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Optimal Monetary Policy in the Tariff-Imposing Economy under Imperfect Pass-Through*

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Abstract

Should the central bank of a tariff-imposing economy raise or cut its nominal interest rate? In a small open economy, Ramsey policy raises the rate under perfect pass-through (PPT) but cuts it under imperfect pass-through (IPT). The rate cut under IPT also obtains in a two-country economy and remains optimal when Home and Foreign goods become more substitutable. The mechanism is the Home Import-Allocation Channel (HIAC): a tariff induces inefficient underconsumption of imports and raises the welfare-relevant law-of-one-price (LOOP) gap. Through uncovered interest parity, monetary easing exerts depreciation pressure, which tends to raise the actual LOOP gap. Under IPT, import-price stickiness slows the adjustment of ex-tariff import prices to the higher Home-currency border price, allowing the Ramsey planner to exploit price rigidity to mitigate tariff-induced underconsumption, subject to competing inflation-stabilization objectives. Under reciprocal tariffs, optimal monetary easing can bring nominal interest rates to the zero lower bound.

Keywords: Import tariffs; optimal monetary policy; pricing behavior; local-currency pricing; zero lower bound.

JEL Classification: E31, E52, F13, F41, F42.

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1 Introduction

Should a tariff-imposing country tighten or ease monetary policy? An import tariff raises the *cum-tariff* relative price of imports, inducing expenditure switching toward Home goods and upward pressure on Home PPI inflation, so the conventional prescription is to tighten. Optimal policy softens this prescription but does not overturn it. Monacelli [39] shows that CPI inflation targeting has a tightening bias relative to Ramsey policy, yet with perfect pass-through the Ramsey planner still raises the nominal rate, if only marginally. Under cooperative optimal policy, Bergin and Corsetti [9] find a rate increase even when exporters set prices in the destination-market currency. Bianchi and Coulibaly [12] characterize optimal policy as expansionary relative to strict producer-price stabilization, a benchmark that does not determine the sign of the nominal-rate response.

We show that the prescription reverses when imports are sold by local importers that set sticky prices in domestic currency. Under such Imperfect Pass-Through (IPT), the Ramsey planner of the tariff-imposing economy *cuts* the nominal interest rate on impact, whereas in the same small open economy (SOE) under Perfect Pass-Through (PPT) the same planner raises it. The cut survives a substantially higher trade elasticity and extends to a two-country economy in which Foreign monetary policy responds endogenously.

The model introduces monopolistically competitive local importers into the SOE framework and calibration of Monacelli [39], following the local-currency-pricing (LCP) structure of Monacelli [38]. Home producers use producer-currency pricing (PCP). Local importers purchase goods at the border and set sticky *ex-tariff* selling prices in Home currency, internalizing the effect of the tariff on their demand. The statutory duty is added to the sticky *ex-tariff* price, so that tariff pass-through and exchange-rate pass-through are separated. The PPT environment removes the importer block and leaves the rest of the model unchanged, so the comparison isolates the role of importer pricing.

The welfare mechanism provides an incentive for monetary easing under IPT. An increase in the Home import tariff raises the *cum-tariff* relative price of imports, inducing inefficient underconsumption of imported goods relative to Home goods and reducing economic welfare. The Home law-of-one-price (LOOP) gap is defined as the ratio of the Home-currency border price of imports to the *ex-tariff* selling price charged by local importers in Home currency. Therefore, for a given Home-currency border price of imports, an increase in the LOOP gap implies a lower *ex-tariff* selling price charged by local importers, helping offset the tariff-induced increase in the *cum-tariff* relative purchaser price of imports and mitigating the underconsumption of imported goods. The Home Import-Allocation Channel (HIAC) captures the resulting increase in the welfare-relevant (WR) Home LOOP gap and the incentive to raise the actual LOOP gap to mitigate this distortion. For a given future nominal exchange rate and Foreign nominal interest rate, uncovered interest parity (UIP) implies that a reduction in the Home nominal interest rate exerts depreciation pressure on the Home currency. This depreciation pressure tends to raise the actual LOOP gap. Nominal depreciation also raises the Home-currency border price of imports, giving importers an incentive to increase their *ex-tariff* selling prices. Under IPT, however, import-price stickiness makes this adjustment sluggish, allowing nominal depreciation to raise the actual LOOP gap despite the importers' pricing response. The HIAC thus provides the Ramsey planner with an incentive to cut the nominal interest rate in response to an increase in the import tariff, subject to the competing inflation-stabilization objectives.

In the SOE, the distinction from PPT is especially transparent. Under PPT, the tariff changes the relative purchaser price of imports immediately, inducing expenditure switching toward Home goods and upward pressure on Home PPI inflation. The planner raises the policy rate. Under IPT, sticky local import prices sharply attenuate the additional expenditure switching induced by exchange-rate movements. The HIAC gives the Ramsey planner an incentive to raise the actual LOOP gap toward its WR level to mitigate the tariff-induced underconsumption of imports, favoring a rate cut. This contrast in the sign of the optimal rate response persists in the SOE High Substitution Case.

The two-country economy adds a Foreign Allocation Channel (FAC). An increase in the Foreign

import tariff or a decline in the Foreign LOOP gap raises the *cum-tariff* relative price of Home goods faced by Foreign households, reducing their consumption of Home goods relative to Foreign goods. For a given Home output, this shifts the consumption of Home goods toward Home households, resulting in inefficient overconsumption of Home goods in Home. The Home WR LOOP gap therefore rises through the FAC to mitigate this distortion.

At the baseline elasticity, a unilateral Home tariff lowers the Home rate on impact under IPT but raises it under PPT, even though the Home currency appreciates in equilibrium. Under higher substitutability, Foreign easing and the associated Home-currency appreciation lead the Home planner to cut its rate even under PPT, but the IPT cut is larger. The sign reversal relative to PPT is therefore a baseline result, whereas the IPT cut holds in every unilateral experiment we consider.

Under reciprocal tariffs, both countries face a loss of export demand and optimally ease in either pass-through environment. Exchange-rate changes induce relatively strong additional expenditure switching under PPT, whereas sticky local-currency import prices attenuate this mechanism under IPT. Accordingly, the size of the optimal interest-rate cuts can differ substantially across environments, and sufficiently large shocks can push nominal interest rates to the zero lower bound (ZLB). These experiments extend the analysis from the direction of optimal unilateral policy under IPT to the international transmission and constraints on monetary accommodation during a tariff war.

The rest of the paper is organized as follows. Section 2 relates our analysis to the literature on tariffs, monetary policy, and exchange-rate pass-through. Section 3 presents the SOE model, the welfare cost function, and the optimal IPT response to a unilateral tariff, using PPT as a comparison. Section 4 extends the analysis to a two-country economy with national Nash–Ramsey policies. Subsection 4.6 studies reciprocal tariffs and ZLB episodes. Section 5 concludes. The Appendix provides the nonlinear equilibrium, Ramsey-policy, and welfare derivations, together with the analytical benchmarks and sensitivity analysis of producer- and importer-price rigidity.

2 Related Literature

2.1 Tariffs and Monetary Policy

The literature on tariffs and optimal monetary policy can be divided into two groups according to which country's monetary-policy response is studied.[1] The first group examines the economy on which a foreign country imposes tariffs. A foreign tariff reduces demand for that economy's exports, presenting its monetary authority with an adverse external-demand shock. Hamano, Pappadà, and Punzi [35], for example, study optimal monetary policy in the tariff-targeted economy, emphasizing exporter dynamics and labor reallocation. Monacelli [39] and Bergin and Corsetti [9] also analyze this side of a tariff shock and find that the optimal response of the tariff-targeted economy is expansionary.[2]

The second group, to which the present paper belongs, studies optimal monetary policy in the *tariff-imposing* economy. The literature focusing on this side of a tariff shock is comparatively limited. Important contributions include Monacelli [39], Bergin and Corsetti [9], Bianchi and Coulibaly [12], and Werning, Lorenzoni, and Guerrieri [42]. These studies either do not determine the sign of the nominal interest-rate response or find that the tariff-imposing country raises its nominal interest rate. In contrast, the Ramsey planner of our tariff-imposing economy cuts its

[1] Several related studies belong to neither group because they do not characterize the optimal monetary-policy response to a given tariff. Auray, Devereux, and Eyquem [4] show how monetary regimes alter equilibrium tariffs and the welfare costs of trade wars, and Auray, Devereux, and Eyquem [5] examine how monetary delegation affects noncooperative tariff choices. Auray, Devereux, and Eyquem [6] compare the real and welfare consequences of tariff shocks under producer-, local-, and dollar-currency invoicing. Hwang and Turnovsky [36] analyze tariff effects under imperfect exchange-rate pass-through, and Amiti, Redding, and Weinstein [2] document the price and welfare effects of the 2018 U.S. tariffs.

[2] Acharya, Challe, and Corsetti [1] study optimal monetary policy after export shocks, including those caused by foreign tariffs, in a small open economy with heterogeneous households. They show that the optimal policy requires more exchange-rate flexibility than flexible inflation targeting provides.

nominal interest rate in response to a unilateral tariff under IPT.

Monacelli [39] shows that the effects of import tariffs in a small-open-economy New Keynesian framework depend on the monetary-policy regime, that CPI inflation targeting has a tightening bias relative to Ramsey policy, and that with perfect pass-through the Ramsey planner raises the nominal rate only marginally. His environment corresponds closely to our perfect pass-through (PPT) benchmark, against which we compare IPT.

Bianchi and Coulibaly [12] study an economy with a production subsidy correcting the steady-state monopoly distortion and emphasize that strict stabilization of domestic Producer Price Index (PPI) inflation is not optimal following an import-tariff increase. Their characterization of the optimal response as expansionary is relative to a strict PPI-inflation-stabilization policy. Our results are consistent with this characterization and sharpen it: under IPT the optimal nominal interest rate itself falls, whereas under PPT in the same SOE the tariff-imposing country raises its nominal interest rate while allowing positive Home PPI inflation.

Werning, Lorenzoni, and Guerrieri [42] show that a tariff on imported intermediate inputs maps exactly into a cost-push shock in the standard closed-economy New Keynesian model, and they characterize the optimal response analytically: rather than keeping inflation at zero, which would require a sharp short-run contraction in output, optimal policy partially accommodates the shock. Bianchi and Coulibaly [12] and Werning, Lorenzoni, and Guerrieri thus share the policy implication that strict inflation stabilization is not optimal following a tariff increase, and our results support the same implication. We complement these studies by characterizing the sign of the nominal interest-rate response itself, which is not the object of their analyses.

Bergin and Corsetti [9] study unilateral and reciprocal tariffs in a model with a richer international trade structure, including global value chains and firm dynamics, and investigate alternative pricing behaviors. In their unilateral-tariff analysis under cooperative optimal policy, the tariff-imposing country raises its nominal interest rate even under local-currency pricing (LCP). Their LCP also generates imperfect exchange-rate pass-through into import prices. Even so, their result contrasts with the optimal cut in our IPT environment.[3] In the case of reciprocal tariffs, by contrast, both Bergin and Corsetti, under cooperation, and our paper, under national Ramsey policies, find that optimal monetary policy involves interest-rate cuts. Our analysis additionally compares how the degree of pass-through affects the magnitude of those cuts and the incidence of the zero lower bound.

In subsequent work, Bergin and Corsetti [10] study the role of exchange-rate adjustment in stabilizing unilateral tariff shocks under cooperative Ramsey policy, together with the role of trade elasticities and price flexibility. Because the cooperative planner chooses both countries' policies, their analysis spans both groups: the optimal response is driven mainly by monetary expansion in the tariff-targeted economy, while the response of the tariff-imposing economy is modest. When the tariff-imposing country's currency serves as the dominant invoicing currency, the optimal monetary stance of that country is more expansionary than strict PPI targeting; this relative accommodation need not imply a decline in the nominal interest rate from its steady-state level. Their mechanism operates through the incentive to stabilize demand faced by foreign exporters that preset prices in the tariff-imposing country's currency. By contrast, our model features sticky prices set by local importing firms, and the Home Import-Allocation Channel gives the Home Ramsey planner an incentive to cut the nominal interest rate on impact.

Relative to these contributions, our deliberately parsimonious model isolates the effects of importer price-setting behavior, with local importers setting sticky *ex-tariff* selling prices, by comparing PPT and IPT under otherwise identical tariff shocks and national Ramsey policy problems. Adding sticky local-currency importer prices to Monacelli's framework reverses the sign of the optimal policy-rate response.

[3] The difference between them and us lies in who sets the sticky local-currency price. Under their LCP, exporting firms set prices in the destination-market currency; under our IPT, importing firms purchase foreign goods and set sticky local-currency selling prices, internalizing the tariff in their price-setting problem. In a simplified analytical version of their model, the Nash and cooperative solutions coincide, and the tariff-imposing country's policy does not respond to its own import tariff. Their quantitative analysis is conducted under cooperation.

2.2 Pricing Behavior, Pass-Through, and Monetary Stabilization

The mechanism is also related to the literature on pricing behavior and exchange-rate pass-through. Betts and Devereux [11] and Devereux and Engel [23] show that local-currency price stickiness weakens the short-run pass-through of exchange-rate movements. Devereux and Engel [24] show that the optimal use of exchange-rate flexibility depends on whether prices are set in producer or local currency, while Corsetti, Dedola, and Leduc [19] emphasize that optimal monetary policy depends on the degree of exchange-rate pass-through into import prices. These results motivate our comparison of PPT and IPT environments. Okano and Eguchi [40] also employ LCP in a small-open-economy policy analysis, although their application concerns fiscal stimulus rather than tariffs.

Our analysis is related to the literature on the role of international relative prices and deviations from the law of one price in optimal monetary policy. De Paoli [22] shows that a national planner in a small open economy has an incentive to use international relative prices rather than mechanically stabilize domestic inflation. Engel [25] shows that under LCP, currency misalignments associated with deviations from the LOOP are inefficient and enter optimal monetary policy. Fujiwara and Wang [31] extend this logic to a noncooperative policy game under LCP. Our paper examines how the LOOP gap can contribute to mitigating the distortion in the relative consumption of imports caused by an import tariff. Under IPT, the HIAC provides the Ramsey planner with an incentive to move the actual LOOP gap toward its welfare-relevant level to mitigate tariff-induced underconsumption of imports.

The dominant-currency-pricing framework of Gopinath et al. [33] offers a complementary explanation of limited exchange-rate pass-through, whereas our model places local importers at the center of the pricing decision. The treatment of the statutory tariff relative to nominal import-price rigidity is also relevant. Chen, Devereux, Shi, and Xu [18] impose the import tax on top of a pre-set price under LCP, whereas Farhi, Gopinath, and Itskhoki [28] place border taxes within an LCP price-setting problem. Our benchmark follows the former convention: the statutory tariff is separated from the sticky *ex-tariff* import price. This distinction also separates tariff pass-through from exchange-rate pass-through. Cavallo, Gopinath, Neiman, and Tang [17] document high tariff pass-through at the border together with more limited retail-price adjustment, while Gopinath, Itskhoki, and Rigobon [34] document incomplete exchange-rate pass-through into U.S. import prices for dollar-priced imports. The retail evidence in Cavallo et al. also suggests compression of retail margins. Flaaen, Hortaçsu, and Tintelnot [29] find substantial consumer-price effects of washing-machine tariffs, including price changes for complementary and domestically produced goods. These studies motivate our distinction between border prices and local selling prices.

2.3 International Policy Interaction and the ZLB

The two-country analysis is related to the literature on international monetary-policy games. Corsetti and Pesenti [20] study the international dimensions of optimal monetary policy, while Bodenstein, Guerrieri, and LaBriola [15] analyze macroeconomic policy games. Faia and Monacelli [27] formulate welfare-maximizing monetary policy in a two-country New Keynesian economy and compare cooperation with Nash competition between national Ramsey planners. We use their national-Ramsey policy-game logic to examine whether the HIAC, identified in the SOE, continues to influence optimal Home monetary policy when Foreign demand and Foreign monetary policy are jointly endogenous. The two-country extension adds the FAC, through which movements in the Foreign LOOP gap and Foreign tariffs alter the international allocation of Home goods and therefore the Home WR LOOP gap.

Finally, the paper is related to the ZLB literature on international monetary transmission, including Fujiwara et al. [30], Bodenstein, Erceg, and Guerrieri [14], and Billi and Galí [13]. The role of the ZLB here is different. We do not add a demand or natural-rate shock to create a liquidity trap. Instead, reciprocal tariffs can make the welfare-maximizing response sufficiently accommodative to exhaust conventional monetary-policy space under either pricing environment.

Import-price rigidity changes the welfare trade-offs and the size of the desired accommodation.

3 Small Open Economy

The small-open-economy model consists of a representative Home household, monopolistically competitive Home producers, and a monetary authority. The household consumes a constant-elasticity-of-substitution (CES) composite of Home and Foreign goods and trades state-contingent claims with the rest of the world. Home producers use PCP and Calvo pricing in both pass-through environments. The government levies an import tariff and rebates tariff revenue lump sum. No employment subsidy is paid to PCP producers. The difference between the two environments is on the import side. Under PPT there is no independent domestic importer-price block and the LOOP holds for the *ex-tariff* import price. Accordingly, the Home welfare cost function contains no WR LOOP-gap deviation term. Under IPT, local importers are added. They use LCP and set sticky *ex-tariff* import prices in Home currency.

3.1 The Model

3.1.1 Perfect Pass-Through Environment

Households and Demand

The Home representative household has lifetime utility

$$\mathcal{U} \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right). \tag{1}$$

Here C_t denotes the Home household's composite consumption and $N_t \equiv \int_0^1 N_t(j)dj$ denotes hours of labor. The parameter $\beta \in (0,1)$ is the discount factor and φ is the inverse Frisch elasticity.

Composite consumption is

$$C_t = \left[(1-\upsilon)^{1/\eta} C_{H,t}^{(\eta-1)/\eta} + \upsilon^{1/\eta} C_{F,t}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}.$$

Here $C_{H,t}$ and $C_{F,t}$ denote consumption of Home and Foreign goods, respectively, $\upsilon \in (0,1)$ is the expenditure weight on Foreign goods, and η is the elasticity of substitution between Home and Foreign goods.

Under PPT, the *ex-tariff* Home-currency import price satisfies

$$P_{F,t} = \mathcal{E}_t P_{F,t}^*. \tag{2}$$

Here $P_{F,t}$ denotes the *ex-tariff* Home-currency import price, $P_{F,t}^*$ denotes the Foreign producer-price index in Foreign currency, and $\mathcal{E}_t$ is the price of Foreign currency in units of Home currency. The Home CPI under the PPT environment is

$$P_t^{PPT} \equiv \left[(1-\upsilon) P_{H,t}^{1-\eta} + \upsilon \left(\widetilde{P}_{F,t}^{PPT} \right)^{1-\eta} \right]^{1/(1-\eta)}. \tag{3}$$

Here P_t^{PPT} denotes the Home Consumer Price Index (CPI), $P_{H,t}$ denotes the Home producer-price index, $\tau_{M,t}$ denotes the Home import-tariff rate, and $\widetilde{P}_{F,t}^{PPT} \equiv (1+\tau_{M,t})\mathcal{E}_t P_{F,t}^*$ denotes the *cum-tariff* import price of Foreign goods under the PPT environment. The CES demand system gives

$$C_{H,t} = (1-\upsilon) \left[\mathcal{G}^{PPT}(\tau_{M,t}, S_t) \right]^{\eta} C_t, \tag{4}$$

where the CPI–PPI ratio is

$$\mathcal{G}^{PPT}\left(\tau_{M,t}, S_t\right) \equiv \frac{P_t^{PPT}}{P_{H,t}}. \tag{5}$$

Here

$$\begin{aligned} S_t &\equiv \frac{P_{F,t}}{P_{H,t}} \\ &= \frac{Z_t}{\Psi_t}, \end{aligned} \tag{6}$$

denotes the *ex-tariff* relative price of Foreign goods relative to the Home PPI (*ex-tariff* relative price of imports, for short), $Z_t \equiv \mathcal{E}_t P^*_{F,t}/P_{H,t}$ denotes the *ex-tariff* relative border price of Foreign goods relative to the Home PPI (*ex-tariff* relative border price of imports). Following Monacelli [38], we refer to $\Psi_t \equiv \mathcal{E}_t P^*_{F,t}/P_{F,t}$ as the LOOP gap.

Under PPT, Eq. (2) implies $\Psi_t = 1$ and hence $S_t = Z_t$. Thus the *ex-tariff* relative price of imports coincides with the *ex-tariff* relative border price of imports.

Import demand is

$$C_{F,t} = \upsilon \left[\frac{(1+\tau_{M,t})S_t}{\mathcal{G}^{PPT}\left(\tau_{M,t}, S_t\right)}\right]^{-\eta} C_t. \tag{7}$$

Combining Eq. (4) with Eq. (7) yields

$$\frac{C_{F,t}}{C_{H,t}} = \frac{\upsilon}{1-\upsilon}\left[(1+\tau_{M,t})S_t\right]^{-\eta}. \tag{8}$$

which suggests that the relative consumption of imports depends on the *ex-tariff* relative price of imports. In addition, once the import tariff is hiked, the relative consumption of imports decreases.

The household's nominal budget constraint is

$$P_{H,t}C_{H,t} + (1+\tau_{M,t})\mathcal{E}_t P^*_{F,t}C_{F,t} + \sum_{h^{t+1}} \xi_{t,t+1}(h^{t+1}|h^t)B_{t+1}(h^{t+1}) = B_t(h^t) + W_t N_t + \mathcal{D}_t + T_t.$$

where $B_{t+1}(h^{t+1})$ is a state-contingent nominal claim purchased at date t, $\xi_{t,t+1}(h^{t+1}|h^t)$ is its date-t state price, W_t is the nominal wage, $\mathcal{D}_t$ denotes nominal dividends from firms, and T_t is the net lump-sum transfer from the government. The import-expenditure term contains both the border value $\mathcal{E}_t P^*_{F,t}C_{F,t}$ and the statutory customs payment $\tau_{M,t}\mathcal{E}_t P^*_{F,t}C_{F,t}$. Hence tariff revenue is

$$TR_t = \tau_{M,t}\mathcal{E}_t P^*_{F,t}C_{F,t}, \qquad T_t = TR_t$$

in the PPT environment, where there is no local-importer subsidy. Thus the statutory duty paid as part of household import expenditure is returned to the household as a lump-sum transfer. By CES duality,

$$P_{H,t}C_{H,t} + (1+\tau_{M,t})\mathcal{E}_t P^*_{F,t}C_{F,t} = P_t^{PPT}C_t,$$

so the same budget constraint can equivalently be written as

$$P_t^{PPT}C_t + \sum_{h^{t+1}} \xi_{t,t+1}(h^{t+1}|h^t)B_{t+1}(h^{t+1}) = B_t(h^t) + W_t N_t + \mathcal{D}_t + T_t. \tag{9}$$

Maximization of Eq. (1) subject to the budget constraint gives the intratemporal condition

$$\frac{W_t}{P_t^{PPT}} = C_t N_t^{\varphi}, \tag{10}$$

and the Euler equation

$$\frac{1}{1+i_t} = \beta \mathrm{E}_t \left[\frac{C_t(\mathcal{G}^{PPT}(\tau_{M,t}, S_t))}{C_{t+1}(\mathcal{G}^{PPT}(\tau_{M,t+1}, S_{t+1}))\Pi_{H,t+1}} \right]. \tag{11}$$

Here i_t denotes the Home nominal interest rate and $\Pi_{H,t} \equiv P_{H,t}/P_{H,t-1}$ denotes gross Home PPI inflation. The Euler equation follows from $\xi_{t,t+1} = \beta(C_t/C_{t+1})(\Pi_{t+1}^{PPT})^{-1}$ and $(1+i_t)^{-1} = \mathrm{E}_t \xi_{t,t+1}$, where $\Pi_t^{PPT} \equiv P_t^{PPT}/P_{t-1}^{PPT}$ denotes gross CPI inflation. The Foreign household satisfies the analogous Euler equation. With $\xi_{t,t+1}$ denoting the Home-currency nominal state price, the Foreign-currency bond satisfies

$$\frac{1}{1+i_t^*} = \mathrm{E}_t \left[\xi_{t,t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right],$$

where i_t^* denotes the Foreign nominal interest rate. This is the stochastic-discount-factor representation of uncovered interest parity (UIP).

Complete international risk sharing implies

$$S_t = C_t \mathcal{G}^{PPT}(\tau_{M,t}, S_t). \tag{12}$$

Production and Pricing Behavior

Each Home firm $j \in [0,1]$ produces a differentiated good according to $Y_t(j) = A_t N_t(j)$, where A_t denotes labor productivity. Let Y_t denote aggregate Home output. Demand for variety j is $Y_t(j) = [P_{H,t}(j)/P_{H,t}]^{-\varepsilon} Y_t$, where $\varepsilon > 1$ is the elasticity of substitution across Home producer varieties. Aggregation gives

$$Y_t = \frac{A_t N_t}{\Delta_{H,t}}, \tag{13}$$

where $\Delta_{H,t} \equiv \int_0^1 [P_{H,t}(j)/P_{H,t}]^{-\varepsilon} dj$ denotes producer-price dispersion. We set $A_t = 1$ throughout the paper.

Home producers face Calvo price rigidity. Following the sequence representation in Monacelli [39], the producer's reset-price condition is

$$\widetilde{p}_{H,t} = \mathcal{M} \frac{\mathcal{Z}_{H,t}}{\mathcal{K}_{H,t}}, \tag{14}$$

where $\widetilde{p}_{H,t} \equiv \overline{P}_{H,t}/P_{H,t}$ is the optimal reset price relative to the current Home PPI, θ is the producer Calvo non-adjustment probability, $MC_t^r \equiv MC_t/P_{H,t}$ is Home real marginal cost, and $\mathcal{M} \equiv \varepsilon/(\varepsilon-1)$ is the desired producer markup. The auxiliary objects are

$$\begin{aligned} \mathcal{K}_{H,t} &\equiv \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k} \left(\prod_{s=1}^{k} \Pi_{H,t+s} \right)^{\varepsilon}, \\ \mathcal{Z}_{H,t} &\equiv \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k} MC_{t+k}^r \left(\prod_{s=1}^{k} \Pi_{H,t+s} \right)^{1+\varepsilon}. \end{aligned} \tag{15}$$

The household nominal stochastic discount factor from t to $t+k$, used in both the producer and importer Calvo pricing problems, is

$$Q_{t,t+k} \equiv \beta^k \frac{C_t(\mathcal{G}^{PPT}(\tau_{M,t}, S_t))}{C_{t+k}(\mathcal{G}^{PPT}(\tau_{M,t+k}, S_{t+k}))} \left(\prod_{s=1}^{k} \Pi_{H,t+s} \right)^{-1}.$$

Using Eq. (10), real marginal cost is

$$MC_t^r = C_t N_t^{\varphi} \mathcal{G}^{PPT}(\tau_{M,t}, S_t). \tag{16}$$

The producer Calvo price-index and dispersion laws are

$$\begin{aligned} 1 &= \theta \Pi_{H,t}^{\varepsilon-1} + (1-\theta)\widetilde{p}_{H,t}^{1-\varepsilon}, \\ \Delta_{H,t} &= \theta \Pi_{H,t}^{\varepsilon} \Delta_{H,t-1} + (1-\theta)\widetilde{p}_{H,t}^{-\varepsilon}. \end{aligned} \tag{17}$$

At first order, Home PPI inflation therefore satisfies

$$\pi_{H,t} = \beta \mathrm{E}_t \pi_{H,t+1} + \kappa_H m c_t^r, \tag{18}$$

where $\pi_{H,t} \equiv \log \Pi_{H,t}$, $mc_t^r \equiv \log(MC_t^r/MC^r)$, MC^r is the deterministic steady-state value of Home real marginal cost, and $\kappa_H \equiv (1-\theta)(1-\beta\theta)/\theta$.

Market Clearing

Home goods-market clearing is

$$Y_t = (1-\upsilon)\left(\mathcal{G}^{PPT}\left(\tau_{M,t}, S_t\right)\right)^{\eta} C_t + \upsilon S_t^{\eta}. \tag{19}$$

The first term is Home household demand in Eq. (4); the second is Foreign demand for Home goods in the SOE limit.

3.1.2 Imperfect Pass-Through Environment

The IPT environment keeps the preferences, CES consumption aggregator, household optimality conditions, complete-risk-sharing condition, Home production technology, and PCP producer-pricing block of the PPT environment. The difference is that local importers set the *ex-tariff* import price in Home currency.

In the IPT environment, a local importer stands between the Foreign producer and the Home household. The importer of record purchases the Foreign good at the border value, remits the statutory duty to the Home government, and passes that statutory component through to the Home household on top of the local-currency *ex-tariff* price.

Following Monacelli [38], we introduce monopolistically competitive local importers that purchase Foreign goods at the border and resell differentiated import varieties to Home households at a markup. These importers set prices in Home currency under Calvo pricing. Consequently, the local-currency import price need not move one-for-one with the border price, and the law of one price (LOOP) no longer holds.

Households and Demand

Similar to the PPT environment, households' utility and optimality conditions have the same form as Eq. (1) and Eqs. (10)–(11), with P_t^{IPT} replacing P_t^{PPT} wherever the Home CPI appears. The IPT complete-risk-sharing identity, however, uses the border relative price Z_t rather than the locally set ex-tariff price S_t, as shown below. The household-facing *cum-tariff* price of Foreign variety ι is

$$\widetilde{P}_{F,t}^{IPT}(\iota) \equiv P_{F,t}(\iota) + \tau_{M,t}\mathcal{E}_t P_{F,t}^*. \tag{20}$$

where $\widetilde{P}_{F,t}^{IPT}(\iota)$ denotes the household-facing *cum-tariff* price of Foreign variety ι under the IPT environment, and $P_{F,t}(\iota)$ denotes the *ex-tariff* Home-currency import price of variety ι. Since the tariff is levied on the border acquisition price $\mathcal{E}_t P_{F,t}^*$, the tariff payment per unit is $\tau_{M,t}\mathcal{E}_t P_{F,t}^*$. The *cum-tariff* purchaser price in Eq. (20) is therefore the sum of the local importer's *ex-tariff* selling price and this tariff payment. Under the PPT environment, the law of one price holds, so that $P_{F,t} = \mathcal{E}_t P_{F,t}^*$ and the *cum-tariff* import price reduces to $(1+\tau_{M,t})P_{F,t}$, as in Monacelli [38]. Under the IPT environment, however, monopolistically competitive local importers set their own

ex-tariff selling prices, and the law of one price does not generally hold. Hence, the *cum-tariff* purchaser price takes the additive form in Eq. (20).[4]

This specification implies complete direct pass-through of the statutory tariff payment into the *cum-tariff* purchaser price, holding the *ex-tariff* selling price and the border acquisition price fixed. It is consistent with the border-price evidence of Cavallo et al. [17], who find nearly complete tariff pass-through into the prices paid by importers at the border. Moreover, since the *ex-tariff* selling price is determined endogenously by the local importer, the specification is not necessarily inconsistent with their finding of limited pass-through into retail prices. Conditional demand for variety ι is

$$C_{F,t}(\iota) = \left(\frac{\widetilde{P}_{F,t}^{IPT}(\iota)}{\widetilde{P}_{F,t}^{IPT}} \right)^{-\varepsilon_M} C_{F,t}, \tag{21}$$

where $\varepsilon_M > 1$ is the elasticity of substitution across import varieties and $\widetilde{P}_{F,t}^{IPT}$ denotes the aggregate *cum-tariff* import price index, being the CPI-relevant import-price index under the IPT environment, defined by

$$\widetilde{P}_{F,t}^{IPT} \equiv \left[\int_0^1 \widetilde{P}_{F,t}^{IPT}(\iota)^{1-\varepsilon_M} d\iota \right]^{1/(1-\varepsilon_M)} \tag{22}$$

Note that the purchaser price is defined variety by variety in Eq. (20), so the aggregate *cum-tariff* import-price index cannot in general be written as $(1+\tau_{M,t})P_{F,t}$ under the imperfect pass-through environment. Equations (21)–(22) imply

$$\int_0^1 \widetilde{P}_{F,t}^{IPT}(\iota) C_{F,t}(\iota) d\iota = \widetilde{P}_{F,t}^{IPT} C_{F,t}.$$

Home-good demand has the CES form of Eq. (4), with $\mathcal{G}_t^{IPT}(\cdot)$ replacing $\mathcal{G}^{PPT}(\tau_{M,t}, S_t)$. Aggregate import demand is

$$C_{F,t} = \upsilon \left(\frac{\widetilde{P}_{F,t}^{IPT}}{P_t^{IPT}} \right)^{-\eta} C_t. \tag{23}$$

Under the IPT environment, Eq. (8) is replaced by

$$\frac{C_{F,t}}{C_{H,t}} = \frac{\upsilon}{1-\upsilon} \left(\frac{\widetilde{P}_{F,t}^{IPT}}{P_{H,t}} \right)^{-\eta}. \tag{24}$$

Similar to the PPT environment, once the import tariff is hiked, the relative consumption of imports decreases.

The Home CPI under the IPT environment is given by

$$P_t^{IPT} \equiv \left[(1-\upsilon) P_{H,t}^{1-\eta} + \upsilon \left(\widetilde{P}_{F,t}^{IPT} \right)^{1-\eta} \right]^{1/(1-\eta)}. \tag{25}$$

instead of Eq. (3). We distinguish the two CPI–PPI mappings by writing $\mathcal{G}^{PPT}(\tau_{M,t}, S_t) \equiv P_t^{PPT}/P_{H,t}$ as in Eq. (5), and $\mathcal{G}_t^{IPT}(\cdot) \equiv P_t^{IPT}/P_{H,t}$. The argument $(\cdot)$ for IPT includes the tariff, the border price, and the complete cross-sectional distribution of local importer prices in Eqs. (20)–(22); $\mathcal{G}_t^{IPT}(\cdot)$ is not, in general, a function of $(\tau_{M,t}, S_t)$ alone. Under complete international

[4] Chen et al. [18] likewise impose the import tax on top of a pre-set LCP price. An alternative convention places border taxes inside the LCP price-setting problem, as in Farhi et al. [28]; Erceg et al. [26] emphasize more generally that the macroeconomic effects of trade taxes depend on the assumed tax pass-through.

risk sharing, the relevant Foreign consumption numeraire is the border price. The IPT identity corresponding to Eq. (12) is therefore

$$Z_t = C_t \mathcal{G}_t^{IPT}(\cdot). \tag{26}$$

The household Euler equation in the IPT environment has the form of Eq. (11), with $\mathcal{G}_t^{IPT}(\cdot)$ in place of $\mathcal{G}^{PPT}(\tau_{M,t}, S_t)$ at date t and the corresponding replacement at date $t+1$. The same substitution applies to the producer and importer stochastic discount factors and to the Home real marginal cost in the IPT environment. The CES consumption aggregator, its expenditure weights, and its elasticity of substitution are identical across the two pass-through environments; only the household-facing *cum-tariff* import price entering that common CES price aggregator differs. Under PPT, $\widetilde{P}_{F,t}^{PPT} = (1+\tau_{M,t})\mathcal{E}_t P_{F,t}^*$, whereas under IPT, $\widetilde{P}_{F,t}^{IPT}$ is the aggregate index defined in Eq. (22) from the household-facing variety prices in Eq. (20).

The household's budget constraint is:

$$P_{H,t}C_{H,t} + \int_0^1 [P_{F,t}(\iota) + \tau_{M,t}\mathcal{E}_t P_{F,t}^*]C_{F,t}(\iota)\, d\iota + \sum_{h^{t+1}} \xi_{t,t+1}(h^{t+1}|h^t)B_{t+1}(h^{t+1}) = B_t(h^t) + W_t N_t + \mathcal{D}_t + T_t.$$

The integral makes the statutory-duty payment explicit on the expenditure side. By Eqs. (20)–(22),

$$\int_0^1 \widetilde{P}_{F,t}^{IPT}(\iota)C_{F,t}(\iota)d\iota = \widetilde{P}_{F,t}^{IPT}C_{F,t}, \qquad P_{H,t}C_{H,t} + \widetilde{P}_{F,t}^{IPT}C_{F,t} = P_t^{IPT}C_t.$$

Hence the same budget constraint can equivalently be written as

$$P_t^{IPT}C_t + \sum_{h^{t+1}} \xi_{t,t+1}(h^{t+1}|h^t)B_{t+1}(h^{t+1}) = B_t(h^t) + W_t N_t + \mathcal{D}_t + T_t,$$

which has the same form as Eq. (9), with P_t^{IPT} replacing P_t^{PPT}, while $\mathcal{D}_t$ includes local-importer dividends. The importer of record remits the statutory duty, and customs revenue is

$$TR_t \equiv \tau_{M,t}\mathcal{E}_t P_{F,t}^* \int_0^1 C_{F,t}(\iota)d\iota. \tag{27}$$

The customs revenue in Eq. (27) is exactly the statutory-duty component paid on the left-hand side of the disaggregated budget constraint and is rebated lump sum through T_t. The tariff-inclusive household import expenditure and lump-sum rebate are also consistent with Bianchi and Coulibaly [12]. The government also pays the time-invariant subsidy τ on the border acquisition cost, so its net lump-sum transfer is

$$T_t = TR_t - \tau\mathcal{E}_t P_{F,t}^* \int_0^1 C_{F,t}(\iota)d\iota.$$

Here τ denotes the subsidy rate paid to local importers; below we set $\tau = 1/\varepsilon_M$. The subsidy receipt $\tau\mathcal{E}_t P_{F,t}^* \int_0^1 C_{F,t}(\iota)d\iota$ is included in importer dividends in $\mathcal{D}_t$, while payment of the subsidy $\tau\mathcal{E}_t P_{F,t}^* \int_0^1 C_{F,t}(\iota)d\iota$ enters T_t with the opposite sign. Thus the statutory tariff is collected from the importer of record, passed through separately to the purchaser in Eq. (20), and returned to the household as a lump-sum rebate.

Production and Pricing Behavior

The IPT environment contains both Home producers and local importers. The Home producer side is unchanged from the PPT environment: production and producer price setting continue to

be governed by Eqs. (13)–(18). In addition, local importers use LCP and face Calvo rigidity. Their exact nonlinear pricing block is given by Eqs. (28)– (32). The *ex-tariff* import-price index satisfies

$$P_{F,t}^{1-\varepsilon_M} = \theta_M P_{F,t-1}^{1-\varepsilon_M} + (1-\theta_M)\overline{P}_{F,t}^{1-\varepsilon_M}, \tag{28}$$

Here θ_M denotes the importer Calvo non-adjustment probability and $\overline{P}_{F,t}$ denotes the *ex-tariff* reset price. Because $P_{F,t}/P_{H,t} = Z_t/\Psi_t$, the *ex-tariff* relative import price obeys

$$\frac{Z_t}{\Psi_t} = \frac{Z_{t-1}}{\Psi_{t-1}}\frac{\Pi_{F,t}}{\Pi_{H,t}}, \tag{29}$$

where $\Pi_{F,t} \equiv P_{F,t}/P_{F,t-1}$ denotes gross *ex-tariff* import inflation.

The importer remits $\tau_{M,t}\mathcal{E}_t P_{F,t}^*$ per imported unit and recovers the same statutory duty from the purchaser. Hence the duty cancels from the importer's net operating cash flow, but it enters the demand faced by a resetting importer. A resetting importer chooses $\overline{P}_{F,t}$ to solve

$$\max_{\overline{P}_{F,t}} \mathrm{E}_t \sum_{k=0}^{\infty} \theta_M^k Q_{t,t+k} \left[\overline{P}_{F,t} - (1-\tau)\mathcal{E}_{t+k}P_{F,t+k}^*\right] C_{F,t+k|t}, \tag{30}$$

The *ex-tariff* price-setting formulation in Eq. (30) is the same local-currency-pricing importer structure as in Monacelli [38], apart from the steady-state subsidy normalization used here. It is consistent with the low nominal exchange-rate pass-through documented by Gopinath, Itskhoki, and Rigobon [34]. Here $C_{F,t+k|t}$ denotes demand at date $t+k$ for an import variety whose *ex-tariff* price was reset at date t and is given by

$$C_{F,t+k|t} = \left[\frac{\overline{P}_{F,t} + \tau_{M,t+k}\mathcal{E}_{t+k}P_{F,t+k}^*}{\widetilde{P}_{F,t+k}}\right]^{-\varepsilon_M} C_{F,t+k}. \tag{31}$$

The tariff-induced reduction in demand is therefore internalized in the reset-price problem. The exact first-order condition is

$$\mathrm{E}_t \sum_{k=0}^{\infty} \theta_M^k Q_{t,t+k} C_{F,t+k|t} \left[1 - \varepsilon_M \frac{\overline{P}_{F,t} - (1-\tau)\mathcal{E}_{t+k}P_{F,t+k}^*}{\overline{P}_{F,t} + \tau_{M,t+k}\mathcal{E}_{t+k}P_{F,t+k}^*}\right] = 0. \tag{32}$$

We set $\tau = 1/\varepsilon_M$, which removes the steady-state importer markup and implies $\Psi = 1$ in the deterministic zero-tariff, zero-inflation steady state.

