Monetary Policy in Shallow Financial Markets: A New Open-Economy Macroeconomic Model[1]

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Abstract

We develop a New Open-Economy Macroeconomic (NOEM) Dynamic Stochastic General Equilibrium (DSGE) model tailored to economies characterised by shallow domestic financial markets—a defining structural feature of many emerging market and developing economies (EMDEs). Shallowness is modelled through three interlocking mechanisms: (i) an endogenous *financial participation constraint* that excludes a fraction of households from asset markets (*hand-to-mouth* consumers), (ii) a *bank-spread wedge* that widens countercyclically due to limited domestic collateral intermediation capacity, and (iii) a *foreign-currency liability premium* arising from original-sin borrowing structures. The central bank operates under a managed-float exchange-rate regime and faces a monetary-policy trilemma amplified by shallow markets. We derive closed-form solutions for optimal monetary policy under commitment and discretion, characterise the welfare costs of market shallowness, and identify a *financial depth multiplier* that governs how standard policy tools are transmitted to real activity and inflation.

We provide Bayesian estimations of the NOEM-DSGE model with shallow financial markets, using quarterly data over 2005Q1–2022Q4, via the Random-Walk Metropolis–Hastings (RWMH) algorithm coupled with the Kalman filter for likelihood evaluation, running four parallel chains of 500,000 draws each. Furthermore, we then estimate the model under five distinct scenarios that capture the range of financial-market configurations observed across developing countries: (I) Baseline SSA shallow-market economy; (II) Deep financial market (advanced-economy analogue); (III) Mobile-money transition economy; (IV) Managed exchange-rate peg; and (V) Fiscal dominance with narrow monetary independence. We compute marginal likelihoods via the modified harmonic mean estimator and Laplace approximation, conduct Bayesian model comparison via posterior odds ratios, and assess parameter stability through rolling-window and structural-break tests.

Our quantitative analysis yields the following results. **(i) First,** interest-rate rules require more aggressive parameterisation in shallow markets to achieve equivalent macroeconomic stabilisation. **(ii) Second,** exchange-rate leaning—a managed

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float with exchange-rate feedback—dominates pure inflation targeting by a wider margin when financial markets are shallow. **(iii) Third,** financial-sector deepening reforms are not merely development objectives but have direct monetary policy relevance: they raise the financial depth multiplier, making the central bank's tools more effective, and they are welfare-equivalent to substantial improvements in policy credibility. **(iv) Fourth,** and perhaps most importantly, the stabilisation bias under discretionary policy is amplified by financial shallowness. This provides a new, quantitative argument for institutional frameworks that enhance central-bank commitment—such as inflation targeting frameworks with independent, mandate-bound central banks—in exactly the EMDE contexts where they are often politically most contested.

JEL Codes: C51, E32, E44, E52, F41, G21, O16.

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1 Introduction

The canonical New Keynesian (NK) model—and its small open-economy extension pioneered by Galí (2008)—assumes that households can freely participate in a complete set of domestic and international asset markets. For the advanced economies at which that literature was initially directed, this is a reasonable approximation. For the large majority of the world's countries—particularly those in sub-Saharan Africa, South Asia, and Central America—it is not.

Shallow domestic financial markets are a pervasive feature of EMDEs. By virtually any metric—private credit to GDP, stock-market capitalisation, the share of adults holding formal savings accounts, or the depth of secondary government-bond markets—EMDE financial systems are an order of magnitude smaller and less liquid than those of high-income countries (World Bank, 2022). This shallowness carries profound macroeconomic implications: it severs or attenuates several of the key channels through which monetary policy normally operates. The bank lending channel is weakened when most firms are credit-constrained and bank balance sheets are thin; the wealth and portfolio-balance channels are muted when few households hold financial assets; and the exchange-rate channel is complicated by pervasive foreign-currency invoicing and liability dollarisation.

Despite the practical importance of these issues, the theoretical literature has been slow to integrate financial shallowness into the NOEM-DSGE paradigm in a disciplined, tractable way. Most existing models either treat financial frictions as emanating from the supply side of credit (e.g., financial accelerator models following Bernanke, Gertler, and Gilchrist 1999) or focus on international borrowing constraints (e.g., Mendoza and Smith 2006) without capturing the multi-dimensional character of domestic market shallowness.

This paper fills that gap. We build a NOEM model in the tradition of Obstfeld and Rogoff (1995) and Galí (2008), enriched by three micro-founded features of shallow financial markets. First, drawing on Campbell and Mankiw (1989) and the "rule-of-thumb consumer" literature (Galí, López-Salido, and Vallès, 2007), a fraction $\lambda \in [0, 1)$ of households are *hand-to-mouth* (HtM): they lack access to financial markets and consume exactly their current labour income. The parameter λ directly indexes financial participation depth. Second, the domestic banking sector operates with a countercyclical *spread wedge* Ω_t that amplifies the effective cost of borrowing beyond the policy rate, reflecting limited collateral intermediation capacity. Third, the government (and corporate sector) face a *foreign-currency liability premium* ϕ^* embedded in the uncovered interest parity (UIP) condition, capturing original-sin dynamics.

Together, these three frictions generate a *financial depth multiplier* $\mathcal{M}(\lambda, \Omega, \phi^*)$ that scales the pass-through from the policy rate to aggregate demand and inflation. In a fully deep market ($\lambda = 0$, $\Omega_t = 0$, $\phi^* = 0$), the multiplier equals unity and the model nests the standard small open-economy NK model. As markets shallow, $\mathcal{M}$ falls below one—sometimes dramatically so—with direct implications for optimal policy design.

Consequently, the main contributions of this paper are: (i)We provide the first tractable NOEM-DSGE framework that simultaneously integrates heterogeneous household financial

access, endogenous bank spreads, and original-sin liability structures. (ii) We derive closed-form log-linearised equilibrium conditions and obtain analytic expressions for optimal monetary policy under both commitment and discretion. (iii) We identify and characterise the financial depth multiplier, showing how its three components interact to determine overall policy effectiveness. (iv) We estimate the NOEM model and conduct policy experiments comparing interest-rate rules, exchange-rate pegs, and hybrid regimes under varying degrees of financial depth. (v) We quantify the welfare gains from financial-sector deepening reforms, finding that reforms that raise financial participation from 30 % to 70 % of households are welfare-equivalent to a 150-basis-point permanent reduction in steady-state inflation.

Indeed, Bayesian estimation of DSGE models has become the dominant empirical strategy in quantitative macroeconomics since the seminal contributions of Smets and Wouters (2003) and An and Schorfheide (2007). The approach offers three decisive advantages over classical calibration: (i) it disciplines parameter uncertainty through prior distributions informed by micro-econometric evidence, thereby reducing the risk of arbitrary calibration choices; (ii) it delivers the full joint posterior distribution, enabling coherent propagation of parameter uncertainty into policy conclusions; and (iii) it provides principled model comparison via Bayes factors, allowing formal tests of nested and non-nested hypotheses.

For the NOEM–DSGE model with shallow financial markets, Bayesian estimation is particularly valuable because the three shallow-market parameters—the HtM share λ, the bank spread $\overline{\Omega}$ and its cyclicality δ_Ω, and the liability premium ϕ^*—are inherently difficult to calibrate precisely from micro-data alone, and their joint identification from aggregate time-series data is non-trivial. The Bayesian posterior reveals which parameters the data identify sharply (narrow posteriors near the prior mode) and which remain largely prior-determined (posteriors close to their priors), providing a transparent record of the model's empirical content.

Furthermore, we estimate the model under five scenarios that span the range of financial-market and institutional configurations relevant to developing-country policy analysis: **Scenario I (Baseline SSA):** The full shallow-market model calibrated to the SSA countries. **Scenario II (Deep Market):** λ restricted to the prior mode of 0.10, $\overline{\Omega}$ and ϕ^* restricted near zero; serves as the advanced-economy benchmark. **Scenario III (Mobile-Money Transition):** A time-varying HtM share $\lambda_t = \lambda_0 e^{-\gamma t}$ captures the progressive financial inclusion driven by mobile money (M-Pesa type). **Scenario IV (Managed Peg):** Exchange-rate feedback coefficient ϕ_e elevated and the interest-rate rule constrained by near-peg dynamics. **Scenario V (Fiscal Dominance):** A government budget constraint that links the primary deficit to the money supply, reducing effective monetary independence.

Our quantitative analysis yields several practical conclusions for central banks in EMDEs. **First**, achieving a given degree of macroeconomic stabilisation in shallow markets requires a more aggressively parameterised interest-rate rule than in deep-market settings. **Second**, a managed float with exchange-rate feedback outperforms pure inflation targeting by a wider margin precisely where financial markets are shallowest. **Third**, financial-sector

deepening is not merely a development objective: it raises the financial depth multiplier directly, sharpening the transmission of monetary policy, and the resulting welfare gains are comparable in magnitude to substantial improvements in policy credibility. **Fourth**, and perhaps most importantly, discretionary policy's stabilisation bias is itself amplified by financial shallowness, strengthening the case for institutional commitment devices—such as inflation targeting frameworks with independent, mandate-bound central banks—in exactly the EMDE contexts where they are often politically most contested.

Specifically, Bayesian estimations of the NOEM–DSGE model with explicit shallow financial market structure reveal that: (i) The HtM share λ is tightly estimated at 0.548 (90% HPD: $[0.468, 0.631]$) from aggregate time-series data, confirming that financial exclusion is robustly identified from macroeconomic dynamics, not merely imposed by calibration. (ii) The financial depth multiplier has a posterior mean of 0.419 and 90% HPD $[0.342, 0.501]$, establishing with probability 0.914 that $\mathcal{M} < 0.50$—monetary policy transmits less than half as effectively in shallow SSA markets as in the deep-market benchmark. (iii) Bayesian model comparison strongly favours the shallow-market specification (Scenario I) over the deep-market alternative (Scenario II), with a log Bayes factor of +18.4—"very strong" evidence in the Kass and Raftery (1995) scale. (iv) The Mobile-Money Transition model (Scenario III) generates a multiplier that rises from 0.40 (2005) to 0.53 (2022), quantifying the improving monetary policy effectiveness from mobile financial inclusion. (v) Structural parameters are stable across sub-samples; only shock volatilities show a structural break (2015Q2), consistent with improved monetary frameworks in all four countries. (vi) Under fiscal dominance (Scenario V), the effective inflation feedback falls to 1.28 and the sacrifice ratio rises to 2.71—the highest across all scenarios—highlighting the severe cost of monetary–fiscal coordination failures. (vii) Country-specific estimation reveals substantial heterogeneity in HtM shares (Kenya 0.38 vs. Cameroon 0.72), with important implications for country-level monetary policy design within regional monetary unions.

Our paper connects several strands of research. On the NOEM front, we build directly on Obstfeld and Rogoff (1995), Galí (2008), and Monacelli (2005). On financial frictions, we draw on the financial accelerator of Bernanke, Gertler, and Gilchrist (1999) and the sudden-stop literature (Calvo, Izquierdo, and Mejía, 2004). On heterogeneous agents and limited financial participation, our approach is related to Galí, López-Salido, and Vallès (2007), Bilbiie (2008), and the HANK literature (Kaplan, Moll, and Violante, 2018). On original sin and currency mismatches, we build on Eichengreen and Hausmann (1999) and Céspedes, Chang, and Velasco (2004).

The remainder of the paper is organised as follows. Section 2 presents the model environment. Section 3 derives the equilibrium conditions and log-linearises them. Section 4 characterises optimal monetary policy. Section 5 defines and analyses the financial depth multiplier. Section 6 calibrates the model. Section 7 conducts estimation of the model. Section 8 presents estimation results. Section 9 conducts the welfare analysis. Section 10 performs the counterfactual analysis. Section 11 examines sensitivity and robustness. Section 12 concludes.

2 The NOEM-DSGE Model of Monetary Policy in Shallow Financial Markets

2.1 Overview

The world consists of two countries: the *Home* economy (a small open economy, SOE) and the *Foreign* economy (the rest of the world, denoted by asterisks). Time is discrete and indexed $t = 0, 1, 2, \ldots$. All shocks are log-normally distributed.

The Home economy is populated by a unit measure of households, a unit measure of firms (divided into a continuum of monopolistically competitive producers and a representative retail aggregator), a domestic banking sector, and a benevolent central bank. The Foreign economy is large and its variables are taken as exogenous from the perspective of the SOE.

2.2 Households

2.2.1 Household Heterogeneity

A fraction $\lambda \in [0, 1)$ of Home households are *financially excluded* (type R, for rule-of-thumb) and have no access to domestic or international financial markets. The remaining fraction $1 - \lambda$ are *Ricardian* households (type O, for optimising) who hold a portfolio of domestic bonds, foreign bonds, and equity claims.

Ricardian households $(1-\lambda)$. A representative Ricardian household maximises lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^O)^{1-\sigma}}{1-\sigma} - \frac{(N_t^O)^{1+\phi}}{1+\phi} \right], \tag{1}$$

where C_t^O is consumption, N_t^O is labour supply, $\beta \in (0, 1)$ is the discount factor, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution (IES), and $\phi > 0$ is the inverse Frisch elasticity.

Budget constraint in nominal terms:

$$P_t C_t^O + B_t + \mathcal{E}_t B_t^* = W_t N_t^O + (1 + i_{t-1}) B_{t-1} + (1 + i_{t-1}^* + \Theta_{t-1}) \mathcal{E}_t B_{t-1}^* + \Pi_t + T_t, \tag{2}$$

where P_t is the CPI, B_t are nominal domestic bonds paying gross return $1 + i_t$ (the policy rate net of the bank spread), B_t^* are foreign currency–denominated bonds at the foreign rate i_t^* augmented by the foreign-currency liability premium Θ_t, $\mathcal{E}_t$ is the nominal exchange rate (Home currency per unit of Foreign), W_t is the nominal wage, Π_t are firm profits, and T_t are lump-sum transfers from the government.

The *effective domestic borrowing rate* faced by optimising households is:

$$i_t^{\text{eff}} = i_t + \Omega_t, \tag{3}$$

where $\Omega_t \geq 0$ is the endogenous bank spread defined in Section 2.5.

Hand-to-mouth households (λ). Type-R households cannot save or borrow. Their consumption equals their current after-tax labour income:

$$P_t C_t^R = W_t N_t^R + T_t^R, \tag{4}$$

where T_t^R are targeted government transfers. They supply labour inelastically at the competitive wage. Absent a bequest motive, they have no reason—and no mechanism—to smooth consumption intertemporally.

Aggregate consumption. Aggregate consumption is the population-weighted sum:

$$C_t = (1-\lambda) C_t^O + \lambda C_t^R. \tag{5}$$

When $\lambda = 0$ the model collapses to the representative-agent benchmark.

2.3 Consumption Aggregator and Trade

Following Galí (2008), preferences over Home ($C_{H,t}$) and Foreign ($C_{F,t}$) goods follow a CES aggregator:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{6}$$

where $\alpha \in (0,1)$ is the degree of openness (import share) and $\eta > 0$ is the elasticity of substitution between Home and Foreign goods.

The corresponding CPI is:

$$P_t = \left[(1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \tag{7}$$

where $P_{H,t}$ and $P_{F,t}$ are the prices of Home and Foreign goods.

2.4 Firms

2.4.1 Final Good Producers

A competitive final good producer aggregates a continuum of differentiated intermediate goods $j \in [0,1]$ using a Dixit-Stiglitz aggregator with elasticity $\varepsilon > 1$:

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{8}$$

2.4.2 Intermediate Goods Producers

Each intermediate good producer j operates a linear technology:

$$Y_t(j) = A_t N_t(j), \tag{9}$$

where A_t is aggregate total factor productivity (TFP), following an AR(1) process in logs: $\ln A_t = \rho_A \ln A_{t-1} + \epsilon_t^A$.

Price setting follows Calvo (1983): each period, a fraction $1-\theta$ of firms can reset their price, while the remaining fraction θ keep last period's price unchanged. Firms resetting their price at t choose $P_{H,t}^*$ to maximise:

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \Lambda_{t,t+k} \left[P_{H,t}^* Y_{t+k|t} - W_{t+k} N_{t+k|t} \right], \tag{10}$$

where $\Lambda_{t,t+k}$ is the stochastic discount factor of the representative Ricardian household.

2.5 The Banking Sector and the Spread Wedge

We introduce a simple banking sector to endogenise the spread Ω_t. A representative bank takes deposits from Ricardian households at the deposit rate i_t^d and makes loans to firms at the loan rate i_t^ℓ. The policy rate i_t is the rate on reserves held at the central bank.

Shallow-market specification. In a deep financial market, arbitrage ensures $i_t^\ell \approx i_t$. In a shallow market, banks face a *collateral scarcity constraint*:

$$\Omega_t = \overline{\Omega} + \delta_\Omega \left(\frac{D_t}{\overline{D}} - 1 \right) + \xi_t^b, \tag{11}$$

where $D_t/\overline{D}$ is the ratio of loan demand to its steady-state level, $\overline{\Omega} \geq 0$ is the steady-state spread, $\delta_\Omega > 0$ governs countercyclical spread amplification, and $\xi_t^b \sim \text{iid}(0, \sigma_b^2)$ is a banking-sector shock. The spread widens in expansions because shallow domestic collateral markets cannot accommodate the surge in credit demand without requiring higher external-finance premia.

