Reputational Goods and Ownership Networks*

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Abstract

We develop a network-based model of the market of *reputational goods* in which the reputations of participants are jointly determined through ownership links. We show that central participants value the reputational goods more, which leads to higher prices in trades between central participants who are not connected to each other, without additional assumptions about preferences or liquidity. Decentralized trade is blind to the reputational externalities and fails to maximize aggregate reputation. We find that systematic deviations of our predictions from conventional Walrasian models are consistent with empirical regularities of the art market, such as the failure of the law of one price.

Keywords: reputational goods; ownership networks; network externalities; law of one price; pairwise stability.

JEL: D85, D62, L14, Z11.

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Introduction

Goods that look interchangeable do not always sell for a similar price; even the expected price range for identical goods might differ considerably depending on the trade participants. This happens in some markets despite goods' durability, abundant liquidity, and ability to resell, and despite ample market depth and the absence of asymmetric information about the goods' innate characteristics. We introduce the conditioning on the identities of market participants, on their placement in the ownership network, and on their reputation.

We propose a model of REPUTATIONAL GOODS where the value of the good depends on the reputation of holders of this and similar goods. This creates a feedback loop, elucidating the value of a consumer's connectedness for the consumer's willingness to pay, and making prices hinge on consumers' identities. Examples of such markets include collectors and artists on the art market, *connoisseurs* and wineries on the premium wine market, universities and professors on the academic labour market, team sports players and sports clubs on the transfer market, researchers and journals on the academic publication market, movie stars and studios, venture capitalists and start-ups, music composers and their patrons, and any recognition-seeking philanthropy. We frame this paper in the art market terminology because the empirical literature on the art trade provides us with ample evidence of non-Walrasian market features to test against our findings.

These reputational spillovers make more connected sellers demand a higher price for a good quality item, and more connected buyers benefit more from acquiring the item; the less the buyer and the seller are connected to each other, the more valuable are the connections attached to the item, so, *ceteris paribus*, less mutually connected participants agree on an even higher price. This mechanism explains the disagreement on the acceptable price ranges for trades on the same market without assuming different preferences or liquidity constraints. Placing more items of the same maker with the same consumer increases the reputation of the maker faster than spreading them across the market would, delivering incentives for clusterization without assuming returns to scale or idiosyncratic preferences.

Every trade redistributes items, affecting the reputations of items and therefore of other holders of these items. Trades are innately bilateral, and the parties only account for the trade's impact on the two participants' own reputations, not for how the trade affects the reputations of others on the market. Decentralized trade therefore might fail to lead to an efficient outcome, even if a pairwise-stable allocation, where no two consumers can trade

to improve the welfare of both, exists. This externality also means that a forward-looking seller's most preferred counterparty might not be the one who is eager to pay the most, but the one who will do more to build up the seller's reputation.

Our model is fundamentally different from "games on the network" literature (Ballester et al., 2006): our links are not exogenous inputs affecting the conditions facing participants of a separate game; they are the traded goods, and they are channels of reputation flows. It is also different from Veblen good or positional good literature (Hopkins and Kornienko, 2004) because value in our model comes not from scarcity, but from the reputation of the other participants via the ownership network. In contrast to the assignment and matching literature, reputational changes are not coming from idiosyncratic preferences or complementarities, but from reputational spillovers across the ownership network.

The Literature Review section contrasts our model with existing models of reputation and/or network interactions. We then review the art market literature to establish stylized facts that we aim to match with our model. We follow with an outline of our model, where we establish fundamentals and study the implications for trade-related decisionmaking. Finally, we discuss our findings, propose venues for future research, and conclude.

Literature Review

In our paper, we develop a story about how reputation is built endogenously. Our endogeneity of value is different from Veblen goods: as an example in that literature, Pesendorfer (1995) discusses the fashion industry as a signalling device in a dating game. The signal is valuable because the high price of a new fashion item allows you to credibly flaunt your type. This explains the constant change in fashion: old fashion becomes omnipresent, gets cheaper due to competition, and stops serving its purpose. In our paper, scarcity does not necessarily drive up prices, but the identities of other consumers might. While conventional Veblen good literature treats status as a function of what the consumer owns, in our paper, status is an endogenous function of what other people connected to you via ownership own. To illustrate the difference, consider this story about the British Guiana ¢1 magenta stamp from the Smithsonian National Postal Museum (2026):

> The distinguished American philatelist Arthur Hind, who owned the known example in the 1920s, was rumoured to have become so obsessed with it that when he came across a second example he bought it and destroyed it, so that his would remain the only one.

The story feels plausible, albeit even the museum says it is not documented, because stamps are pure Veblen goods, while a similar story for art—a collector buying a second art piece and destroying it hoping to increase the value of their own copy—seems unlikely.

Hirsch (1976) introduces positional goods as those that are scarce in an absolute or socially imposed sense. Hopkins and Kornienko (2004) model this as an exogenous reward for the *rank* among consumers of a positional good: such a formulation makes the consumed amount of the positional good inefficient, for strategic reasons similar to payments in an all-pay auction. In our model, the reputation of ownership is endogenous rather than rank-based, and the externality function is bipartite: the value depends on the specific network position of all collectors, and no comparisons with others are involved.

Reputations Tirole (1996) proposes a theory of reputations where individuals' opportunities depend on the average reputation of group members as this is more observable than individual reputation. Board and Meyer-ter-Vehn (2013) and Board and Meyer-ter-Vehn (2022), following the literature surveyed in Bar-Isaac and Tadelis (2008), study how reputation develops, and how it interacts with investment in innate quality. In our paper, by contrast, reputation is generated through ownership of reputational goods, whose reputation is determined, in turn, from the reputation of holders, leading to spillovers across the ownership network. Schönfeld and Reinstaller (2007) discuss network effects in reputations on the art market while modeling competition between art dealers, but do not model where the reputations come from.

Networks A lot of literature on network formation is built around games played on networks. Jackson (2005) mentions a few; in particular Kranton and Minehart (2001) and Wang and Watts (2006) talk about networks connecting buyers and sellers. This is similar to connections between artists and collectors in our model, and the network nature of connections creates price variability, but goods in these models are not durable and the reputation propagation that determines a large number of interactions in our model is therefore absent in theirs. Other models, such as Polanski (2007), have buyer-sellers, similar to our derivative network of connections between collectors, but they are limited by the network, and do not change the network through trade.

Jackson and Wolinsky (1996) is one of the first studies where links are established to improve connections, but it has exogenous "values" and "costs" of connections, while in our model the reputational values of art pieces, trade in which creates links between artists and collectors, are purely endogenous. However the concepts of *pairwise stability* and *efficiency* they introduced are clearly applicable in our model too. The endogenous network

formation literature developed from this paper is vast; a couple of special mentions are König et al. (2014) who offer a centrality-based network formation with link decay and obtain nested split graphs as the outcome. Their results are driven by the same force—centrality-begets-centrality—as hoarding networks that we find. Baumann (2021) extends Jackson and Wolinsky (1996) model to allow participants to manipulate the intensity of the connection. In our model, the intensity of the connection is hard to manipulate directly, but it does affect outcomes immensely, even for those who are not participating in the trade. Sun et al. (2026) studies the problem of the network design, but looks at the problem from the social planner perspective, and arrives at nested split graphs; they limit the amount of links a network can establish in a period.

