Monetary and Fiscal Policy in a Monetary Union: The Role of Financial Integration*

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Abstract

We study how financial integration affects optimal monetary and fiscal policy in a two-country New Keynesian model of a monetary union with financially constrained intermediaries. National fiscal authorities choose transfers, while a common central bank sets the interest rate and conducts quantitative easing (QE). Analytically, national governments redistribute more from savers to borrowers than a coordinated planner because they do not internalize the full cross-border costs of their transfers. Financial integration strengthens the domestic welfare gains from national fiscal transfers while weakening the common central bank's ability to direct the benefits of QE toward individual countries. Quantitatively, these effects persist in a model that relaxes some simplifying assumptions of the analytical analysis.

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1 Introduction

The European Commission's Savings and Investments Union, launched in March 2025, is a major initiative to promote financial integration across the EU.[1] In the euro area, financial integration takes place under a common monetary policy, while fiscal decisions remain national. How does deeper financial integration affect welfare and optimal monetary policy, including quantitative easing (QE), and fiscal policy? Specifically, we focus on how financial integration affects national governments' incentives to redistribute and the common central bank's ability to address country-specific conditions through QE.

We build a two-country New Keynesian model in a monetary union with savers, borrowers, and financial intermediaries. Asset markets are segmented: savers hold one-period deposits, while borrowers issue long-term bonds to finance consumption. Financial intermediaries perform maturity transformation by taking in deposits and holding long-term bonds. They face a leverage constraint that limits the supply of credit. Financial integration allows intermediaries to hold government bonds issued in both countries. National fiscal authorities redistribute resources from savers to borrowers through transfers financed by taxes and government debt. A common central bank sets the policy rate and conducts QE by purchasing government bonds.

Under simplifying assumptions, our model reduces to six equations: an IS equation, two country-specific Phillips curves, a relative-output condition, a terms-of-trade equation, and a monetary policy rule. National fiscal authorities maximize domestic welfare, while the common central bank maximizes union welfare. We derive a micro-founded quadratic union welfare loss function that captures inflation, output gaps, and deviations from efficient consumption allocation within and across countries. This tractable framework

[1] See the European Commission's Savings and Investments Union strategy (https://finance.ec.europa.eu/publications/savings-and-investments-union-strategy-enhance-financial-opportunities-eu-citizens-and-businesses_en), March 19, 2025; and the European Council conclusions (https://www.consilium.europa.eu/en/press/press-releases/2025/03/20/european-council-conclusions-on-competitiveness-european-defence-and-security-and-migration/), March 20, 2025, paragraphs 15–16.

allows us to characterize optimal national fiscal and union monetary policy analytically.

We first show that the fiscal Nash equilibrium features excess redistribution from savers to borrowers. Specifically, starting from the coordinated allocation, each national government has an incentive to unilaterally increase transfers to domestic borrowers to raise domestic welfare. This incentive arises because national governments value the domestic benefits of redistribution but do not internalize its full welfare costs abroad. We characterize the resulting equilibrium and show that borrower consumption exceeds the coordinated level in both countries. For common productivity shocks without cost-push shocks, this equilibrium yields closed-form consumption ratios that characterize the extent of excess redistribution, which increases with trade openness and vanishes in trade autarky.

We further examine whether financial integration strengthens or weakens national governments' incentives to redistribute. We find that greater integration raises the domestic welfare gain from a fiscal transfer, strengthening the incentive to redistribute. When foreign intermediaries absorb part of the debt financing a transfer, less domestic private lending is crowded out, strengthening the benefit to domestic borrowers.

In contrast to fiscal transfers, greater financial integration reduces the domestic welfare gain from central-bank purchases of domestic bonds. Purchases of domestic bonds held by foreign intermediaries relax their leverage constraint, allowing them to lend more to foreign borrowers. As financial integration increases, more of the benefits of QE accrue to foreign borrowers, reducing the domestic welfare gain.

We then characterize how financial integration affects the optimal allocation of QE across countries. The central bank purchases more bonds per resident from the country where lending falls further below its efficient level. Financial integration reduces the welfare gain from this allocation relative to purchases proportional to population, because cross-border bond holdings make QE less effective at expanding lending in the targeted country.

Fiscal redistribution generates a larger cross-country gap in borrower consumption under greater financial integration, even after the central bank optimally reallocates QE. Thus, integration amplifies the relative consumption effect of fiscal redistribution both with and without an optimal QE response.

Together, these results show that financial integration strengthens the domestic benefits of national fiscal redistribution while weakening the common central bank's ability to direct the benefits of QE toward individual countries.

We then quantify the transmission of fiscal transfers and QE and their implications for welfare in a calibrated model, relaxing the simplifying assumptions imposed for analytical tractability. The opposite effects of financial integration on the welfare gains from fiscal transfers and QE carry over to the quantitative exercise. Greater financial integration increases the domestic welfare gain from a fiscal transfer but reduces the gain from QE purchases of domestic government bonds.

Following an adverse credit shock in one country, greater financial integration reduces the welfare advantage of directing QE toward that country rather than allocating purchases in proportion to population.

Finally, a domestic fiscal transfer raises domestic borrower consumption relative to foreign borrower consumption. In response, the central bank purchases foreign government bonds to narrow this gap, but greater financial integration weakens the offset.

The remainder of the paper is organized as follows. Section 2 presents a tractable model and reduces it to a system of six equations. Section 3 derives the efficient allocation and welfare criterion. Sections 4 and 5 present the analytical results: Section 4 studies fiscal policy and financial integration, and Section 5 examines QE and its interaction with fiscal redistribution. Section 6 presents the quantitative analysis, and Section 7 concludes.

Literature. Our paper contributes to the literature on studying monetary and fiscal policy coordination in a currency union. Clarida, Galí, and Gertler (2002) characterize gains

from monetary cooperation in a two-country New Keynesian model. Chari and Kehoe (2007) analyze fiscal free riding when governments do not internalize the inflation costs of their debt. Galí and Monacelli (2008) characterize optimal common monetary policy and national government spending in a currency union. Kempf and Von Thadden (2013) examine when cooperation and commitment matter among private players, national fiscal authorities, and a common central bank.

A related literature studies cross-country insurance and transfers. Farhi and Werning (2017) characterize constrained-efficient risk sharing through a fiscal union, and Dmitriev and Hoddenbagh (2019) derive optimal transfers across countries in a monetary union. Berriel et al. (2024) study the choice between centralized and decentralized fiscal authority, highlighting the tradeoff between internalizing fiscal externalities and using local information. We study a different transfer margin together with the central bank's country-targeted QE policy: national governments redistribute among domestic residents via fiscal transfers, with the common central bank setting country-specific QE policies. This distinction lets us examine how cross-border bond holdings change the domestic benefits and cross-country costs of national redistribution and the central bank's country-targeted QE policies.

Our paper also contributes to the growing literature on financial frictions and fiscal policy in a monetary union. Bi and Traum (2023) study central-bank purchases of local-government debt and show how the policy can affect both local-government financing and private credit. Silva (2023) characterizes optimal fiscal policy in a currency union with sticky prices and financially constrained arbitrageurs, focusing on the response to capital-flow shocks and external debt. Bellifemine, Couturier, and Jamilov (2025) analyze heterogeneous fiscal space, and Maćkowiak and Schmidt (2025) study the fiscal backing needed for price stability in a monetary union.

Our study instead focuses on debt-financed fiscal transfers across domestic households and the way cross-border holdings of government bonds change their effects on private lending. The model builds on work that uses constrained intermediaries to study un-

conventional monetary policy and its interaction with fiscal policy, including Gertler and Karadi (2011), Sims and Wu (2021), Sims, Wu, and Zhang (2023) and Wu and Xie (2025). We extend this approach to a monetary union in which fiscal authorities maximize national welfare while the central bank maximizes union welfare.

Our paper also contributes to the literature on monetary transmission and QE across borders. Wu and Zhang (2019), Alpanda and Kabaca (2020), and Kolasa and Wesołowski (2020) study international spillovers from unconventional monetary policy and asset purchases. Gabaix and Maggiori (2015) link international capital flows to exchange rates through intermediaries' limited risk-bearing capacity, and Wu, Xie, and Zhang (2024) study how intermediaries' cross-border bond holdings affect conventional and unconventional policy transmission. Another thread of studies focuses on how the central bank should allocate purchases across countries (e.g., Bletzinger and von Thadden, 2021; Kabaca et al., 2023). We complement these works by tracing how cross-border holdings alter the country-targeted QE: purchases of one country's bonds may release lending capacity in another country. We also compare this channel with the effect of national fiscal transfers and show how greater integration changes the relative domestic welfare gains from the two policies.

Lastly, our analysis also relates to the welfare implications of international financial integration. Bai and Zhang (2012) show that incomplete contracts and limited enforcement can account for the limited improvement in international risk sharing following financial liberalization. Farhi and Werning (2012) characterize optimal capital controls in a small open economy with a fixed exchange rate. We study how financial integration through government-bond holdings changes the welfare effects of fiscal redistribution and QE within a monetary union, taking the degree of financial integration as given.

2 Model

We build a two-country New Keynesian model in a monetary union. Each country has its own fiscal authority, while a common central bank sets union-wide monetary policy. The home country has population $1-\gamma$, and the foreign country, denoted by an asterisk, has population γ. Each country has two groups of households: savers and borrowers, where borrowers are less patient than savers and face a borrowing constraint. We assume the population of savers and borrowers to be the same in each country.[2]

Within each country, the private sector follows the framework of Sims, Wu, and Zhang (2023) and Wu and Xie (2025). In the baseline case, households and financial intermediaries can only hold bonds issued in their own country, and this assumption is relaxed when we analyze the role of financial integration in Section 4.

2.1 Saver

A representative saver chooses consumption, labor, and deposits to maximize

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[\frac{(C^s_{t+j})^{1-\sigma} - 1}{1-\sigma} - \psi_L \frac{L^{1+\chi}_{t+j}}{1+\chi} \right], \tag{2.1}$$

where C^s_t is the saver's consumption composite and L_t is the labor supply. Parameter β is the subjective discount factor, σ is the inverse of the intertemporal elasticity of substitution, and χ is the inverse of the Frisch elasticity of labor supply. ψ_L is a constant scaling parameter.

The saver's budget constraint is

$$P_t C^s_t + D_t = W_t L_t + I_{t-1} D_{t-1} + P_t T^U_t + Z_t, \tag{2.2}$$

[2]The relative population shares of savers and borrowers are a normalization and do not affect the results.

where P_t is the price of the consumption composite and W_t is the nominal wage. The saver holds one-period riskless deposits, D_{t-1}, from period $t-1$ to t, which pay a nominal interest rate I_{t-1}. The term $T_t^U \equiv \Phi_t^F + \Phi_t^{FI} - T_t^G - X_t^b - X_t^{FI}$ is the net transfer the saver receives. Here, Φ_t^F and Φ_t^{FI} denote dividends from firms and financial intermediaries, respectively; T_t^G denotes lump-sum taxes paid to the local government; and X_t^b and X_t^{FI} denote transfers to borrowers and financial intermediaries, respectively.

We assume that savers participate in a union-wide risk-sharing fund. The fund implements a risk-sharing rule through state-contingent lump-sum payments to savers in each country, such that

$$\frac{(C_t^s)^{-\sigma}}{P_t} = \frac{(C_t^{s*})^{-\sigma}}{P_t^*}. \tag{2.3}$$

As a result, the fund makes lump-sum transfers to savers in the two countries. The last term Z_t in (2.2) denotes the nominal net payment to savers per capita in the home country, with a positive value representing a receipt. The fund does not accumulate assets or issue debt and incurs no resource costs. Therefore, payments balance within the union in each period, that is,

$$(1-\gamma)Z_t + \gamma Z_t^* = 0.$$

The sharing rule is committed to before monetary and fiscal policy choices and is maintained across shock realizations and policy regimes. Individual savers are atomistic and take fund payments as given when choosing consumption, labor, and deposits. Policymakers anticipate how aggregate fund payments adjust to implement the rule.

This arrangement is motivated by the literature on fiscal risk sharing in monetary unions, such as Farhi and Werning (2017) and Dmitriev and Hoddenbagh (2019), which studies state-contingent cross-country transfers as a means of providing insurance and macroeconomic stabilization. We adopt the risk-sharing rule (2.3) among savers as a maintained institutional assumption.

The saver's optimality problem yields

$$\psi_L L_t^{\chi} = (C_t^s)^{-\sigma} w_t, \tag{2.4}$$

$$(C_t^s)^{-\sigma} = \beta I_t \mathbb{E}_t \Big[\frac{(C_{t+1}^s)^{-\sigma}}{\Pi_{t+1}} \Big], \tag{2.5}$$

where $w_t \equiv W_t / P_t$ is the real wage in the home country.

The consumption bundle in the home country satisfies

$$C_t^s = [v^{\frac{1}{\vartheta}} (C_{H,t}^s)^{\frac{\vartheta-1}{\vartheta}} + (1-v)^{\frac{1}{\vartheta}} (C_{F,t}^s)^{\frac{\vartheta-1}{\vartheta}}]^{\frac{\vartheta}{\vartheta-1}}, \tag{2.6}$$

where $v = 1 - \gamma\nu$ is the home saver's expenditure weight on home-produced goods and ν measures trade openness. Similarly, the foreign saver's weight on foreign-produced goods is $v^* = 1 - (1-\gamma)\nu$. When $0 < \nu < 1$, we have $v > 1 - \gamma$ and $v^* > \gamma$, that is, savers in both countries exhibit home bias. When $\nu = 1$, there is no home bias in both countries. Parameter ϑ is the elasticity of substitution between country-specific goods.

The consumption price index (CPI) in the home country is

$$P_t = \Big[v P_{H,t}^{1-\vartheta} + (1-v) P_{F,t}^{1-\vartheta} \Big]^{\frac{1}{1-\vartheta}}.$$

The foreign consumption bundle and price index follow a similar form; see Appendix A.

The saver's cost-minimization problem yields the demands for home- and foreign-produced goods as

$$C_{H,t}^s = v (\frac{P_{H,t}}{P_t})^{-\vartheta} C_t^s, \tag{2.7}$$

$$C_{F,t}^s = (1-v) (\frac{P_{F,t}}{P_t})^{-\vartheta} C_t^s. \tag{2.8}$$

Appendix A reports the symmetric foreign demand equations.

We define the terms of trade as the price of foreign-produced goods relative to home-

produced goods:

$$\mathcal{E}_t = \frac{P_{F,t}}{P_{H,t}}. \tag{2.9}$$

Because both countries use the same currency in the monetary union, free trade within the union implies $P_{H,t} = P^*_{H,t}$ and $P_{F,t} = P^*_{F,t}$.

2.2 Borrower

A representative borrower chooses consumption and long-term bond issuance to maximize

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta_b^j \left[\frac{(C^b_{t+j})^{1-\sigma} - 1}{1-\sigma} \right],$$

where the discount factor $\beta_b < \beta$ implies that the borrower is less patient than the saver and therefore borrows in equilibrium. Here, C^b_t denotes the borrower's consumption. For simplicity, we assume that the borrower cannot supply labor and therefore has no labor income.

Its budget constraint is

$$P_t C^b_t + B^P_{t-1} = Q_t(B^P_t - \kappa B^P_{t-1}) + P_t X^b_t + P_t T^C_t, \tag{2.10}$$

where B^P_t denotes long-term private bonds issued by the home borrower, $Q_t(B^P_t - \kappa B^P_{t-1})$ is the market value of newly issued long-term bonds, X^b_t is a transfer from the domestic saver, and T^C_t is a lump-sum transfer from the country-specific government.

We model long-term bonds as perpetuities with decaying coupon payments, following Woodford (2001). One unit of a perpetuity issued in period t pays 1 dollar in period $t+1$, κ dollars in period $t+2$, κ^2 dollars in period $t+3$, and so on.

Let B_{t-1} denote the total nominal liability on past issues due at time t, which can be

written in terms of past new issues N_{t-j}:

$$B_{t-1} = \sum_{j=1}^{\infty} \kappa^{j-1} N_{t-j}.$$

Consequently, we have

$$N_t = B_t - \kappa B_{t-1}.$$

Let Q_t denote the price of a new issue, and then $\kappa^j Q_t$ is the time-t price of the perpetuity that is issued in period $t-j$. Thus, the market value of newly issued bonds is $Q_t(B_t - \kappa B_{t-1})$. For the borrower, this expression corresponds to the first term on the right-hand side of (2.10).

The borrower's Euler equation is

$$(C_t^b)^{-\sigma} = \beta_b \mathbb{E}_t[\frac{(C_{t+1}^b)^{-\sigma} R_{t+1}}{\Pi_{t+1}}], \tag{2.11}$$

where

$$R_{t+1} = \frac{1 + \kappa Q_{t+1}}{Q_t}$$

is the holding-period return of home-country long-term private bonds.

2.3 Financial Intermediaries

Each financial intermediary operates for one period and purchases only bonds issued in its own country in the baseline model. The mass of intermediaries is proportional to country population. The intermediary receives a transfer $P_t X_t^{FI}$ from the saver. It consists of new equity $P_t \mathcal{X}^{FI}$ and the value of long-term bonds carried by the previous intermediary:

$$P_t X_t^{FI} = P_t \mathcal{X}^{FI} + \kappa Q_t B_{t-1}^{FI}.$$

The intermediary also receives deposits D_t from the saver. It uses these funds to

purchase long-term bonds and hold reserves $RE^{FI}_{H,t}$ at the union central bank. Its balance sheet is

$$Q_t B^{FI}_t + RE^{FI}_{H,t} = D_t + P_t X^{FI}_t,$$

where B^{FI}_t includes both long-term private bonds issued by the borrowers and public bonds issued by the government in the home country. Without losing generality, we assume these two types of bonds are perfect substitutes for the intermediaries, so that they share the same price in equilibrium.

In period $t+1$, the intermediary pays lump-sum dividends Φ^{FI}_{t+1} to the saver. It chooses its asset portfolio to maximize expected discounted dividends subject to

$$Q_t B^{FI}_t \leq \Theta P_t \mathcal{X}^{FI},$$

where Θ is the leverage ratio. Appendix A gives the detailed dividend expression. Because the intermediary operates on behalf of the saver, it discounts dividends using the saver's stochastic discount factor:

$$\Lambda_{t,t+1} = \beta \left(\frac{C^s_{t+1}}{C^s_t} \right)^{-\sigma} \frac{1}{\Pi_{t+1}}.$$

The first-order conditions of the financial intermediary with respect to B^{FI}_t and $RE^{FI}_{H,t}$ are

$$\mathbb{E}_t \Lambda_{t,t+1} \left(R_{t+1} - I_t \right) = \Omega_t, \tag{2.12}$$

$$\mathbb{E}_t \Lambda_{t,t+1} \left(I^{RE}_t - I_t \right) = 0, \tag{2.13}$$

where Ω_t is the Lagrange multiplier on the leverage constraint. We assume the intermediary's leverage constraint is always binding. As a result, $\Omega_t > 0$, which creates a positive spread between the deposit rate and the return on long-term bonds. The financial intermediary in the foreign country faces a similar problem symmetrically.

2.4 Firms

The supply side is the same as in a standard New Keynesian model. A continuum of intermediate firms in the home country, indexed by $j \in [0, 1-\gamma]$, produces differentiated varieties. A representative domestic final-good firm aggregates them into the home-produced composite good.

The final-good firm combines intermediate outputs $Y_t(j)$ using the CES aggregator

$$Y_t = \left[\left(\frac{1}{1-\gamma} \right)^{\frac{1}{\epsilon}} \int_0^{1-\gamma} Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}.$$

Cost minimization yields the demand for each variety as

$$Y_t(j) = \frac{1}{1-\gamma} \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} Y_t, \tag{2.14}$$

and the aggregate price index satisfies

$$P_{H,t} = \left[\frac{1}{1-\gamma} \int_0^{1-\gamma} P_{H,t}(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}. \tag{2.15}$$

Each intermediate firm uses the linear production technology $Y_t(j) = A_t L_t(j)$, where the log-linearized productivity $a_t \equiv \log(A_t)$ follows an AR(1) process. We also assume that the domestic government subsidizes intermediate firms' wage costs at a constant rate $s_p = 1/\epsilon$. Firm j therefore bears the net labor cost $(1 - s_p) W_t L_t(j)$. This subsidy eliminates the distortion due to monopolistic competition in the steady state. A contemporaneous lump-sum tax on savers finances this subsidy.

Intermediate goods firms face monopolistic competition with the demand function given by (2.14) and are subject to Calvo (1983) style of nominal rigidity when setting their prices. In any period, with a probability $1 - \phi$, an intermediate goods firm is able to re-set its price ($0 < \phi < 1$). To save space, Appendix A shows the firms' optimal pricing problem

and price index evolution equation, and Appendix B shows the domestic Phillips curve after log-linearization.

2.5 Monetary and Fiscal Authorities

The central bank in the monetary union sets the policy rate according to a standard Taylor rule:

$$\ln I_t - \ln \bar{I} = \phi_\pi(\ln \Pi_t^{agg} - \ln \bar{\Pi}^{agg}) + \phi_y(\ln Y_t^{agg} - \ln \bar{Y}^{agg}), \tag{2.16}$$

where the union-average inflation and output are

$$\ln \Pi_t^{agg} \equiv (1-\gamma)\ln \Pi_t + \gamma \ln \Pi_t^*,$$
$$\ln Y_t^{agg} \equiv (1-\gamma)\ln Y_t + \gamma \ln Y_t^*.$$

The central bank allocates reserves according to the market value of its holdings of each country's bonds:

$$RE_t = Q_t B_t^{CB}, \qquad RE_t^* = Q_t^* B_t^{CB,*}.$$

Let $QE_t \equiv Q_t B_t^{CB}/P_t$ denote the real purchase of home country issued bonds and $QE_t^* \equiv Q_t^* B_t^{CB,*}/P_t^*$ denote the real purchase of foreign country issued bonds. These regional allocation rules imply the union-wide balance-sheet identity. Reserves and deposits earn the common nominal policy rate.

The central bank rebates its balance-sheet surplus to the national fiscal authorities in equal real per-capita amounts, measured in units of each country's consumption basket, that is, $T_t^{CB} = T_t^{CB*}$. Appendix A provides the rebate details.

Each fiscal authority in the two countries chooses per capita transfers T_t^C to borrowers. A fraction $\eta \in (0,1)$ of the fiscal stimulus is financed contemporaneously by a lump-sum tax on savers, that is,

$$T_t^s \equiv \eta T_t^C,$$

and the remainder is debt financed. Appendix A provides the complete government budget constraint and the government's passive lump-sum tax rule for T_t^G. Both countries use the same financing rule.

For tractability, following Sims, Wu, and Zhang (2023), we further assume that the fiscal authority fully bails out the borrower's legacy liabilities. This assumption implies that the borrower's consumption satisfies

$$C_t^b = \Theta \bar{X}^{FI} + QE_t + T_t^C - (1-\eta)T_t^C, \tag{2.17}$$

which does not depend on any lagged variables. The derivation of (2.17) can be found in Appendix A.

2.6 Market Clearing

Goods-market clearing requires

$$(1-\gamma)Y_t = (1-\gamma)[C_{H,t}^s + C_{H,t}^b] + \gamma[C_{H,t}^{s,*} + C_{H,t}^{b,*}]. \tag{2.18}$$

A symmetric condition follows for the foreign-produced goods[3].

Bond-market clearing requires

$$B_t^G + B_t^P = B_t^{FI} + B_t^{CB}. \tag{2.19}$$

A similar condition follows for the foreign country counterpart.

Steady state for welfare and monetary policy. We use variables with bars to denote steady-state values unless otherwise stated. At the symmetric steady state with zero inflation, we have $\bar{A} = \bar{A}^* = 1$, $\bar{P}_H = \bar{P}_F = \bar{P} = \bar{P}^*$. Let $z \in (0,1)$ be the common steady

[3]We assume government spending $G_t = G_t^* = 0$.

consumption share of borrowers, so that $\bar{C}^s = (1-z)\bar{Y}$ and $\bar{C}^b = z\bar{Y}$. Appendix A gives the remaining steady-state conditions.