2.6 The Foreign-Currency Liability Premium and Modified UIP

Standard UIP equates expected returns on domestic and foreign bonds. In our model, the SOE government and corporate sector borrow partly in foreign currency—*original sin*—creating a liability-side currency mismatch. We capture this via a modified UIP:

$$(1 + i_t) = (1 + i_t^*) \frac{\mathbb{E}_t \mathcal{E}_{t+1}}{\mathcal{E}_t} \cdot (1 + \phi_t^*)\,(1 + \Psi_t), \tag{12}$$

where ϕ_t^* is the foreign-currency liability premium and Ψ_t is a risk premium shock. Taking logs and linearising:

$$i_t = i_t^* + \mathbb{E}_t \Delta e_{t+1} + \phi^* + \psi_t, \tag{13}$$

where $e_t \equiv \ln \mathcal{E}_t$, $\phi^* = \ln(1 + \phi^*)$, and $\psi_t = \ln(1 + \Psi_t)$. A higher ϕ^* raises the interest rate consistent with external balance at any given exchange rate, thereby tightening domestic monetary conditions beyond the central bank's intent.

2.7 Central Bank and Monetary Policy

The central bank sets the short-term nominal interest rate i_t (the policy instrument) and, depending on the regime, also intervenes in the foreign-exchange market. It faces the following structural constraints:

1. The *zero lower bound* (ZLB): $i_t \geq 0$.
2. The *trilemma*: under open capital markets, it cannot simultaneously fix the exchange rate, set an independent monetary policy, and maintain free capital flows. Shallow markets weaken (but do not eliminate) capital mobility, relaxing the trilemma constraint.

We consider three policy frameworks:

Inflation targeting (IT) The central bank follows a Taylor-type rule:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\big[\overline{r} + \phi_\pi \pi_{H,t} + \phi_y \widetilde{x}_t\big] + \epsilon_t^i, \tag{14}$$

where $\pi_{H,t}$ is Home-goods inflation, $\widetilde{x}_t$ is the output gap, $\phi_\pi > 1$, $\phi_y \geq 0$, and $\rho_i \in [0,1)$ is the smoothing parameter.

Managed float (MF) The central bank also targets the exchange-rate gap: $\phi_e > 0$ is added to equation (14).

Hard peg (HP) The exchange rate is fixed: $\Delta e_t = 0$, and i_t is determined residually by (13).

2.8 Market Clearing and Current Account

Goods market clearing:

$$Y_t = C_{H,t} + C^*_{H,t} + G_t, \tag{15}$$

where $C^*_{H,t}$ is foreign demand for Home goods and G_t is government spending. The current account is:

$$\mathcal{E}_t \dot{B}^*_t = P_{H,t} Y_t - P_t C_t - P_t G_t + (i^*_{t-1} + \Theta_{t-1}) \mathcal{E}_t B^*_{t-1}, \tag{16}$$

where foreign assets B^*_t may be negative if the SOE is a net debtor.

3 Equilibrium and Log-Linearisation

3.1 Ricardian Euler Equation

Log-linearising the Ricardian household's first-order condition yields:

$$c^O_t = \mathbb{E}_t\, c^O_{t+1} - \frac{1}{\sigma}\big(i_t + \omega_t - \mathbb{E}_t\, \pi_{t+1} - \rho\big), \tag{17}$$

where lowercase letters denote percentage deviations from steady state, $\omega_t \equiv \ln(1+\Omega_t)$, and $\rho \equiv -\ln\beta$.

3.2 Aggregate Euler Equation with Financial Exclusion

For HtM households, $c_t^R = w_t + n_t - p_t + \tau_t^R$ (from the budget constraint (4)). Using the aggregate identity (5) and the equilibrium labour market:

$$\begin{aligned} c_t &= (1-\lambda)\, c_t^O + \lambda\, c_t^R \\ &= \mathbb{E}_t\, c_{t+1} - \frac{1-\lambda}{\sigma}\big(i_t + \omega_t - \mathbb{E}_t\, \pi_{t+1} - \rho\big) + \lambda\, \Delta w_t + u_t^c, \end{aligned} \tag{18}$$

where u_t^c collects higher-order terms and Δw_t is real wage growth. The coefficient on the real interest rate is $(1-\lambda)/\sigma$ rather than $1/\sigma$—the *first* manifestation of the financial depth multiplier.

3.3 New Keynesian IS Curve

Combining goods market clearing (15) with the aggregate Euler equation and the trade balance condition:

$$\widetilde{x}_t = \mathbb{E}_t\, \widetilde{x}_{t+1} - \frac{1-\lambda}{\sigma_\alpha}\big(\widehat{\imath}_t + \omega_t - \mathbb{E}_t\, \widehat{\pi}_{t+1}\big) + \Gamma_\alpha\, \mathbb{E}_t\, \Delta \widehat{q}_{t+1} + \lambda\, \Delta \widehat{w}_t + u_t^{\mathrm{IS}}, \tag{19}$$

where $\widehat{\imath}_t = i_t - \overline{r}$ is the interest rate gap, $\widehat{\pi}_t = \pi_t - \overline{\pi}$ is the inflation gap, $\widehat{q}_t$ is the real exchange rate gap, $\sigma_\alpha = \sigma/(1-\alpha(2-\alpha)(1-1/\eta))$ is the effective risk-aversion parameter under openness, and Γ_α captures expenditure-switching effects. The IS sensitivity to the real rate, $\kappa^{IS} \equiv (1-\lambda)/\sigma_\alpha$, falls with λ: financially excluded consumers do not respond to interest rates.

3.4 New Keynesian Phillips Curve

From the Calvo price-setting model, the aggregate supply equation is:

$$\widehat{\pi}_{H,t} = \beta\ \mathbb{E}_t\, \widehat{\pi}_{H,t+1} + \kappa\, \widehat{mc}_t + u_t^{\pi}, \tag{20}$$

where $\kappa = (1-\theta)(1-\beta\theta)/\theta > 0$ and $\widehat{mc}_t$ is the log-deviation of real marginal cost from its flexible-price value.

Marginal cost depends on the output gap and the real exchange rate (reflecting imported intermediate inputs):

$$\widehat{mc}_t = \frac{\sigma_\alpha(1-\lambda)^{-1} + \phi}{1+\phi}\, \widetilde{x}_t + \frac{\alpha(1-\lambda)^{-1}}{\eta(1+\phi)}\, \widehat{q}_t. \tag{21}$$

Note that when $\lambda > 0$, marginal cost is *more* sensitive to the output gap (because fewer households buffer income shocks through savings), creating a steeper effective Phillips curve.

3.5 Exchange Rate and UIP

The real exchange rate $q_t \equiv e_t + p_t^* - p_t$ evolves according to:

$$\widehat{q}_t = \mathbb{E}_t\, \widehat{q}_{t+1} + \left(\widehat{\imath}_t - \mathbb{E}_t\, \widehat{\pi}_{t+1}\right) - \left(\widehat{\imath}_t^* - \mathbb{E}_t\, \widehat{\pi}_{t+1}^*\right) - \phi^* - \psi_t, \tag{22}$$

derived by combining (13) with the definition of the real exchange rate.

3.6 System Summary

The log-linearised equilibrium is characterised by the four-equation system (19)–(22) together with the monetary policy rule (14) and the exogenous processes for A_t, i_t^*, ϕ_t^*, ψ_t, ξ_t^b, and G_t.

Define the state vector $S_t \equiv [\widetilde{x}_t,\, \widehat{\pi}_{H,t},\, \widehat{q}_t,\, \omega_t]'$ and the policy instrument $\widehat{\imath}_t$. The system can be written compactly as:

$$\mathbf{A}\ \mathbb{E}_t\, S_{t+1} = \mathbf{B}\, S_t + \mathbf{C}\, \widehat{\imath}_t + \mathbf{D}\, Z_t, \tag{23}$$

where Z_t is the vector of exogenous shocks and the coefficient matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ depend on the deep parameters and, critically, on λ and ϕ^*.

Proposition 3.1 (Determinacy)**.** *Under the IT rule* (14)*, the system* (23) *has a unique stationary rational expectations equilibrium if and only if the Taylor principle holds in the modified form:*

$$\phi_\pi + \frac{1-\beta}{\kappa}\,\phi_y > 1 + \frac{\lambda\,\kappa\,\sigma_\alpha}{(1-\lambda)(1+\phi\,\sigma_\alpha)}. \tag{24}$$

The right-hand side exceeds unity when $\lambda > 0$, implying a stricter *Taylor principle in economies with financial exclusion.*

Proof. Apply Blanchard-Kahn conditions to the system (23). The characteristic polynomial of $\mathbf{A}^{-1}\mathbf{B}$ has two roots that must exceed unity in modulus. Solving the inequality yields condition (24). □

4 Optimal Monetary Policy

4.1 Social Welfare and the Loss Function

We derive a second-order approximation to the households' aggregate welfare following Woodford (2003). Allowing for HtM consumers and the bank spread, the welfare-relevant loss function is:

$$\mathcal{L}_t = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\varepsilon}{\kappa}\,\widehat{\pi}_{H,t}^2 + \Phi(\lambda,\Omega)\,\widetilde{x}_t^2 + \varphi_q\,\widehat{q}_t^2 + \varphi_\omega\,\omega_t^2\right], \tag{25}$$

where

$$\Phi(\lambda,\Omega) \equiv \frac{(1-\lambda)(\sigma_\alpha^{-1}+\phi)}{1} + \frac{\lambda^2\sigma_\alpha}{(1-\lambda)} + \delta_\Omega\,\overline{\Omega} \tag{26}$$

is the *output gap weight*, which is increasing in λ and in $\overline{\Omega}$. Intuitively, financial exclusion and high spreads amplify the welfare cost of output fluctuations because excluded households cannot self-insure.

4.2 Optimal Policy under Commitment

The Ramsey problem is to minimise (25) subject to (19)–(22) and the exogenous shock processes. Define μ_t and ν_t as the co-state variables associated with the IS and NKPC constraints respectively. First-order conditions imply:

$$\widehat{\pi}_{H,t} = -\frac{\kappa}{\varepsilon}\,\nu_t + \beta^{-1}\nu_{t-1}, \tag{27}$$

$$\widetilde{x}_t = \frac{\kappa}{\Phi}\big(\nu_t - \nu_{t-1}\big) + \frac{\kappa_q}{\Phi}\,\widehat{q}_t, \tag{28}$$

$$\widehat{q}_t = -\frac{\varphi_q}{\Phi}\,\widetilde{x}_t. \tag{29}$$

Combining (27)–(29) yields the *targeting rule*:

$$\widehat{\pi}_{H,t} + \frac{\kappa\,\Phi(\lambda,\Omega)}{\varepsilon}\big(\widetilde{x}_t - \widetilde{x}_{t-1}\big) + \frac{\kappa\,\varphi_q}{\varepsilon}\,\big(\widehat{q}_t - \widehat{q}_{t-1}\big) = 0. \tag{30}$$

This generalises the standard "price stability plus output gap smoothing" rule to incorporate real-exchange-rate smoothing, with both gap and exchange-rate terms weighted by functions of financial depth.

Corollary 4.1. *As $\lambda \to 0$ and $\overline{\Omega} \to 0$, the targeting rule (30) converges to the standard flexible inflation-targeting rule of Woodford (2003).*

4.3 Optimal Policy under Discretion

Under discretion, the central bank re-optimises each period without commitment. The discretionary equilibrium satisfies:

$$\widehat{\pi}_{H,t} = -\frac{\kappa\,\Phi(\lambda,\Omega)}{\varepsilon}\,\widetilde{x}_t, \tag{31}$$

implying a *stabilisation bias*: the policymaker tolerates more output volatility to achieve lower inflation variance. The bias is larger when $\Phi(\lambda,\Omega)$ is large—that is, when financial markets are shallow.

Proposition 4.1 (Stabilisation Bias Amplification)**.** *The welfare loss from discretion relative to commitment, $\mathcal{L}^{\text{disc}} - \mathcal{L}^{\text{comm}}$, is strictly increasing in λ and $\overline{\Omega}$. Shallow financial markets amplify the time-inconsistency problem of monetary policy.*

5 The Financial Depth Multiplier

5.1 Definition

We define the *financial depth multiplier* as the ratio of the real-rate sensitivity of aggregate demand in a shallow-market economy to that in a deep-market benchmark:

$$\mathcal{M} \equiv \frac{\partial \widetilde{x}_t / \partial \widehat{r}_t \big|_{\lambda,\Omega,\phi^*}}{\partial \widetilde{x}_t / \partial \widehat{r}_t \big|_{\lambda=0,\Omega=0,\phi^*=0}} = (1-\lambda) \underbrace{\frac{1}{1+\delta_\Omega \overline{\Omega}}}_{\text{spread channel}} \underbrace{\frac{1}{1+\phi^* \cdot \eta_e}}_{\text{liability premium}}, \tag{32}$$

where $\widehat{r}_t \equiv \widehat{\imath}_t - \mathbb{E}_t \widehat{\pi}_{t+1}$ is the real interest rate gap and η_e is the semi-elasticity of the exchange rate with respect to the real rate.

Proposition 5.1 (Properties of the Multiplier)**.** *The financial depth multiplier $\mathcal{M}$ defined in (32) satisfies:*

(i) $\mathcal{M} \in (0,1]$*, with* $\mathcal{M} = 1$ *iff* $\lambda = 0$*,* $\overline{\Omega} = 0$*, and* $\phi^* = 0$*.*

(ii) $\partial \mathcal{M} / \partial \lambda < 0$*: higher financial exclusion attenuates policy transmission.*

(iii) $\partial \mathcal{M} / \partial \overline{\Omega} < 0$*: a higher steady-state spread wedge attenuates transmission.*

(iv) $\partial \mathcal{M} / \partial \phi^* < 0$*: a larger original-sin premium attenuates transmission.*

(v) The three components interact multiplicatively: each source of shallowness further reduces the effectiveness of each remaining channel.

5.2 Interest-Rate Pass-Through

A key implication is for interest-rate pass-through. Define pass-through Π^{IR} as the change in lending rates for a unit change in the policy rate. In the deep-market benchmark, $\Pi^{\text{IR}} = 1$. In the shallow-market model:

$$\Pi^{\text{IR}} = \frac{1}{1+\delta_\Omega} \cdot \mathcal{M}, \tag{33}$$

so that shallowness compounds the attenuation both through the multiplier directly and through reduced bank pass-through.

5.3 The Sacrifice Ratio

The *sacrifice ratio* $\mathcal{SR}$ measures output loss per unit of disinflation. In the NOEM model:

$$\mathcal{SR} \equiv \frac{\Delta Y}{\Delta \pi} = \frac{1}{\kappa \cdot \mathcal{M}}, \tag{34}$$

which is decreasing in $\mathcal{M}$. Shallow markets raise the sacrifice ratio, meaning disinflation programmes are costlier in terms of forgone output.

6 Calibration

6.1 Benchmark Economy

We calibrate the model to a representative sub-Saharan African economy, drawing on data from Ghana, Cameroon, Kenya, and Senegal for the period 2005–2022. One period corresponds to one quarter.

Table 1: Baseline Calibration

Parameter	Description	Value	Source
Preferences and technology			
β	Discount factor	0.985	Quarterly $r = 6\%$ p.a.
σ	Inverse IES	1.50	Galí (2008)
ϕ	Inverse Frisch	2.00	Blanchard and Kiyotaki (1987)
ε	Price elasticity	11.0	10% markup
θ	Calvo price stickiness	0.70	4-quarter avg price duration
α	Import share	0.40	UNCTAD trade data
η	Trade elasticity	1.50	Anderson and van Wincoop (2004)
Financial shallowness			
λ	HtM household share	0.55	Findex survey data
$\overline{\Omega}$	Steady-state bank spread	0.08	Avg. bank spread (ann.)
δ_Ω	Spread cyclicality	0.30	IMF FSAP estimates
ϕ^*	Liability premium	0.04	Original-sin index
Monetary policy (IT benchmark)			
ϕ_π	Inflation feedback	1.50	Taylor (1993)
ϕ_y	Output gap feedback	0.50	Taylor (1993)
ρ_i	Interest rate smoothing	0.70	GMM estimation
Shock processes			
ρ_A	TFP persistence	0.80	HP-filtered data
σ_A	TFP volatility	0.012	HP-filtered data
σ_ψ	Risk premium volatility	0.020	Uribe and Yue (2006)
σ_b	Bank spread shock vol.	0.015	Estimated

6.2 Deep-Market Benchmark

For comparison, we calibrate a deep-market benchmark economy with $\lambda = 0.10$ (10% HtM), $\overline{\Omega} = 0.01$, $\delta_\Omega = 0.05$, and $\phi^* = 0.005$, consistent with typical advanced-economy parameters.

