Our Deficit, Your Problem?
Fiscal Sustainability, Exchange Rates, and Inflation*

C. Christopher Hyland† Tobias Kawalec‡

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Abstract

Fiscal deficits in a financial Hegemon such as the United States transmit to exchange rates and inflation abroad. Using market-value data on U.K. and U.S. sovereign debt over 1975–2025, we recover fiscal deficit innovations and document three facts: U.S. deficit innovations depreciate the dollar against sterling, raise U.K. borrowing costs, and raise U.K. consumer and producer prices persistently. We rationalise these facts in a quantitative two-country model in which future surpluses need not back government debt and Hegemon bonds carry a convenience yield. A Hegemon fiscal expansion tightens the foreign government's intertemporal budget constraint through two channels: a wealth effect on foreign holders of Hegemon debt and, more importantly, a spillover to its financing costs, because no arbitrage ties returns on foreign bonds to returns on Hegemon bonds. We call this transmission *fiscal contagion* and show that the monetary–fiscal policy mix, including the foreign country's own policy stance, determines whether it raises foreign inflation or causes a foreign recession. Estimating the model on U.S. and U.K. data, we find that U.S. fiscal shocks account for about a quarter of the 2021–2023 surge in U.K. inflation. Fiscal sustainability in economies financially integrated with the Hegemon therefore depends in part on fiscal policy at the Hegemon.

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†University of Oxford, Saïd Business School, Exeter College, and Oxford-Man Institute. Email: christopher.hyland.dphil@said.oxford.edu

‡University of Vienna, Department of Economics. Email: tobias.kawalec@univie.ac.at.

1 Introduction

Debates over fiscal sustainability have focused primarily on the domestic inflationary consequences of debt-financed deficits. Yet fiscal expansions can also move exchange rates and therefore revalue government debt in other countries. This channel matters most for the United States, which sits at the centre of the global financial system: its currency anchors international reserves, and its government debt is the benchmark safe asset on balance sheets worldwide. April 2025 showed that this status cannot be taken for granted: the dollar depreciated even as Treasury yields rose, a combination at odds with the conventional safe-haven view of U.S. government debt (Jiang et al., 2025). Over the same period, international holdings of U.S. Treasuries fell by nearly $50 billion (U.S. Treasury, 2026). When investors reallocate out of Treasuries in this way, the price adjustments need not stay within U.S. markets: they can propagate to the exchange rates and discount rates of other sovereigns whose bonds those investors hold as substitutes. If such episodes recur, as current fiscal pressures suggest they may, U.S. fiscal policy becomes a source of fiscal risk for other governments.

In this paper, we study how fiscal deficits in a financial Hegemon affect exchange rates and inflation in a Foreign economy. We complement the goods-market view of the exchange rate with an asset-market view: as in Itskhoki and Mukhin (2021), the exchange rate enters the equilibrium conditions of both goods and asset markets. Households in both countries hold both governments' bonds, but Hegemon bonds also earn a convenience yield: they are valued for their role as a safe asset, not only for their cash flows (Brunnermeier et al., 2024). When one bond becomes more attractive than the other, Foreign households shift their savings towards it. Because the same households hold both bonds, no arbitrage ties the return on Foreign bonds to the return on Hegemon bonds. A Hegemon fiscal shock therefore reaches the Foreign economy in two ways: exchange-rate movements revalue Foreign holdings of Hegemon debt, and, as returns on Hegemon bonds adjust, returns on Foreign bonds must adjust with them, which changes the Foreign government's financing costs. We call this joint transmission *fiscal contagion*: a Hegemon deficit shock can tighten the intertemporal budget constraint of the Foreign government even when Foreign expected primary surpluses are unchanged. Whether it raises Foreign inflation or causes a Foreign recession depends on the monetary–fiscal policy mix in both countries.[1]

The empirical challenge in linking deficits to exchange rates and inflation is that what

[1]The international allocation of bonds in our model is pinned down by convenience demand, international bond holding costs, market clearing, and the accumulation of net foreign assets. Since the marginal convenience yield declines with each country's holdings of Hegemon bonds, the size of fiscal contagion depends on how steeply convenience demand slopes in each country and on portfolio exposure.

matters is the *unfunded* part of a deficit, which investors do not expect future surpluses to back, and such expectations cannot be observed directly. Building on Hall and Sargent (2011), Cochrane (2019), and Jiang et al. (2024c), we use a market-value decomposition of U.K. and U.S. sovereign debt over 1975–2025 to recover the deficits implied by observed debt prices, which reflect how investors expect the debt to be backed. We take the unpredictable component of these deficits as our measure of deficit innovations. The decomposition also shows that the excess return on gilts over Treasuries moves the value of U.K. debt as much as U.K. deficits do.

Using these deficit innovations, we document three facts. First, deficits weaken the issuing country's currency: U.S. deficit innovations depreciate the dollar against sterling, and U.K. deficit innovations depreciate sterling, consistent with unfunded deficits making a country's bonds less attractive to foreign investors. This runs against standard open-economy models, in which a debt-financed fiscal expansion appreciates the issuing country's currency, but is in line with earlier evidence that fiscal expansions depreciate the real exchange rate (Kim and Roubini, 2008; Monacelli and Perotti, 2010; Ravn et al., 2012). Second, U.S. deficit innovations raise U.K. borrowing costs: gilt yields rise by more than Treasury yields, and existing gilts lose value. Third, U.S. deficit innovations raise U.K. consumer and producer prices persistently, whereas U.K. deficit innovations raise them only temporarily. The rise in U.K. producer prices, even as the weaker dollar makes U.S. goods cheaper, shows that the spillover is not simply imported inflation.

To rationalise these facts, we develop a two-country open-economy model in which fiscal shocks in a financial Hegemon, whose sovereign debt carries a convenience yield, propagate internationally through the no-arbitrage conditions that link the two sovereign bond markets. The threat of fiscal inflation in the Hegemon thereby transmits fiscal pressure to other economies, raising the real burden of their debt. In an analytical version of the model, we show how an exchange-rate jump revalues inherited sovereign debt and how the monetary–fiscal policy mix splits the adjustment between the exchange rate and the price level.[2]

Suppose an unfunded Hegemon transfer expansion raises the outstanding stock of Treasuries under a fiscally-led policy mix in both countries. Because future surpluses do not back the new debt, Hegemon households treat it as net wealth and raise demand, and the Hegemon currency depreciates. The depreciation lowers Foreign import prices and so raises the real value of the Foreign government's inherited debt. Because future Foreign

[2]Mechanisms linking Hegemon deficits to exchange rates and to price levels in other countries have received growing attention (e.g., Ding and Jiang, 2024; Mac Mullen, 2025). This work supports a close link between deficits and exchange rates, but it assigns only a limited role to government debt valuation in equilibrium and gives little attention to the nominal devaluation of government debt across borders.

surpluses do not back this higher real debt and the Foreign central bank accommodates, subsequent Foreign inflation must erode it. Convenience yields weaken the no-arbitrage link between Hegemon and Foreign bond returns and so dampen the spillover. In our calibration, most of the spillover runs through this link rather than through the revaluation of Foreign holdings of Hegemon debt. Other policy mixes change the form of the spillover. Under what we label "Hegemon Dominance", an expansionary Hegemon fiscal authority faces a hawkish monetary authority, so the exchange rate rather than Hegemon inflation keeps Hegemon debt valued: the deficit appreciates the Hegemon currency and exports a Foreign boom. If instead Foreign surpluses back Foreign debt and the Foreign central bank follows the Taylor principle, the same deficit causes a Foreign recession rather than inflation.

Estimating the quantitative model on U.S. and U.K. data since 1998 under a fiscally-led policy mix in both countries, we find that, in the posterior mean, U.S. fiscal shocks account for about half of the 2021–2023 surge in U.S. producer-price inflation and for about a quarter of the surge in U.K. producer-price inflation. Had U.S. fiscal shocks from 2021 onwards been half as large, the U.K. producer-price level would have risen by about 1.8 percentage points less over 2021–2023 (90 percent posterior interval: 1.1 to 2.6), because a stronger dollar would have raised the real value of U.K. debt by less. U.K. inflation would have been lower through 2021 and at its peak in 2022. The spillover survives, and is larger, when funded and unfunded deficit shocks coexist as in Bianchi et al. (2023).

Related literature. Our title echoes the book by Rogoff (2025). Our focus, however, is on how sovereign deficits move exchange rates and how they affect the fiscal position of other countries that are closely integrated financially.

Our primary contribution is to the literature on sovereign deficits and their links to inflation and exchange rates. The paper is also close to work on fiscally driven price-level determination, which depends on the broader monetary–fiscal policy mix. This literature was inaugurated by Sargent and Wallace (1981), with Leeper (1991) and Woodford (1995) laying much of the groundwork for models that attribute at least part of inflation to fiscal policy, as reviewed by Leeper and Leith (2016) and Cochrane (2023). Support for fiscally driven inflation has been developed both theoretically (Ascari et al., 2023; Bassetto and Cui, 2018; Bassetto et al., 2024; Bianchi and Melosi, 2022; Bianchi et al., 2023; Caramp and Silva, 2023; Bigio et al., 2024; Corsetti and Maćkowiak, 2024; Kaplan et al., 2024) and empirically (Ascari et al., 2024; Banerjee et al., 2022; Cochrane, 2022a; Reichlin et al., 2023), with Barro and Bianchi (2026) collecting extensive international evidence that the post-Covid inflation can be attributed in significant part to expansionary fiscal policy. Giannone and Primiceri (2026) are particularly close to us: they highlight strong demand-push factors, including

accommodative fiscal and monetary policy, in the post-Covid inflation episode. The importance of the monetary environment for fiscal transmission also appears in Leeper et al. (2017), where government-spending multipliers depend on the monetary–fiscal policy mix, and in Bianchi and Melosi (2017), who link the absence of deflation during the Great Recession to uncertainty about how public debt will be stabilised.

Most of this work is set in closed economies, but a small and growing literature brings these insights, and monetary–fiscal interactions more broadly, to international models. Early contributions linking fiscal price-level determination and exchange rates include Sims (1997), Woodford (1998), Sims (1999), Bergin (2000), and Dupor (2000). Benigno and Missale (2004) and Corsetti and Maćkowiak (2006) apply these ideas to the links between public debt and currency stability, especially during episodes of severe currency turmoil, and Giannitsarou and Scott (2006) and Gourinchas and Rey (2007) study the international valuation of government debt and its effects on inflation and exchange rates in the run-up to the Global Financial Crisis. Relatedly, Bodenstein et al. (2025) show that the gains from international monetary cooperation depend on financial-market structure and net foreign asset positions, further highlighting the role of financial integration in policy spillovers.[3] Empirically, Auerbach and Gorodnichenko (2013) document cross-country output spillovers from government spending.

Alongside the renewed interest in the link between deficits and inflation, research has revisited the role of deficits in exchange-rate determination. Schmitt-Grohé and Uribe (2003) provide an early analysis of exchange-rate adjustments relative to inflation in financing fiscal deficits within emerging markets.[4] Gourinchas et al. (2025) show how segmentation in bond and currency markets links monetary policy to exchange rates and foreign bond yields, which relates to our model in which we link fiscal news to relative bond returns and exchange rates through convenience yields and cross-border holding costs. Valchev (2020) shows that excess returns on currency holdings can be traced to endogenous movements in bond convenience yields (which only Treasuries earn in our model), which in turn depend on the interaction between monetary and fiscal policy.[5] Engel and Wu (2023) show that government-bond liquidity yields help explain exchange-rate movements across G10 currencies. Closest to our contribution are Alberola et al. (2021) and Jiang (2022). Alberola et al. (2021) tie unexpected exchange-rate movements, relative to a frictionless complete-markets benchmark, to non-Ricardian fiscal regimes in which government debt

[3] Fornaro and Romei (2026) show that non-cooperative monetary policy during shortages of tradable goods can cause an excessive global contraction.

[4] Auerbach and Gorodnichenko (2016) find that U.S. defence-spending announcements appreciate the dollar, highlighting the distinction between spending news and the deficit innovations studied here.

[5] Jiang et al. (2024b) and Bianchi and Bigio (2026) document further evidence on convenience yields of U.S. assets.

lacks sufficient fiscal backing. Jiang (2022) draws out implications of fiscal price-level determination for exchange rates, showing that real exchange-rate adjustments are linked to government surplus shocks through the cost of issuing government debt. More broadly, our approach treats U.S. sovereign debt as a reference safe asset through which global financial conditions and business cycles are transmitted, as in Bodenstein et al. (2023). Since we attribute most of the observed variation to the common discount kernel, our argument is also related to Kirsanova et al. (2026).

The paper also relates to the large macro-finance literature on nominal and real exchange-rate determination, which we cannot survey fully here. Itskhoki (2021) provides a partial review, including an overview of the many puzzles that standard complete-markets models fail to resolve. Chari et al. (2002) provide an early quantitative benchmark linking monetary shocks and sticky prices to volatile and persistent real exchange rates (see also Galí and Monacelli, 2005), while Corsetti et al. (2008) show that incomplete asset markets and low trade elasticities can generate large relative-price movements that hinder international consumption risk sharing. Their analysis makes the interaction between relative prices and cross-country wealth central to international transmission, an interaction that our model brings to fiscal shocks and sovereign debt valuation. Of particular relevance is the general work on the disconnect between exchange rates and macroeconomic fundamentals (Duarte, 2003; Bacchetta and Van Wincoop, 2004; Engel and West, 2005; Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021), since our argument links the exchange rate back to one fundamental: aggregate demand managed by the fiscal authority. Our paper therefore also connects to recent work that pushes back against the view that exchange rates are disconnected from fundamentals. Bodenstein et al. (2024) link exchange-rate dynamics to trade balance shocks, which are reduced-form demand disturbances. Kekre and Lenel (2024a) attribute most cross-country exchange-rate variation to demand movements, viewed through the interest-rate differentials they generate, and Mac Mullen (2025) shows that these interest-rate differentials can be partly attributed to fiscal deficits, in line with the evidence and model presented here.

We also contribute to the literature on the international role of Hegemon debt, since we analyse interactions between a financial Hegemon and countries that lack the privilege conferred by financial dominance. Ellison and Scott (2020) provide a long-run perspective on the valuation of U.K. national debt on which our analysis builds, to which we add the international dimension, extend their sample, and focus more closely on inflation. We complement their analysis of foreign debt holdings by tracing how fiscal shocks revalue foreign-issued sovereign debt and transmit to inflation abroad. Jiang et al. (2024c) study the U.S. experience in recent decades and find evidence of a persistent mispricing of U.S.

sovereign debt. As Chen et al. (2025) document, this dominant position of U.S. sovereign debt was inherited from the United Kingdom in the early 20th century. Ding and Jiang (2024) make explicit a number of channels through which monetary and fiscal policy are intertwined in the presence of a dominant financial Hegemon.[6] Jiang et al. (2021) link the valuation of the U.S. dollar to convenience yields enjoyed by foreign investors holding safe U.S. assets, and Jiang (2021) argues that the U.S. fiscal cycle links currency risk premia of the dollar to those of other currencies. Our convenience-yield mechanism builds on Jiang et al. (2023) and Jiang et al. (2024b), who show how convenience yields on dollar-denominated debt enter as a wedge in the Euler equations that determine exchange rates.

Finally, by treating the U.S. as a financial Hegemon that anchors global discount rates, we relate to the literature on the global financial cycle (Rey, 2015; Miranda-Agrippino and Rey, 2020), surveyed by Miranda-Agrippino and Rey (2022).[7] Jiang et al. (2024a) is closely related, building a model of the global financial cycle with international demand for safe dollar assets, which we approximate through our convenience-yield mechanism. Kekre and Lenel (2024b) likewise link demand for safe dollar bonds to exchange rates and global activity. Farhi and Maggiori (2018) study how a financial Hegemon that supplies (quasi-)safe assets anchors the international monetary system. Relative to this literature, which traces the global financial cycle to the Hegemon's monetary stance or to the supply of and demand for safe dollar assets, we identify a distinctly fiscal origin: the cycle can be driven by the perceived backing of the Hegemon's debt, and propagates not merely to asset prices and exchange rates but to the nominal valuation of *foreign* sovereign liabilities through the common discount kernel the Hegemon anchors.

Outline. Section 2 recovers fiscal deficit innovations from the market value of U.K. and U.S. sovereign debt and uses them to document how deficits move exchange rates, borrowing costs, and inflation. Section 3 develops a two-country model that isolates the valuation mechanism. Section 4 describes the quantitative extension of this model and its calibration, studies how fiscal shocks transmit under different policy mixes, separates the spillover channels, and estimates the model on U.S. and U.K. data. Section 5 concludes.

2 Fiscal Impulses and International Spillovers

We now document three empirical results that our model later rationalises, using deficit innovations implied by the market value of sovereign debt. First, deficits weaken the

[6] This literature is in turn connected to the dominant currency paradigm (Gopinath and Itskhoki, 2022).

[7] Relatedly, Bianchi and Coulibaly (2026) study international spillovers through the real interest rate and how macroprudential policies can insulate economies from adverse foreign-policy effects.

issuing country's currency: U.S. deficits depreciate the dollar against sterling, and U.K. deficits depreciate sterling. Second, U.S. deficits raise U.K. borrowing costs: 10-year gilt yields rise and the value of existing gilts falls. Third, U.S. deficits raise U.K. consumer and producer prices persistently.

2.1 Recovering Deficit Innovations from Market-Value Debt

We recover the deficit implied by the market value of government debt from a linearised debt valuation identity. Let $X \in \{H, F\}$ index the two countries: H is the U.S., the financial Hegemon, and F is the U.K., for which Ellison and Scott (2020) provide a long series of the market value of government debt. Our quarterly sample runs from 1975, shortly after the end of the Bretton Woods system, to the end of 2025. Following Campbell and Shiller (1988), Hall and Sargent (2011), and Cochrane (2019), the market value of government debt satisfies the linearised identity

$$\rho_X v^X_{t+1} = v^X_t + r^{\text{nom},X}_{t+1} - \pi^X_{t+1} - g^X_{t+1} - s^X_{t+1}, \tag{1}$$

where ρ_X is a linearisation constant, v^X_t is the log ratio of the market value of debt to GDP at the end of period t, $r^{\text{nom},X}_{t+1}$ is the log nominal return on the portfolio of government bonds, π^X_{t+1} is inflation, g^X_{t+1} is log real GDP growth, and s^X_{t+1} is the scaled real primary surplus relative to GDP, which we back out as the residual that makes the identity hold exactly. We work with market rather than face value, as Hall and Sargent (2011) emphasise, for three reasons. Market values capture the revaluation of outstanding bonds when yields change, which face values ignore. They are also forward-looking: bond prices reflect how investors expect the debt to be backed, whether by future surpluses, growth, low interest costs, or nominal devaluation. Finally, market values are the measure that enters the government's intertemporal budget constraint, which equates the real value of debt with the present value of future surpluses.

To see how U.S. discount rates enter the value of U.K. debt, we write the gilt return as the Treasury return converted into sterling plus a return wedge, $r^{\text{nom},F}_{t+1} = r^{\text{nom},H}_{t+1} - \Delta e_{t+1} + \xi_{t+1}$. Here $e_t \equiv \log E_t$ is the log nominal exchange rate in dollars per pound, so an increase in e_t is a depreciation of the dollar, and $\Delta e_{t+1} \equiv e_{t+1} - e_t$. The wedge ξ_{t+1} is the realised excess return on gilts over Treasuries in a common currency. It reflects differences in expected returns, such as convenience yields, and revaluations from changes in long-term yields, since the two portfolios differ in maturity. In the data, we measure this exchange-rate term with the change in sterling's real effective exchange rate, so the return wedge also absorbs inflation differences between the U.K. and its trading partners and sterling's movements

against currencies other than the dollar. The debt valuation equation for the U.K. then becomes

$$\rho_F v^F_{t+1} = v^F_t + \left(r^{\text{nom},H}_{t+1} - \Delta e_{t+1} + \xi_{t+1}\right) - \pi^F_{t+1} - g^F_{t+1} - s^F_{t+1}. \tag{2}$$

Every term in this equation other than s^F_{t+1} is observable, so iterating it backwards from a base year recovers the surplus path that rationalises the observed market value of government debt. Setting $\rho_F \simeq 1$ for notational simplicity gives

$$v^F_t = v^F_0 + \sum_{j=1}^{t}\left(r^{\text{nom},H}_j - \Delta e_j + \xi_j\right) - \sum_{j=1}^{t}\pi^F_j - \sum_{j=1}^{t} g^F_j - \sum_{j=1}^{t} s^F_j. \tag{3}$$

Equation (3) serves two purposes. First, together with equation (1) for the U.S., it delivers the cumulative surplus of each country, and hence the deficits implied by the market pricing of government debt. Because it is backed out from market prices, this residual reflects how the investors who price the debt view its fiscal backing. We use this measure of deficits in Section 2.2. Second, it separates the contributions of the Treasury return, the exchange rate, and the return wedge to the value of U.K. government debt.

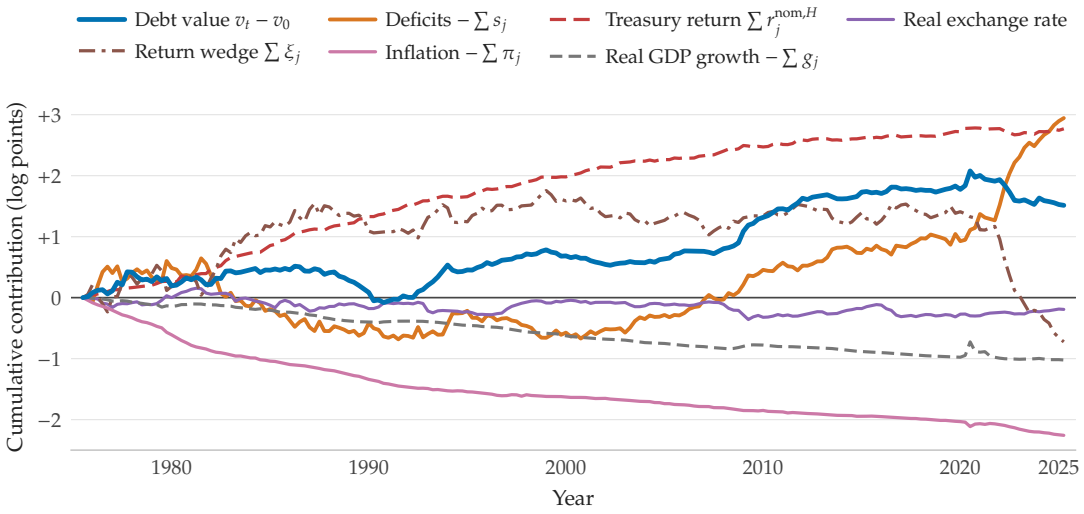


Figure 1: Backward decomposition of the market value of U.K. government debt, following equation (3), in log points relative to 1975. Positive contributions raise the ratio. The exchange-rate component uses sterling's real effective exchange rate.

Figure 1 shows the decomposition for the U.K.[8] Each line plots one of the cumulative

[8]Online Appendix A shows a similar decomposition for the U.S., as well as forward-looking decomposi-

sums in equation (3), measured from 1975, so all lines start at zero and at each date the components add up to the change in log debt/GDP (blue). The period-by-period change in the cumulative deficit (orange) is the implied deficit. The cumulative deficit rises sharply after the Global Financial Crisis and again after Covid. The return wedge (brown) moves the value of U.K. debt as much as deficits do. After 2020, its fall more than offset the rise in deficits, and the market value of debt fell relative to GDP. This fall coincides with large capital losses on gilts as yields rose, which lower the value of existing debt but raise the cost of new borrowing. The return on U.K. debt relative to U.S. debt therefore matters for U.K. debt sustainability alongside the U.K.'s own deficits.

2.2 Exchange Rates, Inflation, and Deficit Shocks

We estimate, by local projections, how U.K. and U.S. deficit innovations move the exchange rate, U.K. borrowing costs, and U.K. prices. We start from the implied surplus s_t^X recovered in Section 2.1. Because realised surpluses respond to the state of the economy, this series is partly predictable from past information, and using it directly would confound fiscal news with the economy's own dynamics. Following Cochrane (2019, 2022b), we therefore estimate a VAR(1) for each country. The VAR includes the implied surplus, the other components of the debt valuation identity in equations (1) and (2), the market value of debt relative to GDP, and yield-curve and cyclical predictors, all of which Online Appendix A.4 lists in detail. The surplus innovation is the residual of the surplus equation, $\check{s}_t^X \equiv s_t^X - \mathbb{E}_{t-1}[s_t^X]$ for $X \in \{H, F\}$, so that deficit innovations are $-\check{s}_t^F$ for the U.K. and $-\check{s}_t^H$ for the U.S. We interpret $\check{s}_t^X$ as news about the fiscal backing of government debt: the change in the market-implied surplus that investors could not have forecast from the previous quarter's information. For each outcome y_t, we estimate the long-differenced local projection

$$y_{t+h} - y_{t-1} = \alpha_h - \beta_h^F \check{s}_t^F - \beta_h^H \check{s}_t^H + \Gamma_h Z_{t-1} + u_{h,t}, \qquad h = 0, 1, \ldots, 16, \tag{4}$$

so that β_h^F and β_h^H measure the cumulative response of y to a U.K. and a U.S. deficit innovation, respectively, from period $t-1$ to $t+h$. Both deficit innovations are scaled to 1 percent of U.K. annual GDP, with U.S. deficits converted into sterling, so the two shocks are of the same size. The controls Z_{t-1} are one lag of each fiscal shock, lagged U.S. CPI inflation (for global inflationary factors), the lagged U.S. output gap, the lagged three-month U.K.–U.S. Treasury basis following Du et al. (2018a), and lagged U.K. real GDP growth and unemployment. We describe each outcome variable y_t in equation (4)

tions of the value of government debt from an estimated VAR(1) model following Cochrane (2019).

when we present its results below.

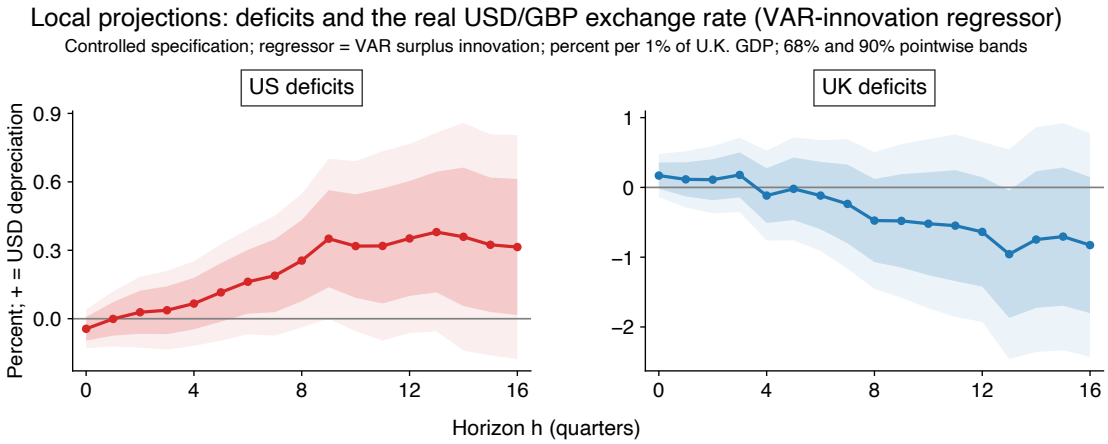


Figure 2: The effect of U.S. and U.K. deficit innovations on the real USD–GBP exchange rate, in percent. A positive value is a real depreciation of the dollar against sterling. Shaded areas indicate 68% and 90% pointwise confidence intervals, respectively, calculated using HAC-robust standard errors.

Figure 2 shows the response of the real exchange rate. Here y_t is the log real exchange rate $q_t \equiv \log Q_t$, where $Q_t = E_t P_t^F / P_t^H$ is the bilateral USD–GBP rate adjusted for CPI inflation in both countries. A U.S. deficit innovation depreciates the dollar against sterling slowly and persistently in real terms, by about 0.35 percent after two to three years, an effect significant at the 68 percent level. After a U.K. deficit innovation, the dollar instead barely moves against sterling on impact but then appreciates gradually in real terms, by almost 1 percent after three years. These responses run against standard open-economy models, in which a debt-financed fiscal expansion raises demand and interest rates at home and so appreciates the issuing country's currency (Kim and Roubini, 2008; Monacelli and Perotti, 2010; Ravn et al., 2012). Both exchange-rate responses are consistent with an unfunded deficit making a country's assets less attractive to foreign investors, so that the resulting outflows depreciate its currency.[9]

To the extent that the deficit innovations capture news about unfunded deficits, they should also move bond prices. Figure 3 shows three measures of U.K. borrowing costs. The first two are ex-ante prices: the 10-year gilt yield, which reflects expected future short rates and term premia and so the cost of new borrowing, and its spread over the U.S. 10-year

[9]Online Appendix D reports complementary reverse regressions following Engel and West (2005), which show that exchange rates, in turn, only weakly predict materialised deficits (as a share of government debt) in the U.K. and the U.S.

yield, which shows whether that cost rises relative to U.S. borrowing costs. The third is the cumulative realised return on the entire stock of gilts, including coupons and capital gains or losses. Because it is measured ex post, it captures the revaluation of existing debt.

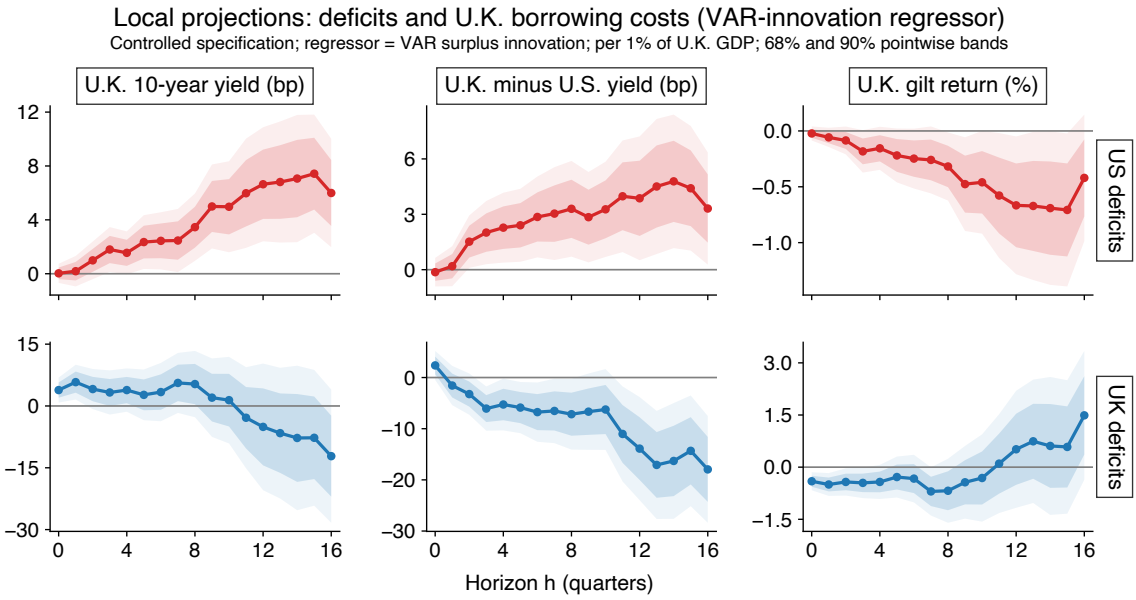


Figure 3: The effect of U.S. and U.K. deficit innovations on U.K. borrowing costs: the U.K. 10-year yield and its spread over the U.S. 10-year yield, in basis points, and the cumulative return on the U.K. gilt portfolio, in percent. Shaded areas indicate 68% and 90% pointwise confidence intervals, respectively, calculated using HAC-robust standard errors.

Investors appear to price the innovations as news about weaker debt sustainability. A U.S. deficit innovation raises the 10-year gilt yield by about 7 basis points after three years, while the U.S. yield rises by only about 3 basis points (see Online Appendix figure C.7), so the spread widens by about 4 basis points. Higher yields also lower the value of existing gilts: over the same three years, the realised return on the gilt portfolio is about 0.7 percent lower than it would otherwise have been, as existing holders take capital losses. The rise in U.K. yields after U.S. deficit innovations is a spillover that our model later explains through correlated discount rates.

Figure 4 shows the response of U.K. prices. Here y_t is the log U.K. CPI or PPI level, $p_t^F \equiv \log P_t^F$, and Z_{t-1} also includes the lagged price level and its lagged first difference. A U.S. deficit innovation raises both U.K. CPI and PPI persistently: CPI by about 0.25 percent and PPI by about 0.4 percent after three to four years. The rise in producer prices shows that the spillover is not simply a matter of imported goods becoming more expensive. A

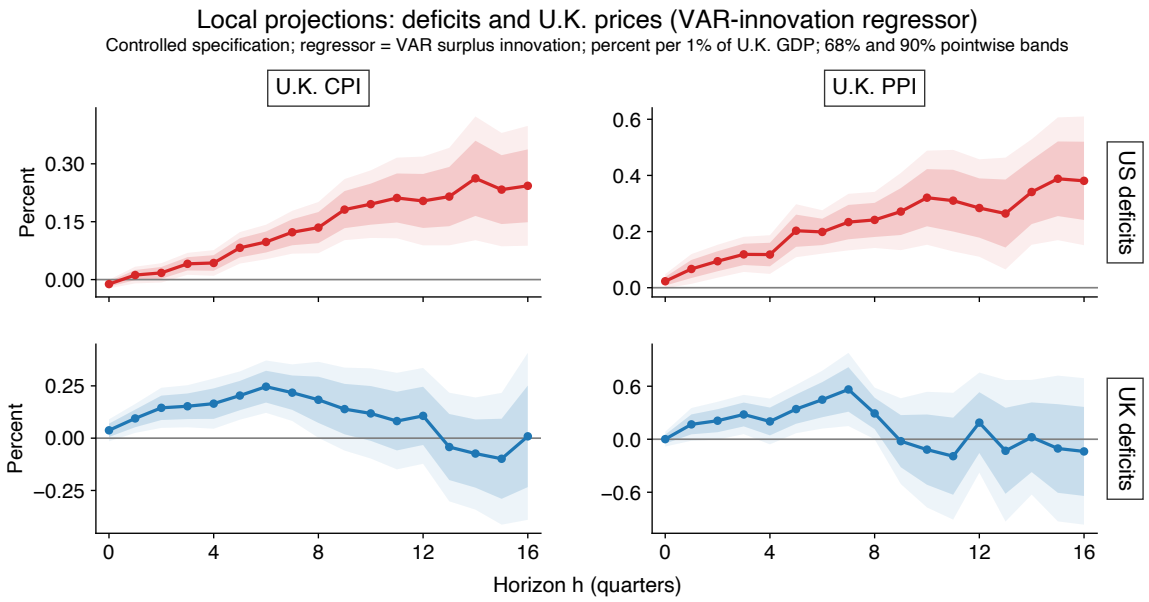


Figure 4: The effect of U.S. and U.K. deficit innovations on the U.K. consumer and producer price levels, in percent. Shaded areas indicate 68% and 90% pointwise confidence intervals, respectively, calculated using HAC-robust standard errors.

U.K. deficit innovation raises U.K. prices too, but only temporarily: CPI rises by up to 0.25 percent after about six quarters and PPI by about 0.55 percent after seven, and both responses fade within about three years. The domestic response matches existing evidence that unfunded deficits raise prices (e.g., Barro and Bianchi, 2026). The persistent spillover of fiscal inflation from U.S. deficits to another advanced economy has not, as far as we know, been documented before.

We run a series of robustness checks on these results. First, we replace our deficit innovations with two established measures of fiscal shocks: government spending shocks identified as in Blanchard and Perotti (2002) for both countries, and the U.S. military spending news of Ben Zeev et al. (2023) (see Online Appendix B for detailed results). The price spillover from U.S. deficits holds under both measures, although the exchange-rate responses are less precise. Second, Online Appendix C varies the controls and the exchange-rate measure, and the main results on exchange rates persist. Finally, we compare the implied deficit flow from which we extract the innovations with independent borrowing forecast revisions in Supplementary Online Appendix G. It moves in the same direction as U.K. private-sector revisions to expected borrowing and as official forecast revisions in large episodes such as the Global Financial Crisis and Covid, although these associations are imprecisely estimated. In the same private-sector data, forecasters who expect more borrowing also revise expected sterling down and expected inflation up, mirroring our

results.

The model we develop next accounts for these facts through two ingredients: monetary–fiscal interactions that determine the perceived backing of government debt, and time-varying convenience yields on U.S. government debt. The convenience yields act as a friction in international bond markets: they drive a wedge between the returns on U.S. and U.K. debt, so that fiscal news moves the exchange rate rather than being absorbed by interest rates alone.

3 A Two-Country Model with a Financial Hegemon

We now develop a two-country New Keynesian model that accounts for the three facts of Section 2: after a U.S. deficit, the dollar depreciates, U.K. borrowing costs rise, and U.K. prices rise persistently. The two countries are a financial Hegemon, country H, which we interpret as the United States, and a Foreign economy, country F, which we take to the data as the United Kingdom. Households in both countries hold both sovereign bonds, but only the Hegemon's bonds, U.S. Treasuries, provide convenience services. This asymmetry captures the Hegemon's privileged position as the issuer of the world's benchmark safe asset. Because households hold both bonds, a Hegemon fiscal shock moves the exchange rate and the discount rate on Foreign debt, as in the first two facts. Time-varying convenience yields govern the strength of this spillover. Whether the resulting revaluation of Foreign debt is backed by Foreign surpluses or must instead be matched by Foreign inflation depends on the monetary–fiscal policy mix, which links these responses to the third fact.

We first present an analytically tractable version of the model, set in continuous time, which shows how an exchange-rate movement revalues sovereign debt in both countries and when Foreign inflation must absorb that revaluation. Section 4 then extends this model to the quantitative version we estimate and conduct policy counterfactuals with.

3.1 Household Problem

Each country has a unit population represented by a household with discount rate ρ.

Hegemon household. The Hegemon household chooses consumption C_t, labour supply N_t, Hegemon-currency bonds B_{Ht}, and Foreign-currency bonds B_{Ft} to maximise

$$\int_0^\infty e^{-\rho t} \{\log C_t - N_t + \nu_H(b_{Ht})\}\, dt, \tag{5}$$

where $b_{Ht} = B_{Ht}/P_t$ and $v_{Ft} = E_t B_{Ft}/P_t$ are its real bond positions in Hegemon consumption units, and E_t is the nominal exchange rate, an increase in which is a depreciation of the Hegemon currency. Only Treasuries provide convenience services, which enter utility through v_H (Krishnamurthy and Vissing-Jorgensen, 2012), so the two bonds are imperfect substitutes. The model is kept tractable through log utility and linear labour disutility for now, and in Section 4 we generalise to convex labour disutility.

The household faces the nominal flow budget constraint

$$P_t C_t + \dot{B}_{Ht} + \tfrac{d}{dt}\Big(E_t B_{Ft}\Big) = i_t B_{Ht} + \Big(i_t^* + \tfrac{\dot{E}_t}{E_t}\Big) E_t B_{Ft} + W_t N_t + \Pi_t - T_t + E_t P_t^* \bar{\mathfrak{T}} - \tfrac{\psi_H P_t}{2\bar{Y}}\Big(v_{Ft} - \bar{v}_F\Big)^2. \quad (6)$$

Here, i_t and i_t^* are the nominal interest rates on Hegemon and Foreign sovereign bonds, $W_t N_t$ is labour income, Π_t is the profit of Hegemon producers net of price-adjustment expenditure, T_t are nominal net taxes, and $E_t P_t^* \bar{\mathfrak{T}}$ is a fixed transfer from Foreign.[10] Foreign bonds earn the Hegemon-currency return $i_t^* + \dot{E}_t/E_t$, so a Hegemon depreciation raises the domestic-currency value of a given Foreign-bond position. The last term is a quadratic cost of holding Foreign bonds away from the target $\bar{v}_F$, a standard device that keeps cross-border positions stationary (Schmitt-Grohé and Uribe, 2003). It is paid in units of the Hegemon consumption basket, where $\psi_H > 0$ and $\bar{Y}$ denotes stationary output and consumption. Consumption is a Cobb–Douglas aggregate of Hegemon and Foreign goods,

$$C_t = \left(\frac{C_{Ht}}{1-\gamma}\right)^{1-\gamma}\left(\frac{C_{Ft}}{\gamma}\right)^{\gamma}, \qquad P_t = P_{Ht}^{1-\gamma} P_{Ft}^{\gamma},$$

where P_{Ht} and P_{Ft} are the Hegemon-currency prices of the two goods, so cost minimisation allocates shares $1-\gamma$ and γ of expenditure to Hegemon and Foreign goods.[11]

Foreign household. The Foreign household's problem (where stars denote Foreign variables) is symmetric. It chooses C_t^*, N_t^*, Foreign-currency bonds B_{Ft}^*, and Treasuries B_{Ht}^*, with real positions $b_{Ft}^* = B_{Ft}^*/P_t^*$ and $v_{Ht}^* = B_{Ht}^*/(E_t P_t^*)$ in Foreign consumption units, and obtains convenience services $\nu_F(v_{Ht}^*)$ from its Treasuries and none from its own government's bonds. Treasuries earn the Foreign-currency return $i_t - \dot{E}_t/E_t$, so a Hegemon depreciation lowers their Foreign-currency value. Holding them away from the target $\bar{v}_H^*$ incurs a quadratic cost with parameter $\psi_F > 0$, paid in units of the Foreign consumption basket, and the household pays the fixed transfer $\bar{\mathfrak{T}}$. Its consumption aggregate mirrors

[10] The transfer supports a symmetric stationary allocation when the two countries hold different net foreign asset positions, which we derive in Supplementary Online Appendix H.4.

[11] The same shares apply to expenditure on portfolio costs, which the budget records separately from consumption.

the Hegemon's, with weight $1-\gamma$ on Foreign goods, so both households spend a share $0 < \gamma < 1/2$ on imported goods.

Convenience yields. The convenience yield is the non-pecuniary return on Treasuries: the marginal utility of convenience services, measured in units of consumption. For the two households,

$$\Phi_t = C_t \, \nu_H'(b_{Ht}), \qquad \Phi_t^* = C_t^* \, \nu_F'(v_{Ht}^*). \tag{7}$$

Both ν_H and ν_F are increasing and concave, so convenience yields fall as households hold more Treasuries: at given consumption, a 1 percent rise in holdings lowers the convenience yield by about ζ_H or ζ_F percent. We choose ν_H and ν_F so that both yields equal $\bar{\Phi}$ at the stationary allocation, with $0 < \bar{\Phi} < \rho$ so that the steady-state real rate on Treasuries, $\rho - \bar{\Phi}$, is positive.[12]

Optimality conditions. Labour supply optimality gives $C_t = W_t/P_t$ and $C_t^* = W_t^*/P_t^*$. Writing $\pi_t \equiv \dot{P}_t/P_t$ and $\pi_t^* \equiv \dot{P}_t^*/P_t^*$ for CPI inflation and $r_t = i_t - \pi_t$ and $r_t^* = i_t^* - \pi_t^*$ for real rates, we obtain the following Euler equations for each household's own bond:

$$\frac{\dot{C}_t}{C_t} = r_t - \rho + \Phi_t, \qquad \frac{\dot{C}_t^*}{C_t^*} = r_t^* - \rho. \tag{8}$$

The convenience yield lowers the pecuniary return that the Hegemon household requires on Treasuries, because it receives part of the return as convenience services. Foreign bonds provide no such convenience services.

Cross-border returns. Each household also holds the other country's bond. Let $Q_t \equiv E_t P_t^*/P_t$ denote the real exchange rate, an increase in which is a real depreciation of the Hegemon currency, and $q_t \equiv \log Q_t$. Combining the Euler equations for these cross-border holdings with equation (8) gives the real UIP conditions of the two households,

$$\begin{aligned} r_t^* &= r_t - \dot{q}_t + \Phi_t + \frac{\psi_H}{\bar{Y}}(v_{Ft} - \bar{v}_F), \\ r_t^* &= r_t - \dot{q}_t + \Phi_t^* - \frac{\psi_F}{\bar{Y}}(v_{Ht}^* - \bar{v}_H^*). \end{aligned} \tag{9}$$

For each household, no arbitrage between the two bonds requires the real return on Foreign bonds to equal the real return on Treasuries, adjusted for the change in the real exchange

[12] We use the isoelastic form $\nu_H(b) = \frac{\bar{\Phi}\bar{b}_H}{(1-\zeta_H)\bar{Y}}\left[(b/\bar{b}_H)^{1-\zeta_H} - 1\right]$, normalised to zero at the stationary holdings $\bar{b}_H$, and symmetrically for ν_F with $\bar{v}_H^*$ and ζ_F.

rate, the convenience yield and the cost of holding foreign bonds.

Because markets are incomplete, a shock can persistently shift wealth between the two countries. We track this shift with relative marginal wealth, defined as $\varpi_t \equiv \log(C_t/C_t^*) - q_t$, which is Hegemon consumption relative to Foreign consumption, measured in the same units. Under complete markets it would be constant (Backus and Smith, 1993). Both households hold Foreign bonds, which provide no convenience services, so comparing their Euler equations for these bonds gives

$$\dot{\varpi}_t = -\frac{\psi_H}{\bar{Y}}(v_{Ft} - \bar{v}_F). \tag{10}$$

Wealth shifts between the two countries only through the returns that households earn on the bond they both hold. When Hegemon households build up Foreign bonds, the cost of holding them means they earn less on these bonds than Foreign households do, so Hegemon consumption falls relative to Foreign consumption.

3.2 Production and Price Setting

Each country produces a traded final good from a continuum of domestic intermediate varieties, with elasticity of substitution $\varepsilon > 1$. We present only the Hegemon's production side, since Foreign is symmetric, with starred variables. Final-good producers are competitive, so demand for each variety and the producer price index are given by

$$Y_H(j) = \left(\frac{P_H(j)}{P_H}\right)^{-\varepsilon} Y_H, \qquad P_H = \left[\int_0^1 P_H(j)^{1-\varepsilon}\, dj\right]^{1/(1-\varepsilon)}, \tag{11}$$

where we drop time subscripts when no confusion arises.

Intermediate producers. Producer j hires domestic labour and operates the linear technology $Y_H(j) = AN(j)$, so its nominal unit cost is W/A and aggregate production in a symmetric allocation is $Y_H = AN$. Because firms are owned by domestic households, Hegemon firms discount nominal profits at $i + \Phi$ rather than at the Treasury rate i. Foreign firms discount at i^*, since Foreign households earn no convenience on their own bond.

