Fiscal Inflation in a Currency Union*

Momo Komatsu
Federal Reserve Board

David Murakami
Bank of England

Ivan Shchapov
International Monetary Fund

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Abstract

We study the propagation of unfunded fiscal shocks in a currency union. For a determinate equilibrium, at most one fiscal authority can issue unfunded debt; we then accommodate multiple partially unfunded countries by introducing monetary-led and fiscal-led parallel economies. Funded government spending shocks cause inflation to diverge across countries, while unfunded shocks raise inflation union-wide, as the common central bank accommodates *fiscal inflation*. Euro area local projections show that positive government spending shocks in Italy raise inflation abroad, while in France and Germany they do not, consistent with Italian shocks being more unfunded than French and German shocks. A Bayesian-estimated quantitative model confirms that unfunded fiscal shocks are a quantitatively significant driver of euro area inflation dynamics. We recover the implied inflation target of the ECB, which averages 2.0 percent over the sample but rises to 2.6 percent in 2021 due to fiscal pressures.

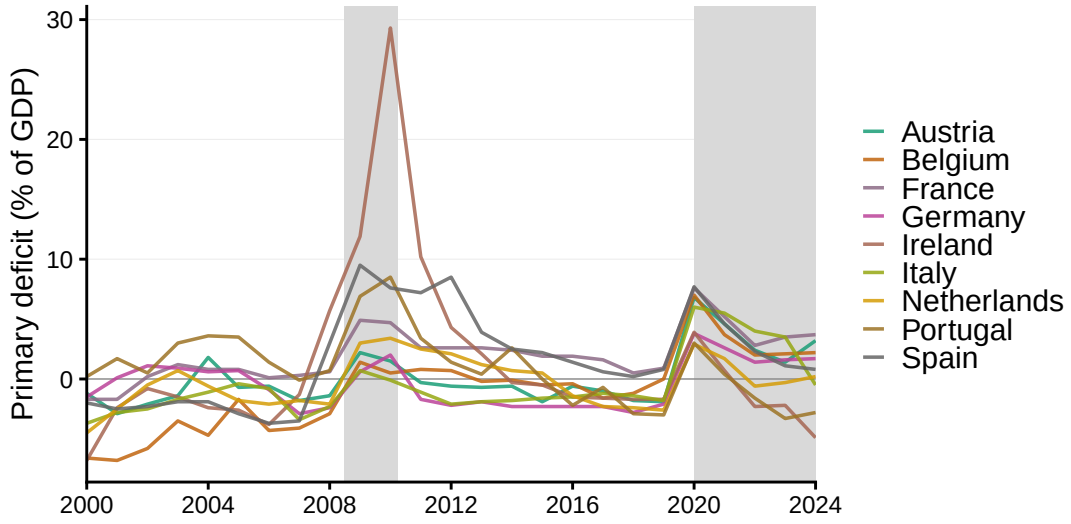
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Komatsu: momo.komatsu@frb.gov | Murakami: david.murakami@bankofengland.co.uk | Shchapov: ishchapov@imf.org

1 Introduction

In the last 25 years, euro area countries have experienced episodes of large primary deficits, as shown in Figure 1. The spikes in the primary deficit coincide with the fiscal expansions during the financial crisis, the COVID-19 crisis, and the cost-of-living crisis (roughly 2008, 2020–2021, and 2021–2023, respectively). The fiscal theory of the price level distinguishes funded from unfunded fiscal expansions. In a closed economy, unfunded expansions are inflationary: the central bank accommodates *fiscal inflation* to satisfy the government's intertemporal budget constraint. In this paper, we investigate the propagation of unfunded fiscal shocks in the euro area and estimate the quantitative importance of *fiscal inflation* of the last 25 years.

Figure 1: Primary deficit as % of GDP in the original euro area countries



Notes: The original euro area countries are the member states of the euro area that adopted the euro on January 1, 1999. We exclude Luxembourg and Finland for data availability reasons. Source: Eurostat.

The first contribution of this paper is to document price level determinacy and the propagation of funded and unfunded shocks in a currency union. We develop a two-country currency union model with bonds and goods trade, and show that for a determinate equilibrium, at most one fiscal authority can issue unfunded debt. Building on Bianchi, Faccini, and Melosi (2023), we introduce partially unfunded debt by decomposing the economy into monetary-led and fiscal-led parallel economies. In the model, funded and unfunded government spending shocks in country A have opposite implications for inflation in country B. A funded government spending shock in A raises demand for domestic goods but crowds out demand for foreign goods. Hence, inflation in country A increases and in country B it decreases on impact. In contrast,

when the government spending shock in A is unfunded, inflation rates in both countries increase because the *fiscal inflation* spills over due to the absence of the central bank response.

Second, we find new evidence on cross-country heterogeneity in fiscal spillovers within the euro area, which, through the lens of the model, we interpret as coming from different degrees of unfundedness. Using local projections and forecast-error fiscal shocks following Auerbach and Gorodnichenko (2012, 2017), we find that in 2004–2025 government spending shocks in Italy increase inflation in other member countries, while the shock in France and Germany do not. These results suggest that Italian government spending shocks are more unfunded than French and German ones.

Last, we develop a quantitative model extension with habit formation, non-tradables, nominal rigidities, capital accumulation, and long-term bonds. We estimate the model for Germany, France, and Italy each against the rest of the euro area. The historical shock decomposition reveals that unfunded fiscal shocks are a quantitatively significant driver of euro area inflation, tracking the major shifts in fiscal stance: contractionary during the austerity phase of the sovereign debt crisis, and expansionary during the COVID pandemic and the subsequent energy crisis. We further recover the model-implied inflation target of the ECB – the time-varying inflation objective driven by unfunded fiscal shocks – which exhibits persistent departures from the ECB's stated two percent mandate: it falls to 1.2 percent in 2014 and rises to 2.6 percent during the post-pandemic episode, though on average it is 2.0 percent.

Related literature. This paper addresses monetary–fiscal interactions in a currency union and therefore bridges two strands of the literature: (i) price-level determinacy and the interaction of monetary and fiscal policy, with Sargent and Wallace (1981), Leeper (1991), Sims (1994), and Woodford (1994) as the pioneers, and (ii) monetary and fiscal policy spillovers in a currency union. The empirical exercise also contributes to the fiscal spillovers literature.

Price-level determinacy and monetary–fiscal interaction have been widely studied, with recent contributions by Cochrane (2021). Our model framework builds on Bianchi, Faccini, and Melosi (2023) (BFM), which introduced a class of general equilibrium models with partially unfunded debt. Among others, Beetsma and Jensen (2005), Galí and Monacelli (2008), Ferrero (2009), and Hjortsoe (2016) study optimal monetary–fiscal policy in a currency union during asymmetric shocks. More recently, authors like Barbier-Gauchard and Betti (2021) study the spillover effects of fiscal policy in a monetary union.

Our paper lies at the intersection of those two strands of literature: we extend the

price determinacy literature to the currency union setting and allow for the analysis of unfunded fiscal shocks. We derive the parameter space for a monetary-led regime and a fiscal-led regime, and combine those as parallel economies following the approach of Bianchi, Faccini, and Melosi (2023) for the closed economy. Bergin (2000) also examines price level determinacy in a currency union, but abstracts from goods and bond trade and from the partially-unfunded debt framework of Bianchi, Faccini, and Melosi (2023), both of which are central to our analysis. The funded vs. unfunded distinction determines the sign of cross-border inflation spillovers, which is a prediction absent from existing currency union models. The quantitative model further allows us to identify episodes of *fiscal inflation* in the historical data and recover the time-varying inflation objective of the ECB implied by the fiscal positions of member states – a quantity that is not directly observable from monetary policy decisions alone.

Fiscal policy spillovers are a growing strand of the literature, with contributors like Goujard (2017), Debrun et al. (2021), Born, Müller, and Pfeifer (2020), and Ascari et al. (2024). Many study the channels of transmission – trade linkages, financial linkages, or demand spillovers. Our paper links these spillovers and transmission channels to whether fiscal shocks are funded or not, and uses a structural model to quantify the contribution of unfunded fiscal shocks to observed euro area inflation dynamics over the last 25 years.

The rest of the paper is organized as follows. Section 2 builds a stylized DSGE model, documents the price-level determinacy conditions of a currency union, and analyzes the propagation of funded and unfunded shocks. In section 3 we present the empirical analysis and interpret the heterogeneous spillovers from fiscal shocks through the lens of the model. In section 4 we estimate a quantitative model, identify episodes of fiscal inflation in the historical data, and recover the model-implied ECB inflation target. Concluding remarks are in Section 5.

2 Stylized model

In this section, we present a stylized model of a currency union and show that for a determinate equilibrium, at most one fiscal authority in the union can pursue an active, unfunded policy. To allow for partially unfunded debt, we follow the approach by Bianchi, Faccini, and Melosi (2023) in which the economy is modelled as a mixture of monetary-led and fiscal-led equilibria. Then, we analyze the propagation mechanisms of funded and unfunded fiscal expansion shocks. The theory provides the structural interpretation of the empirical findings in Section 3.

The stylized model considers a currency union of two countries, A and B, each with

a fixed endowment. The countries are inhabited by households who consume goods from both countries with a home bias. Moreover, households in each country hold bonds issued by the fiscal authority of both countries. The fiscal authority in each country collects taxes and issues bonds, and the monetary authority targets an average of the inflation rates in the two countries with an inflation-targeting rule. We allow for partially unfunded debt as in Bianchi, Faccini, and Melosi (2023). Country B variables are denoted with *, except for prices and the nominal interest rate which are the same across the union. Equations are for the economy in country A unless otherwise stated.

2.1 Households

Households derive utility from a consumption bundle and receive an endowment Y. They can smooth consumption by holding bonds issued in countries A and B, subject to adjustment costs. They face the maximization problem:

$$\max_{\{C_t, B_{At}, B_{Bt}\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \ln C_t, \tag{1}$$

subject to:

$$\begin{aligned} &P_t C_t + B_{At} + B_{Bt} + P_t T_t \\ &\qquad = P_t Y + R_{t-1}\left(B_{A,t-1} + B_{B,t-1}\right) - \frac{\psi_b P_t Y \bar{b}_B}{2}\left(b_{Bt} - \bar{b}_B\right)^2, \end{aligned} \tag{2}$$

where β is the discount factor, C_t is a bundle of consumption goods and P_t is its price, defined as:

$$C_t = \left[(1-\alpha_I)^{1/\mu}(C_{At})^{(\mu-1)/\mu} + \alpha_I^{1/\mu}(C_{Bt})^{(\mu-1)/\mu}\right]^{\mu/(\mu-1)}, \tag{3}$$

$$P_t = \left[(1-\alpha_I)P_{At}^{1-\mu} + \alpha_I P_{Bt}^{1-\mu}\right]^{1/(1-\mu)}, \tag{4}$$

where C_{At} is a good produced in country A, C_{Bt} a good produced in country B, and P_{At} and P_{Bt} are their prices, respectively. α_I is the share of imported goods and μ is the elasticity of substitution between domestic and imported goods. B_{At} is nominal holdings of bonds issued by the government in country A and B_{Bt} is nominal holdings of bonds issued by the government in country B, and R_t is the gross nominal interest rate in the currency union. We define real bond holdings (as shares of output) as $b_{At} \equiv \frac{B_{At}}{P_t Y}$ and $b_{Bt} \equiv \frac{B_{Bt}}{P_t Y}$. Moreover, $\bar{b}_A = \frac{\bar{B}_A}{\bar{P}\bar{Y}}$ and $\bar{b}_B = \frac{\bar{B}_B}{\bar{P}\bar{Y}}$ are steady-state real bond holdings as shares of output. Households face adjustment cost ψ_b for deviating foreign bond holdings from steady state, but can trade domestic bonds without friction. T_t are real lump-sum net taxes.

The cost minimization problem for the consumption bundle results in the per-period relative demand between country A and country B goods, $\frac{C_{At}}{C_{Bt}} = \frac{1-\alpha_I}{\alpha_I}\left(\frac{P_{At}}{P_{Bt}}\right)^{-\mu}$, using de-

mand equations $C_{At} = (1-\alpha_I)\left(\frac{P_{At}}{P_t}\right)^{-\mu} C_t$ and $C_{Bt} = \alpha_I \left(\frac{P_{Bt}}{P_t}\right)^{-\mu} C_t$. The intertemporal optimization problem results in two Euler equations, one for each bond:

$$1 = \beta R_t \mathbb{E}_t \left[\Pi_{t+1}^{-1} \frac{C_t}{C_{t+1}}\right], \tag{5}$$

$$1 = \frac{\beta R_t}{1 + \psi_b \overline{b}_B \left(b_{Bt} - \overline{b}_B\right)} \mathbb{E}_t \left[\Pi_{t+1}^{-1} \frac{C_t}{C_{t+1}}\right], \tag{6}$$

where $\Pi_t = P_t / P_{t-1}$ is gross inflation.

We define the terms of trade as the relative price of the good produced in country B to the good produced in country A as $S_t = \frac{P_{Bt}}{P_{At}}$.

2.2 Fiscal authority

The fiscal authority of both countries issues bonds and collects taxes. The nominal budget constraint for the fiscal authority is thus the following:

$$B_t + P_t T_t = R_{t-1} B_{t-1}, \tag{7}$$

where taxes T_t coincide with the real primary surplus and B_t is the supply of bonds. The government follows a fiscal rule:

$$\frac{\tau_t}{\overline{\tau}} = \left(\frac{s_{b,t-1}}{\overline{s}_b}\right)^{\gamma} e^{\zeta_t}, \tag{8}$$

where $\tau_t = \frac{T_t}{Y}$ is the surplus-to-output ratio and $s_{bt} = \frac{B_t}{P_t Y}$ is the real market value of debt as a share of output. Parameter γ determines how strongly the government adjusts primary surpluses with debt levels. ζ_t is a lump-sum tax shock, following an AR(1) process. In Section 2.6 we explain how we introduce partially unfunded government debt.

2.3 Monetary authority

The central bank of the currency union follows a simple inflation-targeting rule:

$$\frac{R_t}{\overline{R}} = \left(\frac{\Pi_t^{MU}}{\overline{\Pi}^{MU}}\right)^{\phi}, \tag{9}$$

where $\Pi_t^{MU} = \Pi_t^{\Theta} \Pi_t^{*1-\Theta} = \left(\frac{P_t}{P_{t-1}}\right)^{\Theta} \left(\frac{P_t^*}{P_{t-1}^*}\right)^{1-\Theta}$ is gross union-wide inflation. Parameter ϕ determines how strongly the central bank adjusts nominal interest rate with inflation movements.

2.4 Equilibrium

The equilibrium of this economy is characterized by a sequence of prices $\{P_{At}, P_{Bt}, R_t\}$ and allocations $\{C_{At}, C_{Bt}, C^*_{At}, C^*_{Bt}, B_{At}, B_{Bt}, B^*_{At}, B^*_{Bt}\}$ such that goods markets clear for the goods produced in countries A and B, and the bond markets clear for bonds supplied by fiscal authorities in both countries.

The goods market clearing conditions for countries A and B are the following:

$$Y = C_{At} + C^*_{At}, \qquad Y^* = C_{Bt} + C^*_{Bt}. \tag{10}$$

The bond markets clear when the supply of bonds in countries A and B matches demand:

$$B_t = B_{At} + B^*_{At}, \qquad B^*_t = B_{Bt} + B^*_{Bt}. \tag{11}$$

Finally, we can express the dynamics of net foreign assets in country A with the current account definition:

$$CA_t \equiv P_{At}Y - P_tC_t = B_{Bt} - B^*_{At} - R_{t-1}\left(B_{B,t-1} - B^*_{A,t-1}\right) - \frac{\psi_b P_t Y \bar{b}_B}{2}\left(b_{Bt} - \bar{b}_B\right)^2. \tag{12}$$

We linearize the relevant equilibrium conditions, such that the hatted variables are log-deviations from the steady state values. See Appendix B for the full set of equilibrium conditions.

2.5 Existence and uniqueness of a solution

Leeper (1991) shows that for closed economy versions of such models, there are two regions of the parameter space that deliver existence and uniqueness of a stationary solution. In the first region monetary policy is active and stabilizes inflation, and the fiscal authority keeps debt on a stable path: active monetary policy and passive fiscal policy. In the second region fiscal policy does not commit to a stable path of debt, such that monetary policy accommodates inflation driven by fiscal imbalances: passive monetary policy and active fiscal policy. We extend the logic to the currency union and characterize the determinacy regions of the general currency union model.

To characterize determinacy, we first derive the monetary and the fiscal block of the economy. For the monetary block we combine the Fisher equation with the monetary rule followed by the central bank, eq. (9). For the fiscal block, we combine the government budget constraints with the fiscal rules from each country, eq. (8) (in country A) and its counterpart (in country B).

We obtain the Fisher equation by adding the Euler equation of country A with respect to b_{At}, eq. (5), to the Euler equation of country B with respect to b^*_{At}, and applying the goods market clearing conditions (10). We use the log-linearized versions of the

equations from Appendix B:

$$\hat{i}_t = \mathbb{E}_t \hat{\pi}^{MU}_{t+1} - \Theta \psi_b Y \bar{b}_B \hat{b}_{Bt} - (1-\Theta)\psi_b Y \bar{b}^*_A \hat{b}^*_{At} \tag{13}$$

The equation links the nominal interest rate to the expected inflation and portfolio adjustment costs. Combining the Fisher equation with the log-linear monetary rule,

$$\hat{i}_t = \phi \hat{\pi}^{MU}_t, \tag{14}$$

leads to the monetary block of the currency union:

$$\mathbb{E}_t \hat{\pi}^{MU}_{t+1} = \phi \hat{\pi}^{MU}_t, \tag{15}$$

for which we apply the no arbitrage conditions $\hat{b}_{Bt} = 0$ and $\hat{b}^*_{At} = 0$. The coefficient in the monetary rule, ϕ, determines the eigenvalue in the monetary block. For the fiscal block, we combine the log-linear government budget constraints of country A and B,

$$\hat{s}_{bt} = \frac{1}{\beta}\left[\hat{s}_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}_t - (1-\beta)\hat{\tau}_t\right], \tag{16}$$

$$\hat{s}^*_{bt} = \frac{1}{\beta}\left[\hat{s}^*_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}^*_t - (1-\beta)\hat{\tau}^*_t\right], \tag{17}$$

with fiscal rules of each country,

$$\hat{\tau}_t = \gamma \hat{s}_{b,t-1}, \tag{18}$$

$$\hat{\tau}^*_t = \gamma^* \hat{s}^*_{b,t-1}. \tag{19}$$

The fiscal block of the currency union consists of two equations:

$$\hat{s}_{bt} = \frac{1}{\beta}\left[(1-(1-\beta)\gamma)\hat{s}_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}_t\right], \tag{20}$$

$$\hat{s}^*_{bt} = \frac{1}{\beta}\left[(1-(1-\beta)\gamma^*)\hat{s}^*_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}^*_t\right], \tag{21}$$

for which γ and γ^*, the debt stabilization parameters, determine the eigenvalues. Equations (15), (20), and (21) describe the currency union model.

Proposition 1 (Determinacy regions of the currency union)**.** *Consider a currency union with two endowment economies and cross-border bond holdings subject to portfolio adjustment costs, characterized by the monetary block* (15) *and the fiscal block* (20) *and* (21)*. The eigenvalues of this system are:*

$$\lambda_1 = \phi, \qquad \lambda_2 = \frac{1-(1-\beta)\gamma}{\beta}, \qquad \lambda_3 = \frac{1-(1-\beta)\gamma^*}{\beta}. \tag{22}$$

For existence and uniqueness of a bounded rational-expectations equilibrium (Blanchard-Kahn conditions), exactly one eigenvalue must be outside the unit circle. The three determinate regimes are:

Regime	ϕ	γ	γ^*	*Interpretation*
Monetary-led	> 1	≥ 1	≥ 1	*CB controls π; both govts fund debt*
Fiscal-led by A	≤ 1	< 1	≥ 1	*Govt A: unfunded; CB accommodates*
Fiscal-led by B	≤ 1	≥ 1	< 1	*Govt B: unfunded; CB accommodates*

When two or more eigenvalues are outside the unit circle, the equilibrium is non-existent. When no eigenvalues are outside the unit circle, the equilibrium is indeterminate.

Proof. Step 1: Eigenvalues. The system consists of three independent difference equations for $\hat{\pi}_t^{MU}$, $\hat{s}_{bt}$, and $\hat{s}_{bt}^*$, yielding eigenvalues as in eq. (22). Note that $|\lambda_2| > 1$ if and only if $\gamma < 1$ and analogously for λ_3: $|\lambda_2| > 1$ requires $\left|\frac{1-(1-\beta)\gamma}{\beta}\right| > 1$. Since $\beta \in (0,1)$ and $\gamma \geq 0$, we have a positive numerator $1-(1-\beta)\gamma > 0$ for $\gamma < 1/(1-\beta)$. So, the condition is $1-(1-\beta)\gamma > \beta$, which requires $\gamma < 1$.

Step 2: Blanchard-Kahn conditions. The system has one non-predetermined (forward-looking, jump) variable, $\hat{\pi}_t^{MU}$, and two predetermined (backward-looking, state) variables, $\hat{s}_{b,t-1}$ and $\hat{s}_{b,t-1}^*$. The Blanchard-Kahn conditions require the number of eigenvalues outside the unit circle to equal the number of non-predetermined variables, which is one.

Step 3: Determinate regimes.

1. **Monetary-led:** $\phi > 1$, $\gamma \geq 1$, and $\gamma^* \geq 1$, for which λ_1 is the unstable eigenvalue outside the unit circle. The central bank reacts more than one-to-one to inflation, pinning down inflation expectations. Both fiscal authorities adjust taxes with debt changes, stabilizing debt. Tax adjustments to debt changes are sufficiently large that the fiscal authorities stabilize demand by signaling that higher debt levels are matched by future taxation. In a monetary-led economy, monetary policy is *active* and fiscal policy in both countries is *passive*.

2. **Fiscal-led by A:** $\phi \leq 1$, $\gamma < 1$, and $\gamma^* \geq 1$, for which λ_2 is the unstable eigenvalue outside the unit circle. Country A's fiscal authority does not stabilize debt, generating an explosive path for $\hat{s}_{bt}$ that is resolved by inflation through the intertemporal government budget constraint. The central bank accommodates this fiscal-led inflation. Country B's authority stabilizes debt. In this fiscal-led economy, monetary policy is *passive*, fiscal policy in country A is *active*, and fiscal policy in country B is *passive*.

3. **Fiscal-led by B:** By symmetry with the regime fiscal-led by A, interchanging γ and γ^*.

Step 4: Indeterminate and non-existent equilibria. A system with two or more eigenvalues outside the unit circle is over-determined: no bounded equilibrium exists. A system with zero eigenvalues outside the unit circle is indeterminate: multiple bounded equilibria (sunspot equilibria) exist. □

Corollary 1 (At most one *active* authority in a currency union of N countries)**.** *With N fiscal authorities and one central bank, at most one fiscal authority can issue unfunded debt by setting $\gamma < 1$ while maintaining a determinate equilibrium.*

Proof. With N fiscal authorities and one central bank, there are N fiscal eigenvalues and one monetary eigenvalue, but still only one non-predetermined variable, $\hat{\pi}_t^{MU}$, the aggregate inflation rate of the currency union. Hence, at most one of the $N + 1$ eigenvalues can be outside the unit circle, implying at most one fiscal authority can have $\gamma < 1$. □

2.6 Introducing partially unfunded debt

This section explains how we model the monetary-led and fiscal-led economy in the above section in one economy, allowing for debt to be *partially* unfunded. We follow the framework developed by BFM but extend it to a currency union setting: we keep track of unfunded government debt in country A, s_{bt}^F, which is a component of the total government debt, s_{bt}. The superscript F denotes that the variable is fiscal-led. To implement an economy in which country A holds unfunded debt, i.e. the fiscal-led economy, we decompose each variable in the monetary rule and fiscal rules as follows:

$$\text{Fiscal rule } A \qquad \frac{\tau_t}{\overline{\tau}} = \left(\frac{s_{b,t-1}}{s_{b,t-1}^F}\right)^{\gamma^M} \left(\frac{s_{b,t-1}^F}{\overline{s}_b}\right)^{\gamma^F} e^{\zeta_t^M + \zeta_t^F}, \tag{23}$$

$$\text{Fiscal rule } B \qquad \frac{\tau_t^*}{\overline{\tau}^*} = \left(\frac{s_{b,t-1}^*}{s_{b,t-1}^{F*}}\right)^{\gamma^{M*}} \left(\frac{s_{b,t-1}^{F*}}{\overline{s}_b^*}\right)^{\gamma^{F*}}, \tag{24}$$

$$\text{Monetary rule} \qquad \frac{R_t}{\overline{R}} = \left[\left(\frac{\Pi_t}{\Pi_t^F}\right)^{\phi^M} \left(\frac{\Pi_t^F}{\overline{\Pi}}\right)^{\phi^F}\right]^{\Theta} \left[\left(\frac{\Pi_t^*}{\Pi_t^{F*}}\right)^{\phi^{M*}} \left(\frac{\Pi_t^{F*}}{\overline{\Pi}^*}\right)^{\phi^{F*}}\right]^{1-\Theta}, \tag{25}$$

for which superscripts M and M^* denote monetary-led variables and parameters, and F and F^* the fiscal-led ones. In the fiscal-led part of the economy, country A's government is not committed to bring debt to a stable path: $\gamma^F \leq 1$. In contrast, country B's government stabilizes debt in the fiscal-led economy, s_{bt}^{F*}, because only country A holds unfunded debt in this economy: $\gamma^{F*} > 1$. The monetary authority accommodates fiscal-led inflation rates, Π_t^F and Π_t^{F*}, with $\phi^F \leq 1$ and $\phi^{F*} \leq 1$. The remaining part of the economy is monetary-led, so that both countries are on a debt stabilizing path,

$\gamma^M > 1$ and $\gamma^{M*} > 1$, and the central bank reacts more than one-to-one to inflation $\phi^M > 1$ and $\phi^{M*} > 1$. ζ_t^M is monetary-led, hence the funded fiscal shock and ζ_t^F is the fiscal-led, unfunded fiscal shock in country A. Because of symmetry, we do not present the shocks in country B in the stylized model.