2.7 Linear Equilibrium Conditions

We now log-linearize the equilibrium around the symmetric steady state. Lowercase variables denote log deviations from the steady state. For simplicity, we define the policy aggregates as:

$$f_t \equiv qe_t + \eta\tau_t^C, \qquad f_t^* \equiv qe_t^* + \eta\tau_t^{C*},$$
$$M_t^H \equiv vf_t + (1-v)f_t^*, \qquad M_t^F \equiv (1-v^*)f_t + v^*f_t^*,$$

where f_t and f_t^* summarize the policy effects on the demand side, while M_t^H and M_t^F summarize the policy effects on the supply side.

The IS curve in the home country is given by

$$\mathbb{E}_t\Delta y_{t+1} = \frac{\alpha}{\sigma}(i_t - \mathbb{E}_t\pi_{H,t+1}) + \left[\Psi_{ye} - \frac{\alpha(1-v)}{\sigma}\right]\mathbb{E}_t\Delta e_{t+1} + \mathbb{E}_t\Delta M_{t+1}^H, \tag{2.20}$$

where $\alpha \equiv 1-z$, and the relative output condition is given by

$$y_t - y_t^* = K_e e_t + d(f_t - f_t^*), \tag{2.21}$$

where $d \equiv v + v^* - 1$. Here, Ψ_{ye} and K_e are constants, defined in Appendix B.

The Phillips curves in each country are given by

$$\pi_{H,t} = \beta\mathbb{E}_t\pi_{H,t+1} + \kappa_p\left[(\chi + \frac{\sigma}{\alpha})y_t + \zeta_H e_t - (1+\chi)a_t - \frac{\sigma}{\alpha}M_t^H\right], \tag{2.22}$$
$$\pi_{F,t} = \beta\mathbb{E}_t\pi_{F,t+1} + \kappa_p\left[(\chi + \frac{\sigma}{\alpha})y_t^* + \zeta_F e_t - (1+\chi)a_t^* - \frac{\sigma}{\alpha}M_t^F\right], \tag{2.23}$$

where $\{\zeta_H, \zeta_F, \kappa_p\}$ are also constants, defined in Appendix B.

By the definition of the terms of trade, its evolution is determined by the relative producer price inflation, that is,

$$e_t = e_{t-1} - \pi_{H,t} + \pi_{F,t}. \tag{2.24}$$

Given exogenous policy instruments $\{qe_t, qe_t^*, \tau_t^C, g_t, \tau_t^{C*}, g_t^*\}$, equations (2.16) and (2.20)–(2.24) solve for the six endogenous variables $\{y_t, y_t^*, \pi_{H,t}, \pi_{F,t}, e_t, i_t\}$. The extended full log-linearized system of the equilibrium can be found in Appendices A and B.

3 Efficient Allocation and Welfare Loss Function

In this section, we characterize the efficient allocation and derive the quadratic welfare loss function following the approach in Woodford (2003). The period welfare loss separates distortions in the union-average allocation from relative adjustment across countries and from the distribution of consumption within and across countries.

We assume that the union-wide welfare is the population-weighted sum of national welfare, that is,

$$\mathcal{W} = (1-\gamma)\mathcal{W}_H + \gamma\mathcal{W}_F, \tag{3.1}$$

where $\mathcal{W}_H$ and $\mathcal{W}_F$ satisfy

$$\mathcal{W}_H \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - \bar{U}}{\bar{U}_{C^s}\bar{C}^s}, \qquad \mathcal{W}_F \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t^* - \bar{U}^*}{\bar{U}_{C^{s*}}\bar{C}^{s*}}. \tag{3.2}$$

Here, the period national utility is defined as $U_t \equiv u(C_t^s) + \delta u(C_t^b) - \psi_L L_t^{1+\chi}/(1+\chi)$, where $\delta > 0$ measures the relative weight on borrower consumption relative to saver consumption. The foreign national utility is symmetric.

3.1 Efficient Allocation

We first solve the first-best problem of a social planner. The social planner allocates the home- and foreign-produced goods across savers and borrowers and chooses labor in each country, subject only to the two union resource constraints. The home consumption and labor conditions are

$$(C_t^s)^{-\sigma} = \delta (C_t^b)^{-\sigma}, \tag{3.3}$$

$$\psi_L L_t^{\chi} = A_t \lambda_{H,t}, \tag{3.4}$$

where $\lambda_{H,t}$ is the Lagrange multiplier on the home resource constraint. These conditions equate the social marginal utility of consumption to its union resource cost. The foreign conditions are symmetric and are reported in Appendix C, together with the detailed expression of $\lambda_{H,t}$ as a function of C_t^s and C_t^{s*}.

At the steady state, (3.3) implies

$$\delta = \left(\frac{z}{1-z} \right)^{\sigma}. \tag{3.5}$$

We assume that the government in each country chooses a steady-state fiscal transfer between savers and borrowers, such that the resulting borrower consumption share z satisfies condition (3.5). Because we have assumed that the national governments impose a labor subsidy to correct the distortion due to monopolistic competition at the steady state in each country, together with condition (3.5), the steady state is efficient.

For the sake of parsimony, for any variable pair x_t and x_t^*, we denote its union average as $x_t^A = (1-\gamma)x_t + \gamma x_t^*$ and the relative cross-country difference as $x_t^D = x_t - x_t^*$. Then, the log-linear efficient allocation satisfies

$$y_t^{A,e} = \frac{1+\chi}{\sigma+\chi} a_t^A, \tag{3.6}$$

$$e_t^e = \frac{1+\chi}{1+\chi K_{FB}} a_t^D, \tag{3.7}$$

$$y_t^{D,e} = K_{FB} e_t^e, \tag{3.8}$$

where K_{FB} is a constant, summarizing trade openness, home bias, and the elasticity of substitution between home- and foreign-produced goods. Common productivity shocks therefore move efficient union output without requiring a change in relative producer prices. Asymmetric shocks generally require the terms of trade to adjust. Derivations of (3.6)–(3.8) and the detailed expression of K_{FB} can be found in Appendix C.

3.2 Welfare Criterion

We now focus on the utility-based quadratic welfare loss function. Define the within-country consumption difference as $b_t \equiv c_t^s - c_t^b$ and $b_t^* \equiv c_t^{s*} - c_t^{b*}$ for the home and foreign countries, respectively.

Maximizing the second-order approximation of the social planner's objective (3.1) is equivalent to minimizing the following quadratic welfare loss function:

$$\begin{aligned}\mathcal{L}_t = \frac{1}{2(1-z)} \Big\{ & \epsilon\Phi[(1-\gamma)\pi_{H,t}^2 + \gamma\pi_{F,t}^2] + (\sigma+\chi)(y_t^A - y_t^{A,e})^2 \\ & + (1-\gamma)\gamma(\chi + K_{FB}^{-1})(y_t^D - y_t^{D,e})^2 \\ & + \sigma z(1-z)[(1-\gamma)b_t^2 + \gamma(b_t^*)^2] + (1-\gamma)\gamma\frac{\sigma T}{K_{FB}} z^2 (b_t^D)^2 \Big\},\end{aligned} \tag{3.9}$$

where $\Phi > 0$ and $T \geq 0$ are constant weights defined in Appendix C.

The period welfare loss includes terms due to price dispersion, inefficient output, within-country saver-borrower consumption dispersion, and inefficient cross-country consumption allocation. The average block can be stabilized independently by the common interest rate and average fiscal transfer, whereas the welfare loss from the relative dispersion needs country-specific policy instruments. Appendix C derives the welfare

approximation.

4 Fiscal Policy and Financial Integration

We first characterize coordinated policy and the fiscal Nash equilibrium, showing why national governments redistribute more than a union planner. We then introduce cross-border government-bond holdings and examine how financial integration changes the domestic welfare benefit of fiscal transfers. Throughout this section, the common central bank sets the interest rate and does not conduct QE, which we discuss in Section 5.

4.1 National Fiscal Policy and Excess Redistribution

Under coordination, the monetary and fiscal authorities jointly minimize the quadratic loss in equation (3.9). We can rewrite the country-specific terms in (3.9) in the form of union averages and cross-country differences:

$$(1-\gamma)\pi_{H,t}^2 + \gamma\pi_{F,t}^2 = (\pi_t^A)^2 + \Gamma(\pi_t^D)^2,$$
$$(1-\gamma)b_t^2 + \gamma(b_t^*)^2 = (b_t^A)^2 + \Gamma(b_t^D)^2,$$

where $\Gamma = \gamma(1-\gamma)$ is a constant.

Coordinated policies can fully stabilize the union-average inflation and output with the common interest rate and across-country average fiscal transfers, such that

$$\pi_t^A = 0, \qquad y_t^A = y_t^{A,e}, \qquad b_t^A = 0. \tag{4.1}$$

See Appendix D for detailed derivations and the corresponding expressions of required policy instruments. The equilibrium achieves the first best only if the terms of trade remain constant at their efficient steady-state value.[4]

[4]To see this, note that $\pi_t^D = -\Delta e_t$. Thus, fully stabilizing (3.9) therefore requires $\pi_t^D = 0$, and hence e_t must stay constant. This is feasible when the union faces only common shocks, since the efficient terms of

We next turn to decentralized national fiscal policy. Under decentralized fiscal policy, each national government maximizes its own residents' welfare, while the common central bank maximizes union welfare. We first ask whether a national government would choose the coordinated transfer when the other two policy paths are held fixed.

Proposition 1 (Coordination Is Not a Nash Equilibrium)**.** *The coordinated allocation is not a fiscal Nash equilibrium. An increase in the home transfer raises home welfare:*

$$\left.\frac{\partial \mathcal{W}_H}{\partial \tau_t^C}\right|_{\text{coord}} > 0. \tag{4.2}$$

The symmetric result holds for the foreign government.

Proof. See Appendix D.

An increase in national transfers raises domestic borrowers' consumption and thereby improves domestic welfare. At the same time, the transfer acts as a domestic demand shock that propagates abroad and induces aggregate fluctuations in the foreign country. Because the domestic government does not internalize the welfare cost of these foreign fluctuations, it sets transfers above the level a coordinated planner would choose. The same incentive applies to both governments, so the coordinated allocation cannot be sustained through unilateral national choices.

We next characterize the fiscal Nash equilibrium. For tractability, we assume constant marginal disutility of labor, $\chi = 0$. We maintain this assumption throughout the remainder of Sections 4 and 5 and relax it in the quantitative analysis in Section 6.

Under this assumption, fiscal transfers affect neither the central bank's optimal interest rate nor the terms of trade, allowing us to solve the national fiscal best responses.

Lemma 1 (Monetary–fiscal separation)**.** *Under productivity shocks, the terms of trade and the*

trade is then constant. However, when there are time-varying asymmetric productivity shocks, the efficient level of terms of trade implied by (3.7) moves away from a constant. In this case, the efficient level of the terms of trade and $\pi_t^D = 0$ cannot hold simultaneously, which creates a trade-off for coordinated policies.

central bank's optimal interest rate are independent of fiscal transfers and satisfy:

$$(1+\beta+\kappa_p)e_t - e_{t-1} - \beta \mathbb{E}_t e_{t+1} = \kappa_p a_t^D, \tag{4.3}$$

$$i_t = \mathbb{E}_t \Delta a_{t+1}^A, \tag{4.4}$$

Proof. See Appendix D.

Under Lemma 1, fiscal transfers change neither the central bank's optimal interest rate choice nor the terms of trade. We can therefore solve the national transfer choices without a monetary or terms of trade response to those choices.

We next show that national fiscal policy raises borrower consumption above the coordinated level in both countries. Corollary 1 establishes this excess redistribution under productivity and cost-push shocks and gives exact consumption ratios under common productivity shocks.

Corollary 1 (Excess Redistribution under National Fiscal Policy)**.** *Assume $e_{-1} = 0$, and consider productivity and cost-push shocks at $t = 0$. Borrower consumption in both countries exceeds its coordinated level for all $t \geq 0$:*

$$C_t^{b,N} > C_t^{b,C}, \tag{4.5}$$

$$C_t^{b*,N} > C_t^{b*,C}, \tag{4.6}$$

where superscripts N and C denote national Nash and coordinated fiscal policy, respectively.

For common productivity shocks only, $a_t = a_t^*$, this excess redistribution has a closed-form characterization:

$$\frac{C_t^{b,N}}{C_t^{b,C}} = v^{-1/\sigma} > 1, \tag{4.7}$$

$$\frac{C_t^{b*,N}}{C_t^{b*,C}} = (v^*)^{-1/\sigma} > 1. \tag{4.8}$$

The redistribution bias vanishes in autarky, where $v = v^* = 1$.

Proof. See Appendix D.

The excess redistribution arises because national governments do not internalize the foreign labor cost of producing imports. Each national authority therefore accounts for only the domestic share of the resource cost of additional consumption, v at home and v^* abroad, whereas the coordinator accounts for the full union resource cost. Under common productivity shocks, a lower domestic expenditure share therefore implies a larger redistribution bias. The bias vanishes in autarky, where each country bears the full resource cost of its consumption.

4.2 Financial Integration and Fiscal Redistribution

Having established excess redistribution under segmented bond markets, we now ask how cross-border government-bond holdings change the domestic welfare benefit of a fiscal transfer, holding all other policy paths fixed. We conduct the analysis in partial equilibrium, holding bond prices, returns, and the policy-rate path fixed. We derive the analytical results in the remainder of this section and Section 5 in this partial-equilibrium environment, whereas Section 6 examines the mechanisms quantitatively in a dynamic general-equilibrium model with endogenous bond prices and policy responses.

With financial integration, intermediaries can hold government bonds issued in either country, while private lending remains local. Their leverage constraints apply to the combined value of domestic and foreign government bonds and local private bonds. The home intermediary's leverage constraint becomes

$$Q_t B_{H,t}^{P,FI} + Q_t B_{HH,t}^{G,FI} + Q_t^* B_{HF,t}^{G,FI} \leq \Theta P_t \mathcal{X}^{FI}, \tag{4.9}$$

where the first term is local private lending and the last two terms are holdings of home and foreign government bonds.

We assume that of the additional home-country issued bonds not purchased by the union central bank, foreign intermediaries hold the share $\gamma\xi$, where $\xi \in [0,1]$ measures the degree of financial integration. At $\xi = 0$, intermediaries hold only domestic bonds in both countries. At $\xi = 1$, each government's bonds held by intermediaries are divided across countries in proportion to population.

In this partial-equilibrium environment, the binding leverage constraint (4.9) fixes the real value of the intermediary's total bond portfolio. Consequently, government bonds issued to finance a fiscal transfer absorb intermediary balance-sheet capacity and crowd out private lending, and hence borrower consumption.[5] The degree of financial integration determines how this crowding out is distributed across countries.

Lemma 2 (Integration Amplifies Fiscal Transmission)**.** *Greater financial integration strengthens the effect of a home fiscal transfer on home borrower consumption relative to that abroad:*

$$\frac{\partial^2 c_t^{b,D}}{\partial \tau_t^{C,D} \partial \xi} = \frac{1-\eta}{z} > 0. \tag{4.10}$$

Proof. See Appendix D.

Greater financial integration shifts more of the debt financing a home fiscal transfer to foreign intermediaries. The transfer therefore crowds out less private lending at home and more abroad, raising home borrower consumption by more relative to that abroad.

We next ask whether the stronger consumption response increases the domestic welfare benefit of a fiscal transfer. Let $w_{H,t}$ denote the impact change in home welfare.

Proposition 2 (Integration Raises the Welfare Gain from Transfers)**.** *Greater financial inte-*

[5] Because bonds also share the same maturity structure, reallocating this fixed portfolio across issuers and private lending creates neither valuation gains nor a change in intermediary dividends net of equity contributions.

gration increases the marginal domestic welfare gain from a fiscal transfer:

$$\frac{\partial^2 w_{H,t}}{\partial \tau_t^C \, \partial \xi} = \frac{(1-v)(1-\eta)}{1-z} > 0. \tag{4.11}$$

Proof. See Appendix D.

Proposition 2 extends the consumption result in Lemma 2 to welfare. By shifting part of the credit crowding out abroad, financial integration increases the domestic welfare benefit of a transfer and further strengthens the national incentives to redistribute discussed in Section 4.1.

5 QE and Financial Integration

Having examined fiscal transfers, we now ask how financial integration changes the welfare effects of QE. We then study the optimal allocation of QE across countries and how financial integration affects the gains from targeting purchases. Finally, we return to fiscal redistribution and ask whether financial integration continues to amplify its effects when QE responds optimally. We maintain the cross-border bond-holding assumption and partial-equilibrium environment of Section 4.2.

5.1 Financial Integration and QE

We first ask how financial integration shapes the effects of QE through additional purchases of home government bonds, with fiscal transfers unchanged. These purchases relax the leverage constraints of the intermediaries holding the bonds, allowing them to expand private lending and thereby raise borrower consumption.

Lemma 3 (Integration Weakens QE Transmission)**.** *Greater financial integration weakens the effect of purchases of home government bonds on home borrower consumption relative to that*

abroad:

$$\frac{\partial^2 c_t^{b,D}}{\partial q e_t^D \, \partial \xi} = -\frac{1}{z} < 0. \tag{5.1}$$

Proof. See Appendix D.

With greater financial integration, foreign intermediaries hold a larger share of home government bonds. Purchases of these bonds therefore expand private lending more abroad and less at home, weakening the relative increase in home borrower consumption. Lemmas 2 and 3 thus show opposite effects of financial integration on fiscal and QE transmission: fiscal transfers crowd out less domestic private lending, whereas QE expands domestic private lending by less.

We next ask how this shift in consumption affects the domestic welfare benefit of QE.

Proposition 3 (Integration Lowers the Welfare Gain from QE)**.** *Greater financial integration decreases the marginal domestic welfare gain from purchases of home government bonds:*

$$\frac{\partial^2 w_{H,t}}{\partial q e_t \, \partial \xi} = -\frac{1-v}{1-z} < 0. \tag{5.2}$$

Proof. See Appendix D.

Propositions 2 and 3 extend the contrast between fiscal transfers and QE in Lemmas 2 and 3 to domestic welfare gains: financial integration increases the gain from transfers but reduces the gain from QE.

5.2 Optimal Allocation of QE across Countries

Financial integration weakens the domestic effects of purchases of home government bonds. We now ask how it changes the optimal allocation of QE across countries and the welfare gains from targeting purchases.

Unlike Section 5.1, where additional purchases increase total QE, we hold total union QE fixed and vary only its composition across countries. Additional purchases of home government bonds therefore require fewer purchases of foreign government bonds. Population-weighted QE allocates purchases in proportion to country-specific population, that is, $qe_t = qe_t^*$, or $qe_t^D = 0$. Targeted QE allows purchases per resident to differ across countries in response to lending below its efficient level.

Let $\ell_{i,t}$ denote realized private lending per resident before the QE intervention.[6] Let $\ell_{i,t}^e$ denote the lending level that supports efficient borrower consumption conditional on the other policy paths. We define the cross-country difference in credit shortfalls as

$$D_t = (\ell_{H,t}^e - \ell_{H,t}) - (\ell_{F,t}^e - \ell_{F,t}). \tag{5.3}$$

A positive D_t means that the home country has the larger credit shortfall.

We assume that departures from population-weighted QE incur a real quadratic operating cost, $\frac{\kappa_{\text{QE}}}{2}(qe_t^D)^2$, with $\kappa_{\text{QE}} > 0$. This additional assumption creates a trade-off for country-targeted QE. Shifting purchases toward the country with the larger shortfall reduces borrower consumption losses but incurs operating costs. The central bank therefore leaves part of the cross-country shortfall difference uncorrected.

Given other policy instruments, we show that the central bank's conditional welfare objective reduces to

$$\max_{qe_t^D} \mathcal{W}(qe_t^D) \;=\; \max_{qe_t^D} \Big\{ -\frac{\mu}{2z^2}\big[D_t - (1-\xi)qe_t^D\big]^2 - \frac{\kappa_{\text{QE}}}{2}(qe_t^D)^2 \Big\}, \tag{5.4}$$

where $\mu = \sigma z/(1-z) > 0$ is the curvature of normalized borrower welfare. Appendix D derives this objective.

We characterize the optimal QE allocation and how financial integration changes its

[6] Lending is measured relative to steady state and normalized by steady-state output. For the home country, $\ell_{H,t} \equiv [Q_t B_t^P / P_t - \bar{Q}\bar{B}^P/\bar{P}]/\bar{Y}$, with the analogous definition for the foreign country.

welfare gain.

Proposition 4 (Optimal QE and Financial Integration)**.** *Holding $D_t \neq 0$ fixed, for $\xi < 1$ the union central bank optimally purchases more bonds per resident from the country with the larger credit shortfall:*

$$qe_t^{D,\text{opt}} = \frac{\mu(1-\xi)}{\kappa_{\text{QE}} z^2 + \mu(1-\xi)^2} D_t. \tag{5.5}$$

The welfare gain over population-weighted QE decreases with financial integration:

$$\frac{d}{d\xi}\left[\mathcal{W}(qe_t^{D,\text{opt}}) - \mathcal{W}(0)\right] < 0. \tag{5.6}$$

At $\xi = 1$, the welfare gain is zero and $qe_t^{D,\text{opt}} = 0$.

Proof. See Appendix D.

Financial integration spreads the benefit of a country-targeted asset purchase on private lending across both countries. It therefore reduces the welfare advantage of targeting QE to the country with the larger credit shortfall.[7]

5.3 Fiscal Redistribution under Optimal QE

We now ask whether financial integration continues to amplify the effects of fiscal redistribution when the central bank optimally reallocates QE.

Maintaining the fixed total QE and operating-cost specification of Section 5.2, we vary the cross-country transfer difference while holding the union-average transfer fixed. The fiscal transfer difference then generates a cross-country credit shortfall difference. Specifically,

$$D_t = -[\eta + (1-\eta)\xi]\tau_t^{C,D}. \tag{5.7}$$

[7] Without operating costs, unconstrained QE fully eliminates a fixed cross-country shortfall difference for any $\xi < 1$, making the welfare gain independent of financial integration.

Unlike the fixed D_t comparison in Section 5.2, financial integration now changes the shortfall difference generated by a given fiscal transfer difference.

Substituting this shortfall into the optimal QE allocation, we examine the consumption response after the central bank's intervention.

Corollary 2 (Integration Amplifies Fiscal Transmission under Optimal QE)**.** *Greater financial integration strengthens the effect of fiscal transfers on the cross-country borrower consumption differential when QE responds optimally:*

$$\frac{\partial^2 c_t^{b,D}}{\partial \tau_t^{C,D} \, \partial \xi} > 0. \tag{5.8}$$

Proof. See Appendix D.

Corollary 2 shows that the consumption amplification underlying Proposition 2 persists when QE responds optimally. A larger home transfer raises home borrower consumption relative to that abroad. Debt issued to finance the home transfer crowds out private lending in both countries, but only home borrowers receive the transfer. The central bank therefore optimally shifts purchases abroad, partially offsetting the consumption differential. Financial integration strengthens the consumption effect of fiscal transfers and weakens the effect of optimal QE.[8] A higher ξ therefore leaves a larger consumption differential even after the optimal QE response.

6 Quantitative Analysis

This section studies how financial integration shapes the dynamic transmission of fiscal transfers and quantitative easing. We first extend the baseline model, and then examine the dynamic counterparts of the analytical results in Sections 4 and 5.

[8]The share of the fiscal consumption effect offset by optimal QE is $\mu(1-\xi)^2/[\kappa_{\text{QE}} z^2 + \mu(1-\xi)^2]$, which decreases with financial integration and equals zero at $\xi = 1$.

6.1 Quantitative Model with Fixed Ownership Shares

We extend the dynamic model of Section 2 by distinguishing private and government bonds in both ownership and pricing. Private bonds can be held only by intermediaries in the issuing country. Government bonds can be held by both domestic and foreign intermediaries, and the central bank purchases only government bonds. Following Sections 4 and 5, a constant ξ now fixes the shares of each government's debt net of QE held by home and foreign financial intermediaries. The remaining economic environment is unchanged.