Adjusting a variety's price costs $\vartheta_H[\dot{P}_H(j)/P_H(j)]^2 Y_H/2$ units of the Hegemon final good, with $\vartheta_H > 0$, as in Rotemberg (1982). Taking aggregate output and prices as given, each firm chooses its price path to solve

$$\max_{\{P_{Hs}(j)\}_{s\geq t}} \int_t^\infty e^{-\int_t^s (i_u+\Phi_u)\,du}\left\{P_{Hs}(j)Y_{Hs}(j) - W_s N_s(j) - \frac{\vartheta_H}{2}\left[\frac{\dot{P}_{Hs}(j)}{P_{Hs}(j)}\right]^2 P_{Hs}Y_{Hs}\right\} ds. \tag{12}$$

Foreign firms solve the symmetric problem with adjustment-cost parameter ϑ_F. At a symmetric allocation, all firms in a country charge the same price, and aggregate price-adjustment expenditure is $\vartheta_H P_H Y_H \pi_H^2/2$, where $\pi_H \equiv \dot{P}_H/P_H$ is producer-price inflation.

3.3 Policy

Government budget constraints. Each government issues nominal debt in its own currency. Real primary surpluses are $s_t = T_t/P_t$ and $s_t^* = T_t^*/P_t^*$. The Hegemon's nominal budget, $\dot{B}_t = i_t B_t - T_t$, implies $\dot{b}_t = (i_t - \pi_t)b_t - s_t$ in real terms, and the budget of the Foreign fiscal authority is symmetric. Integrating forward and ruling out explosive debt gives the debt valuation conditions

$$b_t = \int_t^\infty e^{-\int_t^u (i_z - \pi_z)\,dz} s_u \, du, \qquad b_t^* = \int_t^\infty e^{-\int_t^u (i_z^* - \pi_z^*)\,dz} s_u^* \, du, \tag{13}$$

which state that the real value of each government's debt equals the present value of its primary surpluses, discounted at the real interest rate on that debt. In steady state, the Hegemon's convenience yield lowers that real rate to $\rho - \bar{\Phi}$, whereas the Foreign rate is ρ.

Fiscal and monetary rules. Let $\hat{b}_t = \log(b_t/\bar{b})$ denote the log deviation of real debt and $\hat{s}_t = (s_t - \bar{s})/\bar{b}$ the surplus deviation as a share of steady-state debt, and similarly for Foreign. Fiscal policy follows a debt feedback rule, while monetary policy responds to CPI inflation:

$$\begin{aligned} \hat{s}_t &= \psi_b \hat{b}_t + \epsilon_t^s, & \hat{s}_t^* &= \psi_b^* \hat{b}_t^*, \\ i_t &= \rho - \bar{\Phi} + \phi_\pi \pi_t, & i_t^* &= \rho + \phi_\pi^* \pi_t^*. \end{aligned} \tag{14}$$

Here, ϵ_t^s is an unanticipated fiscal shock. Central banks respond only to CPI inflation, but we allow for output-gap responses later on in Section 4. Following Leeper (1991), fiscal policy is *passive* when surpluses respond to debt by more than the steady-state real rate, $\psi_b > \rho - \bar{\Phi}$ at the Hegemon and $\psi_b^* > \rho$ abroad, so that they stabilise debt, and *active* when they respond by less. Monetary policy is *active* when its inflation coefficient, ϕ_π or ϕ_π^*, exceeds one, so that the Taylor principle holds, and *passive* when it is below one.

3.4 Aggregate Dynamics and Equilibrium

Goods trade freely, so the law of one price holds for each good:

$$P_{Ht} = E_t P_{Ht}^*, \qquad P_{Ft} = E_t P_{Ft}^*. \tag{15}$$

The terms of trade, $\text{tot}_t \equiv P_{Ft}/P_{Ht}$, is the price of Foreign goods relative to Hegemon goods. Both the terms of trade and the real exchange rate equal one in steady state, and the price indices imply $q_t = (1-2\gamma)\tau_t$, where $\tau_t \equiv \log \text{tot}_t$. Thus, with home bias, the real exchange rate moves less than one-for-one with the terms of trade. CPI inflation is $\pi_t = \pi_{Ht} + \gamma\dot{\tau}_t$ at the Hegemon and $\pi^*_t = \pi^*_{Ft} - \gamma\dot{\tau}_t$ abroad, where π^*_{Ft} is Foreign producer-price inflation.

We write aggregate demand and price setting in output-gap form, relative to the flexible-price allocation, denoted by a superscript n, that would prevail at each date given the actual productivity, debt stocks, net foreign assets and relative marginal wealth. The terms of trade under this allocation are $\tau^n_t = \log(A_t/A^*_t) - \varpi_t$, which improve for the Hegemon when its households are relatively wealthier, since home bias tilts their demand towards Hegemon goods. The natural convenience yields Φ^n_t and Φ^{*n}_t are the convenience yields at flexible-price consumption and Treasury holdings.

The output gaps are $x_t \equiv \log(Y_{Ht}/Y^n_{Ht})$ and $x^*_t \equiv \log(Y^*_{Ft}/Y^{*n}_{Ft})$, and the terms-of-trade gap is $\tilde{\tau}_t \equiv \tau_t - \tau^n_t$. Goods-market clearing gives $x_t = \log(C_t/C^n_t) + \gamma\tilde{\tau}_t$ and $x^*_t = \log(C^*_t/C^{*n}_t) - \gamma\tilde{\tau}_t$. A positive terms-of-trade gap makes Foreign goods relatively expensive and so shifts demand towards Hegemon goods. Because the two allocations share relative marginal wealth, relative output gaps equal the terms-of-trade gap, $x_t - x^*_t = \tilde{\tau}_t$.

Natural real rates. We define the *natural* real rates as the returns on each household's own-country bond that support flexible-price consumption growth in its Euler equation:

$$\begin{aligned} r^n_t &= \rho + \frac{\dot{A}_t}{A_t} - \Phi^n_t - \gamma\dot{\tau}^n_t, \\ r^{*n}_t &= \rho + \frac{\dot{A}^*_t}{A^*_t} + \gamma\dot{\tau}^n_t. \end{aligned} \tag{16}$$

Productivity growth raises the return required to support flexible-price consumption growth, while the convenience yield lowers the Hegemon's required return. Changes in the flexible-price terms of trade affect the two countries' consumption purchasing power in opposite directions. With productivity fixed, natural rates therefore move for only two reasons: changes in convenience yields, and shifts in wealth between the two countries that move the natural terms of trade.

Aggregate demand. We summarise aggregate demand by two IS curves. Differentiating the output gaps and substituting the actual and flexible-price Euler equations for each

household's own bond gives

$$\begin{aligned}\dot{x}_t &= (i_t - \pi_t - r_t^n) + (\Phi_t - \Phi_t^n) + \gamma\dot{\tilde{\tau}}_t,\\ \dot{x}_t^* &= (i_t^* - \pi_t^* - r_t^{*n}) - \gamma\dot{\tilde{\tau}}_t.\end{aligned} \tag{17}$$

The first term in each curve is the gap between the actual and natural real rates, as in a closed-economy New Keynesian model. The Hegemon curve also contains the convenience-yield *gap*: a convenience yield above its flexible-price level raises the total return on Treasuries and so tilts Hegemon consumption towards the future. The last term maps changes in the terms-of-trade gap into demand for domestic output.

Aggregate supply. Two Phillips curves describe aggregate supply. Labour supply and the flexible-price allocation imply that each output gap equals the log deviation of real marginal cost from its flexible-price level and symmetrically abroad. From the linearised price-setting conditions, we arrive at

$$\dot{\pi}_{Ht} = \rho\pi_{Ht} - \kappa_H x_t, \quad \dot{\pi}_{Ft}^* = \rho\pi_{Ft}^* - \kappa_F x_t^*. \tag{18}$$

These are New Keynesian Phillips curves, with slopes $\kappa_H \equiv (\varepsilon - 1)/\vartheta_H$ and $\kappa_F \equiv (\varepsilon - 1)/\vartheta_F$. Both curves discount at the rate ρ, i.e., the convenience yield lowers the Treasury rate but not the rate at which Hegemon firms discount profits. Integrated forward, each curve makes producer-price inflation proportional to the present value of future output gaps.

Government debt dynamics. Linearising the government budgets and substituting the surplus rules gives

$$\begin{aligned}\dot{\hat{b}}_t &= (\rho - \bar{\Phi} - \psi_b)\hat{b}_t + [i_t - \pi_t - (\rho - \bar{\Phi})] - \epsilon_t^s,\\ \dot{\hat{b}}_t^* &= (\rho - \psi_b^*)\hat{b}_t^* + (i_t^* - \pi_t^* - \rho).\end{aligned} \tag{19}$$

Real debt grows when the real rate exceeds its steady-state value, so the real rate that governs private demand in the IS curves also governs debt valuation. Hegemon debt also grows when a fiscal shock lowers the surplus.

The external budget. Bond-market clearing requires each government's debt to equal the two households' holdings, $b_t = b_{Ht} + Q_t v_{Ht}^*$ and $b_t^* = b_{Ft}^* + v_{Ft}/Q_t$. Foreign net foreign assets are $n_t^* = v_{Ht}^* - v_{Ft}/Q_t$, and consolidating the Foreign household and government budgets gives their law of motion. The Foreign trade balance equals $\gamma\bar{Y}\omega_t$, so Foreign runs a trade

surplus when Hegemon households are relatively wealthy.

Equilibrium. With all these prerequisites out of the way, we can define the equilibrium in our two-country model. The equilibrium paths for $(x_t, x_t^*, \pi_{Ht}, \pi_{Ft}^*, \tau_t, \hat{b}_t, \hat{b}_t^*, n_t^*, \varpi_t)$ solve eight differential equations: the IS curves (17), the Phillips curves (18), the debt laws (19), the external budget, and the drift of relative marginal wealth (10). The identity $x_t - x_t^* = \tilde{\tau}_t$ closes the system. In the following, we will study transitions after an unanticipated shock at $t = 0$. As we illustrate in Section 4, the question of whether the competitive equilibrium is unique depends on the policy mix in *both* countries jointly.

3.5 Fiscal Transmission

We initially focus on the fiscally-led policy mix, with active fiscal and passive monetary policy in both countries. The exchange rate links the two countries' fiscal adjustments because it changes the purchasing power of each government's nominal debt.

Hegemon fiscal adjustment and the exchange rate. An expansionary U.S. fiscal shock lowers primary surpluses, and fiscal policy does not offset it. Integrating the government budget forward gives

$$\hat{b}_t = \int_t^\infty e^{-(\rho-\Phi-\psi_b)(u-t)} \epsilon_u^s \, du + (1-\phi_\pi) \int_t^\infty e^{-(\rho-\Phi-\psi_b)(u-t)} \pi_u \, du. \tag{20}$$

A negative surplus shock therefore requires a lower real value of outstanding debt, higher subsequent inflation, or both. With $\phi_\pi < 1$, higher inflation lowers real borrowing costs and slows the growth of real debt. Dollar depreciation helps this adjustment by raising the dollar price of imports. Under sticky producer prices, higher import prices raise the U.S. CPI and lower the real value of outstanding dollar debt, so part of the adjustment *must* occur through the purchasing power of existing debt. The government budget determines how much debt revaluation and inflation are needed in total, whereas the goods-market and no-arbitrage conditions determine the exchange-rate response jointly.

Foreign fiscal valuation. Whenever the dollar depreciates, there is a direct fiscal effect on Foreign. Cheaper U.S. imports lower the U.K. price level, so outstanding sterling debt becomes a larger claim on U.K. consumption, even though the U.K. has undertaken no fiscal expansion of its own. U.K. surpluses do not rise enough to support this higher debt

at unchanged real rates. Jointly, the Foreign government budget and monetary rule imply

$$\hat{b}_t^* = -\int_t^\infty e^{-(\rho-\psi_b^*)(u-t)}(r_u^* - \rho)\,du = (1-\phi_\pi^*)\int_t^\infty e^{-(\rho-\psi_b^*)(u-t)}\pi_u^*\,du. \tag{21}$$

The higher real debt therefore forces U.K. real rates to fall below ρ on average. With $\phi_\pi^* < 1$, the U.K. central bank raises its policy rate less than one-for-one with inflation, so higher U.K. inflation delivers this fall.

Foreign aggregate demand and inflation. The resulting lower real rates are reflective of elevated U.K. inflation in equilibrium. Integrating the Foreign IS and Phillips curves forward gives

$$\begin{aligned}\pi_{Ft}^* &= \frac{\kappa_F}{\rho}\int_t^\infty [1-e^{-\rho(u-t)}](r_u^{*n} - r_u^*)\,du - \gamma\kappa_F\int_t^\infty e^{-\rho(u-t)}\tilde{\tau}_u\,du,\\ \pi_t^* &= \pi_{Ft}^* - \gamma\dot{\tau}_t.\end{aligned} \tag{22}$$

After a U.S. deficit, the two terms pull in opposite directions. The first raises U.K. producer inflation: real rates below their natural level boost U.K. demand. The second lowers it: the depreciation makes U.K. goods relatively expensive, so demand shifts towards U.S. goods and U.K. output falls on impact. Because equation (21) requires U.K. inflation to be positive on average, the demand boost from lower real rates outweighs the loss of competitiveness over time. Since firms set prices looking ahead, the expected boost from low real rates raises U.K. producer prices even while the U.K. output gap is still negative. U.K. CPI falls on impact as import prices drop, and then rises with producer prices.

This is *fiscal contagion*: a U.S. deficit depreciates the dollar, raises the real value of U.K. debt, and so forces U.K. real rates down and U.K. prices up, although U.K. fiscal policy has not changed. The result is the pattern in Section 2: a weaker dollar followed by persistently higher U.K. prices. We now aim to quantify precisely this fiscal contagion.

4 Quantitative Results: Fiscal Spillovers

We now take a quantitative version of the model in Section 3 to the data, which Section 4.1 describes. Section 4.2 simulates Hegemon convenience-yield and fiscal shocks under each policy mix, and Section 4.3 separates the channels through which a Hegemon fiscal shock is transmitted. Section 4.4 estimates the model on U.S. and U.K. data since 1998.

4.1 The Quantitative Model

The quantitative model keeps the structure of Section 3 but generalises convenience services, introduces a time-varying holding wedge, extends the supply, demand, and policy blocks, and lets shocks hit both countries. Supplementary Online Appendix H presents the full specification and derivations.

Convenience yields. Following Jiang et al. (2023, 2024b), convenience services now interact with consumption rather than entering utility additively as in Section 3: the convenience flow from Treasuries shifts the marginal utility of consumption, so changes in that flow affect saving decisions directly. Each country's demand for convenience services is also subject to an exogenous shifter, θ_t at the Hegemon and θ_t^* abroad, so the convenience yield on Treasuries can move for reasons unrelated to fiscal policy, such as financial regulation (Du et al., 2018b; He et al., 2022) and foreign demand for safe dollar assets (Jiang et al., 2021).

Holding wedge. Foreign holdings of Treasuries are now subject to a time-varying proportional wedge $\varkappa_t$, which is rebated lump-sum to Hegemon households. In Section 3, no arbitrage forces both households to value the convenience of Treasuries equally in steady state. The wedge lets these values differ, with $\bar{\varkappa} = \bar{\Phi}^* - \bar{\Phi}$, and its fluctuations act as cross-border financial shocks in the spirit of Itskhoki and Mukhin (2021). The convenience yield that Foreign households earn on Treasuries, net of the holding wedge and of their holding cost, is the model counterpart of the return wedge ξ_t in Section 2.

Supply, demand, and policy. Labour disutility is now convex, with inverse Frisch elasticity ϕ (ϕ^* abroad), and consumption is a CES aggregate with trade elasticity η, so relative prices shift expenditure shares between Hegemon and Foreign goods. Central banks may also respond to output gaps, with coefficients ϕ_x and ϕ_x^*.

Shocks and equilibrium. Seven exogenous shocks cause fluctuations around the steady-state in our economy: productivity, convenience demand, and fiscal shocks (ϵ_t^s and ϵ_t^{s*}) in each country, and the holding wedge. In Sections 4.2 and 4.3, each state jumps after an unanticipated shock and then decays back to steady state at a constant rate. The equilibrium is described by the same variables as in Section 3, and the gross cross-border positions follow from the two households' UIP conditions (which we define in detail in Appendix equations (H.16) and (H.17)).

Illustrative calibration. Sections 4.2 and 4.3 use the benchmark calibration in table 1 to illustrate the model's mechanisms, in particular how fiscal contagion depends on convenience yields and on the policy mix. The left panel of the table gives the four policy configurations, and the right panel the other parameters. The policy coefficients, convenience-yield levels, and shock persistences are illustrative values, which Section 4.4 replaces with estimates.

Param.	AM/PF	AF/PM	HD	H-AF/ F-AM
ϕ_π	1.5	0.0	1.5	0.0
ϕ_π^*	1.5	0.7	0.0	1.5
ϕ_x	0.0	0.0	1.0	0.0
ϕ_x^*	0.0	0.0	0.0	0.0
ψ_b	1.0	-0.2	-0.2	-0.2
ψ_b^*	1.0	-0.2	1.0	1.0

(a) Policy configurations

Param.	Value	Targets/Sources
$\kappa_H = \kappa_F$	0.03	Phillips-curve slopes, calibrated (see text)
$\phi = \phi^*$	2.00	Standard value
$\rho = \rho^*$	0.005	Standard quarterly discount rate
γ	0.25	Import share ($1-\gamma$ is home bias)
η	3	Trade elasticity, calibrated (see text)
$\bar{\Phi}$	0.0002	Steady-state convenience yield, Hegemon
$\bar{\Phi}^*$	0.0001	Steady-state convenience yield, Foreign
ζ_H	2	Convenience-yield sensitivity, Hegemon
ζ_F	8	Convenience-yield sensitivity, Foreign
ω_H	0.68	Share of U.S. debt held in the U.S. (U.S. Treasury, 2025)
ψ_H	10	Hegemon cost of holding Foreign debt, calibrated (see text)
ψ_F	0.000186	Foreign cost of holding Hegemon debt (Schmitt-Grohé and Uribe, 2003), per quarter
$\bar{v}_F/\bar{b}^*$	0.15	Share of Foreign debt held at the Hegemon
$\bar{b}/\bar{Y}$	4.28	U.S. debt in 2025 (107% of annual GDP), relative to quarterly output
$\bar{b}^*/\bar{Y}$	3.84	U.K. debt in 2025 (96% of annual GDP), relative to quarterly output
Persist.	0.80	Quarterly persistence, convenience-demand shocks
Persist.	0.90	Quarterly persistence, fiscal, productivity, and holding-wedge shocks

(b) Benchmark calibration

Table 1: Calibration: policy configurations and benchmark parameters.

In a closed economy, Leeper (1991) distinguishes two regimes: a monetary-led regime, with active monetary and passive fiscal policy, and a fiscally-led regime, with the roles

reversed. With two countries, the active and passive roles can also be split across borders, so we consider four configurations. Under the *monetary-led* mix (AM/PF), both central banks satisfy the Taylor principle, and both fiscal authorities raise surpluses enough to stabilise debt, with ψ_b and ψ_b^* well above the steady-state real rates. Under the *fiscally-led* mix (AF/PM), future surpluses do not back government debt, and both central banks raise nominal rates less than one-for-one with inflation. We also consider two further cases of interest. Under *Hegemon Dominance* (HD), both Hegemon authorities are active, while both Foreign authorities are passive. The *mixed* configuration (H-AF/F-AM) is fiscally-led at the Hegemon and monetary-led abroad: the Hegemon does not raise surpluses to back its debt, while Foreign does. The policy mix matters mainly because it determines whether households treat government debt as net wealth: under passive fiscal policy, higher debt is matched by higher future taxes; under active fiscal policy, it is not.

The Frisch elasticity and the discount rate take standard values. The debt ratios are the 2025 values for the U.S. and the U.K., relative to quarterly output, and the share of U.S. debt held in the U.S. comes from U.S. Treasury (2025). Because that share counts all foreign holders of Treasuries, Foreign stands for the rest of the world as much as for the U.K. in this calibration.

The share of Foreign debt held at the Hegemon, and the levels and sensitivities of the convenience yields, are illustrative values that capture two regularities in the literature on safe-asset demand (Du et al., 2018a; Jiang et al., 2024b): convenience yields are higher at the Hegemon, reflecting the liquidity of the Treasury market, and they respond strongly to bond supply. We do not estimate the Phillips-curve slopes, the trade elasticity, or the holding costs. The slopes and the trade elasticity lie within conventional ranges, and among such values we use those under which the model accounts for the 2021–2023 episode studied in Section 4.4. The Foreign cost ψ_F is the value of Schmitt-Grohé and Uribe (2003) expressed per quarter, so Foreign investors hold Treasuries almost free of cost, while the larger Hegemon cost, $\psi_H = 10$, keeps Hegemon holdings of Foreign bonds close to their target.

4.2 Simulation

We simulate the following two Hegemon shocks under each of the four policy configurations: a fall in the Treasury convenience yield and a deficit-financed fiscal transfer.

4.2.1 Convenience Yield Shocks

We first consider a 1-basis-point fall in the convenience yield that Hegemon households derive from Treasuries, through the demand shifter θ_t. At the benchmark calibration, this halves the steady-state Hegemon convenience yield. Figure 5 shows the responses of output gaps, producer-price inflation, the terms-of-trade gap, and the nominal exchange rate.

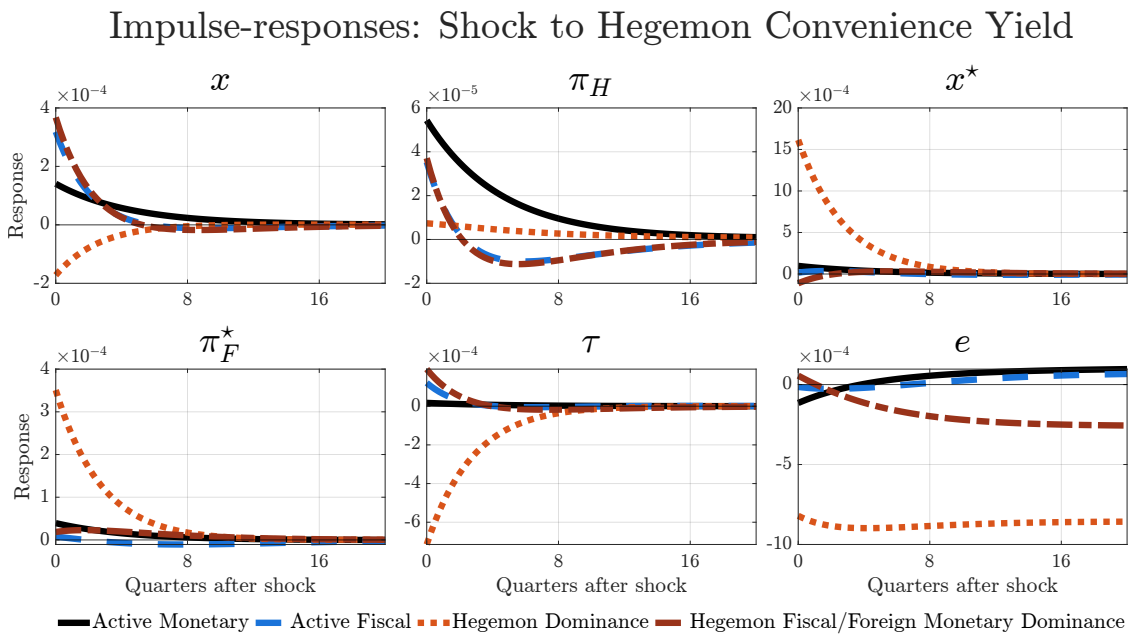


Figure 5: Impulse responses to a 1-basis-point fall in the Hegemon convenience yield under the four policy configurations. Output gaps, the terms-of-trade gap, and the exchange rate are log deviations, while inflation rates are quarterly. In the legend, "Active Monetary" is the monetary-led policy mix (AM/PF), "Active Fiscal" the fiscally-led policy mix (AF/PM), and "Hegemon Fiscal/Foreign Monetary Dominance" the mixed configuration (H-AF/F-AM).

A fall in the convenience yield works like a fall in the demand for safe assets. Because Treasuries now deliver less of a return through convenience services, Hegemon households require a higher pecuniary return to hold them, which raises the natural real rate in equation (16). As long as the policy rate does not rise by as much, the actual real rate falls below the natural rate, and Hegemon households save less and bring consumption forward. The shock is thus the mirror image of a flight to safety, and of the "risk premium" shock of Smets and Wouters (2007), which Fisher (2015) interprets as a shock to the demand for safe and liquid assets. How strongly Hegemon demand responds depends on the Hegemon

central bank. Under the monetary-led policy mix, it raises the Hegemon's policy rate, so output rises by less but inflation by more than in the fiscally-led and mixed configurations, where the policy rate does not respond. In those two configurations, inflation turns slightly negative after about two quarters, so the Hegemon price level ends up lower than before the shock. Under the fiscally-led policy mix, this disinflation coincides with a fall in the real value of Hegemon debt, most of which Hegemon households hold.

Abroad, the shock matters little except under Hegemon Dominance. In the other configurations, Foreign output and inflation barely move, and the exchange rate moves only slightly. Hegemon Dominance differs because both Hegemon authorities are active: since neither surpluses nor inflation at the Hegemon can keep its debt valued, the exchange rate must do so. The Hegemon currency appreciates persistently, which lowers the Hegemon price level and makes Foreign goods relatively cheap. Expenditure switching shifts demand towards Foreign goods, so Hegemon output falls and the Hegemon central bank cuts its policy rate. In the Foreign economy, higher prices of imports from the Hegemon raise inflation, and since the passive Foreign central bank does not respond, Foreign real rates fall and demand booms: Foreign output and inflation rise by an order of magnitude more than under any other configuration.

4.2.2 Fiscal Transfer Shocks

We next consider an expansionary Hegemon fiscal transfer of 1 percent of quarterly steady-state output, a negative innovation to ϵ_t^s. Figure 6 shows the resulting dynamics. Under the monetary-led policy mix, the transfer has essentially no effect: higher future surpluses back government debt exactly, so households save it to meet the future taxes, as under Ricardian equivalence (Barro, 1974). Under the fiscally-led policy mix, future surpluses do not back government debt sufficiently, so Hegemon households treat the new debt as net wealth and spend the transfer (Woodford, 1995). With the Hegemon policy rate unchanged, Hegemon output rises by about 1.8 percent on impact and producer-price inflation by about 0.4 percentage points. The Hegemon currency depreciates by about 0.9 percent, which helps the fiscal adjustment: with producer prices sticky, higher import prices raise the Hegemon price level and lower the real value of debt that surpluses no longer back (Section 3.5). The depreciation also makes Foreign goods relatively expensive and lowers the Foreign-currency value of Foreign holdings of Treasuries, so Foreign output falls on impact.

Foreign inflation nevertheless rises: over five years, Foreign producer prices rise by about 1.3 percent, more than Hegemon producer prices (1.1 percent). The reason is the valuation mechanism of Section 3.5. The depreciation lowers the Foreign price level

through import prices and so raises the real value of Foreign debt. Because future Foreign surpluses do not back this higher real debt, Foreign real rates must fall, and since the Foreign central bank raises its policy rate less than one-for-one with inflation, lower real rates come with higher Foreign inflation, as equation (21) requires. This is how "our" deficit at the Hegemon becomes "your" problem abroad.

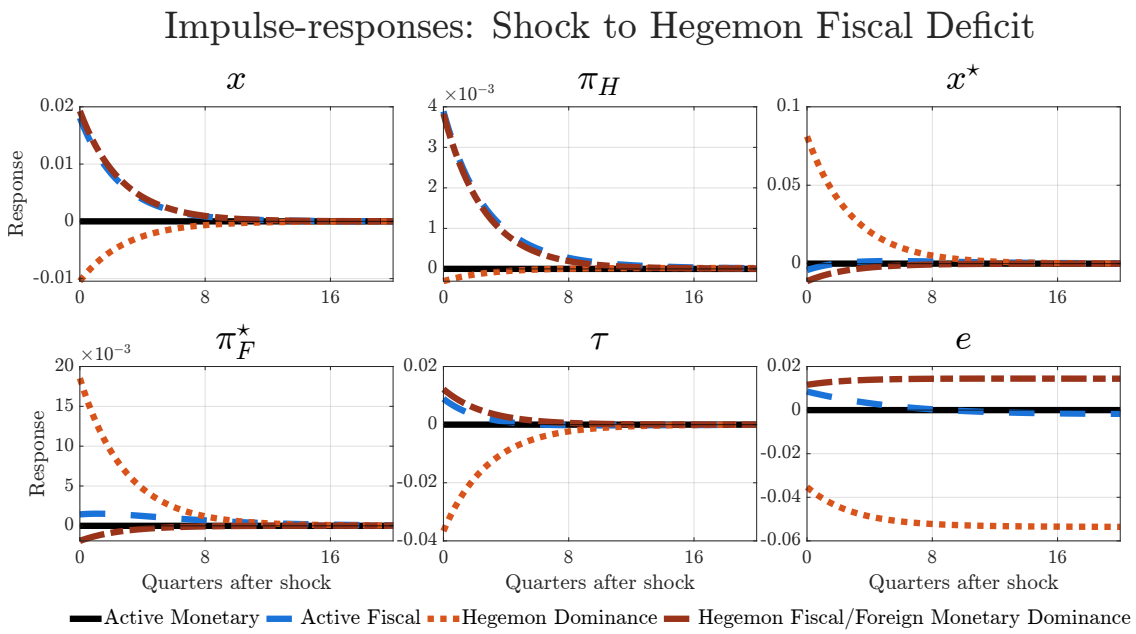


Figure 6: Impulse responses to an expansionary Hegemon fiscal transfer shock of 1 percent of quarterly steady-state output under the four policy configurations. Output gaps, the terms-of-trade gap, and the exchange rate are log deviations, while inflation rates are quarterly. In the legend, "Active Monetary" is the monetary-led policy mix (AM/PF), "Active Fiscal" the fiscally-led policy mix (AF/PM), and "Hegemon Fiscal/Foreign Monetary Dominance" the mixed configuration (H-AF/F-AM).

Hegemon Dominance reverses the exchange-rate response. As in Section 4.2, both Hegemon authorities are active, so the exchange rate rather than Hegemon inflation keeps Hegemon debt valued: the Hegemon central bank leans against inflation, and the currency appreciates instead, by about 3.5 percent on impact and 5.4 percent after five years. Foreign goods become relatively cheap, and the resulting expenditure switching shifts demand away from Hegemon goods, so Hegemon output falls by about 1 percent on impact, the Hegemon central bank lowers its policy rate, and Hegemon inflation barely moves. Foreign households, by contrast, become wealthier as their Treasury holdings appreciate in Foreign currency, while the same expenditure switching raises demand for Foreign goods. Because

the passive Foreign central bank does not respond to the resulting inflation, Foreign output rises by about 8 percent on impact and Foreign inflation by about 1.9 percentage points, and Foreign producer prices rise by about 5.4 percent over five years, four times as much as under the fiscally-led policy mix.

The mixed-policy configuration shows that the inflation spillover needs active fiscal policy in Foreign. Hegemon policy is the same as under the fiscally-led policy mix, so Hegemon output and inflation rise as before, and the Hegemon currency depreciates persistently, by about 1.4 percent after five years. Foreign policy, however, is now monetary-led: Foreign surpluses back the higher real value of Foreign debt, so Foreign real rates no longer need to fall. The depreciation then works only through relative prices: Foreign goods become relatively expensive, demand shifts towards Hegemon goods, and Foreign output falls by about 1 percent on impact and Foreign producer prices by about 0.5 percent over five years. Although the shock originates at the Hegemon, Foreign policy effectively *decides* whether it raises Foreign inflation or causes a Foreign recession.

The size of convenience yields matters for these results, as we show in Supplementary Online Appendix I where we raise both steady-state convenience yields by orders of magnitude. In that case, deficits have real effects even under the monetary-led policy mix: Treasuries are now more than claims on future taxes, and the extra supply lowers their convenience yield, which, as in the convenience-yield shock above, raises the natural rate and stimulates demand. Additionally, with high convenience yields, the Foreign economy is largely insulated under the fiscally-led policy mix, because convenience yields weaken the link between Hegemon and Foreign discount rates, so the cumulative rise in Foreign producer prices falls from 1.3 to about 0.15 percent.

4.3 Decomposing the Spillover Channels

We now use two additional illustrative exercises to see what exactly drives inflation in the two countries and the exchange rate in our model.

Contributions of model features. Figure 7 decomposes the cumulative 20-quarter responses of Hegemon inflation π_H, Foreign inflation π_F^*, and the nominal exchange rate e to the fiscal shock of Section 4.2 into the contributions of three features of the model (convenience yields, international portfolios, and home bias) and a policy component.[13] The decomposition starts from a stripped-down model in which all three features are switched off: convenience yields are scaled close to zero, each country holds almost none

[13]We do not show the results for the Hegemon dominance calibration here, as they are an order of magnitude larger for Foreign inflation.

of the other's debt, and home bias is removed by setting $\gamma = 1/2$. Restoring one feature at a time gives the plotted contribution, measured as the change in the response. The policy component, labelled "Policy" in figure 7, collects the response of the stripped-down model and all interactions among the features. It therefore contains the fiscal-valuation mechanism of Section 3.5, which operates even in the stripped-down model.

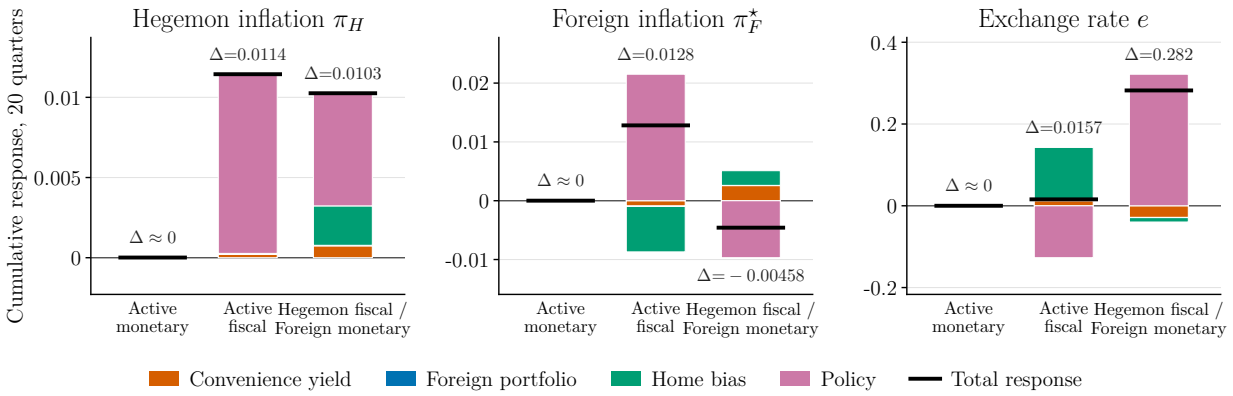


Figure 7: Decomposition of the cumulative 20-quarter response of π_H, π^*_F, and e to a Hegemon transfer shock under the benchmark calibration. The three columns in each panel are the monetary-led policy mix, the fiscally-led policy mix, and the mixed configuration. Each coloured bar is the change in the response when one feature is added to a stripped-down model with all three features switched off. The bar labelled "Policy" contains the response of the stripped-down model and all interactions. Black lines and the numbers above or below each column give the total response. For e, the bars cumulate the level of the exchange rate over 20 quarters.

In each column of figure 7, the four contributions sum to the cumulative impulse response of Section 4.2. As mentioned before, under the monetary-led policy mix, future surpluses back the transfer, so the fiscal innovation has essentially no effect. Under the fiscally-led policy mix, the policy component drives inflation in both countries. It accounts for almost all of the rise in Hegemon inflation and for more than the whole rise in Foreign inflation, of which home bias offsets more than a third and convenience yields a small part, while international portfolios contribute almost nothing. Future surpluses back neither the Hegemon transfer nor the higher real value of Foreign debt after the depreciation, so households in both countries treat government debt as net wealth, and because neither central bank raises its policy rate one-for-one with inflation, real rates fall and inflation rises. For the exchange rate, the home bias and policy components nearly cancel: as the Foreign central bank raises its policy rate with Foreign inflation, the initial depreciation of

the Hegemon currency is reversed, leaving a small cumulative response.

Under the mixed configuration, Foreign policy reverses the sign of the spillover. Hegemon policy is unchanged, so the policy component again accounts for about two-thirds of the rise in Hegemon inflation, with home bias contributing most of the rest. Foreign taxes, however, now rise to back the higher real value of Foreign debt, and the Foreign central bank follows the Taylor principle, so the debt is no longer net wealth for Foreign households, who also lose on their Treasuries as the Hegemon currency depreciates. Without this wealth effect, the loss of competitiveness dominates: Foreign output falls, and the policy component lowers Foreign inflation, which convenience yields and home bias each offset by about a quarter. Cheaper imports lower the Foreign CPI, so the Foreign central bank cuts its policy rate, and the Hegemon currency keeps depreciating, by about 1.4 percent after five years.

Convenience yields work in the opposite direction to the policy component in both configurations, so they dampen the spillover to Foreign inflation whatever its sign. The reason lies in the no-arbitrage condition between the two bonds, which ties the Foreign real rate to the return on Treasuries. With convenience yields, part of any change in that return is absorbed by a change in the convenience yield instead, so the Foreign real rate moves less, and with it Foreign inflation. The effect is small in the benchmark calibration, where convenience yields are small, but large when they are high. In fact, in the high convenience-yield calibration of Section 4.2, the fall in the Foreign real rate under the fiscally-led policy mix is less than half as large, and Foreign inflation barely rises.

Convenience yields and portfolio exposure. Figure 8 shows the cumulative 20-quarter response of Foreign inflation to the Hegemon fiscal shock as Foreign holdings of Treasuries and the Hegemon convenience yield vary, both under the fiscally-led policy mix (left panel) and under Hegemon Dominance (right panel). The horizontal axis is the share of Treasuries held at the Hegemon rather than abroad, so moving to the left means that Foreign households hold more of them. The vertical axis is the steady-state Hegemon convenience yield, with the steady-state Foreign convenience yield held fixed. Moving up means a higher convenience yield, so the steady-state real rate on Treasuries is lower than it would otherwise be. The spillover can reach Foreign households through two channels: a direct revaluation of their Treasury holdings when the exchange rate moves, and a revaluation of Foreign government debt through linked discount rates. The direct revaluation scales with Foreign households' inherited Treasury holdings, so moving along the horizontal axis changes this channel in particular, but portfolio exposure can also affect the discount-rate channel through equilibrium adjustments. Because the axis measures

the share held at the Hegemon, however, it also changes how many Treasuries Hegemon households hold, which matters when convenience yields are high, as we show below.

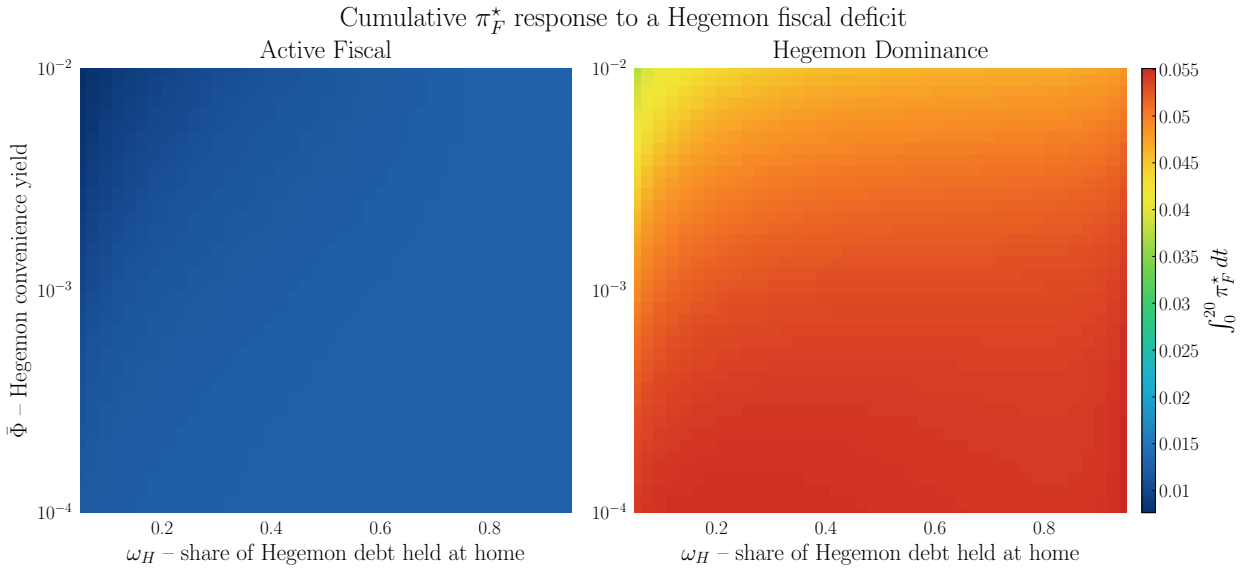


Figure 8: Cumulative 20-quarter response of Foreign inflation, $\int_0^{20} \pi_F^* \, dt$, to a Hegemon deficit of 1 percent of quarterly output, by steady-state Hegemon convenience yield (vertical axis, logarithmic scale) and the share of Hegemon debt held at the Hegemon (horizontal axis). The Foreign convenience yield is held at $\bar{\Phi}^* = 10^{-4}$. Left panel: fiscally-led policy mix (AF/PM). Right panel: Hegemon Dominance (HD).

The figure varies the steady-state Hegemon convenience yield while holding its Foreign counterpart fixed, changing Hegemon households' willingness to absorb portfolio adjustments. Across the grid, the cumulative rise in Foreign producer prices lies between 0.8 and 1.3 percent under the fiscally-led policy mix and between 3.7 and 5.5 percent under Hegemon Dominance. At low convenience yields, this exported inflation varies little with the share of Treasuries that Foreign households hold, so it does not arise mainly from the direct revaluation. The revaluation of Foreign debt through linked discount rates therefore accounts for most of the spillover, consistent with figure 7, in which international portfolios contribute almost nothing to Foreign inflation under the fiscally-led policy mix. The strength of this channel clearly depends on the policy mix, since, across the grid, the spillover is about four times larger under Hegemon Dominance than under the fiscally-led policy mix.

At high convenience yields, exported inflation is lower, most clearly when Foreign households hold *more* Treasuries. When they hold 90 percent of them, raising $\bar{\Phi}$ from 10^{-4}

to 10^{-2} lowers exported inflation from about 1.2 to 0.85 percent under the fiscally-led policy mix and from about 5.4 to 4.1 percent under Hegemon Dominance, whereas under the fiscally-led policy mix it barely moves when Hegemon households hold most Treasuries. This pattern does not reflect the convenience that Foreign households derive from their Treasury holdings, which the figure holds fixed. Instead, it reflects the convenience yield of Hegemon households, who, given the stock of Treasuries, are the counterparties whenever Foreign households trade them. Under the fiscally-led policy mix, Foreign households sell Treasuries to finance spending as Foreign real rates fall, whereas under Hegemon Dominance, they put part of their gain from the appreciation into Treasuries. When Foreign households hold most Treasuries, Hegemon households hold few, so the same trade is a large percentage change in Hegemon holdings. But since a 1 percent rise in holdings lowers the convenience yield by about 2 percent at any $\bar{\Phi}$, the Hegemon convenience yield moves most (in percentage point terms) when $\bar{\Phi}$ is high. Then, the convenience yield falls under the fiscally-led policy mix, as Hegemon households buy Treasuries, and it rises under Hegemon Dominance, as Hegemon households sell Treasuries. Either move makes the trade less attractive, so Hegemon households trade fewer Treasuries. Under the fiscally-led policy mix, Hegemon households raise their Treasury holdings by less than 1 percent over five years at $\bar{\Phi} = 10^{-2}$, against almost 8 percent at $\bar{\Phi} = 10^{-4}$, and spend more on consumption instead, so Foreign households keep more of their Treasuries and spend less. Under Hegemon Dominance, Hegemon households instead sell fewer Treasuries, and the Hegemon currency appreciates on impact by about 2.4 rather than 3.6 percent at $\bar{\Phi} = 10^{-4}$, so Foreign wealth and demand rise by less. At high convenience yields, the spillover in both policy specifications is therefore smallest where Foreign households hold the most Treasuries, although under Hegemon Dominance Foreign households would gain most from the appreciation.

4.4 Estimation

We estimate the model at quarterly frequency from 1998Q1, the start of the first full year after the Bank of England gained operational independence, to 2024Q4. We specify that each exogenous state follow an Ornstein–Uhlenbeck process, keeping the above equilibrium conditions unchanged otherwise:

$$
\begin{aligned}
da_t &= -\varphi_a a_t\,dt + \sigma_a\,dW_{a,t}, & da_t^* &= -\varphi_a^* a_t^*\,dt + \sigma_a^*\,dW_{a^*,t},\\
d\hat{\theta}_t &= -\varphi_\theta \hat{\theta}_t\,dt + \sigma_\theta\,dW_{\theta,t}, & d\hat{\theta}_t^* &= -\varphi_\theta^* \hat{\theta}_t^*\,dt + \sigma_\theta^*\,dW_{\theta^*,t},\\
d\epsilon_t^s &= -\varphi_s \epsilon_t^s\,dt + \sigma_s\,dW_{s,t}, & d\epsilon_t^{s*} &= -\varphi_s^* \epsilon_t^{s*}\,dt + \sigma_s^*\,dW_{s^*,t},\\
d\hat{\varkappa}_t &= -\varphi_\varkappa \hat{\varkappa}_t\,dt + \sigma_\varkappa\,dW_{\varkappa,t}.
\end{aligned}
\tag{23}
$$

Here, a_t and a_t^* are log productivity, hats denote deviations from steady state, and the $W_{j,t}$ are independent standard Brownian motions. A negative fiscal innovation lowers surpluses, so it is a fiscal expansion. The model stays linear, so uncertainty itself does not change decisions, which we view as an approximation that keeps the estimation tractable.

