Foreign Exchange Intervention under Dominant-Currency Settlement: A New-Monetarist Approach

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Abstract

Taiwan invoices 95 percent of its exports in U.S. dollars, keeps 97 percent of export receipts in foreign currency, and buys foreign exchange worth 4.7 percent of GDP a year. In a two-country monetary search model with flexible prices and international trade settled, not only invoiced, in dollars, reserve purchases revalue dollar claims and shift purchasing power from local-currency earners to dollar earners at home and abroad. Anticipated intervention expands dollar-earning sectors and contracts the intervener's domestic sector; a covert one passes through only in proportion to the uninformed party's bargaining share. It is beggar-thyself, not beggar-thy-neighbor, invites no competitive devaluation under random matching, and works through the reserve leg, not money creation; optimal intervention is second-best provision of dollar liquidity by a government that cannot issue it. Taiwan's observed operations imply a welfare weight of about 0.75 on the tradable sector and a gain below 0.1 percent, so intervention is mostly distributional.

JEL Classification: E42, E50, E52, E58, F31, F41
Keywords: exchange rate intervention, international reserves, monetary policy pass-through, competitive devaluation

1 Introduction

Foreign exchange intervention (FXI) is widely used in practice, especially by emerging and small open economies, yet its real effects remain theoretically unsettled. Recent empirical work shows that FXI is pervasive, often effective at smoothing exchange rates, and disproportionately used through foreign-exchange purchases in spot markets, particularly by emerging economies (Fratzscher et al., 2019). At the same time, a large share of world trade

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is invoiced in a dominant currency like the U.S. dollar rather than in the producer's or importer's currency (Gopinath et al., 2020). Together these two facts pose a puzzle, because dominant-currency invoicing limits the pass-through of exchange-rate movements to export prices. Through what mechanism, then, can FXI still generate real effects?

In the conventional view, exchange-rate movements generate real effects because of nominal rigidity. In sticky-price open-economy models of the New Keynesian tradition, prices are preset in nominal terms, and the expenditure switching induced by exchange-rate changes is the central transmission channel. Under the dominant currency paradigm (DCP), documented and formalized by Gopinath et al. (2010) and Gopinath et al. (2020), export prices are sticky in dollars, so a bilateral depreciation against the dollar barely changes the relative price faced by foreign customers, exports respond weakly in the short run, and the expansionary effect of a depreciation operates mainly through expenditure shifting from imported goods to domestic goods.

This paper proposes a complementary mechanism that does not require nominal rigidity. I develop a two-country monetary search model in the spirit of Lagos and Wright (2005) and Rocheteau and Wright (2005), in which domestic transactions are settled in local currency while international transactions are settled in the dominant currency, and all prices are flexible, with terms of trade set by proportional Kalai bargaining (Kalai, 1977), although under symmetric information the mechanism needs neither search nor bargaining (Online Appendix G). In this environment, the dominant currency matters not because prices are preset in it, but because it is the asset with which international transactions must be settled, a role absent from the New Keynesian models of the dominant currency paradigm, in which trade requires no settlement balances and the dollar balances an intervention withdraws play no part. Because dollar trade prices are sticky within a year (Gopinath and Rigobon, 2008), the settlement channel complements the sticky-price channel and governs the incidence of sustained intervention once prices have reset (Section 2).

FXI then works by changing the relative purchasing power of the two currencies. An intervention that depreciates the local currency lowers the real value of local-currency claims and raises the real value of dominant-currency claims. The key implication is a wealth redistribution effect, whereby purchasing power moves away from agents who are paid in local currency and toward agents who receive dominant-currency income. Because tradable-sector producers earn dominant currency while domestic-sector producers earn local currency, the intervention alters sectoral incentives and generates real effects even under fully flexible prices.

This mechanism also provides a straightforward policy implication. A country that cannot issue the dominant currency cannot directly relax the liquidity constraint that limits its

international trade; what its government does control is the supply of its own currency, and a reserve purchase uses that margin to act on the other. Withdrawing dominant currency from private circulation raises the purchasing power of the dominant-currency balances that importers already hold, while the local currency issued to pay for the reserves dilutes the balances that finance domestic trade. FXI is thus a second-best provision of dominant-currency liquidity by a government that cannot issue that currency.

The model rests on three ingredients. First, international trade is invoiced predominantly in a single dominant currency (Gopinath et al., 2020). Second, official intervention is pervasive and concentrated in spot purchases of foreign reserves (Fratzscher et al., 2019). Third, cross-border payments require dominant-currency balances, so importers choose their currency portfolios in anticipation of international transactions, a liquidity-based portfolio choice in the spirit of Geromichalos and Simonovska (2014).[1]

The paper delivers five main results. First, FXI generates a wealth redistribution from local-currency holders in the intervening country to dominant-currency holders both at home and abroad. When the intervention arrives as a pure shock, output does not move on impact, because production decisions predate the intervention, but purchasing power is redistributed immediately: agents holding the depreciating local currency lose, while dominant-currency holders, including the intervening country's own exporters, gain.

Second, when the intervention is anticipated, the redistribution feeds back into production decisions at the announcement date. Producers who expect to be paid in the dominant currency expand output, while those who expect to be paid in the depreciating local currency contract, so dominant-currency-earning sectors expand and the intervening country's domestic sector shrinks.

Third, when only the intervening country's agents foresee the intervention, the production response is governed by a *belief-weighted pass-through* rule, in which the terms of trade in a bilateral match weight each party's belief by the counterparty's bargaining share, so that quantities adjust fully only where the intervention is common knowledge inside the match. The intervener's domestic sector still contracts fully, its exports expand only in proportion to the foreign customers' bargaining share $1-\theta$, and informed exporters capture part of the revaluation of the dominant currency as an information rent, scaled by their own share θ.

Fourth, under random matching the intervention creates no incentive for competitive devaluation. In an extension with a third peripheral country, that country's agents weakly gain and its export volume does not fall at impact, because a public intervention changes no relative price among exporters that settle in the dominant currency, and a covert one, though it improves the terms that the intervener's informed exporters offer, cannot draw

[1] Section 2 documents all three ingredients for Taiwan.

buyers away from other exporters when matching is random. The systematic cost of the policy falls on the intervener's own domestic sector.

Fifth, replacing FXI with a one-time money injection of equal size, without the reserve purchase, eliminates the gains to dominant-currency holders, which identifies the reserve leg as the operative component of the policy.

Two extensions show how far these results reach. Reinvesting the purchased dollars in dollar bonds scales the reserve leg by the bonds' liquidity while the local-currency issuance stays at full size (Proposition 7). When exporters choose the settlement currency and settling in a foreign currency is costly, dominant-currency settlement can be a self-reinforcing equilibrium, coexisting with settlement in each exporter's own currency, in which an anticipated intervention raises the dominant currency's use at impact as well as its value (Proposition 8); at Taiwan's scale the added effect is negligible.

The optimal-policy analysis builds on these results. Intervention raises the government's objective only when the weight ω on the tradable sector exceeds a threshold that falls as dominant-currency liquidity becomes scarcer, and the optimal size is nondecreasing in ω up to the point where the export-side dominant-currency liquidity constraint ceases to bind or the counterparties' dominant-currency balances are exhausted. In Taiwan's calibration a utilitarian government never intervenes at the baseline, and inverting the optimal-size rule at the observed scale of operations yields a weight of about 0.75, and slightly more when reserves are held in highly liquid bonds. The estimate is indicative rather than sharp but consistent with a mercantilist tilt. At that weight the calibrated welfare gain is less than 0.1 percent, so the analysis characterizes a policy whose main consequences are distributional.

These predictions differ from the sticky-price benchmark in three ways, each with a policy implication. The real effects come from the redistribution of purchasing power between currencies rather than from preset prices and expenditure switching, and because a public intervention raises the purchasing power of the dominant currency for every holder rather than the relative price of the intervener's goods, no other exporter's volume falls at impact under random matching, so the retaliation logic behind competitive devaluation loses its footing. The cross-country impact has the opposite sign, since foreign producers who earn the dominant currency share in the gains, the intervener's imports expand rather than contract at the announcement date, and the systematic loser is the intervener's own domestic sector; the comparison with a pure money injection attributes these effects to the reserve leg, which offers an explanation for why central banks intervene through spot reserve purchases rather than through money creation alone. And the full quantity response requires that the intervention be common knowledge inside each match, a condition that informed exporters have no incentive to create, since under voluntary disclosure of a verifiable announcement

they prefer to conceal (Remark 1).

Literature review. This paper relates to four strands of literature. The first is the literature on exchange-rate regimes and FXI. Friedman (1953) argued for flexible exchange rates as a mechanism for relative-price adjustment, while Johnson (1969) emphasized that smaller and less developed economies may prefer exchange-rate arrangements tied to major trading partners. More recent work studies FXI as a policy instrument in environments with financial frictions. Fanelli and Straub (2021) show that FXI can be an optimal stabilization tool when asset-market segmentation generates a pecuniary externality, Amador et al. (2020) study exchange-rate policy at the zero lower bound, and Hassan et al. (2023) develop a risk-based theory of stabilization against anchor currencies. Relative to this literature, my paper studies FXI through the redistribution induced by changes in the real return to currencies that are used asymmetrically in domestic and international transactions, rather than through segmented financial markets or asset-pricing risk.

The second strand is the literature on dominant currency pricing, invoicing, and pass-through. Gopinath et al. (2010) show that the invoicing currency is central for exchange-rate pass-through, Gopinath et al. (2020) formalize the dominant currency paradigm, Amiti et al. (2022) study how firms choose invoicing currencies, Egorov and Mukhin (2023) characterize optimal monetary policy under dollar pricing, and Gopinath and Stein (2021) tie dominant-currency status to the dollar's broader financial role. My paper complements the DCP view with a transmission mechanism that operates when the dominant currency is also the settlement asset of international trade.

The third strand, connected to this emphasis on settlement rather than invoicing alone, is the literature on reserve accumulation and international liquidity. Aizenman and Marion (2004) emphasize the role of reserves in the presence of sovereign risk and costly taxation, Bacchetta et al. (2013) show how reserve accumulation can interact with capital controls in fast-growing economies with constrained households, and Arce et al. (2019) interpret reserves as a macroprudential tool when private agents fail to internalize future borrowing constraints. My paper is complementary to this literature, since reserve purchases appear in the model as the operational counterpart of FXI and the object of interest is how they generate real effects by altering the scarcity and purchasing power of the dominant currency in circulation, not the optimal stock of reserves.

The fourth strand is the monetary-search literature on the use and circulation of currencies and liquid assets. Lagos and Wright (2005) and Rocheteau and Wright (2005) provide the basic framework used here, and Zhang (2014) studies how international currency status can arise from informational advantages. Gomis-Porqueras et al. (2017) endogenize currency

acceptance under the threat of costly counterfeiting, and show that this threat, through the liquidity constraints it induces, can pin down the nominal exchange rate even when the two currencies are perfect substitutes. Geromichalos and Simonovska (2014) derive international portfolio choice from the liquidity of assets in decentralized trade. I take the currency of settlement in each match type as given, until Section 5.8 lets exporters choose it, and embed DCP and FXI into a monetary-search environment in which the dominant currency is held because international matches must be settled in it.

Relative to these literatures, the paper makes four contributions. First, it identifies a transmission mechanism for FXI, the revaluation of settlement balances, that requires no nominal rigidity and makes intervention beggar-thyself rather than beggar-thy-neighbor, and documents its premise for Taiwan (Section 2). Second, it characterizes covert intervention and its information rents through the belief-weighted pass-through rule of Lemma 1. Third, it separates the reserve leg from the money creation that finances it (Proposition 6) and extends the separation to reserves reinvested in partially liquid dominant-currency bonds (Proposition 7). Fourth, it characterizes optimal intervention as second-best provision of dominant-currency liquidity, a shadow Friedman rule in the mercantilist limit (Proposition 9), and backs out from the observed scale of operations the welfare weight that rationalizes it.

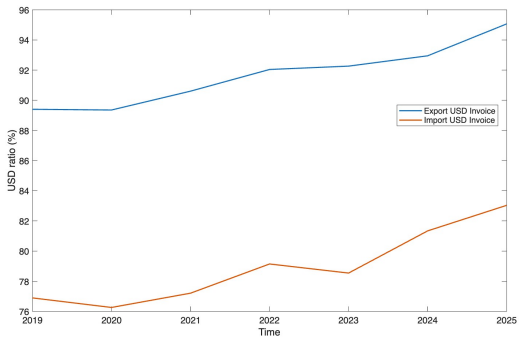
The remainder of the paper proceeds as follows. Section 2 documents the motivating evidence from Taiwan. Section 3 lays out the environment. Section 4 characterizes the monetary equilibrium. Section 5 studies the effects of FXI. Section 6 characterizes the optimal size of an anticipated intervention. Section 7 concludes. Online Appendix A characterizes the fixed-injection regime, Online Appendix B examines robustness to the bargaining protocol, Online Appendix C covers alternative instruments and reversal, Online Appendix D develops the bond extension, Online Appendix E sets out endogenous dominant-currency settlement, Online Appendix F details how bargaining power governs pass-through, and Online Appendix G turns to price taking; all remaining proofs are in Online Appendix H.

2 Motivating evidence: dominant-currency settlement in Taiwan

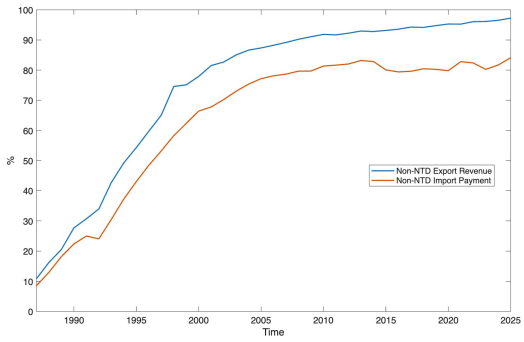
Before turning to the model, this section documents the three ingredients for Taiwan, an economy that combines heavy dollar reliance in trade with one of the world's largest reserve positions. The facts below establish that the environment the model formalizes describes a prototypical intervener.

Invoicing. Figure 1(a) reports the U.S. dollar share of Taiwan's trade by invoicing currency, computed from customs declarations. Between 2019 and 2025 the dollar share rose from 89.4% to 95.1% for exports and from 76.9% to 83.0% for imports, with the local currency playing a negligible role on either side. These numbers place Taiwan at the dollar-intensive end of the invoicing distribution documented by Gopinath et al. (2020) and support the model's first ingredient, that international trade is invoiced in a single dominant currency in both directions of trade.

Settlement. Figure 1(b) reports the currency in which cross-border payments are made and kept. The Central Bank of the Republic of China (Taiwan), henceforth CBC, decomposes exporters' foreign-exchange receipts into the part sold to banks against New Taiwan dollars and the part retained in foreign currency, and symmetrically for importers' payments. In 2025, 97.2% of export receipts were *not* converted into the local currency, and 84.1% of import payments were made out of foreign-currency balances rather than purchased in the spot market at the time of payment. Firms therefore hold dominant-currency balances and settle trade with them directly. This is the model's third ingredient, the portfolio behavior of firms in the tradable sector.



(a) U.S. dollar share of trade invoicing



(b) Trade settled without NTD conversion

Figure 1: Dominant-currency invoicing and settlement in Taiwan. Panel (a): share of exports and imports invoiced in U.S. dollars, from customs declarations (Customs Administration, Ministry of Finance), 2019–2025. Panel (b): share of export receipts not sold for New Taiwan dollars and share of import payments not purchased with New Taiwan dollars (Central Bank of the Republic of China (Taiwan), table EG46Y01), 1987–2025.

Intervention. Figure 2 turns to the policy, using the reserve-based spot-intervention proxies of Adler et al. (2025). Taiwan intervened persistently over 2000–2025: depreciation-side operations, purchases of foreign currency, occur in 56% of months; the average absolute operation is 0.61% of GDP per month; and gross depreciation-side purchases average 4.7% of

GDP per year over the complete years 2000–2024. This pattern fits the model's second ingredient, that intervention is concentrated in the spot market. Depreciation-side operations are therefore the main focus of this paper.

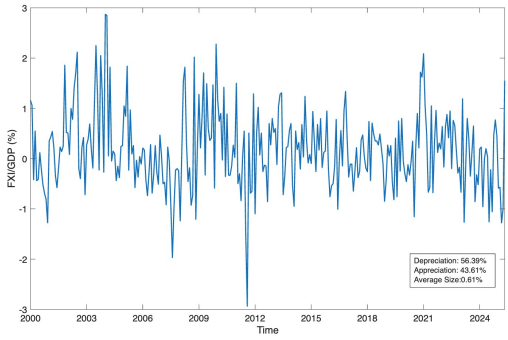


Figure 2: Foreign exchange intervention in Taiwan. Estimated foreign exchange intervention, monthly, 2000–2025: spot-intervention proxy as a share of GDP, from the dataset of Adler et al. (2025); positive values are purchases of foreign currency (depreciation-side operations).

Prices and official portfolios. Two further considerations discipline the modeling choices. First, dollar trade prices are sticky at high frequency. In U.S. micro data, the median price spell in the currency of pricing lasts 10.6 months for imports and 12.8 months for exports (Gopinath and Rigobon, 2008), so nominal rigidity governs how trade responds to exchange-rate movements within about a year. Taiwan's intervention, by contrast, has been sustained for decades, and the reserves it accumulates are persistent, so its incidence plays out over horizons at which dollar trade prices have reset. The model abstracts from nominal rigidity to isolate the channel that operates at those horizons, the revaluation of settlement balances, and its period is one year, about the length of a typical price spell. The abstraction is not innocuous at shorter horizons. If dollar prices were fixed over the period of the operation, the withdrawal would lower real dollar balances instead of raising their purchasing power and would tighten rather than relax the liquidity constraints in dollar-settled trade until prices reset. The redistribution and the sectoral responses characterized in Section 5 therefore describe the economy once prices have adjusted, and they complement rather than contradict the short-run predictions of sticky-price models. Second, the dollar's role in official portfolios mirrors its role in private settlement, accounting for about 57% of allocated global reserves in 2025 (IMF COFER), held overwhelmingly in safe interest-bearing claims rather than currency, the case covered by the bond extension of Section 5.7.

3 Model

The environment is based on Lagos and Wright (2005) with a two-country setup as in Zhang (2014). This model keeps currency portfolios analytically tractable, a characteristic which is important when it comes to the redistribution effect of FXI. Time is discrete and indexed by $t = 0, 1, 2, \ldots$ Each period has two stages. Stage 1 is a decentralized market (DM) where buyers meet sellers bilaterally and trade DM goods. Stage 2 is a centralized market (CM) where agents produce and consume CM goods (numeraire), and rebalance their currency portfolios. All goods are perishable within stage. Currencies are perfectly durable fiat objects. The discount factor across periods is $\beta \in (0, 1)$.

There are two countries denoted by $i \in \{H, F\}$, which stand for home country and foreign country. The countries are symmetric in preferences, technologies, and meeting probabilities for simplicity. Each country is populated by a unit mass of buyers and a unit mass of sellers. Types are fixed over time. Buyers consume DM goods but cannot produce them. Sellers can produce DM goods but do not value them. All agents can consume and produce the CM goods. Producing h units of CM goods costs h units of utility.

To motivate the use of currency, I assume there is no perfect record of individuals' trading history as in Kocherlakota (1998), nor is there perfect enforcement of debt. Hence, one must pay upfront with assets, which is currency, to facilitate transactions. While local currency is used in domestic trade, I assume that international trade is settled in currency F.[2] This is an ad-hoc assumption, relaxed in Section 5.8, that lets me focus on the policy effect of FXI. Empirically, this assumption fits the DCP literature such as Gopinath et al. (2020). Theoretically, accepting non-local currency generates an extra cost, and the use of the currency F as international currency is an endogenous outcome if the cost is low enough, as shown in Zhang (2014). Gomis-Porqueras et al. (2017) likewise endogenize acceptance through the threat of counterfeiting; here each currency is instead priced by its own transaction demand.

There is a government in each country and the government is the sole issuer of local currency. Both currencies are recognizable and cannot be counterfeited. The evolution of currency is $M_t^i = (1+\mu_t^i)M_{t-1}^i$, where M_t^i denotes the nominal stock of country i currency and growth rate μ_t^i in period t. To keep money circulating in the economy, I assume $(1+\mu_t^i) \geq \beta$. The government can give a nominal transfer τ_{it} at the beginning of stage 2. Meanwhile, the government can conduct FXI by issuing local currency to exchange for international currency ΔX_t at a desired nominal exchange rate e_t with its citizens at the beginning of stage 2. Since country F is the issuer of the international currency, only country H would conduct FXI.

[2]The international currency can in principle be issued by a third country. Assuming that country F is the international currency issuer makes it possible to study the effect of FXI on the issuer of the targeted currency; Section 5.5 introduces a third peripheral country while keeping country F as the issuer.

Each government needs to satisfy its period budget constraint.

$$\text{(Government H) } M_t^H - M_{t-1}^H = \tau_{Ht} + e_t \Delta X_t \tag{1}$$

$$\text{(Government F) } M_t^F - M_{t-1}^F = \tau_{Ft} \tag{2}$$

where $\tau_{it} \equiv \sum_j \tau_{ijt}$ denotes the aggregate nominal lump-sum transfer to country i's citizens and τ_{ijt} is the transfer received by an agent of type j.

A buyer's period utility is

$$u(q) + x - h, \tag{3}$$

when the buyer consumes q units of DM goods, consumes x units of the CM goods, and produces h units of CM goods. Similarly, a seller's period utility is

$$-c(q) + x - h, \tag{4}$$

when the seller supplies q units of a DM good. The functions $u(q)$ and $c(q)$ satisfy

$$u(0) = c(0) = 0, \qquad u' > 0, \quad u'' < 0, \quad c' > 0, \quad c'' \geq 0, \quad u'(0) = \infty. \tag{5}$$

Let $\overline{q}$ denote the efficient DM quantity, defined by

$$u'(\overline{q}) = c'(\overline{q}). \tag{6}$$

In stage 1, a buyer in country $i \in \{H, F\}$ has probability $\delta \in (0, 1)$ of meeting a domestic seller and hence the counterparties are referred to as the domestic buyer and seller in country i. Similarly, the buyer has probability $1-\delta$ of meeting a foreign seller, and the counterparties are then referred to as the international buyer (seller) in country i (j). When a buyer and a seller meet, the terms of trade, which specify the quantity q and the means of payment d, are determined by proportional (Kalai) bargaining (Kalai, 1977), under which the seller receives a share $\theta \in (0, 1)$ of the match surplus and the buyer receives the remaining share $1 - \theta$, subject to the buyer's liquidity constraint. The parameter θ measures the seller's bargaining power. If no agreement is reached, nothing happens and both proceed to stage 2 directly.[3]

The timing of events is summarized as follows. At the beginning of stage 1 of period t, each buyer in both countries meets a specific seller with the probabilities above. Then the terms of trade are determined by proportional bargaining and the transaction occurs. At the

[3] Aruoba et al. (2007) show that bargaining protocol affects the equilibrium outcome in this class of models. In Online Appendix B, I analyze the same policy when buyers offer a take-it-or-leave-it deal. The main results remain, except that the covert-intervention export response vanishes in the buyer-offer limit, as discussed there.

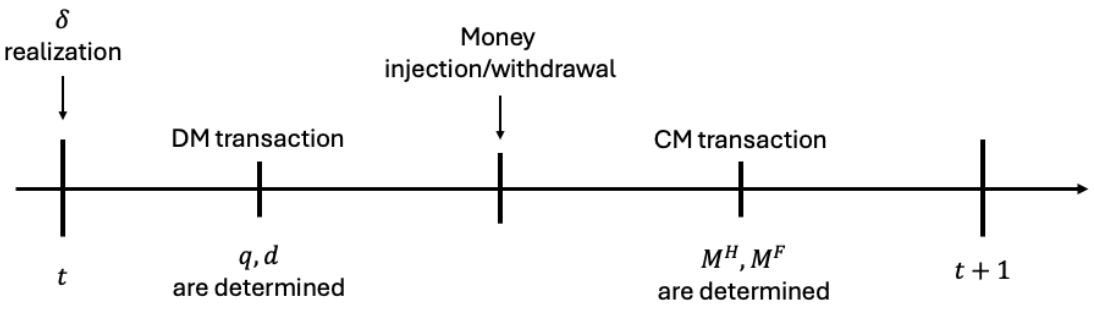


Figure 3: Timeline

beginning of stage 2 of period t, each government can inject or withdraw local currency and country H can implement FXI. Finally, agents readjust their currency holdings by consuming or producing CM goods for the next period. Figure 3 illustrates the sequence of events. Also, for notational ease, variables corresponding to the next period are denoted by subscript $+1$ for the rest of the paper.