6.1.1 Assets and ownership

Prices $Q^P_{j,t}$ and $Q^G_{j,t}$ refer respectively to private and government bonds issued in country j, $j \in \{H, F\}$. Their nominal holding-period returns are

$$R^P_{j,t+1} = \frac{1 + \kappa Q^P_{j,t+1}}{Q^P_{j,t}}, \qquad R^G_{j,t+1} = \frac{1 + \kappa Q^G_{j,t+1}}{Q^G_{j,t}}. \tag{6.1}$$

The intermediary in country j holds domestic private bonds $B^{P,FI}_{j,t}$, domestic government bonds $B^{G,FI}_{j|j,t}$, and foreign government bonds $B^{G,FI}_{\ell|j,t}$. In $B^{G,FI}_{i|j,t}$ the first subscript identifies the issuer and the second the holder.

6.1.2 Fixed government-bond shares

Let $\xi \in [0, 1]$ measure the degree of financial integration. Define government debt available to intermediaries, per resident of the issuing country, by

$$\mathcal{N}_{j,t} = B^G_{j,t} - B^{CB}_{j,t} \geq 0, \qquad j = H, F. \tag{6.2}$$

Of home government bonds not held by the central bank, foreign intermediaries hold the fixed aggregate share $\gamma\xi$. Of foreign government bonds not held by the central bank, home

intermediaries hold $(1-\gamma)\xi$. In country-total units, the four allocation equations are

$$\begin{aligned}(1-\gamma)B^{G,FI}_{H|H,t} &= (1-\gamma\xi)(1-\gamma)\mathcal{N}_{H,t},\\ \gamma B^{G,FI}_{H|F,t} &= (\gamma\xi)(1-\gamma)\mathcal{N}_{H,t}.\end{aligned} \tag{6.3}$$

$$\begin{aligned}(1-\gamma)B^{G,FI}_{F|H,t} &= [(1-\gamma)\xi]\gamma\mathcal{N}_{F,t},\\ \gamma B^{G,FI}_{F|F,t} &= [1-(1-\gamma)\xi]\gamma\mathcal{N}_{F,t}.\end{aligned} \tag{6.4}$$

At $\xi = 0$, government debt is held domestically. At $\xi = 1$, each issuer's debt net of QE is allocated across countries in proportion to population.

6.1.3 Intermediary balance sheets and financial constraints

The balance sheet and equity transfer of the intermediary in country j satisfy

$$Q^{P}_{j,t}B^{P,FI}_{j,t} + \sum_i Q^{G}_{i,t}B^{G,FI}_{i|j,t} + RE_{j,t} = D_{j,t} + P^{j}_{t}X^{FI}_{j,t}, \tag{6.5}$$

$$P^{j}_{t}X^{FI}_{j,t} = P^{j}_{t}\mathcal{X}^{FI}_{j} + \kappa\left[Q^{P}_{j,t}B^{P,FI}_{j,t-1} + \sum_i Q^{G}_{i,t}B^{G,FI}_{i|j,t-1}\right]. \tag{6.6}$$

As in the baseline, the intermediary operates for one period and receives fixed new real equity $\mathcal{X}^{FI}_{j}$ and the surviving securities of its predecessor.

The financial constraint is

$$Q^{P}_{j,t}B^{P,FI}_{j,t} + \xi_G\sum_i Q^{G}_{i,t}B^{G,FI}_{i|j,t} \leq \Theta_{j,t}P^{j}_{t}\mathcal{X}^{FI}_{j}. \tag{6.7}$$

The parameter $\Theta_{j,t}$ measures the leverage that can be taken by the intermediary, and is subject to a credit shock. $\xi_G \in (0,1)$ indicates that diverting government bonds is harder than diverting private bonds for the intermediaries.

The foreign government position incurs the cross-border adjustment cost. Define its

resident-CPI market-value deviation by

$$x_{j,t} = \frac{Q^G_{\ell,t} B^{G,FI}_{\ell|j,t} / P^j_t - \bar{Q}^G \bar{B}^{G,FI}_{\ell|j} / \bar{P}}{\bar{Y}}, \qquad \mathcal{C}^x_{j,t} = \frac{\phi_x}{2} \bar{Y} x^2_{j,t}. \tag{6.8}$$

The intermediary maximizes $\mathbb{E}_t[\Lambda_{j,t,t+1} P^j_{t+1} \Phi^{FI}_{j,t+1}]$ subject to (6.5)–(6.7) and the fixed allocation conditions (6.3). Its dividends are

$$\begin{aligned} P^j_{t+1}\Phi^{FI}_{j,t+1} = {} & (R^P_{j,t+1} - I_t) Q^P_{j,t} B^{P,FI}_{j,t} \\ & + \sum_i (R^G_{i,t+1} - I_t) Q^G_{i,t} B^{G,FI}_{i|j,t} \\ & + (I^{RE}_t - I_t) RE_{j,t} + I_t P^j_t X^{FI}_{j,t} - P^j_{t+1} \mathcal{C}^x_{j,t}. \end{aligned} \tag{6.9}$$

With multiplier $\Omega_{j,t}$ on (6.7), optimality for the two freely chosen securities and reserves gives

$$\mathbb{E}_t[\Lambda_{j,t,t+1}(R^P_{j,t+1} - I_t)] = \Omega_{j,t}, \tag{6.10}$$

$$\mathbb{E}_t[\Lambda_{j,t,t+1}(R^G_{j,t+1} - I_t)] = \xi_G \Omega_{j,t}, \tag{6.11}$$

$$\mathbb{E}_t[\Lambda_{j,t,t+1}(I^{RE}_t - I_t)] = 0. \tag{6.12}$$

Define real market values and real financing capacity by

$$\mathcal{A}^P_{j,t} = \frac{Q^P_{j,t} B^{P,FI}_{j,t}}{P^j_t}, \qquad \mathcal{A}^G_{i|j,t} = \frac{Q^G_{i,t} B^{G,FI}_{i|j,t}}{P^j_t}, \qquad \mathcal{K}_{j,t} = \Theta_{j,t} \bar{X}^{FI}_j.$$

When (6.7) binds,

$$\mathcal{A}^P_{j,t} = \mathcal{K}_{j,t} - \xi_G \sum_i \mathcal{A}^G_{i|j,t},$$

$$\Delta\mathcal{A}^P_{j,t} = \Delta\mathcal{K}_{j,t} - \xi_G \sum_i \Delta\mathcal{A}^G_{i|j,t}.$$

At unchanged financing capacity, a larger government-asset market value therefore reduces private-asset market value.

6.1.4 Fiscal accounts

The borrower in country j still issues only private bonds and, as in the baseline, takes lump-sum transfers as given. Its budget and Euler equation can be rewritten as

$$P_t^j C_{j,t}^b + B_{j,t-1}^P = Q_{j,t}^P (B_{j,t}^P - \kappa B_{j,t-1}^P) + P_t^j (X_{j,t}^b + T_{j,t}^C), \tag{6.13}$$

$$(C_{j,t}^b)^{-\sigma} = \beta_b \mathbb{E}_t[(C_{j,t+1}^b)^{-\sigma} R_{j,t+1}^P / \Pi_{t+1}^j]. \tag{6.14}$$

Retaining the positive-debt steady-state closure, total saver taxes are

$$T_{j,t}^G = \eta T_{j,t}^C + \varrho \frac{Q_{j,t-1}^G B_{j,t-1}^G}{P_{t-1}^j} + T_{j,t}^{sub}. \tag{6.15}$$

The production subsidy $S_{j,t} = s_p W_{j,t} L_{j,t} / P_t^j$, with $s_p = 1/\epsilon$, is financed by $T_{j,t}^{sub} = S_{j,t}$. After canceling these two terms, the government budget is

$$T_{j,t}^C + \frac{B_{j,t-1}^G}{P_t^j} = \frac{Q_{j,t}^G (B_{j,t}^G - \kappa B_{j,t-1}^G)}{P_t^j} + \eta T_{j,t}^C + \varrho \frac{Q_{j,t-1}^G B_{j,t-1}^G}{P_{t-1}^j} + T_{j,t}^{CB}. \tag{6.16}$$

The full-bailout transfer must distinguish the prices of the two inherited liabilities and include the same fixed real intercept:

$$\begin{aligned} X_{j,t}^b = {} & \frac{(1 + \kappa Q_{j,t}^P) B_{j,t-1}^P + (1 + \kappa Q_{j,t}^G) B_{j,t-1}^G}{P_t^j} \\ & - \varrho \frac{Q_{j,t-1}^G B_{j,t-1}^G}{P_{t-1}^j} - T_{j,t}^{CB}. \end{aligned} \tag{6.17}$$

Combining (6.13), (6.16), and (6.17) gives the useful accounting identity

$$C_{j,t}^b = \frac{Q_{j,t}^P B_{j,t}^P + Q_{j,t}^G B_{j,t}^G}{P_t^j} + \eta T_{j,t}^C. \tag{6.18}$$

6.1.5 Central bank, market clearing, and calibration targets

Define real QE in the issuer's consumption units as $QE_{j,t} = Q^G_{j,t} B^{CB}_{j,t} / P^j_t$. The union central bank's balance sheet and remittances satisfy

$$\sum_j n_j Q^G_{j,t} B^{CB}_{j,t} = \sum_j n_j RE_{j,t}, \tag{6.19}$$

$$\sum_j n_j P^j_t T^{CB}_{j,t} = \sum_j n_j R^G_{j,t} Q^G_{j,t-1} B^{CB}_{j,t-1} - I_{t-1} \sum_j n_j RE_{j,t-1}. \tag{6.20}$$

We retain equal real per-capita rebates, $T^{CB}_{H,t} = T^{CB}_{F,t}$. Reserves are allocated according to the nominal market value of the central bank's holdings of each country's government bonds:

$$RE_{j,t} = Q^G_{j,t} B^{CB}_{j,t}.$$

Equivalently, $RE_{j,t} / P^j_t = QE_{j,t}$. These regional rules imply the union balance-sheet identity (6.19). Equal real per-capita central-bank remittances are retained and remain distinct from payments through the saver fund.

Private and government debt clear separately:

$$B^P_{j,t} = B^{P,FI}_{j,t}, \qquad B^G_{j,t} = B^{G,FI}_{j|j,t} + \frac{n_\ell}{n_j} B^{G,FI}_{j|\ell,t} + B^{CB}_{j,t}. \tag{6.21}$$

6.2 Quantitative Results

Calibration. Our calibration builds on the strategy and targets in Wu, Xie, and Zhang (2024). Table 1 reports the parameter values. We vary the financial integration parameter ξ while holding the remaining parameters fixed to examine how cross-border government-bond holdings affect policy transmission.

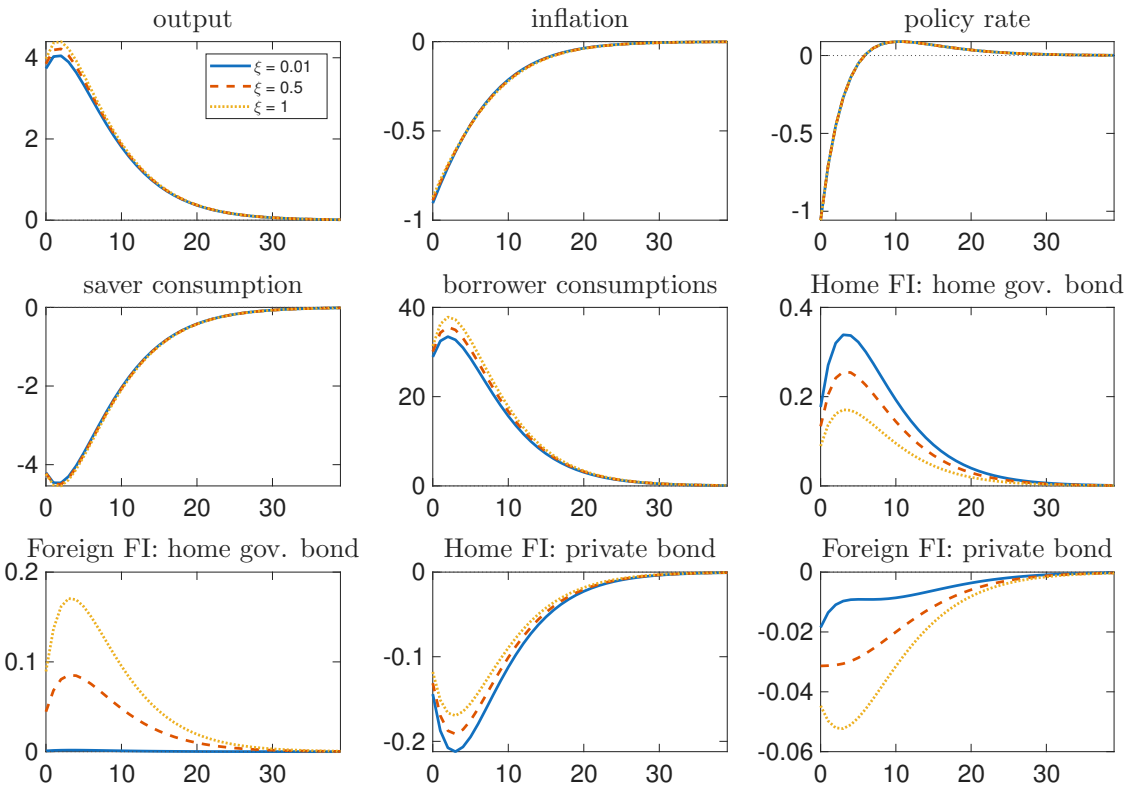


Figure 1: HOME TRANSFER SHOCK

Notes: The figure reports the impulse responses of the model's endogenous variables to a 1% home fiscal-transfer shock. Each panel displays the response of a different variable over a 40-quarter horizon. The blue solid, red dashed, and yellow dotted lines correspond to $\xi \in \{0.01, 0.5, 1\}$, respectively. The x-axis is time, and the y-axis is the percentage change from the steady state.

Financial Integration and the Effects of Monetary and Fiscal Policies. We first examine the dynamic counterparts of Propositions 2 and 3. Figures 1 and 2 report responses to a positive home fiscal transfer shock. We set $\eta = 0.5$, so half of the additional transfer is financed by contemporaneous domestic lump-sum taxes on savers and the remainder by government debt.

Figure 1 compares impulse responses for $\xi = 0.01$, 0.5, and 1, shown by the blue solid, red dashed, and yellow dotted lines, respectively. At all three levels of financial integration, the transfer raises home borrower consumption and output, while home saver consumption falls. The increase in government debt is absorbed by intermediaries in both countries according to the fixed ownership shares. With binding financing constraints, the

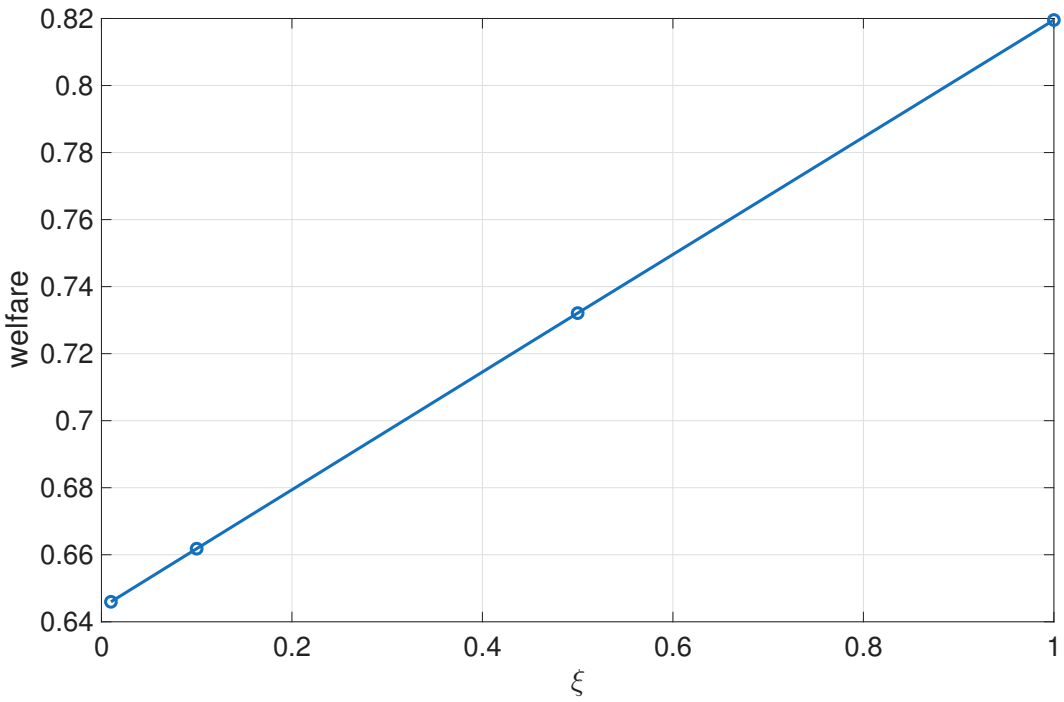


Figure 2: HOME TRANSFER WELFARE

Notes: The figure plots the welfare measure under a home fiscal-transfer shock against the degree of financial integration, ξ.

additional government holdings crowd out private lending.

Greater financial integration shifts a larger share of this crowding out to foreign intermediaries. Home intermediaries absorb less of the additional government debt and reduce their private bond holdings by less. The increases in home borrower consumption and output are therefore larger at higher values of ξ. Foreign intermediaries, in contrast, increase their holdings of home government bonds by more and experience a larger reduction in private bond holdings.

Figure 2 shows that the home welfare gain from the transfer rises with financial integration. Cross-border holdings shift part of the intermediation cost of additional home government debt abroad, reducing the displacement of domestic private lending. This pattern is consistent with Proposition 2 and equation (4.11).

Figures 3 and 4 report the corresponding responses to a home QE shock.

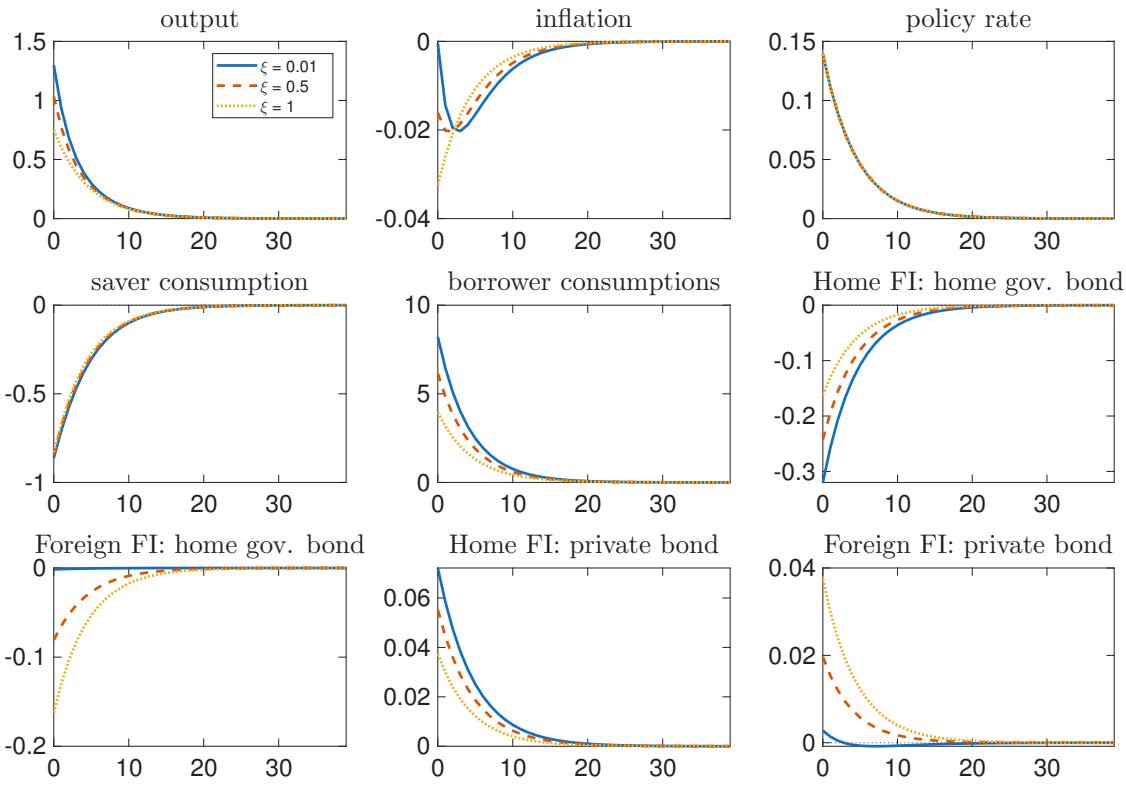


Figure 3: HOME QE SHOCK

Notes: The figure reports the impulse responses of the model's endogenous variables to a 1% home QE shock. Each panel displays the response of a different variable over a 40-quarter horizon. The blue solid, red dashed, and yellow dotted lines correspond to $\xi \in \{0.01, 0.5, 1\}$, respectively. The x-axis is time, and the y-axis is the percentage change from the steady state.

Figure 3 shows that purchases of home government bonds reduce the amount of government debt held by intermediaries and release financing capacity for private lending. Home borrower consumption and output increase, while home saver consumption falls.

Both countries' intermediaries reduce their holdings of home government bonds and initially expand their private bond holdings. As ξ rises, the reduction in home intermediaries' government holdings becomes smaller, while the reduction in foreign intermediaries' holdings becomes larger. The associated increase in private lending therefore shifts toward the foreign country. The expansion in home borrower consumption and output becomes smaller as financial integration increases.

Figure 4 shows that the home welfare gain from QE declines with ξ. This result is

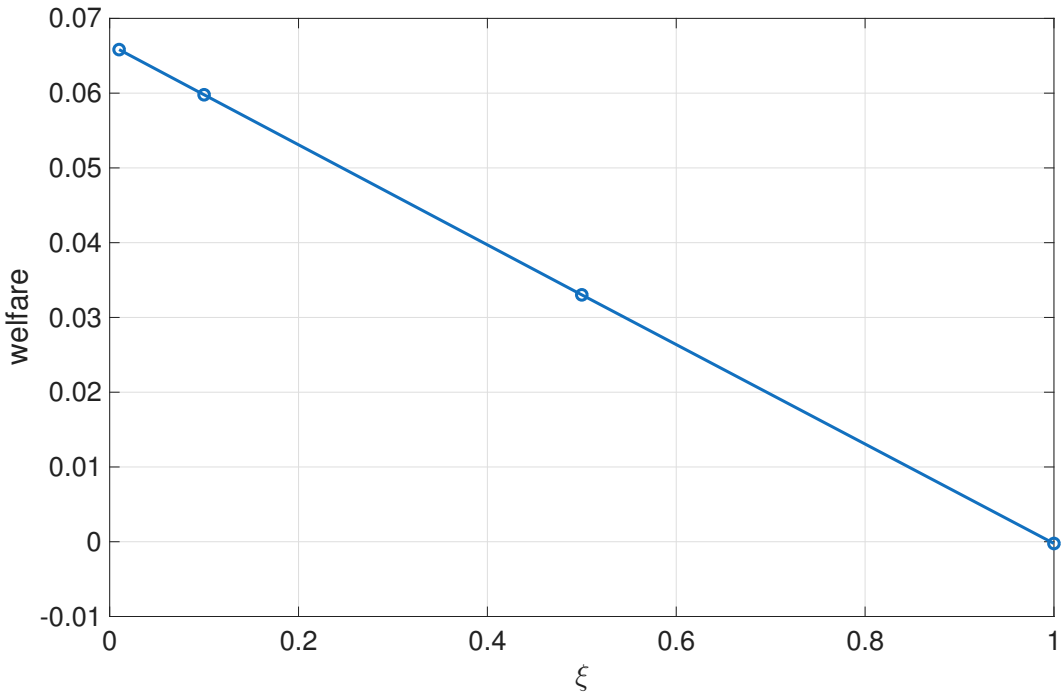


Figure 4: HOME QE WELFARE

Notes: The figure plots the welfare measure under a home QE shock against the degree of financial integration, ξ. Welfare is evaluated using the discounted utility of the home household relative to its steady-state benchmark, normalized by $C_s^{1-\sigma}$.

consistent with Proposition 3 and equation (5.2): financial integration spreads the lending-capacity benefits of home bond purchases across countries, reducing their domestic welfare effect.