Data. We attempt to match as closely as possible six quarterly time series from the data: the U.S. output gap, U.S. and U.K. producer-price inflation, the relative producer-price level in common currency, the observable counterpart of the terms of trade τ_t, and the growth of the real market value of U.S. and U.K. government debt, each deflated by the country's own producer price index.[14] Inflation and debt growth are $100\,\Delta\log$ changes, the relative price level is $100\log(E_t P_{Ft}^*/P_{Ht})$, and all series are demeaned over the sample.[15]

The estimation varies 22 parameters: the policy coefficients ϕ_π, ϕ_x, ϕ_π^*, ϕ_x^*, ψ_b, and ψ_b^*, the steady-state convenience-yield levels $\bar{\Phi}$ and $\bar{\Phi}^*$, and the mean reversion and diffusion of all seven shock processes. All other parameters, including the convenience-yield sensitivities ζ_H and ζ_F, the holding costs ψ_H and ψ_F, and the target cross-border positions, remain at their values in table 1.

Estimation method. We use the method of Christensen et al. (2024), which estimates continuous-time models on discrete data through the exact discretisation of their state-space representation and a Kalman-filter likelihood. Our headline exercise is a Bayesian estimation (Herbst and Schorfheide, 2016).[16] The priors on the quarterly autocorrelations of the shocks are Beta distributions, and those on the quarterly innovation standard deviations are wide inverse-gamma distributions, so that the data determine the relative importance of each shock. The policy coefficients and the convenience-yield levels have normal priors. Four of these priors are informative: those on ϕ_π (centred on one) and

[14] Because the model's debt stocks are measured in consumption units, observed real-debt growth corresponds to $\Delta(\hat{b}_t + \gamma\tau_t)$ at the Hegemon and $\Delta(\hat{b}_t^* - \gamma\tau_t)$ abroad.

[15] These observables correspond to the minimal state variable (MSV) system. We do not use data on the Foreign output gap, for instance, as it is not part of that system.

[16] Supplementary Online Appendix J reports maximum-likelihood estimates and Bayesian estimates that add one of two historical convenience-yield series as an observable. Because no prior disciplines maximum likelihood, its parameter space is narrower than the one used for the Bayesian estimation.

ϕ^*_π (centred on 0.5), and those on the convenience yields $\bar{\Phi}$ and $\bar{\Phi}^*$ (centred on 0.7 and 0.5 percent a year). Table E.1 in the Appendix lists all priors, and Supplementary Online Appendix J.6 describes the sampler and its convergence.

Estimates. Table 2 reports the posterior modes. In the monetary rules, the Hegemon inflation coefficient is just above one, at 1.03, but the output coefficient is negative, at −0.54, which makes the Hegemon rule passive once its response to output is taken into account. The Foreign inflation coefficient in its monetary rule is 0.75. The fiscal coefficients are negative, so future surpluses do not back government debt, and the Foreign coefficient lies at the upper bound of its range. The estimates therefore describe a fiscally-led policy mix in both countries, although for fiscal policy this is imposed by the parameter space. The steady-state convenience yields are about 0.33 percent a quarter at the Hegemon and 0.12 percent abroad, or roughly 1.3 and 0.5 percent a year.

Policy and convenience yield parameters			**Shock mean-reversion and diffusion parameters**		
Parameter	Estimate	Post. s.d.	Parameter	Estimate	Post. s.d.
ϕ_π	1.0338	0.0060	φ_a	$0.0202^\dagger$	0.0464
ϕ^*_π	0.7536	0.0977	φ^*_a	0.8836	0.1821
ϕ_x	-0.5423	0.0429	φ_θ	0.4850	0.4266
ϕ^*_x	-0.4096	0.0490	φ^*_θ	0.4850	0.4276
ψ_b	-0.0372	0.0201	φ_s	$0.0101^\dagger$	0.0008
ψ^*_b	$-0.5000^\dagger$	0.0852	φ^*_s	0.0418	0.0188
$10^3\,\bar{\Phi}$	3.2762	0.4800	$\varphi_\varkappa$	$0.0202^\dagger$	0.0125
$10^3\,\bar{\Phi}^*$	1.2297	0.3834	σ_a	12.8191	2.0584
			σ^*_a	16.7911	2.0990
			σ_θ	0.4247	0.4414
			σ^*_θ	0.4247	0.4246
			σ_s	0.6798	0.2401
			σ^*_s	2.4492	0.4076
			$\sigma_\varkappa$	0.5278	0.1960

Table 2: Estimates for the exact continuous-time Bayesian specification using U.S. and U.K. data since 1998. Estimates are posterior modes, posterior standard deviations are computed from four chains of 1,000,000 draws each (first 250,000 discarded, every tenth draw retained). A dagger (†) marks a mode at a bound of the parameter space (table E.1). The historical decompositions and counterfactuals are evaluated at the posterior median. Convergence diagnostics are reported in Supplementary Online Appendix J.6.

Historical decomposition. Figure 9 decomposes U.S. inflation, U.K. inflation, and the terms of trade since 2015 into the contributions of individual shocks, at the posterior median. All three series are deviations from their 1998–2024 averages, so a positive value means inflation above its sample mean. Inflation is quarterly producer-price inflation, and the terms of trade is the price of U.K. goods relative to U.S. goods in a common currency, so a fall means that U.K. goods have become cheaper. In each quarter, the bars give each shock's contribution and, together with initial conditions, sum to the data.

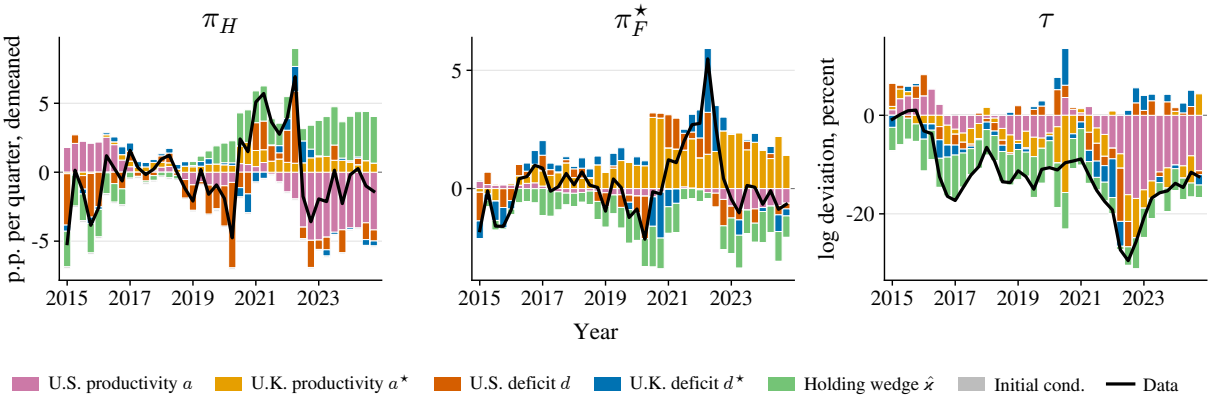


Figure 9: Historical shock decomposition, 2015–2024, main Bayesian specification. Inflation is in percentage points and the terms of trade in percent, and the black line shows the data. In the legend, d and $d^\star$ denote the fiscal shocks ϵ^s and ϵ^{s*}, and $\hat{\kappa}$ the holding wedge $\hat{\varkappa}$. Convenience-demand shocks contribute almost nothing and are not shown.

To summarise the 2021–2023 surge, we add up how far inflation exceeded its sample mean in the quarters of 2021–2023 in which it was above that mean, and report the share of this total that each shock accounts for.[17] U.S. fiscal shocks account for about half of the surge in U.S. inflation (51 percent), and the holding wedge for a similar amount, while U.S. productivity shocks account for most of the subsequent disinflation. The large role of fiscal shocks is in line with Giannone and Primiceri (2026), who find that demand, including expansionary fiscal policy, drove most of the post-pandemic inflation in the U.S. and the euro area. U.K. fiscal shocks account for about a third of the surge in U.K. inflation (35 percent), concentrated between late 2021 and the third quarter of 2022. The largest contributions to U.K. inflation come from U.K. productivity shocks, offset in part by the holding wedge.

U.S. fiscal shocks account for about a quarter of the surge in U.K. inflation (25 percent),

[17] Table J.4 in the Supplementary Online Appendix reports posterior intervals for these shares.

the spillover that this paper highlights. Their contribution is concentrated in 2021 and the first half of 2022 and is partly reversed afterwards. In the terms-of-trade panel, both U.S. and U.K. fiscal shocks lowered the relative price of U.K. goods in 2021 and the first half of 2022 and mostly raised it afterwards, while U.S. productivity shocks account for much of its fall in 2022.

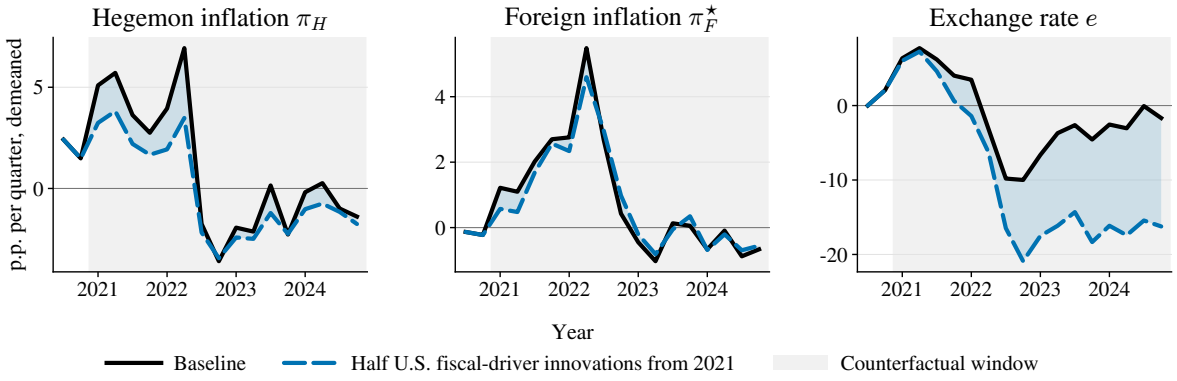


Figure 10: Counterfactual: halving U.S. fiscal-shock innovations from 2021Q1, at the posterior median of the main Bayesian specification. Inflation is demeaned quarterly producer-price inflation, in percentage points, and the exchange rate is the model-implied nominal exchange rate, in percent.

Counterfactual. Figure 10 shows what would have happened had the U.S. fiscal shocks from 2021 onwards been half as large, with earlier shocks left unchanged.[18] U.S. inflation would have been much lower: the cumulative rise in the U.S. price level over 2021–2023 would have been about 14 percentage points smaller. It would also have left the dollar much stronger: the dollar appreciates markedly after 2021 in the counterfactual, and by the end of 2024 it is about 15 percent stronger than in the baseline.

The U.K. price level would have risen by about 1.8 percentage points (posterior mean estimate) less over 2021–2023. The difference is concentrated in 2021 and the first half of 2022, while U.K. inflation was rising: at its peak in the second quarter of 2022, quarterly U.K. inflation would have been almost one percentage point lower. With a smaller U.S. fiscal expansion, the dollar is stronger, so U.S. imports are less cheap and the real value

[18] At the posterior median parameter vector, U.S. fiscal innovations from 2021 contribute 28.36 percentage points to cumulative U.S. inflation over 2021–2023, while earlier innovations contribute −20.11 points. The historical decomposition therefore records a net contribution of 8.25 points, whereas the counterfactual halves the contribution of the later innovations, giving 14.18 points.

of U.K. debt rises by less. U.K. real rates then need to fall less to keep that debt valued, and U.K. prices rise less. From late 2022 the effect is partly reversed, as in the historical decomposition, so U.K. inflation would still have surged, but from a lower path and to a lower peak.

Robustness. Online Appendix F relaxes the data-driven estimation that yielded a fiscally-led policy mix by following Bianchi et al. (2023), separating deficits that future surpluses back (funded) from deficits that are not backed by sufficient expected future surpluses (unfunded), and by allowing central banks to respond actively to funded inflation but accommodate unfunded inflation. In that exercise, U.S. fiscal shocks still raise U.K. inflation, and they do so by even more than in our headline estimates: at the posterior modes, they account for about 58 percent of the 2021–2023 surge in U.K. inflation, against about a quarter in the corresponding headline specifications. Because other shocks offset part of the fiscal contributions, fiscal shocks end up accounting for more than the whole of the domestic surges in this exercise, about 140 percent in the U.S. and 130 percent in the U.K.

5 Conclusion

We have argued that fiscal deficits in a financial Hegemon transmit to exchange rates and inflation abroad. Deficit innovations recovered from the market value of U.K. and U.S. sovereign debt show that U.S. deficit innovations depreciate the dollar against sterling, raise U.K. borrowing costs, and persistently raise U.K. consumer and producer prices. In a two-country New Keynesian model in which Hegemon bonds carry a convenience yield, a Hegemon deficit under a fiscally-led policy mix depreciates its currency, which makes imports cheaper abroad and so raises the real value of Foreign government debt. Because future Foreign surpluses do not back this higher real debt, Foreign real rates must fall and Foreign prices rise, even though Foreign fiscal policy has not changed. Most of this spillover runs through the no-arbitrage link between Hegemon and Foreign bond returns rather than through Foreign holdings of Hegemon debt. The same deficit can raise Foreign inflation, export a Foreign boom, or cause a Foreign recession, depending on whether each government backs its debt with future surpluses and how each central bank responds.

Our main contribution is to show that fiscal policy at the Hegemon matters for inflation abroad, not only at home. Work on fiscally driven inflation has mostly studied closed economies, while work on exchange rates and convenience yields has focused on asset prices rather than on the fiscal positions of other countries. We bring the two together: a Hegemon deficit revalues Foreign government debt, and the monetary–fiscal policy mix in

both countries determines whether this *fiscal contagion* shows up as Foreign inflation or as a Foreign recession. To the best of our knowledge, the persistent spillover of fiscal inflation from U.S. deficits to another advanced economy has not been documented before.

Quantitatively, U.S. fiscal shocks account for about a quarter of the 2021–2023 surge in U.K. inflation. Had they been half as large from 2021 onwards, the price level in the U.K. would have risen by about 1.8 percentage points less over 2021–2023. For countries closely integrated with the U.S. sovereign debt market, fiscal sustainability is therefore not a purely domestic question but depends in part on monetary and fiscal policy at the Hegemon. A Foreign country's own policy mix offers only limited protection, because it imports a stabilisation trade-off: a Foreign economy can avoid imported inflation when its central bank follows the Taylor principle and future surpluses back its debt, but the same deficit then causes a Foreign recession. Whether a Foreign economy can avoid both outcomes, for instance through optimal time-consistent debt management, remains an open question.

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Online Appendix

A Construction of the Deficit Innovations

This Appendix details the deficit innovations used in Section 2.1. We first derive the government debt valuation equation and extend it to the Hegemon/Foreign setting used throughout the paper. We then explain how we recover the deficit series, how we construct the backward and forward decompositions, and how to interpret the resulting innovations. The derivations follow Campbell and Shiller (1988), Hall and Sargent (2011), and Cochrane (2019), extended to two countries.

For the U.S., we use the data of Cochrane (2019), with the macroeconomic aggregates updated to 2025. For the U.K., we use the market value of government debt from Ellison and Scott (2020), updated with the gilt database of Cairns and Wilkie (2024). The quarterly sample begins in 1975, shortly after the end of the Bretton Woods system, and runs to the end of 2025.

A.1 The Market Value of Government Debt

We define the nominal end-of-period market value of debt V_t as

$$V_t \equiv M_t + \sum_{j=0}^{\infty} Q_t^{(t+1+j)} B_t^{(t+1+j)},$$

where M_t is non-interest bearing money, $B_t^{(t+j)}$ is the *face value* of zero-coupon nominal debt outstanding at the end of period t and due at the beginning of period $t+j$, and $Q_t^{(t+j)}$ is the time-t market price of that bond, with $Q_t^{(t)} = 1$. We divide by nominal GDP and take logs to obtain

$$v_t \equiv \log\left(\frac{V_t}{P_t Y_t}\right),$$

where P_t is the price level and Y_t is real GDP. Dividing by the price level makes the market value of debt a real variable, while dividing by real GDP makes it stationary.

We next define the nominal return on the portfolio of government debt, which captures how bond-price changes between the end of period t and the beginning of period $t+1$ affect the value of the debt held between periods. The holding-period return is

$$R_{t+1}^{\text{nom}} \equiv \frac{M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)}}{M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)}},$$

and its logarithm is $r_{t+1}^{\text{nom}} \equiv \log(R_{t+1}^{\text{nom}})$. The denominator is the end-of-period-t market value of all government liabilities held into the next period. The numerator is the market value of the same portfolio once each bond is one period closer to maturity.

Following Hall and Sargent (2011) and Cochrane (2019), we stress the importance of using the market value of debt rather than its commonly reported book value. Market values ensure that discount-rate variation is priced in because bond prices embed time-varying real yields and risk premia. Ignoring these movements would mechanically attribute changes in debt-to-GDP to prospective surpluses or growth. It would also omit capital gains and losses within the debt portfolio, so the period-by-period government budget identity would not hold without additional valuation adjustments.

The nonlinear government flow identity is

$$M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)} = P_{t+1} s_{t+1}^{\text{level}} + M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)}, \tag{A.1}$$

where s_{t+1}^{level} is the real primary surplus and therefore does not include interest payments. At the beginning of period $t+1$, money M_t and bonds $B_t^{(t+1+j)}$ are outstanding. Equation (A.1) is an exact accounting identity. Following Campbell and Shiller (1988) and Cochrane (2019), we log-linearise it to

$$\rho v_{t+1} = v_t + r_{t+1}^{\text{nom}} - \pi_{t+1} - g_{t+1} - s_{t+1}, \tag{A.2}$$

where ρ is a linearisation constant, v_t is the log ratio of the market value of debt to GDP at the end of period t, r_{t+1}^{nom} is the log nominal return on the portfolio of government bonds, π_{t+1} is inflation, g_{t+1} is log real GDP growth, and s_{t+1} is the scaled counterpart of the real primary surplus s_{t+1}^{level}. In the empirical construction, we back out s_{t+1} as the residual that makes equation (A.2) hold period by period:

$$s_{t+1} = v_t + r_{t+1}^{\text{nom}} - \pi_{t+1} - g_{t+1} - \rho v_{t+1}.$$

This ensures exact accounting within the linearised specification, and it puts mark-to-market changes in the debt stock into the recovered surplus rather than into a statistical discrepancy. We therefore interpret s_{t+1} as the primary surplus needed to validate the market value of government debt.

A.2 The Hegemon/Foreign Convention

Throughout, $X \in \{H, F\}$, where H denotes the U.S. financial Hegemon and F denotes the Foreign economy, which we take to be the U.K. Applying equation (A.2) separately to the two countries gives

$$\rho_H v_{t+1}^H = v_t^H + r_{t+1}^{\text{nom},H} - \pi_{t+1}^H - g_{t+1}^H - s_{t+1}^H, \tag{A.3}$$

and

$$\rho_F v_{t+1}^F = v_t^F + r_{t+1}^{\text{nom},F} - \pi_{t+1}^F - g_{t+1}^F - s_{t+1}^F. \tag{A.4}$$

Let $e_t \equiv \log E_t$ denote the log nominal exchange rate, with an increase corresponding to a depreciation of the Hegemon currency and an appreciation of the Foreign currency. We relate the two nominal holding-period returns through a return wedge

$$r_{t+1}^{\text{nom},H} - r_{t+1}^{\text{nom},F} = \Delta e_{t+1} - \xi_{t+1}, \tag{A.5}$$

where ξ_{t+1} is the return wedge: the realised excess return on gilts over Treasuries in a common currency. It may contain predictable risk-premium or convenience-yield components, as well as revaluations from changes in long-term yields. Substituting this into equation (A.4) yields

$$\rho_F v_{t+1}^F = v_t^F + r_{t+1}^{\text{nom},H} - \Delta e_{t+1} + \xi_{t+1} - \pi_{t+1}^F - g_{t+1}^F - s_{t+1}^F. \tag{A.6}$$

Equation (A.6) shows that U.S. discount-rate movements, exchange-rate revaluations, and movements in the return wedge enter the Foreign government debt valuation equation on the same footing as domestic inflation, growth, and primary surpluses. This is the sense in which changes in Hegemon bond returns can matter for the value of other-country government debt.

A.3 Recovering the Deficit Innovations

All components of equations (A.3) and (A.6), apart from the surplus terms, are directly observable or constructed from observable bond prices and quantities. We therefore recover the implied surplus series as

$$s_{t+1}^H = v_t^H + r_{t+1}^{\text{nom},H} - \pi_{t+1}^H - g_{t+1}^H - \rho_H v_{t+1}^H,$$

and

$$s_{t+1}^F = v_t^F + r_{t+1}^{\text{nom},H} - \Delta e_{t+1} + \xi_{t+1} - \pi_{t+1}^F - g_{t+1}^F - \rho_F v_{t+1}^F.$$

The corresponding market-value-debt-consistent deficit measures are $-s_t^X$ for $X \in \{H, F\}$. Iterating the Foreign flow identity from a base period and setting $\rho_F \simeq 1$ for notational simplicity gives

$$v_t^F = v_0^F + \sum_{j=1}^{t} \left(r_j^{\text{nom},H} - \Delta e_j + \xi_j \right) - \sum_{j=1}^{t} \pi_j^F - \sum_{j=1}^{t} g_j^F - \sum_{j=1}^{t} s_j^F.$$

This is an accounting attribution and thus *not* a test of the present-value relation. The orange deficit line in figure 1 is $v_0^F - \sum_{j=1}^{t} s_j^F$, so its period-by-period change satisfies

$$\left(v_0^F - \sum_{j=1}^{t} s_j^F \right) - \left(v_0^F - \sum_{j=1}^{t-1} s_j^F \right) = -s_t^F.$$

To isolate the unanticipated component of this measure, we use the innovation from the same finite-order VAR that underlies the forward decomposition below. For each variable x_t, define

$$\check{x}_t \equiv x_t - \mathbb{E}_{t-1}[x_t].$$

The surplus innovations underlying the empirical exercise are therefore

$$\check{s}_t^F = s_t^F - \mathbb{E}_{t-1}[s_t^F], \qquad \check{s}_t^H = s_t^H - \mathbb{E}_{t-1}[s_t^H]. \tag{A.7}$$

The deficit innovations are $-\check{s}_t^F$ for the U.K. and $-\check{s}_t^H$ for the U.S. When the plotted cumulative deficit path is formed from these deficit innovations, its period-by-period difference is exactly the corresponding deficit innovation. This distinction separates the market-value-consistent deficit measure $-s_t^X$ from its unpredictable component $-\check{s}_t^X$.

A.4 Forward Decomposition and the VAR

The backward decomposition accounts for realised debt dynamics. The forward-looking present-value identity instead puts expected future surpluses, discount rates, inflation, growth, exchange-rate movements, and the return wedge into a single valuation equation.

Iterating equation (A.6) forward and imposing the usual no-bubble condition gives

$$v_t^F = \mathbb{E}_t\left[\sum_{j=1}^{\infty} \rho_F^{j-1} s_{t+j}^F\right] + \mathbb{E}_t\left[\sum_{j=1}^{\infty} \rho_F^{j-1} g_{t+j}^F\right] - \mathbb{E}_t\left[\sum_{j=1}^{\infty} \rho_F^{j-1} r_{t+j}^{\text{nom},H}\right] + \mathbb{E}_t\left[\sum_{j=1}^{\infty} \rho_F^{j-1} \Delta e_{t+j}\right] - \mathbb{E}_t\left[\sum_{j=1}^{\infty} \rho_F^{j-1} \xi_{t+j}\right] + \mathbb{E}_t\left[\sum_{j=1}^{\infty} \rho_F^{j-1} \pi_{t+j}^F\right]. \tag{A.8}$$

Denote the six present values, before applying the displayed signs, by $\mathrm{PV}_t^{(s)}$, $\mathrm{PV}_t^{(g)}$, $\mathrm{PV}_t^{(r)}$, $\mathrm{PV}_t^{(\Delta e)}$, $\mathrm{PV}_t^{(\xi)}$, and $\mathrm{PV}_t^{(\pi)}$, respectively. Equation (A.8) can then be written as

$$v_t^F = \mathrm{PV}_t^{(s)} + \mathrm{PV}_t^{(g)} + \mathrm{PV}_t^{(\pi)} - \mathrm{PV}_t^{(r)} + \mathrm{PV}_t^{(\Delta e)} - \mathrm{PV}_t^{(\xi)}.$$

We estimate these expected components with a VAR(1), following Cochrane (2019, 2022b). The vector x_t contains the nominal holding-period return, real growth, inflation, the implied primary surplus, the market-value debt ratio, the real exchange-rate change Δq_t, the return wedge, and yield-curve and cyclical predictors: Bank Rate and the 10-year gilt yield for the U.K., and the output gap and the 10-year Treasury yield for the U.S. The VAR is specified as

$$x_{t+1} = A x_t + \varepsilon_{t+1}, \qquad \varepsilon_{t+1} \sim \mathcal{N}(0, \Sigma). \tag{A.9}$$

The valuation identity uses nominal exchange-rate changes. Under $Q_t = E_t P_t^F / P_t^H$, these satisfy $\Delta e_t = \Delta q_t + \pi_t^H - \pi_t^F$. Recovering the nominal component $\mathrm{PV}_t^{(\Delta e)}$ from the real component $\mathrm{PV}_t^{(\Delta q)}$ therefore requires forecasts of the inflation differential for the matching exchange-rate quotation and price baskets. For a vector a_k selecting a component represented in the VAR, the discounted expected contribution of component k is

$$\mathrm{PV}_t^{(k)} = a_k' A (I - \rho_F A)^{-1} x_t, \qquad k \in \{s, g, r, \pi, \Delta q, \xi\}.$$

This representation also delivers the one-step-ahead innovations used in equation (A.7). The same statistical model therefore produces both the expected paths entering the forward decomposition and the unpredictable deficit component entering the local projections.

Figure A.1 shows that expected surpluses and inflation contribute materially to the valuation of U.K. government debt, while the discount-rate, exchange-rate, and return-wedge components account for periods in which looking only at expected fiscal flows alone would imply too low or too high a value.

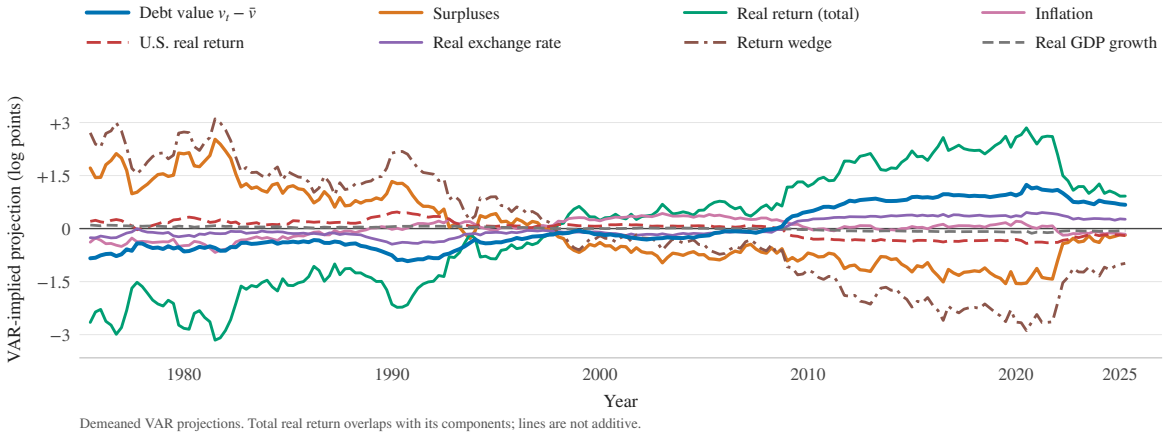


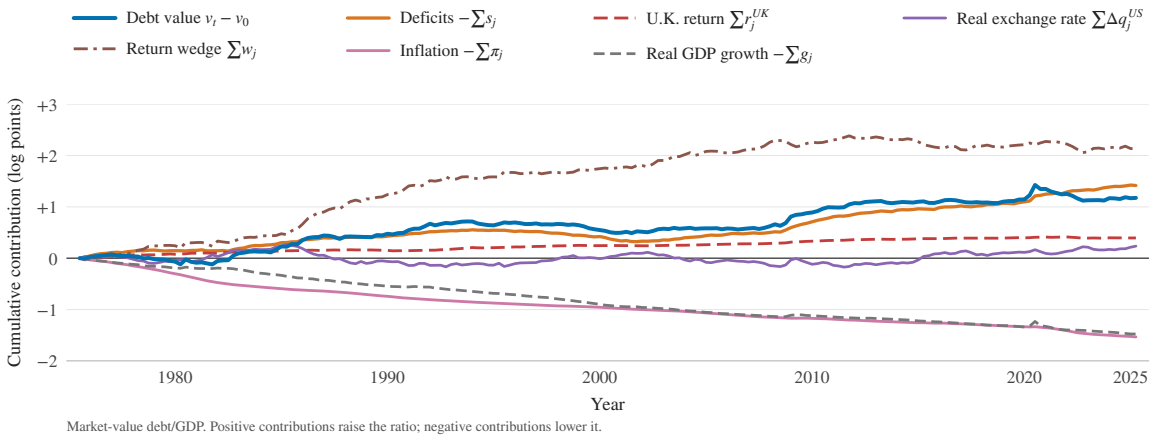
Figure A.1: Forward decomposition of the value of U.K. government debt.

Curve (legend label)	Algebraic object	Economic meaning
Debt value $v_t - \bar{v}$	v_t^F	Market-value debt-to-GDP ratio.
Surpluses	$\mathrm{PV}_t^{(s)}$	Present value of future primary surpluses.
Real return (total)	$-\mathrm{PV}_t^{(r)} + \mathrm{PV}_t^{(\pi)}$	Present value of future real discount rates.
U.S. real return	$\mathrm{PV}_t^{(r)}$	Present value of future U.S. real returns.
Inflation	$\mathrm{PV}_t^{(\pi)}$	Present value of future U.K. inflation.
Real exchange rate	$\mathrm{PV}_t^{(\Delta q)}$	Present value of future real exchange-rate changes.
Return wedge	$\mathrm{PV}_t^{(\xi)}$	Present value of future return wedges.
Real GDP growth	$\mathrm{PV}_t^{(g)}$	Present value of future real growth.

Table A.1: Mapping of figure A.1 to the components of the forward decomposition.

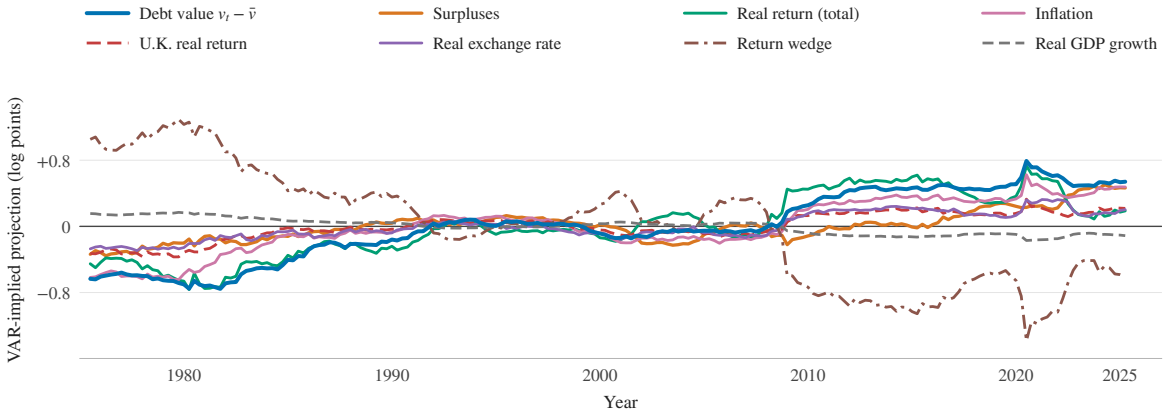
A.5 Backward and Forward Decompositions for the U.S.

Figures A.2 and A.3 show the corresponding backward and forward decompositions of the value of U.S. government debt, constructed as in equations (3) and (A.9).



Market-value debt/GDP. Positive contributions raise the ratio; negative contributions lower it.

Figure A.2: Backward decomposition of the market value of U.S. government debt.



Demeaned VAR projections. Total real return overlaps with its components; lines are not additive.

Figure A.3: Forward decomposition of the value of U.S. government debt.

B Robustness of the Empirical Deficit Estimates

B.1 Comparison with Official Forecast Revisions

Official forecast revisions concern expected deficits, including their predictable part, so we compare them with the implied deficit flow from Section 2.1 rather than with its unpredictable component. The comparison shows how far the recovered measure moves with independently published revisions to expected borrowing.

The official forecasts come from the Office for Budget Responsibility and, earlier in the sample, HM Treasury for the U.K., and from the Congressional Budget Office for the U.S. We compare projections for the same fiscal years across successive forecast vintages. For the U.S., these are primary surplus projections, excluding interest expenditure. For the U.K., we use headline public sector net borrowing (PSNB), including debt interest. The U.K. comparison therefore reflects revisions to expected borrowing that may arise from both the primary fiscal position and debt servicing costs.

Let T_v denote the fiscal year in progress at the publication date of forecast vintage v, and let $\widehat{s}^{v}_{T_v+j}$ denote the surplus projected for fiscal year $T_v + j$. For each consecutive pair of forecast vintages, we construct a discounted sum of revisions to the same fiscal-year targets as

$$Z_v = -\sum_{j \in \mathcal{J}_v} \bar{D}_j \left(\widehat{s}^{v}_{T_v+j} - \widehat{s}^{v-1}_{T_v+j} \right), \qquad \bar{D}_j = (1 + r_D)^{-j}. \tag{B.1}$$

We fix $r_D = 0.03$, so that positive values of Z_v indicate an upwards revision to expected deficits. The set $\mathcal{J}_v$ contains horizons between zero and ten for which the same fiscal-year target is observed in both vintages. Targets absent from either vintage are excluded. The fiscal-year origin is determined by the October start of the U.S. fiscal year and the April start of the U.K. fiscal year. We scale the nominal revision sum by the market value of government debt in the later forecast-vintage quarter and multiply by 100, so that the two measures are in comparable units. For the U.S., $\widehat{s}$ is the primary surplus forecast. For the U.K., it is the negative of forecast headline PSNB. Thus, eleven fiscal years define the maximum discounting window, while the number of observed revisions within that window varies across vintage pairs.

For both countries, let f^X_t denote the valuation-implied quarterly deficit flow expressed as a fraction of the market value of government debt, where $X \in \{H, F\}$ identifies the country. In the raw-flow comparison, we make use of the one-quarter-lagged measure $x^X_t = 100 f^X_{t-1}$. We sum this measure over quarters after the earlier forecast-vintage quarter up to and including the later forecast-vintage quarter. We retain intervals spanning one to

eight quarters and require every quarterly flow in the interval to be observed.

We also construct a discounted AR(8)-based transformation of the contemporaneous flow as

$$\mathrm{PV}_t^X = 100 \sum_{j=0}^{43} \beta^j \widehat{f}_{t+j|t}^X, \qquad \beta = (1 + r_D)^{-1/4}. \tag{B.2}$$

Here, the predictions $\widehat{f}_{t+j|t}^X$ are constructed from the flow's full-sample mean and the estimated AR(8) lag coefficients.[19] The upper panels plot the difference in this transformation between the later and earlier forecast-vintage quarters. The AR(8) coefficients and mean are estimated over the full available quarterly series, so this is an ex-post transformation of the measure. The 44-quarter horizon is rolling with each date, while Z_v compares forecasts for fixed fiscal-year targets observed in both vintages. The comparison therefore concerns the co-movement of two differently constructed fiscal measures.

Figures B.1 and B.2 compare the official forecast revisions with the recovered deficit measure from Section 2 for the U.K. and the U.S., respectively. Our discounted series (orange line) moves in the same direction as some of the larger official forecast revisions, including parts of the GFC and Covid episodes, although there are also substantial differences. These differences may reflect fiscal coverage, forecast horizons, the timing of information, and the construction of the measures.

To look more directly at this relation, table B.1 relates contemporaneous and subsequent Congressional Budget Office (CBO) revisions to our valuation-based measure. The estimated coefficients on our measure are positive, but none is statistically significant at the reported levels, including the contemporaneous comparison in column (1). Column (4), which also conditions on the contemporaneous CBO revision, likewise provides little evidence of incremental predictive information about the next published revision. Thus, the regressions do not establish that our measure anticipates subsequent CBO revisions. Together with the figures, they point to some common episodes of fiscal "stress" that are clearly identified by the VAR-based innovations used in our empirical exercise.

B.2 Spillovers under Other Fiscal Shock Measures

To check that our deficit innovations capture a useful dimension of fiscal developments, we also examine two established measures of fiscal shocks and whether they generate similar international responses. These measures differ from our headline innovations in

[19] For numerical stability, the implementation rescales the companion matrix to a largest-root modulus of 0.98 whenever its largest-root modulus is at least 0.999. The transformation uses the resulting matrix.

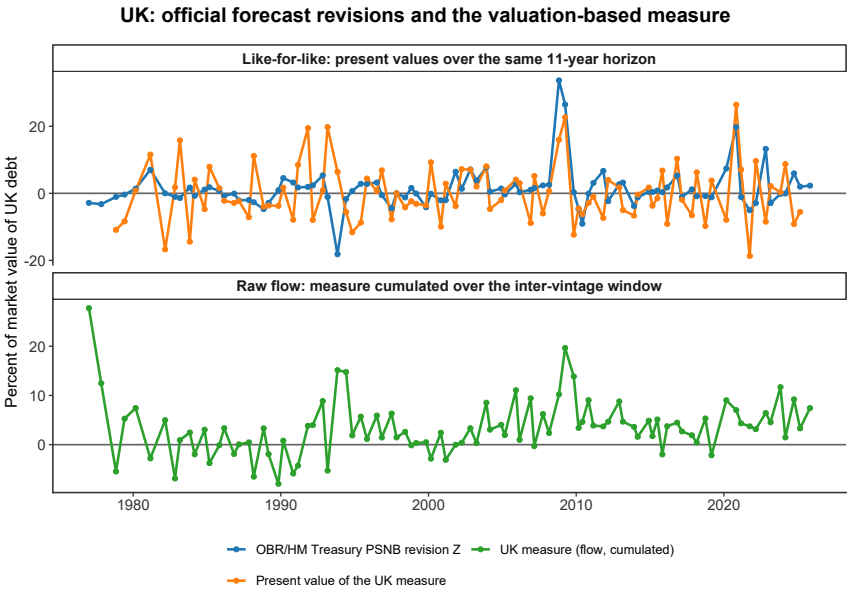


Figure B.1: A comparison of U.K. official borrowing forecast revisions from the U.K.'s Office for Budget Responsibility /HM Treasury with the valuation-based deficit measure from Section 2, expressed as percent of the market value of U.K. government debt. The upper panel compares the official revision measure with the change in the 44-quarter AR(8)-based present value of contemporaneous deficit flows between successive forecast-vintage quarters. The lower panel sums the one-quarter-lagged deficit flow over quarters after the earlier forecast-vintage quarter up to and including the later forecast-vintage quarter. Official revisions use fiscal-year targets observed in both vintages within a maximum eleven-fiscal-year window.

the type of fiscal disturbance that they capture, so we primarily compare the direction and timing of the responses. In the following figures, each fiscal measure is divided by its own standard deviation over the displayed shock-date sample. The impulses therefore correspond to one-standard-deviation expansions within each exercise, with their economic size depending on the fiscal measure and sample considered.

The specifications retain lagged U.K. real GDP growth and unemployment, U.S. inflation and the output gap, the Treasury convenience-yield basis, and the relevant lagged fiscal measures. For the price-level responses, we additionally include the lagged level and change of the respective U.K. price index and use U.S. CPI inflation for CPI and U.S. PPI inflation for PPI. The outcomes are cumulative log changes from the quarter before the fiscal impulse to each response horizon. The sample dates shown in the figures refer to the fiscal impulses, while the observations available at each horizon also depend on the availability of the corresponding outcome. Shaded bands are pointwise 68% and 90%

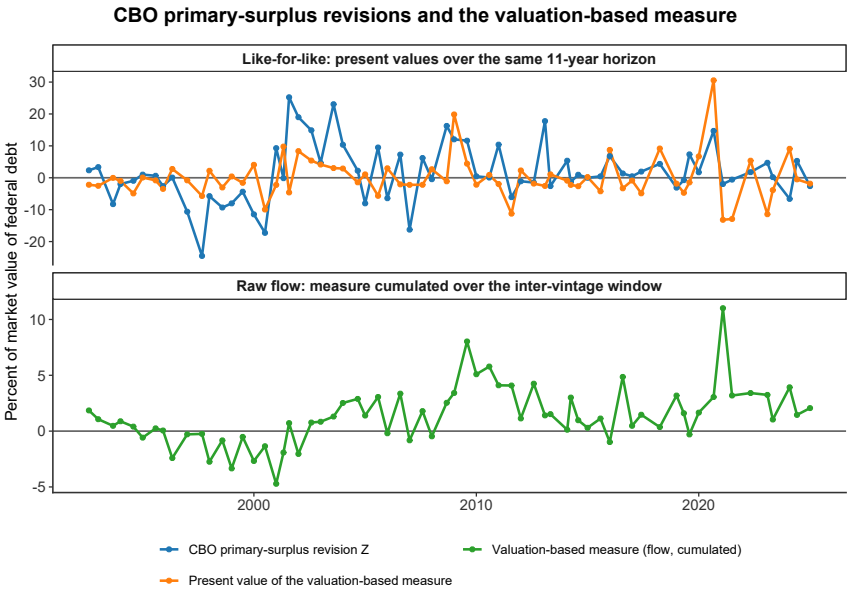


Figure B.2: A comparison of U.S. primary surplus forecast revisions from the CBO with the valuation-based deficit measure from Section 2, expressed as percent of the market value of federal debt. The upper panel compares the official revision measure, oriented so that positive values indicate larger expected deficits, with the change in the 44-quarter AR(8)-based present value of contemporaneous deficit flows between successive forecast-vintage quarters. The lower panel sums the one-quarter-lagged deficit flow over quarters after the earlier forecast-vintage quarter up to and including the later forecast-vintage quarter. Official revisions use fiscal-year targets observed in both vintages within a maximum eleven-fiscal-year window.

intervals.

B.2.1 Military Spending News

We first consider the military news series of Ben Zeev et al. (2023), normalised by lagged nominal trend GDP, which captures news about future military spending. The denominator is the lagged product of the GDP deflator and trend real GDP, and quarters with zero military news remain in the sample. For this exercise, we use the relevant modern sample from 1992Q4 to 2015Q4. Since this measure identifies U.S. fiscal news, we retain the contemporaneous and lagged U.K. headline deficit measure as additional controls, together with lagged U.S. military news. Figure B.3 shows that the responses of U.K. CPI and PPI are generally positive and increase over the medium run, which is directionally consistent with the price spillovers in the headline exercise. The response of the real exchange rate is considerably less precise. The point estimates indicate a depreciation of the dollar

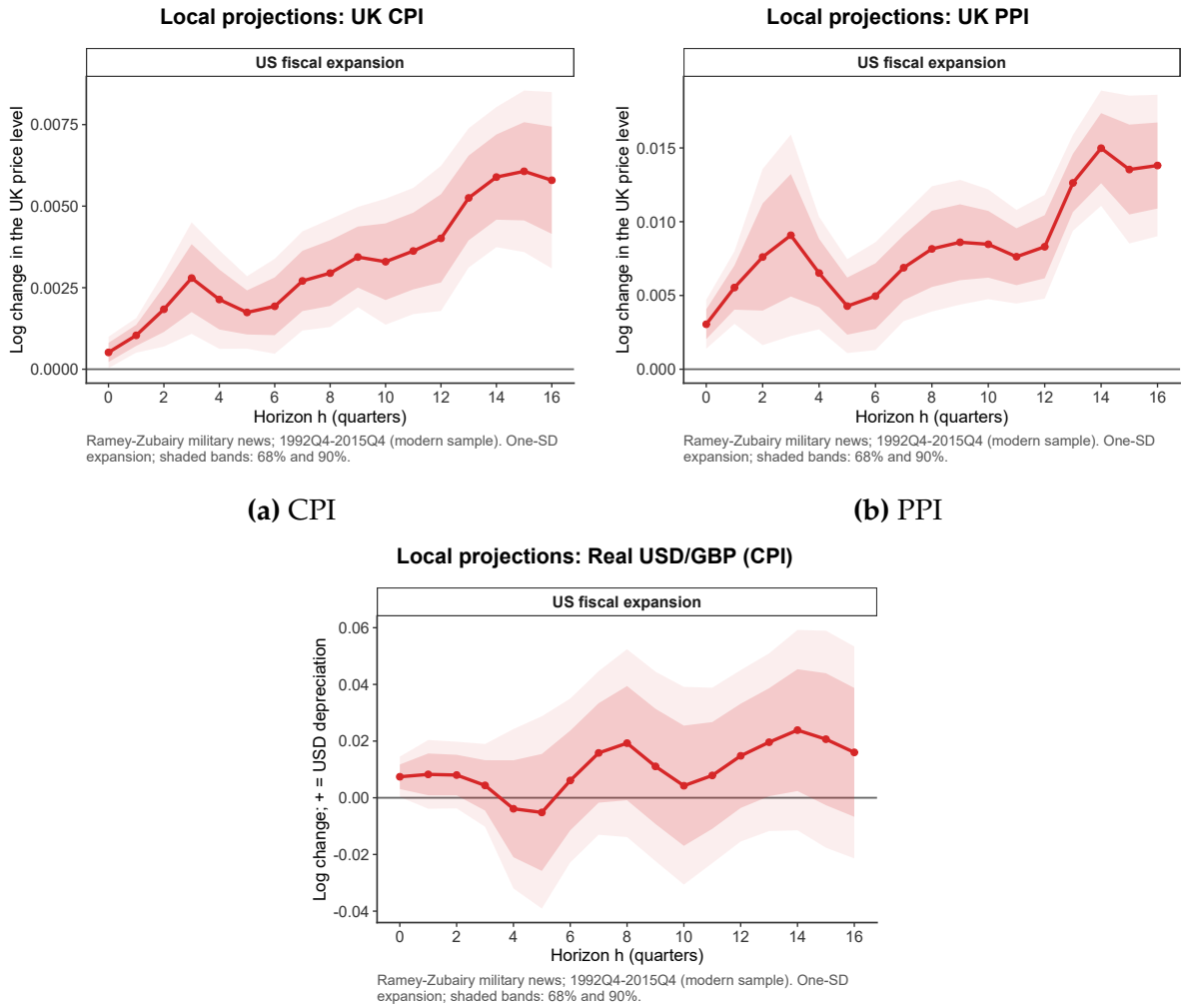
	Dependent variable:			
	Z_v	Z_{v+1}	Z_{v+1}^{long}	Z_{v+1}
	(1)	(2)	(3)	(4)
Model-deficit	0.658	0.271	0.110	0.095
	(0.513)	(0.554)	(0.459)	(0.478)
Z_v				0.264*
				(0.147)
Constant	0.613	1.124	0.660	0.947
	(1.964)	(2.193)	(1.740)	(1.764)
Observations	67	66	66	66
R^2	0.035	0.006	0.001	0.073
Adjusted R^2	0.020	−0.010	−0.014	0.043
Note:	*p<0.1; **p<0.05; ***p<0.01			

Table B.1: Regressing contemporaneous and subsequent published CBO revisions on the valuation-based measure cumulated over the current inter-vintage window. Here, v indexes successive retained CBO vintage-pair observations. Z_v is the contemporaneous revision measure, Z_{v+1} is the revision measure in the next retained observation, and Z_{v+1}^{long} restricts that next observation's revisions to available horizons $j = 2, \ldots, 10$, measured from the fiscal year in progress at its later publication date and discounted at a rate of 3% p.a. Column (4) also conditions on the contemporaneous CBO revision. Standard errors are robust against heteroskedasticity and autocorrelation. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

against sterling at many horizons, but the response temporarily changes sign and the confidence bands are wide. Thus, the military-news measure provides clearer support for the price spillover than for a persistent exchange-rate response. The fiscal impulses in this comparison end in 2015Q4, so the positive price responses arise from military news dated before Brexit and Covid.

