is not time varying under $m=1.3$, computed using cluster-robust standard errors. *p<0.1; **p<0.05; ***p<0.01.

Table C.3: Time variation in the main variables

D Experimental material

D.1 Experimental instructions



Instructions

Welcome! Today you will be participating in an experiment studying economic decision-making. Your identity in this experiment will be **anonymized**, information on your decisions will only be linked to your station ID, and never connected to personally identifiable information. At the end of the experiment, you will be asked to provide your **IBAN** for payment. Your IBAN will not be stored: once we have processed your electronic payment, we will erase payment information. You may use a **calculator** anytime during the experiment.

There will be a set of instructions on the next page, please read these instructions carefully and **answer the four-question quiz** on the following page in order to proceed to the experiment. You can go back and forth between the instruction page and the quiz page as many times as necessary. Once all participants have answered all questions correctly, you will start the experiment. At the end of the experiment, you will be asked to fill out a **short questionnaire** and submit your IBAN.

Thank you for your participation and good luck!

Next

Instructions

Overview of the experiment

In this experiment, you will be trading with other players to get as many chips as you can by playing a real-time **market game** that uses a double auction mechanism (both buyers and sellers will be making offers). These chips determine your payout at the end of the experiment. To get chips, you will need to buy and sell packages of chips with other participants in the experiment. You will have to buy different types of packages of chips with different currencies. These market games are made up of a sequence of periods. Each sequence consists of **an indefinite number of periods**. Basically, how long it lasts is randomized. The number of periods played in each sequence is randomly determined in the following manner: At the end of each period, the computer will roll a virtual 6-sided dice. If the numbers 2, 3, 4, 5 or 6 are rolled from the dice, the sequence continues and you will continue playing the game, but if a '1' is rolled, the sequence ends and the game will restart. This means that all currency holdings and chips you currently have will disappear, and a new sequence will start. In other words, **after each period, the market game continues with a probability** 5/6 ≃ 83% **or ends with a probability** 1/6 ≃ 17%.

We will run **as many sequences as possible during the time for which you have been recruited**. After a sequence has ended, if there is not enough time remaining to start a new sequence, the experiment ends.

Before beginning the first sequence, you will have **2 practice periods** to familiarize yourself with the interface. Your decisions during those 2 practice periods will have no consequences on the actual experiment and will not count towards your earnings. It will be made clear to you and all other players when the actual experiment will start.

The market game

You are part of a group of **players** participating in markets where you buy and sell packages of chips. The rules are the same for all players. You will be split into equally sized groups of **buyers** and **sellers**. Each player will alternate between the roles of seller and buyer from period to period. Whether you start your first period as a seller or a buyer is randomly determined by the computer. For instance, if you are assigned the role of a buyer in period 1, you will then be a seller in period 2, then a buyer again in period 3, and so on until the sequence ends.

You will buy and sell chips in **two markets that each use different currencies**: Market A using Currency A and Market B using Currency B. Since you will alternate roles, buying chips is possible only every second period (when you are a buyer). Whenever you begin a period as a seller, you will be given 5 packages which you cannot use or keep for yourself, your goal as a seller is to sell these packages in order to increase your currency holdings to buy more packages when you are once again a buyer.

Number of chips per package

Chips are **bought and sold in packages**. Packages come in two types depending on which market you buy them from, the first type of package can only be bought from Market A which uses Currency A, the second type can only be bought from Market B which uses Currency B. Packages in **both** auction houses are always worth **100** chips per package.

Package	1st	2nd	3rd	4th	5th	...
Chip content (in A)	100	100	100	100	100	...
Chip content (in B)	100	100	100	100	100	...

Example: You buy 1 package using Currency A and 2 packages using Currency B in the first period of trading. Then you alternate roles to seller in the second period, then in the third period of trading, are a buyer again and you buy 2 more packages in Currency A and 1 more in package in Currency B. In total, you have purchased 3 x 100 = 300 chips using Currency A and 3 x 100 = 300 chips using Currency B, for a total of 300 + 300 = 600 chips so far.