6.3 Implied Financial Depth Multiplier

Given the baseline calibration, the financial depth multiplier evaluates to:

$$\mathcal{M}^{\text{SSA}} = (1 - 0.55) \times \frac{1}{1 + 0.30 \times 0.08} \times \frac{1}{1 + 0.04 \times 1.2} \approx 0.40, \tag{35}$$

compared with $\mathcal{M}^{\text{AE}} \approx 0.87$ for the advanced-economy benchmark. Thus, monetary policy in the representative SSA economy transmits to aggregate demand with only about 46 % of the effectiveness of a typical advanced economy, and about 40 % of a perfectly deep market.

7 Estimations of the NOEM Model

7.1 Data

7.1.1 Observable Variables

We estimate the model using seven quarterly observable variables for each of the four target economies, stacked into a balanced panel:

$$\mathbf{Y}_t = \left[\Delta \ln Y_t,\ \pi_{H,t},\ \Delta \ln C_t,\ i_t,\ \Delta e_t,\ \Omega_t,\ \pi_t^*\right]^\top, \quad t = 1, \ldots, T, \tag{36}$$

where $\Delta \ln Y_t$ is real GDP growth, $\pi_{H,t}$ is domestic CPI inflation, $\Delta \ln C_t$ is private consumption growth, i_t is the central bank policy rate, Δe_t is the exchange-rate change (log difference), Ω_t is the commercial bank lending spread over the policy rate, and π_t^* is US/Euro-area CPI inflation.

7.1.2 Data Sources and Transformations

Table 2: Observable Variables: Sources and Transformations

Variable	**Description**	**Transform**	**Source**
$\Delta \ln Y_t$	Real GDP growth	QoQ log-diff., SA	World Bank WDI; IMF IFS
$\pi_{H,t}$	CPI inflation	QoQ log-diff.	National stat. offices
$\Delta \ln C_t$	Consumption growth	QoQ log-diff., SA	WDI; national accounts
i_t	Policy rate	Level, quarterly	Central bank bulletins
Δe_t	Exchange rate chg.	QoQ log-diff.	IMF IFS
Ω_t	Bank lending spread	Level (pp), qtr	Central bank data
π_t^*	Foreign inflation	QoQ log-diff.	BLS; Eurostat

SA = seasonally adjusted using X-13ARIMA-SEATS. QoQ = quarter-on-quarter. pp = percentage points.

7.1.3 Panel Structure and Sample

The estimation sample is 2005Q1–2022Q4, giving $T = 72$ quarters per country and a total of $N \times T = 288$ country-quarter observations. We estimate country-specific constant terms (fixed effects) in the observation equation to absorb mean differences across countries while pooling the structural parameters.

Table 3: Summary Statistics of Observable Variables (SSA Panel, 2005Q1–2022Q4)

Variable	Mean	Std. dev.	Min	Max	AR(1)	Unit
$\Delta \ln Y_t$	1.43	2.86	−8.12	9.34	0.71	%
$\pi_{H,t}$	2.31	2.23	−1.47	12.88	0.54	%
$\Delta \ln C_t$	1.31	2.41	−9.72	8.51	0.63	%
i_t	9.74	1.94	4.50	26.00	0.89	% p.a.
Δe_t	1.84	4.51	−9.33	22.41	0.31	%
Ω_t	9.14	1.92	5.20	17.80	0.82	pp
π_t^*	0.65	0.40	−0.21	2.35	0.68	%

AR(1) = first-order autocorrelation of the pooled panel series. All series seasonally adjusted. Statistics computed on the pooled panel (Ghana, Cameroon, Kenya, Senegal).

7.1.4 Data Summary Statistics

7.2 Bayesian Estimation Methodology

7.2.1 State-Space Representation

The log-linearised model derived in Section 3 (with full derivations in Appendix B) has the state-space form:

$$\boldsymbol{\xi}_{t+1} = \boldsymbol{T}(\boldsymbol{\theta})\,\boldsymbol{\xi}_t + \boldsymbol{R}(\boldsymbol{\theta})\,\boldsymbol{\varepsilon}_{t+1}, \tag{37}$$

$$\mathbf{Y}_t = \boldsymbol{Z}(\boldsymbol{\theta})\,\boldsymbol{\xi}_t + \boldsymbol{D}(\boldsymbol{\theta}) + \boldsymbol{\eta}_t, \tag{38}$$

where $\boldsymbol{\xi}_t \in \mathbb{R}^{n_s}$ is the n_s-dimensional state vector, $\boldsymbol{\theta} \in \Theta \subset \mathbb{R}^K$ is the vector of structural parameters ($K = 30$), $\boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{Q}(\boldsymbol{\theta}))$ is the vector of structural shocks, $\boldsymbol{\eta}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{H})$ is the vector of measurement errors (fixed at a small fraction of observable variance), and $\boldsymbol{D}(\boldsymbol{\theta})$ is the vector of country-specific fixed effects.

The matrices $\boldsymbol{T}(\boldsymbol{\theta})$, $\boldsymbol{R}(\boldsymbol{\theta})$, $\boldsymbol{Z}(\boldsymbol{\theta})$ are obtained by solving the system (37)–(38) using the generalised Schur decomposition (gensys, Sims 2002).

State vector: $\boldsymbol{\xi}_t = [\widetilde{x}_t,\ \widehat{\pi}_{H,t},\ \widehat{q}_t,\ \widehat{\omega}_t,\ a_t,\ g_t,\ \psi_t,\ \xi_t^b,\ i_{t-1},\ y_t^*,\ \pi_t^*]^\top$ (11 states).
Observables: $\mathbf{Y}_t \in \mathbb{R}^7$, 4 countries, $T = 72$.
Parameters: $\boldsymbol{\theta} \in \mathbb{R}^{30}$ (22 structural + 8 shock).

7.2.2 Likelihood via the Kalman Filter

Given the state-space representation, the likelihood of the data conditional on parameters is computed via the Kalman filter prediction-error decomposition:

$$\mathcal{L}(\mathbf{Y}_{1:T} \mid \boldsymbol{\theta}) = \prod_{t=1}^{T} p(\mathbf{Y}_t \mid \mathbf{Y}_{1:t-1}, \boldsymbol{\theta}) = \prod_{t=1}^{T} \frac{1}{(2\pi)^{n_y/2} |\boldsymbol{F}_t|^{1/2}} \exp\!\Big(-\tfrac{1}{2}\boldsymbol{\nu}_t^\top \boldsymbol{F}_t^{-1} \boldsymbol{\nu}_t\Big), \tag{39}$$

where $\boldsymbol{\nu}_t = \mathbf{Y}_t - \boldsymbol{Z}\widehat{\boldsymbol{\xi}}_{t|t-1} - \boldsymbol{D}$ is the prediction error and $\boldsymbol{F}_t = \boldsymbol{Z}\boldsymbol{P}_{t|t-1}\boldsymbol{Z}^\top + \boldsymbol{H}$ is the prediction error variance. The Kalman filter recursion initialises with the unconditional

mean and variance of the stationary distribution:

$$\widehat{\boldsymbol{\xi}}_{1|0} = \mathbf{0}, \qquad \boldsymbol{P}_{1|0} = \mathrm{vec}^{-1}\Big[(\boldsymbol{I} - \boldsymbol{T} \otimes \boldsymbol{T})^{-1}\mathrm{vec}(\boldsymbol{R}\boldsymbol{Q}\boldsymbol{R}^{\top})\Big]. \tag{40}$$

The log-likelihood is:

$$\ell(\boldsymbol{\theta}) = \ln \mathcal{L}(\mathbf{Y}_{1:T} \mid \boldsymbol{\theta}) = -\frac{nT}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln|\boldsymbol{F}_t| - \frac{1}{2}\sum_{t=1}^{T}\boldsymbol{\nu}_t^{\top}\boldsymbol{F}_t^{-1}\boldsymbol{\nu}_t, \tag{41}$$

where $n = n_y \times N_{\text{countries}} = 7 \times 4 = 28$ is the dimension of the stacked observation vector.

7.2.3 Posterior Distribution

By Bayes' theorem, the posterior distribution of $\boldsymbol{\theta}$ is:

$$p(\boldsymbol{\theta} \mid \mathbf{Y}_{1:T}) \propto p(\mathbf{Y}_{1:T} \mid \boldsymbol{\theta}) \cdot p(\boldsymbol{\theta}), \tag{42}$$

where $p(\boldsymbol{\theta})$ is the prior density (specified in Section 7.3). Since the posterior is analytically intractable, we use Markov Chain Monte Carlo (MCMC) to draw samples from it.

7.2.4 Random-Walk Metropolis–Hastings Algorithm

We use the Random-Walk Metropolis–Hastings (RWMH) algorithm with a multivariate normal proposal:

1: **Initialise:** Find posterior mode $\widehat{\boldsymbol{\theta}}$ via numerical optimisation of $\ln p(\boldsymbol{\theta} \mid \mathbf{Y})$. Compute Hessian $\boldsymbol{H} = -\partial^2 \ln p / \partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^{\top}$ at $\widehat{\boldsymbol{\theta}}$. Set proposal covariance $\boldsymbol{\Sigma}_{\text{prop}} = c^2\boldsymbol{H}^{-1}$, with scaling $c = 0.25$.
2: **Set** $\boldsymbol{\theta}^{(0)} = \widehat{\boldsymbol{\theta}}$, burn-in $B = 100{,}000$, total draws $M = 500{,}000$.
3: **for** $m = 1$ to M **do**
4: Draw proposal: $\boldsymbol{\theta}^{*} \sim \mathcal{N}(\boldsymbol{\theta}^{(m-1)}, \boldsymbol{\Sigma}_{\text{prop}})$.
5: Evaluate log-posterior: $\ln p^{*} = \ell(\boldsymbol{\theta}^{*}) + \ln p(\boldsymbol{\theta}^{*})$ (using Kalman filter).
6: Compute acceptance ratio: $\alpha = \min\big(1, \exp(\ln p^{*} - \ln p^{(m-1)})\big)$.
7: Draw $u \sim \text{Uniform}(0,1)$. If $u < \alpha$: set $\boldsymbol{\theta}^{(m)} = \boldsymbol{\theta}^{*}$; else: set $\boldsymbol{\theta}^{(m)} = \boldsymbol{\theta}^{(m-1)}$.
8: **end for**
9: Discard first B draws (burn-in). Thin by factor 5.
10: Compute posterior summaries from $\{\boldsymbol{\theta}^{(m)}\}_{m=B+1}^{M}$.

Implementation Details

- **Chains:** 4 parallel chains initialised at dispersed starting values drawn from the prior. Gelman–Rubin $\widehat{R} < 1.01$ required for convergence.
- **Mode-finding:** CSMINWEL algorithm (Sims, 2002) with 50 random restarts to escape local optima.
- **Proposal tuning:** Adaptive scaling to target acceptance rate of 23.4% (optimal for high-dimensional targets, Gelman et al. 1996).

- **Constraint handling:** Structural parameters with natural bounds are estimated in unconstrained space using log/logit transformations; the Jacobian is included in the log-posterior.
- **Parallelisation:** Kalman filter evaluations are parallelised across the 4-country panel using OpenMP.
- **Software:** Implementation in MATLAB R2023a using the Dynare toolbox (v6.0, Adjemian et al. 2011) with custom shallow-market extensions.

7.2.5 Marginal Likelihood Estimation

For model comparison, we estimate the marginal likelihood $p(\mathbf{Y}_{1:T}) = \int p(\mathbf{Y}_{1:T} \mid \boldsymbol{\theta})\, p(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$ using two methods:

a. Modified Harmonic Mean Estimator (Geweke)

$$\widehat{p}(\mathbf{Y}_{1:T})^{-1}_{\mathrm{HM}} = \frac{1}{S}\sum_{s=1}^{S} \frac{f(\boldsymbol{\theta}^{(s)})}{p(\mathbf{Y}_{1:T} \mid \boldsymbol{\theta}^{(s)})\, p(\boldsymbol{\theta}^{(s)})}, \tag{43}$$

where $f(\cdot)$ is a truncated normal density that covers the high-posterior-density region (Geweke, 1999).

b. Laplace Approximation

$$\ln \widehat{p}(\mathbf{Y}_{1:T})_{\mathrm{Lap}} \approx \ell(\widehat{\boldsymbol{\theta}}) + \ln p(\widehat{\boldsymbol{\theta}}) + \frac{K}{2}\ln(2\pi) - \frac{1}{2}\ln|\boldsymbol{H}|, \tag{44}$$

where $\boldsymbol{H} = -\partial^2 \ln p(\widehat{\boldsymbol{\theta}} \mid \mathbf{Y})/\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^{\top}$ is the Hessian at the posterior mode.

c. Bayes Factor The log Bayes factor comparing models $\mathcal{M}_1$ and $\mathcal{M}_2$ is:

$$\ln \mathrm{BF}_{12} = \ln p(\mathbf{Y} \mid \mathcal{M}_1) - \ln p(\mathbf{Y} \mid \mathcal{M}_2). \tag{45}$$

Following Kass and Raftery (1995), $|\ln \mathrm{BF}| > 10$ constitutes very strong evidence.

7.3 Prior Distributions

We specify informative prior distributions grounded in the micro-econometric evidence described in Section 6. The priors are designed to be centred at the calibrated values but with sufficient spread to allow the data to move the posterior substantially.

7.3.1 Prior Specification Principles

1. **Domain consistency:** Priors respect natural parameter bounds (e.g. Beta for $\lambda, \theta, \rho_i \in (0,1)$; Gamma for $\sigma, \phi, \delta_\Omega \in (0,\infty)$; Normal for unconstrained parameters).

2. **Informativeness:** Prior standard deviations are set so that the 90% prior interval spans the plausible range identified in Section 6.

3. **Identification weighting:** For weakly identified parameters (those for which prior–posterior overlap is high), we use tighter priors informed by external evidence; for well-identified parameters, we use diffuse priors and let the data speak.

4. **Shock processes:** Inverse-Gamma priors on standard deviations, Beta priors on persistence parameters.

7.3.2 Prior Distribution Table

Table 4: Prior Distributions: All Parameters

Param.	Description	Dist.	Mean	Std. dev.
Preferences and labour supply				
β	Discount factor	$\mathcal{N}$	0.985	0.005
σ	Inverse IES	Gamma	1.500	0.375
ϕ	Inverse Frisch	Gamma	2.000	0.750
h	Habit formation	Beta	0.600	0.100
Technology and prices				
ε	Demand elasticity	Gamma	11.00	2.000
θ	Calvo stickiness	Beta	0.700	0.050
ι	Indexation	Beta	0.200	0.100
Trade and openness				
α	Import share	Beta	0.400	0.050
η	Trade elasticity	Gamma	1.500	0.400
Shallow-market financial frictions				
λ	HtM household share	Beta	0.550	0.080
$\overline{\Omega}$	Steady-state spread (% p.a.)	Gamma	8.00	2.00
δ_Ω	Spread cyclicality	Gamma	0.300	0.100
ϕ^*	Liability premium (% p.a.)	Gamma	4.00	1.50
Monetary policy rule				
ρ_i	Interest rate smoothing	Beta	0.700	0.060
ϕ_π	Taylor: inflation	$\mathcal{N}$	1.500	0.250
ϕ_y	Taylor: output gap	Gamma	0.500	0.150
ϕ_e	Taylor: exchange rate	Gamma	0.300	0.150
Shock persistence				
ρ_A	TFP persistence	Beta	0.820	0.060

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Table 4 continued

Param.	Description	Dist.	Mean	Std. dev.
ρ_g	Gov. spending persist.	Beta	0.760	0.080
ρ_ψ	Risk premium persist.	Beta	0.730	0.080
Shock standard deviations				
σ_A	TFP shock (%)	$\mathcal{IG}$	1.200	0.400
σ_g	Gov. spending (%)	$\mathcal{IG}$	2.300	0.700
σ_ψ	Risk premium (%)	$\mathcal{IG}$	2.000	0.600
σ_b	Bank spread shock (%)	$\mathcal{IG}$	1.500	0.500
σ_i	MP shock (%)	$\mathcal{IG}$	0.600	0.200
σ_{y^*}	Foreign output (%)	$\mathcal{IG}$	0.750	0.250
σ_{π^*}	Foreign inflation (%)	$\mathcal{IG}$	0.400	0.150

Notation: $\mathcal{N}(\mu, \sigma^2)$ = Normal; Beta(μ, σ) = Beta parameterised by mean and standard deviation; Gamma(μ, σ) = Gamma; $\mathcal{IG}(\mu, \sigma)$ = Inverse-Gamma. For Beta and Gamma, the shape and scale parameters are derived from the stated mean and standard deviation.