4 Equilibrium

4.1 Stage 2 problem

I start with the currency portfolio choice in stage 2. Assume the agent $j \in \{B, S\}$, which stands for buyer and seller, in country i holds the currency portfolio (M_{ij}^H, M_{ij}^F), which represents currency H and F holdings, at the beginning of stage 2. The lifetime value function for this agent is

$$W_{ij}(M_{ij}^H, M_{ij}^F) = \max_{M_{ij,+1}^H, M_{ij,+1}^F} x - h + \beta V_{ij}(M_{ij,+1}^H, M_{ij,+1}^F)$$
$$\text{s.t.} \quad x = h + \phi^H(M_{ij}^H - M_{ij,+1}^H) + \phi^F(M_{ij}^F - M_{ij,+1}^F) + \phi^i \tau_{ij}$$

where ϕ^i is the price of currency i in terms of CM goods and V_{ij} is the value function in stage 1. Notice that stage 2 is a centralized market and therefore the law of one price holds. As a result, the nominal exchange rate is defined as $e = \frac{\phi^F}{\phi^H}$, which is the amount of currency H one needs to exchange for one unit of currency F.

After substituting the budget constraint into the value function, I rewrite the stage 2

problem as follows.

$$W_{ij}(M_{ij}^H, M_{ij}^F) = \phi^H M_{ij}^H + \phi^F M_{ij}^F + \underbrace{\max_{M_{ij,+1}^H, M_{ij,+1}^F} \left[\phi^i \tau_{ij} - \phi^H M_{ij,+1}^H - \phi^F M_{ij,+1}^F + \beta V_{ij}(M_{ij,+1}^H, M_{ij,+1}^F)\right]}_{\equiv W_{ij}(0,0)} \tag{7}$$

Here, the future portfolio decision is independent of the current portfolio. The first order conditions are

$$\frac{\partial V_{ij}(M_{ij,+1}^H, M_{ij,+1}^F)}{\partial M_{ij,+1}^k} \leq \frac{\phi^k}{\beta}, \text{ with equality if } M_{ij,+1}^k > 0, \ k \in \{H, F\}. \tag{8}$$

The left hand side is the marginal benefit of holding an additional unit of currency k while the right hand side is the marginal cost of holding an additional unit of currency k. It is possible that the agent decides not to hold any currency if the marginal cost is always higher.

4.2 Stage 1 problem - seller

Assume a seller in country i holds the currency portfolio (M_{iS}^H, M_{iS}^F) at the beginning of stage 1. The lifetime value function for this agent is

$$V_{iS}(M_{iS}^H, M_{iS}^F) = \delta V_{iS}^N(M_{iS}^H, M_{iS}^F) + (1-\delta) V_{iS}^T(M_{iS}^H, M_{iS}^F)$$

where V_{iS}^N, V_{iS}^T stand for the scenarios of matching with a domestic buyer and an international buyer respectively. In each match, the terms of trade (q, d) are determined by proportional bargaining. Using the linearity of W established above, the value to a country-H seller of a deal (q, d) is

$$\begin{aligned} V_{HS}^N(M_{HS}^H, M_{HS}^F) &= -c(q_{HH}) + W_{HS}(M_{HS}^H + d, M_{HS}^F); \\ V_{HS}^T(M_{HS}^H, M_{HS}^F) &= -c(q_{FH}) + W_{HS}(M_{HS}^H, M_{HS}^F + d) \end{aligned}$$

where q_{ij} means the DM goods consumed by a country i buyer and produced by a country j seller, so the buyer's surplus from a deal paid in currency k is $u(q) - \phi^k d$ and the seller's is $\phi^k d - c(q)$.[4] The proportional rule assigns the seller the share θ of the total match surplus

[4]Notice that the means of payment d generally differs across meeting types. With a slight abuse of notation, the same symbol is used in every meeting to keep the expressions concise.

$S(q) \equiv u(q) - c(q)$:

$$\phi^k d - c(q) = \theta\left[u(q) - c(q)\right] \quad \Longleftrightarrow \quad \phi^k d = z(q) \equiv \theta u(q) + (1-\theta)c(q), \tag{9}$$

where z is the *payment function*, the real payment that supports the quantity q. Since $z'(q) = \theta u'(q) + (1-\theta)c'(q) > 0$, the payment function is strictly increasing. For sellers in country H the rule applies with $k = H$ in domestic matches and $k = F$ in international matches; for sellers in country F both match types are settled in currency F, and V_{FS}^N and V_{FS}^T follow in a similar manner. With these conditions, the value function becomes

$$V_{iS}(M_{iS}^H, M_{iS}^F) = \theta\left[\delta\, S(q_{ii}) + (1-\delta)\, S(q_{ji})\right] + W_{iS}(M_{iS}^H, M_{iS}^F), \quad j \neq i.$$

The seller's DM surplus does not depend on her own currency holdings, because the traded quantity is pinned down by the buyer's liquidity and the sharing rule. The surplus terms therefore drop out of the envelope conditions, which are

$$\frac{\partial V_{iS}(M_{iS}^H, M_{iS}^F)}{\partial M_{iS}^k} = \frac{\partial W_{iS}(M_{iS}^H, M_{iS}^F)}{\partial M_{iS}^k} = \phi^k, \ k \in \{H, F\}.$$

Combined with (8), sellers hold a positive amount of currency iff

$$\frac{\partial V_{iS}(M_{iS,+1}^H, M_{iS,+1}^F)}{\partial M_{iS,+1}^k} = \phi_{+1}^k = \frac{\phi^k}{\beta} \Rightarrow 1 = \frac{\phi^k/\phi_{+1}^k}{\beta}. \tag{10}$$

Notice that $\frac{\phi^k}{\phi_{+1}^k} = \Pi_{+1}^k$ is the gross inflation rate in currency k. Therefore, if the inflation is sufficiently large such that $\frac{\Pi_{+1}^k}{\beta} > 1$, then the sellers hold zero currency across periods and spend all their revenue in stage 2. When $\Pi_{+1}^k = \beta$ (the Friedman rule), sellers are indifferent and I assume they carry zero holdings out of the CM without loss of generality.

4.3 Stage 1 problem - buyer

Assume a buyer in country i holds the currency portfolio (M_{iB}^H, M_{iB}^F) at the beginning of stage 1. The lifetime value function for this agent is

$$V_{iB}(M_{iB}^H, M_{iB}^F) = \delta V_{iB}^N(M_{iB}^H, M_{iB}^F) + (1-\delta)V_{iB}^T(M_{iB}^H, M_{iB}^F)$$

where V_{iB}^N, V_{iB}^T stand for the scenarios of matching with a domestic seller and an international seller respectively. In each match, the proportional rule (9) determines the terms of trade, so that the payment supporting a quantity q is $d = z(q)/\phi^k$ in the payment currency k and

the buyer receives the surplus share $(1-\theta)S(q)$, which is strictly increasing in q on $(0,\overline{q}]$. The quantity is therefore the largest one consistent with the buyer's liquidity constraint. For the buyers in country H, the determination of the terms can be formulated as below.

$$\max_{q_{HH}} u(q_{HH}) + W_{HB}\left(M_{HB}^H - \frac{z(q_{HH})}{\phi^H},\ M_{HB}^F\right) \text{ s.t. } M_{HB}^H \geq \frac{z(q_{HH})}{\phi^H};$$
$$\max_{q_{HF}} u(q_{HF}) + W_{HB}\left(M_{HB}^H,\ M_{HB}^F - \frac{z(q_{HF})}{\phi^F}\right) \text{ s.t. } M_{HB}^F \geq \frac{z(q_{HF})}{\phi^F}.$$

With the linearity result from stage 2, the problem becomes

$$\max_{q_{HH}} (1-\theta)\left[u(q_{HH}) - c(q_{HH})\right] + W_{HB}\left(M_{HB}^H,\ M_{HB}^F\right) \text{ s.t. } M_{HB}^H \geq \frac{z(q_{HH})}{\phi^H};$$
$$\max_{q_{HF}} (1-\theta)\left[u(q_{HF}) - c(q_{HF})\right] + W_{HB}\left(M_{HB}^H,\ M_{HB}^F\right) \text{ s.t. } M_{HB}^F \geq \frac{z(q_{HF})}{\phi^F}.$$

The first order conditions are

$$[q_{HH}]:\ (1-\theta)\frac{u'(q_{HH}) - c'(q_{HH})}{z'(q_{HH})} = \frac{\lambda_H^N}{\phi^H}; \quad [q_{HF}]:\ (1-\theta)\frac{u'(q_{HF}) - c'(q_{HF})}{z'(q_{HF})} = \frac{\lambda_H^T}{\phi^F}$$

where λ_H^N and λ_H^T are the Lagrange multipliers on the liquidity constraints. Similarly, the first order conditions for buyers in country F are

$$(1-\theta)\frac{u'(q_{FF}) - c'(q_{FF})}{z'(q_{FF})} = (1-\theta)\frac{u'(q_{FH}) - c'(q_{FH})}{z'(q_{FH})} = \frac{\lambda_F}{\phi^F}$$

Notice that there is only one Lagrange multiplier for country F's buyers because all their transactions are settled in currency F. The condition also implies $q_{FF} = q_{FH} = q_F$. With these conditions, the value functions become

$$V_{HB}(M_{HB}^H, M_{HB}^F) = \max_{q_{HH}, q_{HF}} (1-\theta)\left[\delta S(q_{HH}) + (1-\delta)S(q_{HF})\right] + W_{HB}(M_{HB}^H, M_{HB}^F), \quad (11)$$
$$V_{FB}(M_{FB}^H, M_{FB}^F) = \max_{q_F} (1-\theta)S(q_F) + W_{FB}(M_{FB}^H, M_{FB}^F), \quad (12)$$

subject to the liquidity constraints in stage 1. If a liquidity constraint is binding, then the corresponding currency is used up in the stage 1 meeting. If not, currency is carried into stage 2. Considering the case in which the liquidity constraints are not binding, the envelope conditions are

$$\frac{\partial V_{iB}(M_{iB}^H, M_{iB}^F)}{\partial M_{iB}^k} = \frac{\partial W_{iB}(M_{iB}^H, M_{iB}^F)}{\partial M_{iB}^k} = \phi^k,\ k \in \{H, F\}.$$

Combined with (8), buyers carry a positive amount of currency directly to the next CM iff

$$\frac{\partial V_{iB}(M^H_{iB,+1}, M^F_{iB,+1})}{\partial M^k_{iB,+1}} = \phi^k_{+1} = \frac{\phi^k}{\beta} \Rightarrow 1 = \frac{\phi^k/\phi^k_{+1}}{\beta} = \frac{\Pi^k_{+1}}{\beta}. \tag{13}$$

Again, this result implies that if the inflation is sufficiently large such that $\frac{\Pi^k_{+1}}{\beta} > 1$, then it is too costly for a buyer to carry currency to the next CM without additional incentives. Consequently, if the buyers hold a positive amount of currency, they spend all of their currency holdings if possible when the inflation is sufficiently large.

Now, consider the equilibrium outcome in which the liquidity constraints are binding and the buyers spend all their currency in each match if possible. Then production is

$$q_{HH} = z^{-1}(\phi^H M^H_{HB}),\ q_{HF} = z^{-1}(\phi^F M^F_{HB}),\ q_F = z^{-1}(\phi^F M^F_{FB})$$

and the buyers' value functions become

$$\begin{aligned} V_{HB}(M^H_{HB}, M^F_{HB}) &= (1-\theta)\left\{\delta S[z^{-1}(\phi^H M^H_{HB})] + (1-\delta)S[z^{-1}(\phi^F M^F_{HB})]\right\} \\ &\quad + \phi^H M^H_{HB} + \phi^F M^F_{HB} + W_{HB}(0,0), \\ V_{FB}(M^H_{FB}, M^F_{FB}) &= (1-\theta)S[z^{-1}(\phi^F M^F_{FB})] + \phi^F M^F_{FB} + \phi^H M^H_{FB} + W_{FB}(0,0). \end{aligned}$$

Here the buyer's payoff in a match is written as the surplus share $(1-\theta)S(q)$ plus the market value of the balances spent; the two sum to $u(q)$, since the payment $z(q)$ covers the seller's cost and the seller's surplus share. This expression gives the marginal benefit of holding additional currency to current period $\frac{\partial V_{iB}}{\partial M^j_{iB}}$. Through envelope theorem and (8), the first-order conditions for buyers in both countries are

$$\begin{aligned} \text{Country H: } \frac{\phi^H/\phi^H_{+1}}{\beta} &= 1 + \delta(1-\theta)\frac{u'(q_{HH,+1}) - c'(q_{HH,+1})}{z'(q_{HH,+1})} \\ \frac{\phi^F/\phi^F_{+1}}{\beta} &= 1 + (1-\delta)(1-\theta)\frac{u'(q_{HF,+1}) - c'(q_{HF,+1})}{z'(q_{HF,+1})} \\ \text{Country F: } \frac{\phi^H/\phi^H_{+1}}{\beta} &\geq 1 \\ \frac{\phi^F/\phi^F_{+1}}{\beta} &= 1 + (1-\theta)\frac{u'(q_{F,+1}) - c'(q_{F,+1})}{z'(q_{F,+1})} \end{aligned}$$

where $q_{ij,+1} = z^{-1}(\phi^k_{+1} M^k_{iB,+1})$ in the corresponding payment currency k. The term following 1 on the right-hand side is the **liquidity premium**, which captures the value of money over

and above its fundamental value.

The competitive equilibrium is defined as below.

Definition 1 (Competitive equilibrium)**.** *Given the paths of currency supply* $\{M_t^H,\ M_t^F\}$*, transfers* $\{\tau_{Ht},\ \tau_{Ft}\}$*, and the FXI path* $\{\Delta X_t\}$*, a competitive equilibrium is a path of money prices* $\{\phi_t^H,\ \phi_t^F\}$ *such that all agents optimize, the terms of trade in every DM match satisfy the proportional rule (9), and the CM goods and money markets clear in stage 2.*

I start with the stationary equilibrium without FXI and treat it as a reference point. In a stationary equilibrium without FXI, the money growth rate is constant, which implies that

$$1+\mu_t^H = 1+\mu^H = \frac{\phi^H}{\phi_{+1}^H} \equiv \Pi^H,\ 1+\mu_t^F = 1+\mu^F = \frac{\phi^F}{\phi_{+1}^F} \equiv \Pi^F,\quad \text{for all } t.$$

Furthermore, I focus on the real money balance holding decision in stage 2 instead of nominal holdings and denote $m_{ij,+1}^k = \phi_{+1}^k M_{ij,+1}^k$ as the real money balance holding of currency k for agent j in country i.

Under the assumption that $1+\mu^i \geq \beta$, sellers in both countries hold zero currency and spend all stage 1 earnings in stage 2. Also, buyers in country F hold zero currency H, and buyers in both countries spend all currency in stage 1 if possible; exactly at $1+\mu^i = \beta$ agents are indifferent, and I select the zero-holding portfolio. The stationary equilibrium can be characterized by the following equations, in which $\Pi_{+1}^k = \Pi^k$ for $k \in \{H, F\}$.

$$\frac{\Pi_{+1}^H}{\beta} = 1+\delta\,\ell(q_{HH,+1}) \tag{14}$$

$$\frac{\Pi_{+1}^F}{\beta} = 1+(1-\delta)\,\ell(q_{HF,+1}) \tag{15}$$

$$\frac{\Pi_{+1}^F}{\beta} = 1+\ell(q_{F,+1}) \tag{16}$$

$$q_{HH,+1} = z^{-1}(m_{HB,+1}^H),\ q_{HF,+1} = z^{-1}(m_{HB,+1}^F), \tag{17}$$

$$q_{F,+1} = z^{-1}(m_{FB,+1}^F) = q_{FF,+1} = q_{FH,+1}. \tag{18}$$

Define the liquidity premium function and its derivative as

$$\ell(q) = (1-\theta)\,\frac{u'(q)-c'(q)}{z'(q)},\quad \ell'(q) = (1-\theta)\,\frac{u''(q)c'(q)-u'(q)c''(q)}{[z'(q)]^2} < 0, \tag{19}$$

where the derivative is strictly negative since $u'' < 0$, $c'' \geq 0$. Hence each of equations (14)–(16) pins down a unique quantity given the corresponding inflation rate, so the stationary monetary equilibrium is unique. Existence requires one qualification. Since $u'(0) = \infty$,

$\lim_{q\to 0} \ell(q) = (1-\theta)/\theta$ for $\theta > 0$. As a result, the premium is bounded and the money demand correspondence is empty if inflation is too high relative to the buyer's share. I therefore restrict attention to

Assumption 1 (Bounded premium)**.** $\theta < \bar{\theta} \equiv \left[1 + \max\left\{\frac{\Pi^H/\beta - 1}{\delta}, \frac{\Pi^F/\beta - 1}{1-\delta}\right\}\right]^{-1}$,

which guarantees an interior solution to each of (14)–(16); the condition for (16) is implied by that for (15). As $\theta \to 0$ the bound on the premium grows without limit and the restriction becomes vacuous.

For the dynamics after the intervention in Section 5, I impose one additional regularity condition, standard in the Lagos-Wright model, which guarantees that nominal money demand is well behaved.

Assumption 2 (Regularity)**.** *The function* $A(q) \equiv z(q)\left[1 + \ell(q)\right]$ *is strictly increasing on* $(0, \overline{q}]$.

Two consequences are used repeatedly below. First, for any $\alpha \in [0,1]$, $z(q)\left[1 + \alpha\ell(q)\right] = (1-\alpha)z(q) + \alpha A(q)$ is a convex combination of two strictly increasing functions and is therefore strictly increasing; the cases $\alpha \in \{\delta, 1-\delta, 1\}$ correspond to the three money-demand equations. Second, the identity $\ell(q)z(q) = A(q) - z(q)$ provides a convenient expression for the marginal value of relaxing a liquidity constraint.

Proposition 1 (Friedman rule)**.** *The Friedman rule, i.e.* $1 + \mu^i = \beta = \Pi^i$*, achieves the optimal allocation, in which* $q_{HH} = q_{HF} = q_F = \overline{q}$*. For country* F*, setting* $\mu^F = \beta - 1$ *is enough to make all currency-*F *trades efficient. However, setting* $\mu^H = \beta - 1$ *is insufficient for country* H *to achieve the optimal allocation since* μ^F *is controlled by the foreign government.*

Focusing on stationary equilibria in which the Friedman rule is infeasible yields the following proposition.[5]

Proposition 2 (The lower inflation, the better)**.** *When the Friedman rule is infeasible, the real money balance is decreasing in the inflation rate. Since the traded quantity is increasing in the corresponding real money holding, the quantity is decreasing in the inflation rate as well.*

5 Foreign exchange rate intervention (FXI)

In this section, I investigate the effect of FXI conducted by a non-dominant-currency issuer, which is country H in this paper. I now specify the details of this policy intervention.

[5] In practice, the Friedman rule is rarely implemented; see Andolfatto (2013) for an analysis of the incentive constraints that limit deflationary policy.

Implementation. The government must choose the direction of the intervention, the instrument, a counterparty, and the financing. I focus on depreciative FXI through open market operations based on the observation of Fratzscher et al. (2019). For the counterparty, I assume the government swaps only with international sellers in country H and the intervention is financed by issuing new currency rather than by taxation, which keeps the operation a central-bank tool.[6]

Specifically, the government chooses a target exchange rate, computes the equilibrium relationship between the size of the operation and the resulting exchange rate, and issues the amount of local currency $\Delta Y_t = \widehat{e}_t \Delta X_t$ required to purchase ΔX_t units of currency F and support the target rate $\widehat{e}$ at stage 2, trading with international sellers in country H. The operation raises the stock of currency H and reduces the stock of currency F in private circulation.[7]

Timing. The default assumption for the FXI implementation timing is at the beginning of stage 2, along with other money creation or withdrawal. Conceptually the operation could occur anytime within a period, but since only two events happen each period, the DM transaction and the CM transaction, there are effectively two options for the timing, before or after the DM transaction; Figure 3 shows the default.

I set the FXI after the DM transaction, because the focus of the analysis is on how sellers react to it. If the FXI happened before the DM transaction, sellers would already know the intervention when making their production decisions, and a one-shot operation would act like a currency swap between the government and country H's buyers,[8] moving quantities purely through the demand side of the liquidity constraints. That case, although interesting, aligns less with the argument that FXI boosts exporters' competitiveness in the international market.

Information. Since I assume the FXI occurs after the DM transactions have been completed, the intervention affects production decisions only if it is expected by agents in advance. Therefore, I investigate the effect of FXI under three different information structures

[6]The government could swap with the foreign government directly. For example, the FED established dollar liquidity swap lines with many central banks from December 2007 to February 2010 in order to provide liquidity globally.

[7]In reality, an intervening central bank can reinvest the proceeds in dominant-currency safe assets, so the currency itself returns to private circulation and what leaves private hands is an equal value of safe dominant-currency claims. The operation studied here should therefore be read as withdrawing dominant-currency *liquidity* broadly. Section 5.7 formalizes this interpretation with partially liquid dominant-currency bonds.

[8]For the discussion related to follow-up policy, please refer to Berentsen and Waller (2011) and Chang and Li (2018).

in order.

First, the FXI is implemented as a **pure shock**. That is, the FXI is beyond all agents' expectations. Naturally, DM production is unaffected by FXI. Nevertheless, such an intervention still has welfare implications in the CM, a situation which will be illustrated later.

The second case is the **anticipated** case, which is the total opposite of the first case. In this scenario, all agents know there will be an intervention. One can also take this case as government H announcing at the beginning of stage 1 that there will be an intervention at stage 2. Working backward from the impact of FXI at stage 2, sellers at stage 1 adjust their production decisions accordingly.

The two cases mentioned above are relatively extreme. In reality, it is reasonable to assume that domestic agents have better knowledge about domestic policy. Thus, the final case is the **asymmetric** case, in which only agents in country H know there will be an intervention at stage 2 while agents in country F are unaware of it. Such asymmetry creates heterogeneous policy effects of the same intervention across countries.

Welfare benchmark. Throughout, welfare effects at each date are evaluated relative to the original no-intervention equilibrium path. In the pure-shock case this coincides with comparing realizations to agents' ex-ante expectations; in the anticipated and asymmetric cases the two comparisons can differ, and I make the distinction explicit where it matters.

Inflation target. Other than the FXI itself, how the central banks manage inflation after the FXI also plays a role. I start from a stationary equilibrium in which the money growth rates are μ^H and μ^F, with $1+\mu^i > \beta$, and the nominal money stocks at the end of period -1 are M_{-1}^H and M_{-1}^F, and I consider central banks in both countries that act symmetrically under an **inflation-stabilizing** regime: once a central bank observes a change in its money stock, from the intervention or from any other shock, it adjusts its nominal injections so that its inflation rate stays at the stationary level. Since the intervention and the period-0 injections occur together at stage 2 of period 0, the earliest the central banks can respond is period 1, so the operation is a one-time level shift of both money stocks. Online Appendix A studies the alternative **fixed-injection** regime, under which each central bank commits to a predetermined path of nominal injections and inflation deviates from its stationary level for a time after the operation; Figure A.1 there compares the money supply paths under the two regimes.

Without loss of generality, I set the period of intervention $t_{FXI} = 0$ and denote all variables on the original stationary equilibrium path with a superscript $*$. Among the equilibria consistent with Definition 1, I restrict attention throughout to paths along which the econ-

omy converges back to the stationary monetary equilibrium associated with the long-run money growth rates. Finally, I restrict the size of the operation.

Assumption 3 (Small intervention)**.** *The operation* $(\Delta Y_0, \Delta X_0)$ *is small enough that (i)* ΔX_0 *does not exceed the currency-F receipts of country H's international sellers, at the beginning of stage 2 of period 0, and (ii) all stage-1 liquidity constraints continue to bind, at least weakly, at* $t = 0$ *in every information scenario considered below.*

Since all liquidity constraints bind strictly in the stationary equilibrium away from the Friedman rule, and equilibrium prices are continuous in the size of the operation, Assumption 3 holds for all sufficiently small interventions.

5.1 Price effects common to all information structures

The intervention changes both money stocks, and the change depends only on the operation $(\Delta Y_0, \Delta X_0)$ itself, not on who anticipates it. This subsection therefore derives the price effects and the induced wealth redistribution before the information structures are introduced.

The newly issued currency ΔY_0 is used to purchase the foreign currency ΔX_0, so $M_0^H = (1+\mu^H)M_{-1}^H + \Delta Y_0$ and $M_0^F = (1+\mu^F)M_{-1}^F - \Delta X_0$, and from period 1 on both central banks keep their inflation rates at the stationary level, so $M_t^k = (1+\mu^k)^t M_0^k$ for $t \geq 1$ and $k \in \{H, F\}$.

The buyers at stage 2 of $t = 0$, who observe the operation in the centralized market regardless of the information structure, expect inflation at its stationary level and choose their currency portfolios accordingly, which pins down the new price paths. The following proposition summarizes them.