To characterize the determinacy of the model, we construct the monetary block and the fiscal block of the economy, as in the previous section. We linearize the above three equations to obtain:

$$\hat{\tau}_t = \gamma^M \left(\hat{s}_{b,t-1} - \hat{s}^F_{b,t-1} \right) + \gamma^F \hat{s}^F_{b,t-1} + \zeta_t^M + \zeta_t^F, \tag{26}$$

$$\hat{\tau}^*_t = \gamma^{M*} \left(\hat{s}^*_{b,t-1} - \hat{s}^{F*}_{b,t-1} \right) + \gamma^{F*} \hat{s}^{F*}_{b,t-1}, \tag{27}$$

$$\hat{i}_t = \Theta \left[\phi^M \left(\hat{\pi}_t - \hat{\pi}^F_t \right) + \phi^F \hat{\pi}^F_t \right] + (1-\Theta) \left[\phi^{M*} \left(\hat{\pi}^*_t - \hat{\pi}^{F*}_t \right) + \phi^{F*} \hat{\pi}^{F*}_t \right]. \tag{28}$$

Substituting the linear monetary rule that allows for inflation coming from unfunded debt, equation (28), into the Fisher equation (15), we obtain the monetary block of this economy:

$$\begin{aligned} \mathbb{E}_t \hat{\pi}^{MU}_{t+1} = \phi^M \hat{\pi}^{MU}_t + \Theta \left(\phi^F - \phi^M \right) \hat{\pi}^F_t + (1-\Theta) \left(\phi^{F*} - \phi^M \right) \hat{\pi}^{F*}_t \\ - \Theta \psi_b \Upsilon \bar{b}_B \hat{b}_{Bt} - (1-\Theta) \psi_b \Upsilon \bar{b}^*_A \hat{b}^*_{At}, \end{aligned} \tag{29}$$

where we use $\phi^{M*} = \phi^M$. Similarly to the closed-economy case, the parameter on the monetary rule, ϕ^M, determines the eigenvalue in the monetary block. For the fiscal block, we substitute the two fiscal rules allowing for unfunded debt, equations (26) and (27), into the government budget constraints (16) and (17):

$$\hat{s}_{bt} = \frac{1}{\beta} \begin{bmatrix} \left(1 - (1-\beta)\gamma^M \right) \hat{s}_{b,t-1} + (1-\beta)\gamma^M \hat{s}^F_{b,t-1} \\ -(1-\beta)\gamma^F \hat{s}^F_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}_t - (1-\beta) \left(\zeta_t^M + \zeta_t^F \right) \end{bmatrix}, \tag{30}$$

$$\hat{s}^*_{bt} = \frac{1}{\beta} \begin{bmatrix} \left(1 - (1-\beta)\gamma^{M*} \right) \hat{s}^*_{b,t-1} + (1-\beta)\gamma^{M*} \hat{s}^{F*}_{b,t-1} \\ -(1-\beta)\gamma^{F*} \hat{s}^{F*}_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}^*_t \end{bmatrix}. \tag{31}$$

Again, the debt stabilization parameters γ^M and γ^{M*} characterize the eigenvalues of the fiscal block.

To close the model, we need to pin down the dynamics of fiscal-led inflation $\hat{\pi}^F_t$ and $\hat{\pi}^{F*}_t$ and the fiscal-led government debt $\hat{s}^F_{bt}$ and $\hat{s}^{F*}_{bt}$. To do so, as in BFM, we construct a shadow economy which keeps track of the fiscal-led variables. The monetary block of this shadow economy is:

$$\mathbb{E}_t \hat{\pi}^{MU,F}_{t+1} = \phi^F \hat{\pi}^{MU,F}_t - \Theta \psi_b \Upsilon \bar{b}_B \hat{b}^F_{Bt} - (1-\Theta) \psi_b \Upsilon \bar{b}^*_A \hat{b}^{F*}_{At}. \tag{32}$$

obtained by aggregating the monetary block in each country:

$$\mathbb{E}_t \hat{\pi}^F_{t+1} = \phi^F \hat{\pi}^F_t - \psi_b \Upsilon \bar{b}_B \hat{b}^F_{Bt}, \tag{33}$$

$$\mathbb{E}_t \hat{\pi}^{F*}_{t+1} = \phi^{F*} \hat{\pi}^{F*}_t - \psi_b \Upsilon \bar{b}^*_A \hat{b}^{F*}_{At}, \tag{34}$$

and using $\phi^F = \phi^{F*}$. This parameter governs how the central bank reacts to fiscal-led inflation and determines the eigenvalue of the monetary block in the shadow economy. The fiscal block of the shadow economy is characterized by the government budget constraints combined with the fiscal rules:

$$\hat{s}^F_{bt} = \frac{1}{\beta}\left[\hat{s}^F_{b,t-1} + \hat{i}^F_{t-1} - \hat{\pi}^F_t - (1-\beta)\hat{\tau}^F_t\right], \tag{35}$$

$$\hat{s}^{F*}_{bt} = \frac{1}{\beta}\left[\hat{s}^{F*}_{b,t-1} + \hat{i}^{F*}_{t-1} - \hat{\pi}^{F*}_t - (1-\beta)\hat{\tau}^{F*}_t\right], \tag{36}$$

where

$$\hat{\tau}^F_t = \gamma^F \hat{s}^F_{b,t-1} + \zeta^F_t, \tag{37}$$

$$\hat{\tau}^{F*}_t = \gamma^{F*}\hat{s}^{F*}_{b,t-1} \tag{38}$$

and where $\hat{i}^F_{t-1}$ is the part of the nominal interest rate that responds to fiscal-led inflation. Following the monetary rule (28), this component can be written as $\hat{i}^F_t \equiv \Theta\phi^F\hat{\pi}^F_t + (1-\Theta)\phi^{F*}\hat{\pi}^{F*}_t$. The debt stabilization parameters, γ^F and γ^{F*} determine the eigenvalues in the fiscal block of the shadow economy.

The equations that characterize the currency union model with partially unfunded debt are: the monetary and fiscal block of the actual economy, (29)–(31) and the monetary and fiscal block of the fiscal-led, shadow economy, (32), (35), and (36). There are two non-predetermined variables, $\hat{\pi}^{MU}_t$ and $\hat{\pi}^{MU,F}_t$, and two eigenvalues outside the unit circle in equations (29) and (35) such that Blanchard-Kahn conditions are satisfied and there is a unique and stable equilibrium.

As in BFM, we exploit the linearity of the model to prove that the non-fiscal-led components of inflation and debt – so $\hat{\pi}^M_t = \hat{\pi}_t - \hat{\pi}^F_t$, $\hat{\pi}^{M*}_t = \hat{\pi}^*_t - \hat{\pi}^{F*}_t$, $\hat{s}^M_{bt} = \hat{s}_{bt} - \hat{s}^F_{bt}$, and $\hat{s}^{M*}_{bt} = \hat{s}^*_{bt} - \hat{s}^{F*}_{bt}$ – are indeed the inflation and debt of a fully monetary-led economy. See Appendix C for details.

2.7 Calibration

The calibration of the stylized model is straightforward and follows the literature. A summary of the calibration of the quantitative model is in Table 1, which includes parameters from the stylized model. The discount factor β is 0.99, the degree of trade openness α_I is 0.25 to allow for home bias according to Eurostat (2023), and the elasticity of substitution between goods produced in country A and B μ is 1. The countries are of equal size, which means that $\Theta = 0.5$. All shocks have a persistence of 0.5.

Debt adjustment cost ψ_b is 0.001. We calibrate the steady-state real market value of debt as share of output, $\bar{s}_b$, as 0.80, which is the average debt-to-GDP ratio of France and Germany for the period 2004–2024, the main countries and period we focus on in

the empirical section that follows (Eurostat Data, 2025). The countries are symmetric such that the net foreign asset position is 0 in steady state.

We follow BFM for the parameters specific to the model with partially unfunded debt. The weight on the monetary-led part of inflation, $\phi^M = \phi^{M*} = 2$, whereas the central bank disregards the fiscal-led part of inflation, $\phi^F = \phi^{F*} = 0$. The fiscal governments stabilize the monetary-led part of government debt with $\gamma^M = \gamma^{M*} = 20$ and disregard the fiscal-led part of debt with $\gamma^F = \gamma^{F*} = 0$.

Time-varying inflation and debt target With this calibration, the linear fiscal and monetary rules from eqs (26)–(28) simplify to:

$$\hat{\tau}_t = \gamma^M \left(\hat{s}_{b,t-1} - \hat{s}^F_{b,t-1} \right) + \zeta^M_t + \zeta^F_t, \tag{39}$$

$$\hat{\tau}^*_t = \gamma^{M*} \hat{s}^*_{b,t-1}, \tag{40}$$

$$\hat{i}_t = \Theta \left[\phi^M \left(\hat{\pi}_t - \hat{\pi}^F_t \right) \right] + (1 - \Theta) \left[\phi^{M*} \left(\hat{\pi}^*_t - \hat{\pi}^{F*}_t \right) \right]. \tag{41}$$

$\hat{\pi}^F_t$ is the *fiscal inflation* arising from issuance of unfunded debt $\hat{s}^F_{bt}$. So, the Taylor rule targets inflation deviations from this time-varying inflation target, which is *fiscal inflation*. Similarly, the fiscal rule in A targets debt deviations from their unfunded share of debt.

2.8 Results

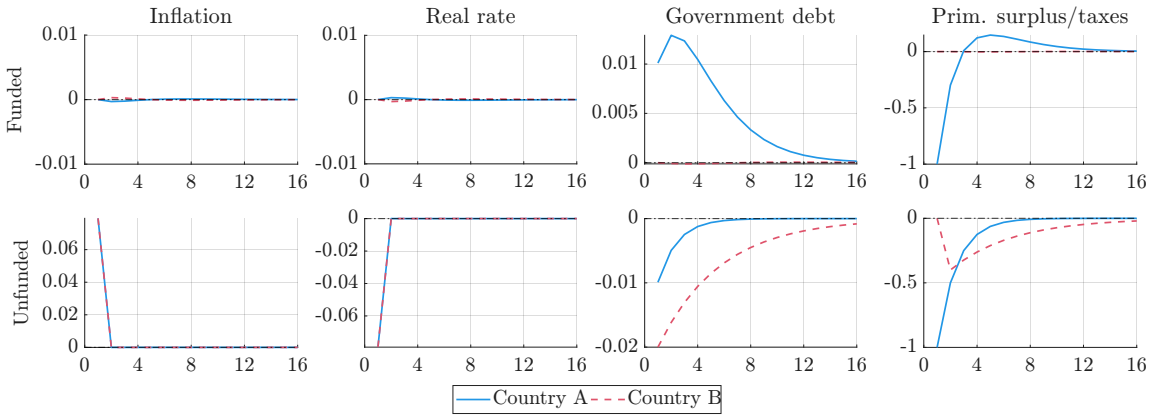
The results of the stylized model provide intuition on how funded and unfunded fiscal shocks propagate through a currency union and generate testable predictions for the empirical analysis that follows. We perform the quantitative exercises, including model estimation and historical shock decompositions in the next Section 4.

First, we consider the results for a lump-sum tax shock, or a shock to the primary surplus, in country A. The results are in Figure 2. The top row shows the results for a funded tax shock, whereas the bottom row shows those for an unfunded tax shock. The funded 1% decline in taxes is followed by an increase in future taxes. Hence, debt increases temporarily. Households anticipate the future tax increase, so demand, and also prices, are unchanged. Therefore, the shock does not spill over to the country B economy.

In contrast, the unfunded 1% decline in taxes is not followed by an increase in future taxes. The intertemporal government budget constraint requires the real value of debt to decrease, achieved by an increase in domestic inflation.[1] Inflation in country B

1. The intertemporal government budget constraint states that the real market value of outstanding debt must equal the present discounted value of future primary surpluses: $\frac{B_{t-1}}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j T_{t+j}$.

Figure 2: Impulse responses to funded and unfunded tax shocks in country A



Notes: Impulse responses to a 1% decline in primary surplus/taxes in country A. The funded shock (top row), i.e. monetary-led, is a shock to ζ_t^M, and the unfunded shock (bottom row) is a shock to ζ_t^F, both with persistence 0.5. The two countries are of equal size. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.

rises by the same amount as in country A, because in the flexible prices and perfect risk sharing, the terms of trade are pinned by real fundamentals only, forcing the nominal price levels of both goods to move proportionally. The real value of debt in country B decreases more than in country A: although inflation is identical in both countries, country A issues additional nominal bonds to finance the tax cut, partially cushioning its real-debt decline, whereas country B's nominal debt expands only through the smaller passive fiscal response. The monetary authority does not react to inflation following an unfunded shock — *fiscal inflation* — that raises the inflation target in the Taylor rule. Hence, the ex-post real rate declines in both countries.[2] This is consistent with BFM's closed-economy finding that the real interest rate responds negatively to unfunded fiscal shocks.

Second, we analyze the results for a government spending shock. Government spending enters the model via the government budget constraint and the goods market clearing.[3] The results are in Figure 3. The funded increase in A's government spending, followed by a future decline, increases government debt in country A. Since government spending absorbs country A's goods, a funded spending shock creates a real

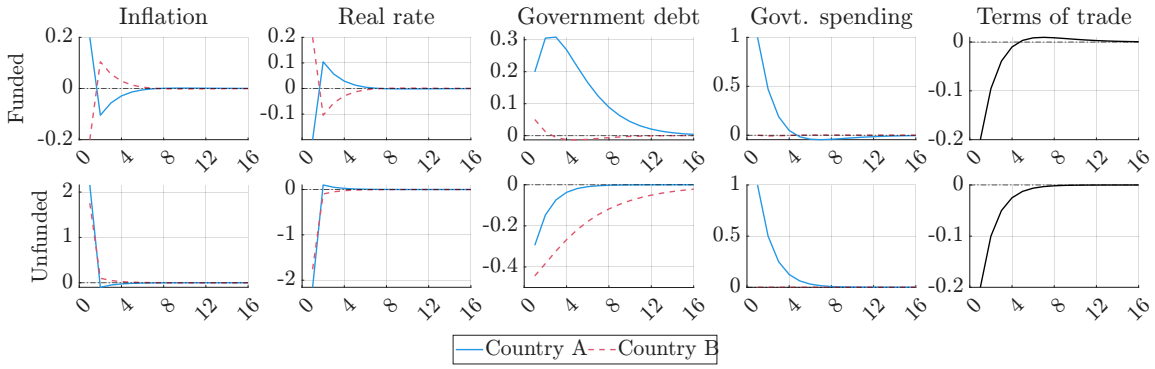
2. We define the real interest rate as $\hat{r}_t \equiv \hat{i}_t - \hat{\pi}_t$ (Appendix B, eq. (B.19)), matching the empirical definition in Section 3. With flexible prices, inflation adjusts within the period, so this contemporaneous measure is the economically relevant one here.

3. We modify the government budget constraint and the goods market clearing condition for country A as follows and do similarly to country B. We assume the steady state government spending-to-GDP ratio to be 0.2.

$$B_t + P_t T_t = R_{t-1} B_{t-1} + G_t \tag{42}$$

$$Y = C_{At} + C^*_{At} + G_t \tag{43}$$

Figure 3: Impulse responses to funded and unfunded government spending shocks in country A



Notes: Impulse responses to a 1% increase in government spending in country A. The funded shock (top row), i.e. monetary-led, and the unfunded shock (bottom row), i.e. fiscal-led, are both with persistence 0.5. The two countries are of equal size. The y-axis is in terms of percentage deviations from steady state. The x-axis is in quarters. Inflation and interest rates are annualized.

demand effect, in contrast to the fully Ricardian funded tax shock. Inflation in country A rises on impact because of the increase in government spending, and the terms of trade appreciate. Since government spending crowds out consumption in country A, demand for country B's goods declines as well, decreasing inflation in B on impact. Hence, inflation rates diverge after a funded government spending shock. After the initial period of the shock, the increase in government debt triggers an increase in lump-sum taxes, since the fiscal rule is passive. So, demand falls and inflation in country A turns negative. Since the central bank targets an average of the inflation rates, the nominal rate remains constant, such that the monetary policy stance is expansionary for country A and contractionary for country B on impact.

After an unfunded government spending shock in country A, domestic inflation increases and the real value of debt decreases, so that the government budget constraint holds. The terms of trade appreciate. In country B, the terms of trade depreciate and the imported good becomes relatively more expensive. Hence, inflation increases in country B and decreases the real value of debt on impact. The passive fiscal rule in B reduces taxes as the real value of debt falls below steady state. The reduction in taxes additionally stimulates domestic demand, further amplifying inflation in country B. Since the monetary authority does not react to *fiscal inflation*, the real rate in both countries declines. So, after an unfunded shock in country A, inflation in both countries increases while the real rate declines.

So, under a funded government spending shock in country A, we see that inflation rates diverge within the currency union, whereas under an unfunded government spending shock, the inflation rates both increase. The model thus delivers a testable prediction: funded government spending shocks generate negative spillover inflation,

while unfunded shocks generate positive spillover inflation across the union. We test this prediction in Section 3 and find that the propagation of Italian shocks is similar to that of unfunded shocks, whereas the propagation of French and German government spending shocks are more ambiguous in fundedness. In Section 4 we build and estimate a medium-scale DSGE model to quantify the mix of funded and unfunded shocks in those countries and explain the different inflation spillovers through those shocks.

3 Empirical analysis

The stylized model predicts that funded government spending shocks generate divergent inflation responses across currency union members, while unfunded shocks cause inflation to co-move. We test these predictions using local projections à la Jordà (2005) and forecast-error fiscal shocks for Germany, France, and Italy—the three largest euro area economies. We establish two facts: fiscal expansions in France and Italy are inflationary in other euro area countries, while expansions in Germany are deflationary. These results suggest that the central bank accommodates inflation from French and Italian fiscal expansions (*fiscal inflation*), but not from German ones.

3.1 Methodology and data

Fiscal shocks. We construct fiscal shocks using the OECD Economic Outlook (OECD, 2025). The Economic Outlook, published twice a year, includes point forecasts of macroeconomic variables—including government spending—for the next few quarters. As in Auerbach and Gorodnichenko (2012, 2017) and Boehm (2020), we construct government spending forecast errors as a share of GDP for each country.[4] We have quarterly data starting from *Economic Outlook No 74* (December 2003), so our sample spans 2004Q1–2026Q1.

We use government consumption in volume (constant-price) terms and extract forecasts for Q1 and Q2 of each year from the second *Economic Outlook* of the previous year, and for Q3 and Q4 from the first *Economic Outlook* of the same year. For example, the 2016Q1–Q2 forecasts come from the second *Economic Outlook* of 2015. We scale the forecasted growth rate by lagged GDP to obtain a government spending forecast as a percentage of GDP. The forecast error $FE_{t|t-1}$ is the realized minus forecasted government-spending growth rate (as a share of GDP), where realizations come from the most recent *Economic Outlook*.

4. Staehr, Tkacevs, and Toiger (2026) apply the same forecast-error identification strategy to EU countries using European Commission (AMECO) primary balance forecast errors rather than OECD government spending forecasts.

Even though $FE_{t|t-1}$ is already the unpredictable innovation in government-spending growth (as in Auerbach and Gorodnichenko (2017)), we further purge predictable components by regressing it on lags of macroeconomic variables, using the residuals as the fiscal shock. Appendix A shows all forecast errors in Figure A.1 and their correlations in Table A.1. Purged government spending shocks are weakly positively correlated with next-quarter debt/GDP growth (r = 0.09–0.14 across countries) as shown in Figure A.2, consistent with spending expansions being partially debt-financed, though the relationship is unsurprisingly noisy given the many other drivers of debt dynamics.

Other data. All control variables are publicly available at the OECD. The harmonized inflation rate and the short-term nominal interest rate are expressed in annualized percentage points. For the union-wide policy rate we use the Wu and Xia (2016) shadow short rate for the euro area, which measures the stance of policy when the effective lower bound binds and is unconstrained by it.[5] The real rate is the difference between the nominal rate and inflation. We focus on the original euro area countries to include countries that have shared a central bank for the longest period.[6]

Local projection specification. We estimate three types of local projections: (i) country-specific, for domestic inflation and real rate; (ii) spillover panel, for inflation and real rate in other countries; and (iii) short-rate, for the union-wide nominal interest rate.

To allow for asymmetric fiscal transmission, we decompose the shock into a positive part $\text{shock}^+_{c,t} = \max(\text{shock}_{c,t}, 0)$ (fiscal expansions) and a negative part $\text{shock}^-_{c,t} = \min(\text{shock}_{c,t}, 0)$ (fiscal contractions), and include both as separate regressors. We focus on the expansion coefficient $\beta^+_{c,h}$; the contraction term enters as a control to ensure $\beta^+_{c,h}$ is identified from expansion variation only.

The country-specific specification (y is domestic inflation or the real rate) is:

$$y_{c,t+h} - y_{c,t} = \beta^+_{c,h}\text{shock}^+_{c,t} + \beta^-_{c,h}\text{shock}^-_{c,t} + \sum_{j=0}^{3} \gamma_{c,h,j} X_{c,t-j} + \varepsilon_{c,t+h}, \tag{44}$$

where $c \in \{\text{Germany, France, Italy}\}$. $y_{c,t+h}$ is the inflation or real rate of country c at time $t+h$. $X_{c,t-j}$ includes the control variables: government spending growth, GDP growth, inflation, and their lags.[7] We estimate (44) separately for domestic inflation and real rate, up to $h = 8$ quarters ahead, with Newey-West HAC standard errors.

5. The shadow rate is available monthly from 2004M9 to 2022M8; we average it to quarterly frequency and splice the observed short rate before and after, following the authors' guidance.

6. Austria, Belgium, Germany, Spain, France, Ireland, Italy, Netherlands, Portugal. We exclude Luxembourg and Finland for data availability reasons.

7. For the real rate local projections, we add the lags of the change of the real rate as controls. Moreover, we include euro area wide GDP and inflation, since those are naturally confounded with the euro area wide short-term interest rate, included in constructing the dependent variable.

The spillover specification estimates inflation and real rate in other euro area countries:

$$y_{i\neq c,t+h} = \beta^{+}_{c,h}\text{shock}^{+}_{c,t} + \beta^{-}_{c,h}\text{shock}^{-}_{c,t} + \sum_{j=0}^{3}\gamma_{i,h,j}X_{i,t-j} + \alpha_{i\neq c} + \varepsilon_{i\neq c,t+h}, \tag{45}$$

where i ranges over all sample countries and $c \in \{\text{Germany, France, Italy}\}$ is the source country. The dependent variable and country fixed effects pertain to receiver countries ($i \neq c$). Controls mirror the country-specific specification, augmented with contemporaneous and lagged euro area-wide inflation and GDP growth from both receiver and source countries. Quarterly time fixed effects are excluded: the source-country shock is identical across receiver countries and collinear with them. The spillover projections use Driscoll-Kraay standard errors with GDP weights.

The short-rate specification replaces $y_{c,t}$ with the euro area-wide short rate i_t:

$$i_{t+h} = \beta^{+}_{c,h}\text{shock}^{+}_{c,t} + \beta^{-}_{c,h}\text{shock}^{-}_{c,t} + \sum_{j=0}^{3}\gamma_{c,h,j}X_{c,t-j} + \varepsilon_{c,t+h}. \tag{46}$$

Controls additionally include i_t, contemporaneous and lagged euro area-wide inflation and GDP growth, and lags of Δi_t. Because the shadow rate is not constrained by the effective lower bound, the short-rate and real-rate projections use the full sample rather than stopping at the 2014Q2 onset of the ELB. Appendix A reports the same projections using the observed short rate on the pre-ELB sample.

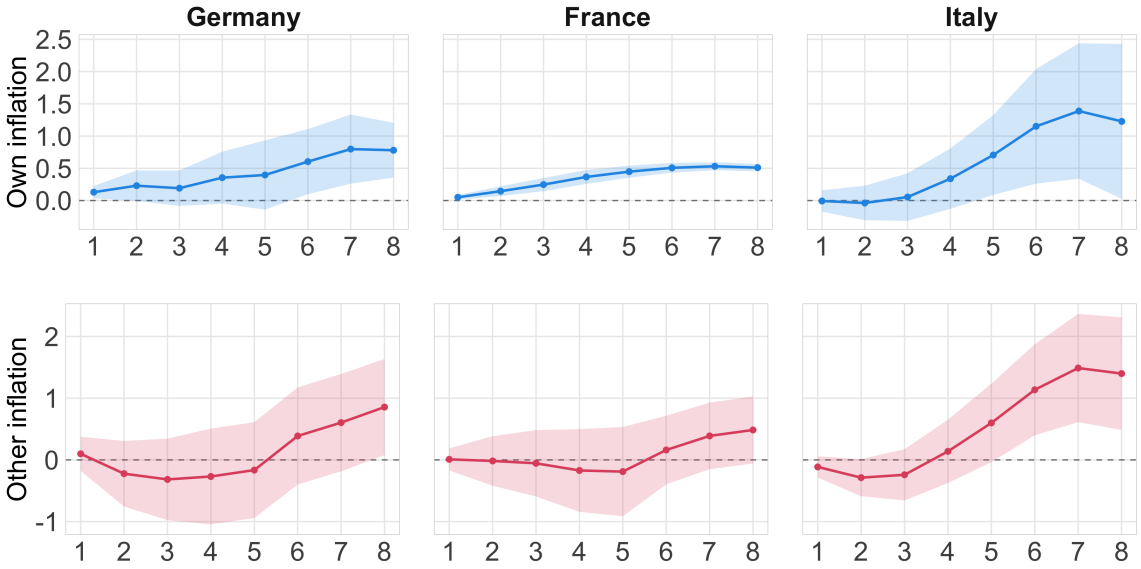
3.2 Results

Figure 4 shows the effect of a fiscal expansion on (i) domestic inflation and (ii) other countries' inflation. Results for (iii) the short-term interest rate, (iv) domestic real rate, and (v) other countries' real rate are in Figure A.3. Column labels indicate the source country.

Figure 4 shows the effect of fiscal expansions in Germany, France, and Italy on domestic (top row) and other-country inflation (bottom row). On the one hand, fiscal expansions are inflationary domestically for all three countries. From the results of the stylized model in Section 2, both funded and unfunded government spending shocks increase inflation domestically through demand effects. Our empirical results are therefore qualitatively in line with the model dynamics.

On the other hand, only Italian expansions raise inflation abroad too, consistent with the propagation of unfunded government spending shocks from the stylized model. German and French shocks are neither inflationary nor deflationary abroad, which could indicate a mix of funded and unfunded expansions, leaving the overall spillover ambiguous.

Figure 4: Impulse responses to a 0.1 pp increase in govt spending (% of GDP)



Notes: Responses are to an unanticipated increase in government consumption of 0.1 percentage points of GDP, approximately the median positive forecast error in the sample. Units are percentage points. The own country and short rate local projections use Newey-West HAC standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2026Q1.

So, interpreting the empirical results through the lens of the model, we find that Italian fiscal expansions are more consistent with unfunded fiscal expansions from the model, showing potential *fiscal inflation* spillovers. German and French fiscal expansions show propagation that is in line with our model when we allow for partially unfunded debt.

Appendix A reports robustness exercises. Figure A.4 uses 2 lags of controls instead of 3. Figure A.5 repeats the exercise with unpurged shocks; Figure A.6 varies the standard error specification. Figure A.7 adds lags of fiscal shocks as controls; Figure A.8 adds contemporaneous weighted fiscal shocks of other countries and their lags. Figure A.9 replaces the shadow rate with the observed short rate on the pre-ELB sample, 2004Q1–2014Q1. Figure A.10 removes within-semester reallocation variation from the forecast errors before re-estimating.[8] All are consistent with the baseline. We also report local projections on sovereign spreads in Figure A.11.