What happens during a period of the market game

- When you **first start a sequence** as a buyer (which will last a random number of periods), you will receive 100 units of Currency A and 100 units of Currency B.
- At the beginning of **every period** when you are a seller, you will receive 5 packages, you must choose to allocate them to be sold on Market A or Market B.
- At the end of **every period** when you are a seller, you lose all packages you failed to sell.
- In each period, you will participate (either as a seller or as a buyer) in two different markets, one using Currency A (red boxes) and one using Currency B (blue boxes).
- Packages that yield diminishing numbers of chips are sold in Market A for Currency A, Packages that yield constant numbers of chips are sold in Market B for Currency B.
- You will carry over currency changes from period to period according to your actions as a buyer or seller.

Growth of the amount of Currency A

In each sequence, at the end of **each period**, the holdings in **currency A of all players**, whether sellers or buyers, are **increased by 30%** . For instance, suppose you were a seller in period 1 and sold packages against 80 units in currency A. You then start period 2 as a buyer with 80 + 80 * 30% = 104 of currency A.

This way, the total quantity of currency A in the experimental market increases by 30% per period. This increase only concerns currency A, the total amount of currency B remains unchanged throughout any sequence.

Phases within a Period and their time limits

1. Allocating Packages Phase: Sellers' choose what allocation of their 5 packages goes to Market A or Market B (**30 seconds**).
2. Trading Phase: Buyers and Sellers make real-time offers for packages (**90 seconds**).

   Note: A trading phase will close before time runs out whenever there are no more packages available for sale or there is no currency left to be spent. Hence, inncludig some waiting time, a period of the market game lasts a maximum of 3 minutes.

Allocating Packages Phase:

- This happens before the trading phase of a period
- Sellers are given 5 packages. They must choose how many of them they want to be offered on Market A and how many can be offered on Market B.

Trading Phase

- Sellers post **asks**, offers to sell a certain number of packages and the price they want to sell.
- Buyers post **bids**; offers to buy a number of packages and the highest price they will pay up to.
- Bids/asks can be changed at any time if they have not yet gone through.
- You can cancel your last bid/ask by clicking the "***Retract last bid***" button and "***Retract last ask***" button

  Note: You can shortcut your participation in the market by clicking on the "***Accept best bid***" button as a seller (this means that you accept to sell at the highest price among all prices offered by the buyers), and by clicking the "***Accept best ask***" as a buyer (this means that you accept to buy at the lowest price among all prices offered by the sellers).

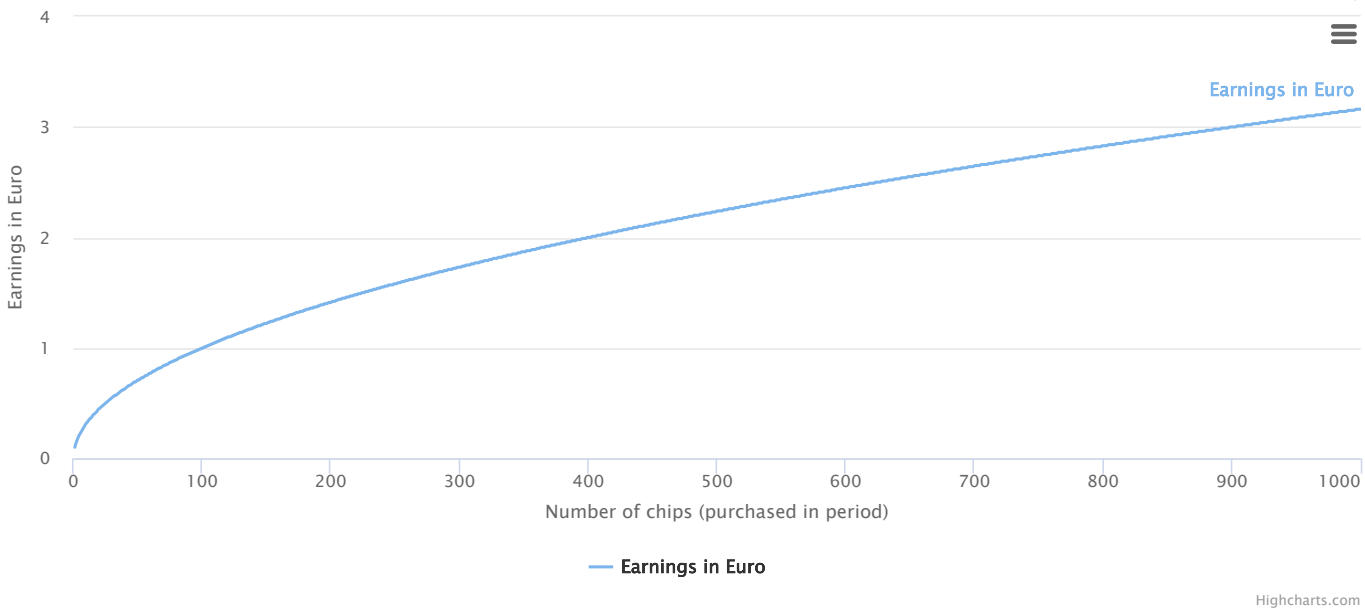
- A trade is finalized every time there is an ask price equal or less than a bid price.
  - The transaction price is the seller's asking price and the number of packages exchanged is the minimum between the number of packages offered by the seller, and desired by the buyer.
  - As a buyer, if no packages are for sale at or below your bid price, you will not buy any packages.
  - As a seller, if no packages are demanded for a price equal or higher to your ask price, you will not sell any packages.

