Optimal Push, Pull, and Failure Funding for Global Health

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Malaria and tuberculosis kill hundreds of thousands of people each year. Yet there is little incentive to develop treatments for diseases concentrated in low-income countries. Governments and funders primarily use push mechanisms, such as grants, and pull mechanisms, such as prizes, to stimulate investment. We add a third instrument: failure insurance, under which the funder reimburses part of a firm's testing costs only if the product fails. Failure insurance is optimal when markets are large enough to reward success but too small to induce initial investment. We formulate the funder's problem as an infinite-dimensional optimization. We use duality to characterize the optimal mix of push, pull, and failure insurance under adverse selection and moral hazard. We then calibrate optimal funding for 29 diseases by combining development costs with disease-specific social value. This benefit-based calibration yields policy conclusions that differ from current practice. Malaria requires a much larger pull incentive than current mechanisms provide. Tuberculosis can warrant failure insurance. Most HIV/AIDS drug development has sufficient commercial incentives for late-stage development.

Key words: mechanism design, adverse selection, moral hazard, global health, innovation, pharmaceutical
History: The current version was updated on September 3, 2026.

1. Introduction

Malaria and tuberculosis each cause over half a million deaths annually (World Health Organization 2024). Yet commercial incentives to develop new drugs for diseases concentrated in low-income countries remain weak. In addition, markets fail to fully capture the broader societal benefits of innovation and reduced disease transmission.

Governments and foundations attempt to address these market failures by funding drug development. For example, in 2022, the Gates Foundation, with assets exceeding $70 billion, invested $624 million in global health research and development (R&D), including over $100 million each for HIV/AIDS, malaria, and tuberculosis (Impact Global Health 2023). Similarly, the Wellcome Trust, with assets exceeding $30 billion, invested $106 million in global health R&D (Impact Global Health 2023).

However, funding new product development presents incentive challenges. Funders typically have less information than firms about a drug's likelihood of success and the firm's capabilities. Funders also cannot fully monitor effort, creating moral hazard. To secure funding, firms may exaggerate a drug's potential and then exert minimal effort, expecting failure regardless. In some cases, these incentive failures lead to outright fraud. For example, in the U.S. Agency for International Development's malaria vaccine program, researchers diverted grant funds and were indicted for theft and conspiracy. The project director later pleaded guilty to accepting kickbacks, filing false tax returns, and making false statements (Desowitz 1991, Kremer 2002). Some view fraud as a major concern in global health funding (García 2019). Even in the absence of fraud, firms may shirk by assigning weaker staff, under-enrolling patients, skimping on follow-up, or neglecting clinical trial quality. Firms can plausibly attribute poor performance to external constraints, especially because trials for tropical diseases often occur in low-income settings with weak infrastructure, and it is costly for the funder to monitor behavior.

Governments and nonprofits primarily rely on push and pull mechanisms to motivate investment. Push mechanisms, such as grants, subsidize R&D costs up front. Pull mechanisms, such as prizes, advance market commitments, and priority review vouchers, reward success and can strengthen accountability by conditioning payment on observable outcomes. A practical difference is timing: pull rewards are typically paid years after testing begins, which means firms must finance development in the interim. Firms tend to discount delayed payments more heavily than large philanthropic funders.[1]

We examine a third mechanism, failure-contingent funding, which we refer to as "insurance."[2] If a project fails, the funder reimburses a share of the firm's testing costs, plus interest. The payment is made only after the firm completes the trial and discloses results to the funder.

In theory, failure-contingent payments can invite gaming. A firm might shirk, or it might work hard, claim failure, receive the payment, and later use the same test results to support approval. Three safeguards reduce this risk. First, insurance covers only part of the testing cost, so the firm still bears losses. Second, failure delays or eliminates commercial returns. In our model, insurance is optimal only when success can generate such returns, so failure has an opportunity cost. Third, the payment can be structured as a conditional reimbursement. The contract can require full disclosure of results, limit reimbursement to one subsidized test per drug–indication pair, and require repayment

[1] For example, the discount rate for firms might be 10.5% (DiMasi et al. 2016), but 3% for funders (Wilkinson et al. 2016). Nevertheless, if the pull mechanism is sufficiently generous, a firm can raise capital based on the expected value of the pull. For instance, Medicines Development for Global Health received funding from Adjuvant Capital to develop its onchocerciasis drug, backed by the anticipated proceeds from a priority review voucher (Ridley et al. 2021).

[2] The funder "insures" development by making payments contingent on failure, which reduces the firm's downside risk. The term does not refer to health insurance for patients, and the instrument is not conventional insurance: there are no premiums, risk pooling, or risk-averse firms.

with interest if the firm later obtains marketing authorization for the same drug and indication. This approval trigger would be observable in regulatory approval records, especially for a neglected-disease product within the funder's scope. Repayment could also be triggered if the firm or licensee uses data from the reimbursed test in a later regulatory submission. The contract could give the funder audit and repayment rights. Breach could trigger interest, penalties, and loss of eligibility for future funding.

Push and insurance both induce testing by firms with lower success probabilities, but they differ in how they allocate rents. Push funding is fixed across types, so high-probability firms receive information rents. In contrast, the expected transfer from insurance falls as the probability of success rises. Insurance therefore gives more support to low-probability firms while limiting rents for high-probability firms. This targeting makes insurance more efficient than push funding under certain model parameter settings.

Insurance resembles a forgivable loan and provides risk sharing. It has several advantages. First, it mitigates adverse selection by reducing incentives to "sandbag" where a firm understates a project's chance of success to qualify for support. Second, paying only upon failure can reduce the funder's expected outlays relative to push funding, because the funder pays nothing if the project succeeds. Finally, insurance need not create moral hazard when reimbursement is capped and the commercial market is moderately attractive; in this case, the firm still prefers to succeed because its expected profits exceed the failure payment.

A funder never needs to combine pull and insurance within the same contract. If the funder promises a payment in both outcomes—success and failure—then part of the payment is guaranteed. That guaranteed component is equivalent to push support, except it is delivered later. Because firms discount delayed payments more heavily than funders do, it is more cost-effective to provide any guaranteed support up front.

We formalize these trade-offs in a principal-agent framework. The model captures both adverse selection (private information about the *ex ante* probability of success) and moral hazard (unobservable effort). We assume that testing costs are known to the funder, based on the experience of funders like the Gates Foundation that support tropical disease product development, as well as data from the Drugs for Neglected Diseases initiative (2019). We formulate the funder's problem as an infinite-dimensional linear program with incentive constraints and derive analytical solutions using duality theory. The results offer policy characterizations across a range of parameters.

Our analysis reveals that diseases fall into distinct funding categories based on consumer welfare and profit potential. First, funders should combine pull and push funding for many tropical diseases, including dengue, malaria, onchocerciasis, and schistosomiasis. Second, push funding alone is optimal for diseases such as cysticercosis and echinococcosis. Third, for diseases like trichomoniasis with

moderate profit potential, funders should combine push funding with insurance. Fourth, for diseases like tuberculosis, funding failure alone is optimal due to moderate commercial value and high public health benefit. Some drugs, such as those for leprosy, should not be funded at all if late-stage testing costs exceed $100 million, because expected welfare gains do not justify the cost. Furthermore, diseases like HIV/AIDS require no public support for late-stage development, given their strong commercial returns.[3]

Our calibrated model challenges current global health funding in four ways. First, it calls for substantially less reliance on push grants. Second, it introduces insurance for diseases such as tuberculosis, a tool which appears absent from existing practice. Third, it recommends greater use of pull mechanisms for high-burden tropical diseases, including dengue, malaria, onchocerciasis, and schistosomiasis. Our results support large pull incentives on the order of $500 million, supplemented by modest push funding. Current pull tools, such as priority review vouchers, are worth far less (Ridley et al. 2021). Malaria in particular justifies a pull commitment exceeding $1 billion. Fourth, it emphasizes disease-specific strategies. Malaria, tuberculosis, and HIV/AIDS should be treated differently despite similar burdens because their commercial prospects diverge. Prior proposals recommended identical large advance market commitments for each disease (Berndt et al. 2007), overlooking commercial differences. Funding for late-stage testing should be rebalanced away from the current HIV/AIDS-first ordering toward malaria and other high-burden diseases with little or no commercial potential.

This study makes both theoretical and practical contributions. On the theoretical side, it is the first to model and calibrate the optimal mix of push, pull, and insurance mechanisms under both adverse selection and moral hazard. It also shows that the funder may optimally reward failure without rewarding success. This is possible when successful development generates a sufficient commercial payoff. On the practical side, our calibration exercise suggests shifting global health funding to greater emphasis on pull and insurance mechanisms, and relatively more late-stage investment in malaria than in HIV/AIDS.

The paper proceeds as follows. The remainder of this section reviews the literature. Section 2 introduces the model, and Section 3 presents the main theoretical results. Section 4 analyzes benchmark cases in which either adverse selection or moral hazard is absent. Section 5 calibrates the model to a set of communicable diseases and associated interventions. Section 6 discusses practical implications. Supplementary tables and figures appear in Appendix EC.1, and proofs of the main-text results are provided in Appendices EC.2–EC.6. Additional extensions, benchmark analyses, and robustness checks, together with their proofs, appear in Appendices EC.7–EC.16.

[3] If the commercial return for HIV/AIDS were lower, for example, for heat-stable formulations in low-income countries, then funding could still be optimal.

1.1. Literature on Principal-Agent Models

Principals may reward failure for three reasons. First, funding failure can motivate firms to reveal poor performance, whether due to low demand (Simester and Zhang 2010) or poor outcomes (Mayer 2022). Second, insurance can promote experimentation. Without it, agents may prefer safer trials (Bolton et al. 2024). Rewarding failure encourages early exploration and improves long-term outcomes (Manso 2011). Furthermore, the principal may reward the agent for exerting effort and truthfully reporting a negative signal (Keppo and Li 2025). Our model differs from this literature because the funder may optimally reward failure without also rewarding success. This is possible because the firm receives an exogenous commercial payoff after success. The third rationale for funding failure is to prevent the agent from underreporting its probability of success, or "sandbagging." Without insurance, underreporting may allow the agent to obtain higher pull funding and earn larger information rents. Khalil et al. (2026) identify a related rationale: the principal may reward the low type after failure to screen types. Their model differs from ours in that it studies dynamic experimentation followed by production with two agent types, excludes discounting, and does not undertake a calibration.

A foundational model of innovation is Aghion and Tirole (1994), which addresses moral hazard in product development. The probability of success depends jointly on the agent's effort and the principal's investment, making it well-suited to innovation policy.

Our work relates to Rietzke and Chen (2020), where agents know their probability of success, make contractible investments, and exert non-contractible effort. We differ in five key ways: we allow rewards for failure; incorporate differing discount rates between the principal and agent; exclude push payments from the firm to the funder, reflecting the context of global health; allow testing costs to exceed social benefits, making inaction optimal in some cases; and provide a full characterization of the optimal mechanism, including explicit optimality conditions and closed-form funding amounts across all model parameters.

We also relate to Lach et al. (2021), who study optimal loans under moral hazard and adverse selection. In their model, successful agents repay loans, mitigating information rents. In contrast, our model assumes no repayment, complicating contract design.

More broadly, our work relates to innovation contract design. Crama et al. (2008) analyze milestone-based contracts with royalties in profitable markets. We focus on products with little or no profit but high social value. Bergmann and Friedl (2008) apply the LEN framework (Holmstrom and Milgrom 1987) to model optimal compensation for a risk-averse R&D agent. Other relevant papers include Hellmann and Thiele (2011) on task selection, Xiao and Xu (2012) on royalty renegotiation, Bhattacharya et al. (2015) on double-sided moral hazard, Schlapp et al. (2015) on effort and information acquisition, and Crama et al. (2017) on licensing contracts with control rights and buy-backs.

1.2. Literature on Drug Development

Average clinical success rates vary across diseases, and this heterogeneity is partly due to selection. Firms avoid low-probability projects when expected commercial returns are limited. For example, the Phase II success rate for infectious diseases is 23%, compared to 11% for cancer drugs (Biotechnology Innovation Organization 2021), perhaps reflecting higher commercial returns for cancer drugs. However, these are averages across many products and indications, so the success-probability threshold that triggers investment for any particular product may differ from the averages.[4]

We assume that costs are known because we focus on sophisticated global-health funders, such as the Gates Foundation, that have access to detailed cost data from sources such as Drugs for Neglected Diseases initiative (2019). These funders are well-positioned to evaluate value and enforce contracts. In contrast, costs may be poorly observed in fast-moving crises or for novel diseases; this was the case during COVID-19 (Snyder et al. 2023). Likewise, costs are often uncertain for rare diseases with limited prior experience (Xu et al. 2026).

Priority review vouchers are the main existing pull mechanism for tropical diseases. Proposed by Ridley et al. (2006) and enacted in 2007, the program awards a transferable FDA voucher for expedited review to firms that develop drugs for eligible tropical diseases. Because vouchers are tradable, small firms can benefit even if they do not have another drug that would gain from expedited review (Gans and Ridley 2013). Vouchers sold for $200 million in 2026 (Ridley and Xu 2026). We show that this amount is an order of magnitude smaller than the incentive needed for malaria, but appropriate for some tropical diseases.[5]

Advance market commitments (AMCs) are another pull mechanism. AMCs promote both innovation and access by committing funders to purchase qualifying products at a pre-agreed price (Berndt et al. 2007, Kremer et al. 2022). AMCs have been used in two cases: a pneumococcal vaccine commitment and Operation Warp Speed during the COVID-19 pandemic (Kremer et al. 2022).

Berndt et al. (2007) proposed a $3 billion AMC for HIV/AIDS, malaria, and tuberculosis based on projected commercial returns for a blockbuster drug. Our approach instead tailors incentives to disease-specific characteristics. For malaria, our recommendation aligns with theirs. For HIV/AIDS and tuberculosis, we propose different strategies reflecting differences in market potential.

While we focus on development incentives, post-approval subsidies may be needed to ensure access in low-income settings if the pull is not explicitly tied to access. Patent pools can help by licensing

[4] Drug development is risky, especially for firms with small portfolios. Investors mitigate this risk through diversification (e.g., megafunds; Lo and Thakor 2022) or by securitizing expected revenues. Firm behavior also matters: improved trial design and reduced patient burden can increase retention and trial success (Song et al. 2023).

[5] A proposed alternative to the priority review voucher is a patent voucher, which rewards drug development with a patent extension for a different drug (Dubois et al. 2026).

intellectual property to generic manufacturers (Wang 2022). Funders can also subsidize retailer stocking or consumer purchases. Retail subsidies are preferable when demand is uncertain and products have long shelf lives (Taylor and Xiao 2014), particularly when funder budgets are large (Ding et al. 2026). When budgets are limited, consumer subsidies may be more effective (Ding et al. 2026).

2. Model

A principal determines whether, when, and how much to fund an agent. The principal in this context refers to a funder such as the Gates Foundation or the Wellcome Trust.

Our model treats the drug developer and its investors as a single agent. It also abstracts from competition among firms. We make this assumption for tractability, but it is plausible for many neglected tropical diseases, which have few products in development.[6]

The firm has a candidate drug for a possible clinical trial followed by submission to a regulator, such as the FDA. We call the clinical trial and the regulatory approval process a "test," the result of which is uncertain.

Before the test, the firm privately knows the probability of success, based on earlier testing of the drug and the firm's capabilities. In order for the test to be successful, the firm needs to exert costly effort by carefully designing the study and recruiting appropriate providers and patients. The effort is not directly observable by the funder. Therefore, our setting involves both adverse selection and moral hazard. Specifically, we model the firm's effort cost as c and the probability of the test being successful given the firm exerts effort as θ, which is the firm's private information (adverse selection). The funder believes that θ is drawn from a distribution with support $\Theta \equiv [\theta_L, \theta_H]$, probability density function f, and cumulative distribution function F (common prior distribution).

If the firm shirks instead, the test will definitely fail, and the corresponding cost to the firm is $\alpha\, c$ for some $\alpha \in [0, 1)$.[7] Here, shirking refers to exerting minimal effort in conducting clinical trials, such as under-investing in infrastructure, hiring inadequately trained personnel, or failing to recruit and follow up with patients appropriately. The value α measures the severity of moral hazard: if $\alpha = 0$, the firm incurs no cost from shirking, and moral hazard is the most severe. If α approaches one, on the other hand, shirking cost becomes the same as effort cost, eliminating the moral hazard issue. One can interpret a strictly positive α as capturing the fixed cost associated with the test, including,

[6] The single-firm assumption is less plausible for HIV and tuberculosis. In 2025, HIV had many drugs in late-stage development. However, this limitation does not affect our policy conclusions because the model predicts no subsidy for HIV. Tuberculosis had two novel drugs in phase III in 2025: contezolid and sudapyridine. Competition could matter for tuberculosis if it lowers expected profit, if the funder faces a binding budget constraint, or if the marginal social value of additional drugs declines. Yet another tuberculosis drug may still have high social value because new mechanisms can help combat drug resistance. Section 5.1 discusses how to interpret the calibration when multiple candidates are under development.

[7] In Appendix EC.7, we show that if the firm's testing cost is linear in effort, the binary effort assumption can be made without loss of generality.

for example, filing paperwork to get permission, opening clinical sites to make the trial accessible, and compensating investigators and participants.[8]

The funder can decide how much to fund the firm before and after the testing result is realized. The funding provided after the test can be conditioned on the test result. Because we analyze a single late-stage trial, the model includes only one signal: success or failure.

Following the revelation principle (Myerson 1981), we study incentive-compatible direct mechanisms without loss of generality. That is, the funder publicly commits to a funding policy at the beginning based on the agent's reported value $\theta \in \Theta$, which specifies: (1) whether to induce testing $q(\theta) \in \{0, 1\}$; (2) an amount of upfront payment $\hat{m}_0(\theta)$ (push); and (3) amounts of subsequent payments conditional on test results. In particular, the payment after a successful test is denoted by $\hat{m}_S(\theta)$ (pull), and after a failed test as $\hat{m}_F(\theta)$ (insurance).

A successful drug brings the funder a value ω, representing consumer welfare. Therefore, when a firm with success probability θ reports truthfully and exerts effort in the test, the funder's expected payoff $u_F(\theta)$ is the potential consumer welfare $\theta\,\omega\,q(\theta)$ minus its expected payments in both periods, i.e.,

$$u_F(\theta) \equiv \left[\theta\,\omega - \hat{m}_0(\theta) - \theta\,\hat{m}_S(\theta) - (1-\theta)\,\hat{m}_F(\theta)\right] q(\theta). \tag{1}$$

We assume that the funder does not discount future payments. In practice, the Gates Foundation's discount rate is not zero but is approximately 3% (Wilkinson et al. 2016). However, assuming a zero discount rate is without loss of generality. It is reasonable to assume that the funder has a lower discount rate because it is not cash-constrained and prioritizes long-term societal benefits. Hence, we can normalize the funder's discount factor to 1 by appropriately rescaling the decision variables. Details are provided in Appendix EC.8.

If the regulator approves the drug, the firm receives a profit of π from the market. The present value of the profit is $\delta\,\pi$, where $\delta \in (0, 1]$ is the firm's discount factor, reflecting the fact that the firm discounts future payments due to cash constraints, the length of testing, and the risks of product failure.

Let $u_1(\theta, \hat{\theta})$ denote the firm's expected payoff when its true type is θ, but it reports $\hat{\theta}$ and exerts effort in the test. With probability $q(\hat{\theta})$, its expected payoff is composed of three components: (1) in period 1, the firm bears the testing cost c and receives an upfront payment of $\hat{m}_0(\hat{\theta})$; (2) the test succeeds with probability θ, and therefore the firm receives a commercial profit π along with a

[8] An equivalent way to frame the cost structure is as follows: the firm pays a fixed cost $\alpha\,c$ upon submitting for testing, and then chooses effort. Low effort incurs no additional cost and leads to certain failure, while high effort adds a cost of $(1-\alpha)\,c$. This framing clarifies that $\alpha\,c$ reflects fixed costs, while $(1-\alpha)\,c$ represents the incremental cost of exerting effort. We thank an anonymous reviewer for this suggestion.

subsequent payment of $\hat{m}_S(\hat{\theta})$; and (3) the trial fails with the remaining probability $1-\theta$, and the firm receives $\hat{m}_F(\hat{\theta})$ as specified by the mechanism. To summarize, we have

$$u_1(\theta,\hat{\theta}) \equiv \Big[-c+\hat{m}_0(\hat{\theta})+\delta\,\theta\,\Big(\hat{m}_S(\hat{\theta})+\pi\Big)+\delta\,(1-\theta)\,\hat{m}_F(\hat{\theta})\Big]\,q(\hat{\theta}). \tag{2}$$

If the firm of type θ misreports as $\hat{\theta}$ and shirks, with probability $q(\hat{\theta})$, it receives an upfront payment $\hat{m}_0(\hat{\theta})$ and bears the partial testing cost $\alpha\,c$, followed by a subsequent payment $\hat{m}_F(\hat{\theta})$ because the test will fail. Its total payoff boils down to

$$u_0(\hat{\theta}) \equiv \Big[-\alpha\,c+\hat{m}_0(\hat{\theta})+\delta\,\hat{m}_F(\hat{\theta})\Big]\,q(\hat{\theta}). \tag{3}$$

A feasible mechanism must satisfy the incentive compatibility constraint, meaning that a firm with type θ should not benefit from misreporting its type as $\hat{\theta}$ or shirking, i.e.,[9]

$$u_1(\theta,\theta) \geq \max\Big\{u_0(\hat{\theta}), u_1(\theta,\hat{\theta})\Big\}, \quad \forall(\theta,\hat{\theta})\in\Theta^2, \tag{IC}$$

where $u_1(\theta,\theta)$ represents the type θ firm's expected payoff when it reports truthfully and exerts effort.

Furthermore, assuming the firm's outside option has a value of zero, the following individual rationality constraint must hold to guarantee the firm's participation

$$u_1(\theta,\theta) \geq 0, \quad \forall\theta\in\Theta. \tag{IR}$$

Finally, we recall the constraints on q,

$$q(\theta)\in\{0,1\}, \quad \forall\theta\in\Theta, \tag{4}$$

and impose the following limited liability constraints, restricting all monetary transfers from the funder to the firm

$$\hat{m}_0(\theta),\ \hat{m}_S(\theta),\ \hat{m}_F(\theta)\geq 0, \quad \forall\theta\in\Theta. \tag{5}$$

The funder's problem can be stated as follows:

$$\max_{\{q(\cdot),\hat{m}_0(\cdot),\hat{m}_S(\cdot),\hat{m}_F(\cdot)\}\in\hat{\Omega}} \int_{\theta_L}^{\theta_H} u_F(\theta)\,\mathrm{d}F(\theta), \tag{6}$$

where the feasible set $\hat{\Omega}$ is determined by constraints (IC), (IR), (4), and (5).

To implement the optimal solution of (6), the funder does not need the firm to report the probability of success θ, and does not dictate the firm's participation. Instead, the funder extends a take-it-or-leave-it contract, specifying payment levels for push, pull, and insurance. The firm can either accept or reject the offer. If the firm accepts, it effectively reveals that its probability of success is above

[9] Consistent with standard practice in the contract theory literature, we focus on mechanisms that induce full effort from a participating agent, while recognizing that in some settings, tolerating shirking could be better.

the threshold for $q(\theta)$ to be 1. If the common prior distribution is uniform, we show that such an indirect mechanism is indeed optimal. Under general distributions, such a take-it-or-leave-it offer performs well. Specifically, it consistently achieves at least 99% of the optimal value under various Beta distributions (see Appendix EC.12 for details).

In order to transform the non-linear optimization problem (6) into a linear one, we introduce the following alternative decision variables,

$$m_0(\theta) \equiv q(\theta)\,\hat{m}_0(\theta), \quad m_S(\theta) \equiv q(\theta)\,\hat{m}_S(\theta), \quad \text{and} \quad m_F(\theta) \equiv q(\theta)\,\hat{m}_F(\theta), \quad \forall \theta \in \Theta. \tag{7}$$

If the firm is induced to test ($q(\theta) = 1$), the quantities m_0, m_S, and m_F are equal to $\hat{m}_0$, $\hat{m}_S$, and $\hat{m}_F$, respectively. If $q(\theta) = 0$, on the other hand, they all take the value 0. We also relax the binary constraint on q such that

$$0 \le q(\theta) \le 1, \quad \forall \theta \in \Theta. \tag{8}$$

Although we allow $q(\theta) \in [0, 1]$ for tractability, we later show that all optimal solutions under the uniform prior distribution satisfy $q(\theta) \in \{0, 1\}$.

We can rewrite both parties' payoff functions in (1), (2), and (3) as

$$u_F(\theta) = \theta\,\omega\,q(\theta) - m_0(\theta) - \theta\,m_S(\theta) - (1-\theta)\,m_F(\theta), \tag{9}$$

$$u_1(\theta, \hat{\theta}) = -c\,q(\hat{\theta}) + m_0(\hat{\theta}) + \delta\,\theta\,\left[m_S(\hat{\theta}) + \pi\,q(\hat{\theta})\right] + \delta\,(1-\theta)\,m_F(\hat{\theta}), \text{ and} \tag{10}$$

$$u_0(\hat{\theta}) = -\alpha\,c\,q(\hat{\theta}) + m_0(\hat{\theta}) + \delta\,m_F(\hat{\theta}), \tag{11}$$

which are all linear functions of the decision variables.

We impose the following feasibility conditions on m's:

$$m_0(\theta),\ m_S(\theta),\ m_F(\theta) \ge 0, \quad \forall \theta \in \Theta. \tag{12}$$

This is a relaxation of the optimization problem (6) because we ignore the constraint that enforces $m_0(\theta) = m_S(\theta) = m_F(\theta) = 0$ if $q(\theta) = 0$. The optimal solution to this relaxed problem satisfies the ignored constraint automatically, and hence it remains optimal for the original problem.

Define the funder's expected utility as

$$U_F(q, m_0, m_S, m_F) \equiv \int_{\theta_L}^{\theta_H} u_F(\theta)\,\mathrm{d}F(\theta), \tag{13}$$

where u_F is specified in (9). The relaxed funder's problem can be formulated as follows:

$$u^* \equiv \max_{\{q(\cdot), m_0(\cdot), m_S(\cdot), m_F(\cdot)\} \in \Omega} U_F(q, m_0, m_S, m_F), \tag{14}$$

where the feasible set Ω is determined by (IC), (IR), (8), and (12).

The following result implies that under the optimal mechanism, the funder never needs to reward success and failure simultaneously.

LEMMA 1. *For any mechanism* $\{q, m_0, m_S, m_F\} \in \Omega$*, there exists a mechanism* $\{q, \tilde{m}_0, \tilde{m}_S, \tilde{m}_F\} \in \Omega$ *such that* $\tilde{m}_S(\theta) \cdot \tilde{m}_F(\theta) = 0$ *for any* $\theta \in \Theta$*, and* $U_F(q, \tilde{m}_0, \tilde{m}_S, \tilde{m}_F) \geq U_F(q, m_0, m_S, m_F)$*.*

That is, any feasible mechanism that offers pull and insurance together is weakly dominated by another one that does not. Intuitively, when both rewards m_S and m_F are strictly positive, conditional on being induced to test ($q = 1$), the funder can increase m_0 by $\delta \cdot \min\{m_S, m_F\}$ while decreasing both m_S and m_F by $\min\{m_S, m_F\}$. This change does not affect the firm's incentive while saving the funder $(1 - \delta) \cdot \min\{m_S, m_F\}$ due to the firm's time discounting.

3. Optimal Mechanism

In this section, we characterize the optimal mechanism that solves the funder's problem (14). First, we show the optimal solution under the following assumption on the common prior distribution of the agent's private information in Sections 3.1 and 3.2.

ASSUMPTION 1. *The common prior of the agent's private information* θ *follows a uniform distribution on* $[0, 1]$*.*

The results obtained in this section under Assumption 1 are intuitive and easy to implement. Then, in Section 3.3, we study the problem for general distributions.

Before going into details, we provide an overview of the optimal mechanisms when $\delta \geq 1/2$. The optimal solution to the funder's problem (14) follows a simple threshold policy under Assumption 1. That is, the funder incentivizes the firm to test if and only if the success rate θ is above a threshold. Furthermore, the optimal compensations $m_0(\theta)$, $m_S(\theta)$, and $m_F(\theta)$ are constants that do not change with θ as long as it is above the threshold. Thus, the funder can implement this optimal mechanism as a simple *take-it-or-leave-it offer*: the firm will accept the offer if and only if its privately known success probability exceeds the threshold.

Figure 1 summarizes this uniform-prior benchmark for $\delta \geq 1/2$. We identify seven regions based on profit and consumer welfare, holding testing cost, shirking cost, and discounting fixed. Below, we describe the optimal policy in each region.

Region (1): Do nothing. Both consumer welfare and profit are too low to justify investment.

Region (2): Pull and push funding. Consumer welfare is high, but profit is low. The funder offers some push funding, but it is capped to prevent shirking. The funder supplements the push with pull.

Region (3): Push only. Profit is higher, so pull funding is no longer needed.

Region (4): Push and insurance. Profit is even higher, so insurance replaces part of push.

Region (5): Insurance only. Profit is high enough that only insurance is needed. Total funding is constant, but firms test progressively riskier drugs as the commercial potential rises.

Region (6): Less insurance. As profit continues to rise, the funder reduces insurance.

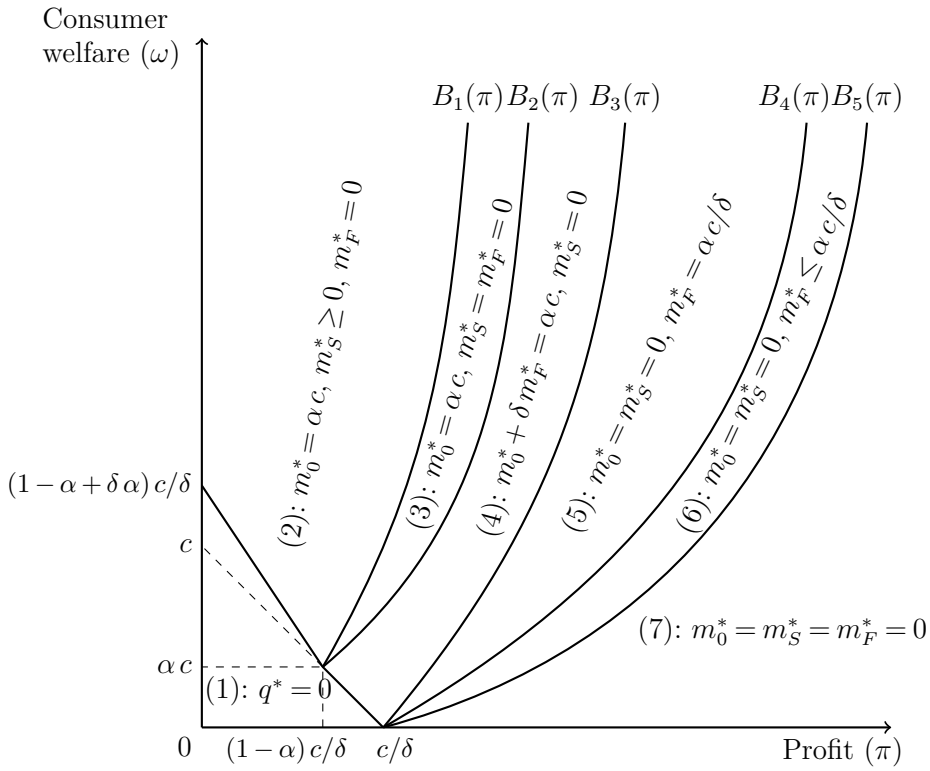


Figure 1 Optimal mechanisms under given profit π and consumer welfare ω if $\delta \geq 1/2$.

Region (7): Do nothing. Profit is so high that the firm will invest without support.

Figure 2 displays the optimal mechanism for increasing π values under a fixed ω. The partition of the x-axis represents the seven cases described above. Figure 2(a) depicts the present value of the maximum payments, and (b) illustrates the testing threshold. As the profit potential π increases, the testing threshold is non-increasing for Regions (2), (3), (5), and (7).

In Appendix EC.9, we quantify the benefit of allowing the insurance funding to reward failure. Our numerical tests show that prohibiting insurance can reduce the funder's objective by up to 20% (Figure EC.5). Therefore, the insurance mechanism discussed in this paper is both conceptually interesting and may yield quantitatively important benefits, depending on the model parameters.

Next, we describe the optimal solution structures in detail in Sections 3.1 and 3.2. In Section 3.3, we relax the assumption of a uniformly distributed θ and demonstrate the robustness of the result.

3.1. Optimal Solutions if the Firm Does Not Heavily Discount the Future

In this subsection, we focus on the case $\delta \geq 1/2$, that is, when the firm does not heavily discount future payoffs. This assumption is empirically plausible: large-scale Phase II or III testing typically takes about two years per phase (DiMasi et al. 2016). Assuming four years of testing and a 16%

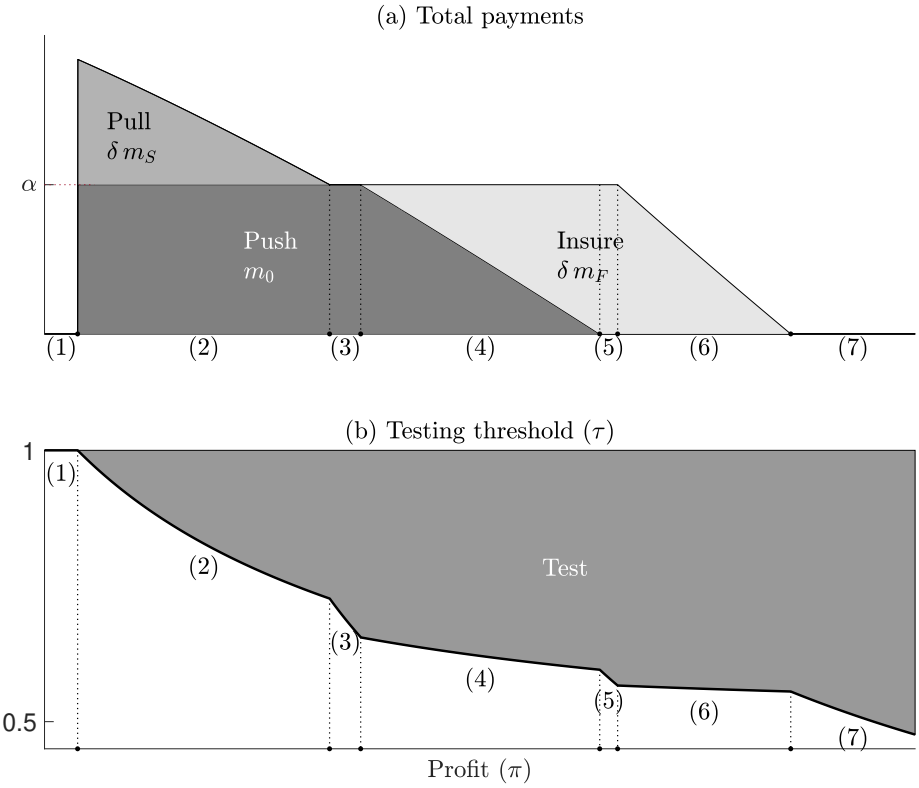


Figure 2 The total payment is the sum of the upfront payment m_0 and either payment upon success $\delta\, m_S$ or payment upon failure $\delta\, m_F$, which captures the maximum present value of payments. The testing threshold (τ) under the optimal mechanism is non-increasing as profit potential rises for these parameter values ($\alpha = 0.5$, $c = 1$, $\delta = 0.6$, and $\omega = 1.2$).

discount rate – the average for a large sample of firms (Gormsen and Huber 2025) – yields a discount factor greater than 1/2. The complementary case ($\delta < 1/2$) is analyzed in Section 3.2.

Proposition 1 considers the case in which both the welfare ω and the profit π are relatively low. In this scenario, the optimal mechanism is not to develop the drug. This result corresponds to Region (1) in Figure 1.[10]

PROPOSITION 1. *If* $\omega + \delta\,\pi \leq c$ *or* $\omega + \pi \leq (1 - \alpha + \delta\,\alpha)\, c/\delta$, *the optimal solution to* (14) *is*

$$q^*(\theta) = m_0^*(\theta) = m_S^*(\theta) = m_F^*(\theta) = 0, \ \forall \theta \in [0,1].$$

In order to show this result, we need to verify optimality in two separate cases: (1) $\omega + \delta\,\pi \leq c$; or (2) $c < \omega + \delta\,\pi \leq (1 - \alpha + \delta\,\alpha)\, c + (1 - \delta)\,\omega$. In case (1), the social surplus $\omega + \delta\,\pi$ is smaller than the

[10] It is without loss of generality to normalize one of the following three parameters, c, π, or ω, to 1. In the Appendix, we set $c = 1$ for simplicity. In this section, we retain all of them to more explicitly express how various quantities depend on these parameters.

testing cost, implying that it is socially inefficient to induce testing. Hence, the optimal strategy for a centralized social planner is to induce no testing and provide no funding. In case (2), even though the social surplus exceeds the testing cost, due to the firm's discounting of future income and the relatively small profit π, the compensation to induce the firm's honest effort would be too high to be worth doing. It is worth noting that if the firm does not discount future income, i.e., $\delta = 1$, case (2) is reduced to $c < \omega + \delta\,\pi \leq c$ and hence vanishes.

To better express the following results, we define the self-funding threshold

$$\bar{\tau} \equiv \frac{c}{\delta\,\pi}, \tag{15}$$

such that for a success rate $\theta > \bar{\tau}$, the expected profit $\delta\,\pi\,\theta$ exceeds the testing cost c. Such a firm always has an incentive to test even without support from the funder.

Next, Proposition 2 focuses on the scenario where the consumer welfare ω is relatively large compared to the firm's profit π. In this case, the optimal mechanism is to provide an upfront payment $\alpha\,c$ and a subsequent one conditional on success, which reflects the funder's intuitive desire to hold the firm accountable. This result corresponds to Region (2) in Figure 1. In particular, define the boundary

$$B_1(\pi) \equiv \pi\,\frac{1-(1-\alpha)\,(1-\alpha-2\,\delta\,\alpha)\,\bar{\tau}^2}{2\,(1-\alpha)^2\,\bar{\tau}^2}, \tag{16}$$

and we have the following result.

Proposition 2. *If* $(1-\alpha)\,\bar{\tau} \leq 1$, $\omega + \pi \geq (1-\alpha+\delta\,\alpha)\,c/\delta$, *and* $\omega \geq B_1(\pi)$, *the following solution is optimal for* (14)*:*

$$q^*(\theta) = \begin{cases} 0, \ \forall \theta \in [0,\tau), \\ 1, \ \forall \theta \in [\tau,1], \end{cases} \qquad m_0^*(\theta) = \begin{cases} 0, \quad \forall \theta \in [0,\tau), \\ \alpha\,c, \ \forall \theta \in [\tau,1], \end{cases}$$

and

$$m_S^*(\theta) = \begin{cases} 0, & \forall \theta \in [0,\tau), \\ \pi\,\left[(1-\alpha)\,\bar{\tau}/\tau - 1\right], & \forall \theta \in [\tau,1], \end{cases} \qquad m_F^*(\theta) = 0, \quad \forall \theta \in [0,1],$$

where $\tau \in [0, (1-\alpha)\,\bar{\tau}]$ *is the unique solution to*

$$2\,\delta\,(\omega+\pi)\,\tau^3 + c\,(\alpha - 2\,\delta\,\alpha - 1)\,\tau^2 - c\,(1-\alpha) = 0.$$

In the (π,ω) parametric space, the policy introduced in Proposition 2 is optimal when the pair is not too close to $(0,0)$ and the welfare ω is above the boundary $B_1(\pi)$. Equivalently, the profit π is not excessively high compared to ω. In such scenarios, even a firm with a high success rate requires compensation to participate. The upfront transfer is set equal to $\alpha\,c$ due to moral hazard: a larger upfront payment would incentivize a firm with a low success rate to misreport as a high type and shirk, resulting in a strictly positive expected payoff. The testing threshold τ is determined by optimality conditions, and the subsequent payment upon success, m_S^*, is designed to ensure that the

threshold type is indifferent between participating and not. Furthermore, as shown in Proposition 2 and Figure 2(a), the total payment $m_0^* + \delta\, m_S^*$ decreases with increasing profit π.

As the profit π continues to increase, successful firms have already received enough incentives from the commercial market. In this case, the optimal policy now only involves an upfront payment of $\alpha\, c$ (push) and no after-test payment (pull). The conditions are formally established in Proposition 3, and demonstrated in Region (3) of Figure 1. Here we define the boundary

$$B_2(\pi) \equiv B_1(\pi) - \pi\, \frac{(1-\delta)\,[1-(1-\alpha)\,\bar{\tau}]}{(1-\alpha)^2\,\bar{\tau}^2} \leq B_1(\pi).$$

PROPOSITION 3. *If* $(1-\alpha)\,\bar{\tau} \leq 1$ *and* $B_2(\pi) \leq \omega < B_1(\pi)$*, the following solution is optimal for* (14)*:*

$$q^*(\theta) = \begin{cases} 0, \ \forall \theta \in [0,\tau), \\ 1, \ \forall \theta \in [\tau,1], \end{cases} \quad m_0^*(\theta) = \begin{cases} 0, & \forall \theta \in [0,\tau), \\ \alpha\, c, & \forall \theta \in [\tau,1], \end{cases} \quad m_S^*(\theta) = m_F^*(\theta) = 0, \quad \forall \theta \in [0,1], \tag{17}$$

where $\tau \equiv (1-\alpha)\,\bar{\tau}$*.*

The two boundaries, $B_1(\pi)$ and $B_2(\pi)$, intersect at $\pi = (1-\alpha)\,c/\delta$, where $\omega = \alpha\, c$. For any given $\omega \geq \alpha\, c$, the condition $B_2(\pi) \leq \omega < B_1(\pi)$ in Proposition 3 can equivalently be stated as requiring the profit π to lie within an interval. It is worth mentioning that when the firm does not discount future cash flow, i.e., $\delta = 1$, we have $B_1(\pi) = B_2(\pi)$, and Region (3) in Figure 1 disappears.