The mechanism through which the tariff raises the importer's own *ex-tariff* price is especially transparent when importer prices are flexible, so that $\overline{P}_{F,t} = P_{F,t}$. From Eqs. (20) and (21), the effective elasticity of demand with respect to the importer's own *ex-tariff* price is

$$\frac{\partial \log C_{F,t}}{\partial \log P_{F,t}} = -\varepsilon_M \frac{P_{F,t}}{P_{F,t} + \tau_{M,t}\mathcal{E}_t P_{F,t}^*}.$$

Because the statutory duty is added to the importer's own price in the purchaser price, a positive tariff makes this demand elasticity smaller in absolute value. Intuitively, once the tariff has already raised the purchaser price, a one-percent increase in $P_{F,t}$ raises the purchaser price by less than one percent. The associated loss of demand is therefore smaller than it would be without the tariff, giving the importer an incentive to raise its own *ex-tariff* price.

For a simple numerical illustration, suppose the border acquisition price is 1 dollar and the importer's *ex-tariff* price is also 1 dollar. Before the tariff, the purchaser price is therefore 1 dollar. A new one-percent tariff adds 0.01 dollar to the purchaser price, raising it to 1.01 dollars. If the importer then raises its own *ex-tariff* price by one percent, from 1 dollar to 1.01 dollars, the purchaser price becomes 1.02 dollars. Relative to the post-tariff purchaser price of 1.01 dollars, however, this is an increase of only $(1.02 - 1.01)/1.01 \simeq 0.99$ percent, rather than one percent.

Hence, for a given demand elasticity ε_M, the associated demand decline is approximately $0.99\varepsilon_M$ percent rather than ε_M percent. The importer therefore loses less demand from a given proportional increase in its own price once the tariff is present, strengthening its incentive to raise the *ex-tariff* price. To derive the flexible-price implication, set $\overline{P}_{F,t} = P_{F,t}$ in Eq. (32). The reset-price condition then becomes

$$1 - \varepsilon_M \frac{P_{F,t} - (1-\tau)\mathcal{E}_t P^*_{F,t}}{P_{F,t} + \tau_{M,t}\mathcal{E}_t P^*_{F,t}} = 0,$$

which can be rearranged as

$$(\varepsilon_M - 1)P_{F,t} = [\varepsilon_M(1-\tau) + \tau_{M,t}]\mathcal{E}_t P^*_{F,t}.$$

Using the fixed importer subsidy $\tau = 1/\varepsilon_M$ gives

$$\Psi_t^{-1} = \frac{P_{F,t}}{\mathcal{E}_t P^*_{F,t}} = 1 + \frac{\tau_{M,t}}{\varepsilon_M - 1}. \tag{33}$$

The inverse LOOP gap Ψ_t^{-1} corresponds to the importer's *ex-tariff* price markup relative to the border acquisition price; Ψ_t is the inverse of this markup measure. Equation (33) gives its value under flexible importer prices. The statutory duty does not enter as a net marginal acquisition cost; it lowers the effective own-price elasticity and thereby induces an opportunistic increase in the *ex-tariff* markup. The subsidy is calibrated to eliminate the zero-tariff steady-state importer markup, but it does not adjust with the tariff and therefore does not eliminate the tariff-induced increase in the importer markup. This flexible-price calculation isolates the opportunistic-pricing effect of the tariff. This effect is closely related to the classic tax-pass-through result under imperfect competition. Delipalla and Keen [21] compare specific and ad valorem taxation under oligopoly, while Anderson, de Palma, and Kreider [3] show in a differentiated-product Bertrand setting that unit excise taxes can be shifted to consumer prices by more than one-for-one, with the incidence depending on the curvature of demand. Weyl and Fabinger [43] provide a general treatment of tax pass-through under imperfect competition. The present mechanism has a distinctive international-pricing implementation: the statutory tariff is proportional to the border acquisition price and is added outside the sticky *ex-tariff* import price, so that the tariff lowers the importer's effective own-price elasticity through the purchaser-price denominator. The resulting incentive to raise the *ex-tariff* import price is nevertheless consistent with this imperfect-competition tax-pass-through literature.

Thus the IPT environment is subject to the exact importer reset-price condition in Eq. (32), in addition to the producer conditions in Eqs. (13)–(18). At first order, Eq. (32) implies

$$\pi_{F,t} = \beta \mathrm{E}_t \pi_{F,t+1} + \kappa_M \left(\psi_t + \frac{\tau_{M,t}}{\varepsilon_M - 1} \right), \tag{34}$$

where $\pi_{F,t} \equiv \log \Pi_{F,t}$, $\psi_t \equiv \log \Psi_t$, and $\kappa_M \equiv (1-\theta_M)(1-\beta\theta_M)/\theta_M$. Equation (34) is used for first-order analytical interpretation, while the nonlinear Ramsey computation retains Eq. (32). Around the zero-tariff steady state, $\log(1+\tau_{M,t}) = \tau_{M,t} + O(\tau_{M,t}^2)$.

Taking logs of Eq. (33) and linearizing around the zero-tariff steady state, the flexible-importer-price benchmark is

$$\psi_t = -\frac{\tau_{M,t}}{\varepsilon_M - 1}.$$

Hence the expression $\psi_t + \tau_{M,t}/(\varepsilon_M - 1)$ on the right-hand side of Eq. (34) is the actual LOOP gap minus its flexible-price benchmark. Import-price inflation depends on this deviation. Thus the tariff term in Eq. (34) reflects the opportunistic *ex-tariff* price increase implied by Eq. (33), not the statutory duty as a net marginal acquisition cost.

Market Clearing

Home goods-market clearing under IPT is

$$Y_t = (1-\upsilon)\left(\frac{P_t^{IPT}}{P_{H,t}}\right)^{\eta} C_t + \upsilon Z_t^{\eta}. \tag{35}$$

The first term uses the exact household CPI in Eq. (25); the second uses the ex-tariff border price Z_t faced by Foreign purchasers of Home goods. The IPT clearing condition differs from the PPT condition in Eq. (19) through its CPI and border-price terms.

3.2 Ramsey Policy

For each pass-through environment, the monetary authority is a social planner that chooses a complete contingent plan under commitment, maximizing Eq. (1). We solve the Ramsey problem from the *timeless perspective*. The lagged Lagrange multipliers entering the initial Ramsey first-order conditions are initialized at their deterministic steady-state values, rather than reset to zero at the date of the tariff shock.

In the PPT environment, the choice variables are C_t, Z_t, N_t, Y_t, $\Pi_{H,t}$, $\widetilde{p}_{H,t}$, $\mathcal{K}_{H,t}$, $\mathcal{Z}_{H,t}$, $\Delta_{H,t}$, and i_t for $t \geq 0$. The constraints are Eqs. (5), (10), (11), and (12) with $\Psi_t = 1$, Eqs. (13)–(17), Eq. (2), Eq. (19) with $\Psi_t = 1$, and the ZLB conditions in Eq. (36).

In the IPT environment, the choice variables are C_t, Z_t, N_t, Y_t, $\Pi_{H,t}$, $\widetilde{p}_{H,t}$, $\mathcal{K}_{H,t}$, $\mathcal{Z}_{H,t}$, $\Delta_{H,t}$, Ψ_t, $\Pi_{F,t}$, $\overline{P}_{F,t}$, $P_{F,t}$, $\widetilde{P}_{F,t}^{IPT}$, and i_t for $t \geq 0$. The constraints are Eqs. (5) and (23), Eqs. (10) and (11) with the IPT CPI substitution stated after Eq. (26), Eq. (26), Eqs. (13)–(17), Eqs. (30)–(31), (28), (32), and (29), Eq. (35), and the ZLB condition in Eq. (36).

The feasibility, non-negativity, and complementary-slackness conditions are

$$i_t \geq 0, \qquad \mu_{i,t} \geq 0, \qquad \mu_{i,t} i_t = 0. \tag{36}$$

Here $\mu_{i,t}$ denotes the Kuhn–Tucker multiplier on the ZLB constraint. The tariff process is exogenous and tariff revenue is rebated lump sum. The Lagrangians and first-order conditions for the two pass-through environments are reported in Appendix C; the first-order conditions are not repeated in the main text. The numerical policy paths are obtained from these nonlinear Ramsey first-order-condition systems using Dynare's perfect-foresight solver with its mixed-complementarity algorithm (`lmmcp`) to impose the ZLB. The ZLB is imposed in all baseline and high-substitution experiments. The sensitivity analysis of producer- and importer-price rigidity, reported in the Appendix, does not impose the ZLB and instead reports notional nominal interest rates. The second-order welfare cost functions below are reporting and interpretation devices rather than policy objectives.

3.3 Calibration and Shocks

The baseline calibration follows Monacelli [39] wherever the model has a direct counterpart. Table 1 summarizes the baseline parameterization.

The tariff process is

$$\tau_{M,t} = \rho_\tau \tau_{M,t-1} + \epsilon_{\tau,t}.$$

We set the one-time innovation $\epsilon_{\tau,0} = 0.20$, following Monacelli [39], as our baseline.[5] Thus the import tariff rises by 20 percentage points on impact. The steady-state tariff is zero. The deterministic quarterly nominal rate is $1/\beta - 1 = 1.0101$ percent, approximately four percent annualized. Hence a response of -101.01 basis points relative to steady state places the actual nominal rate exactly at the ZLB.

[5] Throughout the paper, "baseline" refers to the parameter values in Table 1 together with the tariff innovation $\epsilon_{\tau,0} = 0.20$.

Table 1: Baseline Calibration

Parameter	Value	Interpretation
β	0.99	Discount factor
υ	0.3	Openness / Home Bias parameter
φ	2.2	Inverse Frisch elasticity
η	1.5	Home–Foreign substitution elasticity
ε	3.8	Elasticity across producer varieties
θ	0.8	Producer Calvo parameter
ε_M	3.8	Elasticity across importer varieties
θ_M	0.8	Importer Calvo parameter
ρ_τ	0.7	Tariff persistence

3.4 Welfare Relevant LOOP Gap

To make the planner's stabilization objectives transparent, we report welfare cost functions based on a second-order approximation to household utility following Benigno and Woodford [7, 8]. In superscripts, PPT and IPT denote the two pass-through environments, respectively. Let $y_t \equiv \log(Y_t/\bar{Y})$ and $z_t \equiv \log(Z_t/\bar{Z})$, where $\bar{Y}$ and $\bar{Z}$ are the original zero-tariff deterministic steady-state levels defined in Appendix E.1. Suppressing terms that are independent of the Home policy allocation, under PPT,

$$\mathcal{L}_t^{SOE,PPT} \equiv \frac{1}{2}\left[\widehat{\Lambda}_y^{SOE,PPT}(y_t - \widehat{y}_t^{w,SOE,PPT})^2 + \widehat{\Lambda}_z^{SOE,PPT}(z_t - \widehat{z}_t^{w,SOE,PPT})^2 + \Lambda_{\pi,H}^{SOE,PPT}\pi_{H,t}^2\right],$$

whereas under IPT it can be written as

$$\begin{aligned}\mathcal{L}_t^{SOE,IPT} \equiv \frac{1}{2}\Big[&\widehat{\Lambda}_y^{SOE,IPT}(y_t - \widehat{y}_t^{w,SOE,IPT})^2 + \widehat{\Lambda}_\psi^{SOE,IPT}(\psi_t - \widehat{\psi}_t^{w,SOE,IPT})^2 \\ &+\widehat{\Lambda}_z^{SOE,IPT}(z_t - \widehat{z}_t^{w,SOE,IPT})^2 + \Lambda_{\pi,H}^{SOE,IPT}\pi_{H,t}^2 + \Lambda_{\pi,F}^{SOE,IPT}\pi_{F,t}^2\Big].\end{aligned}$$

Here the hatted Λ coefficients are the completed-square welfare weights associated with y_t, z_t, and, under IPT, ψ_t in the expressions above. The corresponding hatted WR reference values are $\widehat{y}_t^{w,SOE,\cdot}$, $\widehat{z}_t^{w,SOE,\cdot}$, and $\widehat{\psi}_t^{w,SOE,IPT}$. See Appendix E for details on these WR reference values.[6]

The contrast between the two expressions shows how the open-economy relative-price externality enters the welfare criterion. Under PPT, the LOOP holds identically, $\Psi_t = 1$, so the relative-price externality is carried by the WR *ex-tariff* relative border price of imports gap $z_t - \widehat{z}_t^{w,SOE,PPT}$. Under IPT, $P_{F,t}/P_{H,t} = Z_t/\Psi_t$, and the welfare criterion contains both the WR *ex-tariff* relative border price of imports gap $z_t - \widehat{z}_t^{w,SOE,IPT}$ and the WR LOOP-gap deviation $\psi_t - \widehat{\psi}_t^{w,SOE,IPT}$. A nominal exchange-rate movement can therefore change the *ex-tariff* relative border price Z_t while, because the local-currency import price $P_{F,t}$ adjusts only gradually, also generating a movement in the LOOP gap Ψ_t. The same relative-price externality is thus manifested jointly through z_t and ψ_t under IPT.

Using complete international risk sharing and Home goods-market clearing, we eliminate the ex-tariff relative border price z_t from the welfare cost functions. See Appendix E for the derivation. Under PPT, $\Psi_t = 1$, so the welfare-relevant movement in z_t is absorbed into the WR output term. The same period welfare cost can therefore be written as

$$\mathcal{L}_t^{SOE,PPT} \equiv \frac{1}{2}\left[\Lambda_y^{SOE,PPT}(y_t - y_t^{w,SOE,PPT})^2 + \Lambda_{\pi,H}^{SOE,PPT}\pi_{H,t}^2\right], \tag{37}$$

[6]The welfare cost function is normalized by steady-state marginal utility of consumption times steady-state consumption, $U_C(C,N)C$. Hence it measures welfare losses in units of steady-state consumption: for example, $\mathcal{L}_t = 0.01$ means, locally, a welfare loss equivalent to a one-percent loss of steady-state consumption. With the logarithmic utility used here, $U_C(C,N)C = 1$.

whereas under IPT the same elimination of z_t leaves the independently moving LOOP gap generated by sticky local import pricing. The same period welfare cost can be written as

$$\mathcal{L}_t^{SOE,IPT} \equiv \frac{1}{2}\Big[\Lambda_y^{SOE,IPT}(y_t - y_t^{w,SOE,IPT})^2 + \Lambda_\psi^{SOE,IPT}(\psi_t - \psi_t^{w,SOE,IPT})^2 + \Lambda_{\pi,H}^{SOE,IPT}\pi_{H,t}^2 + \Lambda_{\pi,F}^{SOE,IPT}\pi_{F,t}^2\Big]. \tag{38}$$

The objects $y_t^{w,SOE,PPT}$ and $y_t^{w,SOE,IPT}$ are the (logarithmic) WR outputs in the PPT and IPT environments, respectively, and $y_t - y_t^{w,SOE,PPT}$ and $y_t - y_t^{w,SOE,IPT}$ are the corresponding WR output gaps. Under the IPT environment, $\psi_t^{w,SOE,IPT}$ is the (logarithmic) WR LOOP gap. Accordingly, $\psi_t - \psi_t^{w,SOE,IPT}$, which measures the deviation of the actual LOOP gap from its welfare-relevant value, is referred to as the WR LOOP-gap deviation. See Appendix E for details on $y_t^{w,SOE,PPT}$, $y_t^{w,SOE,IPT}$, and $\psi_t^{w,SOE,IPT}$. Thus Eq. (37) contains no separate relative-price term, while Eq. (38) retains the WR LOOP-gap deviation and the import-inflation term.

For the tariff experiment, the welfare-relevant (WR) LOOP gap in the SOE is given by

$$\psi_t^{w,SOE,IPT} = \Gamma_{\text{own}}^{SOE}\tau_{M,t}, \tag{39}$$

where $\Gamma_{\text{own}}^{SOE}$ denotes the structural loading of the Home import tariff on the WR LOOP gap. See Appendix E for its derivation. Under the baseline parameterization, $\Gamma_{\text{own}}^{SOE} = 0.421$. To understand why the WR LOOP gap rises in response to an import tariff, we log-linearize the purchaser-price equation at the individual import-variety level, aggregate across varieties, and use Eq. (24). Around the zero-tariff steady state, this yields:

$$c_{F,t} - c_{H,t} = -\eta(z_t + \tau_{M,t} - \psi_t), \tag{40}$$

with $c_{H,t} \equiv \log\left(\frac{C_{H,t}}{C_H}\right)$ and $c_{F,t} \equiv \log\left(\frac{C_{F,t}}{C_F}\right)$.

Eq. (40) reveals the economic mechanism underlying the HIAC. An increase in the import tariff raises the *cum-tariff* relative price of imports, thereby inducing an inefficiently low level of import consumption relative to Home-goods consumption and reducing economic welfare. For a given *ex-tariff* relative border price z_t, an increase in the LOOP gap ψ_t lowers the relative purchaser price of imports and offsets this distortion. Consequently, the WR LOOP gap rises with the tariff, as shown above, to mitigate the tariff-induced underconsumption of imports (Fig. 1). The HIAC thus provides the Ramsey planner with an incentive to raise the actual LOOP gap toward the WR LOOP gap.

Eq. (34) suggests that an increase in the actual LOOP gap does not lead to an immediate and full adjustment of the *ex-tariff* import price. While an increase in the LOOP gap puts upward pressure on *ex-tariff* import-price inflation, Calvo price rigidity slows the adjustment of the import price to the higher Home-currency border price. Consequently, nominal depreciation can raise the actual LOOP gap despite the importers' incentive to increase their selling prices. As Eq. (40) shows, for a given *ex-tariff* relative border price, a higher LOOP gap mitigates tariff-induced underconsumption of imported goods relative to Home goods and thereby improves economic welfare. The Ramsey planner can thus exploit import-price stickiness to move the actual LOOP gap toward its WR level and thereby alleviate the inefficiently low level of import consumption relative to Home-goods consumption.

For a given future nominal exchange rate and Foreign nominal interest rate, UIP implies that a reduction in the Home nominal interest rate exerts depreciation pressure on the Home currency. This depreciation pressure tends to raise the actual LOOP gap. The HIAC thus provides the Ramsey planner with an incentive to cut the nominal interest rate in response to an increase in the import tariff, subject to the competing inflation-stabilization objectives.

Substituting Eq. (39), the WR LOOP-gap component of the welfare cost becomes

$$\frac{\Lambda_\psi^{SOE,IPT}}{2}\left(\psi_t - \psi_t^{w,SOE,IPT}\right)^2 = \frac{\Lambda_\psi^{SOE,IPT}}{2}\left(\psi_t - \Gamma_{\text{own}}^{SOE}\tau_{M,t}\right)^2.$$

Since both $\Lambda_{\psi}^{SOE,IPT}$ and $\Gamma_{\text{own}}^{SOE}$ are positive, an increase in the import tariff raises the WR LOOP gap. When the actual LOOP gap lies below the WR LOOP gap, increasing it reduces this component of the welfare cost. Together with Eq. (40), this establishes the welfare rationale for the HIAC: a higher LOOP gap mitigates tariff-induced underconsumption of imports.

Table 2: SOE Welfare Weights: Baseline Values

Environment	WR Variable	Weight	Value
PPT	$y_t - y_t^{w,SOE,PPT}$	$\Lambda_{y}^{SOE,PPT}$	1.654
	$\pi_{H,t}$	$\Lambda_{\pi,H}^{SOE,PPT}$	39.378
IPT	$y_t - y_t^{w,SOE,IPT}$	$\Lambda_{y}^{SOE,IPT}$	1.654
	$\psi_t - \psi_t^{w,SOE,IPT}$	$\Lambda_{\psi}^{SOE,IPT}$	0.506
	$\pi_{H,t}$	$\Lambda_{\pi,H}^{SOE,IPT}$	39.378
	$\pi_{F,t}$	$\Lambda_{\pi,F}^{SOE,IPT}$	27.295

Note: All entries are rounded evaluations of the structural welfare coefficients derived in Appendix E.1; the unrounded values are retained in the second-order calculation.

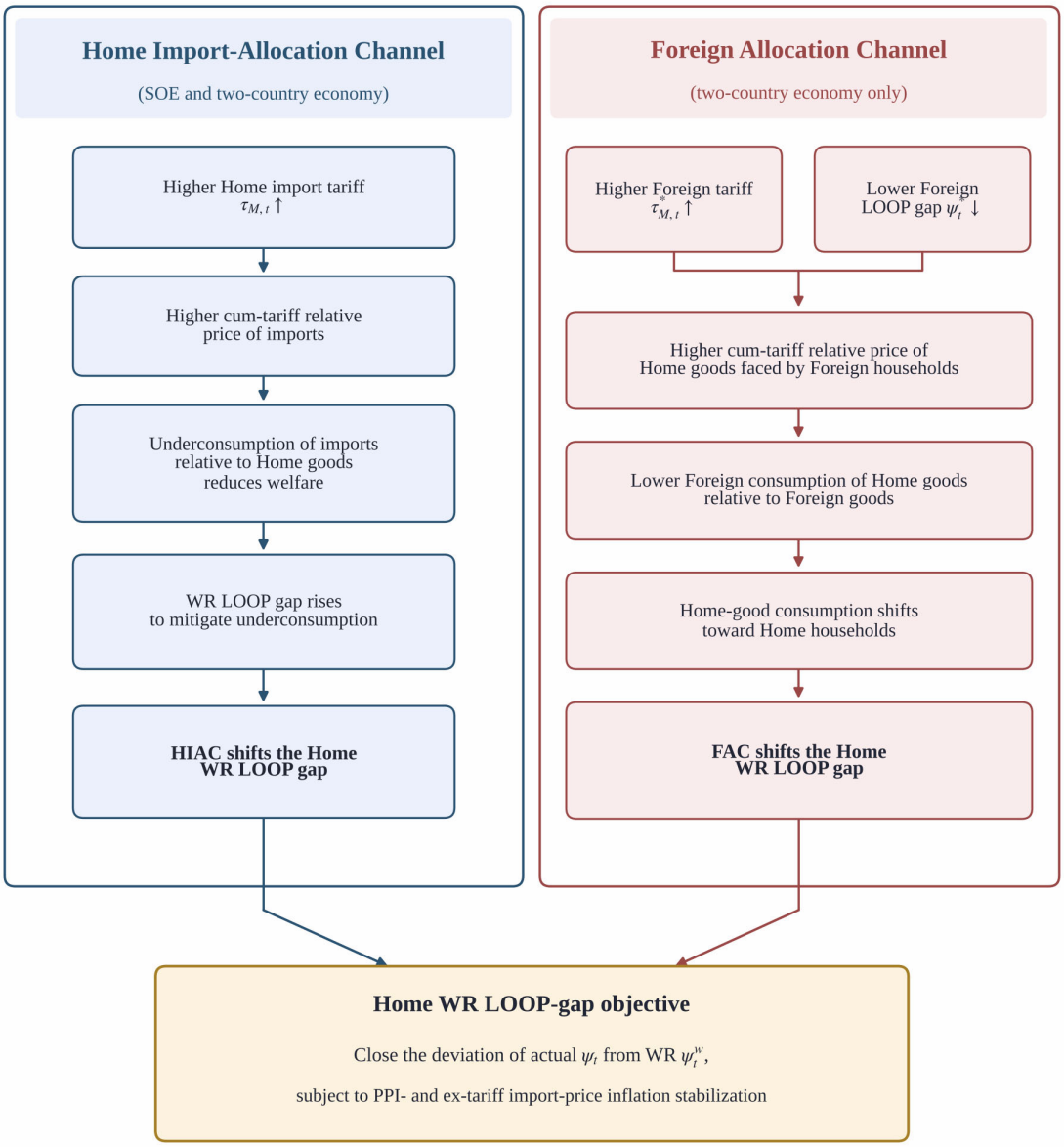


Figure 1: Home Import-Allocation Channel, Foreign Allocation Channel, and the WR LOOP-gap objective

Note: The HIAC operates in both the SOE and the two-country economy, whereas the FAC arises only in the two-country economy.

Engel [25] interprets deviations from the law of one price under LCP as currency misalignment and a source of welfare loss. In our setting, log-linearizing the definition of the LOOP gap yields

$$\psi_t = e_t + p^*_{F,t} - p_{F,t},$$

with $e_t \equiv \log \mathcal{E}_t$, $p_{F,t} \equiv \log P_{F,t}$, and $p^*_{F,t} \equiv \log P^*_{F,t}$. Define the WR nominal exchange rate by

$$e_t^{w,SOE,IPT} \equiv p_{F,t} - p^*_{F,t} + \psi_t^{w,SOE,IPT}. \tag{41}$$

It follows immediately that

$$\psi_t - \psi_t^{w,SOE,IPT} = e_t - e_t^{w,SOE,IPT}. \tag{42}$$

Hence, following Engel's currency-misalignment interpretation, reducing the WR LOOP-gap deviation is exactly equivalent to reducing the nominal exchange-rate misalignment, measured by the deviation of the actual nominal exchange rate from the WR nominal exchange rate.

3.5 Nominal Interest Rate Response to an Increase in the Import Tariff

3.5.1 Baseline Case

Figure 2 compares the SOE responses to a 20-percentage-point Home import-tariff increase. The solid blue line reports IPT, and the dotted green line reports PPT. The central result is that the Home Ramsey planner *lowers* the nominal interest rate under IPT (Panel 1). On impact, the policy rate falls by 32.87 basis points under IPT and rises by 17.03 basis points under PPT.

Under IPT, an increase in the Home import tariff raises the *cum-tariff* relative price of imports, inducing inefficient underconsumption of imported goods relative to Home goods. As Eq. (40) shows, an increase in the LOOP gap mitigates this distortion by lowering the relative purchaser price of imports for a given *ex-tariff* relative border price. Accordingly, the WR LOOP gap rises with the tariff, as shown in Eq. (39) (Fig. 1 and Panel 10, Fig. 2). The HIAC thus gives the Ramsey planner an incentive to move the actual LOOP gap toward its WR level. The planner cuts the nominal interest rate to exert depreciation pressure and reduce the WR LOOP-gap deviation, subject to the PPI inflation and *ex-tariff* import inflation stabilization objectives (Panels 1 and 11, Fig. 2). The rate cut reflects the resulting intertemporal welfare trade-off, not the absence of inflation stabilization objectives. Its magnitude cannot be read directly from the WR LOOP gap, since the actual LOOP-gap path, its deviation from the WR LOOP gap, and the PPI and import-price inflation all enter the nonlinear committed Ramsey allocation.

Under IPT, the Ramsey planner cuts the nominal interest rate to induce nominal depreciation and thereby narrow the WR LOOP-gap deviation (Eq. 42). However, the competing objectives of stabilizing Home PPI inflation and *ex-tariff* import-price inflation limit the extent of monetary easing. Consequently, the rate cut is insufficient to prevent the actual LOOP gap from falling, even as the WR LOOP gap rises (Panels 9 and 10).

PPT serves as a comparison. Its import price adjusts with the nominal exchange rate, so the LOOP holds and the Home welfare cost function contains no WR LOOP-gap deviation term. From Eq. (8), the increase in the cum-tariff relative price of imports shifts spending toward Home goods. Relative consumption of imports declines under both environments (Panel 8), but the strong expenditure-switching effect under PPT raises Home demand, real marginal cost, and pressure on Home PPI inflation. In view of the large PPI-inflation stabilization weight in Eq. (37) and Table 2, the PPT planner raises the nominal rate (Panel 1).

Appendix A provides two analytical benchmarks that clarify the role of nominal price rigidities in these results. Proposition 1 establishes sufficient conditions for an initial interest-rate cut under IPT even with flexible producer prices, whereas Proposition 2 establishes exact equivalence between IPT with flexible import prices and PPT with an appropriately scaled tariff.

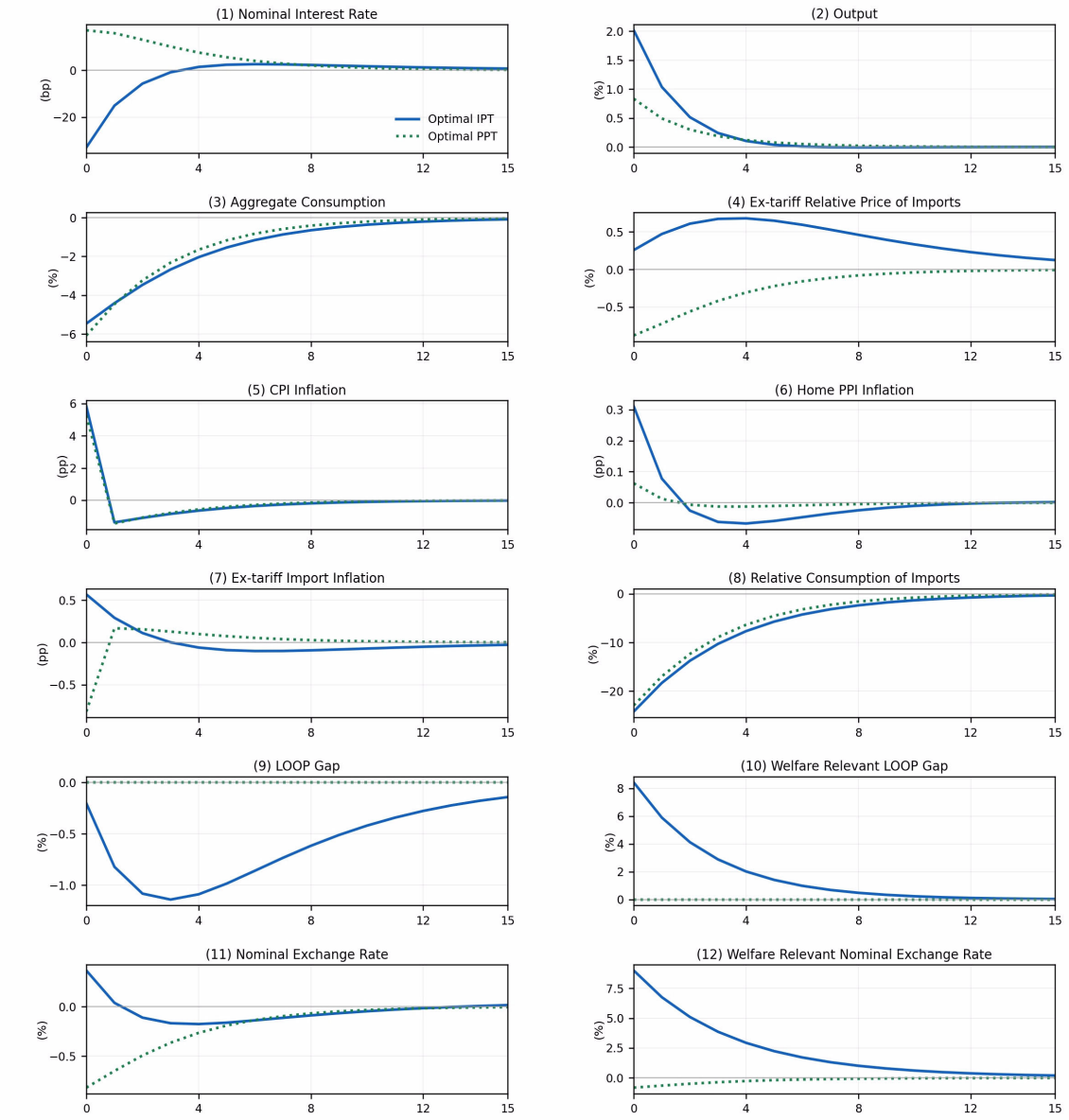


Figure 2: Small open economy: optimal PPT and IPT responses to a 20-percentage-point Home import tariff. Panel 1 reports the nominal interest-rate deviation from steady state in basis points; Panel 8 reports the percentage deviation of relative import consumption from steady state; other responses are in percent or percentage points. Under PPT, the LOOP holds identically.

3.5.2 High Substitution Case

Figure 3 presents the SOE experiment with the substitution elasticity raised to $\eta = 3$, holding the tariff shock and every other parameter fixed. *IPT continues to call for an initial interest-rate cut*, and the cut is larger than under the baseline elasticity (Panel 1). Equation (39) remains the relevant WR LOOP-gap relation. Although the own-tariff coefficient $\Gamma_{\text{own}}^{SOE}$ declines from 0.421 to 0.293, the Home WR LOOP gap still rises with the tariff, preserving the incentive for monetary easing through the HIAC. The magnitude of the optimal rate cut reflects this incentive together with the inflation-stabilization objectives.

As a comparison, under PPT higher substitutability strengthens the tariff-induced expenditure switching into Home goods and the associated upward pressure on Home PPI inflation (Panels 2 and 6). The optimal policy remains a rate increase, although its impact response is smaller and the policy rate rises further in subsequent quarters. The impact responses are -47.08 basis points under IPT and $+3.31$ basis points under PPT. Thus the SOE produces an optimal IPT rate cut under both elasticities, while the policy-sign contrast with PPT also persists.

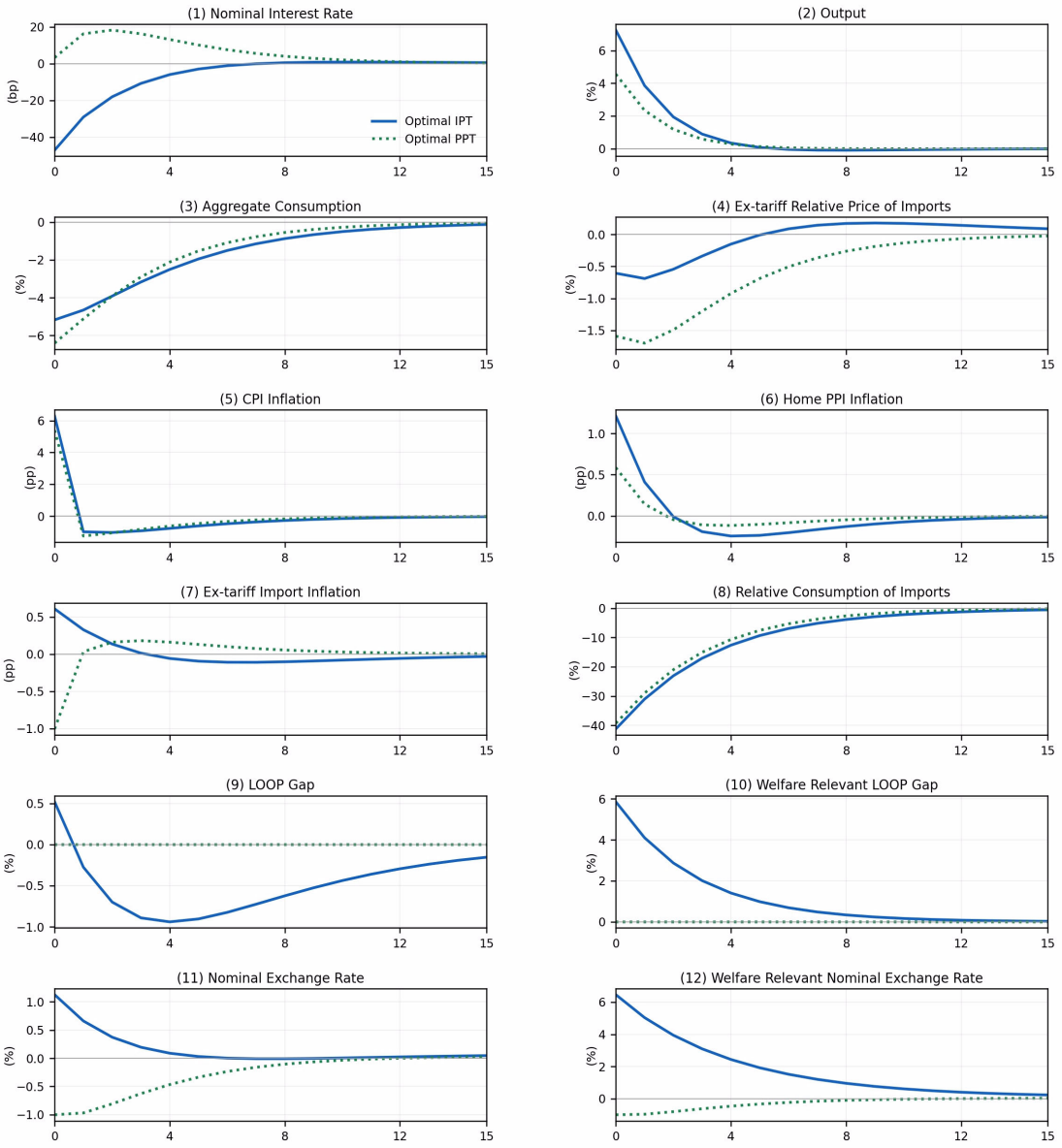


Figure 3: Small open economy: responses under the High Substitution Case ($\eta = 3$), holding other parameters and the unilateral Home tariff shock fixed. The panels, units, and PPT/IPT line styles are as in Fig. 2.

4 Two-Country Economy

4.1 The Model

The small-open-economy model in Sec. 3 takes the Foreign economy as given. We now endogenize that economy and embed the SOE structure in a unit-mass two-country world. Country size is represented by household mass: households are indexed by $h \in [0,1]$, with $h \in [0,\nu)$ in country H and $h \in [\nu,1]$ in country F. Because steady-state per-capita output is common across the two countries, ν is also Home's share of steady-state world output and is therefore calibrated to Home's share of world gross domestic product (GDP). Producer varieties retain the country-specific unit-continuum normalization used in the SOE. To make country-size accounting transparent in the bilateral importer block, Home importer varieties are indexed by $\iota \in [0,\nu)$ and Foreign importer varieties by $\iota \in [\nu,1]$. Importer CES integrals are normalized by the corresponding country mass, ν or $1-\nu$. Hence the country-size ratios in bilateral goods-market clearing convert per-capita demand across countries, while the normalized importer integrals take averages within the importing country. Households are identical within each country, so we continue to write the Home allocation in representative-household form. The Home household retains the preferences in Eq. (1), while the Foreign representative household has lifetime utility

$$\mathcal{U}^* \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t^* - \frac{(N_t^*)^{1+\varphi}}{1+\varphi} \right).$$

Here C_t^* and N_t^* denote Foreign composite consumption and labor, respectively. The Foreign household maximizes $\mathcal{U}^*$ subject to the exact starred counterparts of the Home household constraints and optimality conditions.

The two-country extension makes endogenous the Foreign objects that were external to the SOE transmission mechanism. We retain the SOE notation for Home. Country F now has its own consumption, labor, output, producer prices, Calvo price dispersion, and monetary policy. Each country is specialized in a continuum of differentiated goods produced with the same technology and producer Calvo structure as in Sec. 3.1.1. Home and Foreign have separate monetary authorities with nominal interest rates i_t and i_t^*, respectively, and Home and Foreign governments may levy import tariffs $\tau_{M,t}$ and $\tau_{M,t}^*$, respectively. Tariff revenue is rebated lump sum in the country that collects it. Complete international financial markets link the two national household problems and impose international risk sharing. Hence a Home policy or tariff change can alter the Foreign allocation and, through the common world equilibrium, feed back to Home.

Home bias is introduced by scaling each country's expenditure weight on partner-country goods by the common openness parameter υ. Given the country sizes above, the Home and Foreign expenditure weights are $\omega_F = \upsilon(1-\nu)$, $\omega_H = 1-\omega_F$, $\omega_H^* = \upsilon\nu$, and $\omega_F^* = 1-\omega_H^*$. Here ω_F is Home's expenditure weight on Foreign goods and ω_H is Home's expenditure weight on Home goods. Likewise, ω_H^* is Foreign's expenditure weight on Home goods and ω_F^* is Foreign's expenditure weight on Foreign goods. Thus ω_F and ω_H^* are the bilateral import weights, whereas ω_H and ω_F^* are the domestic-good weights. For $\upsilon < 1$, the bilateral import weights are smaller than the corresponding partner-country economic-size shares, generating home bias.

These expenditure weights enter the two national consumption baskets:

$$\begin{aligned} C_t &= \left[\omega_H^{1/\eta} C_{H,t}^{(\eta-1)/\eta} + \omega_F^{1/\eta} C_{F,t}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}, \\ C_t^* &= \left[(\omega_F^*)^{1/\eta} (C_{F,t}^*)^{(\eta-1)/\eta} + (\omega_H^*)^{1/\eta} (C_{H,t}^*)^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}. \end{aligned} \tag{43}$$

where $C_{F,t}^*$ and $C_{H,t}^*$ denote Foreign consumption of Foreign and Home goods, respectively. Bilateral goods-market clearing therefore replaces the SOE assumption that Foreign demand can be treated as an external object.