Together, Figures 1-4 extend the analytical mechanisms to a dynamic environment with endogenous bond prices, inflation, and the policy rate.

Financial Integration and the Effects of home QE versus Union QE. We next examine the targeting mechanism in Proposition 4. Following an adverse home credit shock, represented by a decline in $\Theta_{H,t}$, we compare two prescribed interventions. Under *home QE*, all additional purchases are concentrated in home government bonds. Under *Union QE*, purchases are allocated across home and foreign government bonds in proportion to population. The two interventions have identical aggregate purchase paths, isolating the

effects of their country allocation.

The home intermediaries' leverage ratio $\Theta_{H,t}$ follows

$$\theta_{H,t} = \rho_\theta \theta_{H,t-1} + \varepsilon_{\theta,t}$$

We consider a negative innovation at $t = 0$, with all subsequent innovations and all other shocks set to zero. We set $\rho_\theta = 0.8$. The foreign leverage remains at its steady-state value. The QE response coefficient is scaled by steady-state intermediary financing capacity:

$$k_{\mathrm{QE},\theta} = \lambda_{\mathrm{QE}} \frac{\bar{\Theta}\bar{X}^{FI}}{\bar{Y}}, \qquad \lambda_{QE} \in \{0.5, 1, 1.5\}.$$

For a given credit shock, λ_{QE} controls the size of the intervention. At each scale, we evaluate $\xi \in \{0.01, 0.1, 0.5, 1\}$.

Let qe_t and qe_t^* denote deviations in the market value of central-bank holdings of home and foreign bonds, respectively, measured per resident of the issuing country and normalized by steady-state output per resident. Under home QE, their paths satisfy

$$qe_t = \rho_{qe} qe_{t-1} + k_{\mathrm{QE},\theta} \varepsilon_{\theta,t},$$
$$qe_t^* = \rho_{qe} qe_{t-1}^*.$$

Under Union QE, the corresponding paths are

$$qe_t = \rho_{qe} qe_{t-1} + (1-\gamma) k_{\mathrm{QE},\theta} \varepsilon_{\theta,t},$$
$$qe_t^* = \rho_{qe} qe_{t-1}^* + (1-\gamma) k_{\mathrm{QE},\theta} \varepsilon_{\theta,t},$$

where $1-\gamma$ and γ are the home and foreign population shares. Because the variables are expressed per resident of each issuing country, equal responses of qe_t and qe_t^* under Union QE allocate the aggregate intervention across the two countries in proportion to

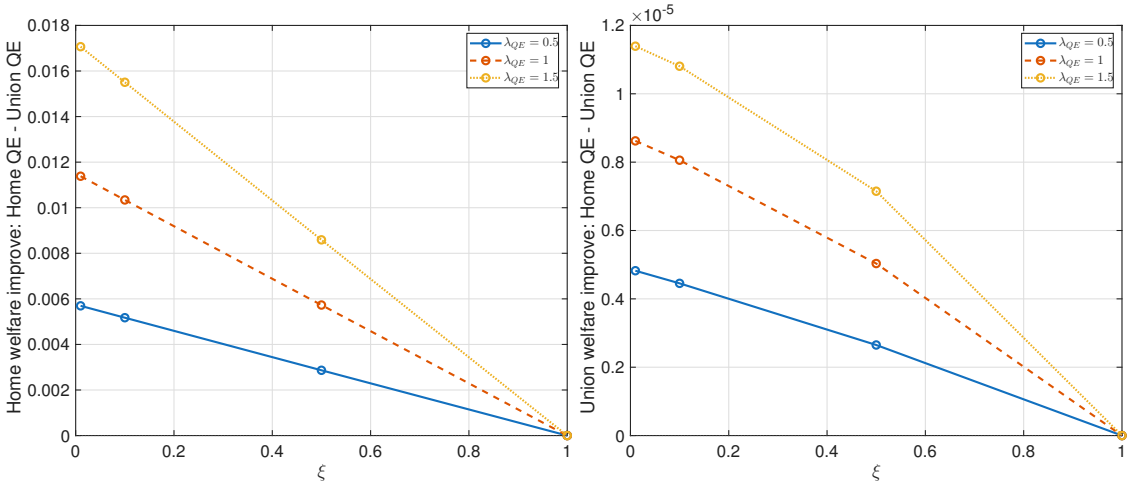


Figure 5: WELFARE UNDER HOME VS. PROPORTIONAL UNION QE

Notes: The left/right panels of the figure plot the difference in discounted home/union welfare between home QE and Union QE following a 1% adverse credit shock at home. Positive values indicate higher welfare under home QE. The horizontal axis measures the degree of financial integration, ξ. Home QE allocates all additional purchases to home government bonds, whereas Union QE allocates purchases across home and foreign government bonds in proportion to population shares. For each QE scale, the two schemes have identical aggregate QE paths. The three curves correspond to scale multipliers $\lambda_{QE} = 0.5, 1, 1.5$.

their populations.

In particular, define aggregate QE per union resident as

$$qe_t^A = (1-\gamma)qe_t + \gamma qe_t^*.$$

Starting from the steady state, both interventions imply

$$qe_t^{A,\text{Home}} = qe_t^{A,\text{Union}} = (1-\gamma)\lambda_{\text{QE}}\frac{\bar{\Theta}\bar{X}^{FI}}{\bar{Y}}\rho_{qe}^t, \qquad t \geq 0.$$

Thus, the aggregate QE paths coincide at every horizon. We set $\rho_{qe} = \rho_\theta = 0.8$, so the intervention and the direct loss of intermediation capacity decay at the same rate.

Figure 5 reports the welfare advantage of home QE over proportional Union QE. The left panel shows home welfare and the right panel shows union welfare. Both advantages decline with financial integration and converge to zero at full integration. When inter-

mediaries in both countries hold each government's debt in proportion to population, concentrating purchases in home bonds no longer concentrates the release of financing capacity at home. This is the dynamic counterpart of the targeting mechanism in Proposition 4. The exercise compares prescribed allocations at a common aggregate scale; it does not solve for the optimal allocation of QE.

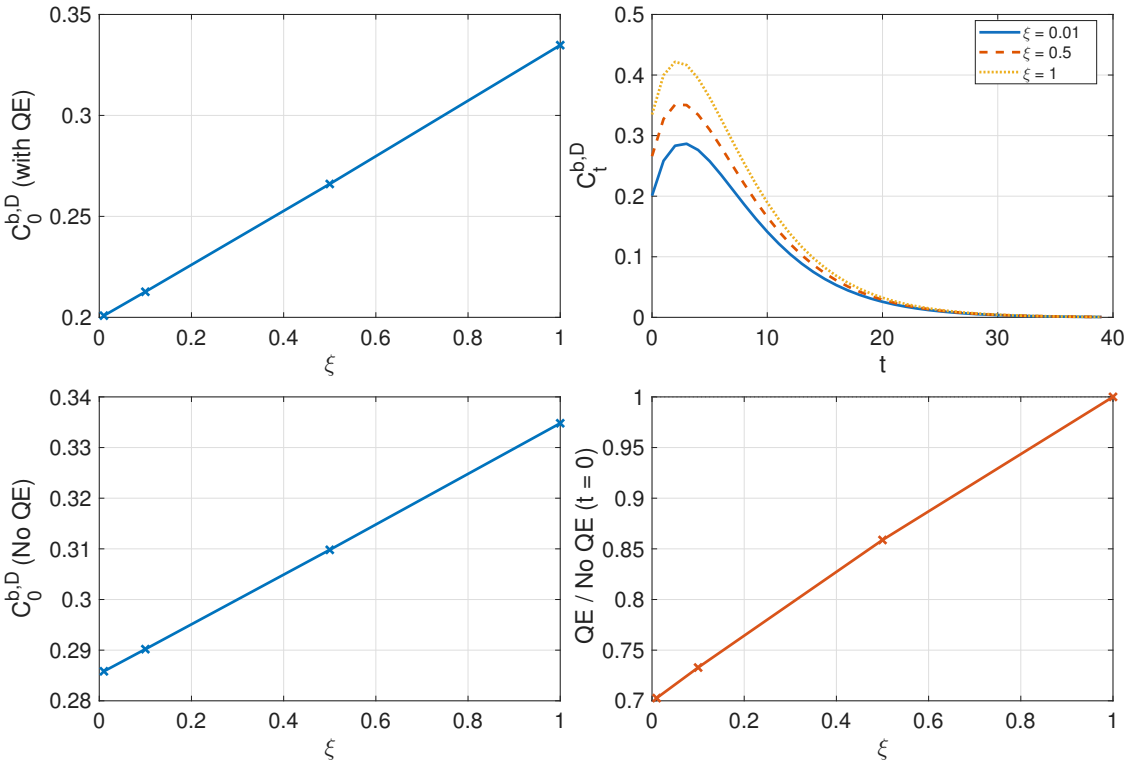


Figure 6: QE OFFSETS TO FISCAL TRANSFER

Notes: The figure plots the difference in home and foreign borrowers' consumption when the home government increases fiscal transfer and the central bank increases QE purchases of foreign government bonds. The top left panel plots the on-impact response of the two countries' borrower consumption difference relative to ξ; the right top panel plots the impulse response of the two countries' borrower consumption difference under $\xi \in \{0.01, 0.5, 1\}$; the bottom left panel plots the on-impact response of the two countries' borrower consumption difference relative to ξ following an increase in home fiscal transfers without additional QE; and the bottom right panel plots the ratio of the values in the top-left panel to those in the bottom-left panel.

Financial Integration and QE Offsets to Fiscal Redistribution. Finally, we examine the mechanism in Corollary 2 using a prescribed QE response to fiscal redistribution. At $t = 0$, the home government increases transfers to borrowers. In the intervention case, the central

bank simultaneously increases its purchases of foreign government bonds by the same amount, measured in the common steady-state output units. We compare this policy mix with the same transfer shock in the absence of additional QE.

The home transfer widens the difference between home and foreign borrower consumption. Purchases of foreign government bonds can reduce this difference by releasing financing capacity for foreign private lending. As financial integration rises, however, a larger share of foreign government bonds is held by home intermediaries. Foreign QE therefore releases more financing capacity at home, weakening its ability to offset the consumption differential.

Figure 6 illustrates these effects. The upper-left panel plots the impact consumption differential with QE, and the lower-left panel shows the corresponding differential without QE. Both increase with ξ. The lower-right panel reports the ratio of the differential with QE to the differential without QE. A ratio below one indicates that foreign QE narrows the gap; its increase toward one shows that this offset becomes weaker as integration rises. At $\xi = 1$, the ratio reaches one and the additional purchases no longer reduce the cross-country consumption difference. The upper-right panel shows that the ordering across integration levels persists during the transition back toward steady state.

7 Conclusion

We study how financial integration affects optimal monetary and fiscal policy in a two-country New Keynesian monetary union with savers, borrowers, and constrained financial intermediaries. National governments redistribute more from savers to borrowers than a coordinated planner because they do not internalize the full cross-border welfare costs of their transfers.

Financial integration has opposite effects on the domestic welfare benefits of fiscal transfers and QE. In our analytical benchmark, cross-border government-bond holdings

increase the domestic welfare gain from a debt-financed transfer by reducing the domestic private lending it displaces. In contrast, central-bank purchases of domestic bonds release lending capacity abroad when foreign intermediaries hold those bonds, reducing the domestic welfare gain from QE.

We also characterize the optimal allocation of a given amount of QE across countries. The central bank purchases more bonds per resident from the country where lending falls further below its efficient level. Financial integration reduces the welfare gain from this allocation relative to purchases proportional to population. When the central bank optimally reallocates QE in response to fiscal redistribution, greater integration leaves a larger cross-country difference in borrower consumption. Integration therefore amplifies the relative consumption effect of fiscal redistribution both with and without an optimal QE response.

Our quantitative analysis relaxes the simplifying assumptions and extends these findings to lifetime welfare. Greater financial integration raises the domestic welfare benefit of fiscal transfers while reducing that of QE. Following a credit shock, the welfare advantage of concentrating QE in the affected country declines with integration for both the country and the union. Integration also weakens the ability of foreign bond purchases to offset the borrower-consumption difference created by a domestic transfer.

Together, these results show that cross-border government-bond holdings strengthen the domestic benefits of national fiscal redistribution while weakening the common central bank's ability to direct the benefits of QE toward individual countries.

References

Alpanda, Sami, and Serdar Kabaca. 2020. "International Spillovers of Large-Scale Asset Purchases." *Journal of the European Economic Association*, 18(1): 342–391.

Bai, Yan, and Jing Zhang. 2012. "Financial Integration and International Risk Sharing." *Journal of International Economics*, 86(1): 17–32.

Bellifemine, Marco, Adrien Couturier, and Rustam Jamilov. 2025. "Monetary Unions with Heterogeneous Fiscal Space." *Journal of International Economics*, 156: 104092.

Berriel, Rafael, Eugenia Gonzalez-Aguado, Patrick J. Kehoe, and Elena Pastorino. 2024. "Is a Fiscal Union Optimal for a Monetary Union?" *Journal of Monetary Economics*, 141: 157–177.

Bi, Huixin, and Nora Traum. 2023. "Unconventional Monetary Policy and Local Fiscal Policy." *European Economic Review*, 156: 104476.

Bletzinger, Tilman, and Leopold von Thadden. 2021. "Designing QE in A Fiscally Sound Monetary Union." *European Economic Review*, 132: 103611.

Calvo, Guillermo A. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics*, 12(3): 383–398.

Chari, Varadarajan V., and Patrick J. Kehoe. 2007. "On the Need for Fiscal Constraints in a Monetary Union." *Journal of Monetary Economics*, 54(8): 2399–2408.

Clarida, Richard, Jordi Galí, and Mark Gertler. 2002. "A Simple Framework for International Monetary Policy Analysis." *Journal of Monetary Economics*, 49(5): 879–904.

Dmitriev, Mikhail, and Jonathan Hoddenbagh. 2019. "Optimal fiscal transfers in a monetary union." *Journal of International Economics*, 117: 91–108.

Farhi, Emmanuel, and Iván Werning. 2012. "Dealing with the Trilemma: Optimal Capital Controls with Fixed Exchange Rates." National Bureau of Economic Research Working Paper 18199.

Farhi, Emmanuel, and Iván Werning. 2017. "Fiscal Unions." *American Economic Review*, 107(12): 3788–3834.

Gabaix, Xavier, and Matteo Maggiori. 2015. "International Liquidity and Exchange Rate Dynamics." *The Quarterly Journal of Economics*, 130(3): 1369–1420.

Galí, Jordi, and Tommaso Monacelli. 2008. "Optimal Monetary and Fiscal Policy in a Currency Union." *Journal of International Economics*, 76(1): 116–132.

Gertler, Mark, and Peter Karadi. 2011. "A Model of Unconventional Monetary Policy." *Journal of Monetary Economics*, 58(1): 17–34.

Kabaca, Serdar, Renske Maas, Kostas Mavromatis, and Romanos Priftis. 2023. "Optimal Quantitative Easing in a Monetary Union." *European Economic Review*, 152: 104342.

Kempf, Hubert, and Leopold Von Thadden. 2013. "When Do Cooperation and Commitment Matter in A Monetary Union?" *Journal of International Economics*, 91(2): 252–262.

Kolasa, Marcin, and Grzegorz Wesołowski. 2020. "International Spillovers of Quantitative Easing." *Journal of International Economics*, 126: 103330.

Maćkowiak, Bartosz, and Sebastian Schmidt. 2025. "Fiscal Backing for Price Stability in A Monetary Union." *American Economic Journal: Macroeconomics*, 17(3): 237–269.

Silva, Dejanir H. 2023. "Optimal Fiscal Consolidation under Frictional Financial Markets." *The Economic Journal*, 133(652): 1537–1585.

Sims, Eric, and Jing Cynthia Wu. 2021. "Evaluating Central Banks' Tool Kit: Past, Present, and Future." *Journal of Monetary Economics*, 118: 135–160.

Sims, Eric, Jing Cynthia Wu, and Ji Zhang. 2023. "The Four Equation New Keynesian Model." *Review of Economics and Statistics*, 105(4): 931–947.

Woodford, Michael. 2001. "Fiscal Requirements for Price Stability." *Journal of Money, Credit and Banking*, 33(3): 669–728.

Woodford, Michael. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton, NJ:Princeton University Press.

Wu, Jing Cynthia, and Ji Zhang. 2019. "Global Effective Lower Bound and Unconventional Monetary Policy." *Journal of International Economics*, 118: 200–216.

Wu, Jing Cynthia, and Yinxi Xie. 2025. "Unconventional Monetary and Fiscal Policy." *Review of Economic Dynamics*, 56: 101259.

Wu, Jing Cynthia, Yinxi Xie, and Ji Zhang. 2024. "The Role of International Financial Integration in Monetary Policy Transmission." *IMF Economic Review*, 72(3): 944–990.

Appendices

A Model Details and Linearization

A.1 foreign-Country Counterparts and Auxiliary Conditions

The foreign saver has the consumption bundle

$$C_t^{s*} = \left[v^{*\frac{1}{\vartheta}} (C_{F,t}^{s*})^{\frac{\vartheta-1}{\vartheta}} + (1-v^*)^{\frac{1}{\vartheta}} (C_{H,t}^{s*})^{\frac{\vartheta-1}{\vartheta}} \right]^{\frac{\vartheta}{\vartheta-1}}$$

and the consumption price index

$$P_t^* = \left[v^* P_{F,t}^{*1-\vartheta} + (1-v^*) P_{H,t}^{*1-\vartheta} \right]^{\frac{1}{1-\vartheta}}.$$

Its demands for home- and foreign-produced goods are

$$C_{H,t}^{s*} = (1-v^*) \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\vartheta} C_t^{s*},$$
$$C_{F,t}^{s*} = v^* \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\vartheta} C_t^{s*}.$$

The financial intermediary's exact dividend payment is

$$P_{t+1}\Phi_{t+1}^{FI} = (R_{t+1} - I_t) Q_t B_t^{FI} + \left(I_t^{RE} - I_t \right) RE_{H,t}^{FI} + I_t P_t X_t^{FI}. \tag{A.1}$$

For the foreign central-bank portfolio, the balance-sheet identity is

$$QE_t^* = RE_t^*. \tag{A.2}$$

foreign bond-market clearing requires

$$B_t^{G,*} + B_t^{P,*} = B_t^{FI,*} + B_t^{CB,*}.$$

foreign goods-market clearing requires

$$\gamma Y_t^* = (1-\gamma)(C_{F,t}^s + C_{F,t}^b) + \gamma(C_{F,t}^{s,*} + C_{F,t}^{b,*}).$$

A.2 Calvo Pricing

Under Calvo pricing, the optimal reset price satisfies

$$\tilde{P}_{H,t} = \frac{\epsilon}{\epsilon - 1} \frac{X_{1,t}}{X_{2,t}},$$

where

$$X_{1,t} = P_{H,t}^{\epsilon} MC_t Y_t + \phi \mathbb{E}_t \Lambda_{t,t+1} X_{1,t+1},$$

and

$$X_{2,t} = P_{H,t}^{\epsilon} Y_t + \phi \mathbb{E}_t \Lambda_{t,t+1} X_{2,t+1}.$$

The home-good price index evolves according to

$$(P_{H,t})^{1-\epsilon} = \phi (P_{H,t-1})^{1-\epsilon} + (1-\phi)(\tilde{P}_{H,t})^{1-\epsilon}.$$

Aggregate firm dividends, in units of the country-specific consumption basket, are

$$\Phi_t^F = \frac{P_{H,t} Y_t - (1 - s_p) W_t L_t}{P_t}.$$

In a symmetric zero-inflation steady state,

$$\bar{P}_H = \frac{\epsilon}{\epsilon - 1}(1 - s_p)\frac{\bar{W}}{\bar{A}} = \frac{\bar{W}}{\bar{A}}, \qquad \frac{\bar{W}}{\bar{P}_H} = \bar{A}.$$

Thus, the steady real marginal cost entering Calvo pricing remains $(\epsilon - 1)/\epsilon$, while the subsidy raises the wage consistent with this pricing condition to its efficient level.

A.3 Central-Bank and Fiscal Accounting

The steady state has equal saver consumption and equal consumption price indices across countries. Net fund payments are zero in this case:

$$\bar{Z}_H = \bar{Z}_F = 0.$$

We normalize fund payments by common steady-state nominal output:

$$z_{j,t} \equiv \frac{Z_{j,t}}{\bar{P}\bar{Y}}.$$

Because steady-state fund payments are zero, this normalization coincides to first order with the deviation of CPI-deflated payments divided by steady-state real output. The linearized saver budget includes $z_{j,t}$, and the fund budget becomes

$$(1-\gamma)z_{H,t} + \gamma z_{F,t} = 0.$$

The central bank's union-wide balance-sheet surplus is rebated according to

$$(1-\gamma)P_t T_t^{CB} + \gamma P_t^* T_t^{CB,*} = (1-\gamma)R_t Q_{t-1} B_{t-1}^{CB} + \gamma R_t^* Q_{t-1}^* B_{t-1}^{CB,*} - I_{t-1}^{RE} RE_{t-1}^{agg}.$$

We assume equal real per-capita remittances across countries, that is,

$$T_t^{CB} = T_t^{CB,*}.$$

Together with the union-wide central-bank budget, this rule determines both country-specific remittances. The home fiscal authority's budget constraint is

$$P_t T_t^C + P_t G_t + P_t \mathcal{S}_t + B_{t-1}^G = Q_t B_t^G - \kappa Q_t B_{t-1}^G + P_t T_t^G + P_t T_t^{CB},$$

where real subsidy spending and its earmarked lump-sum tax satisfy

$$\mathcal{S}_t = s_p \frac{W_t L_t}{P_t}, \qquad T_t^{\text{sub}} = \mathcal{S}_t,$$

and total lump-sum taxes are

$$T_t^G = T_t^s + \varrho Q_{t-1} \frac{B_{t-1}^G}{P_{t-1}} + T_t^{\text{sub}}, \qquad T_t^s = \eta T_t^C.$$

The subsidy and its earmarked tax cancel from the government budget, leaving

$$T_t^C + \frac{B_{t-1}^G}{P_t} = \frac{Q_t (B_t^G - \kappa B_{t-1}^G)}{P_t} + T_t^s + \varrho Q_{t-1} \frac{B_{t-1}^G}{P_{t-1}} + T_t^{CB}.$$

The same cancellation occurs between firm dividends and the subsidy tax in the saver's income:

$$\frac{W_t L_t}{P_t} + \Phi_t^F - T_t^{\text{sub}} = \frac{P_{H,t} Y_t}{P_t}.$$

This is an accounting cancellation, not an absence of incentive effects: firms face subsidized marginal costs, whereas savers pay a lump-sum tax.

The full-bailout transfer is

$$X_t^b = (1+\kappa Q_t)\Pi_t^{-1}(\mathcal{B}_{t-1}^P + \mathcal{B}_{t-1}^G) - \varrho Q_{t-1}\mathcal{B}_{t-1}^G - T_t^{CB},$$

where $\mathcal{B}_{t-1}^P$ and $\mathcal{B}_{t-1}^G$ are the real values of private and public bonds. Because the production subsidy is financed by a separate tax, it does not alter this bailout specification.

A.4 Reference Steady State

In the symmetric zero-inflation reference state, saver labor supply and the production subsidy imply

$$\psi_L \bar{L}^\chi = (\bar{C}^s)^{-\sigma}, \qquad \bar{Y} = \bar{L} = \left[\frac{(1-z)^{-\sigma}}{\psi_L}\right]^{1/(\sigma+\chi)}.$$

Steady transfers and balance sheets are chosen consistently with $\bar{C}^s = (1-z)\bar{Y}$, $\bar{C}^b = z\bar{Y}$, and the financial equilibrium. These private-equilibrium restrictions do not determine the planner's welfare weight.