As we increase the profit π further, it becomes sufficient to induce the firm's effort. However, for a firm with a relatively low success rate to be willing to test, it becomes optimal to "reward failure." We formally present the result in Proposition 4, which captures Region (4) of Figure 1. Define the corresponding boundary as

$$B_3(\pi) \equiv B_2(\pi) - \pi\, \frac{\alpha\,[3\,(2\,\delta-1) + (2+\alpha-4\,\delta\,\alpha-2\,\delta)\,\bar{\tau} + (2\,\delta\,\alpha-2\,\alpha+1)\,\bar{\tau}^2]}{2\,(1-\alpha)^2\,\bar{\tau}} \leq B_2(\pi).$$

PROPOSITION 4. *If* $(1-\alpha)\,\bar{\tau} \leq 1$*,* $\omega + \delta\,\pi > c$*, and* $B_3(\pi) \leq \omega < B_2(\pi)$*, the following solution is optimal for* (14)*:*

$$q^*(\theta) = \begin{cases} 0, \ \forall \theta \in [0,\tau), \\ 1, \ \forall \theta \in [\tau,1], \end{cases} \quad m_0^*(\theta) = \begin{cases} 0, & \forall \theta \in [0,\tau), \\ \delta\,\pi\,\left[\alpha\,\bar{\tau} + (1-\alpha)\,\bar{\tau}/\tau - 1\right], & \forall \theta \in [\tau,1], \end{cases}$$

and

$$m_F^*(\theta) = \begin{cases} 0, & \forall \theta \in [0,\tau), \\ \pi\,\left[1-(1-\alpha)\,\bar{\tau}/\tau\right], & \forall \theta \in [\tau,1], \end{cases} \quad m_S^*(\theta) = 0, \ \forall \theta \in [0,1],$$

where the threshold $\tau \in [0,1]$ *is uniquely determined by*

$$2\,\delta\,(\omega+\pi)\,\tau^3 - [2\,\delta\,\pi\,(1-\delta) + (1-\alpha+2\,\delta\,\alpha)\,c]\,\tau^2 - (1-\alpha)\,(2\,\delta-1)\,c = 0.$$

As illustrated in Figure 2(a), the present value of the total payment that the firm receives is at least $m_0^* + \delta\, m_F^* = \alpha\, c$ in Region (4). Therefore, even if the drug fails, the firm is partially compensated,

which encourages its participation. In other words, rewarding failure provides insurance for the firm. The threshold τ in Proposition 4 guarantees that if the firm's success probability θ is above this threshold, the present value from success ($\delta\,\pi\,\theta$) is sufficiently high to motivate the firm's participation and effort.

Intuitively, the insurance payment m_F helps to mitigate the firm's incentive to underreport its probability of success ("sandbagging"). Without such a payment, the funder has to rely on either m_0 or m_S to induce participation. However, these tools cannot prevent the firm from underreporting the probability θ, provided that the report remains above the testing threshold τ, to obtain a larger push or pull payment. A positive m_F, by contrast, makes such underreporting less attractive. This is because for a fixed expected payment $(1-\theta)\,m_F$ that contributes to the *ex ante* participation, the funder would offer a lower m_F if the firm reports a lower θ.

This further implies that the (IC) constraints may bind for both upward and downward deviations. This two-sided binding pattern differs from the familiar one-sided pattern in standard screening models (Myerson 1981) and also arises in Khalil et al. (2026).

The expression for $m_0^*(\theta) = \delta\,\pi\,[\alpha\,\bar{\tau} + (1-\alpha)\,\bar{\tau}/\tau - 1]$ in Proposition 4 eventually becomes zero for sufficiently high profit π. If we increase π further, m_0^* will stay at zero, while the threshold τ decreases, allowing drugs with an even lower prior chance of success to be tested. In this case, the firm only receives a payment from the funder if the test fails. The detailed result is presented in Proposition 5, which corresponds to Region (5) of Figure 1. Define the boundary as

$$B_4(\pi) \equiv B_3(\pi) - \pi\,\frac{(2\,\delta - 1)\,(1-\bar{\tau})\,(1-\alpha\,\bar{\tau})^2}{2\,(1-\alpha)^2\,\bar{\tau}^2} \leq B_3(\pi).$$

Proposition 5. *If $\bar{\tau} \leq 1$ and $B_4(\pi) \leq \omega < B_3(\pi)$, the following solution is optimal for* (14)*:*

$$q^*(\theta) = \begin{cases} 0, \forall \theta \in [0,\tau), \\ 1, \forall \theta \in [\tau,1], \end{cases} \quad m_F^*(\theta) = \begin{cases} 0, & \forall \theta \in [0,\tau), \\ \alpha\,c/\delta, & \forall \theta \in [\tau,1], \end{cases} \quad m_0^*(\theta) = m_S^*(\theta) = 0, \quad \forall \theta \in [0,1],$$

where $\tau = (1-\alpha)\,\bar{\tau}/(1-\alpha\,\bar{\tau})$.

If the profit π grows even higher, the funder can start lowering its support level and rely more and more on the commercial market. The corresponding optimality conditions are characterized in Proposition 6, depicting Region (6) of Figure 1. The boundary is given by

$$B_5(\pi) \equiv B_4(\pi) - \pi\,\frac{\alpha\,(1-\bar{\tau}^2)}{2\,(1-\alpha)\,\bar{\tau}} \leq B_4(\pi).$$

Proposition 6. *If $\bar{\tau} \leq 1$ and $B_5(\pi) \leq \omega < B_4(\pi)$, the following solution is optimal for* (14)*: $m_0^*(\theta) = m_S^*(\theta) = 0,\ \forall \theta \in [0,1]$,*

$$q^*(\theta) = \begin{cases} 0, \forall \theta \in [0,\tau), \\ 1, \forall \theta \in [\tau,1], \end{cases} \ m_F^*(\theta) = \begin{cases} 0, & \forall \theta \in [0,\tau), \\ \pi\,(\bar{\tau}-\tau)/(1-\tau), & \forall \theta \in [\tau,1], \end{cases}$$

where $\tau \equiv (1+\bar{\tau})/[2\,(\omega/\pi + 1)]$.

Starting from this region, the funder no longer guarantees to cover the cost of shirking αc for the participating firm. The reason is that the profit is high enough so that the funder does not need to offer the highest insurance level ($\alpha c/\delta$) to induce testing of a drug with a relatively low probability of success. In fact, the testing threshold $\tau \equiv (1+\bar{\tau})/[2(\omega/\pi+1)]$ is increasing in π when $\omega > \bar{\tau}\pi$. That is, it can be optimal to exclude more types of firms as the profit π increases. The optimality conditions drive this feature: the savings from tightening the testing threshold outweigh the benefit of testing a drug with a relatively low *ex ante* efficacy level.

Finally, when the profit π is high enough to incentivize testing, the funder does not need to intervene. This is shown in Region (7) of Figure 1 and summarized in Proposition 7 below.

PROPOSITION 7. *If $\omega < B_5(\pi)$, the following solution is optimal for* (14)*:*

$$q^*(\theta) = \begin{cases} 0, \forall \theta \in [0,\tau), \\ 1, \forall \theta \in [\tau,1], \end{cases} \quad m_0^*(\theta) = m_S^*(\theta) = m_F^*(\theta) = 0, \quad \forall \theta \in [0,1], \tag{18}$$

where $\tau = \bar{\tau}$.

We prove the results in this section using duality for the (infinite-dimensional) linear optimization problem (14). In particular, we first follow the standard step on the (IC) constraint from Myerson (1981) to eliminate the decision variable $m_0(\theta)$. The objective function then involves three virtual valuation functions, corresponding to coefficients of $q(\theta)$, $m_S(\theta)$, and $m_F(\theta)$, respectively. We then construct a dual feasible solution for each of the seven cases. The corresponding dual objective values are upper bounds on the optimal objective values in (14). However, these upper bounds can be achieved using the primal solutions proposed in the aforementioned seven propositions, which validates optimality. The detailed proofs are provided in Appendix EC.3.

The following corollary further shows that, as the profit level π increases, the funder generally funds less promising drugs for testing, except in Regions (4) and (6) of Figure 1. The same monotonicity holds across all regions as consumer welfare ω increases. Moreover, increases in either commercial profit π or consumer welfare ω make both parties *ex ante* better off, except for the firm in Region (4) when π increases.

COROLLARY 1. *If $\delta \geq 1/2$, fixing other model parameters,*

(a) the optimal testing threshold τ according to Propositions 1–3, 5, and 7 is non-increasing in π.

(b) the optimal testing threshold τ according to Propositions 1–7 is non-increasing in ω; and

(c) the funder's expected payoff is non-decreasing in both π and ω. The firm's ex ante payoff is non-decreasing in ω. With respect to π, the firm's ex ante payoff is non-decreasing in Regions (1)–(3) and (5)–(7), but not necessarily in Region (4).

Figure 2(b) illustrates the non-increasing testing threshold τ with profit π in Regions (1)-(3), (5), and (7). In this particular parameter setting, the threshold is decreasing in Regions (4) and (6) as well.

3.2. Optimal Solutions if the Firm Heavily Discounts the Future

To address cases where the firm heavily discounts future payments, we extend the model to consider discount factors (δ) below 0.5. This situation may arise if Phase III testing is unusually long or if the firm's discount rate is substantially higher than average. When the firm places less value on future payoffs, mechanisms that rely on delayed rewards, such as pull and insurance, become less effective.

The optimality conditions are unchanged in the cases described in Propositions 1 (no testing) and 2 (push and pull). The proof for the case $\delta < 1/2$ is incorporated into these propositions.

The next proposition summarizes the optimal solution in the remaining cases, including maximal push, less push, and commercial. Define the boundaries as:

$$\tilde{B}_2(\pi) \equiv B_1(\pi) - \pi \, \frac{-(1-2\,\delta)\,(1-\alpha)^2\,\bar{\tau}^2 - 2\,\delta\,(1-\alpha)\,\bar{\tau} + 1}{2\,(1-\alpha)^2\,\bar{\tau}^2}$$

and

$$\tilde{B}_5(\pi) \equiv \tilde{B}_2(\pi) - \pi \, \frac{\alpha\,\delta\,(\bar{\tau}+1)}{(1-\alpha)\,\bar{\tau}} \leq \tilde{B}_2(\pi).$$

PROPOSITION 8. *If $\delta < 1/2$, the optimal policy is given as follows:*

(a) If $\tilde{B}_2(\pi) \leq \omega < B_1(\pi)$, the optimal solution to (14) *is given by* (17).

(b) If $\omega + \delta\,\pi > c$ and $\tilde{B}_5(\pi) \leq \omega < \tilde{B}_2(\pi)$, the optimal solution to (14) *is given by: $m_S^*(\theta) = m_F^*(\theta) = 0,\ \forall \theta \in [0,1]$,*

$$q^*(\theta) = \begin{cases} 0, \forall \theta \in [0,\tau), \\ 1, \forall \theta \in [\tau,1], \end{cases} \quad \textit{and} \quad m_0^*(\theta) = \begin{cases} 0, & \forall \theta \in [0,\tau), \\ c - \delta\,\pi\,\tau, & \forall \theta \in [\tau,1], \end{cases}$$

where $\tau \equiv (1+\bar{\tau})/[\omega/(\delta\,\pi)+2]$.

(c) If $\omega < \tilde{B}_5(\pi)$, the optimal solution to (14) *is given by* (18).

A key difference from the $\delta \geq 1/2$ case is that the funder never uses insurance to induce testing. The intuition is that when the firm places less value on future payoffs, it becomes too costly for the funder to rely on future payments to provide incentives. The pull mechanism still arises because the amount of push m_0 is bounded above by the moral hazard constraint. Consequently, when the commercial profit π is sufficiently low, the funder must use m_S to provide additional incentives beyond m_0.

Figure EC.3 in Appendix EC.1 summarizes the five possible cases when the firm's discount factor δ is below 1/2. For the cases already analyzed in the previous section, we retain the same numbering. We label the new case by (3'), defined in Proposition 8, case (b), where the optimal policy is to grant an upfront payment m_0 smaller than $\alpha\,c$.

Analogous to Corollary 1, we summarize the comparative statics in the following corollary.

COROLLARY 2. *If $\delta < 1/2$, fixing other model parameters,*

(a) the optimal testing threshold τ, as characterized in Propositions 1-2 and in Proposition 8(a) and (c), is non-increasing in π;

(b) the optimal testing threshold τ according to Propositions 1-2 and 8 is non-increasing in ω; and

(c) both the funder's and the firm's expected payoffs are non-decreasing in π and in ω.

Hence, when the firm values future rewards less, pull and insurance mechanisms are less effective. In such settings, the funder should rely primarily on push funding. Pull incentives may still be used when the disease burden is high and push support has already been maximized. Insurance should not be used. As in the baseline model, if the disease burden is low or the commercial market is strong, the funder should not intervene.

The following remark clarifies the relationship between insurance and success-contingent repayment, as well as the role of the nonnegativity restriction on success-contingent transfers.

REMARK 1. Suppose the funder is allowed to collect a payment from the firm following success, so that m_S may take negative values. Then, without loss of optimality, attention can be restricted to mechanisms satisfying $m_F(\theta) = 0$ for all $\theta \in [0,1]$. To see this, consider any feasible mechanism

$$\{q(\cdot), m_0(\cdot), m_S(\cdot), m_F(\cdot)\}$$

and define

$$\tilde{m}_0(\theta) = m_0(\theta) + \delta\, m_F(\theta), \qquad \tilde{m}_S(\theta) = m_S(\theta) - m_F(\theta), \qquad \tilde{m}_F(\theta) = 0.$$

The transformed mechanism

$$\tilde{\mathcal{M}} \equiv \{q(\cdot), \tilde{m}_0(\cdot), \tilde{m}_S(\cdot), 0\}$$

leaves the firm's payoff unchanged under both effort and shirking and therefore preserves all reporting, effort, and participation incentives. Moreover, the funder's payoff for type θ increases by

$$(1-\delta)\, m_F(\theta) \geq 0.$$

Thus, failure-contingent funding is weakly dominated by additional upfront funding combined with a success-contingent repayment. Appendix EC.10 formalizes these comparisons. Proposition EC.3 shows that under the discounted limited-liability condition $\tilde{m}_0(\theta) + \delta\, \tilde{m}_S(\theta) \geq 0$, the two payment representations are equivalent when $\delta = 1$; the $\tilde{\mathcal{M}}$ formulation weakly improves the funder's value when $\delta < 1$, strictly so if an optimal original mechanism uses insurance with positive probability. Corollary EC.1 further shows that an uncapped success-repayment scheme can strictly outperform failure-contingent funding in the interior less-insurance region while preserving the firm's option to reject the contract and self-fund testing. □

3.3. General Common-Prior Distributions

Now we study the model without Assumption 1. That is, we allow the common prior for drug efficacy to take general distributions. The corresponding optimal solution to (14) may be much more complex than the one illustrated in Sections 3.1 and 3.2. In particular, it may involve a multi-menu contract that is hard to implement and therefore less practically relevant. Therefore, in this section, we explore the optimal take-it-or-leave-it mechanisms. For any given testing threshold, we characterize the optimal funding levels in closed form. Our result indicates that under the optimal threshold, the funding levels have characteristics very similar to those of the solutions in Sections 3.1 and 3.2.

Recall the definition of the self-funding threshold $\bar{\tau}$ in (15). If a drug's success rate is higher than $\bar{\tau}$, the firm has sufficient incentive to test. Therefore, the funder only needs to evaluate any potential threshold in $\check{\Theta} \equiv [\theta_L,\ \min\{\bar{\tau}, \theta_H\}]$.

The optimization problem (14) with the following additional constraints computes the best take-it-or-leave-it offer for a given testing threshold $\check{\tau} \in \check{\Theta}$:

$$\begin{aligned} &q(\theta) = m_0(\theta) = m_S(\theta) = m_F(\theta) = 0,\ \forall \theta \in [\theta_L, \check{\tau}),\ \text{and} \\ &q(\theta) = 1,\ m_0(\theta) = \check{m}_0,\ m_S(\theta) = \check{m}_S,\ m_F(\theta) = \check{m}_F,\ \forall \theta \in [\check{\tau}, \theta_H], \end{aligned} \tag{19}$$

for some decision variables $\check{m}_0$, $\check{m}_S$, $\check{m}_F$. More precisely, we consider the following optimization problem,

$$u^{\mathrm{TH}}(\check{\tau}) \equiv \max_{\{\check{m}_0, \check{m}_S, \check{m}_F\} \in \bar{\Omega}(\check{\tau})} \int_{\theta_L}^{\theta_H} u_F(\theta)\, \mathrm{d}F(\theta), \tag{20}$$

where u_F follows (9), and the feasible set $\bar{\Omega}(\check{\tau})$ is determined by (IC), (IR), (8), (12), and (19).

Lemma 2. *For any $\check{\tau} \in \check{\Theta}$, we have*

$$u^{\mathrm{TH}}(\check{\tau}) = \max_{\{\check{m}_0, \check{m}_S, \check{m}_F\} \in \bar{\Omega}'(\check{\tau})} \int_{\check{\tau}}^{\theta_H} [\theta\, \omega - \check{m}_0 - \theta\, \check{m}_S - (1-\theta)\, \check{m}_F]\, \mathrm{d}F(\theta), \tag{21}$$

in which the feasible set $\bar{\Omega}'(\check{\tau})$ is defined by the following constraints:

$$\check{m}_S - \check{m}_F + \pi \geq 0, \tag{22}$$

$$\check{m}_0 + \delta\, \check{\tau}\, \check{m}_S + \delta\, (1-\check{\tau})\, \check{m}_F = c - \delta\, \pi\, \check{\tau}, \tag{23}$$

$$\check{m}_0 + \delta\, \check{m}_F \leq \alpha\, c,\ \textit{and} \tag{24}$$

$$\check{m}_0,\ \check{m}_S,\ \check{m}_F \geq 0. \tag{25}$$

The following proposition identifies the optimal solution to the linear optimization problem (21) in closed form.

Proposition 9. *Define functions*

$$\mathcal{G}(\check{\tau}) \equiv \alpha + [(1-\alpha)/\check{\tau} - 1/\bar{\tau}] \quad \textit{and} \quad \mathcal{H}(\check{\tau}) \equiv \delta\, (1-\check{\tau})\, [1 - F(\check{\tau})] - \int_{\check{\tau}}^{\theta_H} (1-\theta)\, \mathrm{d}F(\theta).$$

The following tuple $(\check{m}_0^*, \check{m}_S^*, \check{m}_F^*)$ is optimal for (20) as well as (21) for a given $\check{\tau} \in \check{\Theta}$:

1. *when* $\mathcal{G}(\check{\tau}) \geq \alpha$, *we have* $\check{m}_0^* = \alpha\, c$, $\check{m}_S^* = \pi\,[\mathcal{G}(\check{\tau}) - \alpha]\,\bar{\tau}$, *and* $\check{m}_F^* = 0$;
2. *when* $0 \leq \mathcal{G}(\check{\tau}) < \alpha$, *and* $\mathcal{H}(\check{\tau}) \geq 0$, *we have* $\check{m}_0^* = \mathcal{G}(\check{\tau})\, c$, $\check{m}_S^* = 0$, *and* $\check{m}_F^* = \pi\,[\alpha - \mathcal{G}(\check{\tau})]\,\bar{\tau}$;
3. *when* $\mathcal{G}(\check{\tau}) < \alpha$ *and* $\mathcal{H}(\check{\tau}) < 0$, *we have* $\check{m}_0^* = \delta\,\pi\,(\bar{\tau} - \check{\tau})$ *and* $\check{m}_S^* = \check{m}_F^* = 0$; *and*
4. *when* $\mathcal{G}(\check{\tau}) < 0$ *and* $\mathcal{H}(\check{\tau}) \geq 0$, *we have* $\check{m}_0^* = \check{m}_S^* = 0$ *and* $\check{m}_F^* = \pi\,(\bar{\tau} - \check{\tau})/(1 - \check{\tau})$.

Note that under Assumption 1 (θ is uniformly distributed on $[0, 1]$), we have

$$\mathcal{H}(\check{\tau}) = \left(\delta - \frac{1}{2}\right)(1 - \check{\tau})^2 \geq 0,\ \forall \delta \geq \frac{1}{2}.$$

This implies that Case 3 of Proposition 9, which requires $\mathcal{H}(\check{\tau}) < 0$, does not arise in the optimal solution under Assumption 1 and $\delta \geq 1/2$. In fact, the condition $\mathcal{G}(\check{\tau}) < \alpha$ in Case 3 directly implies that the corresponding $\check{m}_0^* < \alpha\, c$. That is, the funder only uses push funding at a level strictly less than $\alpha\, c$, similar to case (b) of Proposition 8.

In order to obtain the best take-it-or-leave-it offer, we use the optimal solution described in Proposition 9 together with a line search for the optimal threshold value $\check{\tau}$. Figure EC.1 in Appendix EC.1 displays the optimal threshold policy when the firm's private information follows the Beta distributions as depicted in Figure EC.1(a). Regions (1)–(7) in panels (b)-(f) correspond to the optimal solutions defined in Propositions 1–7 (and hence regions (1)-(7) in Figure 1), respectively. Consistent with the previous section, we use (3') to represent Case 3 of Proposition 9, which involves only an upfront payment less than $\alpha\, c$.

As shown in the figure, the overall structure of the parameter regions where different optimal solutions arise largely mirrors that of the uniform distribution case. We further present Figure EC.2 in Appendix EC.1, which resembles Figure 2 for the uniform distribution case. This figure illustrates that as the commercial profit π increases, policy structures (1) to (7) become optimal in sequence, consistent with the uniform distribution case. Between (3) and (4), however, it may be optimal to provide only push funding less than $\alpha\, c$, as illustrated in (3'). The optimal testing threshold τ decreases in π in panel (b).

Overall, the general structure of the optimal solution established in Sections 3.1 and 3.2 for the uniformly distributed prior appears robust across general prior distributions. In particular, the best take-it-or-leave-it mechanism for general nonuniform distributions takes only the forms of (1)-(7) from Section 3.1 or (3') from Section 3.2. Finally, Figure EC.6 of Appendix EC.12 illustrates that even though a simple take-it-or-leave-it offer may not always be optimal, it consistently performs within 99% of the optimal value.

Appendix EC.13 further considers a distributionally robust variant of the model in which the funder knows only the mean of the success probability. Our analysis shows that the robust take-it-or-leave-it problem can be reduced to two one-dimensional optimization problems.

4. Benchmark Comparisons

To illustrate the roles of adverse selection and moral hazard, we examine counterfactual scenarios in which each element is absent. In Sections 4.1 and 4.2, we characterize the optimal mechanism when either moral hazard or adverse selection is removed. In Appendix EC.14, we examine the benchmark with neither adverse selection nor moral hazard.

4.1. No Moral Hazard

Here we consider a benchmark in which effort is observable and contractible, so deviations involving shirking are unavailable. Lemma EC.3 in Appendix EC.6 formally establishes the intuitive result that this problem is equivalent to setting $\alpha = 1$ in problem (14). Consequently, the following result follows from Propositions 1–8.

COROLLARY 3. *Assuming $\alpha = 1$, it is optimal not to offer any pull funding. That is, $m_S^*(\theta) = 0$ for all $\theta \in [0,1]$. Furthermore, we summarize the optimal solution q^*, m_0^*, and m_F^* to (14) in the following cases:*

(a) If $\omega + \delta\pi \le c$, which satisfies the condition of Proposition 1, it is optimal not to test the drug.

(b) Suppose $\omega + \delta\pi > c$ and $\delta \ge 1/2$.

(i) If $\delta\pi \le c$, this corresponds to Proposition 4, with a testing threshold $\tau = 1 - \dfrac{\omega + \delta\pi - c}{\omega + \pi} \in (0,1)$.

(ii) If $\dfrac{\delta\pi}{1 + 2\omega/\pi} < c < \delta\pi$, this corresponds to Proposition 6, with a testing threshold $\tau = \dfrac{1 + \bar{\tau}}{2(\omega/\pi + 1)}$.

(iii) If $0 \le c \le \dfrac{\delta\pi}{1 + 2\omega/\pi}$, this corresponds to Proposition 7, which implies letting the commercial market take over.

(c) Suppose $\omega + \delta\pi > c$ and $\delta < 1/2$.

(i) If $\dfrac{\delta\pi}{1 + \omega/(\delta\pi)} \le c$, this corresponds to Proposition 8(b), with a testing threshold $\tau = \dfrac{1 + \bar{\tau}}{\omega/(\delta\pi) + 2}$.

(ii) If $0 \le c < \dfrac{\delta\pi}{1 + \omega/(\delta\pi)}$, this corresponds to Proposition 8(c).

According to Corollary 3, Regions (2), (3), and (5) in Figures 1 and EC.3 vanish when there is no moral hazard. In the remaining regions, Cases (b)(i), (b)(ii), and (c)(i) of Corollary 3 require funding for drug development. The first two cases involve rewarding failure ($m_F > 0$). Hence, rewarding failure is a robust strategy even if the funder is not concerned with moral hazard. This is because when effort is observable, the funder does not need to reward success, and funding for failure helps cover testing costs.

More specifically, in Case (b)(i), the reward for failure is exactly the profit π. Therefore, as long as θ is above the threshold τ, the firm's expected payoff does not depend on the test outcome. This highlights the fact that the reward for failure serves as "insurance." In fact, the present value of the total reward exactly equals the testing cost c. That is, the funder extracts all the surplus, and the firm receives no information rent.

The situation is different for Cases (b)(ii) and (c)(i). For the drug with boundary type $\theta = \tau$, the firm's expected net payoff equals zero. Any drug with θ above the threshold τ receives a positive information rent. Overall, the different cases, especially Case (b)(i), imply that the model, even without moral hazard, does not reduce to the standard screening problem studied in Myerson (1981).

Without moral hazard, the funder can contract directly on effort, so pull funding is unnecessary; it instead relies on push funding, insurance, or both. In Case (b)(i), as the profit potential π increases, push funding decreases and drops to zero when $\pi = c/\delta$, whereas insurance equals π and therefore increases. In Case (b)(ii), insurance then declines and becomes zero as π increases, at which point the mechanism transitions to Case (b)(iii). In Case (c), the amount of push funding decreases to zero at the boundary between Cases (c)(i) and (c)(ii).

4.2. No Adverse Selection

Here, we consider a second-best benchmark with moral hazard, but without adverse selection. The success probability θ is therefore public information.

This analysis highlights the value of information. Comparing outcomes in this setting to the baseline model in Section 3 quantifies the value to the funder of learning about whether the firm is likely to succeed with the drug.

In this setting, the value θ is fixed, and hence the decision variables q, m_0, m_S, and m_F are scalars instead of functions. The incentive constraint (IC) is reduced to

$$-c\,q + \delta\,\theta\,(m_S + \pi\,q) + \delta\,(1-\theta)\,m_F \geq -\alpha\,c\,q + \delta\,m_F. \tag{26}$$

The corresponding funder's problem is formulated as follows:

$$\max_{\{q,m_0,m_S,m_F\}\in\Omega_{\mathrm{SB}}} \theta\,\omega\,q - m_0 - \theta\,m_S - (1-\theta)\,m_F, \tag{27}$$

where the feasible set Ω_{SB} is determined by (26) and the following linear inequality constraints:

$$-c\,q + m_0 + \delta\,\theta\,(m_S + \pi\,q) + \delta\,(1-\theta)\,m_F \geq 0, \tag{28}$$

$$0 \leq q \leq 1, \text{ and} \tag{29}$$

$$m_0,\ m_S,\ m_F \geq 0. \tag{30}$$

The next proposition characterizes the funder's optimal mechanism according to (27).

PROPOSITION 10. *The following four cases summarize the optimal solution to the funder's problem without adverse selection* (27)*:*

(a) if $\theta \in [0,\ \max\{c/(\omega+\delta\pi), (\delta\alpha-\alpha+1)\,c/(\delta(\omega+\pi))\})$*, it is optimal not to induce testing and provide no funding, i.e.,*

$$q = m_0 = m_S = m_F = 0;$$

(b) if $\theta \in [(\delta\alpha-\alpha+1)\,c/(\delta(\omega+\pi)),\ (1-\alpha)\,\bar{\tau})$*, it is optimal to induce testing and provide a combination of push and pull funding with an expected present value of* $c-\delta\pi\theta$*, i.e.,*

$$q = 1, \quad m_0 = \alpha c, \quad m_S = \frac{(1-\alpha)\,c}{\delta\theta} - \pi, \quad m_F = 0;$$

(c) if $\theta \in [\max\{c/(\omega+\delta\pi), (1-\alpha)\,\bar{\tau}\},\ \bar{\tau})$*, it is optimal to induce testing and provide push funding of amount* $c-\delta\pi\theta$*, i.e.,*

$$q = 1, \quad m_0 = c-\delta\pi\theta, \quad m_S = m_F = 0;\ \textit{and}$$

(d) if $\theta \in [\bar{\tau}, 1]$*, the firm is willing to test without the funder, i.e.,*

$$q = 1, \quad m_0 = m_S = m_F = 0.$$

Note that the condition $c/(\omega+\delta\pi) > (\delta\alpha-\alpha+1)\,c/(\delta(\omega+\pi))$ in Case (a) is equivalent to $c/(\omega+\delta\pi) > (1-\alpha)\,\bar{\tau}$ in Case (c), which further implies that $(\delta\alpha-\alpha+1)\,c/(\delta(\omega+\pi)) < (1-\alpha)\,\bar{\tau}$, or, the disappearance of Case (b). Therefore, the intervals for θ in the cases presented in Proposition 10 are indeed mutually exclusive and collectively exhaustive.

To summarize, when the success rate θ is public information, the funder should not offer any funding if the drug is *ex ante* very ineffective (Case (a)), and let the commercial market take over if the drug is sufficiently promising (Case (d)). For intermediate levels of θ, the funder should rely on either push only (Case (c)) or pull plus push (Case (b)).

It is also worth noting that in both Cases (b) and (c), the present value of the firm's expected inflow is $m_0 + \delta\theta(m_S+\pi) = c$, which equals the testing cost. In Case (b), the drug is less promising *ex ante*, which implies that the expected profit from potential success is not high enough for the firm to exert effort. Therefore, the funder needs to provide some pull ($m_S > 0$) to guarantee that the incentive constraint (26) is satisfied. Indeed, this constraint binds at the optimum. In Case (c), on the other hand, the probability of success θ is higher, and the expected profit from success is sufficiently high so that the funder no longer needs to provide additional reward for success. In this case, the constraint (26) is not even binding at optimality. In both cases, the push funding ($m_0 > 0$) is to guarantee the firm's participation.

Without adverse selection, the funder has no reason to provide insurance ($m_F = 0$). Insurance is used to counteract the firm's incentive to underreport the drug's probability of success, but in this case, the funder already knows the true probability.

5. Calibration

We calibrate the model using parameter values drawn from the literature on drug development and global health. This section describes our calibration methods and presents the resulting funding recommendations.

5.1. Calibration Methods

We examine whether support is needed to induce development and whether development is worth its expected social cost. The first depends on testing costs, success probabilities, discounting, and profit. The second depends on disease burden and the value of reducing it.

We specify seven key parameters: testing cost, disease burden, value of life, discount factor, success probability, relative shirking cost, and profit. These values illustrate how the model applies in practice. While some parameters are held constant across diseases for simplicity, the framework allows for disease-specific variation.

Testing cost. We assume a Phase III testing cost of either \$100 million or \$400 million (Table 1). The lower estimate is based on DNDi reports citing €60–190 million for a new chemical entity (Drugs for Neglected Diseases initiative 2019). The upper bound reflects Phase III out-of-pocket costs ranging from \$126 million to \$628 million (Wouters et al. 2020).[11] We incorporate capital costs and failure rates separately, bringing the fully capitalized cost per approved product above \$1 billion.

Shirking. If the firm shirks, it still incurs 70% of testing costs ($\alpha = 0.7$), reflecting unavoidable trial costs, such as enrolling patients but reducing staff quality or oversight. We test robustness to $\alpha = 0.35$ and $\alpha = 1$ (no moral hazard). We also examine policies in which the funder is uncertain about α and seeks to maximize its worst-case payoff (Appendix EC.15).

Disease burden. We use 2021 World Health Organization (WHO) data on disability-adjusted life years (DALYs) lost by income group (World Health Organization 2024), focusing on infectious and parasitic diseases. We exclude category-level entries (e.g., "Other STDs") and diseases with highly effective, low-cost vaccines (e.g., measles). Countries are classified as high-income (GNI per capita $\geq$ \$14,000) or otherwise. DALYs are denoted as DALY_h (high-income) and DALY_l (low- and middle-income). We also report whether the disease is classified as neglected (Table EC.1). We assume each successful product reduces disease-specific DALYs by 10%, reflecting modest market share and effectiveness. For example, Naci et al. (2025) report that the median drug adds half a year of full health; if used by 20% of patients, this implies a 10% burden reduction. In 2015, anti-malarials reduced burden by 16% (Artemether-Lumefantrine), 12% (Artesunate-Amodiaquine), and 1% (Artesunate-Sulfadoxine-Primaquine) (Hassoun 2015).

[11] These align with Mulcahy et al. (2025), who estimate total out-of-pocket costs of \$360 million across all phases. If Phase III represents two-thirds, that implies \$240 million—consistent with our range.

The baseline model studies a single product. However, the model can be interpreted as the decision problem for an additional drug, provided that the products are developed sequentially. With simultaneous development, the same interpretation applies under three conditions. First, funding one candidate must have little effect on the funder's overall budget. This is plausible for large funders when support for a single product is small relative to total funding. Second, one product must not greatly reduce the value of another. Our 10% burden-reduction assumption helps address this concern because it leaves room for several differentiated products to create social value. Third, one product must not greatly reduce another product's expected profit. For many diseases, expected profit is negligible, so competition has little effect. For HIV/AIDS, expected profit is high, but the model recommends no late-stage funding. Tuberculosis is the main case where competition may matter, although differentiated antimicrobials can retain high marginal value because new mechanisms help combat drug resistance.

Value of life. For low- and middle-income countries, we assume a value of \$2,000 per DALY saved ($\omega_{\mathrm{l}}/\mathrm{DALY}_{\mathrm{l}}$), based on income level (Kazibwe et al. 2022). Per capita income is \$1,700 in sub-Saharan Africa and \$2,400 in South Asia (World Bank 2025). We test robustness with \$6,000. For high-income countries, we assume \$12,000 per DALY saved ($\omega_{\mathrm{h}}/\mathrm{DALY}_{\mathrm{h}}$), equal to one-quarter of per capita income (\$48,000) (World Bank 2025), with the remainder attributed to profit or variable costs (Figure EC.4).

Discount factor. We assume six years of testing without revenue, followed by ten years of sales. With a 10.5% annual discount rate (DiMasi et al. 2016, Wouters et al. 2024), this implies a discount factor of 0.55 for payments received at the time of approval, with smaller weights applied to later profits. In practice, the discount factor may be higher if the funder reduces the firm's risk or if Phase III testing lasts only four years (DiMasi et al. 2016). We test robustness to both higher and lower discount factors.

Success probability. We assume a uniform prior and test robustness using alternative Beta distributions.

Profit. In low-income countries, we assume zero profit ($\pi_{\mathrm{l}}/\mathrm{DALY}_{\mathrm{l}} = 0$), reflecting cost-based pricing. For example, Coartem and Sirturo are both sold for under \$1 per day. In high-income countries, we assume profit of \$24,000 per DALY averted ($\pi_{\mathrm{h}}/\mathrm{DALY}_{\mathrm{h}}$), about 62% of the United Kingdom's \$39,000 willingness-to-pay threshold. The remaining 38% covers production and operating expenses, consistent with Wouters et al. (2024), who estimate such costs at 25–60%. This profit estimate also aligns with income-based valuation, as it represents about half of average per capita income in high-income countries (\$48,000), with the rest attributed to costs and consumer surplus (Figure EC.4). We round up to \$1 million if the present value of profit is lower.

Notation	Description	Main	Robust
c	Testing cost	\$400 million	\$100 million
α	Relative cost of shirking	70%	35%
$\omega_{\mathrm{l}}/\mathrm{DALY}_{\mathrm{l}}$	Value per DALY for low- and middle-income countries	\$2,000	\$6,000
$\omega_{\mathrm{h}}/\mathrm{DALY}_{\mathrm{h}}$	Value per DALY for high-income countries	\$12,000	\$12,000
$\pi_{\mathrm{l}}/\mathrm{DALY}_{\mathrm{l}}$	Profit per DALY for low- and middle-income countries	\$0	\$0
$\pi_{\mathrm{h}}/\mathrm{DALY}_{\mathrm{h}}$	Profit per DALY for high-income countries	\$24,000	\$24,000
share	Share of profit and DALYs per product	10%	10%
$\hat{\delta}$	Annual discount rate for the firm	10.5%	10.5%
T	Time until profit	6 years	4 years
δ	Firm discount factor, $(1+\hat{\delta})^{-T}$	0.55	0.67
θ	Probability of technical success	$\mathcal{U}_{[0,1]}$	Various
π	Profit	Equation (31)	Equation (31)
ω	Consumer welfare	Equation (32)	Equation (32)
	Time on the market	10 years	10 years

Table 1 Model parameters and their assumed values used in the main analysis and robustness check. Disease burden is measured in disability-adjusted life years (DALYs), as reported by the World Health Organization.

We calculate the present value of profit per drug, π, at the beginning of the 10-year profit period as follows:

$$\pi = \sum_{t=1}^{10} \frac{\text{share} \times \mathrm{DALY}_{\mathrm{h}} \times (\pi_{\mathrm{h}}/\mathrm{DALY}_{\mathrm{h}})}{(1+\hat{\delta})^{t-1}}. \tag{31}$$

We calculate the present value of consumer welfare by summing up ten years with no discount:

$$\omega = 10 \times \text{share} \times [\mathrm{DALY}_{\mathrm{l}} \times (\omega_{\mathrm{l}}/\mathrm{DALY}_{\mathrm{l}}) + \mathrm{DALY}_{\mathrm{h}} \times (\omega_{\mathrm{h}}/\mathrm{DALY}_{\mathrm{h}})]. \tag{32}$$

Based on these parameters, we determine the optimal funding mechanism according to Propositions 1-9.

5.2. Calibration Results

We focus on three high-burden communicable diseases: tuberculosis, malaria, and HIV/AIDS. Although they impose similar global burdens, they require different policy responses. For example, tuberculosis and malaria each caused over 50 million life-years lost in low- and middle-income countries in 2021. However, their burden in high-income countries differs by two orders of magnitude (Table EC.1). These differences influence commercial potential and lead to distinct optimal funding strategies.

Tuberculosis. The optimal policy is insurance: reimbursing a share of testing costs if the product fails. This is because commercial rewards are sufficient for success, given the high burden in high-income countries. If testing costs are very high (e.g., \$2 billion; see Table EC.5) or the firm heavily discounts future revenue ($\delta < 1/2$), the policy shifts to push. If testing costs are low, the funder should not invest.

Malaria. The optimal policy is pull funding on the order of a billion dollars supplemented by push funding that covers a share of testing costs (Table 2, Figure 3). This recommendation is robust to changes in key parameters, including a higher value of life, a lower cost share under shirking (Figure 4), a discount factor below 0.5 (Table EC.2), alternative priors on success probability (Tables EC.3 and EC.4), and a wide range of testing costs (Table EC.5). In particular, pull remains the preferred strategy when Phase III testing costs range from $1 million to $5 billion. For a drug that reduces malaria burden by 10% and requires $100 million for Phase III testing, our model recommends a pull reward of roughly $1 billion. The prize should increase as testing costs rise (Table 2). Even at this lower bound, the optimal pull commitment for malaria far exceeds current practice. Priority review vouchers sold for $200 million in 2026 (Ridley and Xu 2026), an order of magnitude below the optimal level for a malaria drug.

HIV/AIDS. The optimal policy is no funding for Phase III testing, because commercial incentives are sufficient. A product that reduces 10% of the HIV/AIDS burden in high-income countries is expected to generate $18 billion in present-value profit. This aligns with real-world data; for example, Biktarvy generated nearly $13 billion in sales in 2023.[12] If testing costs are under $3.3 billion, public funding is unnecessary. If costs exceed that threshold, the funder should offer insurance to reimburse firms in case of failure (Table EC.5).

Communicable diseases with moderate market potential. For acute hepatitis A, chlamydia, cysticercosis, echinococcosis, food-borne trematodes, and trichomoniasis, insurance is optimal when testing costs are $100 million. For acute hepatitis A, genital herpes, trichomoniasis, and tuberculosis, insurance is optimal even at $400 million. Insurance is recommended for diseases like these with moderate markets, unless there is no adverse selection. In that case, the funder offers a mix of push and pull (Table EC.6).

High-burden tropical diseases with no profit potential. For dengue, lymphatic filariasis, malaria, onchocerciasis, and schistosomiasis, the optimal policy consists of pull funding above $100 million, combined with push funding to cover a portion of testing costs. This result is robust to variation in testing costs, the value of life, time-to-market, the severity of moral hazard, and assumptions about the probability of success. Across different priors, including Beta distributions and a uniform prior, policy recommendations remain consistent (Table EC.3). For these diseases, push funding plus priority review vouchers might be sufficient, if testing costs are sufficiently low.

Low-burden tropical diseases with no profit potential. Blinding trachoma, leprosy, trichuriasis, and trypanosomiasis also lack commercial potential but have smaller disease burdens. These diseases merit public funding only if testing costs are relatively low. If the expected welfare gains do not justify the costs, the optimal policy is not to invest (Table 2).

	Testing cost 400				Testing cost 100			
	Policy	**Push**	**Pull**	**Ins.**	**Policy**	**Push**	**Pull**	**Ins.**
Leprosy	(1) Nothing	0	0	0	(1) Nothing	0	0	0
Gonorrhoea		0	0	0	(2) Pull+push	70	12	0
Trachoma		0	0	0		70	83	0
Trichuriasis		0	0	0		70	104	0
Trypanosomiasis		0	0	0		70	59	0
Leishmaniasis	(2) Pull+push	280	285	0		70	15	0
Rabies		280	588	0		70	163	0
Dengue		280	516	0		70	158	0
Schistosomiasis		280	565	0		70	201	0
Whooping cough		280	902	0		70	343	0
Lymphatic filariasis		280	505	0		70	195	0
Malaria		280	2083	0		70	809	0
Yellow fever		280	330	0		70	155	0
Ascariasis		280	394	0		70	180	0
Hookworm disease		280	348	0		70	165	0
Onchocerciasis		280	540	0		70	236	0
Syphilis		280	547	0	(3) Push	70	0	0
Chagas disease		280	107	0		70	0	0
Acute hepatitis E		280	86	0	(4) Push+insure	60	0	18
Chlamydia		280	58	0		49	0	37
Trematodes		280	12	0	(5) Insure	0	0	127
Cysticercosis	(3) Push	280	0	0	(6) Insure less	0	0	105
Echinococcosis		280	0	0		0	0	102
Acute hepatitis A	(4) Push+insure	214	0	120		0	0	47
Trichomoniasis		218	0	113		0	0	46
Genital Herpes		148	0	240	(7) Commercial	0	0	0
Tuberculosis	(6) Insure less	0	0	243		0	0	0
Acute hepatitis C	(7) Commercial	0	0	0		0	0	0
HIV/AIDS		0	0	0		0	0	0

Table 2 Optimal policies depend on disease characteristics. Units are millions of dollars. "Ins." refers to the funder partially insuring the firm's losses. In this and subsequent tables, "trematodes" refers to food-borne trematodes. Source: authors' calculations using 2021 data from the World Health Organization on disability-adjusted life years lost.