In a two-country model, the LOOP also applies explicitly to Home goods. Under PPT it is

$$P_{H,t} = \mathcal{E}_t P_{H,t}^*,$$

where $P^*_{H,t}$ denotes the Home-goods price in Foreign currency. Define the Foreign LOOP gap by

$$\Psi^*_t \equiv \frac{P_{H,t}/\mathcal{E}_t}{P^*_{H,t}}, \qquad \psi^*_t \equiv \log \Psi^*_t.$$

Using $S_t \equiv P_{F,t}/P_{H,t}$, the Home *ex-tariff* relative price of imports can therefore be written as

$$S_t = \frac{P_{F,t}}{\mathcal{E}_t P^*_{H,t}} \frac{1}{\Psi^*_t},$$

where $P_{F,t}/(\mathcal{E}_t P^*_{H,t})$ is the *ex-tariff* relative consumer price of imports to exports. Under the PPT environment, only PCP producers are present and the LOOP holds in both directions, so $\Psi_t = \Psi^*_t = 1$. Under the IPT environment, local importers using LCP operate in both countries, so Ψ_t and Ψ^*_t are determined endogenously. Home importers retain the flexible-price markup condition in Eq. (33) and the first-order importer Phillips curve in Eq. (34). Since Foreign importers of Home goods face the corresponding monopolistically competitive pricing problem, their flexible-price condition is

$$(\Psi^*_t)^{-1} = \frac{P^*_{H,t}}{P_{H,t}/\mathcal{E}_t} = 1 + \frac{\tau^*_{M,t}}{\varepsilon_M - 1}, \tag{44}$$

and their first-order importer Phillips curve under sticky local-currency prices is

$$\pi^*_{H,t} = \beta \mathrm{E}_t \pi^*_{H,t+1} + \kappa_M \left(\psi^*_t + \frac{\tau^*_{M,t}}{\varepsilon_M - 1} \right). \tag{45}$$

where $\pi^*_{H,t} \equiv \log(P^*_{H,t}/P^*_{H,t-1})$. We denote the Foreign CPI by $P^{*,PPT}_t$ in the PPT environment and by $P^{*,IPT}_t$ in the IPT environment. Under PPT, the Home relative consumption of imports is determined by

$$\frac{C_{F,t}}{C_{H,t}} = \frac{\omega_F}{\omega_H} \left[(1 + \tau_{M,t}) S_t \right]^{-\eta}. \tag{46}$$

The bilateral goods-market conditions follow directly from variety-level clearing. For a Home producer variety j, domestic and export demand imply

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left[C_{H,t} + \frac{1}{\nu} \int_\nu^1 C^*_{H,t}(\iota) d\iota \right]. \tag{47}$$

Aggregating Eq. (47) with the producer CES index gives the exact bilateral goods-market conditions

$$\begin{aligned} Y_t &= C_{H,t} + \left(\frac{1-\nu}{\nu} \right) C^*_{H,t} \frac{1}{1-\nu} \int_\nu^1 \left(\frac{\widetilde{P}^*_{H,t}(\iota)}{\widetilde{P}^*_{H,t}} \right)^{-\varepsilon_M} d\iota, \\ Y^*_t &= C^*_{F,t} + \left(\frac{\nu}{1-\nu} \right) C_{F,t} \frac{1}{\nu} \int_0^\nu \left(\frac{\widetilde{P}^{IPT}_{F,t}(\iota)}{\widetilde{P}^{IPT}_{F,t}} \right)^{-\varepsilon_M} d\iota. \end{aligned} \tag{48}$$

where $C^*_{H,t}(\iota)$ denotes Foreign-country demand for Home goods sold by local importer variety ι, and $\widetilde{P}^*_{H,t}(\iota)$ is the Foreign mirror of the variety-level *cum-tariff* purchaser price in Eq. (20). The corresponding aggregate *cum-tariff* import-price index is

$$\widetilde{P}^*_{H,t} \equiv \left[\frac{1}{1-\nu} \int_\nu^1 \left(\widetilde{P}^*_{H,t}(\iota) \right)^{1-\varepsilon_M} d\iota \right]^{1/(1-\varepsilon_M)}.$$

The exact Foreign mirror applies to a Foreign producer variety. Producer-price dispersion affects labor requirements through Eq. (13), whereas the two integrals in Eq. (48) are physical-quantity factors implied by conditional import demand evaluated at *cum-tariff* purchaser prices. They are not producer-price-dispersion terms; they arise from aggregating physical import quantities across importer varieties.

The two-country economy makes explicit the step from the SOE by endogenizing the Foreign economy. A Home tariff first changes Home absorption, the Home LOOP gap, and Home importer marginal cost. Because Foreign demand and production are now endogenous, the same disturbance also changes Foreign consumption, output, producer and importer prices, and the Foreign policy rate. Those Foreign responses alter Home export demand and international relative prices and therefore feed back into the Home allocation. The resulting spillover and spillback are thus generated inside the common two-country equilibrium rather than appended as reduced-form disturbances.

4.2 National Ramsey Policies

The benchmark two-country policy game follows the national Ramsey approach of Faia and Monacelli [27]. Complete international risk sharing and the relevant goods-market relations are imposed before the national first-order conditions are formed. Hence the international relative price is eliminated before national differentiation and is not a choice variable of either national planner.

For the Ramsey differentiation we substitute the private producer reset-price condition and, in the IPT environment, the exact importer reset-price condition before taking national first-order conditions. In the PPT environment, the Home planner therefore chooses the paths of C_t, $\mathcal{G}^{PPT}(\tau_{M,t}, Z_t/\Psi_t)$, N_t, Y_t, $\Pi_{H,t}$, $\Delta_{H,t}$, and i_t, subject to the producer reset-price map and all remaining private-equilibrium restrictions. In the IPT environment, S_t and $\Pi_{F,t}$ are additionally endogenous in the Home best response. The complete cross-sectional importer-price distribution continues to obey the exact Calvo updating and purchaser-price definitions in Eqs. (20)–(32); it is not replaced by an aggregate multiplicative-tariff shortcut. In particular, Z_t is not a Home Ramsey choice variable.

The Foreign planner solves the exact Home–Foreign mirror problem. Its choice set is the starred counterpart of the preceding Home set, with its own local *ex-tariff* import-price ratio and importer inflation added under IPT. No additional international-relative-price variable is introduced into either national choice set.

The Home social planner commits to a complete contingent Home plan and maximizes $\mathcal{U}$, taking the complete contingent Foreign plan as given and subject to the common nonlinear two-country private-equilibrium system. The Foreign social planner symmetrically maximizes $\mathcal{U}^*$, taking the Home contingent plan as given and subject to the same common private-equilibrium system. The nominal interest rates i_t and i_t^* are choice variables and are subject to the ZLB in Home and Foreign, respectively. Both national Ramsey problems are solved from the *timeless perspective*. The lagged national Lagrange multipliers are initialized at their respective deterministic steady-state values; the initial conditions do not reset past commitments at the date of the tariff shock.

The national Ramsey first-order conditions are derived from these nonlinear problems. The resulting nonlinear Home and Foreign Ramsey first-order-condition systems, together with the common nonlinear two-country private-equilibrium system, are solved jointly at their mutual best response. This nonlinear Nash–Ramsey fixed point generates all two-country policy paths reported below.[7] The national Ramsey Lagrangians and first-order conditions, including the ZLB Kuhn–Tucker conditions, are reported in Appendix C. As in the SOE, the second-order welfare loss functions are not solved to obtain the policy allocation; they are introduced only when interpreting the nominal-interest-rate impulse responses.

[7] The nonlinear deterministic transition paths are computed in Dynare by `perfect_foresight_solver` with the mixed-complementarity algorithm `lmmcp`, imposing the two national ZLB constraints directly.

4.3 Calibration and Shocks

For the two-country numerical evaluation we retain the baseline parameterization in Table 1 and set the Home economic-size share to $\nu = 0.27$ to match the U.S. share of world GDP at current prices in U.S. dollars, using the April 2026 International Monetary Fund (IMF) World Economic Outlook Database [37].[8] Together with $\upsilon = 0.30$, this implies Home's import expenditure weight $\omega_F = \upsilon(1-\nu) = 0.219$. This 21.9 percent share is close to the 22.3 percent average ratio of U.S. imports of goods and services to personal consumption expenditures over 2000–2025 [41]. These values are used in the two-country welfare weights and in the baseline-parameterization IRFs reported below.

4.4 Welfare Relevant LOOP Gap

As in the SOE, the policy allocation is obtained from the nonlinear national Ramsey first-order-condition systems described in Sec. 4.2, not from the second-order welfare representation below. The second-order welfare representation is used only to identify the WR variables and to interpret the nominal-interest-rate responses. In the PPT environment, the Home period welfare cost is

$$\mathcal{L}_{H,t}^{PPT} \equiv \frac{1}{2}\left[\Lambda_y^{PPT}(y_t - y_t^{w,PPT})^2 + \Lambda_z^{PPT}(z_t - z_t^{w,PPT})^2 + \Lambda_{\pi,H}^{PPT}\pi_{H,t}^2\right], \tag{49}$$

whereas in the IPT environment it is

$$\begin{aligned}\mathcal{L}_{H,t}^{IPT} \equiv \frac{1}{2}\Big[&\Lambda_y^{IPT}(y_t - y_t^{w,IPT})^2 + \Lambda_\psi^{IPT}(\psi_t - \psi_t^{w,IPT})^2 + \Lambda_z^{IPT}(z_t - z_t^{w,IPT})^2 \\ &+\Lambda_{\psi^*}^{IPT}(\psi_t^* - \psi_t^{*,w,IPT})^2 + \Lambda_{\pi,H}^{IPT}\pi_{H,t}^2 + \Lambda_{\pi,F}^{IPT}\pi_{F,t}^2 \\ &+\Lambda_{\pi_H^*}^{IPT}(\pi_{H,t}^*)^2\Big].\end{aligned} \tag{50}$$

In Eqs. (49)–(50), $y_t^{w,\cdot}$ is the (logarithmic) WR output and $y_t - y_t^{w,\cdot}$ is the WR output gap. Likewise, $z_t^{w,\cdot}$ is the (logarithmic) WR *ex-tariff* relative border price of imports and $z_t - z_t^{w,\cdot}$ is the WR *ex-tariff* relative border price of imports gap. In the IPT environment, $\psi_t^{w,IPT}$ is the (logarithmic) Home WR LOOP gap and $\psi_t - \psi_t^{w,IPT}$ is the Home WR LOOP-gap deviation. The object $\psi_t^{*,w,IPT}$ is the WR reference value attached to the Foreign LOOP gap ψ_t^* in the *Home* welfare cost function. It is therefore distinct from the Foreign planner's own WR LOOP gap, denoted below by $\psi_t^{w*,IPT}$, which is obtained from the exact Home–Foreign mirror of the national welfare criterion. See Appendices E.2.1 and E.2.2 for details on $y_t^{w,PPT}$, $z_t^{w,PPT}$, $y_t^{w,IPT}$, $\psi_t^{w,IPT}$, $z_t^{w,IPT}$, and $\psi_t^{*,w,IPT}$.

Under PPT, the LOOP holds and Home welfare contains no LOOP-gap term. Under IPT, the Home welfare cost includes the Home WR LOOP-gap deviation, $\frac{1}{2}\Lambda_\psi^{IPT}(\psi_t - \psi_t^{w,IPT})^2$. An increase in the Home import tariff raises the *cum-tariff* relative purchaser price of imports for given *ex-tariff* prices, inducing inefficient underconsumption of imported goods relative to Home goods. For a given *ex-tariff* relative border price, a higher Home LOOP gap lowers the relative purchaser price of imports and mitigates this distortion. The HIAC thus gives the Home Ramsey planner an incentive to move the actual Home LOOP gap toward the Home WR LOOP gap. In the two-country economy, the Home WR LOOP gap also depends on the Foreign import tariff, the Foreign LOOP gap, and the international relative price, as shown in Eq. (51).

In the SOE, complete international risk sharing and Home goods-market clearing permit z_t to be eliminated from the reduced welfare representation. In the two-country economy, z_t remains explicit, and movements in Home relative demand feed back into Foreign demand and the common two-country equilibrium. Table 3 reports the corresponding baseline Home welfare weights in the two-country economy. The negative coefficients on the WR *ex-tariff* relative border price of imports gap term, $\Lambda_z^{PPT} = -0.0930$ and $\Lambda_z^{IPT} = -0.0978$, reflect the same strategic relative-price

[8] In the two-country economy, the baseline also includes $\nu = 0.27$.

consideration. In the national Ramsey problem, relative-price movements can shift expenditure toward Home goods, support Home output and income, and thereby improve Home welfare. This is the national relative-price externality. The negative sign therefore represents a signed strategic welfare contribution rather than a conventional positive stabilization penalty. Accordingly, we use the term "stabilization objective" for the positive-weight quadratic terms; the negative-weight completed-square term is reported as a signed strategic welfare contribution. This does not make the relative price an independent Ramsey choice variable; it remains an equilibrium outcome that monetary policy can influence strategically.

Table 3: Two-country Home Welfare Weights: Baseline Values

Environment	WR Variable	Weight	Value
PPT	$y_t - y_t^{w,PPT}$	Λ_y^{PPT}	2.077
	$z_t - z_t^{w,PPT}$	Λ_z^{PPT}	-0.093
	$\pi_{H,t}$	$\Lambda_{\pi,H}^{PPT}$	47.432
IPT	$y_t - y_t^{w,IPT}$	Λ_y^{IPT}	2.860
	$\psi_t - \psi_t^{w,IPT}$	Λ_ψ^{IPT}	0.406
	$z_t - z_t^{w,IPT}$	Λ_z^{IPT}	-0.098
	$\psi_t^* - \psi_t^{*,w,IPT}$	$\Lambda_{\psi^*}^{IPT}$	0.352
	$\pi_{H,t}$	$\Lambda_{\pi,H}^{IPT}$	65.313
	$\pi_{F,t}$	$\Lambda_{\pi,F}^{IPT}$	21.283
	$\pi_{H,t}^*$	$\Lambda_{\pi_H^*}^{IPT}$	13.519

Note: Coefficients are calculated from the structural-parameter expressions in Appendix E.2 and rounded to three decimals. Negative relative-price weights are signed strategic terms, not positive stabilization penalties.

The definition of the Home WR LOOP gap is given by:

$$\psi_t^{w,IPT} = \Gamma_{\text{own}}\tau_{M,t} + \Gamma_{\text{cross}}(\tau_{M,t}^* - \psi_t^*) + \Gamma_z z_t. \tag{51}$$

Here Γ_{own} is the structural loading of the Home tariff, Γ_{cross} is the structural loading of $\tau_{M,t}^* - \psi_t^*$, and Γ_z is the structural loading of the international relative price z_t. See Appendix E.2.2 for details on Γ_{own}, Γ_{cross}, and Γ_z. Under the baseline parameterization, $\Gamma_{\text{own}} = 0.364$, $\Gamma_{\text{cross}} = 0.060$, and $\Gamma_z = -0.016$. The positive Home-tariff coefficient reflects the same welfare mechanism as in the SOE: a higher Home LOOP gap mitigates tariff-induced underconsumption of imports by lowering their relative purchaser price. The HIAC therefore gives the Home Ramsey planner an incentive to move the actual Home LOOP gap toward the Home WR LOOP gap (Fig. 1).

An increase in the Foreign import tariff $\tau_{M,t}^*$ raises the *cum-tariff* relative price of Home goods faced by Foreign households, reducing their consumption of Home goods relative to Foreign goods. Holding Home output and the other relative prices fixed, a larger share of Home goods is then consumed by Home households. A higher Home LOOP gap lowers the *cum-tariff* relative price of Foreign goods faced by Home households, increasing their relative consumption of Foreign goods and mitigating the resulting inefficient overconsumption of Home goods in Home. The Home WR LOOP gap therefore rises to mitigate this distortion through the FAC (Fig. 1).

A decline in the Foreign LOOP gap works in the same direction as an increase in the Foreign import tariff: it raises the *cum-tariff* relative price of Home goods faced by Foreign households, reducing their consumption of Home goods relative to Foreign goods and thereby raising the Home WR LOOP gap through the FAC (Fig. 1).

With a unilateral Home tariff, $\tau_{M,t}^* = 0$, so the FAC term equals $-\Gamma_{\text{cross}}\psi_t^*$. Under the unilateral-tariff case, the Foreign LOOP gap rises on impact, making the FAC contribution negative. Under the reciprocal-tariff case, by contrast, the Foreign import tariff increase and the decline in the Foreign LOOP gap raise the Home WR LOOP gap through the FAC.

An increase in the Foreign import tariff raises the *cum-tariff* relative purchaser price of Home goods faced by Foreign households, inducing inefficient underconsumption of imports in Foreign. Under IPT, the same import-allocation mechanism gives the Foreign Ramsey planner an incen-

tive to move the actual Foreign LOOP gap toward the Foreign WR LOOP gap to mitigate this distortion. The Foreign WR LOOP gap is given by the Home–Foreign mirror of Eq. (51):

$$\psi_t^{w*,IPT} = \Gamma^*_{\text{own}}\tau^*_{M,t} + \Gamma^*_{\text{cross}}(\tau_{M,t} - \psi_t) + \Gamma^*_z z_t. \tag{52}$$

where the starred Γ coefficients are the exact Home–Foreign mirrors of the corresponding Home loadings in Eq. (51). Under the baseline parameterization, $\Gamma^*_{\text{own}} = 0.328$, $\Gamma^*_{\text{cross}} = 0.024$, and $\Gamma^*_z = 0.007$. Thus the positive Foreign-tariff coefficient reflects the same welfare mechanism as the positive Home-tariff coefficient in Eq. (51). The Home import tariff and the Home LOOP gap enter Eq. (52) because they affect Home households' consumption of Foreign goods relative to Home goods. A higher Home tariff reduces this relative consumption, resulting in inefficient overconsumption of Home goods in Home and underconsumption of Home goods in Foreign. The Foreign WR LOOP gap therefore rises to mitigate this distortion by encouraging Foreign households to consume more Home goods relative to Foreign goods. Conversely, a higher Home LOOP gap increases Home households' relative consumption of Foreign goods, reducing the need for this adjustment and lowering the Foreign WR LOOP gap.

Even in the two-country economy, deviations from the LOOP under the IPT environment can be interpreted as currency misalignment, as discussed in Sec. 3.4. For the Home economy, define the WR nominal exchange rate in the two-country economy as $e_t^w \equiv p_{F,t} - p^*_{F,t} + \psi_t^{w,IPT}$. It follows that the Home WR LOOP-gap deviation satisfies $\psi_t - \psi_t^{w,IPT} = e_t - e_t^w$. Analogously, define the Foreign WR nominal exchange rate by

$$e_t^{w*} \equiv p_{H,t} - p^*_{H,t} - \psi_t^{w*,IPT}. \tag{53}$$

The log-linearized definition of the Foreign LOOP gap, $\psi_t^* = p_{H,t} - e_t - p^*_{H,t}$, then implies

$$\psi_t^* - \psi_t^{w*,IPT} = -(e_t - e_t^{w*}). \tag{54}$$

Thus the same bilateral nominal exchange rate e_t is compared with two country-specific WR nominal exchange rates. Moving e_t toward e_t^w reduces the Home WR LOOP-gap deviation, whereas moving it toward e_t^{w*} reduces the Foreign WR LOOP-gap deviation. In general, e_t^w and e_t^{w*} differ because the Home and Foreign planners face different national welfare trade-offs.

4.5 Nominal Interest Rate Response to an Increase in the Import Tariff

4.5.1 Baseline Case

Figure 4(a) compares the Home responses to a unilateral tariff increase in the two-country economy, and Fig. 4(b) reports the Foreign responses. The solid blue lines represent IPT and the dotted green lines PPT. The main finding is unchanged by endogenizing Foreign monetary policy: *under IPT, the tariff-imposing Home planner cuts its nominal interest rate* (Panel 1). On impact, the Home rate falls by 14.85 basis points under IPT and rises by 8.14 basis points under PPT.

The Home Import-Allocation Channel (HIAC) remains operative in the two-country economy. Under IPT, an increase in the Home import tariff raises the *cum-tariff* relative purchaser price of imports, inducing inefficient underconsumption of imported goods relative to Home goods. The positive Home-tariff component of the Home WR LOOP gap reflects the incentive to mitigate this distortion by moving the actual Home LOOP gap toward its WR level; see Eq. (51), Panel 10 of Fig. 4(a), and Fig. 1.

The Foreign Allocation Channel (FAC) is an additional welfare consideration absent from the SOE, but its realized contribution under a unilateral Home tariff increase has the opposite sign from that in the case of a declining Foreign LOOP gap. Because the Foreign tariff is zero and the Foreign LOOP gap rises (Panel 4 of Fig. 4(b)), the FAC component $\Gamma_{\text{cross}}(\tau^*_{M,t} - \psi^*_t)$ is negative on impact. It therefore partly offsets the positive Home-tariff contribution to the Home WR LOOP gap, rather than reinforcing it.

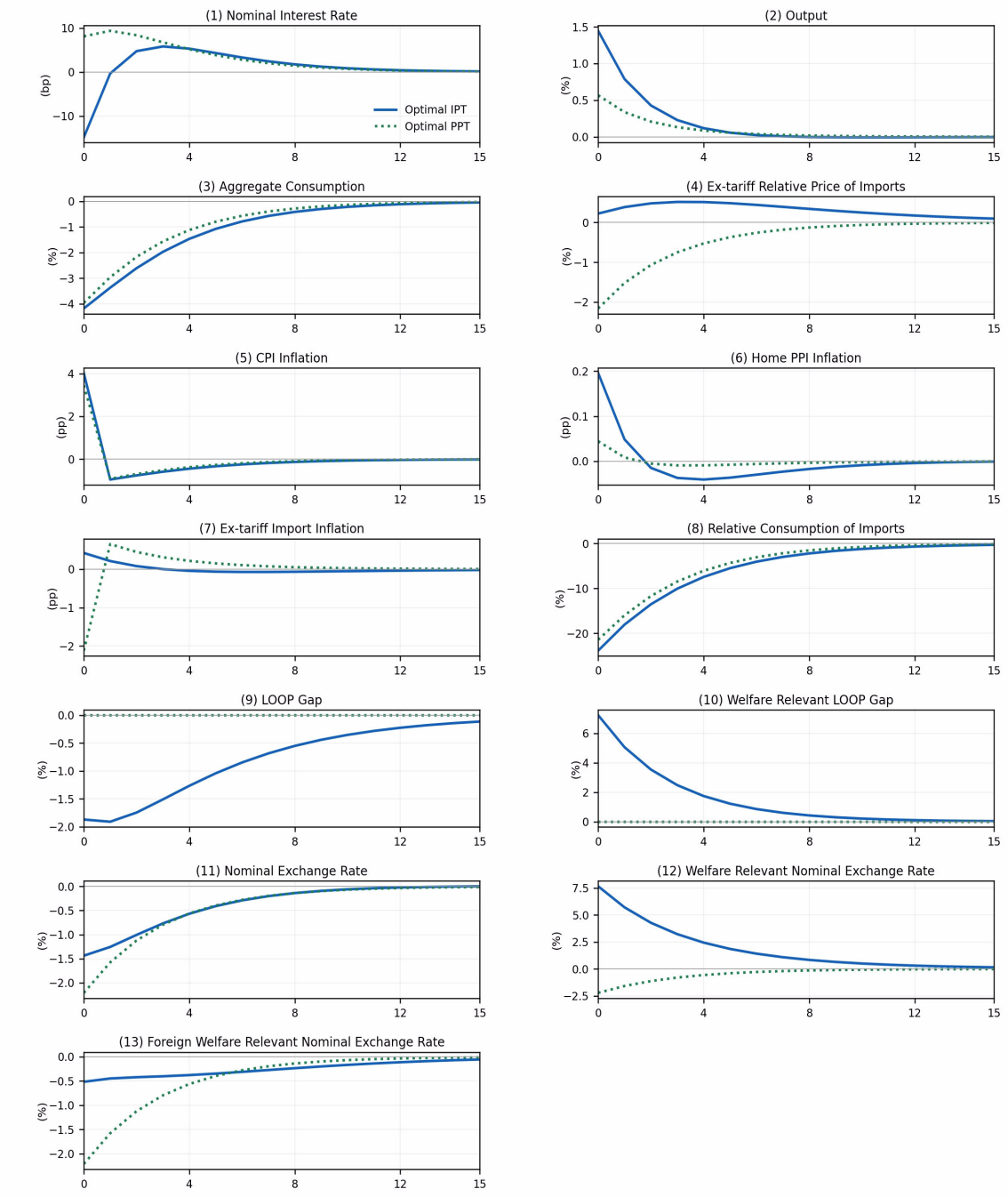


Figure 4: Two-country unilateral tariff: (a) Home responses under optimal PPT and IPT allocations following a 20-percentage-point Home import-tariff shock. Panel 1 reports the interest-rate deviation in basis points and Panel 8 the percentage deviation of relative consumption of imports from steady state. Other deviations and inflation rates are in percent or percentage points.

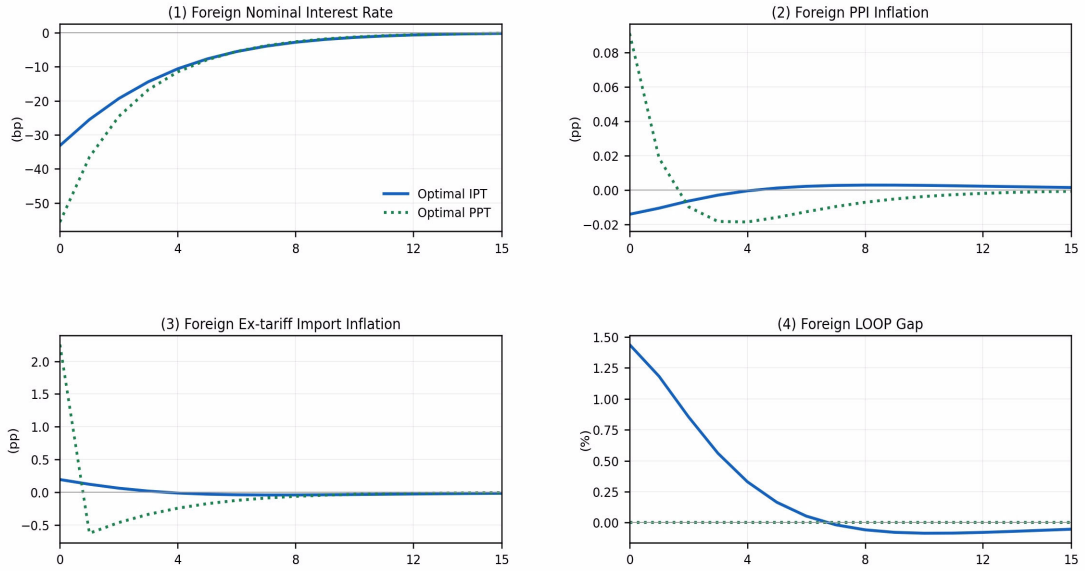


Figure 4: Two-country unilateral tariff: (b) Foreign optimal PPT and IPT responses. Panels 1–4 report Foreign nominal interest rate, Foreign PPI inflation, ex-tariff import inflation, and Foreign LOOP gap, respectively. The interest-rate deviation is in basis points and other series in percent or percentage points.

The optimal Home rate cut under IPT emerges from the full committed Ramsey trade-off, including the WR LOOP-gap deviation, Home PPI inflation, and *ex-tariff* import-price inflation in Eq. (50); it cannot be quantitatively attributed to the FAC or to the initial Home WR LOOP gap alone. In particular, the tariff-induced increase in the Home WR LOOP gap provides an incentive to raise the actual Home LOOP gap, not a separate accounting contribution to the equilibrium policy-rate cut. The entire anticipated path of the WR LOOP-gap deviation, together with the other welfare-relevant variables, matters for the optimal allocation.

The Home currency nevertheless appreciates in equilibrium, and the actual Home LOOP gap remains negative (Panels 9 and 11). An optimal rate cut exerts depreciation pressure *relative to alternative policy choices*; it need not produce equilibrium depreciation when Foreign monetary policy and international relative prices are endogenous. The ex-tariff import-inflation objective also limits the extent to which the Home planner seeks to adjust the LOOP gap.

PPT illustrates a different policy mechanism. Under PPT, an increase in the Home import tariff raises the *cum-tariff* relative price of imports, inducing expenditure switching toward Home goods (Eq. 46). Under the baseline substitution elasticity, the resulting increase in Home demand raises real marginal cost and puts upward pressure on Home PPI inflation, inducing a nominal interest-rate increase (Panel 1, Fig. 4(a)). Hence, in this baseline experiment, IPT and PPT yield opposite signs of the optimal Home policy-rate response. The next subsection shows why this sign comparison must not be generalized to every substitution elasticity.

4.5.2 High Substitution Case

Figures 5(a) and 5(b) report the unilateral tariff experiment when $\eta = 3$. The key IPT finding persists: Home initially cuts its nominal interest rate, by 47.81 basis points (Panel 1). The WR LOOP gap remains positive on impact, although the own-tariff loading Γ_{own} in Eq. (51) declines from 0.364 to 0.053 (Panel 10). At the same time, the FAC coefficient Γ_{cross} rises, and the welfare weight on the Home WR LOOP-gap deviation increases. However, the Foreign LOOP gap rises

under the unilateral tariff increase (Panel 4 of Fig. 5(b)). The higher FAC coefficient therefore amplifies its negative contribution to the Home WR LOOP gap. The Home WR LOOP gap is smaller on impact than in the baseline, even though the initial Home rate cut is larger. Under PPT, the LOOP gap is identically zero, and the Home welfare cost function contains no WR LOOP-gap deviation term. Under IPT, the HIAC gives the Home Ramsey planner an incentive to move the actual Home LOOP gap toward the Home WR LOOP gap to mitigate tariff-induced underconsumption of imports.

The comparative PPT result is different from the baseline. Under PPT, the stronger direct substitution away from Foreign goods raises demand for Home goods and puts upward pressure on Home PPI inflation. However, it also produces a much larger contraction in Foreign demand, inducing a sharp reduction in the Foreign policy rate. Foreign reaches the ZLB on impact (Panel 1, Fig. 5), and the Home currency appreciates strongly (Panel 11, Fig. 5). Because of the PPT environment, Home-currency appreciation immediately lowers the *cum-tariff* relative price of imports faced by Home households, inducing expenditure switching toward Foreign goods. This restrains Home demand and exerts downward pressure on Home PPI inflation. The Home planner therefore cuts its rate even under PPT, by 34.08 basis points on impact. Home PPI inflation nevertheless remains positive: the exchange- rate-induced pressure partially offsets, rather than fully eliminates, the direct effect of the Home tariff. A binding Foreign ZLB accompanies this policy interaction, but the sign of the Home PPT rate cut is not attributable to the ZLB constraint alone.

Under IPT, an increase in the Home import tariff gives the Home Ramsey planner an incentive, through the HIAC, to move the actual Home LOOP gap toward the Home WR LOOP gap to mitigate tariff-induced underconsumption of imports. This increase in the Home import tariff decreases the nominal interest rate in both countries, and the rate cut is larger in Foreign (Panel 1, Fig. 5(a) and (b)). The nominal exchange rate appreciates, and this appreciation raises the Foreign LOOP gap Ψ_t^* (Panel 4, Fig. 5(b)). With the Foreign tariff at zero, this increase makes the FAC contribution to the Home WR LOOP gap negative, partially offsetting the Home tariff's upward effect (Fig. 1). The Home IPT rate cut exceeds the PPT cut on impact by 13.73 basis points. Thus *the IPT rate cut is robust to higher substitution elasticity*, even though a policy-sign reversal relative to PPT is not. In both environments Home subsequently raises the policy rate temporarily above its steady-state level, while Home PPI inflation turns negative after its initial positive response (Panels 1 and 6). Thus the planner permits an initial increase in producer-price inflation and subsequently generates disinflation to restrain the cumulative change in the Home producer-price level.

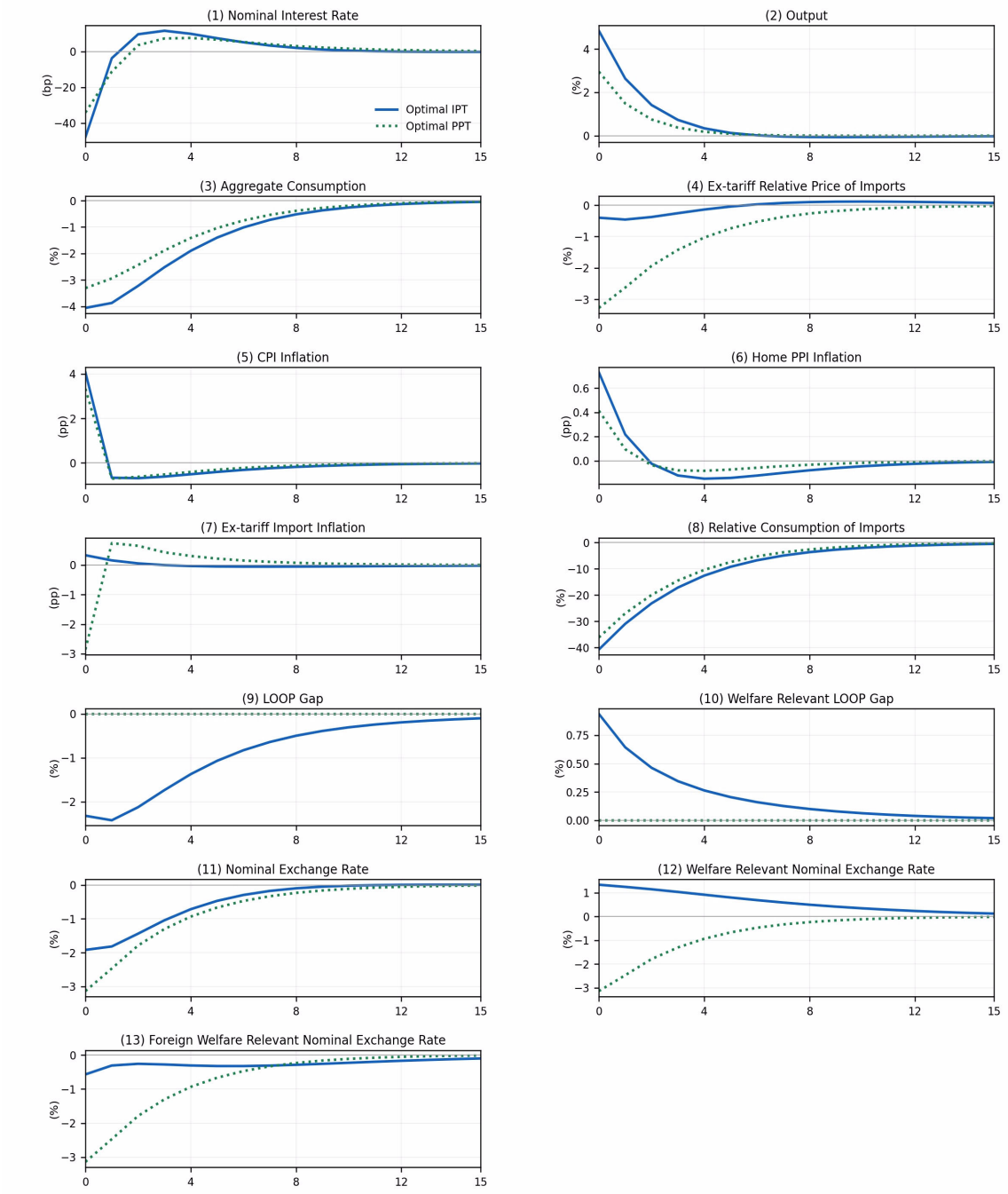


Figure 5: Two-country unilateral tariffs, High Substitution Case ($\eta = 3$): (a) Home PPT and IPT responses. Panel definitions and units match Fig. 4.

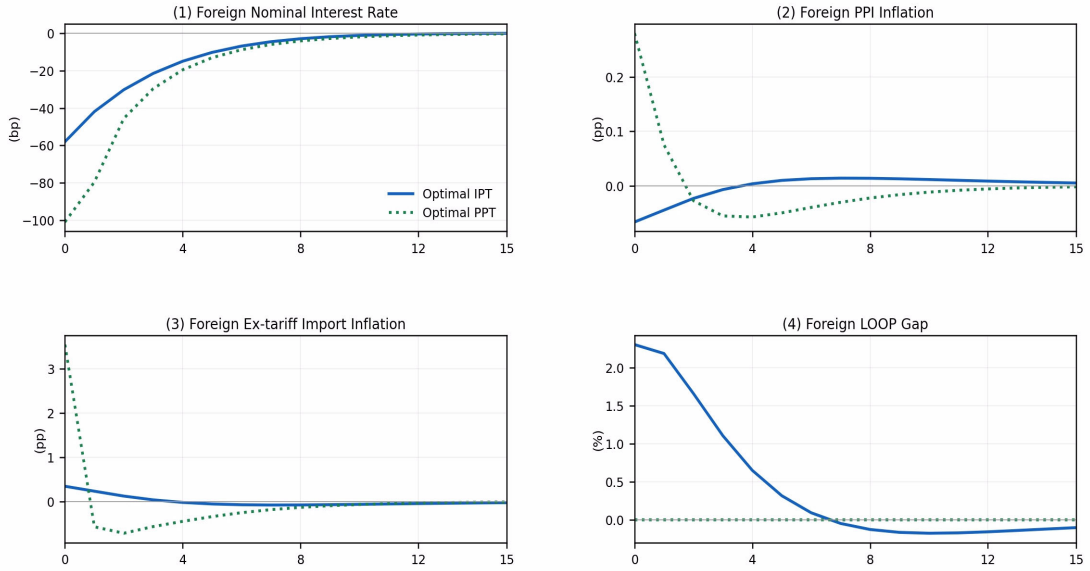


Figure 5: Two-country unilateral tariffs, High Substitution Case ($\eta = 3$): (b) Foreign PPT and IPT responses. Panel definitions and units match Fig. 4.

4.6 Reciprocal Tariff

This subsection asks how the optimal IPT accommodation changes when both countries levy reciprocal import tariffs. The PPT environment provides a comparison for isolating the consequences of nominal import-price rigidity. Home and Foreign receive equal, simultaneous shocks to $\tau_{M,t}$ and $\tau^*_{M,t}$; both shocks have persistence 0.7. All remaining parameters, the two national Nash–Ramsey problems, and the occasionally binding nominal-interest-rate ZLB are those of Sec. 4.

4.6.1 Baseline Case

Figure 6(a) and Fig. 6(b) report the optimal Home and Foreign responses to simultaneous 20-percentage-point increases in the two import tariffs. Under IPT, the Home planner continues to lower its nominal interest rate; unlike the unilateral experiment, the Home rate reaches the ZLB on impact. The Foreign tariff directly reduces Foreign demand for Home goods, while the Home tariff reduces Home demand for Foreign goods. The resulting loss of export demand gives both planners an incentive to ease monetary policy. Under IPT, the HIAC provides an additional incentive for monetary easing by mitigating tariff-induced underconsumption of imports. The export-demand motive is stronger for Home, whose production is more dependent on exports than Foreign production under the baseline country-size calibration. Accordingly, the Home planner eases more strongly than the Foreign planner in both pricing environments (Panel 1 of Fig. 6(a) and Panel 1 of Fig. 6(b)). The resulting equilibrium features Home currency depreciation under both PPT and IPT (Panel 11 of Fig. 6(a)), in contrast to the Home currency appreciation in the unilateral experiment in Sec. 4.5.1.

The magnitude of accommodation depends on import pricing behavior. Under IPT, local-currency import prices respond only gradually to exchange-rate movements, so changes in the *cum-tariff* relative price of imports induce less expenditure switching than under PPT. Under PPT, by contrast, exchange-rate movements are promptly reflected in the *cum-tariff* relative price of imports. Monetary accommodation can therefore generate a relatively strong additional expenditure-switching response, helping a country offset the decline in demand for its exports caused by the trading partner's tariff. Consistent with this mechanism, the optimal monetary eas-

ing is smaller under IPT than under PPT in both countries, except where the Home interest-rate responses are initially truncated by the ZLB.