Clusterization Empirically, prominent collectors match with prominent artists, prominent departments match with prominent researchers, and prominent football clubs match with prominent players. Rosen (1981) explains this with increasing returns to scale when serving multiple people in creative professions; while it is true that in our model everyone agrees on the fundamentals, the returns to scale are not the driver. The literature reviewed by Sattinger (1993) has explicit complementarities that lead to positive assortative matching. Our model shows how network centrality built upon an ownership network leads to higher willingness to pay for central agents; our complementarities are coming out of network interactions, not from increasing returns to scale or production function complementarities. Perhaps the most similar model to ours in that regard is Baetz (2015), who obtains hierarchy in connection while suppressing concentration by decreasing marginal utility of benefits. But in his model the links are sponsored, which does not exactly translate into our model.

Art Market

We illustrate our machinery through the art market because it is well-documented by the empirical literature, yielding us stylized facts to capture with our model.

Prendergast (2019) is an excellent qualitative introduction into the economics of art dealing. Most of our design assumptions are coming from that paper. The network of art trade is large and interconnected; Schich et al. (2017) report that even from 1801-1820 one could not reliably study a submarket of art ignoring connections outside the market. De Silva et al. (2023) reports that dealers with high centrality in networks built on purchases survive longer on the art market than those without. Papers that study returns on asset investment—treating art purchases as if they were stock market position taking, such as

Etro and Stepanova (2021, 2026), Goetzmann (1993), Mei and Moses (2002), Renneboog and Spaenjers (2013)—provide stylized facts that we are trying to match with our model.

There are theoretical works on the industrial organization of the art market (see Etro, 2025, and papers cited in it). We intentionally drop conventional economic issues discussed in that literature (for instance, principal-agent relationships in price setting when contracting to paint the Sistine Chapel) and omit stylized facts well-explained by conventional literature to focus on the impact of the network of ownership.

Stylized Fact 1 Seemingly similar art pieces command wildly different prices on markets, with differences hard to explain away by transaction costs: Pesando (1993) confirms this with Picasso prints in New York, and Pesando and Shum (2007) repeats the exercise on worldwide data. Our model explains why the law of one price does not hold: the potential price range between willingness to pay and willingness to sell for different collectors depends on the identities of both collectors, and therefore might be so different across pairs of collectors participating in the observed transactions that they do not even overlap. Graddy et al. (2026) and Etro and Stepanova (2021) report that evening auctions are yielding much more than day auctions, even for similar art pieces and in the same auction houses. Even exhibitions in museums boost art prices (Hellmanzik, 2016). Stepanova and Sedino (2026) report that prices are increasing in the number of private-sale mentions. This could be explained by liquidity concerns of collectors (a larger market means resell is easier), but their Fig. 4 demonstrates that the premium is driven by sales via prominent galleries specifically. If a liquidity premium explanation were driven just by a larger market, the intermediary identity should not matter, but it does for the mechanism we propose in this paper.

Stylized Fact 2 Many collectors pay more for art pieces than on average: Etro and Stepanova (2026) study the art sold by Knoedler gallery in New York and show that specific collectors paid significant premia. The opposite holds for auction houses; Prendergast (2019) mentions it as a well-known fact that investment in art for resale at an auction is not a financially prudent strategy, while Ashenfelter and Graddy (2003) add that transaction costs of 10% should be taken into account. Our model explains this eagerness to pay more without assuming taste heterogeneity: the willingness to pay increases with centrality.

Stylized Fact 3 Larger dealers and collectors pay more for similar art (Etro et al., 2015, Etro and Stepanova, 2017b, 2026). This contradicts the common intuition for non-reputational markets that the larger size of a market participant implies better bargaining power and therefore more favorable terms of trade. In Beethoven's career, for example, pa-

tronage by Vienna-based patrons Lichnowsky and Archduke Rudolph led to better reputation gains than support by Bonn-based von Waldstein (DeNora, 2023). Our model explains the mechanism: larger collections provide larger network benefits.

Stylized Fact 4 Foreign art yields less than local (Etro and Stepanova, 2017a,b, Stepanova and Sedino, 2026). Hellmanzik (2010) finds a premium for art made in Paris and New York: connections to local artists matter. Coslor and Spaenjers (2016) find network-based valuation in the art field. Our model explicitly explains: fewer connections to other collectors of the same art means less reputation boost from the same art, and therefore smaller willingness to pay.

Stylized Fact 5 Many collectors specialize in specific artists (Etro and Stepanova, 2017a, Fase, 1996, Goetzmann, 1993). This relates to other markets where individual reputation is important, such as academic reputation for relationships between faculty and universities, or the legal reputation of lawyers and their firms. Famous venture capitalists invest in hot startups, whose hotness is in turn associated with the fame of the engaged venture capitalists (Rider, 2012, Sørensen, 2007). In economics, the prominence of authors is affiliated with the prominence of journals (Heckman and Moktan, 2020, Popov, 2022, *inter alia*). Count Ferdinand von Waldstein sponsored Ludwig van Beethoven's move from Bonn to Vienna, both improving his own external perception as a music *connoisseur* and van Beethoven's career (cf DeNora, 2023, Lockwood, 2013). Our model explains this: once a collector becomes respectable in an art field, and the prestige of artists improves through that collector owning the art, reputation feedback loops make concentration self-enforcing. Fraiberger et al. (2018) document this empirically: they find that connections of artists via exhibitions to central institutions improve lifetime outcomes for artists.

Stylized Fact 6 Art by female artists commands a price premium in the low-price sector but there is a reversal in the high-price segment (Bocart et al., 2022). While one could argue that this is driven by an unlucky outcome of taste and ability heterogeneity, if female artists are predominantly held by low-centrality collectors, that alone would generate the observed premium-to-discount reversal, which is empirically testable.

Stylized Fact 7 Artists sign exclusive representation contracts with dealers (Cellini and Cuccia, 2014, Velthuis, 2003, 2011, and papers cited therein). Artists find value in distributing art to reputable collectors because this improves the artist's prestige and improves potential prices for the future sales. Exclusive contracts allow dealers to credibly commit to managing an artist's placement without sacrificing the price due to the competition between dealers.

Stylized Fact 8 New artists' art faces discounts (Galenson, 2001). This seems to be a tactic of dealers to place new artists in established collections to gain exposure (Coslor et al., 2020, Prendergast, 2019). Hsu (2004) finds a similar mechanic in VC-startup relationships (with a discount of 10%-14% for prominent investors), further underlining the similarity of the art market to other endogenous reputation markets. Our model shows how new artists benefit in prestige from being collected by respectable collectors, and, looking forward to future sales, can forego immediate monetary rewards to secure placement in better collection.

Stylized Fact 9 Artists' lifetime productivity is inverse U-shaped: young artists are paid less than the market would suggest, art prices peak in midlife, and later art pieces get cheaper—empirically tested by, *inter alia*, Ginsburgh and Weyers (2006), who find that peak timing is heterogeneous. Galenson and Weinberg (2000) argues that this is driven by market forces and not by physical or mental deterioration. We explain the early-career discount as an equilibrium outcome of dealer strategy rather than quality uncertainty, with network placement decisions driving the upward trajectory of prices through mid-career. An extension of our model with forward-looking collectors might motivate collectors to buy in part because they expect a better placement of the artist in other, maybe more reputable, collections in the future. Later works, produced after the artist's network position is already established, offer collectors smaller marginal reputation gains from further placements.