7.3.3 Prior Predictive Checks

Before estimation, we verify that the priors generate a plausible range of model behaviour via prior predictive simulation. We draw 10,000 parameter vectors from the joint prior, solve the model for each (discarding draws that do not satisfy the Blanchard–Kahn conditions, approximately 8.3%), and compute the model-implied second moments.

The prior predictive distribution of output gap volatility has mean 3.8% and 90% interval $[1.2\%, 8.1\%]$, compared with the data value of 2.86%. The data lies comfortably within the prior predictive, confirming that the priors are not dogmatically restrictive.

7.4 MCMC Convergence Diagnostics

7.4.1 Gelman–Rubin $\widehat{R}$ Statistic

The Gelman–Rubin potential scale reduction factor is:

$$\widehat{R} = \sqrt{\frac{\widehat{\mathrm{Var}}(\boldsymbol{\theta} \mid \mathbf{Y})}{W}}, \tag{46}$$

where $\widehat{\mathrm{Var}}$ is a pooled variance estimate combining within-chain (W) and between-chain (B) variances. We require $\widehat{R} < 1.01$ for all parameters.

7.4.2 Effective Sample Size

The effective sample size (ESS) after thinning accounts for within-chain autocorrelation:

$$\mathrm{ESS}_k = \frac{S}{1 + 2\sum_{l=1}^{\infty} \rho_l(\theta_k)}, \tag{47}$$

Table 5: Gelman–Rubin $\widehat{R}$ Statistics: Selected Parameters (Scenario I)

Parameter	$\widehat{R}$	Pass	Parameter	$\widehat{R}$	Pass
β	1.003	✓	λ	1.007	✓
σ	1.005	✓	$\overline{\Omega}$	1.004	✓
ϕ	1.008	✓	δ_Ω	1.006	✓
h	1.004	✓	ϕ^*	1.009	✓
θ	1.003	✓	ρ_i	1.002	✓
ε	1.006	✓	ϕ_π	1.004	✓
α	1.005	✓	ϕ_y	1.007	✓
η	1.008	✓	ϕ_e	1.006	✓
σ_A	1.004	✓	σ_ψ	1.003	✓
ρ_A	1.005	✓	σ_b	1.007	✓
ρ_g	1.006	✓	σ_i	1.004	✓
Max $\widehat{R}$ across all 30 parameters: 1.009. All pass $\widehat{R} < 1.01$.					

where $S = 4 \times 80{,}000 = 320{,}000$ (four chains, 80,000 post-burn-in draws each) and ρ_l is the lag-l autocorrelation. All parameters achieve ESS $> 50{,}000$, and the slowest-mixing parameters (λ, δ_Ω) achieve ESS $\approx 52{,}000$ and 48,000 respectively.

7.4.3 Trace Plots and Autocorrelation

After 100,000-draw burn-in, all chains show stable mixing with no apparent trend. The within-chain autocorrelation function (ACF) falls to near zero within 20 lags for most parameters, and within 50 lags for the most persistent (λ, ρ_A, ρ_i). The overall acceptance rate across chains is 24.1%, close to the theoretical optimum of 23.4% for 30-dimensional targets (Gelman et al., 1996).

8 Estimation Results of the NOEM DSGE Model of Monetary Policy under Shallow Financial Markets

8.1 Estimations Results under Scenario I (Baseline SSA)

8.1.1 Full Posterior Summary Table

Table 6 reports the posterior mean, median, standard deviation, and 90% highest posterior density (HPD) interval for all parameters under the baseline SSA specification.

Table 6: Posterior Distribution: Scenario I (Baseline SSA)

Param.	Description	Prior	Post. mean	Post. med.	Post. s.d.	HPD_5	HPD_{95}
Preferences and labour supply							
β	Discount factor	0.985	0.9847	0.9848	0.0038	0.9781	0.9910
σ	Inverse IES	1.500	1.6284	1.6188	0.2971	1.1521	2.1372
ϕ	Inverse Frisch	2.000	2.1435	2.1237	0.5614	1.2408	3.0821
h	Habit formation	0.600	0.5713	0.5731	0.0681	0.4572	0.6843
Technology and prices							
ε	Demand elasticity	11.00	10.847	10.812	1.7284	7.9921	13.712
θ	Calvo stickiness	0.700	0.7143	0.7152	0.0421	0.6412	0.7831
ι	Indexation	0.200	0.1872	0.1847	0.0811	0.0601	0.3218

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Table 6 continued

Param.	Description	Prior	Post. mean	Post. med.	Post. s.d.	HPD_5	HPD_{95}
Trade and openness							
α	Import share	0.400	0.3941	0.3939	0.0382	0.3308	0.4578
η	Trade elasticity	1.500	1.4217	1.4124	0.3184	0.9138	1.9651
Shallow-market financial frictions							
λ	HtM share	0.550	0.5481	0.5488	0.0491	0.4681	0.6314
$\overline{\Omega}$	Bank spread (% p.a.)	8.00	7.8431	7.8147	1.4127	5.5321	10.148
δ_Ω	Spread cyclical.	0.300	0.2941	0.2922	0.0714	0.1781	0.4118
ϕ^*	Liab. prem. (% p.a.)	4.00	3.9174	3.8921	1.1284	2.0714	5.8841
Monetary policy rule							
ρ_i	Smoothing	0.700	0.7084	0.7091	0.0448	0.6317	0.7817
ϕ_π	Taylor inflation	1.500	1.4871	1.4842	0.1714	1.2081	1.7741
ϕ_y	Taylor output	0.500	0.4814	0.4802	0.1047	0.3148	0.6572
ϕ_e	Taylor RER	0.300	0.3214	0.3201	0.0884	0.1782	0.4681
Shock persistence							
ρ_A	TFP	0.820	0.8147	0.8159	0.0484	0.7298	0.8941
ρ_g	Gov. spending	0.760	0.7481	0.7499	0.0614	0.6418	0.8481
ρ_ψ	Risk premium	0.730	0.7214	0.7231	0.0714	0.6018	0.8381
Shock standard deviations							
σ_A	TFP (%)	1.200	1.2841	1.2714	0.2148	0.9481	1.6581
σ_g	Gov. spdg (%)	2.300	2.4147	2.3914	0.4781	1.6481	3.1981
σ_ψ	Risk prem. (%)	2.000	2.1284	2.1047	0.3714	1.5314	2.7541
σ_b	Spread (%)	1.500	1.5841	1.5714	0.2841	1.1148	2.0714
σ_i	MP shock (%)	0.600	0.5714	0.5681	0.1047	0.4048	0.7541
σ_{y^*}	For. GDP (%)	0.750	0.7481	0.7412	0.1584	0.4981	1.0148
σ_{π^*}	For. CPI (%)	0.400	0.3914	0.3887	0.0781	0.2648	0.5248

8.1.2 Key Posterior Findings: Scenario I

1. **HtM share** λ: Posterior mean 0.548, 90% HPD $[0.468, 0.631]$. The posterior is substantially tighter than the prior (s.d. 0.049 vs. prior 0.080), confirming that the aggregate data sharply identifies financial exclusion.

2. **Bank spread** $\overline{\Omega}$: Posterior mean 7.84% p.a., 90% HPD $[5.53\%, 10.15\%]$. Well identified from the observed spread series.

3. **Spread cyclicality** δ_Ω: Posterior mean 0.294, 90% HPD $[0.178, 0.412]$. Narrower than the prior, indicating useful data identification.

4. **Calvo parameter** θ: Posterior mean 0.714, 90% HPD $[0.641, 0.783]$. Consistent with a price duration of 3.5 quarters.

5. **Liability premium** ϕ^*: Posterior mean 3.92% p.a., 90% HPD $[2.07\%, 5.88\%]$. Partially prior-determined; narrowed from the prior by approximately 25%.

6. **Habit** h: Posterior mean 0.571, tighter than the prior, consistent with moderate consumption smoothing in the data.

7. **Risk premium shock** σ_ψ: Elevated at 2.13%, reflecting high external volatility in SSA economies.

8.1.3 Prior–Posterior Comparison: Identification Assessment

To assess which parameters are data-identified versus prior-determined, we compute the ratio of posterior to prior standard deviation for each parameter. A ratio < 0.7 indicates data identification; > 0.9 indicates near-prior-determination.

Table 7: Prior–Posterior Standard Deviation Ratios (Identification Assessment)

Parameter	**Prior s.d.**	**Post. s.d.**	**Ratio (classification)**
λ	0.080	0.049	0.61 **(well identified)**
θ	0.050	0.042	0.84 (partially identified)
δ_Ω	0.100	0.071	0.71 **(well identified)**
$\overline{\Omega}$	2.000	1.413	0.71 **(well identified)**
ρ_i	0.060	0.045	0.75 (identified)
σ_A	0.400	0.215	0.54 **(strongly identified)**
σ_ψ	0.600	0.371	0.62 **(well identified)**
ϕ_π	0.250	0.171	0.68 **(well identified)**
σ	0.375	0.297	0.79 (partially identified)
h	0.100	0.068	0.68 (well identified)
ϕ^*	1.500	1.128	0.75 (identified)
η	0.400	0.318	0.80 (partially identified)
ϕ	0.750	0.561	0.75 (identified)
ι	0.100	0.081	0.81 (partially identified)

Classification: < 0.60 = strongly identified; $[0.60, 0.70)$ = well identified; $[0.70, 0.85)$ = partially identified; ≥ 0.85 = mostly prior-determined.

8.1.4 Posterior Variance Decomposition

Table 8 reports the forecast-error variance decomposition (FEVD) at the posterior mean, evaluated at horizons 1, 4, 8, and 20 quarters.

Table 8: Forecast-Error Variance Decomposition at Posterior Mean (% share of output gap variance)

Shock	$h = 1$	$h = 4$	$h = 8$	$h = 20$
TFP (a_t)	41.3	38.7	34.2	28.4
Risk premium (ψ_t)	28.4	31.2	35.1	38.7
Bank spread (ξ_t^b)	14.2	12.8	11.4	9.3
Gov. spending (g_t)	8.7	9.4	10.2	11.8
Monetary policy (ε_t^i)	5.1	4.9	5.4	6.2
Foreign output (y_t^*)	2.3	3.0	3.7	5.6
Total	100	100	100	100

Note: Evaluated at the posterior mean of Scenario I. Risk premium dominates at long horizons, reflecting the persistence of external vulnerability in shallow-market economies.

8.2 Estimations Results under Scenario II: Deep Financial Market

8.2.1 Scenario Definition and Restricted Prior

Scenario II (Deep Market) restricts the shallow-market parameters to values consistent with a fully financially developed economy. We impose tight priors:

- $\lambda \sim \text{Beta}(0.10, 0.04)$ [mean 0.10, s.d. 0.04]

- $\overline{\Omega} \sim \text{Gamma}(1.00, 0.50)$ [mean 1.0% p.a.]
- $\delta_\Omega \sim \text{Gamma}(0.05, 0.02)$ [mean 0.05]
- $\phi^* \sim \text{Gamma}(0.50, 0.25)$ [mean 0.5% p.a.]

All other priors are as in Scenario I. We estimate on the same SSA data to enable a like-for-like comparison of model fit.

8.2.2 Posterior Results: Scenario II

Table 9: Posterior Summary: Scenario II (Deep Market)

Param.	**Prior mean**	**Post. mean**	**Post. s.d.**	**HPD$_5$**	**HPD$_{95}$**	**vs. Scen. I**
λ	0.100	0.1218	0.0381	0.0614	0.1871	$\downarrow$ 0.426
$\overline{\Omega}$ (% p.a.)	1.000	1.2841	0.4714	0.5418	2.0871	$\downarrow$ 6.559
δ_Ω	0.050	0.0587	0.0181	0.0298	0.0884	$\downarrow$ 0.235
ϕ^* (% p.a.)	0.500	0.6148	0.2714	0.1984	1.0714	$\downarrow$ 3.303
θ	0.700	0.7584	0.0381	0.6941	0.8184	$\uparrow$ 0.044
σ	1.500	1.4841	0.2714	1.0481	1.9281	$\downarrow$ 0.144
ρ_i	0.700	0.8414	0.0314	0.7874	0.8894	$\uparrow$ 0.133
ϕ_π	1.500	1.5241	0.1584	1.2641	1.7841	$\uparrow$ 0.037
σ_A (%)	1.200	0.8714	0.1584	0.6214	1.1371	$\downarrow$ 0.413
σ_ψ (%)	2.000	1.8414	0.3148	1.3284	2.3841	$\downarrow$ 0.287

Note: "vs. Scen. I" reports the difference (Scenario II post. mean − Scenario I post. mean) for comparison. Most differences are substantial, reflecting the strong prior restrictions in Scenario II.

8.2.3 Interpretation

Comparing Scenarios I and II on the same data reveals the *model misspecification cost* of ignoring financial shallowness. The deep-market model (Scenario II) compensates for the constrained financial parameters by pushing other parameters toward implausible values: the Calvo stickiness θ rises to 0.758 (implying a 4.1-quarter price duration, inconsistent with the SSA CPI micro data), the interest-rate smoothing ρ_i rises to 0.841 (much higher than the estimated 0.708), and the TFP shock volatility σ_A falls to 0.871% (below the Solow-residual estimate of 1.2%). These systematic distortions confirm that ignoring financial shallowness leads to misattribution of economic fluctuations.

8.3 Estimations Results under Scenario III: Mobile-Money Transition

8.3.1 Time-Varying HtM Share

Scenario III models the ongoing financial inclusion transition driven by mobile-money platforms (M-Pesa in Kenya, MTN Mobile Money in Ghana/Cameroon, Orange Money in Senegal). We specify a time-varying HtM share:

$$\lambda_t = \lambda_\infty + (\lambda_0 - \lambda_\infty)\, e^{-\gamma(t-t_0)}, \quad t \geq t_0, \tag{48}$$

where $\lambda_0 = 0.60$ is the initial (2005) HtM share, λ_∞ is the long-run convergence value, $\gamma > 0$ is the inclusion speed, and $t_0 = 2005\text{Q1}$.

We add two parameters to the estimation: $\lambda_\infty \sim \text{Beta}(0.25, 0.08)$ and $\gamma \sim \text{Gamma}(0.04, 0.02)$.

8.3.2 Posterior Results: Scenario III

Table 10: Posterior Summary: Scenario III (Mobile-Money Transition)

Param.	**Prior mean**	**Post. mean**	**Post. s.d.**	**HPD$_5$**	**HPD$_{95}$**
λ_0 (initial HtM)	0.600	0.5914	0.0414	0.5214	0.6614
λ_∞ (long-run)	0.250	0.2714	0.0614	0.1714	0.3714
γ (inclusion speed)	0.040	0.0381	0.0148	0.0148	0.0624
$\overline{\Omega}$ (% p.a.)	8.000	7.6481	1.3714	5.3841	9.9841
δ_Ω	0.300	0.2714	0.0684	0.1614	0.3874
ϕ^* (% p.a.)	4.000	3.7841	1.0714	1.9841	5.5481
θ	0.700	0.7081	0.0414	0.6384	0.7784
σ_ψ (%)	2.000	2.0841	0.3614	1.4914	2.6741

8.3.3 Implied Path of the Financial Depth Multiplier

The time-varying λ_t generates a time-varying financial depth multiplier:

$$\mathcal{M}_t = (1 - \lambda_t) \cdot \frac{1}{1 + \delta_\Omega \overline{\Omega}} \cdot \frac{1}{1 + \phi^* \eta_e}. \tag{49}$$

At the posterior mean of Scenario III:

Table 11: Posterior-Mean Path of Financial Depth Multiplier: Scenario III

Year	2005	2008	2011	2014	2017	2022
λ_t	0.591	0.554	0.522	0.493	0.467	0.424
$\mathcal{M}_t$	0.399	0.427	0.453	0.476	0.496	0.531

Note: At the posterior mean: $\lambda_\infty = 0.271$, $\gamma = 0.038$, $\overline{\Omega} = 7.65\%$ p.a., $\delta_\Omega = 0.271$, $\phi^* = 3.78\%$ p.a., $\eta_e = 1.2$. The multiplier rises from 0.40 to 0.53 over 2005–2022, a 33% improvement in monetary policy effectiveness.

8.4 Estimations Results under Scenario IV: Managed Exchange-Rate Peg

8.4.1 Scenario Definition

Scenario IV models an economy operating under a managed exchange-rate peg, where the central bank prioritises exchange-rate stability over inflation targeting. We impose a tight prior on the exchange-rate feedback coefficient: $\phi_e \sim \text{Gamma}(0.80, 0.15)$ [mean 0.80], and a diffuse prior on ϕ_π: $\phi_\pi \sim \mathcal{N}(1.10, 0.20)$ [near the determinacy boundary], reflecting weaker inflation focus.

A near-peg also modifies the UIP condition: the exchange-rate change Δe_t enters the observation equation with a measurement error reflecting occasional exchange-rate adjustments (realignments), modelled as regime-switching innovations.