8. For each country and semester (S1: Q1–Q2, S2: Q3–Q4), we compute the quarterly forecast error as before, but replace each quarter's error with the semester average of the two quarterly forecast errors. This keeps the semester-level surprise in government spending but removes within-semester timing reallocation (e.g., if spending was expected in Q3 but realized in Q4). We then apply the same purging step and re-estimate all local projections with the same controls and horizons.

4 Quantitative model

In this section, we build a medium-scale currency union model with partially unfunded debt. The model features habit formation by households, production of tradable and non-tradable goods with nominal rigidities, capital formation, and long-term bonds. The model aims to capture euro area business cycles, enabling estimation of the unobserved *fiscal inflation*. The Online Appendix contains a detailed derivation of the model. Appendix D summarizes the model with the relevant log-linear model equations.

4.1 Households

Households derive utility from consumption C_t and disutility from providing labor L_t. Internal consumption habits link utility to the household's own previous-period consumption. The per-period utility function is:

$$\ln\left(C_t - \varepsilon_t^{tC} - h\left(C_{t-1} - \varepsilon_{t-1}^{tC}\right)\right) - \frac{1}{1+\chi} L_t^{1+\chi} \exp(\varepsilon_t^h), \tag{47}$$

where h is the habit parameter, χ the inverse-Frisch elasticity of labor supply, ε_t^h is an IID labor supply shock, and ε_t^{tC} is an IID forced-savings shock that enters inside the habit term. Letting λ_t denote the marginal utility of current consumption, internal habit implies $\widehat{mrs}_t = -\hat{\lambda}_t + \chi \hat{l}_t + \varepsilon_t^h$. As in Ferroni, Fisher, and Melosi (2024) and Cardani et al. (2022), we use these two shocks to accommodate the COVID-19 pandemic period (2020Q1–2021Q2) in the estimation.

Households derive income from renting out capital K_t at rate R_{Kt} to tradable goods producers and exerting labor for wages W_t. The price of a unit of capital is Q_t and the depreciation rate is δ. Households can invest in capital through perfectly competitive capital producers. They can also save and smooth consumption through one-period government bonds B_t which are in zero net supply, and through long-term government bonds from both country A, B_{At}, and country B, B_{Bt}, in nonzero net supply. The one-period bonds pay interest rate R_t which is set by the central bank. We follow Woodford (2001) and model long-term bonds as a perpetuity with geometrically declining coupons with average maturity of $(1-\rho)^{-1}$ quarters, where $\rho \in [0,1]$ is a constant rate of decay. Households can purchase the long-term bonds from country A and B at price P_t^L and P_t^{L*} respectively. Arbitrage conditions determine the price of these bonds, together with country specific risk premium shocks ξ_t^{rp} and ξ_t^{rp*}:

$$R_t = \mathbb{E}_t R_{t+1}^L e^{\xi_t^{rp}} \equiv \mathbb{E}_t \left[\frac{1 + \rho P_{t+1}^L}{P_t^L}\right] e^{\xi_t^{rp}}, \tag{48}$$

and

$$R_t = \mathbb{E}_t R_{t+1}^{L*} e^{\xi_t^{rp*}} \equiv \mathbb{E}_t \left[\frac{1 + \rho P_{t+1}^{L*}}{P_t^{L*}} \right] e^{\xi_t^{rp*}}. \tag{49}$$

The per-period household budget constraint is:

$$\begin{aligned} &P_t(1+\tau_{Ct})C_t + Q_t K_t + P_t^L B_{At} + P_t^{L*} B_{Bt} + B_t \\ &= W_t(1-\tau_{Lt})L_t + P_t Z_t + \left[R_{Kt}(1-\tau_{Kt})\mu_{Kt} - a(\mu_{Kt}) + Q_t(1-\delta)\right] K_{t-1} + R_{t-1}B_{t-1} \\ &\quad + \left(1 + \rho P_t^L\right) B_{A,t-1} + \left(1 + \rho P_t^{L*}\right) B_{B,t-1} - \frac{\psi_b P_t Y_t \bar{b}_B}{2} \left(b_{Bt} - \bar{b}_B\right)^2, \end{aligned} \tag{50}$$

where Z_t are lump-sum transfers from the government and τ_{Ct}, τ_{Lt}, and τ_{Kt} are consumption tax, labor income tax, and capital income tax, respectively. μ_{Kt} is the capital utilization rate and $a(\mu_{Kt})$ the resource cost. As in the stylized model, domestic long-term bond holdings are frictionless while foreign long-term bond holdings are subject to a quadratic adjustment cost around their steady state.

C_t is a constant elasticity of substitution (CES) aggregate of tradable goods C_{Tt} and non-tradable goods C_{Nt}, and P_t is the price of the aggregate bundle:

$$C_t = \left[(1-\alpha_N)^{1/\mu_N}(C_{Tt})^{(\mu_N-1)/\mu_N} + \alpha_N^{1/\mu_N}(C_{Nt})^{(\mu_N-1)/\mu_N}\right]^{\mu_N/(\mu_N-1)}, \tag{51}$$

$$P_t = \left[(1-\alpha_N)P_{Tt}^{1-\mu_N} + \alpha_N P_{Nt}^{1-\mu_N}\right]^{1/(1-\mu_N)}, \tag{52}$$

where C_{Tt} and C_{Nt} are consumption of tradable and non-tradable goods, respectively, and P_{Tt} and P_{Nt} their prices. α_N is the share of non-tradable goods in the consumption basket and μ_N the elasticity of substitution between tradable and non-tradable goods. The tradable bundle C_{Tt} and its price P_{Tt} correspond to the consumption good and its price in the stylized model.

4.2 Wage setting

We introduce differentiated labor and wage rigidities as in Christiano, Eichenbaum, and Evans (2005). We assume that each household supplies a unique labor type $L_t(l)$ at nominal wage $W_t(l)$, where $l \in [0,1]$. A labor union then transforms the differentiated labor outputs into aggregate labor L_t using a CES aggregator, $L_t = \left(\int_0^1 L_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} dl\right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$, where ϵ_w is the elasticity of substitution across labor types.

With probability $(1-\theta_w)$, each labor type can reset its wage each period. When the labor type is not re-optimizing its wage, the wage evolves via the following indexation, $W_t(l) = W_{t-1}(l)\Pi_{t-1}^{\chi_w}$, where Π_t is gross price inflation and $\chi_w \in [0,1]$ the degree of wage indexation. When the labor type is re-optimizing its wage, it maximizes expected household utility, $\mathbb{E}_t \sum_{s=0}^{\infty}(\beta\theta_w)^s[\ln C_{t+s}^{\text{eff}} - L_{t+s}(l)^{1+\chi}\exp(\varepsilon_{t+s}^h)/(1+\chi)]$, taking

into account labor type demand, wage indexation, the household budget constraint, and the associated marginal utility λ_{t+s}. Thus the wage objective uses actual effective consumption, $C_t^{\text{eff}} = C_t - \varepsilon_t^{tC} - h(C_{t-1} - \varepsilon_{t-1}^{tC})$, rather than an unadjusted consumption proxy. The first-order condition, after applying the symmetry condition $W_t(l) = W_t$ and log-linearizing, gives the standard wage Phillips Curve:

$$\hat{\pi}_t^w = \beta \mathbb{E}_t \hat{\pi}_{t+1}^w + \chi_w \hat{\pi}_{t-1} + \kappa_w \left(\widehat{mrs}_t - \hat{w}_t^{real} \right) - \kappa_w \frac{\overline{\tau}_L}{1 - \overline{\tau}_L} \hat{\tau}_{Lt} + u_t^w, \tag{53}$$

where $\kappa_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\chi)}$ and u_t^w is a wage markup shock. Following Christiano, Eichenbaum, and Evans (2005), we set the gross desired wage markup to 1.05, which implies $\epsilon_w = 1.05/(1.05-1) = 21$. Its country-$B$ analogue enters $\kappa_w^* = \frac{(1-\theta_w^*)(1-\beta\theta_w^*)}{\theta_w^*(1+\epsilon_w\chi^*)}$.

4.3 Firms

4.3.1 Capital producers

Capital producers transform final goods into investment subject to adjustment costs as in Christiano, Eichenbaum, and Evans (2005). The optimization problem for the capital producers is $\max_{I_t} \mathbb{E}_t \sum_{t=0}^{\infty} \Lambda_{t,t+1} \left[Q_t - 1 - \frac{\psi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t$, where $\Lambda_{t,t+1} = \beta\lambda_{t+1}/\lambda_t$ is the stochastic discount factor and ψ_I the investment adjustment cost. Investment turns into capital K_t, which evolves according to the law of motion $K_t = (1-\delta)K_{t-1} + I_t$. We assume that capital is only used in the tradable sector.

4.3.2 Final goods producers

Final goods producers aggregate differentiated intermediate goods from monopolistically competitive firms in both the tradable and non-tradable goods sector, using a CES aggregator: $Y_{Tt} = \left(\int_0^1 Y_{Tt}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$ and $Y_{Nt} = \left(\int_0^1 Y_{Nt}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$ where $i \in [0,1]$ represents the continuum of firms in the tradable goods sector, producing goods $Y_{Tt}(i)$, and $j \in [0,1]$ the continuum of firms in the non-tradable sector, producing goods $Y_{Nt}(i)$. ϵ is the elasticity of substitution between the differentiated goods.

4.3.3 Tradable goods producers

There is a continuum of tradable goods firms $i \in [0,1]$ who produce differentiated intermediate inputs $Y_{Tt}(i)$ for the final goods producer using the following production technology: $Y_{Tt}(i) = A_{Tt}(\mu_{Kt}K_{t-1}(i))^{\alpha} L_{Tt}(i)^{1-\alpha}$, where α is the output elasticity of capital. Cost minimization implies $R_{Kt} = \alpha MC_{Tt} Y_{Tt}/(\mu_{Kt}K_{t-1})$, while the household's utilization choice satisfies $(1-\tau_{Kt})R_{Kt} = a'(\mu_{Kt})$. A_{Tt} is the labor augmenting technology and follows an AR(1) process. The firms are under monopolistic competition

and are subject to Calvo (1983) price rigidities. Hence, the optimization problem of the firm is:

$$\max_{P_{At}(i),L_{Tt}(i),K_t(i)} \mathbb{E}_0 \sum_{s=0}^{\infty} \theta_T^s \Lambda_{t,t+1} \left[\left(\frac{P_{At}(i)}{P_{A,t+s}} - MC_{t+s}(i) \right) Y_{T,t+s}(i) \right], \tag{54}$$

subject to the production function and the demand for differentiated tradable goods. $(1-\theta_T)$ is the fraction of firms that can set their price in each period and $MC_t(i)$ is the real marginal cost for firm i. After applying the symmetry conditions $P_{At}(i) = P_{At}$ and $MC_t(i) = MC_t$ and log-linearizing, the optimization problem gives rise to the standard New Keynesian Phillips Curve (NKPC):

$$\hat{\pi}_{At} = \kappa_T \widehat{mc}_{Tt} + \beta \mathbb{E}_t \hat{\pi}_{A,t+1} + u_t^T \tag{55}$$

where $\kappa_T \equiv \frac{(1-\theta_T\beta)(1-\theta_T)}{\theta_T}$, $\widehat{mc}_{Tt} = \hat{w}_t^{real} + \hat{l}_{Tt} - \hat{y}_{Tt}$ is the log-linear real marginal cost of the firm, and u_t^T is a tradable goods markup shock.

4.3.4 Non-tradable goods producers

There is a continuum of non-tradable firms $j \in [0,1]$ producing differentiated intermediate inputs $Y_{Nt}(j)$ for the final goods producer. In contrast to the tradable goods producers, they only use labor as a production input: $Y_{Nt}(j) = A_{Nt} L_{Nt}(j)^{1-\alpha}$. A_{Nt} is the labor augmenting technology and follows an AR(1) process. Like tradable firms, non-tradable producers face monopolistic competition with Calvo (1983) price rigidities; the labor demand and NKPC are analogous.

4.4 Monetary authority

The monetary policy rule responds to the GDP-weighted union inflation gap and is augmented relative to the stylized model with interest-rate smoothing and a transitory monetary policy shock ε_t^m:

$$\hat{i}_t = \rho_r \hat{i}_{t-1} + (1-\rho_r)\phi^M \left(\hat{\pi}_t^{MU} - \pi_t^{F,MU} \right) + \varepsilon_t^m, \tag{56}$$

where ρ_r governs interest-rate smoothing, $\hat{\pi}_t^{MU} \equiv \Theta\hat{\pi}_t + (1-\Theta)\hat{\pi}_t^*$ is union CPI inflation, and $\pi_t^{F,MU} \equiv \Theta\pi_{A,t}^F + (1-\Theta)\pi_{B,t}^{F*}$ is the union inflation target.[9]

9. The Online Appendix considers a version of the model with an effective lower bound and quantitative easing.

4.5 Fiscal authority

Assuming a zero net supply of one-period government bonds, the nominal government budget constraint is:

$$P_t^L B_{At} + \tau_{Kt} R_{Kt} \mu_{Kt} K_{t-1} + \tau_{Lt} W_t L_t + \tau_{Ct} P_t C_t = \left(1 + \rho P_t^L\right) B_{A,t-1} + P_t G_t + P_t Z_t. \quad (57)$$

The government raises revenue through distortionary taxes on capital income at rate τ_{Kt}, labor income at rate $\tau_{L,t}$, and consumption at rate $\tau_{Ct} = \tau_C$ (held constant), together with lump-sum transfers Z_t. The steady-state tax rates are calibrated to euro area averages: $\bar{\tau}_K = 0.25$, $\bar{\tau}_L = 0.38$, and $\bar{\tau}_C = 0.20$. The fiscal authority adjusts distortionary taxes, government spending-to-output $g_t = G_t/Y_t$, and lump-sum transfers-to-output $z_t = Z_t/Y_t$ in response to the lagged debt gap:

$$\hat{\tau}_{Kt} = \rho_K \hat{\tau}_{K,t-1} + (1 - \rho_K)\gamma_K \left(\hat{s}_{b,t-1} - s^F_{b,A,t}\right), \quad (58)$$

$$\hat{\tau}_{L,t} = \rho_L \hat{\tau}_{L,t-1} + (1 - \rho_L)\gamma_L \left(\hat{s}_{b,t-1} - s^F_{b,A,t}\right), \quad (59)$$

$$\hat{g}_t = \rho_g \hat{g}_{t-1} - (1 - \rho_g)\gamma_g \left(\hat{s}_{b,t-1} - s^F_{b,A,t}\right) + \varepsilon^g_t, \quad (60)$$

$$\hat{z}_t = \rho_z \hat{z}_{t-1} - (1 - \rho_z)\left[\gamma_z \left(\hat{s}_{b,t-1} - s^F_{b,A,t}\right) + \phi_{zy} \hat{y}_t\right] + \frac{1}{\overline{Z}/\overline{Y}} \zeta_t, \quad (61)$$

where ρ_X are persistence parameters and γ_X the debt feedback coefficients on the debt-to-GDP ratio $s_{bt} = \frac{P_t^L B_{At}}{P_t Y_t}$. The debt target evolves as $s^F_{b,A,t} = \rho_s s^F_{b,A,t-1} + \varepsilon^U_{A,t}$, and the starred country-B rules use their corresponding starred parameters and target $s^{F*}_{b,B,t}$. As in Bianchi, Faccini, and Melosi (2023), transfers also respond to output fluctuations, and ζ_t is the funded transfer disturbance.

Relationship to the stylized decomposition. The estimated quantitative model is a single floating-target economy rather than an explicit ML–FLA–FLB superposition. The debt targets summarize the fiscal-led debt component from the stylized decomposition, while the inflation targets enter the single union Taylor rule directly. The FLA and FLB labels therefore do not denote separately solved economies in the estimated model.

4.6 Equilibrium

The equilibrium of this economy is characterized by a sequence of prices

$$\left\{P_{At}, P_{Bt}, P_{Nt}, P^*_{Nt}, W_t, W^*_t, R_t, R_{Kt}, R^*_{Kt}\right\},$$

and allocations

$$\left\{Y_{Tt}, Y^*_{Tt}, Y_{Nt}, Y^*_{Nt}, K_t, K^*_t, \mu_{Kt}, \mu^*_{Kt}, I_t, I^*_t, G_t, G^*_t, C_{Tt}, C^*_{Tt}, C_{Nt}, C^*_{Nt}, L_{Tt}, L^*_{Tt}, L_{Nt}, L^*_{Nt}, B_{At}, B_{Bt}, B^*_{At}, B^*_{Bt}\right\},$$

such that goods, labor, and bond markets clear. We express the dynamics of net foreign assets in country A with the current account definition:

$$\begin{aligned} CA_t \equiv & P_{At}Y_{Tt} - P_{Tt}C_{Tt} - \frac{\psi_b P_t Y_t \bar{b}_B}{2}\left(b_{Bt} - \bar{b}_B\right)^2 - P_{At}I_t\left[1 + \frac{\psi_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] - P_{At}G_t - P_{At}a(\mu_{Kt})K_{t-1} \\ = & B_{Bt} - B^*_{At} - R_{t-1}\left(B_{B,t-1} - B^*_{A,t-1}\right). \end{aligned} \tag{62}$$

4.7 Estimation

We discipline the structural parameters through Bayesian estimation of the two-country model. We estimate the model separately for each country pair, with country A being either France, Germany, or Italy, and country B being the GDP-weighted aggregate of the remaining euro area countries. Roughly a third of our estimation sample is spent at the effective lower bound (ELB), where the observed policy rate no longer measures the stance of monetary policy. We therefore observe the policy stance through the Wu and Xia (2016) shadow rate, which is unconstrained by the bound, evaluate the likelihood of the linear model with the Kalman filter, and simulate the posterior with a random-walk Metropolis–Hastings sampler. The Online Appendix reports two alternatives: an estimation on the observed short rate, which forces the linear rule through the quarters at the bound, and an estimation that imposes an occasionally binding ELB and adds a block for the Eurosystem's asset purchases, whose likelihood is evaluated with the piecewise Kalman filter (Giovannini, Pfeiffer, and Ratto, 2021). The Online Appendix contains more details about all three estimations.

Fiscal economy estimation strategy. We follow Guo, Han, and Smit (2026) and estimate time-varying inflation targets and unfunded debt targets. The monetary rule responds to country inflation-target gaps, while each fiscal rule responds to its country-specific debt-target gap. A single unfunded shock $\varepsilon^U_{c,t}$ drives both targets simultaneously: $s^F_{b,c,t} = \rho_s s^F_{b,c,t-1} + \varepsilon^U_{c,t}$ and $\pi^F_{c,t} = \phi_F \pi^F_{c,t-1} + \phi_{\pi,u}\varepsilon^U_{c,t}$, where Guo, Han, and Smit (2026) analytically derive ρ_s and ϕ_F; we use their solutions, which are isomorphic to the shadow economy but substantially reduce the computational burden. The shock thus relaxes the debt-target gap that fiscal rules respond to and simultaneously raises the central bank's inflation target, so the monetary rule accommodates higher inflation. We calibrate $\phi_{\pi,u} = 0.10$ following Guo, Han, and Smit (2026). See Appendix D and

the Online Appendix for details.

Data and shocks. We use the publicly available large datasets for the euro area, collected by Barigozzi, Lissona, and Tonni (2025), for 2000Q1–2025Q3, where 2025Q3 is the last quarter of the Eurostat transfers data; after growth rates are formed, the estimation sample is 2000Q2–2025Q3 (102 quarters). For each country pair, we construct 21 observables: ten per region (real consumption, real government spending, real investment, hours, real GDP, and gross debt-to-GDP in growth rates, wage and CPI inflation, the long-term interest rate, and transfers growth) and one shared short-term interest rate. The short rate is the Wu and Xia (2016) shadow rate for the euro area, spliced with the observed short rates outside the effective lower bound period. We construct the data for transfers ourselves with data from Eurostat, similarly to Bianchi, Faccini, and Melosi (2023). See the Online Appendix for details.

The shocks we introduce into our model for the estimation are: technology (tradable and non-tradable), monetary policy, funded fiscal disturbances, unfunded fiscal disturbances, risk premia, preferences, price markups (tradable and non-tradable), wage markups, investment, and government spending. We add measurement errors for investments, consumption, hours, wages, output, debt, government spending, the long-term interest rate, the short-term interest rate, and transfers in each region.

Priors. Table 1 shows the parameters that are fixed in the estimation. These are matched to data or standard literature values. The discount factor $\beta = 0.99$ implies a 4% steady-state annualized real rate. Baseline trade openness $\alpha_I = 0.25$ matches the euro area import share (Eurostat, 2023). The country-specific import shares are $\alpha_I^A = 2(1-\Theta)\alpha_I$ and $\alpha_I^B = 2\Theta\alpha_I$; this adjustment preserves balanced bilateral trade and reduces to α_I when $\Theta = 1/2$. The non-tradable consumption share $\alpha_N = 0.6$ and the tradable–non-tradable elasticity $\mu_N = 1$ follow the literature. The relative country size Θ equals country A's GDP share in the estimation sample. Habit persistence $h = 0.70$ follows the literature.

Capital depreciates at $\delta = 0.025$ following Christiano, Eichenbaum, and Evans (2005). The output elasticity of capital $\alpha = 0.40$ and investment adjustment cost $\psi_I = 2.00$ follow the literature. The rate of decay $\rho^L = 0.975$ matches EMU data, implying 10-year average bond maturity. We take the steady-state real market value of debt, $\overline{s}_b = 0.80$, from Eurostat Data (2025).

The coefficient ϕ^M has a prior mean of 2, consistent with active monetary policy. We calibrate the inflation-target loading $\phi_{\pi,u} = 0.10$ and the inflation- and debt-target persistences $\phi_F = \rho_s = 0.98$, following Guo, Han, and Smit (2026). For the fiscal debt-

stabilization parameters, we deviate from BFM's value of 20 and instead calibrate $\gamma^M = \gamma^{M*} = \gamma^G = \gamma^{G*} = 0.30$. BFM's choice of 20 implies very aggressive debt stabilization that is not consistent with the data; 0.30 is sufficient to satisfy Blanchard–Kahn stability conditions in the currency union.

Table 2 shows prior distributions. We estimate region-specific structural parameters for both countries, including Calvo price and wage stickiness (θ_T, θ_N, θ_w), and wage indexation χ_w. A single pooled trade elasticity μ applies to bilateral demand in both regions. The cross-border bond friction ψ_b, capital utilization cost ψ_u, interest rate smoothing ρ_r, and ϕ^M are also common parameters. We fix the inverse Frisch elasticities at $\chi = \chi^* = 1$ (Table 1). The prior means of the Calvo probabilities are 0.75 (Beta), wage indexation is 0.25 (Beta), and ϕ^M is 2 (Normal). We estimate the volatilities of the funded government-spending and transfer shocks, σ_g^{ML} and σ_ζ^{ML} (and their counterparts σ_g^{ML*}, σ_ζ^{ML*}), with diffuse inverse-Gamma priors. The AR(1) persistence parameters of the funded fiscal shocks (ρ_g^{ML}, ρ_ζ^{ML}, ρ_z, and their starred counterparts) are estimated with Beta priors. We also estimate common COVID shock scales σ_h and σ_c for both regions' heteroskedastic labor-supply and forced-savings wedges with Gamma priors. Remaining persistence and standard deviation parameters use standard Beta and inverse-Gamma priors.

Posteriors. Table 2 presents the prior and posterior distributions for the estimation with Germany as country A and the rest of the euro area as country B; the French and Italian estimations are in Tables A.2 and A.3 in Appendix A.2. The posterior median of the Taylor rule inflation coefficient ϕ^M is 2.13 for Germany, 1.71 for France, and 2.09 for Italy, and the lower bounds of the 90 percent highest posterior density intervals, 1.78, 1.46, and 1.73, all exceed one, confirming the active monetary regime that the partially-unfunded-debt framework requires. Interest-rate smoothing ρ_r is 0.78, 0.72, and 0.78. The unfunded shock volatilities $\sigma_{U,A}$ and $\sigma_{U,B}$ have posterior medians of 0.20 and 0.25 for Germany, 0.22 and 0.15 for France, and 0.45 and 0.19 for Italy.

4.8 Results

We present the historical shock decomposition of euro area headline inflation and the model-implied ECB inflation target. The quantitative results complement the reduced-form evidence in Section 3: whereas the local projections identify the sign of fiscal spillovers for individual source countries, the structural model traces the aggregate time-series path of fiscal inflation over the full sample period. Since results are broadly consistent across the three estimations, we present the average for a euro area-wide picture that is robust to the choice of reference country; the individual country-pair

Table 1: Calibrated parameters

Parameter	Description	Value	Source
Households			
β	Discount factor	0.99	literature
α_I	Degree of trade openness	0.25	Eurostat (2023)
α_N	Share of non-tradable goods	0.6	literature
μ_N	Elasticity of subst. trad. vs. non-trad. goods	1	literature
Θ	GDP share of country A	-	data; sample-specific[a]
h	Habit persistence	0.70	literature
ϵ_w	Elasticity across labor types	21	Christiano, Eichenbaum, and Evans (2005)
χ, χ^*	Inverse Frisch elasticity of labor supply	1	calibrated
Firms			
δ	Capital depreciation rate	0.025	Christiano, Eichenbaum, and Evans (2005)
α	Output elasticity of capital	0.40	literature
ψ_I	Investment adjustment cost	2.00	literature
Monetary			
$\phi_{\pi,u}$	Inflation-target shock loading	0.10	Guo, Han, and Smit (2026)
ϕ_F	Inflation-target persistence	0.98	Guo, Han, and Smit (2026)
$\overline{R}$	Steady-state short rate (annualized pp)	-	data; sample mean
Fiscal			
ρ^L	Rate of decay long-term bond	0.975	EMU Criterion
$\overline{s}_b$	Steady-state real market value debt	0.80	Eurostat Data (2025)
γ^M, γ^{M*}	Fiscal rule debt-stabilization (ML)	0.30	calibrated
γ^G, γ^{G*}	Govt. spending debt-stabilization (ML)	0.30	calibrated
ρ_s	Debt-target persistence	0.98	Guo, Han, and Smit (2026)

[a] Θ is set to country A's average GDP share in the estimation sample, computed from the same Barigozzi, Lissona, and Tonni (2025) dataset used for the observables. This gives $\Theta \approx 0.21$ for France, $\Theta \approx 0.29$ for Germany, and $\Theta \approx 0.19$ for Italy.

results are in Appendix A.2, and the results of the two alternative estimations are in the Online Appendix.