Proposition 3 (Impact of the intervention)**.** *Suppose Assumptions 1, 2, and 3 hold, FXI is implemented at stage 2 of period 0, and the central banks are in the inflation-stabilizing regime. Then: (i) for all* $t \geq 1$, $\Pi_t^H = \Pi^{H*}$, $\Pi_t^F = \Pi^{F*}$, *and all DM quantities are at their stationary values; (ii) at* $t = 0$, *prices jump exactly in proportion to the change in the money stocks,*

$$\phi_0^H = \phi_0^{H*}\frac{M_0^{H*}}{M_0^H} < \phi_0^{H*}, \qquad \phi_0^F = \phi_0^{F*}\frac{M_0^{F*}}{M_0^F} > \phi_0^{F*},$$

so the exchange rate depreciates, $e_0 = e_0^*\,(M_0^H/M_0^{H*})(M_0^{F*}/M_0^F) > e_0^*$, *and purchasing power is redistributed from currency-H holders to currency-F holders.*

The impact effect is a pure level jump implied by the quantity theory of money. Each price moves exactly in proportion to the change in the corresponding money stock, and the depreciation is permanent in levels even though every real variable returns to its stationary

value from period 1 on; Lemma 2 in Section 6 writes the two ratios in terms of the size of the operation. The proposition characterizes the price system given the operation; the following corollary records that this characterization does not depend on who anticipates the intervention, which is what permits the present subsection to precede the analysis of the information structures.

Corollary 1 (Invariance to the information structure)**.** *Fix the operation* $(\Delta Y_0, \Delta X_0)$*, and suppose that the announcement of the intervention, if there is one, arrives no earlier than the beginning of stage 1 of period 0. Then under every information structure considered below, the equilibrium price paths* $\{\phi_t^H, \phi_t^F\}_{t\geq 0}$*, the exchange rate path, and the DM quantities for* $t \geq 1$ *are those implied by the operation under the prevailing monetary regime, namely those of Proposition 3 under the inflation-stabilizing regime and of Proposition A.1 under the fixed-injection regime of Online Appendix A. The information structure affects only the DM quantities at* $t = 0$ *and the distribution of the period-0 welfare impact.*

The timing specification in Corollary 1 matters. If the intervention were announced before the CM of period -1, buyers would rebalance their portfolios in anticipation, currency demands would shift already at $t = -1$, and the price paths would differ across information structures. All announcements considered in this paper arrive within period 0, after the last portfolio choice, so what the information structures below vary is who inside period 0 knows, not when money demand reacts.

The wealth redistribution effect. Proposition 3 gives $\phi_0^{H*} > \phi_0^H$ and $\phi_0^{F*} < \phi_0^F$. The change reflects the quantity theory of money: other things equal, a larger stock of a currency lowers its purchasing power. Thus, purchasing power is redistributed from currency-H holders to currency-F holders, which I call the **wealth redistribution effect**.

The swap counterparties of the FXI are the international sellers in country H. The swap itself takes place at the market exchange rate in the frictionless CM, so the sellers are indifferent between swapping with the government and selling their currency F for CM goods directly. However, they benefit because they happen to hold currency F at the very moment its purchasing power rises.

The fixed-injection alternative. If the central banks instead follow a money-growth rule, fixing their injection paths whatever happens, the operation also moves inflation for a time. Currency-H inflation falls below and currency-F inflation rises above its stationary level from period 1, and both converge back monotonically (Proposition A.1 in Online Appendix A). The impact prices move in the same directions as in Proposition 3, so the period-0

results below carry over, but a **transition effect** is added for $t \geq 1$: the temporarily higher currency-F inflation lowers the quantity traded in every currency-F match, which hurts all of country F's agents and country H's international buyers and sellers, while the temporarily lower currency-H inflation raises the quantity in country H's domestic matches (Table A.1). Since that effect could be replicated by engineering the same inflation paths, it is a feature of the regime rather than of the intervention, and I leave it to the appendix.

The shock effect. I call the impact at period $t = 0$ the **shock effect**. It is the component of the welfare impact that depends on the information structure, for it is determined by which agents foresee the operation when stage-1 decisions are made and by the nominal positions agents hold when the operation is executed. The next three subsections characterize it scenario by scenario.

Table 1 collects the results in a common format, decomposing each agent's period-0 welfare change into a *Trade* component, the change in the DM trade surplus ($u(q)$ for buyers, $-c(q)$ for sellers), and a *Reval.* component, the revaluation of nominal positions between the DM and the CM, which comes from the unspent balances for buyers and from the DM receipts for sellers. The net effect is the sum of the two components. The decomposition separates the two channels through which the shock effect operates, the quantity response and the redistribution, and makes visible which of the two each information structure activates. The Reval. entries are transfers between currency holders rather than new surplus, so any aggregate gain or loss must run through the Trade entries.

5.2 Pure-shock case

In the pure-shock case, the FXI is beyond all agents' expectations, so stage-1 decisions at $t = 0$ are made on the original price path, no DM quantity changes, and the Trade component of every agent's period-0 welfare is zero. The entire shock effect in this case comes from the revaluation of nominal positions in the CM.

Proposition 3 implies that currency-H holders are hurt as the purchasing power is lower than it would have been on the original equilibrium path, i.e. $\phi_0^H < \phi_0^{H*}$, and the loss is proportional to the balance held. As a result, country H's domestic sellers and country H's international buyers are hurt by FXI. The former receive currency H as the means of payment in the stage 1 transaction and are hurt because they obtain fewer CM goods than they expected before the FXI. Meanwhile, the latter do not spend their currency H in the stage 1 transaction and are hurt because they need to provide more CM goods, relative to other country H buyers, to rebalance their currency portfolios in stage 2.

On the other hand, the pure-shock FXI benefits currency-F holders including all sellers in country F, country H's international sellers, and country H's domestic buyers. Country F's buyers, who have spent their entire currency F holdings in the DM, are unaffected at $t = 0$.

5.3 Anticipated case

In the anticipated case, government H announces at the beginning of stage 1 of period 0 that it will implement the intervention at stage 2, and commits to the operation $(\Delta Y_0, \Delta X_0)$ of the pure-shock case. By Corollary 1, the price paths, the realized exchange rate $e_0 > e_0^*$, the wealth redistribution effect, and the transition effect are those of Section 5.1.

Unlike the pure-shock case, the sellers know the prices of the currencies will adjust after the FXI when they make their DM production decisions, and so do the buyers when the terms of trade are struck. Nevertheless, the buyers' currency portfolios are predetermined, as the FXI announcement comes after their portfolio decisions. Hence, buyers spend their predetermined balances and DM production is bargained over the correctly anticipated prices as follows.

$$q_{HH,0} = z^{-1}(\phi_0^H M_{HB,0}^H) < z^{-1}(\phi_0^{H*} M_{HB,0}^H) = q_{HH,0}^*, \tag{20}$$

$$q_{HF,0} = z^{-1}(\phi_0^F M_{HB,0}^F) > z^{-1}(\phi_0^{F*} M_{HB,0}^F) = q_{HF,0}^*, \tag{21}$$

$$q_{F,0} = z^{-1}(\phi_0^F M_{FB,0}^F) > z^{-1}(\phi_0^{F*} M_{FB,0}^F) = q_{F,0}^*. \tag{22}$$

The currency-H earners, who are country H's domestic sellers, reduce their production because of the lower price of currency H, ϕ_0^H. The same logic applies to all currency-F earners but in the opposite direction.

Since DM production can be adjusted, the proportional rule holds at the correctly anticipated prices, and the revaluation of each seller's receipts is offset by the adjustment of production only up to the seller's share of the surplus change, leaving a net seller impact of $\theta\,[S(q_0) - S(q_0^*)]$ in each match. The gains from DM trade change according to the adjustment in DM production, so the buyers' welfare changes as well, by $(1-\theta)$ times the surplus change plus the revaluation of the entire predetermined portfolio; equivalently, by the change in $u(q)$ plus the revaluation of unspent balances, the decomposition used in Table 1. The FXI effect in the anticipated case is summarized in the following Proposition 4 and the anticipated block of Table 1.

Proposition 4. *Suppose the intervention is announced at the beginning of stage 1 of period 0 and anticipated by all agents. At $t = 0$, sellers adjust DM production in the direction of*

the revaluation of the currency they are paid in, so that $q_{HF,0} > q^*_{HF,0}$ *and* $q_{F,0} > q^*_{F,0}$*, while* $q_{HH,0} < q^*_{HH,0}$*. For every seller, the change in production cost and the revaluation of the DM receipts cancel up to the seller's share of the surplus change, and the net welfare impact on a seller is* $\theta\left[S(q_0) - S(q^*_0)\right]$ *in her match, positive in every currency-F match, negative in country H's domestic matches. Buyers' welfare moves with their share of the DM surplus, as the currency-F users, international buyers in country H and all buyers in country F, consume more because the purchasing power of currency F is higher, while domestic buyers in country H, who pay in currency H, consume less.*

The anticipated block of Table 1 makes the partial offset visible, as every seller's Trade and Reval. entries are individually nonzero while their sum is the θ-scaled surplus change, positive for currency-F earners and negative for country H's domestic sellers. For country H's buyers, the two components oppose each other and the net sign is ambiguous; the optimal-policy analysis of Section 6 takes a stand on precisely this trade-off by weighing the Trade components of country H's residents, buyers and sellers alike, treating the revaluations as transfers.

5.4 Asymmetric case

Finally, I reconsider the assumption that information about the intervention is symmetric across countries. In reality, agents in country H often obtain information easily from country F, as transparency is one of the preconditions for issuing the dominant currency. However, the reverse might not be true. The policy transparency of a country's own government depends on institutional quality, and it often increases with economic development. As a result, in the asymmetric case, I assume that only agents in country H learn that the intervention will take place at stage 2, while agents in country F are unaware of it and expect the original price path.

Once the intervention is implemented at stage 2, it is observed by everyone in the centralized market, so from the CM of $t = 0$ onward all agents share the same information and Corollary 1 applies, leaving the price paths, the wealth redistribution effect, and the transition effect unchanged. I therefore again restrict attention to the DM and CM at $t = 0$.

The key observation is that, under proportional bargaining, the terms of trade in a bilateral match depend on the beliefs of *both* parties. Each side evaluates the payment at the stage-2 price it expects, so the proportional rule holds with each party's surplus computed under its own belief; uninformed agents are genuinely unaware of the intervention and draw no inference from the terms they face (Online Appendix B discusses this informational convention). This yields the following result.

Lemma 1 (Belief-weighted pass-through)**.** *At $t = 0$, consider a match settled in currency k, and let $\phi^b, \phi^s \in \{\phi_0^k, \phi_0^{k*}\}$ denote the stage-2 price expected by the buyer and the seller, respectively, where ϕ_0^k is the realized price and ϕ_0^{k*} the price on the original path. The traded quantity is*

$$q = z^{-1}\left(\left[\theta\phi^b + (1-\theta)\phi^s\right] M\right),$$

where M is the buyer's holding of the payment currency, so each party's belief enters the terms of trade with a weight equal to the ***counterparty's*** *surplus share.*

In the asymmetric case, the match in which both parties are informed is country H's domestic match, which is settled in currency H; the match in which both parties are uninformed is country F's domestic match, which is settled in currency F. Every international match contains exactly one informed party, so international quantities adjust partially, with the informed buyer's belief carrying weight θ in country H's import matches and the informed seller's belief carrying weight $1-\theta$ in its export matches.

Hence the domestic quantity in country H adjusts as in the anticipated case, country F's domestic quantity remains at its pure-shock level, and the international quantities lie strictly between their pure-shock and anticipated-case levels for $\theta \in (0,1)$. The FXI effect in the asymmetric case is summarized in the following Proposition 5 and the asymmetric block of Table 1.

Proposition 5. *Suppose the intervention is announced at the beginning of stage 1 of period 0 and is observed only by agents in country H, while agents in country F expect the original price path. Then at $t = 0$: (i) $q_{HH,0} = z^{-1}(\phi_0^H M_{HB,0}^H) < q_{HH,0}^*$, as in the anticipated case; (ii) international quantities adjust partially, $q_{HF,0} = z^{-1}\left(\left[\theta\phi_0^F + (1-\theta)\phi_0^{F*}\right] M_{HB,0}^F\right)$ and $q_{FH,0} = z^{-1}\left(\left[\theta\phi_0^{F*} + (1-\theta)\phi_0^F\right] M_{FB,0}^F\right)$, while $q_{FF,0} = q_{F,0}^*$; (iii) international sellers in country H earn an information rent $\theta\,(\phi_0^F - \phi_0^{F*})M_{FB,0}^F$ in the CM, in addition to their share $\theta\left[S(q_{FH,0}) - S(q_{F,0}^*)\right] > 0$ of the partial surplus adjustment; domestic sellers in country H bear their share of the domestic contraction, $\theta\left[S(q_{HH,0}) - S(q_{HH,0}^*)\right] < 0$; (iv) international sellers in country F, matched with informed buyers, gain in the CM the windfall $(1-\theta)(\phi_0^F - \phi_0^{F*})M_{HB,0}^F$ on their currency-F receipts together with their share $\theta\left[S(q_{HF,0}) - S(q_{HF,0}^*)\right] > 0$ of the partial surplus adjustment, while domestic sellers in country F, matched with uninformed buyers, gain the full pure-shock windfall $(\phi_0^F - \phi_0^{F*})M_{FB,0}^F$; (v) uninformed buyers in country F matched with informed country-H sellers consume strictly more for the same payment.*

Two features distinguish the asymmetric case from the anticipated case. First, the intervener's exports expand only in proportion to the customers' bargaining share $1-\theta$ because

an uninformed foreign buyer agrees to a larger quantity only insofar as the informed seller's belief enters the terms of trade. The domestic sector still contracts fully, so the composition of country H's output tilts toward tradables.

Second, information about the intervention commands a rent proportional to the informed party's own bargaining share, as informed sellers matched with uninformed international buyers capture $\theta\,\Delta\phi_0^F M_{FB,0}^F$, and uninformed international sellers matched with informed buyers still receive the windfall $(1-\theta)\Delta\phi_0^F M_{HB,0}^F$, where $\Delta\phi_0^F \equiv \phi_0^F - \phi_0^{F*}$. In this sense, the distribution of information about the intervention determines both the incidence of the revaluation gains and the volume of international trade through the bargaining weights.

Remark 1 (Voluntary disclosure)**.** *Suppose the announcement is verifiable, so that an informed agent can credibly disclose it inside a match, and disclosure is voluntary. Informed buyers strictly prefer to disclose to raise the belief-weighted price from $\theta\phi_0^F + (1-\theta)\phi_0^{F*}$ to ϕ_0^F, which yields more goods for the same payment. Informed sellers strictly prefer to conceal because their concealed payoff exceeds the disclosed payoff $\theta S(q_f)$ by exactly $c(q_f) - c(q_p)$ where q_f and q_p are the disclosed and concealed quantities, $z(q_f) = \phi_0^F M$ and $z(q_p) = \left[\theta\phi_0^{F*} + (1-\theta)\phi_0^F\right] M$. As a result, partial transparency raises the informed country's **imports** fully, but its exports only partially.*

			Pure shock			Anticipated			Asymmetric		
Country	Agent		Trade	Reval.	Net	Trade	Reval.	Net	Trade	Reval.	Net
H	Buyer	Intl. (q_{HF})	$\times$	$-$	$-$	$+$	$-$	?	$+$	$-$	?
		Dom. (q_{HH})	$\times$	$+$	$+$	$-$	$+$	?	$-$	$+$	?
	Seller	Intl. (q_{FH})	$\times$	$+$	$+$	$-$	$+$	$+$	$-$	$+$	$+$
		Dom. (q_{HH})	$\times$	$-$	$-$	$+$	$-$	$-$	$+$	$-$	$-$
F	Buyer	Intl. (q_{FH})	$\times$	$\times$	$\times$	$+$	$\times$	$+$	$+$	$\times$	$+$
		Dom. (q_{FF})	$\times$	$\times$	$\times$	$+$	$\times$	$+$	$\times$	$\times$	$\times$
	Seller	Intl. (q_{HF})	$\times$	$+$	$+$	$-$	$+$	$+$	$-$	$+$	$+$
		Dom. (q_{FF})	$\times$	$+$	$+$	$-$	$+$	$+$	$\times$	$+$	$+$

Rows refer to the agent's realized stage-1 match. *Trade* is the change in the DM trade component, $u(q)$ for buyers and $-c(q)$ for sellers, with signs reported in welfare terms. *Reval.* is the revaluation of nominal positions in the period-0 CM: for buyers, the balances left unspent in the DM (country F's buyers spend their entire portfolio, hence $\times$); for sellers, the DM receipts. *Net* is the sum of the two components and ? indicates that the sign is ambiguous.

Table 1: The shock effect at $t = 0$ across information structures

5.5 Competitive devaluation

The natural next question is whether the intervention invites retaliation. From the discussion above, the shock effect on country F's agents is weakly positive at $t = 0$ in every case and nothing happens to them afterward, so country F has no incentive to retaliate; under the fixed-injection regime its transition losses are an inflation cost that country F, as the issuer of the settlement currency, could not undo by a devaluation of its own (Online Appendix A). The relevant question therefore concerns other peripheral countries.

I extend the environment with a third country K that is symmetric to country H. Country K issues its own currency, its domestic transactions are settled in currency K, all international transactions remain settled in currency F, and country K is allowed to conduct FXI against currency F as well. The probability that a buyer from country $i \in \{H, F, K\}$ meets a seller from country $j \in \{H, F, K\}$ is $\delta_{ij} \geq 0$, with $\sum_j \delta_{ij} = 1$. Because the supply path of currency K and its demand (by country K's buyers for domestic purchases) are unaffected by country H's intervention, the price path of currency K is unchanged, and country H's FXI transmits to country K only through currency F. The characterization in Proposition 3 extends directly, with currency-F money demand now aggregating over the international purchases of country H's and country K's buyers and all purchases of country F's buyers. The impact on agents in countries H and F is the same as in the two-country analysis, since the mechanism operates through the purchasing power redistribution at $t = 0$.

I therefore focus on the impact on country K's agents when country H conducts FXI under the three information structures; in the asymmetric scenario, country K's agents are uninformed, like country F's. The results are summarized in Table 2.

		Pure shock			Anticipated			Asymmetric		
Agent		Trade	Reval.	Net	Trade	Reval.	Net	Trade	Reval.	Net
Buyer	Intl. (q_{Kj})	×	×	×	+	×	+	+	×	+
	Dom. (q_{KK})	×	+	+	×	+	+	×	+	+
Seller	Intl. (q_{jK})	×	+	+	−	+	+	−	+	+
	Dom. (q_{KK})	×	×	×	×	×	×	×	×	×

Components as defined in Table 1. In the asymmetric scenario the nonzero Trade entries apply to matches with informed country-H counterparties; in matches with uninformed counterparties the Trade entries are ×, while the sellers' Reval. and Net entries remain + through the unanticipated appreciation of currency-F receipts, as in the pure-shock block. International buyers' unspent balances are in currency K, whose price path is unchanged, hence ×.

Table 2: The shock effect at $t = 0$ in country K when country H intervenes

From $t = 1$ on, every real variable is back at its stationary value (Proposition 3), so

country K's agents are affected only at $t = 0$. Then, country K's domestic buyers gain from the appreciation of their unspent currency F, and its international buyers, who have spent their currency F and hold only currency K, are unaffected in the CM. In the DM, quantities respond according to Lemma 1, so country K's international buyers obtain the full adjustment in the anticipated scenario and, in the asymmetric scenario, a partial one when matched with informed country-H sellers; their shock effect is weakly positive in every scenario. Country K's international sellers gain in every scenario, through the appreciation of their currency-F receipts when neither party foresaw the intervention, through the windfall $(1-\theta)\Delta\phi_0^F M_{HB,0}^F$ plus their share θ of the partially expanded surplus when matched with informed country-H buyers, and through their share θ of the larger match surplus in the anticipated scenario. Domestic sellers in country K are paid in currency K and are unaffected throughout.

Corollary 2 (No competitive devaluation)**.** *Country H's intervention weakly benefits every agent in country K at $t = 0$ and leaves every agent in country K unaffected thereafter. At $t = 0$, export quantities are unchanged in the pure-shock scenario; in the anticipated scenario all countries' exports expand symmetrically; in the asymmetric scenario country H's exports expand partially, with weight $1-\theta$, while country K's exports expand only in its matches with country H's informed buyers, with weight θ, and are unchanged toward country F. Hence, with matching probabilities fixed, a covert intervention never reduces country K's export quantities at impact, which creates no incentive for country K to devalue in response.*

Country K thus has no loss to undo, and a devaluation of its own would only impose on it the domestic-sector contraction of the earlier analysis; under the fixed-injection regime of Online Appendix A it would in addition amplify the transitional dominant-currency inflation that then costs its international traders.

Under dominant-currency settlement and random matching, then, FXI is beggar-thyself rather than beggar-thy-neighbor, for its systematic cost falls on the intervener's own domestic sector while other peripheral countries' currency-F holders share in the revaluation gains. A covert intervention can raise the intervener's export volume partially, but it does so by delivering more goods to foreign customers for the same dominant-currency payment, not by taking demand from other exporters. Competitive devaluation does not arise from the redistribution channel, in contrast with the sticky-price expenditure-switching narrative in which one country's depreciation diverts demand away from others' exports. As a result, country H's intervention gives country K no competitive reason to intervene in response.[9]

[9]Several peripheral countries may intervene at the same time, and their operations interact through the common stock of currency F, but under random matching the reason is not competitive devaluation.

Remark 2 (Scope of Corollary 2)**.** *The matching probabilities δ_{ij} are exogenous, so the allocation of buyers and sellers across markets cannot respond to the intervention, and Corollary 2 concerns quantities per match rather than who trades with whom. In the pure-shock and anticipated scenarios both countries' exports are settled in currency F and priced by the same proportional rule, so a buyer surrendering M obtains the same quantity from an exporter in either country, and even a buyer free to choose a counterparty would have no reason to switch. In the asymmetric scenario an uninformed buyer obtains $z^{-1}\left(\left[\theta\phi_0^{F*} + (1-\theta)\phi_0^F\right] M\right)$ from an informed seller against $z^{-1}(\phi_0^{F*}M)$ from an uninformed one, so under directed search with observable terms informed exporters would draw buyers away from country K's exporters, unless country K's exporters inferred the intervention from those terms and matched them. Online Appendix G shows the same under price taking, where buyers choose freely among exporters, provided their goods are gross substitutes. The result does not cover the extensive margin on the seller side. Because the intervention raises the surplus of export matches and lowers that of domestic matches in country H, sellers there who could choose their market would move into export markets and compete with country K's exporters for buyers holding currency F, which could lower country K's export volume without any change in relative prices and give country K a reason to intervene as well. Studying that reallocation requires a directed-search version of the model and is left for future research; the no-competitive-devaluation result is established under random matching.*

5.6 One-time injection

The redistribution mechanism operates through two legs: the issuance of currency H and the withdrawal of currency F from private circulation. To isolate the role of the reserve-purchase leg, consider an alternative policy, an unanticipated one-time injection of the same amount ΔY_0 of currency H at stage 2 of period 0, distributed lump-sum, with no purchase of currency F, under the inflation-stabilizing regime.

Proposition 6 (FXI versus one-time injection)**.** *Under the one-time injection, (i) the paths of currency-H prices, inflation, and domestic quantities $\{\phi_t^H, \Pi_t^H, q_{HH,t}\}$ coincide with those of the pure-shock FXI in Proposition 3; (ii) currency F is unaffected: $\phi_t^F = \phi_t^{F*}$, $q_{HF,t} = q_{HF}^*$, and $q_{F,t} = q_F^*$ for all t; (iii) the exchange rate depreciates under both policies, but strictly more under FXI, with $e_0^{\text{FXI}} > e_0^{\text{inj}} > e_0^*$.*

Both policies depreciate the local currency, but only FXI raises the purchasing power of dominant-currency claims. The gains to dominant-currency earners, which include the intervening country's own exporters, and the entire production response on the currency-F

side in the anticipated case arise from the withdrawal of dominant currency from private circulation, not from the creation of local currency per se. This distinguishes reserve purchases from inflationary finance and offers an explanation for why interventions are concentrated in spot purchases of foreign exchange, as documented by Fratzscher et al. (2019). The reserve leg is the component that delivers the effects on the tradable sector.[10]

5.7 Reserves held in dominant-currency bonds

In practice an intervening central bank reinvests the dollars it buys in dollar safe assets, so what leaves private hands is safe dollar claims rather than cash. Online Appendix D adds to the environment a one-period nominal bond issued by government F at price ψ_t, of which a buyer holding b units can transfer at most κb in a currency-F match, $\kappa \in [0,1]$. What disciplines trade is then the effective balance $M^F + \kappa b$, and the relevant stock is the effective supply $A^F \equiv M^F + \kappa B^F$ in private hands, where B^F is the stock of bonds in private circulation. The parameter κ indexes the liquidity of the reserve asset, from an illiquid claim at $\kappa = 0$ to a perfect substitute for currency at $\kappa = 1$; a Treasury-like asset lies close to the upper end.