A.5 Log-Linear Equilibrium Conditions

The home-country variables, with foreign counterparts where applicable, are grouped as follows:

$$\{c_t^s, c_{H,t}^{s,*}, c_{H,t}^s, l_t, w_t, \lambda_t, i_t, \pi_t, \pi_{H,t}, c_t^b, c_{H,t}^{b,*}, c_{H,t}^b, q_t, r_t\},$$
$$\{b_t^{FI}, i_t^{RE}, re_t, d_t, y_t, x_{1,t}, x_{2,t}, \widetilde{mc}_t, \widetilde{p}_{H,t}, \nu_t^p, a_t, b_t^P, b_t^G\},$$
$$\{b_t^{cb}, qe_t, \omega_t, y_t^{agg}, \pi_t^{agg}, \delta_{i,t}, \tau_t^C, e_t\}.$$

Lowercase variables denote log deviations of the corresponding uppercase variables from steady state. For example, $a_t = \ln A_t - \ln A$.[9] Variables without time subscripts denote steady-state values. Policy instruments and asset positions are exceptions: we measure them as level deviations from steady state, normalized by steady-state output:

$$\tau_t^C \equiv \frac{T_t^C - T^C}{Y}, \qquad b_t^P \equiv \frac{B_t^P/P_t - B^P/P}{Y}, \qquad qe_t \equiv \frac{QE_t - QE}{Y},$$

$$d_t \equiv \frac{D_t/P_t - D/P}{Y}, \qquad re_t \equiv \frac{RE_t/P_t - RE/P}{Y},$$

[9] For a nominal variable, the lowercase variable denotes the log deviation of its real counterpart. For example, $w_t = \ln(W_t/P_t) - \ln(W/P)$.

and similarly for b_t^G, b_t^{CB}, and b_t^{FI}. We also define

$$z \equiv \frac{C^b}{C^s + C^b}.$$

A.6 Linearized Equations

The reference steady state includes the constant production subsidy $s_p = 1/\epsilon$ from Section 2. The subsidy-adjusted real marginal cost has steady-state value $(\epsilon - 1)/\epsilon$, and $\bar{W}/\bar{P}_H = \bar{A}$. The marginal-cost and Calvo log-linear equations below therefore retain their coefficients. For the pure quadratic welfare baseline, set $\psi = 0$, $g_t = g_t^* = 0$, and suppress government-purchase shocks in the general processes below.

Exogenous shock processes:

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t}, \tag{A.3}$$

$$qe_t = \rho_{qe} qe_{t-1} + \varepsilon_{qe,t}, \tag{A.4}$$

$$\delta_{i,t} = \rho_\delta \delta_{i,t-1} + \varepsilon_{\delta,t}, \tag{A.5}$$

$$\tau_t^C = \rho_\tau \tau_{t-1}^C + \varepsilon_{\tau,t}. \tag{A.6}$$

Labor supply:

$$\chi l_t = -\sigma c_t^s + w_t. \tag{A.7}$$

Saver's consumption Euler equation:

$$\sigma(\mathbb{E}_t c_{t+1}^s - c_t^s) = i_t - \mathbb{E}_t \pi_{t+1}, \tag{A.8}$$

$$\lambda_t = -\sigma(c_t^s - c_{t-1}^s) - \pi_t. \tag{A.9}$$

Borrower's consumption Euler equation:

$$\sigma(\mathbb{E}_t c_{t+1}^b - c_t^b) = \mathbb{E}_t(r_{t+1} - \pi_{t+1}). \tag{A.10}$$

Return and price of long-term bond:

$$r_t = \frac{\kappa}{R} q_t - q_{t-1}. \tag{A.11}$$

FI's financial constraint:

$$\frac{b^{FI}}{Y} q_t + b_t^{FI} = 0. \tag{A.12}$$

FI's balance-sheet condition after substituting $P_t X_t^{FI} = P_t \bar{X}^{FI} + \kappa Q_t B_{t-1}^{FI}$:

$$\frac{Qb^{FI}}{Y}(1-\kappa)q_t + Qb_t^{FI} - \kappa Qb_{t-1}^{FI} + \frac{\kappa Qb^{FI}}{Y}\pi_t + re_t = d_t. \tag{A.13}$$

FI's FOC:

$$\mathbb{E}_t\lambda_{t+1} + \frac{\beta R}{\Omega}\mathbb{E}_t r_{t+1} - \frac{\beta I}{\Omega} i_t = \omega_t, \tag{A.14}$$

$$i_t^{RE} = i_t. \tag{A.15}$$

Calvo pricing:

$$\tilde{p}_{H,t} = \tilde{x}_{1,t} - \tilde{x}_{2,t}, \tag{A.16}$$

$$\tilde{x}_{1,t} = (1-\phi\beta)(\tilde{mc}_t + y_t) + \phi\beta\mathbb{E}_t[\lambda_{t+1} + \tilde{x}_{1,t+1} + (\epsilon+1)\pi_{H,t+1}], \tag{A.17}$$

$$\tilde{x}_{2,t} = (1-\phi\beta)y_t + \phi\beta\mathbb{E}_t[\lambda_{t+1} + \tilde{x}_{2,t+1} + \epsilon\pi_{H,t+1}], \tag{A.18}$$

$$\tilde{mc}_t = w_t - a_t + (1-v)e_t. \tag{A.19}$$

Here $\tilde{p}_{H,t}$ is the log deviation of $\tilde{P}_{H,t}/P_{H,t}$, and

$$\begin{aligned}
\tilde{x}_{1,t} &= \ln\left(\frac{X_{1,t}}{P_{H,t}^{1+\epsilon}}\right) - \ln\left(\frac{X_1}{P_H^{1+\epsilon}}\right), \\
\tilde{x}_{2,t} &= \ln\left(\frac{X_{2,t}}{P_{H,t}^{\epsilon}}\right) - \ln\left(\frac{X_2}{P_H^{\epsilon}}\right), \\
\tilde{mc}_t &= \ln(MC_t/P_{H,t}) - \ln(MC/P_H).
\end{aligned}$$

Thus marginal cost is normalized by the producer price rather than CPI.

Production function:

$$y_t = a_t + l_t - v_t^p. \tag{A.20}$$

Price dispersion:

$$v_t^p = 0. \tag{A.21}$$

PPI inflation:

$$\pi_{H,t} = \frac{1-\phi}{\phi}\tilde{p}_{H,t}. \tag{A.22}$$

CPI inflation:

$$\pi_t = v\pi_{H,t} + (1-v)\pi_{F,t}, \tag{A.23}$$

$$\pi_t^* = v^*\pi_{F,t}^* + (1-v^*)\pi_{H,t}^*.$$

Since $P_{H,t} = P_{H,t}^*$ and $P_{F,t} = P_{F,t}^*$, $\pi_{H,t} = \pi_{H,t}^*$ and $\pi_{F,t} = \pi_{F,t}^*$.

The full-bailout assumption and the borrower's budget constraint imply that

$$(1-\psi)zc_t^b = qe_t + \tau_t^C - (1-\eta)[\tau_t^C + g_t]. \tag{A.24}$$

We decompose consumption. Consumption components remain log deviations:

$$c_{H,t}^s = \vartheta(1-v)e_t + c_t^s, \tag{A.25}$$

$$c_{H,t}^{s,*} = \vartheta v^* e_t + c_t^{s,*}, \tag{A.26}$$

$$c_{H,t}^b = \vartheta(1-v)e_t + c_t^b, \tag{A.27}$$

$$c_{H,t}^{b,*} = \vartheta v^* e_t + c_t^{b,*}. \tag{A.28}$$

Union-wide inflation and output:

$$\pi_t^{agg} = (1-\gamma)\pi_t + \gamma\pi_t^*, \tag{A.29}$$

$$y_t^{agg} = (1-\gamma)y_t + \gamma y_t^*. \tag{A.30}$$

Policy rate:

$$i_t = \phi_\pi \pi_t^{agg} + \phi_y y_t^{agg} + \delta_{i,t}. \tag{A.31}$$

Central bank bond purchases (QE):

$$\frac{Qb^{CB}}{Y} q_t + Qb_t^{CB} = qe_t, \tag{A.32}$$

$$qe_t = re_t. \tag{A.33}$$

Goods market clearing:

$$\begin{aligned} y_t &= \Psi_{ye} e_t + (1-\psi)(1-z)c_t^s \\ &\quad + v\left[qe_t + \eta\tau_t^C\right] + \frac{\gamma(1-v^*)}{1-\gamma}\left[qe_t^* + \eta\tau_t^{C*}\right]. \end{aligned} \tag{A.34}$$

Bond-market clearing:

$$b_t^P + b_t^G = b_t^{FI} + b_t^{CB}. \tag{A.35}$$

Government budget constraint: The subsidy spending $\mathcal{S}_t$ and its lump-sum financing $T_t^{\text{sub}} = \mathcal{S}_t$ cancel before linearization. Their deviations, if needed for the gross fiscal

accounts, are identical:

$$\frac{T_t^{\text{sub}} - \bar{T}^{\text{sub}}}{\bar{Y}} = \frac{\mathcal{S}_t - \bar{\mathcal{S}}}{\bar{Y}} = \frac{\bar{\mathcal{S}}}{\bar{Y}}(w_t + l_t), \qquad \frac{\bar{\mathcal{S}}}{\bar{Y}} = s_p$$

at the symmetric steady state. The net government budget is therefore unchanged. Define $\tau_t^{CB} = (T_t^{CB} - T^{CB})/Y$. Keeping the country-specific central-bank remittance explicit, and using $R = 1/\beta_b$, gives

$$\begin{aligned}(1-\eta)(\tau_t^C + g_t) = Qb_t^G + (\varrho - R)Qb_{t-1}^G \\ + \frac{Qb^G}{Y}\left[q_t + (\varrho - R)q_{t-1} - R(r_t - \pi_t)\right] + \tau_t^{CB}.\end{aligned} \tag{A.36}$$

The remittances satisfy the union central-bank budget in Section 2. Country-specific remittances follow the equal-real-per-capita rule specified in Appendix A.3. Around the symmetric steady state, this implies $\tau_t^{CB} = \tau_t^{CB,*}$. Under the full-bailout specification, remittances cancel from the borrower-spending equation (A.24). Under the maintained full-bailout specification, τ_t^{CB} cancels from the borrower-spending expression and does not enter the reduced IS and Phillips curves.

The terms of trade satisfies

$$e_t = e_{t-1} + \pi_{F,t} - \pi_{H,t}. \tag{A.37}$$

The foreign-country equations are analogous, except for the consumption Euler equation.

The cross-country consumption relation follows from the risk-sharing rule implemented by the union fund. Log-linearizing that rule around the symmetric steady state gives

$$\sigma(c_t^s - c_t^{s*}) + p_t - p_t^* = 0.$$

The consumption price indices imply

$$p_t - p_t^* = (1 - v - v^*)e_t.$$

As a result,

$$c_t^s - c_t^{s*} = \frac{1}{\sigma}(v + v^* - 1)e_t. \tag{A.38}$$

We use (A.38) to replace the foreign saver's Euler equation in the equilibrium.

B Derivation of the IS and Phillips Curves

Starting from the linearized conditions in Appendix A, we aggregate goods demand and marginal costs, derive the IS and Phillips curves, and assemble the independent reduced system. The final two subsections give the flexible-price benchmark and the corresponding output-gap representation.

B.1 Notation and Coefficient Definitions

We linearize around a symmetric zero-inflation steady state with $P_H = P_F = P = P^*$, equal per-capita output, and equal expenditure shares across countries, with the production subsidy and its lump-sum financing specified in Section 2. All deviations are measured from this subsidized steady state. The subsidy rate is constant and its financing cancels from the net fiscal budget, so it adds no shock or policy term to the IS and Phillips curves. For the welfare baseline in Section 3, set $\psi = 0$, giving $\alpha = 1 - z$; the formulas below retain ψ and government-purchase deviations for extensions. Throughout this section, define

$$\begin{aligned} \alpha &= (1-\psi)(1-z), & d &= v + v^* - 1, & h &= \chi + \sigma/\alpha, \\ f_t &= qe_t + \eta\tau_t^C, & f_t^* &= qe_t^* + \eta\tau_t^{C*}, & & \\ M_t^H &= vf_t + (1-v)f_t^*, & M_t^F &= (1-v^*)f_t + v^* f_t^*. & & \end{aligned}$$

The home-bias parameterization implies $\gamma(1-v^*)/(1-\gamma) = 1-v$ and $(1-\gamma)(1-v)/\gamma = 1-v^*$. The coefficients are

$$\begin{aligned} \Psi_{ye} &= (1-v)\left[\vartheta(v+v^*) - \frac{\alpha d}{\sigma}\right], \\ \Psi_{ye}^* &= -(1-v^*)\left[\vartheta(v+v^*) - \frac{\alpha d}{\sigma}\right], \\ K_e &= \Psi_{ye} - \Psi_{ye}^* + \frac{\alpha d}{\sigma} = \vartheta(v+v^*)(2-v-v^*) + \frac{\alpha d^2}{\sigma}, \\ \zeta_H &= 1 - v - \frac{\sigma\Psi_{ye}}{\alpha}, \qquad \zeta_F = -(1-v^*) - \frac{\sigma\Psi_{ye}^*}{\alpha}, \\ \kappa_p &= \frac{(1-\phi)(1-\phi\beta)}{\phi}. \end{aligned}$$

For any variable z_t, $\Delta z_t = z_t - z_{t-1}$ denotes its first difference.

B.2 Aggregating Goods Market Clearing

We use the symmetric zero-inflation steady state specified in Section 2: $P_H = P_F = P = P^*$, equal per-capita output and consumption, and equal government-spending shares. The notation $\alpha, d, f_t, M_t^H, M_t^F$ and the coefficients $\Psi_{ye}, \Psi_{ye}^*, K_e$ are defined in Section B.1. The linearized price indices imply

$$\begin{aligned} p_t - p_{H,t} &= (1-v)e_t, & p_t - p_{F,t} &= -ve_t, \\ p_t^* - p_{H,t} &= v^* e_t, & p_t^* - p_{F,t} &= -(1-v^*)e_t. \end{aligned}$$

These expressions also apply to the Cobb–Douglas limit $\vartheta = 1$. Combining product demands with goods-market clearing yields

$$\begin{aligned} y_t &= \vartheta(1-v)(v+v^*)e_t + \alpha[vc_t^s + (1-v)c_t^{s*}] + M_t^H, \\ y_t^* &= -\vartheta(1-v^*)(v+v^*)e_t + \alpha[(1-v^*)c_t^s + v^* c_t^{s*}] + M_t^F. \end{aligned} \tag{B.1}$$

Here we have used the borrower-consumption equation to combine borrower spending and government purchases into f_t and f_t^*. Using $c_t^{s*} = c_t^s - de_t/\sigma$ from (A.38),

$$\begin{aligned} y_t &= (1-v)\left[\vartheta(v+v^*) - \frac{\alpha d}{\sigma}\right] e_t + \alpha c_t^s + M_t^H \\ &= \Psi_{ye} e_t + \alpha c_t^s + M_t^H, \end{aligned} \tag{B.2}$$

where $\Psi_{ye} \equiv (1-v)\left[\vartheta(v+v^*) - \frac{\alpha d}{\sigma}\right]$.

Similarly, using $c_t^s = c_t^{s*} + de_t/\sigma$,

$$\begin{aligned} y_t^* &= -(1-v^*)\left[\vartheta(v+v^*) - \frac{\alpha d}{\sigma}\right] e_t + \alpha c_t^{s*} + M_t^F \\ &= \Psi_{ye}^* e_t + \alpha c_t^{s*} + M_t^F, \end{aligned} \tag{B.3}$$

where $\Psi_{ye}^* \equiv -(1-v^*)\left[\vartheta(v+v^*) - \frac{\alpha d}{\sigma}\right]$.

Subtracting gives

$$\begin{aligned} y_t - y_t^* &= \left(\Psi_{ye} - \Psi_{ye}^* + \frac{\alpha d}{\sigma}\right) e_t + d(f_t - f_t^*) \\ &= \left[\vartheta(v+v^*)(2-v-v^*) + \frac{\alpha d^2}{\sigma}\right] e_t + d(f_t - f_t^*). \end{aligned} \tag{B.4}$$

B.3 Deriving Marginal Costs

Nominal marginal cost net of the constant subsidy is $MC_t = (1 - s_p)W_t / A_t$, with $s_p = 1/\epsilon$. Since s_p is constant, its log deviation is zero, so the marginal-cost log-linearization retains the same coefficients. PPI-deflated marginal cost is $\widetilde{MC}_t = MC_t / P_{H,t}$, and w_t denotes the log deviation of the CPI-deflated wage. Hence

$$\begin{aligned}
\tilde{mc}_t &= w_t - a_t + (1 - v)e_t \\
&= \chi(y_t - a_t) + \sigma c_t^s - a_t + (1 - v)e_t \\
&= \left(\chi + \frac{\sigma}{\alpha}\right) y_t - (1 + \chi)a_t + \left(1 - v - \frac{\sigma \Psi_{ye}}{\alpha}\right) e_t - \frac{\sigma}{\alpha} M_t^H \\
&= h y_t + \zeta_H e_t - (1 + \chi)a_t - \frac{\sigma}{\alpha} M_t^H,
\end{aligned} \tag{B.5}$$

where $h \equiv \chi + \frac{\sigma}{\alpha}$ and $\zeta_H \equiv 1 - v - \frac{\sigma \Psi_{ye}}{\alpha}$.

For the foreign country, the producer-price deflator is $P_{F,t}$, which gives

$$\begin{aligned}
\tilde{mc}_t^* &= w_t^* - a_t^* - (1 - v^*)e_t \\
&= \chi(y_t^* - a_t^*) + \sigma c_t^{s*} - a_t^* - (1 - v^*)e_t \\
&= \left(\chi + \frac{\sigma}{\alpha}\right) y_t^* - (1 + \chi)a_t^* - \left(1 - v^* + \frac{\sigma \Psi_{ye}^*}{\alpha}\right) e_t - \frac{\sigma}{\alpha} M_t^F \\
&= h y_t^* + \zeta_F e_t - (1 + \chi)a_t^* - \frac{\sigma}{\alpha} M_t^F,
\end{aligned} \tag{B.6}$$

where $h \equiv \chi + \frac{\sigma}{\alpha}$ and $\zeta_F \equiv -(1 - v^*) - \frac{\sigma \Psi_{ye}^*}{\alpha}$.

B.4 Phillips Curves and IS Curves in Output Levels

Subtracting (A.18) from (A.17) and using (A.16) and (A.22) yields

$$\pi_{H,t} = \kappa_p \tilde{mc}_t + \beta \mathbb{E}_t \pi_{H,t+1}. \tag{B.7}$$

Substitution of (B.5) and (B.6) gives the two output-level Phillips curves (B.11)–(B.12) in Section B.6. Combining the saver Euler equation (A.8) with (B.2) gives the home IS curve (B.8); the foreign derivation uses (B.3) and gives (B.9). Subtracting the two goods-market conditions and using (A.38) yields (B.4), which is the relative-output equation (B.10). The complete output-level system is collected in Section B.6; the flexible-price and gap representations are developed in the final two subsections.

B.5 Policy Rule and Cross-Country Conversions

The six endogenous reduced-system variables are $\{y_t, y_t^*, \pi_{H,t}, \pi_{F,t}, i_t, e_t\}$. The policy and shock paths are $\{qe_t, qe_t^*, \tau_t^C, \tau_t^{C*}, g_t, g_t^*, a_t, a_t^*, \delta_{i,t}\}$; foreign exogenous processes are specified analogously to the home ones. The union inflation index satisfies

$$\begin{aligned}\pi_t^{agg} &= [(1-\gamma)v + \gamma(1-v^*)]\pi_{H,t} + [\gamma v^* + (1-\gamma)(1-v)]\pi_{F,t} \\ &= (1-\gamma)\pi_{H,t} + \gamma\pi_{F,t}.\end{aligned}$$

The common Taylor rule is therefore (B.20). Together, the relative-price identity (B.13) and CPI–PPI conversions (B.14)–(B.15) complete the independent system listed in Section B.6. In the optimized policy problems of Section 4, the chosen interest-rate path replaces the Taylor rule.

B.6 The Complete Reduced System

The home IS curve can be written in CPI or PPI form:

$$\begin{aligned}\mathbb{E}_t\Delta y_{t+1} &= \frac{\alpha}{\sigma}(i_t - \mathbb{E}_t\pi_{t+1}) + \Psi_{ye}\mathbb{E}_t\Delta e_{t+1} + \mathbb{E}_t\Delta M_{t+1}^H \\ &= \frac{\alpha}{\sigma}(i_t - \mathbb{E}_t\pi_{H,t+1}) + \left[\Psi_{ye} - \frac{\alpha(1-v)}{\sigma}\right]\mathbb{E}_t\Delta e_{t+1} + \mathbb{E}_t\Delta M_{t+1}^H.\end{aligned} \tag{B.8}$$

The foreign counterpart is

$$\begin{aligned}\mathbb{E}_t\Delta y_{t+1}^* &= \frac{\alpha}{\sigma}(i_t - \mathbb{E}_t\pi_{t+1}^*) + \Psi_{ye}^*\mathbb{E}_t\Delta e_{t+1} + \mathbb{E}_t\Delta M_{t+1}^F \\ &= \frac{\alpha}{\sigma}(i_t - \mathbb{E}_t\pi_{F,t+1}) + \left[\Psi_{ye}^* + \frac{\alpha(1-v^*)}{\sigma}\right]\mathbb{E}_t\Delta e_{t+1} + \mathbb{E}_t\Delta M_{t+1}^F.\end{aligned} \tag{B.9}$$

Using the consumption relation (A.38), the relative-output condition replaces the foreign IS curve in the independent system:

$$\begin{aligned}y_t - y_t^* &= K_e e_t + d(f_t - f_t^*) \\ &= \left[\vartheta(v+v^*)(2-v-v^*) + \frac{\alpha d^2}{\sigma}\right](e_{t-1} - \pi_{H,t} + \pi_{F,t}) + d(f_t - f_t^*).\end{aligned} \tag{B.10}$$

The Phillips curves are

$$\pi_{H,t} = \beta\mathbb{E}_t\pi_{H,t+1} + \kappa_p\left[hy_t + \zeta_H e_t - (1+\chi)a_t - \frac{\sigma}{\alpha}M_t^H\right], \tag{B.11}$$

$$\pi_{F,t} = \beta \mathbb{E}_t \pi_{F,t+1} + \kappa_p \left[h y_t^* + \zeta_F e_t - (1+\chi) a_t^* - \frac{\sigma}{\alpha} M_t^F \right]. \tag{B.12}$$

The relative-price identity and CPI–PPI conversions are

$$e_t = e_{t-1} - \pi_{H,t} + \pi_{F,t}, \tag{B.13}$$

$$\pi_t = \pi_{H,t} + (1-v)\Delta e_t, \tag{B.14}$$

$$\pi_t^* = \pi_{F,t} - (1-v^*)\Delta e_t. \tag{B.15}$$

In particular, $\mathbb{E}_t \Delta e_{t+1} = \mathbb{E}_t(\pi_{F,t+1} - \pi_{H,t+1})$. Consumption is recovered from

$$y_t = \Psi_{ye} e_t + \alpha c_t^s + M_t^H, \tag{B.16}$$

$$(1-\psi) z c_t^b = q e_t + \tau_t^C - (1-\eta)\tau_t^C, \tag{B.17}$$

$$y_t^* = \Psi_{ye}^* e_t + \alpha c_t^{s*} + M_t^F, \tag{B.18}$$

$$(1-\psi) z c_t^{b*} = q e_t^* + \tau_t^{C*} - (1-\eta)\tau_t^{C*}. \tag{B.19}$$

The union inflation index simplifies to $\pi_t^{agg} = (1-\gamma)\pi_{H,t} + \gamma \pi_{F,t}$, and the common policy rule is

$$\begin{aligned} i_t = {} & \phi_\pi [(1-\gamma)\pi_{H,t} + \gamma \pi_{F,t}] \\ & + \phi_y [(1-\gamma) y_t + \gamma y_t^*] + \delta_{i,t}. \end{aligned} \tag{B.20}$$

Given the paths of $\{q e_t, q e_t^*, \tau_t^C, \tau_t^{C*}, a_t, a_t^*, \delta_{i,t}\}$ and the inherited terms of trade, the six endogenous variables are $\{y_t, y_t^*, \pi_{H,t}, \pi_{F,t}, e_t, i_t\}$. The independent equations are (B.8), (B.10), (B.11), (B.12), (B.13), and (B.20). Equation (B.9) is retained as the foreign IS representation, not as an additional independent restriction.