The contrasting adjustment of the Home ex-tariff relative import price S_t (Panel 4 of Fig. 6(a)) illustrates the difference in pass-through. Under PPT, S_t responds strongly on impact and then declines, whereas its response under IPT is more gradual. Under IPT, the initial Home-currency depreciation raises the actual Home LOOP gap ψ_t (Panel 9). The Home WR LOOP gap also rises, reflecting the HIAC and the positive FAC contribution arising from the Foreign tariff increase and the decline in the Foreign LOOP gap (Eq. (51), Panel 10). The corresponding Foreign WR LOOP gap is given by Eq. (52). Under PPT, the LOOP holds identically, so the actual LOOP gaps vanish. At the baseline tariff rate, the Home nominal interest rate reaches the ZLB on impact under both PPT and IPT, and becomes positive in the following quarter. The observed interest-rate paths alone, however, do not reveal the full difference in the desired degree of monetary easing. Table 4 therefore also reports the minimum *notional* Home nominal interest rate, obtained from the corresponding Ramsey solution without the ZLB constraint. With a 20-percentage-point reciprocal tariff increase, the minimum notional rates are -30.84 basis points under PPT and -19.19 basis points under IPT. These are quarterly nominal interest-rate levels, not deviations from steady state. Thus PPT calls for a larger unconstrained rate cut, although the constrained Home policy rates coincide at zero on impact. The tariff-rate sensitivity of this result is examined in Sec. 4.6.3; Table 4 reports the corresponding notional rates and Home ZLB durations for all four baseline-elasticity tariff magnitudes.

Table 4: Reciprocal tariffs at $\eta = 1.5$: Home notional rates and ZLB durations

Initial tariff	PPT		IPT	
shock (%)	Min. notional rate (bp)	ZLB quarters	Min. notional rate (bp)	ZLB quarters
5	58.84	0	60.35	0
10	23.57	0	27.32	0
15	-6.01	1	1.11	0
20	-30.84	1	-19.19	1

Note: Notional rates are quarterly nominal interest-rate levels, in basis points (not deviations from their steady state); the deterministic steady-state level is 101.01 bp. A ZLB quarter is a quarter in which the constrained optimal nominal interest rate is zero.

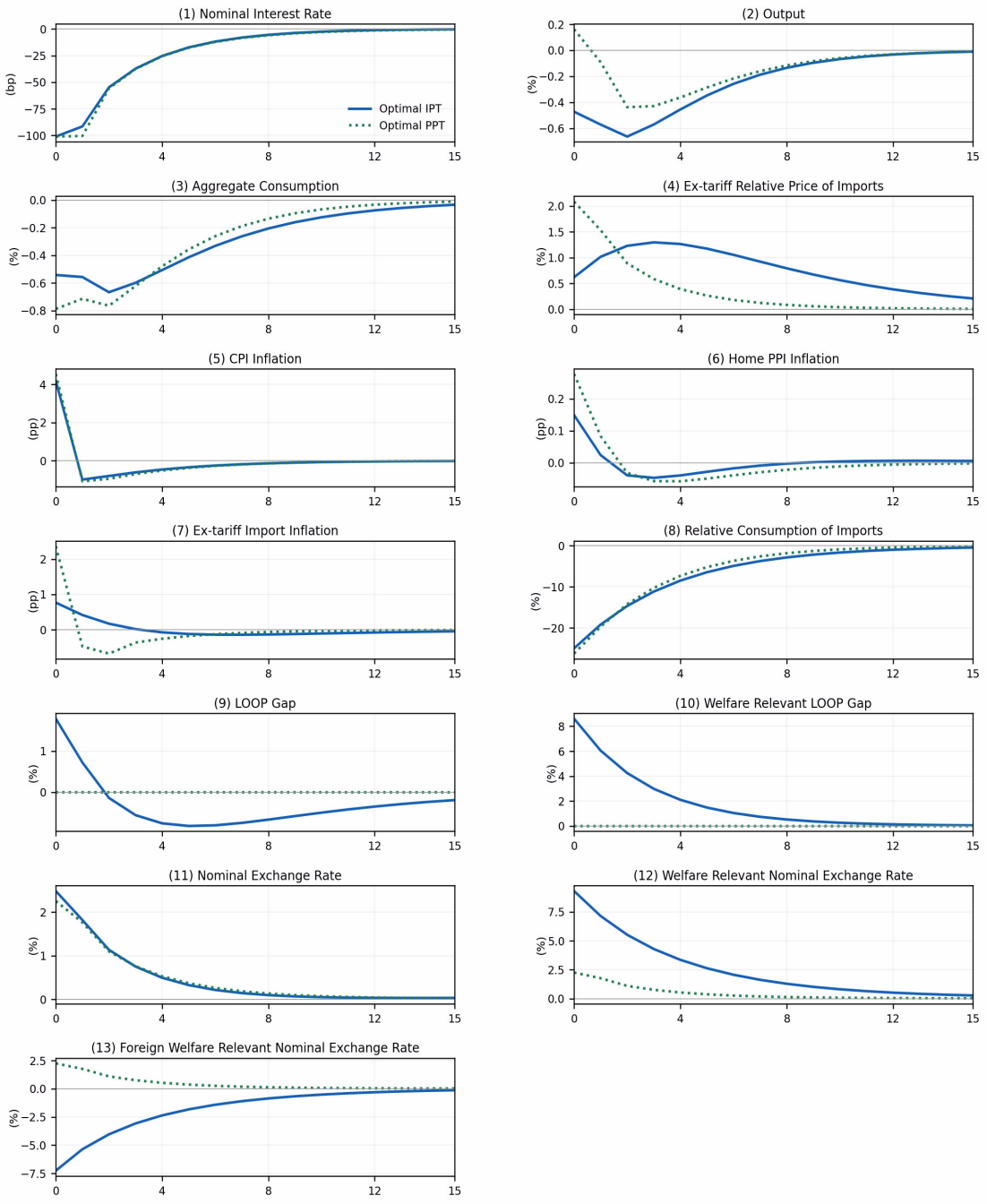


Figure 6: Two-country reciprocal tariffs: (a) Home responses under optimal PPT and IPT allocations following simultaneous 20-percentage-point import-tariff shocks. Panels 1–13 follow the variable selection of the unilateral baseline figure. Panel 1 is in basis points, Panel 8 is in percent deviation from steady state, and other variables are in percent or percentage points.

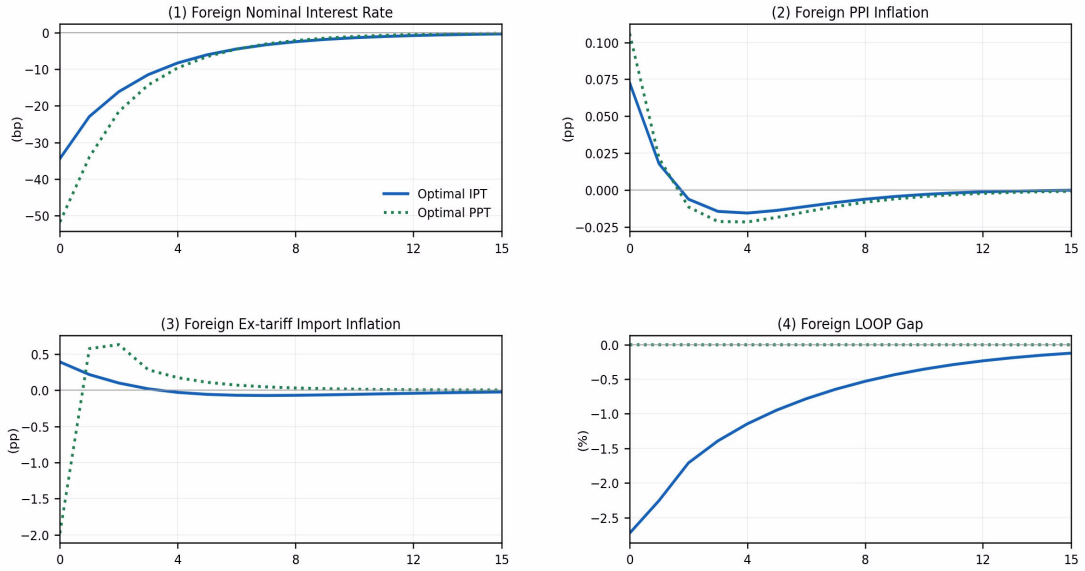


Figure 6: Two-country reciprocal tariffs: (b) Foreign optimal PPT and IPT responses to the baseline reciprocal tariff shock. Panels 1–4 report Foreign nominal interest rate, Foreign PPI inflation, ex-tariff import inflation, and Foreign LOOP gap, respectively. The interest rate is in basis points and other series in percent or percentage points.

4.6.2 High Substitution Case

We retain the reciprocal tariff shocks of Sec. 4.6.1 and raise only the Home–Foreign substitution elasticity from the baseline value $\eta = 1.5$ to $\eta = 3$, the High Substitution Case considered by Monacelli [39]. The experiment examines how stronger expenditure switching affects optimal monetary policy, holding all other structural parameters and the tariff paths unchanged. Figures 7(a) and 7(b) compare optimal PPT and IPT responses at $\eta = 3$; the corresponding baseline paths are reported in Figs. 6(a) and 6(b).

The IPT policy response remains a substantial rate cut. The Home nominal interest rate reaches the ZLB for three quarters, rather than one in the baseline, whereas the Foreign IPT nominal rate remains positive (Panel 1 of Figs. 7(a) and 7(b)). The Home rate cut reflects both the Home WR LOOP gap and the external-demand effects of reciprocal tariffs.

For comparison, under PPT a higher substitution elasticity strengthens expenditure switching in both countries. The Home tariff shifts Home expenditure from Foreign goods toward Home goods, while the Foreign tariff reduces Foreign demand for Home goods. The latter effect creates an incentive for Home monetary easing: under PPT, Home-currency depreciation lowers the *cum-tariff* relative price of Home goods faced by Foreign households, supporting their demand for Home goods. This incentive operates against the inflationary pressure arising from the Home tariff's direct expenditure-switching effect. In the High Substitution Case, the optimal response is nevertheless a sufficiently large rate cut for the Home nominal interest rate to reach the ZLB for four quarters, compared with one quarter in the baseline (Panel 1 of Fig. 7(a)). Foreign also cuts its nominal rate to the ZLB for one quarter, whereas its rate remains positive in the baseline (Panel 1 of Fig. 7(b)). The Home currency depreciates on impact under PPT (Panel 11).

Under IPT, the Home welfare cost function remains the two-country criterion in Eq. (50), with its coefficients and welfare-relevant (WR) reference values re-evaluated at $\eta = 3$. Table 6 reports the corresponding welfare weights, and Table 7 the loadings of the Home WR LOOP gap. A key change is the stronger *Foreign Allocation Channel* (FAC). In Eq. (51), the coefficient on

$\tau^*_{M,t} - \psi^*_t$ rises from 0.060 in the baseline to 0.122 at $\eta = 3$. An increase in $\tau^*_{M,t} - \psi^*_t$ raises the *cum-tariff* relative price of Home goods faced by Foreign households, reducing their consumption of Home goods relative to Foreign goods. Greater substitutability amplifies this reduction in relative consumption and the resulting inefficient shift of Home-good consumption toward Home households. In this reciprocal-tariff experiment, the Foreign LOOP gap falls on impact under both elasticities (Panel 4 of Fig. 7(b)); together with the positive Foreign tariff, its negative sign makes the FAC contribution positive. The increase in the FAC loading strengthens this contribution without requiring a substantially larger initial fall in the Foreign LOOP gap.

At the same time, the own-tariff loading Γ_{own} declines from 0.364 to 0.053. Consequently, the *total* initial increase in the Home WR LOOP gap is smaller at $\eta = 3$ than under the baseline, even though the Foreign-side contribution is larger (Panel 10 of Fig. 7(a)). The welfare weight on the Home WR LOOP-gap deviation, Λ_ψ, nevertheless rises from 0.406 to 1.206 (Table 6). A higher elasticity makes relative demands more sensitive to prices, increasing the welfare consequences of deviations from the WR allocation. The weight on ex-tariff import-price inflation also rises, limiting the extent to which monetary accommodation can be used to adjust the actual LOOP gap.

Consistent with these competing considerations, the initial actual Home LOOP gap increases more under IPT at $\eta = 3$ than in the baseline, while the Home currency depreciates (Panels 9 and 11 of Fig. 7(a)). The Home ZLB episode lasts longer under IPT at $\eta = 3$ because the higher elasticity of substitution amplifies expenditure switching. Foreign monetary policy also eases under IPT, but its nominal rate remains positive, unlike under PPT. Thus, at $\eta = 3$, reciprocal tariffs generate a Home ZLB episode in both pricing environments and a Foreign ZLB episode only under PPT. The constrained Home policy rates alone cannot rank the desired unconstrained interest-rate reductions. We therefore also solve the *same* nonlinear Nash–Ramsey systems with the Home and Foreign ZLB constraints removed, leaving all structural parameters, tariff paths, and timeless-perspective initial conditions unchanged. Table 5 reports the minimum quarterly notional interest-rate *levels*, alongside the constrained ZLB durations. Under PPT the notional Home rate reaches −167.70 basis points, compared with −114.78 basis points under IPT. For Foreign the minima are −39.57 and 30.71 basis points, respectively. Hence PPT requires more accommodation in both economies and produces a Foreign ZLB episode, even though the HIAC provides an additional welfare-based incentive for Home monetary easing under IPT.

Table 5: Reciprocal tariffs at $\eta = 3$: notional minima and ZLB durations

	PPT		IPT	
Economy	Min. notional rate (bp)	ZLB quarters	Min. notional rate (bp)	ZLB quarters
Home	−167.70	4	−114.78	3
Foreign	−39.57	1	30.71	0

Note: Minimum notional rates are quarterly nominal interest-rate *levels*, not deviations from steady state; the steady-state nominal rate is 101.01 bp. Notional solutions omit both national ZLB constraints. ZLB durations refer to the corresponding constrained Nash–Ramsey allocations. The tariff increases by 20 percentage points in each country and follows the baseline persistence 0.7.

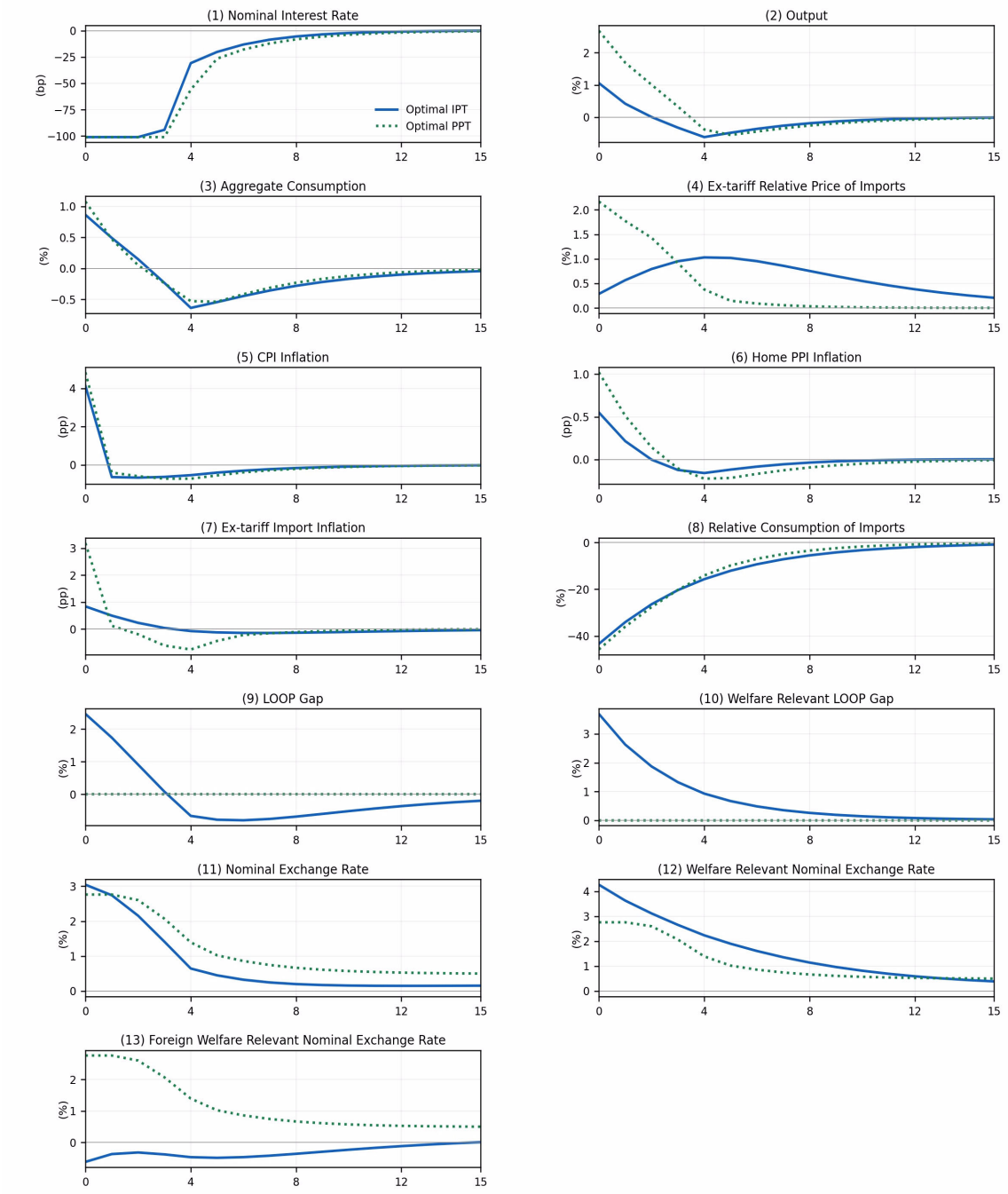


Figure 7: Two-country reciprocal tariffs, High Substitution Case ($\eta = 3$): (a) Home PPT and IPT responses. Panel definitions and units match Fig. 6.

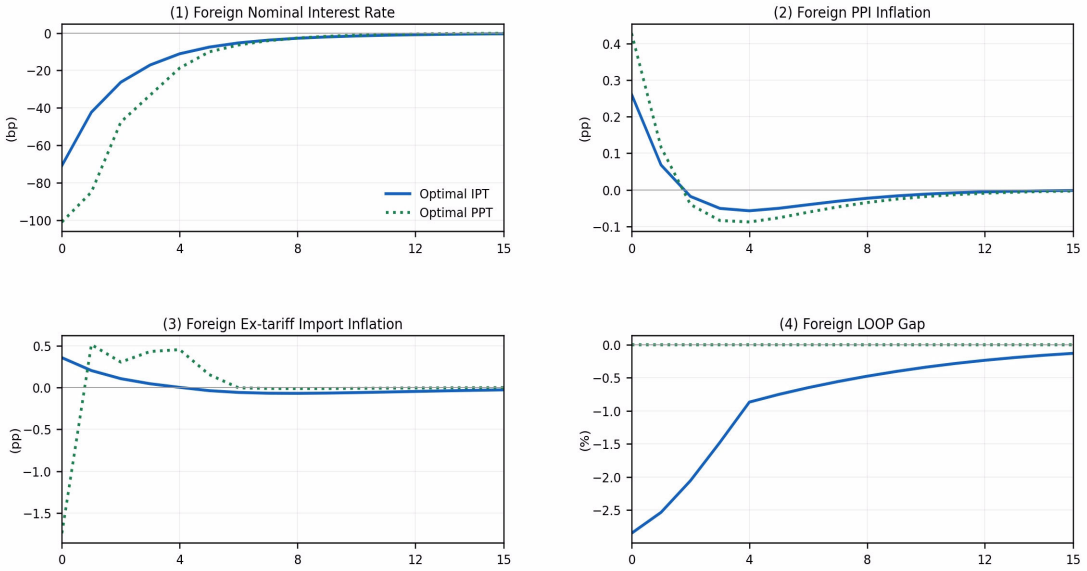


Figure 7: Two-country reciprocal tariffs, High Substitution Case ($\eta = 3$): (b) Foreign PPT and IPT responses. Panel definitions and units match Fig. 6.

4.6.3 ZLB Episode Facing Import Tariff

To assess how the reciprocal-tariff results vary with the magnitude of the shock, we reduce the initial import-tariff increase in each country from 20 to 15, 10, and 5 percentage points, leaving all other features of the model unchanged. Larger reciprocal tariff shocks generally call for larger Home interest-rate reductions and produce greater Home currency depreciation in both pricing environments.

Table 4 compares the minimum *notional* Home interest rates with the duration of the ZLB in the constrained solutions. At a 10-percentage-point tariff increase, the minimum notional rates remain positive under both PPT (23.57 basis points) and IPT (27.32 basis points), and neither environment generates a ZLB episode. At 15 percentage points, the PPT notional minimum falls to -6.01 basis points, whereas the IPT minimum remains positive at 1.11 basis points. Accordingly, the Home ZLB binds for one quarter only under PPT. With a 20-percentage-point shock, the minima fall to -30.84 and -19.19 basis points, respectively, and the ZLB binds for one quarter in both environments. The notional rates therefore confirm that the need for monetary accommodation is stronger under PPT than under IPT, even when the constrained rates are identical on impact.

The absence of a ZLB episode for a 10-percentage-point tariff shock (Table 4) is relevant when comparing the 20-percentage-point baseline experiment with the recent U.S. experience. The Budget Lab at Yale [16] estimated that the average effective U.S. tariff rate was approximately 11–12 percent in August 2025, compared with 2.4 percent before the 2025 tariff announcements. The increase in the effective rate was therefore closer to 10 percentage points than to the 20-percentage-point baseline shock. We do not identify a direct empirical counterpart of the model's symmetric reciprocal-tariff shock from these aggregates.

Consequently, even if the Federal Reserve had implemented the optimal monetary policy prescribed by our model, a reciprocal-tariff shock of roughly 10 percentage points would not have generated a ZLB episode under either pricing environment. Actual retaliation was uneven, and other features of U.S. policy and the international environment differ from those of the experiment. Nevertheless, the absence of a U.S. ZLB episode to date cannot, by itself, establish that the model's ZLB result under a substantially larger reciprocal tariff shock is implausible or merely theoretical.

5 Conclusion

This paper examines the optimal monetary response to import tariffs under imperfect pass-through (IPT). Our central finding is that monetary easing is optimal under IPT in both the small open economy and the two-country economy, even when Home and Foreign goods become more substitutable. Under perfect pass-through (PPT), by contrast, the Ramsey planner raises the nominal interest rate in the small open economy.

The Home Import-Allocation Channel (HIAC) explains the welfare rationale for this result. An increase in the Home import tariff raises the *cum-tariff* relative purchaser price of imports, inducing inefficient underconsumption of imported goods relative to Home goods. This distortion raises the welfare-relevant (WR) law-of-one-price (LOOP) gap, giving the Ramsey planner an incentive to raise the actual LOOP gap toward its WR level.

While an increase in the LOOP gap puts upward pressure on *ex-tariff* import-price inflation, Calvo price rigidity slows the adjustment of the import price to the higher Home-currency border price. Consequently, nominal depreciation can raise the actual LOOP gap despite the importers' incentive to increase their selling prices. For a given *ex-tariff* relative border price, a higher LOOP gap mitigates tariff-induced underconsumption of imported goods relative to Home goods and thereby improves economic welfare. Through uncovered interest parity (UIP), a rate cut exerts depreciation pressure on the Home currency and tends to raise the actual LOOP gap under IPT. This mechanism helps explain why a nominal interest rate cut can be optimal under IPT following an increase in the import tariff, subject to the competing inflation-stabilization objectives.

Import-price stickiness is not merely a source of nominal distortion. It also allows the Ramsey planner to influence the LOOP gap through monetary policy and thereby mitigate tariff-induced underconsumption of imported goods.

Appendix

A Analytical Benchmarks and Price-Rigidity Sensitivity

All allocations reported above solve the nonlinear Ramsey first-order conditions; the welfare cost functions are not used to compute them. To isolate the mechanism behind the IPT rate cut, this Appendix section characterizes the sign of the impact response analytically in two benchmarks, each of which removes one of the two nominal rigidities: producer-price stickiness or import-price stickiness. The first step is an exact property of the nonlinear SOE.

Lemma 1. *In the SOE under IPT, with a constant Foreign price P_F^* and perfect foresight after the tariff innovation, the nonlinear equilibrium satisfies $1 + i_t = \beta^{-1}\mathcal{E}_{t+1}/\mathcal{E}_t$. Hence*

$$\hat{\imath}_0 \equiv \log(1 + i_0) - \log(1/\beta) = e_1 - e_0 = (\psi_1 - \psi_0) + \pi_{F,1}. \tag{A.1}$$

Proof. Since $Z_t = \mathcal{E}_t P_F^*/P_{H,t}$, Eq. (26) gives $C_t P_t^{IPT} = \mathcal{E}_t P_F^*$. Substituting this into the IPT Euler equation yields $(1 + i_t)^{-1} = \beta C_t P_t^{IPT}/(C_{t+1} P_{t+1}^{IPT}) = \beta \mathcal{E}_t/\mathcal{E}_{t+1}$. The last equality in Eq. (A.1) follows from the exact definition $\psi_t = e_t + p_F^* - p_{F,t}$. □

Thus the optimal rate is cut on impact if and only if the Home currency is expected to appreciate between dates 0 and 1. The second step removes producer-price stickiness.

Proposition 1. *Consider the SOE under IPT with flexible producer prices ($\theta = 0$) and the tariff process $\tau_{M,t} = \rho_\tau \tau_{M,t-1} + \epsilon_{\tau,t}$ with $\rho_\tau \in [0, 1)$, hit at date 0 by an unanticipated innovation $\epsilon_{\tau,0} > 0$ from the zero-tariff steady state, with perfect foresight thereafter. Consider the first-order approximation of the nonlinear timeless-perspective Ramsey first-order conditions, and suppose that the ZLB does not bind. Let $Q \equiv Q^{SOE,IPT}$ and $B \equiv B^{SOE,IPT}$ be as in Eq. (E.21), $\Lambda_{\pi,F} \equiv$*

$\Lambda_{\pi,F}^{SOE,IPT}$ *as in Eq. (E.22), and* p_* *and* k_* *as in Eq. (E.16). Define*

$$g \equiv \frac{k_*}{p_* + \varphi}, \qquad \tilde{\Lambda}_\psi \equiv Q_{22} - 2Q_{12}g + Q_{11}g^2,$$
$$\tilde{\Gamma}_{\text{own}} \equiv \frac{B_2 - (B_1 + Q_{12})g + Q_{11}g^2}{\tilde{\Lambda}_\psi}, \qquad \vartheta \equiv \frac{\kappa_M \Lambda_{\pi,F}}{\tilde{\Lambda}_\psi}.$$

and let $\delta \in (0,1)$ *be the root of* $\beta\delta^2 - (1 + \beta + \kappa_M\vartheta)\delta + 1 = 0$ *inside the unit circle. Then*

$$\frac{\hat{\imath}_0}{\tau_{M,0}} = -\tilde{\Gamma}_{\text{own}}(1 - \rho_\tau) + \frac{1 - \vartheta}{\vartheta} \frac{(1 - \delta)(1 - \beta\delta)(\delta + \rho_\tau - 1)}{1 - \beta\delta\rho_\tau} \left(\tilde{\Gamma}_{\text{own}} + \frac{1}{\varepsilon_M - 1} \right). \tag{A.2}$$

In particular, the optimal (notional) nominal interest rate falls on impact whenever $\tilde{\Gamma}_{\text{own}} > 0$ *and* $(\vartheta - 1)(\delta + \rho_\tau - 1) \geq 0$*, where*

$$\delta + \rho_\tau \geq 1 \iff \kappa_M\vartheta \leq \frac{\rho_\tau(1 - \beta + \beta\rho_\tau)}{1 - \rho_\tau}.$$

The proof is given in Appendix F.1. It differentiates the nonlinear Ramsey first-order conditions directly; no quadratic welfare criterion is minimized. The coefficients Q, B, and $\Lambda_{\pi,F}$ appear in Proposition 1 because the coefficient matrix of the differentiated first-order conditions is the Hessian of the nonlinear Ramsey Lagrangian evaluated at the steady-state multipliers (Lemma 2), which is the object computed in Appendix E.1.[9]

The first term captures the tariff-induced change in the WR LOOP gap and its subsequent decay. When $\tilde{\Gamma}_{\text{own}} > 0$, the tariff raises the WR LOOP gap; because it subsequently declines as the tariff shock dissipates, the Home currency is expected to appreciate after impact, and Lemma 1 translates this expected appreciation into a rate cut. The second term captures the interaction between the HIAC-induced incentive to narrow the WR LOOP-gap deviation and the competing objective of stabilizing import-price inflation: under commitment, the planner sets $\psi_t - \tilde{\Gamma}_{\text{own}}\tau_{M,t} = -\vartheta p_{F,t}$ (Appendix F.1). When $\vartheta \geq 1$, the planner weighs import-price stability more heavily than closing the LOOP-gap deviation; when $\delta + \rho_\tau \geq 1$, the import price level is still rising at date 1. Together, the two conditions make the second term reinforce the cut. Because $\kappa_M\Lambda_{\pi,F} = -\varepsilon_M\bar{C}^{1-\eta}\Theta_M^{AS}$ by Eq. (E.22), ϑ does not depend on θ_M; importer stickiness enters only through κ_M in the persistence condition, which is easier to satisfy when import prices are stickier. Under PPT with $\theta = 0$, by contrast, the producer block, risk sharing, and goods-market clearing pin down y_t and z_t for any monetary policy, so there is no welfare motive for any interest-rate response. The IPT rate cut in Proposition 1 is therefore generated by the importer-pricing mechanism alone: it arises without any producer-price rigidity.

At the baseline values of the remaining parameters, $\tilde{\Gamma}_{\text{own}} = 0.414$, $\vartheta = 2.79$, $\delta = 0.687$, and $\kappa_M\vartheta = 0.145$, well below the bound 1.640; for $\eta = 3$, the corresponding values are 0.250, 1.54, 0.757, and 0.080. Both conditions of Proposition 1 therefore hold. In this benchmark, a one-percentage-point tariff increase lowers the notional net nominal rate on impact by 16.25 basis points for $\eta = 1.5$ and by 8.83 basis points for $\eta = 3$.[10]

The opposite benchmark removes import-price stickiness instead.

Proposition 2. *Consider the SOE under IPT with flexible import prices (*$\theta_M = 0$*) and the fixed importer subsidy* $\tau = 1/\varepsilon_M$*. For any tariff path* $\{\tau_{M,t}\}$*, the nonlinear Ramsey allocation under IPT, including the nominal-interest-rate path and the ZLB conditions, coincides with the nonlinear Ramsey allocation under PPT for the tariff path* $\{\tilde{\tau}_{M,t}\}$*, where*

$$\tilde{\tau}_{M,t} \equiv \frac{\varepsilon_M}{\varepsilon_M - 1}\,\tau_{M,t}. \tag{A.3}$$

[9] As a check, differentiating the nonlinear Ramsey first-order conditions directly at the baseline calibration ($\theta = 0.8$) gives $\partial \log(1 + i_0)/\partial\tau_{M,0} = -0.025904$ for $\eta = 1.5$ and -0.05884 for $\eta = 3$, which coincide with the solution of the linear–quadratic problem. With $\theta = 10^{-8}$, the same direct differentiation gives -0.160890975 for $\eta = 1.5$ and -0.087397486 for $\eta = 3$, against -0.160890996 and -0.087397488 from Eq. (A.2).

[10] To first order, the change in the net rate is $di_0 = \beta^{-1}d\hat{\imath}_0$, so these values equal β^{-1} times Eq. (A.2) evaluated at $\tau_{M,0} = 0.01$.

Proposition 2 is exact and does not rely on any approximation; its proof is in Appendix F.2. With flexible import prices, Eq. (33) pins down the LOOP gap as a function of the tariff, independently of monetary policy. Although the tariff-induced distortion in the relative consumption of imports remains, monetary policy cannot mitigate this distortion by adjusting the LOOP gap. The HIAC therefore does not induce a monetary-policy response through LOOP-gap adjustment in this limiting case. The opportunistic-pricing incentive remains, however, and it magnifies the effective tariff by the factor $\varepsilon_M/(\varepsilon_M-1)$. If the importer subsidy were instead indexed to the tariff, $\tau_t = (1+\tau_{M,t})/\varepsilon_M$, the ex-tariff import price would equal the border price and the IPT economy with $\theta_M = 0$ would coincide with PPT for the same tariff path. The sign of the IPT response with flexible import prices is therefore the sign of a PPT response, which can be characterized in closed form to first order.

Corollary 1. *Under the assumptions of Proposition 2, let the tariff follow $\tau_{M,t} = \rho_\tau \tau_{M,t-1} + \epsilon_{\tau,t}$ with $\rho_\tau \in [0,1)$, hit at date 0 by an unanticipated innovation $\epsilon_{\tau,0} > 0$ from the zero-tariff steady state, with perfect foresight thereafter, and consider the first-order approximation of the nonlinear timeless-perspective Ramsey first-order conditions with a non-binding ZLB. Let p_*, k_*, and g be as in Proposition 1, and define*

$$\omega \equiv \frac{B^{SOE,PPT}}{Q^{SOE,PPT}}, \qquad \chi \equiv \frac{\kappa_H(p_* + \varphi)\Lambda_{\pi,H}^{SOE,PPT}}{Q^{SOE,PPT}},$$

with $Q^{SOE,PPT}$ and $B^{SOE,PPT}$ as in Eq. (E.21) and $\Lambda_{\pi,H}^{SOE,PPT}$ as in Eq. (E.22). Let $\delta_H \in (0,1)$ be the root of $\beta\delta^2 - [1+\beta+\kappa_H(p_*+\varphi)\chi]\delta + 1 = 0$ inside the unit circle. Then

$$\frac{\hat{\imath}_0^{IPT}}{\tau_{M,0}} = \frac{\varepsilon_M}{\varepsilon_M - 1}\left[(k_* - p_*\omega)(1-\rho_\tau) + \frac{1-p_*\chi}{\chi}(\omega - g)\frac{(1-\delta_H)(1-\beta\delta_H)(\delta_H+\rho_\tau-1)}{1-\beta\delta_H\rho_\tau}\right]. \quad \text{(A.4)}$$

The bracket is $\hat{\imath}_0^{PPT}/\tilde{\tau}_{M,0}$. The optimal (notional) nominal interest rate under IPT with $\theta_M = 0$ therefore rises on impact if and only if the bracket is positive. A sufficient condition is $p_*\omega < k_*$ together with $(1-p_*\chi)(\omega-g)(\delta_H+\rho_\tau-1) \geq 0$.

Equation (A.4) mirrors Eq. (A.2). Its first term is the decay of the tariff-induced change in the WR *ex-tariff* relative border price, $z_t^w = (p_*\omega - k_*)\tilde{\tau}_{M,t}$. When $p_*\omega < k_*$, the tariff calls for a WR real appreciation; as it decays, the Home currency is expected to depreciate, and Lemma 1 translates this into a rate increase. In Proposition 1, by contrast, the tariff raises the WR LOOP gap and calls for a WR depreciation whose decay implies a rate cut. The second term reflects the anchoring of the producer price level under commitment, $y_t - \omega\tilde{\tau}_{M,t} = -\chi p_{H,t}$ (Appendix F.2).

At the baseline values of the remaining parameters, $p_*\omega - k_* = -0.039$, $\omega = 0.060$, $g = 0.029$, $\chi = 3.71$, and $\delta_H = 0.477$ for $\eta = 1.5$; for $\eta = 3$, the corresponding values are -0.033, 0.397, 0.085, 3.61, and 0.500. The bracket in Eq. (A.4) is positive for both elasticities, although the simple sufficient condition does not hold because $\delta_H + \rho_\tau > 1$. To first order, a 20-percentage-point tariff with flexible import prices therefore raises the notional net rate on impact by 28.8 basis points for $\eta = 1.5$ and by 12.9 basis points for $\eta = 3$. The exact nonlinear values implied by Proposition 2, obtained by solving PPT with the effective tariff $\tilde{\tau}_{M,0} = 0.2714$, are +21.49 and +1.90 basis points, respectively. The sign is not universal: for $\eta = 3$ and $\rho_\tau = 0.9$, the bracket is negative, because PPT itself calls for a cut there.

Between the Two Benchmarks. The main purpose of the parameter map is to connect the two analytical limits and examine their implications in the fully nonlinear Ramsey model for a finite tariff shock. Proposition 1 establishes conditions for a cut in the limit of flexible producer prices with sticky import prices; Proposition 2 establishes exact equivalence to PPT with a magnified tariff at the flexible-import-price boundary. At our calibration, the latter boundary calls for a rate increase. Figures 8 and 9 connect these two limits. They report the fully nonlinear Ramsey responses to a 20-percentage-point tariff with $\rho_\tau = 0.7$ on a grid of producer and importer Calvo

parameters, $\theta \in \{0.1, \ldots, 0.8\}$ and $\theta_M \in \{0, 0.1, \ldots, 0.8\}$, for $\eta = 1.5$ and $\eta = 3$. The ZLB is not imposed, so the figures report the notional rate. For $\theta_M > 0$, each node solves the nonlinear IPT Ramsey problem. For $\theta_M = 0$, each node solves the PPT Ramsey problem with the effective tariff $\tilde{\tau}_{M,0} = (3.8/2.8) \times 0.20 = 0.2714$, which is exactly equivalent by Proposition 2. All 144 nodes converge, with maximum residuals below 2×10^{-12}. At 10 of the 144 nodes (4 for $\eta = 1.5$ and 6 for $\eta = 3$, all with $\theta \leq 0.2$), the optimal notional rate becomes negative at some date; at all other nodes the ZLB would not bind, so the results coincide with those under the ZLB.

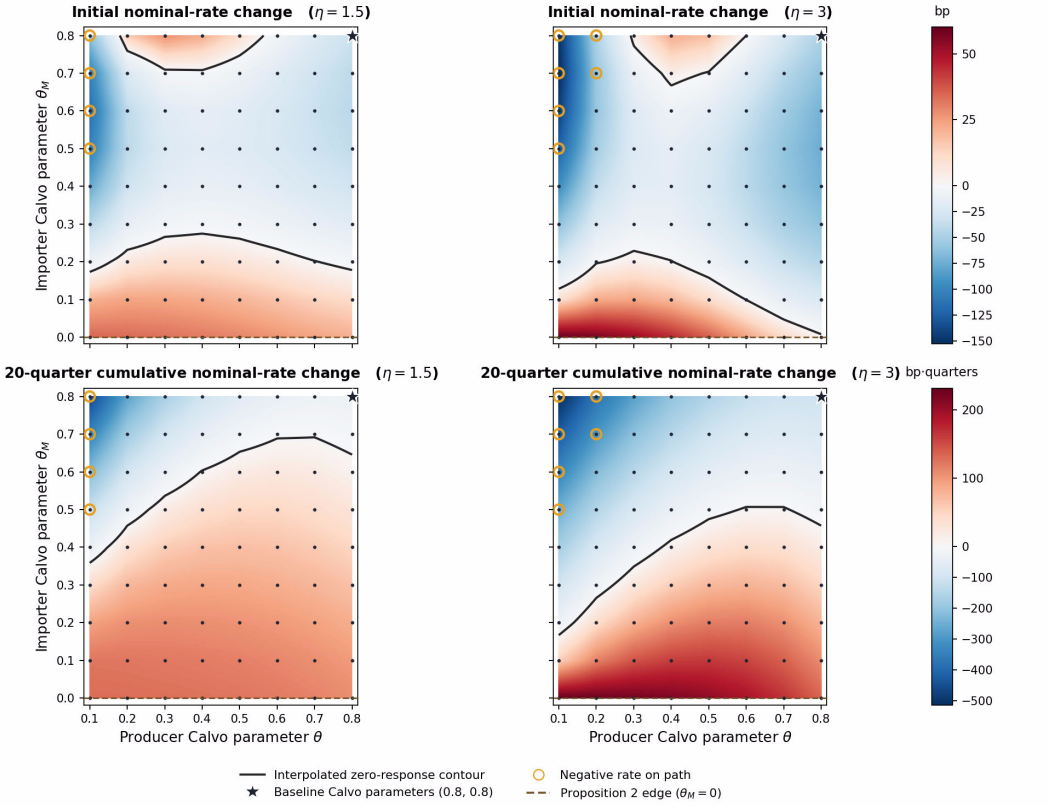


Figure 8: Small open economy under IPT: optimal nominal interest-rate response to a 20-percentage-point Home import tariff across producer and importer price stickiness ($\rho_\tau = 0.7$, ZLB not imposed). Top row: initial response, $10{,}000(i_0 - i_{ss})$, in basis points. Bottom row: cumulative response over the first 20 quarters, in basis-point-quarters. Left: $\eta = 1.5$; right: $\eta = 3$. Dots mark the 144 solved parameter combinations; colors and the black zero contours are bilinear interpolations between them. The row $\theta_M = 0$ (dashed) is solved as PPT with the effective tariff 0.2714, which is exactly equivalent by Proposition 2; it is not the PPT response to a 20-percentage-point tariff. The star marks the baseline Calvo-parameter values $(\theta, \theta_M) = (0.8, 0.8)$. Open circles mark nodes whose optimal notional rate becomes negative at some date. The color scales are centered at zero but differ between negative and positive values.