Example:You are a buyer and your bid is 2 packages at 100 units of Currency A. There is a seller whose ask is 3 packages at 80 units of Currency A. The transaction takes place and you buy from that seller 2 packages at 80 units of Currency A.

- You can submit as many asks and bids as you want as long as the markets are open.
  - As a buyer, you can never spend more currency than what you have allocated to each market. It can happen that you are not able to spend all your currency holdings. In this case, the remaining balance is then transferred to the next period.
  - As a seller, you can never sell more packages than you have allocated to a Market. It can happen that you are not able to sell all your packages. In this case, the unsold packages disappear at the end of that trading phase, which means that they may not be consumed or sold later.

Your earnings at the end of the experiment

The chips you purchase convert into money, hence you can accrue earnings only when you are a buyer. The payoff graphic below indicates how many Euros you can earn (y-axis) as a function of the number of chips you purchase in a given period (x-axis). There is also a slider that you can use to see the exact amount of Euros for each exact number of chips.



The more chips you buy, the more real-life money you can earn at the end of the experiment. However the chips you buy will convert into progressively less currency the more chips you accumulate within a single period.

Your total earnings in each sequence are the sum of your earnings in every period when you are a buyer during that sequence. At the end of the experiment, two sequences will be randomly selected (with equal probability out of all sequences played) and we will pay you **the money you earn during those two sequences, on top of a 9 euro participation compensation for your time and efforts**.

