Supplying Safety to an Aging World: Demographics, Debt, and Risk Premia*

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Abstract

By raising the share of life spent in retirement, demographic aging shortens the duration of individuals' labor earnings relative to their consumption horizons. Since labor income is relatively safe, its shortening duration increases households' desire for safe bonds and lowers their capacity to bear equity risk. Slower population growth reinforces these effects. A consequence of aging is therefore a lower safe rate, with the risk premium rising to make aging investors willing to absorb the risk. In a two-country model in which Foreign ages more severely, Home households (which we identify with the U.S.) optimally supply safe bonds and take leveraged equity positions, as they are in the better *relative* position to do so. As a result, Home's equity premium rises by more than Foreign's, consistent with the evidence. Quantitatively, we find that demographic changes since 1980 have lowered the world safe rate by 71 basis points and raised the U.S. equity premium by 251 basis points (compared with 49 basis points abroad). Aging can therefore lower safe returns globally while raising the compensation required to bear equity risk – particularly in the "younger" country, which is better placed to absorb such risk and becomes the global safe asset issuer.

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1 Introduction

In this paper, we study the equilibrium allocation of risk between two countries that differ in their aging patterns – with one country aging "more severely" than the other, in the sense that its population growth rate declines by more and its residents spend a greater fraction of their lives in retirement. Since labor income is relatively safe, shorter working lives relative to total lifespans raise the desire to save in safe assets, like bonds, that can replace such income in retirement (Merton, 1971). Aging can therefore be expected to lower safe rates, while pushing up the equity premium. This can be seen as a reflection of the fact that an older population is less suited to bear risk, meaning the risk premium must rise to make them willing to do so (Kopecky and Taylor, 2022).

Our particular contribution is to analyze this dynamic in an open economy, where aging is more severe in Foreign than it is in Home (which, empirically, can be associated with the U.S.). In such a setup, we show that the relatively younger Home households are better placed to take on equity risk, whilst issuing safe bonds to the rest of the world, where aging has hit harder. Home (U.S.) households therefore take a leveraged equity position, functioning like "the world's venture capitalist" (Gourinchas and Rey, 2007). Our paper thus links observed patterns in global imbalances, safe rates, and risk premia to cross-country differences in demographic forces. In this sense, our paper can be understood as "Merton meets Gourinchas and Rey".

It has long been debated whether secular movements in safe rates of interest and risk premia reflect changes in growth, market structure, or risk. Over the past few decades, real safe rates have fallen significantly, while the pre-tax return on private capital has remained broadly stable in the U.S. – implying a rising equity premium, at least in the U.S. (Gomme et al., 2015; Farhi and Gourio, 2018; Reis, 2022). The stability of the return on capital sits uneasily with the "global savings glut hypothesis" (Bernanke, 2005), as that should lower risky rates of return alongside risk-free ones. It is similarly hard to reconcile with a simple productivity or capital-deepening story. Consequently, the widening wedge between safe and risky returns is widely interpreted as a rising price of risk.

What has driven this rise? The leading explanations are incomplete in important ways. Farhi and Gourio (2018) show that the joint behavior of safe rates, risky returns, and equity premia can be reconciled by allowing disaster probabilities to drift upward over time. But disaster risk is notoriously hard to measure, and attempts to do so find little evidence that it has risen (Papanikolaou, 2018). Marx et al. (2021) conclude that "a large increase in the risk aversion of investors, or a growing weight of risk-averse investors" is necessary to match the data, but this merely relocates the puzzle: what caused risk aversion to rise?

Invoking secular changes in a structural parameter risks turning it into a free parameter, a criticism that Farhi and Gourio (2018) themselves acknowledge.[1] Eggertsson et al. (2021) attribute these trends to rising markups, but this explanation is sensitive to contested measurement (Basu, 2019; De Loecker et al., 2020; Raval, 2023) and may be quantitatively overstated in the presence of intangible capital (Farhi and Gourio, 2018). Reis (2022) develops a theory based on rising misallocation of investment due to financial frictions. Each of these accounts appeals to forces that are difficult to observe or measure, or that themselves need explaining.

This paper argues that a different, more measurable trend can explain the observed patterns. In particular, we argue that *aging* – characterized by falling population growth rates, longer expected lifetimes, and individuals spending a greater fraction of their lives in retirement – increases the demand for safe financial wealth in a way that lowers the safe rate and pushes up the equity risk premium. Rather than treating a higher price of risk as a primitive, we connect it to demographics – a force that is directly observable and precisely measured, and whose secular trend is widely acknowledged – through safe-asset issuance and investor balance sheets.

Starting on the measurement front, we first document how (a) the risk premium appears to have risen by more in the U.S. than elsewhere, while (b) aging has hit harder outside the U.S., in the sense that the fraction of life spent in retirement has risen by more there. Consequently, non-U.S. individuals' "retirement-duration gaps" – the expected duration of their consumption stream minus the expected duration of their labor earnings – have widened by more.

Motivated by these observations, we develop a model to understand how these patterns might be linked. We first study a small country (consisting of two types of agents: stockholders and nonstockholders) that ages in isolation, with the interest rate held fixed at (r^*). If this country can "import" all the desired safe assets from abroad, demographics will not have any impact on the risk premium. But that no longer holds when some of the bonds need to be supplied domestically, by the limited set of stockholders. In that case, they are asked to take on a leveraged equity position, which – given the fact that they are aging too – does not appeal to them *a priori*. To make them willing to hold equity on a leveraged balance sheet, they therefore need to be compensated with a higher risk premium – thus creating a link from demographic forces to asset prices.

We then move to a two-country model to study how risk is shared in an open economy

[1] Farhi and Gourio (2018) present their explanation as semi-structural, noting that "allowing these parameters to vary over time flexibly suggests they are not microfounded and invariant to policy. Yet we find the results useful because deeper explanations need to be consistent with the changes of parameters implied by our approach" (p. 217).

equilibrium. In our main experiment, we mimic the empirical pattern in which the U.S. has aged less than other parts of the world. This places U.S. households in a better relative position to take on risk and to supply safe assets to the rest of the world (whose *relative* demand for safe assets is stronger). In equilibrium, the "younger" U.S. economy becomes the safe asset issuer while investing more in equity. Although it is in the best relative position to do so, the fact that it ends up loading on risk, while not being totally free from domestic aging pressures, implies that it will be compensated with a higher equity risk premium. Quantitatively, we find that demographic changes since 1980 have lowered the world safe rate by 71 basis points and raised the U.S. equity premium by 251 basis points (compared with 49 basis points abroad).

Related literature. Our paper builds on various previous contributions. Firstly, we deploy the insight of Merton (1971) that human capital is bond-like in nature (see Bodie et al., 1992; Campbell et al., 2001; Viceira, 2001; Cocco et al., 2005 for other prominent contributions to this vast literature; Gomes (2020) offers a survey). Taking this point to an international two-country setup, we show that the country with the larger retirement-duration gap ("Foreign" in our labelling) has a stronger incentive to accumulate the safe asset – to replace the flow of labor income (which, relative to the duration of the consumption stream, is quicker to peter out in Foreign). In equilibrium, Home therefore becomes the safe asset issuer: its residents have a comparative advantage in this role because their relatively safe labor income covers a larger share of the lifetime over which consumption must be financed.

Our paper also relates to the literature aimed at explaining who becomes the global safe asset issuer which, in practice, has been the U.S. in recent history. Caballero et al. (2008), Mendoza et al. (2009), and Maggiori (2017) show how this outcome can result from the U.S. being more financially developed, but do not explain why the U.S. is characterized by a greater level of financial development. In our setup, the country with the shorter retirement-duration gap becomes the safe asset issuer, which is consistent with the observation that U.S. individuals tend to spend a shorter fraction of their lives in retirement.[2] The savings that are being channeled to the U.S. in this way (looking for a relatively safe rate of return, to proxy fading labor income) may furthermore have contributed to the U.S.'s superior level of financial development (Ordóñez and Piguillem, 2019), at which point the channel – originally rooted in differential aging patterns – would obtain an extra kick by also launching the financial development channel. This brings us to the Gourinchas and Rey (2007) characterization of the U.S. as "the world's venture capitalist" – in

[2]In practice, other factors, such as country size (Hassan, 2013) and debt volume (He et al., 2019), are likely to be important as well when it comes to determining who will become the global safe asset issuer, but we abstract from those here.

the sense of the U.S. mostly issuing safe debt, whilst simultaneously investing in more risky options – with our paper explaining this outcome from cross-country differences in the duration embedded in human wealth, which tends to be relatively safe in nature. In particular, our model implies that U.S. households are relatively more able to carry equity-related risks, as their relatively safe labor income lasts for a greater fraction of their expected lifetimes.

More generally, we build on the voluminous literature exploring the impact of demographics on various rates of return. Most research in this area has focused on the depressing effect that aging can have on the safe rate of return (Domeij and Flodén, 2006; Krueger and Ludwig, 2007; Ferrero, 2010; Carvalho et al., 2016; Rachel and Smith, 2017; Eggertsson et al., 2019; Rachel and Summers, 2019; Gagnon et al., 2021; Lisack et al., 2021; Bárány et al., 2023; Carvalho et al., 2025; Rachel, 2025; Auclert et al., 2025, 2026). By building a stochastic model that offers both safe and risky investment options, we are instead interested in the implications aging might have for the *risky* rate of return – and the implied risk premium. There is a small literature studying this question from a closed-economy perspective (Ang and Maddaloni, 2005; Kopecky and Taylor, 2022)[3], but – to the best of our knowledge – ours offers the first open-economy analysis. In doing so, we show how cross-country differences in aging patterns shape global imbalances and differential trends in risk premia.

On the risk-premium front, our paper relates to the broader literature observing that safe rates of return appear to have fallen by more than risky rates – implying a widening risk premium (Gomme et al., 2015; Farhi and Gourio, 2018; Reis, 2022; Blanchard, 2023). A global saving glut would push down the returns on all assets alike, which is hard to reconcile with the stability of risky returns; Caballero et al. (2017a,b) therefore attribute these trends to excess demand specifically for safe assets. Our mechanism provides a demographic source for this excess demand: as households spend more of their lives in retirement, aging raises the demand for safe assets in particular, rather than for assets in general. The widening risk premium is most well-established for the U.S. (to which the aforementioned papers apply), with our analysis suggesting that this trend is also *most pronounced* in the U.S. Our model attributes this international divergence to U.S. households having taken on more risk over the years (acting like "venture capitalists", in the sense of Gourinchas and Rey (2007)), with the higher U.S. risk premium compensating

[3]An earlier literature (Mankiw and Weil, 1989; Abel, 2001; Brooks, 2002; Abel, 2003; Geanakoplos et al., 2004) asked whether aging trends could translate into lower-frequency dynamics in asset prices. The idea was that prices would peak as "boomers" were financially preparing themselves for retirement (accumulating assets), with prices falling again as these households entered retirement, where they might sell their assets to finance consumption. Empirically, however, Poterba (2001) found that retirees do not dissave much in retirement, making an "asset market meltdown" less likely from this perspective.

them for doing so.

Outline. Section 2 presents the motivating evidence. Section 3 develops a partial equilibrium model to illustrate the core mechanism. Sections 4 and 5 introduce and quantify the two-country model. The appendices give the derivations and further details of the quantitative exercises.

2 Motivating evidence

This section presents two motivating observations that our model will later connect in a causal way. Focusing on the period since 1980, from which point on demographics are generally thought to have influenced rates of return (Carvalho et al., 2016; Gagnon et al., 2021; Lisack et al., 2021; Carvalho et al., 2025; Rachel, 2025), we first show that the risk premium appears to have increased more in the U.S. than elsewhere. Second, we document how the fraction of life spent in retirement has risen by more among non-U.S. households. This means that they are, on average, characterized by a greater retirement-duration gap (defined as the expected duration of their consumption stream, which only ends upon death, minus the expected duration of their labor income stream, which ends upon retirement).

2.1 Trends in risk premia

While there are several papers pointing out that the U.S. risk premium has risen over time (Gomme et al., 2015; Farhi and Gourio, 2018; Reis, 2022), less is known about whether this finding extends to other countries.

To shine light on this issue, we turn to the Macro History Database (Jordà et al., 2017, 2019), which provides us with the relevant data for 18 major economies.[4] From it, we calculate country-level risk premia as the total risky rate of return (which combines rates of return on equity and housing – the latter being an important vehicle for retirement savings), minus the t-bill rate as a proxy for the safe rate. The resulting series is naturally noisy, as it is affected by unforeseen shocks, which aren't reflected in the ex-ante risk premium. So to get at underlying trends, we work with a 15-year moving average of current and past outturns (but results are robust to other smoothing approaches).

Our main takeaway from Figure 1 is that, since 1980, the risk premium has risen by more in the U.S. This finding, also visible in Kopecky and Taylor (2022) (who exclude

[4]The dataset includes: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, U.K., U.S.

housing and solely look at the return on equity), follows when basing inference on a PPP-weighted average (see panel (a)), but looking at a simple, equal-weighted average yields the same message (see panel (b)). Over the full 1980–2020 sample, the non-U.S. realized premium actually trends downward. That decline is heavily concentrated in the early part of the sample, however. From 1990 onward, the risk premium rises again – albeit by less than what is observed for the U.S. Consequently, our main takeaway is a message of international divergence, with the rise in risk premia being most pronounced for the U.S.

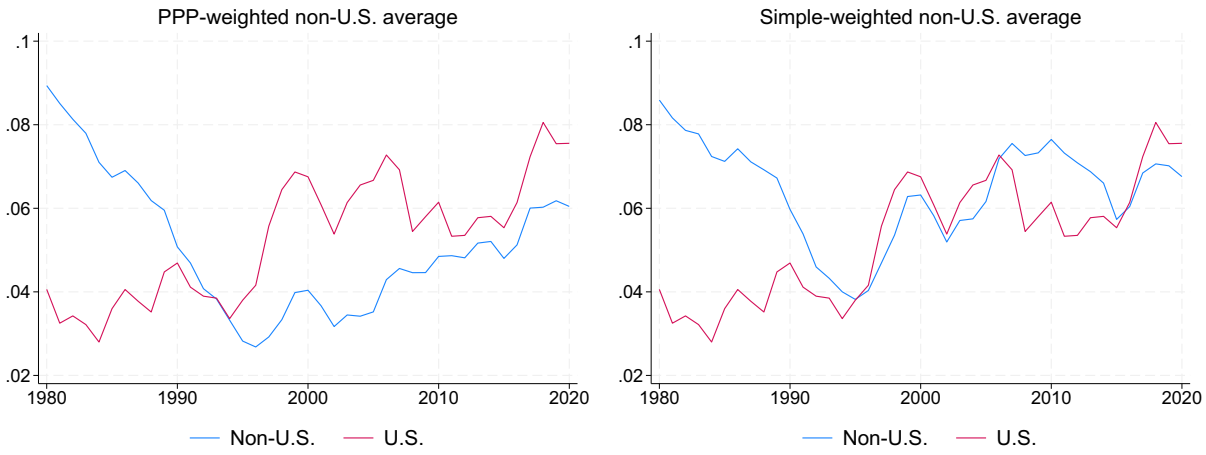


Figure 1: Risk premium developments over 1980-2020: U.S. versus non-U.S.
Notes: Risk premium series are first smoothed at the country level by taking a 15-year moving average of current and past outturns. Data are from the Macro History Database (Jordà et al., 2017, 2019). Non-U.S. composite consists of the non-U.S. countries mentioned in footnote 4.

At this stage, and despite the smoothing applied, one might worry about the reliance on realized returns; conceptually, looking at *expected* returns — which are not distorted by unforeseen shocks that happen to hit in-sample — is more appropriate. This is exactly the exercise done by Corsetti et al. (2025). Because of the heavier data requirements associated with the underlying Gordon growth formula, they are confined to the post-1997 sample period, lose information on the return to housing, as well as part of the cross-country dimension (they have to limit themselves to the G7 economies). Importantly, however, the key message remains: also in ex-ante terms, the risk premium is found to have risen by more in the U.S. compared to other economies.

2.2 Trends in life expectancy and retirement age

A second dimension along which the U.S. and other major economies have been characterized by differential trends, is the fraction of time spent in retirement. This can be analyzed using the OECD's "Pensions at a Glance" database, which contains data on "effective retirement ages" and "expected years after retirement" for the 18 major economies underlying Section 2.1 (as listed in footnote 4). The OECD data are broken down by gender, but below we focus on aggregates obtained after taking a simple average between males and females.

Life expectancy at retirement – calculated as the sum of the effective retirement age and expected years after retirement – has risen significantly across countries: somewhat more in the non-U.S. part of our sample, but the U.S. is not far behind.

There is, however, substantial cross-country heterogeneity in whether those additional years are spent working, or in retirement. Since 1980, the time spent in retirement has risen by 4.8 years in the non-U.S. part of our sample; this is almost equal to their total increase in life expectancy over that period (+5.4 years), meaning that nearly all of the additional life years gained are being "consumed" in retirement. By contrast, U.S. households have been more inclined to stretch the length of their working careers (retiring later in life), meaning that their expected years after retirement have risen by less (by 1.3 years to be precise, compared to life expectancy having risen by 4.4 years since 1980).

As a result of these developments, the fraction of time spent in retirement has risen by most in the non-U.S. part of our sample. There, it rose from 0.210 in 1980 to over 0.25 in 2024 (a relative increase of nearly 20 percent). In the U.S., by contrast, it has remained relatively constant – moving only marginally, from 0.211 to 0.215. This means that the retirement-duration gap, that part of life where consumption needs to be financed from past savings and associated returns, has widened by more in the non-U.S. part of our sample.

An important country not covered by the OECD database is China. But, there too, available evidence suggests that the majority of life years gained have been added to time spent in retirement. Since the 1950s, Chinese statutory retirement ages have been fixed at 55 and 60 years for women and men, respectively – meaning that life years gained over time have primarily been adding to years after retirement.[5] Only in 2025 has China embarked on a reform trajectory to increase the statutory retirement ages by 3 years (over a 15-year transition period), to 58 years for women and 63 years for men.

In the remainder of this paper, we will build a model to show how differences in these

[5] According to the UN World Population Prospects database, Chinese life expectancy at 65 has risen by 4.8 years since 1980 – with most of these life years gained thus having been added to retirement.

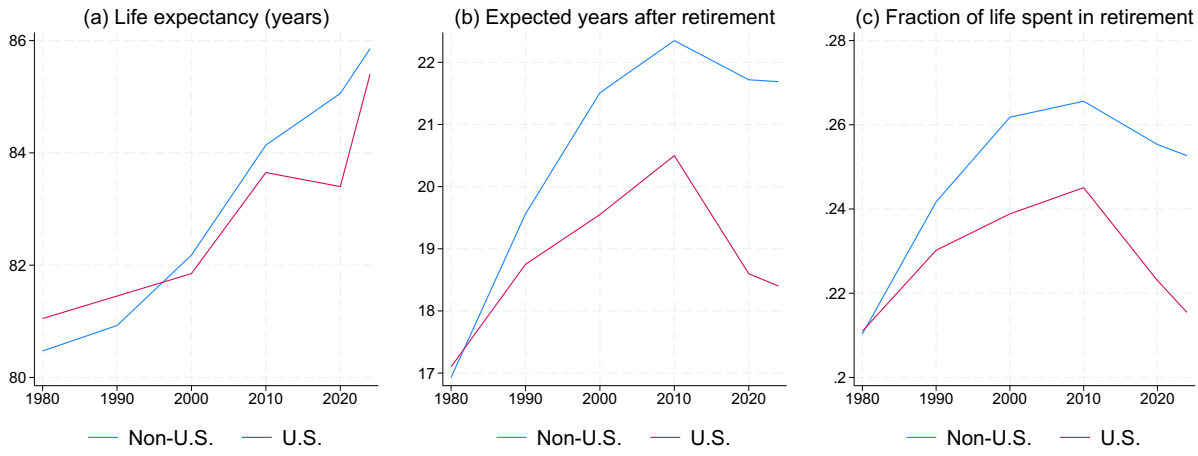


Figure 2: Demographic developments over 1980-2024: U.S. versus non-U.S.
Notes: Data are from the OECD's "Pensions at a Glance" database. Non-U.S. composite consists of the non-U.S. countries mentioned in footnote 4.

demographic trends may be a driver of our first motivating observation – that the U.S. risk premium has risen by more than that in other countries.

3 A fixed-rate benchmark

To clarify the mechanism, we start by studying aging, first defined as individuals spending a larger fraction of their life in retirement, at a fixed world safe rate, r^*. (In a subsequent step, we will also incorporate the impact of lower rates of population growth.) One can think of this analysis as applying to the non-U.S. part of our sample, which, as documented in Section 2.2, has aged more. Section 4 will generalize the analysis to two countries, effectively adding the U.S. and endogenizing the safe rate. A key feature of our environment is limited stock-market participation: nonstockholders can hold only bonds, so stockholders alone must hold the risky asset.

We model aging as a decline in mortality while holding the duration of labor income fixed. This approximates the experience of the non-U.S. part of our sample, where increases in longevity have mostly translated into longer retirements because the effective retirement age has not kept pace with life expectancy. Financing a longer retirement from the same labor-income stream increases demand for safe bonds, as they form the closest substitute for the waning labor-income stream (which is also relatively safe in nature) – an insight that goes back to Merton (1971).

To satisfy this "Mertonian" demand for safe income, new bonds may be issued by local non-U.S. stockholders or by an external party, such as the U.S., which lies outside

this partial-equilibrium analysis until Section 4. This distinction determines who bears risk. If local stockholders issue the bonds, the same risky asset must be held on a more leveraged balance sheet. Because local stockholders are also aging, and so share the Mertonian desire to save in bonds, they are *a priori* unwilling to take on this leverage, and the equity premium must rise to compensate them. If the bonds are supplied externally, local stockholders need not issue additional debt and this balance-sheet channel is absent; as Section 4 shows, it is then the premium in the country *issuing* the safe debt that rises. Appendix A contains the derivations for this section, while Appendices B and C cover the transition dynamics and the population-growth extension.

3.1 Demography, labor income, and household balance sheets

The demographic structure follows Yaari (1965) and Blanchard (1985). Households die at the constant rate $\delta > 0$, have no bequest motive, and hold wealth through actuarially fair annuity accounts. Newborns enter without financial wealth. With stationary population, the density at age $a \geq 0$ is $\delta e^{-\delta a}$, and expected remaining life is always $1/\delta$.

We take the evidence that labor income is safer than capital income (Campbell et al., 2001; Davis and Willen, 2013; Lustig et al., 2013; Guiso and Pistaferri, 2020) to the extreme and assume that labor income is locally riskless.[6] Labor income declines with age at rate $\chi > 0$, so a household can be thought of as retiring smoothly rather than at a discrete date. Writing the aggregate wage bill as W, earnings at age a are

$$w(a) = W\frac{\delta + \chi}{\delta}e^{-\chi a}, \qquad \int_0^\infty w(a)\delta e^{-\delta a}\, da = W,$$

where the factor $(\delta + \chi)/\delta$ normalizes aggregate earnings to W. A useful object for the analysis that follows is the *retirement-duration gap*, which we define as

$$\Delta \equiv \frac{1}{\delta} - \frac{1}{\chi + \delta} = \frac{\chi}{\delta(\chi + \delta)} > 0. \tag{1}$$

This gap compares the expected consumption horizon, $1/\delta$, with the expected duration of labor earnings, $1/(\chi+\delta)$. Aging widens it: households must bridge a longer period, during which they no longer earn much labor income, whilst still having to finance consumption (which they do by drawing on past savings and the associated returns). In particular, if mortality falls while $\chi + \delta$ is held fixed, the gap widens one-for-one with expected lifetime.

[6]The results generalize to the case in which labor income follows a diffusion process. The relevant condition is that wage risk is lower than the risk embedded in asset returns. Introducing wage risk does not alter the main economic mechanism, but substantially increases the algebraic complexity.