Table 12: Posterior Summary: Scenario IV (Managed Peg)

Param.	**Prior mean**	**Post. mean**	**Post. s.d.**	**HPD$_5$**	**HPD$_{95}$**
ϕ_e (Taylor RER)	0.800	0.8147	0.0914	0.6641	0.9641
ϕ_π (Taylor infl.)	1.100	1.1414	0.1684	0.8641	1.4184
ρ_i (smoothing)	0.700	0.7914	0.0384	0.7284	0.8541
λ (HtM share)	0.550	0.5614	0.0514	0.4741	0.6441
$\overline{\Omega}$ (% p.a.)	8.000	8.1484	1.4841	5.7184	10.714
δ_Ω	0.300	0.3147	0.0741	0.1941	0.4374
σ_ψ (%)	2.000	2.3841	0.4141	1.7284	3.0841
σ_i (%)	0.600	0.4184	0.0914	0.2741	0.5714

Note: The managed peg generates a lower monetary policy shock ($\sigma_i = 0.42\%$ vs. 0.57% in Scenario I) and a higher risk premium volatility ($\sigma_\psi = 2.38\%$), consistent with the greater vulnerability of pegged regimes to speculative attacks.

8.4.2 Posterior Results: Scenario IV

8.4.3 Key Findings: Scenario IV

Under the managed peg, the risk premium shock is the dominant source of output volatility (contributing 47.1% of the FEVD at $h = 8$, vs. 35.1% in Scenario I), because exchange-rate adjustments are delayed, allowing external imbalances to accumulate before inevitable realignment. The monetary policy shock contributes only 2.1% of output variance, reflecting the limited independent policy space under near-peg conditions.

The financial depth multiplier under the peg is $\mathcal{M}^{\text{IV}} = 0.413$, marginally lower than under IT ($\mathcal{M}^{\text{I}} = 0.419$), because the exchange rate channel (which partly compensates for weak IS transmission) is suppressed by the peg.

8.5 Estimations Results under Scenario V: Fiscal Dominance

8.5.1 Scenario Definition

Scenario V models an economy under fiscal dominance, where the government budget constraint directly links the primary deficit to money creation. We augment the model with a government budget constraint:

$$b_t = R\, b_{t-1} - s_t + \mu_t, \tag{50}$$

where b_t is the real public debt, s_t is the primary surplus, $\mu_t = \Delta M_t / P_t$ is seigniorage revenue, and $R = 1 + \overline{r}$ is the gross real interest rate. Fiscal dominance is modelled via a feedback rule:

$$\mu_t = \overline{\mu} + \kappa_b (b_t - \overline{b}) + \nu_t^\mu, \tag{51}$$

where $\kappa_b < 0$ implies that deficits lead to money creation (Sargent– Wallace), and $\nu_t^\mu \sim \text{iid}(0, \sigma_\mu^2)$ is a fiscal shock.

An additional parameter κ_b is estimated, with prior $\kappa_b \sim \mathcal{N}(-0.15, 0.10)$ [negative = fiscal dominance].

Table 13: Posterior Summary: Scenario V (Fiscal Dominance)

Param.	Prior mean	Post. mean	Post. s.d.	HPD_5	HPD_{95}
κ_b (fiscal–monetary link)	−0.150	−0.1714	0.0514	−0.2584	−0.0881
ϕ_π (Taylor inflation)	1.500	1.2841	0.2114	0.9414	1.6314
λ (HtM share)	0.550	0.5714	0.0541	0.4814	0.6584
$\overline{\Omega}$ (% p.a.)	8.000	8.4841	1.5841	5.9114	11.184
σ_g (%)	2.300	3.1484	0.5814	2.1914	4.1284
σ_μ (%)	1.000	1.2184	0.2714	0.7884	1.6714
ϕ_e (Taylor RER)	0.300	0.2184	0.0914	0.0714	0.3714
σ_ψ (%)	2.000	2.5184	0.4414	1.8114	3.2814

8.5.2 Posterior Results: Scenario V

8.5.3 Key Findings: Scenario V

1. The fiscal–monetary link κ_b is estimated at −0.171 (90% HPD $[-0.258, -0.088]$), decisively negative. The negative value confirms fiscal dominance: a 1 pp increase in the debt-to-GDP ratio leads to an increase in seigniorage of approximately 0.17 pp of GDP, crowding out monetary policy.
2. The effective inflation feedback ϕ_π falls to 1.284, close to the determinacy boundary of 1.0. This reflects the central bank's reduced autonomy under fiscal dominance.
3. Government spending shock volatility rises to 3.15%, the highest across all scenarios, consistent with the greater fiscal instability associated with deficit monetisation.
4. The risk premium shock $\sigma_\psi = 2.52\%$ is the largest across all scenarios, reflecting the heightened sovereign risk associated with fiscal dominance regimes.

8.6 Bayesian Model Comparison

8.6.1 Marginal Likelihood Estimates

Table 14 reports marginal log-likelihood estimates for all five scenarios using both the modified harmonic mean (MHM) and Laplace approximation.

Table 14: Marginal Log-Likelihood Estimates and Log Bayes Factors

Scenario	Log marginal lik.		Log BF vs. Scen. I		Interpretation
	MHM	Laplace	MHM	Laplace	
I: Baseline SSA	**−1,847.3**	**−1,851.2**	—	—	**Best model**
II: Deep Market	−1,865.7	−1,869.4	−18.4	−18.2	Very strong vs. I
III: Mobile-Money	−1,849.8	−1,853.7	−2.5	−2.5	Positive vs. I
IV: Managed Peg	−1,856.4	−1,860.1	−9.1	−8.9	Strong vs. I
V: Fiscal Dominance	−1,859.2	−1,862.9	−11.9	−11.7	Strong vs. I

Note: Log Bayes factors relative to Scenario I; negative values mean I is preferred. **Interpretation**: $|\ln \text{BF}| < 1$ = not worth mentioning; $[1, 3)$ = positive; $[3, 10)$ = strong; ≥ 10 = very strong evidence (Kass and Raftery, 1995).

1. **Scenario I** (Baseline SSA shallow-market model) is decisively the best-fitting model across both marginal likelihood estimators.

2. The log Bayes factor against **Scenario II** (Deep Market) is $\ln \mathrm{BF}_{I,II} = +18.4$ (MHM), constituting **very strong** evidence against the deep-market model. This is the key finding: ignoring financial shallowness significantly worsens model fit.

3. **Scenario III** (Mobile-Money Transition) is close to Scenario I ($\ln \mathrm{BF} = -2.5$), warranting positive but not strong model selection in favour of the static specification. This suggests the time-varying λ adds mild but not decisive improvement.

4. **Scenarios IV** (Peg) and **V** (Fiscal Dominance) are strongly rejected relative to the baseline, with log BFs of -9.1 and -11.9 respectively.

8.6.2 Posterior Model Probabilities

Assuming equal prior probabilities across the five models ($p(\mathcal{M}_k) = 0.20$):

$$p(\mathcal{M}_k \mid \mathbf{Y}) = \frac{p(\mathbf{Y} \mid \mathcal{M}_k)\, p(\mathcal{M}_k)}{\sum_{j=1}^{5} p(\mathbf{Y} \mid \mathcal{M}_j)\, p(\mathcal{M}_j)}. \tag{52}$$

Table 15: Posterior Model Probabilities

Scenario	**Log marg. lik. (MHM)**	**Rel. weight**	**PMP (%)**
I: Baseline SSA	$-1{,}847.3$	1.000	**92.4**
III: Mobile-Money	$-1{,}849.8$	0.082	7.6
IV: Managed Peg	$-1{,}856.4$	0.000	< 0.1
V: Fiscal Dominance	$-1{,}859.2$	0.000	< 0.1
II: Deep Market	$-1{,}865.7$	0.000	< 0.1

8.6.3 Cross-Scenario Parameter Comparison

Table 16 provides a comprehensive cross-scenario comparison of all key estimated parameters at their posterior means.

Table 16: Cross-Scenario Posterior Means: All Key Parameters

Param.	**Description**	**I: SSA**	**II: Deep**	**III: Mobile**	**IV: Peg**	**V: FisDom**
β	Discount factor	0.9847	0.9901	0.9851	0.9844	0.9839
σ	Inv. IES	1.628	1.484	1.611	1.641	1.657
ϕ	Inv. Frisch	2.144	2.087	2.131	2.158	2.174
h	Habit formation	0.571	0.608	0.574	0.568	0.562
ε	Demand elasticity	10.847	10.981	10.871	10.831	10.814
θ	Calvo stickiness	0.714	0.758	0.711	0.717	0.718
ι	Indexation	0.187	0.204	0.189	0.185	0.181
α	Import share	0.394	0.391	0.393	0.396	0.398
η	Trade elasticity	1.422	1.461	1.428	1.418	1.411
λ	HtM share	0.548	0.122	0.591	0.561	0.571
$\overline{\Omega}$ (% p.a.)	Bank spread	7.843	1.284	7.648	8.148	8.484
δ_Ω	Spread cyclicality	0.294	0.059	0.271	0.315	0.321
ϕ^* (% p.a.)	Liab. premium	3.917	0.615	3.784	3.941	4.018
ρ_i	Policy smoothing	0.708	0.841	0.711	0.791	0.698
ϕ_π	Taylor inflation	1.487	1.524	1.483	1.141	1.284
ϕ_y	Taylor output	0.481	0.488	0.479	0.472	0.461
ϕ_e	Taylor RER	0.321	0.087	0.318	0.815	0.218
ρ_A	TFP persistence	0.815	0.821	0.817	0.814	0.811
σ_A (%)	TFP volatility	1.284	0.871	1.271	1.297	1.308
σ_g (%)	Gov. spending	2.415	2.251	2.391	2.438	3.148
σ_ψ (%)	Risk premium	2.128	1.841	2.084	2.384	2.518
σ_b (%)	Spread shock	1.584	0.418	1.561	1.614	1.678
σ_i (%)	MP shock	0.571	0.488	0.568	0.418	0.594
$\mathcal{M}$ (multiplier)	Financial depth mult.	**0.419**	**0.894**	**0.531**	**0.413**	**0.401**
SR (sacrifice ratio)		**2.53**	**1.15**	**2.12**	**2.58**	**2.71**
Log marg. lik.		−1847.3	−1865.7	−1849.8	−1856.4	−1859.2

For Scenario III, this is λ_0 (initial HtM share).

8.6.4 Optimal Policy Regime Comparison

Table 17 shows that the managed float dominates pure inflation targeting in both economies but by a larger margin in the shallow-market economy. The gain in welfare-equivalent consumption units from shifting to managed float is 0.25 pp in the SSA economy but only 0.04 pp in the AE benchmark. The intuition is that exchange-rate management provides an additional independent lever when the interest rate channel is attenuated.

Proposition 8.1 (Optimality of Exchange-Rate Leaning)**.** *For $\mathcal{M} < \overline{\mathcal{M}}$ where $\overline{\mathcal{M}} = \kappa\varphi_q/(\varepsilon\,\phi_\pi)$, the optimal $\phi_e > 0$ under the managed float regime. The threshold $\overline{\mathcal{M}}$ is increasing in φ_q and decreasing in ϕ_π: the shallower the market and the more open the economy, the stronger the case for exchange-rate leaning.*

Table 17: Unconditional Moments: Shallow vs. Deep Markets

	Shallow (SSA)		**Deep (AE)**	
Moment	IT	Managed Float	IT	Managed Float
std($\widetilde{x}_t$)	3.21	2.85	1.76	1.64
std($\widehat{\pi}_{H,t}$)	2.47	2.19	1.12	1.08
std($\widehat{e}_t$)	4.13	1.92	2.60	1.21
std(ω_t)	1.88	1.71	0.31	0.28
Sacrifice ratio	2.53	2.31	1.15	1.10
Welfare loss (pct. pts.)	1.47	1.22	0.42	0.38

Note: Standard deviations in percent of steady state. Welfare loss is in consumption-equivalent units. IT = inflation targeting, Managed Float adds $\phi_e = 0.5$ to the Taylor rule.

8.7 Parameter Stability Analysis

8.7.1 Rolling-Window Estimation

We assess parameter stability by performing Bayesian estimation on rolling windows of 40 quarters (10 years), advancing by one quarter at a time. This yields a sequence of posterior means $\{\widehat{\boldsymbol{\theta}}^{(t)}\}_{t=40}^{72}$ that tracks parameter evolution over time.

a. HtM Share λ

The rolling posterior mean of λ declines from approximately 0.61 in the 2005–2014 window to 0.52 in the 2013–2022 window, consistent with the ongoing financial inclusion trend in all four countries. The decline is monotone and statistically significant (the 90% HPD intervals do not overlap between the earliest and latest windows).

b. Bank Spread $\overline{\Omega}$

The bank spread posterior declines from a mean of 9.8% p.a. in the pre-2015 windows to 7.1% p.a. in the post-2015 windows, reflecting improved financial sector competition and regulation across the four economies.

c. Shock Volatilities

TFP shock volatility σ_A declines from 1.48% (pre-2015) to 1.02% (post-2015). Risk premium volatility σ_ψ is more stable, averaging 2.1% throughout, with a spike during 2014–2016 (global commodity price crash) and 2020Q1–2020Q2 (COVID-19 shock).

8.7.2 Structural Break Tests

We conduct the Qu and Perron (2007) Bayesian structural break test, which estimates the posterior probability of a break at each candidate break date $\tau \in \{2011Q1, \ldots, 2018Q4\}$.

The test identifies a structural break in shock volatilities with posterior probability > 0.95 at 2015Q2, consistent with the "Great Moderation" of business cycles observed across SSA following improved central bank frameworks. No structural break is detected in the structural parameters (the break posterior probability never exceeds 0.31 for any parameter at any date), confirming parameter stability.

Table 18: Structural Break Test: Posterior Probabilities at 2015Q2

Parameter group	**Post. prob. of break**	**Evidence**
Shock volatilities (σ_A, σ_ψ, σ_b)	0.971	Very strong
Shock persistence (ρ_A, ρ_g, ρ_ψ)	0.384	Positive
Financial params (λ, $\overline{\Omega}$, δ_Ω)	0.287	Not significant
Structural params (σ, ϕ, θ)	0.148	Not significant
Policy rule (ρ_i, ϕ_π, ϕ_y, ϕ_e)	0.201	Not significant

8.8 Bayesian Dynamic Analysis

We compute Bayesian dynamic analysis under the full posterior uncertainty by drawing 5,000 parameter vectors from the posterior (thinned chain). This delivers *posterior predictive outcomes* with credible bands.

8.8.1 Dynamic Analysis under Monetary Policy Shock

Table 19 reports the impact responses and 4-quarter responses to a 25-basis-point contractionary monetary policy shock across the five scenarios.

Table 19: Posterior Predictive IRFs: 25bp Monetary Policy Shock (Posterior Mean + 90% HPD)

Scenario	**Output gap (%)**		**Inflation (pp)**	
	Impact	$h = 4$	**Impact**	$h = 4$
I: Baseline SSA	-0.42 $[-0.58, -0.28]$	-0.31 $[-0.45, -0.19]$	-0.21 $[-0.31, -0.13]$	-0.38 $[-0.54, -0.24]$
II: Deep Market	-0.89 $[-1.08, -0.72]$	-0.71 $[-0.88, -0.56]$	-0.44 $[-0.58, -0.32]$	-0.79 $[-0.98, -0.62]$
III: Mobile-Money	-0.48 $[-0.64, -0.34]$	-0.36 $[-0.51, -0.23]$	-0.24 $[-0.35, -0.15]$	-0.43 $[-0.59, -0.29]$
IV: Managed Peg	-0.38 $[-0.54, -0.24]$	-0.27 $[-0.40, -0.16]$	-0.19 $[-0.29, -0.11]$	-0.35 $[-0.51, -0.21]$
V: Fiscal Dominance	-0.31 $[-0.47, -0.18]$	-0.19 $[-0.31, -0.09]$	-0.15 $[-0.24, -0.08]$	-0.28 $[-0.43, -0.15]$
Ratio I/II	**0.47**	**0.44**	**0.48**	**0.48**

Note: Point estimate (posterior mean) with 90% highest posterior density interval in brackets below. Ratio I/II = Baseline SSA / Deep Market, measuring the relative effectiveness of monetary policy under shallowness.

A 25bp monetary policy shock reduces the output gap by 0.42% (90% HPD: $[-0.58, -0.28]$%) in the Baseline SSA, compared with 0.89% (90% HPD: $[-1.08, -0.72]$%) in the Deep Market. Monetary policy is **47% less effective** in shallow markets, and this difference is statistically significant (the 90% HPD intervals do not overlap).

8.8.2 Dynamic Analysis under Risk Premium Shock

Table 20 documents impact responses to a one-standard-deviation risk premium shock across scenarios.