The optimal funding policies remain stable for most diseases as we vary four key parameters: shirking, the value of a life-year, the firm's discount rate, and the distribution of the probability of success (Figure 4). Although these parameters influence the recommended funding amounts, the core policy prescriptions are largely unchanged. Moreover, the recommended strategies are a significant departure from current practice.

The robustness checks above assess the sensitivity of the recommended policies to alternative parameter assumptions. Appendix EC.16 considers a distinct implementation concern: the true parameters remain at their calibrated values, while the funder designs the mechanism using misspecified beliefs about one or two parameters. Across the displayed cases for malaria, tuberculosis, and HIV/AIDS, the resulting mechanism continues to outperform the corresponding full-push benchmark.

[12] Sales data are based on the authors' calculations using IQVIA MIDAS.

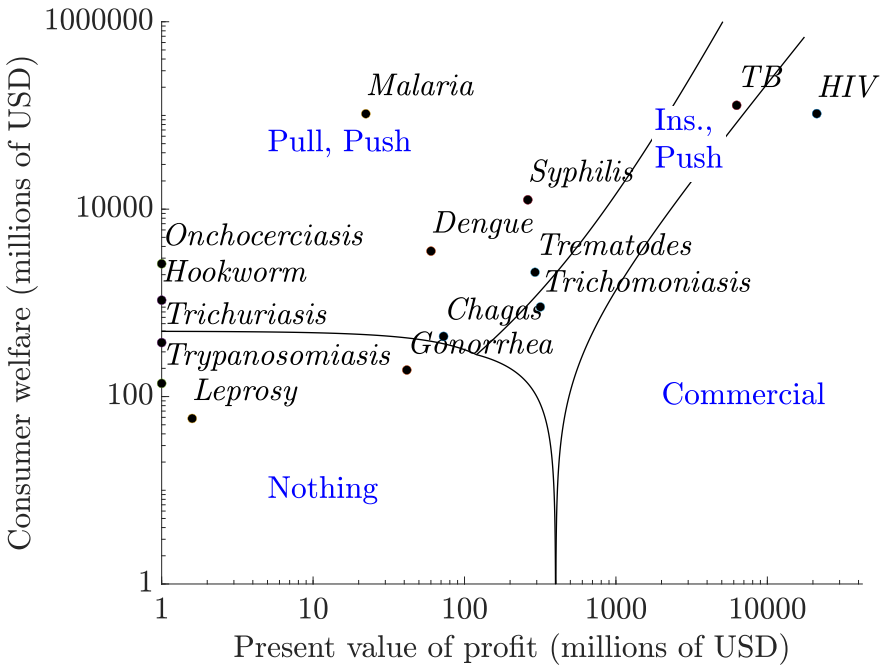


Figure 3 Optimal funding mechanisms for selected communicable diseases. The testing cost is $400 million. To reduce clutter, the figure combines Regions (2) and (3) (both involve some push) and Regions (4)–(6) (all involve insurance). Source: authors' calculations using 2021 data from the World Health Organization on disability-adjusted life years lost.

6. Conclusion

Our analysis shows how funders should optimally allocate three tools: pull funding, push funding, and insurance against failure. We focus on settings where commercial profits are insufficient to cover testing costs, but the social value of the investment is high. To address both moral hazard and adverse selection, we solve an optimization problem in which the funder chooses how to support each firm type. We use duality theory to characterize the optimal solution and derive explicit formulas for when to use each funding tool.

Pull funding, which rewards success, directly mitigates moral hazard and should be used more extensively. Push funding is useful when firms are less patient than funders, but should represent only a small share of total support—contrary to current practice.

Insurance mitigates adverse selection by discouraging firms from obtaining funding by understating a product's potential. However, both insurance and push funding are potentially vulnerable to moral hazard. To address this, the funder should cover only part of the testing costs through either mechanism. Capping support ensures that the firm retains "skin in the game" and remains motivated to exert effort. In addition, insurance should be used only when a viable commercial market exists, so firms have something to gain from success. As a result, insurance may be optimal for products treating acute hepatitis A, trichomoniasis, and tuberculosis, but not malaria.

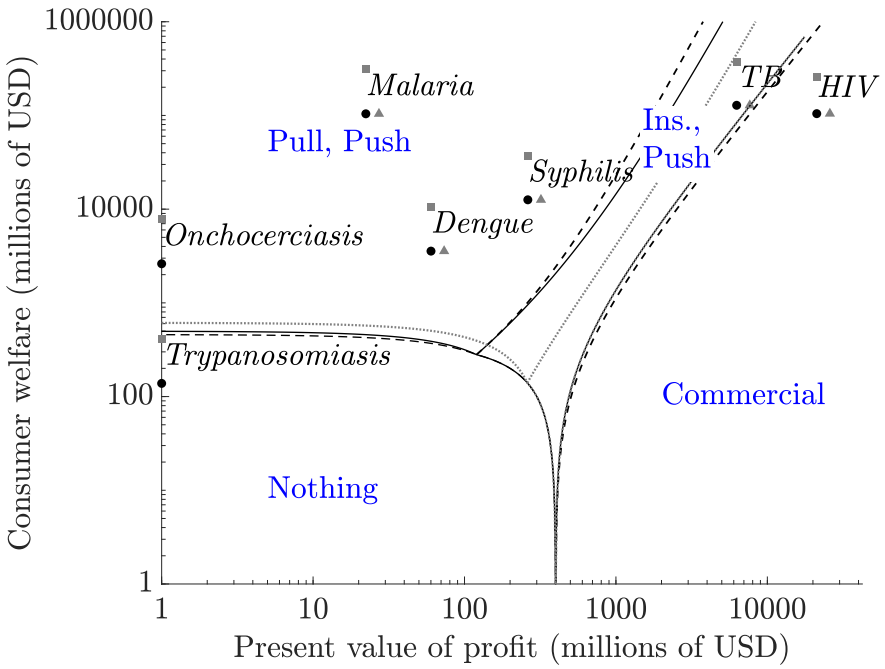


Figure 4 Optimal funding mechanisms for selected communicable diseases are robust to tripling the value of a life-year in low-income countries (gray squares), reducing the time until profit to 4 years (gray triangles and dashed lines), or halving the share of testing cost paid when shirking (dotted lines) relative to the baseline in Figure 3.

For malaria, the optimal policy is primarily pull funding, supplemented by limited push support. Recommended pull funding is on the order of a billion dollars, with larger amounts justified by low success probabilities or high testing costs. Pull funding ensures firms have a strong incentive to succeed despite limited commercial prospects. Optimal push funding is an order of magnitude smaller. An example of pull funding is the priority review voucher, but if the voucher value is $200 million per product (Ridley et al. 2021), then it is insufficient for malaria. Funders could enhance this by offering a "priority plus" reward—such as a prize or purchase guarantee, in addition to the voucher.

For tuberculosis, expected profits may be low due to limited market returns, low success probabilities, or high testing costs. In such cases, the funder should offer insurance or push funding. The same applies to diseases such as hepatitis A and trichomoniasis. When expected profits are high, no external funding is needed because the market can sustain development on its own.

For HIV/AIDS, commercial incentives are strong enough that no public funding for Phase III testing is required under plausible assumptions. Only if out-of-pocket testing costs are an order of magnitude higher does insurance become optimal. Governments and foundations spend more on HIV/AIDS R&D than on any other infectious or parasitic disease (Impact Global Health 2023). Although the results do not support late-stage funding for most HIV/AIDS drug development, they

do not rule out support for targeted products for low-income populations or procurement of existing HIV/AIDS treatments.

Funders should also consider whether a disease has an identified product need. The World Health Organization Target Product Profile Directory provides a useful screen. As of 2026, active World Health Organization profiles cover Chagas disease, dengue, genital herpes, gonorrhoea, HIV/AIDS, leishmaniasis, malaria, trypanosomiasis, and tuberculosis (World Health Organization 2026). For dengue, for example, the need is especially clear because no specific antiviral treatment is available. For malaria, drugs and vaccines already exist, but World Health Organization profiles call for safer chemoprevention, longer-lasting protection, and next-generation vaccines or monoclonal antibodies that prevent infection, reduce transmission, or provide stronger and more durable protection.

We prescribe optimal policies for late-stage testing, where the firm is within six years of potentially earning revenue under reasonable assumptions about the discount rate. Even when the timeline exceeds six years or the discount rate is higher, the policy recommendations for malaria and HIV/AIDS remain essentially unchanged. For tuberculosis, however, if the testing timeline is especially long, the model recommends shifting from deferred support to immediate support through push funding.

Our framework applies when innovation is costly and a welfare-focused funder can verify development costs, trials, and success. These conditions fit many global health products and may also hold in other innovation-intensive sectors. By combining push, pull, and insurance in a targeted way, funders can better align incentives, spur innovation, and improve social outcomes.

Acknowledgments

The authors are grateful for feedback from Kate Antrobus, Michael Kremer, Andrew Mulcahy, Christopher Snyder, and Jean Tirole. The authors thank seminar participants at Boston University, Cornell University, Duke Kunshan University, Duke University, the Gates Foundation, INSEAD, the Massachusetts Institute of Technology, the National University of Singapore, Soochow University, the University of California at Los Angeles, the University of Chicago, the University of Michigan–Ann Arbor, and the University of North Carolina at Chapel Hill, and conference participants at the Allied Social Science Associations Annual Meeting, CSAMSE 2026, Kellogg Operations Rookiepalooza 2026, the INFORMS Advances in Decision Analysis Conference, the INFORMS Annual Meeting, the INFORMS Healthcare Conference, and the Toulouse Health Economics Conference. David Ridley and Chenxi Xu received funding from the Gates Foundation (Investment ID INV-080080). The conclusions and opinions expressed in this work are those of the authors alone.

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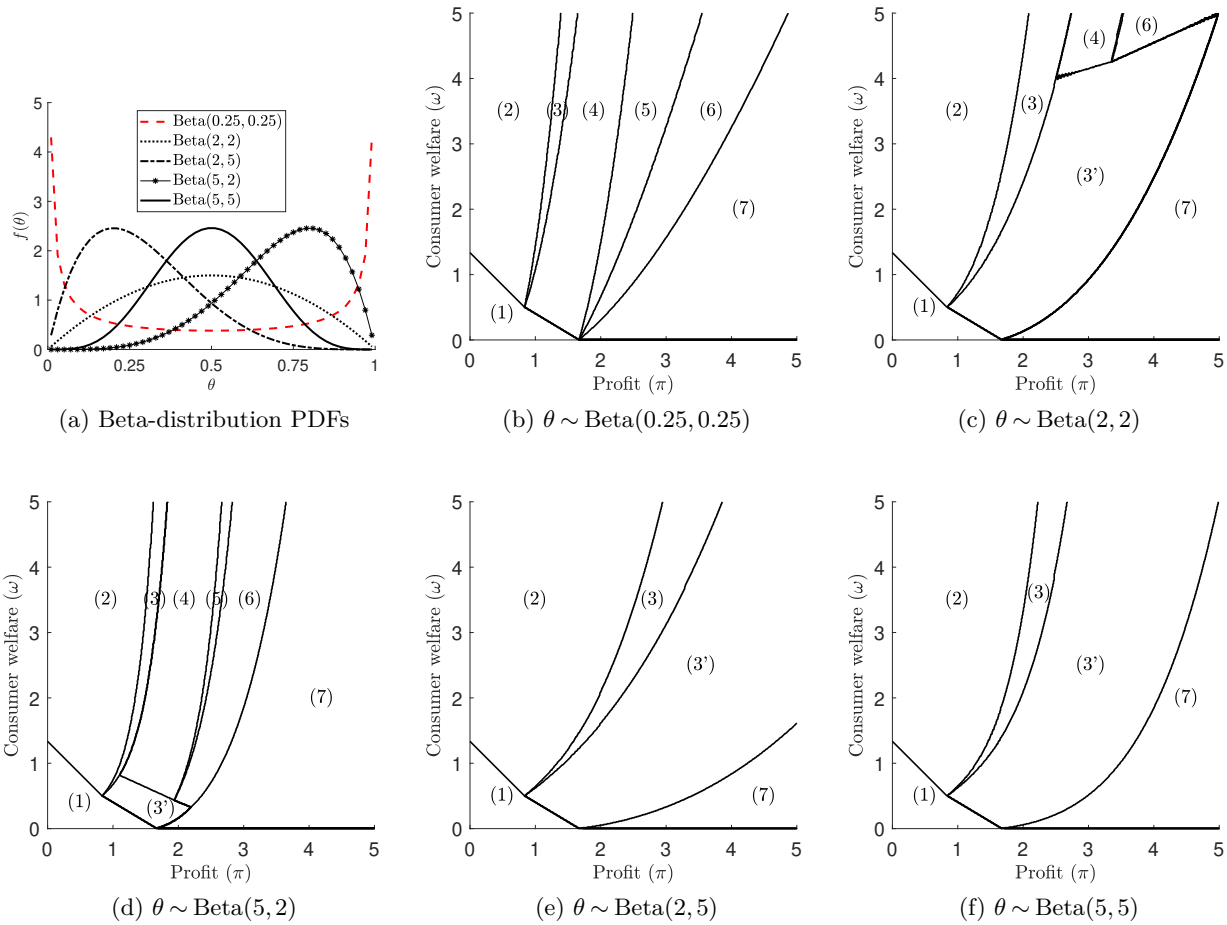
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EC.1. Additional Figures and Tables



(a) Beta-distribution PDFs (b) $\theta \sim \text{Beta}(0.25, 0.25)$ (c) $\theta \sim \text{Beta}(2, 2)$

(d) $\theta \sim \text{Beta}(5, 2)$ (e) $\theta \sim \text{Beta}(2, 5)$ (f) $\theta \sim \text{Beta}(5, 5)$

Figure EC.1 Optimal threshold policies when θ is not uniformly distributed ($\alpha = 0.5$, $c = 1$, and $\delta = 0.6$).

Disease	Type	Neglected	Annual DALYs (000) by income group High ($DALY_h$)	Low+Middle ($DALY_l$)
Acute hepatitis A	Viral	0	74	1792
Acute hepatitis C	Viral	0	206	342
Acute hepatitis E	Viral	0	13	219
Ascariasis	Parasitic	1	0	681
Chagas disease	Parasitic	1	8	170
Chlamydia	Bacterial	0	13	149
Cysticercosis	Parasitic	1	43	1063
Dengue	Viral	1	7	1742
Echinococcosis	Parasitic	1	34	535
Genital herpes	Viral	0	45	213
Gonorrhoea	Bacterial	0	5	68
HIV/AIDS	Viral	0	2419	37749
Hookworm disease	Parasitic	1	0	534
Leishmaniasis	Parasitic	1	13	857
Leprosy	Bacterial	1	0	28
Lymphatic filariasis	Parasitic	1	3	1320
Malaria	Parasitic	0	3	52046
Onchocerciasis	Parasitic	1	0	1307
Rabies	Viral	1	9	2588
Schistosomiasis	Parasitic	1	4	1851
Syphilis	Bacterial	0	30	6111
Trachoma	Bacterial	1	0	134
Trematodes	Parasitic	1	33	861
Trichomoniasis	Parasitic	0	36	236
Trichuriasis	Parasitic	1	0	188
Trypanosomiasis	Parasitic	1	0	69
Tuberculosis	Bacterial	0	716	59867
Whooping cough	Bacterial	0	3	5294
Yellow fever	Viral	0	0	495

Table EC.1 Harm from communicable diseases, measured in disability-adjusted life years (DALYs) lost. "Neglected" indicates whether the World Health Organization classifies the disease as a neglected tropical disease. Source: Authors' calculations using 2021 DALY data from the World Health Organization.

	Malaria	Tuberculosis	HIV	Trypanosomiasis
(1) Nothing	-	-	-	95 to 5,000
(2) Pull+push	1 to 5,000	2,475 to 5,000	-	1 to 95
(3) Push	-	390 to 2,475	-	-
(3') Less Push	-	109 to 390	1,404 to 5,000	-
(7) Commercial	-	1 to 109	1 to 1,404	-

Table EC.2 Critical values of testing costs (in millions of US dollars) for the case in which the discount factor is less than one half. Here, the discount rate is 16.5% and the time to approval is six years. Testing costs are assumed to range from $1 million to $5 billion.

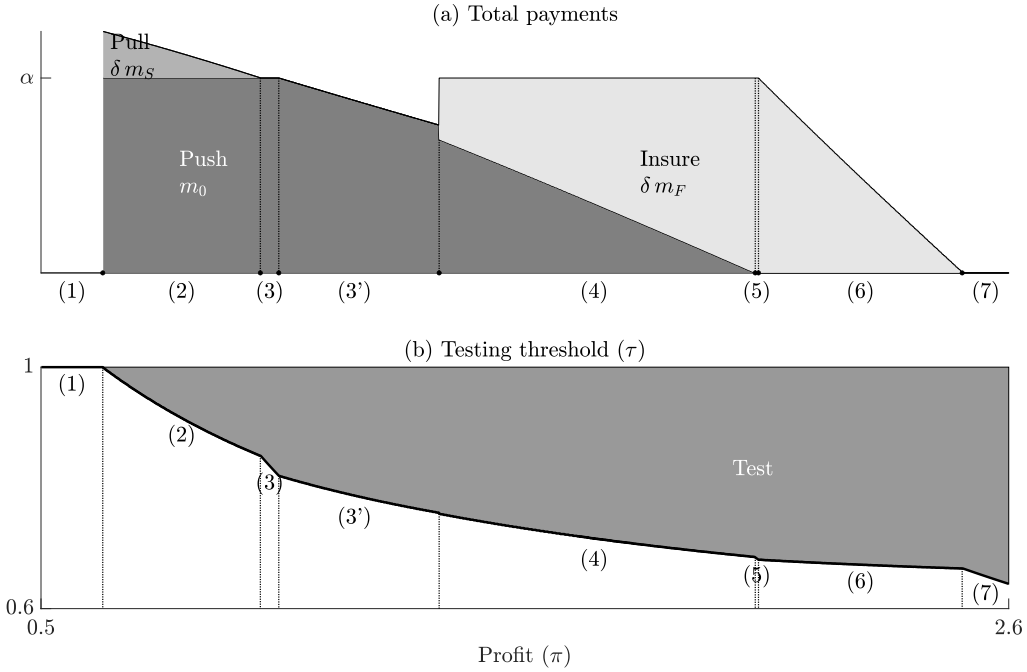


Figure EC.2 The total payment is the sum between the upfront payment m_0 and either payment upon success $\delta\, m_S$ or payment upon failure $\delta\, m_F$, which captures the maximum present value of payments ($\theta \sim$ Beta$(5,2)$, $\alpha = 0.5$, $c = 1$, $\delta = 0.6$, and $\omega = 0.7$).

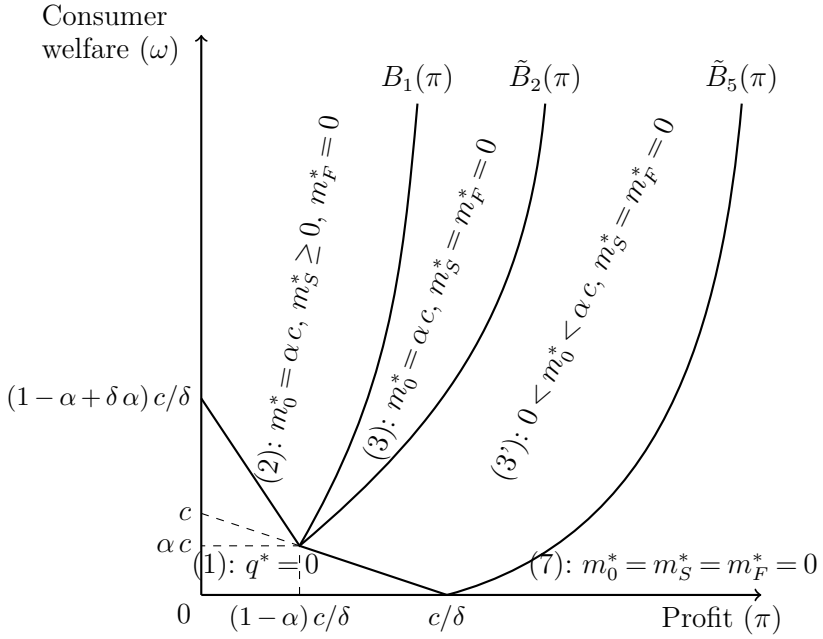


Figure EC.3 Optimal mechanisms under given profit π and consumer welfare ω if $\delta < 1/2$.

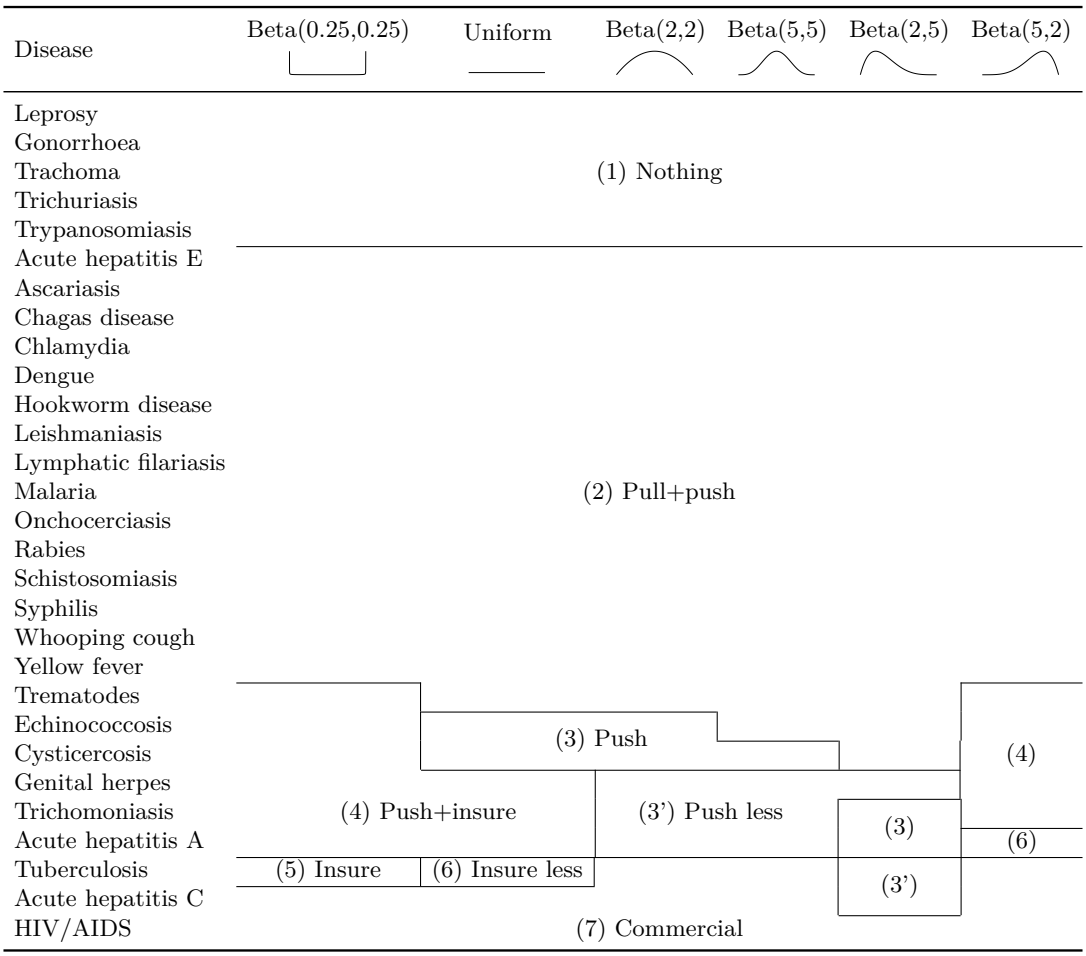


Table EC.3 Optimal policy identifiers for various distributions of the probability of success (θ).

	Malaria			**Tuberculosis**			**HIV**		
	Push	**Pull**	**Insure**	**Push**	**Pull**	**Insure**	**Push**	**Pull**	**Insure**
Baseline	280	2083	0	0	0	243	0	0	0
Testing cost (c): \$100MM	70	809	0	0	0	0	0	0	0
Life value (ω/DALY): \$6,000	280	3042	0	0	0	510	0	0	0
Cost of shirking (α): 0.35	140	3527	0	0	0	243	0	0	0
Time until profit: 4 years	280	1818	0	0	0	110	0	0	0
Mod. flat: Beta(2,2)	280	1780	0	0	0	0	0	0	0
Centered: Beta(5,5)	280	1271	0	0	0	0	0	0	0
Left skew: Beta(2,5)	280	2935	0	124	0	0	0	0	0
Right skew: Beta(5,2)	280	820	0	0	0	0	0	0	0
U-shape: Beta(0.25,0.25)	280	1909	0	0	0	510	0	0	0

Table EC.4 Funding mechanisms for malaria, tuberculosis, and HIV when changing a single assumption about the parameters or prior distributions.

	Malaria	Tuberculosis	HIV	Trypanosomiasis
(1) Nothing	-	-	-	111 to 5,000
(2) Pull+push	1 to 5,000	4,570 to 5,000	-	1 to 111
(3) Push	-	1,955 to 4,570	-	-
(4) Push+insure	-	1,500 to 1,955	-	-
(5) Insure	-	878 to 1,500	-	-
(6) Insure less	-	267 to 878	3,302 to 5,000	-
(7) Commercial	-	1 to 267	1 to 3,302	-

Table EC.5 Critical values of testing costs (millions of US dollars). Testing costs are assumed to range from $1 million to $5 billion.

	No moral hazard			No adverse selection		
	Push	Pull	Insure	Push	Pull	Insure
Gonorrhoea	-	-	-	-	-	-
Trachoma	-	-	-	-	-	-
Leprosy	-	-	-	-	-	-
Trichuriasis	-	-	-	-	-	-
Trypanosomiasis	-	-	-	-	-	-
Genital herpes	3	-	722	222	-	-
Cysticercosis	21	-	691	229	-	-
Trichomoniasis	83	-	577	257	-	-
Echinococcosis	98	-	549	264	-	-
Trematodes	108	-	532	268	-	-
Syphilis	138	-	477	280	9	-
Acute hepatitis E	283	-	212	-	-	-
Chlamydia	288	-	204	-	-	-
Leishmaniasis	290	-	201	280	285	-
Rabies	321	-	145	280	341	-
Chagas disease	327	-	132	-	-	-
Dengue	340	-	109	280	376	-
Schistosomiasis	361	-	70	280	415	-
Whooping cough	374	-	48	280	438	-
Lymphatic filariasis	376	-	44	280	441	-
Malaria	378	-	41	280	445	-
Yellow fever	396	-	7	-	-	-
Ascariasis	398	-	4	280	481	-
Hookworm disease	399	-	1	-	-	-
Onchocerciasis	400	-	-	280	485	-
Tuberculosis	-	-	243	-	-	-
Acute hepatitis A	-	-	638	110	-	-
Acute hepatitis C	-	-	-	-	-	-
HIV/AIDS	-	-	-	-	-	-

Table EC.6 Optimal policies when either moral hazard or adverse selection is absent. Without adverse selection, we assume that the probability of success θ is 0.45. Units are millions of dollars. Disease burden is measured in disability-adjusted life years (DALYs), as reported by the World Health Organization in 2021.

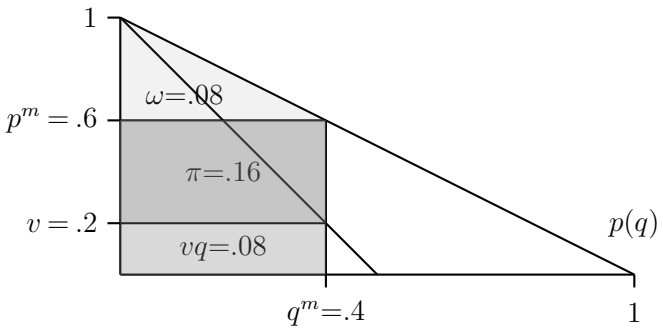


Figure EC.4 Assuming linear demand ($p(q)$), a monopoly price (p^m), and constant variable cost (v) set at 20% of the maximum willingness to pay, total surplus is split as follows: firm profit (π) is 50%, consumer welfare (ω) is 25%, and variable cost is the remainder.

EC.2. Proofs of Section 2

Proof of Lemma 1. For any feasible mechanism $\Xi \equiv \{q(\cdot), m_0(\cdot), m_S(\cdot), m_F(\cdot)\} \in \Omega$, we propose the following mechanism $\tilde{\Xi} \equiv \{\tilde{q}(\cdot), \tilde{m}_0(\cdot), \tilde{m}_S(\cdot), \tilde{m}_F(\cdot)\}$, where

$$\begin{aligned}
&\tilde{q}(\theta) = q(\theta), && \forall \theta \in \Theta,\\
&\tilde{m}_0(\theta) = m_0(\theta) + \delta \min\{m_S(\theta), m_F(\theta)\}, && \forall \theta \in \Theta,\\
&\tilde{m}_S(\theta) = m_S(\theta) - \min\{m_S(\theta), m_F(\theta)\}, && \forall \theta \in \Theta,\\
&\tilde{m}_F(\theta) = m_F(\theta) - \min\{m_S(\theta), m_F(\theta)\}, && \forall \theta \in \Theta.
\end{aligned}$$

With a slight abuse of notation, we use $\hat{u}_1(\theta, \hat{\theta}; \Xi)$ and $\hat{u}_0(\hat{\theta}; \Xi)$ to denote the firm's expected payoff when its true type is θ and it reports $\hat{\theta}$, and when it reports $\hat{\theta}$ and shirks, respectively, both under mechanism Ξ. It is straightforward to verify that the firm's payoff functions remain the same, i.e.,

$$\begin{aligned}
\hat{u}_1(\theta, \hat{\theta}; \tilde{\Xi}) =& -c\,\tilde{q}(\hat{\theta}) + \tilde{m}_0(\hat{\theta}) + \delta\,\theta\left[\tilde{m}_S(\hat{\theta}) + \pi\,\tilde{q}(\hat{\theta})\right] + \delta\,(1-\theta)\,\tilde{m}_F(\hat{\theta})\\
=& -c\,q(\hat{\theta}) + m_0(\hat{\theta}) + \delta\,\theta\left[m_S(\hat{\theta}) + \pi\,q(\hat{\theta})\right] + \delta\,(1-\theta)\,m_F(\hat{\theta})\\
=& \hat{u}_1(\theta, \hat{\theta}; \Xi)
\end{aligned}$$

and

$$\hat{u}_0(\hat{\theta}; \tilde{\Xi}) = -\alpha\,c\,\tilde{q}(\hat{\theta}) + \tilde{m}_0(\hat{\theta}) + \delta\,\tilde{m}_F(\hat{\theta}) = -\alpha\,c\,q(\hat{\theta}) + m_0(\hat{\theta}) + \delta\,m_F(\hat{\theta}) = \hat{u}_0(\hat{\theta}; \Xi).$$

By construction, we know that $\tilde{m}_0(\cdot)$, $\tilde{m}_S(\cdot)$, and $\tilde{m}_F(\cdot)$ are all nonnegative. Therefore, the proposed mechanism $\tilde{\Xi}$ is feasible.

It remains to verify that the funder's expected payoff under truth-telling is weakly higher, which follows from

$$\begin{aligned}
u_F(\theta; \tilde{\Xi}) &= \theta\,\omega\,\tilde{q}(\theta) - \tilde{m}_0(\theta) - \theta\,\tilde{m}_S(\theta) - (1-\theta)\,\tilde{m}_F(\theta)\\
&= \theta\,\omega\,q(\theta) - m_0(\theta) - \theta\,m_S(\theta) - (1-\theta)\,m_F(\theta) + (1-\delta)\min\{m_S(\theta), m_F(\theta)\}\\
&\geq u_F(\theta; \Xi).
\end{aligned}$$

This completes the proof. □

EC.3. Proofs of Section 3.1

By dividing the objective function and both sides of constraints (IC) and (IR) by c, we can, without loss of generality, normalize the model by setting $c = 1$ and replacing the parameters (c, ω, π) with $(1, \omega/c, \pi/c)$.

For notational simplicity, we set $c = 1$ for the remainder of the appendix. Accordingly, the optimality conditions and solutions presented in the main text apply with c replaced by 1. For instance, the boundary B_1 defined in equation (16) simplifies to

$$B_1(\pi) \equiv \frac{\delta^2 \pi^3 - (1-\alpha)(1-\alpha-2\delta\alpha)\pi}{2(1-\alpha)^2}.$$

We omit other resulting simplified expressions for brevity.

Reformulation of the funder's problem (14). We use $u(\theta)$ to denote the firm's expected payoff when it reports its type truthfully and exerts effort, i.e.,

$$u(\theta) \equiv u_1(\theta, \theta), \quad \forall \theta \in \Theta.$$

The (IC) constraint can be rewritten as two linear inequalities, i.e.,

$$u(\theta) \geq u_1(\theta, \hat{\theta}), \quad \forall (\theta, \hat{\theta}) \in \Theta^2 \tag{EC.1}$$

and

$$u(\theta) \geq u_0(\hat{\theta}), \quad \forall (\theta, \hat{\theta}) \in \Theta^2. \tag{EC.2}$$

The following lemma is standard in mechanism design and allows us to eliminate the decision variable m_0 (see Krishna (2009), Chapter 5).

Lemma EC.1. *The incentive constraint* (EC.1) *holds if and only if*

$$m_S(\cdot) - m_F(\cdot) + \pi\, q(\cdot) \text{ nondecreasing} \tag{EC.3}$$

and

$$m_0(\theta) = u(\theta_L) + \delta \int_{\theta_L}^{\theta} A(y)\,\mathrm{d}y + q(\theta) - \delta\,\theta\,[m_S(\theta) + \pi\, q(\theta)] - \delta\,(1-\theta)\,m_F(\theta). \tag{EC.4}$$

Proof of Lemma EC.1. First, we prove that constraint (EC.1) implies (EC.3) and (EC.4). Constraint (EC.1) can be rewritten as

$$u(\theta) \geq u(\hat{\theta}) + \delta\,(\theta - \hat{\theta})\,\left[m_S(\hat{\theta}) - m_F(\hat{\theta}) + \pi\, q(\hat{\theta})\right].$$

Reversing the role of θ and $\hat{\theta}$ yields

$$u(\hat{\theta}) \geq u(\theta) + \delta\,(\hat{\theta} - \theta)\,[m_S(\theta) - m_F(\theta) + \pi\, q(\theta)].$$

Combining the two inequalities above yields

$$\delta\,(\theta - \hat{\theta})\,\left[m_S(\hat{\theta}) - m_F(\hat{\theta}) + \pi\, q(\hat{\theta})\right] \leq u(\theta) - u(\hat{\theta}) \leq \delta\,(\theta - \hat{\theta})\,[m_S(\theta) - m_F(\theta) + \pi\, q(\theta)],$$

implying the monotonicity condition (EC.3).

Invoking the envelope theorem yields

$$u'(\theta) = \delta\,[m_S(\theta) - m_F(\theta) + \pi\, q(\theta)]\,,$$

which implies that

$$u(\theta) = u(\theta_L) + \delta \int_{\theta_L}^{\theta} A(y)\,\mathrm{d}y.$$

Solving for m_0 yields (EC.4).

To prove the other direction, without loss of generality, fix $0 \le \hat{\theta} < \theta \le 1$, and we have

$$\begin{aligned} u(\theta) =&\ u(\theta_L) + \delta \int_{\theta_L}^{\theta} A(y)\,\mathrm{d}y \\ =&\ u(\theta_L) + \delta \int_{\theta_L}^{\hat{\theta}} A(y)\,\mathrm{d}y + \delta \int_{\hat{\theta}}^{\theta} A(y)\,\mathrm{d}y \\ =&\ u(\hat{\theta}) + \delta \int_{\hat{\theta}}^{\theta} A(y)\,\mathrm{d}y \\ \ge&\ u(\hat{\theta}) + \delta\,(\theta - \hat{\theta})\,\left[m_S(\hat{\theta}) - m_F(\hat{\theta}) + \pi\, q(\hat{\theta})\right], \end{aligned}$$

where the last inequality follows from the monotonicity condition (EC.3).

This completes the proof. □

The "envelope" condition (EC.4) allows us to eliminate m_0 and rewrite the funder's problem in terms of the "virtual valuation" functions defined by

$$\begin{aligned} w_q(\theta) &\equiv \theta\,\omega + \delta\,\pi\,\theta - 1 - \delta\,\pi\,\frac{1-F(\theta)}{f(\theta)}, \\ w_S(\theta) &\equiv -(1-\delta)\,\theta - \delta\,\frac{1-F(\theta)}{f(\theta)}, \text{ and} \\ w_F(\theta) &\equiv -(1-\delta)\,(1-\theta) + \delta\,\frac{1-F(\theta)}{f(\theta)}. \end{aligned}$$

For notational simplicity, define

$$A(\theta) \equiv m_S(\theta) - m_F(\theta) + \pi\, q(\theta). \tag{EC.5}$$

The funder's problem (14) can now be reformulated as follows:

$$\max_{\{q(\cdot),m_S(\cdot),m_F(\cdot),u(\theta_L)\}\in\Omega_1} -u(\theta_L) + \int_{\theta_L}^{\theta_H} [w_q(\theta)\,q(\theta) + w_S(\theta)\,m_S(\theta) + w_F(\theta)\,m_F(\theta)]\,\mathrm{d}\theta, \tag{EC.6}$$

where the feasible set Ω_1 is defined by (8), (EC.3), and the following linear inequality constraints: for any $(\theta, \hat{\theta}) \in \Theta^2$,

$$\delta \int_{\theta_L}^{\theta} A(y)\,\mathrm{d}y \ge \delta \int_{\theta_L}^{\hat{\theta}} A(y)\,\mathrm{d}y + (1-\alpha)\,q(\hat{\theta}) - \delta\,\hat{\theta}\,\left[m_S(\hat{\theta}) + \pi\, q(\hat{\theta})\right] + \delta\,\hat{\theta}\,m_F(\hat{\theta}), \tag{EC.7}$$

$$u(\theta_L) + \delta \int_{\theta_L}^{\theta} A(y)\,\mathrm{d}y \ge 0, \tag{EC.8}$$

$$u(\theta_L) + \delta \int_{\theta_L}^{\theta} A(y)\,\mathrm{d}y + q(\theta) - \delta\,\theta\,[m_S(\theta) + \pi\, q(\theta)] - \delta\,(1-\theta)\,m_F(\theta) \ge 0, \text{ and} \tag{EC.9}$$

$$m_S(\theta),\ m_F(\theta) \ge 0. \tag{EC.10}$$

The inequalities (EC.7), (EC.8), and (EC.9) are the moral hazard constraint, the individual rationality constraint, and the feasibility constraint on m_0, respectively.

We next describe the dual feasible solutions used in the proofs under Assumption 1, where $\Theta = [0,1]$ and $f(\theta) = 1$. The argument does not rely on a general strong-duality theorem for infinite-dimensional linear programs. Instead, for each parameter region, we construct a feasible primal mechanism and a dual feasible solution whose induced upper bound coincides with the candidate objective value. Proposition 1 is proved by a direct upper-bound argument; the remaining cases use the dual feasible solutions summarized below.

Define the objective value of a feasible tuple by

$$J(q, m_S, m_F, u(0)) \equiv -u(0) + \int_0^1 [w_q(\theta)\, q(\theta) + w_S(\theta)\, m_S(\theta) + w_F(\theta)\, m_F(\theta)]\, \mathrm{d}\theta.$$

Define the residual associated with the moral-hazard constraint (EC.7) evaluated at the lowest type by

$$R^{\mathrm{MH}}(\theta) \equiv \delta \int_0^\theta A(y)\, \mathrm{d}y + (1-\alpha)\, q(\theta) - \delta\, \theta\, [m_S(\theta) + \pi\, q(\theta)] + \delta\, \theta\, m_F(\theta).$$

Since (EC.7) holds for every pair $(\theta, \hat{\theta})$, setting the true type equal to the lowest type implies $R^{\mathrm{MH}}(\theta) \leq 0$ for every $\theta \in [0,1]$. We also define

$$R^0(\theta) \equiv u(0) + \delta \int_0^\theta A(y)\, \mathrm{d}y + q(\theta) - \delta\, \theta\, [m_S(\theta) + \pi\, q(\theta)] - \delta\, (1-\theta)\, m_F(\theta).$$

The term $R^0(\theta)$ is the upfront payment $m_0(\theta)$ implied by the envelope formula (EC.4); hence feasibility requires $R^0(\theta) \geq 0$.

A dual feasible solution consists of nonnegative multipliers χ_L, $\eta(\theta)$, $\beta(\theta)$, $\sigma(\theta)$, $\gamma(\theta)$, $\xi(\theta)$, and $\nu(\theta)$, such that the following coefficient-matching identity holds for all admissible $(q, m_S, m_F, u(0))$:

$$\begin{aligned} J(q, m_S, m_F, u(0)) = &\int_0^1 \sigma(\theta)\, \mathrm{d}\theta + \chi_L[-u(0)] + \int_0^1 \eta(\theta)\, R^{\mathrm{MH}}(\theta)\, \mathrm{d}\theta \\ &+ \int_0^1 \beta(\theta)[-q(\theta)]\, \mathrm{d}\theta + \int_0^1 \sigma(\theta)[q(\theta) - 1]\, \mathrm{d}\theta + \int_0^1 \gamma(\theta)\ [-R^0(\theta)]\, \mathrm{d}\theta \\ &+ \int_0^1 \xi(\theta)[-m_S(\theta)]\, \mathrm{d}\theta + \int_0^1 \nu(\theta)[-m_F(\theta)]\, \mathrm{d}\theta. \end{aligned} \tag{EC.11}$$

For any feasible mechanism, all residuals on the right-hand side of (EC.11), when multiplied by their associated nonnegative multipliers, are nonpositive, except for the constant term $\int_0^1 \sigma(\theta)\, \mathrm{d}\theta$. Therefore,

$$J(q, m_S, m_F, u(0)) \leq \int_0^1 \sigma(\theta)\, \mathrm{d}\theta.$$

That is, to prove optimality of a candidate mechanism, it suffices to construct nonnegative multipliers satisfying (EC.11) and to verify that the candidate mechanism attains the upper bound $\int_0^1 \sigma(\theta)\, \mathrm{d}\theta$. The remaining moral-hazard and participation constraints, and the monotonicity condition on $A(\theta)$

Table EC.7 Primal constraints and dual multipliers in the weak-duality argument.