Since wages are locally riskless and the safe rate is fixed, aggregate human wealth, the present value of expected labor income, is

$$\mathcal{H}(\delta, \chi, W) = \frac{W}{r^* + \chi + \delta}. \tag{2}$$

This is the human-wealth analogue of the Gordon growth formula. A higher safe rate, a steeper decline in earnings (i.e., faster retirement), or higher mortality all shorten the duration of the wage claim and lower its present value.

At birth, a household is permanently assigned to be a stockholder or a nonstockholder, $k \in \{S, N\}$. Stockholders have population share π_S, so type-specific wage bills and human wealth are

$$W_S = \pi_S W, \qquad W_N = (1 - \pi_S) W, \qquad \mathcal{H}_S = \pi_S \mathcal{H}, \qquad \mathcal{H}_N = (1 - \pi_S)\mathcal{H}. \tag{3}$$

Because type is assigned at birth independently of age and earnings, both types face the same age–earnings profile, so each type's human wealth is simply its population share of the aggregate. The permanent type split captures the concentration of equity ownership and consumption exposure in the data (Mankiw and Zeldes, 1991; Attanasio et al., 2002; Brav et al., 2002).

Let $\mathcal{B}_N$ denote local nonstockholders' bond holdings. Their financial wealth consists entirely of bonds, $\mathcal{F}_N = \mathcal{B}_N$, including bonds issued locally and abroad. Their total safe wealth is $\mathcal{T}_N = \mathcal{B}_N + \mathcal{H}_N$: bond holdings plus human wealth.

The local Lucas tree, which we take to represent equity, is in unit supply and pays $D_t\, dt$. Dividends follow a driftless geometric Brownian motion, driven by the standard Brownian motion Z_t:

$$\frac{dD_t}{D_t} = \sigma_D\, dZ_t, \qquad \sigma_D > 0. \tag{4}$$

Let P be the ex-dividend value of the tree and $\mathcal{B}_S \leq 0$ local stockholders' bond position. Negative values denote borrowing, and debt outstanding is $-\mathcal{B}_S$. Their financial wealth, total wealth, leverage, and risky share are

$$\begin{aligned} \mathcal{F}_S &= P + \mathcal{B}_S, \quad \mathcal{T}_S = P + \mathcal{B}_S + \mathcal{H}_S, \\ \mathcal{L}_S &= \frac{P}{P + \mathcal{B}_S}, \quad \theta_S = \frac{P}{P + \mathcal{B}_S + \mathcal{H}_S}. \end{aligned} \tag{5}$$

The distinction between financial and total wealth matters because the two ratios enter the equity premium separately. Financial leverage, $\mathcal{L}_S$, measures the risky position relative to the tree's value net of safe debt, and governs the leverage wedge; the risky share, θ_S,

measures it relative to total wealth including human wealth.

3.2 Household optimization and bond supply

Household problems. A household of type $k \in \{S, N\}$, either a stockholder (S) or a nonstockholder (N), has CRRA period utility with coefficient $\gamma_k > 0$,

$$v_{\gamma_k}(c) = \begin{cases} \dfrac{c^{1-\gamma_k}}{1-\gamma_k}, & \gamma_k \neq 1, \\ \log c, & \gamma_k = 1, \end{cases}$$

where log utility is the limiting case as $\gamma_k \to 1$. Each type also has its own discount rate ρ_k.

We first write the problem of a surviving household and then aggregate its policy across all households of that type. Conditional on being alive at date t, a household survives for another u years with probability $e^{-\delta u}$, so the expected lifetime utility of a type-k household is

$$\mathcal{U}_k\Big(\{C_{k,t+u}\}_{u\geq 0}\Big) = \mathbb{E}_t \int_0^\infty e^{-(\rho_k+\delta)u}\, v_{\gamma_k}(C_{k,t+u})\, du, \qquad k \in \{S, N\}. \tag{6}$$

Mortality acts like additional impatience: it raises each effective discount rate from ρ_k to $\rho_k + \delta$ and shortens the household's horizon.

Nonstockholders hold only annuitized bonds. Their problem is

$$\begin{aligned} V_N(\mathcal{F}_{N,t}, \mathcal{H}_N) = &\sup_{\{C_{N,t+u}\}_{u\geq 0}} \mathcal{U}_N, \\ \text{subject to} \qquad & d\mathcal{F}_{N,t} = \Big[(r^* + \delta)\mathcal{F}_{N,t} + W_N - C_{N,t}\Big]dt. \end{aligned} \tag{7}$$

Surviving households earn $r^* + \delta$: in addition to the safe rate, they receive the mortality credit $\delta\mathcal{F}_{N,t}$, financed by the wealth of those who die.

A stockholder chooses consumption and the amount $\mathcal{E}_{S,t}$ invested in the risky tree. The residual position, $\mathcal{F}_{S,t} - \mathcal{E}_{S,t}$, is held in bonds and may be negative, in which case the stockholder issues bonds. Because only stockholders can hold the tree, market clearing requires $\mathcal{E}_{S,t} = P_t$. Their bond position is then $\mathcal{F}_{S,t} - \mathcal{E}_{S,t} = \mathcal{B}_{S,t}$, so local debt issuance appears as borrowing on the stockholder balance sheet.

Write the cum-dividend tree return as $dR_t = \mu_R dt + \sigma_R dZ_t$. Risk taking is subject to a leverage penalty

$$\Phi_S = \frac{\psi_S}{2}(\mathcal{L}_S - 1)^2, \qquad \tau_S = \frac{\partial \Phi_S}{\partial \mathcal{L}_S} = \psi_S(\mathcal{L}_S - 1),$$

whose marginal charge τ_S applies to each unit of risky holdings. The proceeds are returned to stockholders in proportion to their total wealth, $\alpha_S \mathcal{T}_{S,t}$ with $\alpha_S = \theta_S \tau_S$.

We interpret τ_S as a reduced-form shadow cost of leveraged risk bearing. It can capture compensation for losses concentrated in crisis states, which are not reflected in the Gaussian return process; regulatory, margin, or capital constraints on leveraged balance sheets; and greater effective aversion to concentrated risk than the baseline preferences alone generate. The calibrated wedge should therefore not be read as a literal fee, but as the additional expected return required for stockholders to hold the risky tree. Building on the recent literature on "intermediary asset pricing" (see Haddad and Muir, 2026 for a review), we use this leverage wedge to match the observed equity risk premium.

Taking τ_S, α_S, and ξ_t as given, the stockholder's problem is[7]

$$\begin{aligned} V_S(\mathcal{F}_{S,t}, \mathcal{H}_S) &= \sup_{\{C_{S,t+u}, \mathcal{E}_{S,t+u}\}_{u \geq 0}} \mathcal{U}_S, \\ \text{subject to} \qquad d\mathcal{F}_{S,t} &= \Big[(r^* + \delta)\mathcal{F}_{S,t} + \mathcal{E}_{S,t}(\mu_R - r^* - \tau_S) + (\alpha_S - \xi_t)\mathcal{T}_{S,t} + W_S - C_{S,t}\Big]dt \\ &\quad + \mathcal{E}_{S,t}\sigma_R \, dZ_t. \end{aligned} \tag{8}$$

Stockholder wealth grows with the annuitized safe return, the tree's excess return net of the leverage charge, the rebate net of the external payment, and labor income, less consumption. In equilibrium, the rebate equals the charge, $\alpha_S \mathcal{T}_{S,t} = P_t \tau_S$, so the wedge affects the incentive to bear risk without withdrawing resources from stockholders.

The consumption rules for each household type are

$$\begin{aligned} C_{N,t} &= m_N \mathcal{T}_{N,t}, \qquad m_N \equiv \delta + \frac{\rho_N + (\gamma_N - 1)r^*}{\gamma_N}, \\ C_{S,t} &= m_S(D_t, \mathcal{B}_{N,t})\mathcal{T}_{S,t}. \end{aligned} \tag{9}$$

Each type consumes a fraction of its total wealth. Because the rules are linear in wealth, they aggregate within type, which is why we work with type totals throughout. The nonstockholder propensity m_N depends only on mortality, preferences, and the imposed safe rate. The stockholder propensity m_S has no closed form under CRRA preferences and is solved numerically together with the tree price. Under log utility the consumption rules reduce to $C_{k,t} = (\rho_k + \delta)\mathcal{T}_{k,t}$.

The stockholder portfolio condition determines the risk premium below, while nonstockholder saving determines safe bond demand. Aggregating the nonstockholder bud-

[7] Stockholders also make an external payment $\mathcal{X}_t = \xi_t \mathcal{T}_{S,t}$ to the rest of the world, where each household takes the rate ξ_t as given.

get across survivors and deaths gives

$$\dot{\mathcal{B}}_{N,t} = r^* \mathcal{B}_{N,t} + W_N - C_{N,t}. \tag{10}$$

Although survivors earn $r^* + \delta$, aggregate bond holdings earn only r^*, because the mortality credit redistributes wealth within the sector. Mortality still affects saving through consumption, with a lower mortality rate lengthening the planning horizon and reducing consumption relative to total wealth.

Fixing earnings duration. Our main longevity experiment is designed to approximate the experience of the non-U.S. part of our sample, where increases in life expectancy have mainly added to years spent in retirement. As a result, the average duration of the labor income stream ("earnings duration") has remained relatively constant – beginning at the start of one's working life, and ending when the individual reaches 60–65 years of age. In our framework, where households retire gradually (to maintain analytical tractability), earnings duration is given by $1/(\chi + \delta)$. Consequently, when we lower mortality δ, thereby increasing expected lifetimes, we raise χ one-for-one, so that $\chi + \delta$, and therewith earnings duration and human wealth $\mathcal{H}$, stays at its initial level:

$$\underline{\chi} + \underline{\delta} = \overline{\chi} + \overline{\delta} \Longleftrightarrow \overline{\chi} = \underline{\chi} + \underline{\delta} - \overline{\delta}, \tag{11}$$

where underlines denote initial values and overlines final values. This removes the windfall that would arise if longer lives also lengthened working careers: the present value of the wage stream, and hence $\mathcal{H}_S$ and $\mathcal{H}_N$, is unchanged. Stationary nonstockholder bond holdings are then

$$\mathcal{B}_N^*(\delta) = \mathcal{H}_N \frac{r^* + \gamma_N \chi - \rho_N}{\gamma_N \delta + \rho_N - r^*}. \tag{12}$$

Suppliers of bonds. Initially, all bonds held by nonstockholders are issued by local stockholders, so $\underline{\mathcal{B}}_N = -\underline{\mathcal{B}}_S = \mathcal{B}_N^*(\underline{\delta})$. A share $\omega \in [0, 1]$ of any subsequent increase is supplied from abroad at the world safe rate, and the remainder by local stockholders, whose bond position becomes

$$\mathcal{B}_S = \underline{\mathcal{B}}_S - (1 - \omega)(\mathcal{B}_N - \underline{\mathcal{B}}_N).$$

We can now establish the following result.

Lemma 1 (Longevity raises bond demand and local borrowing)**.** *Suppose stationary bond holdings are positive and stable, i.e., $r^* + \gamma_N \chi - \rho_N > 0$ and $\gamma_N \delta + \rho_N - r^* > 0$. Holding $\chi + \delta$ fixed,*

so that human wealth $\mathcal{H}_N$ is unchanged, a decline in mortality raises nonstockholders' stationary bond holdings, and local stockholders borrow the share $1-\omega$ of the increase:

$$\frac{d\mathcal{B}_N^*}{d\delta} = -\frac{\gamma_N[\mathcal{B}_N^*(\delta)+\mathcal{H}_N]}{\gamma_N\delta+\rho_N-r^*} < 0, \qquad \frac{d\mathcal{B}_S^*}{d\delta} = -(1-\omega)\frac{d\mathcal{B}_N^*}{d\delta}. \tag{13}$$

Intuitively, a lower δ widens the retirement-duration gap Δ: the same human wealth must now finance consumption over a longer expected lifetime. Households therefore consume a smaller fraction of their wealth and build up bonds to replace the labor income that ends well before they die.

3.3 Local debt issuance and the equity risk premium

Lemma 1 shows that longer lives raise local debt whenever $\omega < 1$. We now show how this debt raises the equity premium. Under CRRA preferences, the stockholder problem (8) yields the portfolio condition that pins down the equity risk premium (ERP) required to hold the tree:

$$\text{ERP} \equiv \mu_R - r^* = \underbrace{\psi_S(\mathcal{L}_S-1)}_{\text{leverage wedge, } \tau_S} + \underbrace{\gamma_S\theta_S\sigma_R^2}_{\text{risk compensation}} + \underbrace{(\gamma_S-1)\sigma_D D v_D \sigma_R}_{\text{intertemporal hedging}}. \tag{14}$$

The first term is the leverage wedge. It is zero when stockholders finance the tree entirely from their own financial wealth ($\mathcal{L}_S = 1$) and rises as the same risky position is supported by more borrowing, at a rate governed by ψ_S. The second term is conventional risk compensation, which increases with risk aversion γ_S, exposure θ_S, and return variance σ_R^2. The final term is the intertemporal hedging term, which reflects stockholders' desire to hedge changes in future investment opportunities, where v_D measures how their continuation value responds to dividends. It can have either sign, vanishes under log utility, and is small in our calibration. Appendix A.6 derives the full pricing system.

Local debt is what links the demand for bonds to the equity premium. When local stockholders issue the additional bonds that nonstockholders demand, they continue to hold the entire tree, so the same risky position is financed with more borrowing. This moves each term of (14): it raises leverage, changes stockholders' risky share, and can alter return volatility. The size of these effects also depends on how the tree price responds, because a change in the price revalues stockholders' balance sheets. We therefore write the price as $P(D, \mathcal{B}_S; \delta, \omega)$ and let $P_B = \partial P/\partial \mathcal{B}_S$ denote its response to the stockholder bond position, so that $P_B > 0$ means additional borrowing *lowers* the tree's price. We begin with leverage.

Proposition 2 (Local debt raises leverage)**.** *Suppose stockholder financial wealth is positive,* $P + \mathcal{B}_S > 0$*. Then*

$$\frac{\partial \mathcal{L}_S}{\partial(-\mathcal{B}_S)} = \frac{P - \mathcal{B}_S P_B}{(P + \mathcal{B}_S)^2} > 0 \tag{15}$$

whenever $\mathcal{B}_S P_B / P < 1$*. In particular, this condition holds at a fixed price and whenever borrowing does not raise the tree price,* $P_B \geq 0$*. Higher local debt therefore raises leverage and, since* $\tau_S = \psi_S(\mathcal{L}_S - 1)$*, the leverage wedge.*

The numerator shows the two ways debt raises leverage. First, at a fixed price, additional debt reduces stockholders' financial wealth without changing their risky position. Second, if borrowing also lowers the tree price, financial wealth falls by as much as the tree itself, which is a larger proportional fall because the tree is partly debt-financed, so leverage rises further. More generally, the condition $\mathcal{B}_S P_B / P < 1$ allows the tree price to rise in response to borrowing, provided that response is not large enough to overturn the balance-sheet effect. A transparent sufficient condition is $P_B \geq -1$: one additional unit of safe debt cannot raise the market value of the tree by more than one unit. This rules out the extreme case in which borrowing triggers an even larger asset-price response and thereby *reduces* leverage. Equivalently, this condition implies that debt issuance cannot mechanically create stockholder net worth.[8]

Next, consider how safe debt changes stockholders' risky asset holdings.

Proposition 3 (Local debt and risky asset holdings)**.** *Suppose stockholders' other wealth, human wealth net of debt, is positive,* $\mathcal{H}_S + \mathcal{B}_S > 0$*. Then*

$$\frac{\partial \theta_S}{\partial(-\mathcal{B}_S)} = \frac{P - (\mathcal{H}_S + \mathcal{B}_S) P_B}{(P + \mathcal{B}_S + \mathcal{H}_S)^2}. \tag{16}$$

Higher local debt raises the share of stockholders' wealth held in the risky asset if and only if it lowers the tree price proportionally less than it lowers their other wealth, $P_B / P < 1/(\mathcal{H}_S + \mathcal{B}_S)$*. In particular, it does so at a fixed tree price.*

Safe debt changes stockholders' risky asset holdings through their balance sheet. The risky share compares the tree with the rest of stockholders' wealth, their human wealth net of debt. When stockholders issue additional safe debt, they continue to hold the entire tree, so the rest of their wealth falls one-for-one and a larger fraction of their wealth is tied up in the risky asset. Issuing safe debt therefore concentrates risk on stockholders'

[8]Note that Proposition 2 does not require $P_B \geq 0$: a modest positive price response to borrowing (in particular: $-1 < P_B < 0$) is compatible with rising leverage. In the quantitative solution, $P_B > 0$ at the steady state, which satisfies the proposition's condition more strongly. But this sign is a property of the calibrated equilibrium rather than a general implication of the proposition.

balance sheets. If borrowing also lowers the tree price, the value of the risky position falls as well, which partly offsets this concentration. The proposition compares the two proportional changes: the risky share rises as long as the tree loses value proportionally less than the rest of stockholders' wealth.

Finally, consider how safe debt affects the riskiness of the tree itself.

Proposition 4 (Local debt and return volatility)**.** *The volatility of the cum-dividend return equals dividend volatility times the price-dividend elasticity,*

$$\sigma_R = \sigma_D \frac{DP_D}{P}. \tag{17}$$

Local debt therefore affects return volatility only through this elasticity,

$$\frac{\partial \sigma_R}{\partial(-\mathcal{B}_S)} = \sigma_D \frac{\partial}{\partial(-\mathcal{B}_S)}\left(\frac{DP_D}{P}\right), \tag{18}$$

so higher local debt raises return volatility if and only if it makes the tree price more sensitive to dividends.

Debt makes the price more sensitive to dividends through a feedback loop on stockholders' balance sheets. Suppose a negative dividend shock lowers the tree price. Because stockholders' debt is fixed, the loss falls entirely on their financial wealth, $P + \mathcal{B}_S$, which therefore falls proportionally more than the price. Leverage and the leverage wedge rise, raising the return stockholders require to hold the tree, and this higher discount rate depresses the price further. The more debt stockholders carry, the thinner their financial wealth relative to the tree, and the more strongly leverage responds to a given price change. Prices then move more than dividends, so return volatility rises even though dividend volatility is unchanged.

Differentiating the premium (14) with respect to local debt collects these channels:

$$\frac{\partial \text{ERP}}{\partial(-\mathcal{B}_S)} = \underbrace{\frac{\partial \tau_S}{\partial(-\mathcal{B}_S)}}_{\text{leverage-wedge channel}} + \underbrace{\gamma_S \sigma_R^2 \frac{\partial \theta_S}{\partial(-\mathcal{B}_S)}}_{\text{asset-allocation channel}} + \underbrace{2\gamma_S \theta_S \sigma_R \frac{\partial \sigma_R}{\partial(-\mathcal{B}_S)}}_{\text{volatility channel}} + \underbrace{(\gamma_S - 1)\sigma_D D \frac{\partial(v_D \sigma_R)}{\partial(-\mathcal{B}_S)}}_{\text{intertemporal hedging}}. \tag{19}$$

The first term is the change in the shadow cost of leverage, which is positive by Proposition 2. The next two split conventional risk compensation: the asset-allocation channel changes stockholders' exposure at a given return variance, and is positive at a fixed tree price by Proposition 3; the volatility channel changes the risk borne at a given exposure, and is positive when debt makes the price more sensitive to dividends (Proposition 4). The

final term allows hedging demand to adjust with investment opportunities and vanishes under log utility.

Lemma 1 and the three propositions together trace how longer lives reach the equity premium. A decline in mortality raises nonstockholders' demand for bonds. When $\omega < 1$, local stockholders meet part of this demand by borrowing, so they hold the same tree with more debt. This raises their leverage and the leverage wedge, concentrates risk on their balance sheets, and can make the tree's return more volatile, each of which raises the return stockholders require to hold the tree. Longer lives therefore raise the equity premium through the debt they lead stockholders to issue. When $\omega = 1$, the rest of the world supplies the additional bonds, local debt is unchanged, and this channel is shut.

3.4 Illustrative calibration

We discipline the household, demographic, and dividend parameters using the non-U.S. part of our sample, where the fraction of a lifetime spent in retirement has risen most (Section 2.2). The initial mortality and earnings-decline rates imply an expected remaining lifetime of about 59 years and a survival-adjusted duration of earnings of about 20 years, so households must finance several decades of consumption after their labor income has largely run down. The external share ω plays a different role. Setting $\omega = 0.70$ places 30 percent of the additional bond demand generated by increased longevity on local non-U.S. stockholder balance sheets at the imposed world safe rate; the remaining 70 percent is supplied externally. This is, of course, an arbitrary choice (not meaning to reflect an empirical estimate of financial openness) with this fixed-rate benchmark experiment solely serving to illustrate the underlying mechanism. We choose an intermediate value between full local supply ($\omega = 0$) and full external supply ($\omega = 1$), and Section 3.6.2 reports results across a wide range of values.

Based on empirical evidence in Chien and Naknoi (2015), we set the stockholder share to 30%, so that equity risk is borne by a minority of households. We set dividend-growth volatility to 12.0%, based on the annual volatility of GDP-weighted real dividend growth for the five non-U.S. G6 economies with available equity observations over 1970–2020.[9]

We set initial local stockholder debt, and hence nonstockholders' initial bond holdings, equal to 145 percent of output. This is somewhat higher than observed central government debt-to-GDP ratios, to reflect that some safe assets are issued by local, quasi-public, or non-governmental entities, such as government-sponsored or government-owned utilities and

[9]The series comes from the Jordà–Schularick–Taylor Macrohistory Database (Jordà et al., 2017, 2019). It contains no usable Canadian equity-return observations. We do not impute them; annual GDP weights are renormalized across France, Germany, Italy, Japan, and the United Kingdom.

highly rated corporations.

Table 1 summarizes the two-step calibration procedure. First, we choose the nonstockholder discount rate ρ_N so that nonstockholders wish to hold exactly this stock of bonds, that is, so that their bond holdings have zero drift in equation (10). Second, given the resulting balance sheet, we choose the leverage sensitivity ψ_S so that the initial equity risk premium equals 5 percent, which we use as a stylized benchmark. The stockholder discount rate ρ_S governs stockholder consumption; we set it to a conventional value of 4%.

Table 1: Non-U.S. calibration

Object	Value	Source
World safe rate, r^*	2.70%	As in Section 5.1
Initial mortality and earnings decline, $\underline{\delta}, \underline{\chi}$	1.69%, 3.39%	Calibrated to longevity trends
Stockholder share, π_S	30%	Non-U.S. evidence
Nonstockholder CRRA coefficient, γ_N	4.00	As in Section 5.1
Stockholder CRRA coefficient, γ_S	1.50	As in Section 5.1
Dividend volatility, σ_D	12.00%	Five-country estimate
Initial local stockholder bond position / initial output, $\underline{\mathcal{B}_S}/\underline{Y}$	-145.00%	Calibration
Initial equity risk premium	5.00%	Target
Nonstockholder discount rate, ρ_N	12.23%	Calibrated to stationary bond holdings
Leverage sensitivity, ψ_S	0.0666	Calibrated to premium target

Notes: $\underline{Y}$ denotes initial output, the fixed reference denominator in the tables. Negative stockholder bond positions denote borrowing. Stockholder debt comprises bonds issued by the stockholder sector, not government debt or an exact counterpart of a single financial-accounts series. Appendix A.5 reports the parameter–target mapping, participation comparisons, and discount-rate and target diagnostics.

Table 2 reports the initial stationary moments. With return volatility of about 13 percent and moderate risk aversion, conventional risk compensation accounts for only 43 percent of the 5 percent premium; the leverage wedge accounts for the remaining 57 percent. Stockholders initially finance about 30 percent of the tree with debt, a leverage ratio of 1.43.

Table 2: Initial stationary moments

Moment	Model value
Equity risk premium, ERP	5.00%
Decomposition of Equity Risk Premium	
Leverage-wedge component	2.85 percentage points
Risk-compensation component	2.14 percentage points
Intertemporal-hedging component	0.00 percentage points
Financial leverage, $\mathcal{L}_S$	1.43
Risky allocation, θ_S	82.07%
Return volatility, σ_R	13.20%

Notes: The decomposition of the equity risk premium corresponds to the three terms in equation (14). Risky allocation is the tree value relative to stockholders' total wealth, including human wealth. Return volatility is the annualized local conditional standard deviation of the cum-dividend return on the non-U.S. risky tree, $\sigma_R = \sigma_D DP_D/P$, evaluated at the stationary state.