B.7 Flexible-Price Benchmark and Output Gaps

For this benchmark, we set the deviations of QE, fiscal transfers and government spending to zero in both countries while retaining the same steady state. The flexible-price economy has its own terms of trade e_t^{flex}; it need not equal the actual e_t. With $\tilde{mc}_t^{\text{flex}} = \tilde{mc}_t^{\text{flex},*} = 0$,

$$\begin{aligned} h y_t^{\text{flex}} &= (1+\chi) a_t - \zeta_H e_t^{\text{flex}}, \\ h y_t^{\text{flex},*} &= (1+\chi) a_t^* - \zeta_F e_t^{\text{flex}}, \\ y_t^{\text{flex}} - y_t^{\text{flex},*} &= K_e e_t^{\text{flex}}. \end{aligned} \tag{B.21}$$

Since $hK_e + \zeta_H - \zeta_F = 1 + \chi K_e$, these equations imply

$$e_t^{\text{flex}} = \frac{1+\chi}{1+\chi K_e}(a_t - a_t^*). \tag{B.22}$$

Define $x_t = y_t - y_t^{\text{flex}}$, $x_t^* = y_t^* - y_t^{\text{flex},*}$, and $\widehat{e}_t = e_t - e_t^{\text{flex}}$. The marginal-cost gaps are

$$\begin{aligned} \tilde{mc}_t &= hx_t + \zeta_H \widehat{e}_t - \frac{\sigma}{\alpha} M_t^H, \\ \tilde{mc}_t^* &= hx_t^* + \zeta_F \widehat{e}_t - \frac{\sigma}{\alpha} M_t^F. \end{aligned} \tag{B.23}$$

These gap representations retain the difference between actual and flexible-price relative prices. Section B.6 uses output levels and does not require this alternative benchmark representation.

B.8 Alternative Output-Gap Representation

We use the flexible-price benchmark in Section B.7, with zero policy deviations and its own e_t^{flex}. Define its real interest rates r_t^{flex} and $r_t^{\text{flex},*}$ through

$$\begin{aligned} \mathbb{E}_t \Delta y_{t+1}^{\text{flex}} &= \frac{\alpha}{\sigma} r_t^{\text{flex}} + \Psi_{ye} \mathbb{E}_t \Delta e_{t+1}^{\text{flex}}, \\ \mathbb{E}_t \Delta y_{t+1}^{\text{flex},*} &= \frac{\alpha}{\sigma} r_t^{\text{flex},*} + \Psi_{ye}^* \mathbb{E}_t \Delta e_{t+1}^{\text{flex}}. \end{aligned} \tag{B.24}$$

These are CPI-based real rates; the two countries need not have the same real rate even with a common nominal rate. Subtracting the benchmark IS equations from their actual counterparts gives

$$\begin{aligned} \mathbb{E}_t \Delta x_{t+1} &= \frac{\alpha}{\sigma}(i_t - \mathbb{E}_t \pi_{t+1} - r_t^{\text{flex}}) + \Psi_{ye} \mathbb{E}_t \Delta \widehat{e}_{t+1} + \mathbb{E}_t \Delta M_{t+1}^H, \\ \mathbb{E}_t \Delta x_{t+1}^* &= \frac{\alpha}{\sigma}(i_t - \mathbb{E}_t \pi_{t+1}^* - r_t^{\text{flex},*}) + \Psi_{ye}^* \mathbb{E}_t \Delta \widehat{e}_{t+1} + \mathbb{E}_t \Delta M_{t+1}^F. \end{aligned} \tag{B.25}$$

The relative-output-gap equation is

$$x_t - x_t^* = K_e \widehat{e}_t + d(f_t - f_t^*). \tag{B.26}$$

If the relative-price gap is eliminated, both foreign output and policy differences must be retained:

$$\mathbb{E}_t \Delta \widehat{e}_{t+1} = \frac{1}{K_e}\left[\mathbb{E}_t \Delta x_{t+1} - \mathbb{E}_t \Delta x_{t+1}^* - d(\mathbb{E}_t \Delta f_{t+1} - \mathbb{E}_t \Delta f_{t+1}^*)\right]. \tag{B.27}$$

Consequently, the equivalent home gap IS equation is

$$\left(1-\frac{\Psi_{ye}}{K_e}\right)\mathbb{E}_t\Delta x_{t+1} = \frac{\alpha}{\sigma}(i_t - \mathbb{E}_t\pi_{t+1} - r_t^{\text{flex}}) - \frac{\Psi_{ye}}{K_e}\mathbb{E}_t\Delta x^*_{t+1} + \left(v - \frac{\Psi_{ye}d}{K_e}\right)\mathbb{E}_t\Delta f_{t+1} + \left(1 - v + \frac{\Psi_{ye}d}{K_e}\right)\mathbb{E}_t\Delta f^*_{t+1}. \tag{B.28}$$

The Phillips curves in gap form are

$$\pi_{H,t} = \beta\mathbb{E}_t\pi_{H,t+1} + \kappa_p\left[hx_t + \zeta_H\widehat{e}_t - \frac{\sigma}{\alpha}M_t^H\right], \tag{B.29}$$

$$\pi_{F,t} = \beta\mathbb{E}_t\pi_{F,t+1} + \kappa_p\left[hx^*_t + \zeta_F\widehat{e}_t - \frac{\sigma}{\alpha}M_t^F\right]. \tag{B.30}$$

The terms-of-trade gap remains present unless further restrictions imply $\widehat{e}_t = 0$. The benchmark does not replace actual relative prices in the output-level system of Section B.6.

C Efficient Allocation and Welfare Derivations

C.1 Efficient allocation

For the welfare derivations, define

$$d = v + v^* - 1 = 1 - \nu, \qquad T = \vartheta\nu(2-\nu), \qquad K_{FB} = T + \frac{d^2}{\sigma}, \qquad \Phi = \frac{\phi}{(1-\beta\phi)(1-\phi)}. \tag{C.1}$$

Let λ_H and λ_F be the multipliers on the home and foreign resource constraints. For a CES consumption basket, define its shadow resource requirement by

$$Q_H = [v\lambda_H^{1-\vartheta} + (1-v)\lambda_F^{1-\vartheta}]^{\frac{1}{1-\vartheta}}, \tag{C.2}$$

$$Q_F = [(1-v^*)\lambda_H^{1-\vartheta} + v^*\lambda_F^{1-\vartheta}]^{\frac{1}{1-\vartheta}}. \tag{C.3}$$

The consumption first-order conditions are

$$(C^s)^{-\sigma} = Q_H, \quad \delta(C^b)^{-\sigma} = Q_H, \quad (C^{s*})^{-\sigma} = Q_F, \quad \delta(C^{b*})^{-\sigma} = Q_F. \tag{C.4}$$

The labor conditions are

$$\psi_L L^\chi = \lambda_H A, \qquad \psi_L(L^*)^\chi = \lambda_F A^*. \tag{C.5}$$

These conditions imply that

$$\lambda_{H,t}^{1-\vartheta} = \frac{v^* \left(C_t^s\right)^{\sigma(\vartheta-1)} - (1-v)\left(C_t^{s*}\right)^{\sigma(\vartheta-1)}}{d}.$$

Together with the two resource constraints, these equations characterize the exact first best. At a symmetric steady state, these conditions imply $C^b/C^s = \delta^{1/\sigma}$ and therefore

$$z^e = \frac{\delta^{1/\sigma}}{1+\delta^{1/\sigma}}^{10}, \qquad \bar{Y}^e = \left[\frac{(1-z^e)^{-\sigma}}{\psi_L}\right]^{1/(\sigma+\chi)}. \tag{C.6}$$

Log-linearizing the allocation rules and the two resource constraints gives

$$(\sigma+\chi) y_t^{A,e} = (1+\chi) a_t^A, \tag{C.7}$$

$$\chi y_t^{D,e} + e_t^e = (1+\chi) a_t^D, \tag{C.8}$$

$$y_t^{D,e} = K_{FB} e_t^e. \tag{C.9}$$

Solving this system gives equations (3.6)– (3.8).

To obtain the welfare approximation, we expand household utility and the resource constraints to second order around the subsidized symmetric steady state. The production subsidy removes the steady-state markup distortion, and the matched welfare weight $\delta = [z/(1-z)]^\sigma$ removes first-order gains from redistributing consumption between savers and borrowers. Price dispersion contributes a second-order term proportional to squared inflation. We substitute the second-order resource constraints into utility, collect population-weighted averages and differences, and complete the output and consumption squares to obtain (3.9). Details can be found in Appendix C.2.

C.2 The welfare loss function

For any two regional variables, define the population-weighted average and the home-minus-foreign difference:

$$u^A = (1-\gamma)u + \gamma u^*, \qquad u^D = u - u^*, \qquad \Gamma = \gamma(1-\gamma).$$

Also define

$$d = 1-\nu, \qquad T = \vartheta\nu(2-\nu), \qquad K = K_{FB} = T + \frac{d^2}{\sigma}.$$

[10] $z^e = z$ under (3.5).

Step 1. Utility and production. Let $u(C) = (C^{1-\sigma} - 1)/(1-\sigma)$, with $u(C) = \log C$ when $\sigma = 1$, and write

$$U = u(C^s) + \delta u(C^b) - \frac{\psi_L L^{1+\chi}}{1+\chi}, \qquad w = \frac{U - \bar{U}}{\bar{U}_{C^s}\bar{C}^s}.$$

The reference steady state satisfies

$$\bar{C}^s = (1-z)\bar{Y}, \quad \bar{C}^b = z\bar{Y}, \quad \bar{U}_{C^s} = \bar{U}_{C^b}, \quad -\bar{U}_L\bar{L} = \bar{U}_{C^s}\bar{Y}.$$

Define

$$m = (1-z)c^s + zc^b, \qquad S = (1-z)(c^s)^2 + z(c^b)^2.$$

Using $\exp(x) = 1 + x + x^2/2$ in utility gives:

$$w = \frac{1}{1-z}\left[m - l + \frac{1-\sigma}{2}S - \frac{1+\chi}{2}l^2\right].$$

For home firms, let $r_j = \log(P_H(j)/P_H)$ and define their mean relative log price and price dispersion by

$$\bar{r}_H = \frac{1}{1-\gamma}\int_0^{1-\gamma} r_j\, dj, \qquad D_H = \frac{1}{1-\gamma}\int_0^{1-\gamma}(r_j - \bar{r}_H)^2\, dj.$$

Expanding the CES price index gives

$$\frac{1}{1-\gamma}\int_0^{1-\gamma}\exp[(1-\epsilon)r_j]\, dj = 1 \quad \Longrightarrow \quad \bar{r}_H = \frac{\epsilon - 1}{2}D_H.$$

Thus

$$\begin{aligned}\Delta_H &= \frac{1}{1-\gamma}\int_0^{1-\gamma}\exp(-\epsilon r_j)\, dj \\ &= 1 - \epsilon\bar{r}_H + \frac{\epsilon^2}{2}D_H = 1 + \frac{\epsilon}{2}D_H.\end{aligned}$$

Since $L = (Y/A)\Delta_H$ and D_H is already second order,

$$l = y - a + \frac{\epsilon}{2}D_H, \qquad l^2 = (y-a)^2.$$

Substitution gives

$$w_0 = \frac{1}{1-z}\left[m - y + a + \frac{1-\sigma}{2}S - \frac{\epsilon}{2}D_H - \frac{1+\chi}{2}(y-a)^2\right].$$

Step 2. Eliminate $m - y$ using the resource constraint. Let $e = \log(P_F/P_H)$, $v = 1 - \gamma\nu$, and $v^* = 1 - (1-\gamma)\nu$. The home CPI satisfies

$$\log \frac{P}{P_H} = (1-v)e + \frac{1-\vartheta}{2} v(1-v)e^2.$$

The total physical quantity of goods per unit of the home consumption basket is

$$\begin{aligned} F_H(e) &= \left(\frac{P}{P_H}\right)^{\vartheta} [v + (1-v)\exp(-\vartheta e)] \\ &= 1 + \frac{\vartheta}{2} v(1-v)e^2. \end{aligned}$$

Similarly, $F_F(e) = 1 + \vartheta v^*(1-v^*)e^2/2$, and

$$\vartheta\left[(1-\gamma)v(1-v) + \gamma v^*(1-v^*)\right] = \Gamma T.$$

Add the two physical resource constraints and use $(1-z)\exp(c^s) + z\exp(c^b) = 1 + m + S/2$. This gives:

$$\begin{aligned} &(1-\gamma)\left(y + \frac{1}{2}y^2\right) + \gamma\left(y^* + \frac{1}{2}(y^*)^2\right) \\ &\quad = (1-\gamma)\left(m + \frac{1}{2}S\right) + \gamma\left(m^* + \frac{1}{2}S^*\right) + \frac{\Gamma T}{2}e^2. \end{aligned}$$

Hence

$$\begin{aligned} &(1-\gamma)(m-y) + \gamma(m^* - y^*) \\ &\quad = \frac{1}{2}\left[(1-\gamma)(y^2 - S) + \gamma((y^*)^2 - S^*)\right] - \frac{\Gamma T}{2}e^2. \end{aligned}$$

Substitute this into the population-weighted sum of utilities:

$$\begin{aligned} 2(1-z)[(1-\gamma)w_0 + \gamma w_0^*] = 2[&(1-\gamma)a + \gamma a^*] \\ &- (1-\gamma)[\epsilon D_H - y^2 + (1+\chi)(y-a)^2 + \sigma S] \\ &- \gamma[\epsilon D_F - (y^*)^2 + (1+\chi)(y^* - a^*)^2 + \sigma S^*] - \Gamma T e^2. \end{aligned}$$

This eliminates the policy-dependent linear terms while keeping their second-order contribution.

Step 3. Replace price dispersion by squared inflation. For either country, the Calvo probability of not resetting prices is ϕ. The reset log price satisfies $\widetilde{p}_t - \bar{p}_{t-1} = \pi_t^P/(1-\phi)$,

where $\bar{p}_{t-1}$ is the mean of old log prices. Therefore,

$$\begin{aligned} D_t &= \phi D_{t-1} + \phi(1-\phi)(\widetilde{p}_t - \bar{p}_{t-1})^2 \\ &= \phi D_{t-1} + \frac{\phi}{1-\phi}(\pi_t^P)^2. \end{aligned}$$

Multiply by β^t and sum:

$$(1-\beta\phi)\mathbb{E}_0 \sum_{t\geq 0} \beta^t D_t = \phi D_{-1} + \frac{\phi}{1-\phi}\mathbb{E}_0 \sum_{t\geq 0} \beta^t (\pi_t^P)^2.$$

For fixed initial dispersion, define

$$\Phi = \frac{\phi}{(1-\beta\phi)(1-\phi)}, \qquad A_\pi = \epsilon\Phi.$$

Union welfare can then be written as $\mathcal{W} = -\mathbb{E}_0 \sum_t \beta^t \mathcal{L}_t +$ policy-independent terms, with

$$\begin{aligned} 2(1-z)\mathcal{L} = A_\pi[(1-\gamma)\pi_H^2 + \gamma\pi_F^2] \\ + (1-\gamma)[-y^2 + (1+\chi)(y-a)^2 + \sigma S] \\ + \gamma[-(y^*)^2 + (1+\chi)(y^*-a^*)^2 + \sigma S^*] + \Gamma T e^2. \end{aligned}$$

The substitution holds for the discounted sums. The inflation rates π_H and π_F correspond to price dispersion within each country.

Step 4. Expand and complete squares to obtain welfare loss. Use the following identities:

$$\begin{aligned} S - m^2 &= z(1-z)(c^s - c^b)^2, \\ (1-\gamma)u^2 + \gamma(u^*)^2 &= (u^A)^2 + \Gamma(u^D)^2, \\ (1-\gamma)uh + \gamma u^* h^* &= u^A h^A + \Gamma u^D h^D. \end{aligned}$$

The last two follow by substituting $u = u^A + \gamma u^D$, $u^* = u^A - (1-\gamma)u^D$ and expanding.

First-order goods-market clearing gives

$$y^A = m^A, \qquad y^D = dm^D + Te.$$

First-order relations suffice inside quadratic terms. Define $q^{\text{alloc}} = m^D - de/\sigma$ and expand:

$$\begin{aligned}(y^D)^2 + \sigma T (q^{\text{alloc}})^2 &= (dm^D + Te)^2 + \sigma T \left(m^D - \frac{d}{\sigma} e \right)^2 \\ &= (d^2 + \sigma T)(m^D)^2 + \left(T^2 + \frac{Td^2}{\sigma} \right) e^2 \\ &= K \left[\sigma (m^D)^2 + Te^2 \right].\end{aligned}$$

Thus

$$\sigma (m^D)^2 + Te^2 = \frac{(y^D)^2}{K} + \frac{\sigma T}{K} (q^{\text{alloc}})^2.$$

Let $b^{\text{gap}} = c^s - c^b$ and $b^{\text{gap},*} = c^{s*} - c^{b*}$. Substitute these identities into the loss and expand $(y - a)^2$:

$$\begin{aligned}2(1-z)\mathcal{L} = {} & A_\pi [(1-\gamma)\pi_H^2 + \gamma \pi_F^2] \\ & + (\sigma + \chi)(y^A)^2 - 2(1+\chi) a^A y^A \\ & + \Gamma \left[(\chi + K^{-1})(y^D)^2 - 2(1+\chi) a^D y^D \right] \\ & + \sigma z (1-z) \left[(1-\gamma)(b^{\text{gap}})^2 + \gamma (b^{\text{gap},*})^2 \right] \\ & + \Gamma \frac{\sigma T}{K} (q^{\text{alloc}})^2 + (1+\chi)[(1-\gamma)a^2 + \gamma (a^*)^2].\end{aligned}$$

Complete squares using $Ay^2 - 2Bay = A(y - Ba/A)^2 - B^2 a^2 / A$. Define

$$y^{A,e} = \frac{1+\chi}{\sigma + \chi} a^A, \qquad y^{D,e} = \frac{1+\chi}{\chi + K^{-1}} a^D, \qquad x^A = y^A - y^{A,e}, \quad x^D = y^D - y^{D,e}.$$

The remaining terms depend only on exogenous productivity and can be removed from the objective. Restoring time subscripts gives the **period welfare loss function**:

$$\begin{aligned}\mathcal{L}_t = \frac{1}{2(1-z)} \Big\{ & A_\pi [(1-\gamma)\pi_{H,t}^2 + \gamma \pi_{F,t}^2] + (\sigma + \chi)(x_t^A)^2 \\ & + \Gamma (\chi + K^{-1})(x_t^D)^2 \\ & + \sigma z (1-z) \left[(1-\gamma)(b_t^{\text{gap}})^2 + \gamma (b_t^{\text{gap},*})^2 \right] \\ & + \Gamma \frac{\sigma T}{K} (q_t^{\text{alloc}})^2 \Big\}.\end{aligned} \tag{C.10}$$

Here $\Gamma = \gamma(1-\gamma)$, $K = K_{FB}$, and $A_\pi = \epsilon \Phi$, so the coefficients match the original equation.

Define $b = b^{\text{gap}}$, $b^* = b^{\text{gap},*}$. To obtain the welfare representation in (3.9), recall that

$$m_t^D = (1-z)c_t^{s,D} + zc_t^{b,D} = c_t^{s,D} - zb_t^D.$$

The sharing condition (A.38) therefore implies

$$q_t^{\text{alloc}} = m_t^D - \frac{d}{\sigma}e_t = \left(c_t^{s,D} - \frac{d}{\sigma}e_t\right) - zb_t^D = -zb_t^D.$$

Substituting this equilibrium relation into (C.10) gives (3.9).

D Policy Derivations

D.1 Union-average stabilization

Define

$$\pi^A = (1-\gamma)\pi_H + \gamma\pi_F, \qquad \pi^D = \pi_H - \pi_F.$$

Expanding squares again,

$$(1-\gamma)\pi_H^2 + \gamma\pi_F^2 = (\pi^A)^2 + \Gamma(\pi^D)^2,$$
$$(1-\gamma)b^2 + \gamma(b^*)^2 = (b^A)^2 + \Gamma(b^D)^2.$$

Therefore, (C.10) becomes

$$\mathcal{L}_0 = \frac{\mathcal{J}^A + \Gamma\mathcal{J}^D}{2(1-z)},$$

$$\mathcal{J}^A = A_\pi(\pi^A)^2 + (\sigma+\chi)(y^A - y^{A,e})^2 + \sigma z(1-z)(b^A)^2,$$
$$\mathcal{J}^D = A_\pi(\pi^D)^2 + (\chi + K^{-1})(y^D - y^{D,e})^2 + \sigma z(1-z)(b^D)^2 + \frac{\sigma T}{K}(q^{\text{alloc}})^2.$$

The constraints must also separate. Use the reduced equilibrium in Section 2.7, with $qe = qe^* = 0$, no binding bounds or adjustment costs on policy instruments, and fiscally admissible transfers. Since $s = c^s$, $m = s - zb$, and $s^D = de/\sigma$,

$$s^A = y^A + zb^A, \qquad q^{\text{alloc}} = -zb^D, \qquad y^D = Ke - dzb^D.$$

Average marginal cost is

$$mc^A = \sigma s^A + \chi y^A - (1+\chi)a^A = (\sigma+\chi)(y^A - y^{A,e}) + \sigma z b^A.$$

The average Phillips curve is therefore:

$$pi_t^A - \beta \mathbb{E}_t \pi_{t+1}^A = \kappa_p \left[(\sigma + \chi)(y_t^A - y_t^{A,e}) + \sigma z b_t^A \right], \qquad \kappa_p = \Phi^{-1}. \tag{D.1}$$

The relative constraints are

$$\pi_t^D - \beta \mathbb{E}_t \pi_{t+1}^D = \kappa_p \left[(1 + \chi K)(e_t - e_t^e) - \chi dz b_t^D \right],$$
$$\pi_t^D = -\Delta e_t, \qquad e_t^e = \frac{y_t^{D,e}}{K}.$$

Both the objective and the constraints separate. Optimize the average component and drop the positive factor $1/(1-z)$:

$$\min_{\{\pi_t^A, y_t^A, b_t^A\}_{t \geq 0}} \frac{1}{2} \mathbb{E}_0 \sum_{t \geq 0} \beta^t \left[A_\pi (\pi_t^A)^2 + (\sigma + \chi)(y_t^A - y_t^{A,e})^2 + \sigma z (1-z)(b_t^A)^2 \right]. \tag{D.2}$$

The constraint is (D.1).

The common interest rate and average fiscal transfer can jointly set all three squares to zero. Goods clearing and the saver Euler equation then give the implementing policies

$$\tau_t^{C,A} = \frac{z}{\eta} \frac{1+\chi}{\sigma+\chi} a_t^A, \tag{D.3}$$

$$i_t = -\frac{\sigma(1+\chi)}{\sigma+\chi} (a_t^A - \mathbb{E}_t a_{t+1}^A), \tag{D.4}$$

proving the union-average statement in the coordination benchmark of Section 4. The relative component is optimized separately.

D.2 National deviation and the settlement benchmark

Proof of Proposition 1. A first-order expansion of the home-country national authority's objective in equation (3.2), combined with first-order goods clearing, gives

$$w_{H,t} = \frac{1-v}{1-z} Z_t, \qquad Z_t = m_t^D - \vartheta(v + v^*) e_t. \tag{D.5}$$

With the efficient steady state, define

$$U_e = 1 + \chi K_e, \qquad A_e = \frac{(1-z)d}{\sigma} - \vartheta(v + v^*). \tag{D.6}$$

Let $f_t^D = \eta(\tau_t^C - \tau_t^{C*})$. The maintained relative equilibrium implies

$$(1+\beta+\kappa_p U_e)e_t - e_{t-1} - \beta \mathbb{E}_t e_{t+1} = -\kappa_p \chi d f_t^D, \qquad Z_t = f_t^D + A_e e_t,$$

where productivity terms vanish at the steady state. Let $r_e \in (0,1)$ be the smaller root of $\beta r_e^2 - (1+\beta+\kappa_p U_e) r_e + 1 = 0$. Normalize the date of the deviation to zero for this derivation. Starting from the same reference at date t gives the same continuation-welfare effect; discounting that effect back to date zero multiplies it by $\beta^t > 0$.