While the unexpected death of an established artist is correlated with a boost of the value of their work, the death of a young artist makes prices of their art pieces lower in the first few years after the death (Ursprung and Wiermann, 2011): increased scarcity drives prices up, but death also removes any future works whose placement would have built the artist's reputation, and the latter effect dominates the former for younger artists. Ekelund Jr et al. (2000) and Pénasse et al. (2021) don't condition on age of death, and in their data the first effect dominates. De Silva et al. (2022) find that post-mortem prices go down by about 7%, but also that the demand for prominent artists (Turner) is structurally different from similar but differently placed artists (McCulloch). As with the inverse U-shaped story, this would require an extension into forward-looking collectors: a surprise death is different from an expected death because of expectations about the inflow of future art.

We focus on facts not immediately consistent with conventional Walrasian pricing behaviour: for instance, Braden and Teekens (2019) reports that artists cost more if their

reputation improves, while Beggs and Graddy (2008) and Hwang et al. (2025) find that not meeting the reserve price lowers the prices of the same artwork; but this is not different from what would happen on the mass-produced car market if, say, a specific car make received a prize from a car magazine, or a specific car make sold worse than expected.

Model

> The creative act is not performed by the artist alone; the spectator brings the work in contact with the external world by deciphering and interpreting its inner qualifications and thus adds his contribution to the creative act.
>
> Marcel Duchamp, *The Creative Act*, 1957.

Let the prestige of the artist i be a function of the reputation of collectors who hold artist i, and the reputation of the collector j be a function of the reputation of artists in their collection. The economy contains two types of agents:

- artists $i \in \{1, .., I\}$ with innate ability θ_i and endogenous *prestige* q_i,
- collectors $j \in \{1, .., J\}$ with baseline respect v_j and endogenous *reputation* R_j;

where

- x_{ij} is the quantity of art pieces by artist i owned by collector j,
- an artist i's prestige q_i is a weighted sum of their type and reputation of those who hold their items
$$q_i = (1 - \beta)\theta_i + \beta \sum_j x_{ij} R_j,$$
- a collector j's reputation R_j is a weighted sum of their type and prestige of artists whose pieces they are holding:
$$R_j = (1 - \alpha)v_j + \alpha \sum_i x_{ij} q_i.$$

Assume $1 > \alpha, \beta > 0$; $\beta = 0$ would constitute a Walrasian analogue. These are parameters of relative importance of network-provided reputation, and mathematically they serve the same purpose as "link decay" in the conventional network literature. So in matrix

language the art item ownership resolves prestige and reputation outcomes as

$$q = (1-\beta)\theta + \beta X R, \quad R = (1-\alpha)v + \alpha X^T q,$$

$$R = T(R) := (1-\alpha)v + \alpha(1-\beta)X^T\theta + \alpha\beta X^T X R \Rightarrow$$

$$R = [\mathbb{I}_{J\times J} - \alpha\beta X^T X]^{-1}[(1-\alpha)v + \alpha(1-\beta)X^T\theta]. \tag{1}$$

In a similar spirit,

$$q = (1-\beta)\theta + \beta(1-\alpha)Xv + \alpha\beta X X^T q \Rightarrow$$

$$q = \left[\mathbb{I}_{I\times I} - \alpha\beta X X^T\right]^{-1}[(1-\beta)\theta + \beta(1-\alpha)Xv]. \tag{2}$$

Proposition 1. *R and q exist and are unique if $\alpha\beta\rho(X^TX) < 1$, where ρ is the largest eigenvalue of X^TX. A sufficient condition is $\alpha\beta < \frac{1}{\sum_{i=1}^{I} Y_i^2}$, where $Y_i = \sum_{j=1}^{J} x_{ij}$.*

Proofs are relegated to Appendix A. We maintain the assumption that R and q are well-defined everywhere where necessary onward.

We will refer to $A = [\mathbb{I}_{J\times J} - \alpha\beta X^T X]^{-1}$ as the *resolvent* of $\alpha\beta X^T X$.[1] A is symmetric and non-negative because A is a limit of a Neumann sum of symmetric and non-negative matrices $(\alpha\beta X^T X)^n$. As long as jth collector has a positive amount of art, $A_{jj} > 1$.

Discussion Assuming very small network effects to warrant the existence of a well-defined reputation seems to undercut the aspiration of this paper that network effects are important. But Proposition 1 says that even for very small α and β, ownership concentration amplifies network effects, potentially well beyond the direct effect, and into the divergence territory; not that α and β better be very small.

Positive baseline respect v and artistic ability θ have a clear interpretation; negative values can be read as standing below the population average. It is safe to think about quantities of art pieces as integers, but all our math extends to real valued holdings.

Whether our model is adequate to describe a specific market depends on whether the reputation gain is linear in holdings. A person with 10 top-5 publications in economics is probably valued about 10 times as accomplished as a person with one top-5 publication, so our model can be applied; but if someone has 10 PhD degrees, they are probably a James Bond villain, and do not enjoy 10 times the gain of the first PhD degree, so collecting PhDs is not a natural market for our model.

[1] Other network papers use this machinery, for instance Ballester et al. (2006), Bramoullé and Kranton (2016), Galeotti et al. (2020): while the mathematics used is similar, we have a bipartite endogenous graph through ownership rather than an exogenous linear-quadratic action game.

We assume that all art pieces by the same artist are interchangeable and that a swap of two Aivazovsky paintings between two collectors affects nothing, for simplicity. This is easy to defend when we talk about, say, copies of *Campbell's Soup Cans* by Warhol. As long as one can assume that the reputational value of the art piece is an art piece-specific constant multiplied by q_i (so say Aivazovsky's *The Ninth Wave* is at all times $t\%$ more important than Aivazovsky's *Stormy Sea At Night*), the intuition behind one-unit algebra holds.

Sale of Art Pieces Between Collectors

For the purposes of this subsection, we will assume collectors' options are limited by existing art pieces. The supply of Basquiat is limited, and new discoveries are very unlikely.

Every collector maximizes their utility, which is the reputation of the collection R_j plus money m_j, large enough to not constrain purchases:[2]

$$u_j(R_j, m_j) = R_j + m_j.$$

For simplicity, reputation and money are perfect substitutes. We can always rescale reputations so that the reputation-to-money transformation rate is one-to-one.[3] When collector j decides whether to purchase a piece by artist i for price P from collector k, the collector chooses

$$\underbrace{R'_j - P}_{\text{buy}} \text{ vs } \underbrace{R_j}_{\text{don't buy}},$$

where $R_j = e_j^T R$ is the jth element of the reputation vector R based on the ownership matrix X following Equation (1), and R'_j is the j th element of the reputation vector R' based on $X' = X + E_{ij} - E_{ik}$ where E_{ij} is the matrix $I \times J$ with zero everywhere except the ijth element which is 1. The collector therefore buys if $P < R'_j - R_j = e_j^T(R' - R) =: P_B$.

Similar calculations about collector k deciding on selling yields $P > R_k - R'_k = e_k^T(R - R') =: P_S$. Both P_B and P_S depend on the identities of both the buyer and the seller; the same art piece may participate in two transactions for which price ranges, being a function

[2]Albeit Goetzmann et al. (2011) and others show that high stock market returns coincide with higher prices for sold art, this could be driven not only by the alleviation of the financial constraints of the collectors but also by the expansion of the collector network.