3.5 Longer retirement spells and the equity risk premium

We now lower mortality permanently to increase the retirement-duration gap by four years, close to the increase in expected years after retirement in the non-U.S. sample since 1980 (Section 2.2). As explained in Section 3.2, we raise χ so that earnings duration, and therewith human wealth, is unchanged; the additional years are spent entirely in retirement, with individuals continuing to retire "at 65" (recall our discussion around equation (11)). Expected remaining lifetime rises from about 59 to 63 years, while the the earnings-weighted average timing of future labor income, $1/(\chi + \delta)$, stays at about 20 years; in terms of the model's parameters, mortality falls from 1.689 to 1.582 percent and χ rises from 3.393 to 3.500 percent.

Nonstockholder bond holdings rise by 19.67 percentage points of initial output, as households save more to finance a longer retirement from a labor income stream of unchanged duration. With $\omega = 0.70$, local stockholders supply 5.90 percentage points of these bonds and the rest of the world the remaining 13.77. Because local stockholders continue to own the entire tree, their leverage rises from 1.43 to 1.49.

Local stockholders are themselves aging: their retirement-duration gap also widens, which strengthens their own "Mertonian" demand for safe income to replace a labor income stream that now fades sooner relative to their consumption horizon. To make them nevertheless willing to hold the more leveraged, riskier tree position, the ERP needs to rise – in this calibration by 50.4 basis points. Table 3 shows that most of this increase, 42.4 basis points, comes from the leverage wedge, with a further 8.6 basis points from

higher return volatility.

Table 3: Impact of increased longevity in the non-U.S. benchmark

Object	Initial	Final (longevity)
Equity risk premium, ERP	5.00%	5.50%
Nonstockholder bond holdings / initial output, $\mathcal{B}_N/\underline{Y}$	145.00%	164.67%
Local stockholder bond position / initial output, $\mathcal{B}_S/\underline{Y}$	-145.00%	-150.90%
Financial leverage, $\mathcal{L}_S$	1.43	1.49
Risky allocation, θ_S	82.07%	82.11%
Return volatility, σ_R	13.20%	13.46%
Decomposition of Equity Risk Premium (basis points)		
Leverage-wedge contribution		42.43
Asset-allocation contribution		0.09
Volatility contribution		8.57
Intertemporal-hedging contribution		-0.67
Total		50.42

Notes: Negative stockholder bond positions denote borrowing. The retirement-duration gap increases by four years. The aggregate wage bill is held fixed and χ is adjusted to hold human wealth fixed. The leverage sensitivity ψ_S is held fixed across equilibria. The decomposition uses arithmetic endpoint averages: changes in the hedging loading enter the hedging contribution, while changes in return volatility enter the volatility contribution (Appendix A.6). Return volatility is the annualized local conditional standard deviation of the cum-dividend return.

3.6 Extensions and robustness

As robustness checks, we extend the benchmark in three directions. First, we recognize that demographic change is driven not only by increases in longevity but also by lower population growth. Next, we discuss how the results vary with who supplies the additional bonds, and we conclude with transitional dynamics.

3.6.1 Slower population growth

Populations age not only because people live longer but also because fewer are born: with slower population growth, the population is older on average even if no one's lifetime changes. Building on Buiter (1988), we add this second source of aging to our longevity experiment. It works through the same mechanism: an older population demands more bonds, local stockholders meet part of this demand by borrowing, and the equity premium rises.

We start population growth at 0.50 percent per year and recalibrate the nonstockholder discount rate and leverage sensitivity to the same bond-holdings and equity-premium

targets as in Section 3.4, holding them fixed thereafter. We first apply the longevity experiment of Section 3.5 at this growth rate and then lower population growth to zero.[10] Appendix C gives the derivations and reports the three allocations.

Slower population growth raises the average age of households, equal to $1/(n + \delta)$, and an older population holds, on average, more bonds per person for two reasons. First, newborn households enter without financial wealth, so fewer births means fewer households that have yet to accumulate savings. Second, older households have seen their earnings decline for longer and therefore have a stronger "Mertonian" demand for safe bonds to replace their fading labor income. These forces make nonstockholder bond holdings rise by a further 21.06 percentage points of initial output, from 162.86 to 183.93 percent. Local stockholders supply 6.32 percentage points of this additional debt while continuing to hold the entire tree, and their leverage rises from 1.49 to 1.58 as the additional borrowing and the fall in the equity price reduce their financial wealth.

An older population therefore finds equity less attractive to hold. Lowering population growth from 0.50 to 0.00 percent raises the ERP from 5.47 to 6.16 percent, adding 69.4 basis points to the 46.9 basis points generated by longer retirement spells alone. Lower population growth rates thus reinforce the effect of spending more of one's life in retirement.[11] Table 4 summarizes the two steps and their combined effect.

3.6.2 External bond supply and the local equity premium

Our benchmark places 30 percent of the additional bond demand on local stockholder balance sheets. As a robustness check, we repeat the longevity experiment of Section 3.5 while varying the share ω of additional bonds supplied by the rest of the world over $[0.40, 1]$, the range over which our numerical solution is validated. The financial calibration and initial debt are held fixed.

Figure 3 shows that the premium response falls as the external share rises. It is 50.4 basis points at the benchmark $\omega = 0.70$ and 31.8 basis points when 80 percent of the additional bonds come from abroad. When all of them come from abroad, the response is essentially zero. Appendix Table 11 decomposes the response at selected values of ω.

The results show that what matters for the local premium is not the demand for bonds itself but who supplies them. The external share determines how much of the additional

[10] The growth rates are illustrative; Section 4 calibrates population growth to the data.

[11] The additional population-growth effect is measured from the post-longevity allocation, with prices and returns evaluated at the steady-state dividend and nonstockholder bond holdings at their zero-drift value. The sequence compares allocations with adjusted balance sheets; it does not trace a demographic transition. The longevity contribution here differs from the 50.4 basis points in Section 3.5 because the initial rate of population growth (and other elements of the calibration) differ.

Table 4: Longevity and population slowdown: Decomposition of Equity Risk Premium

Variable and unit	Longevity increase ($\underline{x} \to x^L$)	Population after longevity ($x^L \to \overline{x}$)	Combined change ($\underline{x} \to \overline{x}$)
A. Changes in prices and balance sheets			
Equity risk premium, ERP, basis points	+46.89	+69.40	+116.29
Financial leverage, $\mathcal{L}_S$, ratio points	+0.06	+0.08	+0.14
Local stockholder debt, $-\mathcal{B}_S$, percent change	+3.70	+4.20	+8.05
Stockholder financial wealth, $\mathcal{F}_S$, percent change	-8.94	-10.84	-18.81
Nonstockholder bond holdings / initial output, $\mathcal{B}_N/\underline{Y}$, percentage points	+17.86	+21.06	+38.93
Risky allocation, θ_S, percentage points	-0.01	+0.07	+0.06
Equity-return volatility, σ_R, percentage points	+0.23	+0.45	+0.69
B. Decomposition of Equity Risk Premium (basis points)			
Leverage wedge	+39.89	+55.24	+95.13
Risky allocation	-0.02	+0.19	+0.17
Return volatility	+7.60	+15.18	+22.79
Intertemporal hedging	-0.58	-1.21	-1.80
Total ERP change	+46.89	+69.40	+116.29

Notes: For any variable x, $\underline{x}$, x^L, and $\overline{x}$ denote the Initial, Longevity, and Final endpoints. Longevity applies the four-year longevity increase at $n = 0.005$; Final additionally lowers n to zero. The middle column is the population increment after longevity. All stocks are per member of the total population. Ratios to output use the same initial per-capita output in every column. Percent changes use each column's starting allocation and compound across sequential steps. Basis-point and ratio-level changes add. Panel B uses the exact endpoint decomposition for each column; its allocation, volatility, and hedging terms need not add across columns because the endpoint-average weights differ. Totals are computed before rounding. The main zero-growth longevity benchmark has a different initial calibration.

demand becomes local debt: the more bonds come from abroad, the less local stockholders need to borrow, and the smaller the rise in their leverage. At $\omega = 1$, bond demand rises by just as much, but local balance sheets are unchanged and the premium barely moves. Who supplies the additional bonds is therefore an important part of the mechanism. The two-country model of Section 4 allows bonds to flow endogenously across countries, so that the division of bond supply between Home and Foreign stockholders is determined in equilibrium rather than imposed.

3.6.3 Transitional dynamics

The comparisons so far are between stationary equilibria, after households have fully adjusted their savings. As a final robustness check, we examine how the response unfolds over time, in the spirit of Farhi and Gourio (2018), since demographic change takes decades and asset prices are forward-looking. We consider the same four-year increase in

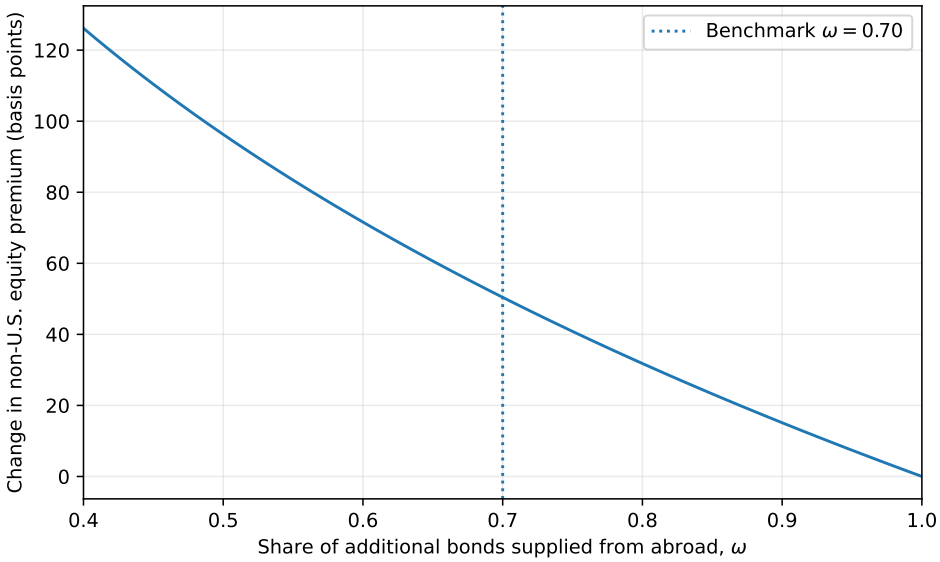


Figure 3: External bond supply and the non-U.S. equity premium

Notes: The figure reports the change in the non-U.S. equity premium following the four-year longevity increase, as the share ω of additional bonds supplied from abroad varies. At $\omega = 1$, all additional bonds are supplied from abroad. The financial calibration and initial debt are held fixed. Results are shown over the range for which the numerical solution is validated.

the retirement-duration gap, now announced unexpectedly at date zero, with the world safe rate fixed and $\omega = 0.70$. Bond holdings and debt cannot jump: they build up only as nonstockholders save and stockholders issue the corresponding debt. The tree price, by contrast, can respond immediately.

The premium responds in two phases. On impact, before any additional bonds have been accumulated, the ERP rises from 5.00 to 5.13 percent, an increase of 13.2 basis points and about 26 percent of the long-run effect. The remainder builds up gradually: the ERP reaches 5.25 percent after ten years and 5.50 percent in the long run, with nonstockholder bond holdings closing half of the gap to their new stationary level in 17.5 years, so the premium adjusts faster than the balance sheets that drive it.[12]

The impact response reflects anticipation. Investors foresee both longer lives and the borrowing that stockholders will undertake to meet the additional bond demand, so the tree price falls at once. With debt still unchanged, the lower price alone reduces stockholders' financial wealth and raises their leverage, and hence the premium. The rest of the adjustment follows the slow accumulation of bonds, as each additional unit of local debt raises leverage further. The stationary result therefore combines an immediate response in asset prices with a gradual increase in stockholder borrowing that takes several

[12]The transition is evaluated at the steady-state dividend $D_t = D_0$. Table 12 and Appendix B give the price and bond-holdings paths, including the external payment.

decades to complete, the horizon over which demographic change itself unfolds. A rising premium may therefore precede the observed build-up of debt.

4 Two-country model

We now extend the fixed-rate benchmark of Section 3 to a two-country model, in which the suppliers of the additional bonds are explicit. In our labeling, Home represents the U.S., while Foreign aggregates the non-U.S. advanced economies studied in Section 2. Both countries retain the household structure of Section 3: nonstockholders hold bonds, while stockholders hold Home and Foreign equity and issue debt, which nonstockholders in either country may hold. The safe rate and the division of debt between Home and Foreign stockholders, both imposed in Section 3, are now determined by world market clearing.

This allows us to ask how differential aging affects the returns required on Home and Foreign equity. When retirement-duration gaps widen more in Foreign, Foreign households develop the stronger "Mertonian" demand for bonds, and Foreign stockholders are the more reluctant to take leveraged equity positions. Home stockholders are therefore better placed, given their "relative youth", to supply the additional bonds and carry the associated equity risk. In our experiment, which combines longer lives with slower population growth, Home stockholders issue most of the additional debt, and the premium on Home equity rises by considerably more than the premium on Foreign equity, in line with the divergence documented in Section 2.1.

4.1 Demography, labor income, and traded assets

We retain the demographic and earnings structure of Section 3.1, now allowing mortality δ_i, earnings decline χ_i, and population growth n_i to differ across Home and Foreign, $i \in \{H, F\}$, so that the two countries can age differently, and hence demand bonds with different intensity. Population growth enters the age distribution and the dilution of wealth by newborns as in Section 3.6.1. Wages remain locally riskless, but their present value now moves with the equilibrium safe rate.

Each country is endowed with an equity tree whose aggregate dividend grows with its population, so that the tree remains proportional to the size of its economy and the dividend per person, $D_{i,t}$, is constant; $P_{i,t}$ denotes its price per person. The wage per person, W_i, is also constant, while χ_i governs only the timing of earnings over the life

cycle. We normalize output per person to one in each country,

$$y_{i,t} = W_{i,t} + D_{i,t} = 1, \tag{20}$$

and write Y_H and Y_F for aggregate output in Home and Foreign at the initial reference date, with world output $Y = Y_H + Y_F$.

Households trade the two trees and a safe bond in zero net supply. The safe rate r_t, fixed in Section 3, is now determined by world bond-market clearing. Write $\text{ERP}_{j,t} \equiv \mu_{R,j,t} - r_t$ for the premium on tree j. The conditional return volatilities are $\sigma_{R,H}$ and $\sigma_{R,F}$, with correlation ϱ_R and covariance $\sigma_{R,HF} = \varrho_R \sigma_{R,H} \sigma_{R,F}$.

4.2 Households, human wealth, and portfolios

We retain the equity-participation structure of Section 3: in each country i, a share $\pi_{S,i}$ of households are stockholders and the rest are nonstockholders. Household subscripts S and N identify stockholders and nonstockholders, followed by country and, where relevant, time indices. All sector wealth, bond holdings, debt, and consumption quantities below are measured per country resident, using the country's entire population.

In a stationary equilibrium, the human wealth of each household type is

$$\mathcal{H}_{S,i,t} = \frac{\pi_{S,i} W_{i,t}}{r_t + \chi_i + \delta_i}, \qquad \mathcal{H}_{N,i,t} = \frac{(1 - \pi_{S,i}) W_{i,t}}{r_t + \chi_i + \delta_i}.$$

This is the capitalization formula (2), now evaluated at the equilibrium safe rate. Wage streams remain riskless, but their present value is not: a fall in the safe rate raises human wealth. Human wealth thus behaves like a bond whose duration is that of expected earnings, $1/(r_t + \chi_i + \delta_i)$: just as bond prices rise when interest rates fall, so does the value of future wages.

Let $\mathcal{B}_{N,i,t}$ denote country-i nonstockholder bond holdings, so that their total wealth is $\mathcal{T}_{N,i,t} = \mathcal{B}_{N,i,t} + \mathcal{H}_{N,i,t}$. They consume the fraction $m_{N,i}(r_t)$ of their total wealth, with m_N as in equation (9) evaluated at the equilibrium safe rate. Aggregating the household budget as before, and allowing for the dilution of wealth per person by newborns, gives

$$\dot{\mathcal{B}}_{N,i,t} = (r_t - n_i)\mathcal{B}_{N,i,t} + (1 - \pi_{S,i}) W_{i,t} - C_{N,i,t}.$$

In a stationary equilibrium, setting this drift to zero yields

$$\mathcal{B}^*_{N,i,t} = \mathcal{H}_{N,i,t} \frac{r_t + \gamma_{N,i}\chi_i - \rho_{N,i}}{\gamma_{N,i}(\delta_i + n_i) + \rho_{N,i} - r_t}. \tag{21}$$

The numerator and denominator are positive in the equilibria we consider. Lower mortality raises desired nonstockholder bond holdings through the same mechanism as in Section 3.2, and lower population growth raises them by reducing dilution, as in Section 3.6.1. What is new is that the safe rate now adjusts. In the two country setting, the safe rate falls as world bond demand rises. A lower return on bonds makes saving in them less attractive, which partly offsets the increase in desired bond holdings.

Country-i stockholders hold both equities and have a bond position $\mathcal{B}_{S,i,t} \leq 0$ where they may borrow from nonstockholders in either country. Let $\mathcal{E}_{ij,t}$ be the value of equity j held by country-i stockholders. Their balance sheet extends equation (5):

$$\mathcal{F}_{S,i,t} = \mathcal{E}_{iH,t} + \mathcal{E}_{iF,t} + \mathcal{B}_{S,i,t}, \qquad \mathcal{T}_{S,i,t} = \mathcal{F}_{S,i,t} + \mathcal{H}_{S,i,t}. \tag{22}$$

Stockholders have the CRRA preferences of Section 3.2, with common curvature γ_S and discount rate $\rho_{S,i}$. The two risky shares are $\theta_{S,i,H,t} = \mathcal{E}_{iH,t}/\mathcal{T}_{S,i,t}$ and $\theta_{S,i,F,t} = \mathcal{E}_{iF,t}/\mathcal{T}_{S,i,t}$, where i identifies the investor's country and the next country index identifies the issuer. In a stationary equilibrium, stockholders consume the fraction $m_{S,i,t}$ of total wealth, $C_{S,i,t} = m_{S,i,t}\mathcal{T}_{S,i,t}$.

Expressed per country resident, the stockholder budget includes dilution of financial wealth by population growth:

$$\dot{\mathcal{F}}_{S,i,t} = (r_t - n_i)\mathcal{F}_{S,i,t} + \mathcal{E}_{iH,t}\mathrm{ERP}_{H,t} + \mathcal{E}_{iF,t}\mathrm{ERP}_{F,t} + \pi_{S,i}W_{i,t} - C_{S,i,t}.$$

In a stationary equilibrium, financial wealth has zero drift. Together with portfolio choice, this condition determines each country's stockholder bond position through equation (22). This replaces the issuance rule of Section 3.2: how much debt Home and Foreign stockholders issue is now an equilibrium outcome rather than an imposed share ω.

4.3 Issuer balance sheets and leverage wedges

Stockholders in each country can hold both countries' trees, financing these equity positions with their own financial wealth and with debt. Since debt finances stockholders' entire equity portfolio, we measure the leverage of country-i stockholders over their holdings of both trees,

$$\mathcal{L}_{S,i,t} = \frac{\mathcal{E}_{iH,t} + \mathcal{E}_{iF,t}}{\mathcal{F}_{S,i,t}} = 1 - \frac{\mathcal{B}_{S,i,t}}{\mathcal{F}_{S,i,t}}, \tag{23}$$

and assume that the wedge on country-i equity rises with this leverage, with slope ψ_i:

$$\tau_{i,t} = \psi_i(\mathcal{L}_{S,i,t} - 1) = -\psi_i \frac{\mathcal{B}_{S,i,t}}{\mathcal{F}_{S,i,t}}, \qquad i \in \{H, F\}. \tag{24}$$

The wedge is thus proportional to country-i stockholders' debt per unit of financial wealth. As before, the charges are rebated, so they affect incentives without absorbing resources. When Home stockholders borrow more relative to their financial wealth, the premium on Home equity therefore rises.

This assumption is a reduced-form way of capturing limits to international risk sharing, whereby domestic risk-bearing capacity matters most at the margin when it comes to pricing domestic equity. In practice, such an outcome could arise for various reasons – ranging from limited cross-border participation and information advantages (Van Nieuwerburgh and Veldkamp, 2009), to institutional mandates, slow-moving capital (Duffie, 2010), or specialized local intermediaries whose balance-sheet constraints affect required returns (Gârleanu and Pedersen, 2011). We do not model these frictions explicitly.

4.4 International market clearing

World asset markets must clear. We aggregate holdings measured per country resident using the fixed initial output shares Y_H/Y and Y_F/Y, which also equal the initial population shares. Holdings of each tree across both countries must equal its market value:

$$\begin{aligned} \frac{Y_H}{Y}\mathcal{E}_{HH,t} + \frac{Y_F}{Y}\mathcal{E}_{FH,t} &= \frac{Y_H}{Y}P_{H,t}, \\ \frac{Y_H}{Y}\mathcal{E}_{HF,t} + \frac{Y_F}{Y}\mathcal{E}_{FF,t} &= \frac{Y_F}{Y}P_{F,t}. \end{aligned} \tag{25}$$

Nonstockholder bond holdings are matched by stockholder borrowing, so bond positions sum to zero across the two countries:

$$\frac{Y_H}{Y}\left(\mathcal{B}_{N,H,t} + \mathcal{B}_{S,H,t}\right) + \frac{Y_F}{Y}\left(\mathcal{B}_{N,F,t} + \mathcal{B}_{S,F,t}\right) = 0. \tag{26}$$

The safe rate adjusts until the bonds that nonstockholders in both countries wish to hold equal the debt that stockholders in both countries are willing to issue. How this debt is divided between Home and Foreign stockholders depends on their balance sheets, and hence on how much each country has aged.

A country's net external bond position is $\mathcal{B}_{N,i,t} + \mathcal{B}_{S,i,t}$: domestically held bonds enter the nonstockholder and stockholder positions with opposite signs and cancel, so a positive

position indicates net lending abroad and a negative one net borrowing. Home's net foreign assets add Foreign equity held at Home and subtract Home equity held abroad, the latter scaled by Y_F/Y_H to express it per Home resident:

$$\text{NFA}_{H,t} = \mathcal{B}_{N,H,t} + \mathcal{B}_{S,H,t} + \mathcal{E}_{HF,t} - \frac{Y_F}{Y_H}\mathcal{E}_{FH,t}. \tag{27}$$

Home can thus be a net debtor in bonds while holding a positive net position in equity, as a "world venture capitalist" would (Gourinchas and Rey, 2007).

4.5 Valuation and risk premia

Stockholders in both countries have the same risk aversion and trade at the same prices, so they choose the same risky shares, $\theta_{S,H,j,t} = \theta_{S,F,j,t} \equiv \theta_{S,j,t}$, $j \in \{H, F\}$. Each country's stockholders therefore own every tree in proportion to their share of world stockholder wealth, and equity-market clearing implies that the risky share of each tree equals its market value relative to world stockholder wealth, $\theta_{S,j,t} = (Y_j/Y)P_{j,t}/\mathcal{T}_{S,W,t}$, where $\mathcal{T}_{S,W,t} = \sum_i (Y_i/Y)\mathcal{T}_{S,i,t}$. The portfolio conditions then give the two market-pricing equations:

$$\begin{aligned} \text{ERP}_{H,t} &= \gamma_S\left(\sigma^2_{R,H}\theta_{S,H,t} + \sigma_{R,HF}\theta_{S,F,t}\right) + \tau_{H,t}, \\ \text{ERP}_{F,t} &= \gamma_S\left(\sigma_{R,HF}\theta_{S,H,t} + \sigma^2_{R,F}\theta_{S,F,t}\right) + \tau_{F,t}. \end{aligned} \tag{28}$$

Each premium has two parts. The first is conventional compensation for portfolio risk, which depends on the variances and covariance of the two returns and on the risky shares stockholders hold. The second is the leverage wedge of the issuing country.