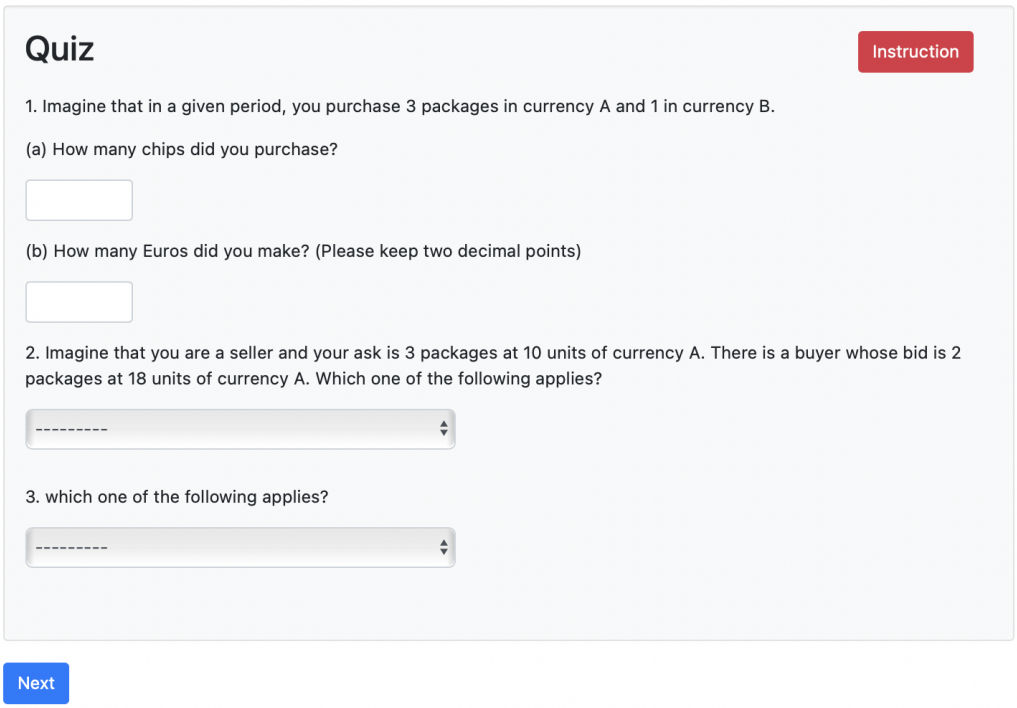
Summary

- You will play several sequences of a market game which lasts a randomized number of periods. After each period, the game ends with a 1/6 ≃ 17% probability. When this happens, all currency holdings are reset to zero and a new sequence starts.
- You will play as many sequences as time allows us. During each sequence, you will play alternating between being buyers and sellers of packages of chips. You can buy and sell chips in Market A using Currency A, or Market B using Currency B.
- At the beginning of each period, when you are a seller, you receive 5 packages of chips to sell to choose to allocate to Market A or Market B, these packages cannot be used or kept by you after each period.
- You start every sequence with 100 units of Currency A and 100 of Currency B. Your changes will carry over from period to period.
- The more chips you purchase in each period, the higher your payoff but earnings do not increase as fast as the number of chips purchased. Only the total earnings from two sequences plus 9 euros will be paid out to you at the end of the

experiment.

Next

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D.2 Graphical user interfaces (GUI)

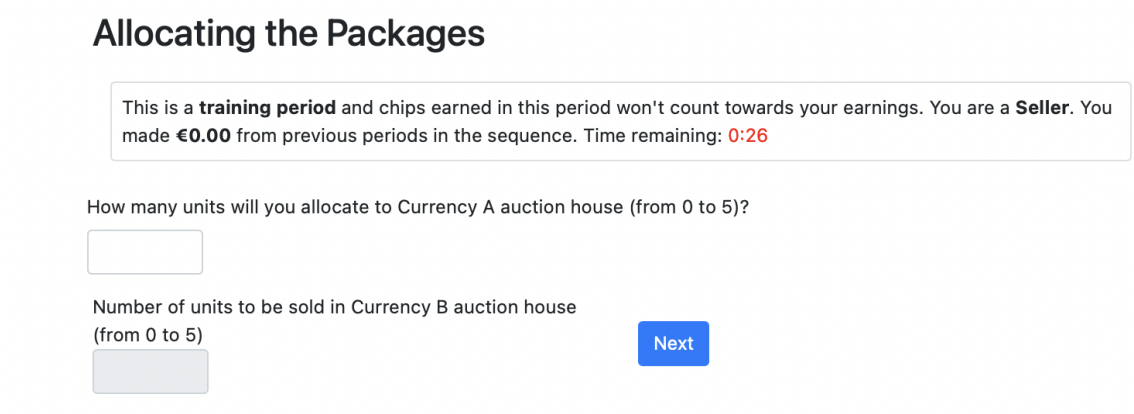


Figure D.1: Screen of the sellers' allocation page of packages between the two markets