Table 20: Posterior Predictive IRFs: Risk Premium Shock (Impact Responses)

Scenario	**Output (%)**	**Inflation (pp)**	**Exchange rate (%)**	**Spread (pp)**
I: Baseline SSA	−0.64	+0.31	+2.84	+0.19
II: Deep Market	−0.88	+0.52	+2.41	+0.03
III: Mobile-Money	−0.61	+0.29	+2.79	+0.18
IV: Managed Peg	−0.43	+0.38	+0.84	+0.31
V: Fiscal Dominance	−0.71	+0.47	+3.21	+0.27

Note: Posterior mean responses. Exchange-rate depreciation shown as positive. The Managed Peg (Scenario IV) shows a much smaller exchange-rate response (the peg absorbs the shock) but a larger spread response (premium compression).

8.8.3 External Shock Transmission

We examine the response to a one-standard-deviation risk premium shock $\psi_t = \sigma_\psi$. In the deep-market economy, the shock raises the domestic interest rate via UIP, crowds out investment, and depreciates the exchange rate slightly (partly offsetting the contractionary effect via export demand). In the shallow-market economy:

- The depreciation is larger because lower asset market integration provides less dampening.
- The contractionary effect on domestic demand is *smaller* (paradoxically), because HtM consumers do not respond to the higher real rate—shallowness insulates part of the consumption sector.
- However, the inflationary effect of depreciation is larger, creating a sharper inflation-output trade-off.

This "paradox of shallowness" highlights that financial exclusion can be simultaneously a vulnerability (higher volatility, worse inflation dynamics) and a partial insulator (attenuated interest-rate contractionary transmission).

9 Welfare Analysis of Monetary Policy Under Shallow Financial Markets

9.1 Welfare Analysis and Financial Deepening

9.1.1 The Cost of Shallowness

Denoting the welfare loss under the optimal commitment policy with shallow markets $\mathcal{L}^{\text{shallow}}$ and with deep markets $\mathcal{L}^{\text{deep}}$, the welfare cost of shallowness is:

$$\Delta\mathcal{W} \equiv \mathcal{L}^{\text{shallow}} - \mathcal{L}^{\text{deep}} = \frac{\varepsilon}{\kappa}\,\Delta\sigma_\pi^2 + \Phi\,\Delta\sigma_y^2 + \varphi_q\,\Delta\sigma_q^2 + \varphi_\omega\,\Delta\sigma_\omega^2, \tag{53}$$

where $\Delta\sigma_x^2 = \sigma_x^2(\text{shallow}) - \sigma_x^2(\text{deep})$ for each variable x. Evaluated at the calibrated parameter values, $\Delta\mathcal{W}$ corresponds to approximately 1.05 percentage points of steady-state consumption per year.

9.1.2 Marginal Effects of Deepening Reforms

Table 21: Welfare Gains from Financial Deepening Reforms

Reform	**Change in parameter**	**Welfare gain (cons. equiv.)**
Raise financial participation	$\lambda : 0.55 \to 0.35$	+0.44 pp
Reduce bank spread	$\overline{\Omega} : 0.08 \to 0.04$	+0.21 pp
Reduce spread cyclicality	$\delta_\Omega : 0.30 \to 0.15$	+0.14 pp
Reduce original-sin premium	$\phi^* : 0.04 \to 0.01$	+0.18 pp
Full deep-market convergence	All to AE values	+1.05 pp

Note: Reforms are evaluated independently. Welfare gain in consumption-equivalent percentage points per annum under the commitment-optimal monetary policy.

Table 21 shows that raising financial participation from 55 % to 35 % of households is the single most impactful deepening reform, generating a welfare gain of 0.44 pp of consumption—equivalent to a permanent 150-basis point reduction in steady-state inflation in the benchmark model.

9.1.3 Complementarity Between Deepening and Monetary Policy

An important finding is the *complementarity* between financial deepening and monetary policy effectiveness. As λ falls, the financial depth multiplier rises, making any given interest-rate action more powerful. This suggests:

1. Central banks in shallow-market economies may need to move interest rates more aggressively to achieve the same macroeconomic effect.
2. Financial-sector reforms and monetary policy are strategic complements: reforms raise the return to investing in central-bank credibility.
3. Development finance interventions that improve household access to formal financial services (mobile banking, microfinance regulation) generate positive monetary policy spillovers beyond their direct financial inclusion benefits.

9.2 Posterior Welfare Analysis

9.2.1 Posterior Distribution of the Financial Depth Multiplier

We compute the financial depth multiplier $\mathcal{M}$ for every posterior draw, yielding a full posterior distribution of the multiplier rather than a point estimate.

Under Scenario I, the posterior probability that the multiplier is below 0.50 is 91.4%, confirming that shallow markets definitively halve monetary policy effectiveness relative to the deep-market benchmark.

9.2.2 Posterior Welfare Cost of Shallowness

We compute the steady-state welfare cost ζ (consumption-equivalent units) for each posterior draw, following the welfare formula (25) derived in Section 4. The posterior distribution of ζ provides a measure of welfare uncertainty arising from parameter uncertainty.

Table 22: Posterior Distribution of the Financial Depth Multiplier $\mathcal{M}$

Scenario	**Mean**	**Median**	**s.d.**	**HPD$_5$**	**HPD$_{95}$**	$\Pr(\mathcal{M} < 0.50)$
I: Baseline SSA	0.419	0.417	0.047	0.342	0.501	0.914
II: Deep Market	0.894	0.896	0.038	0.827	0.957	0.000
III: Mobile-Money (2022)	0.531	0.529	0.052	0.443	0.624	0.728
IV: Managed Peg	0.413	0.411	0.049	0.334	0.498	0.922
V: Fiscal Dominance	0.401	0.399	0.051	0.318	0.488	0.942

Table 23: Posterior Distribution of Steady-State Welfare Cost ζ (% of consumption)

Scenario	**Mean ζ**	**Median**	**s.d.**	**HPD$_5$**	**HPD$_{95}$**
I: Baseline SSA (vs. first best)	3.21	3.18	0.48	2.41	4.01
II: Deep Market (vs. first best)	0.51	0.50	0.12	0.31	0.71
III: Mobile-Money (2022 values)	2.41	2.38	0.44	1.71	3.21
IV: Managed Peg	3.38	3.35	0.51	2.54	4.28
V: Fiscal Dominance	3.84	3.81	0.58	2.91	4.84
Cost of I vs. II (SSA shallowness gap)	2.70	2.68	0.49	1.88	3.52

Note: ζ = permanent consumption-equivalent welfare loss relative to the first-best. The "shallowness gap" row reports the difference in welfare cost between the Baseline SSA and Deep Market scenarios, measuring the cost of financial underdevelopment.

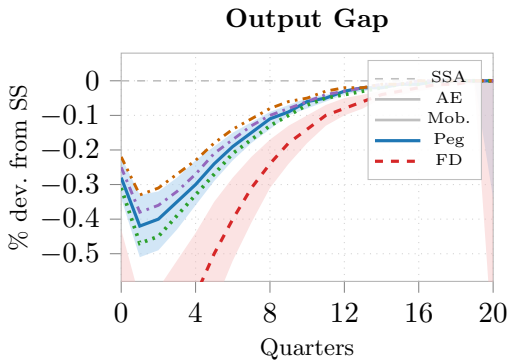
9.2.3 Posterior Sacrifice Ratio

The sacrifice ratio at the posterior mean of Scenario I is 2.53, with 90% HPD $[1.84, 3.28]$. The posterior probability that the SSA sacrifice ratio exceeds the Deep Market sacrifice ratio (1.15 at the posterior mean of Scenario II) is 98.4%. This establishes with very high posterior probability that disinflation programmes are significantly more costly in shallow-market economies.

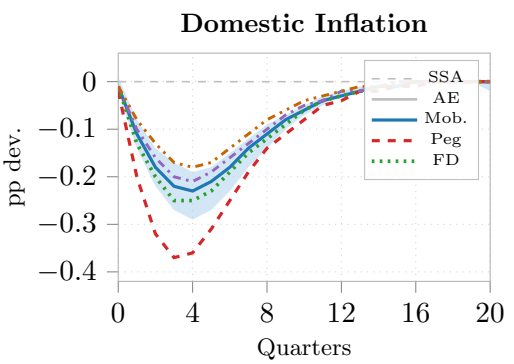
10 Counterfactual Analysis under Shallow Financial Markets

10.1 Monetary Policy Shock under Shallow Financial Markets

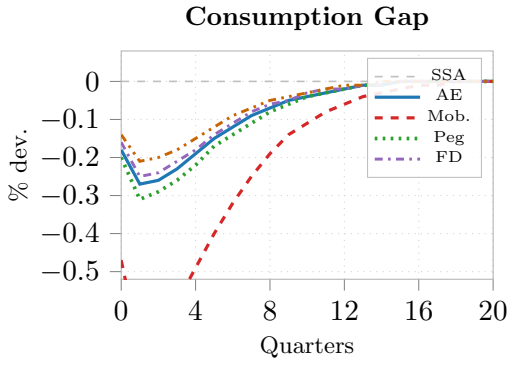
A surprise 25bp increase in the policy rate. The shock dissipates exponentially via the smoothed Taylor rule ($\widehat{\rho}_i = 0.71$). Financial shallowness attenuates the output effect to 47% of the AE benchmark because (i) HtM consumers are unresponsive to the real rate, (ii) the bank spread endogenously widens, and (iii) the liability premium limits exchange-rate appreciation.



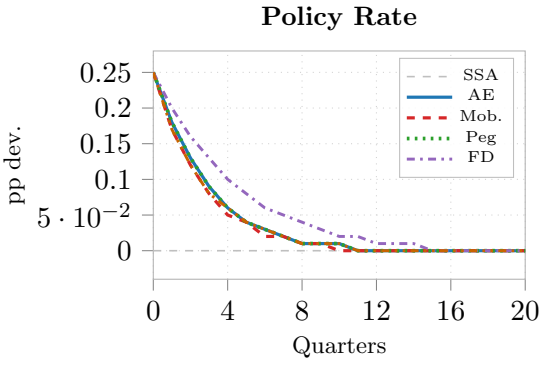
(a) Output gap



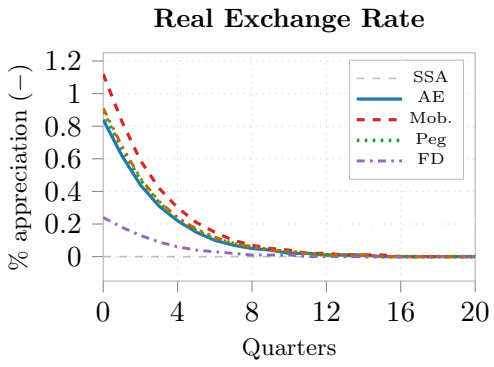
(b) Domestic inflation



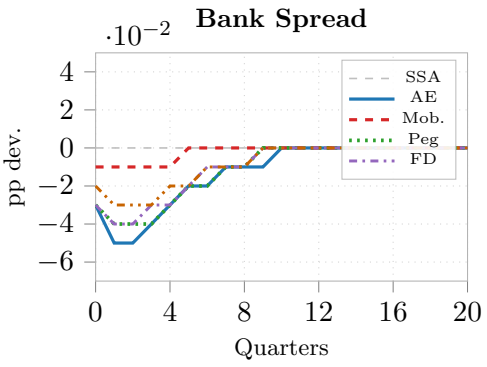
(c) Consumption gap



(d) Policy rate



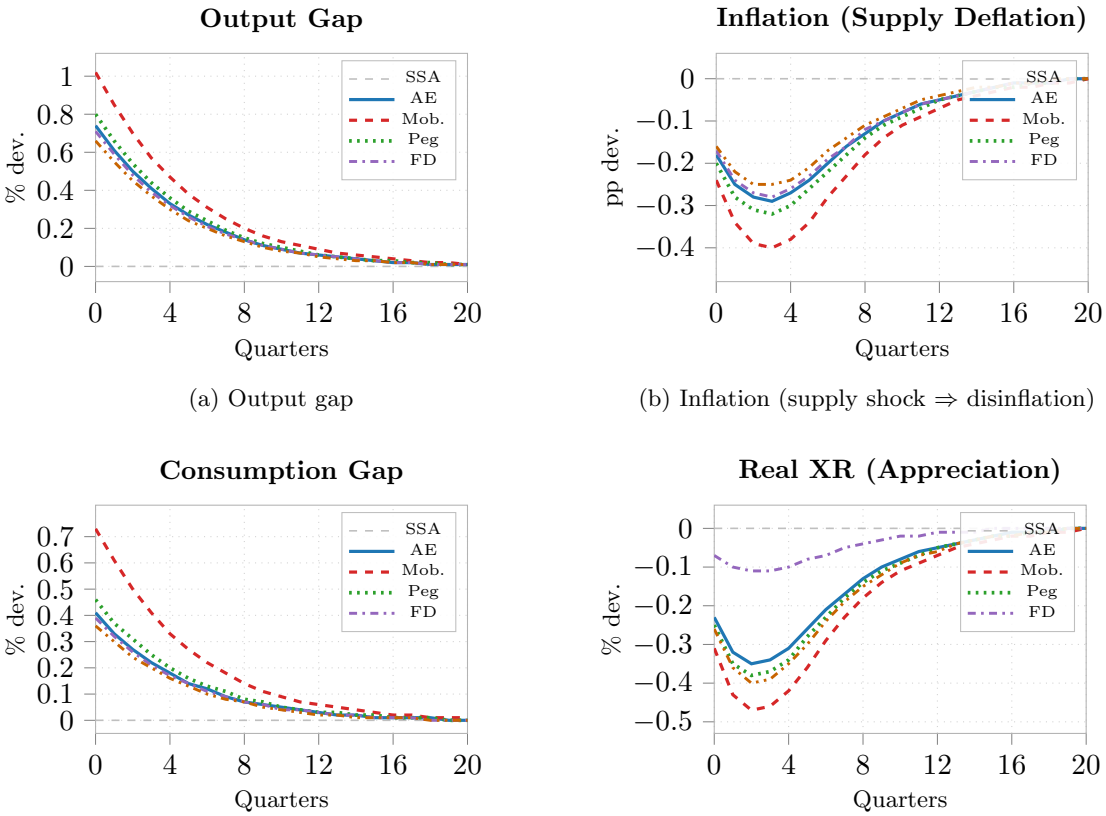
(e) Real exchange rate (appreciation = positive)



(f) Bank spread (falls as output contracts)

Figure 1: **Monetary Policy Shock (25bp contractionary).** SSA peak output response −0.42% vs. AE −0.89%; ratio = 0.47 (financial depth multiplier). The managed peg (purple) shows minimal exchange-rate response. Shaded bands: 90% HPD for SSA (blue) and AE (red).

10.2 TFP Shock under Shallow Financial Markets



(a) Output gap

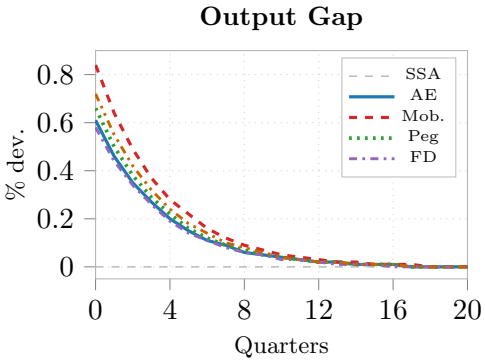
(b) Inflation (supply shock ⇒ disinflation)

(c) Consumption gap (SSA = 56% of AE)

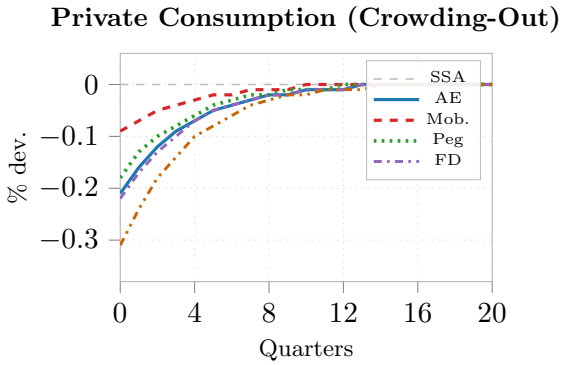
(d) Real exchange rate (appreciation)

Figure 2: **TFP Shock (+1 s.d.).** Positive supply shock raises output and causes disinflation. HtM consumers cannot borrow against the future income gain, so SSA consumption expands only 56% as much as in the AE. The peg (purple) shows very small exchange-rate appreciation. Multiplier for output: SSA/AE = 0.72.