Proposition 7 (Reinvested FXI)**.** *Let the purchased currency be reinvested in bonds of face value $\Delta B_0 \equiv \Delta X_0/\psi_0$, with the rollover surplus rebated as in Online Appendix D. Suppose Assumptions 1 and 2 hold, that Assumption 3 holds with part (i) applied to the purchase ΔX_0 and part (ii) applied to the equilibrium induced by the effective withdrawal $\kappa\Delta B_0$, that ΔB_0 does not exceed the stock of bonds in private circulation, and, under the fixed-injection regime of Online Appendix A, that $\mu^H, \mu^F > 0$. Under either monetary regime and any of the information structures of Section 5, the equilibrium induced by the reinvested operation $(\Delta Y_0, \Delta X_0)$ coincides with the equilibrium induced by the pair of a one-time issuance of ΔY_0 of currency H and a withdrawal of $\kappa\Delta B_0$ of effective currency-F supply, in the economy with A^F in place of M^F; the two legs separate because currency-H demand depends only on the path of M^H and currency-F demand only on the path of A^F. In particular: at $\kappa = 1$, where $\psi_0 = 1$, the pair is exactly the benchmark FXI of Section 5; at $\kappa = 0$ the reserve leg is neutralized and the operation is equivalent, in equilibrium prices and quantities, to a one-time injection of ΔY_0 (Section 5.6) under the same regime and information structure; and for $\kappa \in (0,1)$ every reserve-leg effect, the revaluation ϕ_0^F/ϕ_0^{F*}, the redistribution toward dominant-currency claimants, and the currency-F production responses, is that of a benchmark withdrawal of size $\kappa\Delta B_0$, that is, the fraction $\kappa/\psi_0 \in (\kappa, 1)$ of the currency spent.*

[10] Two further questions about the instrument are deferred to Online Appendix C: whether reallocating the dominant currency already in private hands could substitute for withdrawing it, and whether the effects survive an anticipated reversal of the operation. Neither overturns the mechanism.

Because the issuance operates at full size whatever asset is bought, a less liquid reserve asset raises the domestic cost of each unit of revaluation, which Section 6 quantifies.

5.8 Endogenous dominant-currency settlement

The analysis so far assumes that every international transaction settles in currency F. I now let exporters choose the settlement currency, in the spirit of Zhang (2014). An exporter settles in its own currency at no cost, or in the other country's currency at an acceptance cost γ per international match, sunk before the terms of trade are set. The cost is distributed across exporters according to a continuous distribution G with $G(0) = 0$ and a positive density on $(0, \infty)$, and buyers learn the settlement currency only in the match. Denote by q_α^k the quantity supported by currency-k balances spent with probability α, with $\alpha\,\ell(q_\alpha^k) = \Pi^k/\beta - 1$ when the currency is carried and $q_\alpha^k = 0$ otherwise. If a share λ_X of country H's exporters settles in currency F, a country-F buyer spends currency F with probability $\delta + (1-\delta)\lambda_X$ and currency H with probability $(1-\delta)(1-\lambda_X)$, and since ℓ is bounded by $(1-\theta)/\theta$, a currency spent with probability α is carried only if $\alpha(1-\theta)/\theta > \Pi^k/\beta - 1$. Online Appendix E sets out the environment.

Proposition 8 (Endogenous dominant-currency settlement)**.** *Let $\theta > 0$ and $\Pi^H, \Pi^F > \beta$ with $\delta/(1-\delta) < (\Pi^H - \beta)/(\Pi^F - \beta) < (1-\delta)/\delta$, a band around $\Pi^H = \Pi^F$ that is nonempty if and only if $\delta < \frac{1}{2}$, and suppose Assumptions 1–3 hold. Then: (i) there is an equilibrium in which every exporter of country F settles in currency F, and the share of country H's exporters that settle in currency F solves $\lambda_X = G(\Gamma_X(\lambda_X))$, where the gain from settling in currency F, $\Gamma_X(\lambda) \equiv \theta[S(q^F_{\delta+(1-\delta)\lambda}) - S(q^H_{(1-\delta)(1-\lambda)})]$, is continuous and strictly increasing; (ii) $\lambda_X = 0$ is an equilibrium, and if $G(\Gamma_X(\lambda)) \geq \lambda$ for some $\lambda \in (0, 1]$, there is a largest equilibrium $\bar{\lambda}_X > 0$, the dominant-currency equilibrium; (iii) in the dominant-currency equilibrium, under the inflation-stabilizing regime and the anticipated scenario, an intervention of size $s \in (0, \bar{s}]$ raises the share of country H's exporters that settle in currency F at $t = 0$ above $\bar{\lambda}_X$ and keeps imports settled in currency F, so exports settled in currency F expand on both margins; (iv) under the same conditions, the threshold of Proposition 9(iii) holds with the export term replaced by the $\bar{\lambda}_X$-weighted gain on exports settled in currency F, net of the $(1-\bar{\lambda}_X)$-weighted loss on those settled in currency H, at the equilibrium balances; switching exporters add no first-order term, since the marginal exporter is indifferent.*

Settlement in currency F is self-reinforcing. The more of country H's exporters settle in currency F, the less often their customers spend currency H, and since a currency spent rarely is costly to carry, they hold less of it, which lowers the value of settling in currency H; the dominant-currency equilibrium therefore coexists with one in which country H's exports

settle in its own currency. The complementarity stops once foreign buyers carry no currency H, so exporters whose acceptance cost exceeds the surplus of a sale settled in currency F never switch, and the dominant-currency share stays below one. With $\theta = 0$, sellers capture no surplus and no exporter pays to settle in currency F; for $\theta > 0$, the benchmark is the limit of the dominant-currency equilibrium as acceptance costs vanish. With an exponential G whose mean sets $\bar{\lambda}_X$ to the 97.8 percent of Taiwan's export value not invoiced in New Taiwan dollars, foreign buyers carry no New Taiwan dollars, so exporters that settle in them make no sale; as the mean cost rises, $\bar{\lambda}_X$ falls, and the equilibrium disappears once it reaches the share at which foreign buyers would start carrying New Taiwan dollars, 0.96 at $\theta = 0.25$ and 0.89 at $\theta = 0.5$. The observed operation raises the share of exporters settling in dollars by 0.02–0.03 percentage points, and the threshold and implied weight of Section 6 differ from their benchmark values by less than 0.002 (Online Appendix E).

Returning to the benchmark, the decomposition of FXI into an issuance leg and a reserve leg raises the normative question that Section 6 answers. If the reserve leg is what raises the purchasing power of dominant-currency balances, when is exercising it worth the domestic dilution that finances it?

6 Optimal foreign exchange intervention

The analysis so far treats the size of the intervention as given and establishes what the policy does. This section turns that positive finding into the paper's normative claim and asks what size of intervention a government with an explicit objective should choose.

6.1 Objective function

I focus on the anticipated scenario, in which the intervention affects production at $t = 0$, under the inflation-stabilizing regime, which provides the natural laboratory for the question for two reasons. First, by Proposition 3 the intervention has no transition effect, so its real consequences are confined to period 0 and the residents' period-0 DM surplus is the complete welfare object on the quantity side. Second, the impact prices admit closed forms, which make the mapping from the size of the operation to equilibrium quantities transparent. Under the fixed-injection regime of Online Appendix A the impact trade-off is qualitatively the same, but the transition adds terms of the opposite sign for both groups (domestic buyers gain and international buyers lose for $t \geq 1$), which a complete treatment would weigh against the impact effects.

In the anticipated scenario each match's surplus is split between the buyer, with share

$1-\theta$, and the seller, with share θ, at correctly anticipated prices (Table 1); the domestic welfare object is therefore the DM surplus accruing to country H's residents: the full surplus of domestic matches, the buyer share of import matches, and the seller share of export matches.[11]

Government H attaches a per-capita weight $\omega \in [0,1]$ to the tradable sector, which consists of international buyers and sellers, and $1-\omega$ to the domestic sector, which consists of domestic buyers and sellers. Government H chooses the size of the reserve purchase, measured as the share of the dominant-currency stock in private circulation that the operation withdraws, $s \equiv \Delta X_0 / M_0^{F*} \in [0, \bar{s}]$, to maximize the weighted DM surplus of its residents

$$W(s;\omega) = \omega\,(1-\delta)\,[(1-\theta)\,S(q_T(s)) + \theta\,S(q_X(s))] + (1-\omega)\,\delta\,S(q_N(s))\,, \tag{23}$$

where $q_T(s) \equiv q_{HF,0}$ (imports), $q_X(s) \equiv q_{FH,0}$ (exports), and $q_N(s) \equiv q_{HH,0}$ (domestic matches) are the period-0 DM quantities induced by an anticipated intervention of size s. $\bar{s} > 0$ is the largest size consistent with Assumption 3 and with the existence of the operation (Lemma 2); either part of Assumption 3 can provide the relevant cap, the availability of dominant-currency balances from the government's counterparties or the requirement that all stage-1 liquidity constraints bind, and which one binds is a quantitative question. Measuring the operation as a share of M_0^{F*} rather than in currency units matches the normalization used in the quantitative work below.

The specification in (23) separates demographics from preferences, in that the masses $1-\delta$ and δ enter as objective population weights while ω is the subjective weight the government places on an individual internationally matched resident relative to a domestically matched one. The utilitarian aggregate corresponds to equal per-capita weights, $\omega = \frac{1}{2}$; $\omega > \frac{1}{2}$ captures a mercantilist tilt toward the sector that trades internationally.

Lemma 2 (Reduced form of the intervention)**.** *Let $m^{H*} \equiv z(q^*_{HH,0})$ and $m^{F*} \equiv z(q^*_{HF,0}) + z(q^*_{F,0})$ denote the stationary real balances of the two currencies, and set $R_H \equiv \Pi^{H*} m^{H*}$ and $R_F \equiv \Pi^{F*} m^{F*}$. I normalize the intervention by the total supply of currency F. That is, $s \equiv \Delta X_0 / M_0^{F*}$. Under the inflation-stabilizing regime, for every $s \in [0, \bar{s}]$ there is a unique operation $(\Delta Y_0, e_0)$ with $\Delta Y_0 = e_0\, s\, M_0^{F*}$ consistent with equilibrium, and the period-0 quantities in the anticipated scenario are*

$$q_T(s) = z^{-1}\big(\rho_F(s)\, z(q^*_{HF,0})\big)\,, \quad q_X(s) = z^{-1}\big(\rho_F(s)\, z(q^*_{F,0})\big)\,, \quad q_N(s) = z^{-1}\big(\rho_H(s)\, z(q^*_{HH,0})\big)\,,$$

[11] The period-0 revaluations of nominal positions (the Reval. entries in the anticipated block of Table 1) are transfers between currency holders, whose incidence is already characterized there; they are excluded from the surplus measure. Including them is straightforward given the closed forms below. Country F's and country K's welfare are likewise outside the objective of a national government.

where

$$\rho_F(s) \equiv \frac{\phi_0^F}{\phi_0^{F*}} = \frac{1}{1-s}, \qquad \rho_H(s) \equiv \frac{\phi_0^H}{\phi_0^{H*}} = 1 - \frac{R_F}{R_H}\frac{s}{1-s},$$

with ρ_F strictly increasing, ρ_H strictly decreasing, and $\rho_F(0) = \rho_H(0) = 1$. All maps are continuously differentiable, q_T and q_X are strictly increasing, q_N is strictly decreasing, and

$$q_T'(0) = \frac{z(q^*_{HF,0})}{z'(q^*_{HF,0})}, \qquad q_X'(0) = \frac{z(q^*_{F,0})}{z'(q^*_{F,0})}, \qquad q_N'(0) = -\frac{R_F}{R_H}\frac{z(q^*_{HH,0})}{z'(q^*_{HH,0})}.$$

Moreover, since $q^*_{F,0} > q^*_{HF,0}$, the liquidity constraint of country-F buyers, which binds in export and country-F domestic matches, is the first to stop binding, at

$$s_X \equiv 1 - \frac{z(q^*_{F,0})}{z(\overline{q})},$$

so Assumption 3 requires $\bar{s} \leq \min\{s_{\text{feas}}, s_X\}$: part (ii) delivers the cap s_X, and part (i) the feasibility bound s_{feas}, the currency-F receipts of country H's international sellers at the beginning of stage 2 of period 0 measured as a share of M_0^{F*}; the existence of the operation requires in addition $\bar{s} < R_H/(R_H+R_F)$, the size at which the required issuance ΔY_0 diverges.

The intervention therefore moves the liquidity margins in opposite directions. It raises the purchasing power of the dominant-currency balances that finance both the intervener's imports and its exports, pushing q_T and q_X up toward the efficient quantity $\overline{q}$, and it dilutes the local-currency balances that domestic buyers carry, pushing q_N further below $\overline{q}$. The optimal policy problem is the trade-off between these margins.

Proposition 9 (Optimal intervention)**.** *Suppose Assumptions 1, 2, and 3 hold, the intervention is anticipated, and the central banks are in the inflation-stabilizing regime. Let $s^*(\omega)$ denote the set of maximizers of (23) on $[0, \bar{s}]$. Then: (i) $s^*(0) = \{0\}$ and $s^*(1) = \{\bar{s}\}$. If $\bar{s} = s_X$, then $q_X(\bar{s}) = \overline{q}$ and export matches achieve the efficient level while import matches remain constrained, $q_T(\bar{s}) < \overline{q}$. If instead $\bar{s} = s_{\text{feas}}$, the same government exhausts the dominant-currency balances available from its counterparties and stops with all margins constrained. (ii) W has strictly increasing differences in (s, ω): every selection from $s^*(\omega)$ is nondecreasing in ω, and there exists $\hat{\omega} \in (0,1)$ such that $0 \in s^*(\omega)$ if and only if $\omega \leq \hat{\omega}$. (iii) A marginal intervention raises the objective, $\frac{\partial W}{\partial s}(0;\omega) > 0$, if and only if $\omega > \omega_0 \equiv \frac{\delta D_N}{\delta D_N + (1-\delta)D_I} \in (0,1)$, where*

$$D_N \equiv \frac{A(q^*_{HH,0}) - z(q^*_{HH,0})}{\Pi^{H*} m^{H*}}, \qquad D_I \equiv \frac{(1-\theta)\left[A(q^*_{HF,0}) - z(q^*_{HF,0})\right] + \theta\left[A(q^*_{F,0}) - z(q^*_{F,0})\right]}{\Pi^{F*} m^{F*}},$$

and A is the function of Assumption 2, with $A - z = \ell z$; m^{H*} and m^{F*} are the stationary real

balances of the two currencies defined in Lemma 2. In particular, under utilitarian weights $\omega = \frac{1}{2}$*, a marginal intervention is beneficial if and only if* $(1-\delta)D_I > \delta D_N$*. (iv) Any interior optimum satisfies*

$$\omega\,(1-\delta)\,[(1-\theta)\,S'(q_T(s))\,q_T'(s) + \theta\,S'(q_X(s))\,q_X'(s)] = (1-\omega)\,\delta\,S'(q_N(s))\,|q_N'(s)|\,,$$

so the weighted marginal liquidity gain in international matches equals the weighted marginal liquidity loss in domestic matches.

Three features of Proposition 9 deserve comment. First, part (i) gives the mercantilist limit a precise meaning. Country H cannot issue the dominant currency, so by Proposition 1 it cannot implement the Friedman rule for its international trades on its own, and reserve purchases partially substitute for the missing instrument by raising the purchasing power of the balances its importers, and its exporters' customers, already hold, a second-best provision of dominant-currency liquidity in a currency the intervener cannot print. A government that cares only about the international sector uses this substitute to the full, so FXI operates as a **shadow Friedman rule** for dominant-currency balances, implemented through scarcity rather than through issuance, and first on the margin where dominant-currency liquidity is least scarce. Its cost is the issuance of local currency, which dilutes domestic real balances, and part (iv) equates the two margins at an interior optimum.

Second, part (iii) shows that intervention is not reserved for mercantilist governments. Even under equal per-capita weights, a marginal intervention raises the objective whenever $(1-\delta)D_I > \delta D_N$, and, other things equal, the threshold ω_0 falls when the liquidity wedge at $q^*_{HF,0}$ is large or the real stock of dominant currency $\Pi^{F*}m^{F*}$ is small, that is, when dominant-currency liquidity is scarce. Bargaining power enters through two opposing channels, as the exporter earns θ of the incremental export surplus while the importer cedes θ of the incremental import surplus to foreign sellers, a holdup that lowers the value of the import margin to the intervener.

Third, the objective (23) contains no market-share term, and Section 5.5 showed that the intervention raises the period-0 purchasing power of dominant-currency balances for every user, so under random matching nothing in the optimal policy problem rewards retaliation.

6.2 Baseline calibration

To gauge magnitudes, I parameterize the model with $u(q) = q^{1-\sigma}/(1-\sigma)$ and $c(q) = q$, so that $\overline{q} = 1$, $z(q) = \theta q^{1-\sigma}/(1-\sigma) + (1-\theta)q$, and $\ell(q) = (1-\theta)(q^{-\sigma}-1)/\left[\theta q^{-\sigma} + 1 - \theta\right]$; Assumption 2 is verified numerically at the baseline for all θ considered. Each money-demand

equation $\alpha\,\ell(q) = \Pi/\beta - 1$ admits the closed form $q^{-\sigma} = (1-\theta)(1+L)/\left[(1-\theta) - L\theta\right]$ with $L \equiv (\Pi/\beta - 1)/\alpha$, which requires $\theta < 1/(1+L)$; at the baseline the existence bound of Assumption 1 is $\bar{\theta} = 0.87$.

I calibrate the model to Taiwanese data. A period is one year, and the baseline calibrated parameters are reported in Table 3. The domestic-match probability is anchored by goods trade, with $1-\delta$ set to the import share of domestic absorption over 2019–2024. Bargaining power has no direct observable target and varies across industries and firms, so I report the numerical results under three choices, $\theta \in \{0,\, 0.25,\, 0.5\}$.

Parameter	Value	Target / source
β	0.96	Annual real interest rate of about 4%; standard
σ	0.5	Curvature of u; conventional midpoint
Π^{H*}	1.02	Long-run inflation target
Π^{F*}	1.02	Long-run inflation target
$1-\delta$	0.57	import share of domestic absorption, 2019–2024 (Customs)
θ	{0, 0.25, 0.5}	No direct target; results reported across the range
X_H	\$336bn	FX export receipts, 2019–2024 avg. (CBC, EG46Y01)
M_0^{F*}	\$1.13–1.15tn	Model-implied pool, X_H/s_{feas}; see text
ΔX_0	\$28.0bn	Gross depreciation-side purchases, 2019–2024 avg. (ACMS)
s	0.024–0.025	$\Delta X_0/M_0^{F*}$; see text

The pool M_0^{F*} does not enter the equilibrium computation, which depends only on model-implied real balances; it normalizes the operation size $s = \Delta X_0/M_0^{F*}$. It is not measured directly but computed from the model's ratio of the currency-F stock to export revenue, $1/s_{\text{feas}}$, applied to observed export receipts, as described in the text; the range reflects the three values of θ. Sources: Central Bank of the Republic of China (Taiwan), foreign-exchange statistics table EG46Y01; Customs Administration, Ministry of Finance; spot-intervention proxies of Adler et al. (2025) (ACMS).

Table 3: Calibrated parameters

With linear cost the payment function is invertible in closed form, so the reduced form of Lemma 2 delivers the quantities explicitly: $z(q_T(s)) = z(q^*_{HF,0})/(1-s)$, $z(q_X(s)) = z(q^*_{F,0})/(1-s)$, and $z(q_N(s)) = \left[1 - (R_F/R_H)\,s/(1-s)\right] z(q^*_{HH,0})$, with R_H and R_F as defined there. Along the first-order condition of Proposition 9(iv), the international term falls and the domestic term rises monotonically in s, so W is single-peaked in this parameterization.

The export kink is $s_X = 1 - z(q^*_{F,0})/z(1)$, and the feasibility bound, from the dominant-currency receipts of country H's international sellers at stage 2 of period 0, is $s_{\text{feas}} = (1-\delta)z(q^*_{F,0})/(\Pi^{F*}m^{F*})$. With $\bar{s} = \min\{s_X, s_{\text{feas}}\}$, the existence bound $R_H/(R_H+R_F)$ of Lemma 2 is slack at every parameterization reported.

Computing the pool and the operation share. The normalization $s = \Delta X_0/M_0^{F*}$ requires an empirical counterpart for the stock of dominant currency from which the operation withdraws. Rather than measuring that stock directly, I compute it from the model, taking

country H's export revenue as the reference observable. The computation proceeds in five steps.

(i) In the stationary equilibrium, the currency-F stock at stage 2 of period 0 is held by country H's buyers, each of whom carries real balances $z(q^*_{HF,0})$ whatever match is later realized, and by country F's buyers, each of whom carries $z(q^*_{F,0})$. Valued at the period-0 CM price on the original path, which exceeds the next-period valuation built into $z(\cdot)$ by the factor Π^{F*}, the pool is $\phi_0^{F*} M_0^{F*} = \Pi^{F*}\left[z(q^*_{HF,0}) + z(q^*_{F,0})\right] = R_F$, with R_F as in Lemma 2.

(ii) Country H's export revenue in the same period is the currency-F receipts of its international sellers, of whom a mass $1-\delta$ are matched, each receiving real balances $z(q^*_{F,0})$. Let X_H denote these receipts in units of currency F; valued at the same price, $\phi_0^{F*} X_H = (1-\delta)\, z(q^*_{F,0})$.

(iii) Dividing (i) by (ii) gives the pool as a multiple of export revenue,

$$\frac{M_0^{F*}}{X_H} = \frac{\phi_0^{F*} M_0^{F*}}{\phi_0^{F*} X_H} = \frac{\Pi^{F*}\left[z(q^*_{HF,0}) + z(q^*_{F,0})\right]}{(1-\delta)\, z(q^*_{F,0})} = \frac{1}{s_{\text{feas}}},$$

the reciprocal of the feasibility bound. The ratio depends only on calibrated parameters and is nearly invariant to bargaining power: 3.43, 3.42, and 3.37 at $\theta = 0$, 0.25, and 0.5 (Table 4).

(iv) The empirical counterpart of X_H is the foreign-currency export receipts of Taiwan's exporters (CBC table EG46Y01, the series underlying Figure 1(b)), which averaged \$336bn over 2019–2024. The pool is therefore $M_0^{F*} \approx 3.4 \times \$336\text{bn} \approx \$1.13\text{–}1.15\text{tn}$.

(v) The operation ΔX_0 is the annual gross depreciation-side purchase of foreign currency from the spot-intervention proxy of Adler et al. (2025), which averaged \$28.0bn over 2019–2024, about 4 percent of GDP per year on average. Hence $s = \Delta X_0 / M_0^{F*} = s_{\text{feas}}\,(\Delta X_0 / X_H) \approx 0.024\text{–}0.025$. By construction the operation is the fraction $\Delta X_0 / X_H \approx 0.08$ of the feasibility bound s_{feas}, a pure data ratio, and roughly one fifth of the binding-liquidity cap $\bar{s} = s_X$ (one sixth at $\theta = 0.5$), so it is comfortably inside Assumption 3.

Findings of calibration. Three findings stand out. First, the threshold weight rises with bargaining power, from $\omega_0 = 0.68$ at $\theta = 0$ to 0.70 at $\theta = 0.25$ and 0.72 at $\theta = 0.5$. The

	$\theta = 0$	$\theta = 0.25$	$\theta = 0.5$
$q^*_{HH,0}$	0.762	0.690	0.557
$q^*_{HF,0}$	0.812	0.754	0.644
$q^*_{F,0}$	0.886	0.849	0.779
Threshold ω_0	0.676	0.698	0.722
Binding-liquidity cap s_X	0.114	0.122	0.152
Feasibility bound s_{feas}	0.292	0.292	0.297
Pool-to-exports ratio $1/s_{\text{feas}}$	3.43	3.42	3.37
Observed operation share s	0.024	0.024	0.025
Maximal size $\bar{s}$	0.114	0.122	0.152
Optimal size s^* (welfare gain)			
$\omega = 0.50$ (utilitarian)	0.000 (0.00)	0.000 (0.00)	0.000 (0.00)
$\omega = 0.70$	0.008 (0.00)	0.001 (0.00)	0.000 (0.00)
$\omega = 0.85$	0.069 (0.19)	0.064 (0.24)	0.068 (0.37)
$\omega = 1.00$	0.114 (0.80)	0.122 (1.23)	0.152 (2.31)
Implied ω (welfare gain)			
observed s	0.746 (0.03)	0.763 (0.04)	0.772 (0.05)
Reserves in bonds of pledgeability κ, threshold ω_0 / implied ω (welfare gain)			
$\kappa = 0.90$	0.698 / 0.761 (0.02)	0.719 / 0.778 (0.03)	0.741 / 0.787 (0.05)
$\kappa = 0.75$	0.733 / 0.788 (0.02)	0.753 / 0.803 (0.03)	0.773 / 0.812 (0.04)
$\kappa = 0.50$	0.802 / 0.842 (0.01)	0.818 / 0.853 (0.02)	0.834 / 0.861 (0.02)

In every column the maximal size $\bar{s}$ is the binding-liquidity cap s_X, the point at which the currency-F constraint in export and country-F domestic matches stops binding. The implied ω is the smallest welfare weight whose optimal size $s^*(\omega)$ reaches the observed operation share s of Table 3; its welfare gain is evaluated at that s under that weight. Welfare gains are $100 \times [W(s^*;\omega) - W(0;\omega)]/W(0;\omega)$. The last block reinvests the purchased dollars in bonds of pledgeability κ (Section 5.7) and reports the threshold $\omega_0(\kappa)$ and the implied weight at the observed share (Lemma D.1). All entries are computed at full precision and rounded for display.