B.2.2 Blanchard–Perotti Spending Shocks

For Blanchard and Perotti (2002)-style shocks, we can obtain both U.K. and U.S. fiscal innovations, which allows us to compare domestic and international responses within the same exercise. The displayed figures use quarterly general-government consumption innovations without a foresight correction. For both countries, we use a recursive VAR with four lags, log real government consumption ordered first, followed by log real GDP and the log GDP deflator, with an intercept and a quadratic trend. The spending measure



(a) CPI

(b) PPI

(c) Real exchange rate

Figure B.3: Local projections relating U.S. military news shocks from Ben Zeev et al. (2023), normalised by lagged nominal trend GDP, to U.K. CPI and PPI price levels and the CPI-adjusted USD/GBP real exchange rate. The shock-date sample is 1992Q4–2015Q4 and the impulse is a one-standard-deviation expansion within this sample. The specifications retain contemporaneous and lagged U.K. headline deficit measures and lagged U.S. military news alongside the macroeconomic controls. Responses are cumulative log changes through sixteen quarters. Positive exchange-rate responses indicate a real depreciation of the dollar against sterling. Shaded areas indicate pointwise 68% and 90% confidence bands.

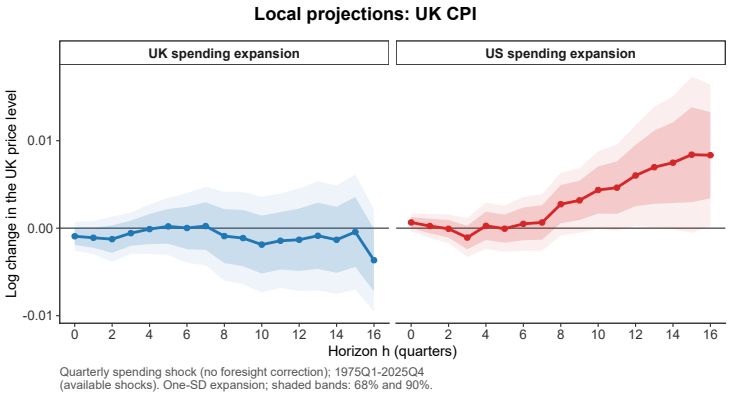
excludes government investment and transfers in both countries. We enter the U.K. and U.S. spending innovations jointly in the local projections, together with their one-quarter lags and the controls described above. In the full sample from 1975Q1 to 2025Q4, figure B.4 shows that U.S. spending expansions are followed by positive U.K. CPI and PPI responses at medium and longer horizons, alongside a real depreciation of the dollar against sterling.

These responses preserve part of the headline U.S. spillover pattern, although their timing differs. The U.K. responses, however, are somewhat less consistent with the headline exercise. The CPI response is weak, the PPI response varies substantially across horizons, and the later exchange-rate response indicates a possible real appreciation of sterling. Thus, the overall degree of agreement with our headline results depends on both the country of origin of the fiscal innovation and the outcome considered. That being said, we view this type of Blanchard and Perotti (2002)-style shock as generally relatively uninformative for the unfunded deficits we are after: since the recovered shocks merely identify spending expansions, their interpretation differs significantly from our headline estimates, as we effectively lack control over the associated revenue response.

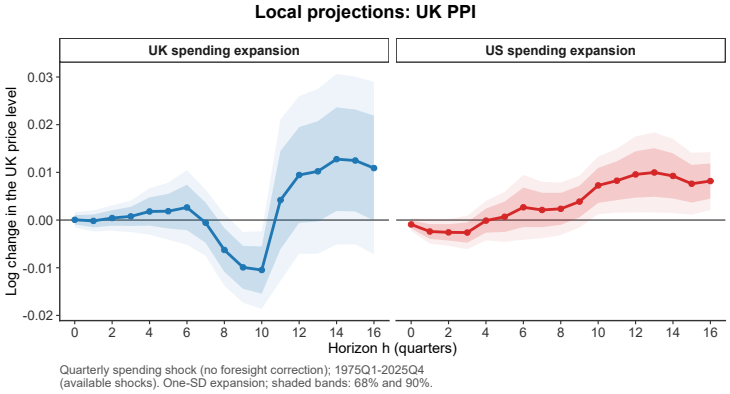
C Additional Local Projection Results

We now look at the effects of U.K. and U.S. deficit innovations from Section 2.2 on exchange rates and price levels, varying the controls and outcome variables in equation (4). Our minimal set of controls contains one-quarter lags of both deficit innovations, U.K. real GDP growth and U.K. unemployment. For price levels, it also contains the lagged level and change of the respective price index. Figure C.1 shows the effects of deficit innovations on the *nominal* USD/GBP exchange rate with this minimal set of controls. Our headline result that deficits depreciate the issuing country's currency broadly persists, although the effect of U.K. deficits is imprecisely estimated. Figure C.2, which uses the main-text controls, gives a similar picture. Figure C.3 shows the response of the CPI-adjusted real exchange rate, where the main-text result again holds, although less precisely. Finally, Figures C.4 and C.5 show that the spillover from U.S. deficits to U.K. prices generally holds with the minimal set of controls, whether the price level is measured by the CPI or the PPI.

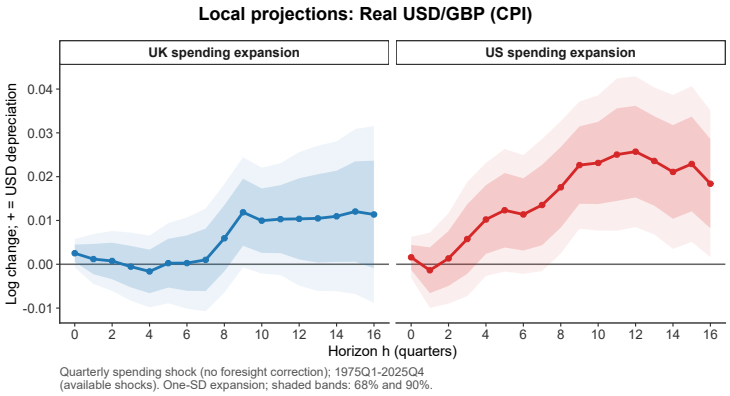
Finally, figures C.6 and C.7 report the full set of borrowing-cost responses behind figure 3: the returns on both bond portfolios together with the return wedge ξ, and the 10-year yields of both countries together with their spread.



(a) CPI



(b) PPI



(c) Real exchange rate

Figure B.4: Local projections relating U.K. and U.S. quarterly general-government consumption innovations to U.K. CPI and PPI price levels and the CPI-adjusted USD/GBP real exchange rate. Shocks use a consumption-first recursive VAR(4), without a foresight correction, and enter the local projections jointly. The shock-date sample is 1975Q1–2025Q4 and the impulses are one-standard-deviation expansions within this sample. Responses are cumulative log changes through sixteen quarters. Positive exchange-rate responses indicate a real depreciation of the dollar against sterling. Shaded areas indicate pointwise 68% and 90% confidence bands.

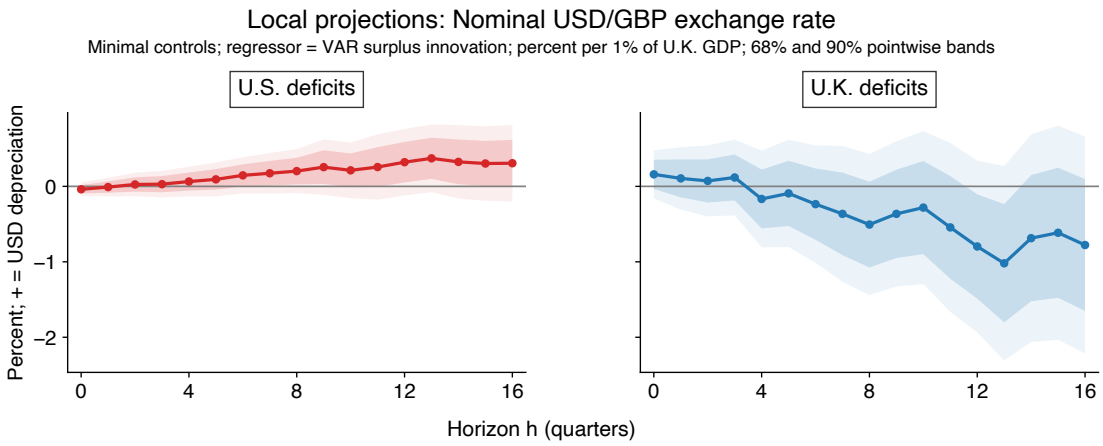


Figure C.1: Local projections of the nominal USD/GBP exchange rate on U.S. and U.K. deficit innovations from Section 2.2, using the minimal set of controls.

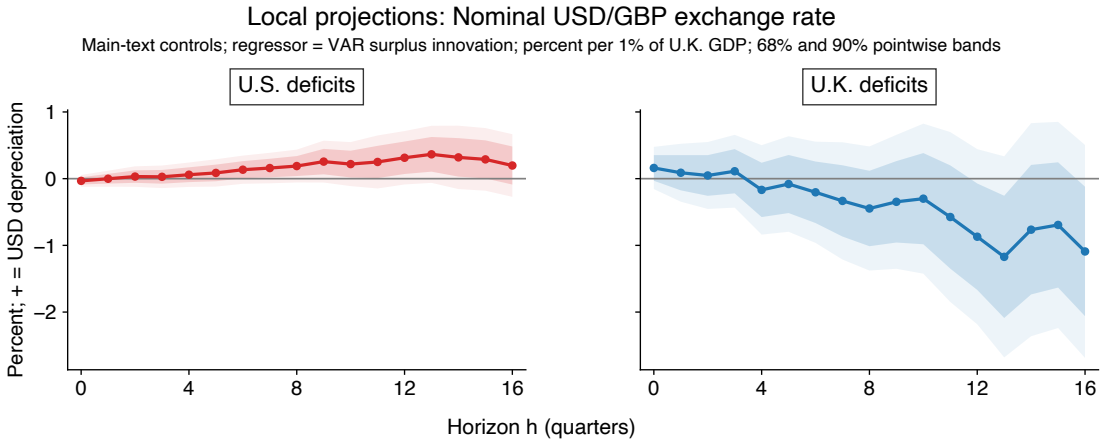


Figure C.2: Local projections of the nominal USD/GBP exchange rate on U.S. and U.K. deficit innovations from Section 2.2, using the same controls as in the main text.

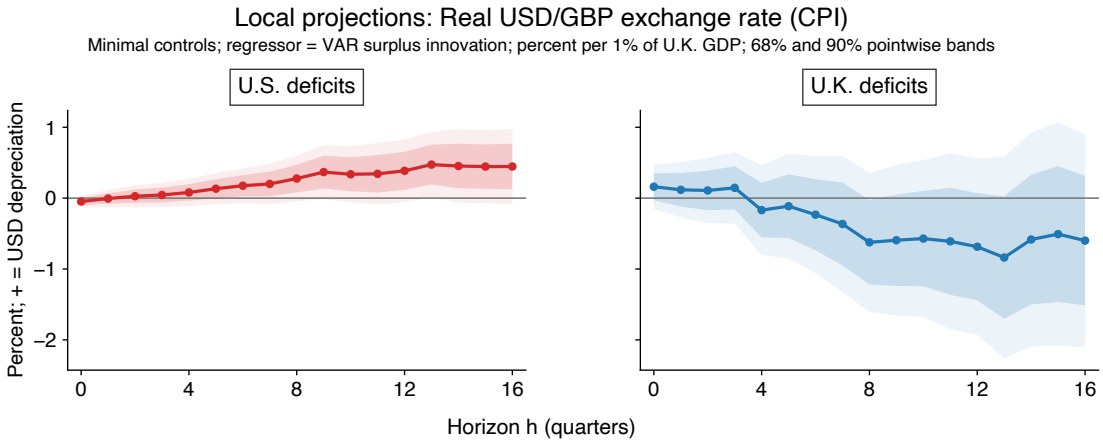


Figure C.3: Local projections of the CPI-adjusted real USD/GBP exchange rate on U.S. and U.K. deficit innovations from Section 2.2, using the minimal set of controls.

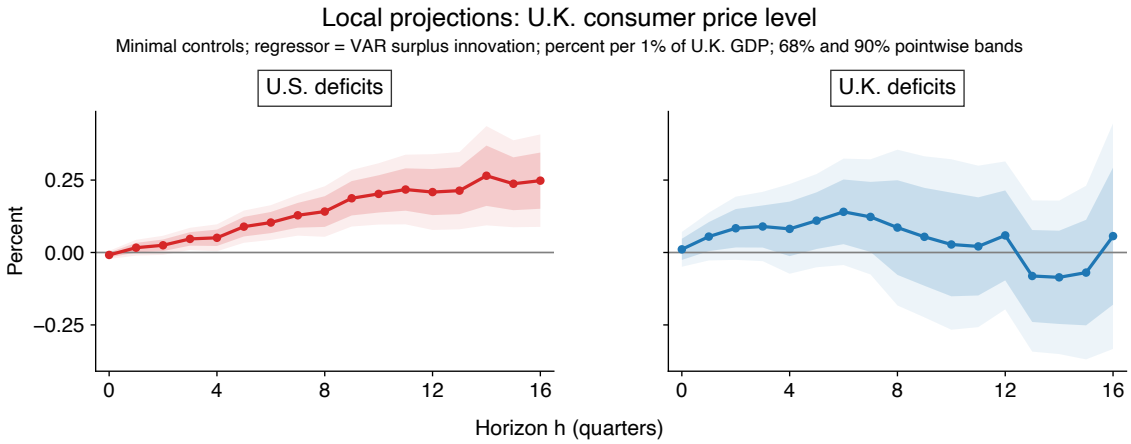


Figure C.4: Local projections of the U.K. CPI price level on U.S. and U.K. deficit innovations from Section 2.2, using the minimal set of controls, including the lagged level and change of the CPI.

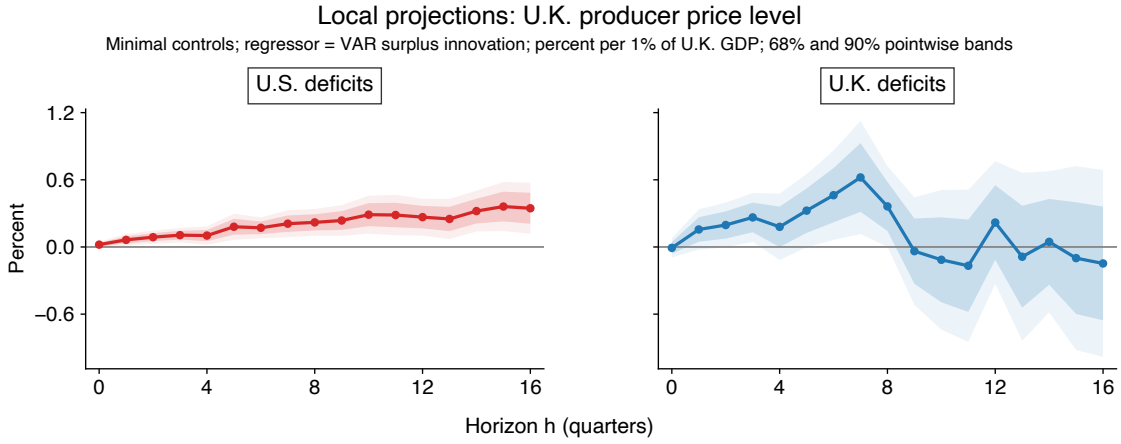


Figure C.5: Local projections of the U.K. PPI price level on U.S. and U.K. deficit innovations from Section 2.2, using the minimal set of controls, including the lagged level and change of the PPI.

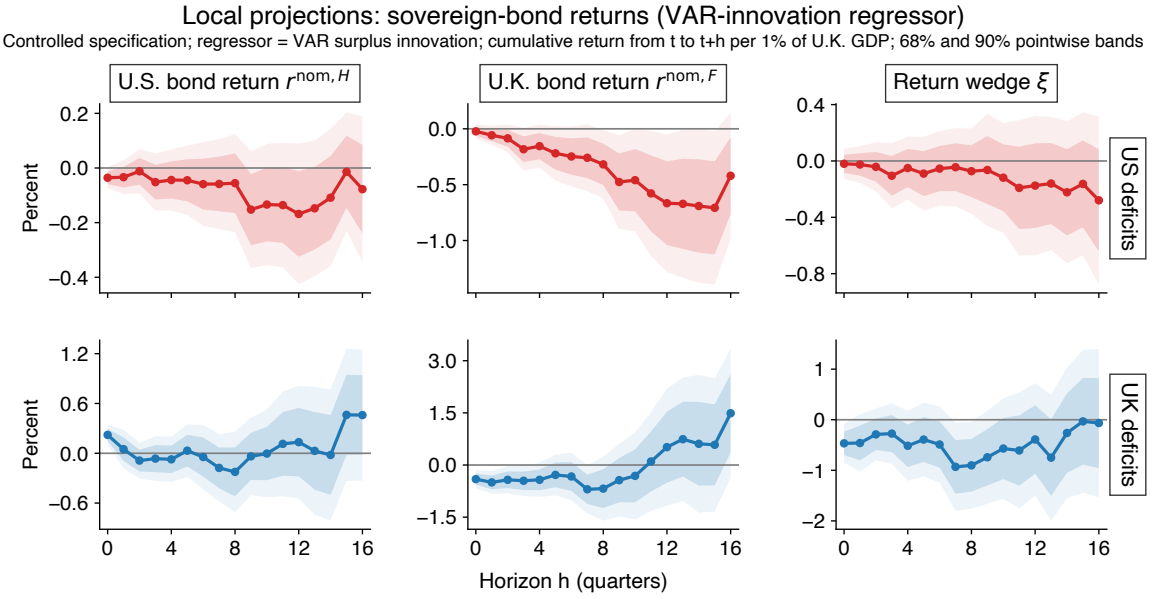


Figure C.6: The effect of U.S. and U.K. deficit innovations on the returns of the full sovereign bond portfolios. The return wedge ξ uses the bilateral nominal dollar–sterling exchange rate.

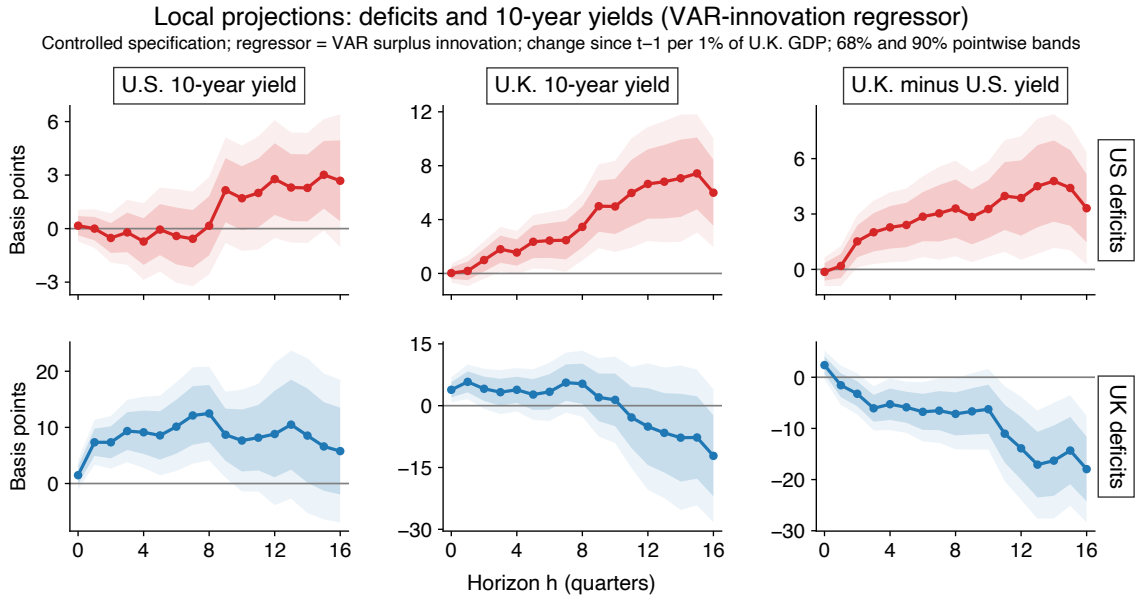


Figure C.7: The effect of U.S. and U.K. deficit innovations on 10-year sovereign yields in the U.S. and the U.K.

D Linking Deficits to Exchange Rates

Complementing Section 2.2, we ask whether changes in real exchange rates help predict realised deficit-to-debt ratios, conditional on recent domestic and foreign fiscal ratios. This exercise follows the predictive direction considered by Engel and West (2005), who examine whether exchange rates contain information about future fundamentals. The use of a fiscal ratio is also related to Jiang (2021), who studies the predictive relationship between the U.S. surplus-to-debt ratio and subsequent currency excess returns. We estimate the following horizon-specific regressions:

$$\begin{aligned}\frac{-s^X_{t+k}}{B^X_{t+k}} = \alpha^{(k)} + \beta^{(k)}\,\Delta q_t^{(-X)/X} + \delta_1^{(k)}\,\frac{-s_t^X}{B_t^X} + \delta_2^{(k)}\,\Delta\left(\frac{-s_t^X}{B_t^X}\right) \\ + \gamma_1^{(k)}\,\frac{-s_t^{-X}}{B_t^{-X}} + \gamma_2^{(k)}\,\Delta\left(\frac{-s_t^{-X}}{B_t^{-X}}\right) + \varepsilon^X_{t+k}, \qquad k = 1,\dots,8.\end{aligned} \tag{D.1}$$

Here $X \in \{H, F\}$ denotes the U.S. or the U.K., and $-X$ denotes the other country. s_t^X denotes the realised surplus flow and B_t^X the contemporaneous debt stock, expressed in the same units, so that $-s_t^X/B_t^X$ is the deficit-to-debt ratio. This local flow notation is different from the scaled surplus variable in equation (1), since the dependent variable is the realised fiscal ratio rather than the innovation $-\check{s}_t^X$ used in the main-text local projections. The log real exchange rate $q_t^{(-X)/X} \equiv \log Q_t^{(-X)/X}$ is the exchange-rate series used in the regression for country X, and $\Delta q_t^{(-X)/X} = q_t^{(-X)/X} - q_{t-1}^{(-X)/X}$. The horizon k is measured in quarters.

The coefficient $\beta^{(k)}$ measures the predictive association between the current exchange-rate change and the fiscal ratio k quarters later, conditional on the current level and first difference of both countries' fiscal ratios. These fiscal controls are equivalent to including their current and previous-quarter levels. The exercise measures conditional in-sample predictability using the observed series. Tables D.1 and D.2 report the results for the U.K. and the U.S.

The exchange-rate coefficient is significant at the 1 percent level for the U.K. at $k = 6$ and at the 5 percent level for the U.S. at $k = 8$, and insignificant at the 10 percent level at all other horizons. Ex-ante observed exchange-rate changes therefore carry little additional information about future deficits on their own.

	Dependent variable:							
	UK deficit-to-debt ratio at $t+k$							
	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
Δq_t	0.0062	−0.1116	0.0669	−0.1206	0.0018	−0.2155***	0.0425	0.0204
	(0.1239)	(0.0870)	(0.0797)	(0.1037)	(0.0917)	(0.0765)	(0.0907)	(0.0663)
$\frac{-s}{B}$-UK	0.0139	0.3992***	0.4582***	0.1726	0.2592**	0.2859***	0.0846	−0.1005
	(0.1231)	(0.1127)	(0.0952)	(0.1178)	(0.1079)	(0.0897)	(0.1012)	(0.1282)
$\Delta\frac{-s}{B}$-UK	−0.1029	−0.2804***	−0.1746**	−0.0363	−0.2173***	−0.0958	0.0046	0.0712
	(0.0660)	(0.0756)	(0.0682)	(0.0998)	(0.0629)	(0.0633)	(0.0691)	(0.0864)
$\frac{-s}{B}$-US	0.6419***	0.1597	0.0661	0.2564	0.2620	0.0625	0.2540	0.4345**
	(0.2341)	(0.1816)	(0.1980)	(0.2232)	(0.1740)	(0.1761)	(0.2239)	(0.2063)
$\Delta\frac{-s}{B}$-US	−0.2992	0.0266	−0.0856	−0.1075	−0.1951	0.0913	−0.2408	−0.1705
	(0.1823)	(0.1771)	(0.1506)	(0.1365)	(0.1675)	(0.1638)	(0.2247)	(0.1784)
Constant	0.0124***	0.0084***	0.0078***	0.0111***	0.0096***	0.0097***	0.0124***	0.0140***
	(0.0031)	(0.0026)	(0.0029)	(0.0030)	(0.0030)	(0.0034)	(0.0035)	(0.0036)
Observations	199	198	197	196	195	194	193	192
R^2	0.0617	0.1277	0.1241	0.0553	0.0788	0.1131	0.0241	0.0206
Adj. R^2	0.0374	0.1050	0.1011	0.0304	0.0544	0.0895	−0.0020	−0.0057

Table D.1: UK: Predictive regressions for the realised deficit-to-debt ratio. Newey-West standard errors in parentheses. Horizons are in quarters. *p<0.1, **p<0.05, ***p<0.01. Significance is reported separately for each coefficient and horizon.

	Dependent variable:							
	US deficit-to-debt ratio at $t+k$							
	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
Δq_t	0.0042	−0.0124	−0.0058	−0.0190	−0.0273	−0.0285	−0.0182	−0.0790**
	(0.0529)	(0.0390)	(0.0395)	(0.0358)	(0.0432)	(0.0412)	(0.0369)	(0.0386)
$\frac{-s}{B}$-UK	0.0172	0.0152	−0.0322	−0.0547	0.0053	−0.0269	−0.0588	−0.0916**
	(0.0437)	(0.0426)	(0.0397)	(0.0359)	(0.0362)	(0.0401)	(0.0395)	(0.0381)
$\Delta\frac{-s}{B}$-UK	−0.0039	−0.0145	0.0388	0.0175	−0.0156	0.0381	0.0192	0.0680***
	(0.0378)	(0.0240)	(0.0262)	(0.0238)	(0.0211)	(0.0274)	(0.0242)	(0.0220)
$\frac{-s}{B}$-US	0.4109***	0.4136***	0.5326***	0.4074***	0.2485**	0.3667***	0.4370***	0.4686***
	(0.0955)	(0.1008)	(0.1077)	(0.1310)	(0.1071)	(0.1051)	(0.1143)	(0.1248)
$\Delta\frac{-s}{B}$-US	−0.1766**	−0.2399***	−0.3205***	−0.0194	−0.2069***	−0.1982***	−0.2522***	−0.2103**
	(0.0779)	(0.0694)	(0.0904)	(0.0695)	(0.0766)	(0.0656)	(0.0966)	(0.0924)
Constant	0.0036***	0.0035***	0.0034**	0.0047***	0.0047***	0.0044***	0.0046***	0.0048***
	(0.0013)	(0.0013)	(0.0014)	(0.0018)	(0.0017)	(0.0017)	(0.0017)	(0.0018)
Observations	198	197	196	195	194	193	192	191
R^2	0.1226	0.1227	0.1757	0.1349	0.0555	0.0845	0.1053	0.1290
Adj. R^2	0.0998	0.0998	0.1541	0.1121	0.0304	0.0600	0.0812	0.1054

Table D.2: US: Predictive regressions for the realised deficit-to-debt ratio. Newey-West standard errors in parentheses. Horizons are in quarters. *p<0.1, **p<0.05, ***p<0.01. Significance is reported separately for each coefficient and horizon.

E Prior Distributions for the Bayesian Estimation

Parameter	Prior imposed on	Distribution	Mean	Std. Dev.	Support
Structural parameters					
ϕ_π	ϕ_π	Normal	1.00	0.1000	$[-0.5, 1.99]$
ϕ^*_π	ϕ^*_π	Normal	0.50	0.1000	$[-0.5, 1.99]$
ϕ_x	ϕ_x	Normal	0.00	0.2887	$[-0.6, 1]$
ϕ^*_x	ϕ^*_x	Normal	0.00	0.2887	$[-0.6, 1]$
ψ_b	ψ_b	Normal	-0.05	0.5887	$[-0.1, 0]$
ψ^*_b	ψ^*_b	Normal	-0.05	0.5887	$[-2, -0.5]$
$\bar{\Phi}$	$\bar{\Phi}$	Normal	0.0018	0.0005	$[10^{-5}, 0.0049]$
$\bar{\Phi}^*$	$\bar{\Phi}^*$	Normal	0.0013	0.0005	$[10^{-5}, 0.02]$
Continuous-time mean-reversion rates					
φ_a	$e^{-h\varphi_a}$	Beta	0.85	0.08	$[0.05, 0.98]$
φ^*_a	$e^{-h\varphi^*_a}$	Beta	0.85	0.08	$[0.05, 0.98]$
φ_θ	$e^{-h\varphi_\theta}$	Beta	0.50	0.18	$[0.05, 0.98]$
φ^*_θ	$e^{-h\varphi^*_\theta}$	Beta	0.50	0.18	$[0.05, 0.98]$
φ_s	$e^{-h\varphi_s}$	Beta	0.50	0.18	$[0.70, 0.99]$
φ^*_s	$e^{-h\varphi^*_s}$	Beta	0.50	0.18	$[0.70, 0.99]$
$\varphi_\varkappa$	$e^{-h\varphi_\varkappa}$	Beta	0.50	0.18	$[0.05, 0.98]$
Continuous-time shock diffusions					
σ_a	$c(\varphi_a)\,\sigma_a$	Inverse Gamma	0.50	0.30	$[10^{-5}, 300]$
σ^*_a	$c(\varphi^*_a)\,\sigma^*_a$	Inverse Gamma	0.50	0.30	$[10^{-5}, 300]$
σ_θ	$c(\varphi_\theta)\,\sigma_\theta$	Inverse Gamma	0.50	0.30	$[10^{-5}, 300]$
σ^*_θ	$c(\varphi^*_\theta)\,\sigma^*_\theta$	Inverse Gamma	0.50	0.30	$[10^{-5}, 300]$
σ_s	$c(\varphi_s)\,\sigma_s$	Inverse Gamma	0.50	0.30	$[10^{-5}, 300]$
σ^*_s	$c(\varphi^*_s)\,\sigma^*_s$	Inverse Gamma	0.50	0.30	$[10^{-5}, 300]$
$\sigma_\varkappa$	$c(\varphi_\varkappa)\,\sigma_\varkappa$	Inverse Gamma	0.50	0.30	$[10^{-5}, 300]$

Table E.1: Prior distributions for the continuous-time Bayesian estimation in Section 4.4, identical across the three Bayesian specifications. ϕ_π is centred on the Taylor threshold, ϕ^*_π on an accommodating U.K. rule, and $\bar{\Phi}$ and $\bar{\Phi}^*$ on convenience yields of about 0.7 and 0.5 percent a year on Treasuries for Hegemon and Foreign investors, respectively (Krishnamurthy and Vissing-Jorgensen, 2012; Du et al., 2018a). Priors on mean-reversion rates are imposed on the quarterly autocorrelations $e^{-h\varphi_j}$, and priors on diffusion parameters on the quarterly innovation standard deviations $c(\varphi_j)\sigma_j$, where h is the sampling interval (one quarter) and $c(\varphi) = \sqrt{[1 - e^{-2\varphi h}]/(2\varphi)}$. Means and standard deviations are those of the distributions before truncation. Each prior is truncated to the support in the last column (for mean-reversion rates, stated for $e^{-h\varphi_j}$; for diffusions, for σ_j).

F Robustness: Funded and Unfunded Deficits as in Bianchi, Faccini, and Melosi (2023)

Our headline policy specification yields a fiscally-led policy mix. To examine whether our mechanism of international fiscal transmission survives a distinction between funded and unfunded disturbances, we implement the policy decomposition of Bianchi et al. (2023). Monetary-led and fiscally-led policy mixes coexist according to the fiscal backing of individual shocks. We retain our two-country model, bilateral portfolios, convenience services, and non-policy calibration, and change the policy rules and fiscal shock processes.

Policy rules. Following Bianchi et al. (2023), we construct a shadow economy that receives only unfunded fiscal shocks. Every linearised variable is the sum of a funded component, denoted by M, and an unfunded component, denoted by U. For instance, $\hat{b}_t = \hat{b}_t^M + \hat{b}_t^U$ and $\hat{\pi}_t = \hat{\pi}_t^M + \hat{\pi}_t^U$. Both components are deviations around the same steady state. The shadow economy determines the fiscal inflation that central banks accommodate. The monetary rules become

$$\begin{aligned} i_t &= \bar{\imath} + \phi_\pi^M\left(\hat{\pi}_t - \hat{\pi}_t^U\right) + \phi_x x_t, \\ i_t^* &= \bar{\imath}^* + \phi_\pi^{M,*}\left(\hat{\pi}_t^* - \hat{\pi}_t^{U,*}\right) + \phi_x^* x_t^*. \end{aligned} \tag{F.1}$$

As in the quantitative rule of Bianchi et al. (2023), the inflation response applies to inflation net of fiscal inflation, while the output response applies to the total output gap. Thus $\hat{\imath}_t^U = \phi_x x_t^U$, and symmetrically abroad, so the accommodation of fiscal inflation still allows an interest-rate response to shadow output. Inflation in these rules is CPI inflation, as in equation (H.47). We retain the constant intercepts and contemporaneous feedback of our model, omitting the interest-rate smoothing and effective lower bound in their quantitative specification.

Let d_t^M and d_t^U denote positive deficit disturbances, so that $\epsilon_t^s = -d_t^M - d_t^U$. The debt-normalised surplus rules are

$$\begin{aligned} \hat{s}_t &= \psi_b^M\left(\hat{b}_t - \hat{b}_t^U\right) + \psi_b^U \hat{b}_t^U - d_t^M - d_t^U, \\ \hat{s}_t^* &= \psi_b^{M,*}\left(\hat{b}_t^* - \hat{b}_t^{U,*}\right) + \psi_b^{U,*} \hat{b}_t^{U,*} - d_t^{M,*} - d_t^{U,*}. \end{aligned} \tag{F.2}$$

Surpluses adjust sufficiently to back funded debt, with $\psi_b^M > \bar{r}$ and $\psi_b^{M,*} > \bar{r}^*$. For unfunded debt, our coefficients measure surplus responses per unit of steady-state debt and therefore differ in normalisation from their fiscal elasticities.

Each country has two independent fiscal processes. For $k \in \{M, U\}$,

$$\begin{aligned} dd_t^k &= -\varphi_d^k d_t^k \, dt + \sigma_d^k \, dW_{d^k,t}, \\ dd_t^{k,*} &= -\varphi_d^{k,*} d_t^{k,*} \, dt + \sigma_d^{k,*} \, dW_{d^{k,*},t}. \end{aligned} \tag{F.3}$$

The model has nine independent Brownian motions and 28 estimated parameters, six more than the headline specification. Table F.1 reports the estimation supports.

Parameter	Maximum likelihood	Bayesian
Funded inflation feedback, $\phi_\pi^M, \phi_\pi^{M,*}$	$[1.0001, 1.99]$	$[1.0001, 1.99]$
Unfunded inflation feedback, $\phi_\pi^U, \phi_\pi^{U,*}$	0 (fixed)	0 (fixed)
Output feedback, ϕ_x, ϕ_x^*	$[-0.6, 1]$	$[-0.6, 1]$
Funded debt feedback, $\psi_b^M, \psi_b^{M,*}$	$[0.00501, 2.01]$	$[0.00501, 2.01]$
U.S. unfunded debt feedback, ψ_b^U	$[-0.1, 0]$	$[-0.1, 0]$
U.K. unfunded debt feedback, $\psi_b^{U,*}$	$[-2, -0.5]$	$[-2, -0.5]$
Fiscal quarterly persistence (both funding types)	$[0.70, 0.99]$	$[0.70, 0.99]$
Other quarterly persistence parameters	$[0.05, 0.98]$	$[0.05, 0.98]$
All shock diffusion coefficients	$[10^{-5}, 10]$	$[10^{-5}, 300]$
U.S. convenience-yield level, $\bar{\Phi}$	$[0.0008, 0.0028]$	$[10^{-5}, 0.0049]$
U.K. convenience-yield level, $\bar{\Phi}^*$	$[0.0003, 0.0023]$	$[10^{-5}, 0.02]$

Table F.1: Parameter restrictions for the robustness exercise. Quarterly persistence is $\exp(-\varphi h)$, with $h = 1$ quarter. The funded debt-feedback lower bound exceeds both steady-state real rates, since $\rho = \rho^* = 0.005$ and $\bar{\Phi} > 0$. Unfunded inflation feedback is fixed and is not included among the 28 estimated parameters.

Estimation. We use the sample 1998Q1–2024Q4 and the same six macroeconomic observables as in the headline exercise. We run three data specifications: one which omits convenience-yield observations, one that includes our "main" convenience-yield measure (Du et al., 2018a), and one that includes an alternative convenience-yield measure (Fleckenstein and Longstaff, 2024). The funded and unfunded components are estimated jointly using the exact quarterly Gaussian likelihood of Christensen et al. (2024), with inflation measured as an interval integral and real debt growth as an endpoint difference.

Priors follow table E.1, subject to the following policy and convenience-yield specification and the supports in table F.1. Funded inflation feedback has a Normal prior with mean 0.98 and standard deviation 1 in each country. All four debt-feedback coefficients have underlying Normal priors with mean -0.05 and standard deviation 0.5887, truncated to their respective supports. Output-feedback priors are Normal with mean zero and standard deviation 0.2887. The priors for $\bar{\Phi}$ and $\bar{\Phi}^*$ are Normal with means 0.0018 and

0.0013, respectively, and standard deviation 0.0005 in both countries.

Each of the four fiscal quarterly persistences has an independent Beta prior with mean 0.50 and standard deviation 0.18, truncated to $[0.70, 0.99]$. Productivity persistence has a Beta prior with mean 0.85 and standard deviation 0.08, and the other non-fiscal persistences have Beta priors with mean 0.50 and standard deviation 0.18, all truncated to $[0.05, 0.98]$. Innovation-scale priors are inverse-gamma type 1 with mean 0.50 and standard deviation 0.30 on the quarterly innovation standard deviation. For an Ornstein–Uhlenbeck process, this standard deviation is $\sigma\sqrt{(1-\exp(-2\varphi h))/(2\varphi)}$. The prior transformation uses each process's own mean-reversion coefficient.

Table F.2 reports the posterior results for the two Bayesian specifications estimated with convenience-yield data. The reported estimates satisfy the local constrained first-order checks, but we do not make a claim of global optimality. Posterior moments use four chains of 250,000 iterations, discarding 50,000 warm-up iterations per chain and thus retaining 800,000 draws per specification.

For both specifications in table F.2, parameters, generated fiscal quantities, and historical and counterfactual quantities pass standard convergence requirements following Vehtari et al. (2021). In our historical and counterfactual diagnostics, we use every hundredth retained draw and exclude structurally zero quantities. The no-convenience-yield specification also passes the parameter and historical/counterfactual checks, but the Monte Carlo standard error for the U.K. total fiscal stationary variance is 5.125% of its posterior standard deviation. We therefore do not report this specification in detail in this appendix, but the results are very similar to the two specifications reported here.

Under the maintained restrictions, funded inflation feedback exceeds one and surplus adjustment backs funded debt. The U.S. funded inflation response and unfunded debt response attain their support boundaries at the modes in both reported specifications.

Historical contributions and spillovers.

The domestic fiscal shares in these two Bayesian specifications are 142.4 and 140.9 percent for U.S. inflation and 130.8 and 130.3 percent for U.K. inflation (the shares exceed 100 percent because other components offset fiscal contributions). Figure F.1 shows historical decompositions for inflation and the terms of trade τ_t under the estimation approach with Bianchi et al. (2023)-style policy rules, showing that U.S. fiscal shocks generally make a positive contribution to U.K. inflation, accounting for around 58 percent of positive, demeaned U.K. inflation over 2021–2023. The corresponding headline shares at posterior-median parameter vectors are 25.6 and 26.9 percent, so allowing funded and unfunded shocks to coexist preserves the positive spillover and increases its estimated share. International transmission continues to operate through the portfolio and debt-valuation

	Main convenience yield			Alternative convenience yield		
Parameter	Mode	Mean	Post. s.d.	Mode	Mean	Post. s.d.
Policy and convenience-yield parameters						
ϕ_π^M	1.0001†	1.1453	0.1389	1.0001†	1.1478	0.1399
$\phi_\pi^{M,*}$	1.4034	1.5561	0.2210	1.4276	1.5597	0.2206
ϕ_x	1.0000†	0.9780	0.0206	1.0000†	0.9784	0.0206
ϕ_x^*	0.7467	0.7367	0.0633	0.7485	0.7412	0.0640
ψ_b^M	0.3664	0.5950	0.2162	0.3698	0.6014	0.2160
$\psi_b^{M,*}$	1.4151	1.4157	0.2669	1.4157	1.4280	0.2774
ψ_b^U	-0.1000†	-0.0602	0.0272	-0.1000†	-0.0607	0.0273
$\psi_b^{U,*}$	-1.6066	-1.6222	0.2070	-1.6175	-1.6253	0.2073
$\bar{\Phi}$	0.00045	0.00044	0.00017	0.00048	0.00046	0.00014
$\bar{\Phi}^*$	0.00158	0.00156	0.00048	0.00167	0.00164	0.00048
Shock mean-reversion parameters						
φ_a	0.7688	0.7674	0.1640	0.7606	0.7607	0.1593
φ_a^*	0.0202†	0.0284	0.0073	0.0202†	0.0279	0.0069
φ_θ	0.1335	0.1368	0.0401	0.0618	0.0680	0.0245
φ_θ^*	0.4849	0.7714	0.4183	0.4849	0.7722	0.4296
φ_d^M	0.3567†	0.2451	0.0707	0.3567†	0.2436	0.0710
$\varphi_d^{M,*}$	0.0832	0.0913	0.0369	0.0831	0.0906	0.0377
φ_d^U	0.1357	0.1517	0.0585	0.1354	0.1533	0.0583
$\varphi_d^{U,*}$	0.3567†	0.2505	0.0758	0.3567†	0.2503	0.0734
$\varphi_\varkappa$	0.3364	0.4789	0.1622	0.3764	0.5087	0.1687
Shock diffusion parameters						
σ_a	7.4270	7.5830	0.7521	7.3955	7.5882	0.7666
σ_a^*	6.7714	7.3503	1.1247	7.1658	7.5690	1.1297
σ_θ	117.3884	138.6428	51.5467	48.8401	56.8869	20.5057
σ_θ^*	0.4247	0.6964	0.4139	0.4247	0.7004	0.4279
σ_d^M	6.1243	6.5767	0.8021	6.1346	6.5992	0.7975
$\sigma_d^{M,*}$	7.1279	7.2678	0.7417	7.1175	7.2824	0.7545
σ_d^U	0.3221	0.3790	0.0995	0.3215	0.3812	0.0976
$\sigma_d^{U,*}$	0.3091	0.3087	0.0511	0.3091	0.3064	0.0511
$\sigma_\varkappa$	2.8676	3.4201	0.7856	3.0082	3.4977	0.8035

Table F.2: Bayesian estimates for the robustness exercise. Modes are constrained posterior modes, and posterior means and standard deviations use four chains with 200,000 retained draws each. Mean-reversion and diffusion coefficients are in quarterly units. † marks a mode at a support boundary to numerical tolerance. Both unfunded inflation responses are fixed at zero. Both reported specifications pass the posterior diagnostic requirements following Vehtari et al. (2021).

equations similar to Section 4.3.

Conditional counterfactuals. The panels in figure F.2 halve the innovations to both U.S. fiscal drivers from 2020Q1, holding the inherited state, other shocks, and policy

coefficients fixed. Earlier fiscal shocks continue to propagate. The intervention starts one year earlier than in our headline exercise. Over 2020Q1–2023Q4, counterfactual cumulative U.K. inflation is lower than baseline by 4.89 and 4.86 percentage points in the main and alternative convenience-yield specifications, while over 2021Q1–2023Q4, the corresponding reductions are 4.71 and 4.70 percentage points. Cumulative U.S. inflation over 2020Q1–2023Q4 is lower by 18.08 and 17.94 percentage points.

The quarterly responses nevertheless change sign. In each of the two convenience-yield specifications, counterfactual U.K. inflation is below baseline in nine of the sixteen quarters of 2020–2023, and the counterfactual exchange rate is below baseline in thirteen. A lower exchange rate corresponds to a stronger dollar under our convention. The exercise therefore supports an international fiscal spillover at the reported posterior modes under coexistence of funded and unfunded policies, with its size and timing depending on fiscal backing and the policy response to unfunded debt.