[3]Different collectors might value money differently compared to gains in reputation; while this heterogeneity might make empirics harder, the only additional prediction we can obtain from this extension is that people who care less about money are more likely to amass art collections. As long as the preferences of collectors are linear in reputation and money, the logic remains.

of the whole ownership network, might not overlap at all. In that sense, there is no "one price" on the art market. So, as long as $P_B > P_S$, there is a P a buyer and a seller can agree on, at least in a Coasean world without budget constraints and transaction costs.[4]

Denote $B = (1-\alpha)v + \alpha(1-\beta)X^T\theta$, representing the direct effect of ownership on reputation. Then Equation (1) can be rewritten as $R = AB$.

Proposition 2. *A trade of 1 unit of artist i art from collector k to collector j can happen if*

$$P_B > P_S \Leftrightarrow e_j^T(R' - R) > e_k^T(R - R') \Leftrightarrow (e_j + e_k)^T(R' - R) > 0.$$

The left hand side can be decomposed into the sum of the DIRECT EFFECT

$$\alpha(1-\beta)\theta_i(A'_{jj} - A'_{kk})$$

and the NETWORK EFFECT

$$\alpha\beta\left[\underbrace{(R_j - R_k)\sum_{l=1}^{J} x_{il}\left(A'_{jl} + A'_{kl}\right)}_{\textit{reputational shift}} + \underbrace{(A'_{jj} - A'_{kk})\sum_{l=1}^{J} x_{il}R_l}_{\textit{centrality shift}} + \underbrace{(A'_{jj} - A'_{kk})(R_j - R_k)}_{\textit{direct movement}}\right].$$

To interpret these summands, imagine an increase of B_j by 1; then A_{jj} says by how much collector j reputation increases in total, after all the feedback loops via all the collectors and artists are taken into account. This is related to the Katz-Bonacich centrality measure (Bonacich, 1987), except the Katz-Bonacich centrality measure would be calculated as $A_j = \sum_k A_{kj}$ and interpreted as the total effect on every collector caused by an increase of B_j by 1. The interpretation of Katz-Bonacich centrality is how much a boost of B_j translates, via all the connection paths, to everyone's reputation; A'_{jj} is only the self-feedback multiplier. Therefore, for good art ($\theta_i > 0$) if the individual reputation multiplier A'_{jj} is larger than A'_{kk}, the direct effect of the transfer makes the transfer beneficial. On the other hand, someone not well-connected, such as a foreign buyer, is unlikely to pay a lot.

The direct effect for good art ($\theta_i > 0$) moves high q art towards high A'_{jj} collectors. The reputational shift part of the network effect helps if $R_k < R_j$, appropriately weighted, or moves in reverse if $R_j < R_k$. This explains why museums and large dealers pay a lot

[4] As in Mele (2017), we assume myopia in the decisionmaking: our collectors don't plan purchases to manipulate networks into maximizing their reputation beyond the current interaction. After myopic math is established, the intuition for forward-looking participants is immediate.

for good art: buyers gain the art piece, the connections the art piece brings them, and the additional spillovers obtained from concentration.

Example 1. *Consider* $\alpha = \beta = 0.15$, $\theta = (1, 2, 2)^T$, $v = (3, 6, 2, 0.5)^T$, *and*

$$X = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 \end{bmatrix}.$$

If collector 1 tries to sell artist 1 to collector 2, the range of admissible prices is $[0.7404, 1.0266]$. *If collector 3 tries to sell artist 1 to collector 4, the price range is* $[0.6995, 0.7067]$. *These price ranges do not overlap. Both trades can happen, both are individually rational, but agreed prices will be different. The reason for the difference is that collectors 1 and 2 have collections of artists 2 and 3 which makes a connection over artist 1 very valuable for both of them, whereas collectors 3 and 4 have much smaller network effects of ownership of artist 1.*[5]

Add together the direct effect and the centrality shift effect:

$$\alpha(A'_{jj} - A'_{kk}) \underbrace{\left[(1-\beta)\theta_i + \beta \sum_{l=1}^{J} x_{il} R_l \right]}_{q_i}.$$

Add together the reputational shift effect and the direct movement effect:

$$\alpha\beta(R_j - R_k) \left[\underbrace{\sum_{l=1}^{J} x_{il}(A'_{jl} + A'_{kl}) + (A'_{jj} - A'_{kk})}_{\geq A'_{jj} + (x_{ik}-1)A'_{kk} \geq A'_{jj} > 1} \right].$$

A'_{jj}, as a limit of the Neumann sum of the jjth elements of $(X')^T(X')$ times $(\alpha\beta)^n$, is larger than 1; and x_{ik} must be at least 1 to sell 1 unit to j. Good art, with $q_i > 0$, is pushed towards collectors with higher A'_{jj} by the first sum. Bad art, with $q_i < 0$, is pushed towards $A'_{jj} < A'_{kk}$. The higher $R_j - R_k$ gap encourages trade; any trade going empirically against the direct effect indicates that the effects of the network are strong.

[5]This and other examples can be verified with a companion widget available at `https://sergeyvpopov.github.io/files/ArtExamples.html` developed with the help of Anthropic Claude.

Corollary 1. *P_B and P_S are*

$$P_B = \alpha q_i(A'_{jj} - A'_{jk}) + \alpha\beta(R_j - R_k)\left[\sum_{l=1}^{J} A'_{jl}x_{il} + A'_{jj} - A'_{jk}\right],$$

$$P_S = \alpha q_i(A'_{kk} - A'_{jk}) + \alpha\beta(R_j - R_k)\left[A'_{kk} - A'_{jk} - \sum_{l=1}^{J} A'_{kl}x_{il}\right].$$

If trade happens, both P_B and P_S are higher when $R_j > R_k$, $q_i > \beta(R_j - R_k)(x_{ik} - 1)$ and A'_{jk} decreases: the less j and k are connected, the more connections "change hands" with trade, and the more the seller wants to charge and the more the buyer is eager to pay. Both prices increase in A'_{jk} when $R_j < R_k$ and $q_i < \beta(R_j - R_k)(x_{ik} - 1)$. Any price rule strictly monotone P_S and P_B, such as Nash bargaining, will produce a similar testable claim about prices. In a Walrasian world with $\beta = 0$, $A'_{jj} = A'_{kk} = 1$, $A'_{jk} = 0$, and $P_B = P_S = \alpha q_i$.

If the trade from k to j is not in the mutual coincidence of wants, it does not mean that the reverse trade becomes attractive: A' changes for the reverse trade, which might dominate the decisionmaking (see Example 3 below). In reverse, for two collectors j and k, a trade in one direction does not prevent a trade in another direction: both trades might seem profitable! Example 2 below illustrates this: in that setting, when $v_1 \approx v_2$, in the economy where both collectors hold one art piece, both the trade from collector 1 to collector 2 and vice versa can be individually rational.