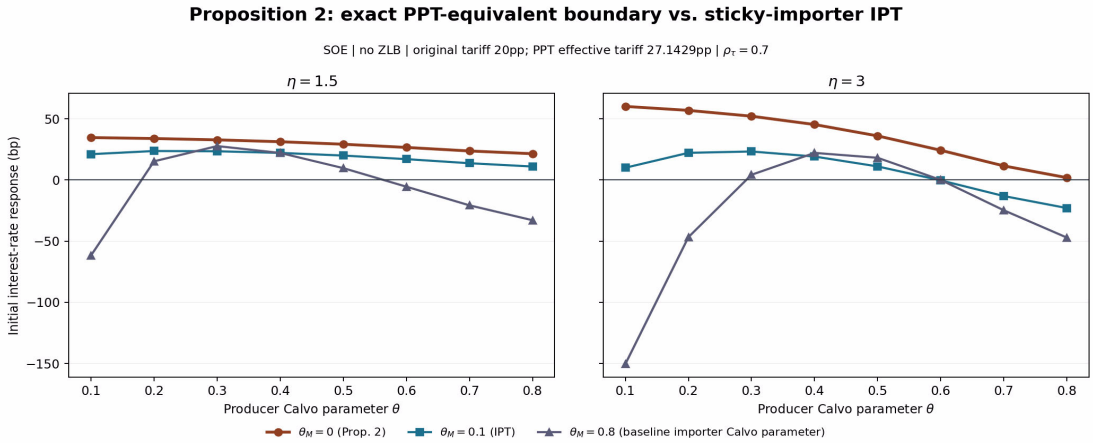


Figure 9: Initial nominal interest-rate response along three rows of Fig. 8: $\theta_M = 0$ (Proposition 2, solved as PPT with the effective tariff 0.2714), $\theta_M = 0.1$, and the baseline importer Calvo parameter $\theta_M = 0.8$. Twenty-percentage-point Home import tariff, $\rho_\tau = 0.7$, ZLB not imposed.

Three features stand out. First, the two edges behave as the propositions predict. At $\theta = 0.1$ and the baseline importer stickiness $\theta_M = 0.8$, the optimal rate is cut by 61.7 basis points for $\eta = 1.5$ and by 150.5 basis points for $\eta = 3$, in line with Proposition 1. Along $\theta_M = 0$, the rate rises for every θ, by between 21.5 and 34.7 basis points for $\eta = 1.5$ and between 1.9 and 60.2 basis points for $\eta = 3$, in line with Proposition 2 and Corollary 1. Convergence toward the $\theta_M = 0$ edge is slow when producer prices are sticky: for $\eta = 3$ and $\theta \geq 0.6$, the response at $\theta_M = 0.1$ is still a cut, so the sign change occurs between $\theta_M = 0.1$ and 0, and the interpolated contour there should not be read literally.

Second, the initial response is a cut over most of the interior, including the baseline: it is negative at 44 of the 48 nodes with $\theta_M \geq 0.3$ for $\eta = 1.5$ and at 43 of 48 for $\eta = 3$. Along the baseline importer stickiness, however, the response is hump-shaped (inverted-U-shaped) in θ (Fig. 9). Starting from the baseline $\theta = 0.8$, lower producer-price stickiness first raises the response and turns the cut into a small increase at intermediate θ, peaking at 27.7 basis points ($\eta = 1.5$, $\theta = 0.3$) and 22.2 basis points ($\eta = 3$, $\theta = 0.4$); the response then falls again, and the cut returns as θ approaches the limit in Proposition 1. The interior hump is a secondary numerical finding that reflects the interaction of producer-price adjustment with the importer-pricing block, not producer-price rigidity in isolation. Along $\theta_M = 0$, where Proposition 2 applies, the initial response is monotonic in θ: it falls from +34.7 to +21.5 basis points as θ rises from 0.1 to 0.8 for $\eta = 1.5$, and from +60.2 to +1.9 for $\eta = 3$. By contrast, with $\theta_M = 0.8$ the response peaks at intermediate θ, as noted above. The difference between these two rows is largest at $\theta = 0.1$, closest to zero at $\theta = 0.3$ for $\eta = 1.5$ and at $\theta = 0.5$ for $\eta = 3$, and larger again at $\theta = 0.8$.

One economic interpretation follows from the structure of the inflation weights in Eq. (E.22). Each weight is inversely proportional to the slope of the corresponding Phillips curve, so that $\kappa_H \Lambda^{SOE,IPT}_{\pi,H}$ and $\kappa_M \Lambda^{SOE,IPT}_{\pi,F}$ depend on neither θ nor θ_M. For a given path of the relevant Phillips-curve driving term—real marginal cost for producer inflation (Eq. (18)) and $\psi_t + \tau_{M,t}/(\varepsilon_M - 1)$ for import-price inflation (Eq. (34))—the inflation contribution to the welfare cost *rises* with flexibility of the corresponding price: the larger inflation response more than offsets the smaller weight. As prices become fully flexible, however, price setting itself drives that term to zero—$mc^r_t = 0$ for producers in the limit of Proposition 1 and $\psi_t = -\tau_{M,t}/(\varepsilon_M - 1)$ for importers at the boundary of Proposition 2—and the corresponding stabilization motive vanishes. When prices are very sticky, inflation responds little. The welfare importance of stabilizing each inflation rate is therefore greatest at intermediate rigidity. This structure is the same as that of the wage-

inflation term in Galí and Monacelli [32], where the volatility of wage inflation rises with wage flexibility while its welfare weight falls, so that welfare losses are hump-shaped in wage rigidity under a currency peg or an exchange-rate-oriented rule.

The HIAC gives the Ramsey planner an incentive to ease monetary policy by moving the actual LOOP gap toward the WR LOOP gap. This incentive is balanced against the objectives of stabilizing import-price inflation and Home PPI inflation. The relative importance of these objectives varies with producer- and importer-price stickiness. Accordingly, when import prices are sticky, the initial response is hump-shaped in θ: at intermediate θ the PPI-stabilization motive offsets most of the easing, and at $\theta_M = 0.8$ enough of it to produce a small increase. Conversely, for given θ the cut is deepest at intermediate importer stickiness—at θ_M between 0.4 and 0.7 for every θ on the grid and both elasticities—and the response rises again toward $\theta_M = 0.8$. Along $\theta_M = 0$, there is no hump: producer-price stickiness still affects the optimal response through the producer-pricing block, but its effect is monotonic in θ on the grid.

The parallel with Galí and Monacelli [32] has limits. In their model, the exchange-rate orientation of policy is imposed by the monetary regime, and their non-monotonicity persists under complete import-price pass-through (their Fig. 12). In our model, the HIAC gives the Ramsey planner an incentive to adjust the LOOP gap through monetary policy when import prices are sticky. At $\theta_M = 0$, however, the LOOP gap is determined by the tariff independently of monetary policy. Their object is welfare under given monetary regimes; ours is the optimal interest-rate path. The argument above interprets the fully nonlinear Ramsey solutions; it is not a decomposition of the Ramsey first-order conditions, and the weights in Eq. (E.22) are not substituted for the original utility objective. The first-order solutions also display a hump along $\theta_M = 0.8$ but remain negative throughout that row; the small positive peaks at intermediate θ in Fig. 9 arise in the fully nonlinear 20-percentage-point experiment.

Third, the cumulative responses in the bottom row distinguish two kinds of rate increase. At $\theta_M = 0.8$, the cumulative 20-quarter response is negative for every θ, between -471 and -29 basis-point-quarters for $\eta = 1.5$ and between -513 and -105 for $\eta = 3$. At intermediate θ, the initial increase is therefore followed by rate cuts large enough to make the cumulative response negative. Near $\theta_M = 0$, by contrast, both the initial and the cumulative responses are positive (between 108 and 130 basis-point-quarters at $\theta_M = 0$ for $\eta = 1.5$ and between 136 and 232 for $\eta = 3$). At $\theta_M = 0$, Eq. (33) pins down the actual LOOP gap as a function of the tariff, independently of monetary policy. The WR LOOP-gap deviation therefore can no longer be adjusted through monetary policy. The Ramsey allocation is equivalent to that under PPT with the adjusted tariff specified in Proposition 2, and monetary policy tightens. The zero contour of the cumulative response rises with θ over most of the grid: cumulative easing prevails when import prices are very sticky or producer prices are nearly flexible, and cumulative tightening prevails when import prices are relatively flexible. The relative stickiness of the two prices alone does not determine the sign, however; for example, the cumulative response is $+38.9$ basis-point-quarters at $(\theta, \theta_M) = (0.4, 0.5)$ for $\eta = 1.5$ and -44.6 at $(0.8, 0.6)$ for $\eta = 3$. At the baseline, the initial cut is followed by later increases, so the cumulative response is small (-33 basis-point-quarters for $\eta = 1.5$ and -107 for $\eta = 3$).

Higher tariff persistence strengthens the initial increase at intermediate producer-price stickiness. At $(\theta, \theta_M) = (0.4, 0.8)$, raising ρ_τ from 0.7 to 0.9 raises the initial response from 22.1 to 46.59 basis points for $\eta = 1.5$ and from 22.2 to 73.33 basis points for $\eta = 3$. The rate cut under IPT is therefore not unconditional. It is robust when import prices are sticky and producer prices are either sticky, as in the baseline, or nearly flexible, as in Proposition 1; it disappears when import prices are nearly flexible, as in Proposition 2; and at intermediate producer-price stickiness, where PPI stabilization matters most, the PPI-inflation stabilization objective can outweigh the HIAC-induced incentive for monetary easing, producing an initial rate increase followed by rate cuts.

B Derivation of the Two-Country Economy Model

This Appendix collects the nonlinear private-equilibrium system of the Two-country Economy in one place. All variables and parameters have exactly the definitions used in the main text. No additional state variable, price concept, tariff convention, or auxiliary economic object is introduced here. Country H has household mass ν and country F has household mass $1-\nu$. The expenditure weights are those in Sec. 4.1, namely $\omega_F = \upsilon(1-\nu)$, $\omega_H = 1-\omega_F$, $\omega_H^* = \upsilon\nu$, and $\omega_F^* = 1-\omega_H^*$.

B.1 PPT Environment

The Home and Foreign representative households have the preferences stated in Sec. 4.1. Their consumption baskets are given by Eq. (43). Under the PPT environment, Eq. (2) and

$$P_{H,t}^* = \frac{P_{H,t}}{\mathcal{E}_t} \tag{B.1}$$

are applicable. Hence the two LOOP gaps are equal to one in levels,

$$\Psi_t \equiv \frac{\mathcal{E}_t P_{F,t}^*}{P_{F,t}} = 1, \qquad \Psi_t^* \equiv \frac{P_{H,t}/\mathcal{E}_t}{P_{H,t}^*} = 1. \tag{B.2}$$

Accordingly, with the logarithmic definitions used in the welfare analysis, $\psi_t \equiv \log \Psi_t = 0$ and $\psi_t^* \equiv \log \Psi_t^* = 0$. Thus the statement that the LOOP gap is zero applies to its log deviation, whereas the corresponding level is one.

The national CPI indices are

$$\begin{aligned} P_t^{PPT} &= \left[\omega_H P_{H,t}^{1-\eta} + \omega_F\{(1+\tau_{M,t})\mathcal{E}_t P_{F,t}^*\}^{1-\eta}\right]^{1/(1-\eta)}, \\ P_t^{*,PPT} &= \left[\omega_F^* (P_{F,t}^*)^{1-\eta} + \omega_H^* \left\{(1+\tau_{M,t}^*)\frac{P_{H,t}}{\mathcal{E}_t}\right\}^{1-\eta}\right]^{1/(1-\eta)}. \end{aligned} \tag{B.3}$$

The two-country $\mathcal{G}^{PPT}(\tau_{M,t}, Z_t/\Psi_t)$ uses the Home expenditure weights ω_H and ω_F in Eq. (B.3), rather than the SOE weights; under PPT, $\Psi_t = 1$. The associated bilateral demand system is

$$\begin{aligned} C_{H,t} &= \omega_H \left(\frac{P_{H,t}}{P_t^{PPT}}\right)^{-\eta} C_t, \\ C_{F,t} &= \omega_F \left(\frac{(1+\tau_{M,t})\mathcal{E}_t P_{F,t}^*}{P_t^{PPT}}\right)^{-\eta} C_t, \\ C_{F,t}^* &= \omega_F^* \left(\frac{P_{F,t}^*}{P_t^{*,PPT}}\right)^{-\eta} C_t^*, \\ C_{H,t}^* &= \omega_H^* \left(\frac{(1+\tau_{M,t}^*)P_{H,t}/\mathcal{E}_t}{P_t^{*,PPT}}\right)^{-\eta} C_t^*. \end{aligned} \tag{B.4}$$

Tariff revenue is rebated lump sum in the importing country, exactly as in the SOE household budget constraint. The Home intratemporal and Euler conditions are Eqs. (10) and (11), with $\Psi_t = 1$; the Foreign household satisfies the exact starred counterparts. In particular,

$$\frac{1}{1+i_t^*} = \beta \mathrm{E}_t \frac{C_t^*(P_t^{*,PPT}/P_{F,t}^*)}{C_{t+1}^*(P_{t+1}^{*,PPT}/P_{F,t+1}^*)\Pi_{F,t+1}^*}. \tag{B.5}$$

Complete international risk sharing gives

$$Z_t = \frac{C_t}{C_t^*}\frac{\mathcal{G}^{PPT}(\tau_{M,t}, Z_t/\Psi_t)}{P_t^{*,PPT}/P_{F,t}^*}, \qquad \Psi_t = 1, \tag{B.6}$$

which is the two-country counterpart of Eq. (12).

In each country, production, producer real marginal cost, the producer reset price, and producer-price dispersion are governed by Eqs. (13)–(17) and their exact starred counterparts. Thus Home production satisfies

$$Y_t = \frac{N_t}{\Delta_{H,t}}, \qquad MC_t^{\tau} = C_t N_t^{\varphi} \mathcal{G}^{PPT}(\tau_{M,t}, Z_t/\Psi_t), \tag{B.7}$$

and the Home reset-price condition is Eq. (14); Foreign production and producer pricing have the identical starred structure.

Because there is no local importer-price dispersion under PPT, bilateral goods-market clearing is

$$Y_t = C_{H,t} + \left(\frac{1-\nu}{\nu}\right) C^*_{H,t}, \qquad Y^*_t = C^*_{F,t} + \left(\frac{\nu}{1-\nu}\right) C_{F,t}. \tag{B.8}$$

Equation (43) and Eqs. (B.1)– (B.8), together with the two producer Calvo blocks and the two monetary-policy conditions, form the nonlinear Two-country PPT private equilibrium used by the national planners.

B.2 IPT Environment

The IPT environment retains the preferences, national CES consumption baskets, household optimality conditions, complete-markets risk-sharing condition, and PCP producer blocks in Appendix B.1. The difference is that Home and Foreign local importers set sticky *ex-tariff* prices in the currency of the importing country. Thus $P_{F,t}$ and $P^*_{H,t}$ are endogenous local importer-price indices. The Home LOOP gap is $\Psi_t \equiv \mathcal{E}_t P^*_{F,t}/P_{F,t}$ as defined in Sec. 3.1.2; its exact Foreign mirror is $\Psi^*_t \equiv (P_{H,t}/\mathcal{E}_t)/P^*_{H,t}$. These LOOP gaps need not equal one away from the deterministic steady state.

The Home household-facing *cum-tariff* price of a Foreign variety and its CES index are Eqs. (20) and (22). The exact Foreign mirror uses the same additive customs-duty convention and the corresponding country-mass normalization.

The CPI indices are therefore

$$\begin{aligned} P_t^{IPT} &= \left[\omega_H P_{H,t}^{1-\eta} + \omega_F (\widetilde{P}_{F,t}^{IPT})^{1-\eta}\right]^{1/(1-\eta)}, \\ P_t^{*,IPT} &= \left[\omega^*_F (P^*_{F,t})^{1-\eta} + \omega^*_H (\widetilde{P}^*_{H,t})^{1-\eta}\right]^{1/(1-\eta)}, \end{aligned} \tag{B.9}$$

Here $\mathcal{G}_t^{IPT}(\cdot) = P_t^{IPT}/P_{H,t}$ and its Foreign counterpart $\mathcal{G}_t^{*,IPT}(\cdot) = P_t^{*,IPT}/P^*_{F,t}$ denote the CPI–PPI ratios constructed from the corresponding exact cum-tariff importer-price indices; see Eqs. (20)–(22). The aggregate bilateral demands are

$$\begin{aligned} C_{H,t} &= \omega_H \left(\frac{P_{H,t}}{P_t^{IPT}}\right)^{-\eta} C_t, \\ C_{F,t} &= \omega_F \left(\frac{\widetilde{P}_{F,t}^{IPT}}{P_t^{IPT}}\right)^{-\eta} C_t, \\ C^*_{F,t} &= \omega^*_F \left(\frac{P^*_{F,t}}{P_t^{*,IPT}}\right)^{-\eta} C^*_t, \\ C^*_{H,t} &= \omega^*_H \left(\frac{\widetilde{P}^*_{H,t}}{P_t^{*,IPT}}\right)^{-\eta} C^*_t. \end{aligned} \tag{B.10}$$

Within the imported-good aggregate, conditional demand is Eq. (21) with the Home-country normalization above, and the Foreign conditional-demand equation is its exact starred counterpart. The Home and Foreign household budget constraints again reduce to $P_t^{IPT}C_t$ and $P_t^{*,IPT}C_t^*$ expenditure by CES duality. The statutory duties are remitted on border customs value and rebated lump sum exactly as in Eq. (27) and its Foreign mirror.

The Home intratemporal and Euler equations retain the forms of Eqs. (10) and (11), with the IPT CPI and $\mathcal{G}_t^{IPT}(\cdot)$ replacing the PPT CPI and $\mathcal{G}^{PPT}(\tau_{M,t}, S_t)$, respectively. The Foreign equations are their exact starred counterparts, and complete international risk sharing remains

$$Z_t = \frac{C_t}{C_t^*}\frac{\mathcal{G}_t^{IPT}(\cdot)}{P_t^{*,IPT}/P_{F,t}^*}. \tag{B.11}$$

The producer blocks remain Eqs. (13)– (17) and their starred counterparts.

Home local importers use the exact nonlinear pricing problem in Eqs. (30)–(32). Their *ex-tariff* price index satisfies Eq. (28), and the local relative import price obeys Eq. (29). Foreign local importers solve the exact Home–Foreign mirror problem. In particular, no multiplicative identity of the form $\widetilde{P}_{F,t}^{IPT} = (1+\tau_{M,t})P_{F,t}$ is imposed when importer prices are sticky; the statutory duty remains additive at the variety level as in Eq. (20).

Bilateral goods-market clearing is given by Eq. (48). Together with the two household blocks, the two PCP producer blocks, and the two LCP importer blocks, the conditions above form the nonlinear Two-country IPT private equilibrium.

C Social Planner's Problem

The policy problems in this section are the nonlinear commitment problems used to generate the allocations. Welfare cost functions are derived separately in Appendix E; they are not minimized to obtain the policy paths.

C.1 Small Open Economy

C.1.1 PPT Environment

In the PPT environment, the social planner maximizes $\mathcal{U}$ subject to Eqs. (10), (11), (12), (13), (14)–(17), (2), (19) with $\Psi_t = 1$, and (36). After Eqs. (14)–(15) have been substituted, the Lagrangian is

$$\begin{aligned}
\mathcal{L}^{SOE,PPT} \;=\; & \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \\
& + \lambda_{p,t}\left[1 - \theta\Pi_{H,t}^{\varepsilon-1} - (1-\theta)\widetilde{p}_{H,t}^{1-\varepsilon}\right] \\
& + \lambda_{\Delta,t}\left[\Delta_{H,t} - \theta\Pi_{H,t}^{\varepsilon}\Delta_{H,t-1} - (1-\theta)\widetilde{p}_{H,t}^{-\varepsilon}\right] \\
& + \lambda_{n,t}\left[N_t - Y_t\Delta_{H,t}\right] \\
& + \lambda_{y,t}\left[Y_t - (1-\upsilon)\mathcal{G}^{PPT}(\tau_{M,t}, Z_t)^{\eta}C_t - \upsilon Z_t^{\eta}\right] \\
& + \lambda_{r,t}\left[Z_t - C_t\mathcal{G}^{PPT}(\tau_{M,t}, Z_t)\right] \\
& + \lambda_{e,t}\left[\frac{1}{1+i_t} - \beta\mathrm{E}_t\frac{C_t\mathcal{G}^{PPT}(\tau_{M,t}, Z_t)}{C_{t+1}\mathcal{G}^{PPT}(\tau_{M,t+1}, Z_{t+1})\Pi_{H,t+1}}\right] + \mu_{i,t}i_t \Bigg\}.
\end{aligned} \tag{C.1}$$

Here $\lambda_{p,t}$, $\lambda_{\Delta,t}$, $\lambda_{n,t}$, $\lambda_{y,t}$, $\lambda_{r,t}$, and $\lambda_{e,t}$ are the multipliers on the price-index, price-dispersion, production, goods-market, risk-sharing, and Euler conditions shown in the same order in Eq. (C.1). The multiplier $\mu_{i,t}$ is the ZLB multiplier defined after Eq. (36).

C.1.2 IPT Environment

In the IPT environment the SOE planner maximizes the same household utility as in Appendix C.1.1, subject to the household and producer conditions of the SOE, the IPT risk-sharing condition (26), and the IPT goods-market condition (35), together with the exact additive-duty importer block in Eqs. (20)–(32), the importer Calvo price-index law in Eq. (28), the relative-import-price law in Eq. (29), and the ZLB. The planner chooses the same Home allocation and producer-price variables as under PPT and, in addition, the local importer-price variables described in Sec. 3.2. The statutory duty is imposed on border customs value and is not replaced by a multiplicative tariff on the sticky *ex-tariff* import price.

C.2 Two-Country Economy

C.2.1 PPT Environment

We now write the two national problems explicitly. Complete international risk sharing in Eq. (B.11) is imposed before either national planner differentiates. It is used to eliminate Z_t from the national differentiation problems and is not a new definition. No separate starred international-relative-price variable is introduced.

In the PPT environment, after Eq. (B.11) has been substituted, the Home CPI–PPI relation is

$$\left(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)\right)^{1-\eta} = \omega_H + \omega_F(1+\tau_{M,t})^{1-\eta}\left[\frac{C_t}{C_t^*}\frac{\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)}{P_t^{*,PPT}/P_{F,t}^*}\right]^{1-\eta},$$

and Home goods-market clearing is

$$\begin{aligned} Y_t &= \omega_H\left(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)\right)^{\eta} C_t \\ &\quad + \frac{1-\nu}{\nu}\omega_H^*\left[\frac{C_t(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right))}{C_t^*(1+\tau_{M,t}^*)}\right]^{\eta} C_t^*. \end{aligned}$$

The Foreign counterparts, used in the Foreign national problem, are

$$\begin{aligned} \left(\frac{P_t^{*,PPT}}{P_{F,t}^*}\right)^{1-\eta} &= \omega_F^* + \omega_H^*(1+\tau_{M,t}^*)^{1-\eta}\left[\frac{C_t^*}{C_t}\frac{P_t^{*,PPT}/P_{F,t}^*}{\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)}\right]^{1-\eta}, \\ Y_t^* &= \omega_F^*\left(\frac{P_t^{*,PPT}}{P_{F,t}^*}\right)^{\eta} C_t^* \\ &\quad + \frac{\nu}{1-\nu}\omega_F\left[\frac{C_t^*(P_t^{*,PPT}/P_{F,t}^*)}{C_t(1+\tau_{M,t})}\right]^{\eta} C_t. \end{aligned}$$

The Home planner takes the complete Foreign contingent plan as given. After the Home reset-price relation has been substituted, its national Lagrangian is

$$\begin{aligned} \mathcal{L}_H^{PPT} = \mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg\{ &\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \\ &+\lambda_{g,t}^H\left[\left(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)\right)^{1-\eta} - \omega_H - \omega_F(1+\tau_{M,t})^{1-\eta}\left[\frac{C_t}{C_t^*}\frac{\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)}{P_t^{*,PPT}/P_{F,t}^*}\right]^{1-\eta}\right] \\ &+\lambda_{y,t}^H\left[Y_t - \omega_H\left(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right)\right)^{\eta} C_t - \frac{1-\nu}{\nu}\omega_H^*\left[\frac{C_t(\mathcal{G}^{PPT}\left(\tau_{M,t}, Z_t/\Psi_t\right))}{C_t^*(1+\tau_{M,t}^*)}\right]^{\eta} C_t^*\right] \\ &+\lambda_{n,t}^H\left[N_t - Y_t\Delta_{H,t}\right] \\ &+\lambda_{p,t}^H\left[1 - \theta\Pi_{H,t}^{\varepsilon-1} - (1-\theta)\widetilde{p}_{H,t}^{1-\varepsilon}\right] \end{aligned}$$

$$+\lambda^H_{\Delta,t}\left[\Delta_{H,t}-\theta\Pi^{\varepsilon}_{H,t}\Delta_{H,t-1}-(1-\theta)\widetilde{p}^{-\varepsilon}_{H,t}\right]$$
$$+\lambda^H_{e,t}\left[\frac{1}{1+i_t}-\beta\mathrm{E}_t\frac{C_t(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right))}{C_{t+1}(\mathcal{G}^{PPT}\left(\tau_{M,t+1},Z_{t+1}/\Psi_{t+1}\right))\Pi_{H,t+1}}\right]+\mu^H_{i,t}i_t\Bigg\}.$$

In this subsection, a superscript H or F on a multiplier identifies the national planner, while the subscript M indicates the corresponding local-importer pricing block.

The Foreign planner takes the complete Home contingent plan as given. Its national Lagrangian is

$$\begin{aligned}
\mathcal{L}^{PPT}_F \;=\; & \mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg\{\log C^*_t-\frac{(N^*_t)^{1+\varphi}}{1+\varphi}\\
&+\lambda^F_{g,t}\left[\left(\frac{P^{*,PPT}_t}{P^*_{F,t}}\right)^{1-\eta}-\omega^*_F-\omega^*_H(1+\tau^*_{M,t})^{1-\eta}\left[\frac{C^*_t}{C_t}\frac{P^{*,PPT}_t/P^*_{F,t}}{\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}\right]^{1-\eta}\right]\\
&+\lambda^F_{y,t}\left[Y^*_t-\omega^*_F\left(\frac{P^{*,PPT}_t}{P^*_{F,t}}\right)^{\eta}C^*_t-\frac{\nu}{1-\nu}\omega_F\left[\frac{C^*_t(P^{*,PPT}_t/P^*_{F,t})}{C_t(1+\tau_{M,t})}\right]^{\eta}C_t\right]\\
&+\lambda^F_{n,t}\left[N^*_t-Y^*_t\Delta^*_{F,t}\right]\\
&+\lambda^F_{p,t}\left[1-\theta(\Pi^*_{F,t})^{\varepsilon-1}-(1-\theta)(\widetilde{p}^*_{F,t})^{1-\varepsilon}\right]\\
&+\lambda^F_{\Delta,t}\left[\Delta^*_{F,t}-\theta(\Pi^*_{F,t})^{\varepsilon}\Delta^*_{F,t-1}-(1-\theta)(\widetilde{p}^*_{F,t})^{-\varepsilon}\right]\\
&+\lambda^F_{e,t}\left[\frac{1}{1+i^*_t}-\beta\mathrm{E}_t\frac{C^*_t(P^{*,PPT}_t/P^*_{F,t})}{C^*_{t+1}(P^{*,PPT}_{t+1}/P^*_{F,t+1})\Pi^*_{F,t+1}}\right]+\mu^F_{i,t}i^*_t\Bigg\}.
\end{aligned}$$

C.2.2 IPT Environment

The two national commitment problems use the exact Two-country private equilibrium in B.2. Complete international risk sharing is imposed before national differentiation, as in Sec. 4.2. The producer reset-price condition in Eq. (14) and the exact importer reset-price condition in Eq. (32) are substituted before the planner FOCs are taken. Hence their reset prices are endogenous implicit functions of the national allocation, while the statutory duty remains additive at the variety level.

The Home planner takes the complete contingent Foreign plan as given and solves

$$\max_{\{C_t,\mathcal{G}^{IPT}_t(\cdot),N_t,Y_t,\Pi_{H,t},\Delta_{H,t},P_{F,t}/P_{H,t},\Pi_{F,t},i_t\}_{t\geq 0}}\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\left(\log C_t-\frac{N_t^{1+\varphi}}{1+\varphi}\right),\tag{C.2}$$

subject to the Home household Euler equation, the complete-markets condition in Eq. (B.11), the Home producer technology and Calvo block, the exact additive-duty importer block in Eqs. (20)–(32), the local import-price law in Eq. (29), the exact bilateral resource constraints in Eq. (48), and

$$i_t\geq 0.\tag{C.3}$$

The notation $\mathcal{G}^{IPT}_t(\cdot)$ in the planner's choice set stands for the CPI–PPI ratio constrained by the exact purchaser-price index in Eqs. (20)–(25), not the PPT formula $\mathcal{G}^{PPT}(\tau_{M,t},Z_t/\Psi_t)$. The complete cross-sectional distributions $\{P_{F,t}(\iota)\}$ and $\{P^*_{H,t}(\iota)\}$ entering the purchaser-price indices and physical- quantity integrals evolve according to the same Calvo reset/non-reset rule as in B.2. They are therefore retained inside the exact private-equilibrium constraints and are not replaced by a separate importer-dispersion state.

For later differentiation it is useful to display the Home Lagrangian after both reset-price relations have been substituted. Using the same multiplier notation as in C.2.1, and adding $\lambda^H_{Mp,t}$ for the Home importer price-index law and $\lambda^H_{q,t}$ for the local relative import-price law, we have

$$\mathcal{L}^{IPT}_H \;=\; \mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg\{\log C_t-\frac{N_t^{1+\varphi}}{1+\varphi}$$

$$
\begin{aligned}
&+\lambda^H_{g,t}\left[\mathcal{G}^{IPT}_t(\cdot)^{1-\eta}-\omega_H-\omega_F\left(\frac{\widetilde{P}^{IPT}_{F,t}}{P_{H,t}}\right)^{1-\eta}\right]\\
&+\lambda^H_{y,t}\Bigg[Y_t-\omega_H\mathcal{G}^{IPT}_t(\cdot)^{\eta}C_t\\
&\qquad-\frac{1-\nu}{\nu}\omega^*_H\left(\frac{\widetilde{P}^*_{H,t}}{P^{*,IPT}_t}\right)^{-\eta}C^*_t\frac{1}{1-\nu}\int_\nu^1\left(\frac{\widetilde{P}^*_{H,t}(\iota)}{\widetilde{P}^*_{H,t}}\right)^{-\varepsilon_M}d\iota\Bigg]\\
&+\lambda^H_{n,t}[N_t-Y_t\Delta_{H,t}]\\
&+\lambda^H_{p,t}\left[1-\theta\Pi^{\varepsilon-1}_{H,t}-(1-\theta)\widetilde{p}^{1-\varepsilon}_{H,t}\right]\\
&+\lambda^H_{\Delta,t}\left[\Delta_{H,t}-\theta\Pi^{\varepsilon}_{H,t}\Delta_{H,t-1}-(1-\theta)\widetilde{p}^{-\varepsilon}_{H,t}\right]\\
&+\lambda^H_{Mp,t}\left[1-\theta_M{\Pi^*_{F,t}}^{\varepsilon_M-1}-(1-\theta_M)\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)^{1-\varepsilon_M}\right]\\
&+\lambda^H_{q,t}\left[\frac{P_{F,t}}{P_{H,t}}-\frac{P_{F,t-1}}{P_{H,t-1}}\frac{\Pi_{F,t}}{\Pi_{H,t}}\right]\\
&+\lambda^H_{e,t}\left[\frac{1}{1+i_t}-\beta\mathrm{E}_t\frac{C_t\mathcal{G}^{IPT}_t(\cdot)}{C_{t+1}\mathcal{G}^{IPT}_{t+1}(\cdot)\Pi_{H,t+1}}\right]+\mu^H_{i,t}i_t\Bigg\}.
\end{aligned}
\tag{C.4}
$$

All occurrences of Z_t and $\mathcal{E}_t$ inside the purchaser-price and reset-price objects in Eq. (C.4) are eliminated with the complete-markets condition before differentiation. In particular, $\overline{P}_{F,t}/P_{F,t}$ is the reset-price ratio implied by the exact private importer FOC (32); no new importer auxiliary variable is introduced here.

The Foreign planner takes the complete contingent Home plan as given and solves the exact starred problem

$$
\max_{\{C^*_t,P^{*,IPT}_t/P^*_{F,t},N^*_t,Y^*_t,\Pi^*_{F,t},\Delta^*_{F,t},P^*_{H,t}/P^*_{F,t},\Pi^*_{H,t},i^*_t\}_{t\geq 0}}\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\left(\log C^*_t-\frac{(N^*_t)^{1+\varphi}}{1+\varphi}\right),
\tag{C.5}
$$

subject to the starred B.2 equilibrium restrictions and $i^*_t\geq 0$. Its Lagrangian is the exact Home–Foreign mirror of Eq. (C.4). The two national commitment problems and the common private equilibrium are solved jointly at their mutual best response.

D Ramsey First-Order Conditions

D.1 Small Open Economy

D.1.1 PPT Environment

Following Monacelli [39] (pp. 74–75), we use the following derivative notation for the producer reset price:

$$
\begin{aligned}
\mathcal{K}'_{H,t-j}(x_t) &\equiv \frac{\partial\mathcal{K}_{H,t-j}}{\partial x_t},\qquad \mathcal{Z}'_{H,t-j}(x_t)\equiv\frac{\partial\mathcal{Z}_{H,t-j}}{\partial x_t},\\
\widetilde{p}'_{H,t-j}(x_t) &\equiv \frac{\partial\widetilde{p}_{H,t-j}}{\partial x_t}=\mathcal{M}\frac{\mathcal{Z}'_{H,t-j}(x_t)\mathcal{K}_{H,t-j}-\mathcal{K}'_{H,t-j}(x_t)\mathcal{Z}_{H,t-j}}{(\mathcal{K}_{H,t-j})^2}.
\end{aligned}
\tag{D.1}
$$

The producer reset price $\widetilde{p}_{H,t-j}$ is defined in Eqs. (14)–(15).

The first-order condition for C_t is

$$\begin{aligned}
0 &= \frac{1}{C_t} - (1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(C_t)\right] \\
&\quad +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(C_t)\right] \\
&\quad -(1-\upsilon)\lambda_{y,t}\mathcal{G}^{PPT}(\tau_{M,t},Z_t)^{\eta} - \lambda_{r,t}\mathcal{G}^{PPT}(\tau_{M,t},Z_t) \\
&\quad -\beta\lambda_{e,t}\mathrm{E}_t\left[\frac{\mathcal{G}^{PPT}(\tau_{M,t},Z_t)}{C_{t+1}\mathcal{G}^{PPT}(\tau_{M,t+1},Z_{t+1})\Pi_{H,t+1}}\right] \\
&\quad +\lambda_{e,t-1}\frac{C_{t-1}\mathcal{G}^{PPT}(\tau_{M,t-1},Z_{t-1})}{C_t^2\mathcal{G}^{PPT}(\tau_{M,t},Z_t)\Pi_{H,t}}.
\end{aligned}$$

The first-order condition for N_t is

$$\begin{aligned}
0 &= -N_t^{\varphi} + \lambda_{n,t} \\
&\quad -(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(N_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(N_t)\right] \\
&\quad +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(N_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(N_t)\right].
\end{aligned}$$

The first-order condition for Y_t is

$$\begin{aligned}
0 &= -\lambda_{n,t}\Delta_{H,t} + \lambda_{y,t} \\
&\quad -(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(Y_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(Y_t)\right] \\
&\quad +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(Y_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(Y_t)\right].
\end{aligned}$$

The first-order condition for Z_t is

$$\begin{aligned}
0 &= -(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(Z_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(Z_t)\right] \\
&\quad +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(Z_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(Z_t)\right] \\
&\quad -\lambda_{y,t}\left[(1-\upsilon)\eta\mathcal{G}^{PPT}(\tau_{M,t},Z_t)^{\eta-1}\frac{\partial\mathcal{G}^{PPT}(\tau_{M,t},Z_t)}{\partial Z_t}C_t + \upsilon\eta Z_t^{\eta-1}\right] \\
&\quad +\lambda_{r,t}\left[1 - C_t\frac{\partial\mathcal{G}^{PPT}(\tau_{M,t},Z_t)}{\partial Z_t}\right] \\
&\quad -\beta\lambda_{e,t}\mathrm{E}_t\left[\frac{C_t\,\partial\mathcal{G}^{PPT}(\tau_{M,t},Z_t)/\partial Z_t}{C_{t+1}\mathcal{G}^{PPT}(\tau_{M,t+1},Z_{t+1})\Pi_{H,t+1}}\right] \\
&\quad +\lambda_{e,t-1}\frac{C_{t-1}\mathcal{G}^{PPT}(\tau_{M,t-1},Z_{t-1})}{C_t\mathcal{G}^{PPT}(\tau_{M,t},Z_t)^2\Pi_{H,t}}\frac{\partial\mathcal{G}^{PPT}(\tau_{M,t},Z_t)}{\partial Z_t}.
\end{aligned}$$

The first-order condition for $\Pi_{H,t}$ is

$$\begin{aligned}
0 \;=\; & -\theta(\varepsilon-1)\lambda_{p,t}\Pi_{H,t}^{\varepsilon-2} - \theta\varepsilon\lambda_{\Delta,t}\Pi_{H,t}^{\varepsilon-1}\Delta_{H,t-1} \\
& -(1-\theta)(1-\varepsilon)\left[\lambda_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(\Pi_{H,t}) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\right] \\
& +(1-\theta)\varepsilon\left[\lambda_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(\Pi_{H,t}) + \sum_{j=1}^{\infty}\beta^{-j}\lambda_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\right] \\
& +\lambda_{e,t-1}\frac{C_{t-1}\mathcal{G}^{PPT}(\tau_{M,t-1},Z_{t-1})}{C_t\mathcal{G}^{PPT}(\tau_{M,t},Z_t)\Pi_{H,t}^2}.
\end{aligned}$$

The first-order condition for $\Delta_{H,t}$ is

$$0 = \lambda_{\Delta,t} - \lambda_{n,t}Y_t - \beta\theta\lambda_{\Delta,t+1}\Pi_{H,t+1}^{\varepsilon}.$$

Finally, the interest-rate condition and the ZLB conditions are

$$-\frac{\lambda_{e,t}}{(1+i_t)^2} + \mu_{i,t} = 0, \qquad i_t \geq 0, \qquad \mu_{i,t} \geq 0, \qquad \mu_{i,t}i_t = 0.$$

D.1.2 IPT Environment

In the IPT environment, the producer block is unchanged and the planner additionally imposes the customs-duty importer block in Eqs. (20)–(22) and Eqs. (30)–(32), together with the local relative-import-price law in Eq. (29). The household budget constraint remains the one stated in Sec. 3, with consumption expenditure P_tC_t, and tariff revenue is given by Eq. (27). The Ramsey first-order conditions used for computation are obtained by differentiating this constrained problem directly. We do not replace the nonlinear importer reset-price condition by the linearized Phillips curve in Eq. (34).