Primal condition	Residual/sign convention used in the weak-duality identity	Multiplier
Lowest-type participation	$-u(0) \le 0$	χ_L
Moral-hazard inequality with true type 0	$R^{\mathrm{MH}}(\theta) \le 0$	$\eta(\theta)$
Lower testing bound	$-q(\theta) \le 0$	$\beta(\theta)$
Upper testing bound	$q(\theta) - 1 \le 0$	$\sigma(\theta)$
Upfront-payment limited liability	$-R^0(\theta) \le 0$	$\gamma(\theta)$
Pull-payment limited liability	$-m_S(\theta) \le 0$	$\xi(\theta)$
Failure-payment limited liability	$-m_F(\theta) \le 0$	$\nu(\theta)$
Other participation and moral-hazard inequalities	All other instances of (EC.7) and (EC.8)	0
Monotonicity of $A(\theta)$	$A(\theta)$ nondecreasing	0

are assigned zero multipliers in the dual feasible solutions; they are nevertheless verified directly for each candidate primal mechanism. Table EC.7 summarizes the primal constraints and their associated dual multipliers.

The boundary functions $B_1(\pi), \ldots, B_5(\pi)$ can also be interpreted as switching loci between adjacent candidate mechanisms. Algebraically, they arise from the feasibility of the candidate primal mechanisms and nonnegativity of the corresponding dual multipliers. Each boundary is the point at which one funding instrument reaches a constraint, and the adjacent mechanisms coincide. At $B_1(\pi)$, the pull payment in Region (2) falls to zero, so the pull-plus-push mechanism coincides with the push-only mechanism in Region (3). At $B_2(\pi)$, the failure payment in Region (4) starts from zero, so the push-only mechanism in Region (3) coincides with the push-plus-insurance mechanism in Region (4). At $B_3(\pi)$, the upfront payment in Region (4) falls to zero, so the mechanism coincides with the insurance-only mechanism in Region (5). At $B_4(\pi)$, the failure payment in Region (6) reaches the cap $\alpha\, c/\delta$, matching the constant-insurance mechanism in Region (5). Finally, at $B_5(\pi)$, the failure payment in Region (6) falls to zero, and the mechanism coincides with the commercial self-funding mechanism in Region (7). Thus, the boundaries are precisely the loci at which the identity of the binding limited-liability or moral-hazard constraint changes.

Proof of Proposition 1. Notice that our candidate primal solution is always feasible.

For part (1), consider any feasible mechanism. For each type θ, let $u(\theta)$ denote the firm's expected payoff when it reports truthfully and exerts effort:

$$u(\theta) \equiv -q(\theta) + m_0(\theta) + \delta\,\theta\,[m_S(\theta) + \pi\, q(\theta)] + \delta\,(1-\theta)\, m_F(\theta).$$

By the individual rationality constraint, $u(\theta) \ge 0$. Solving the preceding expression for $m_0(\theta)$ gives

$$m_0(\theta) = u(\theta) + q(\theta) - \delta\,\theta\,[m_S(\theta) + \pi\, q(\theta)] - \delta\,(1-\theta)\, m_F(\theta).$$

Substituting this expression into the funder's payoff from type θ, we obtain

$$\begin{aligned} u_F(\theta) &= \theta\,\omega\,q(\theta) - m_0(\theta) - \theta\,m_S(\theta) - (1-\theta)\,m_F(\theta) \\ &= [\theta\,(\omega+\delta\,\pi) - 1]\,q(\theta) - u(\theta) - (1-\delta)\,[\theta\,m_S(\theta) + (1-\theta)\,m_F(\theta)] \\ &\leq [\theta\,(\omega+\delta\,\pi) - 1]\,q(\theta) \\ &\leq 0, \end{aligned}$$

where the first inequality follows from $u(\theta) \geq 0$, $m_S(\theta) \geq 0$, $m_F(\theta) \geq 0$, and $\delta \leq 1$, and the second inequality follows from $\theta \leq 1$, $q(\theta) \geq 0$, and the condition $\omega + \delta\,\pi \leq 1$. Therefore, every feasible mechanism gives the funder a payoff weakly below zero for every type. Integrating over $\theta \in [0,1]$, the funder's objective value is at most zero. The candidate mechanism $q(\theta) = m_0(\theta) = m_S(\theta) = m_F(\theta) = 0$ for all $\theta \in [0,1]$ attains objective value zero, establishing optimality.

We now prove part (2). Consider any feasible mechanism and any type $\theta > 0$. The individual rationality constraint gives

$$m_0(\theta) \geq q(\theta) - \delta\,\theta\,[m_S(\theta) + \pi\,q(\theta)] - \delta\,(1-\theta)\,m_F(\theta).$$

Substituting this lower bound for $m_0(\theta)$ into the funder's payoff from type θ yields

$$\begin{aligned} u_F(\theta) &= \theta\,\omega\,q(\theta) - m_0(\theta) - \theta\,m_S(\theta) - (1-\theta)\,m_F(\theta) \\ &\leq [\theta\,\omega + \delta\,\theta\,\pi - 1]\,q(\theta) - \theta\,(1-\delta)\,m_S(\theta) - (1-\theta)(1-\delta)\,m_F(\theta). \end{aligned}$$

The no-shirking constraint evaluated at the same reported type implies

$$\delta\,\theta\,[m_S(\theta) + \pi\,q(\theta) - m_F(\theta)] \geq (1-\alpha)\,q(\theta),$$

and hence

$$m_S(\theta) \geq \left(\frac{1-\alpha}{\delta\,\theta} - \pi\right) q(\theta) + m_F(\theta).$$

Because the coefficient of $m_S(\theta)$ in the preceding upper bound is $-\theta(1-\delta) \leq 0$, substituting this lower bound for $m_S(\theta)$ gives

$$\begin{aligned} u_F(\theta) &\leq [\theta\,\omega + \delta\,\theta\,\pi - 1]\,q(\theta) - \theta\,(1-\delta)\left[\left(\frac{1-\alpha}{\delta\,\theta} - \pi\right) q(\theta) + m_F(\theta)\right] - (1-\theta)(1-\delta)\,m_F(\theta) \\ &= \left[\theta\,(\omega+\pi) - \frac{1-\alpha+\delta\,\alpha}{\delta}\right] q(\theta) - (1-\delta)\,m_F(\theta) \\ &\leq \left[\omega + \pi - \frac{1-\alpha+\delta\,\alpha}{\delta}\right] q(\theta) \\ &\leq 0, \end{aligned}$$

where the last inequality follows from the condition $\delta(\omega+\pi) \le 1-\alpha+\delta\alpha$. For $\theta = 0$, we have

$$u_F(0) = -m_0(0) - m_F(0) \le 0.$$

Hence every feasible mechanism gives the funder a payoff weakly below zero for every type. Integrating over $\theta \in [0,1]$, the funder's objective value is at most zero. The candidate mechanism $q(\theta) = m_0(\theta) = m_S(\theta) = m_F(\theta) = 0$ for all $\theta \in [0,1]$ attains objective value zero, establishing optimality.

This completes the proof. □

Proof of Proposition 2. To verify that the candidate primal solution is feasible, we need to prove that

$$m_S^* = \frac{1-\alpha}{\delta\tau} - \pi \ge 0 \quad \Leftrightarrow \quad \tau \le a \equiv \frac{1-\alpha}{\delta\pi}.$$

Define

$$h(\theta) \equiv 2\delta(\omega+\pi)\theta^3 + (\alpha - 2\delta\alpha - 1)\theta^2 + \alpha - 1.$$

The threshold τ in the proposition is the zero of h. By the maintained condition $(1-\alpha)\bar{\tau} \le 1$, we have $a \le 1$. Moreover,

$$h'(\theta) = 2\theta\left[3\delta(\omega+\pi)\theta + \alpha - 2\delta\alpha - 1\right].$$

The bracketed term is strictly increasing in θ and is negative at $\theta = 0$. Hence, h is either decreasing on the relevant interval or first decreasing and then increasing. Since

$$h(0) = -(1-\alpha) < 0$$

and, using the normalized expression

$$B_1(\pi) = \frac{\delta^2\pi^3 - (1-\alpha)(1-\alpha-2\delta\alpha)\pi}{2(1-\alpha)^2},$$

we can write $h(a)$ directly in terms of the distance from the boundary $B_1(\pi)$:

$$h(a) = \frac{1-\alpha}{\delta^2\pi^3}\left[-\delta^2\pi^3 + (1-\alpha)(1-\alpha-2\delta\alpha)\pi + 2\omega(1-\alpha)^2\right] = \frac{2(1-\alpha)^3}{\delta^2\pi^3}(\omega - B_1(\pi)) \ge 0,$$

where the inequality follows from $\omega \ge B_1(\pi)$. Therefore, the function h is first decreasing and then increasing on $[0,1]$, and there exists a unique zero $\tau \in (0,a]$, implying that $m_S^* \ge 0$.

We next verify the remaining feasibility conditions. Under the candidate mechanism,

$$q^*(\theta) = \mathbf{1}_{\{\theta \ge \tau\}}, \qquad m_0^*(\theta) = \alpha\,\mathbf{1}_{\{\theta \ge \tau\}}, \qquad m_S^*(\theta) = \left(\frac{1-\alpha}{\delta\tau} - \pi\right)\mathbf{1}_{\{\theta \ge \tau\}}, \qquad m_F^*(\theta) = 0.$$

Therefore,

$$A^*(\theta) = m_S^*(\theta) - m_F^*(\theta) + \pi\, q^*(\theta) = \frac{1-\alpha}{\delta\tau}\,\mathbf{1}_{\{\theta \ge \tau\}},$$

which is nondecreasing. In addition,

$$\delta \int_0^\theta A^*(y)\,\mathrm{d}y = (1-\alpha)\left(\frac{\theta}{\tau}-1\right)^+ \geq 0,$$

so the participation constraints hold. The expression for m_0 implied by the envelope formula is

$$\delta \int_0^\theta A^*(y)\,\mathrm{d}y + q^*(\theta) - \delta\,\theta\,[m_S^*(\theta)+\pi\,q^*(\theta)] - \delta\,(1-\theta)m_F^*(\theta) = \begin{cases} 0, & \theta<\tau, \\ \alpha, & \theta\geq\tau, \end{cases}$$

which coincides with the proposed m_0^*. Finally, for any reported type $\hat{\theta}$, the right-hand side of the moral-hazard constraint (EC.7) under the candidate mechanism equals zero. Hence (EC.7) reduces to

$$\delta \int_0^\theta A^*(y)\,\mathrm{d}y \geq 0,$$

which was already verified. Thus the candidate mechanism is feasible.

We now prove optimality. Define

$$\begin{aligned}
\eta(\theta) &\equiv \frac{1}{2\,\delta}\left(\frac{1}{\theta^2}+1\right)-1, && \forall\theta\in(0,1], \\
\sigma(\theta) &\equiv \frac{h(\theta)}{2\,\delta\,\theta^2}, && \forall\theta\in[\tau,1], \\
\beta(\theta) &\equiv -\frac{h(\theta)}{2\,\delta\,\theta^2}, && \forall\theta\in(0,\tau], \\
\nu(\theta) &\equiv 1-\delta, && \forall\theta\in[0,1],
\end{aligned}$$

with $\chi_L \equiv 1$. All other multipliers are set equal to zero. Since $h(\theta)\leq 0$ on $(0,\tau]$, $h(\theta)\geq 0$ on $[\tau,1]$, $\delta\leq 1$, and

$$\eta(\theta) \geq \frac{1}{\delta} - 1 \geq 0,$$

all the displayed multipliers are nonnegative.

A direct calculation gives

$$\int_\theta^1 \eta(y)\,\mathrm{d}y = \frac{1}{2\,\delta}\left(\frac{1}{\theta}-1\right) + \left(\frac{1}{2\,\delta}-1\right)(1-\theta),$$

and hence the following coefficient-matching identities hold:

$$\begin{aligned}
&(1-\alpha-\delta\,\pi\,\theta)\,\eta(\theta) + \delta\,\pi\int_\theta^1 \eta(y)\,\mathrm{d}y - \beta(\theta) = w_q(\theta), && \forall\theta\in(0,\tau], \\
&(1-\alpha-\delta\,\pi\,\theta)\,\eta(\theta) + \delta\,\pi\int_\theta^1 \eta(y)\,\mathrm{d}y + \sigma(\theta) = w_q(\theta), && \forall\theta\in[\tau,1], \\
&-\delta\,\theta\,\eta(\theta) + \delta\int_\theta^1 \eta(y)\,\mathrm{d}y = w_S(\theta), && \forall\theta\in(0,1], \\
&\delta\,\theta\,\eta(\theta) - \delta\int_\theta^1 \eta(y)\,\mathrm{d}y - \nu(\theta) = w_F(\theta), && \forall\theta\in(0,1].
\end{aligned}$$

Because $\eta(\theta)$ and $\beta(\theta)$ may be singular as $\theta \downarrow 0$, we implement the preceding coefficient-matching identities through truncation. Fix any $\varepsilon \in (0,\tau)$, and define

$$H(\theta) \equiv \int_{\theta}^{1} \eta(y)\,\mathrm{d}y.$$

For any admissible feasible mechanism, the expected transfers are finite, so all the following integrals are well defined on the truncated domain. Applying Fubini's theorem on $[\varepsilon,1]$ and using the coefficient-matching identities above gives

$$\begin{aligned}
&\int_{\varepsilon}^{1} \eta(\theta)\left[\delta\int_{0}^{\theta} A(y)\,\mathrm{d}y + (1-\alpha)\,q(\theta) - \delta\,\theta\,[m_S(\theta)+\pi\,q(\theta)] + \delta\,\theta\,m_F(\theta)\right]\mathrm{d}\theta \\
&\quad + \chi_L[-u(0)] + \int_{\varepsilon}^{\tau} \beta(\theta)[-q(\theta)]\,\mathrm{d}\theta + \int_{\tau}^{1} \sigma(\theta)q(\theta)\,\mathrm{d}\theta + \int_{\varepsilon}^{1} \nu(\theta)[-m_F(\theta)]\,\mathrm{d}\theta \\
&= -u(0) + \int_{\varepsilon}^{1} [w_q(\theta)\,q(\theta) + w_S(\theta)\,m_S(\theta) + w_F(\theta)\,m_F(\theta)]\,\mathrm{d}\theta + \delta\,H(\varepsilon)\int_{0}^{\varepsilon} A(y)\,\mathrm{d}y.
\end{aligned}$$

The last term is nonnegative. Indeed, setting $\hat{\theta}=0$ in the moral-hazard constraint (EC.7) gives

$$\delta\int_{0}^{\varepsilon} A(y)\,\mathrm{d}y \geq (1-\alpha)\,q(0) \geq 0,$$

while $H(\varepsilon)\geq 0$ because $\eta \geq 0$. Moreover, all terms on the left-hand side of the preceding identity are nonpositive for a feasible mechanism, except $\int_{\tau}^{1}\sigma(\theta)q(\theta)\,\mathrm{d}\theta$, which is bounded above by $\int_{\tau}^{1}\sigma(\theta)\,\mathrm{d}\theta$. It follows that

$$-u(0) + \int_{\varepsilon}^{1} [w_q(\theta)\,q(\theta) + w_S(\theta)\,m_S(\theta) + w_F(\theta)\,m_F(\theta)]\,\mathrm{d}\theta + \delta\,H(\varepsilon)\int_{0}^{\varepsilon} A(y)\,\mathrm{d}y \leq \int_{\tau}^{1}\sigma(\theta)\,\mathrm{d}\theta.$$

Dropping the nonnegative boundary term yields

$$-u(0) + \int_{\varepsilon}^{1} [w_q(\theta)\,q(\theta) + w_S(\theta)\,m_S(\theta) + w_F(\theta)\,m_F(\theta)]\,\mathrm{d}\theta \leq \int_{\tau}^{1}\sigma(\theta)\,\mathrm{d}\theta.$$

Finally, letting $\varepsilon \downarrow 0$ gives

$$-u(0) + \int_{0}^{1} [w_q(\theta)\,q(\theta) + w_S(\theta)\,m_S(\theta) + w_F(\theta)\,m_F(\theta)]\,\mathrm{d}\theta \leq \int_{\tau}^{1}\sigma(\theta)\,\mathrm{d}\theta.$$

Thus, the objective function is bounded above by

$$\int_{\tau}^{1}\sigma(\theta)\,\mathrm{d}\theta.$$

It remains to show that the candidate mechanism attains this upper bound. Substituting the candidate mechanism into the objective function gives

$$\int_{\tau}^{1}\left[\theta\,\omega - \alpha - \theta\left(\frac{1-\alpha}{\delta\,\tau} - \pi\right)\right]\mathrm{d}\theta.$$

Moreover,

$$\begin{aligned}\int_\tau^1 \sigma(\theta)\,\mathrm{d}\theta - \int_\tau^1 \left[\theta\,\omega - \alpha - \theta\left(\frac{1-\alpha}{\delta\,\tau} - \pi\right)\right]\mathrm{d}\theta &= \frac{1-\alpha}{2\,\delta}\int_\tau^1 \left(\frac{2\,\theta}{\tau} - 1 - \frac{1}{\theta^2}\right)\mathrm{d}\theta \\ &= \frac{1-\alpha}{2\,\delta}\left[\frac{1-\tau^2}{\tau} - (1-\tau) + \left(1-\frac{1}{\tau}\right)\right] \\ &= 0.\end{aligned}$$

Thus the candidate primal mechanism attains the dual upper bound and is optimal.

This completes the proof. □

Proof of Proposition 3. Under the condition $(1-\alpha)\,\bar{\tau} \le 1$, the threshold $\tau = (1-\alpha)\,\bar{\tau}$ lies in $[0,1]$. The candidate mechanism

$$q^*(\theta) = \mathbf{1}_{\{\theta \ge \tau\}}, \qquad m_0^*(\theta) = \alpha\,\mathbf{1}_{\{\theta \ge \tau\}}, \qquad m_S^*(\theta) = m_F^*(\theta) = 0$$

is feasible. Indeed,

$$A^*(\theta) = m_S^*(\theta) - m_F^*(\theta) + \pi\,q^*(\theta) = \pi\,\mathbf{1}_{\{\theta \ge \tau\}},$$

which is nondecreasing. Moreover,

$$\delta\int_0^\theta A^*(y)\;\mathrm{d}y = \delta\,\pi\,(\theta-\tau)^+ \ge 0,$$

so the participation constraints hold. The upfront payment implied by the envelope formula is

$$\delta\int_0^\theta A^*(y)\;\mathrm{d}y + q^*(\theta) - \delta\,\theta\,\pi\,q^*(\theta) = \begin{cases} 0, & \theta < \tau, \\ 1 - \delta\,\pi\,\tau = \alpha, & \theta \ge \tau, \end{cases}$$

which coincides with the proposed m_0^*. Finally, for any reported type $\hat{\theta}$, the right-hand side of the moral-hazard constraint (EC.7) equals zero: it is trivially zero if $\hat{\theta} < \tau$, while if $\hat{\theta} \ge \tau$, it equals

$$\delta\,\pi\,(\hat{\theta} - \tau) + (1-\alpha) - \delta\,\pi\,\hat{\theta} = 1 - \alpha - \delta\,\pi\,\tau = 0.$$

Hence (EC.7) reduces to $\delta\int_0^\theta A^*(y)\;\mathrm{d}y \ge 0$, which was already verified.

We now prove optimality. If $B_1(\pi) = B_2(\pi)$, the region $B_2(\pi) \le \omega < B_1(\pi)$ is empty, so suppose $B_1(\pi) > B_2(\pi)$. Define

$$C \equiv (2\,\delta - 1) + 2\,(1-\delta)\,\underbrace{\frac{\omega - B_2(\pi)}{B_1(\pi) - B_2(\pi)}}_{\in[0,1)}.$$

Because $\delta \ge 1/2$, we have

$$2\,\delta - 1 \le C < 1.$$

Define

$$\begin{aligned}
&\chi_L \equiv 1,\\
&\eta(\theta) \equiv \frac{1}{2\,\delta}\left(\frac{C}{\theta^2}+1\right)-1, && \forall \theta \in (0,1],\\
&\beta(\theta) \equiv G(\theta), && \forall \theta \in (0,\tau],\\
&\sigma(\theta) \equiv -G(\theta), && \forall \theta \in [\tau,1],\\
&\xi(\theta) \equiv \frac{1-C}{2}, && \forall \theta \in [0,1],\\
&\nu(\theta) \equiv \frac{C-(2\,\delta-1)}{2}, && \forall \theta \in [0,1],
\end{aligned}$$

where

$$G(\theta) \equiv (1-\alpha-\delta\,\pi\,\theta)\,\eta(\theta)+\delta\,\pi\int_\theta^1 \eta(y)\;\mathrm{d}y - w_q(\theta).$$

All other multipliers are set equal to zero.

We first verify dual feasibility. Since $C \geq 2\,\delta-1 \geq 0$,

$$\eta'(\theta) = -\frac{C}{\delta\,\theta^3} \leq 0,$$

and

$$\eta(1) = \frac{C-(2\,\delta-1)}{2\,\delta} \geq 0.$$

Thus $\eta(\theta) \geq 0$ for all $\theta \in (0,1]$. The multipliers ξ and ν are nonnegative because $C \leq 1$ and $C \geq 2\,\delta-1$. It remains to verify the signs of β and σ. Direct substitution gives

$$G(\tau) = \frac{(1-\alpha)(1-\tau)}{2\,\delta\,\tau^2}\left[C-\left[(2\,\delta-1)+2\,(1-\delta)\frac{\omega-B_2(\pi)}{B_1(\pi)-B_2(\pi)}\right]\right] = 0.$$

Moreover,

$$G'(\theta) = \frac{C(\alpha-1)}{\delta\,\theta^3} - (\omega+\pi) < 0, \qquad \forall \theta \in (0,1].$$

Therefore, $G(\theta) \geq 0$ on $(0,\tau]$ and $G(\theta) \leq 0$ on $[\tau,1]$, implying $\beta(\theta) \geq 0$ and $\sigma(\theta) \geq 0$.

A direct calculation gives

$$\int_\theta^1 \eta(y)\;\mathrm{d}y = \frac{C}{2\,\delta}\left(\frac{1}{\theta}-1\right)+\left(\frac{1}{2\,\delta}-1\right)(1-\theta).$$

Consequently, the following coefficient-matching identities hold:

$$\begin{aligned}
&(1-\alpha-\delta\,\pi\,\theta)\,\eta(\theta)+\delta\,\pi\int_\theta^1 \eta(y)\;\mathrm{d}y - \beta(\theta) = w_q(\theta), && \forall \theta \in (0,\tau],\\
&(1-\alpha-\delta\,\pi\,\theta)\,\eta(\theta)+\delta\,\pi\int_\theta^1 \eta(y)\;\mathrm{d}y + \sigma(\theta) = w_q(\theta), && \forall \theta \in [\tau,1],\\
&-\delta\,\theta\,\eta(\theta)+\delta\int_\theta^1 \eta(y)\;\mathrm{d}y - \xi(\theta) = w_S(\theta), && \forall \theta \in (0,1],\\
&\delta\,\theta\,\eta(\theta)-\delta\int_\theta^1 \eta(y)\;\mathrm{d}y - \nu(\theta) = w_F(\theta), && \forall \theta \in (0,1].
\end{aligned}$$

Using these identities, for any feasible mechanism we have

$$\begin{aligned}
&\int_0^1 \eta(\theta)\left[\delta\int_0^\theta A(y)\,\mathrm{d}y + (1-\alpha)\,q(\theta) - \delta\,\theta\,[m_S(\theta)+\pi\,q(\theta)] + \delta\,\theta\,m_F(\theta)\right]\mathrm{d}\theta \\
&\quad + \chi_L[-u(0)] + \int_0^\tau \beta(\theta)[-q(\theta)]\,\mathrm{d}\theta + \int_\tau^1 \sigma(\theta)[q(\theta)-1]\,\mathrm{d}\theta + \int_\tau^1 \sigma(\theta)\,\mathrm{d}\theta \\
&\quad + \int_0^1 \xi(\theta)[-m_S(\theta)]\,\mathrm{d}\theta + \int_0^1 \nu(\theta)[-m_F(\theta)]\,\mathrm{d}\theta \\
&= -u(0) + \int_0^1 [w_q(\theta)\,q(\theta) + w_S(\theta)\,m_S(\theta) + w_F(\theta)\,m_F(\theta)]\,\mathrm{d}\theta.
\end{aligned}$$

Because all multipliers are nonnegative and the residuals multiplied by them are nonpositive for every feasible mechanism, the objective function is bounded above by

$$\int_\tau^1 \sigma(\theta)\,\mathrm{d}\theta.$$

It remains to show that the candidate mechanism attains this upper bound. Substituting the candidate mechanism into the objective function gives

$$\int_\tau^1 (\theta\,\omega - \alpha)\,\mathrm{d}\theta.$$

Furthermore, direct calculation yields

$$\int_\tau^1 \sigma(\theta)\,\mathrm{d}\theta - \int_\tau^1 (\theta\,\omega-\alpha)\,\mathrm{d}\theta = \frac{(C+\tau)(1-\tau)(\alpha+\delta\,\pi\,\tau-1)}{2\,\delta\,\tau} = 0,$$

where the last equality follows from $\tau = (1-\alpha)/(\delta\,\pi)$. Therefore, the candidate primal mechanism attains the dual upper bound and is optimal.

This completes the proof. □

Proof of Proposition 4. We first verify that $B_3(\pi) \le B_2(\pi)$ for any $\pi \ge (1-\alpha)/\delta$. Recall that, after normalizing $c=1$, we have $\bar\tau = 1/(\delta\,\pi)$. Since

$$B_2(\pi) - B_3(\pi) = \frac{\alpha\,\pi}{2(1-\alpha)^2\bar\tau}\left[3(2\delta-1) + (2+\alpha-4\delta\alpha-2\delta)\bar\tau + (2\delta\alpha-2\alpha+1)\bar\tau^2\right],$$

it suffices to show that the bracketed term is nonnegative. Equivalently, after multiplying by $\delta^2\pi^2$, define

$$\phi(\pi) \equiv 3\delta^2(2\delta-1)\pi^2 + \delta(2+\alpha-4\delta\alpha-2\delta)\pi + 2\delta\alpha - 2\alpha + 1.$$

When $\pi \ge 1/\delta$, the argument proceeds as follows. If $\delta = 1/2$, then

$$\phi(\pi) = (1-\alpha)\left(1+\frac{\pi}{2}\right) \ge 0.$$

Now suppose $\delta > 1/2$. Then ϕ is convex. Moreover,

$$\phi'(\pi) = 6\,\delta^2(2\,\delta-1)\pi + \delta(2+\alpha-4\,\delta\,\alpha-2\,\delta),$$

and hence

$$\phi'(1/\delta) = \delta\,[10\,\delta - 4 + \alpha(1 - 4\,\delta)] \geq \delta(6\,\delta - 3) \geq 0,$$

where the first inequality uses $\alpha \leq 1$ and $\delta > 1/2$. Therefore, ϕ is increasing on $[1/\delta, \infty)$. Since

$$\phi(1/\delta) = 4\delta - \alpha(1 + 2\delta) \geq 2\delta - 1 \geq 0,$$

we have $\phi(\pi) \geq 0$ for all $\pi \geq 1/\delta$.

It remains to consider $(1-\alpha)/\delta \leq \pi \leq 1/\delta$. Let $x \equiv \delta\,\pi \in [1-\alpha, 1]$. A direct calculation gives

$$B_2(\pi) - (1 - x) = \frac{(\alpha + x - 1)\,[(2\delta - 1)x^2 + (1-\alpha)x + 2\delta(1-\alpha)]}{2\delta(1-\alpha)^2} \geq 0.$$

Similarly,

$$(1 - x) - B_3(\pi) = \frac{(1-x)P(x,\delta)}{2\,\delta\,(1-\alpha)^2},$$

where

$$P(x,\delta) \equiv (2\,\delta - 1)x^2 + (1 + \alpha - 4\,\alpha\,\delta)x + 4\,\alpha^2\,\delta - 2\,\alpha^2 - 4\,\alpha\,\delta + \alpha + 2\,\delta.$$

We claim that $P(x,\delta) \geq 0$ for all $x \in [1-\alpha, 1]$ and $\delta \geq 1/2$. At $\delta = 1/2$,

$$P(x, 1/2) = (1-\alpha)(1+x) \geq 0.$$

Moreover,

$$\frac{\partial P(x,\delta)}{\partial \delta} = 2\left[(x-\alpha)^2 + (1-\alpha)^2\right] \geq 0.$$

Hence $P(x,\delta) \geq P(x, 1/2) \geq 0$. Since $x \leq 1$, this implies

$$(1 - x) - B_3(\pi) \geq 0.$$

Together with $B_2(\pi) \geq 1 - x$, we obtain

$$B_3(\pi) \leq 1 - \delta\,\pi \leq B_2(\pi),$$

which completes the proof that $B_3(\pi) \leq B_2(\pi)$ for $(1-\alpha)/\delta \leq \pi \leq 1/\delta$.

Under the candidate mechanism in Proposition 4, after the normalization $c = 1$, the positive transfers for participating types can be written as

$$m_0^* = \alpha + \frac{1-\alpha}{\tau} - \delta\,\pi, \qquad m_F^* = \pi - \frac{1-\alpha}{\delta\,\tau}, \qquad m_S^* = 0.$$

Thus primal feasibility requires

$$\tau \geq \frac{1-\alpha}{\delta\,\pi}$$

to ensure $m_F^* \geq 0$, and, when $\delta\,\pi > \alpha$,

$$\tau \leq \frac{1-\alpha}{\delta\,\pi - \alpha}$$

to ensure $m_0^* \geq 0$. If $\delta\,\pi \leq \alpha$, the nonnegativity of m_0^* holds automatically.

Define

$$h(\theta) \equiv 2\,\delta\,(\omega+\pi)\,\theta^3 - [2\,\delta\,\pi\,(1-\delta) + (1-\alpha) + 2\,\delta\,\alpha]\,\theta^2 - (1-\alpha)(2\,\delta - 1).$$

The threshold τ in Proposition 4 is the unique positive zero of h. To see this, let

$$K \equiv 2\,\delta\,\pi\,(1-\delta) + (1-\alpha) + 2\,\delta\,\alpha > 0.$$

Then

$$h'(\theta) = 2\,\theta\,[3\,\delta\,(\omega+\pi)\theta - K].$$

Hence $h'(0) = 0$, $h'(\theta) < 0$ for $\theta \in (0, \theta_0)$, and $h'(\theta) > 0$ for $\theta > \theta_0$, where

$$\theta_0 \equiv \frac{K}{3\,\delta\,(\omega+\pi)}.$$

Thus h is decreasing on $[0, \theta_0]$ and increasing on $[\theta_0, \infty)$. Moreover,

$$h(0) = -(1-\alpha)(2\,\delta - 1) \leq 0,$$

and $h(\theta) < 0$ for all sufficiently small positive θ. On the other hand,

$$h(1) = 2\,\delta\,(\omega + \delta\,\pi - 1) > 0.$$

Therefore, the turning point satisfies $\theta_0 < 1$, and h has a unique positive zero $\tau \in (0, 1]$. In addition,

$$h(\theta) \leq 0, \quad \forall \theta \in [0, \tau], \qquad h(\theta) \geq 0, \quad \forall \theta \in [\tau, 1].$$

A direct calculation gives

$$h\left(\frac{1-\alpha}{\delta\,\pi}\right) = \frac{2(1-\alpha)^3}{\delta^2\pi^3}\,[\omega - B_2(\pi)] \leq 0,$$

where the inequality follows from $\omega < B_2(\pi)$. Hence $(1-\alpha)/(\delta\,\pi) \leq \tau$, so $m_F^* \geq 0$.

If $\delta\,\pi > \alpha$, we have

$$h\left(\frac{1-\alpha}{\delta\,\pi - \alpha}\right) = \frac{2\delta(1-\alpha)^3}{(\delta\,\pi - \alpha)^3}\,[\omega - B_3(\pi)] \geq 0,$$

where the inequality follows from $\omega \geq B_3(\pi)$. Hence $\tau \leq (1-\alpha)/(\delta\,\pi - \alpha)$, so $m_0^* \geq 0$. Thus the candidate transfers are nonnegative.

We next verify the remaining feasibility conditions. Under the candidate mechanism,

$$q^*(\theta) = \mathbf{1}_{\{\theta \geq \tau\}}, \qquad m_S^*(\theta) = 0, \qquad m_F^*(\theta) = \left(\pi - \frac{1-\alpha}{\delta\,\tau}\right)\mathbf{1}_{\{\theta \geq \tau\}}.$$

Therefore,

$$A^*(\theta) = m_S^*(\theta) - m_F^*(\theta) + \pi q^*(\theta) = \frac{1-\alpha}{\delta\,\tau}\mathbf{1}_{\{\theta \geq \tau\}},$$

which is nondecreasing. Also,

$$\delta \int_0^\theta A^*(y)\,\mathrm{d}y = (1-\alpha)\left(\frac{\theta}{\tau}-1\right)^+ \geq 0,$$

so the participation constraints hold. The upfront payment implied by the envelope formula is

$$\delta \int_0^\theta A^*(y)\,\mathrm{d}y + q^*(\theta) - \delta\,\theta\,\pi\,q^*(\theta) - \delta(1-\theta)m_F^*(\theta) = \begin{cases} 0, & \theta < \tau, \\ \alpha + \dfrac{1-\alpha}{\tau} - \delta\,\pi, & \theta \geq \tau, \end{cases}$$

which coincides with $m_0^*(\theta)$. Finally, for any reported type $\hat{\theta} \geq \tau$, the right-hand side of the moral-hazard constraint (EC.7) equals

$$(1-\alpha)\left(\frac{\hat{\theta}}{\tau}-1\right) + (1-\alpha) - \delta\,\hat{\theta}\pi + \delta\,\hat{\theta}\left(\pi - \frac{1-\alpha}{\delta\,\tau}\right) = 0,$$

and for $\hat{\theta} < \tau$ it is also zero. Thus the moral-hazard constraint reduces to

$$\delta \int_0^\theta A^*(y)\,\mathrm{d}y \geq 0,$$

which has already been verified. Hence the candidate mechanism is feasible.

We now prove optimality. Define

$$\begin{aligned}
&\chi_L \equiv 1, \\
&\eta(\theta) \equiv \frac{1}{2\delta}\left(\frac{2\delta-1}{\theta^2}+1\right) - 1, && \forall \theta \in (0,1], \\
&\xi(\theta) \equiv 1-\delta, && \forall \theta \in [0,1], \\
&\beta(\theta) \equiv -\frac{h(\theta)}{2\delta\theta^2}, && \forall \theta \in (0,\tau], \\
&\sigma(\theta) \equiv \frac{h(\theta)}{2\delta\theta^2}, && \forall \theta \in [\tau,1].
\end{aligned}$$

All other multipliers are set equal to zero. Since $\delta \geq 1/2$, we have $\eta(\theta) \geq 0$ and $\xi(\theta) \geq 0$. Since $h(\theta) \leq 0$ on $(0,\tau]$ and $h(\theta) \geq 0$ on $[\tau,1]$, the multipliers β and σ are also nonnegative.

A direct calculation gives

$$\int_\theta^1 \eta(y)\,\mathrm{d}y = \frac{2\delta-1}{2\delta}\left(\frac{1}{\theta}+\theta-2\right).$$

Using this expression and the definitions of β, σ, and ξ, we obtain the following coefficient-matching identities:

$$\begin{aligned}
&(1-\alpha-\delta\,\pi\,\theta)\,\eta(\theta) + \delta\,\pi\int_\theta^1 \eta(y)\,\mathrm{d}y - \beta(\theta) = w_q(\theta), && \forall \theta \in (0,\tau], \\
&(1-\alpha-\delta\,\pi\,\theta)\,\eta(\theta) + \delta\,\pi\int_\theta^1 \eta(y)\,\mathrm{d}y + \sigma(\theta) = w_q(\theta), && \forall \theta \in [\tau,1], \\
&-\delta\,\theta\,\eta(\theta) + \delta\int_\theta^1 \eta(y)\,\mathrm{d}y - \xi(\theta) = w_S(\theta), && \forall \theta \in (0,1], \\
&\delta\,\theta\,\eta(\theta) - \delta\int_\theta^1 \eta(y)\,\mathrm{d}y = w_F(\theta), && \forall \theta \in (0,1].
\end{aligned}$$

Therefore, for any feasible mechanism,

$$\begin{aligned}
&\int_0^1 \eta(\theta)\left[\delta\int_0^\theta A(y)\,\mathrm{d}y+(1-\alpha)q(\theta)-\delta\,\theta\,[m_S(\theta)+\pi q(\theta)]+\delta\,\theta\,m_F(\theta)\right]\mathrm{d}\theta\\
&\quad+\chi_L[-u(0)]+\int_0^\tau \beta(\theta)[-q(\theta)]\,\mathrm{d}\theta+\int_\tau^1 \sigma(\theta)[q(\theta)-1]\,\mathrm{d}\theta+\int_\tau^1 \sigma(\theta)\,\mathrm{d}\theta+\int_0^1 \xi(\theta)[-m_S(\theta)]\,\mathrm{d}\theta\\
&=-u(0)+\int_0^1 [w_q(\theta)q(\theta)+w_S(\theta)m_S(\theta)+w_F(\theta)m_F(\theta)]\,\mathrm{d}\theta.
\end{aligned}$$

Because all displayed multipliers are nonnegative and the residuals multiplied by them are nonpositive for every feasible mechanism, the funder's objective is bounded above by

$$\int_\tau^1 \sigma(\theta)\,\mathrm{d}\theta.$$

It remains to verify that the candidate mechanism attains this upper bound. Substituting the candidate mechanism into the objective function gives

$$\int_\tau^1 \left[\theta\,\omega-\left(\alpha+\frac{1-\alpha}{\tau}-\delta\,\pi\right)-(1-\theta)\left(\pi-\frac{1-\alpha}{\delta\,\tau}\right)\right]\mathrm{d}\theta.$$

A direct calculation yields

$$\begin{aligned}
&\int_\tau^1 \sigma(\theta)\,\mathrm{d}\theta-\int_\tau^1 \left[\theta\,\omega-\left(\alpha+\frac{1-\alpha}{\tau}-\delta\,\pi\right)-(1-\theta)\left(\pi-\frac{1-\alpha}{\delta\,\tau}\right)\right]\mathrm{d}\theta\\
&=\frac{\alpha-1}{2\delta\,\tau}\left[-\frac{\tau(2\delta-1)}{\theta}+(\tau+2-2\delta)\theta-\theta^2\right]_{\theta=\tau}^{\theta=1}=0.
\end{aligned}$$

Thus the candidate primal mechanism attains the dual upper bound and is optimal.

This completes the proof. □

Before proving Proposition 5, we record a simple interpolation lemma. The lemma applies when the same candidate mechanism is optimal at two adjacent boundary values of ω, and it allows us to establish optimality over the entire interval of consumer-welfare values by verifying optimality only at the endpoints.

LEMMA EC.2. *Fix all parameters except ω, and let $x\in\mathcal{X}$ be a feasible mechanism that is independent of ω. If x is optimal at ω_L and at ω_H, where $\omega_L\le\omega_H$, then x is optimal for every $\omega\in[\omega_L,\omega_H]$.*

Proof of Lemma EC.2. The feasible set $\mathcal{X}$ is independent of ω, because ω appears only in the funder's objective. For any feasible mechanism $z\equiv\{q_z,m_{0,z},m_{S,z},m_{F,z}\}\in\mathcal{X}$, the funder's objective can be written as

$$J_\omega(z)=\omega\,Q(z)+R(z),$$

where

$$Q(z)\equiv\int_0^1 \theta\,q_z(\theta)\,\mathrm{d}\theta$$

and $R(z)$ collects all terms independent of ω. Hence, for any $\omega = \lambda\,\omega_L + (1-\lambda)\omega_H$, with $\lambda \in [0,1]$, we have

$$J_\omega(z) - J_\omega(x) = \lambda\,[J_{\omega_L}(z) - J_{\omega_L}(x)] + (1-\lambda)\,[J_{\omega_H}(z) - J_{\omega_H}(x)].$$

Since x is optimal at both endpoints, the two bracketed terms are weakly negative for every feasible z. Therefore,

$$J_\omega(z) - J_\omega(x) \le 0$$

for every feasible z, which proves that x is optimal at ω. □

Proof of Proposition 5. Under the condition $\bar{\tau} \le 1$, we have $\delta\,\pi \ge 1$. By the definition of τ in Proposition 5, we have $\tau \in [0,1]$ and

$$\delta\,\pi - \alpha = \frac{1-\alpha}{\tau}.$$

The candidate mechanism is

$$q^*(\theta) = \mathbf{1}_{\{\theta \ge \tau\}}, \qquad m_F^*(\theta) = \frac{\alpha}{\delta}\mathbf{1}_{\{\theta \ge \tau\}}, \qquad m_0^*(\theta) = m_S^*(\theta) = 0.$$

We first verify feasibility. Under the candidate mechanism,

$$A^*(\theta) = m_S^*(\theta) - m_F^*(\theta) + \pi\,q^*(\theta) = \left(\pi - \frac{\alpha}{\delta}\right)\mathbf{1}_{\{\theta \ge \tau\}},$$

which is nondecreasing. Moreover,

$$\delta \int_0^\theta A^*(y)\,\mathrm{d}y = (1-\alpha)\left(\frac{\theta}{\tau} - 1\right)^+ \ge 0,$$

so the participation constraints hold. The upfront payment implied by the envelope formula is

$$\delta \int_0^\theta A^*(y)\,\mathrm{d}y + q^*(\theta) - \delta\,\theta\,\pi\,q^*(\theta) - \delta\,(1-\theta)m_F^*(\theta) = \begin{cases} 0, & \theta < \tau, \\ (1-\alpha)\left(\dfrac{\theta}{\tau} - 1\right) + 1 - \delta\,\pi\,\theta - \alpha(1-\theta), & \theta \ge \tau. \end{cases}$$

Using $\delta\,\pi - \alpha = (1-\alpha)/\tau$, the second line equals zero. Hence $m_0^*(\theta) = 0$, as proposed. Finally, for any reported type $\hat{\theta} \ge \tau$, the right-hand side of the moral-hazard constraint (EC.7) equals

$$(1-\alpha)\left(\frac{\hat{\theta}}{\tau} - 1\right) + (1-\alpha) - \delta\,\hat{\theta}\pi + \alpha\,\hat{\theta} = \left(\frac{1-\alpha}{\tau} - \delta\,\pi + \alpha\right)\hat{\theta} = 0,$$

and for $\hat{\theta} < \tau$ it is also zero. Therefore, the moral-hazard constraint reduces to

$$\delta \int_0^\theta A^*(y)\,\mathrm{d}y \ge 0,$$

which was already verified. Thus the candidate mechanism is feasible.

We now prove optimality by using the two adjacent boundary cases. Let

$$\omega_L \equiv B_4(\pi), \qquad \omega_H \equiv B_3(\pi).$$

If $\omega_L = \omega_H$, the endpoint argument below already proves the result. Hence suppose $\omega_L < \omega_H$.