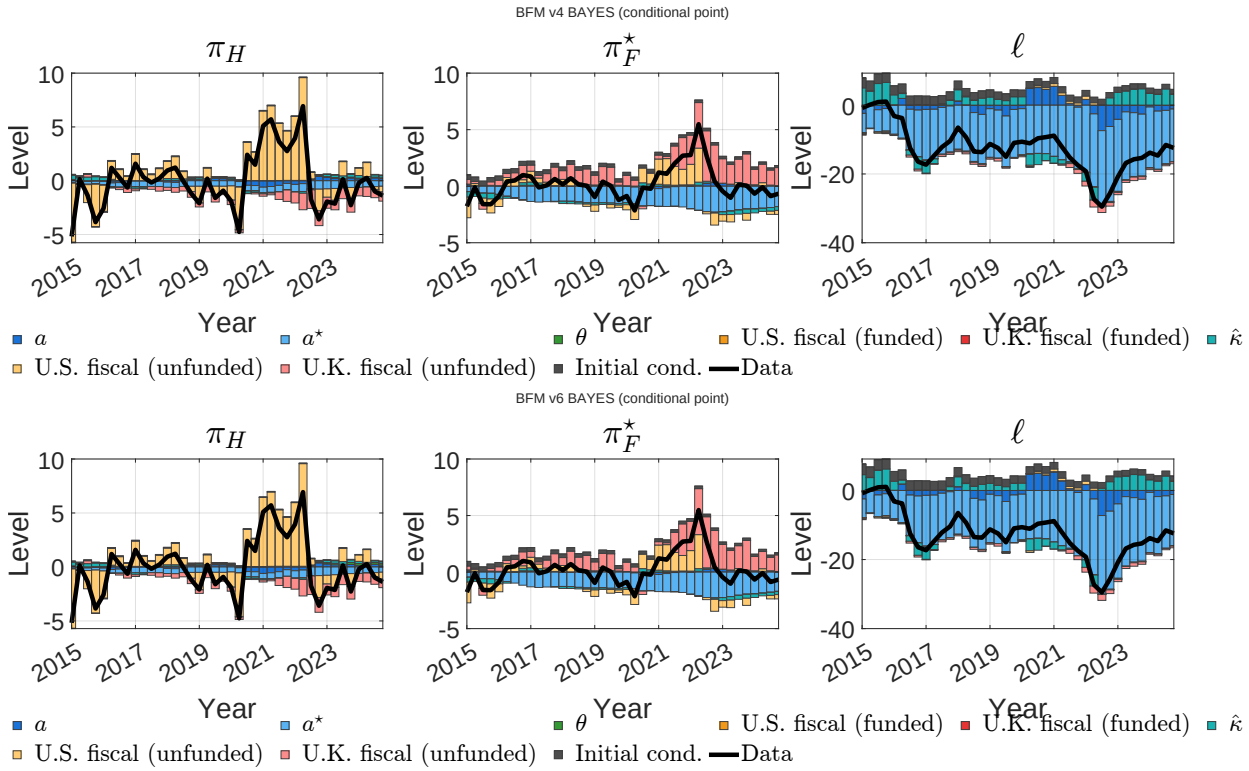


Figure F.1: Historical decompositions of quarterly PPI inflation in both countries and the terms of trade τ_t, 2015Q1–2024Q4. The first row shows the specification estimated with the main convenience-yield measure (Du et al., 2018a), and the second row shows the specification estimated with the alternative convenience-yield measure (Fleckenstein and Longstaff, 2024). Black lines are data, bars are conditional contributions at the posterior modes.

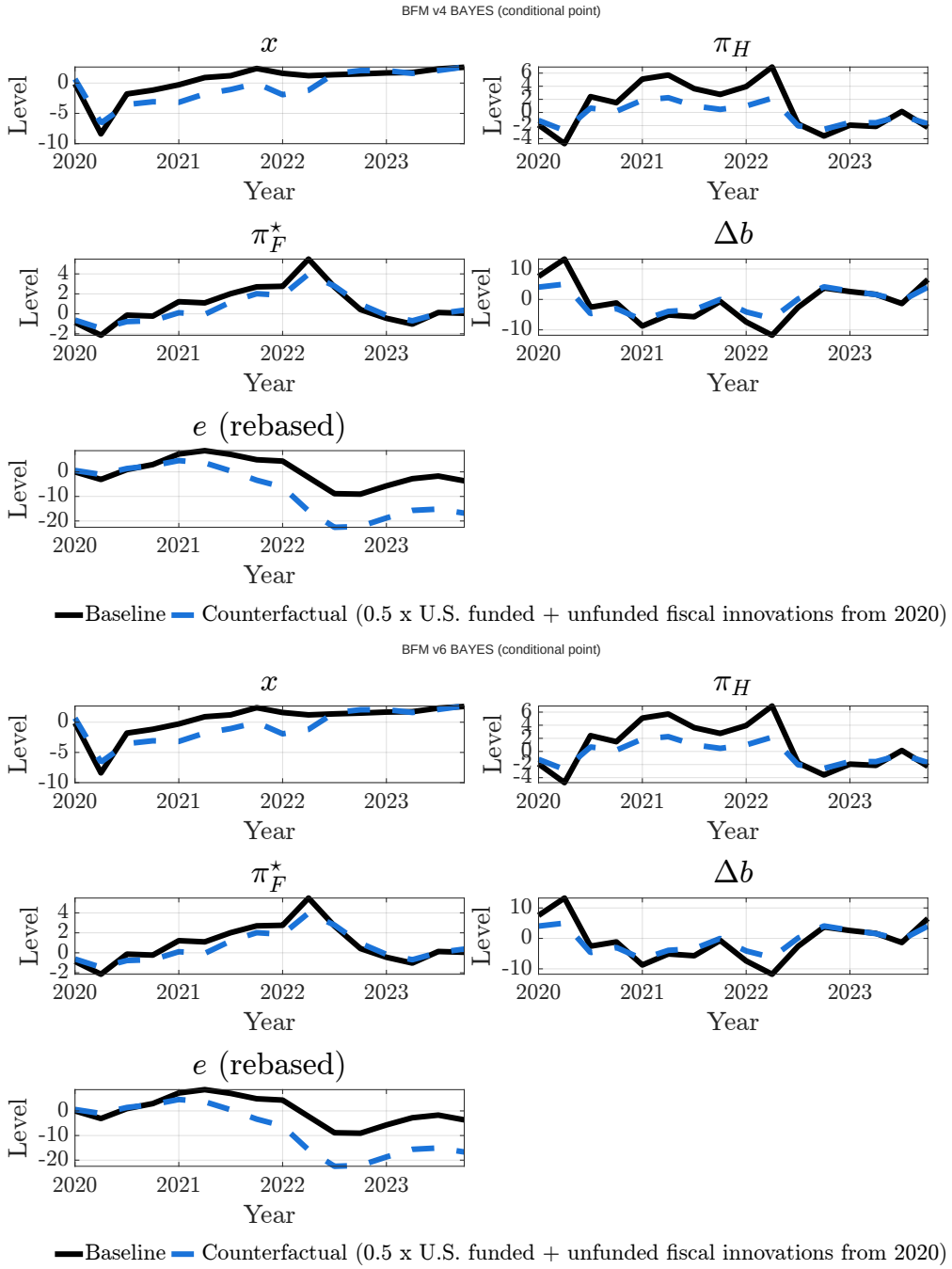


Figure F.2: Conditional fiscal counterfactuals for the Bayesian specifications estimated with the main convenience-yield measure (Du et al., 2018a) (top) and estimated with the alternative convenience-yield measure (Fleckenstein and Longstaff, 2024) (bottom), 2020Q1–2023Q4. Both funded and unfunded U.S. fiscal innovations are halved from 2020Q1. The inherited state, other shocks, and policy coefficients are held fixed. Paths are evaluated at the posterior modes.

(Not For Publication) Supplementary Online Appendix

G Private-Sector Deficit Forecast Revisions

This Appendix complements Section 2 with private-sector data on deficit expectations. We ask how far U.K. forecasters link revisions to expected deficits, exchange rates, and inflation. To do so, we assemble nearly 30 years of individual macroeconomic forecasts by private forecasters in the U.K., a dataset that, to the best of our knowledge, has not been assembled before.

G.1 How Forecasters Link Deficits, Exchange Rates, and Inflation

Data and Estimation

The data consist of all private-sector forecasts collected by HM Treasury (HM Treasury, 2025). Average forecasts are available in machine-readable form, but individual forecasts are not, so we hand-collected all forecaster-level data at a monthly frequency since September 1997.

HM Treasury asks a selection of banks and financial institutions ("City" forecasters) and a larger number of independent forecasting bodies ("non-City" forecasters) for their current-year and next-year predictions of a range of macroeconomic and financial variables. Not all forecasters update their forecasts each month, but the median forecaster who is in the sample for at least three years provides updates multiple times over the year. Of particular interest to us are the forecasts on sovereign borrowing (officially "Public Sector Net Borrowing", or PSNB), exchange rates, interest rates, CPI and RPI inflation, and output growth. PSNB includes debt interest, so its revisions provide information on expected borrowing that may reflect both changes to the primary fiscal position and changes to debt servicing costs.

We begin with descriptive statistics. The sample contains 88 forecasters over almost 28 years (September 1997 to June 2025), with 15,089 forecaster-month observations that contain at least one new forecast.[20]

Figure G.1 shows the number of forecasters in the regression sample in each year. It fluctuates between 25 and 33 for much of the earlier sample and declines from the mid-2010s, to fewer than 20 in the most recent years.

Because forecasts are made for current-year and next-year targets, the number of new

[20]We drop the forecasts of the European Commission, the OECD, and the IMF, which reflect model estimates that rarely respond to new information at a *monthly* frequency.

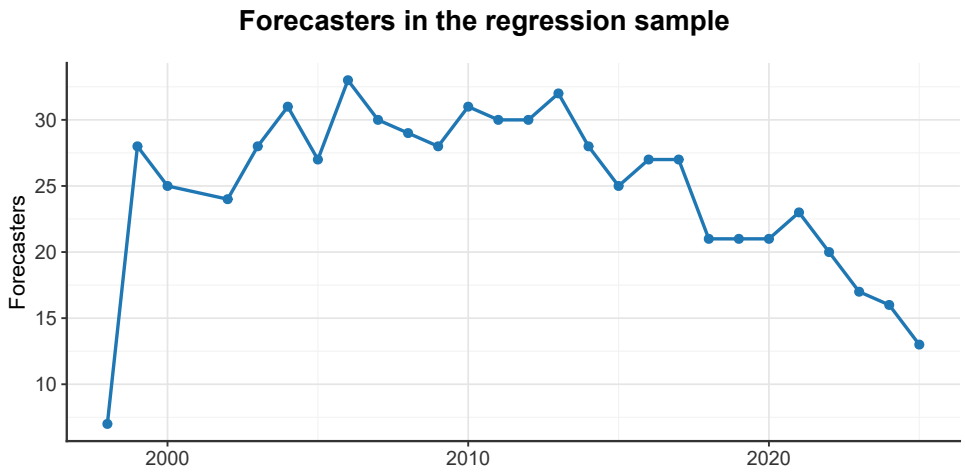


Figure G.1: Number of unique forecasters appearing in the regression sample in each year.

forecasts *within* a year determines how many revisions we observe. Figure G.2 shows how often forecasters provide a new current-year borrowing forecast: typically 5–7 times a year, with some variation across years. Revisions formed from these new forecasts are the basis of our analysis.

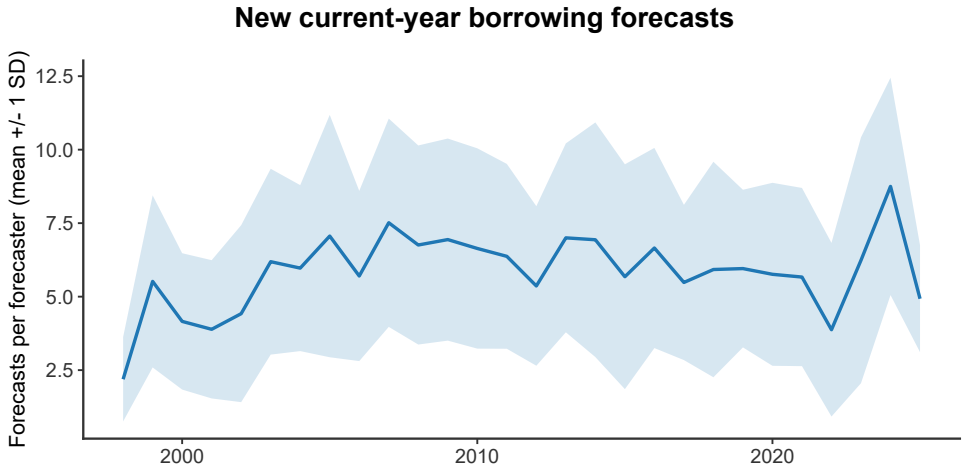


Figure G.2: Number of new current-year borrowing forecasts per forecaster per year, on average. Shaded bands indicate one standard deviation around the mean across forecasters.

Table G.1 summarises the levels of the forecasts. The average PSNB forecast is about 72 billion GBP in nominal terms, average forecast real output growth is slightly below 2

percent, and average CPI and RPI inflation forecasts are about 2.3 and 2.9 percent.

Variable	Mean	Median	SD	Q1	Q3
PSNB (nominal GBP billion)	71.575	52.000	61.325	33.300	106.900
g_Y (%)	1.738	1.900	2.010	1.200	2.600
π_{RPI} (%)	2.892	2.800	1.459	2.300	3.400
π_{CPI} (%)	2.298	2.100	1.256	1.700	2.600
$\mathcal{E}_{\text{Sterling\|Basket}}$ (index points)	84.207	82.100	7.577	79.200	87.300

Table G.1: Summary statistics of forecast levels. PSNB is expressed in nominal GBP billion, growth and inflation forecasts in percent, and the sterling basket index in index points.

Figure G.3 shows the distribution of revisions to PSNB, exchange-rate, and CPI and RPI inflation forecasts over time. Most revisions are close to zero, but their dispersion is substantial. Exchange-rate revisions (lower-right panel) are especially dispersed around the Brexit episode, while PSNB revisions (upper-right panel) show particularly large upward revisions to expected borrowing during Covid.

Econometric specification. Let T denote the calendar-year forecast target, T^s the corresponding fiscal-year target for PSNB, $t-1$ the date of the previous forecast, and t the current forecast date. Every revision uses the same two new forecast dates and holds the target of each variable fixed. We allow a gap of at most three months between forecast dates and estimate

$$\Delta \mathbb{E}_{it} \mathcal{E}_T = \beta_0 + \beta_1 \Delta \mathbb{E}_{it} \text{PSNB}_{T^s} + \beta_2 \Delta \mathbb{E}_{it} g_{Y,T} + \beta_3 \Delta \mathbb{E}_{it} r_{RPI,T} + \Gamma Z_{it} + \varepsilon_{it}. \tag{G.1}$$

In equation (G.1), the left-hand side $\Delta \mathbb{E}_{it} \mathcal{E}_T$ is the revision to the expected year-end sterling basket index, expressed as 100 times the change in its logarithm. The coefficient of interest, β_1, measures the association between revisions to expected borrowing, expressed as a percentage of annualised current-quarter nominal GDP, and revisions to the expected exchange rate. We form the nominal borrowing revision before dividing by annualised nominal GDP at the later forecast date. β_2 and β_3 are the coefficients on revisions of output growth and the real interest-rate proxy constructed using RPI inflation forecasts. Finally, ΓZ_{it} captures further control variables and the fixed effects specified in table G.2, and ε_{it} is the estimation error. These fixed effects include either forecaster effects or joint forecaster–PSNB-target effects, together with calendar/fiscal horizon and month-of-year effects. We control for other forecast revisions because theory links them and because

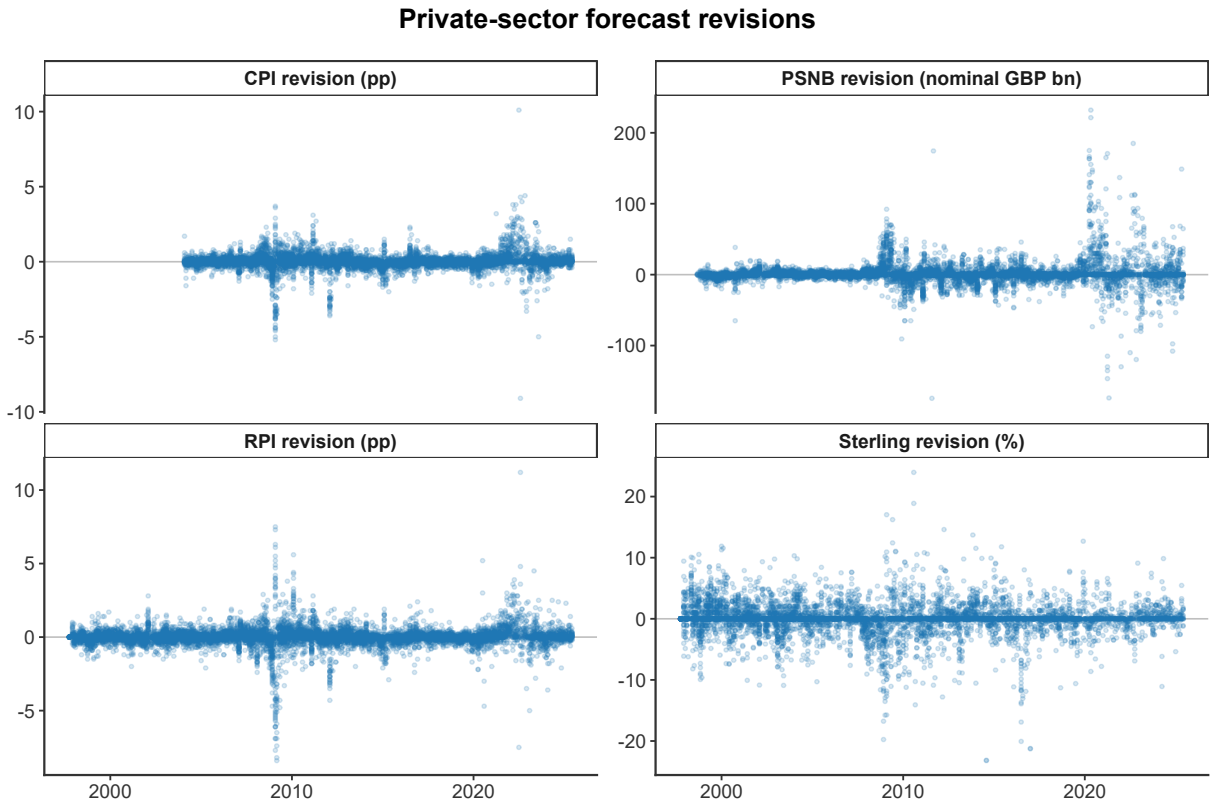


Figure G.3: Distribution of private-sector forecast revisions over time. The panels show CPI and RPI inflation revisions in percentage points, PSNB revisions in nominal GBP billion, and sterling-index revisions in percent.

Crump et al. (2025) show that forecasters revise their forecasts jointly, so that pairwise comparisons miss important associations.

Results

We first show that forecasters on average link upward revisions to expected borrowing with downward revisions to expected sterling. We then consider whether inflation revisions move with borrowing revisions, and how far borrowing revisions align with our headline deficit measure.

Table G.2 relates revisions to year-end exchange-rate forecasts to concurrent revisions of other forecasts. An upward revision to expected PSNB of one percentage point of annual GDP goes with a downward revision of about 0.55–0.58 percent in the expected year-end sterling basket index. The association is statistically significant in every specification, including when the gap between forecast dates is restricted to one month, although

	Outcome: $\Delta\mathbb{E}(\mathcal{E}_{\text{Sterling}\|\text{Basket}})$ (%)				
	(1)	(2)	(3)	(4)	(5)
$\Delta\mathbb{E}$(PSNB) (% of GDP)	-0.5684***	-0.5787***	-0.5614***	-0.5662***	-0.5477***
	(0.1375)	(0.1412)	(0.1478)	(0.1479)	(0.1538)
$\Delta\mathbb{E}(g_Y)$	0.4078**	0.4653**	0.2784*	0.3103**	0.3563*
	(0.2031)	(0.1907)	(0.1647)	(0.1448)	(0.2069)
$\Delta\mathbb{E}(r_{RPI})$	0.1187	0.1669			0.0932
	(0.1470)	(0.1281)			(0.1362)
$\Delta\mathbb{E}(i)$			0.7091	0.8259*	
			(0.4686)	(0.4373)	
$\Delta\mathbb{E}(\pi_{RPI})$			-0.0180	-0.0491	
			(0.1638)	(0.1520)	
Observations	4,042	4,042	4,042	4,042	3,415
R^2	0.14482	0.07059	0.15176	0.08020	0.15234
PSNB partial R^2	0.02117	0.02245	0.02081	0.02170	0.02331
Forecaster–target joint FE	✓		✓		✓
Forecaster FE		✓		✓	
Publication-date FE					
Calendar/fiscal horizon FE	✓	✓	✓	✓	✓
Month-of-year FE	✓	✓	✓	✓	✓
Maximum revision gap (months)	3	3	3	3	1

Table G.2: Forecast revision regressions across the audited forecast panel. PSNB revisions are expressed as percent of annualised current-quarter nominal GDP, sterling revisions are 100 log index changes, and inflation revisions are percentage points. Every outcome and control uses the same two new forecast dates and fixed targets. Joint FE interact forecaster and PSNB fiscal target. Standard errors are two-way clustered by forecaster and publication date; $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$. Full R^2 includes fixed effects and controls, partial R^2 is the incremental fit of PSNB.

borrowing revisions explain only a modest part of the variation in sterling revisions.

Figure G.4 shows the negative relation between exchange-rate and borrowing forecast revisions. Both variables are residualised on the other regressors in column (1) of table G.2, using exactly the same sample. We group the residualised borrowing revisions into 20 bins containing approximately equal numbers of observations and plot the mean residuals within each bin. A higher borrowing forecast corresponds to a downwards revision in expected year-end exchange rates, consistent with the direction of the relation in table G.2.

Figure G.5 shows the corresponding relation for CPI and RPI inflation revisions. Here, both variables are residualised on an intercept and revisions to output growth and nominal interest rates, together with the same set of joint forecaster–PSNB-target, calendar/fiscal horizon, and month-of-year effects. The relations are noisy, but they slope upwards: forecasters who raise their borrowing forecasts also tend to raise their inflation forecasts.

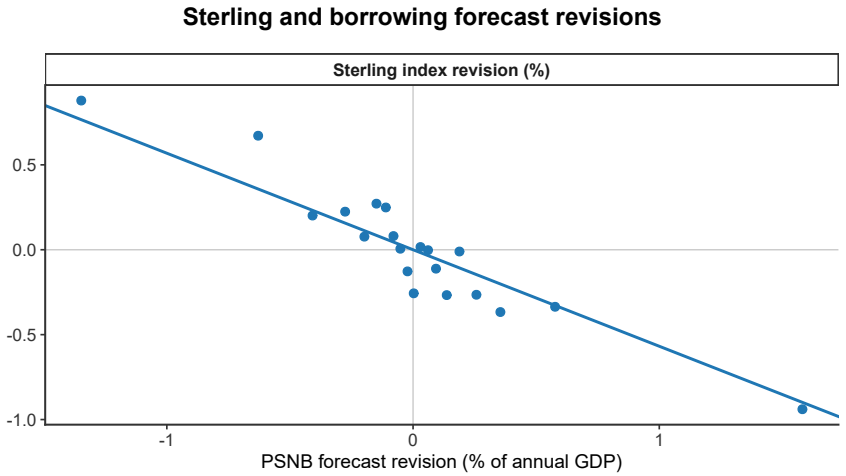


Figure G.4: Binned scatterplot showing the conditional association between PSNB forecast revisions and sterling basket index forecast revisions. Both variables are residualised on the other regressors in column (1) of table G.2, including its controls and fixed effects, on the same regression sample. PSNB revisions are expressed as percent of annual GDP and sterling revisions as 100 log index changes.

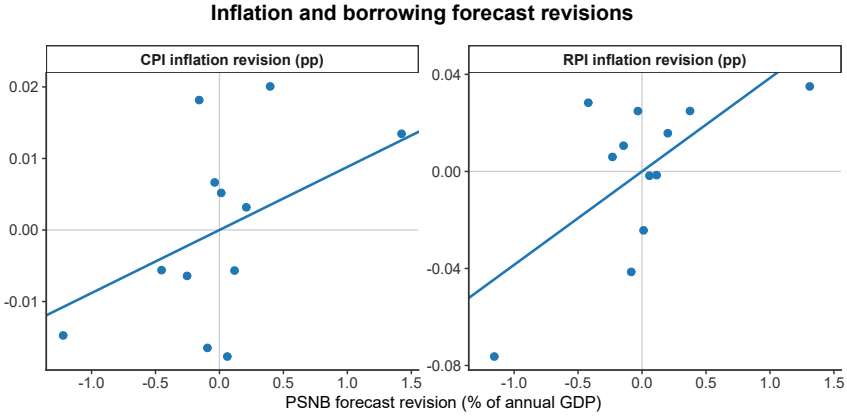


Figure G.5: Binned scatterplots showing the conditional association between PSNB forecast revisions and CPI and RPI inflation forecast revisions. Both variables are residualised on an intercept, output-growth and nominal-interest-rate revisions, joint forecaster–PSNB-target effects, calendar/fiscal horizon effects, and month-of-year effects. PSNB revisions are expressed as percent of annual GDP and inflation revisions in percentage points.

Private-sector forecasters therefore link higher expected borrowing to expected sterling depreciation, consistent in direction with the effect of U.K. deficit innovations in Section 2.2.

G.2 Forecast Revisions and Our Deficit Measure

To assess whether our valuation-implied deficit measure moves with private-sector expectations, we regress quarterly aggregates of the individual PSNB revisions on our recovered deficits:

$$\mathcal{R}^A_{t,h} = \alpha^A_h + \beta^A_{h,F} d^F_t + \beta^A_{h,H} d^H_t + \Gamma^A_h Z_{t-1} + u^A_{t,h}. \tag{G.2}$$

where d^F_t and d^H_t denote the current-quarter valuation-implied deficit flows used in the quarterly data construction, expressed in percentage points of annual U.K. nominal GDP, with the U.S. measure converted into sterling. Positive values indicate larger deficits. The dependent variable is

$$\mathcal{R}^A_{t,h} = A_{i \in \mathcal{I}_{t,h}} \left[\frac{100}{Y^{F,\text{ann}}_t} \left\{ \mathbb{E}_{it}(\text{PSNB}_{T_h(t)}) - \mathbb{E}_{i,t-1}(\text{PSNB}_{T_h(t)}) \right\} \right],$$

where $T_h(t)$ identifies the current ($h = 0$) or next ($h = 1$) U.K. fiscal year, with the current fiscal year beginning in April. The set $\mathcal{I}_{t,h}$ contains forecasters with new forecasts in both adjacent quarters for the same fiscal-year target. Within each quarter, we retain their last new forecast. The horizon is classified at the later forecast date, while the fiscal-year target is held fixed across the two quarters. PSNB and the GDP denominator are expressed in the same currency units, and the nominal revision is formed before scaling by annualised nominal GDP at the later date.

We estimate the equation separately for each fiscal-year horizon, using the mean, median, and 10 percent trimmed mean as the aggregation operator A. The controlled specification includes one-quarter lags of the U.K. and U.S. deficit measures, U.K. real GDP growth and unemployment, U.S. CPI inflation and the output gap, and the Treasury convenience-yield basis. The parsimonious specification omits Z_{t-1} but retains both contemporaneous deficit measures. The figure reports $\hat{\beta}^A_{h,F}$ with bands constructed as $\hat{\beta}^A_{h,F} \pm 1.96\text{SE}$, using Newey-West standard errors with four quarterly lags.

In figure G.7, both median PSNB revisions and the U.K. deficit measure are residualised on the contemporaneous U.S. deficit measure and all lagged controls in the controlled specification, including an intercept. We do so separately for the current and next fiscal-year targets, using exactly the complete regression sample for each target. The slopes of these residual scatterplots therefore correspond to the controlled median coefficients in figure G.6.

Figure G.6 plots the coefficients from specification (G.2), while figure G.7 plots a scatter of the median forecast revisions against our headline deficit measure for the U.K. after

adjusting for the included controls. The point estimates are positive for both fiscal-year targets and all aggregations, but most of the 95 percent confidence intervals include zero. Private-sector revisions therefore move in the same direction as our headline deficit measure, but the estimates are imprecise, and the scatterplots likewise show a positive association with substantial dispersion. Matching revisions within the same forecaster and fiscal-year target limits the role of changes in the composition of forecasters, while the quarterly timing of the deficit measure does not identify the date on which forecasters learned the information.

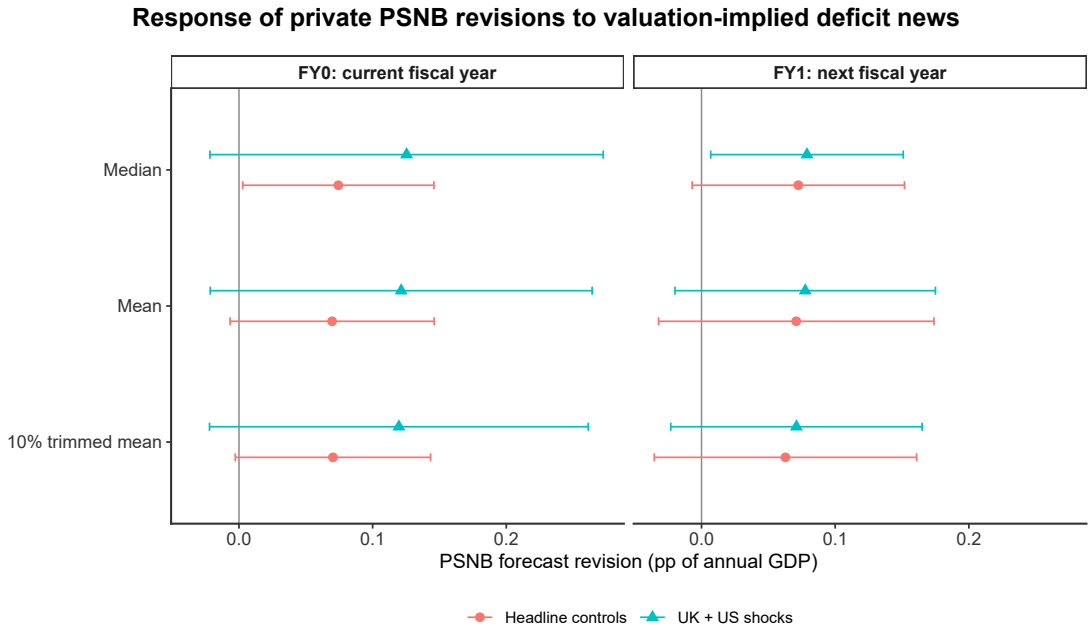


Figure G.6: Coefficients linking quarterly aggregates of PSNB revisions to our valuation-implied U.K. deficit flow, in line with equation (G.2). Coefficients express the PSNB revision in percentage points of annual U.K. GDP per one-percentage-point increase in the U.K. deficit measure. Intervals are pointwise 95% confidence intervals using Newey-West standard errors with four quarterly lags.

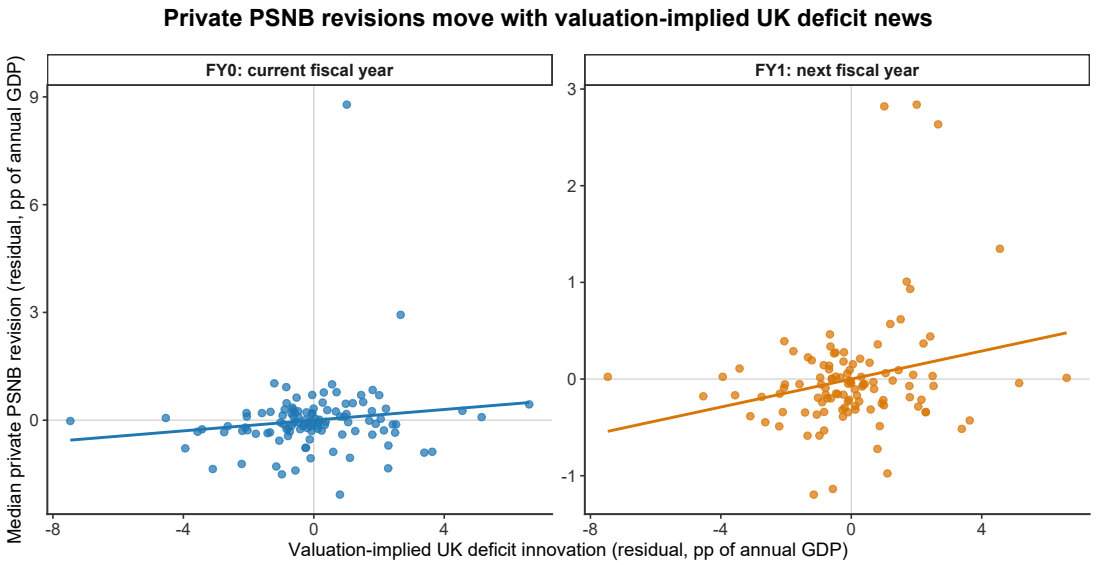


Figure G.7: Scatterplots of median private-sector PSNB revisions against our valuation-implied U.K. deficit flow. Both variables are residualised on the contemporaneous U.S. deficit flow and the lagged controls in equation (G.2). Slopes correspond to the controlled median coefficients in figure G.6.

H The Quantitative Model: Specification and Derivations

This part of the Appendix specifies the quantitative model used in Section 4 and derives its equilibrium conditions. Relative to the analytical model of Section 3, it makes convenience services non-separable from consumption, introduces convenience-demand shocks and a cross-border holding wedge, and allows convex labour disutility ($\phi, \phi^* > 0$), CES expenditure allocation, monetary responses to output gaps, and fiscal disturbances in both countries. In the specification we simulate and estimate, both households hold both sovereign bonds subject to the quadratic holding costs of Section 3. The presentation follows the household and firm problems through policy, the conditional flexible-price reference, aggregate dynamics, and the reduced equilibrium.

Hats on positive level variables denote log deviations from the stationary steady state unless explicitly defined otherwise, while hats on inflation, interest rates and the holding wedge denote level deviations. Thus $\hat{\varkappa}_t \equiv \varkappa_t - \bar{\varkappa}$. We impose $\rho = \rho^*$ throughout. The steady-state firm discount coefficients consequently satisfy $\bar{\delta}_H = \bar{\delta}_F = \rho$, as derived in (H.52).

Define productivity deviations by $a_t \equiv \log(A_t/\bar{A})$ and $a_t^* \equiv \log(A_t^*/\bar{A}^*)$, and aggregate debt deviations by $\hat{b}_t \equiv \log(b_t/\bar{b})$ and $\hat{b}_t^* \equiv \log(b_t^*/\bar{b}^*)$.

H.1 Households, Goods Demand, and International Pricing

Bilateral bond holdings. Households in both countries hold both sovereign bonds, only Hegemon debt provides convenience services, and, following Schmitt-Grohé and Uribe (2003) and Section 3, convex flow costs penalise cross-border positions away from their steady-state levels. Let B_{Ft} denote Hegemon households' holdings of Foreign debt in Foreign currency and define $v_{Ft} \equiv E_t B_{Ft}/P_t$. Foreign holdings of Hegemon debt are $v^*_{Ht} = B^*_{Ht}/(E_t P^*_t)$, as in (H.10). The two holding-cost functions, measured in the respective consumption bundles, are

$$\mathcal{K}_H(v_{Ft}) = \frac{\psi_H}{2\bar{Y}}(v_{Ft} - \bar{v}_F)^2, \qquad \mathcal{K}_F(v^*_{Ht}) = \frac{\psi_F}{2\bar{Y}}(v^*_{Ht} - \bar{v}^*_H)^2, \qquad \psi_H, \psi_F > 0,$$

which describe the costs of the levels of holdings. The convex costs are resource expenditures and do not constitute intermediary revenue.

H.1.1 The Hegemon Household

The representative Hegemon household chooses consumption C_t, labour supply N_t, and holdings of both sovereign bonds, $b_{Ht} \equiv B_{Ht}/P_t$ and v_{Ft}, to maximise

$$\int_0^\infty e^{-\rho t}\Big\{\log C_t - \frac{N_t^{1+\phi}}{1+\phi} + \nu(b_{Ht}, C_t;\, \theta_t)\Big\}\, dt. \tag{H.1}$$

The function $\nu(b_{Ht}, C_t; \theta_t)$ captures the convenience services generated by its Hegemon debt holdings, and θ_t is an exogenous shifter that governs their time-varying value. The household faces the nominal flow budget constraint

$$\begin{aligned} P_t C_t + dB_{Ht}/dt + d(E_t B_{Ft})/dt + P_t \mathcal{K}_H(v_{Ft}) = i_t B_{Ht} + \Big(i^*_t + \tfrac{\dot{E}_t}{E_t}\Big) E_t B_{Ft} \\ + W_t N_t + \Pi_t + \Pi^I_t + E_t P^*_t \tilde{\mathfrak{T}} - T_t, \\ \Pi^I_t \equiv \varkappa_t B^*_{Ht}. \end{aligned} \tag{H.2}$$

Here, $i_t B_{Ht}$ is domestic interest income, $(i^*_t + \dot{E}_t/E_t) E_t B_{Ft}$ is the currency-adjusted return on Foreign debt, $P_t \mathcal{K}_H(v_{Ft})$ is the resource cost of holding it, $W_t N_t$ is labour income, Π_t denotes the profits of Hegemon intermediate producers, and T_t denotes nominal net taxes. The term Π^I_t is a lump-sum rebate of the fees collected on Hegemon debt held abroad, which we keep separate from productive profits because it redistributes income without using goods, and $E_t P^*_t \tilde{\mathfrak{T}}$ is a constant transfer from the Foreign household that supports a symmetric steady state.

Convenience yield. Following Jiang et al. (2023, 2024b), we allow Hegemon government debt to provide liquidity services. These services generate a departure from perfect risk-sharing (Backus and Smith, 1993). We adopt an isoelastic specification in real bond holdings relative to steady state,

$$v(b_{Ht}, C_t;\, \theta_t) = \frac{\bar{\Phi}\,\theta_t\,\bar{b}_H}{(1-\zeta_H)\,C_t}\left[\left(\tfrac{b_{Ht}}{\bar{b}_H}\right)^{1-\zeta_H} - 1\right], \qquad \zeta_H > 0. \tag{H.3}$$

Here, $\bar{b}_H$ normalises bond holdings, $\bar{\Phi}$ determines their steady-state marginal convenience value, ζ_H governs the curvature of convenience demand, and θ_t shifts the perceived liquidity services provided by Hegemon debt. The specification is normalised so that the convenience flow is zero in the stationary steady state. Scaling the flow by $1/C_t$ makes convenience services non-separable from consumption and implies $v_C = -v_t/C_t$, where $v_t \equiv v(b_{Ht}, C_t; \theta_t)$.

The marginal convenience value of real Hegemon bond holdings, scaled by consumption, is

$$\Psi_t \equiv v_b\, C_t = \bar{\Phi}\,\theta_t\left(\tfrac{b_{Ht}}{\bar{b}_H}\right)^{-\zeta_H}, \tag{H.4}$$

where v_b denotes the derivative of the convenience flow v with respect to the *real* quantity of debt held. Its effective contribution to the household's pricing kernel, which enters the Euler equation, is

$$\Phi_t \equiv \frac{\Psi_t}{1 - v_t}.$$

The effective convenience yield Φ_t is the non-pecuniary return that lowers the nominal yield required on Hegemon debt. Time variation in the underlying convenience flow also shifts the pricing kernel directly through $\dot{v}_t/(1-v_t)$.

With $u(C, N, b_H; \theta) = \log C - \frac{N^{1+\phi}}{1+\phi} + v(b_H, C; \theta)$ and v as in (H.3), the consumption scaling gives $v_C = -v_t/C_t$, so that

$$u_C = \frac{1}{C_t} + v_C = \frac{1 - v_t}{C_t}, \qquad u_N = -N_t^{\phi}, \qquad u_{b_H} = v_b = \frac{\Psi_t}{C_t}.$$

Differentiating (H.3) with respect to b_{Ht},

$$v_b = \frac{\bar{\Phi}\theta_t\bar{b}_H}{(1-\zeta_H)C_t}(1-\zeta_H)\left(\tfrac{b_{Ht}}{\bar{b}_H}\right)^{-\zeta_H}\frac{1}{\bar{b}_H} = \frac{\bar{\Phi}\theta_t}{C_t}\left(\tfrac{b_{Ht}}{\bar{b}_H}\right)^{-\zeta_H} = \frac{\Psi_t}{C_t}.$$

Define the Hegemon nominal discount factor

$$\Lambda_t \equiv e^{-\rho t}\,\frac{u_C}{P_t} = e^{-\rho t}\,\frac{1 - v_t}{C_t P_t}. \tag{H.5}$$

An extra unit of *real* Hegemon bonds delivers, per unit time, the return i_t plus the non-pecuniary flow v_b measured in units of the marginal utility of consumption, namely $v_b/u_C = \Psi_t/(1-v_t) = \Phi_t$. The bond first-order condition is therefore

$$-\frac{\dot{\Lambda}_t}{\Lambda_t} = i_t + \Phi_t, \qquad \text{with} \qquad \lim_{T\to\infty} \Lambda_T B_{HT} = 0. \tag{H.6}$$

The second condition rules out explosive bond positions. Define the preference-drift composite

$$\chi_t \equiv \frac{\dot{v}_t}{1-v_t} = -\frac{d}{dt}\log(1-v_t),$$

and analogously $\chi_t^* \equiv \dot{v}_t^*/(1-v_t^*)$. From (H.5), $-\dot{\Lambda}_t/\Lambda_t = \rho - \frac{d}{dt}\log(1-v_t) + \dot{C}_t/C_t + \pi_t$. Thus,

$$i_t + \Phi_t = \rho + \chi_t + \frac{\dot{C}_t}{C_t} + \pi_t \quad \Longleftrightarrow \quad \frac{\dot{C}_t}{C_t} = i_t - \pi_t - \rho + \Phi_t - \chi_t, \tag{H.7}$$

The labour first-order condition $-u_N = u_C W_t/P_t$, and its Foreign counterpart, give

$$C_t N_t^{\phi} = (1-v_t)\frac{W_t}{P_t}, \qquad C_t^*(N_t^*)^{\phi^*} = (1-v_t^*)\frac{W_t^*}{P_t^*}. \tag{H.8}$$

Foreign debt provides no convenience services to the Hegemon household and pays no intermediation wedge. Its Euler equation includes the marginal holding cost,

$$\begin{aligned} -\frac{\dot{\Lambda}_t}{\Lambda_t} &= i_t^* + \frac{\dot{E}_t}{E_t} - \frac{\psi_H}{\bar{Y}}(v_{Ft} - \bar{v}_F) \\ \Longleftrightarrow \quad \frac{\dot{C}_t}{C_t} &= i_t^* + \frac{\dot{E}_t}{E_t} - \pi_t - \rho - \chi_t - \frac{\psi_H}{\bar{Y}}(v_{Ft} - \bar{v}_F), \end{aligned} \tag{H.9}$$

with transversality condition $\lim_{T\to\infty} \Lambda_T E_T B_{FT} = 0$.

H.1.2 The Foreign Household and the Holding Wedge

The Foreign household chooses consumption, labour supply, and holdings of both sovereign bonds. It derives convenience services from Hegemon bonds but none from its own government's debt. Let

$$v_{Ht}^* \equiv \frac{B_{Ht}^*}{E_t P_t^*} \tag{H.10}$$

denote the Foreign household's Hegemon debt holdings measured in Foreign consumption units. This is a gross external asset position. With $Q_t \equiv E_t P_t^*/P_t$, the Foreign economy's net foreign asset position is $n_t^* \equiv v_{Ht}^* - v_{Ft}/Q_t$.

The Foreign household maximises

$$\int_0^\infty e^{-\rho^* t}\Big\{\log C_t^* - \frac{N_t^{*1+\phi^*}}{1+\phi^*} + \nu^*\Big(v_{Ht}^*, C_t^*;\, \theta_t^*\Big)\Big\}\, dt \tag{H.11}$$

subject to the budget constraint (H.13). It enjoys a convenience flow on its real quantity of Hegemon debt holdings, given by

$$\nu^*\Big(v_{Ht}^*, C_t^*;\, \theta_t^*\Big) = \frac{\bar{\Phi}^*\, \theta_t^*\, \bar{v}_H^*}{(1-\zeta_F)\, C_t^*}\left[\left(\tfrac{v_{Ht}^*}{\bar{v}_H^*}\right)^{1-\zeta_F} - 1\right], \qquad \zeta_F > 0, \tag{H.12}$$

Its budget constraint is

$$P_t^* C_t^* + dB_{Ft}^*/dt + d\Big(\frac{B_{Ht}^*}{E_t}\Big)/dt + P_t^* \mathcal{K}_F(v_{Ht}^*) = \Big[i_t^* B_{Ft}^* + \Big(i_t - \tfrac{\dot{E}_t}{E_t} - \varkappa_t\Big)\frac{B_{Ht}^*}{E_t} + W_t^* N_t^* + \Pi_t^* - T_t^* - P_t^* \bar{\mathfrak{T}}\Big]. \tag{H.13}$$

The paid return on Hegemon debt, measured in Foreign currency, is therefore $i_t - \dot{E}_t/E_t - \varkappa_t$. An appreciation of the Hegemon currency raises the Foreign-currency value of a given nominal Treasury position, whereas a depreciation lowers it. The holding wedge $\varkappa_t$ is defined below.

Its marginal and effective convenience yields are

$$\Psi_t^* = \bar{\Phi}^* \theta_t^* \left(\frac{v_{Ht}^*}{\bar{v}_H^*}\right)^{-\zeta_F}, \qquad \Phi_t^* \equiv \frac{\Psi_t^*}{1-\nu_t^*}.$$

Exchange-rate movements consequently affect the Foreign household through two linked margins: they revalue its Treasury wealth and, by changing v_{Ht}^*, alter the marginal convenience value of those holdings.