Table 4: Optimal intervention at the baseline calibration

reason is the holdup effect noted after Proposition 9: as θ rises, a larger share of the import-margin gain accrues to foreign sellers, while the domestic cost always counts in full and the new exporter margin is too close to efficiency to compensate, its wedge $\ell(q^*_{F,0}) = \Pi^{F*}/\beta - 1$ being small at low dominant-currency inflation.

Second, conditional on a mercantilist government, i.e. $\omega = 1$, both the maximal size and its payoff rise with θ, as $\bar{s}$ increases from 0.114 to 0.152 and the welfare increases from 0.8 to 2.3 percent of the weighted period-0 surplus. Higher θ depresses the stationary international quantities, which widens the distance the intervention can close, and the mercantilist government now also values the exporter margin, which reaches efficiency exactly at the cap, $q_X(\bar{s}) = \overline{q}$.

Third, the utilitarian government, i.e. $\omega = 0.5$, never intervenes at the baseline for any θ considered, so the case for intervention in normal times remains mercantilist, and bargaining power reinforces rather than overturns that verdict, since ω_0 moves away from the utilitarian weight as θ rises.

Revealed weights. By assuming the government intervenes optimally, one can infer which welfare weights are consistent with the observed scale of intervention. Because $s^*(\omega)$ is nondecreasing (Proposition 9(ii)) and, in this parameterization, continuous and strictly increasing where it is interior, an operation size strictly between 0 and $\bar{s}$ identifies a single ω. The implied-ω block of Table 4 reports the result. The observed operation share $s \approx 0.024$ is delivered by $\omega \approx 0.75$–0.77 across the θ grid, above the utilitarian benchmark $\omega = 0.5$ and above the intervention threshold $\omega_0 \approx 0.68$–0.72. On the per-capita interpretation of ω adopted in (23), this corresponds to a government that values a unit of surplus accruing to an internationally matched resident roughly three times as much as one accruing to a domestically matched resident.

The estimate should be taken as indicative rather than sharp because the calibration assumes equal population masses in the two countries and relies on model-implied moments that are not mapped directly to data. Furthermore, the government's problem (23) contains a single reason to intervene, the liquidity trade-off between the two settlement margins, whereas the literature offers others that the model deliberately shuts down: precautionary reserve accumulation and self-insurance (Aizenman and Marion, 2004; Bacchetta et al., 2013), the correction of pecuniary externalities in segmented financial markets (Fanelli and Straub, 2021), and stabilization against exchange-rate risk (Hassan et al., 2023). To the extent that such motives contribute to the observed stance, the inferred ω absorbs them, and it should be understood as the weight that rationalizes the observed scale within the present mechanism rather than as a structural measure of preferences that holds across all motives.

Reserves held in bonds. When the purchased dollars are reinvested in bonds of pledgeability κ, Proposition 7 implies that a purchase of share s of the effective dominant-currency stock withdraws only the fraction $k \equiv \kappa/\psi^*$ of the currency spent, where ψ^* is the stationary bond price, while the issuance still pays for the full purchase, so the marginal domestic cost is unchanged. The threshold of Proposition 9(iii) then becomes

$$\omega_0(\kappa) = \frac{\delta D_N}{\delta D_N + (1-\delta)\,(\kappa/\psi^*)\,D_I},$$

which rises as κ falls (Lemma D.1 in Online Appendix D), while the observed share s is unchanged because the model-implied pool measures effective dominant-currency liquidity. For a Treasury-like asset, say $\kappa = 0.9$, the reserve leg retains about 90 percent of its effect, and the threshold and the implied weight rise by about 0.02 and 0.015 (last block of Table 4); at $\kappa = 0.5$ the implied weight reaches 0.84–0.86. The less liquid the assets in which reserves are held, the stronger the tilt toward the tradable sector needed to rationalize the observed operations.

Bargaining power and pass-through. The revaluation ρ_F does not depend on θ, while the depreciation elasticity $d\ln e_0/ds$ rises from 3.23 to 3.34 and the elasticity of the import quantity with respect to the revaluation from 1.00 to 1.55 as θ rises from 0 to 0.5 (Online Appendix F).

7 Conclusion

This paper develops a mechanism through which foreign exchange intervention affects real activity under dominant-currency settlement, without relying on nominal rigidity. In a two-country monetary search model in which domestic transactions are settled in local currency and international transactions are settled in the dominant currency, intervention changes the relative purchasing power of the two currencies and thereby redistributes wealth across sectors and countries. This redistributional channel is the central force behind the paper's results.

The analysis delivers five findings. A pure-shock intervention leaves production unchanged but immediately redistributes purchasing power from local-currency holders to dominant-currency holders at home and abroad. An anticipated intervention feeds the redistribution into production, so dominant-currency-earning sectors expand and the intervener's domestic sector contracts. A covert intervention passes through only partially, with each informed party's belief weighted by the counterparty's bargaining share, so the intervener's

exports expand only in proportion to its customers' share $1-\theta$ and informed exporters earn information rents. And under random matching the intervention invites no competitive devaluation, because a third peripheral country weakly gains at impact and is unaffected thereafter. The comparison with a one-time money injection identifies the reserve-purchase leg as the component that delivers these effects. Two extensions show that the reserve leg weakens as the assets in which reserves are held become less liquid and that, when exporters choose the settlement currency at a cost, dominant-currency settlement can be a self-reinforcing equilibrium that an anticipated intervention strengthens at impact.

The dominant currency matters here not only as an invoicing unit but as a settlement asset. FXI policy then acquires a monetary interpretation. The optimal-policy analysis recasts FXI as second-best provision of dominant-currency liquidity by a government that cannot issue it. Intervention is worthwhile only when the dominant-currency margin is sufficiently distorted or the government places enough weight on its internationally matched residents; the optimal size rises with that weight and is capped by the counterparties' dominant-currency balances or by the point at which the export-side liquidity constraint stops binding, so under symmetric country parameters the efficient import quantity is never reached. In the Taiwan calibration, the observed scale of intervention is consistent with a welfare weight of about 0.75 on internationally matched residents, which points to a mercantilist tilt in the government's objective. Even so, the gain in the government's weighted objective is only a few hundredths of a percent of period-0 surplus, so sustained intervention emerges as a policy whose payoff is primarily distributional.

Set against the sticky-price benchmark, which governs the response within a price spell, these results amount to a complementary theory of the same policy at the horizon of sustained intervention: transmission through the purchasing power of settlement balances rather than preset prices, beggar-thyself rather than beggar-thy-neighbor incidence, a central role for the reserve operation without segmented asset markets, and transparency as a precondition for the full production response.

Several extensions are left for future research: combining the settlement channel with sticky dollar prices; the path of settlement choices along the fixed-injection transition and the selection among settlement equilibria; a directed-search version in which sellers choose between domestic and export markets; the policy game among peripheral countries; repeated or persistent interventions in a richer dynamic environment; a full strategic treatment of inference inside the match, together with generalized Nash bargaining; and richer empirical work on settlement-currency patterns to quantify the redistribution channel.

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Online Appendix

Foreign Exchange Intervention under Dominant-Currency Settlement: A New-Monetarist Approach

Yu-Chen Liu

A The fixed-injection regime

This appendix characterizes the intervention of Section 5 when the central banks follow a money-growth rule instead of stabilizing inflation. Under the fixed-injection regime, each central bank commits to a predetermined path of nominal injections whatever happens. Taking central bank H as an example, it fixes the injection path $\Delta M_t^H = \mu^H(1+\mu^H)^t M_{-1}^H$, and the money created by the intervention is not counted in this category; the same applies to central bank F, for which FXI is not an option. The money supply paths are therefore

$$M_t^H = (1+\mu^H)^{t+1} M_{-1}^H + \Delta Y_0, \qquad M_t^F = (1+\mu^F)^{t+1} M_{-1}^F - \Delta X_0 \qquad \text{for } t \geq 0,$$

so inflation deviates from its stationary level for a time after the operation, whereas under the inflation-stabilizing regime of Section 5 both stocks grow at their stationary rates from period 0 on and differ from the original path only by the level shift at $t = 0$. Figure A.1 compares the two paths. The continuity argument that justifies Assumption 3 also keeps the transition inflation rates of Proposition A.1 inside the existence bound of Assumption 1, since the bound holds strictly at the stationary equilibrium.

Proposition A.1 (Fixed-injection regime)**.** *Suppose Assumptions 1, 2, and 3 hold, FXI is implemented at stage 2 of period 0, the central banks are in the fixed-injection regime, and the stationary money growth rates are strictly positive, $\mu^H, \mu^F > 0$. Then the money growth factors satisfy*

$$\begin{aligned}
&1+\mu_0^H > 1+\mu^H, \qquad 1+\mu_t^H < 1+\mu_{t+1}^H < 1+\mu^H, \ \forall\, t \geq 1, \qquad \lim_{t\to\infty}(1+\mu_t^H) = 1+\mu^H;\\
&1+\mu_0^F < 1+\mu^F, \qquad 1+\mu_t^F > 1+\mu_{t+1}^F > 1+\mu^F, \ \forall\, t \geq 1, \qquad \lim_{t\to\infty}(1+\mu_t^F) = 1+\mu^F,
\end{aligned}$$

and the equilibrium inflation paths satisfy

$$\begin{aligned}
&\Pi_0^H > \Pi^{H*}, \qquad \Pi_t^H < \Pi_{t+1}^H < \Pi^{H*}, \ \forall\, t \geq 1, \qquad \lim_{t\to\infty}\Pi_t^H = \Pi^{H*} = 1+\mu^H;\\
&\Pi_0^F < \Pi^{F*}, \qquad \Pi_t^F > \Pi_{t+1}^F > \Pi^{F*}, \ \forall\, t \geq 1, \qquad \lim_{t\to\infty}\Pi_t^F = \Pi^{F*} = 1+\mu^F.
\end{aligned}$$

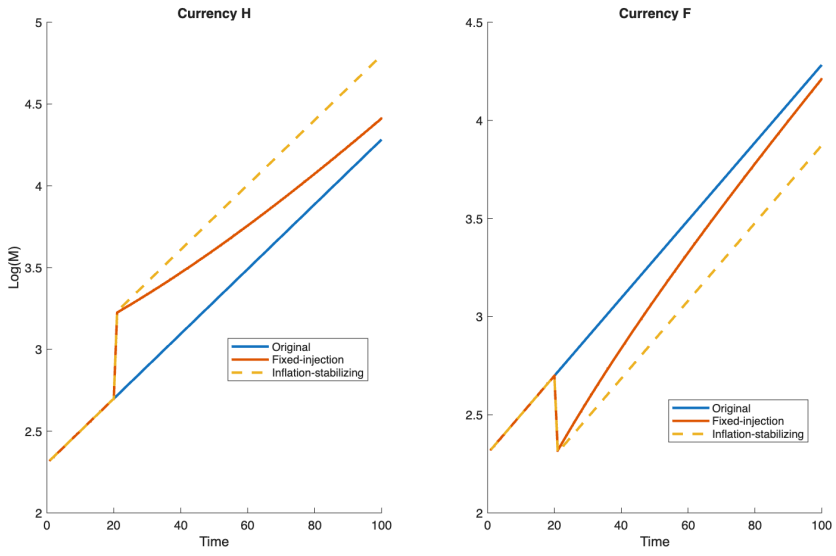


Figure A.1: Money supply paths under two regimes when $\mu^i > 0$

Correspondingly,

$$q_{HH,t} > q_{HH,t+1} > q^*_{HH},\ \forall\, t \geq 1, \qquad \lim_{t\to\infty} q_{HH,t} = q^*_{HH};$$
$$q_{HF,t} < q_{HF,t+1} < q^*_{HF},\ \forall\, t \geq 1, \qquad \lim_{t\to\infty} q_{HF,t} = q^*_{HF};$$
$$q_{F,t} < q_{F,t+1} < q^*_{F},\ \forall\, t \geq 1, \qquad \lim_{t\to\infty} q_{F,t} = q^*_{F}.$$

Finally, the impact prices satisfy $\phi^H_0 < \phi^{H*}_0$ and $\phi^F_0 > \phi^{F*}_0$, so there is a wealth redistribution effect from currency-H holders to currency-F holders.

Proof of Proposition A.1. I first establish the properties of the money growth factors, then the currency-H block, and finally the currency-F block, which is symmetric.

Money growth. For $t \geq 1$,

$$1 + \mu^H_t = \frac{(1+\mu^H)^{t+1} M^H_{-1} + \Delta Y_0}{(1+\mu^H)^t M^H_{-1} + \Delta Y_0} = (1+\mu^H)\left[1 - \frac{\frac{\mu^H}{1+\mu^H}\Delta Y_0}{(1+\mu^H)^t M^H_{-1} + \Delta Y_0}\right],$$

which is strictly less than $1+\mu^H$, strictly increasing in t, and converges to $1+\mu^H$; at $t=0$,

$$1 + \mu^H_0 = \frac{(1+\mu^H) M^H_{-1} + \Delta Y_0}{M^H_{-1}} > 1 + \mu^H.$$

The computation for currency F is identical with $-\Delta X_0$ in place of ΔY_0 and all inequalities reversed.

Currency H: the recursion. In equilibrium, currency H is carried across periods only by country H's buyers, so market clearing implies that the real balances available at the beginning of the DM of period t are $m_t^H \equiv \phi_t^H M_{t-1}^H$, with $q_{HH,t} = z^{-1}(m_t^H)$. The stage-2 first-order condition can be written as

$$\frac{\Pi_{t+1}^H}{\beta} = f(m_{t+1}^H), \qquad f(m) \equiv 1 + \delta\, \ell(z^{-1}(m)),$$

where f is strictly decreasing by (19). Define

$$g(m) \equiv \beta m f(m) = \beta \left[(1-\delta)z(q) + \delta A(q)\right]\Big|_{q=z^{-1}(m)},$$

which is strictly increasing by Assumption 2. Since $m_{t+1}^H/m_t^H = (1+\mu_t^H)/\Pi_{t+1}^H$ by definition, the equilibrium path satisfies

$$g(m_{t+1}^H) = \beta m_{t+1}^H f(m_{t+1}^H) = \beta m_{t+1}^H \frac{\Pi_{t+1}^H}{\beta} = (1+\mu_t^H)\, m_t^H, \qquad t \geq 1, \tag{A.1}$$

while in the stationary equilibrium

$$g(m^{H*}) = (1+\mu^H) m^{H*}, \qquad \text{i.e.} \qquad \beta f(m^{H*}) = \Pi^{H*} = 1+\mu^H;$$

m^{H*} is the unique positive solution of $g(m) = (1+\mu^H)m$ because f is strictly decreasing.

Step 1: $\Pi_t^H < \Pi^{H*}$, equivalently $m_t^H > m^{H*}$, for all $t \geq 1$.
Suppose to the contrary that $m_s^H \leq m^{H*}$ for some $s \geq 1$. Then the first-order condition implies $\Pi_s^H = \beta f(m_s^H) \geq \Pi^{H*} = 1+\mu^H > 1+\mu_s^H$, so

$$g(m_{s+1}^H) = (1+\mu_s^H) m_s^H < \Pi_s^H m_s^H = g(m_s^H),$$

and hence $m_{s+1}^H < m_s^H \leq m^{H*}$ by the monotonicity of g. Iterating, $\{m_t^H\}_{t\geq s}$ is strictly decreasing and bounded above by m^{H*}. If it converged to some $\bar{m} > 0$, taking limits in (A.1) and using $\mu_t^H \to \mu^H$ would give $g(\bar{m}) = (1+\mu^H)\bar{m}$, so $\bar{m} = m^{H*}$ by uniqueness; but the sequence is strictly decreasing, so $\bar{m} < m_{s+1}^H < m^{H*}$, a contradiction. Hence $m_t^H \to 0$: the value of money vanishes and the path never returns to the stationary monetary equilibrium, contradicting the equilibrium selection.

Step 2: monotone convergence from below.
Suppose $\Pi_t^H \leq 1+\mu_t^H$ for some $t \geq 1$. Then $g(m_{t+1}^H) = (1+\mu_t^H)m_t^H \geq \Pi_t^H m_t^H = g(m_t^H)$, so $m_{t+1}^H \geq m_t^H$ and $\Pi_{t+1}^H \leq \Pi_t^H \leq 1+\mu_t^H < 1+\mu_{t+1}^H$. By induction, $\{\Pi_s^H\}_{s\geq t}$ is non-increasing and therefore cannot converge to $\Pi^{H*} = 1+\mu^H > \Pi_t^H$, contradicting the return to the

stationary equilibrium. Hence $\Pi_t^H > 1 + \mu_t^H$ for all $t \geq 1$, which by the same comparison delivers $m_{t+1}^H < m_t^H$ and thus $\Pi_t^H < \Pi_{t+1}^H$ for all $t \geq 1$. The sequence $\{m_t^H\}$ is strictly decreasing and bounded below by m^{H*} (Step 1), and its limit must solve $g(\bar{m}) = (1+\mu^H)\bar{m}$, so $m_t^H \downarrow m^{H*}$ and $\Pi_t^H \uparrow \Pi^{H*}$. The quantity statements follow from $q_{HH,t} = z^{-1}(m_t^H)$ and the strict monotonicity of z^{-1}.

Step 3: impact effect.

Prices up to $t = -1$ are on the original path, so $\Pi_0^H = \phi_{-1}^{H*}/\phi_0^H$. Suppose $\phi_0^H \geq \phi_0^{H*}$. Using $\Pi_i^H < \Pi^{H*}$ for $i \geq 1$ (Steps 1–2),

$$\frac{\phi_t^H}{\phi_t^{H*}} = \frac{\phi_0^H}{\phi_0^{H*}} \prod_{i=1}^{t} \frac{\Pi^{H*}}{\Pi_i^H} \geq \frac{\Pi^{H*}}{\Pi_1^H} > 1, \qquad t \geq 1,$$

and since $M_{t-1}^H > M_{t-1}^{H*}$,

$$m_t^H = \phi_t^H M_{t-1}^H = \frac{\phi_t^H}{\phi_t^{H*}} \frac{M_{t-1}^H}{M_{t-1}^{H*}} \phi_t^{H*} M_{t-1}^{H*} > \frac{\Pi^{H*}}{\Pi_1^H} \phi_t^{H*} M_{t-1}^{H*} = \frac{\Pi^{H*}}{\Pi_1^H} m^{H*} > m^{H*}$$

for all $t \geq 1$, contradicting $m_t^H \to m^{H*}$. Hence $\phi_0^H < \phi_0^{H*}$ and $\Pi_0^H = \phi_{-1}^{H*}/\phi_0^H > \phi_{-1}^{H*}/\phi_0^{H*} = \Pi^{H*}$.

Currency F: two groups. Currency F is carried across periods by two groups: country H's buyers, who use it for international purchases, and country F's buyers. For country H's buyers, denote their beginning-of-period real balances by $m_{HB,t}^F \equiv \phi_t^F M_{HB,t-1}^F$, where $M_{HB,t-1}^F + M_{FB,t-1}^F = M_{t-1}^F$. Their stage-2 first-order condition reads

$$\frac{\Pi_{t+1}^F}{\beta} = f_{HB}(m_{HB,t+1}^F), \qquad f_{HB}(m) \equiv 1 + (1-\delta)\,\ell(z^{-1}(m)),$$

which has exactly the structure of the currency-H condition with the matching probability δ replaced by $1-\delta$: f_{HB} is strictly decreasing, the group's real demand is strictly decreasing in Π_{t+1}^F, and the group analogue of g,

$$g_{HB}(m) \equiv \beta m f_{HB}(m) = \beta\left[\delta z(q) + (1-\delta)A(q)\right]\Big|_{q=z^{-1}(m)},$$

is strictly increasing by Assumption 2. Country F's buyers are identical in structure with

$1-\delta$ replaced by 1, since they pay in currency F in every match:

$$\frac{\Pi^F_{t+1}}{\beta} = f_{FB}(m^F_{FB,t+1}), \qquad f_{FB}(m) \equiv 1 + \ell(z^{-1}(m)),$$
$$g_{FB}(m) \equiv \beta m f_{FB}(m) = \beta A(q)\Big|_{q=z^{-1}(m)},$$

with f_{FB} strictly decreasing and g_{FB} strictly increasing.

Currency F: aggregation. Because both groups face the same inflation rate, their demands move together period by period: $m^F_{HB,t}$ and $m^F_{FB,t}$ are both strictly decreasing in Π^F_t. Aggregate beginning-of-period real balances $m^F_t \equiv \phi^F_t M^F_{t-1} = m^F_{HB,t} + m^F_{FB,t}$ are therefore strictly decreasing in Π^F_t, which defines the strictly decreasing inverse $\Pi^F_t = P(m^F_t)$. Along the demand system,

$$g_F(m^F_t) \equiv m^F_t\, P(m^F_t) = g_{HB}(m^F_{HB,t}) + g_{FB}(m^F_{FB,t}),$$

which is strictly increasing in m^F_t, since each term on the right is strictly increasing in the corresponding group's balances and both groups' balances rise as Π^F_t falls. The identity $m^F_{t+1}/m^F_t = (1+\mu^F_t)/\Pi^F_{t+1}$ then delivers the aggregate recursion

$$g_F(m^F_{t+1}) = (1+\mu^F_t)\, m^F_t,$$

and the argument of Steps 1–3 applies to it in the opposite direction, because $1+\mu^F_t$ decreases to $1+\mu^F$ from above and all inequalities are reversed; in the contradiction branch of Step 1 the candidate sequence is now increasing, and it either converges to a second fixed point of $G(m) = (1+\mu^F)m$, which uniqueness excludes, or grows until the liquidity constraints stop binding, after which $\ell = 0$ implies $\Pi^F_{t+1} = \beta < \Pi^{F*}$ permanently, again contradicting the return to the stationary equilibrium. This yields $\Pi^F_t > \Pi^F_{t+1} > \Pi^{F*}$ for $t \geq 1$ with $\Pi^F_t \downarrow \Pi^{F*}$, and $\phi^F_0 > \phi^{F*}_0$, i.e. $\Pi^F_0 < \Pi^{F*}$. Finally, the aggregate conclusion maps back into each group: since each group's demand is a strictly decreasing function of the common inflation rate, $q_{HF,t} = z^{-1}(m^F_{HB,t})$ and $q_{F,t} = z^{-1}(m^F_{FB,t})$ recover monotonically from below along the transition,

$$q_{HF,t} < q_{HF,t+1} < q^*_{HF} \qquad \text{and} \qquad q_{F,t} < q_{F,t+1} < q^*_F.$$

Finally, $\phi^H_0 < \phi^{H*}_0$ and $\phi^F_0 > \phi^{F*}_0$ constitute the purchasing power redistribution from currency-H holders to currency-F holders. This completes the proof. □

The impact prices move in the same directions as under the inflation-stabilizing regime,

although by different amounts, since period-0 money demand now reflects the expected transition inflation. The shock-effect analysis of Section 5 uses only the sign comparisons $\phi_0^H < \phi_0^{H*}$ and $\phi_0^F > \phi_0^{F*}$ together with Assumption 3, so Lemma 1, Propositions 4 and 5, and Corollary 2 at $t = 0$ carry over. What the regime adds is an effect along the transition.

The transition effect. For $t \geq 1$, the DM production pattern implies that country F's buyers are hurt by the lower production $(q_{FH,t},\ q_{FF,t})$ caused by the temporarily higher Π_t^F. For country H's buyers, the impact depends on the match type. There are negative effects in international trade because of a lower $q_{HF,t}$ and a higher Π_t^F, and positive effects in domestic trade because of a higher $q_{HH,t}$ and a lower Π_t^H. Therefore, the ex-ante welfare impact for country H's buyers depends on the share of domestic transaction, and the larger that share, the more a country-H buyer benefits. Meanwhile, each seller earns the share θ of the match surplus, which implies the impact is qualitatively the same as their counterparty has.

It is noteworthy that the impact for $t \geq 1$ is caused by the change in the inflation path. Technically, although the subsequent impact is created by the FXI at $t = 0$, it could be replicated by engineering the same inflation path. Therefore, I call the impact for $t \geq 1$ the **transition effect**. By Corollary 1 the transition effect is common to all information structures, and Table A.1 reports it once. Since convergence back to the stationary equilibrium is asymptotic, every entry vanishes as $t \to \infty$.