The two parts are shared differently across countries. Because stockholders everywhere hold the same portfolio, conventional risk compensation is borne by world stockholder wealth as a whole, so a change in either country's balance sheet affects the conventional component of both premia. The leverage wedge, by contrast, stays with the issuer: higher Home leverage raises $\tau_{H,t}$ but leaves $\tau_{F,t}$ unchanged. When Home stockholders supply most of the additional bonds, the Home premium therefore rises by more than the Foreign premium. This is how differential aging can generate the divergence in risk premia documented in Section 2.1.

In a stationary equilibrium, each tree is valued by the Gordon growth formula, capitalizing its dividend per person at the required return net of the growth of the issuing population:

$$P_{j,t} = \frac{D_{j,t}}{r_t + \text{ERP}_{j,t} - n_j}, \qquad j \in \{H, F\}. \tag{29}$$

Slower population growth therefore lowers the price of a tree even at an unchanged required return, because its dividend base grows more slowly. A higher premium lowers the price as well, which reduces stockholders' financial wealth and so raises their leverage further.

5 Quantitative analysis

We now calibrate the two-country model and ask how the demographic changes observed since 1980 affect the world safe rate and the premia on Home and Foreign equity. We solve three stationary equilibria: one under the initial demographic parameters, one after longevity increases, and one after population growth also slows. We refer to initial values as $\underline{x}$, use x^L to denote longevity-only values, and refer to final values as $\overline{x}$. In each equilibrium, financial wealth per person has zero drift for both household types, and the safe rate, equity premia, portfolios, and issuer wedges are jointly determined.

5.1 Calibration and targets

We calibrate Home to the U.S. and Foreign to the other seventeen economies in Section 2. Their 2023 PPP-GDP weights imply initial output shares of 43 percent for Home and 57 percent for Foreign, which remain fixed throughout the experiment. As in the fixed-rate benchmark, the model is annual. Initial population growth is 1.0 percent per year at Home and 0.6 percent in Foreign, roughly matching population growth around 1980 in the two regions. Initial mortality and earnings-decline rates, 1.689 and 3.393 percent at Home and 1.597 and 3.446 percent in Foreign, are based on the OECD data of Section 2.2.

We target an initial equity premium of 4.0 percent at Home and 3.0 percent in Foreign, rather than the stylized 5 percent of Section 3, and a world safe rate of 2.7 percent, based on the data of Section 2.1. Equity-return volatility, from the same data, is 13.7 percent at Home and 12.0 percent in Foreign, and we set the correlation between the two returns to zero.

To discipline risky-asset supply, we set market capitalization relative to local output to 1.90 at Home and 1.30 in Foreign, reflecting the larger size of the U.S. equity market. We also set Home nonstockholder bond holdings to 60 percent of output, Home's net external bond position to -45 percent, and Home NFA to -40 percent. These are reference values chosen to capture the U.S. position as a net debtor in bonds with a positive net equity position, rather than precise estimates for any single year. Together they imply that Home holds net foreign equity of 5 percent of output. Foreign's balance sheet then follows from

market clearing.

Stock-market participation is 50 percent at Home and 30 percent in Foreign, consistent with the U.S. and non-U.S. participation rates in Chien and Naknoi (2015). Stockholders in both countries have curvature $\gamma_S = 1.5$, while nonstockholders are more risk averse, with $\gamma_{N,i} = 4$, a standard value of risk aversion in the literature.

Table 5: Baseline calibration and targets

Input or target	Home	Foreign
Output share, Y_i/Y	43.09%	56.91%
Equity-market value / own output, $\underline{P}_i/\underline{y}_i$	1.90	1.30
Initial equity premium, $\underline{\text{ERP}}_i$	4.00%	3.00%
Initial safe rate, $\underline{r}$	2.70% (common)	
Initial population growth, $\underline{n}_i$	1.00%	0.60%
Mortality, $\underline{\delta}_i$	1.69%	1.60%
Earnings decline, $\underline{\chi}_i$	3.39%	3.45%
Stockholder participation, $\pi_{S,i}$	50%	30%
Stockholder curvature, γ_S	1.50	1.50
Nonstockholder curvature, $\gamma_{N,i}$	4.00	4.00
Equity-return volatility, $\sigma_{R,i}$	13.70%	12.00%
Nonstockholder bond holdings / own output, $\underline{\mathcal{B}}_{N,i}/\underline{y}_i$	0.60	Implied by market clearing
Net external bond position / own output, $(\underline{\mathcal{B}}_{N,i} + \underline{\mathcal{B}}_{S,i})/\underline{y}_i$	-0.45	Implied by market clearing
NFA / own output, $\underline{\text{NFA}}_i/\underline{y}_i$	-0.40	Implied by market clearing

Table 5 summarizes the calibration. Each initial balance sheet, together with the part of its premium not accounted for by risk compensation, identifies the issuer slopes $\psi_H = 0.0306$ and $\psi_F = 0.0308$. The remaining household and market conditions determine Foreign nonstockholder bond holdings and the household discount rates.[13] Appendix D gives the full system and the resulting parameters, which we hold fixed throughout the demographic experiment.

[13]The implied discount rates ρ are too high when interpreted literally: about 7.6 percent for stockholders and 14.0 to 14.2 percent for nonstockholders. We use them, however, as effective saving parameters of the retirement-saving block, rather than as estimates of impatience. In particular, they correct for the fact that our model does not feature a non-funded, pay-as-you-go style pension income. If left unaddressed, this implies a counterfactually high demand for private savings. Higher discount rates correct for this omission by lowering saving demand.

5.2 Baseline equilibrium

The initial column of Table 6 reports the baseline equilibrium. Home stockholders owe debt of 105.00 percent of output against financial wealth of 90.00 percent, giving leverage of 2.17, compared with 1.87 for Foreign stockholders. Foreign nonstockholders hold bonds worth 92.97percent of output, well above the 60.00 percent held at Home, and part of this demand is met by Home stockholders. Home is therefore already a net issuer of bonds to the rest of the world at the start of the experiment, which is why its equity carries the larger leverage wedge.

Because stockholders in both countries hold the same portfolio shares, each group owns the same fraction of both trees, equal to its share of world stockholder wealth. Home stockholders therefore own 53.92 percent of both the Home and the Foreign equity market. Since the model abstracts from home bias, these cross-border holdings are considerably larger than in the data, a limitation to bear in mind when interpreting the quantitative results.

5.3 The impact of demographic changes

We apply the demographic changes observed since 1980 in two steps. First, we raise expected years after retirement by 1.30 years at Home and 4.77 years in Foreign, matching the evidence in Section 2.2. As in the longevity experiment of Section 3.5, we lower mortality and raise the rate of earnings decline one-for-one, so that the additional years are spent entirely in retirement.[14] The retirement-duration gap therefore widens almost four times as much in Foreign as at Home. Second, we also lower population growth, from 1.0 to 0.8 percent at Home and from 0.6 to 0.1 percent in Foreign, again in line with the changes since 1980 in the economies described in Section 2.

Table 6 reports the main result. The Home equity premium rises from 4.00 to 4.96 percent after the longevity change and to 6.51 percent once population growth also slows, a combined increase of 250.50 basis points. The Foreign premium rises by only 49.09 basis points, while the world safe rate falls by 71.44 basis points.[15] Demographic change thus

[14] Foreign mortality falls from 1.597 to 1.484 percent and its rate of earnings decline rises from 3.446 to 3.559 percent; at Home, mortality falls from 1.689 to 1.653 percent and earnings decline rises from 3.393 to 3.429 percent.

[15] This is lower than implied by Rachel (2025), who puts the impact of demographics on the equilibrium safe rate at -275 basis points; our quantification is more in line with Auclert et al. (2025), who arrive at -110 basis points. Either way, this does leave a potential gap with respect to the observed fall in safe rates. Beaudry et al. (2025) however show that an OLG-style setup with retirement forces at play leaves room for *monetary policy* to contribute to secular trends in safe rates. Relatedly, Hyland (2026) shows that secular increases in macroeconomic risk can raise risk premia while simultaneously lowering the neutral safe rate, providing another channel through which safe rates may decline independently of demographic forces.

lowers the safe rate everywhere but raises the equity premium far more in the country that has aged less, mirroring the divergence documented in Section 2.1.

Table 6: Longevity and slower population growth: stationary equilibria

Moment	Initial	Longevity	Final
Home equity premium, ERP_H (%)	4.00	4.96	6.51
Foreign equity premium, ERP_F (%)	3.00	3.17	3.49
World safe rate, r (%)	2.70	2.34	1.99
Home nonstockholder bond holdings, $\mathcal{B}_{N,H}/y_H$ (%)	60.00	56.41	50.29
Foreign nonstockholder bond holdings, $\mathcal{B}_{N,F}/y_F$ (%)	92.97	101.85	98.88
Home stockholder bond position, $\mathcal{B}_{S,H}/y_H$ (%)	-105.00	-112.60	-108.75
Foreign stockholder bond position, $\mathcal{B}_{S,F}/y_F$ (%)	-58.89	-59.30	-54.61
Home stockholder financial wealth, $\mathcal{F}_{S,H}/y_H$ (%)	90.00	75.11	53.64
Foreign stockholder financial wealth, $\mathcal{F}_{S,F}/y_F$ (%)	67.32	63.81	52.37
Home stockholder financial leverage, $\mathcal{L}_{S,H}$	2.17	2.50	3.03
Foreign stockholder financial leverage, $\mathcal{L}_{S,F}$	1.87	1.93	2.04

Notes: Stocks are per country resident and relative to own output. Negative stockholder bond positions denote borrowing. The full experiment adds the population slowdown to the longevity change. Financial wealth excludes human wealth. Output per person is unchanged in both countries. The columns compare fully adjusted stationary equilibria; additional life-years do not denote calendar years of transition. The world goods-market residual derived in Appendix D is 0.07 percent of world output in the first two columns and 0.11 percent in the third.

The mechanism begins with the additional saving needed to finance longer retirements. This demand is stronger in Foreign, where expected years after retirement have risen by more. After the longevity change, Foreign nonstockholder bond holdings rise by 8.87 percentage points of Foreign output. The fall in the safe rate reduces Home nonstockholder bond holdings, partly offsetting this increase; this general-equilibrium offset is absent from the fixed-rate benchmark. On net, world nonstockholder bond holdings rise by 3.51 percentage points of world output. Both stockholder sectors borrow more, with Home stockholders accounting for 93 percent of the increase in borrowing. The reason is that the retirement-duration gap widens by less for Home stockholders: they continue to receive the safe labor income stream for a longer fraction of their expected lifetimes, which places them in a *relatively* better position to take on the additional debt. Because Home stockholders take on most of the new debt, it is mainly the Home leverage wedge that rises, while conventional risk compensation is shared across both markets.

Slower population growth then reduces the dilution of accumulated wealth by new entrants, as in Section 3.6.1. In general equilibrium, however, its effects differ from the fixed-rate benchmark. The safe rate falls by a further 35.05 basis points, and at

this lower rate nonstockholder bond holdings fall back, to 50.29 percent of output at Home and 98.88 percent in Foreign, so that world nonstockholder bond holdings end 0.82 percentage points of world output below their initial level. Relative to the longevity-only equilibrium, stockholder bond positions in both countries move back toward zero. The premium nevertheless rises further, because stockholders' financial wealth falls even faster than their debt. A lower safe rate reduces the return stockholders earn on their wealth, so they accumulate less of it, and slower dividend growth lowers equity prices (Section 4.5), with higher premia deepening the fall. Over the two steps, Home stockholders' leverage rises from 2.17 to 3.03, and required returns must rise to make them willing to hold the resulting equity positions.

Foreign stockholders account for the rest of the additional debt in the longevity-only step. Over the full experiment, their financial wealth falls by more than their debt, so their leverage also rises, from 1.87 to 2.04. The Foreign premium therefore rises as well, but by much less.

Table 7 reports the premium changes for each step. Because return volatilities are held fixed (Section 4.1), the volatility channel that raised the premium in Section 3.5 is absent here, so these results likely understate the effect of demographic change on equity premia.

Table 7: Decomposition of Equity Risk Premium

Demographic change	Home	Foreign
Longevity	+96.44	+17.17
Population slowdown after longevity	+154.07	+31.92
Combined	+250.50	+49.09

Notes: Entries are changes in equity premia, in basis points. The population-slowdown row reports the additional change after longevity has increased. The combined row compares the initial allocation with the allocation after both demographic changes.

With dividends per person fixed, the higher required return on Home equity and the slower growth of its dividend base lower its price by 25.88 percent. Foreign equity rises in value by 3.92 percent after the longevity change, because the decline in the safe rate exceeds the increase in its premium, but ends 5.14 percent below its initial value once population growth slows.

Home's net external bond position becomes more negative by 13.47 percentage points of output, while its net foreign equity rises by 16.57 points, more than offsetting the additional borrowing, so NFA improves from −40.00 to −36.89 percent of output. Home thus borrows more in bonds while investing more in foreign equity, deepening the "world

venture capitalist" pattern of Section 4.4. Greater aging abroad, combined with slower population growth, therefore raises the U.S. equity premium by far more than the Foreign premium while leaving the U.S. net foreign asset position little changed.

6 Conclusion

We have argued that demographic aging can simultaneously lower safe rates and raise the compensation required to bear equity risk. As individuals spend a greater fraction of their lives in retirement, their relatively safe labor income runs out sooner relative to the horizon over which consumption must be financed. Aging therefore raises the demand for safe assets in particular, rather than for assets in general. Meeting that demand requires the investors who hold risky assets to issue safe debt and hold their equity on more leveraged balance sheets, which raises the return they require. Slower population growth reinforces this mechanism by pushing up investors' average age.

Our main contribution is to study how this mechanism operates in an open economy. What matters for a country's equity premium is not only how much its residents demand safe assets, but who supplies them. When aging is more severe in Foreign than at Home, Foreign households develop the stronger demand for safety, while the relatively younger Home households are better placed to supply it. Home stockholders therefore issue most of the additional safe debt and bear the associated leverage. Because conventional risk compensation is shared by stockholders worldwide while the leverage wedge stays with the issuer, the premium rises most in the country that supplies the safe assets. Differential aging can therefore connect global imbalances, with the U.S. acting as the world's venture capitalist, to the international divergence in risk premia documented in the data.

Quantitatively, applying the demographic changes observed since 1980 lowers the world safe rate by 71 basis points and raises the U.S. equity premium by 251 basis points, compared with 49 basis points abroad, while leaving the U.S. net foreign asset position little changed. Our results therefore provide a demographic interpretation of two prominent macro-finance trends: the global decline in safe rates and the particularly pronounced rise in the U.S. equity premium.

A Fixed-rate benchmark: Derivations

This appendix derives the household saving and pricing decisions underlying Section 3 and examines how the results depend on the calibration and household preferences. As in the main text, we take the local economy to represent the non-U.S. part of our sample and hold the safe rate r^* fixed. We specify external issuance through a rule and leave the external issuer's decisions outside this partial equilibrium analysis. The state of the economy is $(D, \mathcal{B}_N)$, where D is the dividend and $\mathcal{B}_N$ denotes local nonstockholders' bond holdings. The local stockholder bond position is $\mathcal{B}_S(\mathcal{B}_N; \omega)$, defined below; debt outstanding is $-\mathcal{B}_S$. Thus $\mathcal{B}_N = -\mathcal{B}_S$ when all bonds are supplied locally, $\omega = 0$. With external issuance, $\mathcal{B}_N$ also includes bonds supplied by the rest of the world, denoted $\mathcal{B}_{\text{ROW}}$, so $\mathcal{B}_N + \mathcal{B}_S = \mathcal{B}_{\text{ROW}}$. These financial positions exclude human wealth; nonstockholders' total safe wealth is $\mathcal{T}_N = \mathcal{B}_N + \mathcal{H}_N$.

A.1 Longevity and human wealth

Households die with constant intensity δ, newborns replace those who die, and the stationary density of age $a \geq 0$ is $f(a) = \delta e^{-\delta a}$. With aggregate wage bill W, the wage profile is

$$w(a; \delta, W) = W \frac{\chi + \delta}{\delta} e^{-\chi a}.$$

Integrating over the age distribution recovers the aggregate wage bill W:

$$\int_0^\infty w(a; \delta, W) f(a)\, da = W(\chi + \delta) \int_0^\infty e^{-(\chi+\delta)a}\, da = W. \tag{30}$$

Expected remaining life is $1/\delta$, whereas the survival-adjusted duration of earnings is $1/(\chi + \delta)$. Their difference is the retirement-duration gap in the main text.

At the fixed safe rate, one unit of the remaining aggregate wage flow has the present-value multiplier

$$A^{\text{HW}}(\delta, \chi) = \int_0^\infty e^{-(r^*+\chi+\delta)u}\, du = \frac{1}{r^* + \chi + \delta}. \tag{31}$$

Aggregate human wealth is therefore $\mathcal{H}(\delta, \chi, W) = W A^{\text{HW}}(\delta, \chi)$. Because type is independent of age and earnings,

$$\mathcal{H}_S = \pi_S \mathcal{H}, \qquad \mathcal{H}_N = (1 - \pi_S)\mathcal{H}, \qquad W_S = \pi_S W, \qquad W_N = (1 - \pi_S)W. \tag{32}$$

To model a longer retirement, we lower mortality while keeping the duration of earnings unchanged. Underlines denote the initial regime and overlines the final, lower-

mortality regime; they do not denote numerical bounds. We hold the aggregate wage bill at $\underline{W}$ and raise the rate at which earnings decline:

$$\overline{W} = \underline{W}, \qquad \overline{\chi} = \underline{\chi} + \underline{\delta} - \overline{\delta}. \tag{33}$$

This adjustment keeps $\overline{\chi} + \overline{\delta} = \underline{\chi} + \underline{\delta}$, so human wealth remains unchanged for the economy as a whole and for each household type: $\mathcal{H}$, $\mathcal{H}_S$, and $\mathcal{H}_N$. We choose $\overline{\delta} = (\underline{\delta}^{-1} + 4)^{-1}$. Mortality therefore declines from 1.689 to 1.582 percent, and χ rises from 3.393 to 3.500 percent, increasing the retirement-duration gap by four years. The participation shares, preferences, and leverage-wedge slope remain fixed. Households therefore need to finance four additional years of consumption from a labor-income stream of unchanged duration and present value.

Subtracting the initial stationary nonstockholder budget from the post-longevity budget gives

$$\overline{\mathcal{B}}_N - \underline{\mathcal{B}}_N = \frac{(\underline{\delta} - \overline{\delta})(\underline{\mathcal{B}}_N + \mathcal{H}_N)}{\overline{\delta} + (\rho_N - r^*)/\gamma_N}. \tag{34}$$

Lower mortality reduces consumption out of a given wealth stock because households must spread their resources over a longer remaining lifetime. With current labor income unchanged, saving rises. Desired nonstockholder bond holdings increase by 19.67 percentage points of initial output to 164.7 percent. This saving response follows from the household budget and is independent of the boundary conditions used to solve for equity prices. With $\omega = 0.70$, the increase in local issuance is 5.90 percentage points of initial output; the remaining 13.77 percentage points are supplied externally.

A.2 Bond demand and local debt issuance

Nonstockholders have CRRA utility with curvature γ_N and pure discount rate ρ_N. They hold bonds through actuarially fair annuity accounts, so their individual annuitized return is $r^* + \delta$, and aggregate consumption is $C_N = [\delta + (\rho_N + (\gamma_N - 1)r^*)/\gamma_N](\mathcal{B}_N + \mathcal{H}_N)$. Aggregating across surviving households, deaths, and newborns gives

$$\dot{\mathcal{B}}_N = \mu_{\mathcal{B}_N}(\mathcal{B}_N; \delta) \equiv r^* \mathcal{B}_N + W_N - \left[\delta + \frac{\rho_N + (\gamma_N - 1)r^*}{\gamma_N}\right](\mathcal{B}_N + \mathcal{H}_N). \tag{35}$$

Aggregate nonstockholder bond holdings earn r^*, although surviving households receive $r^* + \delta$. Annuity credits redistribute the wealth of households who die to survivors and therefore cancel in the aggregate budget.

At the initial target $\underline{\mathcal{B}}_N = -\underline{\mathcal{B}}_S$, zero drift identifies the nonstockholder discount rate as

$$\begin{aligned}\rho_N &= \gamma_N \left[\frac{-r^* \underline{\mathcal{B}}_S + \underline{W}_N}{-\underline{\mathcal{B}}_S + \underline{\mathcal{H}}_N} - \underline{\delta} \right] - (\gamma_N - 1) r^* \\ &= \frac{\underline{\mathcal{H}}_N (r^* + \gamma_N \underline{\chi}) - \underline{\mathcal{B}}_S (r^* - \gamma_N \underline{\delta})}{-\underline{\mathcal{B}}_S + \underline{\mathcal{H}}_N}.\end{aligned} \tag{36}$$

The second equality uses $\underline{W}_N = (r^* + \underline{\chi} + \underline{\delta})\underline{\mathcal{H}}_N$. We hold this discount rate fixed when mortality changes. Setting the drift in equation (35) to zero gives the stationary level of nonstockholder bond holdings:

$$\begin{aligned}\mathcal{B}_N^*(\delta) &= \frac{\gamma_N W_N - [\gamma_N \delta + \rho_N + (\gamma_N - 1) r^*] \mathcal{H}_N}{\gamma_N \delta + \rho_N - r^*} \\ &= \mathcal{H}_N \frac{r^* + \gamma_N \chi(\delta) - \rho_N}{\gamma_N \delta + \rho_N - r^*}.\end{aligned} \tag{37}$$

The stationary stock is positive and stable when $\gamma_N \delta + \rho_N - r^* > 0$ and $r^* + \gamma_N \chi - \rho_N > 0$. Holding ρ_N, $\mathcal{H}_N$, and W_N fixed, with $d\chi/d\delta = -1$, gives

$$\frac{d\mathcal{B}_N^*}{d\delta} = -\frac{\gamma_N [\mathcal{B}_N^*(\delta) + \mathcal{H}_N]}{\gamma_N \delta + \rho_N - r^*} < 0. \tag{38}$$

Initially, all bonds held by local nonstockholders are supplied by local stockholders: $\underline{\mathcal{B}}_N = -\underline{\mathcal{B}}_S = \mathcal{B}_N^*(\underline{\delta})$ and $\underline{\mathcal{B}}_{\text{ROW}} = 0$. As desired nonstockholder bond holdings change, we allocate the additional issuance between local stockholders and the rest of the world according to

$$\mathcal{B}_S(\mathcal{B}_N; \omega) = \underline{\mathcal{B}}_S - (1 - \omega)(\mathcal{B}_N - \underline{\mathcal{B}}_N), \qquad \mathcal{B}_{\text{ROW}}(\mathcal{B}_N; \omega) = \omega(\mathcal{B}_N - \underline{\mathcal{B}}_N), \qquad \mathcal{B}_N + \mathcal{B}_S = \mathcal{B}_{\text{ROW}}. \tag{39}$$

Here $\mathcal{B}_S$ is the local stockholder bond position and $\mathcal{B}_{\text{ROW}}$ denotes externally supplied bonds. All additional bonds are supplied locally at $\omega = 0$ and externally at $\omega = 1$. The rule specifies who meets the extra demand for bonds at the fixed world safe rate.

The balance sheets of the two household types are therefore

$$\begin{aligned}\mathcal{F}_S(D, \mathcal{B}_N) &= P(D, \mathcal{B}_N) + \mathcal{B}_S(\mathcal{B}_N; \omega), \quad \mathcal{T}_S(D, \mathcal{B}_N) = \mathcal{F}_S(D, \mathcal{B}_N) + \mathcal{H}_S, \\ \mathcal{F}_N(\mathcal{B}_N) &= \mathcal{B}_N, \qquad\qquad\qquad\qquad \mathcal{T}_N(\mathcal{B}_N) = \mathcal{B}_N + \mathcal{H}_N.\end{aligned} \tag{40}$$

Local stockholders owe $-\mathcal{B}_S$, while local nonstockholders hold bonds worth $\mathcal{B}_N$. Their bond positions sum to zero at $\omega = 0$; with external issuance, part of nonstockholders' safe

income is paid by the rest of the world.