The international holding wedge. Foreign holdings of Hegemon debt are subject to an exogenous proportional intermediation wedge $\varkappa_t$, measured as a rate per unit of the position per unit of time. The Foreign holder receives the net return $i_t - \dot{E}_t/E_t - \varkappa_t$. The bond pricing wedge is collected by a consolidated Hegemon financial sector, whose nominal fee-revenue flow is $\Pi_t^I = \varkappa_t B_{Ht}^*$, and which is rebated lump-sum to the Hegemon household. We introduced the financial sector here as an accounting device to close the model only. Thus, $\varkappa_t$ changes the Foreign investor's marginal return without creating an unaccounted

loss of world income.

We interpret $\varkappa_t$ as a reduced-form wedge between the gross currency-adjusted return on Hegemon debt and the return received by Foreign investors.[21] The wedge plays two roles. First, its steady-state value permits domestic and Foreign investors to assign different marginal convenience values to the same Hegemon bond while satisfying their respective Euler equations. Second, following the international financial-shock interpretation in Itskhoki and Mukhin (2021), deviations $\hat{\varkappa}_t \equiv \varkappa_t - \bar{\varkappa}$ represent cross-border financial disturbances that move international portfolios and the exchange rate independently of fiscal and convenience-yield innovations.

Let $\Lambda_t^* \equiv e^{-\rho^* t}(1-\nu_t^*)/(C_t^* P_t^*)$ denote the Foreign nominal (Foreign-currency) discount factor, and recall $v_{Ht}^* = B_{Ht}^*/(E_t P_t^*)$ from (H.10). Differentiating (H.12) gives $\nu_v^* = \Psi_t^*/C_t^*$ and hence $\nu_v^*/u_{C^*} = \Psi_t^*/(1-\nu_t^*) = \Phi_t^*$. The Foreign household holds two assets. Its own government's debt bears no convenience flow and no holding wedge, so that

$$-\frac{\dot{\Lambda}_t^*}{\Lambda_t^*} = i_t^* \quad \Longleftrightarrow \quad \frac{\dot{C}_t^*}{C_t^*} = i_t^* - \pi_t^* - \rho^* - \chi_t^*, \tag{H.14}$$

Hegemon debt delivers the Foreign-currency return $i_t - \dot{E}_t/E_t$, net of the intermediation wedge $\varkappa_t$ and the marginal holding cost, plus the convenience flow Φ_t^*, so that

$$\begin{aligned} -\frac{\dot{\Lambda}_t^*}{\Lambda_t^*} &= i_t - \frac{\dot{E}_t}{E_t} - \varkappa_t + \Phi_t^* - \frac{\psi_F}{\bar{Y}}(v_{Ht}^* - \bar{v}_H^*) \\ \Longleftrightarrow \quad \frac{\dot{C}_t^*}{C_t^*} &= i_t - \frac{\dot{E}_t}{E_t} - \varkappa_t - \pi_t^* - \rho^* + \Phi_t^* - \chi_t^* - \frac{\psi_F}{\bar{Y}}(v_{Ht}^* - \bar{v}_H^*), \end{aligned} \tag{H.15}$$

Differencing (H.14) and (H.15) gives $i_t^* = i_t - \dot{E}_t/E_t - \varkappa_t + \Phi_t^* - \psi_F(v_{Ht}^* - \bar{v}_H^*)/\bar{Y}$, which is the modified UIP condition (H.17). Because the convenience argument is v_{Ht}^*, already expressed in Foreign consumption units, no exchange rate normalisation constant appears anywhere in the Foreign block. The deterministic transversality conditions are $\lim_{T\to\infty} \Lambda_T^* B_{FT}^* = 0$ and $\lim_{T\to\infty} \Lambda_T^*(B_{HT}^*/E_T) = 0$.

Differencing the Hegemon household's Euler equations (H.7) and (H.9), and the Foreign household's Euler equations, respectively, gives

$$i_t - i_t^* = \frac{\dot{E}_t}{E_t} - \Phi_t - \frac{\psi_H}{\bar{Y}}(v_{Ft} - \bar{v}_F), \tag{H.16}$$

[21] The wedge may capture cross-border intermediation spreads, balance-sheet charges, regulatory fees, and hedging-related payments accruing to the Hegemon financial sector.

$$i_t - i_t^* = \frac{\dot{E}_t}{E_t} + \varkappa_t - \Phi_t^* + \frac{\psi_F}{\bar{Y}}(v_{Ht}^* - \bar{v}_H^*). \tag{H.17}$$

The preference-drift terms cancel within each household, so neither condition depends on χ_t or χ_t^*. Equating the two conditions determines the relative portfolio allocation,

$$\Phi_t - \Phi_t^* + \varkappa_t + \frac{\psi_H}{\bar{Y}}(v_{Ft} - \bar{v}_F) + \frac{\psi_F}{\bar{Y}}(v_{Ht}^* - \bar{v}_H^*) = 0. \tag{H.18}$$

The net convenience valuations can therefore differ outside the steady state, with their difference supported by the marginal holding costs. For a given path of the exchange rate, a higher Foreign convenience yield Φ_t^* lowers the pecuniary return on Hegemon debt required by the Foreign investor, whereas a higher holding wedge $\varkappa_t$ raises it. The composite wedge $\Phi_t^* - \varkappa_t - \psi_F(v_{Ht}^* - \bar{v}_H^*)/\bar{Y}$ is therefore the model counterpart of the return wedge ξ_t used in Section 2. Equation (H.17) makes the Foreign household's marginal valuation of Hegemon debt part of exchange-rate determination. This is the channel through which Foreign demand for Hegemon bonds feeds into exchange rate determination and, ultimately, into the cross-country transmission of fiscal shocks.

H.1.3 Goods Demand and Price Identities

The consumption aggregate is the CES bundle

$$C_t = \left[(1-\gamma)^{1/\eta} C_{Ht}^{(\eta-1)/\eta} + \gamma^{1/\eta} C_{Ft}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}, \tag{H.19}$$

where $1-\gamma$ is the domestic expenditure share (home bias) and $\eta > 0$ is the intratemporal elasticity of substitution. Given total nominal expenditure, cost minimisation yields the demand functions

$$C_{Ht} = (1-\gamma)\left(\tfrac{P_{Ht}}{P_t}\right)^{-\eta} C_t, \qquad C_{Ft} = \gamma\left(\tfrac{P_{Ft}}{P_t}\right)^{-\eta} C_t. \tag{H.20}$$

The corresponding consumer price index (CPI) is

$$P_t = \left[(1-\gamma)P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}\right]^{1/(1-\eta)}. \tag{H.21}$$

We define CPI inflation as $\pi_t \equiv \dot{P}_t/P_t$ and Hegemon producer-price inflation as $\pi_{Ht} \equiv \dot{P}_{Ht}/P_{Ht}$.

The Foreign consumption aggregate is symmetric to (H.19) with weights reversed, giving

$$C_{Ft}^* = (1-\gamma)\left(\tfrac{P_{Ft}^*}{P_t^*}\right)^{-\eta} C_t^*, \qquad C_{Ht}^* = \gamma\left(\tfrac{P_{Ht}^*}{P_t^*}\right)^{-\eta} C_t^*, \tag{H.22}$$

Define the price-index and expenditure composites below, where Ω_t is the relative-

consumption measure in (H.27).

$$\begin{aligned} G_t &\equiv (1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta}, \quad Z_t \equiv (1-\gamma) + \gamma Q_t^{\eta-1} e^{-\Omega_t}, \\ G_t^* &\equiv (1-\gamma) + \gamma \mathrm{tot}_t^{\eta-1}, \quad Z_t^* \equiv (1-\gamma) + \gamma Q_t^{1-\eta} e^{\Omega_t}. \end{aligned} \tag{H.23}$$

Factoring P_{Ht} out of the Hegemon CPI (H.21) and P_{Ft}^* out of its Foreign counterpart gives

$$P_t = P_{Ht}\, G_t^{1/(1-\eta)}, \qquad P_t^* = P_{Ft}^*\, (G_t^*)^{1/(1-\eta)}, \tag{H.24}$$

The law of one price (15) implies $\mathrm{tot}_t^* \equiv P_{Ht}^*/P_{Ft}^* = P_{Ht}/P_{Ft} = \mathrm{tot}_t^{-1}$, so that the stated expression for G_t^* follows. Using $P_{Ft} = E_t P_{Ft}^*$ therefore yields

$$Q_t \equiv \mathrm{tot}_t \left(\frac{G_t^*}{G_t} \right)^{1/(1-\eta)}. \tag{H.25}$$

Taking logs of the exact identity (H.25) gives

$$q_t = \log Q_t = \tau_t + \frac{\log G_t^* - \log G_t}{1-\eta}.$$

Log-linearising this relationship around the symmetric steady state $\overline{\mathrm{tot}} = 1$ gives

$$q_t \simeq (1-2\gamma)\, \tau_t, \qquad q_t^n \simeq (1-2\gamma)\, \tau_t^n, \qquad \tilde{q}_t \simeq (1-2\gamma)\, \tilde{\tau}_t.$$

Similarly, a first-order expansion of (H.24) around the symmetric steady state gives

$$\log P_t - \log P_{Ht} \simeq \gamma\, \tau_t, \qquad \log P_t^* - \log P_{Ft}^* \simeq -\gamma\, \tau_t, \tag{H.26}$$

Taking time derivatives gives the first-order CPI–PPI relationships in (H.74). Finally, $\tau_t = \log P_{Ft} - \log P_{Ht}$ implies $\dot{\tau}_t = \pi_{Ft} - \pi_{Ht}$, and $P_{Ft} = E_t P_{Ft}^*$ implies $\pi_{Ft} = \dot{\hat{e}}_t + \pi_{Ft}^*$, so that $\dot{\hat{e}}_t = \dot{\tau}_t - \hat{\pi}_{Ft}^* + \hat{\pi}_{Ht}$.

H.2 Portfolios, Convenience Services, and Relative Wealth

Define the relative-consumption and relative marginal-wealth measures

$$\Omega_t \equiv \log C_t - \log C_t^* - q_t, \qquad \varpi_t \equiv \Omega_t - \log\left(\frac{1-\nu_t}{1-\nu_t^*} \right). \tag{H.27}$$

The first object measures relative consumption taking into account movements in the real exchange rate. The second adjusts this measure for the effect of convenience services on the two households' marginal utilities of consumption. With additive convenience utility in the analytical specification, the definition of ϖ_t in Section 3.1 requires no such marginal-utility adjustment. We refer to ϖ_t as the relative-wealth state. As in the analytical specification, it would be constant under complete-markets risk sharing.

Differencing the Hegemon and Foreign Euler equations on their respective own-currency bonds and substituting the UIP condition (H.17) yields the international risk-sharing condition (Itskhoki and Mukhin, 2021),

$$\frac{\dot{C}_t}{C_t} - \frac{\dot{C}^*_t}{C^*_t} - \frac{\dot{Q}_t}{Q_t} = (\rho^* - \rho) + \Big(\Phi_t - \Phi^*_t + \varkappa_t\Big) + \frac{\psi_F}{\bar{Y}}(v^*_{Ht} - \bar{v}^*_H) + \Big(\chi^*_t - \chi_t\Big). \tag{H.28}$$

For this derivation, define $\Xi_t \equiv \log[(1-\nu_t)/(1-\nu^*_t)]$. Then, $\dot{\Xi}_t = \chi^*_t - \chi_t$, while (H.27) implies $\varpi_t = \Omega_t - \Xi_t$. Under $\rho = \rho^*$, subtracting $\dot{\Xi}_t$ from $\dot{\Omega}_t$ in (H.28) therefore gives (H.29).

$$\dot{\varpi}_t = \Phi_t - \Phi^*_t + \varkappa_t + \frac{\psi_F}{\bar{Y}}(v^*_{Ht} - \bar{v}^*_H) = -\frac{\psi_H}{\bar{Y}}(v_{Ft} - \bar{v}_F). \tag{H.29}$$

The first expression is the difference between the effective valuation of Hegemon debt by the Hegemon household and its net valuation by the Foreign household, including the marginal holding cost. The second follows from (H.18). These valuation differences therefore accumulate into persistent movements in relative consumption, as in equation (10). The Hegemon holding cost prevents Foreign debt from providing both households with the same frictionless saving margin. The state ϖ_t is not itself a stock of international assets. Its relationship with the Foreign net foreign asset position is determined jointly by bond-market clearing and the international budget constraint developed below.

H.2.1 Bond-Market Clearing and Convenience-Demand Linearisation

Both sovereign bonds are held by both households, so the bond-market clearing conditions are

$$B_t = B_{Ht} + B^*_{Ht}, \qquad B^*_t = B_{Ft} + B^*_{Ft}.$$

Writing $b_t \equiv B_t/P_t$ and using the definition of v^*_{Ht} in (H.10), the Hegemon bond market clearing becomes

$$b_t = b_{Ht} + Q_t\, v^*_{Ht}. \tag{H.30}$$

The real exchange rate Q_t converts the Foreign household's Treasury position into Hegemon consumption units. Foreign bond-market clearing in consumption units is $b^*_t = b^*_{Ft} + v_{Ft}/Q_t$,

where $b^*_{Ft} \equiv B^*_{Ft}/P^*_t$. Gross Treasury holdings determine Foreign convenience yields, while net foreign assets are $n^*_t = v^*_{Ht} - v_{Ft}/Q_t$.

Let

$$\omega_H \equiv \frac{\bar{b}_H}{\bar{b}}, \qquad \omega_F \equiv \frac{\bar{v}^*_H}{\bar{b}} = 1 - \omega_H$$

denote the shares of Hegemon debt held in steady state by Hegemon and Foreign households. We first linearise convenience demand and bond-market clearing. Recall that θ_t and θ^*_t shift the two households' convenience demand for Hegemon debt, b_{Ht} and v^*_{Ht} denote their respective real holdings of that debt (with the latter measured in Foreign consumption units) and Φ_t and Φ^*_t are the corresponding effective convenience yields. Using the hat notation introduced above, write

$$\begin{aligned}
\hat{\theta}_t &\equiv \log(\theta_t/\bar{\theta}), & \hat{\theta}^*_t &\equiv \log(\theta^*_t/\bar{\theta}^*),\\
\hat{b}_{Ht} &\equiv \log(b_{Ht}/\bar{b}_H), & \hat{v}^*_{Ht} &\equiv \log(v^*_{Ht}/\bar{v}^*_H),\\
\hat{v}_{Ft} &\equiv \log(v_{Ft}/\bar{v}_F), & \upsilon &\equiv \bar{v}_F/\bar{v}^*_H,\\
\hat{\Phi}_t &\equiv \log(\Phi_t/\bar{\Phi}), & \hat{\Phi}^*_t &\equiv \log(\Phi^*_t/\bar{\Phi}^*).
\end{aligned}$$

To first order, the two effective convenience yields satisfy

$$\hat{\Phi}_t = \hat{\theta}_t - \mathcal{A}_H \hat{b}_{Ht}, \qquad \hat{\Phi}^*_t = \hat{\theta}^*_t - \mathcal{A}_F \hat{v}^*_{Ht}, \tag{H.31}$$

where

$$\mathcal{A}_H \equiv \zeta_H - \varsigma_H, \qquad \mathcal{A}_F \equiv \zeta_F - \varsigma_F, \qquad \varsigma_H \equiv \frac{\bar{\Phi}\bar{b}_H}{\bar{C}}, \qquad \varsigma_F \equiv \frac{\bar{\Phi}^*\bar{v}^*_H}{\bar{C}^*}.$$

The coefficients $\mathcal{A}_H$ and $\mathcal{A}_F$ measure the net sensitivity of effective convenience yields to bond holdings. The curvature parameters ζ_H and ζ_F capture diminishing marginal convenience services, while ς_H and ς_F capture the associated first-order effect on the marginal utility of consumption. Thus, when $\mathcal{A}_j > 0$, an increase in a household's Treasury position lowers the effective convenience yield it assigns to an additional unit of Hegemon debt.

Write $\hat{b}_{Ht} \equiv \log(b_{Ht}/\bar{b}_H)$. Then, $(b_{Ht}/\bar{b}_H)^{1-\zeta_H} = \exp[(1-\zeta_H)\hat{b}_{Ht}] \approx 1 + (1-\zeta_H)\hat{b}_{Ht}$, so (H.3) gives

$$\nu_t \approx \frac{\bar{\Phi}\theta_t\bar{b}_H}{(1-\zeta_H)C_t}(1-\zeta_H)\hat{b}_{Ht} = \frac{\bar{\Phi}\theta_t\bar{b}_H}{C_t}\,\hat{b}_{Ht}.$$

Because $\hat{b}_{Ht}$ is already first order, variation in the prefactor enters only through products such as $\hat{\theta}_t\hat{b}_{Ht}$, which are second order. Evaluating the prefactor at the steady state and

using $\bar{\nu} = 0$ gives

$$\nu_t \simeq \varsigma_H \hat{b}_{Ht}, \qquad \nu_t^* \simeq \varsigma_F \hat{v}_{Ht}^*, \qquad \varsigma_H \equiv \frac{\bar{\Phi}\bar{b}_H}{\bar{C}}, \quad \varsigma_F \equiv \frac{\bar{\Phi}^*\bar{v}_H^*}{\bar{C}^*}. \tag{H.32}$$

Differentiating, $\chi_t \approx \dot{\nu}_t = \varsigma_H \dot{\hat{b}}_{Ht}$ and $\chi_t^* \approx \varsigma_F \dot{\hat{v}}_{Ht}^*$, and $\Xi_t \approx \nu_t^* - \nu_t$. From (H.4), $\hat{\Psi}_t \equiv \log(\Psi_t/\bar{\Phi}) = \hat{\theta}_t - \zeta_H \hat{b}_{Ht}$, and since $\Phi_t = \Psi_t/(1-\nu_t)$,

$$\hat{\Phi}_t = \hat{\Psi}_t - \log(1-\nu_t) = \hat{\theta}_t - \zeta_H \hat{b}_{Ht} + \varsigma_H \hat{b}_{Ht} = \hat{\theta}_t - \mathcal{A}_H \hat{b}_{Ht}, \tag{H.33}$$

and symmetrically $\hat{\Phi}_t^* = \hat{\theta}_t^* - \mathcal{A}_F \hat{v}_{Ht}^*$, which is (H.31).

H.3 Production, Price Setting, and Output

H.3.1 Production and Resource Accounting

In each country, a unit continuum of differentiated intermediate goods is aggregated by competitive final good producers via a Dixit–Stiglitz technology, $Y_{Ht} = [\int_0^1 Y_{Ht}(j)^{(\varepsilon-1)/\varepsilon} dj]^{\varepsilon/(\varepsilon-1)}$ with $\varepsilon > 1$, and symmetrically abroad. Cost minimisation yields

$$Y_{Ht}(j) = \Big[\frac{P_{Ht}(j)}{P_{Ht}}\Big]^{-\varepsilon} Y_{Ht}, \qquad P_{Ht} \equiv \Big[\int_0^1 P_{Ht}(j)^{1-\varepsilon}\, dj\Big]^{1/(1-\varepsilon)},$$

with a symmetric Foreign sector. Each Hegemon intermediate producer j operates $Y_{Ht}(j) = A_t N_t(j)$. Under flexible prices, cost minimisation and the isoelastic demand give the standard constant markup pricing rule $P_{Ht}(j) = \frac{\varepsilon}{\varepsilon-1}\frac{W_t}{A_t}$, i.e. $\mathrm{mc}_t^n = 1$. This condition characterises the reference allocation only. Under sticky prices $\mathrm{mc}_t \neq 1$ and the marginal cost wedge is carried explicitly in (H.44). Per-period nominal profits, net of adjustment costs, are

$$\Pi_t = P_{Ht}Y_{Ht} - W_t N_t - P_{Ht}\tfrac{\vartheta_H}{2}\pi_{Ht}^2 Y_{Ht}, \qquad \Pi_t^* = P_{Ft}^* Y_{Ft}^* - W_t^* N_t^* - P_{Ft}^*\tfrac{\vartheta_F}{2}(\pi_{Ft}^*)^2 Y_{Ft}^*,$$

so that household labour and profit income together equal

$$W_t N_t + \Pi_t = P_{Ht}Y_{Ht}\Big(1 - \tfrac{\vartheta_H}{2}\pi_{Ht}^2\Big), \qquad W_t^* N_t^* + \Pi_t^* = P_{Ft}^* Y_{Ft}^*\Big(1 - \tfrac{\vartheta_F}{2}(\pi_{Ft}^*)^2\Big). \tag{H.34}$$

Price adjustment and holding costs use resources. The latter are allocated across goods according to the respective consumption bundles, so goods market clearing is

$$\begin{aligned}\Big(1-\tfrac{\vartheta_H}{2}\pi_{Ht}^2\Big)Y_{Ht} &= C_{Ht}+C_{Ht}^*+(1-\gamma)\Big(\tfrac{P_{Ht}}{P_t}\Big)^{-\eta}\mathcal{K}_H(v_{Ft})+\gamma\Big(\tfrac{P_{Ht}^*}{P_t^*}\Big)^{-\eta}\mathcal{K}_F(v_{Ht}^*),\\ \Big(1-\tfrac{\vartheta_F}{2}(\pi_{Ft}^*)^2\Big)Y_{Ft}^* &= C_{Ft}+C_{Ft}^*+\gamma\Big(\tfrac{P_{Ft}}{P_t}\Big)^{-\eta}\mathcal{K}_H(v_{Ft})+(1-\gamma)\Big(\tfrac{P_{Ft}^*}{P_t^*}\Big)^{-\eta}\mathcal{K}_F(v_{Ht}^*).\end{aligned} \tag{H.35}$$

Productive labour and profit income satisfy (H.34). In addition, the cross-border fee paid by Foreign holders leads to the Hegemon financial-sector receipt $\Pi_t^I = \varkappa_t B_{Ht}^*$. Because this receipt is a financial redistribution rather than production income, it is kept separate from Π_t and does not enter either resource constraint. Including the counterpart receipt therefore closes the holding-wedge accounting without changing the government budgets, bond-market-clearing conditions, or resource constraints.

Hegemon firms are owned by Hegemon households, whose nominal discount factor is Λ_t in (H.5). Equity bears no convenience flow, so by (H.6) any nominal claim without a convenience service must be discounted at $-\dot{\Lambda}_t/\Lambda_t = i_t + \Phi_t$. Foreign firms are owned by Foreign households and, by (H.14), discount at i_t^*.

H.3.2 Rotemberg Inflation Dynamics

Lemma 1 (Rotemberg inflation dynamics)**.** *Producer price inflation in the two countries satisfies*

$$\Big[i_t+\Phi_t-\pi_{Ht}-\tfrac{\dot{Y}_{Ht}}{Y_{Ht}}\Big]\pi_{Ht} = \kappa_H(\mathrm{mc}_t-1)+\dot{\pi}_{Ht}, \tag{H.36}$$

$$\Big[i_t^*-\pi_{Ft}^*-\tfrac{\dot{Y}_{Ft}^*}{Y_{Ft}^*}\Big]\pi_{Ft}^* = \kappa_F(\mathrm{mc}_t^*-1)+\dot{\pi}_{Ft}^*, \tag{H.37}$$

where $\kappa_H \equiv (\varepsilon-1)/\vartheta_H$, $\kappa_F \equiv (\varepsilon-1)/\vartheta_F$, *and* $\mathrm{mc}_t \equiv \frac{\varepsilon}{\varepsilon-1}\frac{W_t}{P_{Ht}A_t}$ *is real marginal cost.*

Proof. The state variable is $P_{Ht}(j)$, the control variable is $\dot{P}_{Ht}(j)$, and the co-state variable is μ_t. The Hegemon firm solves

$$V_0(P_{H0}) = \max_{\{P_{Ht}(j)\}_{t\geq 0}} \int_0^\infty \exp\Big\{-\int_0^t\Big[i_s+\Phi_s\Big]ds\Big\}\Big[\Pi_t(P_{Ht}(j))-\Theta_t(j)\Big]dt,$$

with $\Theta_t(j) = \frac{\vartheta_H}{2}\Big(\dot{P}_{Ht}(j)/P_{Ht}(j)\Big)^2 P_{Ht}Y_{Ht}$. We can form the current value Hamiltonian as

$$\mathcal{H} = \Big[P_{Ht}(j)\Big(\tfrac{P_{Ht}(j)}{P_{Ht}}\Big)^{-\varepsilon}Y_{Ht}-\tfrac{W_t}{A_t}\Big(\tfrac{P_{Ht}(j)}{P_{Ht}}\Big)^{-\varepsilon}Y_{Ht}\Big]-\tfrac{\vartheta_H}{2}\Big(\tfrac{\dot{P}_{Ht}(j)}{P_{Ht}(j)}\Big)^2 P_{Ht}Y_{Ht}+\mu_t\,\dot{P}_{Ht}(j).$$

The presence of convenience yields only affects the *discount rate* in the co-state equation, and the instantaneous profit and adjustment cost terms are unchanged. The necessary

first-order conditions for firm optimality, together with the transversality condition $\lim_{T\to\infty} \exp\{-\int_0^T (i_s + \Phi_s)\, ds\}\, \mu_T\, P_{HT}(j) = 0$, are

$$\vartheta_H \frac{\dot{P}_{Ht}(j)}{P_{Ht}(j)} \frac{P_{Ht}}{P_{Ht}(j)} Y_{Ht} = \mu_t, \tag{H.38}$$

$$\dot{\mu}_t = \big(i_t + \Phi_t\big)\mu_t - \mathcal{H}_{P_H}, \tag{H.39}$$

where

$$\mathcal{H}_{P_H} = (1-\varepsilon)\Big(\tfrac{P_{Ht}(j)}{P_{Ht}}\Big)^{-\varepsilon} Y_{Ht} + \varepsilon \frac{W_t}{P_{Ht}(j)} \frac{1}{A_t} \Big(\tfrac{P_{Ht}(j)}{P_{Ht}}\Big)^{-\varepsilon} Y_{Ht} + \vartheta_H \Big(\tfrac{\dot{P}_{Ht}(j)}{P_{Ht}(j)}\Big)^2 \frac{P_{Ht}}{P_{Ht}(j)} Y_{Ht}.$$

We restrict our attention to symmetric domestic firm pricing. Imposing $P_{Ht}(j) = P_{Ht}$ and writing $\pi_{Ht} = \dot{P}_{Ht}/P_{Ht}$, condition (H.38) becomes $\mu_t = \vartheta_H \pi_{Ht} Y_{Ht}$ and condition (H.39) becomes

$$\dot{\mu}_t = (i_t + \Phi_t)\mu_t - \Big[(1-\varepsilon) Y_{Ht} + \varepsilon \tfrac{W_t}{P_{Ht} A_t} Y_{Ht} + \vartheta_H \pi_{Ht}^2 Y_{Ht}\Big]. \tag{H.40}$$

Differentiating $\mu_t = \vartheta_H \pi_{Ht} Y_{Ht}$ gives $\dot{\mu}_t = \vartheta_H(\dot{\pi}_{Ht} Y_{Ht} + \pi_{Ht} \dot{Y}_{Ht})$. Substituting into (H.40) and dividing by $\vartheta_H Y_{Ht}$,

$$\dot{\pi}_{Ht} + \pi_{Ht} \frac{\dot{Y}_{Ht}}{Y_{Ht}} = (i_t + \Phi_t)\pi_{Ht} + \frac{\varepsilon - 1}{\vartheta_H} - \frac{\varepsilon}{\vartheta_H} \frac{W_t}{P_{Ht} A_t} - \pi_{Ht}^2.$$

Since $\frac{\varepsilon-1}{\vartheta_H} - \frac{\varepsilon}{\vartheta_H}\frac{W_t}{P_{Ht}A_t} = -\kappa_H(\mathrm{mc}_t - 1)$ with $\kappa_H = (\varepsilon-1)/\vartheta_H$ and $\mathrm{mc}_t = \frac{\varepsilon}{\varepsilon-1}\frac{W_t}{P_{Ht}A_t}$, rearranging the final expression gives the claimed result in the statement of the lemma for Hegemon producers. The result for Foreign producers follows by a symmetric argument, with discount rate i_t^* in place of $i_t + \Phi_t$, giving (H.37). □

H.3.3 Output from Goods Demand and Labour Supply

Write $\lambda_H \equiv 1/(1+\phi)$ and $\lambda_F \equiv 1/(1+\phi^*)$. The output expressions (H.44)–(H.45) follow from goods demand and labour supply.

Step 1. Hegemon demand and goods market clearing. Using (15) and $Q_t = E_t P_t^*/P_t$,

$$\frac{P_{Ht}^*}{P_t^*} = \frac{P_{Ht}/E_t}{P_t Q_t / E_t} = \frac{P_{Ht}}{P_t}\frac{1}{Q_t} \quad \Longrightarrow \quad \Big(\tfrac{P_{Ht}^*}{P_t^*}\Big)^{-\eta} = \Big(\tfrac{P_{Ht}}{P_t}\Big)^{-\eta} Q_t^{\eta}.$$

Substituting (H.20) and (H.22) into the Hegemon resource constraint in (H.35),

$$\Big(1 - \tfrac{\vartheta_H}{2}\pi_{Ht}^2\Big) Y_{Ht} = \Big(\tfrac{P_{Ht}}{P_t}\Big)^{-\eta}\Big[(1-\gamma)(C_t + \mathcal{K}_H(v_{Ft})) + \gamma Q_t^{\eta}(C_t^* + \mathcal{K}_F(v_{Ht}^*))\Big].$$

The level risk-sharing relation $\log C_t - \log C_t^* - q_t = \Omega_t$ from (H.27) gives $C_t^* = C_t Q_t^{-1} e^{-\Omega_t}$, so that the consumption part of the bracket equals $C_t Z_t$ with Z_t as in (H.23). Using $P_{Ht}/P_t = G_t^{1/(\eta-1)}$ from (H.24),

$$\left(1 - \tfrac{\vartheta_H}{2}\pi_{Ht}^2\right) Y_{Ht} = G_t^{\eta/(1-\eta)}\left[C_t Z_t + (1-\gamma)\mathcal{K}_H(v_{Ft}) + \gamma Q_t^{\eta}\mathcal{K}_F(v_{Ht}^*)\right]$$

and hence

$$C_t = Y_{Ht}\left(1 - \tfrac{\vartheta_H}{2}\pi_{Ht}^2\right)\frac{G_t^{\eta/(\eta-1)}}{Z_t} - \frac{(1-\gamma)\mathcal{K}_H(v_{Ft}) + \gamma Q_t^{\eta}\mathcal{K}_F(v_{Ht}^*)}{Z_t}. \tag{H.41}$$

Step 2. Labour supply with a marginal cost wedge. From the definition of mc_t, $W_t/P_t = \frac{\varepsilon-1}{\varepsilon}\mathrm{mc}_t A_t (P_{Ht}/P_t)$. Combining with (H.8) and $N_t = Y_{Ht}/A_t$,

$$C_t\left(\frac{Y_{Ht}}{A_t}\right)^{\phi} = (1-\nu_t)\frac{\varepsilon-1}{\varepsilon}\,\mathrm{mc}_t\, A_t\, G_t^{1/(\eta-1)}. \tag{H.42}$$

Substituting (H.41) into (H.42) and using $G_t^{1/(\eta-1)-\eta/(\eta-1)} = G_t^{-1}$,

$$Y_{Ht}^{1+\phi} = A_t^{1+\phi}\,\frac{\varepsilon-1}{\varepsilon}\,\mathrm{mc}_t(1-\nu_t)\,G_t^{-1}Z_t\left(1 - \tfrac{\vartheta_H}{2}\pi_{Ht}^2\right)^{-1}\left[1 + \frac{(1-\gamma)\mathcal{K}_H(v_{Ft}) + \gamma Q_t^{\eta}\mathcal{K}_F(v_{Ht}^*)}{C_t Z_t}\right],$$

and taking the $(1+\phi)$-th root gives (H.44).

Step 3. Foreign output. The Foreign block is symmetric. We have $\left(P_{Ft}^*/P_t^*\right)^{-\eta} = \left(P_{Ft}/P_t\right)^{-\eta} Q_t^{\eta}$, and the Foreign resource constraint is

$$\left(1 - \tfrac{\vartheta_F}{2}(\pi_{Ft}^*)^2\right) Y_{Ft}^* = \left(\tfrac{P_{Ft}}{P_t}\right)^{-\eta}\left[\gamma(C_t + \mathcal{K}_H(v_{Ft})) + (1-\gamma)Q_t^{\eta}(C_t^* + \mathcal{K}_F(v_{Ht}^*))\right].$$

Inverting the risk-sharing relation gives $C_t = C_t^* Q_t e^{\Omega_t}$, so that the consumption part is

$$\gamma C_t + (1-\gamma)Q_t^{\eta}C_t^* = C_t^*\left[\gamma Q_t e^{\Omega_t} + (1-\gamma)Q_t^{\eta}\right] = C_t^* Q_t^{\eta}\left[(1-\gamma) + \gamma Q_t^{1-\eta}e^{\Omega_t}\right] = C_t^* Q_t^{\eta} Z_t^*. \tag{H.43}$$

Note that the substitution must be made into the bracket in its pre-substitution form, since otherwise spurious powers $Q_t^{2-\eta}$ and $e^{2\Omega_t}$ appear. Combining (H.43) with $\left(P_{Ft}/P_t\right)^{-\eta} Q_t^{\eta} = \left(P_{Ft}^*/P_t^*\right)^{-\eta} = (G_t^*)^{\eta/(1-\eta)}$ gives $\left(1 - \tfrac{\vartheta_F}{2}(\pi_{Ft}^*)^2\right) Y_{Ft}^* = (G_t^*)^{\eta/(1-\eta)}[C_t^* Z_t^* + \gamma Q_t^{-\eta}\mathcal{K}_H(v_{Ft}) + (1-\gamma)\mathcal{K}_F(v_{Ht}^*)]$,

and repeating the algebra of Steps 1 and 2 with starred variables gives

$$Y_{Ht} = A_t\Big[\tfrac{\varepsilon-1}{\varepsilon}\mathrm{mc}_t(1-\nu_t)\Big]^{\lambda_H} G_t^{-\lambda_H} Z_t^{\lambda_H}\Big(1-\tfrac{\vartheta_H}{2}\pi_{Ht}^2\Big)^{-\lambda_H} \times \left[1+\frac{(1-\gamma)\mathcal{K}_H(v_{Ft})+\gamma Q_t^{\eta}\mathcal{K}_F(v_{Ht}^*)}{C_t Z_t}\right]^{\lambda_H}, \tag{H.44}$$

$$Y_{Ft}^* = A_t^*\Big[\tfrac{\varepsilon-1}{\varepsilon}\mathrm{mc}_t^*(1-\nu_t^*)\Big]^{\lambda_F} (G_t^*)^{-\lambda_F} (Z_t^*)^{\lambda_F}\Big(1-\tfrac{\vartheta_F}{2}(\pi_{Ft}^*)^2\Big)^{-\lambda_F} \times \left[1+\frac{\gamma Q_t^{-\eta}\mathcal{K}_H(v_{Ft})+(1-\gamma)\mathcal{K}_F(v_{Ht}^*)}{C_t^* Z_t^*}\right]^{\lambda_F}. \tag{H.45}$$

H.4 Policy Rules and the Stationary Equilibrium

We close the model by specifying fiscal and monetary policy in each country. Fiscal authorities adjust primary surpluses in response to outstanding government debt, while central banks set nominal interest rates in response to inflation and output gaps. The interaction between these rules determines how fiscal and convenience-demand disturbances are absorbed through debt valuations, inflation, and international portfolios.

Fiscal rules. Let $s_t \equiv T_t/P_t$ and $s_t^* \equiv T_t^*/P_t^*$ denote real primary surpluses at the Hegemon and Foreign, respectively. The government budget constraints and debt valuation conditions are those of the analytical specification, equation (13). Convenience yields affect valuation only through equilibrium interest rates, since they do not enter the government budget constraints directly. The effect is nonetheless first order for the Hegemon: the steady-state real rate on Hegemon debt is $\bar{r} = \rho - \bar{\Phi}$ rather than ρ, so a given surplus stream supports a larger debt stock. The level of $\bar{\Phi}$ therefore matters through this convenience rent rather than through the Phillips curve discount rate.

Each fiscal authority chooses a feedback rule for the primary surplus (Leeper, 1991),

$$\hat{s}_t = \psi_b \hat{b}_t + \epsilon_t^s, \qquad \hat{s}_t^* = \psi_b^* \hat{b}_t^* + \epsilon_t^{s*}, \qquad \hat{s}_t \equiv \frac{s_t - \bar{s}}{\bar{b}}, \quad \hat{s}_t^* \equiv \frac{s_t^* - \bar{s}^*}{\bar{b}^*}, \tag{H.46}$$

where ϵ_t^s and ϵ_t^{s*} are the exogenous components of the debt-normalised surpluses, with steady state zero. A negative innovation lowers the primary surplus. In steady state, $\bar{s} = \bar{r}\bar{b}$, $\bar{s}^* = \bar{r}^*\bar{b}^*$ with $\bar{r} = \rho - \bar{\Phi}$ and $\bar{r}^* = \rho^*$. Fiscal policy is *passive* when $\psi_b > \bar{r}$, so that surpluses adjust enough to stabilise debt, and *active* when $\psi_b < \bar{r}$, so that the government commits to deficit finance. The equality $\psi_b = \bar{r}$ is a boundary case with zero direct debt feedback in the linearised debt law. The Foreign block is symmetric.

Monetary rules. Both central banks follow standard continuous-time Taylor rules with constant intercepts,

$$i_t = \bar{\imath} + \phi_\pi \hat{\pi}_t + \phi_x x_t, \qquad i_t^* = \bar{\imath}^* + \phi_\pi^* \hat{\pi}_t^* + \phi_x^* x_t^*, \tag{H.47}$$

where $\hat{\pi}_t$ and $\hat{\pi}_t^*$ denote CPI inflation relative to the zero inflation target, and x_t and x_t^* are the respective output gaps. The intercepts are the steady-state nominal rates, $\bar{\imath} = \rho - \bar{\Phi}$ and $\bar{\imath}^* = \rho^*$.

We hold these intercepts fixed rather than indexing them to the time-varying natural rates. A constant intercept rule is closer to the specification the empirical Taylor rule literature estimates, and it avoids requiring the central bank to observe and respond to the drift of the terms of trade gap. The monetary authority is considered *active* when $\phi_\pi > 1$ under $\phi_x = 0$ (the Taylor principle holds) and *passive* when $\phi_\pi < 1$ under $\phi_x = 0$.

Fiscal and monetary policy jointly determine how the government valuation conditions (13) are satisfied. Under an active-monetary, passive-fiscal regime, monetary policy stabilises inflation while primary surpluses adjust to stabilise debt. Under an active-fiscal, passive-monetary regime, future surpluses do not adjust sufficiently, so the price level moves to reconcile the real value of nominal debt with the present value of surpluses. The quantitative analysis considers both symmetric and asymmetric combinations of these policy regimes across the two countries.

H.4.1 Stationary Equilibrium

Set $\bar{A} = \bar{A}^* = 1$, $\bar{\theta} = \bar{\theta}^* = 1$, $\bar{\pi} = \bar{\pi}^* = 0$ and $\overline{\dot{E}/E} = 0$, and normalise price levels so that $\bar{Q} = \bar{E}\bar{P}^*/\bar{P} = 1$. The level of the steady-state nominal exchange rate $\bar{E}$ is then a free normalisation: all results are stated in log deviations $\hat{e}_t = \log(E_t/\bar{E})$, so its value plays no role. We set $\bar{E} = 0.8$, which implies $\bar{P}^*/\bar{P} = 1/\bar{E}$. The convenience yield specifications (H.3) and (H.12) are normalised so that

$$\bar{\nu} = \bar{\nu}^* = 0, \qquad \bar{\Phi} = \frac{\bar{\Psi}}{1 - \bar{\nu}} = \bar{\Psi}, \qquad \bar{\Phi}^* = \frac{\bar{\Psi}^*}{1 - \bar{\nu}^*} = \bar{\Psi}^*.$$

At $\bar{Q} = 1$, we impose $0 < \bar{v}_F < \bar{b}^*$ and $0 < \bar{v}_H^* < \bar{b}$, so that both households hold both bonds. Since marginal holding costs vanish in steady state, evaluating the four Euler equations (H.7), (H.9), (H.14) and (H.15) at zero consumption growth gives

$$\bar{\imath} = \rho - \bar{\Phi}, \qquad \bar{\imath}^* = \rho^*, \qquad \bar{\imath} = \rho^* + \bar{\varkappa} - \bar{\Phi}^*, \tag{H.48}$$

Under $\rho = \rho^*$, these conditions imply

$$\bar{\varkappa} = \bar{\Phi}^* - \bar{\Phi}, \qquad \bar{\imath} = \rho - \bar{\Phi}, \qquad \bar{\imath}^* = \rho. \tag{H.49}$$

The steady-state holding wedge absorbs the difference between the two households' marginal convenience valuations, which would have to coincide otherwise.

At $\overline{\text{tot}} = 1$ we have $\bar{G} = \bar{G}^* = 1$, and with $\bar{\Omega} = 0$ also $\bar{Z} = \bar{Z}^* = 1$. Setting $\pi_{Ht} = 0$ in (H.36) gives $\overline{\text{mc}} = \overline{\text{mc}}^* = 1$, so (H.44)-(H.45) reduce to

$$\bar{Y}_H = \left(\tfrac{\varepsilon-1}{\varepsilon}\right)^{\lambda_H}, \qquad \bar{Y}_F^* = \left(\tfrac{\varepsilon-1}{\varepsilon}\right)^{\lambda_F},$$

while goods market clearing and $\bar{\Omega} = 0$ require $\bar{Y}_H = \bar{C} = \bar{C}^* = \bar{Y}_F^* \equiv \bar{Y}$.

The steady-state fiscal positions satisfy $\bar{s} = \bar{r}\bar{b}$ and $\bar{s}^* = \bar{r}^*\bar{b}^*$ with

$$\bar{r} = \bar{\imath} - \bar{\pi} = \rho - \bar{\Phi}, \qquad \bar{r}^* = \bar{\imath}^* - \bar{\pi}^* = \rho^*, \qquad \bar{r}_v = \bar{\imath} - \bar{\varkappa} - \overline{\dot{E}/E} - \bar{\pi}^* = \rho^* - \bar{\Phi}^*. \tag{H.50}$$

The steady-state discount coefficients in the two Rotemberg problems are

$$\bar{\delta}_H \equiv \bar{\imath} + \bar{\Phi} - \overline{\dot{Y}_H/Y_H} - \bar{\pi}_H, \qquad \bar{\delta}_F \equiv \bar{\imath}^* - \overline{\dot{Y}_F^*/Y_F^*} - \bar{\pi}_F^*, \tag{H.51}$$

and stationarity, zero trend inflation, (H.50) and $\rho = \rho^*$ imply

$$\bar{\delta}_H = \rho, \qquad \bar{\delta}_F = \rho^*, \qquad \text{hence} \quad \bar{\delta}_H = \bar{\delta}_F = \rho = \rho^*. \tag{H.52}$$

The Hegemon discounts its surpluses at $\rho - \bar{\Phi} < \rho$, so that a given surplus stream supports a larger debt stock. This convenience rent is the first-order role of the *level* $\bar{\Phi}$ in the model. The Foreign investor's real return on Treasuries net of the wedge, $\bar{r}_v = \rho^* - \bar{\Phi}^*$, is exactly the rate at which its own convenience yield compensates the shortfall relative to its rate of time preference. By contrast, the firm discount rates in (H.51) are $\bar{\delta}_H = \bar{\imath} + \bar{\Phi} = \rho$ and $\bar{\delta}_F = \bar{\imath}^* = \rho^*$, so that the convenience yield in $\bar{\delta}_H$ is exactly offset by the lower nominal rate it induces and has no effect on the Phillips curve's steady-state discount coefficient.

Steady state and the transfer normalisation. At the steady state $\bar{P}_F^*/\bar{P}^* = 1$ and $\bar{Y}_F^* = \bar{C}^* = \bar{Y}$, the net external-return term in (H.77) is $\bar{r}_v\bar{v}_H^* - \bar{r}^*\bar{v}_F$ and is exactly offset by $-\bar{\mathfrak{T}}$. To retain a symmetric real allocation with asymmetric steady-state portfolios, the budget constraints (H.2) and (H.13) carry the constant lump-sum transfer $\bar{\mathfrak{T}} = \bar{r}_v\bar{v}_H^* - \bar{r}^*\bar{v}_F$, with $\bar{r}_v = \rho^* - \bar{\Phi}^*$ and $\bar{r}^* = \rho^*$ from (H.50). The transfer also closes the Hegemon side, since consolidating (H.2) with the Hegemon government budget and bond-market clearing shows that the

Hegemon household pays net external interest $(\bar{\imath} - \bar{\varkappa})\bar{v}^*_H - \bar{\imath}^*\bar{v}_F = \bar{r}_v\bar{v}^*_H - \bar{r}^*\bar{v}_F$ in steady state, which the transfer receipt exactly offsets, so that $\bar{C} = \bar{Y}$ on both sides.

H.5 The Conditional Reference and Gap Relationships

Relative prices switch expenditure across goods when $\eta \neq 1$. We write $\mathcal{S} \equiv 2\gamma(1-\gamma)(\eta-1)$ for this expenditure-switching coefficient, which vanishes in the Cobb–Douglas case.

Throughout, a superscript n denotes the conditional flexible price allocation evaluated at the same conditioning variables as the sticky price economy. In particular, the flexible price allocation is evaluated at the same productivity levels A_t, A^*_t, liquidity shifters θ_t, θ^*_t, holding wedge $\varkappa_t$, aggregate real debt stocks b_t, b^*_t, gross Foreign Treasury position v^*_{Ht}, and the relative-wealth state ϖ_t. It therefore removes nominal rigidity without eliminating the convenience-service wedges. This conditional reference is not a separate flexible-price dynamic equilibrium: the actual gross positions are determined jointly by the portfolio condition and the international budget constraint. We measure activity and relative prices relative to the flexible-price allocation. Define

$$x_t \equiv \log\left(\frac{Y_{Ht}}{Y^n_{Ht}}\right), \quad x^*_t \equiv \log\left(\frac{Y^*_{Ft}}{Y^{*n}_{Ft}}\right),$$
$$\tau_t \equiv \log \mathrm{tot}_t, \qquad \tau^n_t \equiv \log \mathrm{tot}^n_t, \qquad \tilde{\tau}_t \equiv \tau_t - \tau^n_t.$$

Here, Y_{Ht} and Y^*_{Ft} denote actual output in the Hegemon and Foreign economies, while Y^n_{Ht} and Y^{*n}_{Ft} denote the corresponding outputs in the conditional flexible-price allocation. Thus, x_t and x^*_t are the Hegemon and Foreign output gaps.