The ZLB condition remains

$$-\frac{\lambda_{e,t}}{(1+i_t)^2} + \mu_{i,t} = 0, \qquad i_t \geq 0, \quad \mu_{i,t} \geq 0, \quad \mu_{i,t}i_t = 0.$$

D.2 Two-Country Economy

D.2.1 PPT Environment

For the Home reset price the derivative notation is that in Eq. (D.1). The Foreign reset price uses the exact starred counterpart,

$$(\widetilde{p}^*_{F,t-j})'(x_t) = \mathcal{M}\frac{(\mathcal{Z}^*_{F,t-j})'(x_t)\mathcal{K}^*_{F,t-j} - (\mathcal{K}^*_{F,t-j})'(x_t)\mathcal{Z}^*_{F,t-j}}{(\mathcal{K}^*_{F,t-j})^2}.$$

The Home first-order condition for C_t is

$$\begin{aligned}
0 = & \frac{1}{C_t} - (1-\theta)(1-\varepsilon)\left[\lambda^H_{p,t}\widetilde{p}_{H,t}^{-\varepsilon}\widetilde{p}'_{H,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon}\widetilde{p}'_{H,t-j}(C_t)\right] \\
& +(1-\theta)\varepsilon\left[\lambda^H_{\Delta,t}\widetilde{p}_{H,t}^{-\varepsilon-1}\widetilde{p}'_{H,t}(C_t) + \sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}_{H,t-j}^{-\varepsilon-1}\widetilde{p}'_{H,t-j}(C_t)\right] \\
& -\lambda^H_{g,t}\omega_F(1+\tau_{M,t})^{1-\eta}(1-\eta)\frac{1}{C_t}\left[\frac{C_t}{C^*_t}\frac{\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}{P^{*,PPT}_t/P^*_{F,t}}\right]^{1-\eta} \\
& -\lambda^H_{y,t}\omega_H\left(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)\right)^{\eta} \\
& -\lambda^H_{y,t}\frac{1-\nu}{\nu}\omega^*_H\eta\frac{C^*_t}{C_t}\left[\frac{C_t(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right))}{C^*_t(1+\tau^*_{M,t})}\right]^{\eta} \\
& -\beta\lambda^H_{e,t}\mathrm{E}_t\left[\frac{\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}{C_{t+1}(\mathcal{G}^{PPT}\left(\tau_{M,t+1},Z_{t+1}/\Psi_{t+1}\right))\Pi_{H,t+1}}\right]
\end{aligned}$$

$$+\lambda^H_{e,t-1}\frac{C_{t-1}(\mathcal{G}^{PPT}\left(\tau_{M,t-1},Z_{t-1}/\Psi_{t-1}\right))}{C_t^2(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right))\Pi_{H,t}}.$$

The Home first-order condition for $\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)$ is

$$\begin{aligned}
0=&-(1-\theta)(1-\varepsilon)\left[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right))+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right))\right]\\
&+(1-\theta)\varepsilon\left[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right))+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right))\right]\\
&+(1-\eta)\lambda^H_{g,t}\left(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)\right)^{-\eta}\\
&-\lambda^H_{g,t}\omega_F(1+\tau_{M,t})^{1-\eta}(1-\eta)\left(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)\right)^{-1}\left[\frac{C_t}{C_t^*}\frac{\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}{P_t^{*,PPT}/P^*_{F,t}}\right]^{1-\eta}\\
&-\eta\lambda^H_{y,t}\omega_H\left(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)\right)^{\eta-1}C_t\\
&-\eta\lambda^H_{y,t}\frac{1-\nu}{\nu}\omega^*_H\left(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)\right)^{-1}C_t^*\left[\frac{C_t(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right))}{C_t^*(1+\tau^*_{M,t})}\right]^{\eta}\\
&-\beta\lambda^H_{e,t}\mathrm{E}_t\left[\frac{C_t}{C_{t+1}(\mathcal{G}^{PPT}\left(\tau_{M,t+1},Z_{t+1}/\Psi_{t+1}\right))\Pi_{H,t+1}}\right]\\
&+\lambda^H_{e,t-1}\frac{C_{t-1}(\mathcal{G}^{PPT}\left(\tau_{M,t-1},Z_{t-1}/\Psi_{t-1}\right))}{C_t(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right))^2\Pi_{H,t}}.
\end{aligned}$$

The remaining Home conditions in the PPT environment are

$$\begin{aligned}
0=&-N_t^{\varphi}+\lambda^H_{n,t}-(1-\theta)(1-\varepsilon)\left[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(N_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(N_t)\right]\\
&+(1-\theta)\varepsilon\left[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(N_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(N_t)\right],\\
0=&-\lambda^H_{n,t}\Delta_{H,t}+\lambda^H_{y,t}-(1-\theta)(1-\varepsilon)\left[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(Y_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(Y_t)\right]\\
&+(1-\theta)\varepsilon\left[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(Y_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(Y_t)\right],\\
0=&-\theta(\varepsilon-1)\lambda^H_{p,t}\Pi^{\varepsilon-2}_{H,t}-\theta\varepsilon\lambda^H_{\Delta,t}\Pi^{\varepsilon-1}_{H,t}\Delta_{H,t-1}\\
&-(1-\theta)(1-\varepsilon)\left[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(\Pi_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\right]\\
&+(1-\theta)\varepsilon\left[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(\Pi_{H,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\right]\\
&+\lambda^H_{e,t-1}\frac{C_{t-1}(\mathcal{G}^{PPT}\left(\tau_{M,t-1},Z_{t-1}/\Psi_{t-1}\right))}{C_t(\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right))\Pi^2_{H,t}},\\
0=&\lambda^H_{\Delta,t}-\lambda^H_{n,t}Y_t-\beta\theta\lambda^H_{\Delta,t+1}\Pi^{\varepsilon}_{H,t+1},\\
0=&-\frac{\lambda^H_{e,t}}{(1+i_t)^2}+\mu^H_{i,t}.
\end{aligned}$$

The Home ZLB conditions are

$$i_t\geq 0,\qquad \mu^H_{i,t}\geq 0,\qquad \mu^H_{i,t}i_t=0.$$

The Foreign first-order conditions in the PPT environment are written explicitly as

$$0=\frac{1}{C_t^*}-(1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(C_t^*)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(C_t^*)\right]$$

$$
\begin{aligned}
&+(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(C^*_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(C^*_t)\right]\\
&-\lambda^F_{g,t}\omega^*_H(1+\tau^*_{M,t})^{1-\eta}(1-\eta)\frac{1}{C^*_t}\left[\frac{C^*_t}{C_t}\frac{P^{*,PPT}_t/P^*_{F,t}}{\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}\right]^{1-\eta}\\
&-\lambda^F_{y,t}\omega^*_F\left(\frac{P^{*,PPT}_t}{P^*_{F,t}}\right)^{\eta}\\
&-\lambda^F_{y,t}\frac{\nu}{1-\nu}\omega_F\eta\frac{C_t}{C^*_t}\left[\frac{C^*_t(P^{*,PPT}_t/P^*_{F,t})}{C_t(1+\tau_{M,t})}\right]^{\eta}\\
&-\beta\lambda^F_{e,t}\mathrm{E}_t\left[\frac{P^{*,PPT}_t/P^*_{F,t}}{C^*_{t+1}(P^{*,PPT}_{t+1}/P^*_{F,t+1})\Pi^*_{F,t+1}}\right]\\
&+\lambda^F_{e,t-1}\frac{C^*_{t-1}(P^{*,PPT}_{t-1}/P^*_{F,t-1})}{(C^*_t)^2(P^{*,PPT}_t/P^*_{F,t})\Pi^*_{F,t}},\\
0=&-(1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(P^{*,PPT}_t/P^*_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(P^{*,PPT}_t/P^*_{F,t})\right]\\
&+(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(P^{*,PPT}_t/P^*_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(P^{*,PPT}_t/P^*_{F,t})\right]\\
&+(1-\eta)\lambda^F_{g,t}\left(\frac{P^{*,PPT}_t}{P^*_{F,t}}\right)^{-\eta}\\
&-\lambda^F_{g,t}\omega^*_H(1+\tau^*_{M,t})^{1-\eta}(1-\eta)\left(\frac{P^{*,PPT}_t}{P^*_{F,t}}\right)^{-1}\left[\frac{C^*_t}{C_t}\frac{P^{*,PPT}_t/P^*_{F,t}}{\mathcal{G}^{PPT}\left(\tau_{M,t},Z_t/\Psi_t\right)}\right]^{1-\eta}\\
&-\eta\lambda^F_{y,t}\omega^*_F\left(\frac{P^{*,PPT}_t}{P^*_{F,t}}\right)^{\eta-1}C^*_t\\
&-\eta\lambda^F_{y,t}\frac{\nu}{1-\nu}\omega_F\left(\frac{P^{*,PPT}_t}{P^*_{F,t}}\right)^{-1}C_t\left[\frac{C^*_t(P^{*,PPT}_t/P^*_{F,t})}{C_t(1+\tau_{M,t})}\right]^{\eta}\\
&-\beta\lambda^F_{e,t}\mathrm{E}_t\left[\frac{C^*_t}{C^*_{t+1}(P^{*,PPT}_{t+1}/P^*_{F,t+1})\Pi^*_{F,t+1}}\right]\\
&+\lambda^F_{e,t-1}\frac{C^*_{t-1}(P^{*,PPT}_{t-1}/P^*_{F,t-1})}{C^*_t(P^{*,PPT}_t/P^*_{F,t})^2\Pi^*_{F,t}},
\end{aligned}
$$

together with

$$
\begin{aligned}
0=&-(N^*_t)^{\varphi}+\lambda^F_{n,t}-(1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(N^*_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(N^*_t)\right]\\
&+(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(N^*_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(N^*_t)\right],\\
0=&-\lambda^F_{n,t}\Delta^*_{F,t}+\lambda^F_{y,t}-(1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(Y^*_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(Y^*_t)\right]\\
&+(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(Y^*_t)+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(Y^*_t)\right],\\
0=&-\theta(\varepsilon-1)\lambda^F_{p,t}(\Pi^*_{F,t})^{\varepsilon-2}-\theta\varepsilon\lambda^F_{\Delta,t}(\Pi^*_{F,t})^{\varepsilon-1}\Delta^*_{F,t-1}\\
&-(1-\theta)(1-\varepsilon)\left[\lambda^F_{p,t}(\widetilde{p}^*_{F,t})^{-\varepsilon}(\widetilde{p}^*_{F,t})'(\Pi^*_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{p,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon}(\widetilde{p}^*_{F,t-j})'(\Pi^*_{F,t})\right]\\
&+(1-\theta)\varepsilon\left[\lambda^F_{\Delta,t}(\widetilde{p}^*_{F,t})^{-\varepsilon-1}(\widetilde{p}^*_{F,t})'(\Pi^*_{F,t})+\sum_{j=1}^{\infty}\beta^{-j}\lambda^F_{\Delta,t-j}(\widetilde{p}^*_{F,t-j})^{-\varepsilon-1}(\widetilde{p}^*_{F,t-j})'(\Pi^*_{F,t})\right]
\end{aligned}
$$

$$+\lambda^F_{e,t-1}\frac{C^*_{t-1}(P^{*,PPT}_{t-1}/P^*_{F,t-1})}{C^*_t(P^{*,PPT}_t/P^*_{F,t})(\Pi^*_{F,t})^2},$$

$$0=\lambda^F_{\Delta,t}-\lambda^F_{n,t}Y^*_t-\beta\theta\lambda^F_{\Delta,t+1}(\Pi^*_{F,t+1})^{\varepsilon},$$

$$0=-\frac{\lambda^F_{e,t}}{(1+i^*_t)^2}+\mu^F_{i,t}.$$

The Foreign ZLB conditions are

$$i^*_t\geq 0,\qquad \mu^F_{i,t}\geq 0,\qquad \mu^F_{i,t}i^*_t=0.$$

D.2.2 IPT Environment

We differentiate the exact intertemporal Lagrangian (C.4) after complete risk sharing and the two private reset-price conditions have been substituted. A prime on $\widetilde{p}_{H,t}$ or on $\overline{P}_{F,t}/P_{F,t}$ below denotes the total derivative of that exact reset-price map with respect to the indicated Ramsey choice; hence all current and future terms implied by commitment are retained. The cross-sectional purchaser-price integrals are differentiated directly and are not replaced by an importer-dispersion variable.

The Home consumption FOC is

$$\begin{aligned}
0=&\frac{1}{C_t}-\lambda^H_{g,t}\omega_F(1-\eta)\left(\frac{\widetilde{P}^{IPT}_{F,t}}{P_{H,t}}\right)^{-\eta}\frac{\partial(\widetilde{P}^{IPT}_{F,t}/P_{H,t})}{\partial C_t}-\lambda^H_{y,t}\omega_H\mathcal{G}^{IPT}_t(\cdot)^{\eta}\\
&-\lambda^H_{y,t}\frac{1-\nu}{\nu}\omega^*_H\frac{\partial}{\partial C_t}\left[\left(\frac{\widetilde{P}^*_{H,t}}{P^{*,IPT}_t}\right)^{-\eta}C^*_t\frac{1}{1-\nu}\int_\nu^1\left(\frac{\widetilde{P}^*_{H,t}(\iota)}{\widetilde{P}^*_{H,t}}\right)^{-\varepsilon_M}d\iota\right]\\
&-(1-\theta)(1-\varepsilon)\Big[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(C_t)\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(C_t)\Big]\\
&+(1-\theta)\varepsilon\Big[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(C_t)\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(C_t)\Big]\\
&-(1-\theta_M)(1-\varepsilon_M)\Big[\lambda^H_{Mp,t}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)'(C_t)\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{Mp,t-j}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)'(C_t)\Big]\\
&-\beta\lambda^H_{e,t}\mathrm{E}_t\left[\frac{\mathcal{G}^{IPT}_t(\cdot)}{C_{t+1}\mathcal{G}^{IPT}_{t+1}(\cdot)\Pi_{H,t+1}}\right]+\lambda^H_{e,t-1}\frac{C_{t-1}\mathcal{G}^{IPT}_{t-1}(\cdot)}{C_t^2\mathcal{G}^{IPT}_t(\cdot)\Pi_{H,t}}.
\end{aligned}\tag{D.2}$$

The FOC for the Home CPI–PPI ratio is

$$\begin{aligned}
0=&(1-\eta)\lambda^H_{g,t}\mathcal{G}^{IPT}_t(\cdot)^{-\eta}-\lambda^H_{g,t}\omega_F(1-\eta)\left(\frac{\widetilde{P}^{IPT}_{F,t}}{P_{H,t}}\right)^{-\eta}\frac{\partial(\widetilde{P}^{IPT}_{F,t}/P_{H,t})}{\partial\mathcal{G}^{IPT}_t(\cdot)}\\
&-\eta\lambda^H_{y,t}\omega_H\mathcal{G}^{IPT}_t(\cdot)^{\eta-1}C_t-\lambda^H_{y,t}\frac{1-\nu}{\nu}\omega^*_H\frac{\partial}{\partial\mathcal{G}^{IPT}_t(\cdot)}\left[\left(\frac{\widetilde{P}^*_{H,t}}{P^{*,IPT}_t}\right)^{-\eta}C^*_t\frac{1}{1-\nu}\int_\nu^1\left(\frac{\widetilde{P}^*_{H,t}(\iota)}{\widetilde{P}^*_{H,t}}\right)^{-\varepsilon_M}d\iota\right]\\
&-(1-\theta)(1-\varepsilon)\Big[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(\mathcal{G}^{IPT}_t)\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(\mathcal{G}^{IPT}_t)\Big]\\
&+(1-\theta)\varepsilon\Big[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(\mathcal{G}^{IPT}_t)\\
&\quad+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(\mathcal{G}^{IPT}_t)\Big]\\
&-(1-\theta_M)(1-\varepsilon_M)\Big[\lambda^H_{Mp,t}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)'(\mathcal{G}^{IPT}_t)
\end{aligned}$$

$$+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{Mp,t-j}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)'(\mathcal{G}^{IPT}_t)\Big]$$
$$-\beta\lambda^H_{e,t}\mathrm{E}_t\left[\frac{C_t}{C_{t+1}\mathcal{G}^{IPT}_{t+1}(\cdot)\Pi_{H,t+1}}\right]+\lambda^H_{e,t-1}\frac{C_{t-1}\mathcal{G}^{IPT}_{t-1}(\cdot)}{C_t\mathcal{G}^{IPT}_t(\cdot)^2\Pi_{H,t}}. \tag{D.3}$$

Here $\widetilde{p}'_{H,t}(\mathcal{G}^{IPT}_t)$ and $(\overline{P}_{F,t}/P_{F,t})'(\mathcal{G}^{IPT}_t)$ mean total derivatives with respect to the displayed CPI–PPI ratio, not derivatives with respect to a newly defined variable.

The labor and output FOCs are

$$\begin{aligned}0=&-N^{\varphi}_t+\lambda^H_{n,t}-(1-\theta)(1-\varepsilon)\Big[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(N_t)\\&+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(N_t)\Big]\\&+(1-\theta)\varepsilon\Big[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(N_t)\\&+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(N_t)\Big],\end{aligned} \tag{D.4}$$

$$\begin{aligned}0=&-\lambda^H_{n,t}\Delta_{H,t}+\lambda^H_{y,t}-(1-\theta)(1-\varepsilon)\Big[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(Y_t)\\&+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(Y_t)\Big]\\&+(1-\theta)\varepsilon\Big[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(Y_t)\\&+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(Y_t)\Big].\end{aligned} \tag{D.5}$$

For the local *ex-tariff* import-price ratio,

$$\begin{aligned}0=&-\lambda^H_{g,t}\omega_F(1-\eta)\left(\frac{\widetilde{P}^{IPT}_{F,t}}{P_{H,t}}\right)^{-\eta}\frac{\partial(\widetilde{P}^{IPT}_{F,t}/P_{H,t})}{\partial(P_{F,t}/P_{H,t})}\\&-\lambda^H_{y,t}\frac{1-\nu}{\nu}\omega^*_H\frac{\partial}{\partial(P_{F,t}/P_{H,t})}\left[\left(\frac{\widetilde{P}^*_{H,t}}{P^{*,IPT}_t}\right)^{-\eta}C^*_t\frac{1}{1-\nu}\int_{\nu}^{1}\left(\frac{\widetilde{P}^*_{H,t}(\iota)}{\widetilde{P}^*_{H,t}}\right)^{-\varepsilon_M}d\iota\right]\\&-(1-\theta_M)(1-\varepsilon_M)\Big[\lambda^H_{Mp,t}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)'(P_{F,t}/P_{H,t})\\&+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{Mp,t-j}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)'(P_{F,t}/P_{H,t})\Big]\\&+\lambda^H_{q,t}-\beta\lambda^H_{q,t+1}\frac{\Pi_{F,t+1}}{\Pi_{H,t+1}}.\end{aligned} \tag{D.6}$$

The two inflation FOCs are

$$\begin{aligned}0=&-\theta(\varepsilon-1)\lambda^H_{p,t}\Pi^{\varepsilon-2}_{H,t}-\theta\varepsilon\lambda^H_{\Delta,t}\Pi^{\varepsilon-1}_{H,t}\Delta_{H,t-1}\\&-(1-\theta)(1-\varepsilon)\Big[\lambda^H_{p,t}\widetilde{p}^{-\varepsilon}_{H,t}\widetilde{p}'_{H,t}(\Pi_{H,t})\\&+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{p,t-j}\widetilde{p}^{-\varepsilon}_{H,t-j}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\Big]\\&+(1-\theta)\varepsilon\Big[\lambda^H_{\Delta,t}\widetilde{p}^{-\varepsilon-1}_{H,t}\widetilde{p}'_{H,t}(\Pi_{H,t})\\&+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{\Delta,t-j}\widetilde{p}^{-\varepsilon-1}_{H,t-j}\widetilde{p}'_{H,t-j}(\Pi_{H,t})\Big]\\&-(1-\theta_M)(1-\varepsilon_M)\Big[\lambda^H_{Mp,t}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)'(\Pi_{H,t})\\&+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{Mp,t-j}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)'(\Pi_{H,t})\Big]\end{aligned}$$

$$+\lambda^H_{q,t}\frac{P_{F,t-1}}{P_{H,t-1}}\frac{\Pi_{F,t}}{\Pi^2_{H,t}}+\lambda^H_{e,t-1}\frac{C_{t-1}\mathcal{G}^{IPT}_{t-1}(\cdot)}{C_t\mathcal{G}^{IPT}_t(\cdot)\Pi^2_{H,t}}, \tag{D.7}$$

$$\begin{aligned}0=&-\theta_M(\varepsilon_M-1)\lambda^H_{Mp,t}\Pi^{\varepsilon_M-2}_{F,t}\\&-(1-\theta_M)(1-\varepsilon_M)\Big[\lambda^H_{Mp,t}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t}}{P_{F,t}}\right)'(\Pi_{F,t})\\&+\sum_{j=1}^{\infty}\beta^{-j}\lambda^H_{Mp,t-j}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)^{-\varepsilon_M}\left(\frac{\overline{P}_{F,t-j}}{P_{F,t-j}}\right)'(\Pi_{F,t})\Big]-\lambda^H_{q,t}\frac{P_{F,t-1}}{P_{H,t-1}}\frac{1}{\Pi_{H,t}}.\end{aligned} \tag{D.8}$$

The producer-dispersion and nominal-rate conditions are

$$0=\lambda^H_{\Delta,t}-\lambda^H_{n,t}Y_t-\beta\theta\lambda^H_{\Delta,t+1}\Pi^{\varepsilon}_{H,t+1}, \tag{D.9}$$

$$0=-\frac{\lambda^H_{e,t}}{(1+i_t)^2}+\mu^H_{i,t},\qquad i_t\geq 0,\quad \mu^H_{i,t}\geq 0,\quad \mu^H_{i,t}i_t=0. \tag{D.10}$$

There is no FOC with respect to a separate importer-dispersion state: in the current micro-importer model the exact purchaser-price cross section in B.2 is retained directly. The Foreign FOCs are obtained from Eqs. (D.2)–(D.10) by the exact Home–Foreign mirror transformation and satisfy

$$-\frac{\lambda^F_{e,t}}{(1+i^*_t)^2}+\mu^F_{i,t}=0,\qquad i^*_t\geq 0,\quad \mu^F_{i,t}\geq 0,\quad \mu^F_{i,t}i^*_t=0. \tag{D.11}$$

Together with the common B.2 private equilibrium, these conditions are the two national best-response systems used in the IPT simulations.

E Welfare Cost Function

The second-order criteria in this section are derived around the distorted deterministic steady state using the Benigno–Woodford linear–quadratic method. Policy-dependent first-order terms are eliminated with the first-order equilibrium restrictions before the remaining second-order terms are reduced and completed into squares. The nonlinear Ramsey allocations in the main text are generated by Appendices C and D, not by minimizing these quadratic criteria.

Reproduction Map and Conventions. The reproducible calculation has five operations, always performed in this order: (i) insert the steady-state restrictions and the stationary Benigno–Woodford multipliers into the *existing* nonlinear household, price-index, goods-market, and Calvo conditions; (ii) eliminate the policy-dependent first-order welfare terms by coefficient matching; (iii) retain the second-order terms of those exact restrictions, including tariff–LOOP-gap interactions and price dispersion; (iv) take the negative Hessian and the tariff cross-derivatives with respect to the displayed policy-dependent coordinates; and (v) complete squares using Eqs. (E.5)–(E.6). The following steps document the structural restrictions, differentiation, and coefficient calculations for the SOE and two-country PPT and IPT welfare expansions. These second-order calculations are used to interpret the original nonlinear Ramsey allocations; they do not determine them.

The reader can locate the inputs and output of each stage as follows:

Environment	First-order elimination and second-order source	Final coefficients and checks
SOE PPT	C.1.1, D.1.1; Eq. (E.18)	Eqs. (E.12), (E.23); E.1 derivation
SOE IPT	C.1.2, D.1.2; Eqs. (E.17), (32)	Eqs. (E.14), (E.23); E.1 derivation
Two-country PPT	C.2.1, D.2.1; Eqs. (E.33), (E.35)	Eqs. (E.37), (E.40)
Two-country IPT	C.2.2, D.2.2; Eqs. (E.49), (E.50)	Eqs. (E.52), (E.54)

A variable without a time subscript denotes its deterministic steady-state value, lower-case real variables denote log deviations, and $O(3)$ denotes terms of total degree three and above. The notation s.o.t.i.p. means second-order terms independent of the national planner's endogenous allocation; it does not imply that those terms are numerically zero. The symbol Θ_χ denotes the stationary multiplier on the CPI identity.

Structural-Parameter Closure. All welfare cost function coefficients in this section are functions of the primitive structural parameters already defined in the main text. For the SOE and two-country economies, these are

$$\beta, \quad \varphi, \quad \eta, \quad \varepsilon, \quad \varepsilon_M, \quad \theta, \quad \theta_M, \quad \upsilon, \quad \nu, \tag{E.1}$$

Their meanings are collected here; none is an independently calibrated welfare coefficient:

Symbol	Definition
β	Household discount factor.
φ	Inverse Frisch elasticity in household labor disutility.
η	Substitution elasticity between Home and Foreign consumption goods.
ε, ε_M	Substitution elasticities across producer varieties and importer varieties, respectively.
θ, θ_M	Calvo non-reset probabilities of Home producers and Home importers, respectively.
υ	Openness/Home-bias parameter entering bilateral expenditure shares.
ν	Home share of the two-country household mass (not needed for the SOE).

The bilateral expenditure weights are not additional parameters:

$$\omega_F = \upsilon(1-\nu), \qquad \omega_H = 1-\omega_F, \qquad \omega_H^* = \upsilon\nu, \qquad \omega_F^* = 1-\omega_H^*. \tag{E.2}$$

Likewise,

$$\mathcal{M} = \frac{\varepsilon}{\varepsilon-1}, \qquad L = \mathcal{M}^{-1} = \frac{\varepsilon-1}{\varepsilon}, \qquad \kappa_H = \frac{(1-\theta)(1-\beta\theta)}{\theta}, \qquad \kappa_M = \frac{(1-\theta_M)(1-\beta\theta_M)}{\theta_M}. \tag{E.3}$$

The steady-state importer subsidy is not an additional structural parameter: throughout the IPT environment it is fixed at $\tau = 1/\varepsilon_M$, as in Sec. 3.1.1. The tariff rates $\tau_{M,t}$ and $\tau_{M,t}^*$ and their persistence parameters determine the exogenous conditioning paths, but the shock persistence does not enter any per-period welfare weight. Hence the tariff processes are not free parameters of the welfare cost function. Every Θ coefficient below is determined by the displayed Benigno–Woodford first-order coefficient-matching system; every real quadratic coefficient is an entry of the Hessian and tariff-loading matrices of the corresponding second-order augmented welfare polynomial; and every Λ is obtained by the displayed sequential Schur completion. The welfare weights and WR values are derived from the structural parameters.

Here Θ denotes a stationary multiplier on an existing first-order restriction, $\widehat{C}$ and C are pre-reduction and reduced second-order welfare coefficients, Q and B are negative-Hessian and tariff-loading matrices, Λ is a completed-square coefficient, and b and Γ denote coefficients in the welfare-relevant reference values. None is an independently calibrated structural parameter. The SOE shares d_H and λ_M are defined where first used below. The exogenous tariff persistence ρ_τ does not enter a per-period welfare weight.

For the sequential square completion used below, retain the algebraic representation already used in the previous derivation. If a reduced policy-dependent quadratic loss is written as

$$\mathcal{L}_t \;=\; \frac{1}{2}\sum_{k=1}^{n}\Lambda_k^{(0)} z_{k,t}^2 - \sum_{k<j} C_{kj}^{(0)} z_{k,t} z_{j,t} - \sum_{k=1}^{n} h_{k,t}^{(0)} z_{k,t} + \text{s.o.t.i.p.}, \tag{E.4}$$

then at step k the completed-square reference value is

$$z_{k,t}^{w} = \frac{h_{k,t}^{(k-1)} + \sum_{j=k+1}^{n} C_{kj}^{(k-1)} z_{j,t}}{\Lambda_k^{(k-1)}}. \tag{E.5}$$

For $j, l > k$, the remaining coefficients are updated according to

$$\begin{aligned}
\Lambda_j^{(k)} &= \Lambda_j^{(k-1)} - \frac{[C_{kj}^{(k-1)}]^2}{\Lambda_k^{(k-1)}}, \\
C_{jl}^{(k)} &= C_{jl}^{(k-1)} + \frac{C_{kj}^{(k-1)} C_{kl}^{(k-1)}}{\Lambda_k^{(k-1)}}, \\
h_{j,t}^{(k)} &= h_{j,t}^{(k-1)} + \frac{h_{k,t}^{(k-1)} C_{kj}^{(k-1)}}{\Lambda_k^{(k-1)}}.
\end{aligned} \tag{E.6}$$

These are algebraic completion formulas only; they do not introduce additional private-equilibrium variables.

E.1 Small Open Economy

Representation with the *Ex-Tariff* Relative Border Price Retained. The two unnumbered welfare representations at the beginning of Sec. 3.4 retain the WR *Ex-tariff* Relative Border Price of Imports Gap explicitly. The period welfare cost itself remains $\mathcal{L}_t^{SOE,\cdot}$.

E.1.1 PPT Environment

For PPT, write the real part of the second-order welfare polynomial, up to terms independent of the endogenous allocation, as

$$\begin{aligned}
\mathcal{L}_{t,\text{real}}^{SOE,PPT} &= \frac{1}{2}\left[-2\widehat{C}_{yy}^{SOE,PPT} y_t^2 - 2\widehat{C}_{zz}^{SOE,PPT} z_t^2\right] - \widehat{C}_{yz}^{SOE,PPT} y_t z_t \\
&\quad - \widehat{C}_{ym}^{SOE,PPT} y_t \tau_{M,t} - \widehat{C}_{zm}^{SOE,PPT} z_t \tau_{M,t} + \text{s.o.t.i.p.},
\end{aligned} \tag{E.7}$$

where s.o.t.i.p. denotes terms independent of the endogenous allocation to the order considered. The coefficients $\widehat{C}_{yy}^{SOE,PPT}$, $\widehat{C}_{zz}^{SOE,PPT}$, $\widehat{C}_{yz}^{SOE,PPT}$, $\widehat{C}_{ym}^{SOE,PPT}$, and $\widehat{C}_{zm}^{SOE,PPT}$ are the literal coefficients on the corresponding quadratic or tariff-interaction terms in this welfare polynomial. They are not free coefficients. Equivalently, they are the corresponding Hessian and tariff-loading derivatives of the negative Benigno–Woodford augmented second-order real welfare contribution generated by the exact C.1.1 constraints and the stationary multipliers in Eq. (E.18). Since the constraints and those multipliers are functions only of Eqs. (E.1)– (E.3), each hatted coefficient is a structural function of the primitive parameters.

The first-order approximations of Eqs. (12) and (19) imply

$$y_t = \left[p_*^{-1}\right] z_t + k_*/p_* \left(\tau_{M,t} - \psi_t\right). \tag{E.8}$$

Under PPT, $\psi_t = 0$. Using Eq. (E.8) to remove z_t from the output–relative-price cross term before completing squares gives

$$\begin{aligned}
\widehat{\Lambda}_y^{SOE,PPT} &\equiv -2\widehat{C}_{yy}^{SOE,PPT} - \frac{2\widehat{C}_{yz}^{SOE,PPT}}{p_*^{-1}}, \\
\widehat{y}_t^{w,SOE,PPT} &\equiv \frac{1}{\widehat{\Lambda}_y^{SOE,PPT}}\left[\widehat{C}_{ym}^{SOE,PPT} - \frac{k_*/p_*}{p_*^{-1}}\widehat{C}_{yz}^{SOE,PPT}\right]\tau_{M,t}, \\
\widehat{\Lambda}_z^{SOE,PPT} &\equiv -2\widehat{C}_{zz}^{SOE,PPT}, \\
\widehat{z}_t^{w,SOE,PPT} &\equiv \frac{\widehat{C}_{zm}^{SOE,PPT}}{\widehat{\Lambda}_z^{SOE,PPT}}\tau_{M,t}.
\end{aligned} \tag{E.9}$$

Thus the WR output appearing in the first unnumbered welfare representation in Sec. 3.4 contains no z_t.

E.1.2 IPT Environment

For IPT, the corresponding real polynomial is

$$\begin{aligned}\mathcal{L}_{t,\text{real}}^{SOE,IPT} &= \frac{1}{2}\left[-2\widehat{C}_{yy}^{SOE,IPT}y_t^2 - 2\widehat{C}_{\psi\psi}^{SOE,IPT}\psi_t^2 - 2\widehat{C}_{zz}^{SOE,IPT}z_t^2\right] \\ &\quad -\widehat{C}_{y\psi}^{SOE,IPT}y_t\psi_t - \widehat{C}_{yz}^{SOE,IPT}y_tz_t - \widehat{C}_{\psi z}^{SOE,IPT}\psi_tz_t \\ &\quad -\widehat{C}_{ym}^{SOE,IPT}y_t\tau_{M,t} - \widehat{C}_{\psi m}^{SOE,IPT}\psi_t\tau_{M,t} - \widehat{C}_{zm}^{SOE,IPT}z_t\tau_{M,t} + \text{s.o.t.i.p.} \end{aligned} \tag{E.10}$$

The hatted C coefficients again denote the literal coefficients on the corresponding terms in the welfare polynomial. Equivalently, they are entries of the Hessian and tariff-loading derivatives of the negative Benigno–Woodford augmented second-order real welfare contribution generated by C.1.2 and the stationary multipliers in Eqs. (E.18)–(E.17). They are therefore structural functions rather than additional parameters. Applying Eq. (E.8) to the y_tz_t term before sequentially completing the y_t, ψ_t, and z_t squares gives

$$\begin{aligned}
\widehat{\Lambda}_y^{SOE,IPT} &\equiv -2\widehat{C}_{yy}^{SOE,IPT} - \frac{2\widehat{C}_{yz}^{SOE,IPT}}{p_*^{-1}}, \\
\widehat{y}_t^{w,SOE,IPT} &\equiv \frac{1}{\widehat{\Lambda}_y^{SOE,IPT}}\left\{\left[\widehat{C}_{ym}^{SOE,IPT} - \frac{k_*/p_*}{p_*^{-1}}\widehat{C}_{yz}^{SOE,IPT}\right]\tau_{M,t}\right. \\
&\qquad \left. + \left[\widehat{C}_{y\psi}^{SOE,IPT} + \frac{k_*/p_*}{p_*^{-1}}\widehat{C}_{yz}^{SOE,IPT}\right]\psi_t\right\}, \\
\widehat{\Lambda}_\psi^{SOE,IPT} &\equiv -2\widehat{C}_{\psi\psi}^{SOE,IPT} - \frac{1}{\widehat{\Lambda}_y^{SOE,IPT}}\left[\widehat{C}_{y\psi}^{SOE,IPT} + \frac{k_*/p_*}{p_*^{-1}}\widehat{C}_{yz}^{SOE,IPT}\right]^2, \\
\widehat{\psi}_t^{w,SOE,IPT} &\equiv \frac{1}{\widehat{\Lambda}_\psi^{SOE,IPT}}\left\{\left[\widehat{C}_{\psi m}^{SOE,IPT} + \frac{1}{\widehat{\Lambda}_y^{SOE,IPT}}\left(\widehat{C}_{y\psi}^{SOE,IPT} + \frac{k_*/p_*}{p_*^{-1}}\widehat{C}_{yz}^{SOE,IPT}\right)\right.\right. \\
&\qquad \left.\left. \times\left(\widehat{C}_{ym}^{SOE,IPT} - \frac{k_*/p_*}{p_*^{-1}}\widehat{C}_{yz}^{SOE,IPT}\right)\right]\tau_{M,t} + \widehat{C}_{\psi z}^{SOE,IPT}z_t\right\}, \\
\widehat{\Lambda}_z^{SOE,IPT} &\equiv -2\widehat{C}_{zz}^{SOE,IPT} - \frac{(\widehat{C}_{\psi z}^{SOE,IPT})^2}{\widehat{\Lambda}_\psi^{SOE,IPT}}, \\
\widehat{z}_t^{w,SOE,IPT} &\equiv \frac{1}{\widehat{\Lambda}_z^{SOE,IPT}}\left\{\widehat{C}_{zm}^{SOE,IPT} + \frac{\widehat{C}_{\psi z}^{SOE,IPT}}{\widehat{\Lambda}_\psi^{SOE,IPT}}\left[\widehat{C}_{\psi m}^{SOE,IPT} + \frac{1}{\widehat{\Lambda}_y^{SOE,IPT}}\left(\widehat{C}_{y\psi}^{SOE,IPT} + \frac{k_*/p_*}{p_*^{-1}}\widehat{C}_{yz}^{SOE,IPT}\right)\right.\right. \\
&\qquad \left.\left. \times\left(\widehat{C}_{ym}^{SOE,IPT} - \frac{k_*/p_*}{p_*^{-1}}\widehat{C}_{yz}^{SOE,IPT}\right)\right]\right\}\tau_{M,t}.
\end{aligned} \tag{E.11}$$

The WR output in Eq. (E.11) therefore depends on the tariff and the LOOP gap, but not on z_t.

Representation After Eliminating the *Ex-Tariff* Relative Border Price. The WR variables used in the numbered SOE welfare-cost equations in Sec. 3.4 are defined here. Around the zero-tariff steady state, the first-order relation $\log(1+\tau_{M,t}) = \tau_{M,t}$ applies. We therefore use the tariff rate $\tau_{M,t}$ directly and introduce no auxiliary tariff variable. In the present SOE experiments there is no export-tariff disturbance. After the first-order elimination of z_t, the unhatted coefficients $C_{yy}^{SOE,PPT}$, $C_{ym}^{SOE,PPT}$, C_{yy}^{IPT}, $C_{y\psi}^{IPT}$, C_{ym}^{IPT}, $C_{\psi\psi}^{IPT}$, and $C_{\psi m}^{IPT}$ are the corresponding entries of the reduced structural Hessian and tariff-loading vector. Hence they are algebraic transforms of the hatted structural coefficients above, not additional parameters. Thus

$$y_t^{w,SOE,PPT} \equiv \frac{C_{ym}^{SOE,PPT}}{\Lambda_y^{SOE,PPT}}\tau_{M,t}, \tag{E.12}$$

$$y_t^{w,SOE,IPT} \equiv \frac{C_{ym}^{IPT}}{\Lambda_y^{SOE,IPT}}\tau_{M,t} + \frac{C_{y\psi}^{IPT}}{\Lambda_y^{SOE,IPT}}\psi_t. \tag{E.13}$$

After completing the output square, collecting the remaining terms in ψ_t gives the WR LOOP gap directly as

$$\psi_t^{w,SOE,IPT} \equiv \frac{C_{\psi m}^{IPT} + \dfrac{C_{y\psi}^{IPT} C_{ym}^{IPT}}{\Lambda_y^{SOE,IPT}}}{\Lambda_\psi^{SOE,IPT}} \tau_{M,t}. \tag{E.14}$$

Here $C_{ym}^{SOE,PPT}$, C_{ym}^{IPT}, $C_{\psi m}^{IPT}$, and $C_{y\psi}^{IPT}$ are the corresponding cross coefficients in the second-order welfare expansion; the coefficient notation is retained from the underlying symbolic welfare derivation. Equation (E.14) depends only on the exogenous tariff innovation and therefore is the reference value entering the LOOP-gap square in Eq. (38). The compact main-text coefficient $\Gamma_{\text{own}}^{SOE}$ is defined as the entire structural coefficient multiplying $\tau_{M,t}$ on the right-hand side of Eq. (E.14).

The coefficients Θ_n, Θ_H^{AS}, and Θ_M^{AS} are undetermined coefficients used to eliminate the first-order terms in the Benigno–Woodford expansion. Table 2 instead reports the resulting welfare weights. For the PPT environment, temporary coefficients Θ_χ, Θ_Y, Θ_n, $\Theta_{mc,H}$, and Θ_H^{AS} are attached to the corresponding first-order equilibrium restrictions. In the IPT environment, $\Theta_{mc,M}$ and Θ_M^{AS} are additionally attached to the importer marginal-cost identity and the importer aggregate-supply restriction.

For the SOE welfare expansion, the structural point of expansion must be the original *distorted* zero-tariff deterministic steady state. Let $\bar{Z}$, $\bar{Y}$, $\bar{C}$, and $\bar{\Xi}$ denote, respectively, the steady levels of Z_t, Y_t, C_t, and $P_t/P_{H,t}$. The original nonlinear equilibrium restrictions give

$$\bar{\Xi}^{1-\eta} = (1-\upsilon) + \upsilon \bar{Z}^{1-\eta}, \qquad \bar{C} = \bar{Z}/\bar{\Xi}, \qquad \bar{Y} = (1-\upsilon)\bar{Z}\bar{\Xi}^{\eta-1} + \upsilon \bar{Z}^{\eta}, \qquad \bar{Z}\bar{Y}^{\varphi} = L. \tag{E.15}$$

Thus neither $\bar{Z} = 1$ nor $\bar{\Xi} = 1$ may be imposed in coefficient calculations. The quantities in Eq. (E.15) are determined entirely by the original structural parameters, not separately calibrated. For the first-order reduction of the exact goods-market and risk-sharing conditions, define the following steady-state structural coefficients:

$$\begin{aligned} w_F &\equiv \frac{\upsilon \bar{Z}^{1-\eta}}{(1-\upsilon) + \upsilon \bar{Z}^{1-\eta}}, \qquad H \equiv (1-\upsilon)\bar{Z}\bar{\Xi}^{\eta-1}, \qquad X \equiv \upsilon \bar{Z}^{\eta}, \\ p_* &\equiv \frac{\bar{Y}}{H[1+(\eta-1)w_F] + \eta X}, \qquad k_* \equiv \frac{(\eta-1)Hw_F}{H[1+(\eta-1)w_F] + \eta X}. \end{aligned} \tag{E.16}$$

Here w_F is the steady-state expenditure share on imports, H is Home-good absorption, and X is exports; they are derived steady-state quantities, not new independent private-equilibrium variables. Implicit differentiation of the original exact restrictions gives $z_t = p_* y_t + k_*(\psi_t - \tau_{M,t})$ (with $\psi_t = 0$ for PPT).