At the upper boundary $\omega = \omega_H$, the candidate mechanism in Proposition 4 coincides with the candidate mechanism above. Indeed, the Proposition 4 threshold is the unique zero of

$$h(\theta;\omega) \equiv 2\,\delta\,(\omega+\pi)\theta^3 - [2\,\delta\,\pi(1-\delta) + (1-\alpha) + 2\,\delta\,\alpha]\,\theta^2 - (1-\alpha)(2\,\delta - 1).$$

A direct calculation gives

$$h(\tau;\omega) = \frac{2\,\delta(1-\alpha)^3}{(\delta\,\pi - \alpha)^3}\,[\omega - B_3(\pi)]\,.$$

Thus, when $\omega = B_3(\pi)$, the Proposition 4 threshold is τ. The Proposition 4 transfers also reduce to those above:

$$m_0^* = \alpha + \frac{1-\alpha}{\tau} - \delta\,\pi = 0,$$

and

$$m_F^* = \pi\left(1 - \frac{1-\alpha}{\delta\,\pi\,\tau}\right) = \frac{\alpha}{\delta}.$$

Therefore, the candidate mechanism in Proposition 5 is optimal at $\omega = \omega_H$.

At the lower boundary $\omega = \omega_L$, the candidate mechanism in Proposition 6 also coincides with the candidate mechanism above. Equivalently, using the normalized expression

$$B_4(\pi) = \frac{(\delta\,\pi + 1)(\delta\,\pi - \alpha)}{2\,\delta(1-\alpha)} - \pi,$$

the Proposition 6 threshold satisfies

$$\frac{\delta\,\pi + 1}{2\,\delta[B_4(\pi) + \pi]} = \frac{1-\alpha}{\delta\,\pi - \alpha} = \tau.$$

Moreover, the Proposition 6 failure payment becomes

$$\pi\,\frac{\bar{\tau} - \tau}{1-\tau} = \frac{\alpha}{\delta}.$$

Thus, at $\omega = B_4(\pi)$, the Proposition 6 mechanism reduces to the same insurance-only mechanism. The dual feasible solution used in the proof of Proposition 6 remains feasible at this boundary, with the relevant limited-liability condition binding. Hence the candidate mechanism is optimal at $\omega = \omega_L$.

The candidate mechanism in Proposition 5 does not depend on ω within the interval $[B_4(\pi), B_3(\pi)]$. We have shown that it is optimal at both endpoints $\omega_L = B_4(\pi)$ and $\omega_H = B_3(\pi)$. Therefore, by Lemma EC.2, the same mechanism is optimal for every $\omega \in [B_4(\pi), B_3(\pi)]$.

This completes the proof. □

Proof of Proposition 6. Under the condition $\bar{\tau} \le 1$, we have $\delta\,\pi \ge 1$. Using the normalized expressions,

$$B_5(\pi) = \frac{\pi(\delta\,\pi - 1)}{2}$$

and

$$B_4(\pi) = \frac{\delta^2\pi^2 - \delta(1-\alpha)\pi - \alpha}{2\,\delta(1-\alpha)}.$$

By definition,

$$\tau = \frac{\delta\,\pi + 1}{2\,\delta(\omega + \pi)}.$$

The candidate failure payment can be written as

$$m_F^* = \pi\,\frac{\bar{\tau} - \tau}{1 - \tau} = \frac{1 - \delta\,\pi\,\tau}{\delta(1-\tau)}.$$

We first verify feasibility. The condition $\tau \le 1$ is equivalent to

$$1 - \delta\,\pi - 2\,\delta\,\omega \le 0,$$

which follows from $\delta\,\pi \ge 1$. Moreover,

$$1 - \delta\,\pi\,\tau = \frac{2(\omega+\pi) - \pi(\delta\,\pi + 1)}{2(\omega+\pi)} = \frac{\omega - B_5(\pi)}{\omega + \pi} \ge 0,$$

where the last inequality follows from $\omega \ge B_5(\pi)$. Hence $m_F^* \ge 0$.

Under the candidate mechanism,

$$q^*(\theta) = \mathbf{1}_{\{\theta \ge \tau\}}, \qquad m_0^*(\theta) = m_S^*(\theta) = 0, \qquad m_F^*(\theta) = m_F^*\,\mathbf{1}_{\{\theta \ge \tau\}}.$$

Thus

$$A^*(\theta) = m_S^*(\theta) - m_F^*(\theta) + \pi\,q^*(\theta) = (\pi - m_F^*)\,\mathbf{1}_{\{\theta \ge \tau\}},$$

which is nondecreasing because $m_F^* \le \pi$, equivalently $1/\delta - \tau\pi \le \pi(1-\tau)$, which follows from $\delta\,\pi \ge 1$. Also,

$$\delta \int_0^\theta A^*(y)\,\mathrm{d}y = \delta(\pi - m_F^*)(\theta - \tau)^+ \ge 0,$$

so the participation constraints hold. The upfront payment implied by the envelope formula equals zero for $\theta < \tau$, while for $\theta \ge \tau$ it is

$$\delta(\pi - m_F^*)(\theta - \tau) + 1 - \delta\,\theta\,\pi - \delta(1-\theta)m_F^* = 1 - \delta\,\pi\,\tau - \delta(1-\tau)m_F^* = 0,$$

where the last equality follows from the definition of m_F^*. Hence $m_0^*(\theta) = 0$, as proposed.

Finally, for any reported type $\hat{\theta} < \tau$, the right-hand side of the moral-hazard constraint (EC.7) is zero. For any reported type $\hat{\theta} \ge \tau$, the right-hand side equals

$$\delta(\pi - m_F^*)(\hat{\theta} - \tau) + (1-\alpha) - \delta\,\hat{\theta}\pi + \delta\,\hat{\theta}m_F^* = 1 - \alpha - \delta\,\pi\,\tau + \delta\,\tau\,m_F^* = \frac{1 - \delta\,\pi\,\tau - \alpha(1-\tau)}{1-\tau} \le 0.$$

Therefore, the moral-hazard constraint reduces to

$$\delta \int_0^\theta A^*(y)\,\mathrm{d}y \geq 0,$$

which was already verified. Thus the candidate mechanism is feasible.

We now prove optimality. Define

$$\begin{aligned}
&\chi_L \equiv \frac{1}{2\,\delta}, \\
&\gamma(\theta) \equiv 1 - \frac{1}{2\,\delta}, && \forall \theta \in [0,1], \\
&\beta(\theta) \equiv \frac{\delta\,\pi + 1}{2\,\delta} - \theta(\omega + \pi), && \forall \theta \in [0,\tau], \\
&\sigma(\theta) \equiv -\frac{\delta\,\pi + 1}{2\,\delta} + \theta(\omega + \pi), && \forall \theta \in [\tau,1], \\
&\xi(\theta) \equiv \frac{1}{2}, && \forall \theta \in [0,1].
\end{aligned}$$

All other multipliers are set equal to zero. Since $\delta \geq 1/2$, we have $\gamma(\theta) \geq 0$. Moreover, β is decreasing, σ is increasing, and $\beta(\tau) = \sigma(\tau) = 0$. Therefore, we have $\beta(\theta) \geq 0$ on $[0,\tau]$ and $\sigma(\theta) \geq 0$ on $[\tau,1]$.

The definitions above imply the following coefficient-matching identities:

$$\begin{aligned}
&\chi_L + \int_0^1 \gamma(\theta)\,\mathrm{d}\theta = 1, \\
&-(1 - \delta\,\pi\,\theta)\gamma(\theta) - \delta\,\pi \int_\theta^1 \gamma(y)\,\mathrm{d}y - \beta(\theta) = w_q(\theta), && \forall \theta \in [0,\tau], \\
&-(1 - \delta\,\pi\,\theta)\gamma(\theta) - \delta\,\pi \int_\theta^1 \gamma(y)\,\mathrm{d}y + \sigma(\theta) = w_q(\theta), && \forall \theta \in [\tau,1], \\
&\delta\,\theta\,\gamma(\theta) - \delta \int_\theta^1 \gamma(y)\,\mathrm{d}y - \xi(\theta) = w_S(\theta), && \forall \theta \in [0,1], \\
&\delta(1-\theta)\gamma(\theta) + \delta \int_\theta^1 \gamma(y)\,\mathrm{d}y = w_F(\theta), && \forall \theta \in [0,1].
\end{aligned}$$

Therefore, for any feasible mechanism,

$$\begin{aligned}
&\chi_L[-u(0)] + \int_0^\tau \beta(\theta)[-q(\theta)]\,\mathrm{d}\theta + \int_\tau^1 \sigma(\theta)[q(\theta) - 1]\,\mathrm{d}\theta + \int_\tau^1 \sigma(\theta)\,\mathrm{d}\theta + \int_0^1 \xi(\theta)[-m_S(\theta)]\,\mathrm{d}\theta \\
&\quad - \int_0^1 \gamma(\theta) \left[u(0) + \delta \int_0^\theta A(y)\,\mathrm{d}y + q(\theta) - \delta\,\theta[m_S(\theta) + \pi\,q(\theta)] - \delta(1-\theta)m_F(\theta) \right] \mathrm{d}\theta \\
&= -u(0) + \int_0^1 [w_q(\theta)q(\theta) + w_S(\theta)m_S(\theta) + w_F(\theta)m_F(\theta)]\,\mathrm{d}\theta.
\end{aligned}$$

Because all multipliers are nonnegative and the residuals multiplied by them are nonpositive for any feasible mechanism, the objective function is bounded above by

$$\int_\tau^1 \sigma(\theta)\,\mathrm{d}\theta.$$

A direct calculation gives

$$\int_{\tau}^{1} \sigma(\theta)\,\mathrm{d}\theta = \frac{1-\tau}{2}\left[\omega(1+\tau)+\pi\,\tau-\frac{1}{\delta}\right].$$

It remains to verify that the candidate mechanism attains this upper bound. Substituting the candidate mechanism into the objective function yields

$$\int_{\tau}^{1}\left[\theta\,\omega-(1-\theta)\frac{1/\delta-\tau\,\pi}{1-\tau}\right]\mathrm{d}\theta.$$

This equals

$$\frac{1-\tau}{2}\left[\omega(1+\tau)+\pi\,\tau-\frac{1}{\delta}\right],$$

which is exactly the dual upper bound. Hence, the candidate primal mechanism attains the upper bound and is optimal.

This completes the proof. □

Proof of Proposition 7. In the relevant region, the normalized boundary is

$$B_5(\pi)=\frac{\pi}{2}(\delta\,\pi-1).$$

Since $\omega\geq 0$, the condition $\omega<B_5(\pi)$ implies $\delta\,\pi>1$, and hence $\tau=\bar{\tau}\in(0,1)$.

Consider the candidate commercial mechanism

$$q^*(\theta)=\mathbf{1}_{\{\theta\geq\tau\}},\qquad m_0^*(\theta)=m_S^*(\theta)=m_F^*(\theta)=0,\qquad \forall\theta\in[0,1],$$

where $\tau=\bar{\tau}=1/(\delta\,\pi)$. This mechanism is feasible. Indeed,

$$A^*(\theta)=\pi\,\mathbf{1}_{\{\theta\geq\tau\}},$$

which is nondecreasing, and

$$\delta\int_0^{\theta}A^*(y)\,\mathrm{d}y=\delta\,\pi\,(\theta-\tau)^+\geq 0.$$

The upfront payment implied by the envelope formula is zero: for $\theta<\tau$ this is immediate, while for $\theta\geq\tau$,

$$\delta\,\pi(\theta-\tau)+1-\delta\,\pi\,\theta=1-\delta\,\pi\,\tau=0.$$

Finally, for any reported type $\hat{\theta}\geq\tau$, the right-hand side of the moral-hazard constraint (EC.7) equals

$$\delta\,\pi(\hat{\theta}-\tau)+(1-\alpha)-\delta\,\pi\,\hat{\theta}=1-\alpha-\delta\,\pi\,\tau=-\alpha\leq 0,$$

and for $\hat{\theta}<\tau$ it equals zero. Hence the moral-hazard constraint reduces to

$$\delta\int_0^{\theta}A^*(y)\,\mathrm{d}y\geq 0,$$

which was already verified.

We now prove optimality. Let

$$\omega_0 \equiv B_5(\pi) = \frac{\pi}{2}(\delta\,\pi - 1).$$

At $\omega = \omega_0$, Proposition 6 applies. Its threshold is

$$\frac{\delta\,\pi + 1}{2\,\delta(\omega_0 + \pi)} = \frac{1}{\delta\,\pi} = \tau,$$

and its failure payment is

$$\pi\,\frac{\bar{\tau} - \tau}{1 - \tau} = 0.$$

Therefore, at $\omega = \omega_0$, the optimal mechanism in Proposition 6 coincides with the commercial mechanism above. Hence the commercial mechanism is optimal at ω_0.

Fix any $\omega \in [0, \omega_0)$. Let x denote the commercial mechanism above, and let $z = \{q, m_0, m_S, m_F\}$ be any feasible mechanism. Define

$$Q(z) \equiv \int_0^1 \theta\,q(\theta)\,\mathrm{d}\theta$$

and

$$M(z) \equiv \int_0^1 [m_0(\theta) + \theta\,m_S(\theta) + (1-\theta)m_F(\theta)]\,\mathrm{d}\theta.$$

Then the funder's objective under welfare ω can be written as

$$J_\omega(z) = \omega\,Q(z) - M(z).$$

For the commercial mechanism x,

$$Q(x) = \int_\tau^1 \theta\,\mathrm{d}\theta, \qquad M(x) = 0, \qquad J_\omega(x) = \omega\,Q(x).$$

We compare any feasible z with x. If $Q(z) \le Q(x)$, then, since $M(z) \ge 0$,

$$J_\omega(z) = \omega\,Q(z) - M(z) \le \omega\,Q(x) = J_\omega(x).$$

If $Q(z) > Q(x)$, optimality of x at ω_0 implies

$$\omega_0\,Q(z) - M(z) \le \omega_0\,Q(x),$$

or equivalently,

$$M(z) \ge \omega_0\,[Q(z) - Q(x)].$$

Therefore,

$$\begin{aligned} J_\omega(z) &= \omega\,Q(z) - M(z) \\ &\le \omega\,Q(z) - \omega_0[Q(z) - Q(x)] \\ &= \omega\,Q(x) + (\omega - \omega_0)[Q(z) - Q(x)] \\ &\le \omega\,Q(x) \\ &= J_\omega(x), \end{aligned}$$

where the last inequality follows from $\omega < \omega_0$ and $Q(z) > Q(x)$. Thus, no feasible mechanism can outperform the commercial mechanism for any $\omega < \omega_0$.

This completes the proof. □

Proof of Corollary 1. We first study the testing threshold. In Region (1), the no-testing policy is optimal, so the threshold can be interpreted as $\tau = 1$, which is constant.

In Region (2), the testing threshold τ is determined by

$$h_2(\tau;\omega,\pi) \equiv 2\,\delta\,(\omega+\pi)\tau^3 + (\alpha - 2\,\delta\,\alpha - 1)\tau^2 + \alpha - 1 = 0.$$

As shown in the proof of Proposition 2, the relevant root lies on the increasing branch of h_2, so

$$\frac{\partial h_2}{\partial \tau}(\tau;\omega,\pi) > 0.$$

Therefore, for $x \in \{\omega,\pi\}$, the implicit function theorem gives

$$\frac{\partial \tau}{\partial x} = -\frac{2\,\delta\,\tau^3}{\partial h_2/\partial \tau} \leq 0.$$

Thus the threshold in Region (2) is non-increasing in both ω and π.

In Region (3), the threshold is

$$\tau = \frac{1-\alpha}{\delta\,\pi},$$

which is decreasing in π and constant in ω.

In Region (4), the threshold τ is determined by

$$h_4(\tau;\omega,\pi) \equiv 2\,\delta\,(\omega+\pi)\tau^3 - [2\,\delta\,\pi(1-\delta) + 2\,\delta\,\alpha - \alpha + 1]\,\tau^2 - (1-\alpha)(2\,\delta - 1) = 0.$$

As shown in the proof of Proposition 4, the relevant root lies on the increasing branch of h_4, so

$$\frac{\partial h_4}{\partial \tau} = 2\,\tau\,[3\,\delta\,(\omega+\pi)\tau - [2\,\delta\,\pi(1-\delta) + 2\,\delta\,\alpha - \alpha + 1]] > 0.$$

The derivative with respect to ω is

$$\frac{\partial h_4}{\partial \omega} = 2\,\delta\,\tau^3,$$

and hence

$$\frac{\partial \tau}{\partial \omega} = -\frac{2\,\delta\,\tau^3}{\partial h_4/\partial \tau} \leq 0.$$

The derivative with respect to π, however, is

$$\frac{\partial h_4}{\partial \pi} = 2\,\delta\,\tau^3 - 2\,\delta(1-\delta)\tau^2 = 2\,\delta\,\tau^2[\tau - (1-\delta)].$$

Therefore,

$$\frac{\partial \tau}{\partial \pi} = -\frac{2\,\delta\,\tau^2[\tau - (1-\delta)]}{\partial h_4/\partial \tau}.$$

Since $\partial h_4/\partial\tau > 0$, the threshold in Region (4) is non-increasing in π if and only if

$$\tau \geq 1-\delta.$$

This condition is not always satisfied. To see when it holds, evaluating h_4 at $1-\delta$ yields

$$\begin{aligned} h_4(1-\delta;\omega,\pi) &= 2\,\delta\,(\omega+\pi)(1-\delta)^3 - [2\,\delta\,\pi(1-\delta) + 2\,\delta\,\alpha - \alpha + 1]\,(1-\delta)^2 - (1-\alpha)(2\,\delta-1) \\ &= 2\,\delta\,\omega(1-\delta)^3 - (2\,\delta\,\alpha - \alpha + 1)\,(1-\delta)^2 - (1-\alpha)(2\,\delta-1) \\ &= \delta\left[2\,\omega(1-\delta)^3 - \left[(1-\alpha)\delta + 2\,\alpha(1-\delta)^2\right]\right]. \end{aligned}$$

Because $h_4(\theta) \leq 0$ for $\theta \leq \tau$ and $h_4(\theta) \geq 0$ for $\theta \geq \tau$, we have, for $\delta < 1$,

$$\tau \geq 1-\delta \quad\Longleftrightarrow\quad h_4(1-\delta;\omega,\pi) \leq 0 \quad\Longleftrightarrow\quad \omega \leq \omega^\dagger \equiv \frac{(1-\alpha)\delta + 2\,\alpha(1-\delta)^2}{2(1-\delta)^3}.$$

The qualification is non-vacuous. Under the normalization $c = 1$, the point $(\pi,\omega) = (1/\delta, 0)$ is a boundary point of Region (4), and $\omega^\dagger > 0$ for $\delta < 1$. Hence, for π slightly above $1/\delta$, Region (4) contains parameter values with $\omega < \omega^\dagger$. A direct calculation gives

$$B_2(\pi) > \omega^\dagger \quad\Longleftrightarrow\quad \delta\,\pi > \frac{1-\alpha}{1-\delta},$$

and

$$B_3(\pi) < \omega^\dagger \quad\Longleftrightarrow\quad \delta\,\pi < \frac{1-\alpha\delta}{1-\delta}.$$

Since

$$\frac{1-\alpha\delta}{1-\delta} - \frac{1-\alpha}{1-\delta} = \alpha,$$

there is a nonempty interval of x values whenever $\alpha > 0$. In particular, for

$$\max\left\{1, \frac{1-\alpha}{1-\delta}\right\} < \delta\,\pi < \frac{1-\alpha\delta}{1-\delta},$$

we have

$$B_3(\pi) < \omega^\dagger < B_2(\pi),$$

so Region (4) also contains parameter values with $\omega > \omega^\dagger$.

If $\delta = 1$, then $1-\delta = 0$, so $\tau \geq 1-\delta$ holds automatically and the threshold is non-increasing in π throughout Region (4). If $\delta < 1$, the sign changes with the cutoff $\omega^\dagger$: the threshold is locally non-increasing in π for $\omega \leq \omega^\dagger$ and locally increasing for $\omega > \omega^\dagger$. Hence the threshold is not always non-increasing in π within Region (4).

In Region (5), the threshold is

$$\tau = \frac{1-\alpha}{\delta\,\pi - \alpha},$$

which is decreasing in π and constant in ω.

In Region (6), the threshold is

$$\tau = \frac{\delta\,\pi + 1}{2\,\delta\,(\omega + \pi)}.$$

Thus

$$\frac{\partial \tau}{\partial \omega} = -\frac{\delta\,\pi + 1}{2\,\delta\,(\omega + \pi)^2} < 0.$$

Moreover,

$$\frac{\partial \tau}{\partial \pi} = \frac{\delta\,\omega - 1}{2\,\delta\,(\omega + \pi)^2},$$

so no monotonicity claim in π is made for Region (6).

Finally, in Region (7), the threshold is

$$\tau = \frac{1}{\delta\,\pi},$$

which is decreasing in π and constant in ω. This proves parts (a) and (b).

We next consider expected payoffs. Since ω enters only the funder's objective and has a nonnegative coefficient $\theta q(\theta)$, while the feasible set is independent of ω, the funder's optimal expected payoff is non-decreasing in ω. The firm's ex ante expected payoff is also non-decreasing in ω. To see this, use the envelope representation of the firm's truthful rent. For a threshold mechanism with threshold τ, the marginal participating type receives zero rent, and for any $\theta \geq \tau$,

$$u_1^*(\theta, \theta) = \delta \int_\tau^\theta A^*(y)\ \mathrm{d}y,$$

where

$$A^*(y) = m_S^*(y) - m_F^*(y) + \pi q^*(y).$$

In Regions (2) and (4), for all participating types,

$$A^*(y) = \frac{(1-\alpha)c}{\delta\,\tau},$$

and hence

$$u_1^*(\theta, \theta) = (1-\alpha)c\left(\frac{\theta}{\tau} - 1\right), \qquad \theta \geq \tau.$$

This expression is weakly increasing as τ decreases. In Region (6),

$$A^*(y) = \pi - m_F^* = \frac{\pi(1-\bar{\tau})}{1-\tau},$$

and therefore

$$u_1^*(\theta, \theta) = \delta\,\pi(1-\bar{\tau})\,\frac{\theta - \tau}{1-\tau}, \qquad \theta \geq \tau.$$

For any fixed $\theta \in [\tau, 1]$,

$$\frac{\partial}{\partial \tau}\left[\delta\,\pi(1-\bar{\tau})\,\frac{\theta-\tau}{1-\tau}\right] = \delta\,\pi(1-\bar{\tau})\,\frac{\theta - 1}{(1-\tau)^2} \leq 0.$$

Thus, when ω increases and the optimal threshold weakly decreases, the rent of every type that was already participating weakly increases, and newly included types receive nonnegative rent. In Regions (3), (5), and (7), the optimal threshold and payments do not depend on ω, so the firm's payoff is unchanged. Hence the firm's interim payoff, and therefore its ex ante expected payoff, is non-decreasing in ω.

For the funder, fix the optimal policy at commercial profit π_0, and consider a higher profit $\pi_1 > \pi_0$. If the original policy uses positive funding, the funder can maintain the same testing threshold at π_1 with weakly lower transfers, because the firm now receives a larger commercial return from success. The participation and effort constraints are therefore weakly easier to satisfy. If the original policy uses no funding, the funder can still provide no funding after π increases. In the commercial region, the self-funding threshold $\bar{\tau} = c/(\delta\,\pi)$ decreases, so more types test without requiring any payment from the funder; in the no-testing region, the funder can always replicate the previous no-payment outcome. Thus, after an increase in π, the funder can replicate the old outcome at weakly lower cost or obtain additional commercially funded participation at zero cost. Since the optimal mechanism under the higher π can do at least as well as this replicated policy, the funder's expected payoff is non-decreasing in π.

For Regions (1)–(3), (5), and (7), the firm's ex ante expected payoff is non-decreasing in π because the testing threshold weakly decreases and the firm's interim payoff weakly increases for every previously participating type.

The firm's ex ante expected payoff is not necessarily non-decreasing in π in Region (4). To see this, note that in Region (4),

$$m_S^*(\theta) = 0, \qquad m_F^*(\theta) = \pi - \frac{1-\alpha}{\delta\,\tau}, \qquad \theta \geq \tau,$$

so

$$m_S^*(\theta) - m_F^*(\theta) + \pi q^*(\theta) = \frac{1-\alpha}{\delta\,\tau}, \qquad \theta \geq \tau.$$

Hence the interim rent of type $\theta \geq \tau$ is

$$u_1(\theta,\theta) = \delta \int_\tau^\theta \frac{1-\alpha}{\delta\,\tau}\,\mathrm{d}y = (1-\alpha)\left(\frac{\theta}{\tau} - 1\right),$$

and the firm's ex ante expected payoff is

$$\int_\tau^1 u_1(\theta,\theta)\,\mathrm{d}\theta = \frac{(1-\alpha)(1-\tau)^2}{2\,\tau}.$$

This expression is decreasing in τ. Since, as shown above, $\partial\tau/\partial\pi > 0$ whenever $\tau < 1-\delta$, the firm's ex ante expected payoff can decrease with π in that part of Region (4).

In Region (6), the firm's ex ante expected payoff is

$$\mathbb{E}[u_1(\theta)] = \int_\tau^1 \left[-1 + \delta\,\theta\,\pi + \delta\,(1-\theta)\,m_F^*\right]\,\mathrm{d}\theta = \frac{(\delta\,\pi - 1)(2\,\delta\,\omega + \delta\,\pi - 1)}{4\,\delta\,(\omega + \pi)}.$$

Differentiating with respect to π gives

$$\frac{\mathrm{d}}{\mathrm{d}\pi}\,\mathbb{E}[u_1(\theta)] = \frac{1}{4\,\delta}\left[\delta^2 + \frac{\delta^2\omega^2 - 1}{(\omega+\pi)^2}\right] \geq 0,$$

where the inequality follows from

$$\delta^2(\omega+\pi)^2 - 1 \geq \delta^2\pi^2 - 1 \geq 0$$

in Region (6).

This completes the proof. □

EC.4. Proofs of Section 3.2

Proof of Proposition 8. We first prove case (b). Let

$$\tau \equiv \frac{1+\bar{\tau}}{\omega/(\delta\,\pi) + 2} = \frac{\delta\,\pi + 1}{\omega + 2\,\delta\,\pi}.$$

The condition $\omega + \delta\,\pi > 1$ implies $\tau < 1$. The proposed upfront payment is

$$m_0^* = 1 - \delta\,\pi\,\tau.$$

Using the expression for τ, we have

$$m_0^* = \frac{\omega - \tilde{B}_5(\pi)}{\omega + 2\,\delta\,\pi} \geq 0,$$

where the inequality follows from $\omega \geq \tilde{B}_5(\pi)$. Hence the limited-liability constraint for m_0 holds.

Under the candidate mechanism in case (b),

$$q^*(\theta) = \mathbf{1}_{\{\theta \geq \tau\}}, \qquad m_0^*(\theta) = m_0^*\,\mathbf{1}_{\{\theta\geq\tau\}}, \qquad m_S^*(\theta) = m_F^*(\theta) = 0.$$

Thus

$$A^*(\theta) = \pi\,\mathbf{1}_{\{\theta\geq\tau\}},$$

which is nondecreasing, and

$$\delta \int_0^\theta A^*(y)\,\mathrm{d}y = \delta\,\pi\,(\theta - \tau)^+ \geq 0.$$

The upfront payment implied by the envelope formula equals zero for $\theta < \tau$, while for $\theta \geq \tau$ it equals

$$\delta\,\pi(\theta - \tau) + 1 - \delta\,\pi\,\theta = 1 - \delta\,\pi\,\tau = m_0^*.$$

Finally, for any reported type $\hat{\theta} \geq \tau$, the right-hand side of the moral-hazard constraint (EC.7) equals

$$\delta\,\pi(\hat{\theta}-\tau)+(1-\alpha)-\delta\,\pi\,\hat{\theta}=1-\alpha-\delta\,\pi\,\tau=m_0^*-\alpha.$$

Moreover,

$$m_0^*-\alpha=\frac{(1-\alpha)[\omega-\tilde{B}_2(\pi)]}{\omega+2\,\delta\,\pi}\leq 0,$$

where the inequality follows from $\omega \leq \tilde{B}_2(\pi)$. For any reported type $\hat{\theta} < \tau$, the right-hand side of (EC.7) is zero. Therefore, the moral-hazard constraint reduces to

$$\delta \int_0^\theta A^*(y)\,\mathrm{d}y \geq 0,$$

which has already been verified. Thus the candidate mechanism in case (b) is feasible.

We now prove its optimality. Define

$$\begin{aligned}
&\chi_L \equiv 1,\\
&\beta(\theta) \equiv -w_q(\theta), \quad \forall \theta \in [0,\tau],\\
&\sigma(\theta) \equiv w_q(\theta), \quad \forall \theta \in [\tau,1],\\
&\xi(\theta) \equiv -w_S(\theta), \quad \forall \theta \in [0,1],\\
&\nu(\theta) \equiv -w_F(\theta), \quad \forall \theta \in [0,1].
\end{aligned}$$

All other multipliers are set equal to zero. Since $\tau=(\delta\,\pi+1)/(\omega+2\,\delta\,\pi)$, we have $w_q(\tau)=0$. Thus $\beta(\theta)\geq 0$ on $[0,\tau]$ and $\sigma(\theta)\geq 0$ on $[\tau,1]$. Because $\delta<1/2$, we also have $w_S(\theta)\leq 0$ and $w_F(\theta)\leq 0$ for all $\theta\in[0,1]$, so ξ and ν are nonnegative.

For any feasible mechanism, we have

$$\begin{aligned}
&\chi_L[-u(0)]+\int_0^\tau \beta(\theta)[-q(\theta)]\,\mathrm{d}\theta+\int_\tau^1 \sigma(\theta)[q(\theta)-1]\,\mathrm{d}\theta+\int_\tau^1 \sigma(\theta)\,\mathrm{d}\theta\\
&\quad+\int_0^1 \xi(\theta)[-m_S(\theta)]\,\mathrm{d}\theta+\int_0^1 \nu(\theta)[-m_F(\theta)]\,\mathrm{d}\theta\\
&=-u(0)+\int_0^1 [w_q(\theta)q(\theta)+w_S(\theta)m_S(\theta)+w_F(\theta)m_F(\theta)]\,\mathrm{d}\theta.
\end{aligned}$$

Because all multipliers are nonnegative and the residuals multiplied by them are nonpositive for any feasible mechanism, the objective function is bounded above by

$$\int_\tau^1 \sigma(\theta)\,\mathrm{d}\theta.$$

The candidate mechanism attains this upper bound because, under the candidate,

$$-u(0)+\int_0^1 [w_q(\theta)q^*(\theta)+w_S(\theta)m_S^*(\theta)+w_F(\theta)m_F^*(\theta)]\,\mathrm{d}\theta=\int_\tau^1 w_q(\theta)\,\mathrm{d}\theta=\int_\tau^1 \sigma(\theta)\,\mathrm{d}\theta.$$

Hence the candidate mechanism in case (b) is optimal.

We next prove case (c). If $\tilde{B}_5(\pi) \le 0$, then the region $\omega < \tilde{B}_5(\pi)$ is empty because $\omega \ge 0$. Hence suppose $\tilde{B}_5(\pi) > 0$, which implies $\delta\,\pi > 1$. Consider the mechanism

$$q^*(\theta) = \mathbf{1}_{\{\theta \ge \bar{\tau}\}}, \qquad m_0^*(\theta) = m_S^*(\theta) = m_F^*(\theta) = 0, \qquad \forall \theta \in [0,1].$$

The feasibility of this mechanism is identical to the feasibility verification in Proposition 7: since $\bar{\tau} = 1/(\delta\,\pi)$, the envelope-implied upfront payment is zero for all θ, and the moral-hazard constraint reduces to $\delta \int_0^\theta \pi\,\mathbf{1}_{\{y \ge \bar{\tau}\}}\,\mathrm{d}y \ge 0$.

Let

$$\omega_0 \equiv \tilde{B}_5(\pi) = \delta\,\pi(\delta\,\pi - 1).$$

At $\omega = \omega_0$, the threshold in case (b) becomes

$$\frac{\delta\,\pi + 1}{\omega_0 + 2\,\delta\,\pi} = \frac{\delta\,\pi + 1}{\delta\,\pi(\delta\,\pi - 1) + 2\,\delta\,\pi} = \frac{1}{\delta\,\pi} = \bar{\tau},$$

and the upfront payment in case (b) becomes

$$1 - \delta\,\pi\,\bar{\tau} = 0.$$

Therefore, the case-(b) mechanism at $\omega = \omega_0$ coincides with the commercial mechanism above, so the commercial mechanism is optimal at ω_0.

The argument in the proof of Proposition 7 now applies verbatim. The feasible set does not depend on ω, and for any feasible mechanism z, the funder's objective can be written as

$$J_\omega(z) = \omega\,Q(z) - M(z),$$

where $Q(z) = \int_0^1 \theta\,q(\theta)\,\mathrm{d}\theta$ and $M(z) = \int_0^1 [m_0(\theta) + \theta\,m_S(\theta) + (1-\theta)m_F(\theta)]\,\mathrm{d}\theta \ge 0$. Since the commercial mechanism is optimal at ω_0, the same comparison as in Proposition 7 shows that it remains optimal for every $\omega < \omega_0$. Hence the commercial mechanism is optimal throughout case (c).

It remains to prove case (a). Let

$$\tau = \frac{1-\alpha}{\delta\,\pi}.$$

Whenever the interval $\tilde{B}_2(\pi) \le \omega < B_1(\pi)$ is nonempty, we have $\tau \in [0,1]$. The candidate mechanism is

$$q^*(\theta) = \mathbf{1}_{\{\theta \ge \tau\}}, \qquad m_0^*(\theta) = \alpha\,\mathbf{1}_{\{\theta \ge \tau\}}, \qquad m_S^*(\theta) = m_F^*(\theta) = 0.$$

Under this mechanism,

$$A^*(\theta) = \pi\,\mathbf{1}_{\{\theta \ge \tau\}},$$

which is nondecreasing, and

$$\delta \int_0^{\theta} A^*(y)\, \mathrm{d}y = \delta\, \pi(\theta - \tau)^+ \geq 0.$$

The envelope-implied upfront payment equals zero for $\theta < \tau$, while for $\theta \geq \tau$ it equals

$$\delta\, \pi(\theta - \tau) + 1 - \delta\, \pi\, \theta = 1 - \delta\, \pi\, \tau = \alpha.$$

For any reported type $\hat{\theta} \geq \tau$, the right-hand side of the moral-hazard constraint (EC.7) equals

$$\delta\, \pi(\hat{\theta} - \tau) + (1 - \alpha) - \delta\, \pi\, \hat{\theta} = 1 - \alpha - \delta\, \pi\, \tau = 0,$$

and for $\hat{\theta} < \tau$ it also equals zero. Thus the moral-hazard constraint reduces to

$$\delta \int_0^{\theta} A^*(y)\, \mathrm{d}y \geq 0.$$

The candidate mechanism in case (a), given by (17), does not depend on ω within the interval $[\tilde{B}_2(\pi), B_1(\pi)]$. We have shown that it is optimal at both endpoints $\omega_L = \tilde{B}_2(\pi)$ and $\omega_H = B_1(\pi)$. Therefore, by Lemma EC.2, the same mechanism is optimal for every $\omega \in [\tilde{B}_2(\pi), B_1(\pi)]$. This proves case (a).

Combining the three cases completes the proof. □

Proof of Corollary 2. The threshold monotonicity results for Regions (1)–(3) and (7) follow from the same arguments as in the proof of Corollary 1. Although Corollary 1 is stated for $\delta \geq 1/2$, the relevant threshold formulas in Regions (1)–(3) and (7) are unchanged when $\delta < 1/2$, and the monotonicity arguments for those regions do not rely on $\delta \geq 1/2$.

It remains to check the new Region (3′) in Proposition 8(b). In this region,

$$\tau = \frac{\delta\, \pi + c}{\omega + 2\, \delta\, \pi}.$$

Since

$$\frac{\partial \tau}{\partial \omega} = -\frac{\delta\, \pi + c}{(\omega + 2\, \delta\, \pi)^2} < 0,$$

the threshold is decreasing in ω in Region (3′). However,

$$\frac{\partial \tau}{\partial \pi} = \frac{\delta(\omega - 2c)}{(\omega + 2\, \delta\, \pi)^2},$$

so the threshold in Region (3′) need not be non-increasing in π. This proves parts (a) and (b).

We next prove part (c). For the funder, the same argument as in the proof of Corollary 1 applies: the optimal mechanism under the higher π can do at least as well as this replicated policy, and hence, the funder's expected payoff is non-decreasing in π. Similarly, since ω appears only in the funder's objective and has a nonnegative coefficient $\theta q(\theta)$, increasing ω leaves the feasible set unchanged

and weakly increases the payoff from any fixed feasible mechanism. Therefore, the funder's optimal expected payoff is non-decreasing in both π and ω.

For the firm, the payoff monotonicity in Regions (1)–(3) and (7) follows from the same argument as in Corollary 1.

It remains to verify the new Region (3′). In this region,

$$m_0^* = c - \delta\,\pi\,\tau, \qquad m_S^* = m_F^* = 0, \qquad \tau = \frac{\delta\,\pi + c}{\omega + 2\,\delta\,\pi}.$$

The firm's interim payoff for a participating type is

$$-c + m_0^* + \delta\,\theta\,\pi = \delta\,\pi(\theta - \tau), \qquad \theta \geq \tau.$$

Therefore, the firm's ex ante expected payoff is

$$\mathbb{E}[u_1(\theta)] = \int_\tau^1 \delta\,\pi(\theta - \tau)\,\mathrm{d}\theta = \frac{\delta\,\pi}{2}\left(\frac{\omega + \delta\,\pi - c}{\omega + 2\,\delta\,\pi}\right)^2.$$

Differentiating with respect to π yields

$$\frac{\mathrm{d}}{\mathrm{d}\pi}\,\mathbb{E}[u_1(\theta)] = \frac{\delta(\omega + \delta\,\pi - c)\,[\omega(\omega + \delta\,\pi - c) + 2\,\delta\,\pi(\delta\,\pi + c)]}{2(\omega + 2\,\delta\,\pi)^3} \geq 0,$$

where the inequality follows from the Region (3′) condition $\omega + \delta\,\pi > c$. Differentiating with respect to ω yields

$$\frac{\partial}{\partial\omega}\,\mathbb{E}[u_1(\theta)] = \frac{\delta\,\pi(\omega + \delta\,\pi - c)(\delta\,\pi + c)}{(\omega + 2\,\delta\,\pi)^3} \geq 0.$$

Thus, the firm's ex ante expected payoff is non-decreasing in both π and ω in Region (3′).

Finally, adjacent mechanisms coincide at the relevant region boundaries: Region (2) coincides with Region (3) at $B_1(\pi)$, Region (3) coincides with Region (3′) at $\tilde{B}_2(\pi)$, and Region (3′) coincides with Region (7) at $\tilde{B}_5(\pi)$. Hence, the payoff functions are continuous across boundaries. Combining the within-region monotonicity results proves that both the funder's and the firm's ex ante expected payoffs are non-decreasing in π and in ω.

This completes the proof. □

EC.5. Proofs of Section 3.3

Proof of Lemma 2. It is straightforward to verify that the funder's expected payoff is reduced to

$$\int_{\check{\tau}}^{\theta_H} [\theta\,\omega - \check{m}_0 - \theta\,\check{m}_S - (1-\theta)\,\check{m}_F]\,\mathrm{d}F(\theta)$$

under the threshold mechanism (19).

Invoking Lemma EC.1, we know that the (IC) constraint is equivalent to (EC.2), (EC.3), and (EC.4). By substituting the decision variables with the threshold mechanism defined in (19), the incentive constraints are further reduced as follows:

$$\check{m}_S - \check{m}_F + \pi \geq 0, \text{ and} \tag{22}$$

$$\check{m}_0 + \delta\,\check{\tau}\,\check{m}_S + \delta\,(1-\check{\tau})\,\check{m}_F = c - \delta\,\pi\,\check{\tau}, \tag{23}$$

$$\check{m}_0 + \delta\,\check{m}_F \leq \alpha\,c, \tag{24}$$

$$-c + \check{m}_0 + \delta\,\theta\,(\check{m}_S + \pi) + \delta\,(1-\theta)\,\check{m}_F \geq -\alpha\,c + \check{m}_0 + \delta\,\check{m}_F, \quad \forall \theta \in [\theta_L, \theta_H], \tag{EC.12}$$

where (22) and (23) come from (EC.3) and (EC.4), respectively. Constraints (24) and (EC.12) are derived from the moral hazard constraint (EC.2), ensuring that firms of types below and above $\check{\tau}$ have no incentive to (misreport and) shirk. Constraint (EC.12) is implied by (22) and (24).

The participation constraint (IR) and the feasibility condition for q hold automatically by construction. The nonnegativity condition (12) boils down to (25).

This completes the proof. □

Proof of Proposition 9. It is straightforward to verify the feasibility of our candidate solution. Given $\check{\tau}$, the term $\omega \int_{\check{\tau}}^{\theta_H} \theta\,\mathrm{d}F(\theta)$ is constant with respect to the decision variables. For notational simplicity, we omit this constant from the objective function. The simplified "objective function" is thus

$$\int_{\check{\tau}}^{\theta_H} [-\check{m}_0 - \theta\,\check{m}_S - (1-\theta)\,\check{m}_F]\,\mathrm{d}F(\theta).$$

In case 1, define

$$\phi = -\frac{1}{\delta\,\check{\tau}} \int_{\check{\tau}}^{\theta_H} \theta\,\mathrm{d}F(\theta), \quad \eta = -\left[1 - F(\check{\tau})\right] + \frac{1}{\delta\,\check{\tau}} \int_{\check{\tau}}^{\theta_H} \theta\,\mathrm{d}F(\theta), \text{ and } \nu = (1-\delta)\left[1 - F(\check{\tau})\right] \geq 0.$$

The nonnegativity of η follows from its monotonicity in $\check{\tau}$ and

$$\left[-\left[1 - F(\check{\tau})\right] + \frac{1}{\delta\,\check{\tau}} \int_{\check{\tau}}^{\theta_H} \theta\,\mathrm{d}F(\theta)\right]_{\check{\tau}=\theta_H} = 0.$$

We have

$$\phi\,[\check{m}_0 + \delta\,\check{\tau}\,\check{m}_S + \delta\,(1-\check{\tau})\,\check{m}_F] + \eta\,(\check{m}_0 + \delta\,\check{m}_F) - \nu\,\check{m}_F = (\phi+\eta)\,\check{m}_0 + \delta\,\check{\tau}\,\phi\,\check{m}_S + [\delta\,(1-\check{\tau})\,\phi + \delta\,\eta - \nu]\,\check{m}_F,$$

and hence, the objective function is bounded by $(c - \delta\,\pi\,\check{\tau})\,\phi + \alpha\,c\,\eta$. Substituting our candidate policy into the objective function attains this upper bound, establishing its optimality.