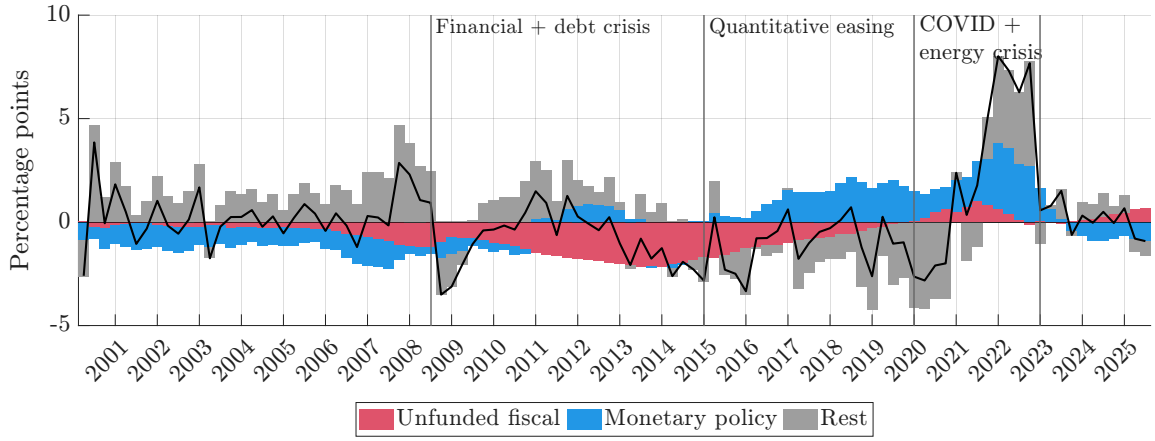
First, Figure 5 presents the historical shock decomposition of euro area headline inflation, decomposing observed quarterly inflation into contributions from unfunded fiscal shocks, monetary policy shocks, and all remaining disturbances. Before the global financial crisis, unfunded fiscal and monetary policy shocks both held inflation slightly below its sample mean. Following the onset of the crisis in 2008, the contribution of unfunded fiscal shocks turned increasingly negative, reached its largest negative values around 2014, and faded only gradually thereafter. Negative unfunded fiscal shocks correspond to fiscal consolidation: for instance, tax increases not reversed by future cuts. These shocks are consistent with the austerity measures implemented after the global financial crisis. From the launch of quantitative easing in 2015, monetary policy shocks turned expansionary: measured by the shadow rate, the stance of policy at the bound was looser than the rule prescribes, and the monetary contribution remained positive until 2023. The contribution of unfunded fiscal shocks turned positive with the COVID-19 pandemic and peaked in 2022, when unfunded fiscal and monetary policy shocks both contributed to higher inflation. The accommodative monetary stance during this period is consistent with the 2021 Strategy Review of the European Central Bank (European Central Bank, 2021), announcing a symmetric two percent inflation target, replacing the previous "close to, but below two percent". Most of the 2022 surge, however, is attributed to the remaining disturbances. From 2024, monetary

Table 2: Priors and posteriors of the two-country Bayesian estimation (Germany vs. rest of EA)

		Prior			Posterior		
Param.	Description	Type	Mean	Std.	Mode	Median	90% HPD CI
Region A structural							
χ_w	Wage indexation	B	0.25	0.10	0.1524	0.1450	[0.0512, 0.2484]
θ_T	Calvo probability tradable prices	B	0.75	0.10	0.4019	0.4020	[0.3040, 0.4961]
θ_N	Calvo probability non-tradable prices	B	0.75	0.10	0.9331	0.9360	[0.9002, 0.9678]
θ_w	Calvo probability wages	B	0.75	0.10	0.1882	0.1880	[0.1320, 0.2438]
Region B structural							
χ_w^*	Wage indexation B	B	0.25	0.10	0.1585	0.1512	[0.0516, 0.2618]
θ_T^*	Calvo probability tradable prices B	B	0.75	0.10	0.6007	0.6011	[0.5069, 0.6909]
θ_N^*	Calvo probability non-tradable prices B	B	0.75	0.10	0.7813	0.7786	[0.7019, 0.8599]
θ_w^*	Calvo probability wages B	B	0.75	0.10	0.1443	0.1432	[0.0993, 0.1885]
Common structural							
μ	Elasticity of substitution across tradable goods	G	1.50	0.30	1.7472	1.7482	[1.7155, 1.7780]
ψ_b	Bond portfolio adjustment cost	G	0.10	0.05	0.1865	0.1805	[0.0829, 0.2863]
ψ_u	Capital utilization cost	B	0.50	0.15	0.0247	0.0227	[0.0074, 0.0412]
Monetary / target policy							
ϕ^{ML}	Taylor rule inflation coefficient (ML)	N	2.00	0.25	2.1346	2.1319	[1.7765, 2.4929]
ρ_r	Taylor rule interest rate smoothing (ML)	B	0.50	0.25	0.7835	0.7845	[0.7499, 0.8186]
Shock persistence							
ρ_a	Technology shock persistence	B	0.80	0.10	0.9742	0.9745	[0.9641, 0.9842]
ρ_{rp}	Risk premium shock persistence	B	0.80	0.10	0.5051	0.4493	[0.3077, 0.8844]
ρ_d	Preference shock persistence	B	0.50	0.15	0.6232	0.6280	[0.5141, 0.7387]
ρ_u	Tradable markup shock persistence	B	0.80	0.10	0.4829	0.4853	[0.3901, 0.5819]
ρ_{uN}	NT markup shock persistence	B	0.80	0.10	0.8860	0.8901	[0.8249, 0.9490]
ρ_w	Wage markup shock persistence	B	0.80	0.10	0.7123	0.7180	[0.5700, 0.8580]
ρ_Z	Transfer rule persistence A (BFM Eq. 19)	B	0.80	0.10	0.9078	0.9107	[0.8581, 0.9601]
ρ_Z^*	Transfer rule persistence B (BFM Eq. 19)	B	0.80	0.10	0.9351	0.9361	[0.9016, 0.9689]
ρ_k	MEI shock persistence	B	0.50	0.15	0.1952	0.1919	[0.0997, 0.2862]
$\rho_g^{ML,A}$	Govt spending persistence A (ML)	B	0.80	0.10	0.9343	0.9356	[0.8924, 0.9776]
$\rho_g^{ML,B}$	Govt spending persistence B (ML)	B	0.80	0.10	0.9339	0.9360	[0.8930, 0.9764]
$\rho_{\zeta,ML}$	Funded transfer shock persistence A (ML)	B	0.50	0.25	0.2509	0.2460	[0.0760, 0.4212]
$\rho_{\zeta^*,ML}$	Funded transfer shock persistence B (ML)	B	0.50	0.25	0.0699	0.0576	[0.0005, 0.1390]
Shock volatilities							
σ_a	St.dev. technology shock	IG	0.50	inf	0.6546	0.6497	[0.4871, 0.8165]
σ_{aN}	St.dev. NT technology shock	IG	0.50	inf	1.8816	1.8771	[1.6734, 2.0998]
σ_{rp}	St.dev. risk premium shock	IG	0.25	inf	0.0380	0.0376	[0.0302, 0.0453]
σ_d	St.dev. preference shock	IG	0.50	inf	1.5593	1.5583	[1.2916, 1.8300]
σ_u	St.dev. tradable markup shock	IG	0.50	inf	1.1816	1.1762	[0.9660, 1.3900]
σ_{uN}	St.dev. NT markup shock	IG	0.50	inf	0.2559	0.2489	[0.1666, 0.3451]
σ_w	St.dev. wage markup shock	IG	0.50	inf	0.2803	0.2767	[0.2103, 0.3431]
σ_k	St.dev. MEI shock	IG	0.50	inf	8.3682	8.3496	[7.3317, 9.3878]
σ_ξ	St.dev. monetary policy shock	IG	0.25	inf	0.2770	0.2752	[0.2295, 0.3255]
$\sigma_\zeta^{ML,A}$	St.dev. transfer shock A (ML, funded)	IG	0.50	inf	0.1724	0.1714	[0.1509, 0.1935]
$\sigma_\zeta^{ML,B}$	St.dev. transfer shock B (ML, funded)	IG	0.50	inf	0.1787	0.1780	[0.1561, 0.2005]
$\sigma_g^{ML,A}$	St.dev. govt spending shock A (ML)	IG	0.50	inf	0.7562	0.7521	[0.6599, 0.8456]
$\sigma_g^{ML,B}$	St.dev. govt spending shock B (ML)	IG	0.50	inf	0.8192	0.8150	[0.7194, 0.9199]
σ_h	St.dev. COVID labor supply shock	G	5.0	2.50	18.4087	18.0789	[11.9272, 24.5242]
σ_{tC}	St.dev. COVID consumption (forced-savings) shock	G	5.0	2.50	7.2286	7.1158	[5.2272, 9.2155]
Target-shock volatilities							
$\sigma_{U,A}$	St.dev. unfunded debt-target shock A	IG	0.50	inf	0.2087	0.2025	[0.1180, 0.2990]
$\sigma_{U,B}$	St.dev. unfunded debt-target shock B	IG	0.50	inf	0.2557	0.2549	[0.1983, 0.3119]

Notes: Results of the estimation with the Wu and Xia (2016) shadow rate as the observed short rate (21 observables, 43 estimated parameters). N = Normal, B = Beta, G = Gamma, IG = Inverse-Gamma, AR coeff. = autoregressive coefficient, St.dev. = standard deviation, HPD = highest posterior density. The estimations for France and Italy are in Tables A.2 and A.3 in Appendix A.2.

Figure 5: Shock decomposition of euro area inflation



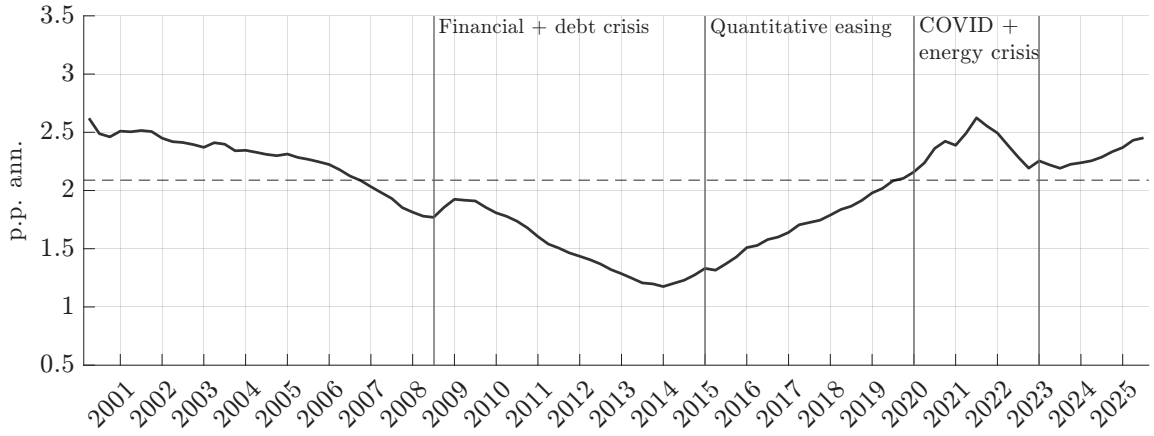
Notes: Historical shock decomposition of euro area headline inflation (annualized quarterly, pp deviations from the sample mean), averaged across three Bayesian estimations with the Wu and Xia (2016) shadow rate as the observed short rate, with Germany, France, and Italy individually as country A and the remaining euro area as country B (estimation sample: 2000Q2–2025Q3). Euro area inflation is Θ times inflation in country A plus $1-\Theta$ times inflation in country B. Unfunded fiscal shocks (red) pool the home and foreign unfunded innovations $(\varepsilon_A^U, \varepsilon_B^U)$, each of which moves its country's debt target and inflation target jointly. Monetary policy shocks are the interest-rate innovation ε^m; all remaining structural shocks are collected in "Rest." The black line is observed inflation. Contributions are computed from the Kalman smoother at the posterior mean.

policy shocks turned contractionary, while unfunded fiscal shocks continued to add to inflation until the end of the sample.

Second, Figure 6 presents the model-implied target inflation rate of the European Central Bank. The model-implied target is the time-varying inflation objective inferred from unfunded fiscal shocks as discussed in Section 2.7. This is a positive concept: it is the rate of inflation that the central bank effectively accommodated, not a claim about the ECB's stated or intended objectives. Averaged across the three estimations, the implied target starts at 2.62 percent in 2000Q2, declines through the 2000s and falls below two percent in 2007Q2. Following the financial crisis and the subsequent austerity, it falls further, to a trough of 1.18 percent in 2014Q1. It recovers after the launch of quantitative easing, is back above two percent from 2019Q2, rises during the COVID pandemic and the subsequent energy crisis, and peaks at 2.62 percent in 2021Q3. Its sample mean is 1.99 percent, consistent with the target rate of the European Central Bank.

Together, the two figures establish that unfunded fiscal shocks are a quantitatively relevant driver of euro area inflation: they account for about a fifth to a quarter of the mean absolute contribution of all shocks to domestic inflation over the sample, and the implied ECB target departed persistently from two percent, falling to 1.2 percent in 2014 and rising to 2.6 percent in 2021.

Figure 6: Model-implied ECB target inflation rate



Notes: Model-implied ECB target inflation rate (annualized, percentage points), simple average of three Bayesian estimations with the Wu and Xia (2016) shadow rate as the observed short rate, with Germany, France, and Italy individually as country A and the remaining euro area as country B (estimation sample: 2000Q2–2025Q3). The implied target is the time-varying inflation objective $\pi_t^{F,MU}$ recovered from the smoothed path of the unfunded innovations ε_A^U and ε_B^U; see Section 2.7. Individual country-pair estimates are in Appendix A.2.

5 Conclusion

This paper studies how unfunded fiscal shocks propagate in a currency union with one central bank and multiple fiscal authorities. We combine evidence from local projections for euro area countries with a two-country New Keynesian model that allows for partially unfunded debt. The central aim is to identify episodes of *fiscal inflation* and quantify the effect it had on observed euro area inflation.

First, we investigate the price determinacy conditions of a currency union and the propagation mechanism of funded and unfunded shocks in a stylized model. We show that at most one fiscal authority can issue unfunded debt when countries share a common central bank. Moreover, we find that funded fiscal government spending shocks increase inflation at home while decreasing inflation abroad, because it shifts demand toward domestic goods. Unfunded government spending shocks increase inflation in both countries because the central bank accommodates *fiscal inflation*.

Our empirical analysis documents substantial heterogeneity in spillovers, consistent with the predictions of the model. Fiscal expansions in Italy are associated with inflationary effects in other countries, while fiscal expansions in France and Germany do not. These patterns suggest that the central bank accommodates fiscal inflation originating in Italy, implying fiscal expansions in those countries are less funded than fiscal expansions in France or Germany.

The quantitative model extends the stylized model to a richer environment with habit formation, nominal rigidities, non-tradables, capital accumulation, and long-term

bonds, with the aim to identify episodes of *fiscal inflation* in the euro area business cycles. Bayesian estimation across three country pairings—Germany, France, and Italy each against the rest of the euro area—reveals that unfunded fiscal shocks are a quantitatively significant driver of euro area inflation, with their contribution tracking the major shifts in fiscal stance: contractionary during the austerity phase of the sovereign debt crisis, and expansionary during the COVID pandemic and the subsequent energy crisis. We also recover the model-implied ECB inflation target—the time-varying inflation objective driven by unfunded fiscal shocks—which exhibits persistent departures from the ECB's stated two percent mandate, from a trough of 1.2 percent in 2014 to a peak of 2.6 percent during the post-pandemic episode. Together, these findings show that the fiscal positions of member states are a significant source of inflationary pressure in the euro area, underscoring the importance of fiscal backing for price stability in a monetary union.

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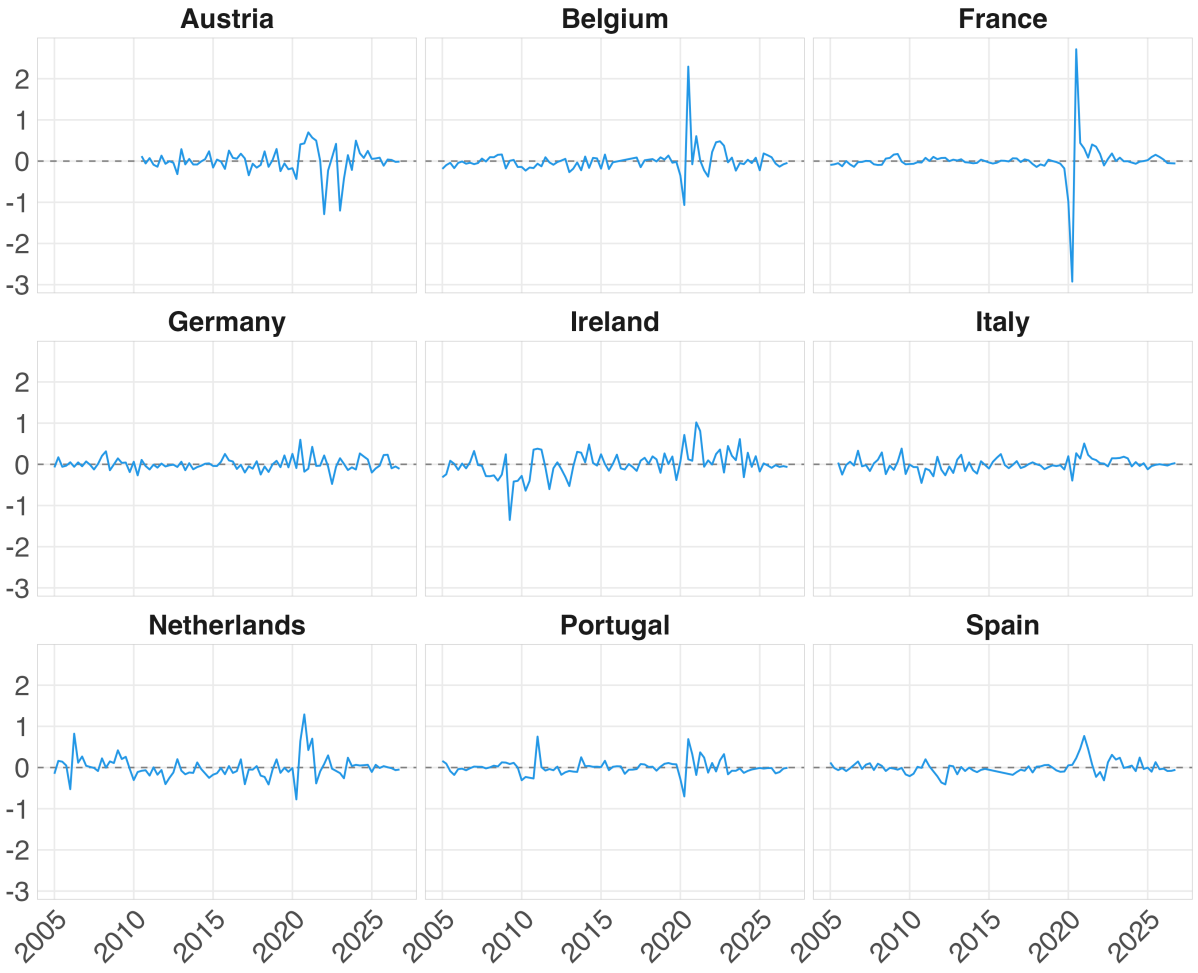
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Appendix

A Additional figures

A.1 Local projections

Figure A.1: Forecast errors of government spending (percentage point of GDP)



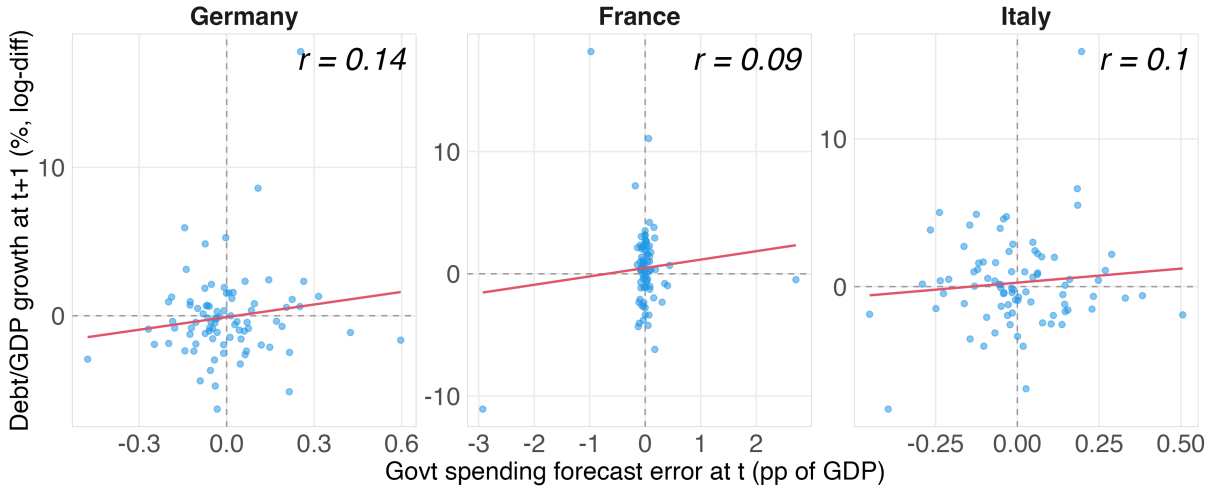
Notes: Purged government spending forecast errors for each country in the sample, 2004Q1–2026Q1. Each series is the residual from a within-country regression of raw forecast errors on lagged macroeconomic controls (output growth, inflation, interest rates), removing predictable variation. Units are percentage points of lagged GDP.

Table A.1: Correlation of forecast errors

	Austria	Belgium	France	Germany	Ireland	Italy	Netherlands	Portugal	Spain
Austria	1.00	0.20	0.29	-0.02	-0.00	0.25	0.33	0.28	0.31
Belgium	0.20	1.00	0.79	0.31	0.04	0.31	0.39	0.54	0.25
France	0.29	0.79	1.00	0.26	-0.12	0.33	0.43	0.63	0.14
Germany	-0.02	0.31	0.26	1.00	0.06	0.33	0.24	0.17	0.10
Ireland	-0.00	0.04	-0.12	0.06	1.00	0.03	0.07	0.02	0.39
Italy	0.25	0.31	0.33	0.33	0.03	1.00	0.37	0.25	0.43
Netherlands	0.33	0.39	0.43	0.24	0.07	0.37	1.00	0.36	0.39
Portugal	0.28	0.54	0.63	0.17	0.02	0.25	0.36	1.00	0.28
Spain	0.31	0.25	0.14	0.10	0.39	0.43	0.39	0.28	1.00

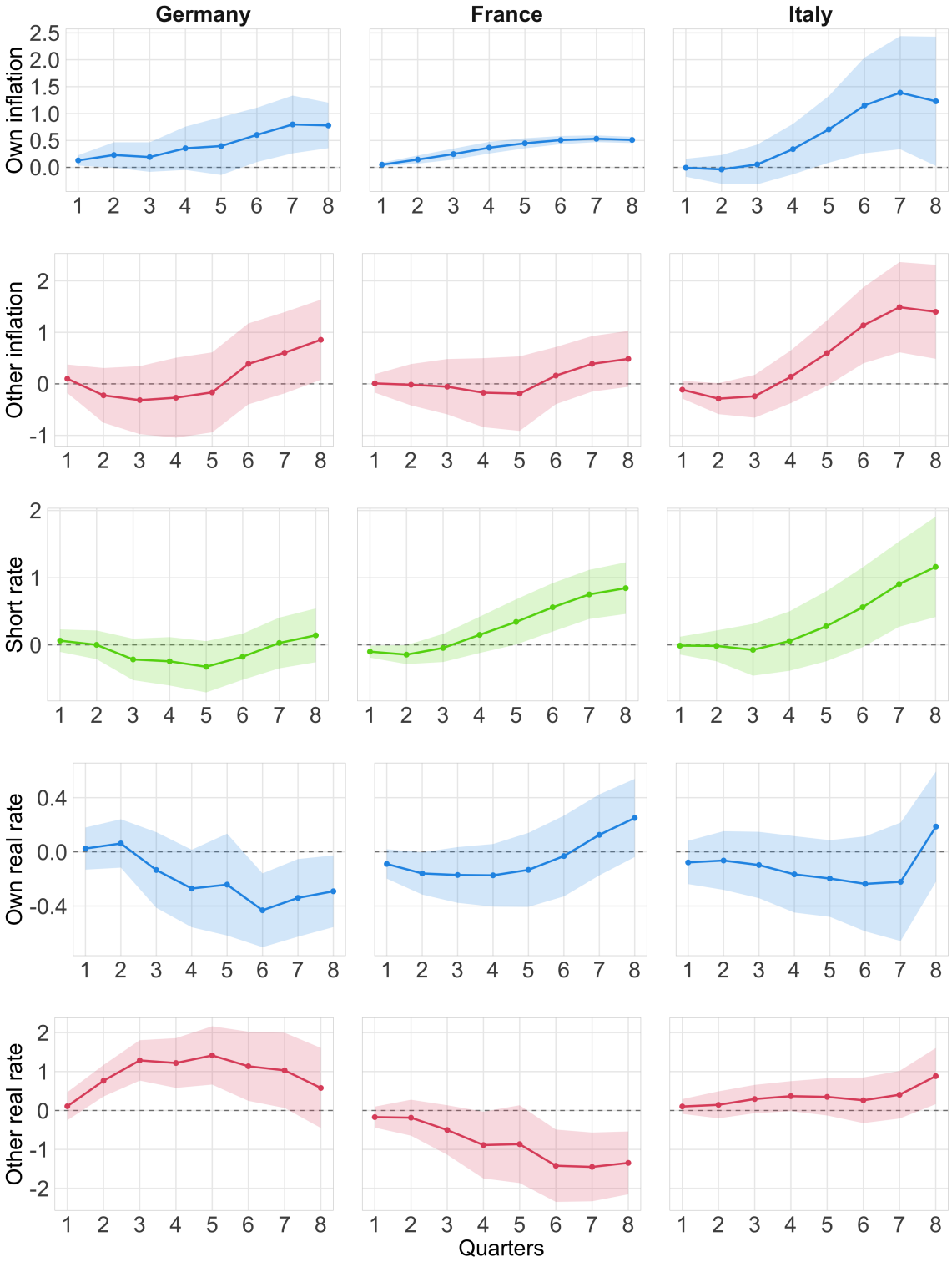
Notes: Pairwise Pearson correlation coefficients of purged government spending forecast errors across countries. Computed over the common sample using pairwise-complete observations.