Country	Agent		$t \geq 1$
H	Buyer	Intl.(q_{HF})	$-$
		Dom.(q_{HH})	$+$
	Seller	Intl.(q_{FH})	$-$
		Dom.(q_{HH})	$+$
F	Buyer	Intl.(q_{FH})	$-$
		Dom.(q_{FF})	$-$
	Seller	Intl.(q_{HF})	$-$
		Dom.(q_{FF})	$-$

Entries refer to match types. All entries vanish as $t \to \infty$.

Table A.1: The transition effect (all information structures)

Under the inflation-stabilizing regime, by contrast, every real variable is back at its stationary value from period 1 (Proposition 3) and every entry of Table A.1 is zero. The transition effect is therefore a consequence of the fixed-injection regime, whereas the shock effect and the wealth redistribution effect are intrinsic to the intervention itself. Along

the transition, country K's international buyers and sellers of Section 5.5 lose from the temporarily higher currency-F inflation, exactly as country F's agents do, while its other agents are unaffected because currency K's path is unchanged; a devaluation by country K would amplify, rather than reverse, that inflation.

Remark A.1 (Decomposition and full sterilization)**.** *If central bank H additionally adjusts within period 0, reducing its regular injection by ΔY_0, then $M_0^H = M_0^{H*}$ and the currency-H side of the intervention is neutralized entirely, while the currency-F effects remain, so the operation reduces to the reserve leg alone. This is the mirror image of the one-time injection of Section 5.6, which retains the issuance leg alone. This scheme finances the reserve purchase through the fiscal transfer τ_{H0} rather than through issuance, taking the operation outside the pure central-bank implementation studied in the text; I therefore note it only as the polar benchmark. FXI therefore decomposes into an issuance leg and a reserve leg, and each polar sterilization scheme isolates one of them. Analogously, if central bank F re-injects ΔX_0 within period 0, the currency-F side is neutralized by the foreign central bank itself and only the issuance leg remains.*

B Robustness: the take-it-or-leave-it boundary

A simplified benchmark in the monetary-search literature is to assume that buyers make take-it-or-leave-it (TIOLI) offers; the proportional protocol used throughout nests that benchmark at $\theta = 0$. Wherever the two parties to a match share the same information, the nesting is exact: at $\theta = 0$ the payment function collapses to the cost function, $z(q) = c(q)$, and the stationary equilibrium of Section 4, the pure-shock and anticipated cases, the transition effect, and the optimal-policy analysis of Section 6 all coincide with their TIOLI counterparts; the exporter term drops out of the objective (23), and the $\theta = 0$ column of Table 4 is the TIOLI benchmark.

Under one-sided information the boundary is exact for informed buyers but discontinuous for informed sellers. As $\theta \to 0$ the proportional rule takes the quantity in a seller-informed match to its fully adjusted level while the information rent $\theta \Delta\phi_0^F M$ vanishes, whereas the literal buyer-offer protocol pins the quantity at its pure-shock level and hands the informed seller the full windfall $\Delta\phi_0^F M$ by accepting, because the proposer's belief alone sets the terms. The discontinuity matters only for the covert-intervention export response, which under the proportional rule responds with weight $1 - \theta$ on the realized price, to first order the fraction $1 - \theta$ of the announced response, and disappears altogether under the buyer-offer protocol. Everything else survives the boundary: the price paths, the wealth redistribution effect, and the transition effect are protocol-independent; transparency remains a precondition for the

full production response under every protocol; and generalized Nash bargaining delivers the same qualitative ranking as the proportional solution, with partial adjustment under one-sided information. Throughout, uninformed agents are modeled as genuinely unaware, so they draw no inference from the agreed terms; a full strategic treatment of such inference is left for future research.

C Alternative instruments and reversal

This appendix takes up the two instrument-design questions deferred from Section 5: whether reallocating existing private dominant-currency balances can substitute for the reserve purchase, and whether an anticipated reversal undoes the operation's effects.

C.1 Reallocating idle dominant-currency balances

Because currency portfolios are chosen before the match is realized, a fraction δ of country H's buyers enter the CM of period 0 holding dominant-currency balances they did not spend. This suggests an alternative instrument: rather than withdrawing currency F from private circulation, the government could channel these idle balances to its internationally matched buyers, for a fee. Such a facility would raise $q_{HF,0}$ without issuing currency H, and would therefore appear to dominate FXI, since it avoids the dilution of ϕ_0^H that constitutes the entire cost side of the trade-off in Section 6. The scope for reallocation is moreover substantial: if all idle balances were redirected, dominant-currency liquidity per international match would rise by the factor $1/(1-\delta)$: each international buyer's balance $M_{HB,0}^F$ would be topped up by the idle balances $\delta M_{HB,0}^F$ spread over the $1-\delta$ international matches, an increment of $\delta/(1-\delta)$ per match.

The facility is nonetheless not a substitute for FXI, for two reasons. First, it operates on one side of the international transaction only. It leaves $M_{FB,0}^F$ untouched, so $q_{FH,0}$ and $q_{FF,0}$ are unchanged and country H's exports do not expand; and because it redistributes nominal balances without altering the stock of currency F in private circulation, it does not move ϕ_0^F. The entire currency-F production response of the anticipated case, equation (22), is therefore absent. What the facility delivers is a relaxation of the intervener's own import constraint, not the revaluation of dominant-currency claims that drives the results of Section 5.

Second, and more fundamentally, lending balances in stage 1 against repayment in stage 2 is precisely the credit arrangement ruled out by the absence of record-keeping and enforcement that makes currency essential here (Kocherlakota, 1998). The natural repair is to

collateralize the operation. An internationally matched buyer holds idle currency H, so the facility could be run as an intra-stage swap rather than a loan. The facility must nonetheless be costly, because a frictionless swap window would let agents rebalance after learning their match type, undoing the ex-ante portfolio commitment on which the redistribution mechanism rests. The fee that makes the facility a plausible policy is thus also what preserves the environment in which FXI can be effective. Related analyses of banking arrangements that reallocate idle balances in monetary search models are Berentsen et al. (2007) and Chang and Li (2018).

C.2 Reversal and the limits of neutrality

Suppose the government commits at $t = 0$ to undoing the operation at $t = T \geq 1$, selling ΔX_0 back to the private sector and retiring ΔY_0, and that agents foresee the reversal. The relevant neutrality benchmark is the irrelevance of open-market operations in the spirit of Wallace (1981). That irrelevance fails here for the same reason it fails throughout the monetary search literature, namely that currencies are held to relax binding liquidity constraints in the DM, so a change in their relative supplies is not a change in the composition of a portfolio of otherwise equivalent assets.

Three observations follow. First, any reversal with $T \geq 1$ leaves at least one decentralized market in which agents carry the perturbed stocks, so the real effects cannot vanish; the only reversal that removes them is a within-period one, which is the sterilization exercise of Remark A.1 and which isolates one leg of the operation rather than eliminating both.

Second, the anticipated reversal dampens the impact effect without reversing it. Under the fixed-injection regime of Online Appendix A, because ϕ_T^H returns to its original path, the return to holding currency H between 0 and T is higher than under a permanent intervention, so currency-H demand at $t = 0$ is higher and ϕ_0^H falls by less; symmetrically, ϕ_0^F rises by less. The inequalities $\phi_0^H < \phi_0^{H*}$ and $\phi_0^F > \phi_0^{F*}$ of Proposition 3 continue to hold, since they follow from the period-0 money stocks alone, so every sign reported in Tables 1 and 2 and in Table A.1 is invariant to T while magnitudes shrink as T falls.

Third, the shock effect is partly self-reversing but the production response is not. Since types are fixed over time, an agent who gains from the revaluation at $t = 0$ largely bears the offsetting loss at $t = T$; with quasi-linear preferences these are additive transfers, so abstracting from the re-optimization of portfolios in the intervening periods, a gain Δ at $t = 0$ and an offsetting loss at $t = T$ net to $(1 - \beta^T)\Delta$, which vanishes as the reversal is brought forward. The DM quantities of the anticipated case admit no such cancellation, because production at $t = 0$ responds to ϕ_0^H and ϕ_0^F, which are realized before any reversal

can occur.

The headline results are therefore robust to reversal in sign, and what a reversal removes is the transition effect, which arises only under the fixed-injection regime of Online Appendix A. This is also the empirically relevant case, since intervention is disproportionately conducted through purchases rather than sales (Fratzscher et al., 2019) and the reserve stocks of the economies that motivate the analysis are persistent (Setser, 2019). The one-sidedness of observed intervention is itself what forecloses the equivalence.

D Reinvested reserves and dominant-currency bonds

This appendix sets out the environment behind Proposition 7 in Section 5.7, proves it, and derives the reduced form behind the reserve-asset results of Section 6. When the purchased currency F is immediately reinvested in dominant-currency bonds, the operation withdraws safe dominant-currency claims rather than currency, and its reserve leg is equivalent to a benchmark withdrawal scaled by the bonds' liquidity weight, while the currency-H issuance operates at full size. That swaps of assets with different liquidity properties are non-neutral is a general feature of monetary models with liquid bonds (Williamson, 2012; Rocheteau et al., 2018).

Environment. Add to the environment of Section 3 a one-period nominal bond issued by government F. A bond purchased in the CM of period t at price ψ_t, in units of currency F, pays one unit of currency F in the CM of period $t+1$; the stock outstanding grows at the currency-F rate, $B^F_t = (1+\mu^F)B^F_{t-1}$, with the net interest financed through the lump-sum taxes τ_{Ft}. Bonds are partially liquid in currency-F matches. In the DM, a buyer holding b maturing bonds can transfer at most κb of them, $\kappa \in [0,1]$, at their redemption value ϕ^F, a pledgeability constraint that can be microfounded by imperfect recognizability (Lester et al., 2012). The liquidity constraint in a currency-F match therefore becomes $z(q) \leq \phi^F \left(M^F + \kappa b\right)$, so what disciplines trade is the *effective* balance $M^F + \kappa b$.

Bond pricing. For a buyer whose currency-F liquidity margin has matching probability α ($\alpha = 1-\delta$ for country H's buyers, $\alpha = 1$ for country F's), the CM first-order conditions for money and bonds are

$$\frac{\phi^F}{\beta\phi^F_{+1}} = 1 + \alpha\,\ell(q_{+1}), \qquad \frac{\psi\,\phi^F}{\beta\phi^F_{+1}} = 1 + \kappa\,\alpha\,\ell(q_{+1}),$$

and dividing the second by the first, using $\alpha\ell(q_{+1}) = \Pi^F_{+1}/\beta - 1$ from the money margin,

$$\psi_t = \frac{\beta}{\Pi^F_{t+1}}\left[1 + \kappa\left(\frac{\Pi^F_{t+1}}{\beta} - 1\right)\right]. \tag{D.1}$$

Because both buyer groups' money margins deliver the same value of $\alpha\ell$, the price (D.1) makes both groups indifferent at the margin, so both are willing to hold bonds and the split of the private bond stock across groups, like the split of each group's portfolio between money and bonds, is indeterminate; only effective balances matter. Sellers hold no bonds: their gross return would be $\beta/(\psi\Pi^F_{+1}) = \left[1 + \kappa(\Pi^F_{+1}/\beta - 1)\right]^{-1} \leq 1$, with strict inequality for $\kappa > 0$ away from the Friedman rule, exactly as they hold no currency; at $\kappa = 0$ or under the Friedman rule they are indifferent and, as with currency, I assign them zero holdings without loss. At $\kappa = 1$, $\psi_t = 1$ and bonds are money; at $\kappa = 0$, $\psi_t = \beta/\Pi^F_{t+1}$ and bonds pay the full illiquid nominal rate $\Pi^F_{+1}/\beta - 1$. For $\kappa \in (0, 1)$ the bond's nominal yield $1/\psi_t - 1$ is strictly below that illiquid benchmark, and the spread between the two is the model's convenience yield on dominant-currency safe assets, increasing in κ.

Since the money first-order conditions are unchanged, the stationary quantities of Section 4 are unchanged, and the currency-F price level satisfies

$$\phi^F_t\, A^F_t = \Pi^{F*} m^{F*}, \qquad A^F_t \equiv M^F_t + \kappa B^F_t,$$

where B^F_t henceforth denotes bonds in *private circulation*, parallel to the convention for M^F_t in Section 5.1 (government F's issuance keeps the stock outstanding growing at μ^F; any bonds held by government H are excluded), so that A^F_t is the effective (liquidity-weighted) supply of dominant-currency assets in private hands, and $m^{F*} = z(q^*_{HF,0}) + z(q^*_{F,0})$ as in the text.

The reinvested operation. At stage 2 of period 0, government H issues ΔY_0, purchases ΔX_0 units of currency F from its international sellers, and immediately reinvests the proceeds in bonds bought from private holders in the secondary market, of face value $\Delta B_0 = \Delta X_0/\psi_0$, maintaining a constant face-value position thereafter. The purchased currency returns to private circulation through the private bond sellers, so in period 0 the private money stock is unchanged while the private bond stock falls by ΔB_0, and the effective supply falls by $\kappa\Delta B_0$.

Rolling the position over requires one accounting convention. In each CM $t \geq 1$, government H collects ΔB_0 of currency F in redemptions and spends $\psi_t \Delta B_0$ on new bonds, a currency-F surplus of $(1 - \psi_t)\Delta B_0$ per period (zero at $\kappa = 1$). I assume the government

returns this surplus to private circulation, rebating it directly in currency F or, equivalently, selling it at market rates and rebating the currency-H proceeds through τ_{Ht}; by quasi-linearity the two are the same. This is exactly what keeps M_t^F on its original path, because relative to no intervention private agents forgo ΔB_0 of redemption inflows and save $\psi_t \Delta B_0$ of rollover purchases, a net shortfall of $(1-\psi_t)\Delta B_0$ that the rebate restores, while the private bond stock stays ΔB_0 below its no-intervention path. The effective supply is therefore $\kappa \Delta B_0$ lower from period 0 onward. Finally, under the fixed-injection regime $\Delta B_0 = \Delta X_0/\psi_0$ is an equilibrium object rather than a policy parameter, since ψ_0 depends on Π_1^F through (D.1) and Π_1^F on the effective withdrawal itself; for ΔX_0 small the fixed point has a unique solution near $\Delta X_0/\psi^*$, with ψ^* the stationary bond price, by the implicit function theorem, because the map $\Delta B \mapsto \Delta B\, \psi_0(\kappa \Delta B)$ is continuously differentiable under Assumption 2 with derivative $\psi^* > 0$ at zero. Under the inflation-stabilizing regime $\Pi_1^F = \Pi^{F*}$, so $\psi_0 = \psi^*$ and ΔB_0 is explicit.

Proof of Proposition 7. The currency-H side is identical across the two operations, since the path of M_t^H changes by the same ΔY_0 as in Section 5.1 and currency-H demand does not involve the bond market. On the currency-F side, the DM constraints depend on portfolios only through effective balances $\phi^F(M^F + \kappa b)$, the CM budget values maturing bonds at par, and by quasi-linearity the composition of wealth is irrelevant to demand; the money first-order conditions are therefore unchanged, and the bond first-order condition determines ψ_t residually through (D.1). Each buyer group's demand for effective real balances is thus the same strictly decreasing function of the inflation rate as its money demand in the benchmark economy, and market clearing equates aggregate effective demand to $\phi_t^F A_t^F$. The equilibrium system determining $\{\phi_t^F, q_{HF,t}, q_{F,t}\}$ is that of Section 5.1 with A_t^F in place of M_t^F; under the fixed-injection regime the rebate convention above keeps M_t^F on its original path, so $A_t^F = (1+\mu^F)^{t+1} A_{-1}^F - \kappa \Delta B_0$ for $t \geq 0$, the same path structure as the benchmark, and the transition characterization of Proposition A.1 and the level-jump characterization of Proposition 3 apply verbatim, the latter with $A_0^F = (1+\mu^F)A_{-1}^F - \kappa \Delta B_0$ and $A_t^F = (1+\mu^F)^t A_0^F$ for $t \geq 1$ under the inflation-stabilizing regime. For the period-0 information structures, Lemma 1 extends verbatim to two-asset payments: transferred currency and maturing bonds redeem at the same CM price ϕ_0^F, so each party values both at its own believed price, the proportional rule yields the quantity solving $z(q) = \left[\theta \phi^b + (1-\theta)\phi^s\right](M^F + \kappa b)$, and part (ii) of Assumption 3 makes both transfer constraints bind; the period-0 results therefore apply with predetermined effective balances in place of predetermined money balances. Since the reinvested operation changes A_t^F by $-\kappa \Delta B_0$, exactly what a withdrawal of that effective size does, and changes M_t^H by ΔY_0, exactly what the issuance leg does, the equivalence with the pair follows. At $\kappa = 1$, (D.1) gives $\psi_0 = 1$, so $\Delta B_0 = \Delta X_0$, the rollover surplus vanishes,

and the pair is the benchmark FXI of size ΔX_0, whose issuance is $\Delta Y_0 = e_0 \Delta X_0$. At $\kappa = 0$, $A_t^F = M_t^F$ is unaffected by the bond purchase, so the currency-F side is untouched and only the issuance ΔY_0 operates: prices and quantities are those of the one-time injection, the identity of the initial recipients being irrelevant to prices by quasi-linearity, as in the proof of Proposition 6. For $\kappa \in (0, 1)$ the effective withdrawal is $\kappa \Delta B_0 = (\kappa/\psi_0)\Delta X_0$, which scales ρ_F and everything downstream of it accordingly. □

Two further observations. First, along the fixed-injection transition the temporarily elevated Π_{t+1}^F lowers ψ_t by (D.1) less than it raises the illiquid nominal rate, so the convenience-yield spread on dominant-currency safe assets widens temporarily; inside the model, the intervention makes dominant-currency safe assets scarcer and more valuable given the exogenous bond-supply path, the counterpart of the evidence that official demand for U.S. Treasuries moves their convenience yield (Krishnamurthy and Vissing-Jorgensen, 2012; Jiang et al., 2021). Second, the equivalence disciplines the interpretation of the quantitative results. If bonds are highly, but not perfectly, liquid, the effective size of an observed operation is close to its face value, while a low κ shrinks it, so the liquidity weight of the assets actually purchased is a quantitative input that empirical work on settlement balances could pin down; the κ block of Table 4 shows how the calibrated threshold and implied weight depend on it.

Reinvested reserves in the optimal-policy problem. The reduced form of Lemma 2 extends directly.

Lemma D.1 (Reduced form with reinvested reserves)**.** *Under the inflation-stabilizing regime and the anticipated scenario, let the purchased currency be reinvested in bonds of pledgeability $\kappa \in (0, 1]$, and measure the purchase as the share $s \equiv \Delta X_0 / A_0^{F*}$ of the effective supply. Then $\psi_0 = \psi^* \equiv (\beta/\Pi^{F*}) \left[1 + \kappa(\Pi^{F*}/\beta - 1)\right]$, and the period-0 quantities are those of Lemma 2 with*

$$\rho_F(s) = \frac{1}{1 - ks}, \qquad \rho_H(s) = 1 - \frac{R_F}{R_H} \frac{s}{1 - ks}, \qquad k \equiv \frac{\kappa}{\psi^*} \in (0, 1],$$

for $s < R_H/(R_F + kR_H)$, the size at which the required issuance diverges; the export-side constraint stops binding at $s = s_X/k$. Consequently

$$\frac{\partial W}{\partial s}(0; \omega) = \frac{\Pi^{F*} m^{F*}}{1 - \theta} \left[\omega(1 - \delta)\, k\, D_I - (1 - \omega)\, \delta D_N\right],$$

and a marginal intervention raises the objective if and only if $\omega > \omega_0(\kappa) \equiv \delta D_N / [\delta D_N + (1 - \delta) k D_I]$, which is strictly decreasing in κ, equals ω_0 of Proposition 9 at $\kappa = 1$, and tends to one as $\kappa \to 0$.

Proof. Under the inflation-stabilizing regime $\Pi_1^F = \Pi^{F*}$, so (D.1) gives $\psi_0 = \psi^*$, and the purchase removes $\kappa\Delta B_0 = k\Delta X_0$ of effective supply, so $A_0^F = A_0^{F*}(1-ks)$. By Proposition 7 the currency-F side is that of a withdrawal of this size in the economy with A^F in place of M^F, whose price level satisfies $\phi_0^F A_0^F = \Pi^{F*} m^{F*} = R_F$ by the level-jump argument of Proposition 3; since $\phi_0^{F*} A_0^{F*} = R_F$ as well, $\rho_F(s) = A_0^{F*}/A_0^F = 1/(1-ks)$. On the currency-$H$ side, $\phi_0^H M_0^H = R_H$ with $M_0^H = M_0^{H*} + e_0\Delta X_0$ and $e_0 = \phi_0^F/\phi_0^H$, so $\rho_H(s)R_H + \phi_0^F\Delta X_0 = R_H$, and $\phi_0^F\Delta X_0 = R_F\, s/(1-ks)$ gives ρ_H, which is positive if and only if $s < R_H/(R_F + kR_H)$. The money first-order conditions, and hence the stationary identities used in Lemma 2, are unchanged, so the quantities follow as there, and the export-side constraint stops binding when $\rho_F(s)z(q^*_{F,0}) = z(\overline{q})$, that is, at $ks = s_X$. Finally, $\rho'_F(0) = k$ and $\rho'_H(0) = -R_F/R_H$, so the computation in the proof of Proposition 9(iii) goes through with the import and export terms multiplied by k. Since $k = (\Pi^{F*}/\beta)\,\kappa/\left[1 + \kappa(\Pi^{F*}/\beta - 1)\right]$ is strictly increasing in κ, with $k = 1$ at $\kappa = 1$ and $k \to 0$ as $\kappa \to 0$, the properties of $\omega_0(\kappa)$ follow. □

The calibration of Section 6 uses this lemma. Because $\phi_0^{F*} A_0^{F*} = R_F$, the model-implied pool measures effective dominant-currency liquidity, so the observed share s is the same as in the benchmark; the maximal size is $\min\{s_{\text{feas}}, s_X/k\}$, the existence bound $R_H/(R_F + kR_H)$ being slack at every reported parameterization, and the implied weight is the smallest ω whose optimal size reaches the observed s.

E Endogenous dominant-currency settlement

This appendix sets out the environment of Section 5.8, proves Proposition 8, derives the threshold of its part (iv), and reports the calibration.

Environment. Each period, at the beginning of stage 1 and after any announcement, every seller chooses the currency in which it settles its international match, should it meet a foreign buyer. Settling in the seller's own currency is free; settling in the other country's currency costs γ, incurred on meeting a foreign buyer and sunk before the terms of trade are set, so the proportional rule (9) is unchanged. The cost is a fixed characteristic of the seller, distributed across the sellers of each country according to a continuous distribution G with $G(0) = 0$ and a positive density on $(0, \infty)$. Settlement is exclusive, so the buyer pays only in the currency the seller has chosen and learns that currency only in the match. Let λ_X be the share of country H's sellers that settle in currency F and $1 - \lambda_M$ the share of country F's sellers that settle in currency H. Table E.1 gives the probability with which each buyer spends each currency. A balance spent with probability α satisfies, by the envelope

argument of Section 4,

$$\frac{\Pi^k_{+1}}{\beta} \geq 1 + \alpha\, \ell(q^k_{\alpha,+1}),$$

with equality if the currency is carried. Since ℓ is strictly decreasing with $\ell(q) \to (1-\theta)/\theta$ as $q \to 0$, the currency is carried, and q^k_α is interior, if and only if $\alpha(1-\theta)/\theta > \Pi^k/\beta - 1$; otherwise $q^k_\alpha \equiv 0$ and matches settled in that currency do not trade. Assumption 1 makes q^H_α interior for $\alpha \geq \delta$ and q^F_α interior for $\alpha \geq 1-\delta$; with $\Pi^H = \Pi^F$ and $\delta < \frac{1}{2}$, it makes q^k_α interior for $\alpha \geq \delta$, so each buyer carries its own currency. Sellers hold no currency, as before.

Buyer	Currency H	Currency F
Country H	$\delta + (1-\delta)(1-\lambda_M)$	$(1-\delta)\lambda_M$
Country F	$(1-\delta)(1-\lambda_X)$	$\delta + (1-\delta)\lambda_X$

A buyer spends its own currency in domestic matches and each currency in the international matches whose seller settles in it. The benchmark is $\lambda_X = \lambda_M = 1$.