The variable τ_t is the log level of the terms of trade, $\mathrm{tot}_t = P_{Ft}/P_{Ht}$, while τ^n_t is its value in the conditional flexible-price allocation. Consequently, $\tilde{\tau}_t$ is the terms-of-trade gap. A positive $\tilde{\tau}_t$ means that Foreign goods are more expensive relative to Hegemon goods than in the reference allocation. Unlike the steady-state deviation τ_t, the gap $\tilde{\tau}_t$ removes the time-varying reference level generated by productivity, fiscal positions, international portfolios, and relative wealth. Its state dependence is derived below.

H.5.1 Portfolio Revaluation Relative to the Reference

Define

$$\Gamma_\Phi \equiv \bar{\Phi}\,\mathcal{A}_H \frac{(1-2\gamma)\omega_F}{\omega_H}, \qquad \Gamma_\Xi \equiv \varsigma_H \frac{(1-2\gamma)\omega_F}{\omega_H}.$$

As for bond market clearing, totally differentiating (H.30) at $\bar{b} = \bar{b}_H + \bar{v}^*_H$ and $\bar{Q} = 1$ gives

$$\hat{b}_t = \omega_H \hat{b}_{Ht} + \omega_F \Big[\hat{v}^*_{Ht} + (1-2\gamma)(\tau^n_t + \tilde{\tau}_t)\Big]. \tag{H.53}$$

The reference allocation is conditioned on the same $\hat{b}_t$ and the same $\hat{v}^*_{Ht}$, so it satisfies $\hat{b}_t = \omega_H \hat{b}^n_{Ht} + \omega_F[\hat{v}^*_{Ht} + (1-2\gamma)\tau^n_t]$. Subtracting,

$$0 = \omega_H\Big(\hat{b}_{Ht} - \hat{b}^n_{Ht}\Big) + \omega_F(1-2\gamma)\,\tilde{\tau}_t \quad \Longrightarrow \quad \hat{b}_{Ht} - \hat{b}^n_{Ht} = -\frac{\omega_F(1-2\gamma)}{\omega_H}\,\tilde{\tau}_t, \tag{H.54}$$

Because the Foreign Treasury position is held fixed in this conditional comparison, only the Hegemon Treasury position differs between the two allocations. Finally, applying (H.33) and (H.32),

$$\begin{aligned}
\Phi_t - \Phi^n_t &= \bar{\Phi}\Big(\hat{\Phi}_t - \hat{\Phi}^n_t\Big) = -\bar{\Phi}\mathcal{A}_H\Big(\hat{b}_{Ht} - \hat{b}^n_{Ht}\Big) = \bar{\Phi}\mathcal{A}_H\frac{(1-2\gamma)\omega_F}{\omega_H}\tilde{\tau}_t = \Gamma_\Phi\tilde{\tau}_t,\\
\Phi^*_t - \Phi^{*n}_t &= -\bar{\Phi}^*\mathcal{A}_F\Big(\hat{v}^*_{Ht} - \hat{v}^*_{Ht}\Big) = 0,\\
\nu_t - \nu^n_t &= \varsigma_H\Big(\hat{b}_{Ht} - \hat{b}^n_{Ht}\Big) = -\Gamma_\Xi\tilde{\tau}_t, \qquad \nu^*_t - \nu^{*n}_t = 0,
\end{aligned} \tag{H.55}$$

and therefore $\tilde{\Xi}_t \equiv \Xi_t - \Xi^n_t = (\nu^*_t - \nu^{*n}_t) - (\nu_t - \nu^n_t) = \Gamma_\Xi\tilde{\tau}_t$. Since $\varpi^n_t = \varpi_t$ by construction of the reference allocation, $\tilde{\Omega}_t = \tilde{\Xi}_t = \Gamma_\Xi\tilde{\tau}_t$, which completes (H.57). Differentiating (H.54) also gives the drift relation used in the IS derivation,

$$\chi_t - \chi^n_t = \varsigma_H\Big(\dot{\hat{b}}_{Ht} - \dot{\hat{b}}^n_{Ht}\Big) = -\Gamma_\Xi\dot{\tilde{\tau}}_t, \qquad \chi^*_t - \chi^{*n}_t = 0. \tag{H.56}$$

Collecting the resulting gap relations gives

$$\Phi_t - \Phi^n_t = \Gamma_\Phi\tilde{\tau}_t, \qquad \Phi^*_t - \Phi^{*n}_t = 0, \qquad \Omega_t - \Omega^n_t = \Gamma_\Xi\tilde{\tau}_t, \tag{H.57}$$

The coefficient Γ_Φ maps the revaluation-induced portfolio adjustment into the Hegemon effective convenience-yield gap, whereas Γ_Ξ maps it into the international risk-sharing gap. The change in the convenience component of Hegemon marginal utility also appears as a change in Ω_t.

These two coefficients summarise the portfolio revaluation channel in the equilibrium system. The convenience-yield effect Γ_Φ enters the Hegemon IS curve, while the risk-sharing effect Γ_Ξ shifts relative demand and generates the asymmetric terms-of-trade coefficients in the two Phillips curves derived below.

H.5.2 Coefficients for the Gap Relationships

The following coefficients shorten the gap and natural-rate expressions derived below.

The gap coefficients used in the derivations are

$$\begin{aligned} K^H_{\tilde\tau} &\equiv \mathcal{S} + (1-\gamma)\Gamma_\Xi, \qquad K^F_{\tilde\tau} \equiv \mathcal{S} - \gamma\Gamma_\Xi, \\ C_{\tilde\tau} &\equiv \gamma + \mathcal{S} - \gamma\Gamma_\Xi, \qquad \mathcal{X}_{\tilde\tau} \equiv 1 + 2\mathcal{S} + (1-2\gamma)\Gamma_\Xi, \\ \mathcal{W}_H &\equiv C_{\tilde\tau} + \Gamma_\Xi + \gamma(\phi_\pi - 1), \qquad \mathcal{W}_F \equiv C_{\tilde\tau} + \gamma(\phi^*_\pi - 1). \end{aligned} \tag{H.58}$$

Finally, the natural-rate derivation uses

$$\begin{gathered} \varrho_H \equiv (1-\gamma)\frac{\phi}{1+\phi}, \quad \varrho_F \equiv (1-\gamma)\frac{\phi^*}{1+\phi^*}, \quad \beta_H \equiv \gamma\frac{\phi}{1+\phi}, \quad \beta_F \equiv \gamma\frac{\phi^*}{1+\phi^*}, \\ \Upsilon_H \equiv -\gamma - \mathcal{S}(1-\lambda_H), \qquad \Upsilon_F \equiv \gamma + \mathcal{S}(1-\lambda_F). \end{gathered}$$

H.5.3 Output, Marginal Costs, and Consumption Gaps

Log-linearised output. Define $y_{Ht} \equiv \log(Y_{Ht}/\bar Y_H)$ and $y^*_{Ft} \equiv \log(Y^*_{Ft}/\bar Y^*_F)$. At the symmetric steady state, $\bar G = \bar G^* = \bar Z = \bar Z^* = 1$ and

$$\begin{aligned} \log G_t &\simeq \gamma(1-\eta)\tau_t, & \log G^*_t &\simeq \gamma(\eta-1)\tau_t, \\ \log Z_t &\simeq \gamma\Big[(\eta-1)q_t - \Omega_t\Big], & \log Z^*_t &\simeq \gamma\Big[(1-\eta)q_t + \Omega_t\Big]. \end{aligned} \tag{H.59}$$

The price adjustment cost factors are second-order, so they drop out. Since $\mathcal{K}_j = \mathcal{K}'_j = 0$ at the steady state, the holding-cost factors are also only of second-order, while $\log(1-\nu_t) \approx -\nu_t$ up to first-order. Taking log deviations of (H.44)–(H.45), using the first-order relation $q_t \simeq (1-2\gamma)\tau_t$ and $\gamma(\eta-1)(1+(1-2\gamma)) = 2\gamma(1-\gamma)(\eta-1) = \mathcal{S}$, gives

$$y_{Ht} = a_t + \lambda_H\Big[\widehat{\mathrm{mc}}_t - \nu_t + \mathcal{S}\tau_t - \gamma\Omega_t\Big], \tag{H.60}$$

$$y^*_{Ft} = a^*_t + \lambda_F\Big[\widehat{\mathrm{mc}}^*_t - \nu^*_t - \mathcal{S}\tau_t + \gamma\Omega_t\Big]. \tag{H.61}$$

Marginal cost gaps. The reference allocation sets $\widehat{\mathrm{mc}}^n_t = \widehat{\mathrm{mc}}^{*n}_t = 0$ and is evaluated at the same $a_t, a^*_t, \omega_t, \hat b_t, \hat v^*_{Ht}$. Subtracting it from (H.60) and using (H.55),

$$x_t = \lambda_H\Big[\widehat{\mathrm{mc}}_t + \Gamma_\Xi\tilde\tau_t + \mathcal{S}\tilde\tau_t - \gamma\Gamma_\Xi\tilde\tau_t\Big] = \lambda_H\Big[\widehat{\mathrm{mc}}_t + \Big(\mathcal{S} + (1-\gamma)\Gamma_\Xi\Big)\tilde\tau_t\Big],$$

and symmetrically $x^*_t = \lambda_F[\widehat{\mathrm{mc}}^*_t - (\mathcal{S} - \gamma\Gamma_\Xi)\tilde\tau_t]$. Inverting gives

$$\widehat{\mathrm{mc}}_t = (1+\phi)x_t - K^H_{\tilde\tau}\tilde\tau_t, \qquad \widehat{\mathrm{mc}}^*_t = (1+\phi^*)x^*_t + K^F_{\tilde\tau}\tilde\tau_t, \tag{H.62}$$

with $K^H_{\tilde\tau}$ and $K^F_{\tilde\tau}$ as in (H.58).

Consumption gaps. Define $c_t \equiv \log(C_t/\bar C)$ and $c^*_t \equiv \log(C^*_t/\bar C^*)$. Taking log deviations of (H.41), dropping the second-order holding-cost terms, and using (H.59),

$$c_t = y_{Ht} - \tfrac{\eta}{1-\eta}\gamma(1-\eta)\tau_t - \gamma\Big[(\eta-1)(1-2\gamma)\tau_t - \Omega_t\Big] = y_{Ht} - (\gamma+\mathcal{S})\tau_t + \gamma\Omega_t, \tag{H.63}$$

where the simplification uses $\mathcal{S} = 2\gamma(1-\gamma)(\eta-1)$. Symmetrically, $c^*_t = y^*_{Ft} + (\gamma+\mathcal{S})\tau_t - \gamma\Omega_t$. Subtracting the reference allocation and using $\tilde\Omega_t = \Gamma_\Xi\tilde\tau_t$ gives

$$\tilde c_t = x_t - C_{\tilde\tau}\tilde\tau_t, \qquad \tilde c^*_t = x^*_t + C_{\tilde\tau}\tilde\tau_t, \qquad C_{\tilde\tau} \equiv \gamma + \mathcal{S} - \gamma\Gamma_\Xi. \tag{H.64}$$

Allocation restriction. By definition $\tilde\Omega_t = \tilde c_t - \tilde c^*_t - \tilde q_t$ with $\tilde q_t \simeq (1-2\gamma)\tilde\tau_t$ to first order. Substituting (H.64),

$$\Gamma_\Xi\tilde\tau_t = x_t - x^*_t - \Big(2C_{\tilde\tau} + (1-2\gamma)\Big)\tilde\tau_t, \qquad 2C_{\tilde\tau} + (1-2\gamma) = 2\gamma + 2\mathcal{S} - 2\gamma\Gamma_\Xi + 1 - 2\gamma = 1 + 2\mathcal{S} - 2\gamma\Gamma_\Xi,$$

so that $x_t - x^*_t = \Big[1 + 2\mathcal{S} + (1-2\gamma)\Gamma_\Xi\Big]\tilde\tau_t = \mathcal{X}_{\tilde\tau}\tilde\tau_t$, which is (H.68).

H.5.4 Natural Terms of Trade and Real Rates

Natural terms of trade. Setting $\widehat{\mathrm{mc}}_t = \widehat{\mathrm{mc}}^*_t = 0$ in (H.60)-(H.61), the production side gives

$$y^n_{Ht} - y^{*n}_{Ft} = a_t - a^*_t - \lambda_H\nu^n_t + \lambda_F\nu^{*n}_t + \mathcal{S}(\lambda_H+\lambda_F)\tau^n_t - \gamma(\lambda_H+\lambda_F)\Omega^n_t. \tag{H.65}$$

Independently, the reference counterpart of (H.63) and its Foreign analogue, together with $\Omega^n_t = \log C^n_t - \log C^{*n}_t - q^n_t$ and the first-order relation $q^n_t \simeq (1-2\gamma)\tau^n_t$, give

$$\Omega^n_t = \Big(y^n_{Ht} - y^{*n}_{Ft}\Big) - 2(\gamma+\mathcal{S})\tau^n_t + 2\gamma\Omega^n_t - (1-2\gamma)\tau^n_t \Longrightarrow y^n_{Ht} - y^{*n}_{Ft} = \Big(1+2\mathcal{S}\Big)\tau^n_t + (1-2\gamma)\,\Omega^n_t, \tag{H.66}$$

where we used $2(\gamma+\mathcal{S}) + (1-2\gamma) = 1 + 2\mathcal{S}$. Equating (H.65) and (H.66),

$$(1 + 2\mathcal{S} - \mathcal{S}(\lambda_H+\lambda_F))\,\tau^n_t = a_t - a^*_t - \lambda_H\nu^n_t + \lambda_F\nu^{*n}_t - \mathcal{D}\,\Omega^n_t, \quad \mathcal{D} = (1-2\gamma) + \gamma(\lambda_H+\lambda_F).$$

Substituting $\Omega^n_t = \Xi^n_t + \varpi_t = \nu^{*n}_t - \nu^n_t + \varpi_t$ and collecting terms gives

$$(1 + 2\mathcal{S} - \mathcal{S}(\lambda_H+\lambda_F))\,\tau^n_t = a_t - a^*_t + (\mathcal{D} - \lambda_H)\nu^n_t + (\lambda_F - \mathcal{D})\nu^{*n}_t - \mathcal{D}\,\varpi_t.$$

Finally, $v_t^n = \varsigma_H \hat{b}_{Ht}^n$, $v_t^{*n} = \varsigma_F \hat{v}_{Ht}^*$ and the reference version of (H.53),

$$\hat{b}_{Ht}^n = \frac{1}{\omega_H}\Big[\hat{b}_t - \omega_F \hat{v}_{Ht}^* - \omega_F(1-2\gamma)\,\tau_t^n\Big],$$

give, after collecting the τ_t^n terms and using $\varsigma_H \omega_F (1-2\gamma)/\omega_H = \Gamma_\Xi$, the state mappings below.

The following auxiliary relations determine the natural terms of trade, recover their actual level and the Hegemon household's Treasury position, and link the two countries' output gaps:

$$\tau_t^n = \mathcal{M}^{-1}\Big[(a_t - a_t^*) + L_b \hat{b}_t + L_v \hat{v}_{Ht}^* - \mathcal{D}\varpi_t\Big], \tag{H.67}$$

$$\tau_t = \tau_t^n + \tilde{\tau}_t, \qquad \hat{b}_{Ht} = \omega_H^{-1}\Big[\hat{b}_t - \omega_F \hat{v}_{Ht}^* - \omega_F(1-2\gamma)\tau_t\Big],$$

$$x_t^* = x_t - \Big[1 + 2\mathcal{S} + (1-2\gamma)\Gamma_\Xi\Big]\tilde{\tau}_t, \tag{H.68}$$

where the composite coefficients are $\mathcal{D} \equiv (1-2\gamma) + \gamma(\lambda_H + \lambda_F)$, $\mathcal{M}\ 1 + 2\mathcal{S} - \mathcal{S}(\lambda_H + \lambda_F) + (\mathcal{D} - \lambda_H)\Gamma_\Xi$, $L_b \equiv \frac{(\mathcal{D}-\lambda_H)\varsigma_H}{\omega_H}$, and $L_v \equiv (\lambda_F - \mathcal{D})\varsigma_F - L_b\omega_F$. The coefficient $\mathcal{D}$ governs the effect of relative wealth on the natural terms of trade, while $\mathcal{M}$ scales its response to the underlying fundamentals. The coefficients L_b and L_v measure the effects of aggregate Hegemon debt and Foreign Treasury holdings, respectively.

Natural rates. The conditional Hegemon natural rate is defined as the real rate that (H.7) implies along the reference path,

$$r_t^n \equiv \rho + \frac{\dot{C}_t^n}{C_t^n} - \Phi_t^n + \chi_t^n, \qquad \bar{r}^n = \rho - \bar{\Phi} = \bar{r}, \qquad \hat{r}_t^n = \dot{c}_t^n - \bar{\Phi}\hat{\Phi}_t^n + \chi_t^n. \tag{H.69}$$

Substituting the reference version of (H.60) into (H.63),

$$c_t^n = a_t - \lambda_H v_t^n + \Upsilon_H\,\tau_t^n + \gamma(1-\lambda_H)\Omega_t^n, \qquad \Upsilon_H \equiv -\gamma - \mathcal{S}(1-\lambda_H),$$

Taking drifts, using $\chi_t^n = \dot{v}_t^n$ and $\dot{\Omega}_t^n = \dot{\Xi}_t^n + \dot{\omega}_t = (\chi_t^{*n} - \chi_t^n) + \dot{\omega}_t$,

$$\dot{c}_t^n = \dot{a}_t - \lambda_H \chi_t^n + \Upsilon_H \dot{\tau}_t^n + \gamma(1-\lambda_H)\Big[\chi_t^{*n} - \chi_t^n + \dot{\omega}_t\Big]. \tag{H.70}$$

Inserting (H.70) into (H.69) and collecting the χ_t^n terms,

$$\hat{r}_t^n = \dot{a}_t - \bar{\Phi}\hat{\Phi}_t^n + \underbrace{(1-\lambda_H)(1-\gamma)}_{\varrho_H}\chi_t^n + \underbrace{\gamma(1-\lambda_H)}_{\beta_H}\chi_t^{*n} + \beta_H\dot{\omega}_t + \Upsilon_H\dot{\tau}_t^n, \tag{H.71}$$

where $1-\lambda_H = \phi/(1+\phi)$. The Foreign natural rate is defined from (H.14) as $r_t^{*n} \equiv \rho^* + \dot{c}_t^{*n} + \chi_t^{*n}$, with $\bar{r}^{*n} = \rho^* = \bar{r}^*$. The reference Foreign consumption level is

$$c_t^{*n} = a_t^* - \lambda_F \nu_t^{*n} + \Upsilon_F \tau_t^n - \gamma(1-\lambda_F)\Omega_t^n, \qquad \Upsilon_F \equiv \gamma + \mathcal{S}(1-\lambda_F),$$

and the same steps give

$$\hat{r}_t^{*n} = \dot{a}_t^* + \beta_F \chi_t^n + \varrho_F \chi_t^{*n} - \beta_F \dot{\omega}_t + \Upsilon_F \dot{\tau}_t^n. \tag{H.72}$$

The conditional-reference objects entering (H.71)–(H.72) are $\chi_t^n = \varsigma_H \hat{\dot{b}}_{Ht}^n$, $\chi_t^{*n} = \varsigma_F \hat{\dot{\vartheta}}_{Ht}^*$, $\hat{\Phi}_t^n = \hat{\theta}_t - \mathcal{A}_H \hat{b}_{Ht}^n$, and τ_t^n from (H.67). We differentiate these reference objects along the actual conditioning paths, including $\dot{\omega}_t$. They therefore depend indirectly on the actual equilibrium and its policy configuration. Equation (H.73) provides an equivalent derivation from flexible-price labour supply.

An equivalent expression for natural rates follows from flexible-price labour supply. Write $N_t^n = Y_{Ht}^n/A_t$ and $N_t^{*n} = Y_{Ft}^{*n}/A_t^*$ for reference employment, and recall that productivity growth is $\dot{a}_t = \dot{A}_t/A_t$ and $\dot{a}_t^* = \dot{A}_t^*/A_t^*$. Differentiating flexible-price labour supply gives, to first order around the stationary equilibrium,

$$r_t^n = \rho + \dot{a}_t - \phi \frac{\dot{N}_t^n}{N_t^n} - \Phi_t^n - \gamma \dot{\tau}_t^n, \qquad r_t^{*n} = \rho + \dot{a}_t^* - \phi^* \frac{\dot{N}_t^{*n}}{N_t^{*n}} + \gamma \dot{\tau}_t^n. \tag{H.73}$$

With convex labour disutility, natural employment growth changes the real return required to support the reference consumption path, and CES demand affects both this employment response and the natural terms of trade. Setting $\phi = \phi^* = 0$ and $\eta = 1$ recovers the analytical specification's natural-rate formulas. Equations (H.71)–(H.72) give the state-based formulas used in computation.

H.6 Aggregate Dynamics and International Accounting

H.6.1 CPI Inflation and Exchange-Rate Identities

With $e_t \equiv \log E_t$ and $\bar{e} \equiv \log \bar{E}$, the deviation of the log nominal exchange rate from its steady state is $\hat{e}_t \equiv e_t - \bar{e} = \log(E_t/\bar{E})$. To first order, the price indices relate CPI inflation and nominal exchange-rate depreciation to producer-price inflation and terms-of-trade growth according to

$$\hat{\pi}_t = \hat{\pi}_{Ht} + \gamma \dot{\tau}_t, \qquad \hat{\pi}_t^* = \hat{\pi}_{Ft}^* - \gamma \dot{\tau}_t, \qquad \dot{\hat{e}}_t = \dot{\tau}_t - \hat{\pi}_{Ft}^* + \hat{\pi}_{Ht}. \tag{H.74}$$

H.6.2 IS Curves

Before substituting the monetary rules, the two output-gap IS equations are

$$\dot{x}_t = (r_t - r_t^n) + \Gamma_\Phi \tilde{\tau}_t + [\gamma + \mathcal{S} + (1-\gamma)\Gamma_\Xi]\dot{\tilde{\tau}}_t.$$

$$\dot{x}_t^* = (r_t^* - r_t^{*n}) - [\gamma + \mathcal{S} - \gamma\Gamma_\Xi]\dot{\tilde{\tau}}_t.$$

Subtracting the reference relation in (H.69) from the actual Euler equation (H.7),

$$\dot{\tilde{c}}_t = (i_t - \pi_t) - r_t^n + \left(\Phi_t - \Phi_t^n\right) - \left(\chi_t - \chi_t^n\right) = (i_t - \pi_t) - r_t^n + \Gamma_\Phi \tilde{\tau}_t + \Gamma_\Xi \dot{\tilde{\tau}}_t,$$

using (H.55) and (H.56). The monetary rule (H.47) gives $(i_t - \pi_t) - \bar{r} = (\phi_\pi - 1)\hat{\pi}_t + \phi_x x_t$ exactly, since the rule has a constant intercept, and $r_t^n - \bar{r} = \hat{r}_t^n$. Using $\tilde{c}_t = x_t - C_{\tilde{\tau}}\tilde{\tau}_t$ from (H.64) and $\hat{\pi}_t = \hat{\pi}_{Ht} + \gamma(\dot{\tau}_t^n + \dot{\tilde{\tau}}_t)$ from (H.74),

$$\dot{x}_t - C_{\tilde{\tau}}\dot{\tilde{\tau}}_t = (\phi_\pi - 1)\hat{\pi}_{Ht} + \gamma(\phi_\pi - 1)\left(\dot{\tau}_t^n + \dot{\tilde{\tau}}_t\right) + \phi_x x_t - \hat{r}_t^n + \Gamma_\Phi \tilde{\tau}_t + \Gamma_\Xi \dot{\tilde{\tau}}_t,$$

which rearranges to the policy-substituted Hegemon IS equation (H.83) with $\mathcal{W}_H = C_{\tilde{\tau}} + \Gamma_\Xi + \gamma(\phi_\pi - 1)$ as previously defined.

Repeating the argument for Foreign, and using $\chi_t^* - \chi_t^{*n} = 0$ and $\Phi_t^* - \Phi_t^{*n} = 0$ from (H.55), gives

$$\dot{x}_t^* = (\phi_\pi^* - 1)\hat{\pi}_{Ft}^* + \phi_x^* x_t^* - \hat{r}_t^{*n} - \mathcal{W}_F \dot{\tilde{\tau}}_t - \gamma(\phi_\pi^* - 1)\dot{\tau}_t^n, \qquad \mathcal{W}_F \equiv C_{\tilde{\tau}} + \gamma(\phi_\pi^* - 1). \tag{H.75}$$

The asymmetry $\mathcal{W}_H - \mathcal{W}_F = \Gamma_\Xi + \gamma(\phi_\pi - \phi_\pi^*)$ has two sources, namely differing monetary responses and the fact that only the Hegemon Treasury position differs in the conditional reference comparison.

Using the consumption-gap relations (H.64), the portfolio closures (H.57), the allocation restriction (H.68), the natural-rate definitions (H.71)–(H.72), the terms-of-trade equation (H.86), and the relative-wealth law (H.29), the Foreign IS curve (H.75) is implied by the Hegemon IS curve (H.83) and is therefore not a separate equilibrium condition.

H.6.3 Phillips Curves

Write (H.36) as $\dot{\pi}_{Ht} = \delta_{Ht}\pi_{Ht} - \kappa_H(\mathrm{mc}_t - 1)$ with $\delta_{Ht} \equiv i_t + \Phi_t - \pi_{Ht} - \dot{Y}_{Ht}/Y_{Ht}$ and $\bar{\delta}_H = \rho$. Setting $\pi_{Ht} = \bar{\pi}_H + \hat{\pi}_{Ht}$ and $\delta_{Ht} = \bar{\delta}_H + \hat{\delta}_{Ht}$,

$$\dot{\hat{\pi}}_{Ht} = \bar{\delta}_H\bar{\pi}_H + \bar{\delta}_H\hat{\pi}_{Ht} + \bar{\pi}_H\hat{\delta}_{Ht} + \hat{\delta}_{Ht}\hat{\pi}_{Ht} - \kappa_H\widehat{\mathrm{mc}}_t.$$

The product $\hat{\delta}_{Ht}\hat{\pi}_{Ht}$ is second order and is dropped at first order, and the zero-inflation target $\bar{\pi}_H = 0$ removes the remaining two terms, so that $\dot{\hat{\pi}}_{Ht} = \bar{\delta}_H \hat{\pi}_{Ht} - \kappa_H \widehat{\mathrm{mc}}_t$. Since $\overline{\mathrm{mc}} = 1$, the level deviation and the log deviation of marginal cost coincide to first order, and since the reference allocation satisfies $\mathrm{mc}^n_t = 1$ identically, the deviation *is* the gap. Substituting (H.62) gives (H.84), and the same steps with $\delta_{Ft} = i^*_t - \pi^*_{Ft} - \dot{Y}^*_{Ft}/Y^*_{Ft}$ and $\bar{\delta}_F = \rho^*$ give (H.85).

The terms-of-trade gap affects marginal costs through expenditure switching and the portfolio-induced change in relative consumption. The coefficients $\mathcal{S}$ and Γ_Ξ separate these two channels.

H.6.4 Terms of Trade Dynamics

Write the modified UIP condition (H.17) in deviations. Using $\bar{\imath} - \bar{\imath}^* = \bar{\varkappa} - \bar{\Phi}^*$ from (H.48), $\overline{\dot{E}/E} = 0$ and $\Phi^*_t = \bar{\Phi}^*(1 + \hat{\Phi}^*_t)$,

$$\hat{\imath}_t - \hat{\imath}^*_t - \hat{\varkappa}_t + \bar{\Phi}^*\hat{\Phi}^*_t - \frac{\psi_F \bar{v}^*_H}{\bar{Y}}\hat{v}^*_{Ht} = \dot{\hat{e}}_t, \qquad \hat{\imath}_t \equiv i_t - \bar{\imath}.$$

Substituting the monetary rules $\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_x x_t$ and $\hat{\imath}^*_t = \phi^*_\pi \hat{\pi}^*_t + \phi^*_x x^*_t$, the CPI–PPI identities $\hat{\pi}_t = \hat{\pi}_{Ht} + \gamma\dot{\tau}_t$, $\hat{\pi}^*_t = \hat{\pi}^*_{Ft} - \gamma\dot{\tau}_t$ and $\dot{\hat{e}}_t = \dot{\tau}_t - \hat{\pi}^*_{Ft} + \hat{\pi}_{Ht}$ from (H.74),

$$\phi_\pi\big(\hat{\pi}_{Ht} + \gamma\dot{\tau}_t\big) - \phi^*_\pi\big(\hat{\pi}^*_{Ft} - \gamma\dot{\tau}_t\big) + \phi_x x_t - \phi^*_x x^*_t + \bar{\Phi}^*\hat{\Phi}^*_t - \hat{\varkappa}_t - \frac{\psi_F \bar{v}^*_H}{\bar{Y}}\hat{v}^*_{Ht} = \dot{\tau}_t + \hat{\pi}_{Ht} - \hat{\pi}^*_{Ft},$$

and collecting the $\dot{\tau}_t$ terms gives (H.86), with $\hat{\Phi}^*_t = \hat{\theta}^*_t - \mathcal{A}_F \hat{v}^*_{Ht}$.

H.6.5 Debt Dynamics

Real government debt $b_t = B_t/P_t$ obeys $\dot{b}_t = (i_t - \pi_t)b_t - s_t$, from the nominal flow constraint $\dot{B}_t = i_t B_t - T_t$ and $s_t = T_t/P_t$. Write $b_t = \bar{b}\, e^{\hat{b}_t}$ and $s_t = \bar{s} + \bar{b}\,\hat{s}_t$ with $\bar{s} = \bar{r}\bar{b}$, so that

$$\dot{\hat{b}}_t = (i_t - \pi_t) - (\bar{r} + \hat{s}_t)\, e^{-\hat{b}_t} \simeq \bar{r}\,\hat{b}_t + \Big[(i_t - \pi_t) - \bar{r}\Big] - \hat{s}_t,$$

where the first-order form drops the second-order product $\hat{s}_t\hat{b}_t$. The monetary rule (H.47) gives $(i_t - \pi_t) - \bar{r} = (\phi_\pi - 1)\hat{\pi}_t + \phi_x x_t$ exactly, since the rule has a constant intercept, and the CPI–PPI relation (H.74) gives $\hat{\pi}_t = \hat{\pi}_{Ht} + \gamma\dot{\tau}_t$. Substituting these together with the fiscal rule $\hat{s}_t = \psi_b \hat{b}_t + \epsilon^s_t$ from (H.46) delivers (H.87). The Foreign budget law-of-motion (H.88) follows from the same steps with $\bar{r}^*$, $\hat{\pi}^*_t = \hat{\pi}^*_{Ft} - \gamma\dot{\tau}_t$ and $\hat{s}^*_t = \psi^*_b \hat{b}^*_t + \epsilon^{s*}_t$.

H.6.6 The International Budget and Relative Wealth

Let $b^*_{Ft} = B^*_{Ft}/P^*_t$ and $w^*_t = b^*_{Ft} + v^*_{Ht}$ denote Foreign household real financial wealth in Foreign consumption units. Dividing (H.13) by P^*_t and using the quotient rule,

$$\dot{w}^*_t = (i^*_t - \pi^*_t)b^*_{Ft} + \Big(i_t - \tfrac{\dot{E}_t}{E_t} - \varkappa_t - \pi^*_t\Big)v^*_{Ht} + \frac{W^*_t N^*_t + \Pi^*_t - T^*_t}{P^*_t} - C^*_t - \mathcal{K}_F(v^*_{Ht}) - \bar{\mathfrak{T}}. \tag{H.76}$$

Foreign bond market clearing is $b^*_t = b^*_{Ft} + v_{Ft}/Q_t$, so that $w^*_t - b^*_t = n^*_t = v^*_{Ht} - v_{Ft}/Q_t$. Subtracting the Foreign government's constraint from (H.76), the domestic fiscal terms cancel, leaving

$$\dot{n}^*_t = \Big(i_t - \tfrac{\dot{E}_t}{E_t} - \varkappa_t - \pi^*_t\Big)v^*_{Ht} - (i^*_t - \pi^*_t)\frac{v_{Ft}}{Q_t} + \frac{W^*_t N^*_t + \Pi^*_t}{P^*_t} - C^*_t - \mathcal{K}_F(v^*_{Ht}) - \bar{\mathfrak{T}}, \tag{H.77}$$

Using (H.34), this becomes the nonlinear balance of payments equation

$$\dot{n}^*_t = \Big(i_t - \tfrac{\dot{E}_t}{E_t} - \varkappa_t - \pi^*_t\Big)v^*_{Ht} - (i^*_t - \pi^*_t)\frac{v_{Ft}}{Q_t} + \frac{P^*_{Ft}}{P^*_t}\Big(1 - \tfrac{\vartheta_F}{2}(\pi^*_{Ft})^2\Big)Y^*_{Ft} - C^*_t - \mathcal{K}_F(v^*_{Ht}) - \bar{\mathfrak{T}}. \tag{H.78}$$

The term $-\varkappa_t v^*_{Ht}$ in the Foreign external return has the Hegemon counterpart

$$\frac{\Pi^I_t}{P_t} = \varkappa_t \frac{B^*_{Ht}}{P_t} = \varkappa_t Q_t v^*_{Ht}.$$

Thus, the wedge is clearly a cross-country financial transfer. Multiplying the Foreign payment $\varkappa_t v^*_{Ht}$ by the real exchange rate Q_t gives exactly the Hegemon payment. At the steady state, $\bar{\Pi}^I = \bar{\varkappa}\bar{B}^*_H$ and, under $\bar{Q} = 1$, $\bar{\Pi}^I/\bar{P} = \bar{\varkappa}\bar{v}^*_H$. If $\bar{\varkappa} = 0$, both the payment and receipt are zero in steady state, although shocks to $\varkappa_t$ can still move returns and international wealth.

Given the two goods market clearing conditions, the two government budget constraints and the two bond market clearing conditions, (H.77) is the one household budget constraint that Walras' law does not deliver. It is therefore an independent equilibrium condition, determining the level of relative consumption.

Linearisation. To first order, $n^*_t - \bar{n}^* = \bar{v}^*_H \hat{v}^*_{Ht} - \bar{v}_F[\hat{v}_{Ft} - (1-2\gamma)\tau_t]$, so that

$$\dot{n}^*_t = \bar{v}^*_H \dot{\hat{v}}^*_{Ht} - \bar{v}_F\Big[\dot{\hat{v}}_{Ft} - (1-2\gamma)\dot{\tau}_t\Big].$$

The terms on the right of (H.78) expand to

$$\begin{aligned}\Big(i_t - \tfrac{\dot{E}_t}{E_t} - \varkappa_t - \pi^*_t\Big)v^*_{Ht} &= \bar{r}_v\bar{v}^*_H + \bar{v}^*_H\Big(\hat{\imath}_t - \dot{\hat{e}}_t - \hat{\varkappa}_t - \hat{\pi}^*_t\Big) + \bar{r}_v\bar{v}^*_H\hat{v}^*_{Ht},\\ -(i^*_t - \pi^*_t)\frac{v_{Ft}}{Q_t} &= -\bar{r}^*\bar{v}_F - \bar{v}_F(\hat{\imath}^*_t - \hat{\pi}^*_t) - \bar{r}^*\bar{v}_F\Big[\hat{v}_{Ft} - (1-2\gamma)\tau_t\Big],\\ \frac{P^*_{Ft}}{P^*_t}Y^*_{Ft} - C^*_t &= \bar{Y}\Big[\gamma\tau_t + y^*_{Ft} - c^*_t\Big], \qquad -\bar{\mathfrak{T}} = -\bar{r}_v\bar{v}^*_H + \bar{r}^*\bar{v}_F,\end{aligned} \tag{H.79}$$

where the trade-balance line uses the first-order relation $\log P^*_{Ft} - \log P^*_t \simeq \gamma\tau_t$ from (H.26) and the fact that the adjustment cost factor is second order. From (H.63) applied to Foreign, $y^*_{Ft} - c^*_t = -(\gamma + \mathcal{S})\tau_t + \gamma\Omega_t$, so that the linearised trade balance is

$$\frac{\mathrm{TB}^*_t}{\bar{Y}} = \gamma\tau_t + y^*_{Ft} - c^*_t = -\mathcal{S}\,\tau_t + \gamma\Omega_t,$$

For $\eta > 1$, a rise in the relative price of Foreign goods worsens the Foreign trade balance, and a rise in Hegemon relative to Foreign consumption improves it. The real return term expands, using the monetary rule and (H.74), as

$$\hat{\imath}_t - \dot{\hat{e}}_t - \hat{\pi}^*_t = \phi_\pi\Big(\hat{\pi}_{Ht} + \gamma\dot{\tau}_t\Big) + \phi_x x_t - \Big(\dot{\tau}_t + \hat{\pi}_{Ht} - \hat{\pi}^*_{Ft}\Big) - \Big(\hat{\pi}^*_{Ft} - \gamma\dot{\tau}_t\Big) = (\phi_\pi - 1)\hat{\pi}_{Ht} + \phi_x x_t + \Big[\gamma\phi_\pi - (1-\gamma)\Big]\dot{\tau}_t.$$

The liability return uses $\hat{\imath}^*_t - \hat{\pi}^*_t = (\phi^*_\pi - 1)\hat{\pi}^*_{Ft} + \phi^*_x x^*_t - \gamma(\phi^*_\pi - 1)\dot{\tau}_t$. Dividing (H.79) by $\bar{v}^*_H$, retaining the $-\hat{\varkappa}_t$ term, and using $\upsilon = \bar{v}_F/\bar{v}^*_H$ delivers (H.89). Finally, from (H.32) and (H.27),

$$\Omega_t = \Xi_t + \varpi_t = \Big(\nu^*_t - \nu_t\Big) + \varpi_t = \varsigma_F\hat{v}^*_{Ht} - \varsigma_H\hat{b}_{Ht} + \varpi_t,$$

An equivalent form in terms of the reference position follows from (H.54), namely $\Omega_t = \varsigma_F\hat{v}^*_{Ht} - \varsigma_H\hat{b}^n_{Ht} + \varpi_t + \Gamma_\Xi\tilde{\tau}_t$.

The relative-wealth law. Linearising (H.29) gives the reduced relative-wealth law (H.90). Linearising (H.18), substituting (H.33) and its Foreign counterpart, and using $\bar{\Phi} - \bar{\Phi}^* + \bar{\varkappa} = 0$ gives the static portfolio condition (H.91). The marginal holding costs support differences in the households' net convenience valuations.

H.7 The Reduced Equilibrium and Its Solution

This subsection collects the reduced equilibrium with bilateral holdings and quadratic holding costs, which gives the system we solve and estimate in Section 4.

The model is driven by seven exogenous states: productivity in each country, the two convenience-demand shifters, the two fiscal disturbances, and the cross-border holding

wedge. Between innovations, these states revert deterministically towards their steady-state values:

$$\begin{aligned} \dot{a}_t &= -\varphi_a a_t, \quad \dot{a}^*_t = -\varphi^*_a a^*_t, \quad \dot{\hat{\theta}}_t = -\varphi_\theta \hat{\theta}_t, \quad \dot{\hat{\theta}}^*_t = -\varphi^*_\theta \hat{\theta}^*_t, \\ \dot{\epsilon}^s_t &= -\varphi_s \epsilon^s_t, \quad \dot{\epsilon}^{s*}_t = -\varphi^*_s \epsilon^{s*}_t, \quad \dot{\hat{\varkappa}}_t = -\varphi_\varkappa \hat{\varkappa}_t. \end{aligned} \tag{H.80}$$

The coefficients $\varphi_j > 0$ govern the corresponding speeds of mean reversion.

We collect these states in the vector

$$z_t = (a_t, a^*_t, \hat{\theta}_t, \hat{\theta}^*_t, \epsilon^s_t, \epsilon^{s*}_t, \hat{\varkappa}_t)'.$$

and compute impulse responses to an unanticipated one-time (MIT) innovation to component j, imposed at $t = 0$ as

$$[z_{0^+}]_j = [z_{0^-}]_j + \Delta z_j, \qquad [z_{0^+}]_k = [z_{0^-}]_k \quad \text{for } k \neq j, \tag{H.81}$$

where $[z_t]_j$ is the jth component of z_t and Δz_j is the size of the innovation. A fiscal expansion has $\Delta z_j < 0$ in the Hegemon or Foreign surplus component. Conditional on this impact, no further innovations occur. If the economy is initially in its stationary steady state, so that $z_{0^-} = 0$, the shocked component subsequently follows

$$[z_t]_j = \Delta z_j \, e^{-\varphi_j t}, \qquad t \geq 0.$$

A local competitive equilibrium consists of household consumption, labour and bond holdings, firm production and prices, and policy paths satisfying the problems and market-clearing conditions above. At first order, it is represented by paths for

$$(x_t, \hat{\pi}_{Ht}, \hat{\pi}^*_{Ft}, \tilde{\tau}_t, \hat{b}_t, \hat{b}^*_t, \hat{v}^*_{Ht}, \varpi_t, \hat{v}_{Ft}). \tag{H.82}$$

conditional on the exogenous processes in (H.80) and the innovation in (H.81). These paths are piecewise continuously differentiable away from the impact date. Foreign output, consumption, labour, convenience yields and the exchange rate are recovered from the auxiliary relations. The Foreign IS equation is implied by the other equilibrium conditions.

For $t > 0$, the equilibrium is characterised by eight differential equations and one static portfolio condition: the Hegemon IS curve, the two Phillips curves, the terms-of-trade equation, the two government debt laws, the Foreign balance of payments, the relative-wealth law of motion, and the relative portfolio allocation:

$$\dot{x}_t = (\phi_\pi - 1)\hat{\pi}_{Ht} + \phi_x x_t + \Gamma_\Phi \tilde{\tau}_t - \hat{r}^n_t + \Big[\mathcal{S} + (1-\gamma)\Gamma_\Xi + \gamma\phi_\pi\Big]\dot{\tilde{\tau}}_t + \gamma(\phi_\pi - 1)\dot{\tau}^n_t, \tag{H.83}$$

$$\dot{\hat{\pi}}_{Ht} = \rho\hat{\pi}_{Ht} - \kappa_H\Big\{(1+\phi)x_t - \Big[\mathcal{S} + (1-\gamma)\Gamma_\Xi\Big]\tilde{\tau}_t\Big\}, \tag{H.84}$$

$$\dot{\hat{\pi}}^*_{Ft} = \rho\hat{\pi}^*_{Ft} - \kappa_F\Big\{(1+\phi^*)x^*_t + \Big[\mathcal{S} - \gamma\Gamma_\Xi\Big]\tilde{\tau}_t\Big\}, \tag{H.85}$$

$$\Big[1 - \gamma(\phi_\pi + \phi^*_\pi)\Big]\dot{\tau}_t = (\phi_\pi - 1)\hat{\pi}_{Ht} - (\phi^*_\pi - 1)\hat{\pi}^*_{Ft} + \phi_x x_t - \phi^*_x x^*_t + \bar{\Phi}^*\Big(\hat{\theta}^*_t - \mathcal{A}_F\hat{v}^*_{Ht}\Big) - \hat{\varkappa}_t - \frac{\psi_F\bar{v}^*_H}{\bar{Y}}\hat{v}^*_{Ht}, \tag{H.86}$$

$$\dot{\hat{b}}_t = (\bar{r} - \psi_b)\hat{b}_t + (\phi_\pi - 1)\hat{\pi}_{Ht} + \gamma(\phi_\pi - 1)\dot{\tau}_t + \phi_x x_t - \epsilon^s_t, \tag{H.87}$$

$$\dot{\hat{b}}^*_t = (\bar{r}^* - \psi^*_b)\hat{b}^*_t + (\phi^*_\pi - 1)\hat{\pi}^*_{Ft} - \gamma(\phi^*_\pi - 1)\dot{\tau}_t + \phi^*_x x^*_t - \epsilon^{s*}_t, \tag{H.88}$$

$$\dot{\hat{v}}^*_{Ht} = \upsilon\Big[\dot{\hat{v}}_{Ft} - (1-2\gamma)\dot{\tau}_t\Big] + \bar{r}_v\hat{v}^*_{Ht} - \upsilon\bar{r}^*\Big[\hat{v}_{Ft} - (1-2\gamma)\tau_t\Big] + (\phi_\pi - 1)\hat{\pi}_{Ht} + \phi_x x_t + \Big[\gamma\phi_\pi - (1-\gamma)\Big]\dot{\tau}_t - \hat{\varkappa}_t - \upsilon\Big[(\phi^*_\pi - 1)\hat{\pi}^*_{Ft} + \phi^*_x x^*_t - \gamma(\phi^*_\pi - 1)\dot{\tau}_t\Big] + \left(\frac{\bar{v}^*_H}{\bar{Y}}\right)^{-1}\Big[-\mathcal{S}\tau_t + \gamma\Omega_t\Big], \tag{H.89}$$

$$\dot{\omega}_t = -\frac{\psi_H\bar{v}_F}{\bar{Y}}\hat{v}_{Ft}, \tag{H.90}$$

$$0 = \bar{\Phi}\Big[\hat{\theta}_t - \mathcal{A}_H\hat{b}_{Ht}\Big] - \bar{\Phi}^*\Big[\hat{\theta}^*_t - \mathcal{A}_F\hat{v}^*_{Ht}\Big] + \hat{\varkappa}_t + \frac{\psi_H\bar{v}_F}{\bar{Y}}\hat{v}_{Ft} + \frac{\psi_F\bar{v}^*_H}{\bar{Y}}\hat{v}^*_{Ht}. \tag{H.91}$$

We impose that $1 - \gamma(\phi_\pi + \phi^*_\pi) \neq 0$. Since $\psi_H\bar{v}_F/\bar{Y} > 0$, (H.91) determines $\hat{v}_{Ft}$. After eliminating this position, we require the reduced derivative matrix to be nonsingular. The mappings in (H.31), (H.67), (H.68) and (H.74) then close the system. Equilibrium paths also satisfy the household and government transversality conditions, as is standard.