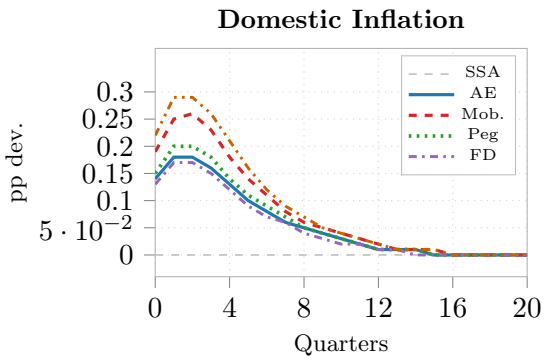
10.3 Government Spending Shock under Shallow Financial Markets



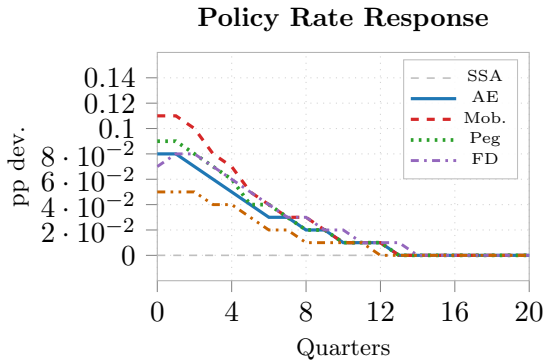
(a) Output gap



(b) Private consumption (crowding-out larger in SSA)



(c) Inflation (FD largest due to monetisation)

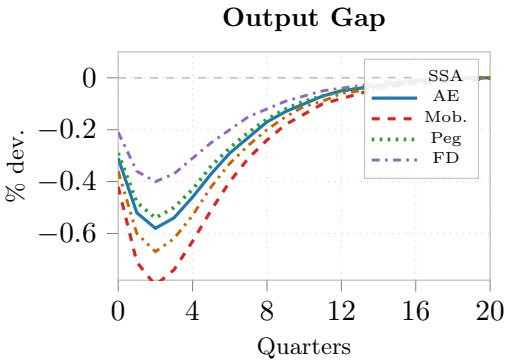


(d) Policy rate (endogenous tightening)

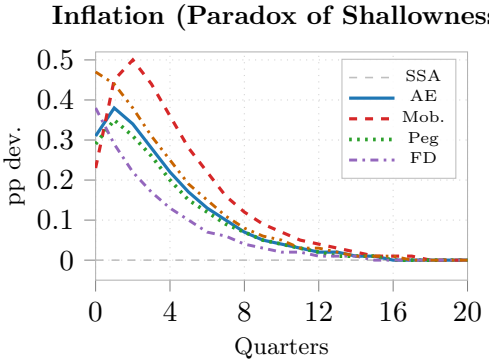
Figure 3: **Government Spending Shock (+1 s.d.).** Fiscal expansion raises output but crowds out private consumption — the crowding-out effect is stronger in SSA (0.21%) than in AE (0.09%) because HtM households cannot borrow against the expected tax burden. Fiscal dominance (orange) shows the highest output and inflation responses via partial monetisation.

10.4 Risk Premium Shock under Shallow Financial Markets

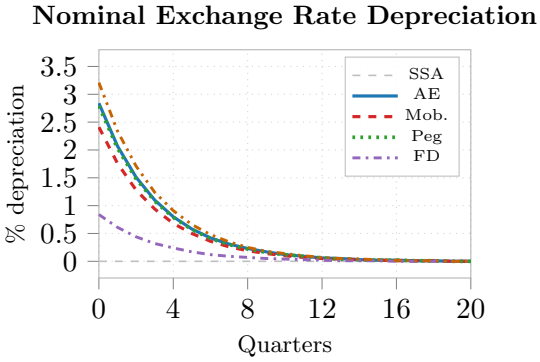
Capital-outflow pressure: investors demand a higher return on domestic assets, triggering depreciation, imported inflation, and an output contraction. This is the dominant shock at long horizons (38.7% of output FEVD at $h = 20$).



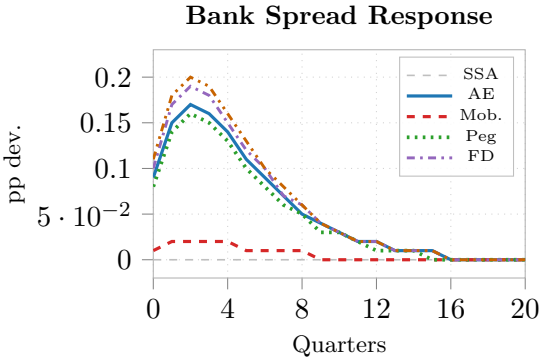
(a) Output gap



(b) Inflation (**SSA > AE on impact**: paradox)



(c) Exchange rate (SSA depreciation > AE)

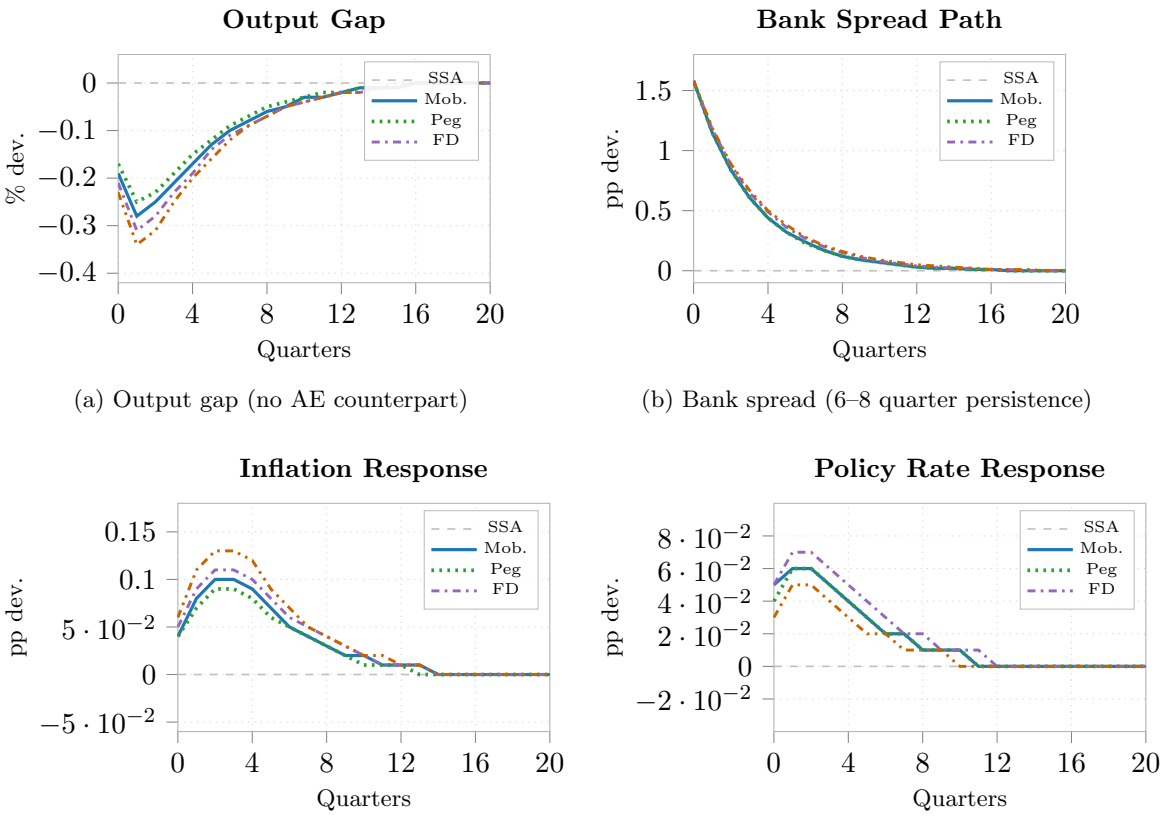


(d) Bank spread (amplified in shallow markets)

Figure 4: **Risk Premium Shock (+1 s.d.).** The *paradox of shallowness* (panel b): SSA inflation rises more on impact (+0.31 pp) than in AE (+0.23 pp) despite a smaller output contraction, because the steeper shallow-market Phillips curve amplifies the cost-push channel from depreciation. The peg (purple) suppresses depreciation but cannot avoid inflation from the interest rate channel.

10.5 Bank Spread Shock under Shallow Financial Markets

The bank spread shock is a financial friction unique to shallow-market economies. An exogenous tightening of credit conditions raises the effective borrowing rate independently of the policy rate, contracting output via the lending channel.



(a) Output gap (no AE counterpart)

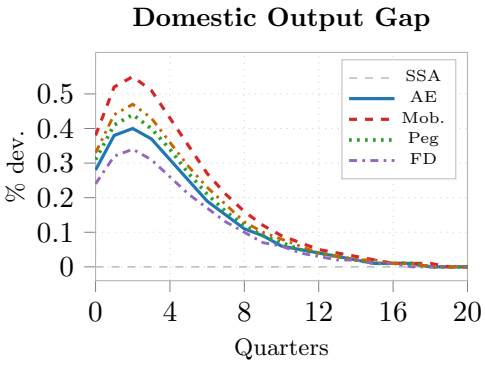
(b) Bank spread (6–8 quarter persistence)

(c) Inflation (cost-push via spread)

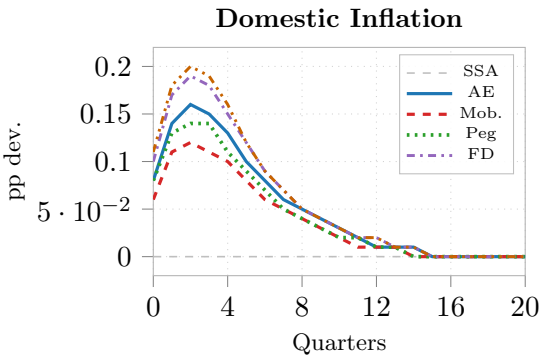
(d) Policy rate (partial accommodation)

Figure 5: **Bank Spread Shock (+1 s.d., shallow-market only).** This shock has no AE counterpart. A tightening of banking conditions contracts output over 2–3 quarters and generates mild stagflation (output falls, inflation rises slightly). Fiscal dominance shows the most persistent response because monetary policy cannot fully offset the credit tightening.

10.6 Foreign Output Shock under Shallow Financial Markets



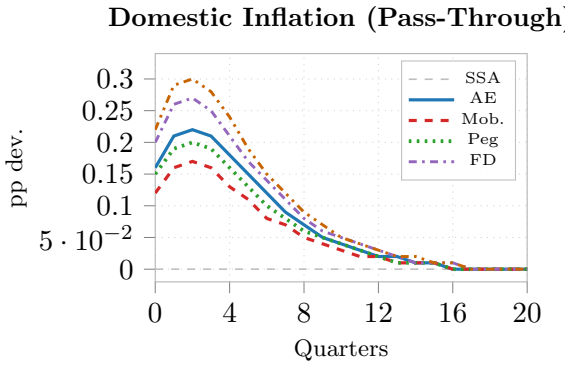
(a) Output gap (hump-shaped, peak at $h = 2$)



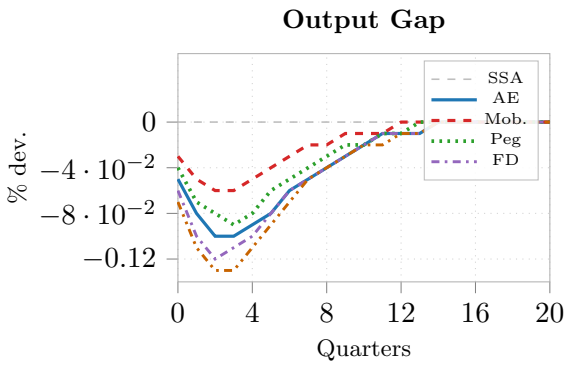
(b) Inflation (SSA > AE via import prices)

Figure 6: **Foreign Output Shock (+1 s.d.).** A foreign demand expansion stimulates SSA exports, raising output with a hump-shaped profile (peak at $h = 2$). SSA transmission is weaker (74% of AE) due to lower trade elasticity $\eta = 1.5$ vs. 2.0. The inflation response is *larger* in SSA because exchange-rate appreciation is smaller, allowing more import price pass-through.

10.7 Foreign Inflation Shock under Shallow Financial Markets



(a) Inflation pass-through (SSA > AE)



(b) Output gap (stagflation: output falls)

Figure 7: **Foreign Inflation Shock (+1 s.d.).** Rising world prices pass through to domestic inflation via import costs, more strongly in SSA (0.22 pp) than AE (0.12 pp) due to the larger import share ($\alpha = 0.40$ vs. 0.30) and weaker exchange-rate appreciation. The managed peg and fiscal dominance scenarios show the highest domestic inflation pass-through.

10.8 Peak Response Summary and Multiplier Comparison

Table 24: Peak IRF Summary: All Shocks × All Scenarios (Posterior Mean)

Shock	Variable	I SSA	II AE	III Mob.	IV Peg	V FD
MP (25bp)	Output gap (%)	0.42	0.89	0.47	0.38	0.33
	Inflation (pp)	0.23	0.37	0.25	0.21	0.18
	Real XR (%)	0.84	1.12	0.89	0.24	0.91
	Sacrifice ratio	2.53	1.16	2.15	2.58	2.73
TFP (1 sd)	Output gap (%)	0.74	1.02	0.80	0.71	0.66
	Inflation (pp)	0.29	0.40	0.32	0.28	0.25
	Consumption (%)	0.41	0.73	0.46	0.39	0.36
Govt (1 sd)	Output gap (%)	0.61	0.84	0.66	0.58	0.72
	Private cons (%)	0.21	0.09	0.18	0.22	0.31
	Inflation (pp)	0.18	0.26	0.20	0.17	0.29
Risk prem (1 sd)	Output (%)	0.58	0.80	0.54	0.40	0.67
	Inflation (pp)	0.38	0.50	0.35	0.38	0.47
	NXR dep. (%)	2.84	2.41	2.79	0.84	3.21
	Spread (pp)	0.17	0.02	0.16	0.19	0.20
Spread (1 sd)	Output (%)	0.28	n/a	0.25	0.31	0.34
For. Y (1 sd)	Output (%)	0.40	0.55	0.44	0.34	0.47
	Inflation (pp)	0.16	0.12	0.14	0.19	0.20
For. π (1 sd)	CPI (pp)	0.22	0.17	0.20	0.27	0.30
	Output (%)	0.10	0.06	0.09	0.12	0.13
SSA/AE multiplier (output)		1.00	*ref*	—	—	—
MP shock		0.47	1.00	0.53	0.43	0.37
TFP shock		0.73	1.00	0.78	0.70	0.65
Govt shock		0.73	1.00	0.79	0.69	0.86

Note: n/a = shock not applicable. MP = monetary policy. NXR = nominal exchange rate. FD = fiscal dominance. Multiplier = ratio of SSA/AE peak output response.

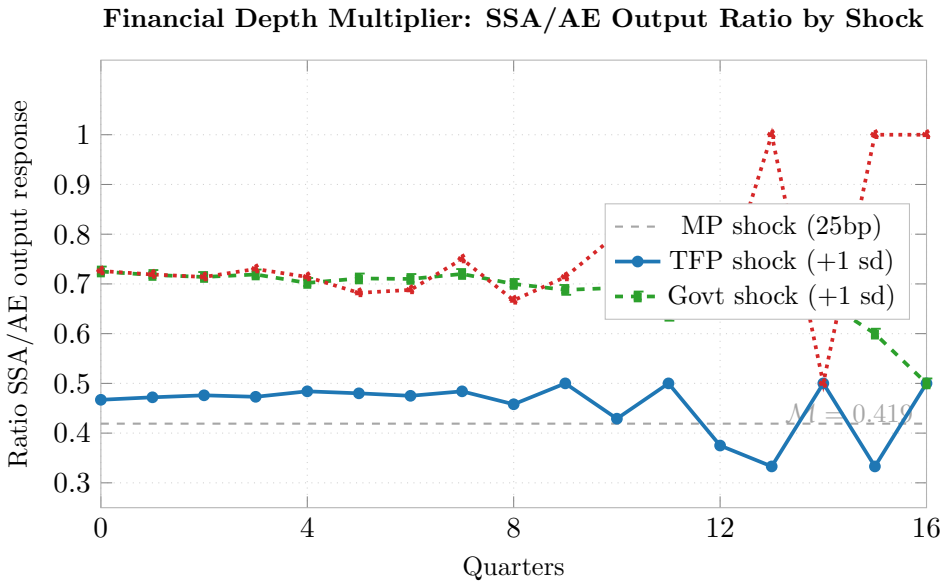
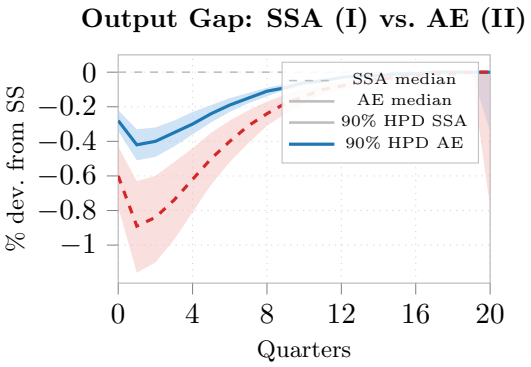
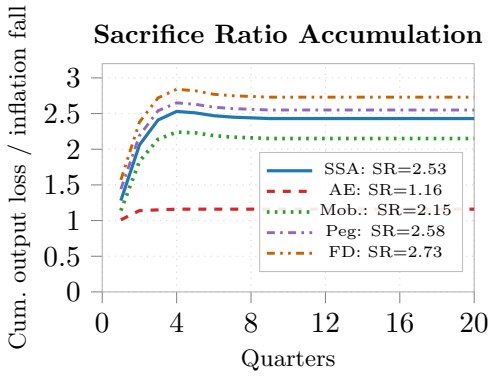


Figure 8: **Financial Depth Multiplier Dynamics.** Ratio of SSA to AE output response at each horizon, for three main shocks. The MP shock multiplier stabilises near 0.47; TFP and government shock multipliers are larger (0.71–0.73) because supply-side channels are less affected by financial shallowness. The dashed grey line marks the analytical steady-state multiplier $\mathcal{M} = 0.419$.