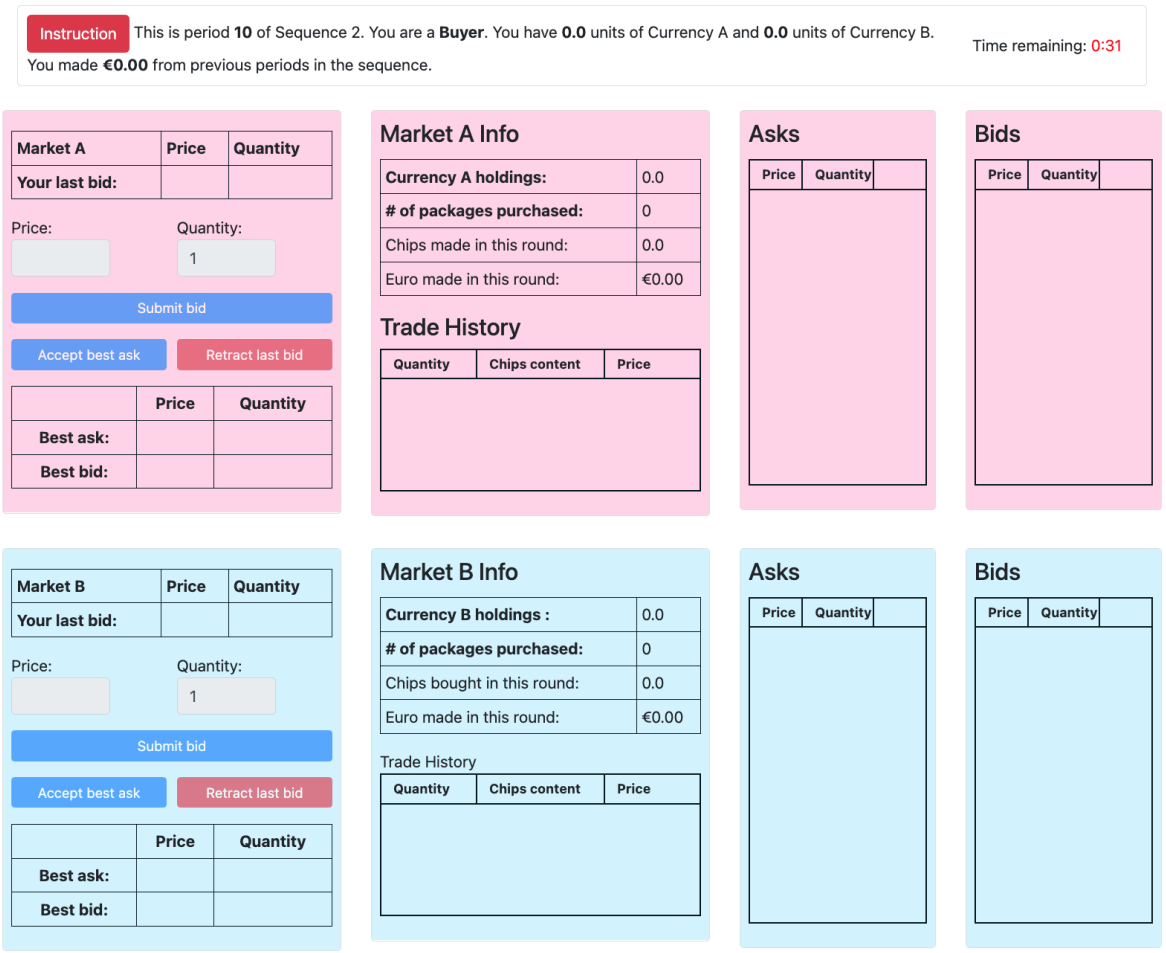


Figure D.2: Graphical user interface of the buyer

Notes: The Dollar-denominated and Peso-denominated trading interfaces are, respectively, on the top (market A in red) and on the bottom (market B in blue). Subjects submit their bids (and symmetrically their asks for the sellers) on the left-hand side column. The middle panel provides the details of the subject's trading history on each market so far in the given round, and the right-hand side panel displays all the asks and bids in each market as soon as they posted. The instructions and the guidance of the experimenter during the training periods make this interface clear to the subjects.

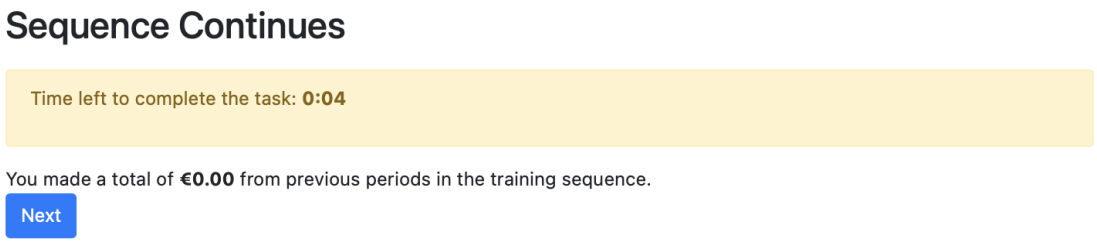


Figure D.3: Screen indicating a continuing sequence



Figure D.4: Word cloud of participants' self-reported strategies (post-experiment questionnaire)

Figure D.4 presents a word cloud illustrating subjects' self-reported behaviors and strategies from the open-ended post-experiment questionnaire. The word cloud aggregates responses across all sessions, with larger words indicating more frequent mentions.

Subjects demonstrated a clear awareness of the monetary incentives embedded in the experimental tasks, as evidenced by recurrent terms such as "currency" and "money." The responses also highlight market equilibration processes driven by prices rather than quantities, reflected in words like "prices," "highest," and "lowest."

Additionally, trading strategies emphasized the importance of timing under competitive pressure and strategic uncertainty. Participants mentioned both quick decision-making (e.g., "first," "fast," "quick," "started") and deliberate delays (e.g., "wait," "later," "last," "keep," "end").