Figure A.2: Correlation of government spending shocks and debt-to-GDP growth



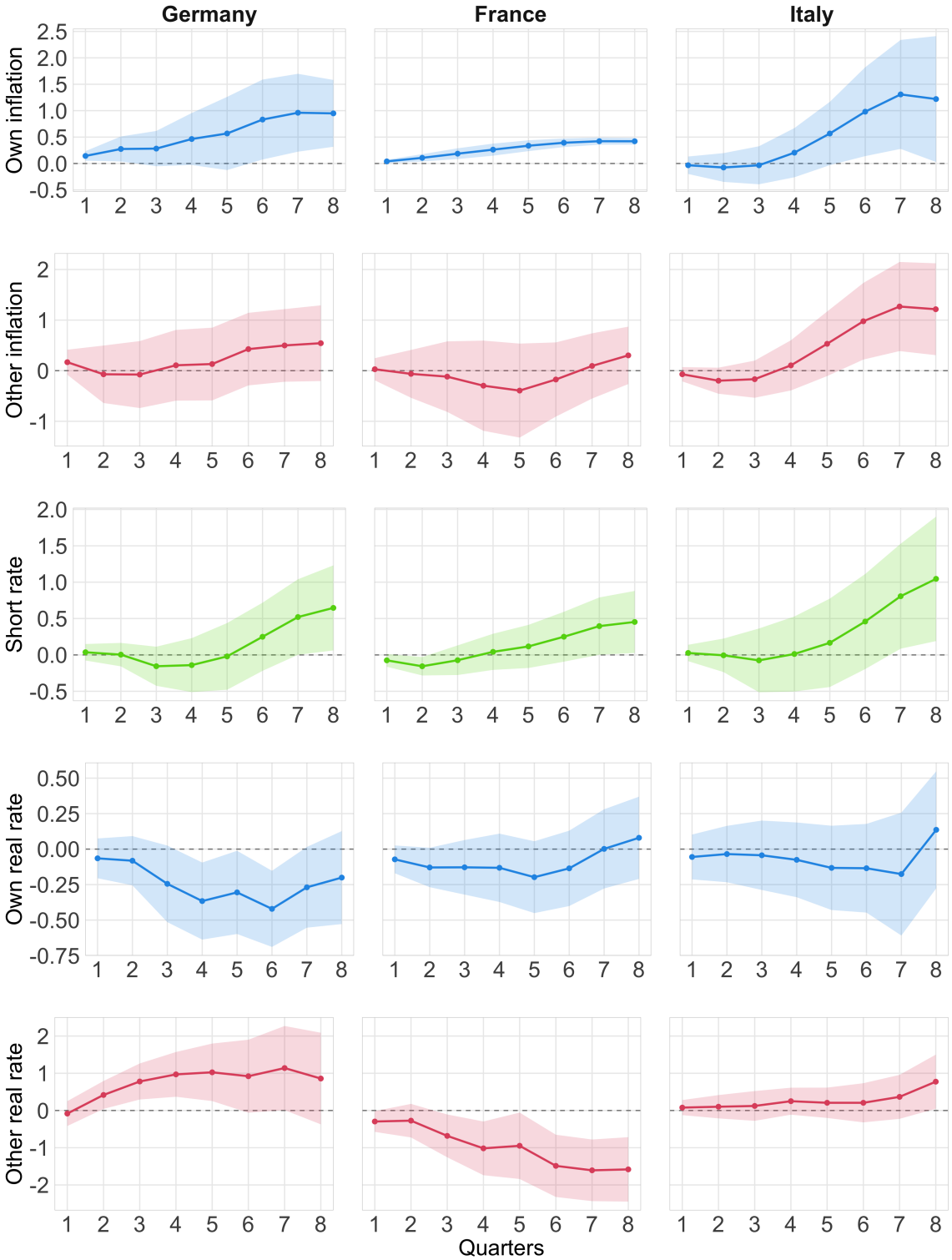
Notes: Scatter plots of purged government spending forecast errors at time t against the log-change in gross government debt-to-GDP at $t+1$, for Germany, France, and Italy (2004Q1–2026Q1). The debt-to-GDP ratio is constructed from the EA-MD-QD dataset by Barigozzi, Lissona, and Tonni (2025) following the same deflation procedure used in estimation. The correlation coefficient r is reported in each panel. The fitted line is estimated by OLS.

Figure A.3: Impulse responses to a 0.1 pp increase in govt spending (% of GDP)



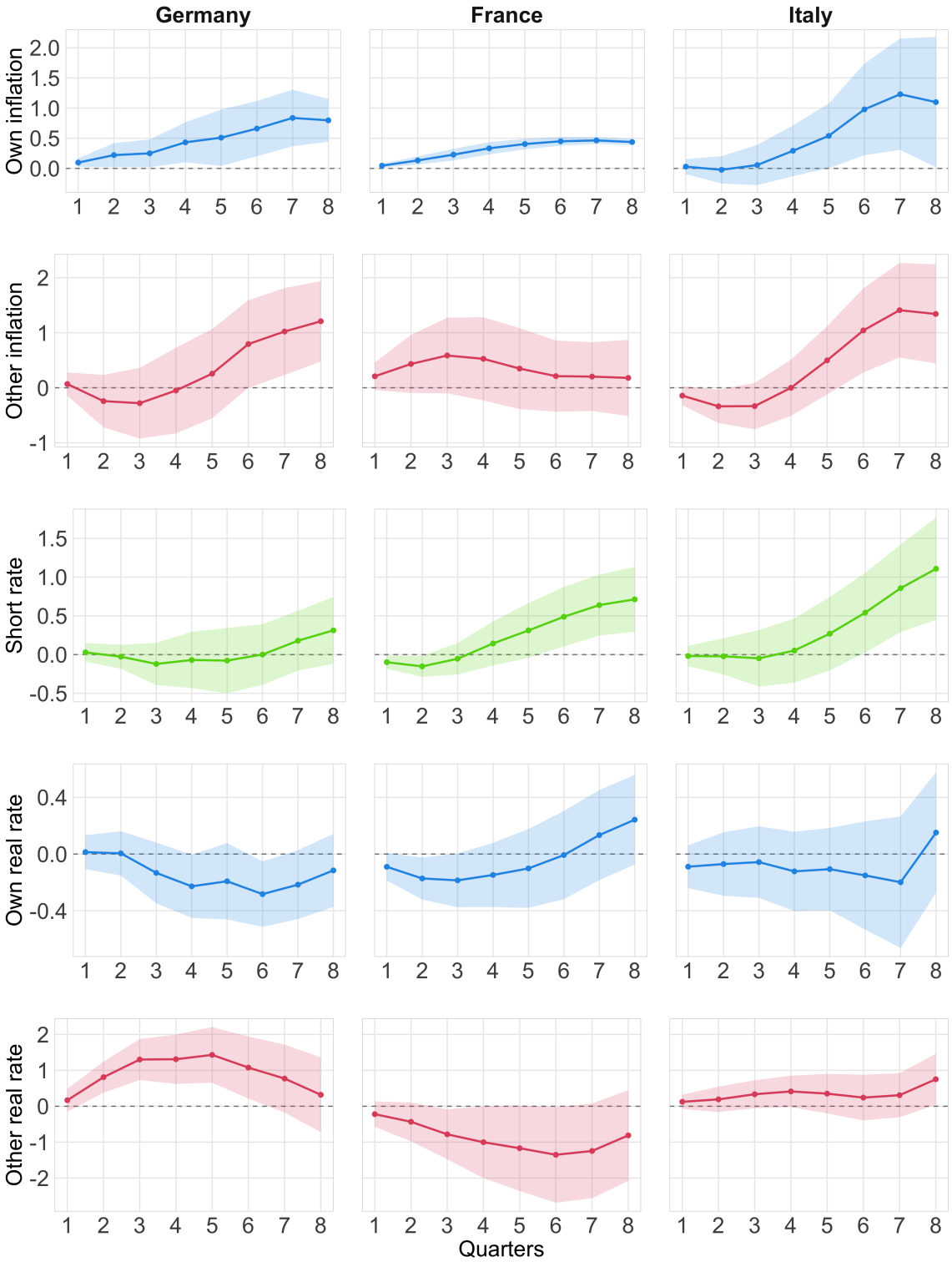
Notes: The own country and short rate local projections use Newey-West HAC standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2026Q1. Units are percentage points.

Figure A.4: Impulse responses to a 0.1 pp increase in govt spending (% of GDP) with 2 lags of controls



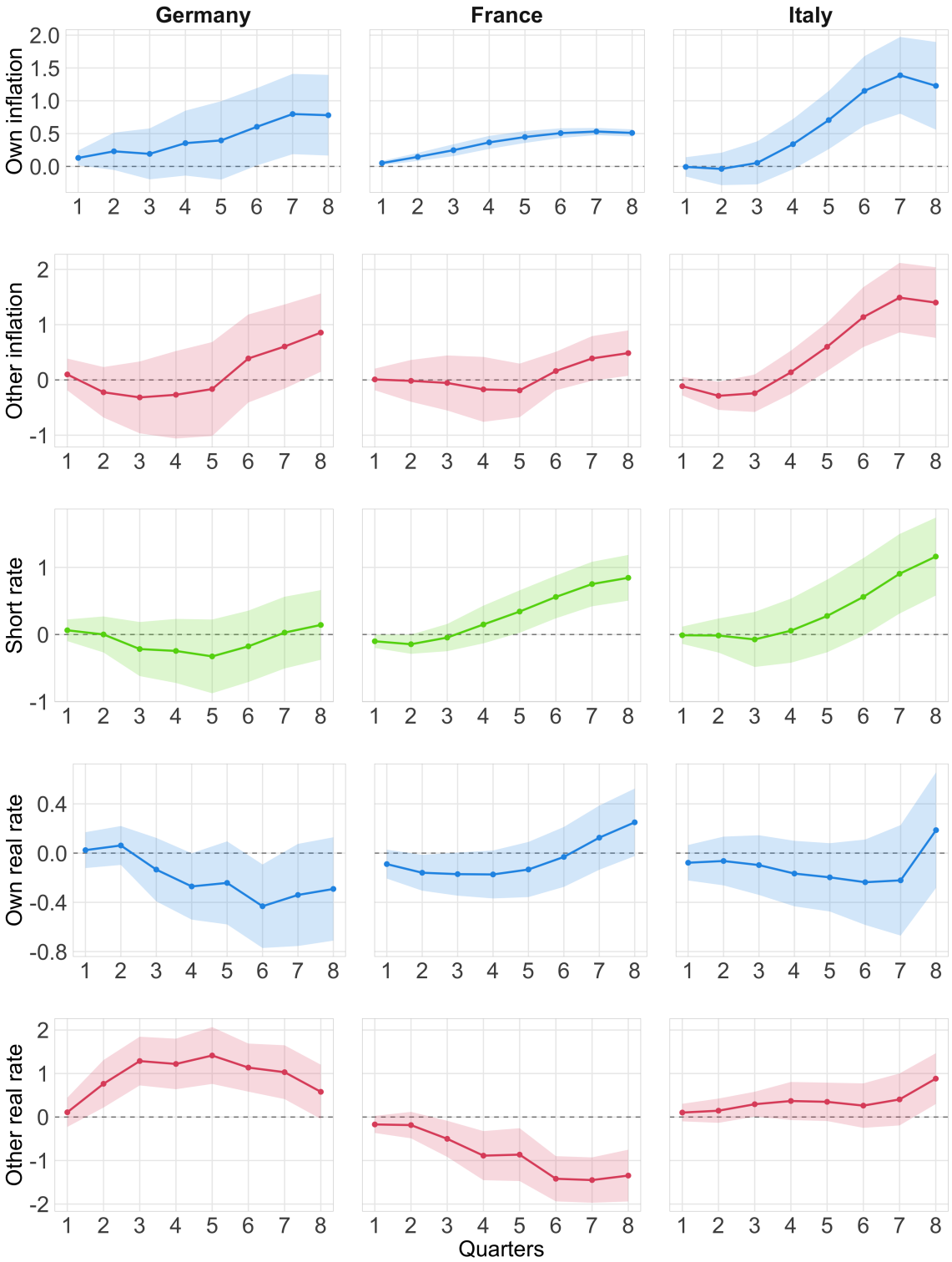
Notes: Same specification as Figure A.3 except using 2 lags (rather than 3) of each control variable. The own country and short rate local projections use Newey-West HAC standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2026Q1. Units are percentage points.

Figure A.5: Impulse responses to a 0.1 pp increase in govt spending (% of GDP) with unpurged shocks



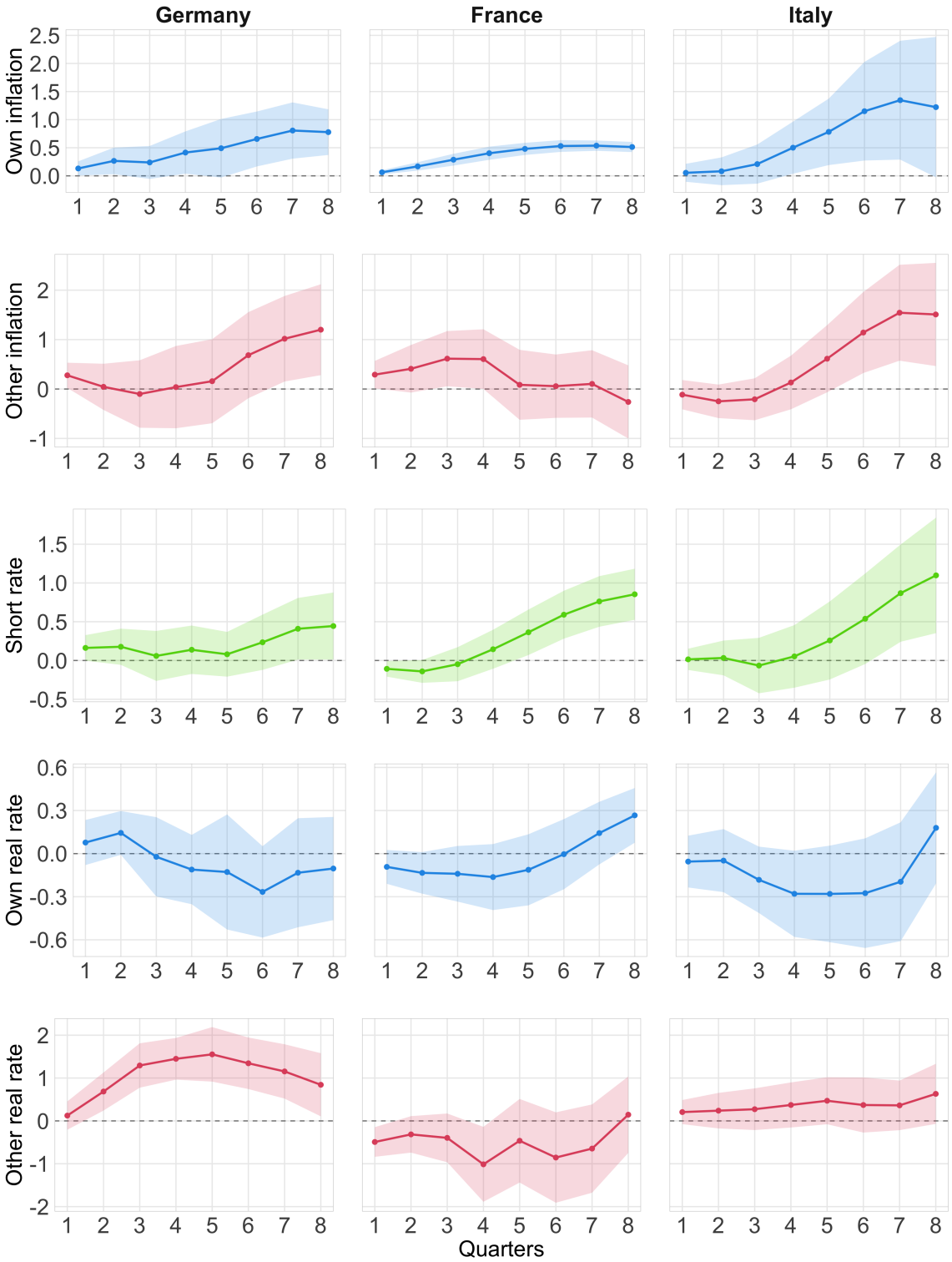
Notes: The own country and short rate local projections use Newey-West HAC standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2026Q1. Units are percentage points.

Figure A.6: Impulse responses to a 0.1 pp increase in govt spending (% of GDP) with different standard errors



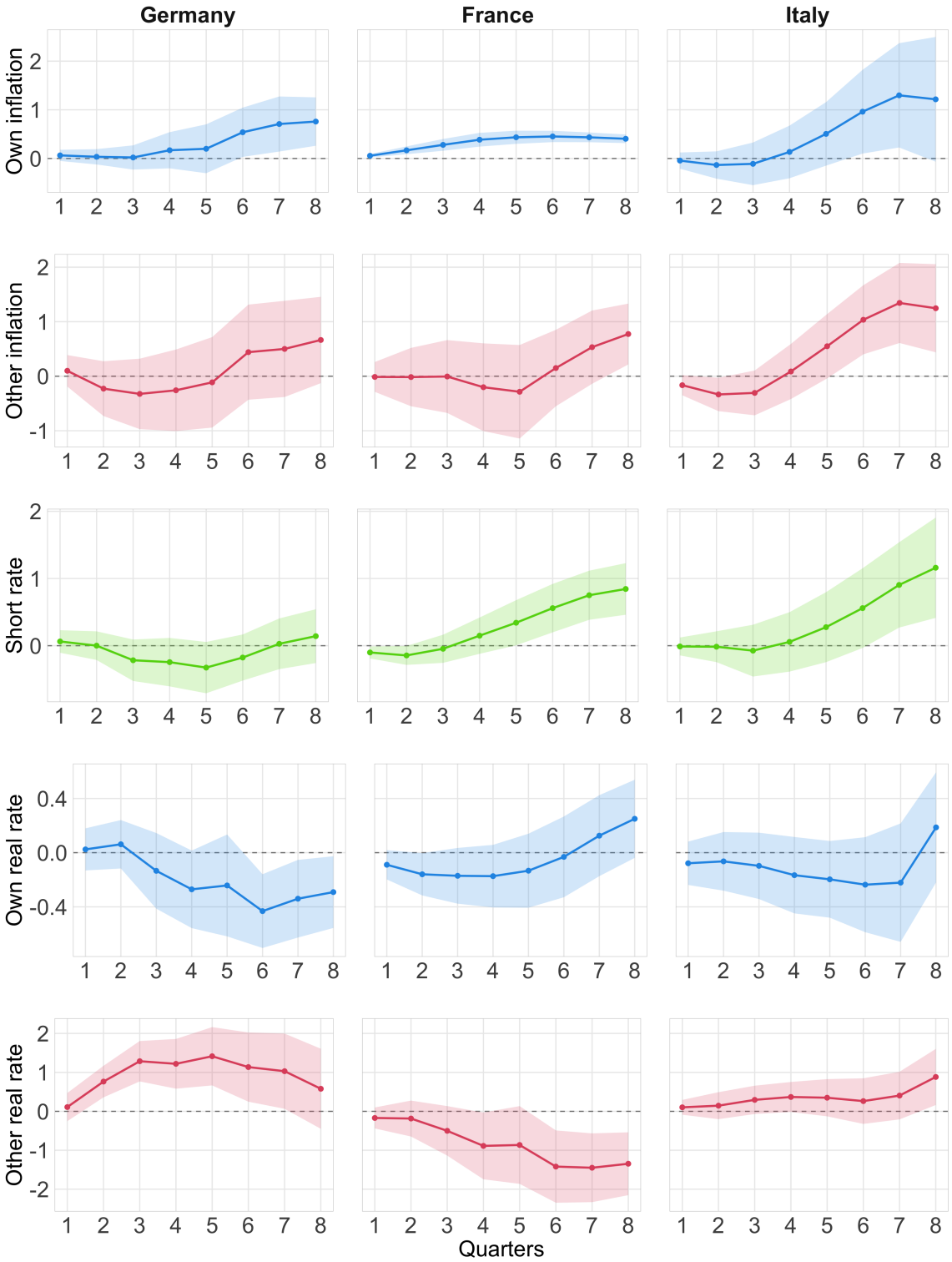
Notes: The own country and short rate local projections use heteroskedastic standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with two-way standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2026Q1. Units are percentage points.

Figure A.7: Impulse responses to a 0.1 pp increase in govt spending (% of GDP) with lags of past shocks as extra controls



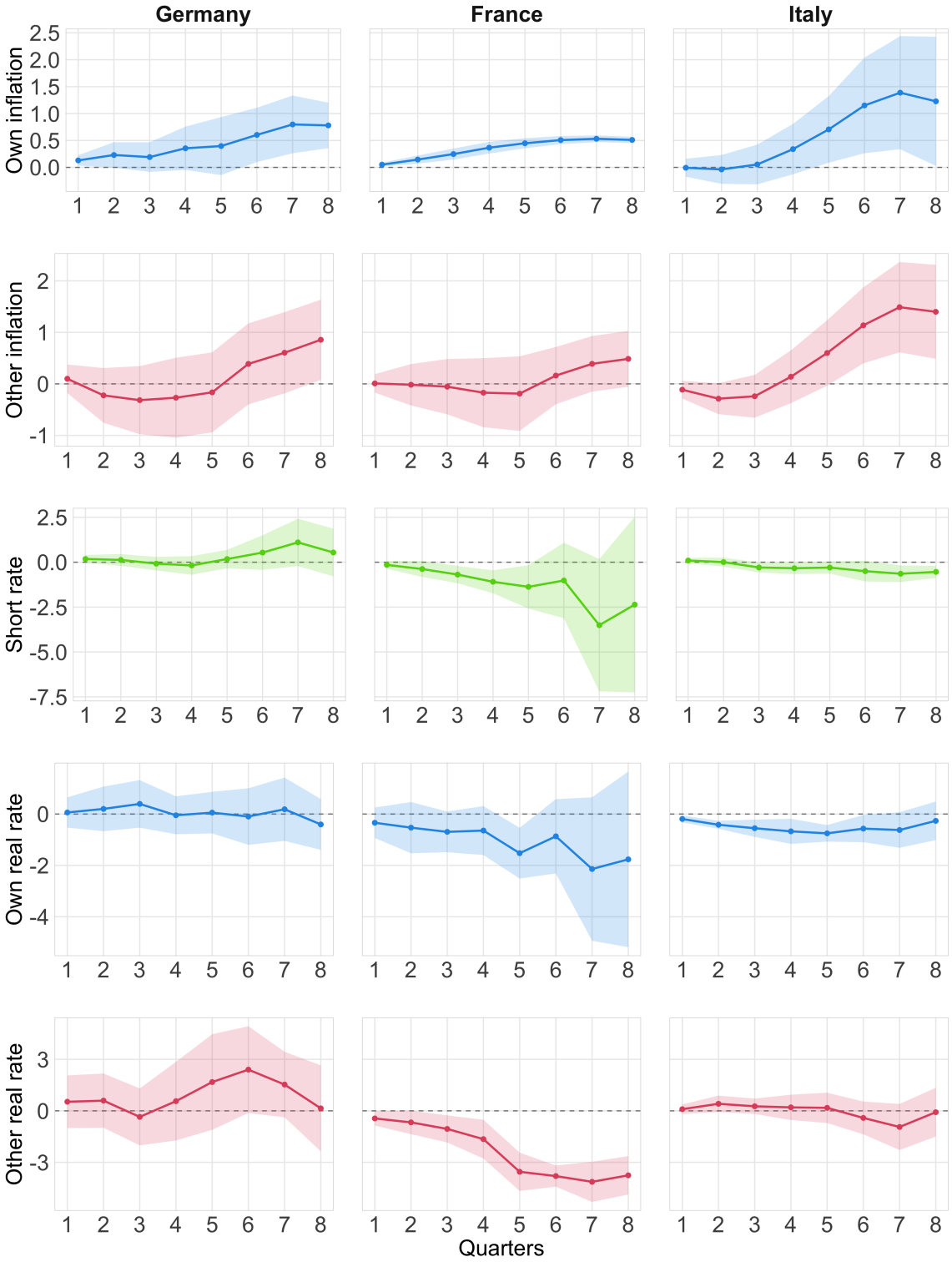
Notes: The own country and short rate local projections use Newey-West HAC standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2026Q1. Units are percentage points.

Figure A.8: Impulse responses to a 0.1 pp increase in govt spending (% of GDP) with contemporaneous weighted other-country fiscal shocks and its lags as extra controls



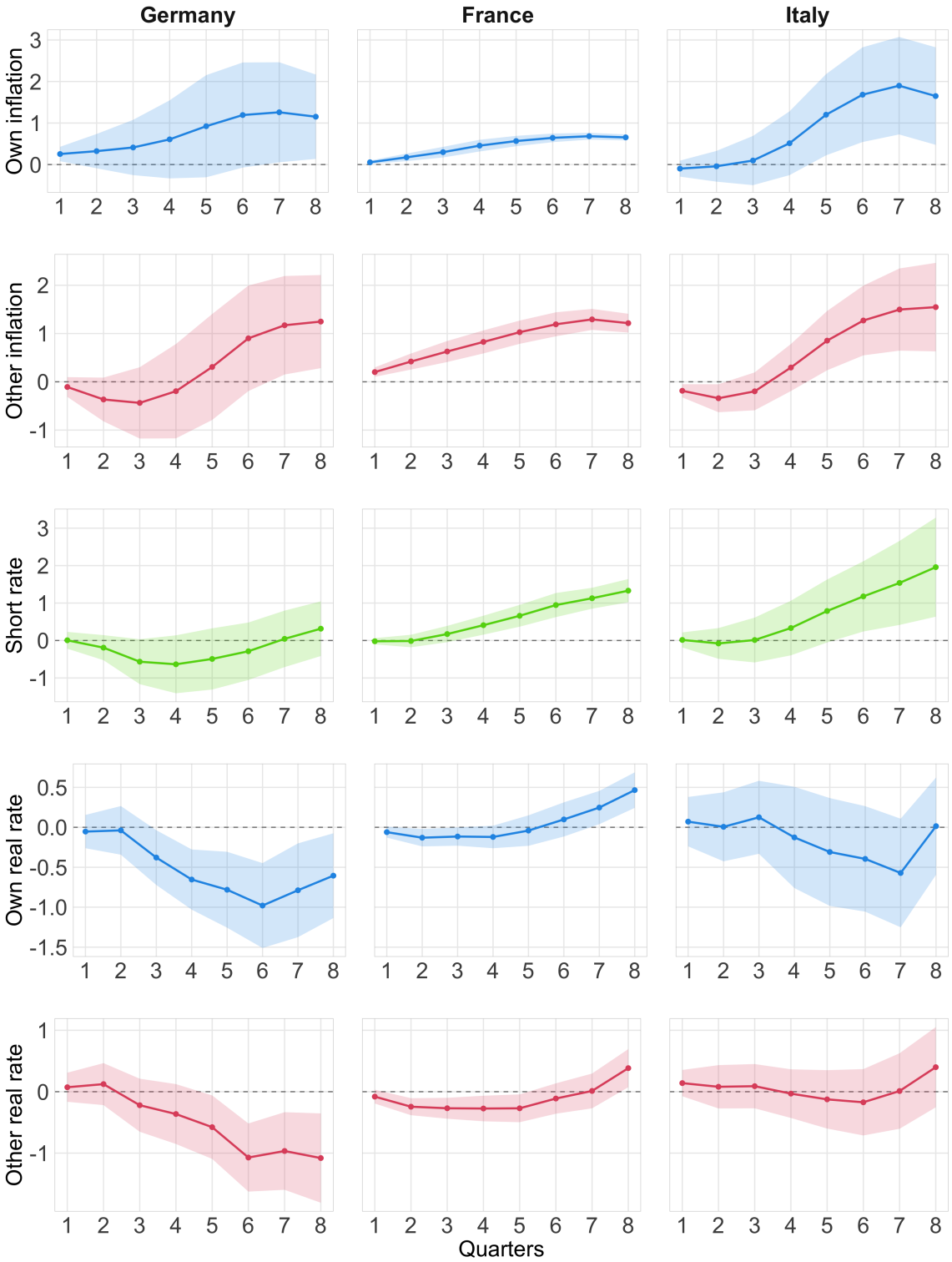
Notes: The own country and short rate local projections use Newey-West HAC standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2026Q1. Units are percentage points.