In case 2, define

$$\begin{aligned}
\phi &= \frac{1}{\delta\,\check{\tau}} \left[-\delta\,\left[1 - F(\check{\tau})\right] + \int_{\check{\tau}}^{\theta_H} (1-\theta)\,\mathrm{d}F(\theta)\right], \\
\eta &= \frac{1-\check{\tau}}{\check{\tau}}\,\left[1 - F(\check{\tau})\right] - \frac{1}{\delta\,\check{\tau}} \int_{\check{\tau}}^{\theta_H} (1-\theta)\,\mathrm{d}F(\theta), \text{ and} \\
\xi &= (1-\delta)\,\left[1 - F(\check{\tau})\right] \geq 0,
\end{aligned}$$

where the nonnegativity of η follows from $\mathcal{H}(\check{\tau}) \geq 0$. We have

$$\phi\left[\check{m}_0 + \delta\,\check{\tau}\,\check{m}_S + \delta\,(1-\check{\tau})\,\check{m}_F\right] + \eta\,(\check{m}_0 + \delta\,\check{m}_F) - \xi\,\check{m}_S = (\phi+\eta)\,\check{m}_0 + (\delta\,\check{\tau}\,\phi - \xi)\,\check{m}_S + \left[\delta\,(1-\check{\tau})\,\phi + \delta\,\eta\right]\check{m}_F,$$

and hence, the objective function is bounded by $(c - \delta\,\pi\,\check{\tau})\,\phi + \alpha\,c\,\eta$. Substituting our candidate policy into the objective function attains this upper bound, establishing its optimality.

In case 3, define

$$\begin{aligned}
\phi &= -\left[1 - F(\check{\tau})\right], \\
\xi &= -\delta\,\check{\tau}\,\left[1 - F(\check{\tau})\right] + \int_{\check{\tau}}^{\theta_H} \theta\,\mathrm{d}F(\theta) \geq 0, \text{ and} \\
\nu &= -\delta\,(1-\check{\tau})\,\left[1 - F(\check{\tau})\right] + \int_{\check{\tau}}^{\theta_H} (1-\theta)\,\mathrm{d}F(\theta),
\end{aligned}$$

where the nonnegativity of ν follows from $\mathcal{H}(\check{\tau}) \leq 0$. The nonnegativity of ξ follows from that of η in case 1. We have

$$\phi\left[\check{m}_0 + \delta\,\check{\tau}\,\check{m}_S + \delta\,(1-\check{\tau})\,\check{m}_F\right] - \xi\,\check{m}_S - \nu\,\check{m}_F = \phi\,\check{m}_0 + (\delta\,\check{\tau}\,\phi - \xi)\,\check{m}_S + \left[\delta\,(1-\check{\tau})\,\phi - \nu\right]\check{m}_F,$$

and hence, the objective function is bounded by $(c - \delta\,\pi\,\check{\tau})\,\phi$. Substituting our candidate policy into the objective function attains this upper bound, establishing its optimality.

In case 4, define

$$\begin{aligned}
\phi &= -\frac{1}{\delta\,(1-\check{\tau})} \int_{\check{\tau}}^{\theta_H} (1-\theta)\,\mathrm{d}F(\theta), \\
\gamma &= \left[1 - F(\check{\tau})\right] - \frac{1}{\delta\,(1-\check{\tau})} \int_{\check{\tau}}^{\theta_H} (1-\theta)\,\mathrm{d}F(\theta), \text{ and} \\
\xi &= \frac{1}{1-\check{\tau}} \left[\int_{\check{\tau}}^{\theta_H} \theta\,\mathrm{d}F(\theta) - \check{\tau}\,\left[1 - F(\check{\tau})\right]\right],
\end{aligned}$$

where the nonnegativity of γ and ξ follows from $\mathcal{H}(\check{\tau}) \geq 0$ and its monotonicity, respectively. We have

$$\phi\left[\check{m}_0 + \delta\,\check{\tau}\,\check{m}_S + \delta\,(1-\check{\tau})\,\check{m}_F\right] - \gamma\,\check{m}_0 - \xi\,\check{m}_S = (\phi-\gamma)\,\check{m}_0 + (\delta\,\check{\tau}\,\phi - \xi)\,\check{m}_S + \delta\,(1-\check{\tau})\,\phi\,\check{m}_F,$$

and hence, the objective function is bounded by $(c - \delta\,\pi\,\check{\tau})\,\phi$. Substituting our candidate policy into the objective function attains this upper bound, establishing its optimality.

This completes the proof. □

EC.6. Proofs of Section 4

LEMMA EC.3. *Let Ω^{AS} denote the feasible set determined by* (EC.1), (IR), (8), *and* (12). *Let $\Omega(1)$ denote the feasible set of the original problem* (14) *evaluated at $\alpha = 1$. For every mechanism in Ω^{AS}, there exists a mechanism in $\Omega(1)$ that yields a weakly higher payoff for the funder. Consequently, the problem with only adverse selection and the original problem at $\alpha = 1$ have the same optimal value.*

Proof of Lemma EC.3. Because contractible effort eliminates deviations involving shirking, $\Omega(1) \subseteq \Omega^{AS}$. Hence, the value of the contractible-effort problem is weakly greater than that of the hidden-action problem at $\alpha = 1$. We establish the reverse inequality by showing that every mechanism feasible under contractible effort can be transformed into a weakly better mechanism that also satisfies the hidden-action constraints at $\alpha = 1$.

Fix any mechanism

$$\mathcal{M} \equiv \{q(\cdot), m_0(\cdot), m_S(\cdot), m_F(\cdot)\} \in \Omega^{AS},$$

and let

$$u(\theta) \equiv -cq(\theta) + m_0(\theta) + \delta\theta\big[m_S(\theta) + \pi q(\theta)\big] + \delta(1-\theta)m_F(\theta)$$

denote the firm's truthful utility. Recall the function $A(\theta)$ defined in (EC.5) and the envelope condition established in Lemma EC.1:

$$u(\theta) = u(0) + \delta \int_0^\theta A(y)\,\mathrm{d}y. \tag{EC.13}$$

If $A(0) \geq 0$, the monotonicity of A implies that $A(\theta) \geq 0$ for every $\theta \in [0,1]$. For any true type θ and reported type $\hat{\theta}$, when $\alpha = 1$,

$$u_1(\theta, \hat{\theta}) - u_0(\hat{\theta}) = \delta\theta A(\hat{\theta}) \geq 0.$$

Reporting incentive compatibility conditional on effort therefore implies

$$u(\theta) = u_1(\theta,\theta) \geq u_1(\theta,\hat{\theta}) \geq u_0(\hat{\theta}), \qquad \forall (\theta,\hat{\theta}) \in [0,1]^2.$$

Thus, $\mathcal{M} \in \Omega(1)$.

It remains to consider the case $A(0) < 0$. Define

$$\theta_0 \equiv \sup\{\theta \in [0,1] : A(\theta) \leq 0\}.$$

Let

$$A^-(\theta) \equiv \max\{-A(\theta), 0\}, \qquad A^+(\theta) \equiv \max\{A(\theta), 0\}.$$

Because A is nondecreasing, A^- is nonincreasing and A^+ is nondecreasing. Define

$$\tilde{\mathcal{M}} \equiv \{\tilde{q}(\cdot), \tilde{m}_0(\cdot), \tilde{m}_S(\cdot), \tilde{m}_F(\cdot)\},$$

where

$$\tilde{u}(\theta) \equiv u(\theta) - \delta \int_\theta^1 A^-(y)\,\mathrm{d}y, \tag{EC.14}$$

$$\tilde{q}(\theta) \equiv q(\theta), \qquad \tilde{m}_S(\theta) \equiv m_S(\theta), \tag{EC.15}$$

$$\tilde{m}_F(\theta) \equiv m_F(\theta) - A^-(\theta), \tag{EC.16}$$

$$\tilde{m}_0(\theta) \equiv \tilde{u}(\theta) + cq(\theta) - \delta\theta\big[m_S(\theta) + \pi q(\theta)\big] - \delta(1-\theta)\tilde{m}_F(\theta). \tag{EC.17}$$

The transformed outcome-dependent slope is

$$\tilde{A}(\theta) \equiv \tilde{m}_S(\theta) - \tilde{m}_F(\theta) + \pi\tilde{q}(\theta) = A(\theta) + A^-(\theta) = A^+(\theta) \geq 0.$$

Moreover, (EC.13) and (EC.14) imply

$$\tilde{u}(\theta) - \tilde{u}(0) = \delta \int_0^\theta \left[A(y) + A^-(y)\right] \mathrm{d}y = \delta \int_0^\theta \tilde{A}(y)\, \mathrm{d}y.$$

Because $\tilde{A}$ is nondecreasing, the standard envelope characterization implies that the transformed mechanism remains incentive compatible conditional on effort.

We next verify participation. Since A is nondecreasing, u is convex. The construction in (EC.14) satisfies

$$\tilde{u}(\theta) = \begin{cases} u(\theta_0), & \theta \leq \theta_0, \\ u(\theta), & \theta \geq \theta_0. \end{cases}$$

Thus, the transformed mechanism preserves participation.

We next verify limited liability. Equation (EC.16) can be written as

$$\tilde{m}_F(\theta) = \begin{cases} m_S(\theta) + \pi q(\theta), & A(\theta) < 0, \\ m_F(\theta), & A(\theta) \geq 0, \end{cases}$$

so $\tilde{m}_F(\theta) \geq 0$. The allocation rule and success transfer are unchanged and therefore remain feasible. From (EC.14)–(EC.17),

$$\tilde{m}_0(\theta) - m_0(\theta) = -\delta \int_\theta^1 A^-(y)\, \mathrm{d}y + \delta(1-\theta)A^-(\theta) \geq 0,$$

where the inequality follows because A^- is nonincreasing and hence

$$\int_\theta^1 A^-(y)\, \mathrm{d}y \leq (1-\theta)A^-(\theta).$$

Therefore, $\tilde{m}_0(\theta) \geq m_0(\theta) \geq 0$, and all limited-liability constraints remain satisfied.

It remains to compare the funder's payoff. For a type θ, the funder's payoff under a mechanism $\mathcal{M}$ is

$$u_F(\theta; \mathcal{M}) = \theta\omega q(\theta) - m_0(\theta) - \theta m_S(\theta) - (1-\theta)m_F(\theta).$$

Because q and m_S are unchanged, (EC.16) and the preceding expression for $\tilde{m}_0 - m_0$ give

$$\begin{aligned} u_F(\theta; \tilde{\mathcal{M}}) - u_F(\theta; \mathcal{M}) &= -\left[\tilde{m}_0(\theta) - m_0(\theta)\right] - (1-\theta)\left[\tilde{m}_F(\theta) - m_F(\theta)\right] \\ &= \delta \int_\theta^1 A^-(y)\, \mathrm{d}y + (1-\delta)(1-\theta)A^-(\theta) \\ &\geq 0. \end{aligned}$$

Thus, the transformation weakly increases the funder's payoff for every type and therefore weakly increases its expected payoff under any prior F.

Finally, consider the firm's effort decision under hidden action when $\alpha = 1$. Let $\tilde{u}_1$ and $\tilde{u}_0$ denote the firm's expected payoffs under the mechanism $\tilde{\mathcal{M}}$ when exerting effort and shirking, respectively. For any true type θ and report $\hat{\theta}$, the payoff difference between exerting effort and shirking after that report is

$$\tilde{u}_1(\theta, \hat{\theta}) - \tilde{u}_0(\hat{\theta}) = \delta\theta\tilde{A}(\hat{\theta}) \geq 0.$$

Thus, exerting effort weakly dominates shirking after every report, and we have

$$\tilde{u}(\theta) \geq \tilde{u}_1(\theta, \hat{\theta}) \geq \tilde{u}_0(\hat{\theta}), \qquad \forall(\theta, \hat{\theta}) \in [0,1]^2.$$

Hence, $\tilde{\mathcal{M}} \in \Omega(1)$. We have therefore shown that every mechanism in Ω^{AS} is weakly dominated by a mechanism in $\Omega(1)$, implying that the value of the hidden-action problem at $\alpha = 1$ is weakly greater than the value of the contractible-effort problem. Combined with $\Omega(1) \subseteq \Omega^{AS}$, this establishes equality of the two optimal values. □

Proof of Corollary 3 By Lemma EC.3, the contractible-effort benchmark has the same optimal value as the original problem (14) evaluated at $\alpha = 1$. It therefore suffices to specialize the primitive optimality and feasibility conditions underlying Propositions 1–8 to $\alpha = 1$. Some of the boundary functions $B_i(\pi)$ and $\widetilde{B}_i(\pi)$ contain $1 - \alpha$ in the denominator and hence cannot be evaluated directly at $\alpha = 1$. These boundary functions, however, are only convenient representations obtained by solving joint inequalities in (π, ω) for ω. The underlying inequalities remain well defined at $\alpha = 1$. We illustrate this point using one boundary pair from the $\delta \geq 1/2$ characterization and one boundary pair from the $\delta < 1/2$ characterization.

First, the no-testing condition in Proposition 1 is

$$\omega + \delta\,\pi \leq c \qquad \text{or} \qquad \omega + \pi \leq \frac{(1 - \alpha + \delta\alpha)c}{\delta}.$$

When $\alpha = 1$, the second inequality becomes $\omega + \pi \leq c$, which implies $\omega + \delta\pi \leq c$. Hence, the no-testing region reduces to

$$\omega + \delta\,\pi \leq c,$$

as stated in part (a).

Now suppose $\omega + \delta\pi > c$ and $\delta \geq 1/2$. Specializing the Region (4) candidate mechanism to $\alpha = 1$ gives

$$\tau = \frac{c + (1-\delta)\pi}{\omega + \pi} = 1 - \frac{\omega + \delta\pi - c}{\omega + \pi}, \qquad m_0^* = c - \delta\pi, \qquad m_S^* = 0, \qquad m_F^* = \pi.$$

The maintained condition $\omega + \delta\pi > c$ implies $\tau \in (0,1)$, while $m_0^* \geq 0$ and $m_F^* \leq c/\delta$ are both equivalent to $\delta\pi \leq c$. This gives part (b)(i).

For an example involving the ordinary boundary functions, consider the less-insurance mechanism in Region (6):

$$\tau = \frac{c+\delta\pi}{2\delta(\omega+\pi)}, \qquad m_F^* = \frac{c-\delta\pi\tau}{\delta(1-\tau)}.$$

For $\alpha < 1$, the boundary conditions $B_5(\pi) \leq \omega < B_4(\pi)$ are obtained by solving the primitive feasibility conditions

$$0 \leq m_F^* \leq \frac{\alpha c}{\delta}$$

for ω. After substituting the expression for τ, these conditions are equivalent to

$$c(2\omega+\pi) - \delta\pi^2 \geq 0$$

and

$$2\delta(1-\alpha)c\omega + \delta(1-\alpha)c\pi + \alpha c^2 - \delta^2\pi^2 \leq 0.$$

When $\alpha < 1$, solving these inequalities for ω gives, respectively,

$$\omega \geq \frac{\pi(\delta\pi - c)}{2c} = B_5(\pi)$$

and

$$\omega \leq \frac{\delta^2\pi^2 - \delta(1-\alpha)c\pi - \alpha c^2}{2\delta(1-\alpha)c} = B_4(\pi).$$

At $\alpha = 1$, the same primitive inequalities instead reduce to

$$c \geq \frac{\delta\pi}{1+2\omega/\pi} \qquad \text{and} \qquad c \leq \delta\pi.$$

Thus, when

$$\frac{\delta\pi}{1+2\omega/\pi} < c < \delta\pi,$$

the less-insurance mechanism has $0 < m_F^* < c/\delta$, giving part (b)(ii). At

$$c = \frac{\delta\pi}{1+2\omega/\pi},$$

the failure payment becomes zero and the mechanism coincides with commercial self-funding. For lower c, the commercial mechanism is optimal, yielding part (b)(iii). The assignment of the equality case is immaterial because the adjacent mechanisms coincide there.

Finally, suppose $\omega + \delta\pi > c$ and $\delta < 1/2$. Consider the less-push mechanism in Proposition 8(b):

$$\tau = \frac{c+\delta\pi}{\omega+2\delta\pi}, \qquad m_0^* = c - \delta\pi\tau.$$

For $\alpha < 1$, the boundary conditions

$$\widetilde{B}_5(\pi) \leq \omega < \widetilde{B}_2(\pi)$$

are obtained by solving

$$0 \le m_0^* \le \alpha c$$

for ω. Substituting the expression for τ, these inequalities become

$$c(\omega + \delta\pi) - \delta^2\pi^2 \ge 0$$

and

$$(1-\alpha)c\omega + (1-2\alpha)c\delta\pi - \delta^2\pi^2 \le 0.$$

When $\alpha < 1$, solving them for ω gives

$$\omega \ge \frac{\delta\pi(\delta\pi - c)}{c} = \widetilde{B}_5(\pi)$$

and

$$\omega \le \frac{\delta\pi[\delta\pi + (2\alpha-1)c]}{(1-\alpha)c} = \widetilde{B}_2(\pi).$$

At $\alpha = 1$, the second primitive inequality is automatically satisfied, whereas the first becomes

$$c \ge \frac{\delta\pi}{1 + \omega/(\delta\pi)}.$$

Together with $\omega + \delta\pi > c$, this gives part (c)(i). If

$$c < \frac{\delta\pi}{1 + \omega/(\delta\pi)},$$

the optimal upfront payment is zero and the mechanism coincides with commercial self-funding, giving part (c)(ii). At equality, the two mechanisms coincide.

Applying the same procedure to the remaining boundaries shows that Regions (2), (3), and (5) have no nondegenerate interior at $\alpha = 1$. All the surviving optimal mechanisms satisfy $m_S^*(\theta) = 0$ for every θ, which completes the proof. □

Proof of Proposition 10. We prove the desired result in the following steps.

When $\theta \ge c/(\delta\pi)$, ignore constraints (26) and (28), and the optimal solution to the relaxed problem is given by

$$q = 1, \quad m_0 = m_S = m_F = 0,$$

satisfying the two ignored constraints automatically.

Next, we prove the following two lemmas to simplify the problem.

LEMMA EC.4. *Without loss of generality, we can focus on mechanisms satisfying $m_F = 0$.*

Proof. Given any mechanism $(\hat{q}, \hat{m}_0, \hat{m}_S, \hat{m}_F)$ with $\hat{m}_F > 0$, we propose the following mechanism $(\tilde{q}, \tilde{m}_0, \tilde{m}_S, \tilde{m}_F)$, where

$$\begin{aligned}
&\tilde{q} = \hat{q},\\
&\tilde{m}_0 = \hat{m}_0 + \delta\,(1-\theta)\,\hat{m}_F,\\
&\tilde{m}_S = \hat{m}_S,\\
&\tilde{m}_F = 0.
\end{aligned}$$

The new mechanism is feasible and generates a (weakly) higher expected payoff for the principal. This completes the proof. □

LEMMA EC.5. *When $\theta < c/(\delta\,\pi)$, the agent's expected payoff is always* 0.

Proof. Suppose the agent's expected payoff is strictly positive when $\theta < c/(\delta\,\pi)$. The principal can always lower the compensation levels m's to make (IR) hold with equality:

1. if $m_0 > 0$, the principal can always lower m_0 without affecting the moral hazard constraint; and
2. if the agent's expected payoff is still strictly positive after setting $m_0 = 0$, it is always feasible to decrease m_S to $\tilde{m}_S$ to make (IR) hold with equality. The moral hazard constraint is automatically satisfied, because

$$-(1-\alpha)\,c\,q + \delta\,\theta\,(\tilde{m}_S + \pi\,q) \geq -c\,q + \delta\,\theta\,(\tilde{m}_S + \pi\,q) = 0.$$

This completes the proof.

When $\theta < c/(\delta\,\pi)$, we can use constraint (IR) to eliminate m_0, i.e.,

$$m_0 = c\,q - \delta\,\theta\,(m_S + \pi\,q).$$

The funder's problem can be reformulated as follows:

$$\begin{aligned}
\max_{q, m_S}\ & (\theta\,\omega + \delta\,\theta\,\pi - c)\,q - \theta\,(1-\delta)\,m_S, &\\
\text{subject to}\ & -[\delta\,\theta\,\pi - (1-\alpha)\,c]\,q - \delta\,\theta\,m_S \leq 0, & (\eta)\\
& -q \leq 0, & (\beta)\\
& q \leq 1, & (\sigma)\\
& (\delta\,\theta\,\pi - c)\,q + \delta\,\theta\,m_S \leq 0, & (\gamma)\\
& -m_S \leq 0, & (\xi)
\end{aligned}$$

where the Greek letters at the end of each inequality represent the shadow prices associated with these constraints.

When $\max\{c/(\omega+\delta\pi),(1-\alpha)c/(\delta\pi)\}\leq\theta\leq c/(\delta\pi)$, define

$$\sigma\equiv\theta\omega+\delta\theta\pi-c,\quad \xi\equiv\theta(1-\delta).$$

It is straightforward to verify that both σ and ξ are nonnegative. We have

$$\sigma\cdot q-\xi\cdot m_S=(\theta\omega+\delta\theta\pi-c)q-\theta(1-\delta)m_S,$$

and hence, the objective function is bounded by

$$\sigma=\theta\omega+\delta\theta\pi-c.$$

Plugging our candidate solution into the objective function yields the upper bound, establishing optimality.

When $(\delta\alpha-\alpha+1)c/(\delta(\omega+\pi))\leq\theta\leq(1-\alpha)c/(\delta\pi)$, define

$$\eta\equiv\frac{1-\delta}{\delta},\quad \sigma\equiv\frac{1}{\delta}\left[\delta\theta(\omega+\pi)-(\delta\alpha-\alpha+1)c\right].$$

It is straightforward to verify that both η and σ are nonnegative. We have

$$\eta\cdot\left[-\left[\delta\theta\pi-(1-\alpha)c\right]q-\delta\theta m_S\right]+\sigma\cdot q=(\theta\omega+\delta\theta\pi-c)q-\theta(1-\delta)m_S,$$

and hence, the objective function is bounded by

$$\sigma=\frac{1}{\delta}\left[\delta\theta(\omega+\pi)-(\delta\alpha-\alpha+1)c\right].$$

Plugging our candidate solution into the objective function yields

$$(\theta\omega+\delta\theta\pi-c)-\theta(1-\delta)\left(\frac{(1-\alpha)c}{\delta\theta}-\pi\right)=\frac{1}{\delta}\left[\delta\theta(\omega+\pi)-(\delta\alpha-\alpha+1)c\right],$$

attaining the upper bound.

When $\theta\leq c/(\omega+\delta\pi)$, define

$$\beta\equiv-\theta\omega-\delta\theta\pi+c,\quad \xi\equiv\theta(1-\delta).$$

It is straightforward to verify that both β and ξ are nonnegative. We have

$$-\beta\cdot q-\xi\cdot m_S=(\theta\omega+\delta\theta\pi-c)q-\theta(1-\delta)m_S,$$

and hence, the objective function is bounded by 0. Plugging our candidate solution into the objective function yields the upper bound.

When $\theta\leq(\delta\alpha-\alpha+1)c/(\delta(\omega+\pi))$, define

$$\eta\equiv\frac{1-\delta}{\delta},\quad \beta\equiv-\frac{1}{\delta}\left[\delta\theta(\omega+\pi)-(\delta\alpha-\alpha+1)c\right].$$

It is straightforward to verify that both η and β are nonnegative. We have

$$\eta \cdot [-[\delta\,\theta\,\pi - (1-\alpha)\,c]\;q - \delta\,\theta\,m_S] - \beta \cdot q = (\theta\,\omega + \delta\,\theta\,\pi - c)\,q - \theta\,(1-\delta)\,m_S,$$

and hence, the objective function is bounded by 0. Plugging our candidate solution into the objective function yields the upper bound.

This completes the proof. □

EC.7. Continuous Effort Level

In this section, we show that the assumption of binary effort can be made without loss of generality when the testing cost is linear in the firm's effort level.

Given its private information θ, the firm may exert an intermediate level of effort that yields a lower probability of success $\tilde{\theta} \le \theta$. We assume that, conditional on the firm's type θ and its chosen probability of success $\tilde{\theta}$, the associated cost $\tilde{c}$ is a convex combination of the cost of exerting no effort ($\alpha\,c$) and the cost of exerting full effort (c). Formally,

$$\tilde{c}(\theta,\tilde{\theta}) = \frac{\tilde{\theta}}{\theta}\,c + \left(1 - \frac{\tilde{\theta}}{\theta}\right)\,\alpha\,c.$$

We have $\tilde{c}(\theta,0) = \alpha\,c$ and $\tilde{c}(\theta,\theta) = c$.

Let $\tilde{u}(\theta,\hat{\theta},\tilde{\theta})$ denote the firm's expected payoff when its true type is θ, but it reports $\hat{\theta}$ and exerts effort such that the success rate is $\tilde{\theta} \le \theta$ in the test. We have

$$\tilde{u}(\theta,\hat{\theta},\tilde{\theta}) \equiv \Big[-\tilde{c}(\theta,\tilde{\theta}) + \hat{m}_0(\hat{\theta}) + \delta\,\tilde{\theta}\,\Big(\hat{m}_S(\hat{\theta}) + \pi\Big) + \delta\,(1-\tilde{\theta})\,\hat{m}_F(\hat{\theta})\Big]\;q(\hat{\theta}). \tag{EC.18}$$

We focus on mechanisms that induce the firm's full effort. A feasible mechanism must satisfy the incentive compatibility constraint, meaning that a firm with type θ should not benefit from misreporting its type as $\hat{\theta}$ or shirking (i.e., $\tilde{\theta} < \theta$), i.e.,

$$\tilde{u}(\theta,\theta,\theta) \ge \tilde{u}(\theta,\hat{\theta},\tilde{\theta}), \quad \forall(\theta,\hat{\theta},\tilde{\theta}) \in \Theta^3 \;\text{ with }\; \tilde{\theta} \le \theta, \tag{EC.19}$$

where $\tilde{u}(\theta,\theta,\theta)$ represents the type θ firm's expected payoff when it reports truthfully and exerts effort.

The following lemma establishes that, without loss of generality, it suffices to restrict the right-hand side of (EC.19) to (2) and (3).

Lemma EC.6. *The incentive compatibility constraint* (EC.19) *holds if and only if condition* (IC) *is satisfied.*

Proof of Lemma EC.6 The proof of the necessary part ((EC.19) $\implies$ (EC.1) and (EC.2)) is straightforward. Setting $\tilde{\theta} = \theta$ and 0 on the right-hand side of (EC.19) yields (EC.1) and (EC.2), respectively.

It remains to prove the sufficient part ((EC.1) and (EC.2) $\implies$ (EC.19)). Multiplying constraint (EC.1) by $\frac{\tilde{\theta}}{\theta}$ and constraint (EC.2) by $1 - \frac{\tilde{\theta}}{\theta}$ and then summing the two yields

$$
\begin{aligned}
&- c\,q(\theta) + m_0(\theta) + \delta\,\theta\,[m_S(\theta) + \pi\,q(\theta)] + \delta\,(1-\theta)\,m_F(\theta) \\
&\geq \frac{\tilde{\theta}}{\theta}\left[-c\,q(\hat{\theta}) + m_0(\hat{\theta}) + \delta\,\theta\left[m_S(\hat{\theta}) + \pi\,q(\hat{\theta})\right] + \delta\,(1-\theta)\,m_F(\hat{\theta})\right] + \left(1 - \frac{\tilde{\theta}}{\theta}\right)\left[-\alpha\,c\,q(\hat{\theta}) + m_0(\hat{\theta}) + \delta\,m_F(\hat{\theta})\right] \\
&= \tilde{u}(\theta, \hat{\theta}, \tilde{\theta}),
\end{aligned}
$$

which completes the proof. □

EC.8. Funder's Discount Rate

Let δ_0 and δ_1 denote the funder's and the firm's discount factors. Given a committed mechanism

$$\mathcal{M} \equiv \{q(\theta), m_0(\theta), \tilde{m}_S(\theta), \tilde{m}_F(\theta)\},$$

the payoff functions of both parties can be written as follows:

$$
\begin{aligned}
u_F(\theta) &= -\,m_0(\theta) + \delta_0\,\theta\,[\tilde{\omega}\,q(\theta) - \tilde{m}_S(\theta)] - \delta_0\,(1-\theta)\,\tilde{m}_F(\theta), \\
u_1(\theta, \hat{\theta}) &= -\,c\,q(\hat{\theta}) + m_0(\hat{\theta}) + \delta_1\,\theta\left[\tilde{m}_S(\hat{\theta}) + \tilde{\pi}\,q(\hat{\theta})\right] + \delta_1\,(1-\theta)\,\tilde{m}_F(\hat{\theta}), \\
u_0(\hat{\theta}) &= -\,\alpha\,c\,q(\hat{\theta}) + m_0(\hat{\theta}) + \delta_1\,\tilde{m}_F(\hat{\theta}).
\end{aligned}
$$

Let

$$\delta \equiv \frac{\delta_1}{\delta_0}, \quad \omega \equiv \delta_0\,\tilde{\omega}, \quad \pi \equiv \delta_0\,\tilde{\pi}, \quad m_S(\theta) \equiv \delta_0\,\tilde{m}_S(\theta), \quad m_F(\theta) \equiv \delta_0\,\tilde{m}_F(\theta).$$

The payoff functions can be expressed as (9), (10), and (11) in Section 2. Therefore, without loss of generality, we normalize the funder's discount factor to 1.

EC.9. No Reward for Failure

In this section, we quantify the value of allowing failure-contingent payments under Assumption 1. We compare the optimal value u^* of the unrestricted funder's problem (14) with the optimal value of a restricted problem in which failure payments are prohibited. Specifically, we impose the additional constraint

$$m_F(\theta) = 0, \qquad \forall \theta \in [0, 1],$$

and let $\hat{u}$ denote the optimal value of this restricted problem. We measure the value of allowing failure-contingent payments by the performance ratio

$$\text{Performance Ratio} \equiv \frac{\hat{u}}{u^*}.$$

Equivalently, $1 - \hat{u}/u^*$ is the percentage loss in the funder's objective from prohibiting failure-contingent payments. When $u^* = \hat{u} = 0$, we set the performance ratio equal to 100% for completeness in the figures.

Before presenting the numerical comparison, we first characterize the restricted optimum. The characterization is closely related to Proposition 8. Although Proposition 8 was stated for the unrestricted problem when $\delta < 1/2$, the same maximal-push, less-push, and commercial mechanisms arise here for any $\delta \in (0, 1]$ once the funder is not allowed to use m_F.

PROPOSITION EC.1. *Suppose Assumption 1 holds and the additional constraint*

$$m_F(\theta) = 0, \qquad \forall \theta \in [0, 1]$$

is imposed. Then Propositions 1 and 2 remain unchanged. In the remaining parameter region, the optimal restricted mechanism has the same form as the mechanisms in Proposition 8(a)–(c), with the same boundary functions $\tilde{B}_2(\pi)$ and $\tilde{B}_5(\pi)$, but without requiring $\delta < 1/2$.

Proof of Proposition EC.1. The optimal mechanisms in Propositions 1 and 2 already satisfy $m_F^*(\theta) = 0$ for every θ. Hence, imposing $m_F \equiv 0$ does not change feasibility or optimality in those regions.

It remains to consider the regions that, in the unrestricted problem, would involve push only, failure funding, or commercial self-funding. We show that the restricted problem has the same maximal-push, less-push, and commercial structure as Proposition 8(a)–(c). The candidate mechanisms in Proposition 8(a)–(c) all satisfy $m_F^*(\theta) = 0$ for every θ, so they are feasible for the restricted problem.

The only new optimality argument needed is for the less-push region in Proposition 8(b). In the proof of Proposition 8(b), the dual feasible solution is

$$\chi_L = 1, \qquad \beta(\theta) = -w_q(\theta), \quad \forall \theta \in [0, \tau],$$
$$\sigma(\theta) = w_q(\theta), \quad \forall \theta \in [\tau, 1], \qquad \xi(\theta) = -w_S(\theta), \quad \forall \theta \in [0, 1],$$

and, in the unrestricted problem, also $\nu(\theta) = -w_F(\theta)$. In the restricted problem, m_F is not a decision variable, so the multiplier ν and the coefficient-matching condition for m_F are no longer needed. The remaining multipliers are nonnegative for any $\delta \in (0, 1]$: the signs of β and σ follow from the definition of

$$\tau = \frac{1 + \bar{\tau}}{\omega/(\delta\,\pi) + 2},$$

and $w_S(\theta) \le 0$ for all $\theta \in [0,1]$, so $\xi(\theta) = -w_S(\theta) \ge 0$. Thus, the same weak-duality argument, after removing the m_F term, proves optimality of the less-push mechanism for the restricted problem.

At $\omega = \tilde{B}_2(\pi)$, the less-push mechanism in Proposition 8(b) coincides with the push-only mechanism in Proposition 8(a): the threshold becomes

$$\tau = (1-\alpha)\bar{\tau}$$

and the upfront payment becomes

$$c - \delta\,\pi\,\tau = \alpha\,c.$$

At $\omega = B_1(\pi)$, the pull-plus-push mechanism in Proposition 2 also coincides with the same push-only mechanism because the pull payment becomes zero. Therefore, by Lemma EC.2, the push-only mechanism is optimal throughout

$$\tilde{B}_2(\pi) \le \omega < B_1(\pi).$$

Finally, at $\omega = \tilde{B}_5(\pi)$, the less-push mechanism in Proposition 8(b) coincides with the commercial mechanism in Proposition 8(c): the threshold becomes

$$\tau = \bar{\tau}$$

and the upfront payment becomes

$$c - \delta\,\pi\,\bar{\tau} = 0.$$

The same monotonicity argument used in the proof of Proposition 7 then implies that the commercial mechanism remains optimal for every lower value of ω.

This completes the proof. □

The first row of Figure EC.5 reports the results under the uniform prior. Comparing the columns with $(\alpha, \delta) = (0.5, 0.6)$ and $(0.9, 0.6)$ shows that, holding the discount factor fixed, the parameter region in which the performance ratio is strictly below 100% expands as α increases. The minimum performance ratio decreases from 93.32% to 87.14%. Thus, failure-contingent funding can become more valuable when shirking entails a higher unavoidable cost.

Comparing the columns with $(\alpha, \delta) = (0.5, 0.6)$ and $(0.5, 0.9)$ shows the effect of the firm's discount factor. As δ increases from 0.6 to 0.9, the minimum performance ratio decreases from 93.32% to 82.99%. When the firm discounts future payments less heavily, failure-contingent payments become a more effective funding instrument, and prohibiting them can therefore generate a larger loss.

To quantify the welfare loss under a non-uniform common prior F, we next characterize the optimal take-it-or-leave-it offer without failure-contingent payments.

PROPOSITION EC.2. *Fix a threshold $\check{\tau} \in \check{\Theta}$ such that the restricted feasible set is nonempty, and impose $\check{m}_F = 0$. Then an optimal solution is*

$$\check{m}_0^* = \min\{\alpha\, c,\, c - \delta\pi\check{\tau}\}, \qquad \check{m}_S^* = \max\left\{\frac{(1-\alpha)c}{\delta\check{\tau}} - \pi,\, 0\right\}. \tag{EC.20}$$

Equivalently:

1. *if $\mathcal{G}(\check{\tau}) \geq \alpha$, then*
$$\check{m}_0^* = \alpha\, c, \qquad \check{m}_S^* = \frac{(1-\alpha)c}{\delta\check{\tau}} - \pi;$$
2. *if $\mathcal{G}(\check{\tau}) < \alpha$, then*
$$\check{m}_0^* = c - \delta\pi\check{\tau}, \qquad \check{m}_S^* = 0.$$

Proof of Proposition EC.2. After imposing $\check{m}_F = 0$, constraint (22) is redundant because $\check{m}_S \geq 0$ and $\pi \geq 0$. For $\check{\tau} > 0$, constraint (23) becomes

$$\check{m}_0 + \delta\check{\tau}\check{m}_S = c - \delta\pi\check{\tau}.$$

The endpoint case follows by continuity whenever it belongs to the feasible threshold set. Because $\check{\tau} \in \check{\Theta}$, we have $c - \delta\pi\check{\tau} \geq 0$. Eliminating $\check{m}_0$ yields

$$\check{m}_0 = c - \delta\pi\check{\tau} - \delta\check{\tau}\check{m}_S.$$

Therefore, the constraints $\check{m}_0 \geq 0$, $\check{m}_0 \leq \alpha\, c$, and $\check{m}_S \geq 0$ are equivalent to

$$\max\left\{\frac{(1-\alpha)c}{\delta\check{\tau}} - \pi,\, 0\right\} \leq \check{m}_S \leq \frac{c}{\delta\check{\tau}} - \pi.$$

As in the proof of Proposition 9, the welfare term

$$\omega \int_{\check{\tau}}^{\theta_H} \theta \,\mathrm{d}F(\theta)$$

is constant with respect to the transfers and can be omitted. Substituting

$$\check{m}_0 = c - \delta\pi\check{\tau} - \delta\check{\tau}\check{m}_S$$

into the remaining part of the objective gives

$$-\left[1 - F(\check{\tau})\right]\check{m}_0 - \check{m}_S \int_{\check{\tau}}^{\theta_H} \theta \,\mathrm{d}F(\theta) = -\left(c - \delta\pi\check{\tau}\right)\left[1 - F(\check{\tau})\right] + \left\{\delta\check{\tau}\left[1 - F(\check{\tau})\right] - \int_{\check{\tau}}^{\theta_H} \theta \,\mathrm{d}F(\theta)\right\}\check{m}_S.$$

Because every participating type satisfies $\theta \geq \check{\tau}$,

$$\int_{\check{\tau}}^{\theta_H} \theta \,\mathrm{d}F(\theta) \geq \check{\tau}\left[1 - F(\check{\tau})\right] \geq \delta\check{\tau}\left[1 - F(\check{\tau})\right].$$

Hence, the coefficient of $\check{m}_S$ is nonpositive, and the objective is weakly decreasing in $\check{m}_S$. It is therefore optimal to choose the smallest feasible success-contingent payment:

$$\check{m}_S^* = \max\left\{\frac{(1-\alpha)c}{\delta\check{\tau}} - \pi,\, 0\right\}.$$

Substituting this expression into the equality constraint yields

$$\check{m}_0^* = c - \delta\pi\check{\tau} - \delta\check{\tau}\check{m}_S^* = \min\left\{\alpha\, c,\, c - \delta\pi\check{\tau}\right\}.$$

Finally,

$$\frac{c}{\delta}\left[\mathcal{G}(\check{\tau}) - \alpha\right] = \frac{(1-\alpha)c}{\delta\check{\tau}} - \pi.$$

Thus, if $\mathcal{G}(\check{\tau}) \geq \alpha$, then

$$\check{m}_0^* = \alpha\, c, \qquad \check{m}_S^* = \frac{(1-\alpha)c}{\delta\check{\tau}} - \pi.$$

If $\mathcal{G}(\check{\tau}) < \alpha$, then

$$\check{m}_0^* = c - \delta\pi\check{\tau}, \qquad \check{m}_S^* = 0.$$

This completes the proof. □

Beginning with the second row, Figure EC.5 shows that the value of failure-contingent funding depends on the shape of the common prior. Under the U-shaped Beta$(.25, .25)$ prior, prohibiting m_F generates appreciable losses in all three parameter settings: the minimum performance ratios are 91.38%, 81.42%, and 90.68%, respectively. By contrast, under the more concentrated Beta$(2, 2)$ and Beta$(5, 5)$ priors and the left-skewed Beta$(2, 5)$ prior, the loss is negligible when $\delta = 0.6$, with minimum performance ratios at or above 99.5%. Failure funding becomes substantially more valuable for these priors when δ rises to 0.9, reducing the minimum performance ratio to between 81.46% and 87.21%. The right-skewed Beta$(5, 2)$ prior exhibits an intermediate pattern when $\delta = 0.6$, but its minimum performance ratio also falls to 82.14% when $\delta = 0.9$. Across the non-uniform priors considered, the largest loss from prohibiting failure payments is therefore 18.58%, attained under the Beta$(.25, .25)$ prior when $\alpha = 0.9$ and $\delta = 0.6$. Overall, the figure shows that the quantitative value of failure-contingent funding is not specific to the uniform prior, although its magnitude depends on the distribution of success probabilities and on the discount factors.