Use Θ_H^{AS} and Θ_M^{AS} for the stationary multipliers on the reset-ex-tariff-price-weighted producer and importer aggregate-supply restrictions, respectively. Setting the coefficients on the first-order policy-dependent y_t and ψ_t terms in the augmented welfare contribution to zero yields

$$\begin{aligned} \Theta_H^{AS} &= \frac{\bar{Y}^{1+\varphi} - (1-w_F)p_*}{(\bar{Y}/\bar{Z})(\varphi + p_*)}, \\ \Theta_M^{AS} &= -\frac{(1-w_F)k_* + w_F + (\bar{Y}/\bar{Z})k_*\Theta_H^{AS}}{\bar{C}^{1-\eta}}, \qquad \text{(IPT only)}. \end{aligned} \tag{E.17}$$

The producer coefficient is the same under PPT and IPT. The associated multiplier on the $n_t = y_t + \log \Delta_{H,t}$ identity is

$$\Theta_n = \bar{Y}^{1+\varphi} - \varphi(\bar{Y}/\bar{Z})\Theta_H^{AS}. \tag{E.18}$$

The remaining first-order auxiliary-identity multipliers are fixed by the same stationarity system, and do not enter the reduced welfare coefficients. In particular, formulas for Θ_Y, Θ_χ, and the aggregate-supply multipliers derived under a unit-normalized steady state do not apply to the nonunit steady state considered here.

Reproducible Importer Coefficient Calculation. The following steps specify the second-order object behind Eq. (E.14); the import tariff continues to be the *additive* duty in Eq. (20). With no first-order importer price dispersion the relevant purchaser-price ratio is

$$\frac{\widetilde{P}_{F,t}^{IPT}}{P_{H,t}} = \bar{Z}(e^{z_t-\psi_t} + \tau_{M,t}e^{z_t}),$$

at zero importer-price dispersion. In particular, interactions of the first-order tariff with second-order importer-price dispersion contribute only from third order. Substituting $(1+\tau_{M,t})\bar{Z}e^{z_t-\psi_t}$ would omit the second-order term $\tau_{M,t}\psi_t$ from its logarithm. The two corresponding exact real conditions used to eliminate consumption and the border price in the second-order expansion are Eqs. (26) and (35).

In the discounted importer reset FOC (32), the *ex-tariff reset price* $\overline{P}_{F,t}$ is fixed across all future terms of a given cohort. Multiplying that FOC by this fixed reset price and summing across resetting cohorts produces a legitimate present-value aggregate-supply restriction. Multiplication by the future *cum-tariff purchaser price* is not legitimate, since that price changes with future statutory tariffs. At zero price dispersion the correct reset-ex-tariff-price-weighted importer restriction, normalized by the steady-state import share, is proportional to

$$\left(\frac{P_t^{IPT}}{P_{H,t}}\right)^{\eta-1}\left(\frac{\widetilde{P}_{F,t}^{IPT}}{P_{H,t}}\right)^{-\eta}\frac{P_{F,t}}{P_{H,t}}\frac{\tau_{M,t}+(\varepsilon_M-1)(1-e^{-\psi_t})}{(\varepsilon_M-1)(e^{-\psi_t}+\tau_{M,t})}.$$

The ex-tariff price factor is $P_{F,t}/P_{H,t} = \bar{Z}e^{z_t-\psi_t}$; it is *not* the cum-tariff price. To verify the importer-price-dispersion term independently, the normalization of the ex-tariff CES price index implies, with $\log(P_{F,t}(\iota)/P_{F,t})$ denoting the cohort log-price deviation,

$$\int_0^1\left(\frac{P_{F,t}(\iota)}{P_{F,t}}\right)^{-\varepsilon_M}d\iota = 1+\frac{\varepsilon_M}{2}\int_0^1\left[\log\frac{P_{F,t}(\iota)}{P_{F,t}}\right]^2 d\iota + O(3).$$

Thus the fixed-reset-ex-tariff-price-weighted aggregate restriction has the importer-dispersion coefficient $\varepsilon_M/2$. The Calvo price-index law then converts its discounted dispersion sum into an importer-inflation-square coefficient involving $1/\kappa_M$. This yields the import-inflation weight in Eq. (E.23), without changing the real Hessian. These manipulations are valid for the discounted reset FOC up to the usual inherited-cohort initial boundary terms.

For completeness, the real augmented second-order expression before differentiation is the following sum of existing household utility (normalized using L as in Eq. (E.3)) and the two reset-price-weighted aggregate-supply restrictions:

$$\begin{aligned}&\log\bar{Z}+z_t-\log\frac{P_t^{IPT}}{P_{H,t}}-\frac{\bar{Y}^{1+\varphi}}{1+\varphi}e^{(1+\varphi)y_t}+\Theta_H^{AS}\frac{\bar{Y}}{\bar{Z}}e^{y_t-z_t}(e^{\varphi y_t+z_t}-1)\\&+\Theta_M^{AS}\left(\frac{P_t^{IPT}}{P_{H,t}}\right)^{\eta-1}\left(\frac{\widetilde{P}_{F,t}^{IPT}}{P_{H,t}}\right)^{-\eta}\bar{Z}e^{z_t-\psi_t}\frac{\tau_{M,t}+(\varepsilon_M-1)(1-e^{-\psi_t})}{(\varepsilon_M-1)(e^{-\psi_t}+\tau_{M,t})}.\end{aligned}$$

The two exact real identities above first eliminate C_t and then implicitly eliminate z_t, including its *second* derivatives. The first derivatives in y_t and ψ_t of the resulting augmented expression are zero at the baseline steady state, providing an independent check on the undetermined coefficients.

Using the stationary multipliers above, evaluate the negative Hessian of this augmented contribution after implicitly eliminating z_t to *second order* from the exact goods-market identity. For the baseline IPT calibration, the policy-variable block $Q^{SOE,IPT}$ and tariff-loading vector $B^{SOE,IPT}$ are shown in Eq. (E.21) below. The omitted tariff–tariff Hessian entry is independent of Home policy. In the notation of Eq. (E.14),

$$\Lambda_y^{SOE,IPT} = 1.6542172374, \qquad C_{y\psi}^{IPT} = 0.0036494190,$$

$$
\begin{aligned}
&C_{ym}^{IPT} = 0.1358745616, \qquad C_{\psi m}^{IPT} = 0.2128527417,\\
&\Lambda_{\psi}^{SOE,IPT} = 0.5062809752,\\
&\Gamma_{\text{own}}^{SOE} = \frac{0.2128527417}{0.5062809752} + \frac{0.0036494190 \cdot 0.1358745616}{1.6542172374 \cdot 0.5062809752},\\
&\Gamma_{\text{own}}^{SOE} = 0.4204241362 + 0.0005920764 = 0.4210162126.
\end{aligned}
$$

The direct tariff–LOOP-gap interaction accounts for most of $\Gamma_{\text{own}}^{SOE}$, while the output-interaction term contributes the remainder. Both contributions are derived at the original distorted steady state rather than calibrated independently.

Differentiation of the Reduced SOE Criteria. The following calculation derives the coefficients of the reduced SOE welfare-cost functions in Eqs. (37)–(38) from the Hessian definitions in Appendices E.1.1 and E.1.2. At zero importer-price dispersion, Eqs. (20) and (25) imply

$$
\begin{aligned}
\left(\frac{P_t^{IPT}}{P_{H,t}}\right)^{1-\eta} &= (1-\upsilon) + \upsilon\left[\bar{Z}(e^{z_t-\psi_t} + \tau_{M,t}e^{z_t})\right]^{1-\eta},\\
\left(\frac{P_t^{PPT}}{P_{H,t}}\right)^{1-\eta} &= (1-\upsilon) + \upsilon\left[(1+\tau_{M,t})\bar{Z}e^{z_t}\right]^{1-\eta}.
\end{aligned} \tag{E.19}
$$

In either environment, the exact risk-sharing and Home goods-market identities in Sec. 3.1.2 and its PPT counterpart give

$$
y_t = \log\left[(1-\upsilon)\bar{Z}e^{z_t}\left(\frac{P_t}{P_{H,t}}\right)^{\eta-1} + \upsilon(\bar{Z}e^{z_t})^{\eta}\right] - \log\bar{Y}, \tag{E.20}
$$

where $P_t = P_t^{PPT}$ or P_t^{IPT} as appropriate. Equations (E.19)–(E.20) are not additional equilibrium restrictions: they simply restate the respective original nonlinear price index, risk-sharing, and goods-market conditions, with consumption eliminated. In particular, the additive-duty IPT price in Eq. (E.19) differs from $(1+\tau_{M,t})\bar{Z}e^{z_t-\psi_t}$.

For the second-order real welfare contribution, differentiate the augmented expression immediately above with the stationary multipliers from Eqs. (E.18)–(E.17). In the PPT case omit only the importer aggregate-supply term and use the PPT relative purchaser price in Eq. (E.19). First eliminate z_t *implicitly* through Eq. (E.20) to second order, and only then take the Hessian in $(y_t, \tau_{M,t})$ for PPT or $(y_t, \psi_t, \tau_{M,t})$ for IPT. Both the first *and* second derivatives of the implicitly determined z_t must be retained; using only the log-linear market-clearing identity omits genuine second-order terms. The actual importer reset price is the fixed *ex-tariff* price in Eq. (32), as described above.

Writing the policy-coordinate negative Hessians and tariff loading vectors as Q and B, the baseline parameterization now gives

$$
\begin{aligned}
Q^{SOE,PPT} &= \begin{pmatrix}1.6542172374\end{pmatrix}, \quad B^{SOE,PPT} = \begin{pmatrix}0.0991577246\end{pmatrix},\\
Q^{SOE,IPT} &= \begin{pmatrix}1.6542172374 & -0.0036494190\\ -0.0036494190 & 0.5062890263\end{pmatrix},\\
B^{SOE,IPT} &= \begin{pmatrix}0.1358745616\\ 0.2128527417\end{pmatrix}.
\end{aligned} \tag{E.21}
$$

Here Q is minus the Hessian of the augmented real welfare contribution in policy variables, and B is its policy-variable–tariff cross-derivative, after the implicit elimination of z_t. Both are determined from the original exact conditions at the true zero-tariff steady state, not from independently calibrated weights. The PPT completion gives

$$
\Lambda_y^{SOE,PPT} = 1.6542172374, \qquad y_t^{w,SOE,PPT} = \frac{0.0991577246}{1.6542172374}\tau_{M,t}.
$$

For IPT, square completion with the output term first yields

$$\begin{aligned}
\Lambda_{y}^{SOE,IPT} &= Q_{11}^{SOE,IPT}, \\
\Lambda_{\psi}^{SOE,IPT} &= Q_{22}^{SOE,IPT} - \frac{(Q_{12}^{SOE,IPT})^2}{Q_{11}^{SOE,IPT}}, \\
\Gamma_{\text{own}}^{SOE} &= \frac{B_2^{SOE,IPT} - Q_{21}^{SOE,IPT} B_1^{SOE,IPT}/Q_{11}^{SOE,IPT}}{\Lambda_{\psi}^{SOE,IPT}}.
\end{aligned}$$

This yields $\Lambda_{\psi}^{SOE,IPT} = 0.5062809752$ and $\Gamma_{\text{own}}^{SOE} = 0.4210162126$.

The original nonlinear producer-price and importer-price Calvo laws determine the second-order producer and importer *physical* dispersion terms. A dispersion increase $\Delta_{H,t} - 1$ enters the reduced augmented welfare contribution with coefficient $-\bar{Y}^{1+\varphi} + (1+\varphi)(\bar{Y}/\bar{Z})\Theta_H^{AS}$. For the importers, the cohort-level ex-tariff-price-weighted reset FOC and the normalized ex-tariff CES price index give a dispersion coefficient $\bar{C}^{1-\eta}\Theta_M^{AS}$. The standard discounted Calvo dispersion law then implies

$$\begin{aligned}
\Lambda_{\pi,H}^{SOE,PPT} &= \Lambda_{\pi,H}^{SOE,IPT} = \frac{\varepsilon}{\kappa_H}\left[\bar{Y}^{1+\varphi} - (1+\varphi)(\bar{Y}/\bar{Z})\Theta_H^{AS}\right], \\
\Lambda_{\pi,F}^{SOE,IPT} &= -\frac{\varepsilon_M}{\kappa_M}\bar{C}^{1-\eta}\Theta_M^{AS}.
\end{aligned} \tag{E.22}$$

The corresponding baseline values are

$$\begin{aligned}
\Lambda_{y}^{SOE,IPT} &= 1.6542172374, \qquad \Lambda_{\psi}^{SOE,IPT} = 0.5062809752, \\
\Lambda_{\pi,H}^{SOE,IPT} &= 39.3775970086, \qquad \Lambda_{\pi,F}^{SOE,IPT} = 27.2946757381.
\end{aligned}$$

For the high-substitution specification $\eta = 3$, holding all other primitive parameters fixed, the corresponding weights are $\Lambda_{y}^{SOE,IPT} = 0.7493034093$, $\Lambda_{\psi}^{SOE,IPT} = 1.2406311889$, $\Lambda_{\pi,H}^{SOE,IPT} = 19.3033398990$, and $\Lambda_{\pi,F}^{SOE,IPT} = 37.2065139728$, with $\Gamma_{\text{own}}^{SOE} = 0.2927934031$.

To verify the reported SOE welfare coefficients, we compute the exact zero-tariff steady state from Eq. (E.15), eliminate the ex-tariff relative border price z_t through implicit differentiation, and check the resulting Hessian against finite-difference calculations. The standard Calvo inflation-square factor is separately verified from the nonlinear reset-price index.

The SOE real weights are defined by the real Hessian and sequential completion in Eq. (E.21). The inflation weights are defined by Eq. (E.22), while $\Lambda_{y}^{SOE,PPT} = Q_{11}^{SOE,PPT}$ and $\Lambda_{y}^{SOE,IPT} = Q_{11}^{SOE,IPT}$.

$$\Lambda_{\psi}^{SOE,IPT} \equiv Q_{22}^{SOE,IPT} - \frac{(Q_{12}^{SOE,IPT})^2}{Q_{11}^{SOE,IPT}}. \tag{E.23}$$

The second-order welfare criteria are expansions around the distorted deterministic steady state. The linear–quadratic construction follows Benigno and Woodford [7, 8]: first-order equilibrium restrictions are used to eliminate terms that would otherwise make a naive second-order utility expansion incorrect around a distorted steady state, and price-dispersion laws generate the inflation-square terms. The WR variables in Eqs. (37), (38), (49), (50) are obtained by exact sequential completion of squares. They are reference values in the welfare criterion, not new private-equilibrium variables.

The numerical allocations reported in the main text are not solutions to the quadratic approximations. They are obtained by solving the nonlinear Ramsey first-order-condition systems in Dynare using the perfect-foresight solver with its mixed-complementarity algorithm (`lmmcp`), which imposes the occasionally binding ZLB directly. No OccBin or piecewise-linear solution method is used. The loss functions are used to interpret those allocations and to identify which nominal and real distortions are welfare-relevant.

E.2 Two-Country Economy

This subsection derives the two real WR variables appearing in Eq. (49), the four real WR variables appearing in Eq. (50), and every welfare weight in Eqs. (49)–(50) from the structural parameters in Eq. (E.1). The construction common to the two pass-through environments is stated first; Appendices E.2.1 and E.2.2 then specialize it. Throughout the first-order representation used to construct these WR variables, we use $\log(1+\tau_{M,t}) = \tau_{M,t}$ and $\log(1+\tau^*_{M,t}) = \tau^*_{M,t}$, without introducing separate tariff variables.

Derivation of Eqs. (49)–(50) from the Structural Parameters. The derivation consists of the following seven steps.

Step 1: Steady state and notation. At the zero-tariff, zero-inflation deterministic steady state, $Z = \Psi = \Psi^* = 1$, $P/P_H = P^*/P^*_F = 1$, and $C = C^* = Y = Y^* = N = N^*$. The producer reset condition, Eq. (14), gives $MC^r = \mathcal{M}^{-1} = L$, and Eq. (16) then gives $N^{1+\varphi} = L$. Because $\{(1-\nu)/\nu\}\omega^*_H = \omega_F$, Home exports absorb the share ω_F of Home output and Home households absorb the share ω_H. Lower-case letters denote log deviations from this steady state: c_t, c^*_t, y_t, n_t, $z_t \equiv \log Z_t$, ψ_t, ψ^*_t, $g_t \equiv \log(P_t/P_{H,t})$, $g^*_t \equiv \log(P^*_t/P^*_{F,t})$, $s_t \equiv \log(P_{F,t}/P_{H,t})$, and $s^*_t \equiv \log(P^*_{H,t}/P^*_{F,t})$. Here P_t denotes the CPI of the environment under consideration: $g_t = \log \mathcal{G}^{PPT}(\tau_{M,t}, Z_t)$ under PPT and $g_t = \log \mathcal{G}^{IPT}_t(\cdot)$ under IPT.

Step 2: National problem and exact identities. As in Appendix C, the Home planner chooses the Home allocation, namely c_t, g_t, y_t, n_t, the Home producer reset price and, under IPT, also s_t and the Home importer reset price, while taking the Foreign plan (c^*_t, g^*_t and, under IPT, s^*_t and the Foreign importer reset price) as given. Complete international risk sharing, Eq. (B.11), is substituted before differentiation and is exact in logs:

$$z_t = c_t + g_t - c^*_t - g^*_t. \tag{E.24}$$

Under IPT, the definitions of the two LOOP gaps give the exact identities $\psi_t = z_t - s_t$ and $\psi^*_t = -z_t - s^*_t$. Let $\widetilde{p}_{F,t} \equiv \log(\widetilde{P}_{F,t}/P_{H,t})$ and $\widetilde{p}^*_{H,t} \equiv \log(\widetilde{P}^*_{H,t}/P^*_{F,t})$ denote the logarithms of the *cum-tariff* relative purchaser prices of imports faced by Home and Foreign households, respectively. Then

$$\begin{aligned} &\text{PPT:} \quad \widetilde{p}_{F,t} = z_t + \tau_{M,t}, \qquad \widetilde{p}^*_{H,t} = -z_t + \tau^*_{M,t}, \\ &\text{IPT:} \quad \widetilde{p}_{F,t} = s_t + \log\left(1 + \tau_{M,t} e^{\psi_t}\right), \qquad \widetilde{p}^*_{H,t} = s^*_t + \log\left(1 + \tau^*_{M,t} e^{\psi^*_t}\right) + O(3). \end{aligned} \tag{E.25}$$

The IPT line follows from the additive-duty purchaser price in Eq. (20), $\widetilde{P}^{IPT}_{F,t}(\iota)/P_{F,t} = P_{F,t}(\iota)/P_{F,t} + \tau_{M,t}\Psi_t$, and its Foreign mirror. Because the *ex-tariff* index and the *cum-tariff* index are CES aggregates with the same elasticity ε_M, cross-sectional importer-price dispersion cancels from $\log(\widetilde{P}^{IPT}_{F,t}/P_{F,t})$ up to third order. In particular, the second-order expansion $\widetilde{p}^*_{H,t} = s^*_t + \tau^*_{M,t} + \tau^*_{M,t}\psi^*_t + O(3)$ contains the interaction $\tau^*_{M,t}\psi^*_t$, which is retained below.

Step 3: Per-period national Lagrangian. Because the steady-state multipliers are constant from the timeless perspective, the Home Lagrangian of Appendix C can be written, up to terms independent of policy, as $\mathrm{E}_0 \sum_t \beta^t \mathcal{J}_{H,t}$ with

$$\begin{aligned} \mathcal{J}_{H,t} \;=\; & c_t - \frac{L}{1+\varphi} e^{(1+\varphi)n_t} + \Theta_\chi \left[g_t - \frac{1}{1-\eta} \log\left\{ \omega_H + \omega_F e^{(1-\eta)\widetilde{p}_{F,t}} \right\} \right] \\ & + \Theta_Y \left[y_t - \log\left\{ \omega_H e^{c_t + \eta g_t} + \omega_F e^{c^*_t - \eta(\widetilde{p}^*_{H,t} - g^*_t)} \widetilde{\Delta}^*_{H,t} \right\} \right] + \mathcal{C}_{H,t} + \mathcal{C}_{M,t}. \end{aligned} \tag{E.26}$$

Here $n_t = y_t + \log \Delta_{H,t}$ by Eq. (13); the first bracket represents the Home CPI restriction, Eq. (B.3) or Eq. (B.9); the second represents the Home goods-market-clearing condition, Eq. (B.8) or Eq. (48), after substituting the bilateral demands in Eqs. (B.4) and (B.10); $\widetilde{\Delta}^*_{H,t} \equiv (1-\nu)^{-1} \int_\nu^1 (\widetilde{P}^*_{H,t}(\iota)/\widetilde{P}^*_{H,t})^{-\varepsilon_M} d\iota$ is the physical-quantity factor in Eq. (48) ($\widetilde{\Delta}^*_{H,t} = 1$ under PPT);

and $\mathcal{C}_{H,t}$ and $\mathcal{C}_{M,t}$ are the Calvo contributions of Home producers and Home importers derived in Step 4 ($\mathcal{C}_{M,t} = 0$ under PPT). The multiplier on the Euler equation is zero at a steady state with $i > 0$. Expressing the CPI and goods-market restrictions in levels rather than logs, or retaining z_t as a separate variable with a multiplier on Eq. (E.24), yields the same reduced Hessian on the first-order equilibrium set.

Step 4: Calvo blocks. Consider a reset-price condition of the form $\mathrm{E}_t \sum_{k\geq 0}(\theta\beta)^k G(u_{t,k}, X_{t+k}) = 0$, where $u_{t,k} \equiv \log(\overline{P}_t/P_{t+k})$ is the price of a seller that reset at t relative to the index at $t+k$, X_{t+k} collects date-$(t+k)$ variables, and the index satisfies $\int (P_t(j)/P_t)^{1-\varepsilon} dj = 1$. Attaching the constant multiplier Θ_R and re-indexing by calendar date $s = t + k$ gives

$$\sum_t \beta^t \Theta_R \mathrm{E}_t \sum_k (\theta\beta)^k G(u_{t,k}, X_{t+k}) = \frac{\Theta_R}{1-\theta} \sum_s \beta^s \int G(\rho_s(j), X_s)\, dj + \text{t.i.p.}, \tag{E.27}$$

where $\rho_s(j) \equiv \log(P_s(j)/P_s)$, because the cohort that last reset at $s-k$ has mass $(1-\theta)\theta^k$. Let $V_s \equiv \int \rho_s(j)^2 dj$. The price-index normalization implies $\int \rho_s(j) dj = (\varepsilon-1)V_s/2 + O(3)$. The dispersion index satisfies $\log \int (P_s(j)/P_s)^{-\varepsilon} dj = \varepsilon V_s/2 + O(3)$. Finally, the Calvo law of motion gives $\sum_s \beta^s V_s = \kappa^{-1} \sum_s \beta^s \pi_s^2 + \text{t.i.p.}$, where $\kappa = (1-\theta)(1-\beta\theta)/\theta$. Since products of $\int \rho_s(j) dj$ with X_s are third order,

$$\frac{\Theta_R}{1-\theta} \sum_s \beta^s \int G(\rho_s(j), X_s)\, dj = \frac{\Theta_R}{1-\theta} \sum_s \beta^s \left[G(0, X_s) + \frac{1}{2}\{(\varepsilon-1)G_u + G_{uu}\} V_s \right] + \text{t.i.p.} + O(3). \tag{E.28}$$

For Home producers, Eq. (14) is equivalent to the reset condition with

$$G_H(u, X_t) = e^{a_{H,t}} e^{-\varepsilon u} \left(e^u - e^{mc_{H,t}}\right), \qquad a_{H,t} \equiv y_t - c_t - g_t, \qquad mc_{H,t} = c_t + \varphi n_t + g_t,$$

where $a_{H,t} = \log\{Y_t P_{H,t}/(C_t P_t)\}$ comes from the household stochastic discount factor and $mc_{H,t} \equiv \log(\mathcal{M}MC_t^r)$ from Eq. (16). Since $G_u = 1$ and $G_{uu} = 1 - 2\varepsilon$ at the steady state, defining $\Theta_H^{AS} \equiv -\Theta_{R,H}/(1-\theta)$ gives

$$\mathcal{C}_{H,t} = \Theta_H^{AS} \left[mc_{H,t} + a_{H,t} mc_{H,t} + \frac{1}{2} mc_{H,t}^2 + \frac{\varepsilon}{2} V_{H,t} \right] + O(3). \tag{E.29}$$

For Home importers, Eq. (32) with $\tau = 1/\varepsilon_M$ is equivalent to the reset condition with

$$G_M(u, X_t) = e^{a_{M,t}} e^u \left[\frac{e^u + \tau_{M,t} e^{\psi_t}}{1 + \tau_{M,t} e^{\psi_t}} \right]^{-\varepsilon_M} \left[1 - \varepsilon_M \frac{e^u - (1 - \varepsilon_M^{-1}) e^{\psi_t}}{e^u + \tau_{M,t} e^{\psi_t}} \right],$$

where $a_{M,t} \equiv c_{F,t} + s_t - c_t - g_t = \log\{C_{F,t} P_{F,t}/(C_t P_t)\}$ and $c_{F,t} = c_t - \eta(\widetilde{p}_{F,t} - g_t)$ by Eq. (23). At the steady state $G_u = -(\varepsilon_M - 1)$ and $G_{uu} = (\varepsilon_M - 1)(2\varepsilon_M - 1)$, so defining $\Theta_M^{AS} \equiv (\varepsilon_M - 1)\Theta_{R,M}/(1-\theta_M)$ gives

$$\mathcal{C}_{M,t} = \Theta_M^{AS} \left[e^{a_{M,t}} \frac{e^{\psi_t} - 1 + \tau_{M,t} e^{\psi_t}/(\varepsilon_M - 1)}{1 + \tau_{M,t} e^{\psi_t}} + \frac{\varepsilon_M}{2} V_{M,t} \right] + O(3). \tag{E.30}$$

The linear part of Eq. (E.30) is $\Theta_M^{AS}\{\psi_t + \tau_{M,t}/(\varepsilon_M - 1)\}$, the driving term of Eq. (34). Finally, $\widetilde{\Delta}_{H,t}^*$ in Eq. (E.26) is the quantity factor of the Foreign importer block; by the same argument, $\log \widetilde{\Delta}_{H,t}^* = \varepsilon_M V_{H,t}^*/2 + O(3)$ and $\sum_s \beta^s V_{H,s}^* = \kappa_M^{-1} \sum_s \beta^s (\pi_{H,s}^*)^2 + \text{t.i.p.}$

Step 5: Steady-state multipliers. The steady-state multipliers are determined by the first-order stationarity conditions of the Home national Lagrangian $\mathcal{J}_{H,t}$ with respect to the Home choice variables. Equivalently, n_t and $mc_{H,t}$ can be kept as separate variables with multipliers Θ_n on $n_t = y_t + \log \Delta_{H,t}$ and $\Theta_{mc,H}$ on $mc_{H,t} = c_t + \varphi n_t + g_t$; the conditions with respect to n_t and $mc_{H,t}$ are then $-L + \Theta_n - \varphi\Theta_{mc,H} = 0$ and $\Theta_{mc,H} + \Theta_H^{AS} = 0$. Under IPT, $\Theta_{mc,M}$ is attached in

the same way to the importer cost ratio $\psi_t = z_t - s_t$, so that $\Theta_{mc,M} + \Theta_M^{AS} = 0$. The resulting linear systems are Eq. (E.33) for PPT and the explicit IPT solution in Eq. (E.49).

Step 6: Second-order criterion. Given the multipliers, the policy-dependent second-order Home welfare contribution is $\frac{1}{2}x_t'\nabla^2\mathcal{J}_H x_t$ evaluated on the first-order equilibrium set, where x_t collects the log deviations in Step 1 and the two tariffs. Terms linear in the Foreign plan (c_t^*, g_t^*, s_t^*) are independent of the Home planner's choices and are included in s.o.t.i.p.. The Foreign-plan variables are expressed as log deviations and enter the remaining second-order terms through the first-order mapping in Eq. (E.48). The coefficients on $V_{H,t}$, $V_{M,t}$ and $V_{H,t}^*$ give the three sticky-inflation weights:

$$\Lambda_{\pi,H} = \frac{\varepsilon}{\kappa_H}(\Theta_n - \Theta_H^{AS}), \qquad \Lambda_{\pi,F} = -\frac{\varepsilon_M}{\kappa_M}\Theta_M^{AS}, \qquad \Lambda_{\pi_H^*} = \frac{\varepsilon_M}{\kappa_M}\omega_F\Theta_Y, \tag{E.31}$$

where the last two are present only under IPT. On the first-order equilibrium set $n_t = y_t$ and $mc_{H,t} = (1+\varphi)y_t + (c_t + g_t - y_t)$, while Eq. (E.48) shows that $c_t + g_t - y_t$, the importer terms, the CPI term, and the goods-market term do not depend on y_t. Hence the producer term reduces to $\Theta_H^{AS}[(1+\varphi)^2 y_t^2 - (c_t + g_t - y_t)^2]/2$, output separates from all other coordinates, $y_t^{w,\cdot} = 0$, and in both environments

$$\Lambda_y = (1+\varphi)L + (1+\varphi)^2\Theta_{mc,H} = (1+\varphi)(\Theta_n - \Theta_H^{AS}) = \frac{(1+\varphi)\kappa_H}{\varepsilon}\Lambda_{\pi,H}. \tag{E.32}$$

Step 7: Square completion and numerical evaluation. Writing the real part as $\frac{1}{2}v_t'Qv_t - v_t'B\boldsymbol{\tau}_t +$ s.o.t.i.p. with $v_t = (y_t, z_t)'$ under PPT and $v_t = (y_t, \psi_t, z_t, \psi_t^*)'$ under IPT, the weights and WR variables follow from the sequential completion in Eqs. (E.5)–(E.6). The calculation was performed independently using exact rational arithmetic and the nonlinear restrictions in Appendices B and C. Its restricted Hessian coincides exactly with the explicit polynomials in Eqs. (E.35) and (E.50), and Eq. (E.28) was checked on exact Calvo cross-sections, with approximation errors shrinking at the cubic rate.

E.2.1 PPT Environment

Under PPT, $\psi_t = \psi_t^* = 0$, so $s_t = z_t$, $s_t^* = -z_t$, and Eq. (E.25) gives $\widetilde{p}_{F,t} = z_t + \tau_{M,t}$ and $\widetilde{p}_{H,t}^* = \tau_{M,t}^* - z_t$. The Home choice variables are c_t, g_t, y_t, n_t, and $mc_{H,t}$. Attaching Θ_χ^{PPT}, Θ_Y^{PPT}, Θ_n^{PPT}, $\Theta_{mc,H}^{PPT}$, and $\Theta_H^{AS,PPT}$ to the Home CPI, Home goods-market, labor, producer-marginal-cost, and producer aggregate-supply restrictions, the absence of first-order terms in c_t, g_t, y_t, n_t, and $mc_{H,t}$ requires

$$\begin{aligned}
0 &= 1 - \omega_F\Theta_\chi^{PPT} - (\omega_H + \eta\omega_F)\Theta_Y^{PPT} - \Theta_{mc,H}^{PPT},\\
0 &= \omega_H\Theta_\chi^{PPT} - \eta\Theta_Y^{PPT} - \Theta_{mc,H}^{PPT},\\
0 &= \Theta_Y^{PPT} - \Theta_n^{PPT},\\
0 &= -L + \Theta_n^{PPT} - \varphi\Theta_{mc,H}^{PPT},\\
0 &= \Theta_{mc,H}^{PPT} + \Theta_H^{AS,PPT}.
\end{aligned} \tag{E.33}$$

Under PPT, c_t and g_t move z_t one-for-one through Eq. (E.24). Therefore c_t enters the CPI restriction with weight ω_F and Home exports with weight $\eta\omega_F$, whereas g_t enters the CPI restriction with net weight $1-\omega_F = \omega_H$ and goods-market clearing with weight $\eta(\omega_H + \omega_F) = \eta$. No separate relative-price multiplier arises, because z_t has been eliminated by complete risk sharing exactly as in the IPT system of E.2.2. Solving Eq. (E.33) gives

$$\begin{aligned}
\Theta_Y^{PPT} = \Theta_n^{PPT} &= \frac{L + \varphi\omega_H}{1 + \varphi\left[\omega_H^2 + \eta(1-\omega_H^2)\right]},\\
\Theta_{mc,H}^{PPT} &= \frac{\Theta_Y^{PPT} - L}{\varphi}, \qquad \Theta_\chi^{PPT} = \frac{\eta\Theta_Y^{PPT} + \Theta_{mc,H}^{PPT}}{\omega_H},
\end{aligned}$$

$$\Theta_H^{AS,PPT} = -\Theta_{mc,H}^{PPT}. \tag{E.34}$$

Under the baseline parameterization,

$$\begin{aligned}
&\Theta_Y^{PPT} = \Theta_n^{PPT} = 0.676499, \qquad \Theta_{mc,H}^{PPT} = -0.027429,\\
&\Theta_\chi^{PPT} = 1.264173, \qquad \Theta_H^{AS,PPT} = 0.027429.
\end{aligned}$$

Specializing Eq. (E.26) to PPT and applying Step 6, the policy-dependent real second-order Home welfare contribution is

$$\begin{aligned}
\mathcal{W}_{H,t}^{(2),real,PPT} &= -\frac{(1+\varphi)L}{2} y_t^2 - \frac{\omega_H \omega_F (1-\eta)\Theta_\chi^{PPT}}{2}(z_t + \tau_{M,t})^2 \\
&\quad - \frac{\omega_H \omega_F \Theta_Y^{PPT}}{2}\left[(c_t + \eta g_t) - \{c_t^* - \eta(\tau_{M,t}^* - z_t - g_t^*)\}\right]^2 \\
&\quad + \Theta_{mc,H}^{PPT}\left[mc_{H,t}^{(1)}(c_t + g_t - y_t) - \frac{1}{2}(mc_{H,t}^{(1)})^2\right] + \text{s.o.t.i.p.},
\end{aligned} \tag{E.35}$$

with $g_t = \omega_F(z_t + \tau_{M,t})$, $g_t^* = \omega_H^*(\tau_{M,t}^* - z_t)$, c_t and c_t^* given by Eq. (E.48) at $\psi_t = \psi_t^* = 0$, and $mc_{H,t}^{(1)} = c_t + \varphi y_t + g_t$. The four terms represent, respectively, the direct utility expansion, the second-order Home CPI contribution, the Home goods-market-clearing contribution reflecting Home and Foreign demand for Home goods, and the producer Calvo contribution in Eq. (E.29). Using $\omega_H - \omega_H^* = 1 - \upsilon$ and $\omega_H + \omega_F^* = 2 - \upsilon$, the first-order equilibrium set implies

$$\begin{aligned}
c_t + g_t - y_t &= -(\eta-1)(2-\upsilon)\omega_F z_t - (\eta-1)\omega_H\omega_F\tau_{M,t} \\
&\quad + (\eta\omega_F^* + \omega_H^*)\omega_F\tau_{M,t}^*, \\
(c_t + \eta g_t) - \{c_t^* - \eta(\tau_{M,t}^* - z_t - g_t^*)\} &= -(\eta-1)(1-\upsilon)z_t + (\eta-1)\omega_F\tau_{M,t} \\
&\quad + (\eta\omega_F^* + \omega_H^*)\tau_{M,t}^*, \\
mc_{H,t}^{(1)} &= (1+\varphi)y_t + (c_t + g_t - y_t).
\end{aligned} \tag{E.36}$$

Substituting Eq. (E.36) into Eq. (E.35) and writing $\mathcal{L}_{H,t}^{PPT,real} = \frac{1}{2} v_t' Q^{PPT} v_t - v_t' B^{PPT}(\tau_{M,t}, \tau_{M,t}^*)'$ with $v_t = (y_t, z_t)'$ gives

$$\begin{aligned}
Q_{11}^{PPT} &= (1+\varphi)L + (1+\varphi)^2\Theta_{mc,H}^{PPT}, \qquad Q_{12}^{PPT} = 0, \qquad B_{11}^{PPT} = B_{12}^{PPT} = 0, \\
Q_{22}^{PPT} &= (\eta-1)\omega_H\omega_F\left[(\eta-1)(1-\upsilon)^2\Theta_Y^{PPT} - \Theta_\chi^{PPT}\right] - (\eta-1)^2(2-\upsilon)^2\omega_F^2\Theta_{mc,H}^{PPT}, \\
B_{21}^{PPT} &= (\eta-1)\omega_H\omega_F\left[\Theta_\chi^{PPT} + (\eta-1)\omega_F\left\{(1-\upsilon)\Theta_Y^{PPT} + (2-\upsilon)\Theta_{mc,H}^{PPT}\right\}\right], \\
B_{22}^{PPT} &= (\eta-1)(\eta\omega_F^* + \omega_H^*)\omega_F\left[(1-\upsilon)\omega_H\Theta_Y^{PPT} - (2-\upsilon)\omega_F\Theta_{mc,H}^{PPT}\right].
\end{aligned} \tag{E.37}$$

Since $Q_{12}^{PPT} = 0$, the output–relative-price cross term vanishes. Completing squares yields the WR variables entering Eq. (49):

$$y_t^{w,PPT} = 0, \tag{E.38}$$

$$z_t^{w,PPT} = \frac{B_{21}^{PPT}\tau_{M,t} + B_{22}^{PPT}\tau_{M,t}^*}{\Lambda_z^{PPT}}, \tag{E.39}$$

and the corresponding PPT weights are

$$\begin{aligned}
&\Lambda_y^{PPT} = Q_{11}^{PPT} = (1+\varphi)(\Theta_n^{PPT} - \Theta_H^{AS,PPT}), \qquad \Lambda_z^{PPT} = Q_{22}^{PPT}, \\
&\Lambda_{\pi,H}^{PPT} = \frac{\varepsilon}{\kappa_H}(\Theta_n^{PPT} - \Theta_H^{AS,PPT}).
\end{aligned} \tag{E.40}$$

Inserting the structural parameters from Table 1 and $\nu = 0.27$ gives

$$Q^{PPT} = \begin{pmatrix} 2.077023 & 0 \\ 0 & -0.092987 \end{pmatrix}, \qquad B^{PPT} = \begin{pmatrix} 0 & 0 \\ 0.112109 & 0.060738 \end{pmatrix},$$

$$\begin{aligned} (\Lambda_y^{PPT}, \Lambda_z^{PPT}, \Lambda_{\pi,H}^{PPT}) &= (2.077023,\ -0.092987,\ 47.432009), \\ z_t^{w,PPT} &= -1.205647\,\tau_{M,t} - 0.653193\,\tau_{M,t}^*. \end{aligned}$$

These numerical values agree, up to rounding, with the PPT coefficients reported in Table 3. Equation (E.39) defines the (logarithmic) WR *ex-tariff* relative border price of imports. Since $\Lambda_z^{PPT} < 0$, the associated completed square is a signed strategic welfare contribution, as discussed in Sec. 4.4.