For any smooth function $g(D, \mathcal{B}_N)$, the physical generator is

$$\mathcal{A}g = \mu_{\mathcal{B}_N}(\mathcal{B}_N; \delta) g_{\mathcal{B}_N} + \frac{1}{2}\sigma_D^2 D^2 g_{DD}. \tag{41}$$

The dividend has zero drift. Both $\mathcal{B}_{\text{ROW}}$ and $\mathcal{B}_S$ are determined by $\mathcal{B}_N$, so neither requires an additional state variable.

A.3 Stockholder pricing under log utility

We begin with log utility, under which stockholders consume a constant fraction of total wealth and portfolio choice has no intertemporal-hedging term. The main text allows CRRA curvature γ_S and sets $\gamma_S = 1.5$ in the calibration. With $\gamma_S = 1$, stockholders maximize

$$\mathbb{E}_t \int_0^\infty e^{-(\rho_S + \delta)u} \log C_{S,t+u}\, du.$$

Their financial leverage and tree exposure relative to total wealth are

$$\mathcal{L}_S = \frac{P}{P + \mathcal{B}_S}, \qquad \theta_S = \frac{P}{P + \mathcal{B}_S + \mathcal{H}_S}. \tag{42}$$

The log consumption rule is

$$C_S = (\rho_S + \delta)\mathcal{T}_S. \tag{43}$$

Stockholders face the fully rebated quadratic leverage penalty

$$\Phi_S = \frac{\psi_S}{2}(\mathcal{L}_S - 1)^2, \qquad \tau_S = \frac{\partial \Phi_S}{\partial \mathcal{L}_S} = \psi_S(\mathcal{L}_S - 1). \tag{44}$$

The marginal wedge raises the return stockholders require to take a leveraged position. The charge on risky holdings is $\tau_S \mathcal{E}_S$, returned through the proportional rebate $\alpha_S \mathcal{T}_S = \tau_S P$, so the charge uses no aggregate resources. The linear marginal expression applies both above and below unit leverage.

Let $u(D, \mathcal{B}_N) = \log P(D, \mathcal{B}_N)$. Itô's lemma gives the tree-return volatility

$$\sigma_R = \sigma_D D u_D = \sigma_D \frac{D P_D}{P}. \tag{45}$$

Under log utility, the stockholder price of aggregate risk is

$$\lambda_S = \theta_S \sigma_R. \tag{46}$$

Combining the tree return with the portfolio first-order condition gives a single equation for the price:

$$0 = \frac{D}{P} + \mathcal{A}u + \frac{1}{2}\sigma_D^2 D^2 u_D^2 - r^* - \psi_S(\mathcal{L}_S - 1) - \theta_S \sigma_R^2. \tag{47}$$

The corresponding equity premium is

$$\mathrm{ERP}_{\log} \equiv \mu_R - r^* = \psi_S(\mathcal{L}_S - 1) + \theta_S \sigma_R^2. \tag{48}$$

The external payment and household decisions. Consider an atomistic stockholder with financial wealth f_h and gross human wealth h_h. Human wealth is the present value of contractual wages and satisfies $dh_h = [(r^* + \delta)h_h - w_h]dt$. Total resources are $\mathcal{T}_h^{\mathrm{ind}} = f_h + h_h$, and the risky position is e_h. The household pays $\xi_t \mathcal{T}_h^{\mathrm{ind}}$ to the rest of the world, taking ξ_t, τ_S, and α_S as given. Its financial budget is $df_h = [(r^* + \delta)f_h + e_h(\mathrm{ERP} - \tau_S) + (\alpha_S - \xi_t)\mathcal{T}_h^{\mathrm{ind}} + w_h - c_h]dt + e_h \sigma_R dZ$. Adding the value of future wages to financial wealth gives the law of motion for total resources:

$$d\mathcal{T}_h^{\mathrm{ind}} = \Big[(r^* + \delta + \alpha_S - \xi_t)\mathcal{T}_h^{\mathrm{ind}} + e_h(\mathrm{ERP} - \tau_S) - c_h\Big]dt + e_h \sigma_R \, dZ. \tag{49}$$

With $V_h^{\mathrm{ind}}(\mathcal{T}_h^{\mathrm{ind}}, D, \mathcal{B}_N) = \log \mathcal{T}_h^{\mathrm{ind}}/(\rho_S + \delta) + K(D, \mathcal{B}_N)$, the first-order conditions are $c_h = (\rho_S + \delta)\mathcal{T}_h^{\mathrm{ind}}$ and $\mathrm{ERP} - \tau_S = (e_h/\mathcal{T}_h^{\mathrm{ind}})\sigma_R^2$. Neither condition involves ξ_t. The payment enters only K, which satisfies a linear equation containing $-\xi_t/(\rho_S + \delta)$. It therefore affects the household's value function while leaving its consumption and risky portfolio shares unchanged under log utility. Aggregating over stockholders leaves the pricing equation (47) and premium (48) unchanged at a given state. This result requires both log utility and a payment proportional to total resources; welfare still depends on the payment.

The balancing rate follows from the aggregate stockholder balance sheet $\mathcal{F}_S = P + \mathcal{B}_S$. Its drift is $\mathcal{A}P + \dot{\mathcal{B}}_S$, whereas the aggregate budget gives $r^*\mathcal{F}_S + P\,\mathrm{ERP} + W_S - C_S - \mathcal{X}$ with $P\,\mathrm{ERP} = D + \mathcal{A}P - r^*P$. Equating the two yields $C_S + \mathcal{X} = D + W_S + r^*\mathcal{B}_S - \dot{\mathcal{B}}_S$. Substituting $C_S = m_S \mathcal{T}_S$, $\mathcal{X} = \xi \mathcal{T}_S$, and the prescribed path $\dot{\mathcal{B}}_{S,t} = -(1 - \omega)\dot{\mathcal{B}}_{N,t}$ gives the transfer rate

$$\xi_t = \frac{D_t + W_{S,t} + r^*\mathcal{B}_{S,t} + (1 - \omega)\dot{\mathcal{B}}_{N,t}}{\mathcal{T}_{S,t}} - m_S(D_t, \mathcal{B}_{N,t}), \tag{50}$$

where $W_{S,t} = \pi_S W_t$ and, at a given state, $\dot{\mathcal{B}}_{N,t} = \mu_{\mathcal{B}_N}$. The rate is implied by the issuance rule and adjusts with the state and demographic regime; it is not a calibrated parameter. The diffusion of the balance sheet, $P\sigma_R$, equals the portfolio diffusion $\mathcal{E}_S \sigma_R$ because $\mathcal{E}_S = P$. Omitting the payment from the pricing calculation leaves a household budget residual

equal to $\mathcal{X}$, which the transfer closes.

Adding the nonstockholder budget gives the national resource identity $C_{S,t}+C_{N,t}+\mathcal{X}_t = W_t + D_t + r^*\mathcal{B}_{\text{ROW},t} - \dot{\mathcal{B}}_{\text{ROW},t}$; under full local incidence, $C_{S,t} + C_{N,t} + \mathcal{X}_t = W_t + D_t$.

When all additional bonds are supplied externally, $\omega = 1$, the local stockholder bond position remains fixed at $\underline{\mathcal{B}}_S$. Equation (47) then admits a $\mathcal{B}_N$-independent solution: all appearances of $\mathcal{B}_N$ outside $\mathcal{A}u$ disappear and $u_{\mathcal{B}_N} = 0$ eliminates the bond-holdings drift. Consequently the price, leverage, price-dividend elasticity, return volatility, and log-utility ERP are unchanged by the longevity experiment. The transfer adjusts because lower mortality changes stockholder consumption. The price invariance relies on log utility and the specified issuance rule; with general CRRA preferences, mortality can also affect prices through consumption and hedging demand.

A.4 Local issuance and the equity premium

Longer retirement raises desired nonstockholder bond holdings. Its effect on local stockholders depends on how much of that demand they meet. Set $z_\delta = -\delta$, so an increase in z_δ is an increase in longevity. From equation (38),

$$\mathcal{B}^*_{N,z_\delta} = \frac{\gamma_N(\mathcal{B}^*_N + \mathcal{H}_N)}{\gamma_N\delta + \rho_N - r^*} > 0, \qquad \mathcal{B}_{S,z_\delta} = -(1-\omega)\mathcal{B}^*_{N,z_\delta}. \tag{51}$$

At fixed P and fixed human wealth,

$$\left.\frac{\partial\theta_S}{\partial\mathcal{B}_S}\right|_P = -\frac{P}{\mathcal{T}_S^2} < 0, \qquad \left.\frac{\partial\mathcal{L}_S}{\partial\mathcal{B}_S}\right|_P = -\frac{P}{\mathcal{F}_S^2} < 0. \tag{52}$$

At a given tree price, additional debt reduces stockholders' net wealth while leaving their risky holdings unchanged. Both leverage and risky exposure relative to total wealth therefore rise. Write $P_B = \partial P/\partial\mathcal{B}_S$ for the price derivative with respect to the local stockholder bond position, holding dividends, demographic parameters, and ω fixed. This differs from the state derivative $P_{\mathcal{B}_N}$ with respect to nonstockholder bond holdings: $P_{\mathcal{B}_N} = -(1-\omega)P_B$ for $\omega < 1$. Allowing the tree price to adjust gives

$$\begin{aligned}\frac{d\mathcal{L}_S}{d\mathcal{B}_S} &= \frac{\mathcal{B}_S P_B - P}{(P+\mathcal{B}_S)^2},\\ \frac{d\theta_S}{d\mathcal{B}_S} &= \frac{(\mathcal{H}_S+\mathcal{B}_S)P_B - P}{(P+\mathcal{B}_S+\mathcal{H}_S)^2}.\end{aligned} \tag{53}$$

With additional borrowing, the leverage wedge rises if $\mathcal{B}_S P_B/P < 1$, while risky exposure rises if $(\mathcal{H}_S+\mathcal{B}_S)P_B - P < 0$. Quantitatively, we find that the leverage condition holds. At

the initial state, the fall in the tree price partly offsets the effect of additional debt on risky exposure. Across the full longevity experiment, risky exposure increases only slightly once prices and household decisions have adjusted.

The third channel operates through the price–dividend elasticity. Since

$$\sigma_R = \sigma_D \frac{DP_D}{P}, \qquad \frac{d\sigma_R}{d\mathcal{B}_S} = \sigma_D \frac{d}{d\mathcal{B}_S}\left(\frac{DP_D}{P}\right), \tag{54}$$

local debt raises return volatility when it makes the price more sensitive to a dividend shock in percentage terms. Whether the elasticity rises is determined by the pricing solution at that state.

For log utility, holding price and volatility fixed gives the direct effect of longevity on the premium:

$$\mathrm{ERP}_{\log, z_\delta}\Big|_{P,\sigma_R} = (1-\omega)\mathcal{B}^*_{N,z_\delta}\left[\psi_S \frac{P}{\mathcal{F}_S^2} + \sigma_R^2 \frac{P}{\mathcal{T}_S^2}\right] > 0 \quad \text{for } \omega < 1. \tag{55}$$

The effect is positive whenever local stockholders supply some of the additional bonds. The full equilibrium response also depends on how prices, risky exposure, and return volatility adjust.

Combining the leverage, allocation, and volatility channels gives

$$\frac{d\,\mathrm{ERP}_{\log}}{d\mathcal{B}_S} = \underbrace{\frac{d\tau_S}{d\mathcal{B}_S}}_{\text{leverage-wedge channel}} + \underbrace{\sigma_R^2 \frac{d\theta_S}{d\mathcal{B}_S}}_{\text{asset-allocation channel}} + \underbrace{2\theta_S\sigma_R \frac{d\sigma_R}{d\mathcal{B}_S}}_{\text{volatility channel}}. \tag{56}$$

Multiplying this expression by $d\mathcal{B}^*_S/d\delta$ gives $\mathrm{ERP}^{\mathrm{debt}}_\delta$, the effect of mortality operating through local debt (Section 3.3). Because $z_\delta = -\delta$, $\mathcal{B}_{S,z_\delta} = -d\mathcal{B}^*_S/d\delta$, so the corresponding effect of increased longevity is $-\mathrm{ERP}^{\mathrm{debt}}_\delta$. This component captures the change in issuance and its pricing consequences; other effects of mortality enter the full equilibrium response separately.

To distinguish the direct debt effect from revaluation, we differentiate equation (48)

along stationary states:

$$\begin{aligned}
\mathrm{ERP}_{\log,z_\delta} &= A_B \mathcal{B}_{S,z_\delta} + A_P P_{z_\delta} + A_\sigma \sigma_{R,z_\delta}, \\
A_B &= -\psi_S \frac{P}{\mathcal{F}_S^2} - \sigma_R^2 \frac{P}{\mathcal{T}_S^2}, \\
A_P &= \psi_S \frac{\mathcal{B}_S}{\mathcal{F}_S^2} + \sigma_R^2 \frac{\mathcal{H}_S + \mathcal{B}_S}{\mathcal{T}_S^2}, \\
A_\sigma &= 2\theta_S \sigma_R.
\end{aligned} \tag{57}$$

Equation (55) gives the first term, which holds price and volatility fixed. The other terms capture changes in the tree price and return volatility.

Return volatility is dividend volatility times the price-dividend elasticity, as shown in (45). Along the stationary state,

$$\frac{d\sigma_R}{dz_\delta} = \sigma_D D_0 \left(u_{D,z_\delta} + u_{D\mathcal{B}_N} \mathcal{B}^*_{N,z_\delta} \right). \tag{58}$$

In our quantitative experiment, longevity raises the price-dividend elasticity and hence return volatility. The pricing equation does not impose this sign in general.

For two log-utility equilibria with the same ψ_S, the change in the premium is

$$\Delta \,\mathrm{ERP}_{\log} = \psi_S \Delta \mathcal{L}_S + \Delta(\theta_S \sigma_R^2). \tag{59}$$

Using the exact symmetric-product identity $\Delta(xy) = y^{\mathrm{mid}}\,\Delta x + x^{\mathrm{mid}}\,\Delta y$, where $x^{\mathrm{mid}} = (\underline{x} + \overline{x})/2$ denotes the arithmetic average of the two endpoints being compared, gives

$$\Delta \,\mathrm{ERP}_{\log} = \underbrace{\psi_S \Delta \mathcal{L}_S}_{\text{leverage-wedge contribution}} + \underbrace{(\sigma_R^2)^{\mathrm{mid}}\,\Delta\theta_S}_{\text{asset-allocation contribution}} + \underbrace{\theta_S^{\mathrm{mid}}\,\Delta(\sigma_R^2)}_{\text{volatility contribution}}. \tag{60}$$

Here $(\sigma_R^2)^{\mathrm{mid}}$ is the average of the two endpoint variances, rather than the square of average volatility. The identity accounts for the full change between equilibria, including interactions between exposure and variance. For the CRRA calibration, the corresponding identity also includes curvature and hedging, as set out in Appendix A.6. At a stockholder share of 30%, the leverage wedge accounts for 42.4 basis points of the 50.4-basis-point increase. The asset-allocation contribution is 0.1 basis points, the volatility contribution is 8.6 basis points, and intertemporal hedging contributes -0.7 basis points. The allocation contribution is small because the fall in the tree price largely offsets the direct effect of additional debt on θ_S. Equation (52) isolates that direct effect by holding the price fixed.

A.5 Calibration and identification

We set the safe rate to 2.7 percent, the stockholder share to 30%, stockholder curvature to $\gamma_S = 1.5$, initial stockholder debt to 145% of output, dividend volatility to 12.0%, and $\rho_S = 4\%$. Targeting $\underline{\mathcal{B}}_N = -\underline{\mathcal{B}}_S$ in (36) gives $\rho_N = 12.23\%$. Conditional on this initial balance sheet, $\psi_S = 0.0666$ matches an initial equity premium of 5 percent. The parameter ψ_S measures the additional return required as leverage rises. Risk aversion is specified separately, and the full rebate of the leverage charge preserves aggregate resources.

In the longevity experiment, mortality falls from 1.689 to 1.582 percent. The retirement-duration gap increases by four years. The earnings-timing adjustment in (33) holds human wealth fixed. Desired nonstockholder bond holdings rise to 164.7 percent of initial output, while local stockholder debt rises to 150.9 percent under $\omega = 0.70$.

The available five-country non-U.S. aggregate has an annual dividend-growth volatility of 11.6 percent; equal weighting and later samples produce values between about 13 and 15 percent. Recalibrating the wedge at the 11.6-percent estimate and at 14 percent gives longevity responses of 55.4 and 32.9 basis points, respectively. At 10 percent, the full four-year experiment raises the premium by 83.2 basis points, holding the external issuance share and pricing boundaries fixed. In these recalibrations, a lower σ_D requires a steeper leverage wedge to match the initial premium. This can make the premium substantially more sensitive to additional debt.

The initial stockholder-debt target identifies ρ_N by requiring nonstockholder financial wealth to be stationary, and the initial premium identifies ψ_S. These targets leave ρ_S undetermined: neither the pricing nor the goods-market restriction identifies it separately. Under log utility, the stockholder discount rate determines consumption out of wealth but drops out of the pricing equation when admissibility constraints do not bind. We set it to 4 percent and examine 16 percent as a sensitivity exercise.

Under CRRA, ρ_S affects continuation values, so we solve prices and consumption policies again when changing it. A higher ρ_S raises stockholder consumption and reduces the external payment needed to balance the budget. At 16 percent, the implied rate is negative, requiring an inward payment of 50.9 percent of output. These prices therefore require inward transfers, outside the benchmark restriction to outward payments.

Changes in stock market participation affect both household balance sheets and the parameters needed to match the initial targets. If ρ_N is recalibrated to preserve $\mathcal{B}_N^*(\underline{\delta}) = -\underline{\mathcal{B}}_S$, then

$$-\left.\frac{d\mathcal{B}_N^*}{d\delta}\right|_{\underline{\delta},\,\underline{\mathcal{B}}_S \text{ targeted}} = \frac{(-\underline{\mathcal{B}}_S + \mathcal{H}_N)^2}{\mathcal{H}_N(\underline{\chi} + \underline{\delta})}. \tag{61}$$

The response therefore depends on more than the nonstockholder share $1-\pi_S$. To see how participation also affects the leverage wedge needed to match the initial premium, hold the initial valuation objects, including the continuation-value derivative, fixed. Denote the common initial premium target by $\mathrm{ERP}^{\mathrm{target}}$. The required wedge slope then satisfies

$$\psi_S^{\mathrm{cal}}(\pi_S) = \frac{\mathrm{ERP}^{\mathrm{target}} - \gamma_S \dfrac{P}{P + \mathcal{B}_S + \pi_S \mathcal{H}} \sigma_R^2 - (\gamma_S - 1)\sigma_D D v_D \sigma_R}{\mathcal{L}_S - 1}. \tag{62}$$

When $\mathcal{L}_S > 1$, a larger stockholder share lowers conventional risk compensation at these fixed valuation objects. A steeper wedge is then needed to match the same premium. In equilibrium, prices and the other valuation objects also adjust, so we solve the model again for each participation share.

Table 8 reports the participation and discount-rate comparisons. In each participation row, we set ρ_N to preserve the initial nonstockholder bond-holdings target and retarget ψ_S to the same 5-percent initial premium. Moving from a 45-percent stockholder share to the benchmark share of 30% raises the response from 47.8 to 50.4 basis points. Because participation affects both bond demand and the calibrated wedge, the factor $1 - \pi_S$ alone cannot predict the size of the response.

Table 8: Participation and discount-rate robustness

Specification	π_S	ρ_S	ρ_N	ψ_S	$\underline{\mathrm{ERP}}$	$\overline{\mathrm{ERP}}$ / ΔERP	$\mathcal{X}/Y$ (%)	Sign
Maintained benchmark	30%	4.00%	12.23%	0.0666	5.00%	5.50% / 50.42 bp	19.81/22.46	outward
Previous participation	45%	4.00%	11.40%	0.0748	5.00%	5.48% / 47.83 bp	23.71/26.45	outward
Equal discount rates	30%	12.23%	12.23%	0.0666	5.00%	5.50% / 50.59 bp	-28.69/-23.41	inward
Inherited ρ_S	30%	16.00%	12.23%	0.0666	5.00%	5.50% / 50.64 bp	-50.89/-44.40	inward

Notes: The equal-discount and 16-percent-discount rows hold the benchmark ψ_S fixed and solve prices and consumption policies again; the initial premium is therefore allowed to change. The last two columns report the external transfer relative to output at the two reference states and its sign over the checked dividend domain. Both discount-rate comparisons require inward payments at the reference states, so their prices require transfers outside the benchmark restriction to outward payments.

Table 9: Balance-sheet mechanics across participation calibrations

						Decomposition of Equity Risk Premium			
	$\mathcal{B}_N/\underline{Y}$ (%)	$\mathcal{B}_S/\underline{Y}$ (%)	$\mathcal{L}_S$	θ_S (%)	σ_R (%)	Wedge	Allocation	Volatility	Hedging
π_S = 30%	145.00/164.67	-145.00/-150.90	1.43/1.49	82.07/82.11	13.20/13.46	42.43	0.09	8.57	-0.67
π_S = 45%	145.00/162.20	-145.00/-150.16	1.42/1.48	67.83/67.19	13.37/13.65	42.47	-1.74	7.68	-0.58

Notes: Each participation calibration matches the initial nonstockholder bond-holdings and premium targets. All four contributions use the CRRA endpoint decomposition.

At the fixed wedge coefficient, moving from $\rho_S = 4\%$ to either $\rho_S = \rho_N$ or 16 percent changes prices and the longevity response only slightly, while stockholder consumption remains positive. With CRRA preferences, prices depend on ρ_S through continuation values.

Table 10 shows that longer retirement raises the equity premium at initial ERP targets of 4, 4.5, and 5 percent. Each row recalibrates ψ_S at the initial demographic state and then holds it fixed throughout the same longevity experiment.

Table 10: Sensitivity to the initial equity-premium target

					Decomposition of Equity Risk Premium			
ERP target	ψ_S	$\underline{\text{ERP}}$	$\overline{\text{ERP}}$	ΔERP	Leverage	Allocation	Volatility	Hedging
4.00%	0.0538	4.00%	4.24%	24.25	19.95	0.88	3.63	-0.21
4.50%	0.0609	4.50%	4.85%	35.08	29.30	0.58	5.59	-0.39
5.00%	0.0666	5.00%	5.50%	50.42	42.43	0.09	8.57	-0.67

Notes: Each row recalibrates ψ_S to its initial premium target and then holds it fixed in the longevity experiment. ERP endpoints are in percent; the total and four channel contributions are in basis points.

Pricing domain and numerical solution. We evaluate stationary outcomes where nonstockholder bond holdings have zero drift. We solve the price function on $D/D_0 \in [0.87, 2.20]$, imposing unit price elasticity at both dividend boundaries, and report prices and returns at the reference dividend. These boundary conditions are the same in the benchmark and in the comparison across external issuance shares.

We follow the solution connected to the calibrated initial state as longevity increases, reporting results over $\omega \in [0.40, 1]$. The CRRA endpoints are validated directly over this range. A separate calculation for the log-utility calibration, in which debt is varied along the price-solution branch, finds a turning point at local stockholder debt of 156.97 percent of initial output. In that calibration, the four-year shock corresponds to $\omega \simeq 0.391$, and $\omega = 0.70$ leaves 6.07 percentage points of initial output below the turning point. This limit applies to the particular log-utility solution branch and dividend interval. It does not

locate a turning point in the CRRA system, estimate a borrowing limit, or rule out other solution branches. All reported endpoints use the same boundary conditions, without imposing a separate lower-price anchor.