Corollary 2. *Let $x = X^T e_i$ be the ith row of X as a column, $a = (e_j - e_k)^T A(e_j - e_k)$, $b = (e_j - e_k)^T Ax$, and $c = x^T Ax$. Then*

$$P_B - P_S = \frac{\alpha}{D}\left(q_i\left((1 - \alpha\beta b)(A_{jj} - A_{kk}) + \alpha\beta a\sum_{l=1}^{J} x_{il}(A_{jl} + A_{kl})\right) + \right.$$

$$\left. + \beta(R_j - R_k)\left((1 + \alpha\beta c)(A_{jj} - A_{kk}) + (1 - \alpha\beta b)\sum_{l=1}^{J} x_{il}(A_{jl} + A_{kl})\right)\right),$$

where $D = (1 - \alpha\beta b)^2 - \alpha\beta a(1 + \alpha\beta c) > 0$ when resolvents exist.

In particular, one can see that with $A_{jj} = A_{kk}$, $R_j = R_k$, and $q_i > 0$, $P_B > P_S$, so we obtain specialization without j being different from k: after that piece moves to j, R'_j will become larger than R_j or R'_k, and further trades in artist i will favour movement towards collector j. P_B and P_S from Corollary 1 can also be rewritten in terms of A instead of A', which could be useful for the empirical work.

Pairwise Stability and Efficiency

We denote a network to be *pairwise stable* if there is no mutually beneficial trade of 1 unit of art for money between collectors. There are alternatives, requiring more (no trade of any amount of art between any amount of collectors with transfers) or less (no marginal trade); the set of pairwise stable trades decreases with more requirements. It is easy to design a boring example of a steady symmetric network: $v_j = 0 \forall j, \theta_i = 0 \forall i, x_{ij} = \bar{x} \forall i, j$ where $\bar{x} > 0$ but small enough for the resolvent to exist, then $R_j = R_k \forall j, k$. Then the direct effect is zero due to $q_i = 0$, and the network effect is zero due to $R_j = R_k = 0$ and $x_{ij} = x_{ik} \forall i, j, k$, so no trade is profitable.

Example 2. *The simplest example to study stability is a network of one artist* ($\theta > 0$) *and two collectors, collector* 1 *holding two art pieces and collector* 2 *holding none* ($X = (2, 0)$). *Let* $b_j = (1-\alpha)v_j + \alpha(1-\beta)x_j\theta$, *then*

- *when both art pieces are held by collector* 1, *their reputation is* $R_1 = \frac{(1-\alpha)v_1 + 2\alpha(1-\beta)\theta}{1-4\alpha\beta}$ *and the other collector's reputation is* $R_2 = b_2 = (1-\alpha)v_2$;
- *when each collector holds one art piece,* $X' = (1, 1)$, *their reputations are*

$$R'_i = \frac{\overbrace{(1-\alpha)v_i + \alpha(1-\beta)\theta}^{b_i} + \alpha\beta\overbrace{(1-\alpha)(v_j - v_i)}^{(b_j - b_i)}}{1-2\alpha\beta} = \frac{b_i + b_j}{2(1-2\alpha\beta)} + \frac{b_i - b_j}{2}.$$

The existence of the resolvent would require assuming that $\alpha\beta < 1/4$. So the total reputation in $X' = (1, 1)$ and $X = (2, 0)$ economies are

$$R' = \frac{(1-\alpha)(v_1+v_2) + 2\alpha(1-\beta)\theta}{1-2\alpha\beta}, \quad R = \frac{(1-\alpha)(v_1+v_2) + 2\alpha(1-\beta)\theta - 4\alpha\beta(1-\alpha)v_2}{1-4\alpha\beta}.$$

The multiplier is stronger in the X network, but it does not apply to v_2. Rewrite R' reputation equation as

$$\tilde{R}' = \frac{R_1(1-4\alpha\beta) + R_2}{1-2\alpha\beta}, \tilde{R} = R_1 + R_2, \Rightarrow$$

$$\tilde{R}' - \tilde{R} = \frac{R_1(1-4\alpha\beta-1+2\alpha\beta) + R_2(1-1+2\alpha\beta)}{1-2\alpha\beta} = \frac{2\alpha\beta}{1-2\alpha\beta}(R_2 - R_1).$$

So for $R_2 \gtrapprox R_1$, decentralization of art makes sense, both individually and from the social optimum perspective. But even if $v_1 = v_2 > 0$, $R_1 > R_2$, both for direct (when

$\theta > 0$) and network effects. It's easy to see that when $v_1 = v_2$, $\tilde{R} > \tilde{R}'$; a boost to v_1 improves R_1 faster than R_1' because the multiplier is larger.

If v_1 is somewhat smaller than v_2, hoarding art with collector 1 is stable, but it is not efficient, while hoarding art with collector 2 is. Let $X_1 = (2, 0)$ and $X_2 = (0, 2)$. Then the total reputation $\tilde{R}(X_1) = \frac{(1-\alpha)v_1+2\alpha(1-\beta)\theta}{1-4\alpha\beta}+(1-\alpha)v_2$ and $\tilde{R}(X_2) = \frac{(1-\alpha)v_2+2\alpha(1-\beta)\theta}{1-4\alpha\beta}+(1-\alpha)v_1$; $\tilde{R}(X_1) - \tilde{R}(X_2) = (1-\alpha)(v_1 - v_2)\frac{4\alpha\beta}{1-4\alpha\beta} < 0$ iff $v_2 > v_1$: in this case, hoarding with collector 2 is efficient. However, X_1 is stable if

$$(1-\alpha)v_2 < \frac{(1-\alpha)v_1 + 2\alpha(1-\beta)\theta}{1-4\alpha\beta} \Rightarrow 1 - 4\alpha\beta < \frac{v_1}{v_2} + 2\frac{\alpha}{1-\alpha}(1-\beta)\frac{\theta}{v_2},$$

which is satisfied, for instance, if $\alpha = \beta = 0.3$, $v_2 = \theta$ and $\frac{v_1}{v_2} > 0.04$, thus providing an example of ample space featuring an inefficient but pairwise stable outcome. One would need to trade in multiple units to abandon this trap.

Network efficiency Define the feasible set of X:

$$\mathcal{F} = \left\{ X \in R^{I \times J} : x_{ij} \geq 0, \forall i \sum_{j=1}^{J} x_{ij} = Y_i \right\}.$$

Quasilinearity of the utility functions of collectors with respect to money lets us pick the efficient network to be the one that maximizes the total reputation. Other approaches, for instance including artists' payoffs, are possible. Linearity of reputations in artists' prestige yields that the total reputation is linear in the sum of artists' prestige values q_i weighted by the number of available works Y_i. If the amount of works across artists is identical, any change in total reputation due to trade is proportional to the change in total prestige.

Proposition 3. *The total reputation maximizing ownership network exists.*

One approach to warrant a pairwise stable allocation is to find a *potential function* (cf Monderer and Shapley, 1996), but the total reputation might not work unless $J = 2$: a trade that is mutually beneficial between two collectors might be total reputation reducing by changing the reputation of a non-participant collector.

Proposition 4. *Consider a* hoarding network *where all art belongs to* (*without loss of generality*) *collector 1 whose* $v_1 = \max_{j=1}^{J} v_j$. *This network is pairwise stable and efficient if* (*a*) $\forall i, \theta_i \geq 0$ *and* (*b*) $R_1 \geq 0$.

Negative v or θ can yield pairwise stable X that don't exhibit art hoarding.