Consider a one-period home deviation $\tau_0^C = \varepsilon$, with $\tau_t^C = 0$ for $t > 0$, holding the common interest-rate and foreign transfer paths fixed. Its bounded first-order relative-price response is

$$e_t = -\kappa_p \chi d \, r_e^{t+1} \eta \varepsilon.$$

Substituting e_t into equation (D.5) and then taking the discounted summation yield the first-order welfare effect of the transfer shock τ_0^C:

$$\left.\frac{\partial \mathcal{W}_H}{\partial \tau_0^C}\right|_{\text{coord}} = \frac{\eta(1-v)}{1-z}\left[1 - \frac{\chi d A_e}{U_e}(1-r_e)\right] > 0. \tag{D.7}$$

If $A_e \leq 0$, the term in brackets is at least one. If $A_e > 0$, its positivity follows from $U_e - \chi d A_e = 1 + \chi\vartheta(v+v^*) > 0$. The symmetric foreign deviation is also strictly profitable. Thus the coordinated reference is not a Nash equilibrium for the general parameters in Proposition 1. When $\chi = 0$ or $d = 0$, (D.7) reduces to the simpler expression $\eta(1-v)/(1-z) > 0$.

Extension to sufficiently small shocks. Let $\mathbf{s} = \{a_t, a_t^*, u_t, u_t^*\}_{t\geq 0}$ denote the paths of productivity and cost-push shocks, with cost-push wedges entering the country-specific Phillips curves additively. Around the steady state, the allocation and its marginal conditional continuation-welfare response to a unilateral transfer deviation depend continuously on sufficiently small bounded shock paths. At zero shocks, (D.7) establishes a strictly positive home derivative, and the analogous foreign derivative is also strictly positive. Continuity therefore gives a neighborhood of zero shocks in which both derivatives remain positive, with the common interest-rate path and the other country's transfer path held fixed. A sufficiently small positive one-period deviation at date t raises the deviating country's conditional expected welfare. Discounting continuation welfare back to date zero multiplies its gain by $\beta^t > 0$ and does not change its sign. This proves the shock extension in Proposition 1.

D.3 Monetary–fiscal separation and the fiscal Nash benchmark

Proof of Lemma 1. Set $\chi = 0$. The relative Phillips curve and the terms-of-trade identity are

$$p_t - \beta \mathbb{E}_t p_{t+1} = \kappa_p (e_t - a_t^D), \qquad p_t = -\Delta e_t. \tag{D.8}$$

Substitution gives (4.3). Fiscal transfers do not appear, so the bounded path of e_t and relative inflation is invariant to the fiscal regime. For $a_t^D = \rho_D a_{t-1}^D + \varepsilon_t^D$, $|\rho_D| < 1$, this path is

$$e_t = r_0 e_{t-1} + \frac{\kappa_p r_0}{1 - \beta \rho_D r_0} a_t^D, \tag{D.9}$$

$$r_0 = \frac{1 + \beta + \kappa_p - \sqrt{(1 + \beta + \kappa_p)^2 - 4\beta}}{2\beta} \in (0, 1). \tag{D.10}$$

For the average block, let $c_t^{s,A}$ denote the population-weighted average of saver consumption log deviations and let f_t^A be the effective average transfer. Goods clearing and the consumption-gap definition imply

$$y_t^A = (1 - z) c_t^{s,A} + f_t^A, \qquad b_t^A = c_t^{s,A} - \frac{f_t^A}{z}. \tag{D.11}$$

At $\chi = 0$, substituting these identities into the average welfare loss and completing the squares gives the following expression, up to policy-independent terms and a positive common factor:

$$\epsilon \Phi (\pi_t^A)^2 + \sigma (1 - z) \left(c_t^{s,A} - \frac{a_t^A}{\sigma} \right)^2 + \frac{\sigma}{z} \left(f_t^A - \frac{z a_t^A}{\sigma} \right)^2. \tag{D.12}$$

For fixed fiscal paths, the final square is outside the central bank's control. The central bank therefore sets the first two squares to zero. The saver Euler equation then recovers equation (4.4). This proves Lemma 1.

Proof of Corollary 1. *No-shock benchmark.* Under the conditions in Corollary 1, let $\Lambda > 0$ be the common marginal union resource value in the symmetric no-shock allocation. Under coordination, borrower consumption satisfies

$$\delta (C^{b,C})^{-\sigma} = \Lambda, \qquad \delta (C^{b*,C})^{-\sigma} = \Lambda. \tag{D.13}$$

Holding the other two policy choices fixed, the primitive national welfare and resource

constraints therefore give

$$\frac{\partial \mathcal{W}_H}{\partial C^b} = \delta u'(C^b) - v\Lambda, \qquad \frac{\partial \mathcal{W}_F}{\partial C^{b*}} = \delta u'(C^{b*}) - v^*\Lambda. \tag{D.14}$$

The two interior fiscal best responses satisfy

$$\delta (C^{b,N})^{-\sigma} = v\Lambda, \qquad \delta (C^{b*,N})^{-\sigma} = v^*\Lambda. \tag{D.15}$$

Because $u(\cdot)$ is strictly concave and the marginal resource costs are constant, these first-order conditions define the unique fiscal best responses. Dividing (D.15) by (D.13) gives

$$\left(\frac{C^{b,N}}{C^{b,C}}\right)^{-\sigma} = v, \qquad \left(\frac{C^{b*,N}}{C^{b*,C}}\right)^{-\sigma} = v^*, \tag{D.16}$$

which establishes the no-shock ratios in Corollary 1.

The full-bailout equation with $G = QE = 0$ gives $C^b = \text{constant} + \eta T^C$, and symmetrically abroad, so each best response is implementable by a fiscal transfer. Under $\chi = 0$, marginal labor disutility is constant and additional borrower demand changes output and labor without changing Λ. Lemma 1 establishes that the common central bank's best response features zero inflation and the coordinated saver allocation. Given $e_{-1} = 0$, its relative-price equation gives the bounded no-shock solution $e = \pi^D = 0$. Settlement positions across countries offset one another and sum to zero for the union. The central bank and both fiscal authorities therefore satisfy their best-response conditions simultaneously, establishing the stated Nash allocation.

Finally, the special case nests Proposition 1. Evaluating (D.14) at coordination gives

$$\left.\frac{\partial \mathcal{W}_H}{\partial C^b}\right|_C = (1-v)\Lambda > 0, \qquad \left.\frac{\partial \mathcal{W}_H}{\partial T^C}\right|_C = \eta(1-v)\Lambda > 0. \tag{D.17}$$

With the normalization $\Lambda = 1/(1-z)$, this is precisely the $\chi = 0$ derivative in (D.7). Proposition 1 therefore gives the positive slope at coordination, while Corollary 1 follows that slope to the unique national fiscal optimum.

Common productivity shocks. Under common productivity shocks and no cost-push wedges, $a_t^D = 0$. Given $e_{-1} = 0$, the bounded relative-price solution in Lemma 1 has $e_t = \pi_t^D = 0$. Common shocks change the marginal resource value $\Lambda_t > 0$ without changing the settlement incidence shares. At each date, the coordinated and national

first-order conditions are

$$\delta(C_t^{b,C})^{-\sigma} = \Lambda_t, \qquad \delta(C_t^{b,N})^{-\sigma} = v\Lambda_t,$$

and symmetrically to the foreign country, with v^*. Dividing these conditions gives (4.7) and (4.8) along the common productivity-shock path, including the no-shock benchmark.

Sufficiently small one-time asymmetric and cost-push shocks. Let $\mathbf{s}_{t_0}$ denote a one-time vector of productivity and cost-push shocks at date t_0. For $t \geq t_0$, define

$$R_{H,t}(\mathbf{s}_{t_0}) = \frac{C_t^{b,N}(\mathbf{s}_{t_0})}{C_t^{b,C}(\mathbf{s}_{t_0})}, \qquad R_{F,t}(\mathbf{s}_{t_0}) = \frac{C_t^{b*,N}(\mathbf{s}_{t_0})}{C_t^{b*,C}(\mathbf{s}_{t_0})}.$$

At zero shocks, the no-shock benchmark gives $R_{H,t}(\mathbf{0}) = v^{-1/\sigma} > 1$ and $R_{F,t}(\mathbf{0}) = (v^*)^{-1/\sigma} > 1$. Let

$$\Delta = \min\{v^{-1/\sigma} - 1, (v^*)^{-1/\sigma} - 1\} > 0.$$

On the maintained locally stable equilibrium branch, the allocations are continuous in the shock vector and the one-time impulse responses are uniformly bounded over subsequent dates. Their convergence to the no-shock allocation therefore yields a neighborhood $\mathcal{N}$ of zero such that, for $\mathbf{s}_{t_0} \in \mathcal{N}$ and every $t \geq t_0$,

$$|R_{H,t}(\mathbf{s}_{t_0}) - R_{H,t}(\mathbf{0})| < \Delta/2, \qquad |R_{F,t}(\mathbf{s}_{t_0}) - R_{F,t}(\mathbf{0})| < \Delta/2.$$

Both ratios remain strictly above one. This proves (4.5) and (4.6) at impact and along the subsequent equilibrium path, completing the proof of Corollary 1.

D.4 QE transmission and targeting with cross-border bond holdings

Proof of Lemmas 2 and 3. Let $\mathcal{G}_t$ be the first-order deviation in the market value of home government debt, $Q_t B_t^G / P_t$, normalized by steady-state output, and let $\mathcal{T}_t^{CB}$ denote central-bank remittances in the same units. At fixed prices and returns, the fiscal budget gives

$$\mathcal{G}_t = (\bar{R} - \varrho)\mathcal{G}_{t-1} + (1 - \eta)\tau_t^C - \mathcal{T}_t^{CB}. \tag{D.18}$$

Combining the borrower budget, the full-bailout transfer, and the government budget gives $C_t^b = Q_t(B_t^P + B_t^G)/P_t + \eta T_t^C$. Let $\mathcal{B}_{H,t}$ denote the government bonds of either issuer held by home intermediaries after QE, measured in market value per resident and in the same normalized deviation units. Define $\mathcal{B}_{F,t}$ analogously for the foreign country. The

fixed-share allocation is

$$\begin{pmatrix} \mathcal{B}_{H,t} \\ \mathcal{B}_{F,t} \end{pmatrix} = \begin{pmatrix} 1-\gamma\xi & \gamma\xi \\ (1-\gamma)\xi & 1-(1-\gamma)\xi \end{pmatrix} \begin{pmatrix} \mathcal{G}_t - qe_t \\ \mathcal{G}_t^* - qe_t^* \end{pmatrix}.$$

Unlike the endogenous bond portfolios in Wu, Xie, and Zhang (2024), these fixed shares isolate the incidence of cross-holdings. Subtracting the two rows gives

$$\mathcal{B}_{H,t} - \mathcal{B}_{F,t} = (1-\xi)(\mathcal{G}_t^D - qe_t^D). \tag{D.19}$$

The binding leverage constraint fixes total intermediary bond value, so local private lending changes by $-\mathcal{B}_{H,t}$. The borrower budget therefore gives $zc_t^b = \eta\tau_t^C + \mathcal{G}_t - \mathcal{B}_{H,t}$, and symmetrically abroad. Subtracting the two equations yields

$$zc_t^{b,D} = \eta\tau_t^{C,D} + \xi\mathcal{G}_t^D + (1-\xi)qe_t^D. \tag{D.20}$$

At the impact date t, inherited deviations and contemporaneous remittance changes are zero, so $\mathcal{G}_t^D = (1-\eta)\tau_t^{C,D}$. Substitution gives

$$c_t^{b,D} = \frac{\eta + (1-\eta)\xi}{z}\tau_t^{C,D} + \frac{1-\xi}{z}qe_t^D. \tag{D.21}$$

Differentiating proves (4.10) and (5.1). The transfer multiplier increases with ξ, while the QE multiplier decreases with ξ.

Proof of Propositions 2 and 3. Under the portfolio and return restrictions preceding Proposition 2, the dividend identity (A.1) applies to the intermediary's total bond portfolio, including bonds of both issuers and local private lending. The binding leverage constraint and fixed real equity fix its total bond value. Common fixed bond prices and returns make the portfolio's composition irrelevant for its payoff; the reserve-deposit spread is zero. Fixed prices and the common maturity structure also leave the value of inherited positions, and hence the equity contribution, unchanged. Thus the reallocation creates no additional first-order net intermediary payout to savers. The utility function in (3.2) and goods-market clearing remain unchanged.

In the efficient equilibrium, the first-order home welfare change without productivity shocks is $w_{H,t} = (m_t - y_t)/(1-z)$, where $m_t = (1-z)c_t^s + zc_t^b$. Under $\chi = 0$, the relative-price equation in Lemma 1 contains neither transfers nor QE. The symmetric no-shock initial condition and the maintained bounded equilibrium selection give $e_t = 0$ and

$c_t^{s,D} = 0$. Goods-market clearing then gives $y_t = vm_t + (1-v)m_t^*$, so

$$w_{H,t} = \frac{1-v}{1-z} m_t^D = \frac{(1-v)z}{1-z} c_t^{b,D}.$$

Thus the same national welfare expansion applies to impact welfare. Substituting (D.21), the marginal impact welfare effects of one-period policy changes at date t, evaluated at the efficient steady state with all other policy paths fixed, are

$$\frac{\partial w_{H,t}}{\partial \tau_t^C} = \frac{1-v}{1-z}[\eta + (1-\eta)\xi],$$
$$\frac{\partial w_{H,t}}{\partial qe_t} = \frac{1-v}{1-z}(1-\xi).$$

Differentiating with respect to ξ proves (4.11) and (5.2). The strict fiscal inequality requires $\eta < 1$: some transfer financing must take the form of debt. To offset the impact effect of a transfer on the borrower-consumption difference, purchases must satisfy

$$\Delta qe_t^D = -\frac{\eta + (1-\eta)\xi}{1-\xi} \Delta \tau_t^{C,D}, \qquad \xi < 1.$$

The required tilt is unbounded as $\xi \uparrow 1$ for a nonzero transfer perturbation. At $\xi = 1$, the coefficient on qe_t^D in (D.21) is zero, so QE cannot offset the effect of the fiscal deviation on the borrower consumption differential.

Dynamic budget and equilibrium restrictions. Unchanged net intermediary payouts do not eliminate the other terms in the saver's budget. In particular, $T_t^U = \Phi_t^F + (\Phi_t^{FI} - X_t^{FI}) - T_t^G - X_t^b$ still includes taxes and bailout transfers, while deposits are an additional state. Public debt and central-bank remittances must satisfy (D.18) at subsequent dates. For example, set QE and its remittances to zero and consider $\tau_0^{C,D} = \varepsilon$ with subsequent transfer deviations zero. Writing $a_G = \bar{R} - \varrho$, the budgets imply $\mathcal{G}_t^D = (1-\eta)a_G^t \varepsilon$ and, for $t \geq 1$, $zc_t^{b,D} = \xi(1-\eta)a_G^t \varepsilon$. Thus the later welfare terms generally do not vanish: even under the maintained $e_t = c_t^{s,D} = 0$ restriction, discounted summation would give

$$\left.\frac{\partial \mathcal{W}_H}{\partial \tau_0^C}\right|_0 = \frac{1-v}{1-z}\left[\eta + \frac{\xi(1-\eta)}{1-\beta a_G}\right], \qquad |\beta a_G| < 1.$$

This is a conditional budget calculation, not an equilibrium solution. With fixed bond returns and inflation, the deterministic linearized borrower Euler equation also requires $c_{t+1}^b = c_t^b$, which a temporary consumption response generally violates. A dynamic

extension must solve that equation jointly with fiscal solvency, the allocation of central-bank remittances across countries, and settlement financing. The impact comparisons in Propositions 2 and 3 do not resolve those requirements.

The conditional union QE problem. Let ℓ_i^{QE} be private lending per resident after the QE intervention, ℓ_i baseline lending before the intervention, and ℓ_i^e efficient lending conditional on the other policy paths. Measure lending in the same linearized units as the existing purchase variables qe_t and qe_t^*. The fixed cross-holding shares and binding intermediary constraints imply

$$\begin{pmatrix} \ell_H^{\mathrm{QE}} \\ \ell_F^{\mathrm{QE}} \end{pmatrix} = \begin{pmatrix} \ell_H \\ \ell_F \end{pmatrix} + \begin{pmatrix} 1-\gamma\xi & \gamma\xi \\ (1-\gamma)\xi & 1-(1-\gamma)\xi \end{pmatrix} \begin{pmatrix} qe_t \\ qe_t^* \end{pmatrix}. \tag{D.22}$$

For the one-period extension, a lending gap $\ell_i^{\mathrm{QE}} - \ell_i^e$ changes borrower consumption in log deviations by $(\ell_i^{\mathrm{QE}} - \ell_i^e)/z$. Since $\bar{C}^b = z\bar{Y}$, the corresponding consumption-level change is $\bar{Y}(\ell_i^{\mathrm{QE}} - \ell_i^e)$. With $\chi = 0$, marginal resource costs are constant. At the symmetric efficient reference, the linear utility gain equals the resource cost. The change in normalized borrower surplus is therefore

$$\frac{\delta}{\bar{U}_{C^s}\bar{C}^s}\left\{u\big(\bar{C}^b + \bar{Y}(\ell_i^{\mathrm{QE}} - \ell_i^e)\big) - u(\bar{C}^b) - u'(\bar{C}^b)\bar{Y}(\ell_i^{\mathrm{QE}} - \ell_i^e)\right\}.$$

Its second-order expansion gives the local welfare loss $\mu(\ell_i^{\mathrm{QE}} - \ell_i^e)^2/(2z^2)$, where

$$\mu = -\frac{\delta u''(\bar{C}^b)(\bar{C}^b)^2}{\bar{U}_{C^s}\bar{C}^s} = \frac{\sigma z}{1-z} > 0.$$

The equality uses $\delta u'(\bar{C}^b) = u'(\bar{C}^s)$ and the CRRA utility function. Symmetry gives a common μ. The additional real portfolio-adjustment cost has curvature $\kappa_{\mathrm{QE}} > 0$, expressed in the same normalized welfare units. The quadratic approximation to conditional union welfare is

$$\mathcal{W}(qe_t^D) = C_0 - \frac{\mu}{2z^2}\sum_{i\in\{H,F\}} n_i(\ell_i^{\mathrm{QE}} - \ell_i^e)^2 - \frac{\kappa_{\mathrm{QE}}}{2}\left[n_H qe_t^2 + n_F(qe_t^*)^2\right], \qquad n_H = 1-\gamma, \quad n_F = \gamma. \tag{D.23}$$

The purchase cost is an additional assumption, rather than a term derived from (3.9). Define the pre-QE shortfall difference as $D = (\ell_H^e - \ell_H) - (\ell_F^e - \ell_F)$.

Proof of Proposition 4. Let $n_H = 1-\gamma$ and $n_F = \gamma$. At fixed $qe_t^A = n_H qe_t + n_F qe_t^*$, write $qe_t = qe_t^A + n_F qe_t^D$ and $qe_t^* = qe_t^A - n_H qe_t^D$. The incidence matrix in (D.22) gives

$$\ell_H^{\mathrm{QE}} = \ell_H + qe_t^A + n_F(1-\xi)qe_t^D,$$
$$\ell_F^{\mathrm{QE}} = \ell_F + qe_t^A - n_H(1-\xi)qe_t^D.$$

Define $d_i = \ell_i^e - \ell_i - qe_t^A$, so that $d_H - d_F = D$. The weighted sum-of-squares identity yields

$$\mathcal{W}(qe_t^D) = C(qe_t^A) - \frac{n_H n_F}{2}\left\{\frac{\mu}{z^2}[D-(1-\xi)qe_t^D]^2 + \kappa_{\mathrm{QE}}(qe_t^D)^2\right\},$$

where $C(qe_t^A)$ is independent of the purchase difference. Removing this constant and the positive population weight gives (5.4). The negative second derivative is $-n_H n_F[\kappa_{\mathrm{QE}} + \mu(1-\xi)^2/z^2]$. The first-order condition therefore gives (5.5).

Completing the square gives the welfare gain

$$\mathcal{W}(qe_t^{D,\mathrm{opt}}) - \mathcal{W}(0) = \frac{\gamma(1-\gamma)\mu^2(1-\xi)^2}{2z^2[\kappa_{\mathrm{QE}}z^2 + \mu(1-\xi)^2]}D^2. \tag{D.24}$$

The derivative of (D.24) with respect to ξ is negative for $D \neq 0$ and $\xi < 1$, since $x/[\kappa_{\mathrm{QE}}z^2 + \mu x]$ increases in $x = (1-\xi)^2$. This proves (5.6). At $\xi = 1$, the lending allocation depends only on qe_t^A, so the strictly positive purchase cost selects $qe_t^{D,\mathrm{opt}} = 0$.

Proof of Corollary 2. Write $k = \eta + (1-\eta)\xi$ for this proof. Equation (D.21) gives $zc_t^{b,D} = k\tau_t^{C,D} + (1-\xi)qe_t^D$. Since the efficient consumption differential is zero, the lending shortfall difference before QE is $D = -k\tau_t^{C,D}$: the shortfall includes the direct consumption effect of transfers as well as the lending displaced by their financing. Thus the relative part of the conditional welfare criterion, up to a positive population weight and an additive constant, is

$$-\frac{\mu}{2z^2}[k\tau_t^{C,D} + (1-\xi)qe_t^D]^2 - \frac{\kappa_{\mathrm{QE}}}{2}(qe_t^D)^2.$$

Its second derivative with respect to qe_t^D is $-\mu(1-\xi)^2/z^2 - \kappa_{\mathrm{QE}} < 0$. Substituting $D = -k\tau_t^{C,D}$ into (5.5) gives

$$qe_t^{D,\mathrm{opt}} = -\frac{\mu(1-\xi)[\eta + (1-\eta)\xi]}{\kappa_{\mathrm{QE}}z^2 + \mu(1-\xi)^2}\tau_t^{C,D}. \tag{D.25}$$

Substitution into the borrower resource equation gives

$$c_t^{b,D} = \frac{\kappa_{QE} z k}{\kappa_{QE} z^2 + \mu(1-\xi)^2} \tau_t^{C,D},$$

The derivative of its transfer coefficient with respect to integration, as stated in (5.8), is

$$\frac{\kappa_{QE} z\{(1-\eta)[\kappa_{QE} z^2 + \mu(1-\xi)^2] + 2\mu(1-\xi)k\}}{[\kappa_{QE} z^2 + \mu(1-\xi)^2]^2} > 0.$$

For $\xi < 1$, the residual consumption-response coefficient above is strictly below the coefficient k/z with QE held fixed, so optimal QE provides a partial offset. At $\xi = 1$, optimal purchases satisfy $qe_t^{D,\text{opt}} = 0$ and $c_t^{b,D} = \tau_t^{C,D}/z$. All comparisons hold the local welfare curvatures and the efficient consumption target fixed.

D.5 National QE as a separate benchmark

A national purchaser values the lending capacity released at home but not the capacity released abroad. The union central bank values both. This benchmark changes QE implementation while maintaining the local welfare criterion in (D.23); it does not characterize the original national-welfare game in Corollary 1.

Proposition 5 (National QE underprovides a positive cross-border spillover)**.** *Consider national implementation of QE, with each authority maximizing its own country's term in* (D.23) *while holding the other purchase fixed. Set $\gamma = 1/2$ and suppose both countries have the same positive pre-QE credit shortfall $\Delta > 0$. In the symmetric interior equilibrium, national purchases and union-optimal purchases are*

$$qe_t^N = \frac{\mu(1-\xi/2)}{\kappa_{QE} z^2 + \mu(1-\xi/2)}\Delta, \tag{D.26}$$

$$qe_t^U = \frac{\mu}{\kappa_{QE} z^2 + \mu}\Delta. \tag{D.27}$$

Hence $qe_t^N < qe_t^U$ for $\xi > 0$. At $\xi = 0$, the two policies coincide.