Table 11: Four-year post-retirement longevity increase: decomposition of the equity risk premium

ω	ΔERP (bp)	Wedge	Allocation	Volatility	Hedging
0.40	126.16	105.79	-0.53	22.76	-1.86
0.70	50.42	42.43	0.09	8.57	-0.67
0.80	31.80	26.77	0.11	5.32	-0.41
1.00	0.01	0.01	-0.00	-0.00	0.01

Notes: All contributions are in basis points. Each row uses the same initial financial calibration and four-year increase in the retirement-duration gap, holding human wealth fixed. The wedge, allocation, volatility, and hedging contributions sum to the premium change. They describe changes between equilibria; measuring the effect of removing the wedge would require solving a separate counterfactual.

A.6 General CRRA preferences

A.6.1 Nonstockholder saving under CRRA preferences

For a CRRA nonstockholder with coefficient $\gamma_N > 0$ and discount rate ρ_N, the ratio of consumption to total safe wealth is

$$\frac{C_N}{\mathcal{B}_N + \mathcal{H}_N} = \frac{\rho_N + \gamma_N\delta + (\gamma_N - 1)r^*}{\gamma_N} = \delta + \frac{\rho_N + (\gamma_N - 1)r^*}{\gamma_N}. \tag{63}$$

In the longevity experiment, we hold W_N fixed and adjust earnings timing to keep $\mathcal{H}_N$ fixed as mortality falls. The stationary nonstockholder bond-holdings target and its mortality derivative are then

$$\begin{aligned} \mathcal{B}^*_{N,\mathrm{CRRA}}(\delta) &= \mathcal{H}_N \frac{r^* + \gamma_N\chi(\delta) - \rho_N}{\gamma_N\delta + \rho_N - r^*}, \\ \frac{d\mathcal{B}^*_{N,\mathrm{CRRA}}}{d\delta} &= -\frac{\gamma_N[\mathcal{B}^*_{N,\mathrm{CRRA}}(\delta) + \mathcal{H}_N]}{\gamma_N\delta + \rho_N - r^*}. \end{aligned} \tag{64}$$

Lemma (Equivalent aggregate saving under CRRA and log utility)**.** *Fix r^*. If a CRRA pair satisfies $[\rho_N + (\gamma_N - 1)r^*]/\gamma_N = \rho_N^{\log}$, then its consumption propensity, aggregate nonstockholder bond-holdings drift, stationary target under earnings-timing compensation, mortality derivative, and reduced transition coincide with those of a log nonstockholder with discount rate $\rho_N^{\log}$, for every δ. This is an equivalence of the aggregate financial block, not of welfare.*

Proof. Substitute $[\rho_N + (\gamma_N - 1)r^*]/\gamma_N = \rho_N^{\log}$ into (63). Then the ratio of consumption to total

safe wealth is $\delta + \rho_N^{\log}$, so the consumption rule and the drift (35) are identical pointwise in $(\mathcal{B}_N, \delta)$. The stationary target, its derivative, and the transition follow from that common linear law. □

At the benchmark balance sheet, $\gamma_N = 4$ and $\rho_N = 12.23\%$ match the same initial nonstockholder bond-holdings target as the log reference case. At the fixed safe rate, the calibration therefore preserves both aggregate bond demand and its response to a longer retirement.

A.6.2 CRRA stockholders

We now derive stockholder pricing for the $\gamma_S \neq 1$ case introduced in Section 3.1. Demographics, nonstockholder saving, and the incidence rule are as in the benchmark. We write the unnormalized CRRA value function as

$$V_S(\mathcal{T}_S, D, \mathcal{B}_N) = \frac{[\mathcal{T}_S G(D, \mathcal{B}_N)]^{1-\gamma_S}}{1-\gamma_S}. \tag{65}$$

The consumption first-order condition is

$$m_S = G^{1-1/\gamma_S}, \qquad C_S = m_S \mathcal{T}_S. \tag{66}$$

With this normalization, the consumption condition has no additional factor involving $\rho_S + \delta$. Let $u = \log P$, $v = \log G$, and $\varrho_S = \rho_S + \delta$. Itô's lemma gives

$$\sigma_R = \sigma_D D u_D, \qquad \sigma_{C,S} = \theta_S \sigma_R + \frac{\gamma_S - 1}{\gamma_S} \sigma_D D v_D. \tag{67}$$

The compensation stockholders require per unit of aggregate risk is

$$\lambda_S = \gamma_S \theta_S \sigma_R + (\gamma_S - 1)\sigma_D D v_D. \tag{68}$$

The coupled stationary equations for u and v are

$$0 = \frac{D}{P} + \mathcal{A}u + \frac{1}{2}\sigma_D^2 D^2 u_D^2 - r^* - \tau_S - \lambda_S \sigma_R \tag{69}$$

and

$$\begin{aligned} 0 = \frac{m_S - \varrho_S}{1-\gamma_S} + \mathcal{A}v + \frac{1}{2}(1-\gamma_S)\sigma_D^2 D^2 v_D^2 \\ + r^* + \delta + \tau_S \theta_S - \xi - m_S + \frac{\gamma_S}{2}\theta_S^2 \sigma_R^2. \end{aligned} \tag{70}$$

Before applying (66), the first term in the continuation equation is $\{(m_S/G)^{1-\gamma_S} - \varrho_S\}/(1-\gamma_S)$. The first-order condition implies $(m_S/G)^{1-\gamma_S} = m_S$, giving (70).

The external transfer rate ξ is given by (50), using the state-dependent consumption propensity $m_S = G^{1-1/\gamma_S}$. The proportional payment reduces total-wealth drift by ξ, which explains its negative sign in the continuation equation. Households take the rate as given when choosing consumption and portfolios, so it does not enter their first-order conditions. We solve jointly for the price, continuation value, consumption propensity, and transfer rate, using the log-utility solution at $\gamma_S = 1$.

The CRRA equity premium is

$$\text{ERP} = \psi_S(\mathcal{L}_S - 1) + \gamma_S \theta_S \sigma_R^2 + (\gamma_S - 1)\sigma_D D v_D \sigma_R. \tag{71}$$

The last term reflects the covariance of returns with changes in future investment opportunities. It vanishes at $\gamma_S = 1$. We solve the log-utility problem separately because the representation in (70) is singular at that limit.

Proposition (Direct longevity effect under CRRA)**.** *Suppose* $r^* + \gamma_N \chi - \rho_N > 0$, $\gamma_N \delta + \rho_N - r^* > 0$, $\psi_S \geq 0$, *and* $\sigma_R > 0$. *Let* $0 < \pi_S < 1$, $P > 0$, $\mathcal{F}_S = P + \mathcal{B}_S > 0$, *and* $\mathcal{T}_S = P + \mathcal{B}_S + \pi_S \mathcal{H} > 0$. *Under earnings-timing compensation, holding* P, σ_R, *and* v_D *fixed, the direct mortality effect is*

$$\begin{aligned} \left.\frac{\partial \,\text{ERP}}{\partial \delta}\right|_{P,\sigma_R,v_D} = & -(1-\omega)\frac{\gamma_N[\mathcal{B}^*_{N,\text{CRRA}} + (1-\pi_S)\mathcal{H}]}{\gamma_N \delta + \rho_N - r^*} \\ & \times \left[\psi_S \frac{P}{(P+\mathcal{B}_S)^2} + \gamma_S \sigma_R^2 \frac{P}{(P+\mathcal{B}_S+\pi_S\mathcal{H})^2}\right] \leq 0. \end{aligned} \tag{72}$$

The inequality is strict for $\omega < 1$ *and is an equality at* $\omega = 1$. *Thus lower mortality raises the direct balance-sheet component of the ERP whenever non-U.S. stockholders supply some of the additional bonds. Their greater debt issuance increases leverage and risky exposure, requiring more compensation to hold the tree at the fixed price and volatility.*

Proof. Substitute $\mathcal{H}_N = (1-\pi_S)\mathcal{H}$ into (64), and use $d\mathcal{B}^*_S/d\delta = -(1-\omega)d\mathcal{B}^*_{N,\text{CRRA}}/d\delta$. At fixed P,

$$\frac{\partial \mathcal{L}_S}{\partial \mathcal{B}_S} = -\frac{P}{(P+\mathcal{B}_S)^2}, \qquad \frac{\partial \theta_S}{\partial \mathcal{B}_S} = -\frac{P}{(P+\mathcal{B}_S+\pi_S\mathcal{H})^2}.$$

Differentiate (71). The intertemporal-hedging term has no direct $\mathcal{B}_S$ derivative when P, σ_R, and v_D are held fixed. Combining the expressions gives (72). □

The proposition isolates the direct effect at fixed model parameters. It does not determine the sign of a comparison that recalibrates ρ_N or ψ_S, or the total finite response of

θ_S. The total CRRA response also accounts for changes in prices, volatility, and hedging. Along a stationary reference state,

$$\begin{aligned}
\mathrm{ERP}_{z_\delta} &= A_B \mathcal{B}_{S,z_\delta} + A_P P_{z_\delta} + A_\sigma \sigma_{R,z_\delta} + A_v v_{D,z_\delta}, \\
A_B &= -\psi_S \frac{P}{\mathcal{F}_S^2} - \gamma_S \sigma_R^2 \frac{P}{\mathcal{T}_S^2}, \\
A_P &= \psi_S \frac{\mathcal{B}_S}{\mathcal{F}_S^2} + \gamma_S \sigma_R^2 \frac{\mathcal{H}_S + \mathcal{B}_S}{\mathcal{T}_S^2}, \\
A_\sigma &= 2\gamma_S \theta_S \sigma_R + (\gamma_S - 1)\sigma_D D v_D, \qquad A_v = (\gamma_S - 1)\sigma_D D \sigma_R.
\end{aligned} \tag{73}$$

For two CRRA equilibria at the same reference dividend, the direct-risk component has the exact decomposition

$$\Delta(\gamma_S \theta_S \sigma_R^2) = \gamma_S (\sigma_R^2)^{\mathrm{mid}} \, \Delta\theta_S + \gamma_S \theta_S^{\mathrm{mid}} \, \Delta(\sigma_R^2), \tag{74}$$

while the intertemporal-hedging component satisfies

$$\Delta[(\gamma_S - 1)\sigma_D D v_D \sigma_R] = (\gamma_S - 1)\sigma_D D \left(\sigma_R^{\mathrm{mid}} \Delta v_D + v_D^{\mathrm{mid}} \Delta \sigma_R \right). \tag{75}$$

We assign the first term in the last parenthesis to intertemporal hedging: it captures the change in the hedging loading at average return volatility. We combine the second term with the ordinary volatility contribution.

Equations (71)–(75) show how the balance-sheet mechanism operates with general CRRA preferences. In our calibration and robustness exercises, we use $\gamma_S = 1.5$ and $\gamma_N = 4$. The longevity experiment raises the equity premium by 50.4 basis points at these preferences. Away from log utility, the transfer also affects prices through the continuation value, so removing it requires solving a different conditional pricing problem.

The leverage wedge accounts for most of the quantitative response. If we remove it without recalibrating the model, the initial equity premium is only 1.89% and longevity raises it by 1.4 basis points, with the other primitives held fixed. The large benchmark response therefore comes from the interaction between greater debt issuance and the leverage sensitivity of required returns.

These results describe equilibrium locally around the benchmark states. Global existence would require further analysis: the dividend process is not ergodic, and the model includes neither default nor a global solvency constraint.

B Transition dynamics in the fixed-rate benchmark

The stationary comparison in the main text combines an immediate change in asset prices with a gradual adjustment in household saving. Prices respond when the decline in mortality is announced; nonstockholders then accumulate the additional bonds needed to finance a longer retirement. We trace these responses using the same permanent longevity increase as in the main text, adjusting earnings decline to keep human wealth fixed. Local non-U.S. stockholders issue the share $1-\omega$ of additional bonds, with $\omega = 0.70$. All financial parameters, including the 30% participation share, are those calibrated in Section 3.4.

B.1 The accumulation of nonstockholder bond holdings

With nonstockholder human wealth, $\mathcal{H}_N$, held fixed, a longer retirement raises the bond holdings nonstockholders wish to accumulate. After mortality falls to $\overline{\delta}$, aggregate nonstockholder bond holdings follow

$$\dot{\mathcal{B}}_{N,t} = -\kappa_B(\mathcal{B}_{N,t} - \overline{\mathcal{B}}^*_N), \qquad \kappa_B = \overline{\delta} + \frac{\rho_N - r^*}{\gamma_N} > 0, \tag{76}$$

where their new long-run level is

$$\overline{\mathcal{B}}^*_N = \mathcal{H}_N \frac{r^* + \gamma_N \overline{\chi} - \rho_N}{\gamma_N \overline{\delta} + \rho_N - r^*}, \qquad \overline{\chi} = \underline{\chi} + \underline{\delta} - \overline{\delta}. \tag{77}$$

The solution is

$$\mathcal{B}_{N,t} = \overline{\mathcal{B}}^*_N + (\underline{\mathcal{B}}_N - \overline{\mathcal{B}}^*_N)e^{-\kappa_B t}. \tag{78}$$

Nonstockholders build their additional bond holdings through saving, so these holdings cannot jump when mortality changes. The speed of adjustment follows from the household budget constraint and CRRA consumption rule, without an additional adjustment friction. The local stockholder bond position satisfies $\mathcal{B}_{S,t} = \underline{\mathcal{B}}_S - (1-\omega)(\mathcal{B}_{N,t} - \underline{\mathcal{B}}_N)$, while $\mathcal{B}_{\mathrm{ROW},t} = \omega(\mathcal{B}_{N,t} - \underline{\mathcal{B}}_N)$. These identities hold throughout the transition, with $\omega = 0.70$.

The implied half-life is about 17.5 years. Thus, household balance sheets adjust gradually even though the demographic change is permanent from date zero.

B.2 Impact repricing and subsequent accumulation

The tree price responds immediately to the change in mortality and the anticipated accumulation of stockholder debt. At the announcement, nonstockholder bond holdings,

local debt, and current dividends are predetermined. We solve jointly for the price and consumption propensity along the deterministic nonstockholder bond-holdings path, using their new stationary solutions as terminal conditions. The time-dependent CRRA system adds u_t and v_t to equations (69) and (70), respectively, and replaces $\mathcal{A}$ with the dividend generator. These time derivatives already account for movement along $\mathcal{B}_{N,t}$. At each date, the transfer rate follows rule (50), using the actual stockholder bond-position drift $\dot{\mathcal{B}}_{S,t} = -(1-\omega)\dot{\mathcal{B}}_{N,t}$. Stockholder financial wealth, $P_t + \mathcal{B}_{S,t}$, therefore jumps with the tree price while debt is still unchanged. The equity premium along the transition satisfies

$$\mathrm{ERP}_t = \psi_S(\mathcal{L}_{S,t} - 1) + \gamma_S \theta_{S,t} \sigma_{R,t}^2 + (\gamma_S - 1)\sigma_D D_t v_{D,t} \sigma_{R,t}. \tag{79}$$

The initial fall in the tree price reduces stockholders' financial wealth and raises their leverage. As nonstockholders subsequently accumulate bonds, stockholders issue the corresponding local debt, further reducing the wealth available to bear the tree's risk.

Table 12: Transition following the longevity increase

	Initial	Impact	Year 10	Long run
Nonstockholder bond holdings / initial output (%)	145.00	145.00	151.44	164.67
Local stockholder bond position, $\mathcal{B}_S/\underline{Y}$ (%)	-145.00	-145.00	-146.93	-150.90
Financial leverage	1.43	1.45	1.46	1.49
Equity risk premium (%)	5.00	5.13	5.25	5.50
External payment / output (%)	19.81	22.00	22.14	22.46

Notes: Mortality falls permanently by 0.107 percentage point and χ rises by the same amount to keep human wealth fixed, while wages remain unchanged. The retirement-duration gap increases by four years. The external share is $\omega = 0.70$. "Impact" denotes the value just after the announcement, before either nonstockholder bond holdings or local debt changes. The transfer is relative to contemporaneous output.

The premium rises before nonstockholders have accumulated any additional bonds. In Table 12, we find an increase of 13.2 basis points on impact, about 26 percent of the eventual 50.4-basis-point response. The remaining 37.2 basis points arise as nonstockholders accumulate bonds and stockholders issue the corresponding debt. The stationary comparison in the main text captures this long-run response.

B.3 Scope of the transition

The transition following the unanticipated, permanent decline in mortality combines the exact reduced-form law for aggregate nonstockholder bond holdings with forward-

looking asset prices. We retain stochastic dividend risk when pricing the tree and report the response along the reference dividend path $D_t = D_0$; this path is conditional on dividends and is not an unconditional mean over their future realizations. The calculation does not track the full cohort-age distribution after the shock. The safe rate remains fixed, and local non-U.S. stockholders issue the share $1 - \omega$ of additional bonds, with $\omega = 0.70$. The exercise therefore describes the timing of adjustment within the fixed-rate benchmark; it does not give the full general-equilibrium response to demographic change.

C Population growth and cohort composition

Slower population growth makes a population older by reducing the share of newborn households, even when individual life expectancy is unchanged. We combine this source of aging with the longevity experiment, as described in Section 3.6.1. We first increase longevity at a population-growth rate of 0.50 percent, adjusting earnings decline to keep human wealth fixed. We then lower population growth to 0.00 percent, holding mortality and earnings decline at their post-longevity values. To derive the balance-sheet effects, consider a fixed demographic regime with mortality rate δ and birth rate per person b. Net population growth is

$$n = b - \delta, \qquad \dot{N}_t = nN_t.$$

With constant mortality and population growth, the stable age distribution is

$$f_n(a) = (\delta + n)e^{-(\delta+n)a}.$$

The mean age is $1/(\delta+n)$, while an individual's expected remaining lifetime is $1/\delta$. Lower population growth therefore raises the average age without lengthening any household's expected remaining life. The longevity experiment changes both.

Earnings decline with age at rate χ. To hold the per-capita wage bill W fixed as the age distribution changes, we scale the earnings profile according to

$$w(a; n) = W\frac{\delta + n + \chi}{\delta + n}e^{-\chi a}.$$

Newborn households enter with no financial assets, so their arrival dilutes bond holdings per person. We measure nonstockholder bond holdings per member of the total population, denoted $\mathcal{B}_{N,t}$, and obtain

$$\dot{\mathcal{B}}_{N,t} = (r^* - n)\mathcal{B}_{N,t} + W_N - C_{N,t}.$$

Human wealth is $\mathcal{H}_N = W_N/(r^* + \delta + \chi)$. Under the CRRA preferences used in the benchmark, stationary nonstockholder bond holdings are

$$\mathcal{B}_N^*(\delta, n) = \mathcal{H}_N \frac{r^* + \gamma_N \chi - \rho_N}{\gamma_N(\delta + n) + \rho_N - r^*}, \qquad \frac{\partial \mathcal{B}_N^*}{\partial n} < 0.$$

Slower population growth reduces this dilution and raises nonstockholder bond holdings per person. The share of these additional bonds supplied by local non-U.S. stockholders increases their debt. Its effect on leverage and required returns also depends on how population growth affects the value of the tree and its dividends.

Tree normalization and the external payment. Let N denote population, Q the value of the single aggregate tree, and D^{agg} its aggregate dividend. The corresponding values per person are $P = Q/N$ and $D = D^{\text{agg}}/N$. We use the same population denominator for household wealth, debt, consumption, and transfers. Mortality credits and individual consumption propensities depend on δ, rather than $\delta + n$. We hold the initial debt-issuance anchor fixed in per-capita units, so $\dot{\mathcal{B}}_S = -(1-\omega)\mu_{\mathcal{B}_N}$ and $\dot{\mathcal{B}}_{\text{ROW}} = \omega \mu_{\mathcal{B}_N}$. Even when per-capita debt is constant, aggregate net issuance per person is $-n\mathcal{B}_S$, because total debt must grow with the population.

We retain the benchmark aggregate dividend process, $dD^{\text{agg}}/D^{\text{agg}} = \sigma_D dZ$. Per-capita dividends consequently satisfy $dD/D = -n\,dt + \sigma_D dZ$. There is a single aggregate ownership claim, and newborn households receive no freely issued equity. The return on this claim satisfies

$$\frac{dQ + D^{\text{agg}}\,dt}{Q} = \frac{dP}{P} + \left(n + \frac{D}{P}\right)dt, \qquad \mu_R = \frac{D}{P} + n + \frac{\mathcal{A}_n P}{P}. \tag{80}$$

Consequently the per-capita stockholder budget is

$$d\mathcal{F}_S = \Big[(r^* - n)\mathcal{F}_S + P\,\text{ERP} + W_S - C_S - \xi \mathcal{T}_S\Big]dt + P\sigma_R dZ. \tag{81}$$

Combining this with $\mathcal{F}_S = P + \mathcal{B}_S$ yields the cash-flow formula

$$C_S + \mathcal{X} = D + W_S + (r^* - n)\mathcal{B}_S + (1-\omega)\mu_{\mathcal{B}_N}, \tag{82}$$

$$\xi = \frac{D + W_S + (r^* - n)\mathcal{B}_S + (1-\omega)\mu_{\mathcal{B}_N}}{\mathcal{T}_S} - m_S, \qquad \mathcal{X} = \xi \mathcal{T}_S, \tag{83}$$

$$C_S + C_N + \mathcal{X} = W + D + (r^* - n)\mathcal{B}_{\text{ROW}} - \omega \mu_{\mathcal{B}_N}. \tag{84}$$

Population growth enters both the return on the aggregate tree and the stockholder budget

through the $-n\mathcal{F}_S$ dilution term. Combining these effects gives the $-n\mathcal{B}_S$ issuance term in the cash-flow formula; a $-nP$ term would omit the population correction to the tree return.

We evaluate prices with nonstockholder bond holdings at their zero-drift value. Let $z_D = \log(D/D_0)$, $u = \log P$, and $g_D = -n$, as implied by the aggregate dividend process. For the log-stockholder reference case, the pricing equation becomes

$$0 = De^{-u} + n + g_D u_{z_D} + \tfrac{1}{2}\sigma_D^2(u_{z_D z_D} - u_{z_D} + u_{z_D}^2) - r^* - \tau_S - \theta_S \sigma_D^2 u_{z_D}^2. \tag{85}$$

We use the benchmark unit-elasticity dividend boundaries. In the CRRA system, population growth adds $g_D \partial_{z_D}$ to the generator and $+n$ to the price equation. The individual continuation equation contains $-\xi$, with no additional mortality credit or impatience term involving n: population growth changes cohort weights, rather than individual survival probabilities.

Bond holdings have zero drift in these comparisons, but per-capita dividends continue to vary. The comparisons therefore do not describe invariant distributions or equilibria with constant per-capita dividends. A change in n alters bond holdings through dilution, whereas the longevity experiment changes mortality and adjusts χ to keep human wealth fixed. We use the aggregate dividend process above and the external payment in equation (83) throughout the comparison.

C.1 Joint experiment and stationary implications

We calibrate the initial allocation $(\underline{\delta}, \underline{\chi}, \underline{n})$ with positive population growth, $\underline{n} = 0.005$, and the same nonstockholder bond-holdings target as in the benchmark, $\underline{\mathcal{B}}_N/\underline{Y} = 1.45$, where $\underline{Y}$ is the fixed initial output per person. Underlines denote the initial allocation, a superscript L the longevity-only allocation, and overlines the final allocation after both demographic changes. The condition that initial nonstockholder bond holdings have zero drift identifies the nonstockholder discount rate:

$$\rho_N = \gamma_N \left[\frac{(r^* - \underline{n})\underline{\mathcal{B}}_N + W_N}{\underline{\mathcal{B}}_N + \mathcal{H}_N} - \underline{\delta} \right] - (\gamma_N - 1) r^*. \tag{86}$$

This gives $\rho_N = 11.83517$ percent per year. We then match the initial 5-percent equity premium, including $\underline{n}$ in the pricing equation, which gives $\psi_S = 0.06619937$. Positive initial population growth therefore requires a different calibration from the zero-growth benchmark. We hold both parameters fixed throughout the experiment. The longevity increase adds four years to expected life while holding $\delta + \chi$ fixed, so human wealth is

unchanged:

$$\delta^L = (\underline{\delta}^{-1} + 4)^{-1}, \qquad \chi^L = \underline{\chi} + \underline{\delta} - \delta^L, \qquad n^L = \underline{n}.$$

Starting from this allocation, the joint experiment sets

$$(\overline{\delta}, \overline{\chi}, \overline{n}) = (\delta^L, \chi^L, 0).$$

Population growth falls by 0.5 percentage point to 0.00 percent per year. Relative to the longevity allocation at $\underline{n}$, this lowers the birth rate from $b^L = \delta^L + \underline{n}$ to $\overline{b} = \delta^L + \overline{n}$, with mortality unchanged. Consequently, individual mortality credits remain δ^L, and nonstockholders' consumption propensity remains $\delta^L + [\rho_N + (\gamma_N - 1)r^*]/\gamma_N$. Stockholders' consumption propensity adjusts with asset prices. The per-capita wage bill and human wealth are the same across all three allocations, as are preferences and financial parameters.