EC.10. Failure Payments and Success-Contingent Repayments

Recall that u^* denotes the optimal value of the funder's expected payoff in the original problem. When the funder may require repayment from the firm, subject to the repayment not exceeding the upfront payment in discounted terms, let u^R denote the resulting optimal value:

$$u^R \equiv \max_{\{q(\cdot), m_0(\cdot), m_S(\cdot), m_F(\cdot)\} \in \Omega_{\text{rep}}} U_F(q, m_0, m_S, m_F), \tag{EC.21}$$

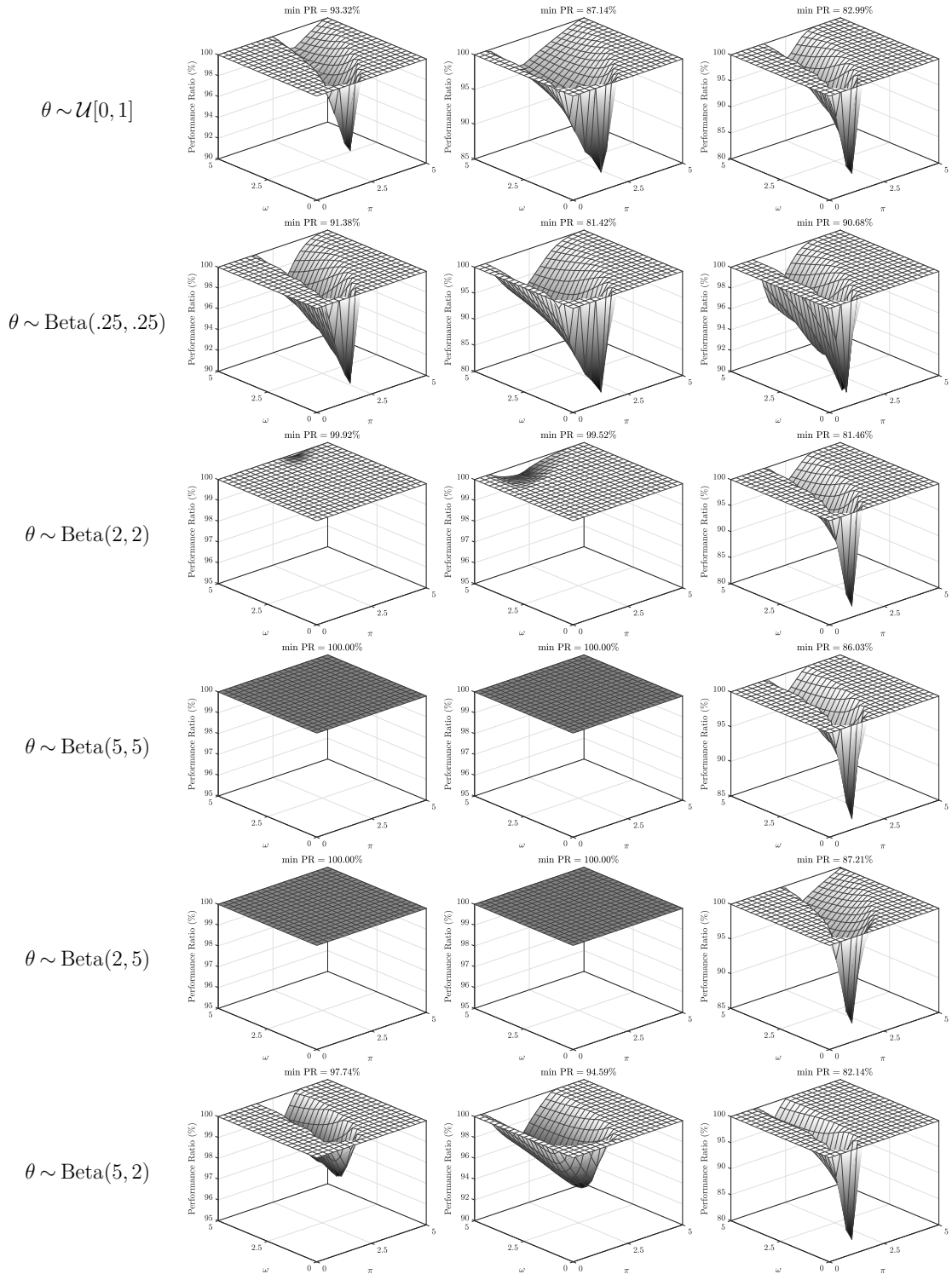


Figure EC.5 Performance ratios of the optimal take-it-or-leave-it offer without failure-contingent payments relative to the optimal take-it-or-leave-it offer with failure-contingent payments. Each row corresponds to a common prior distribution of θ, and each column corresponds to a combination of α and δ; $c = 1$.

where Ω_{rep} is the feasible set defined by (IC), (IR), (8), and

$$m_0(\theta) \geq 0, \quad m_F(\theta) = 0, \quad m_0(\theta) + \delta m_S(\theta) \geq 0, \quad \forall \theta \in \Theta.$$

The following proposition compares the original failure-payment environment with this capped success-repayment environment.

PROPOSITION EC.3. *(i) If $\delta = 1$, then*

$$u^R = u^*.$$

(ii) If $\delta < 1$, then, for any optimal mechanism

$$\{q^*, m_0^*, m_S^*, m_F^*\}$$

in the original problem,

$$u^R \geq u^* + (1-\delta) \int_\Theta m_F^*(\theta) \, \mathrm{d}F(\theta).$$

In particular, $u^R > u^*$ if an optimal original mechanism uses a positive failure payment with positive probability.

Proof of Proposition EC.3. All transformations below are applied pointwise in the reported type. Consider any feasible mechanism

$$\mathcal{M} \equiv \{q, m_0, m_S, m_F\}$$

in the original problem. Define the transformed mechanism

$$\tilde{\mathcal{M}} \equiv \{q, \tilde{m}_0, \tilde{m}_S, \tilde{m}_F\},$$

where

$$\tilde{m}_0 \equiv m_0 + \delta \, m_F, \qquad \tilde{m}_S \equiv m_S - m_F, \qquad \tilde{m}_F \equiv 0. \tag{EC.22}$$

The transformed transfers satisfy

$$\tilde{m}_0 \geq 0$$

and

$$\tilde{m}_0 + \delta \, \tilde{m}_S = m_0 + \delta \, m_S \geq 0.$$

Direct substitution shows that the transformation preserves the firm's payoff for every true type, report, and effort choice, and therefore preserves all participation, reporting, and effort constraints. Moreover,

$$U_F(q, \tilde{m}_0, \tilde{m}_S, \tilde{m}_F) = U_F(q, m_0, m_S, m_F) + (1-\delta) \int_\Theta m_F(\theta) \, \mathrm{d}F(\theta). \tag{EC.23}$$

If $\{q^*, m_0^*, m_S^*, m_F^*\}$ is an optimal mechanism in the original problem, applying (EC.22) to this mechanism gives a feasible mechanism in the success-repayment problem. Hence,

$$u^R \geq u^* + (1-\delta) \int_\Theta m_F^*(\theta)\, \mathrm{d}F(\theta),$$

which proves part (ii). When $\delta = 1$, the same transformation implies

$$u^R \geq u^*.$$

To establish the reverse inequality when $\delta = 1$, consider any feasible mechanism

$$\tilde{\mathcal{M}} = \{q, \tilde{m}_0, \tilde{m}_S, 0\}$$

in the success-repayment problem. Define

$$m_F \equiv [-\tilde{m}_S]^+, \qquad m_S \equiv [\tilde{m}_S]^+, \qquad m_0 \equiv \tilde{m}_0 - [-\tilde{m}_S]^+,$$

where $[x]^+ \equiv \max\{x, 0\}$. Since

$$\tilde{m}_0 + \tilde{m}_S \geq 0,$$

the resulting transfers are nonnegative. Moreover,

$$m_0 + m_F = \tilde{m}_0 \qquad \text{and} \qquad m_0 + m_S = \tilde{m}_0 + \tilde{m}_S.$$

Thus, the two representations specify the same total transfers following failure and success. When $\delta = 1$, they therefore generate identical payoffs for the firm and the funder under every true type, report, and effort choice. Every feasible mechanism in the success-repayment problem consequently has a payoff-equivalent representation in the original problem, which implies

$$u^* \geq u^R.$$

Combining this inequality with $u^R \geq u^*$ gives

$$u^* = u^R.$$

□

The firm retains the option to reject the funder's offer and test using its own resources, in which case its payoff is

$$\underline{u}(\theta) \equiv \max\{0, -c + \delta\, \theta\, \pi\}.$$

By the preceding characterization, the optimal testing rule is bang–bang. Consider first a nondegenerate optimum, and choose a report $\hat{\theta}$ such that $q(\hat{\theta}) = 1$. For any type satisfying $-c + \delta\theta\pi > 0$, incentive compatibility in the original failure-payment problem gives

$$\begin{aligned} u_1(\theta,\theta) &\geq u_1(\theta,\hat{\theta}) \\ &= -c + \delta\theta\pi + m_0(\hat{\theta}) + \delta\theta m_S(\hat{\theta}) + \delta(1-\theta)m_F(\hat{\theta}) \\ &\geq -c + \delta\theta\pi = \underline{u}(\theta), \end{aligned}$$

where the last inequality follows from limited liability.

The same argument applies to the capped success-repayment problem. In that problem,

$$u_1^R(\theta,\theta) \geq -c + \delta\theta\pi + m_0(\hat{\theta}) + \delta\theta m_S(\hat{\theta}).$$

If $m_S(\hat{\theta}) \geq 0$, the last two terms are nonnegative. If $m_S(\hat{\theta}) < 0$, then

$$m_0(\hat{\theta}) + \delta\theta m_S(\hat{\theta}) \geq m_0(\hat{\theta}) + \delta m_S(\hat{\theta}) \geq 0,$$

where the final inequality is the repayment cap. Thus,

$$u_1^R(\theta,\theta) \geq \underline{u}(\theta).$$

For types with $-c + \delta\theta\pi \leq 0$, the result follows from the ordinary participation constraint.

Finally, when the optimal mechanism has $q = 0$, the no-testing condition implies $\delta\pi \leq c$, so that $\underline{u}(\theta) = 0$ for every type. Therefore, an optimal mechanism in each problem satisfies participation relative to self-funded testing. Explicitly imposing this condition consequently does not change the values u^* and u^R.

Once the repayment cap is removed, however, limited liability no longer implies this participation condition. We therefore retain the firm's self-testing option explicitly in the uncapped repayment problem:

$$u_1(\theta,\theta) \geq \underline{u}(\theta) \equiv \max\{0, -c + \delta\,\theta\,\pi\}, \qquad \forall\theta \in \Theta. \tag{IR-ST}$$

To examine the value of relaxing the discounted repayment cap, let u^U denote the optimal value of the uncapped success-repayment problem:

$$u^U \equiv \max_{\{q(\cdot),m_0(\cdot),m_S(\cdot),m_F(\cdot)\}\in\Omega_U} U_F(q, m_0, m_S, m_F), \tag{EC.24}$$

where Ω_U is the feasible set defined by (IC), (IR-ST), (8), and

$$m_0(\theta) \geq 0, \quad m_F(\theta) = 0, \quad \forall\theta \in \Theta.$$

Thus, unlike the capped repayment problem, the uncapped problem does not impose

$$m_0(\theta) + \delta\, m_S(\theta) \geq 0.$$

Because every feasible mechanism in the capped repayment problem is also feasible in the uncapped problem,

$$u^U \geq u^R.$$

The following corollary establishes a strict comparison with the original failure-payment problem in the interior less-insurance region.

COROLLARY EC.1. *Consider an interior parameter region covered by Proposition 6, in which the original optimal mechanism has threshold $\tau \in (0,1)$ and a constant failure payment $\bar{m}_F$ for participating types satisfying*

$$0 < \bar{m}_F < \frac{\alpha c}{\delta}.$$

Then

$$u^U > u^*.$$

In particular, when $\delta = 1$,

$$u^U > u^R = u^*.$$

Proof of Corollary EC.1. Consider the optimal less-insurance mechanism in Proposition 6. For participating types, it has

$$m_0^* = m_S^* = 0, \qquad m_F^* = \bar{m}_F.$$

The marginal participating type's participation constraint binds:

$$c = \delta \tau \pi + \delta (1-\tau) \bar{m}_F. \tag{EC.25}$$

We construct a feasible mechanism in the uncapped success-repayment environment that strictly improves on this less-insurance mechanism. For $\varepsilon > 0$, define

$$\theta_\varepsilon \equiv \frac{\bar{m}_F + \tau \varepsilon}{\bar{m}_F + \varepsilon}.$$

Since $\tau \in (0,1)$ and $\bar{m}_F > 0$, we have

$$\tau < \theta_\varepsilon < 1 \qquad \text{and} \qquad \theta_\varepsilon \to 1 \text{ as } \varepsilon \downarrow 0.$$

Together with $\bar{m}_F < \pi$, $\delta \bar{m}_F < \alpha c$, and $\bar{\tau} < 1$, this implies that there exists sufficiently small $\varepsilon > 0$ such that

$$\pi - \bar{m}_F - \varepsilon > 0, \qquad \delta (\bar{m}_F + \tau \varepsilon) < \alpha c, \qquad \theta_\varepsilon > \bar{\tau}.$$

Consider the following mechanism:

$$q^\varepsilon(\theta) = \mathbf{1}_{\{\theta \geq \tau\}}, \qquad \tilde{m}_F^\varepsilon(\theta) = 0,$$

and

$$(\tilde{m}_0^{\varepsilon}(\theta), \tilde{m}_S^{\varepsilon}(\theta)) = \begin{cases} (0,0), & \theta < \tau, \\ (\delta\,(\bar{m}_F + \tau\,\varepsilon), -\bar{m}_F - \varepsilon)\,, & \tau \le \theta < \theta_\varepsilon, \\ (0,0), & \theta_\varepsilon \le \theta \le 1. \end{cases}$$

Thus intermediate types receive an upfront payment and make a repayment following success, whereas sufficiently high types test using their own resources.

Under the repayment option, a type-θ firm's payoff from exerting effort is

$$-c + \delta\,(\bar{m}_F + \tau\,\varepsilon) + \delta\,\theta\,(\pi - \bar{m}_F - \varepsilon) = \delta\,(\pi - \bar{m}_F - \varepsilon)(\theta - \tau),$$

where the equality follows from (EC.25). If the firm instead rejects the repayment option and self-funds testing, its payoff from testing is $-c + \delta\,\theta\,\pi$. The difference between the payoff from the repayment option and that from self-funded testing is

$$\delta\,(\bar{m}_F + \varepsilon)(\theta_\varepsilon - \theta).$$

Since $\theta_\varepsilon > \bar{\tau}$, types in $[\tau, \theta_\varepsilon]$ weakly prefer the repayment option to their outside option: for $\theta < \bar{\tau}$, the outside option is zero and the repayment option yields a nonnegative payoff, whereas for $\theta \ge \bar{\tau}$, the claim follows from the payoff difference above. Types in $[\theta_\varepsilon, 1]$ weakly prefer self-funded testing, while types below τ prefer not to test. Hence the incentive constraints, including (IR-ST), are satisfied.

Under the repayment option, the payoff from shirking is

$$-\alpha\,c + \delta\,(\bar{m}_F + \tau\,\varepsilon) < 0,$$

whereas the intended effort payoff is nonnegative for every assigned type. Thus the effort constraints are also satisfied.

The repayment option violates the repayment cap imposed previously:

$$\tilde{m}_0^{\varepsilon} + \delta\,\tilde{m}_S^{\varepsilon} = \delta\,(\bar{m}_F + \tau\,\varepsilon) - \delta\,(\bar{m}_F + \varepsilon) = -\delta\,(1 - \tau)\varepsilon < 0.$$

Thus the discounted success repayment exceeds the upfront advance.

It remains to compare the funder's payoff with that under the original failure-payment mechanism. For a type $\theta \in [\tau, \theta_\varepsilon)$, the original mechanism's expected payment is

$$(1 - \theta)\,\bar{m}_F,$$

whereas the new mechanism's expected payment is

$$\delta\,(\bar{m}_F + \tau\,\varepsilon) - \theta\,(\bar{m}_F + \varepsilon).$$

The reduction in the funder's expected payment is therefore

$$(1-\theta)\,\bar{m}_F - [\delta\,(\bar{m}_F + \tau\,\varepsilon) - \theta\,(\bar{m}_F + \varepsilon)] = (1-\delta)\bar{m}_F + \varepsilon(\theta - \delta\,\tau) \geq 0,$$

where the inequality follows from $\theta \geq \tau$ and $\delta \leq 1$. This inequality is strict for almost every $\theta \in [\tau, \theta_\varepsilon)$. For a type $\theta \in [\theta_\varepsilon, 1]$, the new mechanism makes no payment, so the reduction relative to the original mechanism is

$$(1-\theta)\bar{m}_F > 0$$

for every $\theta < 1$. The two mechanisms induce the same testing allocation, while the new mechanism strictly reduces the funder's expected payment. Consequently,

$$u^U > u^*$$

for every $\delta \in (0, 1]$ satisfying the conditions of the corollary. When $\delta = 1$, Proposition EC.3 gives $u^R = u^*$, and hence

$$u^U > u^R = u^*.$$

This completes the proof. □

EC.11. Resubmission and Clawback

Suppose the funder can claw back the failure-contingent funding $m_F(\hat{\theta})$ upon a later success. Consider a firm of type θ that reports $\hat{\theta}$. If it exerts effort upfront, its expected payoff is

$$u_1(\theta, \hat{\theta}) = -c\,q(\hat{\theta}) + m_0(\hat{\theta}) + \delta\,\theta\,[\pi\,q(\hat{\theta}) + m_S(\hat{\theta})] + \delta\,(1-\theta)\,m_F(\hat{\theta}).$$

If instead it first shirks, receives the failure-contingent payment, and then later exerts effort, its expected payoff is

$$-\alpha\,c\,q(\hat{\theta}) + m_0(\hat{\theta}) + \delta\,m_F(\hat{\theta}) + \delta\,\left[-c\,q(\hat{\theta}) + \delta\,\theta\,[\pi\,q(\hat{\theta}) - m_F(\hat{\theta})]\right],$$

where the term $\pi\,q(\hat{\theta}) - m_F(\hat{\theta})$ reflects the clawback upon later success. The payoff difference between exerting effort upfront and following this fail-then-resubmit strategy is

$$\begin{aligned}
&\left[-c\,q(\hat{\theta}) + m_0(\hat{\theta}) + \delta\,\theta\,[\pi\,q(\hat{\theta}) + m_S(\hat{\theta})] + \delta\,(1-\theta)\,m_F(\hat{\theta})\right] \\
&\quad - \left[-\alpha\,c\,q(\hat{\theta}) + m_0(\hat{\theta}) + \delta\,m_F(\hat{\theta}) + \delta\,\left[-c\,q(\hat{\theta}) + \delta\,\theta\,[\pi\,q(\hat{\theta}) - m_F(\hat{\theta})]\right]\right] \\
&= (\alpha + \delta - 1)\,c\,q(\hat{\theta}) + \delta\,\theta\,m_S(\hat{\theta}) + \delta\,(1-\delta)\,\theta\,[\pi\,q(\hat{\theta}) - m_F(\hat{\theta})].
\end{aligned}$$

The firm would choose to exert effort in the resubmission stage only if the continuation value is nonnegative:

$$-c\,q(\hat{\theta}) + \delta\,\theta\,[\pi\,q(\hat{\theta}) - m_F(\hat{\theta})] \geq 0.$$

Equivalently,

$$\delta\,\theta\,[\pi\,q(\hat{\theta}) - m_F(\hat{\theta})] \geq c\,q(\hat{\theta}).$$

Using this inequality in the payoff difference above gives

$$\begin{aligned}
&(\alpha+\delta-1)c\,q(\hat{\theta}) + \delta\,\theta\,m_S(\hat{\theta}) + \delta(1-\delta)\,\theta\,[\pi\,q(\hat{\theta}) - m_F(\hat{\theta})] \\
&\geq (\alpha+\delta-1)c\,q(\hat{\theta}) + \delta\,\theta\,m_S(\hat{\theta}) + (1-\delta)c\,q(\hat{\theta}) \\
&= \alpha\,c\,q(\hat{\theta}) + \delta\,\theta\,m_S(\hat{\theta}) \\
&\geq 0.
\end{aligned}$$

Thus, whenever the fail-then-resubmit strategy has a nonnegative continuation value, it is weakly dominated by exerting effort upfront. If the continuation value is negative, the firm would not resubmit after shirking, and the deviation reduces to the standard one already ruled out by the moral-hazard constraint. Hence, an enforceable clawback preserves the static incentive logic of the model.

EC.12. Suboptimality of the Take-It-or-Leave-It Mechanism

Figure EC.6 compares the performance of the best take-it-or-leave-it offer with that of the unrestricted optimal mechanism. The theoretical performance ratio is

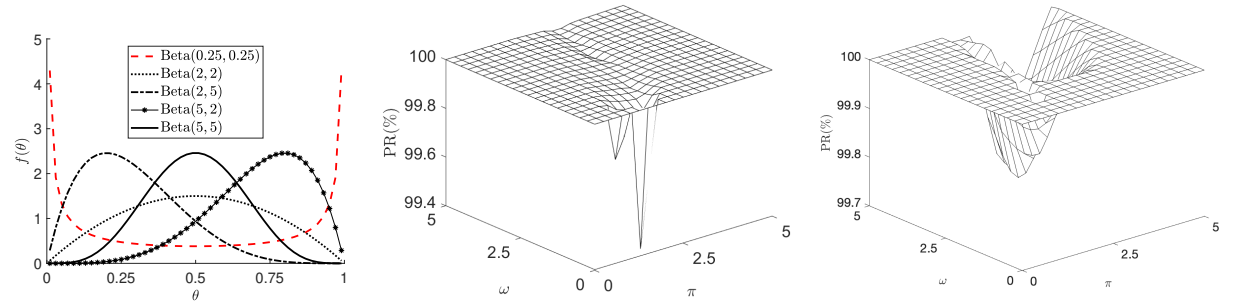
$$\mathrm{PR} \equiv \frac{\max_{\check{\tau}\in[0,1]} u^{\mathrm{TH}}(\check{\tau})}{u^*},$$

where u^* is the value of the unrestricted funder's problem (14). In the numerical implementation, we approximate u^* by u^{LP}, the value of the finite-dimensional LP obtained by discretizing the unrestricted problem over the θ-grid. Thus, the plotted ratio replaces u^* with u^{LP}. If the unrestricted numerical LP solution is itself implementable as a take-it-or-leave-it offer, the best take-it-or-leave-it offer attains the same finite-grid value, and we set $\mathrm{PR} = 1$. We also set $\mathrm{PR} = 1$ when both the unrestricted LP value and the best take-it-or-leave-it value are zero.

We consider the five Beta distributions in Figure EC.1. Panels (b)–(f) show that the best take-it-or-leave-it offer achieves at least 99% of the unrestricted numerical-LP benchmark in all cases considered.

We also repeat this comparison at the disease-specific calibration points under the same five Beta priors. For each disease-prior pair, we compute u^{LP} and compare it with the value of the best take-it-or-leave-it offer. Table EC.8 reports the largest positive gaps.

The largest calibration-point gap is 0.37%, for genital herpes under the Beta$(5,2)$ prior. Thus, the take-it-or-leave-it mechanism is not only close to optimal on the broader (π,ω) grids in Figure EC.6, but also essentially optimal at the disease-specific calibration points under the non-uniform priors considered.



(a) Density functions (b) $\theta \sim \text{Beta}(0.25, 0.25)$ (c) $\theta \sim \text{Beta}(2, 2)$

(d) $\theta \sim \text{Beta}(5, 2)$ (e) $\theta \sim \text{Beta}(2, 5)$ (f) $\theta \sim \text{Beta}(5, 5)$

Figure EC.6 Suboptimality of the simple threshold policy when θ is not uniformly distributed.

Disease	Beta(.25, .25)	Beta(2, 2)	Beta(5, 5)	Beta(2, 5)	Beta(5, 2)
Acute hepatitis A	$< 0.01\%$	0.04%	0.18%	0.02%	0.10%
Genital herpes	$< 0.01\%$	0.00%	$< 0.01\%$	$< 0.01\%$	0.37%
Cysticercosis	$< 0.01\%$	0.00%	0.01%	$< 0.01\%$	0.06%
Trichomoniasis	$< 0.01\%$	0.00%	0.00%	0.03%	0.24%
Echinococcosis	$< 0.01\%$	0.00%	$< 0.01\%$	$< 0.01\%$	0.06%

Table EC.8 Largest calibration-point suboptimality gaps of the best take-it-or-leave-it mechanism under alternative Beta priors. Positive values below 0.01% are reported as $< 0.01\%$; all omitted diseases have gaps below 0.01% under every prior.

EC.13. Distributionally Robust Optimization

In this section, we study a distributionally robust version of the problem in which the funder only knows the mean of the firm's success probability θ. Specifically, an adversary chooses a distribution from

$$\mathcal{F} = \left\{ F \in \mathcal{P} : \int_0^1 \theta \, \mathrm{d}F(\theta) = \mu \right\},$$

where $\mathcal{P}$ is the set of probability distributions on $[0, 1]$. We focus on the nondegenerate case $\mu \in (0, 1)$; the cases $\mu = 0$ and $\mu = 1$ are immediate.

The funder again focuses on take-it-or-leave-it mechanisms defined in (19). Let $\check{u}(\theta \mid \mathcal{M})$ denote the funder's expected payoff using mechanism $\mathcal{M} \equiv \{\check{\tau}, \check{m}_0, \check{m}_S, \check{m}_F\}$ when the drug's prior probability

of success is θ, i.e.,

$$\check{u}(\theta \mid \mathcal{M}) = \begin{cases} \theta\,\omega - \check{m}_0 - \theta\,\check{m}_S - (1-\theta)\,\check{m}_F, & \theta \in [\check{\tau}, 1], \\ 0, & \theta \in [0, \check{\tau}). \end{cases} \tag{EC.26}$$

We also allow the no-testing mechanism, denoted by $\mathcal{M}^{\emptyset}$, which induces no testing and gives the funder payoff zero for every θ.

The robust optimization problem is as follows:

$$Z \equiv \sup_{\mathcal{M} \in \Omega_R} \check{u}^{\mathcal{F}}(\mathcal{M}), \tag{EC.27}$$

where

$$\check{u}^{\mathcal{F}}(\mathcal{M}) \equiv \inf_{F \in \mathcal{F}} \int_0^1 \check{u}(\theta \mid \mathcal{M})\,\mathrm{d}F(\theta),$$

and the feasible set Ω_R is determined by constraints (22)–(25) and

$$0 < \check{\tau} \leq 1,$$

together with the degenerate no-testing mechanism $\mathcal{M}^{\emptyset}$.

The following lemma implies that it is without loss of generality to consider incentive-compatible take-it-or-leave-it offers $\mathcal{M} \in \Omega_R$ that generate a nonnegative payoff for the funder for every realization of the agent's private information, i.e., $\check{u}(\theta \mid \mathcal{M}) \geq 0$ for all $\theta \in [0, 1]$. This result is crucial in characterizing the adversary's response to the announced mechanism.

Lemma EC.7. *For any mechanism $\mathcal{M} \in \Omega_R$ with $\check{u}(\theta \mid \mathcal{M}) < 0$ for some $\theta \in [0, 1]$, there exists a mechanism $\tilde{\mathcal{M}} \in \Omega_R$ such that*

$$\check{u}(\theta \mid \tilde{\mathcal{M}}) \geq \max\left\{\check{u}(\theta \mid \mathcal{M}), 0\right\}, \qquad \forall \theta \in [0, 1].$$

The following lemma characterizes the adversary's choice of distribution when the funder commits to a mechanism $\mathcal{M} \in \Omega_R$.

Lemma EC.8. *Given a mechanism $\mathcal{M} \equiv \{\check{\tau}, \check{m}_0, \check{m}_S, \check{m}_F\} \in \Omega_R$ satisfying $\check{u}(\theta \mid \mathcal{M}) \geq 0$ for any $\theta \in [0, 1]$,*

1. *if $\check{\tau} > \mu$, define*

$$F^*(\theta) \equiv \begin{cases} 0, \ \forall \theta \in [0, \mu), \\ 1, \ \forall \theta \in [\mu, 1], \end{cases}$$

and we have

$$\check{u}^{\mathcal{F}}(\mathcal{M}) = \int_0^1 \check{u}(\theta \mid \mathcal{M})\,\mathrm{d}F^*(\theta) = 0; \tag{EC.28}$$

2. *if* $0 < \check{\tau} \le \mu$, *define*

$$F_\epsilon^*(\theta|\mathcal{M}) \equiv \begin{cases} 0, & \forall \theta \in [0, \check{\tau} - \epsilon), \\ (1-\mu)/(1-\check{\tau}+\epsilon), & \forall \theta \in [\check{\tau} - \epsilon, 1), \\ 1, & \theta = 1, \end{cases}$$

and we have

$$\check{u}^{\mathcal{F}}(\mathcal{M}) = \lim_{\epsilon \downarrow 0} \int_0^1 \check{u}(\theta \mid \mathcal{M}) \, \mathrm{d}F_\epsilon^*(\theta|\mathcal{M}) = \frac{\mu - \check{\tau}}{1 - \check{\tau}} \, \check{u}(1 \mid \mathcal{M}). \tag{EC.29}$$

If $\check{\tau} = 0$, *the same value is attained by the distribution that places probability* $1-\mu$ *on* 0 *and probability* μ *on* 1.

Based on the worst-case distributions described in Lemma EC.8, we can characterize the robust take-it-or-leave-it offer. Proposition EC.4 is the main theoretical result of this subsection. It shows that the optimal robust take-it-or-leave-it offer can be computed by solving two one-dimensional optimization problems. The reason is that, by Lemma 1, it is without loss of generality to consider mechanisms that do not reward success and failure simultaneously.

Proposition EC.4. *The value* Z *defined in* (EC.27) *is characterized by*

$$Z = \max\{Z_1, Z_2, 0\},$$

where Z_1 *and* Z_2 *are determined by the following one-dimensional optimization problems:*

$$\begin{aligned} Z_1 = \max_{\check{\tau} \in (0,\mu]} \quad & \frac{\mu - \check{\tau}}{1 - \check{\tau}} \left[\omega + (\delta\,\check{\tau} - 1)\check{m}_S + \delta\,\pi\,\check{\tau} - c\right], \\ \textit{subject to} \quad & \check{m}_S = \max\left\{\frac{(1-\alpha)c}{\delta\,\check{\tau}} - \pi, 0\right\} \le \min\left\{\frac{c}{\delta\,\check{\tau}} - \pi, \frac{\check{\tau}\,\omega + \delta\,\pi\,\check{\tau} - c}{(1-\delta)\,\check{\tau}}\right\} \end{aligned} \tag{EC.30}$$

and

$$\begin{aligned} Z_2 = \max_{\check{\tau} \in (0,\mu]} \quad & \frac{\mu - \check{\tau}}{1 - \check{\tau}} \left[\omega + \delta\,\pi\,\check{\tau} + \delta(1-\check{\tau})\check{m}_F - c\right], \\ \textit{subject to} \quad & 0 \le \check{m}_F = \min\left\{\frac{c - \delta\,\pi\,\check{\tau}}{\delta(1-\check{\tau})}, \frac{\delta\,\pi\,\check{\tau} - (1-\alpha)c}{\delta\,\check{\tau}}, \frac{\check{\tau}\,\omega + \delta\,\pi\,\check{\tau} - c}{(1-\delta)(1-\check{\tau})}\right\}. \end{aligned} \tag{EC.31}$$

If an optimization problem has an empty feasible set, its value is interpreted as $-\infty$. *When* $\delta = 1$, *the constraints involving division by* $1-\delta$ *are interpreted in their undivided form before taking the limit.*

Let $\check{\tau}_1$ *and* $\check{\tau}_2$ *denote optimal solutions to* (EC.30) *and* (EC.31), *respectively.*

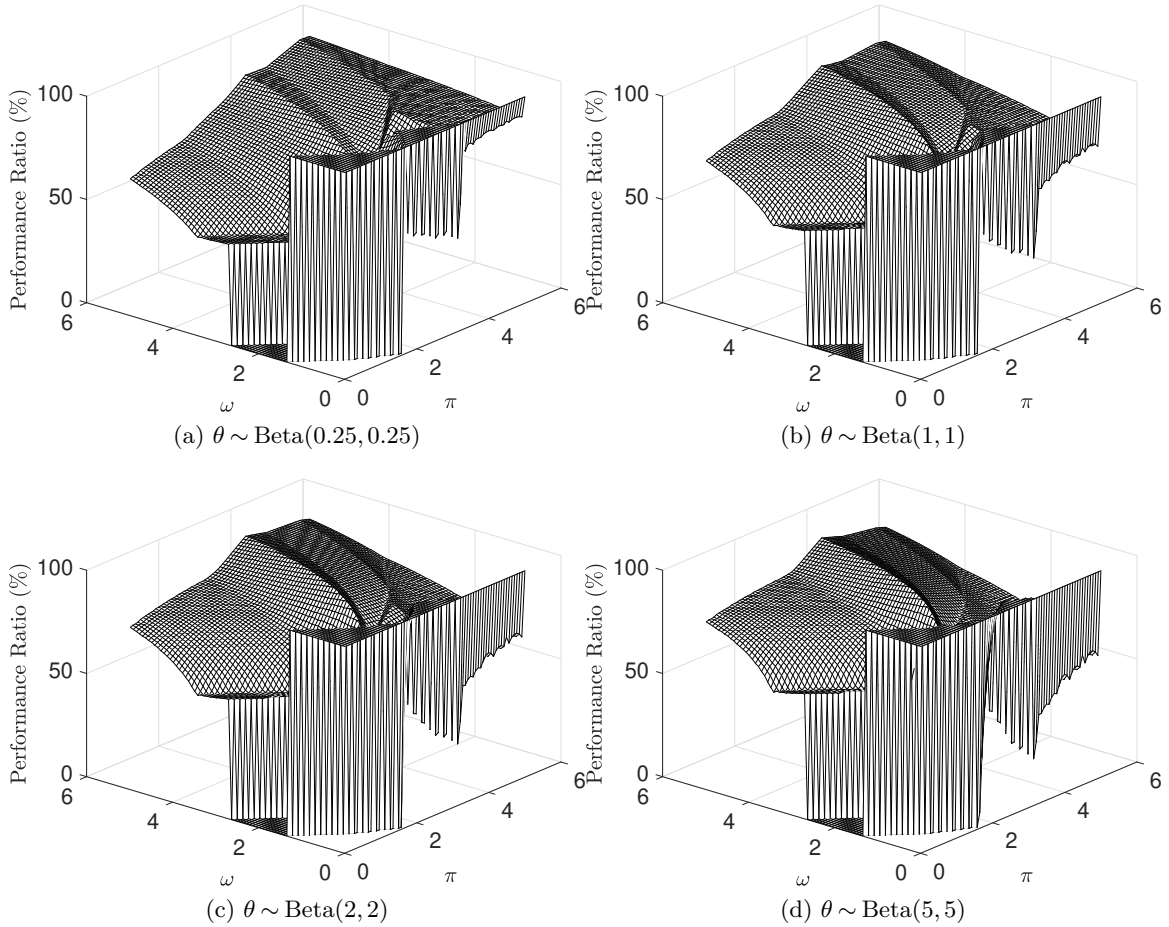
1. *If* $Z = Z_1$, *the following mechanism is optimal for* (EC.27)*:*

$$\check{\tau}^* = \check{\tau}_1, \qquad \check{m}_S^* = \check{m}_S(\check{\tau}_1), \qquad \check{m}_F^* = 0, \qquad \check{m}_0^* = c - \delta\,\check{\tau}_1(\check{m}_S^* + \pi).$$

2. *If* $Z = Z_2$, *the following mechanism is optimal for* (EC.27)*:*

$$\check{\tau}^* = \check{\tau}_2, \qquad \check{m}_S^* = 0, \qquad \check{m}_F^* = \check{m}_F(\check{\tau}_2), \qquad \check{m}_0^* = c - \delta\,\pi\,\check{\tau}_2 - \delta\,\check{m}_F^*(1-\check{\tau}_2).$$

3. *If* $Z = 0$, *the degenerate no-testing mechanism* $\mathcal{M}^\emptyset$ *is optimal.*



(a) $\theta \sim \text{Beta}(0.25, 0.25)$ (b) $\theta \sim \text{Beta}(1, 1)$

(c) $\theta \sim \text{Beta}(2, 2)$ (d) $\theta \sim \text{Beta}(5, 5)$

Figure EC.7 Performance ratio of the distributionally robust take-it-or-leave-it policy under alternative true Beta priors. The robust policy is computed using only the mean $\mu = 0.5$, while the benchmark uses the true distribution. The performance ratio is the expected payoff of the robust policy divided by the expected payoff of the distribution-aware optimal take-it-or-leave-it policy. Parameters are $\alpha = 0.5$, $c = 1$, and $\delta = 0.6$.

Figure EC.7 evaluates the loss from using the distributionally robust policy when the true prior distribution of θ is one of four Beta distributions with mean $\mu = 0.5$. The numerator is the funder's expected payoff from the robust policy characterized in Proposition EC.4; the denominator is the expected payoff from the distribution-aware optimal take-it-or-leave-it policy characterized in Proposition 9. The figure shows that the robust policy performs well over most of the (π, ω) parameter space, especially when commercial profit is moderate or high. The main performance losses occur when commercial profit is small. In particular, when both π and ω are small, the distributionally robust policy may choose to forgo testing altogether; if the distribution-aware benchmark would still induce testing under the true prior, the performance ratio drops to zero. More generally, when π is small, the robust policy is relatively conservative because commercial returns provide little support for participation and effort incentives, so relying only on the mean of θ can lead the funder to choose

a higher threshold. As π increases, commercial incentives make testing easier to sustain, and the robust policy typically achieves a performance ratio of more than 80%. Overall, the figure suggests that mean-only distributional information is often sufficient to obtain high expected performance, but full distributional information is most valuable when commercial profit is low.

EC.13.1. Proofs of Appendix EC.13

LEMMA EC.9. *For any $\mathcal{M} \equiv \{\check{\tau}, \check{m}_0, \check{m}_S, \check{m}_F\} \in \Omega_R$, there exists $\mathcal{M}' \equiv \{\check{\tau}', \check{m}'_0, \check{m}'_S, \check{m}'_F\} \in \Omega_R$ such that*

$$\begin{aligned} &\check{m}'_S \cdot \check{m}'_F = 0, \\ &\check{m}'_S \le \omega, \ \textit{and} \\ &\check{u}^{\mathcal{F}}(\mathcal{M}') \ge \check{u}^{\mathcal{F}}(\mathcal{M}). \end{aligned}$$

Furthermore, the funder's payoff function $\check{u}(\cdot \mid \mathcal{M}')$ is nondecreasing on $[\check{\tau}', 1]$.

Proof of Lemma EC.9. The first condition follows from Lemma 1, which improves the funder's payoff pointwise and preserves feasibility.

After applying Lemma 1, suppose the resulting mechanism has $\check{m}_S > \omega$. Since $\check{m}_S \check{m}_F = 0$, this implies $\check{m}_F = 0$. Therefore, for any participating type $\theta \in [\check{\tau}', 1]$,

$$\check{u}(\theta \mid \mathcal{M}) = \theta(\omega - \check{m}_S) - \check{m}_0 \le 0,$$

and nonparticipating types generate payoff zero. Hence, $\check{u}(\theta \mid \mathcal{M}) \le 0$ for every θ. The degenerate no-testing mechanism $\mathcal{M}^{\emptyset}$ gives payoff zero for every type and therefore weakly dominates $\mathcal{M}$, implying that

$$\check{u}^{\mathcal{F}}(\mathcal{M}_0) \ge \check{u}^{\mathcal{F}}(\mathcal{M}).$$

For the resulting mechanism $\mathcal{M}'$, we have $\check{m}'_S \le \omega$. The monotonicity of $\check{u}$ follows from

$$\check{u}'(\theta \mid \mathcal{M}') = \underbrace{\omega - \check{m}'_S}_{\ge 0} + \check{m}'_F \ge 0.$$

This completes the proof. □

LEMMA EC.10. *For any mechanism $\mathcal{M} \equiv \{\hat{\tau}, \hat{m}_0, \hat{m}_S, \hat{m}_F\} \in \Omega_R$ with $\check{u}(\hat{\tau} \mid \mathcal{M}) < 0$, there exists $\tilde{\mathcal{M}} \equiv \{\tilde{\tau}, \tilde{m}_0, \tilde{m}_S, \tilde{m}_F\} \in \Omega_R$ satisfying either $\tilde{m}_0 = 0$ or $\check{u}(\theta \mid \tilde{\mathcal{M}}) \ge 0$ for any $\theta \in [0, 1]$, such that $\check{u}(\theta \mid \tilde{\mathcal{M}}) \ge \check{u}(\theta \mid \mathcal{M})$ for any $\theta \in [0, 1]$.*

Proof of Lemma EC.10. If $\hat{m}_0 = 0$, the original mechanism already satisfies the desired property. Hence suppose $\hat{m}_0 > 0$.

For a nondegenerate threshold mechanism, feasibility implies $\hat{m}_S - \hat{m}_F + \pi > 0$. For any $\hat{m}_0' \in [0, \hat{m}_0]$, define

$$\acute{\tau}(\hat{m}_0') \equiv \frac{c - \hat{m}_0' - \delta\,\hat{m}_F}{\delta\,(\hat{m}_S - \hat{m}_F + \pi)},$$

and

$$\acute{\mathcal{M}}(\hat{m}_0') \equiv \{\acute{\tau}(\hat{m}_0'), \hat{m}_0', \hat{m}_S, \hat{m}_F\}.$$

Lowering $\hat{m}_0$ and adjusting the threshold according to the threshold-type participation equation preserves the other feasibility constraints whenever the resulting threshold remains in $[0, 1]$.

For any $\theta \geq \acute{\tau}(\hat{m}_0')$, we have

$$\breve{u}(\theta \mid \acute{\mathcal{M}}(\hat{m}_0')) = \theta\,\omega - \hat{m}_0' - \theta\,\hat{m}_S - (1-\theta)\hat{m}_F.$$

If $\omega - \hat{m}_S + \hat{m}_F = 0$, then this active payoff is constant in θ. Since it is negative at the original threshold, the degenerate no-testing mechanism pointwise dominates $\mathcal{M}$, and the result follows. Hence suppose $\omega - \hat{m}_S + \hat{m}_F > 0$, and define the zero of the active payoff by

$$\tau^*(\hat{m}_0') = \frac{\hat{m}_0' + \hat{m}_F}{\omega - \hat{m}_S + \hat{m}_F}.$$

Because $\breve{u}(\hat{\tau} \mid \mathcal{M}) < 0$, we have

$$\acute{\tau}(\hat{m}_0) < \tau^*(\hat{m}_0).$$

If $\acute{\tau}(0) \geq \tau^*(0)$, continuity implies that there exists $\tilde{m}_0 \in [0, \hat{m}_0]$ such that

$$\acute{\tau}(\tilde{m}_0) = \tau^*(\tilde{m}_0).$$

If $\acute{\tau}(0) < \tau^*(0)$, set $\tilde{m}_0 = 0$. In either case,

$$\acute{\tau}(\tilde{m}_0) \leq \tau^*(\tilde{m}_0).$$

If $\acute{\tau}(\tilde{m}_0) \leq 1$, under mechanism $\acute{\mathcal{M}}(\tilde{m}_0)$, we have

$$\breve{u}(\theta \mid \acute{\mathcal{M}}(\tilde{m}_0)) = \begin{cases} \theta\,\omega - \tilde{m}_0 - \theta\,\hat{m}_S - (1-\theta)\,\hat{m}_F, & \forall \theta \in [\acute{\tau}(\tilde{m}_0), 1], \\ 0, & \forall \theta \in [0, \acute{\tau}(\tilde{m}_0)). \end{cases}$$

1. When $\theta \in [0, \acute{\tau}(\hat{m}_0))$, we have

$$\breve{u}(\theta \mid \mathcal{M}) = \breve{u}(\theta \mid \acute{\mathcal{M}}(\tilde{m}_0)) = 0.$$

2. When $\theta \in [\acute{\tau}(\hat{m}_0), \acute{\tau}(\tilde{m}_0))$, we have

$$\begin{aligned}\check{u}(\theta \mid \mathcal{M}) &= \theta\,\omega - \hat{m}_0 - \theta\,\hat{m}_S - (1-\theta)\,\hat{m}_F \\ &\le \theta\,\omega - \tilde{m}_0 - \theta\,\hat{m}_S - (1-\theta)\,\hat{m}_F \\ &\le 0\end{aligned}$$

and

$$\check{u}(\theta \mid \acute{\mathcal{M}}(\tilde{m}_0)) = 0.$$

3. When $\theta \in [\acute{\tau}(\tilde{m}_0), 1]$, we have

$$\check{u}(\theta \mid \acute{\mathcal{M}}(\tilde{m}_0)) - \check{u}(\theta \mid \mathcal{M}) = \hat{m}_0 - \tilde{m}_0 \ge 0.$$

Therefore, for any $\theta \in [0,1]$, we have

$$\check{u}(\theta \mid \mathcal{M}) \le \check{u}(\theta \mid \acute{\mathcal{M}}(\tilde{m}_0)),$$

and $\acute{\mathcal{M}}(\tilde{m}_0)$ is a feasible mechanism satisfying the desired properties.