Table E.1: Probabilities of spending each currency under endogenous settlement

Settlement choices. A seller of country H that settles in currency F earns $\theta S(q)$ in an international match, where q is the quantity that its customer's currency-F balances support, and pays γ; settling in currency H earns $\theta S(q')$, where q' is the quantity that the customer's currency-H balances support. With the stationary quantities, it settles in currency F if and only if $\gamma < \Gamma_X(\lambda_X)$, with Γ_X as in Proposition 8. Symmetrically, a seller of country F settles in currency H if and only if $\gamma < \Gamma_M(\lambda_M) \equiv \theta[S(q^H_{\delta+(1-\delta)(1-\lambda_M)}) - S(q^F_{(1-\delta)\lambda_M})]$. A stationary equilibrium therefore requires $\lambda_X = G(\Gamma_X(\lambda_X))$ and $1-\lambda_M = G(\Gamma_M(\lambda_M))$, two conditions that are independent of each other at given inflation rates. With m^{k*} now summed over all holders of currency k, $R_k \equiv \Pi^{k*} m^{k*}$ becomes

$$\begin{aligned} R_H &= \Pi^{H*}\big[z(q^H_{\delta+(1-\delta)(1-\lambda_M)}) + z(q^H_{(1-\delta)(1-\lambda_X)})\big], \\ R_F &= \Pi^{F*}\big[z(q^F_{(1-\delta)\lambda_M}) + z(q^F_{\delta+(1-\delta)\lambda_X})\big]. \end{aligned}$$

Since country H's international sellers receive currency F only from the customers of the sellers that settle in it, part (i) of Assumption 3 caps the operation at $s_{\text{feas}} = (1-\delta)\lambda_X z(q^F_{\delta+(1-\delta)\lambda_X})/R_F$, which is also the ratio used to compute the observed share, as in Section 6. Part (ii) requires every quantity settled in currency F to stay at or below $\overline{q}$, which at the calibrated equilibria first fails at $s_X = 1 - z(q^F_{\delta+(1-\delta)\lambda_X})/z(\overline{q})$. Let $\bar{s}$ be the largest size at which Assumption 3 holds and $\rho_H > 0$. Under the inflation-stabilizing regime the level-jump argument of Proposition 3 uses only aggregate money-market clearing, so for

$s \in [0, \bar{s}]$, $\rho_F(s) = 1/(1-s)$ and $\rho_H(s) = 1 - (R_F/R_H)s/(1-s)$, as in Lemma 2. Extending the equilibrium selection of Section 5 to the settlement shares, the economy returns from $t = 1$ on to the settlement equilibrium it started in; since the period-1 problem is then the stationary one, this continuation is an equilibrium.

Proof of Proposition 8. Part (i). Since ℓ is strictly decreasing and $\Pi^k > \beta$, $q^k_\alpha < \overline{q}$ is nondecreasing in α, strictly where the currency is carried, and continuous, also where the currency stops being carried, because $q^k_\alpha \to 0$ as α decreases to $\theta(\Pi^k/\beta - 1)/(1-\theta)$. Since S is strictly increasing on $[0, \overline{q}]$, the first term of Γ_X is nondecreasing in λ, strictly where currency F is carried, and the second is nonincreasing, strictly where currency H is carried. At every λ at least one of the two currencies is carried, because the two spending probabilities sum to one while Assumption 1 implies $(1-\theta)/\theta > (\Pi^H/\beta - 1) + (\Pi^F/\beta - 1)$, since $\max\{a/\delta, b/(1-\delta)\} \geq a + b$ for $a, b \geq 0$. Currency F is carried on an upper interval of λ and currency H on a lower one, so Γ_X is strictly increasing. A seller of country H compares $\theta S(q) - \gamma$ with $\theta S(q')$, and since G is continuous, ties have probability zero, so $\lambda_X = G(\Gamma_X(\lambda_X))$, with $G(\Gamma) = 0$ for $\Gamma \leq 0$. On the import side, at $\lambda_M = 1$ the buyers of country H spend currency H with probability δ and currency F with probability $1-\delta$, both carried by Assumption 1, and the lower bound of the inflation band gives $\ell(q^H_\delta) = (\Pi^H/\beta - 1)/\delta > (\Pi^F/\beta - 1)/(1-\delta) = \ell(q^F_{1-\delta})$, so $q^H_\delta < q^F_{1-\delta}$, $\Gamma_M(1) < 0$, and no seller of country F pays to settle in currency H.

Part (ii). At $\lambda_X = 0$ the buyers of country F spend currency F with probability δ and currency H with probability $1-\delta$. Currency H is then carried by Assumption 1, since $\delta < \frac{1}{2}$, and the upper bound of the inflation band gives $(\Pi^F/\beta - 1)/\delta > (\Pi^H/\beta - 1)/(1-\delta)$, so $q^F_\delta < q^H_{1-\delta}$, with $q^F_\delta = 0$ if currency F is not carried. Hence $\Gamma_X(0) = \theta[S(q^F_\delta) - S(q^H_{1-\delta})] < 0$ and $G(\Gamma_X(0)) = 0$. The map $\Phi(\lambda) \equiv G(\Gamma_X(\lambda))$ is continuous and nondecreasing from $[0,1]$ into $[0,1]$. If $\Phi(\lambda_1) \geq \lambda_1$ for some $\lambda_1 \in (0,1]$, the set $\{\lambda \in [\lambda_1, 1] : \Phi(\lambda) \geq \lambda\}$ is nonempty and closed, and its largest element $\bar{\lambda}_X$ satisfies $\Phi(\bar{\lambda}_X) \geq \bar{\lambda}_X$. A strict inequality is impossible, because for $\bar{\lambda}_X < 1$ continuity would give $\Phi(\lambda) > \lambda$ just above $\bar{\lambda}_X$, and for $\bar{\lambda}_X = 1$, $\Phi(1) \leq 1$. Hence $\bar{\lambda}_X = \Phi(\bar{\lambda}_X)$, and every equilibrium lies in $[0, \bar{\lambda}_X]$.

Part (iii). In the anticipated scenario the sellers choose after the announcement, while the buyers' balances are those of the stationary equilibrium. Let $\alpha^F_X \equiv \delta + (1-\delta)\bar{\lambda}_X$ and $\alpha^H_X \equiv (1-\delta)(1-\bar{\lambda}_X)$ be the probabilities with which a buyer of country F spends the two currencies at $\bar{\lambda}_X$. For $s \in (0, \bar{s}]$, a seller of country H that settles in currency F trades $q^F_X(s) \equiv z^{-1}(\rho_F(s) z(q^F_{\alpha^F_X})) > q^F_{\alpha^F_X}$, and one that settles in currency H trades $q^H_X(s) \equiv z^{-1}(\rho_H(s) z(q^H_{\alpha^H_X})) \leq q^H_{\alpha^H_X}$, which is zero if currency H is not carried; by part (ii) of Assumption 3, all quantities are at most $\overline{q}$. Since S is strictly increasing on $[0, \overline{q}]$, the gain at $t = 0$, $\Gamma_{X,0}(s) \equiv \theta[S(q^F_X(s)) - S(q^H_X(s))]$, exceeds $\Gamma_X(\bar{\lambda}_X)$, and because G has a positive density, the share of country H's exporters that settle in currency F, $\lambda_{X,0} = G(\Gamma_{X,0}(s))$, exceeds $\bar{\lambda}_X =$

$G(\Gamma_X(\bar{\lambda}_X))$. On the import side, $\Gamma_{M,0}(s) = \theta[S(z^{-1}(\rho_H(s)z(q^H_\delta))) - S(z^{-1}(\rho_F(s)z(q^F_{1-\delta})))] < \Gamma_M(1) < 0$, so imports remain settled in currency F. The volume of exports settled in currency F per seller of country H, $(1-\delta)\lambda_{X,0}q^F_X(s)$, therefore exceeds $(1-\delta)\bar{\lambda}_X q^F_{\alpha^F_X}$ on both margins.

Part (iv) is Lemma E.1. □

The inflation band enters only the import side of part (i) and part (ii). If $(\Pi^H - \beta)/(\Pi^F - \beta)$ exceeds $(1-\delta)/\delta$, then $\Gamma_X(0) > 0$ and $\lambda_X = 0$ is not an equilibrium; if it falls short of $\delta/(1-\delta)$, then $\Gamma_M(1) > 0$ and some of country F's exporters settle in currency H. In the pure-shock scenario the settlement choices precede the operation and do not respond at $t = 0$. In the asymmetric scenario the informed sellers of country H also anticipate the information rent $\theta(\phi^F_0 - \phi^{F*}_0)M$ of Proposition 5 on sales settled in currency F, so the share that settles in currency F rises as well. Under the fixed-injection regime, the temporarily higher currency-F inflation of Proposition A.1 lowers the buyers' currency-F balances along the transition and hence Γ_X, a force toward less settlement in currency F after the impact; characterizing that path is left for future research. If instead each buyer knew its suppliers' settlement currency when choosing a portfolio, as in established supply relationships, the probabilities of Table E.1 would not depend on the settlement shares and the complementarity behind the multiple equilibria would vanish.

The optimal-policy threshold. In the objective (23), the seller's share $\theta S(q_X)$ of an export match becomes the expected net surplus of an internationally matched seller of country H, $V_X(s) \equiv \theta S(q^H_X(s)) + \int \max\{\Gamma_{X,0}(s) - \gamma, 0\}\, dG(\gamma)$, which nets out the costs of settling in currency F.

Lemma E.1 (Threshold under endogenous settlement)**.** *In the dominant-currency equilibrium, under the inflation-stabilizing regime and the anticipated scenario, let $\ell z|_q \equiv \ell(q)z(q)$, with $\ell z|_0 \equiv 0$, and let α^F_X and α^H_X be as in the proof of Proposition 8. Then*

$$\frac{\partial W}{\partial s}(0;\omega) = \frac{R_F}{1-\theta}\left[\omega(1-\delta)\tilde{D}_I - (1-\omega)\,\delta D_N\right],$$

where $D_N \equiv \ell z|_{q^H_\delta}/R_H$, as in Proposition 9 but with the aggregate balances above, and

$$\tilde{D}_I \equiv \frac{1}{R_F}\left\{(1-\theta)\,\ell z|_{q^F_{1-\delta}} + \theta\left[\bar{\lambda}_X\,\ell z|_{q^F_{\alpha^F_X}} - (1-\bar{\lambda}_X)\tfrac{R_F}{R_H}\,\ell z|_{q^H_{\alpha^H_X}}\right]\right\}.$$

If $\tilde{D}_I > 0$, a marginal intervention raises the objective if and only if $\omega > \tilde{\omega}_0 \equiv \delta D_N/[\delta D_N + (1-\delta)\tilde{D}_I]$. As G concentrates at zero, $\bar{\lambda}_X \to 1$ and these reduce to Proposition 9(iii).

Proof. With every import settled in currency F, the import and domestic terms are those of Proposition 9. For the export term, $V_X'(s) = \theta S'(q_X^H(s))\, q_X^{H\prime}(s) + G(\Gamma_{X,0}(s))\, \Gamma_{X,0}'(s)$, since the boundary term of the integral vanishes, the integrand being zero at $\gamma = \Gamma_{X,0}(s)$, the cost of the indifferent seller. With $\Gamma_{X,0}'(s) = \theta[S'(q_X^F(s))\, q_X^{F\prime}(s) - S'(q_X^H(s))\, q_X^{H\prime}(s)]$ and $G(\Gamma_X(\bar\lambda_X)) = \bar\lambda_X$, at $s = 0$

$$V_X'(0) = \bar\lambda_X\, \theta S'(q^F_{\alpha_X^F})\, q_X^{F\prime}(0) + (1-\bar\lambda_X)\, \theta S'(q^H_{\alpha_X^H})\, q_X^{H\prime}(0).$$

As in the proof of Proposition 9(iii), $q'(0) = \rho'(0)\, z(q)/z'(q)$ with $\rho_F'(0) = 1$ and $\rho_H'(0) = -R_F/R_H$, and $S'(q)z(q)/z'(q) = \ell(q)z(q)/(1-\theta)$; when currency H is not carried, $q_X^H \equiv 0$ and its term vanishes. Collecting terms gives the expression, and the limit follows because $\bar\lambda_X \to 1$ sends $q^F_{\alpha_X^F}$ to q_F^* and removes the currency-H term. □

Calibration. At the baseline calibration, G is exponential with mean $\bar\gamma$, chosen so that the share of country H's exporters that settle in currency F is $\bar\lambda_X = 0.9776$, one minus the 2.24 percent of Taiwan's exports invoiced in New Taiwan dollars (the average of the annual shares over 2019–2024, with third currencies treated as currency F). The case $\theta = 0$ is excluded, since then no exporter settles in currency F. At $\theta \in \{0.25, 0.5\}$ a buyer of country F would spend New Taiwan dollars in only $(1-\delta)(1-\bar\lambda_X) \approx 1.3$ percent of its matches, so it carries none, and a seller that settles in New Taiwan dollars makes no export sale. The target thus maps a share of export value into a share of exporters: the model has no export value settled in New Taiwan dollars, and the target only pins down $\bar\gamma$, which equals 26 percent of the seller's surplus in a dollar-settled match and on which the results barely depend (Table E.2). The dominant-currency equilibrium coexists with $\lambda_X = 0$ and with an unstable interior equilibrium, and as $\bar\gamma$ rises, $\bar\lambda_X$ falls until the equilibrium disappears at $1 - \theta(\Pi/\beta - 1)/[(1-\theta)(1-\delta)]$, the share below which foreign buyers would carry New Taiwan dollars, which is 0.963 at $\theta = 0.25$ and 0.890 at $\theta = 0.5$; no stable equilibrium has an intermediate share. Below that share, a higher λ makes foreign buyers spend New Taiwan dollars less often and carry fewer of them, so the value of settling in them falls fast; at $\theta = 0.5$, Γ_X rises from 0.13 at $\lambda = 0.70$ to 0.49 at 0.89. Above it, no sale is settled in New Taiwan dollars and foreign buyers already spend currency F in at least 94 percent of their matches, so Γ_X rises by only 0.2 percent up to $\lambda = 1$, and exporters whose cost exceeds it never switch. On the import side the model settles every import in currency F, while 4.86 percent of Taiwan's imports are invoiced in New Taiwan dollars. At Taiwan's share the observed operation raises export volume at impact by 3.1 and 3.8 percent for $\theta = 0.25$ and 0.5, the extensive margin accounting for less than 1 percent of the rise, and the threshold and

the implied weight differ from their benchmark values by less than 0.002. In the equilibrium with $\lambda_X = 0$, country H's exporters earn no currency F, so $s_{\text{feas}} = 0$ and the operation must buy it from other holders, which by quasi-linearity does not affect prices; an operation of the observed relative size then contracts country H's exports, now settled in the depreciating currency, and makes settlement in currency F profitable for no exporter.

	$\bar{\lambda}_X$	Unstable equilibrium	$\tilde{\omega}_0$	Implied ω (gain)	$\Delta\lambda_{X,0}$ (pp)
$\theta = 0.25$					
Benchmark	1	—	0.698	0.763 (0.04)	—
	0.990	0.907	0.698	0.762 (0.04)	0.009
Taiwan	0.978	0.940	0.699	0.762 (0.04)	0.017
$\theta = 0.5$					
Benchmark	1	—	0.722	0.772 (0.05)	—
	0.990	0.686	0.722	0.772 (0.06)	0.017
Taiwan	0.978	0.756	0.722	0.772 (0.05)	0.032
	0.950	0.824	0.723	0.771 (0.05)	0.055
	0.900	0.882	0.725	0.769 (0.05)	0.082

Baseline calibration of Table 3, inflation-stabilizing regime, anticipated scenario. Each row sets the mean of the exponential distribution G so that the dominant-currency equilibrium has the share $\bar{\lambda}_X$ of country H's exporters settling in currency F; the Taiwan rows use $\bar{\lambda}_X = 0.9776$, and the benchmark row is the limit $\bar{\gamma} \to 0$. The dominant-currency equilibrium requires $\bar{\lambda}_X \geq 0.963$ at $\theta = 0.25$ and $\bar{\lambda}_X \geq 0.890$ at $\theta = 0.5$. The unstable equilibrium is the interior equilibrium below $\bar{\lambda}_X$; the equilibrium $\lambda_X = 0$ exists in every row. $\Delta\lambda_{X,0}$ is the rise in the share of country H's exporters that settle in currency F at $t = 0$ under the observed operation, in percentage points. Welfare gains are in percent, as in Table 4. All entries are computed at full precision and rounded for display.

Table E.2: Endogenous dominant-currency settlement at the baseline calibration

F Bargaining power and pass-through

This appendix develops the pass-through results summarized in Section 6.

The parameter θ does not merely reallocate surplus; it governs how the intervention passes through to prices, quantities, and information. Three channels can be distinguished, and they move in different directions.

Pass-through to prices. The revaluation of the dominant currency is invariant to bargaining power, since by Lemma 2 $\rho_F(s) = 1/(1-s)$ depends only on the share of the dominant-currency stock withdrawn, because the period-0 price level is pinned down by predetermined portfolios and the post-intervention stock, not by the terms of trade inside matches. The equilibrium *depreciation*, by contrast, does depend on θ, because

$d \ln e_0/ds\big|_{s=0} = 1 + R_F/R_H$ and the ratio R_F/R_H of dominant-currency to local-currency real stocks varies with θ through money demand. At the baseline the effect is modest, with the impact depreciation elasticity rising from 3.23 to 3.34 as θ rises from 0 to 0.5.

Pass-through to quantities. A given revaluation moves traded quantities with elasticity

$$\varepsilon_\theta(q) \equiv \frac{d \ln q}{d \ln \rho_F} = \frac{z(q)}{q\, z'(q)},$$

which equals 1 at $\theta = 0$ with linear cost and is strictly increasing in θ: with $c(q) = q$,

$$\varepsilon_\theta(q) = \frac{1 + \theta\,[u(q)/q - 1]}{1 + \theta\,[u'(q) - 1]},$$

and $u(q)/q > u'(q)$ by strict concavity with $u(0) = 0$, so $\varepsilon_\theta > 1$ for $\theta > 0$ and $\partial \varepsilon_\theta / \partial \theta > 0$ at every $q < \overline{q}$.[12] Seller bargaining power therefore *amplifies* the allocative pass-through of the intervention, with $\varepsilon_\theta(q^*_{HF,0})$ rising from 1.00 to 1.55 at the baseline as θ rises from 0 to 0.5. The economics is a flattening of the payment function, for with $\theta > 0$ the marginal payment z' falls as q rises toward $\overline{q}$, so a given increase in the purchasing power of balances buys a larger quantity response.

Note that θ has two faces, amplifying the response of quantities to the operation while dampening the welfare value of that response to the intervener (Proposition 9), because a share θ of the import-margin gain accrues to foreign sellers.

Pass-through of information. Under asymmetric information, Lemma 1 shows that the intervention passes into the terms of trade with the weight of the *counterparty's* bargaining share, so an informed buyer's knowledge moves quantities with weight θ and an informed seller's with weight $1-\theta$. The covert-intervention export response is the fraction $1-\theta$ of the announced one, the import response is the fraction θ, and the information rents scale with the informed party's own share. As $\theta \to 0$ the informed-buyer channel shuts down on the import side while the informed-seller channel reaches full pass-through on the export side.[13] Bargaining power is thus the elasticity of quantities with respect to *private* information about policy, in the same way that ε_θ is the elasticity with respect to the *public* revaluation.

In sum, the pass-through of FXI depends on bargaining power, and the dependence runs in different directions across channels. The CM price-level pass-through of the operation is invariant to θ, though the equilibrium depreciation is not, the quantity pass-through increases

[12] For general convex c, a sufficient condition for $\varepsilon_\theta \geq \varepsilon_0$ is that the average-to-marginal ratio of u exceeds that of c, $u(q)/[q\,u'(q)] \geq c(q)/[q\,c'(q)]$, which holds whenever u is more concave than c.

[13] Online Appendix B shows that this limit depends on the bargaining protocol.

with θ, the welfare pass-through to the intervener decreases with θ, and the information pass-through splits between the parties according to their shares.

G Competitive trade without search

This appendix studies the intervention in an environment without search, as a robustness check on the settlement channel and an extension of Section 5.5. Final-goods producers buy domestic and tradable inputs in competitive markets in every period, with no uncertainty about whom they trade with, and they choose freely among exporters; sellers have convex costs. The two-stage structure is kept, so balances chosen in stage 2 are spent in stage 1 of the next period, and the intervention takes place at stage 2. I do not redo the optimal-policy analysis or the calibration here.

Environment. There are three countries, $i \in \{H, K, F\}$, each with unit masses of final-goods producers, domestic sellers, and exporters. Country F issues the dominant currency, and countries H and K are peripheral, as in Section 5.5. In stage 1, the final-goods producers of country i buy a domestic input q_{Ni} from its domestic sellers, paid in currency i, and a tradable input Q_i in a world market, paid in currency F, and produce $\mathcal{G}(q_{Ni}, Q_i)$ units of the stage-2 good, where $\mathcal{G}$ is Cobb–Douglas with constant returns to scale. The tradable input is the composite of the exports q_{Xj} of the three countries,

$$Q = \Big[\sum_j a_j^{1/\eta} \, q_{Xj}^{(\eta-1)/\eta} \Big]^{\eta/(\eta-1)},$$

with weights $a_j > 0$ that sum to one and an elasticity of substitution $\eta > 0$ between the goods of different exporting countries, with the Cobb–Douglas limit at $\eta = 1$. Every stage-1 market is competitive, each country's exports have a single currency-F price p_{Xj}, and the composite costs $P \equiv \big[\sum_j a_j p_{Xj}^{1-\eta} \big]^{1/(1-\eta)}$. Every seller produces at the cost $c(q) = q^2/2$, so a seller that expects the stage-2 value ϕ^s of the currency it is paid in supplies $q = \phi^s p$ at the price p and, if its expectation is correct, earns $q^2/2$. Stage 2 is as before, and the government of country H buys currency F from its exporters.

Equilibrium. In stage 2 of period $t-1$, a final-goods producer of country $i \neq F$ chooses balances (M^i, M^F) to maximize $\beta[\mathcal{G}(q_{Ni}, Q_i) + \phi_t^i(M^i - p_{Ni} q_{Ni}) + \phi_t^F(M^F - PQ_i)] - \phi_{t-1}^i M^i - \phi_{t-1}^F M^F$, subject to $p_{Ni} q_{Ni} \leq M^i$ and $PQ_i \leq M^F$ in stage 1 of period t; a final-goods producer of country F holds only currency F and splits it between the two inputs. With $\Pi^k > \beta$, a unit of currency left unspent costs more than it returns, so final-goods producers spend

their balances in full, and those of country F spend constant shares on the two inputs. I consider a stationary monetary equilibrium under the inflation-stabilizing regime of Section 5, select the continuation that returns to it from $t = 1$, and maintain Assumption 3, whose part (ii) now requires final-goods producers to spend their balances in full at $t = 0$ and so bounds the size of the operation. For an operation of size $s \equiv \Delta X_0 / M_0^{F*} < R_H/(R_H + R_F)$, the impact prices follow from money-market clearing at stage 2 of period 0 as in Lemma 2, so $\phi_0^F/\phi_0^{F*} = \rho_F(s) > 1 > \rho_H(s) = \phi_0^H/\phi_0^{H*}$, with R_F and R_H now the stationary real balances of currencies F and H times their inflation factors, while the value of currency K is unchanged. I write $\rho \equiv \rho_F(s)$.