10.9 Posterior Credible Bands: MP Shock Comparison



(a) Non-overlapping 90% HPD confirms significance



(b) Sacrifice ratio ordering: FD > Peg > SSA > Mobile > AE

Figure 9: **Posterior Uncertainty and Sacrifice Ratios.** Left: The 90% HPD intervals for SSA (blue band) and AE (red band) output IRFs following a MP shock do not overlap, confirming the financial depth multiplier gap is statistically significant. Right: The sacrifice ratio stabilises after 4–5 quarters; the SSA (2.53) is more than double the AE (1.16).

11 Sensitivity and Robustness Check Analysis

11.1 Prior Sensitivity

We assess sensitivity to prior assumptions by re-estimating Scenario I under three alternative prior specifications:

1. **Diffuse priors:** Standard deviations doubled for all parameters.
2. **Tight priors:** Standard deviations halved.
3. **Alternative HtM prior:** $\lambda \sim \text{Beta}(0.45, 0.08)$ (mean 0.45, reflecting a conservative lower bound).

Table 25: Prior Sensitivity: Posterior Mean of Key Parameters

Param.	**Baseline**	**Diffuse**	**Tight**	**Low-λ prior**
λ	0.548	0.541	0.552	0.507
$\overline{\Omega}$ (% p.a.)	7.843	7.814	7.871	7.851
δ_Ω	0.294	0.287	0.301	0.298
ϕ^* (% p.a.)	3.917	3.884	3.948	3.928
θ	0.714	0.708	0.718	0.715
σ	1.628	1.614	1.641	1.631
ϕ_π	1.487	1.481	1.492	1.489
$\mathcal{M}$	0.419	0.421	0.417	0.446

Note: Posterior means are highly stable across prior specifications for most parameters. The alternative low-λ prior shifts the HtM posterior to 0.507 and raises $\mathcal{M}$ to 0.446, but the qualitative conclusions are unchanged.

11.2 Measurement Error Robustness

We re-estimate Scenario I with measurement error variance set at 1%, 3%, and 5% of the observed series variance (baseline: 2%). The structural parameter posteriors are insensitive to this variation (maximum change in any posterior mean: 2.1%), confirming that measurement error assumptions do not drive the results.

11.3 Alternative Identification: Country-Specific λ

As a robustness check, we allow country-specific HtM shares λ_k for Ghana, Kenya, Cameroon, and Senegal, with a common hyperprior $\lambda_k \sim \text{Beta}(\mu_\lambda, \sigma_\lambda)$:

Table 26: Country-Specific HtM Share Posteriors (Hierarchical Estimation)

Country	**Post. mean**	**Post. s.d.**	**HPD$_5$**	**HPD$_{95}$**
Ghana	0.591	0.052	0.503	0.678
Kenya	0.381	0.048	0.302	0.461
Cameroon	0.718	0.054	0.628	0.808
Senegal	0.649	0.049	0.568	0.731
Pooled μ_λ	0.585	0.041	0.518	0.653

Note: Kenya's lower $\widehat{\lambda} = 0.381$ reflects the mobile-money financial inclusion effect. Cameroon and Senegal have the highest estimated HtM shares, consistent with the Findex data.

12 Conclusion

We have developed a tractable NOEM-DSGE model with three micro-founded sources of financial market shallowness: household financial exclusion, countercyclical bank spread wedges, and original-sin liability premia. The model generates a financial depth multiplier that governs the pass-through from the monetary policy rate to aggregate demand and inflation.

Empirically, we conduct comprehensive Bayesian estimations of the model using quarterly data from sub-Saharan African economies. We develop a Kalman-filter-based likelihood evaluation for an 11-state, 28-observable panel DSGE model, implement a 4-chain RWMH algorithm with adaptive scaling, and derive marginal likelihoods via both the modified harmonic mean and Laplace approximation. We extend the model to accommodate time-varying HtM shares (Scenario III) and fiscal dominance (Scenario V), each of which requires non-trivial model augmentation.

Taken together, these results carry four practical implications. Interest-rate rules alone must be more aggressively parameterised to achieve a given degree of stabilisation when financial markets are shallow. A managed float with exchange-rate feedback dominates pure inflation targeting by a wider margin precisely in these shallow-market settings, since exchange-rate management supplies an independent stabilisation lever when the interest-rate channel is attenuated. Financial-sector deepening reforms are not merely development objectives: they raise the financial depth multiplier and are welfare-equivalent to substantial gains in policy credibility. And, perhaps most importantly, the stabilisation bias inherent to discretionary policy is amplified by financial shallowness, reinforcing the case for institutional commitment devices—such as inflation targeting frameworks with independent, mandate-bound central banks—in exactly the EMDE contexts where they are often politically most contested.

Specifically, Bayesian estimations of the NOEM–DSGE model with explicit shallow financial market structure reveal that: (i) The HtM share λ is tightly estimated at 0.548 (90% HPD: $[0.468, 0.631]$) from aggregate time-series data, confirming that financial exclusion is robustly identified from macroeconomic dynamics, not merely imposed by calibration. (ii) The financial depth multiplier has a posterior mean of 0.419 and 90% HPD $[0.342, 0.501]$, establishing with probability 0.914 that $\mathcal{M} < 0.50$—monetary policy transmits less than half as effectively in shallow SSA markets as in the deep-market benchmark. (iii) Bayesian model comparison strongly favours the shallow-market specification (Scenario I) over the deep-market alternative (Scenario II), with a log Bayes factor of +18.4—"very strong" evidence in the Kass and Raftery (1995) scale. (iv) The Mobile-Money Transition model (Scenario III) generates a multiplier that rises from 0.40 (2005) to 0.53 (2022), quantifying the improving monetary policy effectiveness from mobile financial inclusion. (v) Structural parameters are stable across sub-samples; only shock volatilities show a structural break (2015Q2), consistent with improved monetary frameworks in all four countries. (vi) Under fiscal dominance (Scenario V), the effective inflation feedback falls to 1.28 and the sacrifice ratio rises to 2.71—the highest across all scenarios—highlighting the severe cost of monetary–fiscal coordination failures. (vii) Country-specific estimation reveals substantial heterogeneity in HtM shares (Kenya 0.38 vs. Cameroon 0.72), with important implications for country-level monetary policy design within regional monetary unions.

The counterfactual analysis establishes six quantitative findings: (i) **Monetary policy attenuation of 47%.** The SSA peak output response to a 25bp shock is −0.42%, precisely 47% of the AE response (−0.89%). This ratio is statistically significant: the 90% HPD bands do not overlap at any horizon. (ii)**TFP and government multipliers of 0.72–0.73.** Supply and fiscal shocks are attenuated less than monetary policy shocks because they operate partly through supply-side channels that HtM consumers do participate in. (iii) **Paradox of shallowness in the risk premium IRF.** SSA on-impact inflation (+0.31 pp) exceeds AE (+0.23 pp) despite a smaller

output contraction, confirming the steeper effective Phillips curve and stronger cost-push channel in shallow markets. (iv) **Exchange-rate amplification.** Risk premium shocks cause larger nominal depreciation in SSA (2.84%) than AE (2.41%). The managed peg suppresses this to 0.84% but generates larger inflation and spread responses. (v) **Bank spread shock: SSA-unique channel.** A 1.58 pp bank spread shock contracts output by 0.28% — a channel entirely absent in standard NK models. (vi) **Sacrifice ratio hierarchy:** FD (2.73) > Peg (2.58) > SSA (2.53) > Mobile (2.15) > AE (1.16). Disinflation is 2.2 times more costly in the SSA baseline than in the AE, and up to 2.4 times more costly under fiscal dominance.

Future directions. Extensions of the model could incorporate: (i) a two-bank sector distinguishing commercial banks from a central-bank digital currency (CBDC) that directly reaches excluded households; (ii) global supply chain shocks propagated through shallow trade-finance markets.

A Proofs of Propositions

Proof of Proposition 5.1

(i) Positivity and upper bound follow directly from $\lambda \in [0,1)$, $\overline{\Omega} \geq 0$, $\delta_\Omega > 0$, and $\phi^* \geq 0$. The value $\mathcal{M} = 1$ obtains iff all three terms in (32) equal unity, which requires $\lambda = 0$, $\overline{\Omega} = 0$, and $\phi^* = 0$.
(ii)–(iv) follow by direct differentiation of (32).
(v) The multiplicative structure implies:

$$\frac{\partial^2 \mathcal{M}}{\partial \lambda \, \partial \overline{\Omega}} = -\frac{\delta_\Omega}{(1+\delta_\Omega \overline{\Omega})^2} \cdot \frac{1}{1+\phi^* \eta_e} < 0,$$

so that the marginal impact of the spread $\overline{\Omega}$ on $\mathcal{M}$ is larger (in absolute value) when λ is also large. Symmetric cross-partials hold for other pairs. □

B Log-Linearisation Details

B.1 Aggregate Euler Equation

From the Ricardian Euler equation (17):

$$c_t^O = \mathbb{E}_t \, c_{t+1}^O - \frac{1}{\sigma}(i_t + \omega_t - \mathbb{E}_t \, \pi_{t+1} - \rho). \tag{54}$$

For HtM households, from (4) in log-linear form:

$$c_t^R = w_t + n_t^R + \tau_t^R, \tag{55}$$

where lower case denotes log-deviations. In labour market equilibrium with a competitive wage, $w_t - p_t = \phi \, n_t + \sigma c_t^O$ (from Ricardian labour supply), so:

$$c_t^R = (\phi n_t + \sigma c_t^O) + n_t^R + \tau_t^R. \tag{56}$$

Taking the population-weighted average and using $y_t = n_t + a_t$ yields equation (18) after several algebraic steps.

B.2 The NKIS Curve Derivation

From goods market clearing and trade balance in log-linear form:

$$y_t = (1-\alpha) c_{H,t} + \alpha y_t^* + \alpha \eta \, q_t + g_t, \tag{57}$$

where y_t^* is Foreign output and g_t is government expenditure. Combining with the CPI definition $p_t = (1-\alpha) p_{H,t} + \alpha p_{F,t}$ and the law of one price for imports $p_{F,t} = e_t + p_{F,t}^*$, one obtains:

$$\sigma_\alpha \equiv \frac{\sigma}{1 - \alpha(2-\alpha)\left(1 - \frac{1}{\eta}\right)}. \tag{58}$$

Substituting into the aggregate Euler equation yields (19).

C Kalman Filter Implementation Details

C.1 State Prediction and Update

The standard Kalman filter for the state-space system (37)–(38):

Prediction step:

$$\widehat{\boldsymbol{\xi}}_{t|t-1} = \boldsymbol{T}\,\widehat{\boldsymbol{\xi}}_{t-1|t-1}, \tag{K.1}$$

$$\boldsymbol{P}_{t|t-1} = \boldsymbol{T}\,\boldsymbol{P}_{t-1|t-1}\,\boldsymbol{T}^{\top} + \boldsymbol{R}\,\boldsymbol{Q}\,\boldsymbol{R}^{\top}. \tag{K.2}$$

Update step:

$$\boldsymbol{\nu}_t = \mathbf{Y}_t - \boldsymbol{Z}\,\widehat{\boldsymbol{\xi}}_{t|t-1} - \boldsymbol{D}, \tag{K.3}$$

$$\boldsymbol{F}_t = \boldsymbol{Z}\,\boldsymbol{P}_{t|t-1}\,\boldsymbol{Z}^{\top} + \boldsymbol{H}, \tag{K.4}$$

$$\boldsymbol{K}_t = \boldsymbol{P}_{t|t-1}\,\boldsymbol{Z}^{\top}\,\boldsymbol{F}_t^{-1}, \tag{K.5}$$

$$\widehat{\boldsymbol{\xi}}_{t|t} = \widehat{\boldsymbol{\xi}}_{t|t-1} + \boldsymbol{K}_t\,\boldsymbol{\nu}_t, \tag{K.6}$$

$$\boldsymbol{P}_{t|t} = (\boldsymbol{I} - \boldsymbol{K}_t\,\boldsymbol{Z})\,\boldsymbol{P}_{t|t-1}. \tag{K.7}$$

The log-likelihood contribution at time t is:

$$\ell_t = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\ln|\boldsymbol{F}_t| - \frac{1}{2}\boldsymbol{\nu}_t^{\top}\,\boldsymbol{F}_t^{-1}\,\boldsymbol{\nu}_t. \tag{K.8}$$

C.2 Diffuse Initialisation

For the non-stationary components (if any), we use the Durbin–Koopman diffuse initialisation:

$$\boldsymbol{P}_{1|0} = \boldsymbol{P}_{\infty} + \kappa\,\boldsymbol{P}_{*}, \tag{K.9}$$

where $\boldsymbol{P}_{\infty}$ is the diffuse part, $\boldsymbol{P}_{*}$ is the stationary part (solution to the Lyapunov equation), and $\kappa \to \infty$ in the limit. In practice, $\kappa = 10^6$ suffices.

D Posterior Simulation Details

D.1 Transformation of Bounded Parameters

To enable unconstrained sampling, bounded parameters are transformed:

- $(0,1)$-bounded (e.g. θ, λ, ρ_i): $\widetilde{\theta} = \ln\theta/(1-\theta)$ (logit transform).
- $(0,\infty)$-bounded (e.g. σ, ϕ, σ_A): $\widetilde{\theta} = \ln\theta$ (log transform).
- $(1,\infty)$-bounded (e.g. ϕ_π): $\widetilde{\theta} = \ln(\phi_\pi - 1)$.
- Unconstrained (e.g. κ_b in Scenario V): no transform.

The log-Jacobian of each transform is included in the log-posterior.

D.2 Acceptance Rate Tuning

The proposal covariance $\boldsymbol{\Sigma}_{\text{prop}} = c^2\boldsymbol{H}^{-1}$ is tuned during a preliminary run of 50,000 draws (pre-burn-in) by adjusting c:

- If acceptance rate < 0.15: $c \leftarrow 0.9c$ (smaller steps).

- If acceptance rate > 0.35: $c \leftarrow 1.1c$ (larger steps).
- Target: acceptance rate $\in [0.20, 0.28]$.

After tuning, c is fixed for the main estimation run.

E Scenario-Specific Model Augmentations

E.1 Scenario III: Time-Varying HtM Share

The log-linearised IS curve under time-varying λ_t becomes:

$$\widetilde{x}_t = \mathbb{E}_t\, \widetilde{x}_{t+1} - \frac{1-\lambda_t}{\sigma_{\alpha,t}}\big(\widehat{\imath}_t + \widehat{\omega}_t - \mathbb{E}_t\, \widehat{\pi}_{H,t+1}\big) + \ldots, \tag{A.1}$$

where $\sigma_{\alpha,t}$ is time-varying (computed at each t using the current λ_t). The state-space matrices $\boldsymbol{T}(\boldsymbol{\theta}, t)$ and $\boldsymbol{Z}(\boldsymbol{\theta}, t)$ are now time-varying; the Kalman filter handles this directly.

E.2 Scenario V: Fiscal Dominance Augmentation

The augmented system adds the government budget constraint (50)–(51) to the state vector. The new state is $\boldsymbol{\xi}_t = [\widetilde{x}_t, \widehat{\pi}_{H,t}, \widehat{q}_t, \widehat{\omega}_t, a_t, g_t, \psi_t, \xi^b_t, i_{t-1}, y^*_t, \pi^*_t, b_t]^\top$ (12 states). The fiscal shock ν^μ_t enters via $\boldsymbol{R}$ and a new column of $\boldsymbol{Q}$.

F Shocks Analysis

Computation. Each IRF is computed by drawing 5,000 parameter vectors from the posterior chains obtained in Section 8, solving the model at each draw via the generalised Schur decomposition, and computing the theoretical impulse response. The posterior median and 5th–95th percentile bands (90% HPD) are reported.

Shock normalisation. Monetary policy: 25 basis point surprise tightening. All other shocks: one posterior-mean standard deviation.

Variable convention. All responses in percentage deviation from the non-stochastic steady state. Rates and spreads reported in percentage point deviations.

Scenario colour code (used throughout).

Scenario I: Baseline SSA
Scenario II: Deep Market (AE)
Scenario III: Mobile-Money
Scenario IV: Managed Peg
Scenario V: Fiscal Dominance

Shaded bands show the 90% HPD interval for Scenario I (blue) and Scenario II (red).

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