Figure A.9: Impulse responses to a 0.1 pp increase in govt spending (% of GDP) without using the shadow rate



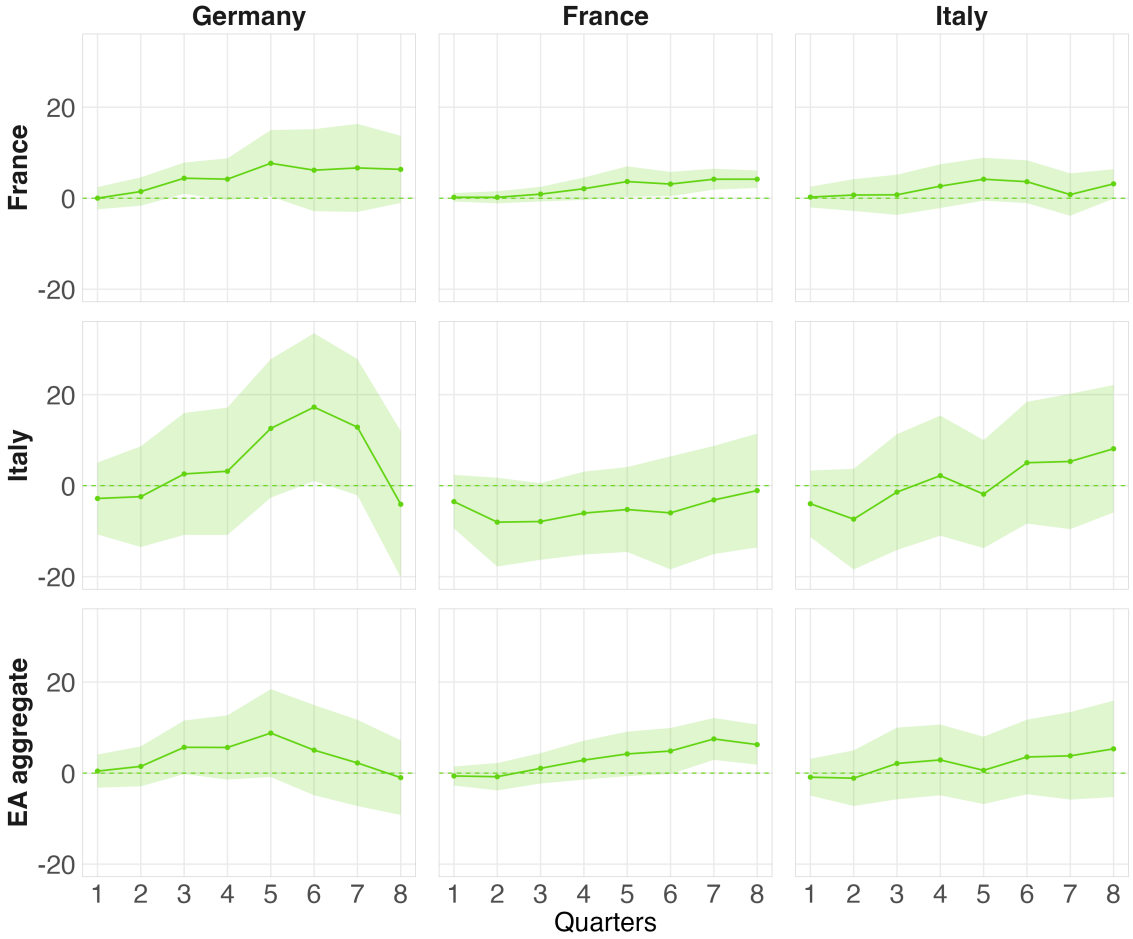
Notes: The own country and short rate local projections use Newey-West HAC standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2014Q2 (before the zero lower bound period). Units are percentage points.

Figure A.10: Impulse responses to a 0.1 pp increase in govt spending (% of GDP) with timing fix



Notes: The own country and short rate local projections use Newey-West HAC standard errors. For spillover effects to other countries, the results are pooled estimates from panel local projections with Driscoll-Kraay standard errors and countries weighted by GDP. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2026Q1. Units are percentage points.

Figure A.11: Impulse responses of sovereign spreads to a 0.1 pp increase in govt spending (% of GDP)



Notes: The dependent variable is the 10-year sovereign bond spread vis-à-vis Germany in basis points. Local projections use Newey-West HAC standard errors. Shaded area shows 90% confidence interval. Sample period is 2004Q1–2026Q1. Controls include source country controls, receiving country controls (if not the same as source country), German controls, and euro area aggregate controls.

A.2 Country-pair results of the estimation

This appendix reports the country-pair results of the estimation in Section 4, with the Wu and Xia (2016) shadow rate as the observed short rate. Tables A.2 and A.3 report the priors and posteriors for France and Italy as country A (Germany is in Table 2), and Figures A.12–A.14 the corresponding shock decompositions and model-implied inflation targets. The results of the estimation on the observed short rate and of the estimation with the ELB and asset purchases are in the Online Appendix.

Table A.2: Priors and posteriors of the two-country Bayesian estimation (France vs. rest of EA)

		Prior			Posterior		
Param.	Description	Type	Mean	Std.	Mode	Median	90% HPD CI
Region A structural							
χ_w	Wage indexation	B	0.25	0.10	0.1632	0.1554	[0.0540, 0.2693]
θ_T	Calvo probability tradable prices	B	0.75	0.10	0.8957	0.9028	[0.8260, 0.9680]
θ_N	Calvo probability non-tradable prices	B	0.75	0.10	0.8291	0.8299	[0.7917, 0.8674]
θ_w	Calvo probability wages	B	0.75	0.10	0.1625	0.1618	[0.1162, 0.2104]
Region B structural							
χ_w^*	Wage indexation B	B	0.25	0.10	0.1010	0.0959	[0.0331, 0.1649]
θ_T^*	Calvo probability tradable prices B	B	0.75	0.10	0.9833	0.9839	[0.9725, 0.9943]
θ_N^*	Calvo probability non-tradable prices B	B	0.75	0.10	0.8723	0.8730	[0.8433, 0.9025]
θ_w^*	Calvo probability wages B	B	0.75	0.10	0.2480	0.2485	[0.1902, 0.3078]
Common structural							
μ	Elasticity of substitution across tradable goods	G	1.50	0.30	1.6335	1.6322	[1.5865, 1.6770]
ψ_b	Bond portfolio adjustment cost	G	0.10	0.05	0.1035	0.0967	[0.0306, 0.1754]
ψ_u	Capital utilization cost	B	0.50	0.15	0.0760	0.0748	[0.0418, 0.1110]
Monetary / target policy							
ϕ^{ML}	Taylor rule inflation coefficient (ML)	N	2.00	0.25	1.7318	1.7070	[1.4552, 1.9872]
ρ_r	Taylor rule interest rate smoothing (ML)	B	0.50	0.25	0.7223	0.7215	[0.6849, 0.7576]
Shock persistence							
ρ_a	Technology shock persistence	B	0.80	0.10	0.9757	0.9772	[0.9644, 0.9882]
ρ_{rp}	Risk premium shock persistence	B	0.80	0.10	0.3613	0.3609	[0.2390, 0.4827]
ρ_d	Preference shock persistence	B	0.50	0.15	0.7019	0.7092	[0.6112, 0.7980]
ρ_u	Tradable markup shock persistence	B	0.80	0.10	0.3240	0.3256	[0.2477, 0.4044]
ρ_{uN}	NT markup shock persistence	B	0.80	0.10	0.6293	0.6317	[0.5217, 0.7368]
ρ_w	Wage markup shock persistence	B	0.80	0.10	0.7140	0.7216	[0.5581, 0.8698]
ρ_Z	Transfer rule persistence A (BFM Eq. 19)	B	0.80	0.10	0.9448	0.9466	[0.9146, 0.9778]
ρ_Z^*	Transfer rule persistence B (BFM Eq. 19)	B	0.80	0.10	0.9189	0.9204	[0.8735, 0.9644]
ρ_k	MEI shock persistence	B	0.50	0.15	0.1970	0.1905	[0.0938, 0.2958]
$\rho_g^{ML,A}$	Govt spending persistence A (ML)	B	0.80	0.10	0.8770	0.8914	[0.7965, 0.9736]
$\rho_g^{ML,B}$	Govt spending persistence B (ML)	B	0.80	0.10	0.9382	0.9390	[0.9079, 0.9682]
$\rho_{\zeta,ML}$	Funded transfer shock persistence A (ML)	B	0.50	0.25	0.0638	0.0531	[0.0006, 0.1306]
$\rho_{\zeta^*,ML}$	Funded transfer shock persistence B (ML)	B	0.50	0.25	0.2116	0.1976	[0.0314, 0.3683]
Shock volatilities							
σ_a	St.dev. technology shock	IG	0.50	inf	0.6214	0.6201	[0.3881, 0.8457]
σ_{aN}	St.dev. NT technology shock	IG	0.50	inf	1.3692	1.3606	[1.1671, 1.5607]
σ_{rp}	St.dev. risk premium shock	IG	0.25	inf	0.0377	0.0374	[0.0310, 0.0444]
σ_d	St.dev. preference shock	IG	0.50	inf	1.0468	1.0430	[0.8010, 1.2845]
σ_u	St.dev. tradable markup shock	IG	0.50	inf	0.7608	0.7563	[0.6301, 0.8887]
σ_{uN}	St.dev. NT markup shock	IG	0.50	inf	0.1658	0.1623	[0.1096, 0.2204]
σ_w	St.dev. wage markup shock	IG	0.50	inf	0.1465	0.1438	[0.1047, 0.1868]
σ_k	St.dev. MEI shock	IG	0.50	inf	6.4398	6.4501	[5.6744, 7.2525]
σ_ξ	St.dev. monetary policy shock	IG	0.25	inf	0.3211	0.3193	[0.2708, 0.3732]
$\sigma_\zeta^{ML,A}$	St.dev. transfer shock A (ML, funded)	IG	0.50	inf	0.1779	0.1768	[0.1555, 0.1993]
$\sigma_\zeta^{ML,B}$	St.dev. transfer shock B (ML, funded)	IG	0.50	inf	0.1550	0.1539	[0.1352, 0.1731]
$\sigma_g^{ML,A}$	St.dev. govt spending shock A (ML)	IG	0.50	inf	1.7443	1.7412	[1.5308, 1.9588]
$\sigma_g^{ML,B}$	St.dev. govt spending shock B (ML)	IG	0.50	inf	0.4933	0.4917	[0.4322, 0.5523]
σ_h	St.dev. COVID labor supply shock	G	5.0	2.50	25.0911	24.8165	[17.1211, 32.9693]
σ_{tC}	St.dev. COVID consumption (forced-savings) shock	G	5.0	2.50	6.6205	6.4667	[4.8855, 8.2968]
Target-shock volatilities							
$\sigma_{U,A}$	St.dev. unfunded debt-target shock A	IG	0.50	inf	0.2308	0.2166	[0.1191, 0.3374]
$\sigma_{U,B}$	St.dev. unfunded debt-target shock B	IG	0.50	inf	0.1538	0.1500	[0.1029, 0.1999]

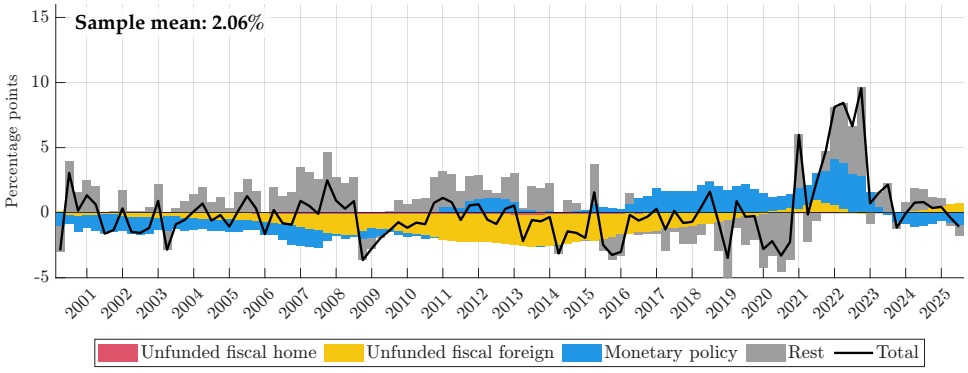
Notes: Results of the estimation with the Wu and Xia (2016) shadow rate as the observed short rate (21 observables, 43 estimated parameters). N = Normal, B = Beta, G = Gamma, IG = Inverse-Gamma, AR coeff. = autoregressive coefficient, St.dev. = standard deviation, HPD = highest posterior density.

Table A.3: Priors and posteriors of the two-country Bayesian estimation (Italy vs. rest of EA)

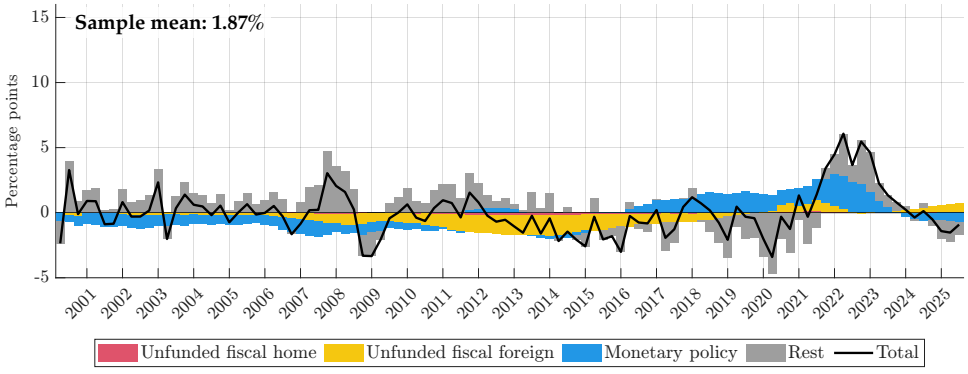
		Prior			Posterior		
Param.	Description	Type	Mean	Std.	Mode	Median	90% HPD CI
Region A structural							
χ_w	Wage indexation	B	0.25	0.10	0.1723	0.1632	[0.0578, 0.2791]
θ_T	Calvo probability tradable prices	B	0.75	0.10	0.9286	0.9566	[0.8986, 0.9921]
θ_N	Calvo probability non-tradable prices	B	0.75	0.10	0.7567	0.7550	[0.6906, 0.8163]
θ_w	Calvo probability wages	B	0.75	0.10	0.1329	0.1320	[0.0797, 0.1846]
Region B structural							
χ_w^*	Wage indexation B	B	0.25	0.10	0.1219	0.1181	[0.0377, 0.2055]
θ_T^*	Calvo probability tradable prices B	B	0.75	0.10	0.4746	0.4713	[0.3919, 0.5533]
θ_N^*	Calvo probability non-tradable prices B	B	0.75	0.10	0.7858	0.7891	[0.7132, 0.8643]
θ_w^*	Calvo probability wages B	B	0.75	0.10	0.1779	0.1769	[0.1245, 0.2311]
Common structural							
μ	Elasticity of substitution across tradable goods	G	1.50	0.30	1.7926	1.7961	[1.7253, 1.8656]
ψ_b	Bond portfolio adjustment cost	G	0.10	0.05	0.1518	0.1434	[0.0568, 0.2447]
ψ_u	Capital utilization cost	B	0.50	0.15	0.0581	0.0564	[0.0227, 0.0918]
Monetary / target policy							
ϕ^{ML}	Taylor rule inflation coefficient (ML)	N	2.00	0.25	2.0933	2.0852	[1.7308, 2.4516]
ρ_r	Taylor rule interest rate smoothing (ML)	B	0.50	0.25	0.7843	0.7848	[0.7509, 0.8179]
Shock persistence							
ρ_a	Technology shock persistence	B	0.80	0.10	0.9582	0.9624	[0.9317, 0.9854]
ρ_{rp}	Risk premium shock persistence	B	0.80	0.10	0.3777	0.3726	[0.2482, 0.4981]
ρ_d	Preference shock persistence	B	0.50	0.15	0.7208	0.7252	[0.6438, 0.8011]
ρ_u	Tradable markup shock persistence	B	0.80	0.10	0.5072	0.5097	[0.4196, 0.5987]
ρ_{uN}	NT markup shock persistence	B	0.80	0.10	0.7329	0.7313	[0.5886, 0.8792]
ρ_w	Wage markup shock persistence	B	0.80	0.10	0.6838	0.6867	[0.5430, 0.8216]
ρ_Z	Transfer rule persistence A (BFM Eq. 19)	B	0.80	0.10	0.8493	0.8544	[0.7658, 0.9350]
ρ_Z^*	Transfer rule persistence B (BFM Eq. 19)	B	0.80	0.10	0.9462	0.9480	[0.9153, 0.9785]
ρ_k	MEI shock persistence	B	0.50	0.15	0.2447	0.2460	[0.1380, 0.3527]
$\rho_g^{ML,A}$	Govt spending persistence A (ML)	B	0.80	0.10	0.8982	0.8994	[0.8322, 0.9643]
$\rho_g^{ML,B}$	Govt spending persistence B (ML)	B	0.80	0.10	0.9591	0.9610	[0.9305, 0.9881]
$\rho_{\zeta,ML}$	Funded transfer shock persistence A (ML)	B	0.50	0.25	0.1036	0.0860	[0.0012, 0.2026]
$\rho_{\zeta^*,ML}$	Funded transfer shock persistence B (ML)	B	0.50	0.25	0.1878	0.1785	[0.0276, 0.3358]
Shock volatilities							
σ_a	St.dev. technology shock	IG	0.50	inf	0.4066	0.3866	[0.2232, 0.5732]
σ_{aN}	St.dev. NT technology shock	IG	0.50	inf	1.3652	1.3700	[1.1588, 1.5809]
σ_{rp}	St.dev. risk premium shock	IG	0.25	inf	0.0377	0.0374	[0.0309, 0.0443]
σ_d	St.dev. preference shock	IG	0.50	inf	1.3980	1.3961	[1.1402, 1.6634]
σ_u	St.dev. tradable markup shock	IG	0.50	inf	1.1450	1.1362	[0.9044, 1.3720]
σ_{uN}	St.dev. NT markup shock	IG	0.50	inf	0.4013	0.3974	[0.2519, 0.5382]
σ_w	St.dev. wage markup shock	IG	0.50	inf	0.2311	0.2276	[0.1494, 0.3133]
σ_k	St.dev. MEI shock	IG	0.50	inf	7.3154	7.2751	[6.3255, 8.2439]
σ_ξ	St.dev. monetary policy shock	IG	0.25	inf	0.2665	0.2649	[0.2204, 0.3130]
$\sigma_\zeta^{ML,A}$	St.dev. transfer shock A (ML, funded)	IG	0.50	inf	0.2378	0.2358	[0.2084, 0.2662]
$\sigma_\zeta^{ML,B}$	St.dev. transfer shock B (ML, funded)	IG	0.50	inf	0.1483	0.1478	[0.1300, 0.1667]
$\sigma_g^{ML,A}$	St.dev. govt spending shock A (ML)	IG	0.50	inf	0.8892	0.8845	[0.7789, 0.9983]
$\sigma_g^{ML,B}$	St.dev. govt spending shock B (ML)	IG	0.50	inf	0.7328	0.7308	[0.6435, 0.8234]
σ_h	St.dev. COVID labor supply shock	G	5.0	2.50	18.9646	18.7843	[12.1302, 26.1531]
σ_{tC}	St.dev. COVID consumption (forced-savings) shock	G	5.0	2.50	7.2024	7.0866	[5.3327, 9.0046]
Target-shock volatilities							
$\sigma_{U,A}$	St.dev. unfunded debt-target shock A	IG	0.50	inf	0.4783	0.4499	[0.1607, 0.7514]
$\sigma_{U,B}$	St.dev. unfunded debt-target shock B	IG	0.50	inf	0.1883	0.1893	[0.1329, 0.2432]

Notes: Results of the estimation with the Wu and Xia (2016) shadow rate as the observed short rate (21 observables, 43 estimated parameters). N = Normal, B = Beta, G = Gamma, IG = Inverse-Gamma, AR coeff. = autoregressive coefficient, St.dev. = standard deviation, HPD = highest posterior density.

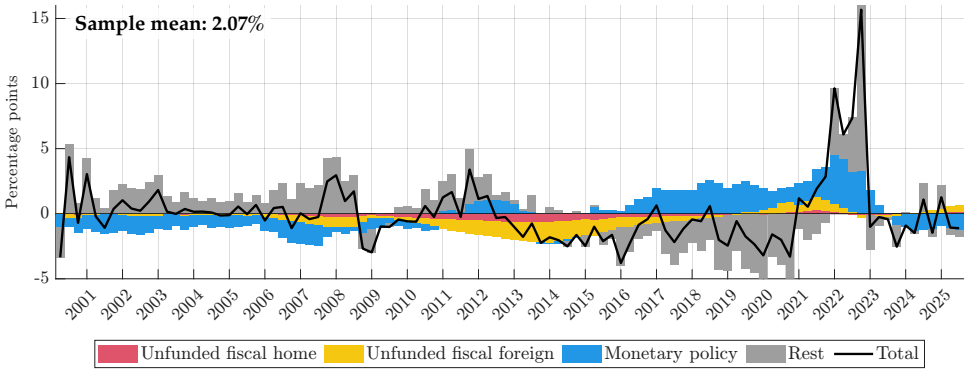
Figure A.12: Shock decomposition of inflation in Germany, France, and Italy (estimation with the shadow rate)



(a) Inflation in Germany



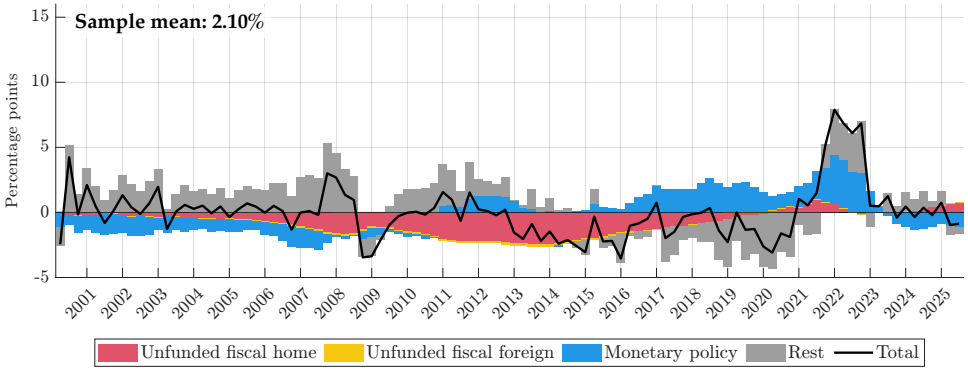
(b) Inflation in France



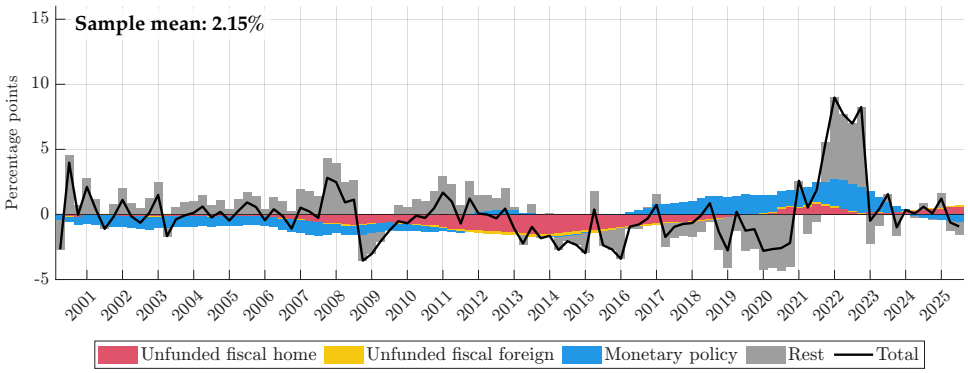
(c) Inflation in Italy

Notes: Historical shock decomposition of domestic inflation in Germany, France, and Italy (annualized quarterly, pp deviations from the sample mean) from the estimation with the shadow rate. Each panel comes from the estimation with that country as country A and the remaining euro area as country B (estimation sample: 2000Q2–2025Q3). Unfunded fiscal shocks home (red) and foreign (yellow) are the unfunded innovations ε_A^U and ε_B^U, each of which moves its country's debt and inflation targets jointly; monetary policy shocks are ε^m; all remaining shocks are "Rest." The black line is observed inflation.

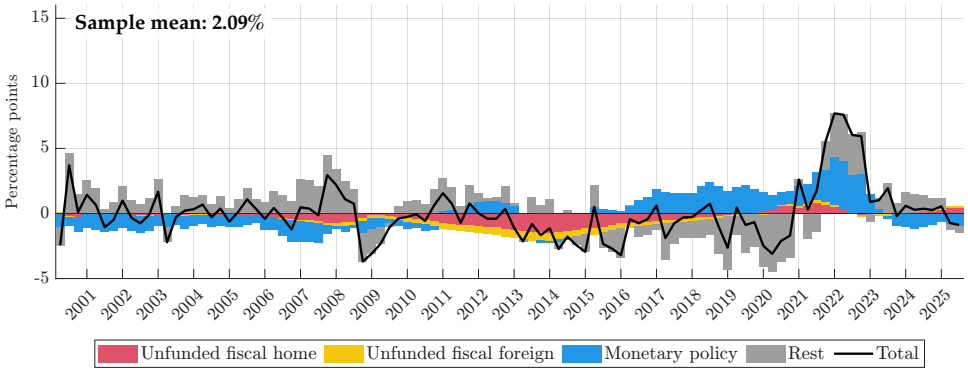
Figure A.13: Shock decomposition of inflation in rest of euro area (estimation with the shadow rate)



(a) Inflation in the rest of the euro area (vs. Germany)



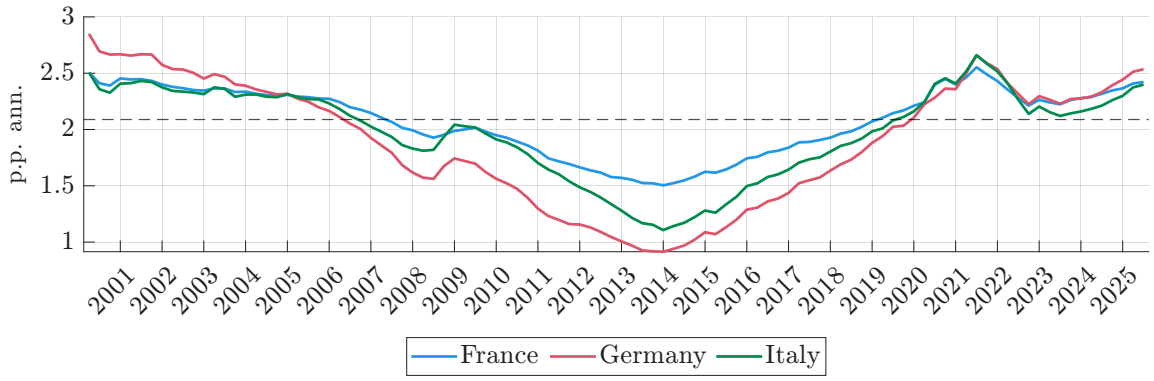
(b) Inflation in the rest of the euro area (vs. France)



(c) Inflation in the rest of the euro area (vs. Italy)

Notes: Historical shock decomposition of inflation in the rest of the euro area (annualized quarterly, pp deviations from the sample mean) from the estimation with the shadow rate. Each panel comes from the estimation with the named country as country A and the remaining euro area as country B (estimation sample: 2000Q2–2025Q3). Unfunded fiscal shocks home (red) and foreign (yellow) are the unfunded innovations ε_A^U and ε_B^U; monetary policy shocks are ε^m; all remaining shocks are "Rest." The black line is observed inflation.