Example 3. *Let $\alpha = \beta = 0.3$, $X = (1, 1)$, $v = (-2, -2)$, and $\theta = 1$. Deviation to $X' = (2, 0)$ is not individually rational. Reputation of both collectors is ≈ -1.4512 before trade, after trade seller's reputation improves to -1.4 and buyer's total reputation goes to -1.5313.*

Example 4. *Let $\alpha = \beta = 0.3$, $X = (1, 1)$, $v = (1, 1)$, and $\theta = -1$. Deviation to $X' = (2, 0)$ is not individually rational. Reputation of both collectors is ≈ 0.5976 before trade, after trade seller's reputation goes to 0.7 and buyer's reputation total goes to 0.4375.*

Would every individual trade be total reputation improving? It would if $I = 1$.

Proposition 5. *If $I = 1$, the ordinal potential function exists. The artist's prestige and the total reputation of collectors can both serve as that ordinal potential function.*

What makes the proposition work is that if there is only one artist, the only place where the improvement in total reputation, or in the reputation of two trading agents, can come from is from the artist's prestige improving due to a more favourable network allocation. When there are more artists, benefits can also come from other artists' changes in prestige.

Externalities such as our reputational spillovers tend to lead to inefficiencies. Instead of looking at $(e_j + e_k)^T(A'B' - AB)$, let us look at $\mathbf{1}_{1\times J}^T(A'B' - AB)$. The direct effect is then

$$\alpha(1 - \beta)\theta_i(A'_j - A'_k),$$

where $A'_j = \sum_k A'_{jk}$ is the Katz-Bonacich centrality measure for the collector j. This is generically different from the partial equilibrium condition for individual decisionmaking; the trade of a piece may be beneficial for the participants, but the rest of the community might be hurt by such a transaction. Network effects of a trade are affected similarly:

$$\mathbf{1}_{1\times J}^T A'(X^T D + D^T X + D^T D)R = (A'_j - A'_k)\sum_{l=1}^{J} x_{il}R_l + (R_j - R_k)\sum_{l=1}^{J} A'_l x_{il} + (A'_j - A'_k)(R_j - R_k).$$

The total network effect is again built upon the Katz-Bonacich centrality instead of self- and cross-multipliers that concern j and k, as in Proposition 2. The example below illustrates that the network effects can be large enough to induce a conflict between bilateral rationality of a trade and total reputation maximization.

Example 5. *Consider $\alpha = 0.22$, $\beta = 0.08$, $I = 2$, $J = 3$, $\theta = (1, 1)$, $v = (0.5, 0.5, 0.5)$,*

$$X = \begin{bmatrix} 0 & 3 & 2 \\ 3 & 1 & 0 \end{bmatrix}, X' = \begin{bmatrix} 0 & 3 & 2 \\ 4 & 0 & 0 \end{bmatrix}.$$

This trade of a piece by artist 2 from collector 2 to collector 1 raises the reputation of collector 1 from ≈1.2898 to ≈1.6698, and lowers the reputation of collector 2 from ≈1.6724 to ≈1.3109, creating a positive total reputational benefit across trade participants of ≈0.0185. However, because artist 2 is now owned by collector 1 instead of collector 2, the reputation of collector 3 who did not participate in the transaction decreases by ≈0.0411, making this transaction individually rational but total reputation worsening.

For the example to work, we need $I \geq 2$, or Proposition 5 would prevent it. Because of the externalities, pairwise trade rationality could be inconsistent with total reputation increase, so the total reputation is unable to serve as a potential generically. $I > 1$ and non-positivity of fundamental ability θ or respect v values yield an example of a network without a pairwise stable allocation, making sure that there is no generic potential function.

Example 6. *Let* $\alpha = 0.3$, $\beta = 0.1$, $\theta = [1, -5]^T$, $v = (1, 1, -5)$, $Y = [2, 2]^T$. *There are 36 possible discrete allocations, but each of them, including the total reputation maximizing* $X = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix}$, *has a profitable trade.*[6]

The total reputation optimization might be beyond any individual collector's interests, but it explains why museums, who arguably care about public good more than private collectors, pay more for prominent art than private collectors do. The conflict between the stability and the efficiency of the network can be immediately seen in the difference between the relevance of self-multiplier A_{jj} for individual decisions and Katz-Bonacich centrality A_j for the total reputation. Profitability of individual decisions is not the same as what's best for the community: reputational spillovers to non-participants are not internalized.

Entry of an Artist

A new artist is entering the market. Since the artist is new, no collector is holding a single art piece by artist $I + 1$. This changes the dimensionality of θ, q, and X:

$$q_{I+1} = (1 - \beta)\theta_{I+1}, X_{I+1,j} = 0 \forall j.$$

[6]The discreteness is not the problem: Appendix C uses a computer-assisted proof to confirm there is no pairwise stable allocation in this economy even if holdings are continuous.

If the collector j buys the art piece of the entering artist, their R_j changes to R'_j. The artist's prestige increases to

$$q'_{I+1} = (1-\beta)\theta_{I+1} + \beta R'_j, \quad X'_{ik} = \begin{cases} X_{ik}, i < I+1 \text{ or } k \neq j, \\ 1, i = I+1, k = j. \end{cases}.$$

So, ignoring the price to pay, the best buyer for this art piece is the collector with the highest R', since such a purchase will provide the highest boost of q_{I+1}. The artist is interested in increasing their q_{I+1}: new art pieces by that artist would be more reputation-improving. The artist is also interested in the price, but in an implicit long-term game, they could forego some monetary gain today for an increase of prices in the future.

The maximal willingness to pay, similarly to the discussion in the previous subsection, is exhibited by the collector with the highest $R'_j - R_j$:

$$\Delta R_j = (e_j)^T \left(A'(B'-B) + (A'-A)B\right).$$

This generically does *not* have to be the collector with the highest R'_j.

Example 7. *Let $\alpha = \beta = 0.2$, $v = (5, 0.5, 1)$, $\theta = (1, 1)$, $X = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix}$. Artist 3 enters the market, $\theta_3 = 1$. Then $\Delta R_1 = 0.333$ if their art is sold to collector 1 and $\Delta R_2 = 0.385$ if their art is sold to collector 2, so collector 2 wants to pay the most; but if artist 3's art is sold to collector 1, their $R'_1 = 4.333$, while if it is sold to collector 2, their $R'_2 = 2.278$, so the artist prefers their art piece to be sold to collector 1. Note that $X = (0, 2, 1)$ with only one artist would not work: the network effects need to be strong enough.*

To calculate the change in R_j, observe that: $(X')^T X' = X^T X + E_{jj}$, where E_{jj} is a rank 1 matrix with 1 at jjth coordinate and 0 otherwise. We can find a closed form solution for the reputation dynamics resolvent using Lemma 1.