To isolate the additional saving generated by lower population growth, write $\mathcal{B}_N^L = \mathcal{B}_N^*(\delta^L, \underline{n})$ and $\kappa_B^L = \delta^L + (\rho_N - r^*)/\gamma_N > 0$. With the numerator of stationary bond demand unchanged, the reduction in dilution gives

$$\overline{\mathcal{B}}_N = \mathcal{B}_N^L \frac{\kappa_B^L + \underline{n}}{\kappa_B^L + \overline{n}}, \qquad \overline{\mathcal{B}}_N - \mathcal{B}_N^L = \frac{(\underline{n} - \overline{n})\mathcal{B}_N^L}{\kappa_B^L + \overline{n}} > 0, \tag{87}$$

where $\kappa_B^L + \overline{n} > 0$. Both demographic changes increase nonstockholder bond holdings relative to the same initial per-capita anchor, $\underline{\mathcal{B}}_N = -\underline{\mathcal{B}}_S$. At both the longevity-only and final allocations, the rule remains

$$\mathcal{B}_S = \underline{\mathcal{B}}_S - (1 - \omega)(\mathcal{B}_N - \underline{\mathcal{B}}_N), \qquad \mathcal{B}_{\text{ROW}} = \omega(\mathcal{B}_N - \underline{\mathcal{B}}_N). \tag{88}$$

Thus $\overline{\mathcal{B}}_S - \mathcal{B}_S^L = -(1 - \omega)(\overline{\mathcal{B}}_N - \mathcal{B}_N^L)$. The local share applies to additional bond demand; the bonds supplied externally during the longevity step remain obligations of external issuers.

In this calibration, lower population growth reinforces the effect of longer retirement: nonstockholder bond holdings and local stockholder debt rise, increasing the risk borne by stockholders. Table 13 reports the three allocations at $D = D_0$, with nonstockholder bond holdings at their zero-drift value. We use the benchmark aggregate dividend process, the dividend interval $D/D_0 \in [0.87, 2.20]$ with unit-elasticity boundaries, and the external payment in equation (83). The external share remains $\omega = 0.70$.

Table 13: Longevity increase followed by lower population growth

Object	Initial	Longevity	Final
Equity risk premium (percent)	5.00	5.47	6.16
Nonstockholder bond holdings / initial output (percent)	145.00	162.86	183.93
Local stockholder bond position, $\mathcal{B}_S/\underline{Y}$ (percent)	-145.00	-150.36	-156.68
Equity value / initial output (percent)	478.93	454.43	427.78
Stockholder financial wealth / initial output (percent)	333.93	304.07	271.10
Financial leverage	1.43	1.49	1.58
Risky allocation (percent)	81.94	81.93	82.00
Equity-return volatility (percent)	13.14	13.37	13.82

Notes: The longevity column increases the retirement-duration gap by four years at 0.50-percent population growth. Mortality falls from 1.69 to 1.58 percent and earnings decline rises from 3.39 to 3.50 percent per year. The final column additionally lowers population growth to 0.00 percent, keeping mortality and earnings decline at their post-longevity values. The safe rate remains 2.70 percent. We hold fixed the per-capita wage bill and human wealth, financial parameters, initial issuance anchor, and $\omega = 0.70$. The aggregate dividend process is unchanged, so per-capita dividend drift is $-n$. Stocks and flows are measured per person. Stocks are scaled by initial per-capita output. Prices and returns are evaluated at the reference dividend, with nonstockholder bond holdings at their zero-drift value.

Relative to the longevity allocation, we find that reduced dilution raises nonstockholder bond holdings by 21.06 percentage points of initial output and local stockholder debt by 6.32 points. At the same time, the equity price falls by 5.86 percent and stockholder financial wealth by 10.84 percent. Stockholders must therefore finance the tree with more debt and less financial wealth: leverage rises from 1.49 to 1.58. The premium required to bear this risk increases by a further 69.4 basis points, from 5.47 to 6.16 percent. Relative to the original baseline, the combined nonstockholder bond-holdings increase is 38.93 percentage points of initial output and the combined premium increase is 116.3 basis points.

Sources of the premium increase. We separate the effects of leverage, risky allocation, return volatility, and intertemporal hedging using the symmetric-product decomposition. For any two of these CRRA allocations,

$$\Delta\text{ERP} = \psi_S \Delta\mathcal{L}_S + \gamma_S(\sigma_R^2)^{\text{mid}}\,\Delta\theta_S + \gamma_S\theta_S^{\text{mid}}\,\Delta\sigma_R^2 + (\gamma_S - 1)\sigma_D D_0(\sigma_R^{\text{mid}}\Delta v_D + v_D^{\text{mid}}\Delta\sigma_R), \quad (89)$$

where the superscript mid denotes arithmetic averages of the endpoints of the particular comparison and ψ_S remains fixed. In particular, $(\sigma_R^2)^{\text{mid}}$ averages endpoint variances. Higher leverage accounts for most of the premium increase following lower population growth, contributing 55.2 basis points. Return volatility also raises the premium, while the risky-allocation contribution is small and positive. Intertemporal hedging partly offsets these effects. This 69.4-basis-point increase includes the adjustment of prices and balance sheets from their post-longevity values. Together with the 46.9-basis-point longevity response, this gives the 116.3-basis-point combined increase. Panel B of Table 4 reports the decomposition for each comparison. As in Appendix A.6, the Δv_D term enters hedging and the $\Delta\sigma_R$ term enters volatility. The total premium changes add across the two steps. The allocation, volatility, and hedging contributions need not add separately, because the decomposition uses different endpoint-average weights in each comparison.

Lower population growth in isolation. We also lower population growth from $\underline{n} = 0.005$ to zero while holding mortality and earnings decline at their initial values. This comparison uses the same initial calibration and issuance anchor, with $\omega = 0.70$. The premium rises by 41.50 basis points, compared with 69.4 basis points when population growth falls after longevity. The difference of 27.91 basis points measures the interaction between the two demographic changes.

Table 14: Separate and combined effects of the demographic changes

Variable and unit	Longevity only ($\underline{x} \to x^L$)	Population only ($\underline{x} \to x^G$)	Both together ($\underline{x} \to \overline{x}$)
Equity risk premium, basis points	+46.89	+41.50	+116.29
Financial leverage, ratio points	+0.06	+0.05	+0.14
Local stockholder debt, $-\mathcal{B}_S$, percent change	+3.70	+3.78	+8.05
Stockholder financial wealth, percent change	-8.94	-6.77	-18.81
Equity price, percent change	-5.11	-3.58	-10.68
Nonstockholder bond holdings / initial output, percentage points	+17.86	+18.25	+38.93
Risky allocation, percentage points	-0.01	+0.25	+0.06
Equity-return volatility, percentage points	+0.23	+0.27	+0.69

Notes: For any object x, $\underline{x}$ denotes its value at the extension's initial allocation, x^G at the population-only allocation, x^L at the longevity-only allocation, and $\overline{x}$ at the final allocation after both changes. Every column starts at $\underline{x}$. We solve jointly for the combined effects. All stock/output differences use the same initial per-capita output denominator. Percent changes are relative to each variable's initial level. The safe rate is imposed and unchanged.

External supply and numerical scope. Greater external bond supply reduces the debt local stockholders must issue, dampening the premium response to both demographic changes. Table 15 shows this comparison at $\omega = 0.75$ with the positive-growth initial calibration held fixed. The main external-supply figure considers longevity alone, starting from zero population growth.

Table 15: Equity-premium decomposition under alternative external supply (basis points)

ω	Longevity	Population after longevity	Combined
0.70	46.89	69.40	116.29
0.75	38.01	50.46	88.48

In a separate log-stockholder calculation with $\omega = 0.50$, the joint saving equation requires local debt of 164.46 percent of initial output, while the computed price branch turns at approximately 158.02 percent. We therefore report no joint price or premium for that case. Our benchmark external share, $\omega = 0.70$, is an assumption about who supplies bonds.

We verify the reported endpoints using independent price-level and finite-difference checks and sector budget identities. Financial wealth is positive and external payments are outward on the sampled domain. We also check the final joint premium on a 3,201-node grid. These calculations impose unit price elasticity at $D/D_0 = 0.87$ and $D/D_0 = 2.20$, so the numerical agreement is conditional on those boundaries. It does not establish boundary independence, global existence, or uniqueness.

C.2 Cohort transition

The joint results compare allocations after balance sheets have adjusted. To trace that adjustment, we would also need to track the changing age distribution and the present value of future wages. For a population-growth transition from the post-longevity allocation, mortality and earnings decline would remain at δ^L and χ^L. The equations below therefore use $\delta = \delta^L$ and $\chi = \chi^L$. We describe the system needed for this cohort transition but do not solve it here. It is distinct from the reduced longevity transition in Appendix B.

Age composition, human wealth, and prices. A change in births gradually alters the age distribution and, hence, the earnings received by the average household. To track this adjustment and value future earnings, write $w(a,t) = k_w(t)e^{-\chi a}$ and define $M_t =$

$\int e^{-\chi a} f_t(a)\,da$ and $J_t = \int_0^\infty e^{-(r^*+\delta+\chi)s} k_w(t+s)\,ds$. Then

$$\dot{M}_t = (\delta + n_t) - (\delta + n_t + \chi)M_t, \qquad \dot{J}_t = (r^* + \delta + \chi)J_t - k_w(t), \qquad \mathcal{H}_t = M_t J_t.$$

With a fixed age-zero wage, J_t is constant. Holding the per-capita wage bill fixed instead requires $k_w(t) = W/M_t$, so human wealth depends on the future wage profile. Computing a transition therefore requires a specified path for n_t; the allocation comparisons above do not impose one.

Given the aggregate tree convention, the conditional price transition must reflect the actual $\mathcal{B}_{N,t}$ and human-wealth paths, together with the price time derivative. The log pricing equation is

$$0 = u_t + De^{-u} + n_t + g_D(t)u_{z_D} + \tfrac{1}{2}\sigma_D^2(u_{z_D z_D} - u_{z_D} + u_{z_D}^2) - r^* - \tau_S - \theta_S \sigma_D^2 u_{z_D}^2.$$

Here u_t accounts for movement along the deterministic demographic and nonstockholder bond-holdings paths; adding a separate $\mu_{\mathcal{B}_N} u_{\mathcal{B}_N}$ term would count the same movement twice. The transfer in equation (83) must likewise use the actual debt drift. Solving this system would allow prices to respond to the whole anticipated path, which a sequence of stationary price comparisons cannot capture.

D Two-country model: stationary derivations

We derive the household wealth equations, issuer leverage wedges, and market-clearing conditions used in Sections 4 and 5. We then show how the calibration identifies the parameters and how demographic changes affect each premium. We evaluate all expressions in stationary equilibria and suppress time subscripts. As in Section 4.1, Y_H and Y_F denote fixed aggregate output at the initial reference date, with $Y = Y_H + Y_F$. Their shares Y_H/Y and Y_F/Y weight per-person quantities in world sums. Lowercase $y_i = 1$ denotes output per person; all other stocks and flows are also per country resident unless stated otherwise. Initial, longevity-only, and final values use the marks $\underline{x}$, x^L, and $\overline{x}$, respectively, as in Section 5.

D.1 Human wealth

For household type $k \in \{S, N\}$, let $\pi_{S,i}$ denote the stockholder population share and $\pi_{N,i} = 1 - \pi_{S,i}$ the nonstockholder share. The wage flow of type k per country resident is $\pi_{k,i} W_i$. Human wealth is the present value of this earnings stream. In a stationary equilibrium, earnings decline at rate χ_i and cease at death, which occurs at rate δ_i. Discounting at the safe rate r gives

$$\begin{aligned} \mathcal{H}_{k,i} &= \pi_{k,i} W_i \int_0^\infty e^{-(r+\chi_i+\delta_i)u}\, du \\ &= \frac{\pi_{k,i} W_i}{r + \chi_i + \delta_i}, \end{aligned} \tag{90}$$

which gives the human-wealth expressions in Section 4.2.

D.2 Nonstockholder consumption and bond holdings

Nonstockholders save in actuarially fair annuities to finance consumption over their remaining lives. With CRRA utility and a constant safe rate, homogeneity of the value function gives the consumption–wealth ratio

$$m_{N,i} = \frac{\rho_{N,i} + \gamma_{N,i}\delta_i + (\gamma_{N,i} - 1)r}{\gamma_{N,i}} = \delta_i + q_i(r), \tag{91}$$

where $q_i(r) \equiv [\rho_{N,i} + (\gamma_{N,i} - 1)r]/\gamma_{N,i}$. This gives the nonstockholder consumption rule of Section 4.2. At the aggregate level, annuity credits to survivors offset the financial wealth of households who die. Newborns enter without financial wealth, so population growth

dilutes financial wealth per person at rate n_i. Nonstockholder bond holdings per country resident therefore evolve according to

$$\dot{\mathcal{B}}_{N,i} = (r - n_i)\mathcal{B}_{N,i} + (1 - \pi_{S,i})W_i - (\delta_i + q_i)(\mathcal{B}_{N,i} + \mathcal{H}_{N,i}). \tag{92}$$

Using the relation between wages and human wealth,

$$(1 - \pi_{S,i})W_i = (r + \chi_i + \delta_i)\mathcal{H}_{N,i} \tag{93}$$

and setting the drift to zero gives

$$\begin{aligned} 0 &= (r - n_i - \delta_i - q_i)\mathcal{B}_{N,i} + (r + \chi_i - q_i)\mathcal{H}_{N,i}, \\ \mathcal{B}^*_{N,i} &= \mathcal{H}_{N,i}\frac{r + \chi_i - q_i}{\delta_i + n_i + q_i - r}. \end{aligned} \tag{94}$$

This establishes equation (21).

In the initial equilibrium, the nonstockholder bond-holdings target $s^{\text{target}}_{N,i} = \underline{\mathcal{B}}_{N,i}/\underline{y}_i$ implies $\underline{\mathcal{B}}_{N,i} = s^{\text{target}}_{N,i}\underline{y}_i$. Solving the zero-drift condition for the effective discount term gives

$$\underline{q}_i = \frac{(\underline{r} - \underline{n}_i)\underline{\mathcal{B}}_{N,i} + (1 - \pi_{S,i})W_i}{\underline{\mathcal{B}}_{N,i} + \underline{\mathcal{H}}_{N,i}} - \underline{\delta}_i. \tag{95}$$

The primitive discount rate is then

$$\rho_{N,i} = \gamma_{N,i}\underline{q}_i - (\gamma_{N,i} - 1)\underline{r}. \tag{96}$$

Holding $\rho_{N,i}$ and $\gamma_{N,i}$ fixed,

$$\begin{aligned} q_i(r) - q_i(\underline{r}) &= \frac{\gamma_{N,i} - 1}{\gamma_{N,i}}(r - \underline{r}), \\ \frac{d[r - q_i(r)]}{dr} &= \frac{1}{\gamma_{N,i}}, \end{aligned} \tag{97}$$

The safe rate affects the return on saving and can also change the effective discount term. These expressions describe the resulting saving incentive, as discussed in Section 4.2.

D.3 Stockholder portfolios

For a stockholder in country i, let $\theta_{S,i,H}$ and $\theta_{S,i,F}$ be Home- and Foreign-equity exposure per unit of total wealth. In our stationary portfolio specification, the intertemporal hedging

term of Appendix A.6 is absent and portfolio choice is myopic: stockholders trade off expected excess returns against risk and the issuer leverage wedges. Dropping terms independent of portfolio choice, the objective is

$$\begin{aligned}&\theta_{S,i,H}\mathrm{ERP}_H + \theta_{S,i,F}\mathrm{ERP}_F \\ &\quad - \frac{\gamma_S}{2}\Big[\sigma^2_{R,H}(\theta_{S,i,H})^2 + 2\sigma_{R,HF}\theta_{S,i,H}\theta_{S,i,F} + \sigma^2_{R,F}(\theta_{S,i,F})^2\Big] \\ &\quad - \theta_{S,i,H}\tau_H - \theta_{S,i,F}\tau_F.\end{aligned} \tag{98}$$

The portfolio wedges are rebated to stockholders. Let

$$\alpha_{S,i} = \theta_{S,i,H}\tau_H + \theta_{S,i,F}\tau_F \tag{99}$$

be the equilibrium rebate per unit of stockholder total wealth. Each household takes $\alpha_{S,i}$, τ_H, and τ_F as given. The aggregate transfer $\alpha_{S,i}\mathcal{T}_{S,i}$ equals the charge on the sector's risky positions. The wedges therefore affect marginal portfolio choice while leaving aggregate resources unchanged. Differentiating the objective gives

$$\begin{aligned}0 &= \mathrm{ERP}_H - \gamma_S\Big(\sigma^2_{R,H}\theta_{S,i,H} + \sigma_{R,HF}\theta_{S,i,F}\Big) - \tau_H, \\ 0 &= \mathrm{ERP}_F - \gamma_S\Big(\sigma_{R,HF}\theta_{S,i,H} + \sigma^2_{R,F}\theta_{S,i,F}\Big) - \tau_F.\end{aligned} \tag{100}$$

Both groups of stockholders face the same wedge on each country's equity, regardless of where they reside. When $\sigma^2_{R,H}\sigma^2_{R,F} - \sigma^2_{R,HF} > 0$, solving these two conditions gives

$$\begin{aligned}\theta_{S,i,H} &= \frac{\sigma^2_{R,F}(\mathrm{ERP}_H - \tau_H) - \sigma_{R,HF}(\mathrm{ERP}_F - \tau_F)}{\gamma_S(\sigma^2_{R,H}\sigma^2_{R,F} - \sigma^2_{R,HF})}, \\ \theta_{S,i,F} &= \frac{\sigma^2_{R,H}(\mathrm{ERP}_F - \tau_F) - \sigma_{R,HF}(\mathrm{ERP}_H - \tau_H)}{\gamma_S(\sigma^2_{R,H}\sigma^2_{R,F} - \sigma^2_{R,HF})}, \qquad i \in \{H, F\}.\end{aligned} \tag{101}$$

Since stockholders also have the same risk aversion in both countries, their portfolio shares coincide: $\theta_{S,i,H} = \theta_{S,H}$ and $\theta_{S,i,F} = \theta_{S,F}$ for $i \in \{H, F\}$. Substituting these shares into equation (100) gives the market-pricing equations (28).

D.4 Stockholder financial wealth

Stockholders consume $C_{S,i} = m_{S,i}\mathcal{T}_{S,i}$, where

$$m_{S,i} = \delta_i + \frac{1}{\gamma_S}\Big\{\rho_{S,i} + (\gamma_S - 1)\Big[r + \theta_{S,i,H}\mathrm{ERP}_H + \theta_{S,i,F}\mathrm{ERP}_F - \frac{\gamma_S}{2}\Big(\sigma^2_{R,H}(\theta_{S,i,H})^2 + 2\sigma_{R,HF}\theta_{S,i,H}\theta_{S,i,F} + \sigma^2_{R,F}(\theta_{S,i,F})^2\Big)\Big]\Big\}. \tag{102}$$

Stockholders finance consumption from earnings and returns on their equity and safe positions. Their aggregate budget per person is

$$\dot{\mathcal{F}}_{S,i} = (r - n_i)\mathcal{F}_{S,i} + \mathcal{E}_{iH}\mathrm{ERP}_H + \mathcal{E}_{iF}\mathrm{ERP}_F + \pi_{S,i}W_i - C_{S,i}. \tag{103}$$

We write equity holdings as $\mathcal{E}_{iH} = \mathcal{T}_{S,i}\theta_{S,i,H}$ and $\mathcal{E}_{iF} = \mathcal{T}_{S,i}\theta_{S,i,F}$, total wealth as $\mathcal{T}_{S,i} = \mathcal{F}_{S,i} + \mathcal{H}_{S,i}$, and consumption as $C_{S,i} = m_{S,i}\mathcal{T}_{S,i}$, with $m_{S,i}$ given by equation (102). Substituting these expressions into the budget and setting the drift to zero gives

$$0 = (r - n_i)\mathcal{F}_{S,i} + (\theta_{S,i,H}\mathrm{ERP}_H + \theta_{S,i,F}\mathrm{ERP}_F)(\mathcal{F}_{S,i} + \mathcal{H}_{S,i}) + \pi_{S,i}W_i - m_{S,i}(\mathcal{F}_{S,i} + \mathcal{H}_{S,i}). \tag{104}$$

Using $\pi_{S,i}W_i = (r + \chi_i + \delta_i)\mathcal{H}_{S,i}$, we collect the terms in financial and human wealth:

$$0 = -[m_{S,i} + n_i - r - \theta_{S,i,H}\mathrm{ERP}_H - \theta_{S,i,F}\mathrm{ERP}_F]\mathcal{F}_{S,i} + [r + \chi_i + \delta_i - m_{S,i} + \theta_{S,i,H}\mathrm{ERP}_H + \theta_{S,i,F}\mathrm{ERP}_F]\mathcal{H}_{S,i}. \tag{105}$$

Solving for $\mathcal{F}_{S,i}$ yields

$$\mathcal{F}^*_{S,i} = \mathcal{H}_{S,i}\frac{r + \chi_i + \delta_i - m_{S,i} + \theta_{S,i,H}\mathrm{ERP}_H + \theta_{S,i,F}\mathrm{ERP}_F}{m_{S,i} + n_i - r - \theta_{S,i,H}\mathrm{ERP}_H - \theta_{S,i,F}\mathrm{ERP}_F}. \tag{106}$$

With $\gamma_S = 1$, $m_{S,i} = \rho_{S,i} + \delta_i$. In our calibration, financial wealth is small relative to human wealth and responds strongly to changes in the denominator. A lower safe rate reduces the return on wealth by more than it reduces $m_{S,i}$ when $\gamma_S > 1$, lowering stockholder financial wealth through the channel described in Section 5.3. Direct equity positions follow from $\mathcal{E}_{iH} = \mathcal{T}_{S,i}\theta_{S,i,H}$ and $\mathcal{E}_{iF} = \mathcal{T}_{S,i}\theta_{S,i,F}$. The stockholder bond position then follows from equation (22).

D.5 Stockholder bond positions and issuer leverage

The stockholder bond position $\mathcal{B}_{S,i}$ is negative when stockholders borrow. Financial wealth is the value of equity holdings plus this bond position:

$$\mathcal{F}_{S,i} = \mathcal{E}_{iH} + \mathcal{E}_{iF} + \mathcal{B}_{S,i}. \tag{107}$$

We consider equilibria with $\mathcal{B}_{S,i} \leq 0$ and $\mathcal{F}_{S,i} > 0$. Dividing by financial wealth and rearranging gives

$$\frac{\mathcal{E}_{iH} + \mathcal{E}_{iF}}{\mathcal{F}_{S,i}} = 1 - \frac{\mathcal{B}_{S,i}}{\mathcal{F}_{S,i}} \equiv \mathcal{L}_{S,i}. \tag{108}$$

Taking a differential gives

$$d\mathcal{L}_{S,i} = -\frac{d\mathcal{B}_{S,i}}{\mathcal{F}_{S,i}} + \frac{\mathcal{B}_{S,i}}{(\mathcal{F}_{S,i})^2}\, d\mathcal{F}_{S,i}, \tag{109}$$

and the issuer wedge changes according to

$$d\tau_i = \psi_i\, d\mathcal{L}_{S,i}. \tag{110}$$

Leverage rises when borrowing, measured by $-\mathcal{B}_{S,i}$, increases relative to stockholder financial wealth. Higher leverage raises the wedge and the return required to hold that issuer's equity. Both countries have the same unit anchor and, in our calibration, nearly equal slopes, so differences in this part of the premium response largely reflect changes in their balance sheets.