Figure A.14: Model-implied ECB target inflation rate by country pair (estimation with the shadow rate)



Notes: Model-implied ECB target inflation rate (annualized, percentage points) from the three estimations with the shadow rate, each with one of Germany, France, or Italy as country A and the remaining euro area as country B (estimation sample: 2000Q2–2025Q3). The implied target is the union inflation objective $\pi_t^{F,MU}$ recovered from the smoothed path of the unfunded innovations. The three series confirm that the main pattern—a trough around 2014 and a rise during the COVID and energy crisis episodes—is robust to the choice of reference country; the estimation with the ELB is in the Online Appendix.

B Equations stylized model

We summarize the stylized model with the following equations, linearized around the steady state as $\hat{x}_t = \ln(X_t) - \ln(\overline{X}) \approx \frac{X_t - \overline{X}}{\overline{X}}$.

$$\text{Agg. Euler equation} \quad \hat{c}_t^{MU} = \mathbb{E}_t \hat{c}_{t+1}^{MU} - \left(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}^{MU}\right) - \Theta\psi_b \Upsilon \overline{b}_B \hat{b}_{Bt} - (1-\Theta)\psi_b \Upsilon \overline{b}_A^* \hat{b}_{At}^* \tag{B.1}$$

$$\text{Govt. budget constraint A} \quad \hat{s}_{bt} = \frac{1}{\beta}\left[\hat{s}_{b,t-1} + \hat{i}_{t-1} - \hat{\pi}_t - (1-\beta)\hat{\tau}_t\right] \tag{B.2}$$

$$\text{Govt. budget constraint B} \quad \hat{s}_{bt}^* = \frac{1}{\beta}\left[\hat{s}_{b,t-1}^* + \hat{i}_{t-1} - \hat{\pi}_t^* - (1-\beta)\hat{\tau}_t^*\right] \tag{B.3}$$

$$\text{Fiscal rule A} \quad \hat{\tau}_t = \gamma \hat{s}_{b,t-1} + \zeta_t \tag{B.4}$$

$$\text{Fiscal rule B} \quad \hat{\tau}_t^* = \gamma^* \hat{s}_{b,t-1}^* + \zeta_t^* \tag{B.5}$$

$$\text{Monetary rule} \quad \hat{i}_t = \phi \hat{\pi}_t^{MU} \tag{B.6}$$

$$\text{Market clearing A} \quad 0 = (1-\alpha_I)\hat{c}_t + \alpha_I \hat{c}_t^* + 2\mu\alpha_I(1-\alpha_I)\hat{s}_t \tag{B.7}$$

$$\text{Agg. goods market clearing} \quad 0 = \Theta\hat{c}_t + (1-\Theta)\hat{c}_t^* \tag{B.8}$$

$$\text{Bonds market clearing A} \quad \overline{s}_b \hat{s}_{bt} = \overline{b}_A \hat{b}_{At} + \overline{b}_A^* \hat{b}_{At}^* \tag{B.9}$$

$$\text{Bonds market clearing B} \quad \overline{s}_b^* \hat{s}_{bt}^* = \overline{b}_B \hat{b}_{Bt} + \overline{b}_B^* \hat{b}_{Bt}^* \tag{B.10}$$

$$\text{No arbitrage A} \quad \hat{b}_{Bt} = 0 \tag{B.11}$$

$$\text{No arbitrage B} \quad \hat{b}_{At}^* = 0 \tag{B.12}$$

$$\text{Terms of trade} \quad \alpha_I\left(\hat{s}_t - \hat{s}_{t-1}\right) = \hat{\pi}_t - \hat{\pi}_{Tt} \tag{B.13}$$

$$\text{Current account} \quad -\hat{c}_t - \alpha_I \hat{s}_t = \hat{ca}_t \tag{B.14}$$

$$\text{Dynamics of NFA} \quad \hat{ca}_t = \overline{b}_B \hat{b}_{Bt} - \overline{b}_A^* \hat{b}_{At}^* - \frac{1}{\beta}\left(\overline{b}_B \hat{b}_{B,t-1} - \overline{b}_A^* \hat{b}_{A,t-1}^*\right) \tag{B.15}$$

$$\text{Agg. inflation} \quad \hat{\pi}_t^{MU} = \Theta\hat{\pi}_t + (1-\Theta)\hat{\pi}_t^* \tag{B.16}$$

$$\text{Inflation A} \quad \hat{\pi}_t = (1-\alpha_I)\hat{\pi}_{Tt} + \alpha_I \hat{\pi}_{Tt}^* \tag{B.17}$$

$$\text{Inflation B} \quad \hat{\pi}_t^* = \alpha_I \hat{\pi}_{Tt} + (1-\alpha_I)\hat{\pi}_{Tt}^* \tag{B.18}$$

$$\text{Real interest rate } A \quad \hat{r}_t = \hat{i}_t - \hat{\pi}_t \tag{B.19}$$

$$\text{Real interest rate } B \quad \hat{r}_t^* = \hat{i}_t - \hat{\pi}_t^* \tag{B.20}$$

where $\hat{x}_t^{MU} \equiv \Theta\hat{x}_t + (1-\Theta)\hat{x}_t^*$ and bond holdings are in real terms (as shares of output), e.g. $\hat{b}_{At} = \frac{\frac{B_{At}}{P_t Y} - \frac{\overline{B}_A}{\overline{PY}}}{\frac{\overline{B}_A}{\overline{PY}}}$.

Steady state. From the Euler equations, we obtain the steady-state nominal interest rate:

$$\overline{R} = \frac{1}{\beta} \tag{B.21}$$

For simplicity, we impose that in steady state the prices of the goods are unitary:

$$\overline{P}_A = \overline{P}_B = 1 \tag{B.22}$$

From the relative demand equations and the goods market clearing conditions, we get the steady-state consumption values, assuming that the endowments $Y = Y^* = 1$:

$$\overline{C}_A = \overline{C}_B^* = 1 - \alpha_I \tag{B.23}$$

$$\overline{C}_B = \overline{C}_A^* = \alpha_I \tag{B.24}$$

$$\overline{C} = \overline{C}^* = 1 \tag{B.25}$$

With these steady-state values, we can get the government budget constraint in steady state:

$$\overline{B} = \frac{\beta}{1-\beta}\overline{T} \tag{B.26}$$

We calibrate the debt-to-GDP ratio $\frac{\overline{B}}{\overline{Y}} = \overline{B} = 0.80$. See the Calibration section in the main text for more details.

C Solving the stylized model with partially unfunded debt

In the model with partially unfunded debt in country A, we can keep all equations the same as the model without, except the fiscal rules and the monetary rule which we replace with equations (26), (27), and (28). Moreover, for every equation we add the equivalent equation for the shadow economies.

To prove that the equations for the model with partially unfunded debt are correct, we need to prove that the following is true: first, that $\hat{s}^M_{bt} = \hat{s}_{bt} - \hat{s}^F_{bt}$ and $\hat{s}^{M*}_{bt} = \hat{s}^*_{bt} - \hat{s}^{F*}_{bt}$, i.e. the difference of the overall stock of debt and fiscal-led stock of debt is the monetary-led stock of debt. Second, that $\hat{\pi}^M_t = \hat{\pi}_t - \hat{\pi}^F_t$ and $\hat{\pi}^{M*}_t = \hat{\pi}^*_t - \hat{\pi}^{F*}_t$, i.e. the inflation rate the central bank tries to stabilize in each economy is the overall inflation net of the fiscal-led part of inflation.

To prove those claims, we follow BFM and construct another parallel economy to characterize the part of inflation the central bank has control over, $\hat{\pi}^M_t$ and $\hat{\pi}^{M*}_t$:

$$\Theta \mathbb{E}_t \hat{\pi}^M_{t+1} + (1-\Theta)\mathbb{E}_t \hat{\pi}^{M*}_{t+1} = \hat{i}^M_t - \Theta \psi_b Y \overline{b}_B \hat{b}_{Bt} - (1-\Theta)\psi_b Y \overline{b}^*_A \hat{b}^*_{At} \tag{C.1}$$

$$\hat{s}^M_{bt} = \frac{1}{\beta}\left[\hat{s}^M_{b,t-1} + \hat{i}^M_{t-1} - \hat{\pi}^M_t - (1-\beta)\hat{\tau}^M_t\right] \tag{C.2}$$

$$\hat{s}^{M*}_{bt} = \frac{1}{\beta}\left[\hat{s}^{M*}_{b,t-1} + \hat{i}^M_{t-1} - \hat{\pi}^{M*}_t - (1-\beta)\hat{\tau}^{M*}_t\right] \tag{C.3}$$

$$\hat{\tau}^M_t = \gamma^M \hat{s}^M_{b,t-1} + \zeta^M_t \tag{C.4}$$

$$\hat{\tau}^{M*}_t = \gamma^{M*} \hat{s}^{M*}_{b,t-1} + \zeta^{M*}_t \tag{C.5}$$

$$\hat{i}^M_t = \Theta \phi^M \hat{\pi}^M_t + (1-\Theta)\phi^{M*}\hat{\pi}^{M*}_t \tag{C.6}$$

We obtain the monetary and fiscal blocks of this parallel economy as with other economies in the main text:

$$\begin{aligned}\Theta \mathbb{E}_t \hat{\pi}^M_{t+1} + (1-\Theta)\mathbb{E}_t \hat{\pi}^{M*}_{t+1} = \Theta \phi^M \hat{\pi}^M_t + (1-\Theta)\phi^{M*}\hat{\pi}^{M*}_t \\ - \Theta \psi_b Y \overline{b}_B \hat{b}_{Bt} - (1-\Theta)\psi_b Y \overline{b}^*_A \hat{b}^*_{At}\end{aligned} \tag{C.7}$$

$$\hat{s}^M_{bt} = \frac{1}{\beta}\left[(1-(1-\beta)\gamma^M)\hat{s}^M_{b,t-1} + \hat{i}^M_{t-1} - \hat{\pi}^M_t - (1-\beta)\zeta^M_t\right] \tag{C.8}$$

$$\hat{s}^{M*}_{bt} = \frac{1}{\beta}\left[(1-(1-\beta)\gamma^{M*})\hat{s}^{M*}_{b,t-1} + \hat{i}^M_{t-1} - \hat{\pi}^{M*}_t - (1-\beta)\zeta^{M*}_t\right] \tag{C.9}$$

We prove $\hat{\pi}^M_t = \hat{\pi}_t - \hat{\pi}^F_t$ by subtracting the equations for the monetary block of the monetary-led economy (C.7) and fiscal-led economy (33) and (34) from the monetary

block equation of the overall economy (29):[10]

$$\Theta\mathbb{E}_t\left[\hat{\pi}_{t+1}-\hat{\pi}^M_{t+1}-\hat{\pi}^F_{t+1}\right]+(1-\Theta)\mathbb{E}_t\left[\hat{\pi}^*_{t+1}-\hat{\pi}^{M*}_{t+1}-\hat{\pi}^{F*}_{t+1}\right]$$
$$=\Theta\phi^M\left(\hat{\pi}_t-\hat{\pi}^M_t-\hat{\pi}^F_t\right)+(1-\Theta)\phi^{M*}\left(\hat{\pi}^*_t-\hat{\pi}^{M*}_t-\hat{\pi}^{F*}_t\right) \quad \text{(C.10)}$$

where we multiply the fiscal-led economy equations with the respective country weights before subtracting. Let $d_t \equiv \hat{\pi}_t - \hat{\pi}^M_t - \hat{\pi}^F_t$ and $d^*_t \equiv \hat{\pi}^*_t - \hat{\pi}^{M*}_t - \hat{\pi}^{F*}_t$. Since $\phi^M = \phi^{M*} > 1$, the equation above reads $\mathbb{E}_t\left[\Theta d_{t+1} + (1-\Theta)d^*_{t+1}\right] = \phi^M\left[\Theta d_t + (1-\Theta)d^*_t\right]$, an unstable forward-looking difference equation whose unique bounded solution sets the union-weighted combination to zero for all t: $\Theta d_t + (1-\Theta)d^*_t = 0$. This single restriction does not pin down d_t and d^*_t separately, since inflation rates generally differ across countries. We therefore use, as in BFM, the linearity of the model and the uniqueness of its equilibrium. Assign each exogenous disturbance to exactly one of the two parallel economies: the funded shocks, such as ζ^M_t, to the monetary-led economy and the unfunded shocks, such as ζ^F_t, to the fiscal-led one (with $\zeta^{M*}_t = 0$, since country B's shocks are not presented). For every variable x of the model, let $\tilde{x}_t \equiv \hat{x}^M_t + \hat{x}^F_t$ be the sum of its values in the two parallel economies. All equations other than the fiscal and monetary rules are linear, identical across the three economies, and free of the policy coefficients, so $\tilde{x}_t$ satisfies them under the disturbances of the actual economy. It also satisfies the actual economy's policy rules. Summing the two parallel economies' fiscal rules for country A gives $\hat{\tau}^M_t + \hat{\tau}^F_t = \gamma^M\hat{s}^M_{b,t-1} + \gamma^F\hat{s}^F_{b,t-1} + \zeta^M_t + \zeta^F_t$, which is (26) evaluated at $\tilde{s}_{b,t-1} = \hat{s}^M_{b,t-1} + \hat{s}^F_{b,t-1}$; the same holds for country B's rule (27). Summing $\hat{i}^M_t$ and $\hat{i}^F_t$ gives $\Theta\left[\phi^M\hat{\pi}^M_t + \phi^F\hat{\pi}^F_t\right] + (1-\Theta)\left[\phi^{M*}\hat{\pi}^{M*}_t + \phi^{F*}\hat{\pi}^{F*}_t\right]$, which is the monetary rule (28) evaluated at $\tilde{\pi}_t = \hat{\pi}^M_t + \hat{\pi}^F_t$ and $\tilde{\pi}^*_t = \hat{\pi}^{M*}_t + \hat{\pi}^{F*}_t$. Each parallel economy has a unique bounded equilibrium by Proposition 1: the monetary-led one because $\phi^M > 1$ and $\gamma^M, \gamma^{M*} > 1$, and the fiscal-led one because $\phi^F \leq 1$, $\gamma^F < 1$, and $\gamma^{F*} > 1$. Hence $\tilde{x}_t$ is a bounded solution of the actual economy that starts from the common steady state. The actual economy's equilibrium is unique (Section 2.6), so $\hat{x}_t = \tilde{x}_t$ for every variable. In particular, $d_t = d^*_t = 0$, i.e. $\hat{\pi}^M_t = \hat{\pi}_t - \hat{\pi}^F_t$ and $\hat{\pi}^{M*}_t = \hat{\pi}^*_t - \hat{\pi}^{F*}_t$, for all t.

Now, we prove that $\hat{s}^M_{bt} = \hat{s}_{bt} - \hat{s}^F_{bt}$. Substituting the fiscal rule of country A and monetary rule that allow for partially unfunded debt, equations (26) and (28) respectively, into the country A government budget constraint (16), we obtain:

$$\beta\hat{s}_{bt} = \left(1-(1-\beta)\gamma^M\right)\hat{s}_{b,t-1} + (1-\beta)\gamma^M\hat{s}^F_{b,t-1} - (1-\beta)\gamma^F\hat{s}^F_{b,t-1} - \hat{\pi}_t - (1-\beta)\left(\zeta^M_t + \zeta^F_t\right)$$
$$+\Theta\left[\phi^M\left(\hat{\pi}_{t-1} - \hat{\pi}^F_{t-1}\right) + \phi^F\hat{\pi}^F_{t-1}\right] + (1-\Theta)\left[\phi^{M*}\left(\hat{\pi}^*_{t-1} - \hat{\pi}^{F*}_{t-1}\right) + \phi^{F*}\hat{\pi}^{F*}_{t-1}\right]. \quad \text{(C.11)}$$

10. Impose the no-arbitrage conditions to simplify the adjustment cost term.

We obtain the equivalent equations for the monetary-led and fiscal-led parallel economies:

$$\beta\hat{s}^M_{bt} = (1-(1-\beta)\gamma^M)\hat{s}^M_{b,t-1} + \Theta\phi^M\hat{\pi}^M_{t-1} + (1-\Theta)\phi^{M*}\hat{\pi}^{M*}_{t-1} - \hat{\pi}^M_t - (1-\beta)\zeta^M_t \quad \text{(C.12)}$$

$$\beta\hat{s}^F_{bt} = (1-(1-\beta)\gamma^F)\hat{s}^F_{b,t-1} + \Theta\phi^F\hat{\pi}^F_{t-1} + (1-\Theta)\phi^{F*}\hat{\pi}^{F*}_{t-1} - \hat{\pi}^F_t - (1-\beta)\zeta^F_t \quad \text{(C.13)}$$

Subtract $\beta\hat{s}^M_{bt}$ and $\beta\hat{s}^F_{bt}$ from $\beta\hat{s}_{bt}$ to obtain:

$$\begin{aligned}\beta\left(\hat{s}_{bt} - \hat{s}^M_{bt} - \hat{s}^F_{bt}\right) = \left(1-(1-\beta)\gamma^M\right)\left(\hat{s}_{b,t-1} - \hat{s}^M_{b,t-1} - \hat{s}^F_{b,t-1}\right) - \left(\hat{\pi}_t - \hat{\pi}^M_t - \hat{\pi}^F_t\right) \\ + \Theta\phi^M\left(\hat{\pi}_{t-1} - \hat{\pi}^M_{t-1} - \hat{\pi}^F_{t-1}\right) + (1-\Theta)\phi^{M*}\left(\hat{\pi}^*_{t-1} - \hat{\pi}^{M*}_{t-1} - \hat{\pi}^{F*}_{t-1}\right) \quad \text{(C.14)}\end{aligned}$$

Using $d_t = 0$ and $d^*_t = 0$ established above, i.e. $\hat{\pi}^M_t = \hat{\pi}_t - \hat{\pi}^F_t$ and $\hat{\pi}^{M*}_t = \hat{\pi}^*_t - \hat{\pi}^{F*}_t$ for all t, the recursion becomes homogeneous:

$$\hat{s}_{bt} - \hat{s}^M_{bt} - \hat{s}^F_{bt} = \frac{1-(1-\beta)\gamma^M}{\beta}\left(\hat{s}_{b,t-1} - \hat{s}^M_{b,t-1} - \hat{s}^F_{b,t-1}\right).$$

All three economies coincide at the common non-stochastic steady state prior to the shock, so $\hat{s}_{b,-1} = \hat{s}^M_{b,-1} = \hat{s}^F_{b,-1} = 0$. By induction on the recursion above, $\hat{s}_{bt} - \hat{s}^M_{bt} - \hat{s}^F_{bt} = 0$ for all $t \geq 0$, hence $\hat{s}^M_{bt} = \hat{s}_{bt} - \hat{s}^F_{bt}$ for all periods. The proof for $\hat{s}^{M*}_{bt} = \hat{s}^*_{bt} - \hat{s}^{F*}_{bt}$ is analogous.

D Equations quantitative model

We summarize the quantitative model with internal consumption habit using the following equations, linearized around the steady state as in Appendix B.

Demand

Effective consumption A
$$\hat{c}_t^{\text{eff}} = \frac{1}{1-h}\left(\hat{c}_t - \tilde{\varepsilon}_t^{tC}\right) - \frac{h}{1-h}\left(\hat{c}_{t-1} - \tilde{\varepsilon}_{t-1}^{tC}\right) \tag{D.1}$$

Effective consumption B
$$\hat{c}_t^{\text{eff}*} = \frac{1}{1-h^*}\left(\hat{c}_t^* - \tilde{\varepsilon}_t^{tC*}\right) - \frac{h^*}{1-h^*}\left(\hat{c}_{t-1}^* - \tilde{\varepsilon}_{t-1}^{tC*}\right) \tag{D.2}$$

Marginal utility A
$$\hat{\lambda}_t = -\frac{1}{1-\beta h}\left(\hat{c}_t^{\text{eff}} - \beta h \mathbb{E}_t \hat{c}_{t+1}^{\text{eff}}\right) + \frac{1}{1-\beta h}\left(\xi_t^d - \beta h \mathbb{E}_t \xi_{t+1}^d\right) \tag{D.3}$$

Marginal utility B
$$\hat{\lambda}_t^* = -\frac{1}{1-\beta h^*}\left(\hat{c}_t^{\text{eff}*} - \beta h^* \mathbb{E}_t \hat{c}_{t+1}^{\text{eff}*}\right) + \frac{1}{1-\beta h^*}\left(\xi_t^{d*} - \beta h^* \mathbb{E}_t \xi_{t+1}^{d*}\right) \tag{D.4}$$

Labor supply A
$$-\hat{\lambda}_t + \chi \hat{l}_t + \varepsilon_t^h = \widehat{mrs}_t \tag{D.5}$$

Labor supply B
$$-\hat{\lambda}_t^* + \chi^* \hat{l}_t^* + \varepsilon_t^{h*} = \widehat{mrs}_t^* \tag{D.6}$$

Agg. Euler equation
$$\mathbb{E}_t \hat{\lambda}_{t+1}^{MU} = \hat{\lambda}_t^{MU} - \left(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}^{MU}\right) \tag{D.7}$$

Agg. capital Euler equation
$$\hat{q}_t^{MU} = \mathbb{E}_t \hat{\lambda}_{t+1}^{MU} - \hat{\lambda}_t^{MU} + \beta(1-\delta)\mathbb{E}_t \hat{q}_{t+1}^{MU} + [1-\beta(1-\delta)]\mathbb{E}_t\left[\hat{r}_{K,t+1}^{MU} - \frac{\overline{\tau}_K}{1-\overline{\tau}_K}\left(\Theta \hat{\tau}_{K,t+1} + (1-\Theta)\hat{\tau}_{K,t+1}^*\right)\right] \tag{D.8}$$

No arbitrage capital and ST bonds A
$$-\left(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}\right) = \hat{q}_t - \beta(1-\delta)\mathbb{E}_t \hat{q}_{t+1} - [1-\beta(1-\delta)]\mathbb{E}_t\left[\hat{r}_{K,t+1} - \frac{\overline{\tau}_K}{1-\overline{\tau}_K}\hat{\tau}_{K,t+1}\right] \tag{D.9}$$

No arbitrage across LT bonds A
$$\hat{b}_{Bt} = 0 \tag{D.10}$$

No arbitrage across LT bonds B
$$\hat{b}_{At}^* = 0 \tag{D.11}$$

No arbitrage LT and ST bonds A
$$\hat{i}_t = \mathbb{E}_t \hat{r}_{t+1}^L + \xi_t^{rp} \tag{D.12}$$

No arbitrage LT and ST bonds B
$$\hat{i}_t = \mathbb{E}_t \hat{r}_{t+1}^{L*} + \xi_t^{rp*} \tag{D.13}$$

Return on LT bonds A
$$\hat{r}_t^L = \rho^L \beta \hat{p}_t^L - \hat{p}_{t-1}^L \tag{D.14}$$

Return on LT bonds B
$$\hat{r}_t^{L*} = \rho^L \beta \hat{p}_t^{L*} - \hat{p}_{t-1}^{L*} \tag{D.15}$$

Demand trad./non-trad. A
$$\hat{c}_{Tt} - \hat{c}_{Nt} = -\mu_N\left(\hat{p}_{Tt} - \hat{p}_{Nt}\right) \tag{D.16}$$

Demand trad./non-trad. B
$$\hat{c}_{Tt}^* - \hat{c}_{Nt}^* = -\mu_N\left(\hat{p}_{Tt}^* - \hat{p}_{Nt}^*\right) \tag{D.17}$$

Wages

$$\text{Wage Phillips curve } A \quad \hat{\pi}_t^w = \beta \mathbb{E}_t \hat{\pi}_{t+1}^w + \chi_w \hat{\pi}_{t-1} + \kappa_w \left(\widehat{mrs}_t - \hat{w}_t^{real} \right) - \kappa_w \frac{\overline{\tau}_L}{1-\overline{\tau}_L} \hat{\tau}_{Lt} + u_t^w \tag{D.18}$$

$$\text{Wage Phillips curve } B \quad \hat{\pi}_t^{w*} = \beta \mathbb{E}_t \hat{\pi}_{t+1}^{w*} + \chi_w^* \hat{\pi}_{t-1}^* + \kappa_w^* \left(\widehat{mrs}_t^* - \hat{w}_t^{*,real} \right) - \kappa_w^* \frac{\overline{\tau}_L}{1-\overline{\tau}_L} \hat{\tau}_{Lt}^* + u_t^{w*} \tag{D.19}$$

$$\text{Wage inflation indexation } A \qquad \hat{\pi}_t^w = \hat{w}_t^{real} - \hat{w}_{t-1}^{real} + \hat{\pi}_t \tag{D.20}$$

$$\text{Wage inflation indexation } B \qquad \hat{\pi}_t^{w*} = \hat{w}_t^{*,real} - \hat{w}_{t-1}^{*,real} + \hat{\pi}_t^* \tag{D.21}$$

Following Christiano, Eichenbaum, and Evans (2005), the gross desired wage markup is 1.05, so $\epsilon_w = 21$, and

$$\kappa_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\chi)}, \qquad \kappa_w^* = \frac{(1-\theta_w^*)(1-\beta\theta_w^*)}{\theta_w^*(1+\epsilon_w\chi^*)}. \tag{D.22}$$

Supply

Tobin's Q A
$$\hat{q}_t = \psi_I \left[\widehat{inv}_t - \widehat{inv}_{t-1} - \beta \left(\mathbb{E}_t \widehat{inv}_{t+1} - \widehat{inv}_t \right) \right] - \xi_t^k \tag{D.23}$$

Tobin's Q B
$$\hat{q}_t^* = \psi_I \left[\widehat{inv}_t^* - \widehat{inv}_{t-1}^* - \beta \left(\mathbb{E}_t \widehat{inv}_{t+1}^* - \widehat{inv}_t^* \right) \right] - \xi_t^{k*} \tag{D.24}$$