E.2.2 IPT Environment

The baseline IPT structural quadratic in this subsection follows from the construction in Steps 1–7 above. Its stationary multipliers are given explicitly in Eq. (E.49), and the second-order real polynomial is displayed in Eq. (E.50). The interaction term $\eta\omega_F\Theta_Y\tau_{M,t}^*\psi_t^*$ between the Foreign import tariff and the Foreign LOOP gap follows directly from the second-order expansion of the Foreign purchaser price in Eq. (E.25). Every $\widehat{C}_{uv}^{IPT}$ is an entry of the structural Hessian or its tariff-loading columns; none is a free welfare parameter. In particular, if Q^{IPT} and B^{IPT} denote the Hessian and the two tariff-loading columns defined in Eq. (E.51), then all $b_\cdot^H$ coefficients below are generated mechanically by Eqs. (E.5)–(E.6). For example,

$$b_{ym}^H = \frac{B_{11}^{IPT}}{Q_{11}^{IPT}}, \qquad b_{ym^*}^H = \frac{B_{12}^{IPT}}{Q_{11}^{IPT}}, \tag{E.41}$$

and the remaining $b_\cdot^H$ are the corresponding second-, third-, and fourth-step Schur-completion loadings. We eliminate Foreign output using complete risk sharing and the first-order world equilibrium. Before collecting the reduced welfare coefficients, we express all local relative-price terms using $z_t = \log(\mathcal{E}_t P_{F,t}^*/P_{H,t})$. Using the identity $\log(P_{F,t}/P_{H,t}) = z_t - \psi_t$ (Sec. 4.1), we express the reduced real Home welfare criterion in terms of y_t, ψ_t, z_t, and ψ_t^*. Sequential completion of squares in the reduced two-country Home welfare criterion yields the following WR variables:

$$\begin{aligned} y_t^{w,IPT} &= b_{ym}^H\tau_{M,t} + b_{ym^*}^H\tau_{M,t}^*, && \text{(E.42)} \\ \psi_t^{w,IPT} &= b_{\psi z}^H z_t + b_{\psi\psi^*}^H\psi_t^* + b_{\psi m}^H\tau_{M,t} + b_{\psi m^*}^H\tau_{M,t}^*, && \text{(E.43)} \\ z_t^{w,IPT} &= b_{z\psi^*}^H\psi_t^* + b_{zm}^H\tau_{M,t} + b_{zm^*}^H\tau_{M,t}^*, \\ \psi_t^{*,w,IPT} &= b_{\psi^* m}^H\tau_{M,t} + b_{\psi^* m^*}^H\tau_{M,t}^*. && \text{(E.44)} \end{aligned}$$

For the compact main-text notation in Eq. (51), define $\Gamma_z \equiv b_{\psi z}^H$, $\Gamma_{\text{own}} \equiv b_{\psi m}^H$, and $\Gamma_{\text{cross}} \equiv b_{\psi m^*}^H = -b_{\psi\psi^*}^H$. The last equality shows that the Foreign import tariff and the Foreign LOOP gap enter the Home WR LOOP gap through $\tau_{M,t}^* - \psi_t^*$. The starred coefficients in Eq. (52) follow from the same derivation with Home and Foreign labels interchanged. The corresponding real-weight definitions are

$$\begin{aligned} \Lambda_y^{IPT} &\equiv -2\widehat{C}_{yy}^{IPT}, \qquad \Lambda_\psi^{IPT} \equiv -2\widehat{C}_{\psi\psi}^{IPT}, \\ \Lambda_z^{IPT} &\equiv -2\widehat{C}_{zz}^{IPT} - \frac{(\widehat{C}_{\psi z}^{IPT})^2}{\Lambda_\psi^{IPT}}, \\ \Lambda_{\psi^*}^{IPT} &\equiv -2\widehat{C}_{\psi^*\psi^*}^{IPT} - \frac{(\widehat{C}_{\psi\psi^*}^{IPT})^2}{\Lambda_\psi^{IPT}} \end{aligned}$$

$$-\frac{1}{\Lambda_z^{IPT}}\left[\widehat{C}_{z\psi^*}^{IPT}+\frac{\widehat{C}_{\psi z}^{IPT}\widehat{C}_{\psi\psi^*}^{IPT}}{\Lambda_\psi^{IPT}}\right]^2. \tag{E.45}$$

Using the baseline multipliers in Eq. (E.49), the welfare weights on Home PPI inflation, Home ex-tariff import-price inflation, and Foreign PPI inflation are given by

$$\Lambda_{\pi,H}^{IPT}=\frac{\varepsilon}{\kappa_H}(\Theta_n-\Theta_H^{AS}),\qquad \Lambda_{\pi,F}^{IPT}=-\frac{\varepsilon_M}{\kappa_M}\Theta_M^{AS},\qquad \Lambda_{\pi_H^*}^{IPT}=\frac{\varepsilon_M}{\kappa_M}\Theta_Y\omega_F. \tag{E.46}$$

Here the coefficients $\widehat{C}_{uv}^{IPT}$ are obtained from the reduced second-order Home welfare polynomial after the relative-price transformation. Sequential completion of squares in the order y,ψ,z,ψ^* then yields the coefficients $b_\cdot^H$ as functions of the structural parameters (Eqs. (E.5)–(E.6)). In this representation, $y_t^{w,IPT}$ is the (logarithmic) WR output, $\psi_t^{w,IPT}$ is the (logarithmic) Home WR LOOP gap, and $z_t^{w,IPT}$ is the (logarithmic) WR *ex-tariff* relative border price of imports. The variable $\psi_t^{*,w,IPT}$ is the WR value associated with the Foreign LOOP gap ψ_t^* in the Home welfare cost function, distinct from the Foreign planner's own WR LOOP gap $\psi_t^{w*,IPT}$. In particular, Eq. (E.43) makes explicit that the Home WR LOOP gap depends not only on the Home tariff but also on the endogenous international relative price z_t and the Foreign LOOP gap ψ_t^*. These WR variables are obtained by completed squares and are not additional private-equilibrium variables.

Explicit Second-Order Home Welfare Polynomial and Coefficients for the Two-Country IPT Baseline. The following steps make the baseline coefficients in Eqs. (E.45)–(E.46) reproducible from the primitive parameters. The calculation is for the *Home national* welfare criterion; the Foreign national criterion is obtained by interchanging country labels, replacing ν by $1-\nu$, and using $-z_t$ as the Foreign ex-tariff border-price coordinate.

Begin with the period utility in Eq. (1). Since $N^{1+\varphi}=L$ in the zero-tariff steady state, its direct second-order expansion, prior to the Benigno–Woodford elimination of policy-dependent linear terms, is

$$u_{H,t}-u_H=c_t-Ly_t-\frac{(1+\varphi)L}{2}y_t^2-L\log\Delta_{H,t}+O(3). \tag{E.47}$$

Here $c_t\equiv\log(C_t/C)$, $y_t\equiv\log(Y_t/Y)$, and $\log\Delta_{H,t}$ starts at second order around the zero-inflation steady state. Equation (E.47) alone is *not* a valid second-order policy loss around a distorted steady state. The Benigno–Woodford construction first cancels c_t-Ly_t using the first-order equilibrium restrictions and the coefficients in Eq. (E.49). The remaining producer- and importer-Calvo dispersion terms contribute the inflation weights below.

The real welfare calculation uses the following first-order identities. Here g_t and g_t^* denote the Home and Foreign CPI–PPI log ratios, $\log\mathcal{G}_t^{IPT}(\cdot)$ and $\log\mathcal{G}_t^{*,IPT}(\cdot)$, respectively:

$$\begin{aligned}
g_t&=\omega_F(z_t-\psi_t+\tau_{M,t}),\\
g_t^*&=\omega_H^*(-z_t-\psi_t^*+\tau_{M,t}^*),\\
c_t&=y_t+\omega_F(z_t-g_t+g_t^*)-\eta\{\omega_H g_t+\omega_F(g_t^*+z_t+\psi_t^*-\tau_{M,t}^*)\},\\
c_t^*&=c_t-z_t+g_t-g_t^*,\\
mc_{H,t}^{(1)}&=c_t+\varphi y_t+g_t.
\end{aligned} \tag{E.48}$$

The first two identities follow from a first-order Taylor expansion of the Home and Foreign CPI indices in Appendix B.2. The third and fourth identities follow by log-linearizing the international risk-sharing and goods-market-clearing conditions in Appendix B.2. The fifth follows from the Home household labor-supply condition and the definition of producer real marginal cost. Thus every expression on the right-hand side of Eq. (E.48) is linear in the four real coordinates and two tariffs.

The baseline stationary-multiplier restrictions have the explicit solution

$$\begin{aligned}
\Theta_Y = \Theta_n &= \frac{1 + L/(\omega_H \varphi)}{\omega_H + \eta\omega_F + 1/(\omega_H \varphi)}, \\
\Theta_{mc,H} &= \frac{\Theta_Y - L}{\varphi}, \\
\Theta_\chi &= \eta\Theta_Y + \frac{\Theta_{mc,H}}{\omega_H}, \\
\Theta_{mc,M} &= \omega_F \Theta_\chi, \\
\Theta_H^{AS} &= -\Theta_{mc,H}, \qquad \Theta_M^{AS} = -\Theta_{mc,M}.
\end{aligned} \tag{E.49}$$

All right-hand sides are functions of the primitive parameters in Eq. (E.1); in particular, L and ω_H, ω_F are fixed by Eqs. (E.2) and (E.3). The policy-dependent real second-order *welfare contribution* (the negative of the real welfare loss, before completing squares) is

$$\begin{aligned}
\mathcal{W}_{H,t}^{(2),real} \;=\; & -\frac{(1+\varphi)L}{2} y_t^2 - \frac{\omega_H \omega_F (1-\eta)\Theta_\chi}{2} (z_t - \psi_t + \tau_{M,t})^2 \\
& - \frac{\omega_H \omega_F \Theta_Y}{2} \left[(c_t + \eta g_t) - \{ c_t^* - \eta(-z_t - \psi_t^* + \tau_{M,t}^* - g_t^*) \} \right]^2 \\
& + \Theta_{mc,H} \left[mc_{H,t}^{(1)} (c_t + g_t - y_t) - \frac{1}{2} (mc_{H,t}^{(1)})^2 \right] \\
& - \frac{\omega_F \Theta_\chi}{\varepsilon_M - 1} \Bigg[\{ (\varepsilon_M - 1)\psi_t + \tau_{M,t} \} \{ -\eta(z_t - \psi_t + \tau_{M,t} - g_t) + (z_t - \psi_t) - g_t \} \\
& \qquad + \frac{\varepsilon_M - 1}{2} \psi_t^2 + \tau_{M,t}\psi_t - \frac{\varepsilon_M + 1}{2} \tau_{M,t}^2 \Bigg] \\
& + \eta\omega_F \Theta_Y \tau_{M,t}^* \psi_t^* + \text{s.o.t.i.p.}
\end{aligned} \tag{E.50}$$

Each line has a direct structural origin in Eq. (E.26): the first term is the expansion of utility; the second is the second-order term of the Home CPI aggregator; the third is the second-order term of Home goods-market clearing; the fourth is the producer Calvo block in Eq. (E.29); and the fifth is the importer Calvo block in Eq. (E.30), combined with the second-order part $\tau_{M,t}\psi_t$ of $\widetilde{p}_{F,t}$ in Eq. (E.25) as it enters the CPI restriction. The last term is the second-order part $\tau_{M,t}^*\psi_t^*$ of the Foreign purchaser price $\widetilde{p}_{H,t}^*$ in Eq. (E.25), which enters Home exports with the weight $-\eta\omega_F$ and is priced by Θ_Y. The interaction between the Foreign import tariff and the Foreign LOOP gap enters the tariff-loading matrix through B_{42}. It affects the loading of $\psi_t^{*,w,IPT}$ on $\tau_{M,t}^*$, while leaving the Home WR LOOP-gap loading unchanged. In particular, the importer term retains the additive-duty price structure of Eq. (20); it is not obtained by multiplying ex-tariff sticky import prices by $1 + \tau_{M,t}$.

Set $v_t = (y_t, \psi_t, z_t, \psi_t^*)'$ and $\boldsymbol{\tau}_t = (\tau_{M,t}, \tau_{M,t}^*)'$. The real loss obtained from Eq. (E.50) is

$$\mathcal{L}_{H,t}^{real} = \frac{1}{2} v_t' Q v_t - v_t' B \boldsymbol{\tau}_t + \text{s.o.t.i.p.}, \quad Q_{ij} = -\frac{\partial^2 \mathcal{W}_{H,t}^{(2),real}}{\partial v_{i,t} \partial v_{j,t}}, \quad B_{ij} = \frac{\partial^2 \mathcal{W}_{H,t}^{(2),real}}{\partial v_{i,t} \partial \boldsymbol{\tau}_{j,t}}. \tag{E.51}$$

Together, Eqs. (E.2), (E.3), (E.49), and (E.48)–(E.51) determine every entry of Q and B from the structural parameters.

Since $Q_{1j} = 0$ for $j > 1$ under the baseline parameterization, sequential Schur completion in

the order y, ψ, z, ψ^* yields

$$\begin{aligned}
\Lambda_y^{IPT} &= Q_{11}, \qquad \Lambda_\psi^{IPT} = Q_{22}, \\
\Lambda_z^{IPT} &= Q_{33} - \frac{Q_{23}^2}{Q_{22}}, \\
\Lambda_{\psi^*}^{IPT} &= Q_{44} - \frac{Q_{24}^2}{Q_{22}} - \frac{\left(Q_{34} - Q_{23}Q_{24}/Q_{22}\right)^2}{Q_{33} - Q_{23}^2/Q_{22}}, \\
\Gamma_{\text{own}} &= \frac{B_{21}}{Q_{22}}, \qquad \Gamma_{\text{cross}} = \frac{B_{22}}{Q_{22}} = \frac{Q_{24}}{Q_{22}}, \qquad \Gamma_z = -\frac{Q_{23}}{Q_{22}}.
\end{aligned} \tag{E.52}$$

The remaining WR variables are obtained from the same matrices:

$$\begin{aligned}
y_t^{w,IPT} &= 0, \\
z_t^{w,IPT} &= \frac{1}{\Lambda_z^{IPT}} \left[\left(B_{31} - \frac{Q_{23}B_{21}}{Q_{22}} \right) \tau_{M,t} + \left(B_{32} - \frac{Q_{23}B_{22}}{Q_{22}} \right) \tau_{M,t}^* - \left(Q_{34} - \frac{Q_{23}Q_{24}}{Q_{22}} \right) \psi_t^* \right], \\
\psi_t^{*,w,IPT} &= \frac{1}{\Lambda_{\psi^*}^{IPT}} \sum_{j=1}^{2} \left[B_{4j} - \frac{Q_{24}B_{2j}}{Q_{22}} - \frac{\left(Q_{34} - Q_{23}Q_{24}/Q_{22}\right)\left(B_{3j} - Q_{23}B_{2j}/Q_{22}\right)}{\Lambda_z^{IPT}} \right] \boldsymbol{\tau}_{j,t}.
\end{aligned} \tag{E.53}$$

Since $Q_{1j} = 0$ for $j > 1$ and $B_{1j} = 0$ for $j = 1, 2$, completing squares yields $y_t^{w,IPT} = 0$.

Inserting the structural parameters from Table 1 and $\nu = 0.27$ produces the matrices

$$Q = \begin{pmatrix} 2.860011 & 0 & 0 & 0 \\ 0 & 0.406343 & 0.006318 & 0.024430 \\ 0 & 0.006318 & -0.097730 & 0.070886 \\ 0 & 0.024430 & 0.070886 & 0.302753 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 0 \\ 0.147982 & 0.024430 \\ 0.160665 & 0.070886 \\ 0.024430 & 0.580243 \end{pmatrix}. \tag{E.54}$$

These intermediate matrices permit direct verification of the welfare coefficients reported in Table 3 through sequential completion of squares. The structural pivots are

$$(\Lambda_y^{IPT}, \Lambda_\psi^{IPT}, \Lambda_z^{IPT}, \Lambda_{\psi^*}^{IPT}) = (2.860011,\ 0.406343,\ -0.097828,\ 0.352099), \tag{E.55}$$

and the three Calvo-inflation coefficients, computed from Eq. (E.46), are

$$(\Lambda_{\pi,H}^{IPT}, \Lambda_{\pi,F}^{IPT}, \Lambda_{\pi_H^*}^{IPT}) = (65.312754,\ 21.282917,\ 13.518748). \tag{E.56}$$

The Home WR LOOP-gap coefficients are $(\Gamma_{\text{own}}, \Gamma_{\text{cross}}, \Gamma_z) = (0.364180, 0.060123, -0.015548)$. The Foreign national mirror uses $\nu' = 1 - \nu$ and $z_t' = -z_t$; its corresponding values are $(\Gamma_{\text{own}}^*, \Gamma_{\text{cross}}^*, \Gamma_z^*) = (0.328075, 0.024116, 0.006695)$, where the last sign has been converted back to the Home z_t coordinate. These values agree, up to rounding, with the coefficients reported in Sec. 4.4. Equation (E.53) gives the remaining two WR references,

$$z_t^{w,IPT} = -1.618801\,\tau_{M,t} - 0.720721\,(\tau_{M,t}^* - \psi_t^*), \qquad \psi_t^{*,w,IPT} = 0.368275\,\tau_{M,t} + 1.788101\,\tau_{M,t}^*.$$

In line with Eq. (E.32), the multipliers $\Theta_n = \Theta_Y = 0.844719$ and $\Theta_H^{AS} = -0.049035$ give $\Lambda_y^{IPT} = (1+\varphi)(\Theta_n - \Theta_H^{AS}) = 2.860011$ and $\Lambda_{\pi,H}^{IPT} = (\varepsilon/\kappa_H)(\Theta_n - \Theta_H^{AS}) = 65.312754$.

The negative Λ_z^{IPT} is a signed national strategic contribution. Schur completion does not make this quadratic positive definite, and is therefore not a positive-definite stabilization penalty.

Welfare Weights for the Baseline and High Substitution Case. The following tables present the coefficient values for the two-country IPT baseline and its High Substitution Case.

Table 6: Home welfare weights across IPT calibrations

Variable	Baseline	High Substitution
$y_t - y_t^w$	2.860	2.221
$\psi_t - \psi_t^w$	0.406	1.206
$z_t - z_t^w$	−0.098	−0.480
$\psi_t^* - \psi_t^{*,w}$	0.352	1.438
$\pi_{H,t}$	65.313	50.711
$\pi_{F,t}$	21.283	33.686
$\pi_{H,t}^*$	13.519	11.320

Note: The IPT superscripts on the WR variables are omitted from the row labels.

Table 7: Loadings of the Home WR LOOP gap across IPT calibrations

Term	Baseline	High Substitution
$\tau_{M,t}$ (Γ_{own})	0.364	0.053
$\tau_{M,t}^* - \psi_t^*$ (Γ_{cross})	0.060	0.122
z_t (Γ_z)	−0.016	−0.059

Note: Each entry is the coefficient on the corresponding term in the Home WR LOOP gap; the structural formulas are given in Appendix E.2.2.

High Substitution Case. The High Substitution Case applies the same welfare-cost derivation at $\eta = 3$; no separate derivation is required. The resulting Home welfare weights are reported in Table 6.

Baseline Reciprocal Tariff Case. The baseline reciprocal-tariff experiment uses the same two-country IPT welfare-cost function and structural parameters as the Baseline Case of the unilateral tariff. Under reciprocal tariffs, both $\tau_{M,t}$ and $\tau_{M,t}^*$ are nonzero, while the welfare weights remain unchanged. Hence no separate welfare-cost derivation is required.

F Proofs of Propositions 1 and 2

F.1 Proof of Proposition 1

Throughout, lower-case price and allocation variables denote log deviations from the zero-tariff deterministic steady state. For convenience, define $c \equiv 1/(\varepsilon_M - 1)$ and $x_t \equiv (y_t, \psi_t)'$. We normalize $p_F^* = 0$, so that $e_t = \psi_t + p_{F,t}$, and set the pre-shock values $p_{F,-1} = e_{-1} = \psi_{-1} = 0$.

Step 1: Differentiating the Nonlinear Ramsey Conditions with $\theta = 0$. With $\theta = 0$, every producer resets its price each period, so $\Delta_{H,t} = 1$ and the producer pricing block reduces to $MC_t^r = \mathcal{M}^{-1}$. By Eqs. (16) and (26), $MC_t^r = Z_t Y_t^{\varphi}$, so the block is $Z_t Y_t^{\varphi} = L$, whose first-order approximation is $z_t + \varphi y_t = 0$. Combining it with the first-order relation $z_t = p_* y_t + k_*(\psi_t - \tau_{M,t})$ of Appendix E.1 gives

$$y_t = g(\tau_{M,t} - \psi_t), \qquad z_t = -\varphi g(\tau_{M,t} - \psi_t), \qquad g \equiv \frac{k_*}{p_* + \varphi}. \tag{F.1}$$

The following lemma expresses the linearization of the original nonlinear Ramsey first-order conditions in terms of the Hessian of the Ramsey Lagrangian.

Lemma 2. *Write the nonlinear timeless-perspective Ramsey problem under perfect foresight as the maximization of $\sum_{t\geq 0} \beta^t U(X_t)$ subject to $F(W_t) = 0$ for $t \geq 0$, where $W_t \equiv$*

$(X'_{t-1}, X'_t, X'_{t+1}, \tau_{M,t})'$, *with Lagrangian* $\mathcal{R} \equiv \sum_{t\geq 0} \beta^t [U(X_t) + \Lambda'_t F(W_t)]$ *and lagged multipliers fixed at their steady-state values. Let* $(\bar{X}, \bar{\Lambda})$ *be the zero-tariff steady state of the Ramsey first-order conditions, and let* $\mathcal{H}$ *denote the Hessian of the period-t term* $U(X_t) + \bar{\Lambda}' F(W_t)$ *with respect to the* stacked *vector* W_t *at that steady state.* $\mathcal{H}$ *therefore contains the intertemporal second derivatives* $\partial^2(\bar{\Lambda}'F)/\partial X_{t-1}\partial X_t$, $\partial^2(\bar{\Lambda}'F)/\partial X_t \partial X_{t+1}$, *and* $\partial^2(\bar{\Lambda}'F)/\partial X_{t-1}\partial X_{t+1}$. *Then the first-order approximation of the nonlinear Ramsey first-order conditions and constraints, in* $(dX_t, d\Lambda_t)$, *is identical to the system of first-order conditions and constraints of*

$$\max \sum_{t\geq 0} \beta^t \frac{1}{2} dW'_t \mathcal{H}\, dW_t \quad \textit{subject to} \quad F_W\, dW_t = 0, \qquad t \geq 0,$$

with $d\Lambda_t$ *as the multipliers and* $d\Lambda_{-1} = 0$.

Proof. X_t enters the period-$(t-1)$, period-t, and period-$(t+1)$ terms of $\mathcal{R}$. Its nonlinear first-order condition is $\partial\mathcal{R}/\partial X_t = 0$, which collects the derivatives of these three terms. Differentiating this condition at $(\bar{X}, \bar{\Lambda})$ with respect to $(dX_s)_{s\geq -1}$, $(d\Lambda_s)_{s\geq -1}$, and $(d\tau_{M,s})_{s\geq 0}$ gives the second derivatives of the three period terms, evaluated at $\bar{\Lambda}$, multiplying $dX_{t-2}, \ldots, dX_{t+2}$ and the tariffs, plus the transposed constraint Jacobians multiplying $d\Lambda_{t-1}, d\Lambda_t, d\Lambda_{t+1}$. Because each period term is a function of W_t alone, these second derivatives are exactly the blocks of $\mathcal{H}$, including its intertemporal blocks. The result is the stationarity condition of the displayed quadratic problem with multipliers $d\Lambda_t$. Differentiating $F(W_t) = 0$ gives the displayed linear constraints. The timeless-perspective initial condition fixes the lagged multipliers at $\bar{\Lambda}$, so $d\Lambda_{-1} = 0$. □

The coefficients Θ in Eqs. (E.17) and (E.18) are determined by the steady-state first-order conditions of the original Ramsey Lagrangian. In the normalization used in Appendix E.1, they are the steady-state multipliers $\bar{\Lambda}$ on the corresponding constraints, including the reset-price-weighted aggregate-supply restrictions. Rescaling a constraint rescales its multiplier inversely and leaves $\bar{\Lambda}'F$, and hence $\mathcal{H}$, unchanged. The matrices Q and B in Eq. (E.21) are minus the blocks of $\mathcal{H}$ in the policy coordinates and the policy–tariff cross-derivatives, after the private-equilibrium identities are used to eliminate C_t and z_t. Eliminating variables through equality constraints that hold exactly does not change the reduced Hessian of the Lagrangian. With $\theta = 0$, intertemporal second derivatives arise in the importer block: in the Calvo price-index and dispersion laws and in the importer reset-price condition, which involves future variables. After the private-equilibrium restrictions are eliminated and the Calvo aggregation in Appendix E.1 is applied, the dynamic second-order terms that remain are those in import-price inflation, $\frac{1}{2}\Lambda_{\pi,F}\pi^2_{F,t}$ in Eq. (E.22), with $\pi_{F,t} = p_{F,t} - p_{F,t-1}$. No producer-inflation term appears, because with $\theta = 0$ there is no producer price dispersion. By Lemma 2, the differentiated nonlinear Ramsey first-order conditions therefore form a linear system whose coefficient matrices are those of the quadratic form

$$\mathcal{L}_t = \frac{1}{2} x'_t Q x_t - x'_t B \tau_{M,t} + \frac{1}{2}\Lambda_{\pi,F}\pi^2_{F,t},$$

together with the differentiated private-equilibrium conditions, with the lagged multipliers at zero deviation. $\mathcal{L}_t$ is used here only as a compact notation for the coefficients of the differentiated original first-order conditions. It is not an objective that is assumed, and it is not the objective of the policy computations in the paper, which solve the nonlinear Ramsey first-order conditions. The differentiated private-equilibrium conditions are Eq. (F.1), the importer Phillips curve (34), the relative-price identity $s_t = s_{t-1} + \pi_{F,t} - \pi_{H,t}$ with $s_t = z_t - \psi_t$, and the Euler equation. Because $\pi_{H,t}$ carries no welfare weight and is otherwise unrestricted, the relative-price identity only determines $\pi_{H,t}$; the Euler equation only determines i_t (Lemma 1). Neither restricts $(\psi_t, \pi_{F,t})$. The differentiated system therefore reduces to the conditions for $\{\psi_t, \pi_{F,t}\}_{t\geq 0}$ associated with Eq. (34) alone.

Step 2: Reduction of the Coefficients. Substituting $x_t = (g(\tau_{M,t} - \psi_t), \psi_t)'$ into $\mathcal{L}_t$ gives

$$\frac{1}{2} x'_t Q x_t - x'_t B \tau_{M,t} \quad = \quad \frac{1}{2}\left(Q_{11}g^2 - 2Q_{12}g + Q_{22}\right)\psi_t^2 - \left[B_2 - (B_1 + Q_{12})g + Q_{11}g^2\right]\psi_t \tau_{M,t} + \text{t.i.p.}$$

$$= \frac{1}{2}\tilde{\Lambda}_\psi \left(\psi_t - \tilde{\Gamma}_{\text{own}}\tau_{M,t}\right)^2 + \text{t.i.p.},$$

where t.i.p. collects terms independent of policy. Since $\tilde{\Lambda}_\psi = v'Qv$ with $v = (-g, 1)'$ and Q is positive definite, $\tilde{\Lambda}_\psi > 0$; also $\Lambda_{\pi,F} > 0$, so $\vartheta > 0$.

Step 3: The Differentiated First-Order Conditions. Let μ_t denote the deviation of the original Ramsey multiplier on the importer pricing block, expressed in the normalization of Eq. (34) written as $\pi_{F,t} - \beta\pi_{F,t+1} - \kappa_M(\psi_t + c\tau_{M,t}) = 0$. By Steps 1 and 2, the differentiated first-order conditions for ψ_t and $\pi_{F,t}$, $t \geq 0$, are

$$\tilde{\Lambda}_\psi \left(\psi_t - \tilde{\Gamma}_{\text{own}}\tau_{M,t}\right) - \kappa_M\mu_t = 0, \qquad \Lambda_{\pi,F}\pi_{F,t} + \mu_t - \mu_{t-1} = 0,$$

where, from the timeless perspective, the lagged multiplier μ_{-1} is at its steady-state value, which is zero in deviation form. Define $\xi_t \equiv \psi_t - \tilde{\Gamma}_{\text{own}}\tau_{M,t}$, so that $\xi_{-1} = \kappa_M\mu_{-1}/\tilde{\Lambda}_\psi = 0$. Eliminating μ_t gives $\Lambda_{\pi,F}\pi_{F,t} = -(\tilde{\Lambda}_\psi/\kappa_M)(\xi_t - \xi_{t-1})$. Summing from date 0 and using $p_{F,-1} = 0$ yields

$$\psi_t - \tilde{\Gamma}_{\text{own}}\tau_{M,t} = -\vartheta p_{F,t}, \qquad t \geq 0. \tag{F.2}$$

Step 4: The Import Price Path. Substituting Eq. (F.2) into Eq. (34) with $\pi_{F,t} = p_{F,t} - p_{F,t-1}$ gives

$$\beta p_{F,t+1} - (1 + \beta + \kappa_M\vartheta)p_{F,t} + p_{F,t-1} = -\kappa_M \left(\tilde{\Gamma}_{\text{own}} + c\right)\tau_{M,t}. \tag{F.3}$$

Let $f(\lambda) \equiv \beta\lambda^2 - (1 + \beta + \kappa_M\vartheta)\lambda + 1 = (1 - \lambda)(1 - \beta\lambda) - \kappa_M\vartheta\lambda$. Since $f(0) = 1 > 0$ and $f(1) = -\kappa_M\vartheta < 0$, f has one root $\delta \in (0, 1)$, and its other root is $1/(\beta\delta) > 1$. Imposing boundedness eliminates the explosive root and yields the unique solution

$$p_{F,t} = \delta p_{F,t-1} + \delta\kappa_M \left(\tilde{\Gamma}_{\text{own}} + c\right) \sum_{k \geq 0} (\beta\delta)^k \tau_{M,t+k}.$$

With no further innovations after the date-0 shock, the AR(1) tariff process implies $\tau_{M,t+k} = \rho_\tau^k \tau_{M,t}$, so $p_{F,t} = \delta p_{F,t-1} + A\tau_{M,t}$, where $A \equiv \delta\kappa_M(\tilde{\Gamma}_{\text{own}} + c)/(1 - \beta\delta\rho_\tau) > 0$. Hence $p_{F,0} = A\tau_{M,0}$, $p_{F,1} = (\delta + \rho_\tau)A\tau_{M,0}$, and $\pi_{F,1} = (\delta + \rho_\tau - 1)A\tau_{M,0}$.

Step 5: The Impact Interest-Rate Response. From $e_t = \psi_t + p_{F,t}$ and Eq. (F.2), $e_t = \tilde{\Gamma}_{\text{own}}\tau_{M,t} + (1 - \vartheta)p_{F,t}$. Lemma 1 then gives

$$\hat{\imath}_0 = e_1 - e_0 = -\tilde{\Gamma}_{\text{own}}(1 - \rho_\tau)\tau_{M,0} + (1 - \vartheta)(\delta + \rho_\tau - 1)A\tau_{M,0}.$$

Because $f(\delta) = 0$ is equivalent to $\kappa_M\vartheta\delta = (1 - \delta)(1 - \beta\delta)$,

$$(1 - \vartheta)A = \frac{1 - \vartheta}{\vartheta} \frac{(1 - \delta)(1 - \beta\delta)}{1 - \beta\delta\rho_\tau} \left(\tilde{\Gamma}_{\text{own}} + c\right),$$

which yields Eq. (A.2).

Step 6: The Sign. Since $\tau_{M,0} = \epsilon_{\tau,0} > 0$, $\rho_\tau < 1$, and $\tilde{\Gamma}_{\text{own}} > 0$, the first term of Eq. (A.2) is strictly negative. In the second term, $(1 - \delta)(1 - \beta\delta)/(1 - \beta\delta\rho_\tau) > 0$ and $\tilde{\Gamma}_{\text{own}} + c > 0$, while $(1 - \vartheta)(\delta + \rho_\tau - 1)/\vartheta \leq 0$ under the stated condition. Hence $\hat{\imath}_0 < 0$. Finally, f is convex with $f(0) > 0$, $f(\delta) = 0$, and $f(1) < 0$, so for $\lambda \in [0, 1]$, $f(\lambda) \geq 0$ if and only if $\lambda \leq \delta$. Applying this to $\lambda = 1 - \rho_\tau$, $\delta + \rho_\tau \geq 1$ if and only if $f(1 - \rho_\tau) \geq 0$, that is, $\rho_\tau\{1 - \beta(1 - \rho_\tau)\} \geq \kappa_M\vartheta(1 - \rho_\tau)$, which is the stated bound. □

Remarks. The proposition concerns the notional interest rate. In the flexible-producer-price benchmark, a 20-percentage-point tariff shock implies a notional rate decline exceeding 101.01 basis points, the distance from the steady-state nominal interest rate to the ZLB. Because the result is a first-order property, its sign does not depend on the size of $\epsilon_{\tau,0}$; the finite-shock responses in Sec. 3.5.1 are computed from the nonlinear Ramsey first-order conditions.

F.2 Proof of Proposition 2 and Corollary 1

Step 1: Importer Pricing with $\theta_M = 0$. With $\theta_M = 0$, only the $k = 0$ term of the importer reset-price condition (32) remains, and every importer resets its price each period. With $\tau = 1/\varepsilon_M$, the condition gives, exactly,

$$P_{F,t}(\iota) = \overline{P}_{F,t} = \left(1 + \frac{\tau_{M,t}}{\varepsilon_M - 1}\right) \mathcal{E}_t P_F^* \qquad \text{for all } \iota,$$

as in Eq. (33). Hence there is no importer price dispersion, and by Eq. (20) every purchaser price equals $P_{F,t} + \tau_{M,t}\mathcal{E}_t P_F^* = (1 + \tilde{\tau}_{M,t})\mathcal{E}_t P_F^*$, with $\tilde{\tau}_{M,t}$ defined in Eq. (A.3). Equation (22) then gives $\widetilde{P}_{F,t}^{IPT} = (1 + \tilde{\tau}_{M,t})\mathcal{E}_t P_F^*$.

Step 2: Equivalence of the Two Ramsey Problems. Under PPT, $\widetilde{P}_{F,t}^{PPT} = (1 + \tau_{M,t})\mathcal{E}_t P_F^*$ and the LOOP holds, so $S_t = Z_t$. By Step 1, the IPT household-facing import price is the PPT one with $\tau_{M,t}$ replaced by $\tilde{\tau}_{M,t}$; hence $P_t^{IPT}/P_{H,t} = \mathcal{G}^{PPT}(\tilde{\tau}_{M,t}, Z_t)$. Substituting this into the remaining IPT constraints listed in Sec. 3.2, namely import demand, the intratemporal and Euler conditions, risk sharing (26), the producer block (13)–(17), goods-market clearing (35), and the ZLB conditions (36), yields exactly the PPT constraints evaluated at $\tilde{\tau}_{M,t}$, with S_t replaced by Z_t. The importer variables Ψ_t, $\Pi_{F,t}$, $\overline{P}_{F,t}$, $P_{F,t}$, and $\widetilde{P}_{F,t}^{IPT}$ are explicit functions of $(\mathcal{E}_t, \tau_{M,t})$ given by Step 1 and appear in no other constraint, so eliminating them leaves the feasible set of the remaining variables unchanged. Tariff revenue, the importer subsidy, and importer dividends enter only the lump-sum transfers, which are not Ramsey constraints under complete markets. The objective (1) is the same in both environments. After eliminating the importer-specific variables, the IPT Ramsey problem under $\{\tau_{M,t}\}$ is equivalent to the PPT Ramsey problem under $\{\tilde{\tau}_{M,t}\}$, including the ZLB constraints and timeless-perspective initial conditions. Consequently, the two problems yield identical optimal paths for the common allocation and monetary-policy variables. This proves Proposition 2.

Step 3: First-Order PPT Response. By Proposition 2, $\hat{\imath}_0^{IPT}(\{\tau_{M,t}\}) = \hat{\imath}_0^{PPT}(\{\tilde{\tau}_{M,t}\})$, so it suffices to derive $\hat{\imath}_0^{PPT}/\tilde{\tau}_{M,0}$ and multiply by $\varepsilon_M/(\varepsilon_M - 1)$. Applying Lemma 2 to the PPT problem, the differentiated nonlinear Ramsey first-order conditions have the coefficients of the quadratic form $\frac{1}{2}Q^{SOE,PPT}y_t^2 - B^{SOE,PPT}y_t\tilde{\tau}_{M,t} + \frac{1}{2}\Lambda_{\pi,H}^{SOE,PPT}\pi_{H,t}^2$, together with the differentiated private-equilibrium conditions. Under PPT, $\psi_t = 0$, so $z_t = p_* y_t - k_*\tilde{\tau}_{M,t}$ and real marginal cost is $mc_t = z_t + \varphi y_t = (p_* + \varphi)(y_t - g\tilde{\tau}_{M,t})$. Writing the producer Phillips curve (18) as $\pi_{H,t} - \beta\pi_{H,t+1} - \kappa_H(p_* + \varphi)(y_t - g\tilde{\tau}_{M,t}) = 0$ and letting μ_t denote the deviation of the original Ramsey multiplier on the producer pricing block in this normalization, the differentiated first-order conditions for y_t and $\pi_{H,t}$, $t \geq 0$, are

$$Q^{SOE,PPT}\left(y_t - \omega\tilde{\tau}_{M,t}\right) - \kappa_H(p_* + \varphi)\mu_t = 0, \qquad \Lambda_{\pi,H}^{SOE,PPT}\pi_{H,t} + \mu_t - \mu_{t-1} = 0,$$

with $\mu_{-1} = 0$. The relative-price and Euler conditions only determine i_t. Eliminating μ_t and summing from date 0 with $p_{H,-1} = 0$ gives

$$y_t - \omega\tilde{\tau}_{M,t} = -\chi p_{H,t}, \qquad t \geq 0. \tag{F.4}$$

Step 4: The Producer Price Path. Substituting Eq. (F.4) into the Phillips curve with $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ gives

$$\beta p_{H,t+1} - [1 + \beta + \kappa_H(p_* + \varphi)\chi]p_{H,t} + p_{H,t-1} = -\kappa_H(p_* + \varphi)(\omega - g)\tilde{\tau}_{M,t}.$$

The characteristic polynomial $f_H(\lambda) = (1-\lambda)(1-\beta\lambda) - \kappa_H(p_* + \varphi)\chi\lambda$ satisfies $f_H(0) = 1 > 0$ and $f_H(1) < 0$, because $\chi > 0$. It therefore has one root $\delta_H \in (0,1)$, and the other root is $1/(\beta\delta_H) > 1$. As in the proof of Proposition 1, imposing boundedness eliminates the explosive root and yields $p_{H,t} = \delta_H p_{H,t-1} + A_H\tilde{\tau}_{M,t}$, where $A_H \equiv \delta_H\kappa_H(p_* + \varphi)(\omega - g)/(1 - \beta\delta_H\rho_\tau)$. Hence $\pi_{H,1} = (\delta_H + \rho_\tau - 1)A_H\tilde{\tau}_{M,0}$.

Step 5: The Impact Interest-Rate Response. Lemma 1 holds under PPT as well, because Eq. (12) with $S_t = Z_t$ gives $C_t P_t^{PPT} = \mathcal{E}_t P_F^*$. With $p_F^* = 0$, $e_t = z_t + p_{H,t}$. Using $z_t = p_* y_t - k_*\tilde{\tau}_{M,t}$ and Eq. (F.4), $e_t = (p_*\omega - k_*)\tilde{\tau}_{M,t} + (1 - p_*\chi)p_{H,t}$. Therefore

$$\hat{\imath}_0^{PPT} = e_1 - e_0 = (k_* - p_*\omega)(1 - \rho_\tau)\tilde{\tau}_{M,0} + (1 - p_*\chi)(\delta_H + \rho_\tau - 1)A_H\tilde{\tau}_{M,0}.$$

Since $f_H(\delta_H) = 0$ is equivalent to $\kappa_H(p_* + \varphi)\chi\delta_H = (1 - \delta_H)(1 - \beta\delta_H)$,

$$(1 - p_*\chi)A_H = \frac{1 - p_*\chi}{\chi}(\omega - g)\frac{(1 - \delta_H)(1 - \beta\delta_H)}{1 - \beta\delta_H\rho_\tau}.$$

Multiplying by $\tilde{\tau}_{M,0}/\tau_{M,0} = \varepsilon_M/(\varepsilon_M - 1)$ yields Eq. (A.4). Under the sufficient condition, the first term is strictly positive and the second is non-negative, so $\hat{\imath}_0^{IPT} > 0$. □

The first-order solution computed directly from the IPT problem converges to Eq. (A.4) as $\theta_M \to 0$. The resulting values of $\hat{\imath}_0^{IPT}/\tau_{M,0}$ are 0.014258 for $\eta = 1.5$ and 0.006397 for $\eta = 3$. These agree, up to rounding, with $\varepsilon_M/(\varepsilon_M - 1)$ times the corresponding PPT values of 0.010506 and 0.004713, respectively.

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