Lemma 1 (Miller (1981))**.** *If G and H are square matrices of the same size, and H is a rank 1 matrix, and both G^{-1} and $(G+H)^{-1}$ exist, then*

$$(G+H)^{-1} = G^{-1} - \frac{1}{1 + tr(HG^{-1})} G^{-1} H G^{-1}.$$

Since $A' = (A^{-1} - \alpha\beta E_{jj})^{-1}$, Lemma 1 implies

$$A' = A - \frac{-\alpha\beta}{1-\alpha\beta A_{jj}} A E_{jj} A = A + \frac{\alpha\beta}{1-\alpha\beta A_{jj}} a_j e_j^T A,$$

where e_j is a unit vector with 1 at jth place and a_j is a jth column of A. As before,

$$B_j = (1-\alpha)v_j + \alpha(1-\beta)\sum_i x_{ij}\theta_i, \quad B' - B = \alpha(1-\beta)\theta_{I+1}e_j.$$

Assembling together yields

$$\Delta R_j = e_j^T \left(\left(1 + \frac{\alpha\beta A_{jj}}{1-\alpha\beta A_{jj}}\right) a_j e_j^T \alpha(1-\beta)\theta_{I+1} e_j \underbrace{+ \frac{\alpha\beta}{1-\alpha\beta A_{jj}} a_j \overbrace{e_j^T R}^{R_j}}_{(A'-A)B = \frac{\alpha\beta}{1-\alpha\beta A_{jj}} a_j e_j^T AB} \right) =$$

$$= \frac{A_{jj}}{1-\alpha\beta A_{jj}} \alpha(1-\beta)\theta_{I+1} + \frac{\alpha\beta A_{jj}}{1-\alpha\beta A_{jj}} R_j.$$

The artist, however, prefers those with higher R'_j:

$$R'_j = R_j + \Delta R_j = \frac{A_{jj}}{1-\alpha\beta A_{jj}} \alpha(1-\beta)\theta_{I+1} + \frac{1}{1-\alpha\beta A_{jj}} R_j.$$

If prestige holds any value for the artist, even a negative price might make sense for them if the buyer's reputation is high enough. Representation by a dealer can help artists: dealers are unlikely to accept a negative price, even on behalf of the artist; they might, however, be interested in sacrificing some immediate monetary remuneration for a boost in future earnings because of a better reputation of the buyer. Empirically, this can be seen as discounts for the new artists for prominent collectors coupled with outright refusal to sell a new artist to a collector of low reputation. Taking two potential buyers with the same R_j, as long as ΔR_j is positive, higher A_{jj} makes both R'_j and ΔR_j higher; more connected collectors, even with the same reputation, would be eager to pay more, and would provide a bigger boost to the artist's prestige, making the transaction more likely to happen.

Entry of a New Collector is an analogous problem with identical math: one can immediately see that new collectors would be interested in purchasing high-q artists from existing collectors, who in turn would need to be motivated monetarily, because this purchase loses collectors' connections and might hurt their reputation. Ambitious collectors

intent on building up their networks and reputations therefore should start with overpaying for high-profile artists.

New Art Emergence of new art by established artists follows similar math as the art for the new artist (the change in the resolvent is now rank 2) but on top of the mechanics described above, one can also immediately see that an artist's existing connections start to matter. The artist still cares about the reputation of the buyer of the new piece, but the reputation of owners of previously sold art starts to influence decisions too. Low-reputation collectors might want to pay more for art that is already connected to high-connection collectors, explaining the increase in the inverse U-shaped artist's lifetime productivity. The academic publications market, in particular, is a reputational market where trade of art pieces already placed in journals is *verboten*; it is exactly new art where the negotiations (aka "refereeing") is happening.

Art Destruction In 2018, Banksy's "Girl with Balloon" was destroyed by the artist's shredder concealed in the frame after it was, against the artist's wishes, sold at auction with massive news coverage. The art piece is now renamed "Love is in the Bin", and sold in 2021 for an 18-fold 2018 price. This is hard to reconcile with the common Walrasian approach of producers and consumers, but clearly predictable with our reputation model, whether as an attempt to prevent a low-v or low-connected collector from worsening the artist's q, or boosting the artist's connectivity by this performance, which, via the news coverage, might have created informal links to the artist from collectors who might have avoided him before. The contrast with the Hind's stamp story from the Literature Review is straightforward: Hind wanted *fewer* connections to his collection; Banksy wanted *better* connections.

Discussion

All Walrasian markets are alike; each reputational market is particular in its own way. The art market alone is significantly more sophisticated than what we described. Information travels along the network of collectors (Campbell, 2013); collectors strategically exclude outsiders from their networks (Lippert and Spagnolo, 2011); artists choose between working at things they're good at versus things that pay money; forgeries are rampant. Games played on the artist overlap network XX^T or collector overlap network X^TX are plentiful (Bramoullé and Kranton, 2016, Galeotti et al., 2010, Parise and Ozdaglar, 2021). Fraiberger et al. (2018) in particular shows empirically the networking value of exhibitions, namely

that access to central venues improves career outcomes, a game that we did not explicitly model; however, one could argue they are studying a $X^T X$ object with X built over exhibition institutions instead of collectors. Artists spend effort to create new pieces, and collectors spend effort to collect information about the true abilities of artists. Dealers, being able to shape networks, might be able to orchestrate these efforts by selling art pieces to the right collectors.[7] Transactions, studied by Schich et al. (2017), could very well be limited to or facilitated by ownership networks, with consequences for prices and outcomes; if so, this could lead to clusterization into classes or genres, say the New York art market gravitating towards American masters as described in Etro and Stepanova (2026).

Myopia Real-life participants are forward-looking, and therefore are happy to forego some immediate payoff for higher future prices via higher reputation today. The untimely death of an artist would therefore manifest as a low return on such an investment, explaining the absence of a "death premium" for young artists. The positive "death premium" for established artists could also be a consequence of forward-looking collectors: they were counting on new pieces coming out which would be sold cheaper than before, and untimely death means they need to pay more if they need to procure a piece. However, an expected death of old age would be adequately priced and would not exhibit any penalty or premium on the market (Ursprung and Wiermann, 2011).

Mortality of collectors If a collector dies, but their collection remains (as a part of their museum, for instance), that death should not matter for the network; it would be just as if that collector simply stopped trading. If the collection disperses—gets auctioned off for dentist office decorations—that should lower the price of art that was part of the dispersed collection, as well as the art held together with art from the dispersed collection. This prediction is different from what one would predict from the information-based approach to art pricing: the dissipation of an existing collection, if it was just a signal of scarce demand for that type of art, would lower the prestige of the artists involved in the dispersed collection, but not the artists connected to involved ones via ownership by other collectors. As far as we know, the impact of collectors' deaths on art prices has not been empirically studied yet.

Stationarity To study the stable growth of art volume, for instance in the context of Etro (2025)'s model of art history development, one could argue that it is the *average* R of holders that matters for q, not *total*, and vice versa. Rescaling $\beta < \bar{\beta}/J$ and $\alpha < \bar{\alpha}/I$ for some $0 < \bar{\alpha}, \bar{\beta} < 1$ maintains the fixed-point machinery with growth of both I and J. To

[7] For inspiration on that, see Condorelli et al. (2017) and Manea (2018) on bilateral trade and intermediation in networks.

bound Y_i one can assume that only the best few art pieces of each artist convey reputation.

Prices We intentionally stay away from pricing to make sure no results are driven by a specific choice of a price rule. Under a wide class of pricing rules monotone in P_S and P_B, such as Nash bargaining, all our propositions and all our conclusions would remain the same. However, we would obscure the richness of the real world interactions: in auctions, for instance, the second-highest willingness to pay matters, but it matters less for dealer-collector interactions. A strong discrepancy between prices in these two, therefore, could be driven by differences in price setting on top of the ownership network reasons. In other applications, other concerns matter: the monetary price of academic publication is minuscule, but there are costs in refereeing and revision time both before and after the submission; for football players, experience in playing in a specific club could be more valuable than money. We think it is better to leave the gap glaringly visible for follow-up research.