H.7.1 Impact Restrictions and Terminal Conditions

At $t = 0$, total nominal sovereign liabilities are inherited, so $B_{0^+} = B_{0^-}$ and $B^*_{0^+} = B^*_{0^-}$. Admissible Rotemberg price paths satisfy $P_{H,0^+} = P_{H,0^-}$ and $P^*_{F,0^+} = P^*_{F,0^-}$. The nominal exchange rate and forward-looking consumption and inflation variables may jump. The

induced impact valuations are

$$b_{0^+} = \frac{B_{0^-}}{P_{0^+}}, \qquad b^*_{0^+} = \frac{B^*_{0^-}}{P^*_{0^+}}, \qquad n^*_{0^+} = \frac{B^*_{H,0^-}}{E_{0^+}P^*_{0^+}} - \frac{B_{F,0^-}}{P^*_{0^+}}.$$

They satisfy bond-market clearing and the price-index mappings. Real debt and net foreign assets therefore inherit the valuation effects of the exchange-rate jump. The portfolio condition (H.18) must hold immediately after an innovation, so the two gross cross-border positions cannot in general both be inherited in nominal terms. Households rebalance their gross positions through self-financing trades, which the holding costs allow because they depend on levels rather than on rates of change. Starting from the stationary steady state, the first-order net-wealth restriction is

$$\hat{v}^*_{H,0^+} - \upsilon\, \hat{v}_{F,0^+} + (1-\gamma)(1+\upsilon)\, \tau_{0^+} = 0, \tag{H.92}$$

with $\upsilon = \bar{v}_F/\bar{v}^*_H$. An exchange-rate jump therefore revalues Foreign net foreign assets, while the gross positions adjust to satisfy the portfolio condition.

Relative marginal wealth obeys a differential law along a perfectly foreseen transition. At an unexpected innovation, its impact value also reflects the revision in household marginal utilities. With the marginal utilities of consumption $u_C = (1-\nu_t)/C_t$ and $u_{C^*} = (1-\nu^*_t)/C^*_t$, which are the current-value real multipliers on the household budgets,

$$\varpi_t = \log u_{C^*} - \log u_C - q_t. \tag{H.93}$$

Thus ϖ_{0^+} is determined jointly with the post-innovation consumption plans, initial budgets, and terminal conditions. Continuity of ϖ_t at an unanticipated shock would be an additional restriction on relative marginal wealth. It does not follow from inherited nominal portfolios or from integrating the perfect-foresight Euler equations over $t > 0$. For a foreseen transition without new information, the differential law applies throughout.

We select locally admissible paths using these initial restrictions, household solvency and transversality conditions, and the terminal government conditions. With transitory disturbances, we restrict attention to solutions converging to the stationary equilibrium. In numerical implementations, the classification of ϖ_t at impact must follow equation (H.93). Our solution therefore treats ϖ_t as a jump variable: its impact value is determined jointly with the other forward-looking variables, and continuity at an unanticipated shock is not imposed.

I Counterfactually High Convenience Yields

Convenience yields much larger than in the benchmark change the dynamics of the model in two ways: (a) fiscal deficits have real effects even under a monetary-led policy mix, and (b) the effects of Hegemon Dominance on the Foreign economy become much larger.

Both the level and the shape of the convenience-yield specification matter for model dynamics, operating through changes to the non-pecuniary value of government bonds as well as through the budget path itself. In the benchmark calibration, an increase in bond supply lowers the convenience yield earned by households in both countries.

We now set the steady-state convenience yields two to three orders of magnitude higher, at $\bar{\Phi} = \bar{\Phi}^* = 0.1$ per quarter, and raise the rate of time preference to $\rho = 0.11$, so that the steady-state Hegemon real rate $\rho - \bar{\Phi}$ remains positive. We keep the rest of the model unchanged, including the elasticities of the convenience yield functions, placing us in a regime in which both households derive *substantial* utility from Hegemon bonds. Because the rate of time preference changes as well, the comparison with the benchmark reflects both higher convenience yields and greater impatience. We view this specification as an illustration of what higher convenience-yield levels can do in our framework.

We first analyse the response to the same 1-basis-point fall in the Hegemon convenience yield as in the main text, which is now small relative to its steady-state level. Figure I.1 presents the responses.

Under the monetary-led, fiscally-led, and mixed policy configurations, the responses resemble those of the benchmark: the Hegemon output gap and inflation rise on impact, while the Foreign economy barely moves. Under the monetary-led policy mix, the Hegemon central bank raises its rate, and the Hegemon currency depreciates. The mixed configuration coincides almost exactly with the fiscally-led policy mix, which shows that, when convenience yields are high, the Hegemon fiscal authority, which issues the global safe asset, largely determines the responses. Under Hegemon Dominance, the Foreign effects are again much larger than under any other configuration: the Hegemon currency appreciates persistently, Hegemon output falls, and Foreign output and inflation rise.

We next turn to the expansionary Hegemon transfer shock of 1 percent of quarterly output. Figure I.2 presents the results.

Under the monetary-led policy mix, the fiscal expansion now has real effects: the Hegemon output gap and inflation rise, and cumulative Hegemon inflation over five years is about 0.8 percent, even though the Hegemon central bank raises its policy rate. The Hegemon currency depreciates gradually, while the Foreign economy is comparatively insulated, with a slight fall in cumulative Foreign inflation. Under the fiscally-led policy

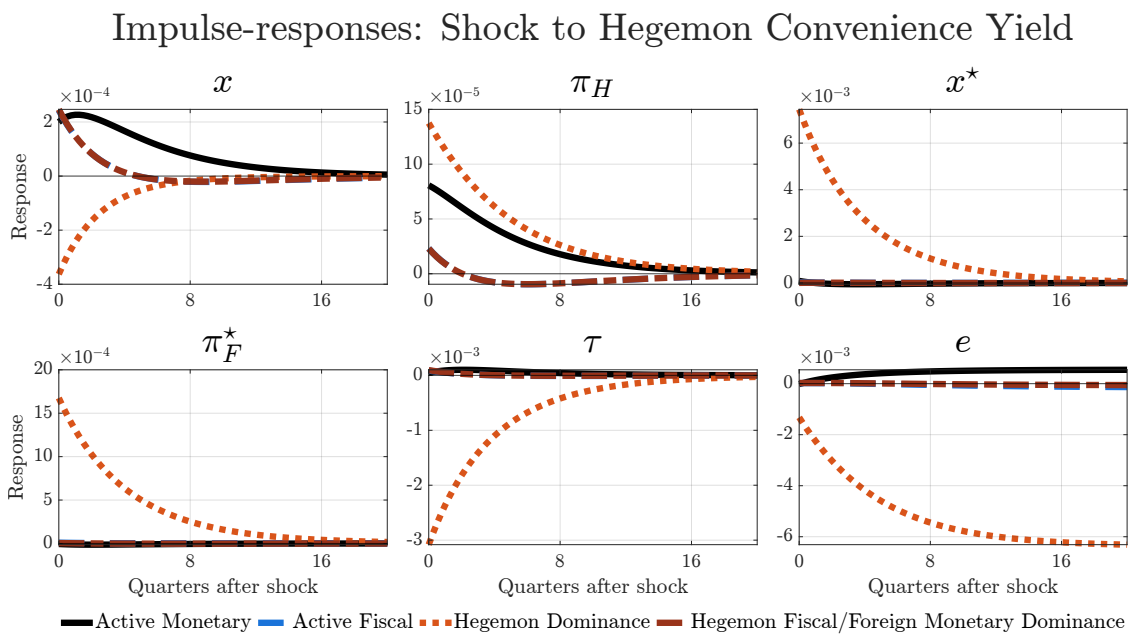


Figure I.1: Impulse responses to a 1-basis-point fall in the Hegemon convenience yield under the high convenience-yield calibration ($\bar{\Phi} = \bar{\Phi}^* = 0.1$, $\rho = 0.11$). The legend is as in figure 5.

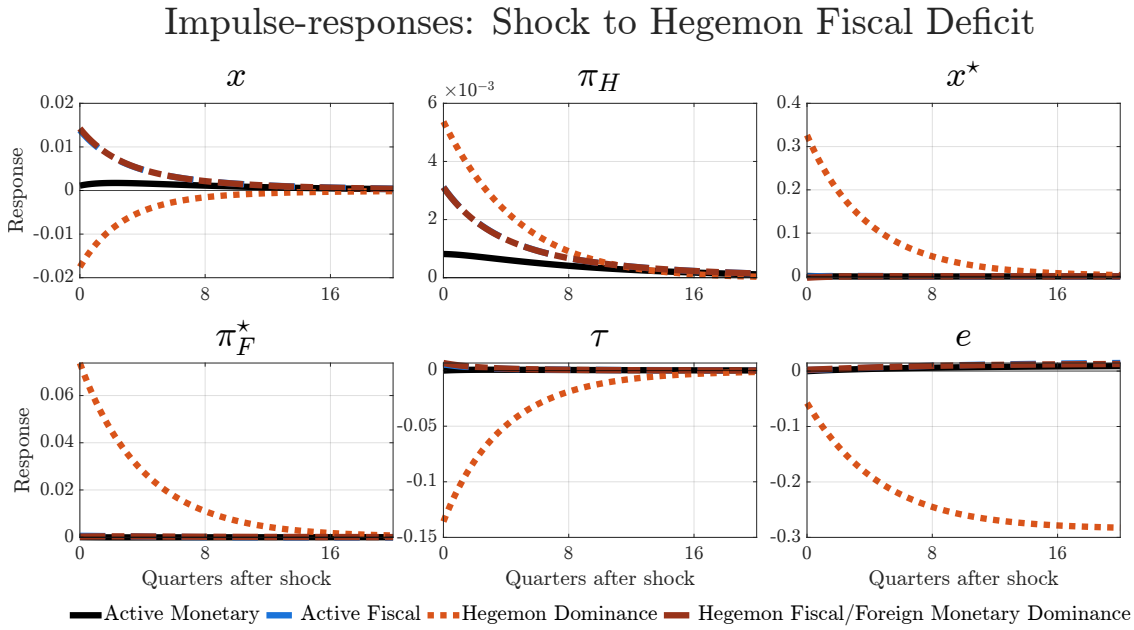


Figure I.2: Impulse responses to an expansionary Hegemon transfer shock of 1 percent of quarterly output under the high convenience-yield calibration ($\bar{\Phi} = \bar{\Phi}^* = 0.1$, $\rho = 0.11$). The legend is as in figure 5.

mix, the transfer is expansionary at the Hegemon, raising cumulative Hegemon inflation by about 1.6 percent, but Foreign inflation barely moves: its cumulative rise falls from about 1.3 percent in the benchmark to 0.15 percent, while the Hegemon currency depreciates persistently. High convenience yields thus limit the fiscal contagion that the model is designed to isolate. Under Hegemon Dominance, by contrast, the spillover becomes much larger: the Hegemon currency appreciates by about 28 percent after five years, Foreign households gain as their Treasury holdings appreciate in Foreign currency, and the cumulative rise in the Foreign price level reaches about 31 percent. Under the mixed configuration, the active Foreign central bank again keeps the Foreign output gap and inflation close to steady state.

Figure I.3 decomposes the responses to the transfer shock under the high convenience-yield calibration, using the method of Section 4.3.

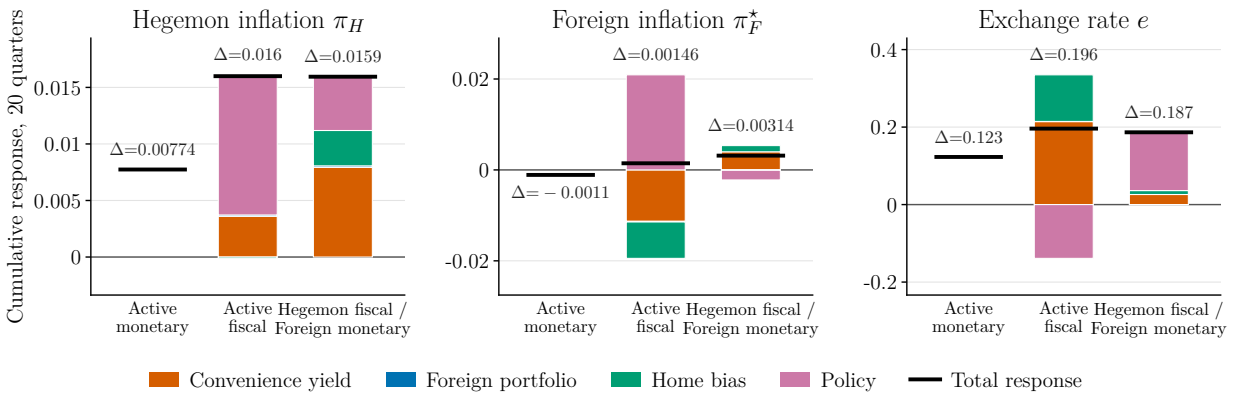


Figure I.3: Decomposition of the cumulative 20-quarter response of π_H, π_F^*, and e to a Hegemon transfer shock under the *high convenience-yield* calibration, computed as in figure 7. Black lines and the numbers above or below each column give the total response. Under the monetary-led policy mix, only the total is available.

Under the fiscally-led and mixed policy configurations, convenience yields now matter more for nominal variables. For Hegemon inflation, they account for about a fifth of the response under the fiscally-led policy mix and for about half under the mixed configuration. For Foreign inflation under the fiscally-led policy mix, convenience yields and home bias together offset almost all of the policy component, which is why the Foreign economy is largely insulated. Under the mixed configuration, convenience yields are the largest positive contribution to Foreign inflation. Under the fiscally-led policy mix, convenience yields thus buffer the price-level spillover abroad.

J Alternative Estimation Approaches

Here we report estimates that complement those in Section 4. Besides the Bayesian specification of the main text, we estimate the model by maximum likelihood (ML), and we add convenience-yield data as an observable, using two alternative series. One way in which we obtain the convenience yield data series is as follows: we first measure the U.S. convenience yield as the difference between AAA corporate bonds and the yield on 10-year Treasuries, and we subsequently measure the U.S. convenience yield as the sum of the aforementioned spread and CIP deviations between the U.S. and the U.K. as estimated by Du et al. (2018a). Alternatively, we also use the Treasury richness series of Fleckenstein and Longstaff (2024) for the U.S. convenience yield. This gives six specifications: ML and Bayesian estimation, each without convenience yield data, with the main convenience yield series and with the alternative series. All six share the model, the calibration in table 1, the exact continuous-time likelihood and the macroeconomic data.

J.1 Parameter Spaces and Starting Values

The three Bayesian specifications share the priors and the parameter space in table E.1. The steady-state U.S. convenience yield stays below the discount rate ($\bar{\Phi} \leq 0.0049 < \rho$), so that the U.S. steady-state real interest rate $\rho - \bar{\Phi}$ is positive. The U.K. convenience yield is at most 0.02, and the quarterly persistence of the shocks is at most 0.99 for fiscal shocks and 0.98 for the others.

Maximum likelihood has no prior to discipline it, and we restrict its parameter space further. Its bounds contain the central 95 percent regions of the four informative Bayesian priors, $\phi_\pi \in [0.8, 1.2]$, $\phi_\pi^* \in [0.3, 0.7]$, $\bar{\Phi} \in [0.0008, 0.0028]$, and $\bar{\Phi}^* \in [0.0003, 0.0023]$, together with the Bayesian restrictions on ϕ_x, ϕ_x^*, ψ_b, ψ_b^*, and the shock persistence, and diffusion coefficients of at most 10. Without such restrictions, the likelihood is maximised where near-unit-root shock processes, most notably for productivity, produce large and mutually offsetting contributions to the historical decomposition. Table J.5 provides these estimates.

Note that the likelihood and the posterior have several local modes. For every specification, we therefore start the optimiser from several points: default starting values, the ML estimate for the same observables, and modes from earlier estimation rounds. We report the mode with the highest likelihood (ML) or posterior density (Bayesian). For the Bayesian specifications, the selected mode is also the starting point of the posterior sampler (see Supplementary Online Appendix J.6).

J.2 ML without Convenience Yields

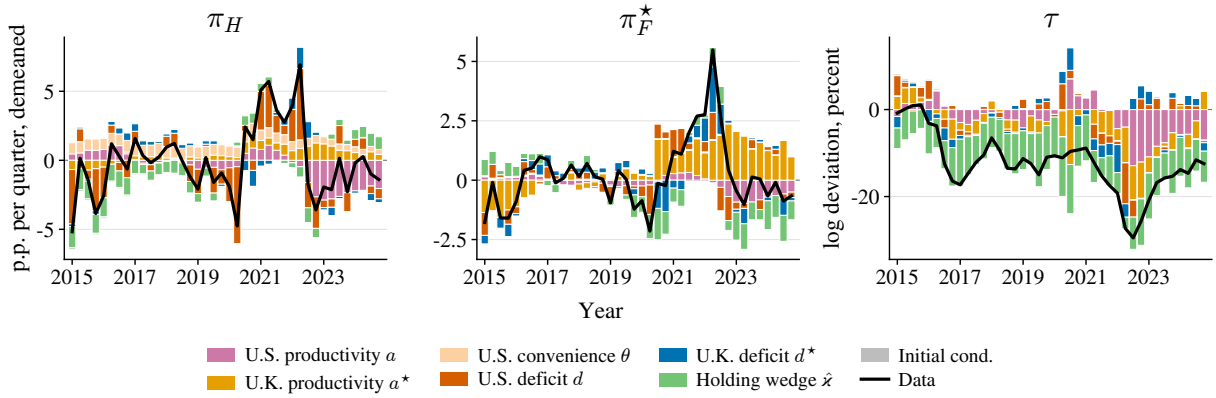


Figure J.1: Historical shock decomposition implied by the model when using ML, without using convenience yield estimates to discipline estimation.

J.3 ML with Convenience Yields

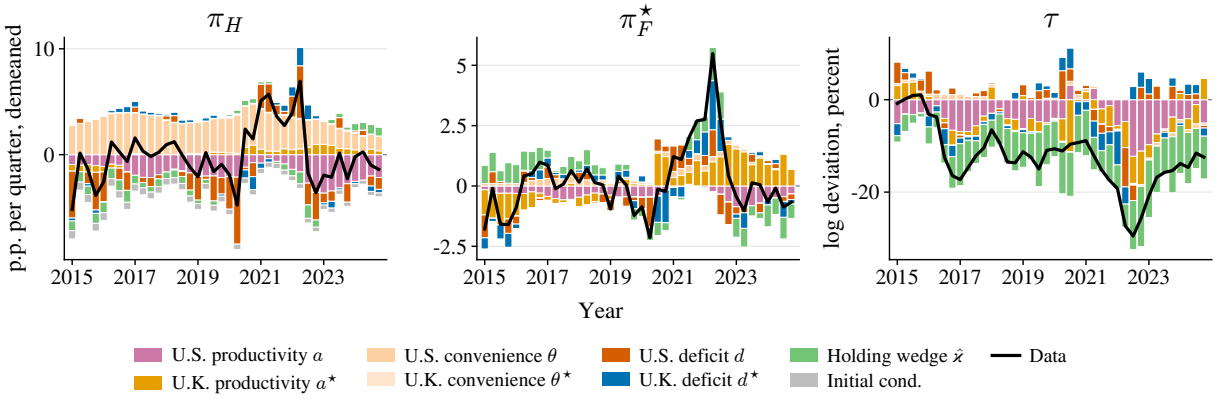


Figure J.2: Historical shock decomposition implied by the model when using ML, using convenience yield estimates to discipline estimation.

J.4 ML with Alternative Convenience Yield Definition

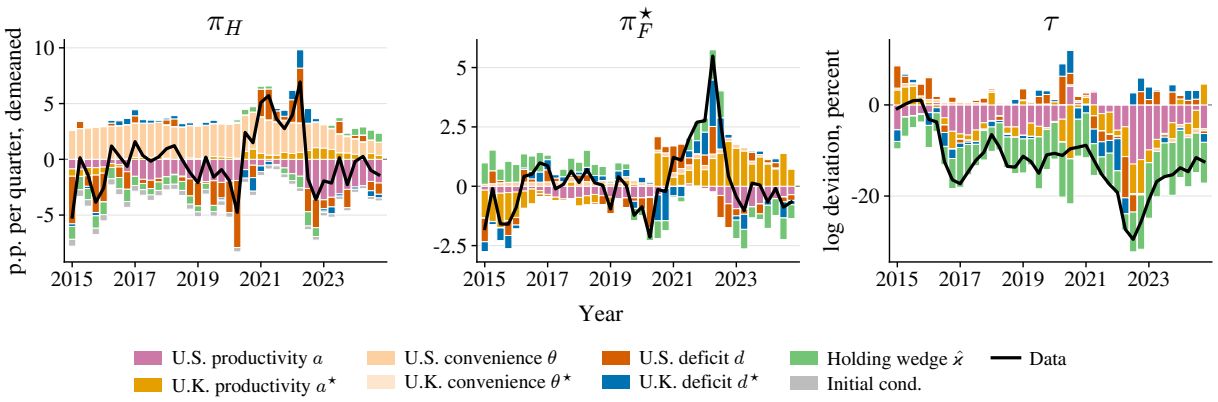


Figure J.3: Historical shock decomposition implied by the model when using ML, when using an alternative convenience yield specification following Fleckenstein and Longstaff (2024).

J.5 Bayesian Estimation with Convenience Yield Data

We here present the results of the two Bayesian specifications that add convenience yield data as an observable. Priors and parameter space are those of the main specification (table E.1).

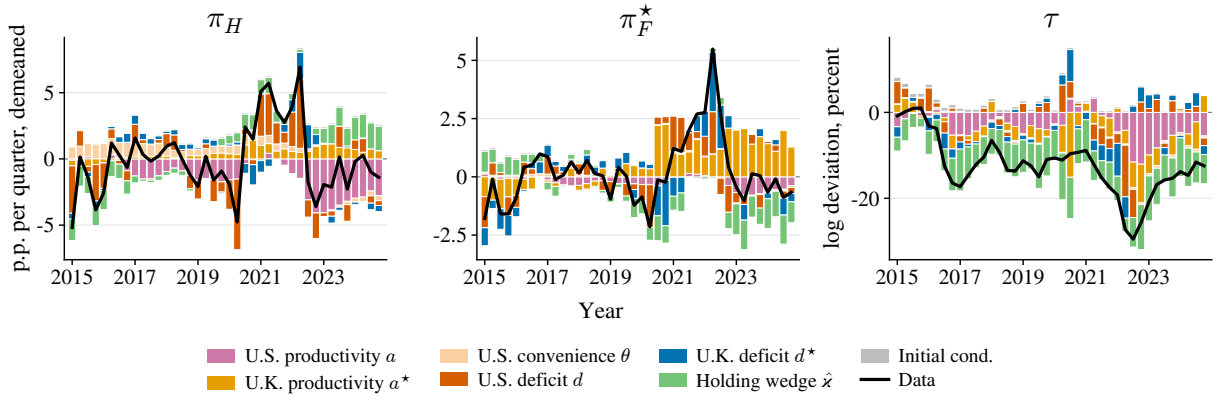


Figure J.4: Historical shock decomposition implied by the model when using Bayesian estimation, main convenience yield specification.

Policy and convenience yield parameters			Shock mean-reversion and diffusion parameters		
Parameter	Estimate	Post. s.d.	Parameter	Estimate	Post. s.d.
ϕ_π	1.0259	0.0474	φ_a	0.1602	0.0984
ϕ_π^*	0.6626	0.0903	φ_a^*	0.8804	0.1893
ϕ_x	-0.4152	0.0482	φ_θ	0.0252	0.0291
ϕ_x^*	-0.4126	0.0508	φ_θ^*	0.4850	0.4259
ψ_b	-0.0009	0.0217	φ_s	$0.0101^\dagger$	0.0031
ψ_b^*	$-0.5000^\dagger$	0.0999	φ_s^*	0.0452	0.0192
$10^3\,\bar{\Phi}$	1.5321	0.3877	$\varphi_\varkappa$	0.0292	0.0168
$10^3\,\bar{\Phi}^*$	1.5479	0.4925	σ_a	8.4394	2.1502
			σ_a^*	17.3606	2.0358
			σ_θ	36.8109	34.8118
			σ_θ^*	0.4247	0.4354
			σ_s	0.2787	0.2895
			σ_s^*	2.4445	0.4598
			$\sigma_\varkappa$	0.7291	0.2668

Table J.1: Estimates from the alternative exact continuous-time Bayesian specification using U.S. and U.K. data since 1998 and convenience yield data as an additional input. Estimates are posterior modes, and posterior standard deviations are computed from six chains of 2,500,000 draws each (first 625,000 discarded, every tenth draw retained). A dagger (†) marks a mode at a bound of the parameter space. The plotted historical decompositions and counterfactuals are evaluated at the posterior-median parameter vector. Convergence diagnostics are reported in Appendix J.6.

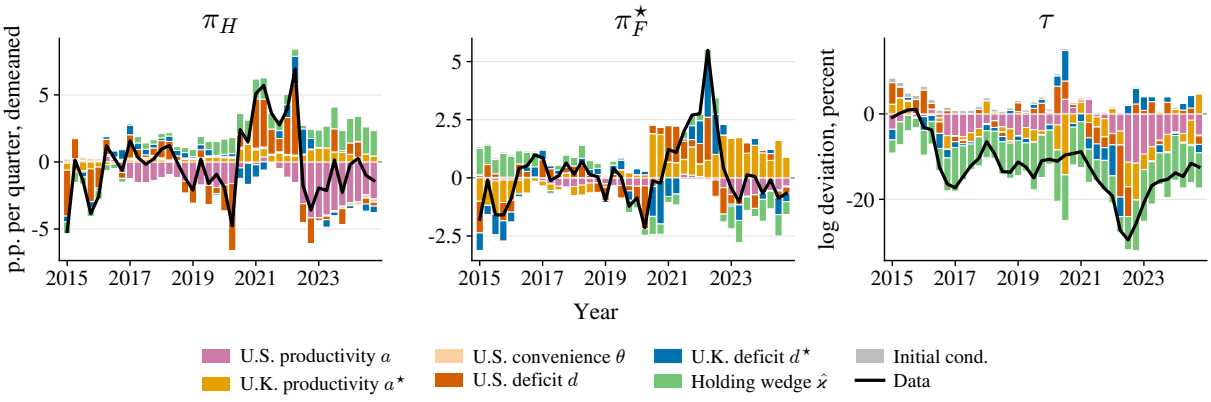


Figure J.5: Historical shock decomposition implied by the model when using Bayesian estimation, alternative convenience yield specification.

J.6 Convergence of the Posterior Samplers

Sampler. For each Bayesian specification, we draw from the exact posterior with random-walk chains: four chains of 1,000,000 draws for the main specification, and six chains

Policy and convenience yield parameters			Shock mean-reversion and diffusion parameters		
Parameter	Estimate	Post. s.d.	Parameter	Estimate	Post. s.d.
ϕ_π	1.0241	0.0970	φ_a	0.1714	0.1501
ϕ^*_π	0.6452	0.0951	φ^*_a	0.8623	0.1938
ϕ_x	-0.3859	0.0585	φ_θ	0.0286	0.0362
ϕ^*_x	-0.4260	0.0555	φ^*_θ	0.4850	0.4302
ψ_b	-0.0011	0.0225	φ_s	$0.0101^\dagger$	0.0056
ψ^*_b	$-0.5000^\dagger$	0.1274	φ^*_s	0.0454	0.0198
$10^3\,\bar{\Phi}$	0.7653	0.1569	$\varphi_\varkappa$	0.0251	0.0191
$10^3\,\bar{\Phi}^*$	1.6532	0.5263	σ_a	8.4823	2.2888
			σ^*_a	17.1820	2.0104
			σ_θ	31.5646	20.5441
			σ^*_θ	0.4247	0.4445
			σ_s	0.2918	0.3014
			σ^*_s	2.4533	0.5629
			$\sigma_\varkappa$	0.8073	0.3387

Table J.2: Alternative convenience yield data. Estimates from the alternative exact continuous-time Bayesian specification using U.S. and U.K. data since 1998 and alternative convenience yield data as an additional input. Estimates are posterior modes, posterior standard deviations are computed from six chains of 2,500,000 draws each (first 625,000 discarded, every tenth draw retained). A dagger (†) marks a mode at a bound of the parameter space (see table E.1). The plotted historical decompositions and counterfactuals are evaluated at the posterior-median parameter vector. Convergence diagnostics are reported in Appendix J.6.

of 2,500,000 draws for each of the two specifications with convenience-yield data. Each diffusion enters through the logarithm of its quarterly innovation standard deviation. It mixes moves of the full parameter vector with moves of single mean-reversion/diffusion pairs. Proposals that leave the parameter space, or that do not deliver a unique bounded equilibrium, are rejected. The first chain starts at the posterior mode. The other chains start from points drawn around the mode with twice the proposal scale, so that the chains begin in different regions of the posterior. During the first quarter of each chain, the proposal covariance is learned from the draws and its scale is adapted towards an acceptance rate of 0.25. We discard these draws and retain every tenth subsequent draw, leading to 300,000 draws for the main specification and 1,125,000 for each of the other two.

Criteria. We regard a posterior as converged when every one of its 22 parameters satisfies the criteria of Vehtari et al. (2021) (as also used, e.g., by Bouscasse and Hong (2026)), computed from all chains of a sampler: a rank-normalised $\hat{R}$ of at most 1.01, and effective sample sizes (ESS) of at least 400 each. Our conclusions rest on functions of the parameters,

so we apply the same criteria to nine generated quantities. These are:

- the shares of U.S. and U.K. fiscal shocks in the 2021–2023 surges of U.S. and U.K. inflation;
- the cumulative counterfactual effects of halving U.S. fiscal innovations on U.S. inflation, U.K. inflation and the exchange rate;
- the mean contribution of U.S. fiscal shocks to U.K. inflation over 2021–2024;
- the counterfactual effects on the U.K. price level and on the nominal exchange rate at the end of the sample.

Each evaluation requires a Kalman smoother, a historical decomposition and a counterfactual. We evaluate them at 500 evenly spaced retained draws per chain, giving 2,000 for the main specification and 3,000 for each specification with convenience-yield data.

Results. Table J.3 reports the diagnostics we obtain from this analysis. All three posteriors meet every criterion. The largest $\hat{R}$ is 1.0022 for the parameters and 1.0019 for the generated quantities. The smallest ESS is 1,072 for the parameters and 1,091 for the generated quantities, against a threshold of 400.

The test of Geweke (1992), which compares the first 10 percent with the last 50 percent of each chain, is less clear-cut. Its statistic exceeds 2 in absolute value for 23 of 88, 77 of 132, and 65 of 132 parameter–chain pairs, where chance alone would produce about four to seven. With effective sample sizes of this order, however, the test seems to have power against very small differences. The difference between early and late means never exceeds 0.32 posterior standard deviations, and its median across parameter–chain pairs is 0.03–0.07.

Table J.4 summarises the posterior distribution of the headline quantities, which is similar across the three Bayesian specifications. In the posterior mean, U.S. fiscal shocks account for 51–68 percent of the 2021–2023 surge in U.S. inflation and for 25–29 percent of the surge in U.K. inflation, and U.K. fiscal shocks for 27–35 percent of the U.K. surge. Halving U.S. fiscal innovations from 2021 lowers cumulative U.S. inflation over 2021–2023 by 8 to 14 percentage points and cumulative U.K. inflation by 1.8 to 1.9 percentage points. The 90 percent posterior intervals of all three counterfactual effects exclude zero in every specification.

	Bayesian no CY (main)	Bayesian CY	Bayesian alt. CY
Chains × draws per chain	4 × 1,000,000	6 × 2,500,000	6 × 2,500,000
Retained draws	300,000	1,125,000	1,125,000
Acceptance rate after burn-in	0.217–0.303	0.205–0.342	0.204–0.323
Share of rejected inadmissible proposals	0.293–0.345	0.279–0.345	0.273–0.325
Parameters (22)			
Max. split-$\hat{R}$	1.0018	1.0022	1.0020
Min. bulk ESS	2,039	1,580	1,072
Min. tail ESS	2,211	1,415	1,257
Max. spread of chain means (post. s.d.)	0.12	0.20	0.20
Max. MCSE of posterior mean (post. s.d.)	0.022	0.025	0.031
Geweke $\lvert z\rvert > 2$ (parameter–chain pairs)	23 of 88	77 of 132	65 of 132
Generated quantities (9; 500 draws per chain)			
Max. split-$\hat{R}$	1.0019	1.0015	1.0013
Min. bulk ESS	1,553	1,998	1,206
Min. tail ESS	1,735	1,274	1,091
Converged ($\hat{R} \leq 1.01$, ESS ≥ 400)	yes	yes	yes

Table J.3: Convergence diagnostics of the posterior samplers. $\hat{R}$ is the rank-normalised and folded split-$\hat{R}$ and ESS the bulk and tail effective sample sizes of Vehtari et al. (2021), computed from all chains of a sampler. Ranges report the minimum and maximum over chains. Spread of chain means: largest difference between the chain-specific posterior means of a parameter, in posterior standard deviations. MCSE: Monte Carlo standard error. Geweke: Geweke (1992) test comparing the first 10% with the last 50% of each chain.

	Bayesian no CY (main)	Bayesian CY	Bayesian alt. CY
Share of the 2021–2023 inflation surge explained by			
U.S. fiscal shocks, U.S. inflation	0.513 [0.394, 0.640]	0.642 [0.542, 0.730]	0.677 [0.540, 0.762]
U.S. fiscal shocks, U.K. inflation	0.247 [0.169, 0.331]	0.283 [0.204, 0.385]	0.288 [0.215, 0.395]
U.K. fiscal shocks, U.K. inflation	0.347 [0.277, 0.424]	0.304 [0.238, 0.372]	0.266 [0.194, 0.342]
Cumulative effect of halving U.S. fiscal innovations, 2021Q1–2023Q4			
U.S. inflation	13.96 [11.04, 17.31]	10.16 [7.06, 13.00]	8.30 [5.80, 10.56]
U.K. inflation	1.78 [1.07, 2.59]	1.87 [1.23, 2.72]	1.87 [1.25, 2.74]
Exchange rate	76.9 [63.4, 91.0]	51.4 [30.2, 69.6]	40.2 [24.1, 54.5]

Table J.4: Posterior distribution of the headline quantities: posterior means and, in brackets, 5th and 95th percentiles over posterior draws. Shares: sum of the shock's contribution over the quarters of 2021Q1–2023Q4 in which demeaned inflation is positive, divided by the sum of inflation over those quarters. Cumulative effects: baseline minus counterfactual, summed over 2021Q1–2023Q4.

J.7 Additional IRFs

We present additional impulse responses to the Hegemon fiscal transfer shock under the four policy configurations, for both the benchmark and the high convenience-yield calibrations.

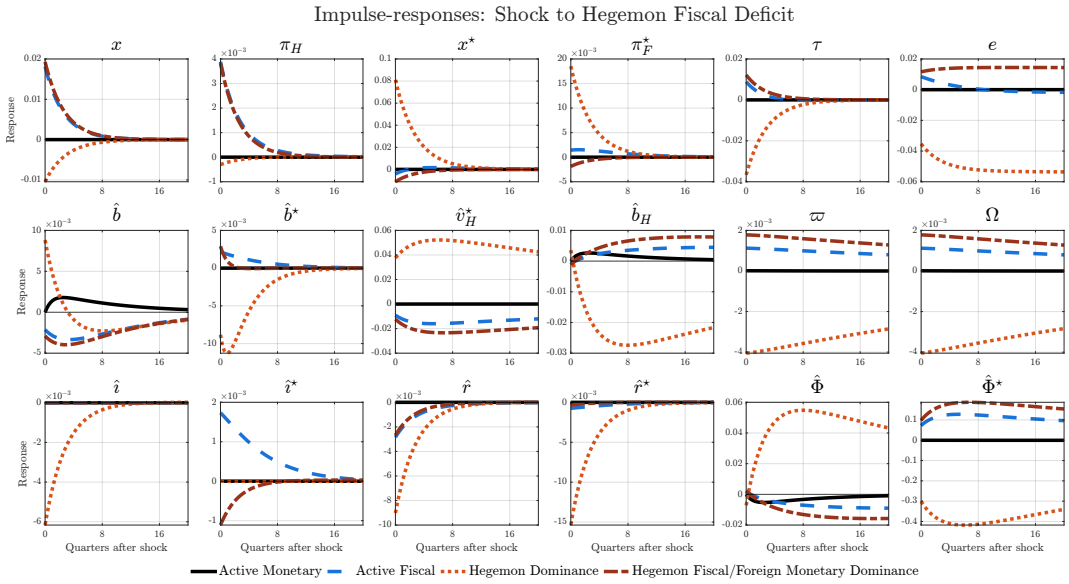


Figure J.6: Additional impulse responses to an expansionary Hegemon fiscal transfer shock: low convenience-yield calibration.

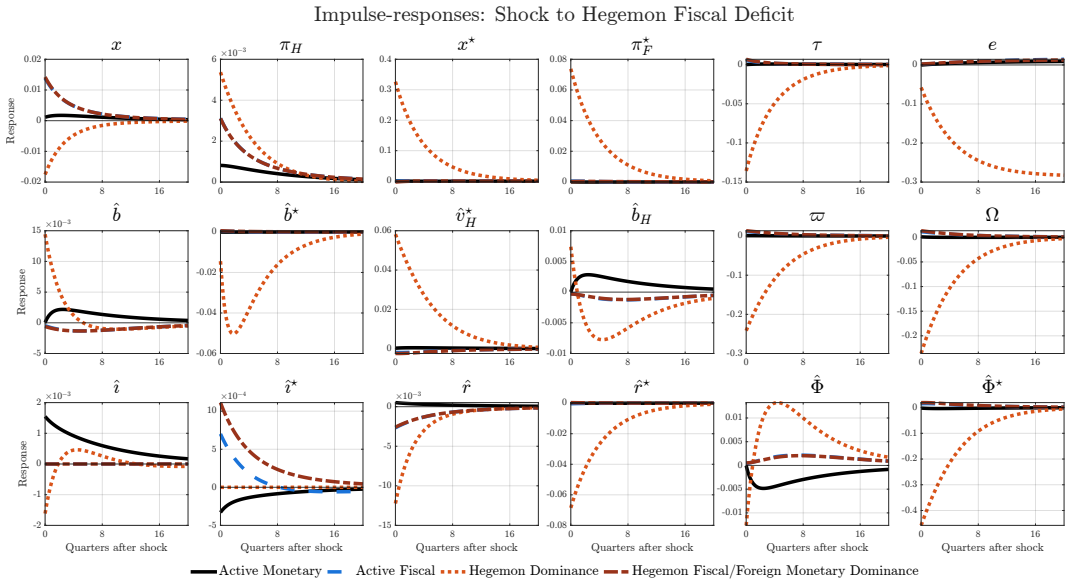


Figure J.7: Additional impulse responses to an expansionary Hegemon fiscal transfer shock: high convenience-yield calibration.

J.8 Comparison across All Specifications

Table J.5 reports the estimates of all six specifications and table J.6 the headline quantities they imply. The six specifications fall into two groups.

- **Bayesian.** The three Bayesian estimates are similar. The U.S. inflation response ϕ_π is just above one (1.02–1.03) and the output response ϕ_x negative (-0.54 to -0.39). U.S. fiscal feedback is close to zero (ψ_b between -0.04 and 0), and U.K. fiscal feedback lies at the upper bound of its range ($\psi_b^* = -0.5$). U.S. fiscal shocks explain 51 to 70 percent of the 2021–2023 U.S. inflation surge and about a quarter of the U.K. surge, and U.K. fiscal shocks 27 to 35 percent of the U.K. surge.
- **ML.** The three ML estimates have $\phi_\pi \approx 1.03$ and $\phi_x \approx -0.5$, and several parameters lie on the bounds of the ML parameter space, among them ϕ_π^*, ϕ_x^*, ψ_b, and $\bar{\Phi}^*$ in all three, and $\bar{\Phi}$ in two.
  - U.S. fiscal shocks explain a similar share of the U.S. inflation surge (0.50–0.59).
  - The share of the U.K. inflation surge attributed to U.S. fiscal shocks is smaller (about 0.11), and so is the share attributed to U.K. fiscal shocks (0.25–0.26).
  - Halving U.S. fiscal innovations lowers U.K. inflation by less (0.65–0.69 against 1.64–1.74 percentage points).

At the estimates, the cumulative effects of halving U.S. fiscal innovations on U.S. inflation, U.K. inflation and the exchange rate are positive in all six specifications. In the Bayesian posteriors, the 90 percent intervals of all three effects exclude zero (table J.4).

	(1) ML no CY	(2) ML CY	(3) Bayesian no CY	(4) Bayesian CY	(5) ML alt. CY	(6) Bayesian alt. CY
Monetary policy						
ϕ_π	1.0314	1.0292	1.0338	1.0259	1.0298	1.0241
ϕ_π^*	$0.7000^\dagger$	$0.7000^\dagger$	0.7536	0.6626	$0.7000^\dagger$	0.6452
ϕ_x	−0.5037	−0.4674	−0.5423	−0.4152	−0.4768	−0.3859
ϕ_x^*	$-0.6000^\dagger$	$-0.6000^\dagger$	−0.4096	−0.4126	$-0.6000^\dagger$	−0.4260
Fiscal policy						
ψ_b	$0.0000^\dagger$	$0.0000^\dagger$	−0.0372	−0.0009	$0.0000^\dagger$	−0.0011
ψ_b^*	−0.8505	−0.7378	$-0.5000^\dagger$	$-0.5000^\dagger$	−0.7998	$-0.5000^\dagger$
Convenience yields						
$10^3\,\bar{\Phi}$	$2.8000^\dagger$	$2.8000^\dagger$	3.2762	1.5321	2.4734	0.7653
$10^3\,\bar{\Phi}^*$	$2.3000^\dagger$	$2.3000^\dagger$	1.2297	1.5479	$2.3000^\dagger$	1.6532
Shock mean reversion						
φ_a	0.3575	0.2568	$0.0202^\dagger$	0.1602	0.3097	0.1714
φ_a^*	0.6017	0.7644	0.8836	0.8804	0.7198	0.8623
φ_θ	$0.0202^\dagger$	$0.0202^\dagger$	0.4850	0.0252	$0.0202^\dagger$	0.0286
φ_θ^*	$2.9931^\dagger$	$0.0202^\dagger$	0.4850	0.4850	$0.0202^\dagger$	0.4850
φ_s	$0.0101^\dagger$	$0.0101^\dagger$	$0.0101^\dagger$	$0.0101^\dagger$	$0.0101^\dagger$	$0.0101^\dagger$
φ_s^*	0.0267	0.0473	0.0418	0.0452	0.0378	0.0454
$\varphi_\varkappa$	0.0897	0.1826	$0.0202^\dagger$	0.0292	0.1290	0.0251
Shock diffusion						
σ_a	$10.0000^\dagger$	9.9144	12.8191	8.4394	$10.0000^\dagger$	8.4823
σ_a^*	$10.0000^\dagger$	$10.0000^\dagger$	16.7911	17.3606	$10.0000^\dagger$	17.1820
σ_θ	$10.0000^\dagger$	$10.0000^\dagger$	0.4247	36.8109	$10.0000^\dagger$	31.5646
σ_θ^*	0.0011	$10.0000^\dagger$	0.4247	0.4247	$10.0000^\dagger$	0.4247
σ_s	0.2636	0.2562	0.6798	0.2787	0.2628	0.2918
σ_s^*	4.1029	4.1124	2.4492	2.4445	4.0775	2.4533
$\sigma_\varkappa$	1.9528	3.2765	0.5278	0.7291	2.4716	0.8073
Log-likelihood	−1717.3	−1714.4	−1687.0	−1548.8	−1522.6	−1465.9

Table J.5: Parameter estimates across specifications. ML columns report maximum-likelihood estimates on the ML parameter space (Section J.1), Bayesian columns posterior modes. Specifications are estimated without a convenience-yield observable (no CY), with the main convenience-yield series (CY) or with the alternative series (alt. CY). φ and σ are the quarterly mean-reversion rates and the diffusion coefficients of the shock processes in equation (23). †At a bound of the respective parameter space (within 0.1% of the interval width and within 1% of the bound). The last row reports the log-likelihood at each estimate.

	(1) ML no CY	(2) ML CY	(3) Bayesian no CY	(4) Bayesian CY	(5) ML alt. CY	(6) Bayesian alt. CY
Share of the 2021–2023 inflation surge explained by						
U.S. fiscal shocks, U.S. inflation	0.592	0.502	0.507	0.645	0.518	0.704
U.S. fiscal shocks, U.K. inflation	0.106	0.109	0.235	0.256	0.112	0.269
U.K. fiscal shocks, U.K. inflation	0.248	0.260	0.345	0.307	0.248	0.268
Cumulative effect of halving U.S. fiscal innovations, 2021Q1–2023Q4						
U.S. inflation	10.87	11.34	14.18	10.27	10.77	8.70
U.K. inflation	0.65	0.69	1.64	1.65	0.69	1.74
Exchange rate	76.0	81.6	80.1	55.3	76.1	45.9

Table J.6: Headline quantities across specifications, evaluated at the ML estimates in table J.5 and, for the Bayesian specifications, at the posterior median (the point at which the historical decompositions and counterfactuals are drawn). Definitions as in table J.4.

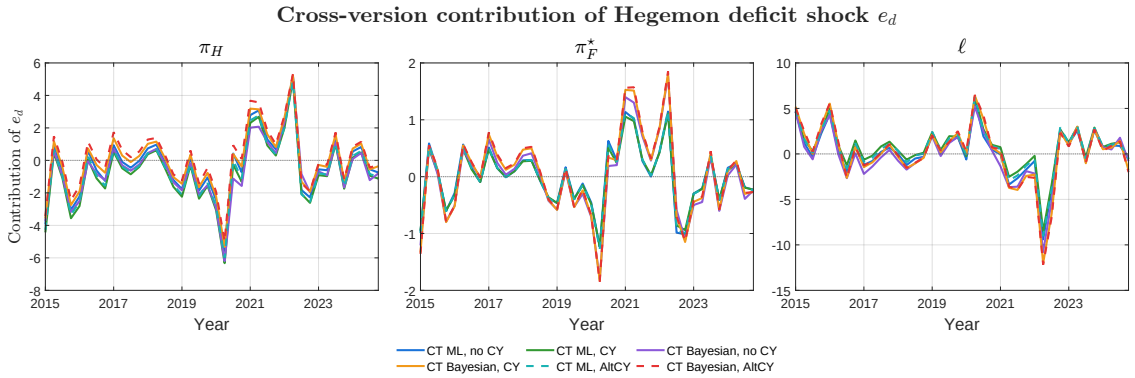


Figure J.8: Comparison of all six estimation approaches: ML without CY data, ML with main CY data, Bayesian without CY data, Bayesian estimation with main CY data, ML with alternative CY data, Bayesian estimation with alternative CY data.