D.6 Tree prices and market clearing

Under our stationary valuation assumption, a tree with dividend D_j and price P_j per person has expected total return $D_j/P_j + n_j$, because its aggregate dividend grows with the issuing population. Its expected excess return is ERP_j, so

$$\frac{D_j}{P_j} + n_j = r + \mathrm{ERP}_j, \tag{111}$$

which gives equation (29). There is one unit of each tree per resident of the issuing country, giving the equity-clearing condition in equation (25). Summing the safe positions of the four household sectors, weighted by initial output shares, gives world bond clearing in equation (26).

Country i's net external bond position is $\mathcal{B}_{N,i} + \mathcal{B}_{S,i}$. Home's net foreign assets per resident then equal its net external bond position plus foreign equity held at Home, less

Home equity held abroad:

$$\mathrm{NFA}_H = \mathcal{B}_{N,H} + \mathcal{B}_{S,H} + \mathcal{E}_{HF} - \frac{Y_F}{Y_H}\mathcal{E}_{FH}. \tag{112}$$

World bond clearing and the two equity-market conditions imply $\frac{Y_H}{Y}\mathrm{NFA}_H + \frac{Y_F}{Y}\mathrm{NFA}_F = 0$.

Summing the four household zero-drift conditions, weighted by the initial output shares, gives

$$\begin{aligned} 0 = \sum_i \frac{Y_i}{Y}\Big[&(r - n_i)(\mathcal{B}_{N,i} + \mathcal{B}_{S,i}) \\ &+ (r - n_i + \mathrm{ERP}_H)\mathcal{E}_{iH} + (r - n_i + \mathrm{ERP}_F)\mathcal{E}_{iF} \\ &+ W_i - C_{S,i} - C_{N,i}\Big]. \end{aligned} \tag{113}$$

Using bond clearing, equity clearing, and $P_j(r + \mathrm{ERP}_j - n_j) = D_j$, the asset-income terms reduce to $\sum_j \frac{Y_j}{Y} D_j - \sum_i \frac{Y_i}{Y} n_i \mathrm{NFA}_i$. Hence

$$\sum_i \frac{Y_i}{Y}(C_{S,i} + C_{N,i}) = \sum_i \frac{Y_i}{Y} y_i - \sum_i \frac{Y_i}{Y} n_i \mathrm{NFA}_i. \tag{114}$$

This identity underlies the goods-market residual discussed in Section 5. Goods markets clear exactly when the countries have equal population growth rates or zero net foreign assets. Otherwise, the final term measures the gap in world goods-market clearing for these stationary comparisons, reported in Section 5.3.

D.7 Calibration

We target the initial safe rate, both equity premia, and both market-capitalization ratios. With output per person normalized to one, the market-value target $\underline{P}_i/\underline{y}_i$ and the valuation equation (29) determine the dividend and wage per person:

$$D_i = \underline{P}_i\,(\underline{r} + \underline{\mathrm{ERP}}_i - \underline{n}_i), \qquad W_i = 1 - D_i. \tag{115}$$

The fixed output shares are Y_H/Y for Home and $Y_F/Y = 1 - Y_H/Y$ for Foreign. These wage and dividend flows per person remain fixed after the demographic change.

Home's net external bond, NFA, and nonstockholder bond-holdings targets determine its initial stockholder balance sheet. The bond target is $\underline{\mathcal{B}}_{N,H} + \underline{\mathcal{B}}_{S,H} = -0.45\underline{y}_H$. Nonstockholder bond holdings of $0.60\underline{y}_H$ therefore imply $\underline{\mathcal{B}}_{S,H} = -1.05\underline{y}_H$. The initial

equity portfolio of Home stockholders satisfies

$$\begin{aligned}\underline{\mathcal{E}}_{HH} + \underline{\mathcal{E}}_{HF} &= \underline{P}_H + \underline{\mathrm{NFA}}_H - \left(\underline{\mathcal{B}}_{N,H} + \underline{\mathcal{B}}_{S,H}\right),\\ \underline{\mathcal{F}}_{S,H} &= \underline{\mathcal{E}}_{HH} + \underline{\mathcal{E}}_{HF} + \underline{\mathcal{B}}_{S,H} = \underline{P}_H + \underline{\mathrm{NFA}}_H - \underline{\mathcal{B}}_{N,H}.\end{aligned} \tag{116}$$

These targets give $\underline{\mathcal{F}}_{S,H}/\underline{y}_H = 0.90$. Initial Home leverage is $\underline{\mathcal{L}}_{S,H} = 1 - (-1.05)/0.90 = 2.17$.

Because both stockholder groups hold the same portfolio shares, Home's share of world equity equals its share of world stockholder total wealth. World stockholder total wealth per world resident is therefore

$$\underline{\mathcal{T}}_{S,W} = \underline{\mathcal{T}}_{S,H}\,\frac{\Upsilon_H \underline{P}_H + \Upsilon_F \underline{P}_F}{\Upsilon\left(\underline{\mathcal{E}}_{HH} + \underline{\mathcal{E}}_{HF}\right)}, \qquad \underline{\mathcal{T}}_{S,H} = \underline{\mathcal{F}}_{S,H} + \underline{\mathcal{H}}_{S,H}. \tag{117}$$

Foreign stockholder total wealth follows from $\underline{\mathcal{T}}_{S,F} = (\Upsilon \underline{\mathcal{T}}_{S,W} - \Upsilon_H \underline{\mathcal{T}}_{S,H})/\Upsilon_F$. Subtracting human wealth gives financial wealth, $\underline{\mathcal{F}}_{S,F} = \underline{\mathcal{T}}_{S,F} - \underline{\mathcal{H}}_{S,F}$. The common portfolio shares follow from Section 4.5, giving equity holdings $\underline{\mathcal{E}}_{ij} = \underline{\mathcal{T}}_{S,i}\underline{\theta}_{S,j}$. We then obtain stockholder bond positions from equation (22). World bond clearing determines Foreign nonstockholder bond holdings:

$$\underline{\mathcal{B}}_{N,F} = -\frac{\Upsilon_H \underline{\mathcal{B}}_{S,H} + \Upsilon_F \underline{\mathcal{B}}_{S,F} + \Upsilon_H \underline{\mathcal{B}}_{N,H}}{\Upsilon_F}. \tag{118}$$

At these targets, both stockholder groups have negative bond positions, positive financial wealth, and positive holdings of both trees.

We identify each issuer slope from the part of its initial premium attributable to the leverage wedge:

$$\begin{aligned}\psi_H &= \left[\underline{\mathrm{ERP}}_H - \gamma_S(\sigma^2_{R,H}\underline{\theta}_{S,H} + \sigma_{R,HF}\underline{\theta}_{S,F})\right]\frac{\underline{\mathcal{F}}_{S,H}}{-\underline{\mathcal{B}}_{S,H}},\\ \psi_F &= \left[\underline{\mathrm{ERP}}_F - \gamma_S(\sigma_{R,HF}\underline{\theta}_{S,H} + \sigma^2_{R,F}\underline{\theta}_{S,F})\right]\frac{\underline{\mathcal{F}}_{S,F}}{-\underline{\mathcal{B}}_{S,F}}.\end{aligned} \tag{119}$$

The targets imply $\psi_H = 0.0306$ and $\psi_F = 0.0308$. Each slope therefore follows from the initial premium, the return moments, and the initial balance sheet.

Finally, the initial stockholder budgets identify the discount rates. The consumption–wealth ratio required for stationary financial wealth is

$$\underline{m}_{S,i} = \frac{(\underline{r} - \underline{n}_i)\underline{\mathcal{F}}_{S,i} + \underline{\mathcal{E}}_{iH}\underline{\mathrm{ERP}}_H + \underline{\mathcal{E}}_{iF}\underline{\mathrm{ERP}}_F + \pi_{S,i}W_i}{\underline{\mathcal{T}}_{S,i}}, \tag{120}$$

and inverting equation (102) gives

$$\rho_{S,i} = \gamma_S(\underline{m}_{S,i} - \underline{\delta}_i) - (\gamma_S - 1)\Big[\underline{r} + \underline{\theta}_{S,H}\underline{\mathrm{ERP}}_H + \underline{\theta}_{S,F}\underline{\mathrm{ERP}}_F - \frac{\gamma_S}{2}\Big(\sigma^2_{R,H}(\underline{\theta}_{S,H})^2 + 2\sigma_{R,HF}\underline{\theta}_{S,H}\underline{\theta}_{S,F} + \sigma^2_{R,F}(\underline{\theta}_{S,F})^2\Big)\Big]. \tag{121}$$

Equation (95) determines the nonstockholder discount coefficients. Table 16 reports these parameters. We hold all calibrated coefficients fixed throughout the demographic experiment.

Table 16: Calibrated parameters in the stationary two-country model

Derived parameter	Home	Foreign
Wage per person	0.89	0.93
Dividend per person	0.11	0.07
Stockholder discount rate (%)	7.62	7.61
Nonstockholder discount coefficient (%)	13.97	14.24
Initial issuer wedge (%)	3.56	2.70
Slope ψ_i	0.0306	0.0308

D.8 Longevity and slower population growth

Our longevity experiment lowers mortality in country i so that expected remaining life increases by ℓ_i years, with $\ell_H = 1.30$ and $\ell_F = 4.77$. We raise the rate of earnings decline by the same amount as mortality falls, so that the additional years of life are spent in retirement:

$$\delta_i^L = \left(\frac{1}{\underline{\delta}_i} + \ell_i\right)^{-1}, \qquad \chi_i^L = \underline{\chi}_i + \underline{\delta}_i - \delta_i^L. \tag{122}$$

The sum $\chi_i + \delta_i$ is unchanged, so the survival-adjusted duration of earnings remains $1/(\underline{\chi}_i + \underline{\delta}_i)$. The retirement-duration gap therefore increases by exactly ℓ_i years. Population growth is unchanged in the longevity-only step, $n_i^L = \underline{n}_i$. In the second step it falls to $\overline{n}_i$, while mortality and earnings decline remain at their longevity-only values: $\overline{\delta}_i = \delta_i^L$ and $\overline{\chi}_i = \chi_i^L$. Both wage bills and both dividend flows per person remain fixed.

Holding the duration of earnings fixed preserves human wealth at a given safe rate, but the equilibrium safe rate can change. Between the initial and final endpoints, this revalues the earnings stream according to

$$\frac{\overline{\mathcal{H}}_{k,i}}{\underline{\mathcal{H}}_{k,i}} = \frac{\underline{r} + \underline{\chi}_i + \underline{\delta}_i}{\overline{r} + \underline{\chi}_i + \underline{\delta}_i}, \qquad k \in \{S, N\}. \tag{123}$$

In our quantitative experiment, the decline in the safe rate raises Foreign human wealth by 4.93 percent after the longevity change and by 10.16 percent over both steps.

Under the age–earnings normalization in Appendix C, entry wages are proportional to $(\chi_i + \delta_i + n_i)/(\delta_i + n_i)$. In Foreign they rise by 5.42 and 26.41 percent after the first and second steps, respectively, while average wages per person remain fixed. The experiment therefore preserves earnings per person and their survival-adjusted duration, while allowing their distribution across cohorts and equilibrium present value to adjust. All reported stocks are per country resident in units of own output, which is unchanged between endpoints.

D.9 Decomposing the change in risk premia

Each premium changes through two channels: the issuer's leverage wedge and the allocation of risk across stockholder portfolios. Differencing the market-pricing equations between any two stationary equilibria gives

$$\begin{aligned} \Delta \mathrm{ERP}_H &= \Delta\tau_H + \gamma_S\Big(\sigma^2_{R,H}\Delta\theta_{S,H} + \sigma_{R,HF}\Delta\theta_{S,F}\Big), \\ \Delta \mathrm{ERP}_F &= \Delta\tau_F + \gamma_S\Big(\sigma_{R,HF}\Delta\theta_{S,H} + \sigma^2_{R,F}\Delta\theta_{S,F}\Big). \end{aligned} \tag{124}$$

The first term captures the leverage channel. The remaining terms capture changes in aggregate risky allocation, scaled by stockholder risk aversion and the relevant return variance or covariance. We hold return variances and covariance fixed, so changes in these moments contribute zero. The decomposition follows from market clearing, and its contributions add exactly to the change in each premium. Table 7 reports the corresponding total premium changes for each step and for the combined change.

D.10 Numerical solution

We solve for five unknowns: the safe rate, both tree prices, and financial wealth in each stockholder sector. Two market-pricing conditions, two stationary stockholder budgets, and world bond clearing determine them. The common portfolio shares impose equity clearing, and nonstockholder bond holdings follow from the stationary saving rule. We vary the demographic parameters gradually to connect the initial solution to the longevity endpoint and then to the endpoint with slower population growth, holding financial parameters and resources per person fixed.

References

Abel, A. B. (2001): "Will Bequests Attenuate the Predicted Meltdown in Stock Prices When Baby Boomers Retire?" *The Review of Economics and Statistics*, 83, 589–595.

——— (2003): "The Effects of a Baby Boom on Stock Prices and Capital Accumulation in the Presence of Social Security," *Econometrica*, 71, 551–578.

Ang, A. and A. Maddaloni (2005): "Do Demographic Changes Affect Risk Premiums? Evidence from International Data," *The Journal of Business*, 78, 341–380.

Attanasio, O. P., J. Banks, and S. Tanner (2002): "Asset Holding and Consumption Volatility," *Journal of Political Economy*, 110, 771–792.

Auclert, A., H. Malmberg, F. Martenet, and M. Rognlie (2026): "Demographics, Wealth, and Global Imbalances in the Twenty-First Century," *Review of Economic Studies*, forthcoming.

Auclert, A., H. Malmberg, M. Rognlie, and L. Straub (2025): "The Race Between Asset Supply and Asset Demand," NBER Working Paper 34470, National Bureau of Economic Research.

Bárány, Z. L., N. Coeurdacier, and S. Guibaud (2023): "Capital Flows in an Aging World," *Journal of International Economics*, 140, 103707.

Basu, S. (2019): "Are Price-Cost Markups Rising in the United States? A Discussion of the Evidence," *Journal of Economic Perspectives*, 33, 3–22.

Beaudry, P., P. Cavallino, and T. Willems (2025): "Monetary Policy Along the Yield Curve: Why Can Central Banks Affect Long-Term Real Rates?" NBER Working Paper 32511, National Bureau of Economic Research.

Bernanke, B. S. (2005): "The Global Saving Glut and the U.S. Current Account Deficit," Speech 03102, Federal Reserve Board.

Blanchard, O. J. (1985): "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 93, 223–247.

——— (2023): *Fiscal Policy under Low Interest Rates*, Cambridge, MA: The MIT Press.

Bodie, Z., R. C. Merton, and W. F. Samuelson (1992): "Labor Supply Flexibility and Portfolio Choice in a Life Cycle Model," *Journal of Economic Dynamics and Control*, 16, 427–449.

Brav, A., G. M. Constantinides, and C. C. Geczy (2002): "Asset Pricing with Heterogeneous Consumers and Limited Participation: Empirical Evidence," *Journal of Political Economy*, 110, 793–824.

Brooks, R. (2002): "Asset-Market Effects of the Baby Boom and Social-Security Reform," *American Economic Review*, 92, 402–406.

Buiter, W. H. (1988): "Death, Birth, Productivity Growth and Debt Neutrality," *The Economic Journal*, 98, 279–293.

Caballero, R. J., E. Farhi, and P.-O. Gourinchas (2008): "An Equilibrium Model of "Global Imbalances" and Low Interest Rates," *American Economic Review*, 98, 358–393.

——— (2017a): "Rents, Technical Change, and Risk Premia: Accounting for Secular Trends in Interest Rates, Returns on Capital, Earnings Yields, and Factor Shares," *American Economic Review*, 107, 614–620.

——— (2017b): "The Safe Assets Shortage Conundrum," *Journal of Economic Perspectives*, 31, 29–46.

Campbell, J. Y., J. F. Cocco, F. J. Gomes, and P. J. Maenhout (2001): "Investing Retirement Wealth: A Life-Cycle Model," in *Risk Aspects of Investment-Based Social Security Reform*, ed. by J. Y. Campbell and M. Feldstein, Chicago: University of Chicago Press, 439–482.

Carvalho, C., A. Ferrero, F. Mazin, and F. Nechio (2025): "Demographics and Real Interest Rates Across Countries and Over Time," *FRBSF Working Paper Series*.

Carvalho, C., A. Ferrero, and F. Nechio (2016): "Demographics and Real Interest Rates: Inspecting the Mechanism," *European Economic Review*, 88, 208–226.

Chien, Y. and K. Naknoi (2015): "The Risk Premium and Long-Run Global Imbalances," *Journal of Monetary Economics*, 76, 299–315.

Cocco, J. F., F. J. Gomes, and P. J. Maenhout (2005): "Consumption and Portfolio Choice over the Life Cycle," *The Review of Financial Studies*, 18, 491–533.

Corsetti, G., S. Lloyd, E. Marin, and D. Ostry (2025): "U.S. Risk and Treasury Convenience," CEPR Discussion Paper 20657, Centre for Economic Policy Research.

Davis, S. J. and P. Willen (2013): "Occupation-Level Income Shocks and Asset Returns: Their Covariance and Implications for Portfolio Choice," *Quarterly Journal of Finance*, 3, 1350011.

De Loecker, J., J. Eeckhout, and G. Unger (2020): "The Rise of Market Power and the Macroeconomic Implications," *Quarterly Journal of Economics*, 135, 561–644.

Domeij, D. and M. Flodén (2006): "Population Aging and International Capital Flows," *International Economic Review*, 47, 1013–1032.

Duffie, D. (2010): "Presidential Address: Asset Price Dynamics with Slow-Moving Capital," *The Journal of Finance*, 65, 1237–1267.

Eggertsson, G. B., N. R. Mehrotra, and J. A. Robbins (2019): "A Model of Secular Stagnation: Theory and Quantitative Evaluation," *American Economic Journal: Macroeconomics*, 11, 1–48.

Eggertsson, G. B., J. A. Robbins, and E. G. Wold (2021): "Kaldor and Piketty's Facts: The Rise of Monopoly Power in the United States," *Journal of Monetary Economics*, 124,

S19–S38.

Farhi, E. and F. Gourio (2018): "Accounting for Macro-Finance Trends: Market Power, Intangibles, and Risk Premia," *Brookings Papers on Economic Activity*, 49, 147–250.

Ferrero, A. (2010): "A Structural Decomposition of the U.S. Trade Balance: Productivity, Demographics and Fiscal Policy," *Journal of Monetary Economics*, 57, 478–490.

Gagnon, E., B. K. Johannsen, and D. López-Salido (2021): "Understanding the New Normal: The Role of Demographics," *IMF Economic Review*, 69, 357–390.

Gârleanu, N. and L. H. Pedersen (2011): "Margin-Based Asset Pricing and Deviations from the Law of One Price," *The Review of Financial Studies*, 24, 1980–2022.

Geanakoplos, J., M. Magill, and M. Quinzii (2004): "Demography and the Long-Run Predictability of the Stock Market," *Brookings Papers on Economic Activity*, 2004, 241–326.

Gomes, F. (2020): "Portfolio Choice Over the Life Cycle: A Survey," *Annual Review of Financial Economics*, 12, 277–304.

Gomme, P., B. Ravikumar, and P. Rupert (2015): "Secular Stagnation and Returns on Capital," *Federal Reserve Bank of St. Louis Economic Synopses*, 1–3.

Gourinchas, P.-O. and H. Rey (2007): "From World Banker to World Venture Capitalist: U.S. External Adjustment and the Exorbitant Privilege," in *G7 Current Account Imbalances: Sustainability and Adjustment*, ed. by R. H. Clarida, Chicago: University of Chicago Press, 11–66.

Guiso, L. and L. Pistaferri (2020): "The Insurance Role of the Firm," *The Geneva Risk and Insurance Review*, 45, 1–23.

Haddad, V. and T. Muir (2026): "Intermediaries and Asset Prices," *Journal of Economic Literature*, 64, 876–906.

Hassan, T. A. (2013): "Country Size, Currency Unions, and International Asset Returns," *The Journal of Finance*, 68, 2269–2308.

He, Z., A. Krishnamurthy, and K. Milbradt (2019): "A Model of Safe Asset Determination," *American Economic Review*, 109, 1230–1262.

Hyland, C. C. (2026): "Taming Black Swans: Monetary Policy in the Presence of Tail Risks," Working Paper, University of Oxford.

Jordà, Ò., K. Knoll, D. Kuvshinov, M. Schularick, and A. M. Taylor (2019): "The Rate of Return on Everything, 1870–2015," *The Quarterly Journal of Economics*, 134, 1225–1298.

Jordà, Ò., M. Schularick, and A. M. Taylor (2017): "Macrofinancial History and the New Business Cycle Facts," *NBER Macroeconomics Annual*, 31, 213–263.

Kopecky, J. and A. M. Taylor (2022): "The Murder-Suicide of the Rentier: Population Aging and the Risk Premium," NBER Working Paper 26943, National Bureau of Economic Research.

Krueger, D. and A. Ludwig (2007): "On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare," *Journal of Monetary Economics*, 54, 49–87.

Lisack, N., R. Sajedi, and G. Thwaites (2021): "Population Aging and the Macroeconomy," *International Journal of Central Banking*, 17, 43–80.

Lustig, H., S. Van Nieuwerburgh, and A. Verdelhan (2013): "The Wealth-Consumption Ratio," *Review of Asset Pricing Studies*, 3, 38–94.

Maggiori, M. (2017): "Financial Intermediation, International Risk Sharing, and Reserve Currencies," *American Economic Review*, 107, 3038–3071.

Mankiw, N. G. and D. N. Weil (1989): "The Baby Boom, the Baby Bust, and the Housing Market," *Regional Science and Urban Economics*, 19, 235–258.

Mankiw, N. G. and S. P. Zeldes (1991): "The Consumption of Stockholders and Nonstockholders," *Journal of Financial Economics*, 29, 97–112.

Marx, M., B. Mojon, and F. R. Velde (2021): "Why have interest rates fallen far below the return on capital?" *Journal of Monetary Economics*, 124, S57–S76.

Mendoza, E. G., V. Quadrini, and J.-V. Ríos-Rull (2009): "Financial Integration, Financial Development, and Global Imbalances," *Journal of Political Economy*, 117, 371–416.

Merton, R. C. (1971): "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," *Journal of Economic Theory*, 3, 373–413.

Ordóñez, G. and F. Piguillem (2019): "Macro and Financial Implications of Aging," NBER Working Paper 26337, National Bureau of Economic Research.

Papanikolaou, D. (2018): "Comment on "Accounting for Macro-Finance Trends: Market Power, Intangibles, and Risk Premia"," *Brookings Papers on Economic Activity*, 236–245.

Poterba, J. M. (2001): "Demographic Structure and Asset Returns," *The Review of Economics and Statistics*, 83, 565–584.

Rachel, L. (2025): "What Next for r*? A Capital Market Equilibrium Perspective on the Natural Rate of Interest," *Brookings Papers on Economic Activity*, 56, 131–213.

Rachel, L. and T. D. Smith (2017): "Are Low Real Interest Rates Here to Stay?" *International Journal of Central Banking*, 13, 1–42.

Rachel, L. and L. H. Summers (2019): "On Secular Stagnation in the Industrialized World," *Brookings Papers on Economic Activity*, 50, 1–76.

Raval, D. (2023): "Testing the Production Approach to Markup Estimation," *Review of Economic Studies*, 90, 2592–2611.

Reis, R. (2022): "Has Monetary Policy Cared Too Much about a Poor Measure of r*?" *Brookings Papers on Economic Activity*, 1–55.

Van Nieuwerburgh, S. and L. Veldkamp (2009): "Information Immobility and the Home

Bias Puzzle," *The Journal of Finance*, 64, 1187–1215.

Viceira, L. M. (2001): "Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income," *Journal of Finance*, 56, 433–470.

Yaari, M. E. (1965): "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer," *The Review of Economic Studies*, 32, 137–150.