Proof of Proposition 5. With equal population shares, the effect of a country's own purchases on its lending is $1 - \xi/2$; the cross-country effect is $\xi/2$. Each national authority minimizes its own quadratic credit loss and purchase cost. Holding the other purchase

fixed, its first-order condition is

$$\kappa_{\mathrm{QE}} q e_t^N = \frac{\mu}{z^2}(1-\xi/2)(\ell_i^e - \ell_i^{\mathrm{QE}}).$$

At a symmetric equilibrium, $\ell_i^e - \ell_i^{\mathrm{QE}} = \Delta - q e_t^N$. Solving gives (D.26). A union purchaser recognizes that a common purchase releases one unit of lending capacity in each country, so its symmetric first-order condition is $\kappa_{\mathrm{QE}} q e_t^U = \mu(\Delta - q e_t^U)/z^2$, which gives (D.27). Both welfare objectives are strictly concave. The map $x \mapsto \mu x/(\kappa_{\mathrm{QE}} z^2 + \mu x)$ is strictly increasing for $x > 0$. Since $1 - \xi/2 < 1$ when $\xi > 0$, national QE is strictly below union QE.

E Linearized Model with Fixed Ownership Shares

E.1 Notation and normalization

This appendix gives the complete fixed-share variant of the distinct-price quantitative system. We use the country and holder notation of Section 6.1. The parameter ξ fixes government ownership shares; ξ_G retains its separate role as the government financing weight. The reference state is symmetric, with $\bar{P}^H = \bar{P}^F = \bar{P}$, $\bar{Y}^H = \bar{Y}^F = \bar{Y}$, and zero inflation. Private and government prices are equal across countries in steady state but differ across asset classes: $\bar{Q}_H^P = \bar{Q}_F^P = \bar{Q}^P$ and $\bar{Q}_H^G = \bar{Q}_F^G = \bar{Q}^G$.

Lowercase prices, returns, consumption, output, labor, and the financing limit denote log deviations. Let $\theta_{j,t} = \log(\Theta_{j,t}/\bar{\Theta})$, $\omega_{j,t} = \log(\Omega_{j,t}/\bar{\Omega})$, and $p_t^j = \log(P_t^j/\bar{P})$. Debt positions and real flows instead denote level deviations divided by $\bar{Y}$. In particular,

$$b_{j,t}^{P,FI} = \frac{B_{j,t}^{P,FI}/P_t^j - \bar{B}_j^{P,FI}/\bar{P}}{\bar{Y}},$$
$$b_{i|j,t}^{G,FI} = \frac{B_{i|j,t}^{G,FI}/P_t^i - \bar{B}_{i|j}^{G,FI}/\bar{P}}{\bar{Y}}.$$

Government face positions are deflated by the *issuer's* CPI, even when the holder is foreign. Issuance $b_{j,t}^P, b_{j,t}^G$ and central-bank face positions $b_{j,t}^{CB}$ use the same issuer-CPI convention. In contrast, market-value positions are measured in the *holder's* CPI:

$$\bar{\mathcal{V}}_j^P = \frac{\bar{Q}^P \bar{B}_j^{P,FI}}{\bar{P}\bar{Y}}, \qquad \bar{\mathcal{V}}_{i|j}^G = \frac{\bar{Q}^G \bar{B}_{i|j}^{G,FI}}{\bar{P}\bar{Y}}.$$

Write $a^P_{j,t}$ and $a^G_{i|j,t}$ for their level deviations divided by $\bar{Y}$. Then

$$a^P_{j,t} = \bar{Q}^P b^{P,FI}_{j,t} + \bar{\mathcal{V}}^P_j q^P_{j,t}, \tag{E.1}$$

$$a^G_{i|j,t} = \bar{Q}^G b^{G,FI}_{i|j,t} + \bar{\mathcal{V}}^G_{i|j}(q^G_{i,t} + p^i_t - p^j_t). \tag{E.2}$$

The foreign-cost argument is exactly $x_{j,t} = a^G_{\ell|j,t}$ to first order. For issuance and central-bank assets, define

$$\bar{f}^P = \frac{\bar{B}^P}{\bar{P}\bar{Y}}, \quad \bar{f}^G = \frac{\bar{B}^G}{\bar{P}\bar{Y}}, \quad \bar{A}^P = \bar{Q}^P \bar{f}^P, \quad \bar{A}^G = \bar{Q}^G \bar{f}^G, \quad \bar{A}^{CB} = \frac{\bar{Q}^G \bar{B}^{CB}}{\bar{P}\bar{Y}}.$$

Thus $\bar{A}^G$ is total government debt, whereas $\bar{\mathcal{V}}^G_{j|j}$ is only the domestic intermediary's own-government position.

E.2 Households and distinct bond returns

For each country j, saver labor supply, the saver Euler equation, and the nominal stochastic discount factor satisfy

$$\chi l_{j,t} = -\sigma c^s_{j,t} + w_{j,t}, \tag{E.3}$$

$$\sigma(\mathbb{E}_t c^s_{j,t+1} - c^s_{j,t}) = i_t - \mathbb{E}_t \pi^j_{t+1}, \tag{E.4}$$

$$\lambda_{j,t+1} = -\sigma(c^s_{j,t+1} - c^s_{j,t}) - \pi^j_{t+1}. \tag{E.5}$$

The foreign saver Euler equation is replaced by

$$\sigma(c^s_{H,t} - c^s_{F,t}) + p^H_t - p^F_t = 0.$$

Borrower consumption is priced by the private-bond return:

$$\sigma(\mathbb{E}_t c^b_{j,t+1} - c^b_{j,t}) = \mathbb{E}_t(r^P_{j,t+1} - \pi^j_{t+1}). \tag{E.6}$$

There are four distinct bond-price and four return variables across the union. For $k \in \{P, G\}$ and $j \in \{H, F\}$,

$$r^k_{j,t} = \frac{\kappa}{\bar{R}^k} q^k_{j,t} - q^k_{j,t-1}. \tag{E.7}$$

The coefficients use the return of the corresponding asset.

E.3 Intermediary optimality and accounts

Let $s_P = \bar{R}^P - \bar{I} > 0$ and $s_G = \bar{R}^G - \bar{I} > 0$. At the reference state,

$$\bar{\Omega} = \beta s_P, \qquad s_G = \xi_G s_P.$$

Under the maintained decentralization, only the foreign-government position is an individual allotment. Domestic government and private holdings remain choices at given prices. Their two asset conditions are

$$\mathbb{E}_t \lambda_{j,t+1} + \frac{\bar{R}^P}{s_P} \mathbb{E}_t r^P_{j,t+1} - \frac{\bar{I}}{s_P} i_t = \omega_{j,t}, \tag{E.8}$$

$$\mathbb{E}_t \lambda_{j,t+1} + \frac{\bar{R}^G}{s_G} \mathbb{E}_t r^G_{j,t+1} - \frac{\bar{I}}{s_G} i_t = \omega_{j,t}. \tag{E.9}$$

There are four such conditions across the union. The two former foreign-government Euler equations are replaced by the two fixed allotments in (E.22) below. Although the same $\omega_{j,t}$ appears in the normalized conditions, the private and government equations have different steady spreads and return coefficients. The factor ξ_G is absorbed using $\xi_G \bar{\Omega} = \beta s_G$.

The within-country implication remains

$$\bar{R}^G \mathbb{E}_t r^G_{j,t+1} - \xi_G \bar{R}^P \mathbb{E}_t r^P_{j,t+1} = (1 - \xi_G) \bar{I} i_t. \tag{E.10}$$

At the symmetric reference state, the allotment shadow value is $\bar{\mu}_j = 0$. Its normalized *level* deviation $m^\mu_{j,t} = (\mu_{j,t} - \bar{\mu}_j)/(\beta s_G)$ is determined residually by

$$m^\mu_{j,t} = \mathbb{E}_t \lambda_{j,t+1} + \frac{\bar{R}^G}{s_G} \mathbb{E}_t r^G_{\ell,t+1} - \frac{\bar{I}}{s_G} i_t - \omega_{j,t} - \frac{\phi_x}{s_G} a^G_{\ell|j,t}. \tag{E.11}$$

Consequently,

$$\bar{R}^G \mathbb{E}_t (r^G_{\ell,t+1} - r^G_{j,t+1}) = \phi_x a^G_{\ell|j,t} + s_G m^\mu_{j,t}. \tag{E.12}$$

These last two equations are diagnostic identities, not additional pricing restrictions. In particular, imposing $m^\mu_{j,t} = 0$ would incorrectly restore unrestricted foreign-bond choice. Reserves earn the policy rate, $i^{RE}_t = i_t$. No Euler condition is imposed on prohibited foreign private bonds.

The binding financing constraint becomes

$$a^P_{j,t} + \xi_G \sum_i a^G_{i|j,t} = \left(\bar{\mathcal{V}}^P_j + \xi_G \sum_i \bar{\mathcal{V}}^G_{i|j} \right) \theta_{j,t}. \tag{E.13}$$

The steady weighted asset stock on the right must be recalculated together with new equity. Using the old unweighted coefficient would be inconsistent.

After substituting inherited equity, the balance sheet is

$$\begin{aligned} d_{j,t} = {} & \bar{Q}^P (b^{P,FI}_{j,t} - \kappa b^{P,FI}_{j,t-1}) + \bar{\mathcal{V}}^P_j [(1-\kappa) q^P_{j,t} + \kappa \pi^j_t] + re_{j,t} \\ & + \sum_i \Big\{ \bar{Q}^G (b^{G,FI}_{i|j,t} - \kappa b^{G,FI}_{i|j,t-1}) \\ & \qquad + \bar{\mathcal{V}}^G_{i|j} [(1-\kappa)(q^G_{i,t} + p^i_t - p^j_t) + \kappa \pi^i_t] \Big\}. \end{aligned} \tag{E.14}$$

Real deposits and reserves, $d_{j,t}$ and $re_{j,t}$, are normalized by $\bar{Y}$. Define total steady equity, including inherited bonds, by

$$\bar{x}^{FI}_j = \frac{\bar{X}^{FI}_j}{\bar{Y}} + \kappa \left(\bar{\mathcal{V}}^P_j + \sum_i \bar{\mathcal{V}}^G_{i|j} \right).$$

Its deviation is

$$\begin{aligned} x^{FI}_{j,t} = \kappa \Big\{ & \bar{Q}^P b^{P,FI}_{j,t-1} + \bar{\mathcal{V}}^P_j (q^P_{j,t} - \pi^j_t) \\ & + \sum_i \left[\bar{Q}^G b^{G,FI}_{i|j,t-1} + \bar{\mathcal{V}}^G_{i|j} (q^G_{i,t} + p^i_{t-1} - p^j_t) \right] \Big\}. \end{aligned} \tag{E.15}$$

For a foreign asset, $p^i_{t-1} - p^j_t$ includes both relative-price and inflation effects. It cannot be replaced by $-\pi^j_t$.

Dividends, divided by $\bar{Y}$, obey

$$\begin{aligned} \mathrm{div}_{j,t} = {} & s_P (a^P_{j,t-1} - \bar{\mathcal{V}}^P_j \pi^j_t) + \bar{\mathcal{V}}^P_j (\bar{R}^P r^P_{j,t} - \bar{I} i_{t-1}) \\ & + \sum_i \left[s_G (a^G_{i|j,t-1} - \bar{\mathcal{V}}^G_{i|j} \pi^j_t) + \bar{\mathcal{V}}^G_{i|j} (\bar{R}^G r^G_{i,t} - \bar{I} i_{t-1}) \right] \\ & + \bar{I} x^{FI}_{j,t-1} + \bar{I} \bar{x}^{FI}_j (i_{t-1} - \pi^j_t). \end{aligned} \tag{E.16}$$

The direct derivative of the quadratic adjustment cost is zero at its steady target. Since the foreign position is assigned, its marginal cost appears only in the shadow-value identity (E.11), not in a free portfolio-choice condition. Thus ϕ_x has no effect on the independent

first-order equilibrium in this variant. The resource cost must still be included in a second-order welfare calculation.

E.4 Borrower, bailout, and government budgets

At the reference state, $\bar{C}^b = z\bar{Y}$ and steady government purchases and fiscal transfers are zero. The borrower budget is

$$\begin{aligned} zc^b_{j,t} + b^P_{j,t-1} - \bar{f}^P \pi^j_t = \bar{Q}^P b^P_{j,t} - \kappa \bar{Q}^P b^P_{j,t-1} \\ + \bar{A}^P[(1-\kappa)q^P_{j,t} + \kappa \pi^j_t] + x^b_{j,t} + \tau^C_{j,t}. \end{aligned} \tag{E.17}$$

Linearizing the full-bailout transfer gives

$$\begin{aligned} x^b_{j,t} = (1+\kappa\bar{Q}^P)(b^P_{j,t-1} - \bar{f}^P\pi^j_t) + \kappa \bar{A}^P q^P_{j,t} \\ + (1+\kappa\bar{Q}^G)(b^G_{j,t-1} - \bar{f}^G\pi^j_t) + \kappa \bar{A}^G q^G_{j,t} \\ - \varrho\left(\bar{Q}^G b^G_{j,t-1} + \bar{A}^G q^G_{j,t-1}\right) - \tau^{CB}_{j,t}. \end{aligned} \tag{E.18}$$

The government budget and non-subsidy tax are

$$\begin{aligned} (1-\eta)\tau^C_{j,t} = \bar{Q}^G b^G_{j,t} + (\varrho - \bar{R}^G)\bar{Q}^G b^G_{j,t-1} \\ + \bar{A}^G\left[q^G_{j,t} + (\varrho - \bar{R}^G)q^G_{j,t-1} - \bar{R}^G(r^G_{j,t} - \pi^j_t)\right] + \tau^{CB}_{j,t}, \end{aligned} \tag{E.19}$$

$$t^{ns}_{j,t} = \eta(\tau^C_{j,t} + g_{j,t}) + \varrho(\bar{Q}^G b^G_{j,t-1} + \bar{A}^G q^G_{j,t-1}). \tag{E.20}$$

The passive tax uses the previous period's *real* government-debt value; there is no current-inflation term in its bracket. Subsidy spending and its earmarked tax cancel before linearization.

For checking the accounts, (6.18) implies

$$zc^b_{j,t} = \bar{Q}^P b^P_{j,t} + \bar{A}^P q^P_{j,t} + \bar{Q}^G b^G_{j,t} + \bar{A}^G q^G_{j,t} + \eta\tau^C_{j,t}. \tag{E.21}$$

It is a consequence of the preceding budgets and should not be added as an independent equation.

E.5 Market clearing, central bank, and saver budgets

Equations (6.3)–(6.4) are written in country-total units in the main text. All $b^{G,FI}_{i|j,t}$ below retain the per-holder-resident convention and the issuer's CPI deflator. Dividing country

totals by the holder's population gives the two linearized foreign allotments, with no relative-price term:

$$b^{G,FI}_{\ell|j,t} = n_\ell \xi \left(b^G_{\ell,t} - b^{CB}_{\ell,t} \right), \qquad j = H, F. \tag{E.22}$$

The debt markets otherwise clear as before:

$$b^P_{j,t} = b^{P,FI}_{j,t}, \qquad b^G_{j,t} = b^{G,FI}_{j|j,t} + \frac{n_\ell}{n_j} b^{G,FI}_{j|\ell,t} + b^{CB}_{j,t}, \quad j = H, F. \tag{E.23}$$

For clarity, the equivalent four-position representation in *per-holder-resident* units is

$$b^{G,FI}_{H|H,t} = (1 - \gamma\xi)(b^G_{H,t} - b^{CB}_{H,t}), \tag{E.24}$$

$$b^{G,FI}_{H|F,t} = (1 - \gamma)\xi(b^G_{H,t} - b^{CB}_{H,t}), \tag{E.25}$$

$$b^{G,FI}_{F|H,t} = \gamma\xi(b^G_{F,t} - b^{CB}_{F,t}), \tag{E.26}$$

$$b^{G,FI}_{F|F,t} = [1 - (1 - \gamma)\xi](b^G_{F,t} - b^{CB}_{F,t}). \tag{E.27}$$

Use either the two foreign allotments plus the two government-clearing equations, or these four position equations. Do not impose both sets as independent restrictions. The two private-market equations remain unchanged.

To express the same rule in market values, define

$$a^{G,\text{net}}_{j,t} = \bar{Q}^G(b^G_{j,t} - b^{CB}_{j,t}) + \bar{A}^{G,\text{net}} q^G_{j,t}, \qquad \bar{A}^{G,\text{net}} = \bar{A}^G - \bar{A}^{CB}. \tag{E.28}$$

Then

$$a^G_{j|j,t} = (1 - n_\ell \xi) a^{G,\text{net}}_{j,t}, \tag{E.29}$$

$$a^G_{j|\ell,t} = n_j \xi \left[a^{G,\text{net}}_{j,t} + \bar{A}^{G,\text{net}}(p^j_t - p^\ell_t) \right]. \tag{E.30}$$

The relative-CPI term is required for a foreign holder's market value. The fixed common-currency ownership share does not justify omitting it. These value formulas follow from the face-position rules and (E.2); they are not extra independent equations.

There are no private-bond terms in central-bank assets. QE values and the union reserve balance satisfy

$$qe_{j,t} = \bar{Q}^G b^{CB}_{j,t} + \bar{A}^{CB} q^G_{j,t}, \tag{E.31}$$

$$re_{H,t} = qe_{H,t}, \qquad re_{F,t} = qe_{F,t}, \tag{E.32}$$

The symmetric reference state sets the reserve value in each country to $\bar{r}^{RE} = \bar{A}^{CB}$. Around this state the central-bank profit identity is

$$\begin{aligned}\sum_j n_j \tau^{CB}_{j,t} = \bar{R}^G \sum_j n_j \left[\bar{Q}^G b^{CB}_{j,t-1} + \bar{A}^{CB}(q^G_{j,t-1} + r^G_{j,t} - \pi^j_t)\right] \\ - \bar{I} \sum_j n_j \left[re_{j,t-1} + \bar{r}^{RE}(i_{t-1} - \pi^j_t)\right], \qquad \tau^{CB}_{H,t} = \tau^{CB}_{F,t}.\end{aligned} \tag{E.33}$$

Only government returns enter this equation. The symmetry of steady reserves, QE, and per-capita remittances cancels the price-level terms when the nominal union identity is linearized.

Let $e_t = \log(P_{F,t}/P_{H,t})$ and define $v_H = -(1-v)$ and $v_F = 1 - v^*$. The saver budget in country j is

$$\begin{aligned}(1-z)c^s_{j,t} + d_{j,t} = y_{j,t} + v_j e_t + \bar{I} d_{j,t-1} + \bar{I}\bar{d}_j(i_{t-1} - \pi^j_t) \\ + \mathrm{div}_{j,t} - t^{ns}_{j,t} - x^b_{j,t} - x^{FI}_{j,t} + z_{j,t}.\end{aligned} \tag{E.34}$$

Use one regional saver budget in the independent equilibrium system; the other follows from the aggregate accounts and market clearing. The steady deposit coefficient $\bar{d}_j = \bar{D}_j/(\bar{P}\bar{Y})$ must be recomputed after changing the financing weights.

The fund payments satisfy

$$(1-\gamma)z_{H,t} + \gamma z_{F,t} = 0.$$

We retain one regional saver budget in the independent system. The other follows from the aggregate accounts, market clearing, and the fund budget.

E.6 Production, prices, goods markets, and policy

For completeness, the remaining first-order quantitative blocks are stated here. Let $\pi^p_{H,t}, \pi^p_{F,t}$ denote producer-price inflation, to distinguish them from π^H_t, π^F_t, which are CPI inflation. For each country, production and the Calvo recursions are

$$y_{j,t} = a_{j,t} + l_{j,t}, \tag{E.35}$$

$$\widetilde{mc}_{j,t} = w_{j,t} - a_{j,t} - v_j e_t, \tag{E.36}$$

$$\widetilde{p}_{j,t} = \widetilde{x}_{1,j,t} - \widetilde{x}_{2,j,t}, \tag{E.37}$$

$$\widetilde{x}_{1,j,t} = (1-\varphi\beta)(\widetilde{mc}_{j,t} + y_{j,t} + u^p_{j,t}) + \varphi\beta\mathbb{E}_t[\lambda_{j,t+1} + \widetilde{x}_{1,j,t+1} + (\epsilon+1)\pi^p_{j,t+1}], \tag{E.38}$$

$$\widetilde{x}_{2,j,t} = (1-\varphi\beta)y_{j,t} + \varphi\beta\mathbb{E}_t[\lambda_{j,t+1} + \widetilde{x}_{2,j,t+1} + \epsilon\pi^p_{j,t+1}], \tag{E.39}$$

$$\pi^p_{j,t} = \frac{1-\varphi}{\varphi}\widetilde{p}_{j,t}. \tag{E.40}$$

Price dispersion has zero first-order deviation at the zero-inflation reference state. The baseline home cost-push experiment sets $u^p_{F,t} = 0$. CPI inflation, terms of trade, and relative CPIs obey

$$\pi^H_t = v\pi^p_{H,t} + (1-v)\pi^p_{F,t}, \qquad \pi^F_t = (1-v^*)\pi^p_{H,t} + v^*\pi^p_{F,t}, \tag{E.41}$$

$$e_t = e_{t-1} + \pi^p_{F,t} - \pi^p_{H,t}, \qquad p^H_t - p^F_t = (1-v-v^*)e_t. \tag{E.42}$$

With zero steady government purchases, the goods markets are

$$\begin{aligned} y_{H,t} = {} & \vartheta(1-v)(v+v^*)e_t + (1-z)[vc^s_{H,t} + (1-v)c^s_{F,t}] \\ & + z[vc^b_{H,t} + (1-v)c^b_{F,t}]. \end{aligned} \tag{E.43}$$

$$\begin{aligned} y_{F,t} = {} & -\vartheta(1-v^*)(v+v^*)e_t + (1-z)[(1-v^*)c^s_{H,t} + v^*c^s_{F,t}] \\ & + z[(1-v^*)c^b_{H,t} + v^*c^b_{F,t}]. \end{aligned} \tag{E.44}$$

The Taylor rule is

$$i_t = \phi_\pi \sum_j n_j \pi^j_t + \phi_y \sum_j n_j y_{j,t} + \delta_{i,t}. \tag{E.45}$$

E.7 Calibration

Table 1: PARAMETERS

Parameter or target	Value	Interpretation
γ, ν, ϑ	0.5, 0.5, 3	Population, openness, trade elasticity
β, z	0.995, 0.33	Saver discount factor and borrower consumption share
σ, χ, ψ_L	1, 1, 1.36	Preferences
ϵ, φ	11, 0.75	Goods elasticity and Calvo probability
κ	0.975	Common contractual coupon decay
$\bar{\Theta}$	4	Weighted-asset leverage limit
$\bar{R}^G$	1.007509907	Retained quarterly government-return anchor
$(\bar{R}^P)^4 - (\bar{R}^G)^4$	0.03	Annual private–government yield difference
ξ_G	0.255122048	Government financing weight
ξ	0.4	Fixed-share integration parameter; varied over $[0, 1]$
β_b	0.985450121	$1/\bar{R}^P$
$\bar{A}^{CB}$	0.12	QE/quarterly output; 3% of annualized output
$\frac{\bar{Q}^G \bar{b}^G}{\bar{Y}}$	0.2	Government debt-to-GDP ratio
Domestic/foreign shares	0.8/0.2	At every date for $\gamma = 0.5, \xi = 0.4$, after subtracting QE
η, ϱ	0.5, 0.2	Baseline tax-financing share and passive tax feedback
ϕ_π, ϕ_y	1.5, 0.25	Taylor-rule coefficients
ρ_a	0.95	Productivity persistence
$\rho_{qe}, \rho_\tau, \rho_\delta, \rho_{ff}, \rho_{cp}$	0.8	Policy, credit, and cost-push persistence
ϕ_x	0	foreign-position resource cost