If $\acute{\tau}(\tilde{m}_0) > 1$, then the active payoff is nonpositive for all $\theta \in [0,1]$, and the degenerate no-testing mechanism pointwise dominates $\mathcal{M}$; in this case, set $\tilde{\mathcal{M}} = \mathcal{M}^{\emptyset}$.

This completes the proof. □

Lemma EC.11. *For any mechanism $\mathcal{M} \equiv \{\hat{\tau}, 0, \hat{m}_S, \hat{m}_F\} \in \Omega_R$ with $\check{u}(\hat{\tau} \mid \mathcal{M}) < 0$, there exists $\tilde{\mathcal{M}} \equiv \{\tilde{\tau}, 0, \tilde{m}_S, \tilde{m}_F\} \in \Omega_R$ such that*

$$\check{u}(\theta \mid \tilde{\mathcal{M}}) \ge \max\{\check{u}(\theta \mid \mathcal{M}), 0\}, \quad \forall \theta \in [0,1].$$

Proof of Lemma EC.11. Because $\hat{m}_0 = 0$, $\hat{m}_S \hat{m}_F = 0$, and $\check{u}(\hat{\tau} \mid \mathcal{M}) < 0$, there are two cases.

First, suppose $\hat{m}_S > 0$ and $\hat{m}_F = 0$. Then

$$\check{u}(\hat{\tau} \mid \mathcal{M}) = \hat{\tau}(\omega - \hat{m}_S) < 0,$$

which implies $\omega < \hat{m}_S$. This contradicts $\hat{m}_S \le \omega$. Therefore, this case cannot arise after applying Lemma EC.9.

Second, suppose $\hat{m}_F > 0$ and $\hat{m}_S = 0$. The conditions $\check{u}(\hat{\tau} \mid \mathcal{M}) < 0$ and $\mathcal{M} \in \Omega_R$ imply

$$\hat{\tau}\,\omega - (1-\hat{\tau})\hat{m}_F < 0$$

and

$$-c + \delta\,\pi\,\hat{\tau} + \delta(1-\hat{\tau})\hat{m}_F = 0.$$

For any $\hat{m}'_F \in [0, \hat{m}_F]$, define

$$\acute{\tau}(\hat{m}'_F) = \frac{c - \delta\,\hat{m}'_F}{\delta(\pi - \hat{m}'_F)},$$

and

$$\acute{\mathcal{M}}(\hat{m}'_F) = \{\acute{\tau}(\hat{m}'_F), 0, 0, \hat{m}'_F\}\,.$$

The threshold $\acute{\tau}(\hat{m}'_F)$ is chosen so that the threshold-type participation constraint binds.

Under $\acute{\mathcal{M}}(\hat{m}'_F)$, for any $\theta \in [\acute{\tau}(\hat{m}'_F), 1]$, the funder's payoff is

$$\check{u}(\theta \mid \acute{\mathcal{M}}(\hat{m}'_F)) = \theta\,\omega - (1-\theta)\hat{m}'_F.$$

Let

$$\tau^*(\hat{m}'_F) \equiv \frac{\hat{m}'_F}{\omega + \hat{m}'_F}$$

denote the zero of this active payoff. Because $\check{u}(\hat{\tau} \mid \mathcal{M}) < 0$, we have

$$\acute{\tau}(\hat{m}_F) < \tau^*(\hat{m}_F).$$

On the other hand,

$$\acute{\tau}(0) = \frac{c}{\delta\pi} > 0 = \tau^*(0).$$

By continuity, there exists $\tilde{m}_F \in [0, \hat{m}_F]$ such that

$$\acute{\tau}(\tilde{m}_F) = \tau^*(\tilde{m}_F).$$

Let

$$\tilde{\mathcal{M}} = \acute{\mathcal{M}}(\tilde{m}_F).$$

If $\acute{\tau}(\tilde{m}_F) > 1$, then the active payoff is nonpositive for every $\theta \in [0,1]$, and the degenerate no-testing mechanism pointwise dominates the original mechanism. Otherwise, $\tilde{\mathcal{M}} \in \Omega_R$, and the funder's active payoff under $\tilde{\mathcal{M}}$ is zero at its threshold and nonnegative above it.

Finally, since $\tilde{m}_F \le \hat{m}_F$, the active payoff under $\tilde{\mathcal{M}}$ is weakly higher than the original active payoff for every type that participates under both mechanisms. If a type participates under $\mathcal{M}$ but not under $\tilde{\mathcal{M}}$, then it lies below the zero of the original active payoff and hence gives the original mechanism a nonpositive payoff; the new mechanism gives it payoff zero. Therefore,

$$\check{u}(\theta \mid \tilde{\mathcal{M}}) \ge \max\{\check{u}(\theta \mid \mathcal{M}), 0\}\,, \quad \forall \theta \in [0,1].$$

This completes the proof. □

Proof of Lemma EC.7. Let $\mathcal{M}^0 \in \Omega_R$ denote the original mechanism. Applying Lemma 1, we obtain a feasible mechanism that pointwise weakly dominates $\mathcal{M}^0$ and satisfies $\check{m}_S \check{m}_F = 0$. By Lemma EC.9, if this mechanism has $\check{m}_S > \omega$, then it is pointwise weakly dominated by the degenerate no-testing mechanism, in which case the desired conclusion follows immediately. Hence, it remains to consider mechanisms satisfying $\check{m}_S \check{m}_F = 0$, $\check{m}_S \le \omega$, and having a nondecreasing active payoff. We denote this mechanism by $\mathcal{M}$.

If $\check{u}(\theta \mid \mathcal{M}) \ge 0$ for every θ, then the current mechanism already satisfies the desired property. Otherwise, because $\check{u}(\theta \mid \mathcal{M}) = 0$ below the threshold and the active payoff is nondecreasing on the participating region, the threshold type must generate a negative payoff:

$$\check{u}(\check{\tau} \mid \mathcal{M}) < 0.$$

Lemma EC.10 then yields a feasible mechanism that pointwise weakly dominates $\mathcal{M}$ and either has nonnegative funder payoff for all types or has zero upfront payment. In the former case, the desired conclusion follows. In the latter case, Lemma EC.11 yields a further feasible mechanism whose payoff weakly dominates both the previous payoff and zero pointwise. Combining these pointwise improvements, we obtain a mechanism $\tilde{\mathcal{M}} \in \Omega_R$ such that

$$\check{u}(\theta \mid \tilde{\mathcal{M}}) \ge \max\left\{\check{u}(\theta \mid \mathcal{M}^0), 0\right\}, \quad \forall \theta \in [0,1].$$

This completes the proof. □

Proof of Lemma EC.8. Define

$$\check{\Omega} \equiv \left\{ \mathcal{M} \equiv \{\check{\tau}, \check{m}_0, \check{m}_S, \check{m}_F\} \in \Omega_R \;\middle|\; \begin{array}{l} \check{u}(\check{\tau} \mid \mathcal{M}) \ge 0 \\ \check{m}_S \cdot \check{m}_F = 0 \\ \check{m}_S \le \omega \end{array} \right\}.$$

Lemmas EC.9–EC.11 allow us to focus on $\mathcal{M} \in \check{\Omega}$ without loss of optimality.

Consider the case in which $\check{\tau} > \mu$. Because $\check{u}(\theta \mid \mathcal{M}) \ge 0$ for any $\theta \in [0,1]$, we know that $\check{u}^{\mathcal{F}}(\mathcal{M}) \ge 0$. Under distribution F^*, no firms will participate, and hence the funder's expected payoff is 0, attaining the lower bound.

Now consider $\check{\tau} \le \mu$. Since $\mu \in (0,1)$, we have $\check{\tau} < 1$. The active payoff is affine on $[\check{\tau}, 1]$. Hence, for every $\theta \in [\check{\tau}, 1]$,

$$\check{u}(\theta \mid \mathcal{M}) = \frac{1-\theta}{1-\check{\tau}}\, \check{u}(\check{\tau} \mid \mathcal{M}) + \frac{\theta - \check{\tau}}{1-\check{\tau}}\, \check{u}(1 \mid \mathcal{M}).$$

Because $\check{u}(\check{\tau} \mid \mathcal{M}) \ge 0$, this implies

$$\check{u}(\theta \mid \mathcal{M}) \ge \frac{\theta - \check{\tau}}{1-\check{\tau}}\, \check{u}(1 \mid \mathcal{M}), \qquad \forall \theta \in [\check{\tau}, 1].$$

For $\theta < \check{\tau}$, the right-hand side below is zero and $\check{u}(\theta \mid \mathcal{M}) \ge 0$. Therefore,

$$\check{u}(\theta \mid \mathcal{M}) \ge \frac{(\theta - \check{\tau})^+}{1-\check{\tau}}\, \check{u}(1 \mid \mathcal{M}), \qquad \forall \theta \in [0,1].$$

Thus, for any $F \in \mathcal{F}$,

$$\int_0^1 \check{u}(\theta \mid \mathcal{M})\,\mathrm{d}F(\theta) \geq \frac{\check{u}(1 \mid \mathcal{M})}{1-\check{\tau}} \int_0^1 (\theta-\check{\tau})^+\,\mathrm{d}F(\theta) \geq \frac{\check{u}(1 \mid \mathcal{M})}{1-\check{\tau}} \int_0^1 (\theta-\check{\tau})\,\mathrm{d}F(\theta) = \frac{\mu-\check{\tau}}{1-\check{\tau}}\,\check{u}(1 \mid \mathcal{M}).$$

It remains to show that the lower bound is attainable. If $\check{\tau}=0$, the distribution that places probability $1-\mu$ on 0 and probability μ on 1 has mean μ and yields the value $\mu\,\check{u}(1 \mid \mathcal{M})$, which equals the displayed lower bound. If $\check{\tau}>0$, then for $0<\epsilon<\check{\tau}$, the distribution $F^*_\epsilon(\cdot \mid \mathcal{M})$ places probability $(1-\mu)/(1-\check{\tau}+\epsilon)$ on $\check{\tau}-\epsilon$ and the remaining probability on 1. Its mean is μ. Since $\check{\tau}-\epsilon<\check{\tau}$, the low type does not participate and gives the funder payoff zero. Hence

$$\int_0^1 \check{u}(\theta \mid \mathcal{M})\;\mathrm{d}F^*_\epsilon(\theta \mid \mathcal{M}) = \frac{\mu-\check{\tau}+\epsilon}{1-\check{\tau}+\epsilon}\,\check{u}(1 \mid \mathcal{M}) \xrightarrow{\epsilon\downarrow 0} \frac{\mu-\check{\tau}}{1-\check{\tau}}\,\check{u}(1 \mid \mathcal{M}).$$

This completes the proof. □

Proof of Proposition EC.4. When $\check{\tau}>\mu$, the funder's expected payoff in the worst-case scenario is 0.

When $\check{\tau}\leq\mu$, following Lemma EC.8, with probability $(\mu-\check{\tau})/(1-\check{\tau})$, the firm is of type $\theta=1$ and will conduct the trial. The funder solves the following optimization problem:

$$\begin{aligned}
\max_{\check{\tau},\check{m}_0,\check{m}_S,\check{m}_F}\quad & \frac{\mu-\check{\tau}}{1-\check{\tau}}\,(\omega-\check{m}_0-\check{m}_S)\\
\text{subject to}\quad & -c+\check{m}_0+\delta\,\check{\tau}(\check{m}_S+\pi)+\delta(1-\check{\tau})\check{m}_F=0,\\
& \check{m}_S-\check{m}_F+\pi\geq 0,\\
& \check{m}_0+\delta\,\check{m}_F\leq\alpha\,c,\\
& 0<\check{\tau}\leq\mu,\\
& \check{m}_0,\check{m}_S,\check{m}_F\geq 0,\\
& \check{\tau}\,\omega-\check{m}_0-\check{\tau}\,\check{m}_S-(1-\check{\tau})\check{m}_F\geq 0.^{13}
\end{aligned}$$

By Lemma EC.9, the funder's problem can be separated into two subproblems – one with $\check{m}_S=0$ and the other with $\check{m}_F=0$. The funder will adopt the option that yields the higher expected payoff.

$$\begin{aligned}
\max_{\check{\tau},\check{m}_0,\check{m}_S}\quad & \frac{\mu-\check{\tau}}{1-\check{\tau}}\,(\omega-\check{m}_0-\check{m}_S),\\
\text{subject to}\quad & -c+\check{m}_0+\delta\,\check{\tau}\,(\check{m}_S+\pi)=0,\\
& \check{m}_0\leq\alpha\,c,\\
& 0\leq\check{\tau}\leq\mu,\\
& \check{m}_0,\check{m}_S\geq 0,\\
& \check{\tau}\,\omega-\check{m}_0-\check{\tau}\,\check{m}_S\geq 0,
\end{aligned}\tag{EC.32}$$

[13] This constraint guarantees the nonnegativity of the funder's payoff function $\check{u}$.

and

$$\begin{aligned}
\max_{\check{\tau},\check{m}_0,\check{m}_F} \quad & \frac{\mu-\check{\tau}}{1-\check{\tau}}\,(\omega-\check{m}_0),\\
\text{subject to} \quad & -c+\check{m}_0+\delta\,\pi\,\check{\tau}+\delta\,(1-\check{\tau})\,\check{m}_F=0,\\
& -\check{m}_F+\pi\geq 0,\\
& \check{m}_0+\delta\,\check{m}_F\leq \alpha\,c,\\
& 0\leq\check{\tau}\leq\mu,\\
& \check{m}_0,\check{m}_F\geq 0,\\
& \check{\tau}\,\omega-\check{m}_0-(1-\check{\tau})\,\check{m}_F\geq 0.
\end{aligned} \tag{EC.33}$$

In subproblem (EC.32), we first use the equality constraint to eliminate $\check{m}_0$, i.e.,

$$\check{m}_0=c-\delta\,\check{\tau}\,(\check{m}_S+\pi).$$

The optimization problem can be rewritten as

$$Z_1\equiv\max_{\check{\tau}\in[0,\mu]}\ \frac{\mu-\check{\tau}}{1-\check{\tau}}\,h_1(\check{\tau}),$$

where

$$\begin{aligned}
h_1(\check{\tau})=\quad \max_{\check{m}_S\geq 0} \quad & \omega+\delta\,\check{\tau}\,(\check{m}_S+\pi)-\check{m}_S-c,\\
\text{subject to} \quad & 0\leq c-\delta\,\check{\tau}\,(\check{m}_S+\pi)\leq\alpha\,c,\\
& c-\delta\,\check{\tau}\,(\check{m}_S+\pi)\leq\check{\tau}\,(\omega-\check{m}_S).
\end{aligned}$$

The feasibility conditions for $\check{m}_S$ can be rewritten as

$$\max\left\{\frac{(1-\alpha)\,c}{\delta\,\check{\tau}}-\pi,0\right\}\leq\check{m}_S\leq\min\left\{\frac{c}{\delta\,\check{\tau}}-\pi,\frac{\check{\tau}\,\omega+\delta\,\pi\,\check{\tau}-c}{(1-\delta)\,\check{\tau}}\right\}.$$

Because $\delta\,\check{\tau}-1<0$, when

$$\max\left\{\frac{(1-\alpha)\,c}{\delta\,\check{\tau}}-\pi,0\right\}\leq\min\left\{\frac{c}{\delta\,\check{\tau}}-\pi,\frac{\check{\tau}\,\omega+\delta\,\pi\,\check{\tau}-c}{(1-\delta)\,\check{\tau}}\right\},$$

the optimal $\check{m}_S$ is determined by

$$\check{m}_S^*=\max\left\{\frac{(1-\alpha)\,c}{\delta\,\check{\tau}}-\pi,0\right\}.$$

Consider the other subproblem (EC.33). Similarly, we use the equality constraint to eliminate $\check{m}_0$, i.e.,

$$\check{m}_0=c-\delta\,\pi\,\check{\tau}-\delta\,(1-\check{\tau})\,\check{m}_F.$$

The funder's problem can be reformulated as follows:

$$Z_2\equiv\max_{\check{\tau}\in[0,\mu]}\ \frac{\mu-\check{\tau}}{1-\check{\tau}}\,h_2(\check{\tau}),$$

where

$$h_2(\check{\tau}) = \max_{\check{m}_F \in [0,\pi]} \quad \omega + \delta\,\pi\,\check{\tau} + \delta\,(1-\check{\tau})\,\check{m}_F - c,$$
$$\text{subject to} \quad c - \delta\,\pi\,\check{\tau} - \delta\,(1-\check{\tau})\,\check{m}_F + \delta\,\check{m}_F \le \alpha\,c,$$
$$c - \delta\,\pi\,\check{\tau} - \delta\,(1-\check{\tau})\,\check{m}_F \ge 0,$$
$$-c + \check{\tau}\,\omega + \delta\,\pi\,\check{\tau} - (1-\delta)\,(1-\check{\tau})\,\check{m}_F \ge 0.$$

The feasibility conditions for $\check{m}_F$ can be rewritten as

$$0 \le \check{m}_F \le \min\left\{\frac{c-\delta\,\pi\,\check{\tau}}{\delta\,(1-\check{\tau})}, \frac{\delta\,\pi\,\check{\tau}-(1-\alpha)\,c}{\delta\,\check{\tau}}, \frac{-c+\check{\tau}\,\omega+\delta\,\pi\,\check{\tau}}{(1-\delta)\,(1-\check{\tau})}, \pi\right\}.$$

The objective function is increasing in $\check{m}_F$, and hence, when

$$0 \le \min\left\{\frac{c-\delta\,\pi\,\check{\tau}}{\delta\,(1-\check{\tau})}, \frac{\delta\,\pi\,\check{\tau}-(1-\alpha)\,c}{\delta\,\check{\tau}}, \frac{-c+\check{\tau}\,\omega+\delta\,\pi\,\check{\tau}}{(1-\delta)\,(1-\check{\tau})}, \pi\right\}.$$

the optimal $\check{m}_F$ is given by

$$\check{m}_F^* = \min\left\{\frac{c-\delta\,\pi\,\check{\tau}}{\delta\,(1-\check{\tau})}, \frac{\delta\,\pi\,\check{\tau}-(1-\alpha)\,c}{\delta\,\check{\tau}}, \frac{-c+\check{\tau}\,\omega+\delta\,\pi\,\check{\tau}}{(1-\delta)\,(1-\check{\tau})}, \pi\right\}.$$

Because

$$\frac{\delta\,\pi\,\check{\tau}-(1-\alpha)\,c}{\delta\,\check{\tau}} - \pi = -\frac{(1-\alpha)\,c}{\delta\,\check{\tau}} \le 0,$$

we have

$$\check{m}_F^* = \min\left\{\frac{c-\delta\,\pi\,\check{\tau}}{\delta\,(1-\check{\tau})}, \frac{\delta\,\pi\,\check{\tau}-(1-\alpha)\,c}{\delta\,\check{\tau}}, \frac{-c+\check{\tau}\,\omega+\delta\,\pi\,\check{\tau}}{(1-\delta)\,(1-\check{\tau})}\right\}.$$

After solving for the optimal testing threshold $\check{\tau}$, all the remaining decision variables can be recovered accordingly.

Let $\check{\tau}_1$ and $\check{\tau}_2$ denote the optimal testing thresholds for subproblems (EC.32) and (EC.33), respectively. Recall that the funder's expected payoff is always nonnegative. The robustly optimal threshold policy is explicitly characterized as follows.

1. If $Z_1 \ge \max\{Z_2, 0\}$, the optimal mechanism to (EC.27) involves rewarding success:

$$\check{\tau}^* = \check{\tau}_1, \quad \check{m}_S^* = \check{m}_S(\check{\tau}_1), \quad \check{m}_F^* = 0, \quad \check{m}_0^* = c - \delta\,\check{\tau}_1\,(\check{m}_S^* + \pi).$$

2. If $Z_2 \ge \max\{Z_1, 0\}$, the optimal mechanism to (EC.27) involves rewarding failure:

$$\check{\tau}^* = \check{\tau}_2, \quad \check{m}_S^* = 0, \quad \check{m}_F^* = \check{m}_F(\check{\tau}_2), \quad \check{m}_0^* = c - \delta\,\pi\,\check{\tau}_2 - \delta\,\check{m}_F^*\,(1-\check{\tau}_2).$$

3. If $\max\{Z_1, Z_2\} < 0$, the optimal mechanism to (EC.27) is to provide no additional incentives:

$$\check{\tau}^* = \min\left\{\frac{c}{\delta\,\pi}, 1\right\}, \quad \check{m}_0^* = \check{m}_S^* = \check{m}_F^* = 0.$$

This completes the proof. □

EC.14. No Moral Hazard and No Adverse Selection

For completeness, we present the first-best optimal solution with neither adverse selection nor moral hazard. The funder's problem reduces to

$$\max_{\{q,m_0,m_S,m_F\}\in\Omega_{\mathrm{FB}}} \theta\,\omega\,q - m_0 - \theta\,m_S - (1-\theta)\,m_F, \tag{EC.34}$$

where the feasible set Ω_{FB} is determined by (28), (29), and (30). We present the optimal funding mechanism in the following proposition.

PROPOSITION EC.5. *The following three cases summarize the optimal solution to the funder's problem without asymmetric information* (EC.34)*:*

(a) if $\theta \in [0, c/(\omega+\delta\,\pi))$, it is optimal not to induce testing, i.e.,

$$q = m_0 = m_S = m_F = 0;$$

(b) if $\theta \in [c/(\omega+\delta\,\pi), \bar{\tau})$, it is optimal to induce testing with a push amount of $c-\delta\,\theta\,\pi$, i.e.,

$$q = 1, \quad m_0 = c - \delta\,\theta\,\pi, \quad m_S = m_F = 0;$$

(c) if $\theta \in [\bar{\tau}, 1]$, the firm is willing to test without the funder, i.e.,

$$q = 1, \quad m_0 = m_S = m_F = 0.$$

Proof of Proposition EC.5. If $\theta \geq c/(\delta\,\pi)$, the firm is capable of self-funding, and the optimal mechanism is to grant no external funding.

If $\theta < c/(\delta\,\pi)$, the individual rationality constraint (28) must bind; otherwise, the funder can strictly improve its own payoff by reducing the funding amount. We can solve (28) as an equality for m_0, i.e.,

$$m_0 = c\,q - \delta\,\theta\,(m_S + \pi\,q) - \delta\,(1-\theta)\,m_F,$$

and substitute it back into the objective function, yielding

$$(\theta\,\omega + \delta\,\theta\,\pi - c)\,q - (1-\delta)\,\theta\,m_S - (1-\delta)\,(1-\theta)\,m_F.$$

If $\theta\,\omega + \delta\,\theta\,\pi - c \leq 0$, it is optimal to set $q = 0$, i.e., not to fund the test.

If $\theta\,\omega + \delta\,\theta\,\pi - c > 0$, ignoring the nonnegativity constraint on m_0, it is optimal to set $q = 1$, $m_S = m_F = 0$, and the resulting $m_0 = c - \delta\,\theta\,\pi > 0$, satisfying the previously ignored constraint.

This completes the proof. □

When there are no incentive issues, i.e., both the probability of success and the agent's effort are publicly observable, the result is intuitive. The funder induces testing if and only if the expected societal value is higher than the cost, that is, $\theta\,(\omega+\delta\,\pi) \geq c$, corresponding to Cases (b) and (c). In this case, if the firm's expected payoff is insufficient to cover the testing cost (Case (b)), the funder pays the shortfall using push funding to ensure the firm's participation.

EC.15. Uncertainty about the Cost of Shirking

Suppose the funder does not have an accurate estimate of the firm's cost of shirking, α, and only knows that it lies within an interval $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. For each α, let $\hat{\Omega}(\alpha)$ denote the feasible set defined in Section 2 by constraints (IC), (IR), (4), and (5), evaluated at that value of α. Because the funder selects a single mechanism before α is realized, the mechanism must be feasible for every $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. Accordingly, define the robust feasible set as

$$\hat{\Omega}^{ROB} \equiv \bigcap_{\alpha \in [\underline{\alpha}, \bar{\alpha}]} \hat{\Omega}(\alpha).$$

The funder then solves the robust optimization problem

$$u^{MAXMIN} \equiv \max_{\{q(\cdot), \hat{m}_0(\cdot), \hat{m}_S(\cdot), \hat{m}_F(\cdot)\} \in \hat{\Omega}^{ROB}} \min_{\alpha \in [\underline{\alpha}, \bar{\alpha}]} \int_{\theta_L}^{\theta_H} u_F(\theta) \, \mathrm{d}F(\theta). \tag{EC.35}$$

The following proposition states that the funder's robust optimal strategy is to implement an optimal funding mechanism corresponding to $\alpha = \underline{\alpha}$.

Proposition EC.6. *The maximin problem in* (EC.35) *is equivalent to the funder's optimization problem under* $\alpha = \underline{\alpha}$. *Consequently, any optimal mechanism derived under* $\alpha = \underline{\alpha}$ *is robust optimal.*

Proof of Proposition EC.6. Consider any $\underline{\alpha} \le \alpha_1 \le \alpha_2 \le \bar{\alpha}$. For every report $\hat{\theta}$, the firm's payoff from shirking satisfies

$$u_0^{\alpha_2}(\hat{\theta}) = u_0^{\alpha_1}(\hat{\theta}) - (\alpha_2 - \alpha_1)\, c\, q(\hat{\theta}) \le u_0^{\alpha_1}(\hat{\theta}).$$

Thus, an increase in α weakly reduces the payoff from shirking, while all other incentive, participation, allocation, and limited-liability constraints remain unchanged. Therefore,

$$\hat{\Omega}(\alpha_1) \subseteq \hat{\Omega}(\alpha_2).$$

The feasible sets are consequently nested, implying that

$$\hat{\Omega}^{ROB} = \bigcap_{\alpha \in [\underline{\alpha}, \bar{\alpha}]} \hat{\Omega}(\alpha) = \hat{\Omega}(\underline{\alpha}).$$

Moreover, the funder's objective function does not directly depend on α. Hence, for any mechanism in $\hat{\Omega}^{ROB}$, the inner minimization in (EC.35) leaves the funder's expected payoff unchanged. It follows that

$$u^{MAXMIN} = \max_{\{q(\cdot), \hat{m}_0(\cdot), \hat{m}_S(\cdot), \hat{m}_F(\cdot)\} \in \hat{\Omega}(\underline{\alpha})} \int_{\theta_L}^{\theta_H} u_F(\theta) \, \mathrm{d}F(\theta),$$

which is exactly the funder's optimization problem under $\alpha = \underline{\alpha}$. Therefore, any optimal mechanism corresponding to $\alpha = \underline{\alpha}$ solves the robust problem. This completes the proof. □

Let $\tilde{u}(\alpha, \hat{\alpha})$ denote the funder's expected payoff when the actual relative cost of shirking is α, while the funder implements the optimal policy derived under $\hat{\alpha}$. The performance ratio is then defined as

$$\text{Performance Ratio} = \frac{\tilde{u}(\alpha, \underline{\alpha})}{\tilde{u}(\alpha, \alpha)} \times 100\%,$$

where the denominator represents the funder's expected payoff when the true value of α is known.

Figure EC.8 presents the performance ratio in a scenario where the funder applies the mechanism optimized for $\underline{\alpha} = 0.35$, whereas the true cost of shirking is considerably larger ($\alpha = 0.7$). In the lower-left corner, we have $\tilde{u}(\alpha, \underline{\alpha}) = \tilde{u}(\alpha, \alpha) = 0$, so the performance ratio is undefined and thus omitted from the figure. The line indicates the boundary B_5, which separates Regions (6) and (7); to the right of this curve, the commercial market takes over. As shown in the figure, the performance ratio is high for the six representative diseases: it exceeds 80% for dengue, is above 90% for malaria and syphilis, and reaches almost 100% for tuberculosis and exactly 100% for HIV/AIDS. For Trypanosomiasis, the optimal policy is not to induce testing under either value of α.

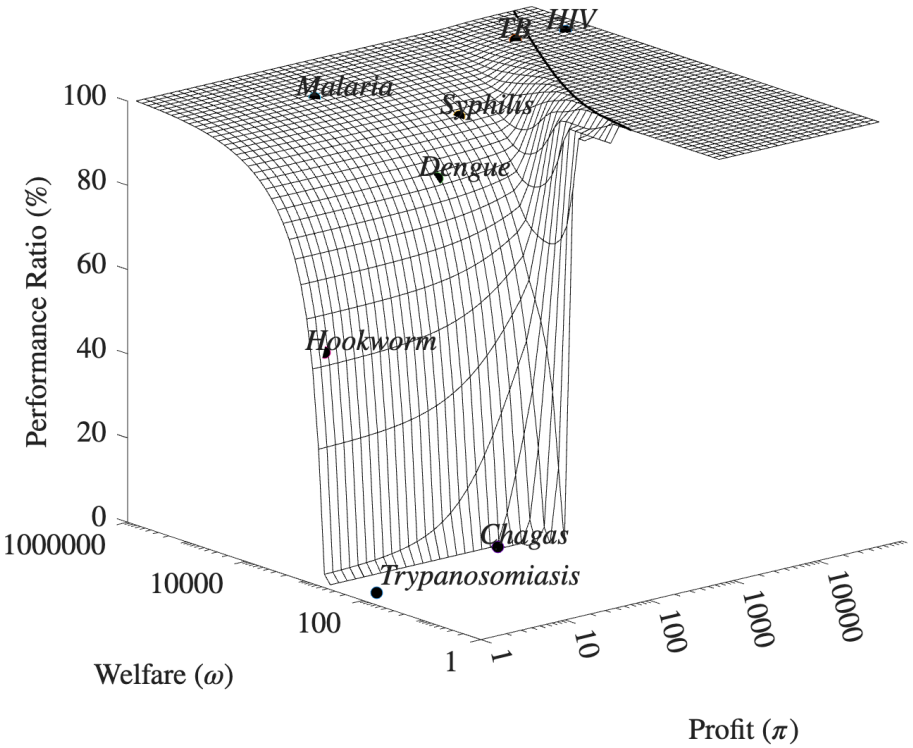


Figure EC.8 Performance ratio when the funder implements the optimal mechanism for $\underline{\alpha} = 0.35$, while the true value is $\alpha = 0.7$, with $c = 1$ and $\delta = 0.55$.

EC.16. Payoff Robustness to Parameter Misspecification

In this section, we conduct the following robustness exercise, where the true model parameters are fixed at their calibrated values, while the funder designs the mechanism using misspecified beliefs. We consider misspecification of the five key parameters α, c, δ, π, and ω. For voucher-eligible diseases, we include the \$200 million priority review voucher value in commercial profit π throughout the comparison. Thus, the benchmark is full push, $m_0 = c$ and $m_S = m_F = 0$, plus the voucher for voucher-eligible diseases, and full push alone for diseases that are not voucher-eligible, such as HIV/AIDS. When the funder misspecifies π, we perturb only the commercial-profit component and treat the voucher value as fixed.

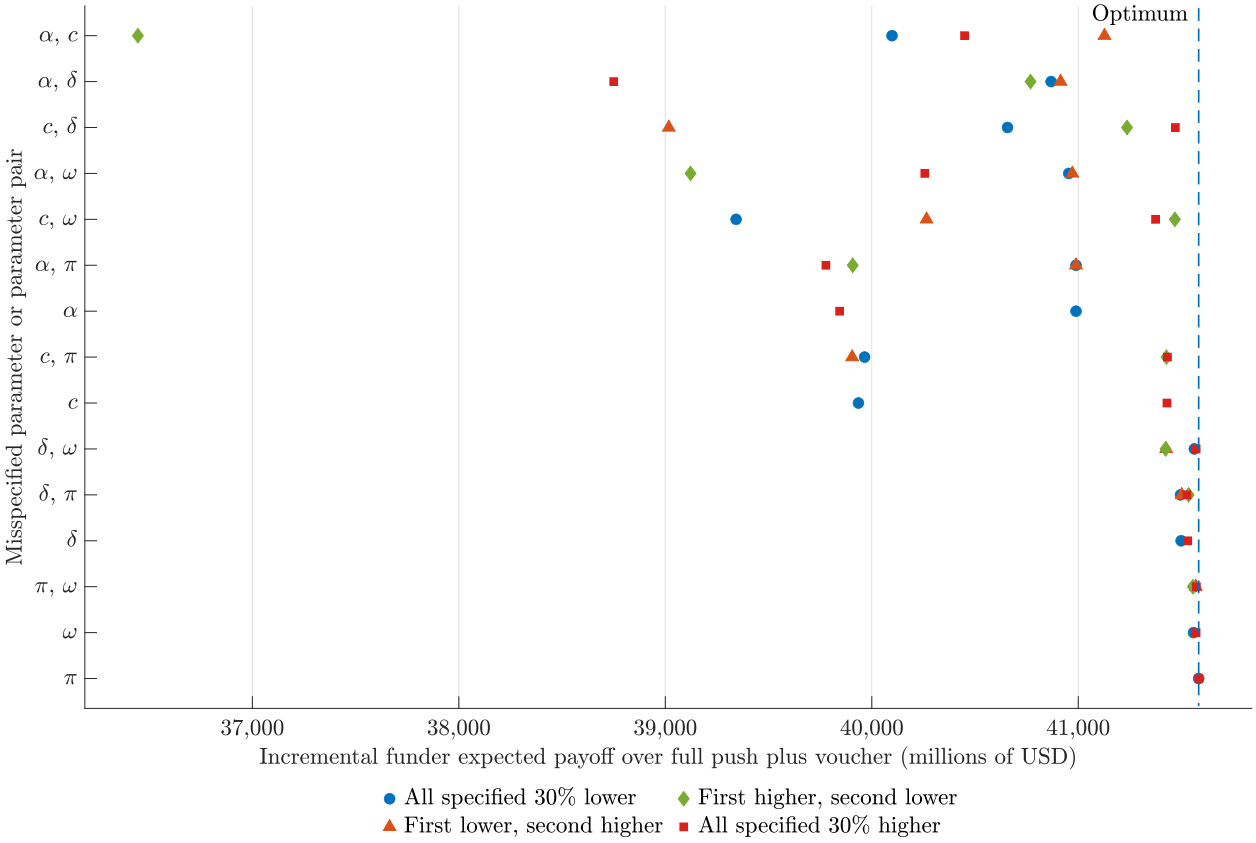


Figure EC.9 Robustness to one- and two-parameter misspecification for malaria. The horizontal axis shows the gain relative to the status quo, which is push funding that fully covers testing costs plus a \$200 million priority review voucher. Rows are ordered by the lowest incremental payoff in each row. The calibrated optimum is the dashed line. The status quo is at zero.

Figures EC.9–EC.11 combine the one- and two-parameter robustness exercises. Each row corresponds either to a single misspecified parameter or to a pair of misspecified parameters. For a single parameter, the two markers represent beliefs that are 30% below or above the calibrated value; for

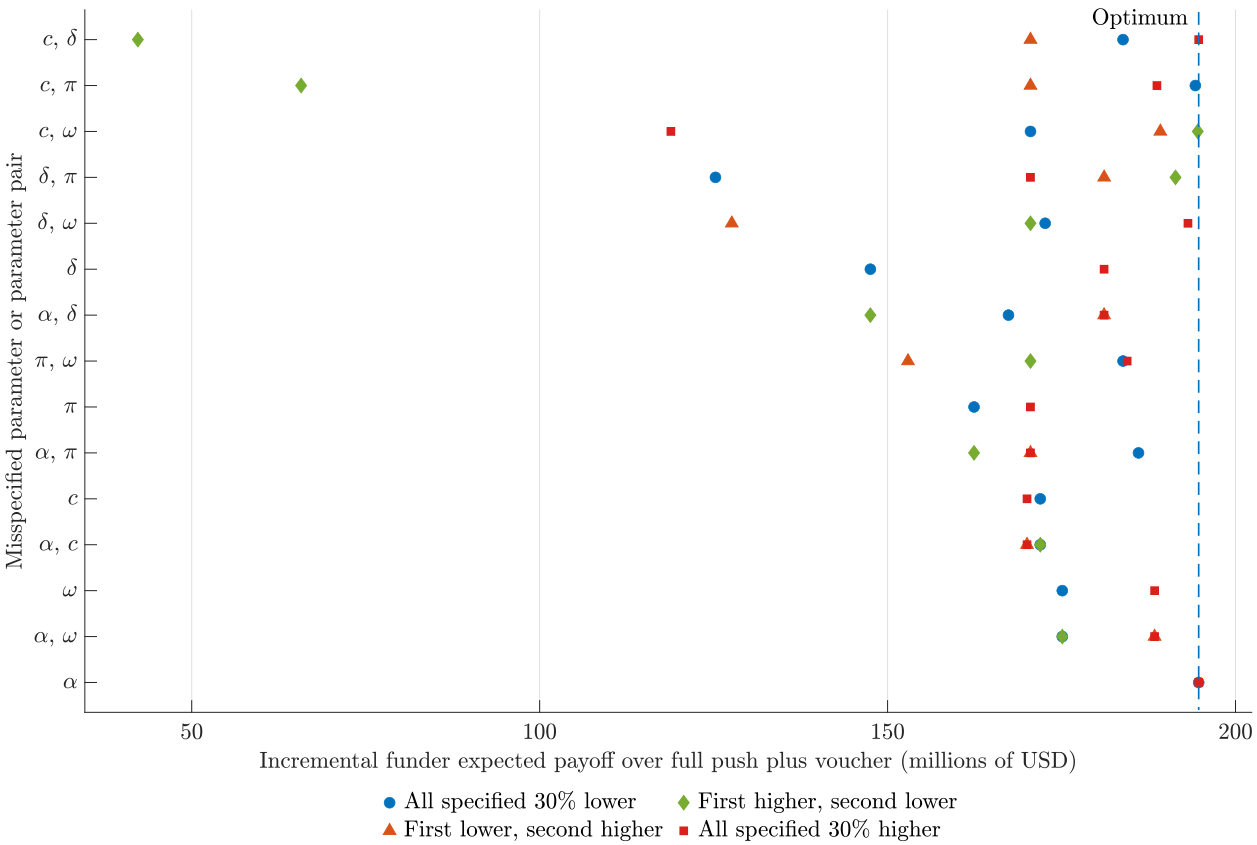


Figure EC.10 Robustness to one- and two-parameter misspecification for tuberculosis. The horizontal axis shows the gain relative to the status quo which is push funding that fully covers testing costs plus a $200 million priority review voucher. Rows are ordered by the lowest incremental payoff in each row. The calibrated optimum is the dashed line. The status quo is at zero.

a parameter pair, the four markers represent the low–low, low–high, high–low, and high–high belief combinations. The rows are ordered by the lowest realized incremental payoff within each row, placing the most adverse misspecification at the top. The horizontal axis reports the realized incremental funder payoff of the proposed mechanism relative to the corresponding full-push benchmark, while the dashed vertical line reports the incremental payoff under the calibrated optimal mechanism. Payoffs are evaluated under the calibrated true parameters. Across the displayed cases for malaria, tuberculosis, and HIV/AIDS, the proposed mechanism continues to outperform the corresponding full-push benchmark, although the magnitude of the gain varies across parameters and parameter pairs. Overall, the figures indicate that the quantitative advantage of the proposed mechanism is robust to substantial one- and two-parameter belief misspecification.

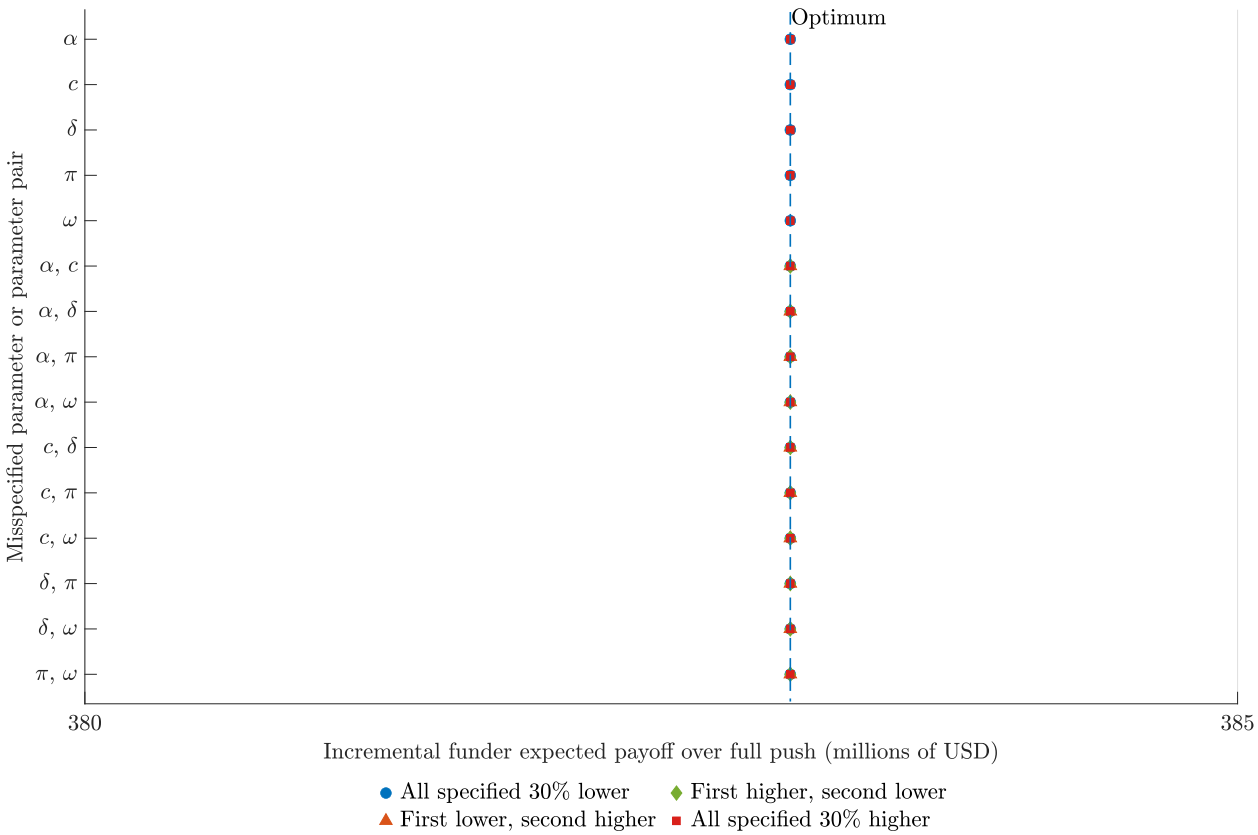


Figure EC.11 Robustness to one- and two-parameter misspecification for HIV/AIDS. The horizontal axis shows the gain relative to the status quo which is push funding that fully covers testing costs. Rows are ordered by the lowest incremental payoff in each row. The calibrated optimum is the dashed line. The status quo is at zero.

References

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