Lemma G.1 (Terms of trade under price taking)**.** *In a stage-1 market in which buyers spend the nominal amount E and sellers expect the value ϕ^s of the payment currency, the traded quantity is $q = (\phi^s E)^{1/2}$, whatever buyers expect. In the world market for exports, with D the total currency-F spending on the tradable input and $Z \equiv \sum_j a_j^{2/(1+\eta)} (\phi_j^s)^{(\eta-1)/(1+\eta)}$, country j's exports receive the share $\chi_j = a_j^{2/(1+\eta)} (\phi_j^s)^{(\eta-1)/(1+\eta)}/Z$ of D, and*

$$q_{Xj} = a_j^{1/(1+\eta)} (\phi_j^s)^{\eta/(1+\eta)} \left(\frac{D}{Z}\right)^{1/2}, \qquad \frac{p_{Xj}}{p_{Xj'}} = \left(\frac{a_j\, \phi_{j'}^s}{a_{j'}\, \phi_j^s}\right)^{1/(1+\eta)},$$
$$P = D^{1/2} Z^{(1+\eta)/[2(1-\eta)]} \quad \text{for } \eta \neq 1.$$

Proof. Supply $q = \phi^s p$ and spending $pq = E$ give $q^2 = \phi^s E$. Country j's exporters receive the share $\chi_j \equiv a_j (p_{Xj}/P)^{1-\eta}$ of D, so $\phi_j^s p_{Xj}^2 = a_j p_{Xj}^{1-\eta} P^{\eta-1} D$, which gives $p_{Xj}^{1+\eta} = a_j D P^{\eta-1}/\phi_j^s$ and the price ratios. Substituting into the definition of P gives $P^{2(1-\eta)/(1+\eta)} = D^{(1-\eta)/(1+\eta)} Z$, from which P, $q_{Xj} = \phi_j^s p_{Xj}$, and χ_j follow. □

Proposition G.1 (Symmetric information without search)**.** *In the pure-shock and anticipated scenarios, at $t = 0$: (i) in the pure-shock scenario no stage-1 price or quantity changes, and in stage 2 every seller paid in currency F gains $\rho - 1$ times the original value of its receipts and country H's domestic sellers lose $1 - \rho_H$ times theirs, while final-goods producers, who hold no currency at that point, and country K's domestic sellers are unaffected; (ii) in the anticipated scenario every currency-F price falls by the factor $\rho^{-1/2}$, so relative prices and expenditure shares are unchanged, the output and surplus of every seller paid in currency F rise by the factors $\rho^{1/2}$ and ρ, and those of country H's domestic sellers fall by the factors $\rho_H^{1/2}$ and ρ_H; (iii) in both scenarios every agent in country K weakly gains, and no country's export volume falls, although final-goods producers choose freely among exporters.*

Proof. (i) Nobody foresees the operation, so stage-1 decisions are those of the stationary equilibrium, and in stage 2 each seller's receipts are worth ρ or ρ_H times their original

value, or their original value in currency K; final-goods producers have spent their balances. (ii) Every seller paid in currency F expects $\rho\phi_0^{F*}$, and country H's domestic sellers expect $\rho_H\phi_0^{H*}$. By Lemma G.1 the currency-F price ratios are unchanged, and so are expenditure shares; since balances are predetermined and spent in full, spending in every market is unchanged, and Lemma G.1 gives the factors, a seller with a correct expectation earning $q^2/2$. (iii) Country K's exporters gain in both scenarios, its domestic sellers are paid in currency K, and its final-goods producers obtain the same domestic input and the same or, in the anticipated scenario, $\rho^{1/2}$ times the tradable input. □

Proposition G.2 (Covert intervention without search)**.** *Suppose the intervention is observed in advance only by agents in country H, and let χ_H be the share of spending on the tradable input that goes to country H's exports in the stationary equilibrium. At $t = 0$: (i) the output of country H's domestic sellers falls by the factor $\rho_H^{1/2}$, and the other domestic markets are unchanged; (ii) the currency-F price of country H's exports falls relative to that of every other country by the factor $\rho^{-1/(1+\eta)}$, so their share of spending rises if and only if $\eta > 1$, and their volume rises; (iii) every other country's export volume changes by the factor $[1 + \chi_H(\rho^{(\eta-1)/(1+\eta)} - 1)]^{-1/2}$, so it falls if $\eta > 1$, is unchanged if $\eta = 1$, and rises if $\eta < 1$; (iv) every agent outside country H weakly gains, including the exporters whose volume falls.*

Proof. Country H's exporters expect $\rho\phi_0^{F*}$, its domestic sellers $\rho_H\phi_0^{H*}$, and all other sellers paid in currency F expect ϕ_0^{F*}. Balances are predetermined and spent in full, and country F's final-goods producers spend constant shares, so spending on each input, and with it D, is unchanged, and (i) follows from Lemma G.1. The lemma gives the price ratio in (ii), and since only country H's term in Z changes, $Z/Z^* = 1 + \chi_H(\rho^{(\eta-1)/(1+\eta)} - 1)$, which gives (iii) because q_{Xj} is proportional to $Z^{-1/2}$ for $j \neq H$. Country H's volume changes by the factor $\rho^{\eta/(1+\eta)}(Z/Z^*)^{-1/2}$, which exceeds one, for $\eta \leq 1$ because Z does not rise and for $\eta > 1$ because $Z/Z^* \leq \rho^{(\eta-1)/(1+\eta)}$, which leaves at least $\rho^{1/2}$; its share rises if and only if $\eta > 1$ because its relative price falls. For (iv), an uninformed exporter that sells q at the price q/ϕ_0^{F*} realizes $\rho q^2 - q^2/2$, against $q^{*2}/2$ in the stationary equilibrium, and $(2\rho - 1)Z^*/Z > 1$ because $2(\rho - 1) > \chi_H(\rho^{(\eta-1)/(1+\eta)} - 1)$, as $(\eta - 1)/(1 + \eta) < 1$ and $\chi_H < 1$. Country F's domestic sellers gain on their receipts with unchanged output, country K's are paid in currency K, and every final-goods producer outside country H obtains the same domestic input and more of the tradable input, whose price index P falls, by Lemma G.1 for $\eta \neq 1$ and directly for $\eta = 1$, since then only p_{XH} changes. □

Propositions G.1 and G.2 sharpen Section 5.5. A public intervention raises the value of currency F for every exporter alike, so it lowers every country's currency-F export price in

the same proportion, and buyers who choose freely have no reason to switch; this part of Corollary 2 does not rely on random matching. A covert intervention instead lets country H's informed exporters undercut the exporters of other countries and, when $\eta > 1$, divert demand from them, a competitive devaluation that arises entirely from the information asymmetry. Country K's exporters still gain at impact, because the revaluation of their receipts outweighs the lower price, so the welfare statement of Corollary 2 survives while, when $\eta > 1$, its statement about export volumes does not, and a government of country K that values export volume would have a reason to respond to a covert intervention but not to a public one. The diversion rests on the convention of Online Appendix B that uninformed agents draw no inference. Exporters that inferred the intervention from the lower price of country H's goods would supply as in the anticipated scenario, while inference by buyers would change nothing, since only sellers' expectations enter prices.

With a CES technology instead of a Cobb–Douglas one, country F's final-goods producers, who pay for both inputs in currency F, also shift spending between them. With an elasticity of substitution above one they shift toward the cheaper tradable input, which raises the demand for every country's exports without changing their relative prices and dampens, and can reverse, the fall in part (iii) of Proposition G.2; with an elasticity below one the shift reinforces that fall.

The environment also differs from the main model in other respects. Final-goods producers spend all their balances, so the intervention revalues only sellers' receipts, and the revaluation of unspent balances behind the buyers' entries in Table 1 disappears. Sellers set quantities from their own expectations, so in a market with given spending only sellers' beliefs enter the terms of trade, the limit $\theta \to 0$ of Lemma 1, although in the world market uninformed exporters' volumes still respond through the reallocation of spending. Disclosure to buyers changes nothing, so the concealment motive of Remark 1 has no counterpart inside a trade, but it reappears across countries. When $\eta > 1$, country H's exporters gain from keeping foreign exporters uninformed, since their surplus is $\rho^{(\eta-1)/(1+\eta)} Z^*/Z$ times its value under an announced intervention, while country H's final-goods producers would rather have them informed. Sellers' surplus comes from convex costs rather than from a bargaining share, so the gain of exporters and the loss of country H's domestic sellers do not require bargaining power; sectors are fixed here, but if sellers could choose them, the seller-side reallocation that Remark 2 leaves open would follow. If exporters chose the settlement currency, the complementarity behind Proposition 8 would not arise, since final-goods producers know whom they buy from.

H Omitted proofs

Proof of Proposition 1. From equations (11) and (12), the match surpluses are maximized at $q_{HH} = q_{HF} = q_F = \overline{q}$, at which point $\ell(\overline{q}) = 0$. Substituting this back into equations (14)–(16) yields $\Pi^H = \Pi^F = \beta$, which is the Friedman rule. Conversely, $\Pi^H = \beta$ forces $\ell(q_{HH}) = 0$ in (14), and $\Pi^F = \beta$ forces $\ell(q_{HF}) = \ell(q_F) = 0$ in (15)–(16); since ℓ is strictly decreasing with $\ell(\overline{q}) = 0$, the joint Friedman rule delivers $q_{HH} = q_{HF} = q_F = \overline{q}$, and all currency-$F$ trades are efficient whenever country F alone sets $\mu^F = \beta - 1$. The efficient allocation obtains for any θ; bargaining power affects only the division of the surplus, with sellers receiving $\theta S(\overline{q})$ in every match. Finally, since country H does not control Π^F, setting $\Pi^H = \beta$ alone is insufficient to achieve the efficient allocation. □

Proof of Proposition 2. Take (14) as an example. Differentiating with respect to Π^H gives

$$\frac{1}{\beta} = \delta\, \ell'(q_{HH})\, \frac{\partial\, q_{HH}}{\partial\, \Pi^H},$$

and $\ell' < 0$ by (19), so $\partial q_{HH}/\partial \Pi^H < 0$. Since $m^H_{HB} = z(q_{HH})$ with z strictly increasing, real balances fall with inflation as well:

$$\frac{\partial\, m^H_{HB}}{\partial\, \Pi^H} = z'(q_{HH})\, \frac{\partial\, q_{HH}}{\partial\, \Pi^H} < 0.$$

Thus, high inflation reduces real money balances and the traded quantity, which implies a lower surplus within the DM match for both parties, since buyers receive $(1-\theta)S(q)$ and sellers $\theta S(q)$ with S strictly increasing on $(0, \overline{q})$. The same logic applies to equations (15) and (16) and gives the same conclusion. This completes the proof. □

Proof of Proposition 3. For $t \geq 1$ the growth factor of currency H is constant at $1 + \mu^H$, so the recursion (A.1), derived in the proof of Proposition A.1 in Online Appendix A, reads $g(m^H_{t+1}) = (1+\mu^H) m^H_t$. Since $g(m) = \beta m f(m)$ and f is strictly decreasing with $\beta f(m^{H*}) = 1 + \mu^H$, it follows that

$$g(m) - (1+\mu^H)m = \beta m \left[f(m) - f(m^{H*}) \right],$$

which is strictly positive for $m < m^{H*}$ and strictly negative for $m > m^{H*}$. Suppose $m^H_1 < m^{H*}$. Then $g(m^H_2) = (1+\mu^H) m^H_1 < g(m^H_1)$, so $m^H_2 < m^H_1$ by the monotonicity of g (Assumption 2), and by induction the sequence $\{m^H_t\}$ is strictly decreasing. Its limit would have to solve $g(\bar{m}) = (1+\mu^H)\bar{m}$, whose unique positive solution m^{H*} exceeds every element of the sequence; hence $m^H_t \to 0$ and the path fails the equilibrium selection. Suppose instead

$m_1^H > m^{H*}$. Then $g(m_2^H) = (1+\mu^H)m_1^H > g(m_1^H)$, so the sequence is strictly increasing; it is bounded above by $z(\overline{q})$, hence converges, and its limit would have to be the unique fixed point m^{H*}, which lies below m_1^H, a contradiction, so this path also fails the selection. Hence $m_t^H = m^{H*}$ for all $t \geq 1$, which delivers $\Pi_t^H = \beta f(m^{H*}) = \Pi^{H*}$ and $q_{HH,t} = q_{HH}^*$ for all $t \geq 1$. The same argument applies to currency F: as shown group by group in the proof of Proposition A.1, the real demands of country H's buyers and country F's buyers are both strictly decreasing in the common inflation rate, so aggregate balances satisfy the recursion $g_F(m_{t+1}^F) = (1+\mu^F)m_t^F$ with g_F strictly increasing; constancy of the aggregate at m^{F*} follows, and constancy of the common inflation rate pins down each group's balances and quantities at their stationary values. This proves (i).

For (ii), $m_1^H = \phi_1^H M_0^H = m^{H*}$ gives $\phi_1^H = m^{H*}/M_0^H$, and $\Pi_1^H = \Pi^{H*}$ gives $\phi_0^H = \Pi^{H*} m^{H*}/M_0^H$. On the original path, the same computation gives $\phi_0^{H*} = \Pi^{H*} m^{H*}/M_0^{H*}$, so $\phi_0^H/\phi_0^{H*} = M_0^{H*}/M_0^H < 1$. The computation for currency F is identical and delivers $\phi_0^F/\phi_0^{F*} = M_0^{F*}/M_0^F > 1$. The exchange-rate expression follows from $e_0 = \phi_0^F/\phi_0^H$. □

Proof of Corollary 1. Because preferences are quasi-linear, stage-2 currency demand depends only on the expected future inflation. The operation is executed at stage 2 of period 0 and is observed by everyone in the centralized market, so from the CM of period 0 onward all agents share the same information in every scenario; and because the announcement arrives after the CM of period -1, the portfolios carried into period 0 are chosen on the original path in every scenario. The money supply paths are pinned down by the operation itself. Hence the sequence of stage-2 conditions that determines $\{\phi_t^H, \phi_t^F\}_{t\geq 0}$ is identical across scenarios, and the DM quantities for $t \geq 1$ are functions of these prices alone. □

Proof of Proposition 4. The quantities follow from equations (20)–(22) and Proposition 3. For sellers, the sharing rule $z(q) = \phi_0^k d$ at the realized price of the payment currency k implies that the period-0 payoff is

$$-c(q) + \phi_0^k d = z(q) - c(q) = \theta S(q),$$

while on the original path it is $\theta S(q_0^*)$. The net impact is therefore $\theta\,[S(q_0) - S(q_0^*)]$, whose sign follows from the quantity rankings. For buyers, the DM component is $u(q) - \phi_0^k d = S(q) - \theta S(q) = (1-\theta)S(q)$, so the buyer-side impact combines $(1-\theta)\,[S(q_0) - S(q_0^*)]$ with the revaluation of the buyer's entire predetermined portfolio, equivalently the change in $u(q)$ plus the revaluation of unspent balances as in Table 1, with signs following the same quantity rankings. □

Proof of Lemma 1. With each side's surplus evaluated at its own expected price, the pro-

portional rule reads

$$(1-\theta)\left[\phi^s d - c(q)\right] = \theta\left[u(q) - \phi^b d\right],$$

which rearranges to

$$z(q) = \left[\theta\phi^b + (1-\theta)\phi^s\right] d.$$

By Assumption 3 the buyer's liquidity constraint binds at the relevant perceived prices, so $d = M$. Since z^{-1} is strictly increasing, the traded quantity is strictly increasing in the belief-weighted price: it is at its anticipated-case level exactly when both beliefs equal the realized price, at its pure-shock level exactly when both equal the original-path price, and strictly in between when the two beliefs differ, for any $\theta \in (0,1)$. □

Proof of Proposition 5. Parts (i) and (ii) follow from Lemma 1: the informed-informed matches are country H's domestic matches, the uninformed-uninformed matches are country F's domestic matches, and each international match contains exactly one informed party, whose belief is weighted by the counterparty's share.

For (iii), the informed international seller's realized payoff decomposes as

$$\phi_0^F M_{FB,0}^F - c(q_{FH,0}) = \theta S(q_{FH,0}) + \phi_0^F M_{FB,0}^F - z(q_{FH,0}),$$

and substituting the belief-weighted payment $z(q_{FH,0}) = \left[\theta\phi_0^{F*} + (1-\theta)\phi_0^F\right] M_{FB,0}^F$ from Lemma 1 gives

$$\phi_0^F M_{FB,0}^F - z(q_{FH,0}) = \theta\left(\phi_0^F - \phi_0^{F*}\right) M_{FB,0}^F,$$

the information rent; the domestic statement is the anticipated-case computation.

For (iv), the same decomposition in an import match, where $z(q_{HF,0}) = [\theta\phi_0^F + (1-\theta)\phi_0^{F*}]\, M_{HB,0}^F$, gives

$$\phi_0^F M_{HB,0}^F - z(q_{HF,0}) = (1-\theta)\left(\phi_0^F - \phi_0^{F*}\right) M_{HB,0}^F;$$

in country F's domestic matches quantities are at pure-shock levels and the revaluation of receipts is the pure-shock windfall.

Part (v) holds because the buyer pays $d = M_{FB,0}^F$ in both cases while $q_{FH,0}$ exceeds its pure-shock level. The price paths are unchanged by Corollary 1. □

Proof of Corollary 2. At $t = 0$, country K's domestically matched buyers hold unspent currency-F balances, which appreciate, while its internationally matched buyers hold only unspent currency K, having spent their currency F, whose price path is unchanged because neither the supply of currency K nor its demand is affected by the operation; by Lemma

1, their DM purchases weakly expand for the same payment whenever a match contains an informed party. International sellers in country K are paid in currency F: when they do not anticipate the intervention their receipts appreciate unexpectedly, and when they do they scale production up to the true price and collect the share θ of a weakly larger match surplus; domestic sellers are paid in currency K and are unaffected. Hence every agent in country K weakly gains at $t = 0$. From $t = 1$ on, the argument of Proposition 3, with country K's international buyers added as a third group of currency-F demanders of the same structure, returns every real variable to its stationary value, so no agent in country K is affected thereafter.

The export statements follow from Lemma 1: a currency-F quantity moves with the weight that the informed party's belief carries in the terms of trade. In the pure-shock scenario no party is informed and no export quantity moves; in the anticipated scenario both parties are informed in every match and all exports expand to their fully adjusted levels; in the asymmetric scenario only country H's residents are informed, so country H's exports expand with its customers' weight $1-\theta$, country K's exports to country H's informed buyers expand with weight θ, and its exports to country F are unchanged. No export quantity of country K falls at $t = 0$ in any scenario. Finally, retaliation cannot help country K, which suffers no loss, while its own reserve purchase would impose on it the domestic dilution characterized in Proposition 3. Since country K's export quantities do not fall at $t = 0$ and it has no loss to undo, the incentive to devalue in response is absent. □

Proof of Proposition 6. Currency-H money demand depends only on the currency-H inflation path, and the supply path of currency H is identical across the two policies, so (i) follows from the currency-H block of Proposition 3. The supply of currency F in private circulation and its demand are unchanged under the injection, so (ii) holds. For (iii), $e_0 = \phi_0^F/\phi_0^H$, where ϕ_0^H is identical across the two policies by (i), while $\phi_0^F > \phi_0^{F*}$ under FXI and $\phi_0^F = \phi_0^{F*}$ under the injection. Therefore

$$e_0^{\text{FXI}} > e_0^{\text{inj}} > e_0^*,$$

where the second inequality holds because the injection lowers ϕ_0^H below ϕ_0^{H*} while leaving ϕ_0^F unchanged. Because preferences are quasi-linear, the identity of the injection's recipients affects the distribution of wealth but not equilibrium prices. □

Proof of Lemma 2. By the proof of Proposition 3, whose price characterization extends to the anticipated scenario by Corollary 1 (period-0 money demand is predetermined and the announcement arrives after the last portfolio choice), the period-0 prices are

$$\phi_0^F = \frac{R_F}{M_0^F}, \qquad \phi_0^H = \frac{R_H}{M_0^H},$$

with $R_H = \Pi^{H*} m^{H*}$ and $R_F = \Pi^{F*} m^{F*}$ as in the statement, $M_0^F = (1-s) M_0^{F*}$, and $M_0^H = M_0^{H*} + \Delta Y_0$. Dividing ϕ_0^F by its stationary counterpart $\phi_0^{F*} = R_F / M_0^{F*}$ gives $\rho_F(s) = 1/(1-s)$, strictly increasing. The operation requires $\Delta Y_0 = e_0 \, s \, M_0^{F*}$ with $e_0 = \phi_0^F / \phi_0^H$, so

$$M_0^H = M_0^{H*} + \frac{\phi_0^F}{\phi_0^H} \, s \, M_0^{F*} = M_0^{H*} + \frac{R_F}{R_H} \, \frac{s}{1-s} \, M_0^H,$$

which yields the unique solution

$$M_0^H(s) = \frac{M_0^{H*}}{1 - \frac{R_F}{R_H} \frac{s}{1-s}},$$

well defined, continuously differentiable, and strictly increasing for $s < R_H/(R_H + R_F)$; shrink $\bar{s}$ if necessary so that this bound and Assumption 3 hold on $[0, \bar{s}]$. Hence

$$\rho_H(s) = \frac{M_0^{H*}}{M_0^H(s)} = 1 - \frac{R_F}{R_H} \, \frac{s}{1-s}$$

is strictly decreasing as stated.

Equations (20)–(22), together with the stationary identities

$$\phi_0^{F*} M_{HB,0}^F = z(q_{HF,0}^*), \qquad \phi_0^{F*} M_{FB,0}^F = z(q_{F,0}^*), \qquad \phi_0^{H*} M_{HB,0}^H = z(q_{HH,0}^*),$$

deliver the expressions for q_T, q_X, and q_N. Differentiating,

$$\rho_F'(0) = 1, \qquad \rho_H'(0) = -\frac{R_F}{R_H},$$

and the chain rule gives the derivatives at $s = 0$.

For the cap, country-F buyers' constraint stops binding when

$$\rho_F(s) \, z(q_{F,0}^*) = z(\overline{q}),$$

which solves to s_X. Since $\ell(q_{F,0}^*) = \Pi^{F*}/\beta - 1 < (\Pi^{F*}/\beta - 1)/(1-\delta) = \ell(q_{HF,0}^*)$ and ℓ is strictly decreasing, $q_{F,0}^* > q_{HF,0}^*$, so the export-side constraint reaches slackness strictly before the import-side one. □

Proof of Proposition 9. Write the marginal value of the operation as

$$\frac{\partial W}{\partial s}(s; \omega) = \omega(1-\delta)\mathcal{A}(s) + (1-\omega)\delta\mathcal{B}(s),$$

with

$$\mathcal{A}(s) \equiv (1-\theta)S'(q_T(s))\,q_T'(s) + \theta S'(q_X(s))\,q_X'(s), \qquad \mathcal{B}(s) \equiv S'(q_N(s))\,q_N'(s).$$

Since $q_N(s) \leq q^*_{HH,0} < \overline{q}$ and $q_N' < 0$, $\mathcal{B}(s) < 0$ on $[0, \bar{s}]$; since $q_T(s) < \overline{q}$ on $[0, \bar{s}]$ and $q_X(s) \leq \overline{q}$ by Assumption 3, with equality at most at $\bar{s} = s_X$, $\mathcal{A}(s) > 0$ on $[0, \bar{s})$ and, since $q_T(\bar{s}) < \overline{q}$ and $\theta < 1$, $\mathcal{A}(\bar{s}) > 0$ as well.

Part (i). At $\omega = 0$, $\frac{\partial W}{\partial s} = \delta\mathcal{B} < 0$, so W is strictly decreasing and $s^* = \{0\}$. At $\omega = 1$, $\frac{\partial W}{\partial s} = (1-\delta)\mathcal{A} > 0$ on $[0, \bar{s})$, so W is strictly increasing and $s^* = \{\bar{s}\}$. When $\bar{s} = s_X$, Lemma 2 gives $\rho_F(s_X)\, z(q^*_{F,0}) = z(\overline{q})$, that is, $q_X(\bar{s}) = \overline{q}$, while $q_T(\bar{s}) < \overline{q}$ because $q^*_{HF,0} < q^*_{F,0}$; when $\bar{s} = s_{\text{feas}} < s_X$, all margins remain constrained.

Part (ii). Since

$$\frac{\partial}{\partial \omega}\frac{\partial W}{\partial s} = (1-\delta)\mathcal{A}(s) - \delta\mathcal{B}(s) > 0 \qquad \text{for all } s,$$

W has strictly increasing differences, and by standard monotone comparative statics every selection from $s^*(\omega)$ is nondecreasing. Consequently the set $\{\omega : 0 \in s^*(\omega)\}$ is an interval containing 0. It is closed because W is continuous in (s, ω) and the maximizer correspondence is upper hemicontinuous. It excludes a neighborhood of 1 because $W(\bar{s}; 1) > W(0; 1)$ and W is continuous in ω. Moreover, since $\mathcal{B}$ is continuous and strictly negative on the compact set $[0, \bar{s}]$, there is $b > 0$ with $\mathcal{B} \leq -b$, and $\mathcal{A}$ is bounded above by some $\bar{\mathcal{A}}$; for

$$\omega < \frac{\delta b}{(1-\delta)\bar{\mathcal{A}} + \delta b},$$

$\frac{\partial W}{\partial s} < 0$ everywhere, so $s^*(\omega) = \{0\}$. Hence $\hat{\omega} \equiv \sup\{\omega : 0 \in s^*(\omega)\} \in (0, 1)$ and the interval property delivers the stated characterization. Note that $\hat{\omega} \leq \omega_0$: if $\partial W/\partial s(0; \omega) > 0$ then $0 \notin s^*(\omega)$, so $\omega > \omega_0$ implies $\omega > \hat{\omega}$.

Part (iii). By Lemma 2 and the identity $(1-\theta)S'(q)z(q)/z'(q) = \ell(q)z(q) = A(q) - z(q)$,

$$\frac{\partial W}{\partial s}(0; \omega) = \frac{1}{1-\theta}\left\{\omega\,(1-\delta)\left[(1-\theta)\,\ell z\big|_{q^*_{HF,0}} + \theta\,\ell z\big|_{q^*_{F,0}}\right] - (1-\omega)\,\delta\,\frac{R_F}{R_H}\,\ell z\big|_{q^*_{HH,0}}\right\}.$$

Using $R_H = \Pi^{H*} m^{H*}$, $R_F = \Pi^{F*} m^{F*}$, and the definitions of D_I and D_N, this becomes

$$\frac{\partial W}{\partial s}(0; \omega) = \frac{\Pi^{F*} m^{F*}}{1-\theta}\left[\omega(1-\delta)D_I - (1-\omega)\delta D_N\right],$$

and the sign claims follow, with $D_I, D_N > 0$ because $\ell(q)z(q) > 0$ for $q < \overline{q}$.

Part (iv) is the first-order condition for an interior maximum. □

References for the Online Appendix

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