Capital accumulation A
$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta \left(\widehat{inv}_t + \xi_t^k \right) \tag{D.25}$$

Capital accumulation B
$$\hat{k}_t^* = (1-\delta)\hat{k}_{t-1}^* + \delta \left(\widehat{inv}_t^* + \xi_t^{k*} \right) \tag{D.26}$$

Capital services A
$$\hat{k}_t^s = \hat{k}_{t-1} + \hat{\mu}_{K,t} \tag{D.27}$$

Capital services B
$$\hat{k}_t^{s*} = \hat{k}_{t-1}^* + \hat{\mu}_{K,t}^* \tag{D.28}$$

Production function trad. A
$$\hat{y}_{Tt} = \hat{a}_{Tt} + \alpha \hat{k}_t^s + (1-\alpha)\hat{l}_{Tt} \tag{D.29}$$

Production function trad. B
$$\hat{y}_{Tt}^* = \hat{a}_{Tt}^* + \alpha \hat{k}_t^{s*} + (1-\alpha)\hat{l}_{Tt}^* \tag{D.30}$$

Production function non-trad. A
$$\hat{y}_{Nt} = \hat{a}_{Nt} + (1-\alpha)\hat{l}_{Nt} \tag{D.31}$$

Production function non-trad. B
$$\hat{y}_{Nt}^* = \hat{a}_{Nt}^* + (1-\alpha)\hat{l}_{Nt}^* \tag{D.32}$$

Capital demand A
$$\widehat{mc}_{Tt} + \hat{y}_{Tt} - \hat{k}_t^s = \hat{r}_{Kt} \tag{D.33}$$

Capital demand B
$$\widehat{mc}_{Tt}^* + \hat{y}_{Tt}^* - \hat{k}_t^{s*} = \hat{r}_{Kt}^* \tag{D.34}$$

FOC utilization A
$$\hat{\mu}_{K,t} = \frac{1-\psi_u}{\psi_u} \left(\hat{r}_{Kt} - \frac{\overline{\tau}_K}{1-\overline{\tau}_K} \hat{\tau}_{Kt} \right) \tag{D.35}$$

FOC utilization B
$$\hat{\mu}_{K,t}^* = \frac{1-\psi_u}{\psi_u} \left(\hat{r}_{Kt}^* - \frac{\overline{\tau}_K}{1-\overline{\tau}_K} \hat{\tau}_{Kt}^* \right) \tag{D.36}$$

Marginal cost trad. A
$$\widehat{mc}_{Tt} = \hat{w}_t^{real} + \hat{l}_{Tt} - \hat{y}_{Tt} \tag{D.37}$$

Marginal cost trad. B
$$\widehat{mc}_{Tt}^* = \hat{w}_t^{*,real} + \hat{l}_{Tt}^* - \hat{y}_{Tt}^* \tag{D.38}$$

NKPC trad. A
$$\hat{\pi}_{At} = \beta \mathbb{E}_t \hat{\pi}_{A,t+1} + \kappa_T \widehat{mc}_{Tt} + u_t^T \tag{D.39}$$

NKPC trad. B
$$\hat{\pi}_{Bt} = \beta \mathbb{E}_t \hat{\pi}_{B,t+1} + \kappa_T \widehat{mc}_{Tt}^* + u_t^{T*} \tag{D.40}$$

NKPC non-trad. A
$$\hat{\pi}_{Nt} = \beta \mathbb{E}_t \hat{\pi}_{N,t+1} + \kappa_N \widehat{mc}_{Nt} + u_t^N \tag{D.41}$$

NKPC non-trad. B
$$\hat{\pi}_{Nt}^* = \beta \mathbb{E}_t \hat{\pi}_{N,t+1}^* + \kappa_N \widehat{mc}_{Nt}^* + u_t^{N*} \tag{D.42}$$

Marginal cost non-trad. A
$$\widehat{mc}_{Nt} = \hat{w}_t^{real} + \hat{l}_{Nt} - \hat{y}_{Nt} \tag{D.43}$$

Marginal cost non-trad. B
$$\widehat{mc}_{Nt}^* = \hat{w}_t^{*,real} + \hat{l}_{Nt}^* - \hat{y}_{Nt}^* \tag{D.44}$$

Fiscal (superscript F denotes a time-varying debt target exempted from fiscal stabilization)

$$\text{Govt. budget constraint } A \quad \hat{s}_{bt} = \frac{1}{\beta}\left[\hat{s}_{b,t-1} + \hat{i}_t^L - \hat{\pi}_t\right] - \omega_{K,b}\left(\hat{\tau}_{Kt} + \hat{r}_{Kt} + \hat{k}_{t-1} + \hat{\mu}_{Kt}\right) - \omega_{L,b}\left(\hat{\tau}_{Lt} + \hat{w}_t^{real} + \hat{l}_t\right) - \omega_{C,b}\hat{c}_t + \omega_{G,b}\hat{g}_t + \omega_{Z,b}\hat{z}_t \quad \text{(D.45)}$$

$$\text{Govt. budget constraint } B \quad \hat{s}_{bt}^* = \frac{1}{\beta}\left[\hat{s}_{b,t-1}^* + \hat{i}_t^{L*} - \hat{\pi}_t^*\right] - \omega_{K,b}\left(\hat{\tau}_{Kt}^* + \hat{r}_{Kt}^* + \hat{k}_{t-1}^* + \hat{\mu}_{Kt}^*\right) - \omega_{L,b}\left(\hat{\tau}_{Lt}^* + \hat{w}_t^{*,real} + \hat{l}_t^*\right) - \omega_{C,b}\hat{c}_t^* + \omega_{G,b}\hat{g}_t^* + \omega_{Z,b}\hat{z}_t^* \quad \text{(D.46)}$$

The coefficients are steady-state revenue or expenditure shares relative to the corresponding debt-to-output ratio:

$$\omega_{K,b} \equiv \frac{\overline{\tau}_K \overline{R}_K \overline{K}}{\overline{Y}\,\overline{s}_b}, \qquad \omega_{L,b} \equiv \frac{\overline{\tau}_L \overline{W}^{real} \overline{L}}{\overline{Y}\,\overline{s}_b}, \qquad \omega_{C,b} \equiv \frac{\overline{\tau}_C \overline{C}}{\overline{Y}\,\overline{s}_b}, \quad \text{(D.47)}$$

$$\omega_{G,b} \equiv \frac{\overline{G}}{\overline{Y}\,\overline{s}_b}, \qquad \omega_{Z,b} \equiv \frac{\overline{Z}}{\overline{Y}\,\overline{s}_b}, \quad \text{(D.48)}$$

with starred analogues defined using country-B steady states. The capital-revenue coefficient is based on the nonlinear tax base $\tau_{Kt} R_{Kt} \mu_{Kt} K_{t-1}$.

$$\text{Transfer rule } A \quad \hat{z}_t = \rho_z \hat{z}_{t-1} - (1-\rho_z)\left[\gamma\left(\hat{s}_{b,t-1} - s_{b,A,t}^F\right) + \phi_{zy}\hat{y}_t\right] + \frac{1}{\overline{Z}/\overline{Y}}\zeta_t \quad \text{(D.49)}$$

$$\text{Transfer rule } B \quad \hat{z}_t^* = \rho_z^* \hat{z}_{t-1}^* - (1-\rho_z^*)\left[\gamma^*\left(\hat{s}_{b,t-1}^* - s_{b,B,t}^{F*}\right) + \phi_{zy}^*\hat{y}_t^*\right] + \frac{1}{\overline{Z}^*/\overline{Y}^*}\zeta_t^* \quad \text{(D.50)}$$

$$\text{Capital tax rule } A \quad \hat{\tau}_{K,t} = \rho_K \hat{\tau}_{K,t-1} + (1-\rho_K)\gamma_K\left(\hat{s}_{b,t-1} - s_{b,A,t}^F\right) \quad \text{(D.51)}$$

$$\text{Capital tax rule } B \quad \hat{\tau}_{K,t}^* = \rho_K \hat{\tau}_{K,t-1}^* + (1-\rho_K)\gamma_K^*\left(\hat{s}_{b,t-1}^* - s_{b,B,t}^{F*}\right) \quad \text{(D.52)}$$

$$\text{Labor tax rule } A \quad \hat{\tau}_{L,t} = \rho_L \hat{\tau}_{L,t-1} + (1-\rho_L)\gamma_L\left(\hat{s}_{b,t-1} - s_{b,A,t}^F\right) \quad \text{(D.53)}$$

$$\text{Labor tax rule } B \quad \hat{\tau}_{L,t}^* = \rho_L \hat{\tau}_{L,t-1}^* + (1-\rho_L)\gamma_L^*\left(\hat{s}_{b,t-1}^* - s_{b,B,t}^{F*}\right) \quad \text{(D.54)}$$

$$\text{Govt. spending rule } A \quad \hat{g}_t = \rho_g \hat{g}_{t-1} - (1-\rho_g)\gamma_G\left(\hat{s}_{b,t-1} - s_{b,A,t}^F\right) + \varepsilon_t^g \quad \text{(D.55)}$$

$$\text{Govt. spending rule } B \quad \hat{g}_t^* = \rho_g^* \hat{g}_{t-1}^* - (1-\rho_g^*)\gamma_G^*\left(\hat{s}_{b,t-1}^* - s_{b,B,t}^{F*}\right) + \varepsilon_t^{g*} \quad \text{(D.56)}$$

$$\text{Debt target } A \quad s_{b,A,t}^F = \rho_s\, s_{b,A,t-1}^F + \varepsilon_{A,t}^U \quad \text{(D.57)}$$

$$\text{Debt target } B \quad s_{b,B,t}^{F*} = \rho_s\, s_{b,B,t-1}^{F*} + \varepsilon_{B,t}^U \quad \text{(D.58)}$$

$\varepsilon_{A,t}^U$ and $\varepsilon_{B,t}^U$ are unfunded fiscal shocks that move the country-specific debt target. A positive shock raises the debt target, so that the fiscal rule excuses this unfunded debt from debt stabilization.

Monetary (superscript F denotes a time-varying inflation target)

$$\text{Taylor rule} \quad \hat{i}_t = \rho_r \hat{i}_{t-1} + (1-\rho_r)\phi^M \left[\Theta(\hat{\pi}_t - \pi^F_{A,t}) + (1-\Theta)(\hat{\pi}^*_t - \pi^{F*}_{B,t})\right] + \varepsilon^m_t \tag{D.59}$$

$$\text{Inflation target } A \quad \pi^F_{A,t} = \phi_F\, \pi^F_{A,t-1} + \phi_{\pi,u}\, \varepsilon^U_{A,t} \tag{D.60}$$

$$\text{Inflation target } B \quad \pi^{F*}_{B,t} = \phi^*_F\, \pi^{F*}_{B,t-1} + \phi^*_{\pi,u}\, \varepsilon^U_{B,t} \tag{D.61}$$

The same unfunded innovation $\varepsilon^U_{c,t}$ that raises country c's debt target raises its inflation target, with loading $\phi_{\pi,u}$: a positive innovation increases the inflation rate accommodated by the central bank. Following Guo, Han, and Smit (2026), we calibrate $\phi_{\pi,u} = \phi^*_{\pi,u} = 0.10$ and $\phi_F = \phi^*_F = \rho_s = \rho^*_s = 0.98$, while the two country-specific volatilities $\sigma_{u,A}$ and $\sigma_{u,B}$ are estimated with diffuse inverse-Gamma priors. The unfunded innovations are mutually independent across countries. Starred fiscal-rule parameters are shown separately above; restrictions that pool a country-B parameter with its country-A analogue are imposed as explicit calibration restrictions.

Equilibrium Let $\omega_C \equiv \overline{C}/\overline{Y}$, $\omega_{CT} \equiv (1-\alpha_N)\omega_C$, $\omega_T \equiv \overline{Y}_T/\overline{Y}$, and $\omega_N \equiv \overline{Y}_N/\overline{Y} = \alpha_N\omega_C$. Country-size-adjusted import shares are $a_A \equiv 2(1-\Theta)\alpha_I$ and $a_B \equiv 2\Theta\alpha_I$. Define

$$\omega^A_{C,d} \equiv (1-a_A)\frac{\omega_{CT}}{\omega_T}, \qquad \omega^A_{C,x} \equiv a_B \frac{1-\Theta}{\Theta}\frac{\omega_{CT}}{\omega_T}, \tag{D.62}$$

$$\omega^B_{C,d} \equiv (1-a_B)\frac{\omega_{CT}}{\omega_T}, \qquad \omega^B_{C,x} \equiv a_A \frac{\Theta}{1-\Theta}\frac{\omega_{CT}}{\omega_T}, \tag{D.63}$$

$$\eta^A_s \equiv \mu a_A(2-a_A-a_B)\frac{\omega_{CT}}{\omega_T}, \qquad \eta^B_s \equiv \mu a_B(2-a_A-a_B)\frac{\omega_{CT}}{\omega_T}. \tag{D.64}$$

$$\text{Market clearing trad. } A \quad \hat{y}_{Tt} = \omega^A_{C,d}\hat{c}_{Tt} + \omega^A_{C,x}\hat{c}^*_{Tt} + \eta^A_s \hat{s}_t + \omega^A_I \widehat{inv}_t + \omega^A_G \hat{g}_t + \omega^A_u \hat{\mu}_{K,t} \tag{D.65}$$

$$\text{Market clearing trad. } B \quad \hat{y}^*_{Tt} = \omega^B_{C,d}\hat{c}^*_{Tt} + \omega^B_{C,x}\hat{c}_{Tt} - \eta^B_s \hat{s}_t + \omega^B_I \widehat{inv}^*_t + \omega^B_G \hat{g}^*_t + \omega^B_u \hat{\mu}^*_{K,t} \tag{D.66}$$

$$\text{Current account} \quad \omega_T \hat{y}_{Tt} - \omega_{CT}\hat{c}_{Tt} - \frac{\overline{I}}{\overline{Y}}\widehat{inv}_t - \frac{\overline{G}}{\overline{Y}}\hat{g}_t - \frac{a'(1)\overline{K}}{\overline{Y}}\hat{\mu}_{K,t} - \omega_T a_A \hat{s}_t = \widehat{ca}_t \tag{D.67}$$

$$\text{Market clearing non-trad. } A \quad \hat{y}_{Nt} = \hat{c}_{Nt} \tag{D.68}$$

$$\text{Market clearing non-trad. } B \quad \hat{y}^*_{Nt} = \hat{c}^*_{Nt} \tag{D.69}$$

Here $\omega^j_I \equiv \overline{I}^j/\overline{Y}^j_T$, $\omega^j_G \equiv \overline{G}^j/\overline{Y}^j_T$, and $\omega^j_u \equiv a'(1)\overline{K}^j/\overline{Y}^j_T$. Bilateral trade balance follows from $\Theta a_A = (1-\Theta)a_B$. The consumption, investment, and government coefficients in

each tradable resource constraint sum to one; utilization has zero steady-state level but a nonzero first-order resource cost.

$$\text{Labor market clearing } A \qquad \hat{l}_t = \omega_T \hat{l}_{Tt} + \omega_N \hat{l}_{Nt} \tag{D.70}$$

$$\text{Labor market clearing } B \qquad \hat{l}^*_t = \omega_T \hat{l}^*_{Tt} + \omega_N \hat{l}^*_{Nt} \tag{D.71}$$

$$\text{Dynamics of NFA} \qquad \hat{ca}_t = \overline{b}_B \hat{b}_{Bt} - \overline{b}^*_A \hat{b}^*_{At} - \frac{1}{\beta}\left(\overline{b}_B \hat{b}_{B,t-1} - \overline{b}^*_A \hat{b}^*_{A,t-1}\right) \tag{D.72}$$

$$\text{Bonds market clearing A} \qquad \overline{s}_b \hat{s}_{bt} = \overline{b}_A \hat{b}_{At} + \overline{b}^*_A \hat{b}^*_{At} \tag{D.73}$$

$$\text{Bonds market clearing B} \qquad \overline{s}^*_b \hat{s}^*_{bt} = \overline{b}_B \hat{b}_{Bt} + \overline{b}^*_B \hat{b}^*_{Bt} \tag{D.74}$$

Definitions

$$\text{Agg. output } A \qquad \hat{y}_t = \omega_T \hat{y}_{Tt} + \omega_N \hat{y}_{Nt} \tag{D.75}$$

$$\text{Agg. output } B \qquad \hat{y}^*_t = \omega_T \hat{y}^*_{Tt} + \omega_N \hat{y}^*_{Nt} \tag{D.76}$$

$$\text{Terms of trade} \qquad a_A\left(\hat{s}_t - \hat{s}_{t-1}\right) = \hat{\pi}_{Tt} - \hat{\pi}_{At} \tag{D.77}$$

$$\text{Agg. consumption } A \qquad \hat{c}_t = (1-\alpha_N)\hat{c}_{Tt} + \alpha_N \hat{c}_{Nt} \tag{D.78}$$

$$\text{Agg. consumption } B \qquad \hat{c}^*_t = (1-\alpha_N)\hat{c}^*_{Tt} + \alpha_N \hat{c}^*_{Nt} \tag{D.79}$$

$$\text{Tradable inflation } A \qquad \hat{\pi}_{Tt} = (1-a_A)\hat{\pi}_{At} + a_A \hat{\pi}_{Bt} \tag{D.80}$$

$$\text{Tradable inflation } B \qquad \hat{\pi}^*_{Tt} = a_B \hat{\pi}_{At} + (1-a_B)\hat{\pi}_{Bt} \tag{D.81}$$

$$\text{CPI inflation } A \qquad \hat{\pi}_t = (1-\alpha_N)\hat{\pi}_{Tt} + \alpha_N \hat{\pi}_{Nt} \tag{D.82}$$

$$\text{CPI inflation } B \qquad \hat{\pi}^*_t = (1-\alpha_N)\hat{\pi}^*_{Tt} + \alpha_N \hat{\pi}^*_{Nt} \tag{D.83}$$

$$\text{Union marginal utility} \qquad \hat{\lambda}^{MU}_t = \Theta\hat{\lambda}_t + (1-\Theta)\hat{\lambda}^*_t \tag{D.84}$$

$$\text{Union Tobin's Q} \qquad \hat{q}^{MU}_t = \Theta\hat{q}_t + (1-\Theta)\hat{q}^*_t \tag{D.85}$$

$$\text{Union capital return} \qquad \hat{r}^{MU}_{Kt} = \Theta\hat{r}_{Kt} + (1-\Theta)\hat{r}^*_{Kt} \tag{D.86}$$

$$\text{Union CPI inflation} \qquad \hat{\pi}^{MU}_t = \Theta\hat{\pi}_t + (1-\Theta)\hat{\pi}^*_t \tag{D.87}$$

$$\text{Union inflation target} \qquad \pi^{F,MU}_t = \Theta\pi^F_{A,t} + (1-\Theta)\pi^{F*}_{B,t} \tag{D.88}$$

$$\text{Real interest rate } A \qquad \hat{r}^{real}_t = \hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} \tag{D.89}$$

$$\text{Real interest rate } B \qquad \hat{r}^{*,real}_t = \hat{i}_t - \mathbb{E}_t\hat{\pi}^*_{t+1} \tag{D.90}$$

Structural shock AR(1) processes

Technology trad. A	$\hat{a}_{Tt} = \rho_a \hat{a}_{T,t-1} + \varepsilon_t^T$	(D.91)
Technology trad. B	$\hat{a}^*_{Tt} = \rho_a \hat{a}^*_{T,t-1} + \varepsilon_t^{T*}$	(D.92)
Technology non-trad. A	$\hat{a}_{Nt} = \rho_a \hat{a}_{N,t-1} + \varepsilon_t^N$	(D.93)
Technology non-trad. B	$\hat{a}^*_{Nt} = \rho_a \hat{a}^*_{N,t-1} + \varepsilon_t^{N*}$	(D.94)
Risk premium shock A	$\xi_t^{rp} = \rho_{rp} \xi_{t-1}^{rp} + \varepsilon_t^{rp}$	(D.95)
Risk premium shock B	$\xi_t^{rp*} = \rho_{rp} \xi_{t-1}^{rp*} + \varepsilon_t^{rp*}$	(D.96)
Preference shock A	$\xi_t^d = \rho_d \xi_{t-1}^d + \varepsilon_t^d$	(D.97)
Preference shock B	$\xi_t^{d*} = \rho_d \xi_{t-1}^{d*} + \varepsilon_t^{d*}$	(D.98)
Tradable goods markup shock A	$u_t^T = \rho_T u_{t-1}^T + \varepsilon_t^{uT}$	(D.99)
Tradable goods markup shock B	$u_t^{T*} = \rho_T u_{t-1}^{T*} + \varepsilon_t^{uT*}$	(D.100)
Non-tradable goods markup shock A	$u_t^N = \rho_N u_{t-1}^N + \varepsilon_t^{uN}$	(D.101)
Non-tradable goods markup shock B	$u_t^{N*} = \rho_N u_{t-1}^{N*} + \varepsilon_t^{uN*}$	(D.102)
Wage markup shock A	$u_t^w = \rho_w u_{t-1}^w + \varepsilon_t^w$	(D.103)
Wage markup shock B	$u_t^{w*} = \rho_w u_{t-1}^{w*} + \varepsilon_t^{w*}$	(D.104)
MEI shock A	$\xi_t^k = \rho_k \xi_{t-1}^k + \varepsilon_t^k$	(D.105)
MEI shock B	$\xi_t^{k*} = \rho_k \xi_{t-1}^{k*} + \varepsilon_t^{k*}$	(D.106)
Monetary policy shock	$\xi_t^m = \varepsilon_t^m$	(D.107)
COVID labor supply A	$\varepsilon_t^h \sim \mathcal{N}(0, \sigma_h^2 \mathbb{1}_{t \in \text{COVID}})$	(D.108)
COVID labor supply B	$\varepsilon_t^{h*} \sim \mathcal{N}(0, \sigma_h^2 \mathbb{1}_{t \in \text{COVID}})$	(D.109)
COVID forced savings A	$\tilde{\varepsilon}_t^{tC} \sim \mathcal{N}(0, \sigma_{tC}^2 \mathbb{1}_{t \in \text{COVID}})$	(D.110)
COVID forced savings B	$\tilde{\varepsilon}_t^{tC*} \sim \mathcal{N}(0, \sigma_{tC}^2 \mathbb{1}_{t \in \text{COVID}})$	(D.111)

where $\tilde{\varepsilon}_t^{tC} \equiv \varepsilon_t^{tC} / \overline{C}$ is the forced-savings wedge normalized to steady-state consumption, following Cardani et al. (2022). The scales σ_h and σ_{tC} are shared across countries.

Fiscal policy shock AR(1) processes

Funded transfer disturbance A	$\zeta_t = \rho_\zeta \zeta_{t-1} + \varepsilon_t^\zeta$	(D.112)
Funded transfer disturbance B	$\zeta_t^* = \rho_\zeta^* \zeta_{t-1}^* + \varepsilon_t^{\zeta*}$	(D.113)
Unfunded (debt and inflation targets) A	$\varepsilon_{A,t}^U \sim \mathcal{N}(0,\ \sigma_{u,A}^2)$	(D.114)
Unfunded (debt and inflation targets) B	$\varepsilon_{B,t}^U \sim \mathcal{N}(0,\ \sigma_{u,B}^2)$	(D.115)

D.0.1 Steady state

We characterize the steady state around which we log-linearize. We impose zero steady-state inflation ($\overline{\Pi} = 1$) and normalize the steady-state price level as $\overline{P} = 1$. The steady-state gross nominal interest rate is $\overline{R} = \beta^{-1}$.

Long-term bond price. With geometrically decaying coupons with rate ρ^L, the steady-state long-term bond price satisfies

$$\overline{P}^L = \beta\left(1 + \rho^L \overline{P}^L\right) \quad \Longrightarrow \quad \overline{P}^L = \frac{\beta}{1 - \beta\rho^L}. \tag{D.116}$$

Defining the real market value of government debt as $\overline{s}_b \equiv \frac{\overline{P}^L \overline{B}}{\overline{P}\,\overline{Y}}$, the steady-state government budget constraint implies the primary-surplus requirement

$$\overline{T} = \frac{1-\beta}{\beta}\overline{s}_b \quad \Longleftrightarrow \quad \overline{s}_b = \frac{\beta}{1-\beta}\overline{T}. \tag{D.117}$$

Since Z denotes transfers to households, its steady-state output share is the residual

$$\frac{\overline{Z}}{\overline{Y}} = \overline{\tau}_K \frac{\overline{R}_K \overline{K}}{\overline{Y}} + \overline{\tau}_L \frac{\overline{W}^{real}\overline{L}}{\overline{Y}} + \overline{\tau}_C \frac{\overline{C}}{\overline{Y}} - \frac{\overline{G}}{\overline{Y}} - \frac{1-\beta}{\beta}\overline{s}_b. \tag{D.118}$$

Capital and investment shares. In steady state Tobin's Q is one, and the capital Euler equation implies

$$1 = \beta\left((1 - \overline{\tau}_K)\overline{R}_K + 1 - \delta\right) \quad \Longrightarrow \quad \overline{R}_K = \frac{\beta^{-1} - (1-\delta)}{1 - \overline{\tau}_K}. \tag{D.119}$$

Using the tradable-sector capital demand condition $\overline{R}_K = \alpha\overline{MC}_T \frac{\overline{Y}_T}{\overline{K}}$ and the normalization $\overline{MC}_T = 1$, we obtain

$$\frac{\overline{K}}{\overline{Y}_T} = \frac{\alpha\overline{MC}_T}{\overline{R}_K}. \tag{D.120}$$

The capital law of motion implies $\overline{I} = \delta\overline{K}$. Let $\omega_I^Y \equiv \overline{I}/\overline{Y}$, $\omega_G^Y \equiv \overline{G}/\overline{Y}$, and $\omega_C \equiv 1 - \omega_I^Y - \omega_G^Y$. Because investment and government purchases use tradables while nontradable output clears nontradable consumption,

$$\omega_N = \alpha_N \omega_C, \qquad \omega_T = (1-\alpha_N)\omega_C + \omega_I^Y + \omega_G^Y = 1 - \omega_N. \tag{D.121}$$

Combining these identities with $\overline{K}/\overline{Y} = \alpha\omega_T/\overline{R}_K$ gives

$$\omega_T = \frac{1 - \alpha_N + \alpha_N\omega_G^Y}{1 - \alpha_N\delta\alpha/\overline{R}_K}, \qquad \omega_I^Y = \delta\frac{\alpha\omega_T}{\overline{R}_K}. \tag{D.122}$$

The tradable-resource coefficients are $\omega_I = \omega_I^Y/\omega_T$, $\omega_G = \omega_G^Y/\omega_T$, and $\omega_u = a'(1)\overline{K}/\overline{Y}_T = (1 - \overline{\tau}_K)\alpha$.

Government spending share. We calibrate $\omega_G^Y = \overline{G}/\overline{Y} = 0.20$. The coefficient entering tradable market clearing is therefore $\omega_G = \omega_G^Y/\omega_T$.