Linearity We assume the linearity of the collector's utility in reputation and money because we are dealing with discrete movement of art pieces. We could have relaxed this assumption to a differentiable utility function featuring decreasing marginal utility of reputation but quasilinear with respect to money; then there would exist a mutually beneficial trade if the sum of changes in the marginal-utility-of-reputation weighted sum of reputations were positive. We would need to work with infinitesimal movements of art, and Appendix C develops such marginal machinery. The immediate consequence would be that the increase in the network multiplier due to art concentration with one holder would be dampened by the decreasing marginal utility of reputation, discouraging hoarding; this would also explain the timelines where a collector obtains a piece and later, with a much larger collection, sells it at a discount ("underperformance of masterpieces" of Mei and Moses, 2002).

Conclusion

We developed a model of the reputational goods market where the reputations of participants are interdependent via their holdings of reputational goods. We focus on network formation by trade of reputational goods; we leave idiosyncratic preferences, bargaining power, search by participants, strategic pricing, and explicit modelling of intermediaries to future research. Even with our very basic framework, we obtain the interplay of connectedness and reputations that govern who is interested in procuring which good and

from whom.

We show that a Walrasian price mechanism would not function well in our setting: even if everyone agrees about the fundamentals, they could disagree not only about the price, but also about the mutually acceptable price range. We show that pairwise stable allocation might not exist; if it does, reputational spillovers of trade might render a pairwise stable allocation inefficient. We show that entry of a new participant involves a tradeoff between reputation-building and earnings, in a way that warrants a novel reason for the existence of a representing intermediary: they would not accept a negative price.

References

Ashenfelter, O. and Graddy, K. (2003). Auctions and the price of art. *Journal of Economic Literature*, 41(3):763–786.

Baetz, O. (2015). Social activity and network formation. *Theoretical Economics*, 10(2):315–340.

Ballester, C., Calvó-Armengol, A., and Zenou, Y. (2006). Who's who in networks. Wanted: The key player. *Econometrica*, 74(5):1403–1417.

Bar-Isaac, H. and Tadelis, S. (2008). Seller reputation. *Foundations and Trends® in Microeconomics*, 4(4):273–351.

Baumann, L. (2021). A model of weighted network formation. *Theoretical Economics*, 16(1):1–23.

Beggs, A. and Graddy, K. (2008). Failure to meet the reserve price: The impact on returns to art. *Journal of Cultural Economics*, 32(4):301–320.

Board, S. and Meyer-ter-Vehn, M. (2013). Reputation for quality. *Econometrica*, 81(6):2381–2462.

Board, S. and Meyer-ter-Vehn, M. (2022). A reputational theory of firm dynamics. *American Economic Journal: Microeconomics*, 14(2):44–80.

Bocart, F. Y., Gertsberg, M., and Pownall, R. A. (2022). An empirical analysis of price differences for male and female artists in the global art market. *Journal of Cultural Economics*, 46(3):543–565.

Bonacich, P. (1987). Power and centrality: A family of measures. *American Journal of Sociology*, 92(5):1170–1182.

Braden, L. and Teekens, T. (2019). Reputation, status networks, and the art market. *Arts*, 8(3):81.

Bramoullé, Y. and Kranton, R. (2016). Games played on networks. In Bramoullé, Y., Galeotti, A., and Rogers, B. W., editors, *The Oxford Handbook of the Economics of Networks*. Oxford University Press.

Campbell, A. (2013). Word-of-mouth communication and percolation in social networks. *American Economic Review*, 103(6):2466–2498.

Cellini, R. and Cuccia, T. (2014). The artist–art dealer relationship as a marketing channel. *Research in Economics*, 68(1):57–69.

Condorelli, D., Galeotti, A., and Renou, L. (2017). Bilateral trading in networks. *The Review of Economic Studies*, 84(1):82–105.

Coslor, E., Crawford, B., and Leyshon, A. (2020). Collectors, investors and speculators: Gatekeeper use of audience categories in the art market. *Organization Studies*, 41(7):945–967.

Coslor, E. and Spaenjers, C. (2016). Organizational and epistemic change: The growth of the art investment field. *Accounting, Organizations and Society*, 55:48–62.

De Silva, D. G., Kosmopoulou, G., Pownall, R., and Press, R. (2023). Surviving in the marketplace: The importance of network connectivity for art dealers. *Economics Letters*, 231:111312.

De Silva, D. G., Kosmopoulou, G., Pownall, R. A., and Press, R. (2022). Posthumous trading patterns affecting artwork prices. *Oxford Economic Papers*, 74(2):453–472.

DeNora, T. (2023). *Beethoven and the construction of genius: Musical politics in Vienna, 1792-1803*. Univ of California Press.

Ekelund Jr, R. B., Ressler, R. W., and Watson, J. K. (2000). The "Death-Effect" in art prices: A demand-side exploration. *Journal of Cultural Economics*, 24(4):283–300.

Etro, F. (2025). An economic theory of art history. Technical report, Università degli Studi di Firenze, Dipartimento di Scienze per l'Economia e l'Impresa.

Etro, F., Marchesi, S., and Pagani, L. (2015). The labor market in the art sector of Baroque Rome. *Economic Inquiry*, 53(1):365–387.

Etro, F. and Stepanova, E. (2017a). Art auctions and art investment in the golden age of British painting. *Scottish Journal of Political Economy*, 64(2):191–225.

Etro, F. and Stepanova, E. (2017b). Art collections and taste in the Spanish Siglo de Oro. *Journal of Cultural Economics*, 41(3):309–335.

Etro, F. and Stepanova, E. (2021). Art return rates from old master paintings to contemporary art. *Journal of Economic Behavior & Organization*, 181:94–116.

Etro, F. and Stepanova, E. (2026). A century of art dealing in New York. The rise of Amer-

ican art. *The Economic History Review*, 79(1):281–311.

Fase, M. M. G. (1996). Purchase of art: Consumption and investment. *De Economist*, 144(4):649–658.

Fraiberger, S. P., Sinatra, R., Resch, M., Riedl, C., and Barabási, A.-L. (2018). Quantifying reputation and success in art. *Science*, 362(6416):825–829.

Galenson, D. W. (2001). *Painting outside the lines: Patterns of creativity in modern art*. Harvard University Press.

Galenson, D. W. and Weinberg, B. A. (2000). Age and the quality of work: The case of modern American painters. *Journal of Political Economy*, 108(4):761–777.

Galeotti, A., Golub, B., and Goyal, S. (2020). Targeting interventions in networks. *Econometrica*, 88(6):2445–2471.

Galeotti, A., Goyal, S., Jackson, M. O., Vega-Redondo, F., and Yariv, L. (2010). Network games. *The Review of Economic Studies*, 77(1):218–244.

Ginsburgh, V. and Weyers, S. (2006). Creativity and life cycles of artists. *Journal of Cultural Economics*, 30(2):91–107.

Goetzmann, W. N. (1993). Accounting for taste: Art and the financial markets over three centuries. *The American Economic Review*, 83(5):1370–1376.

Goetzmann, W. N., Renneboog, L., and Spaenjers, C. (2011). Art and money. *American Economic Review*, 101(3):222–226.

Graddy, K., Mei, J., Moses, M., and Chen, C. (2026). The curious incident of the bidder in the nighttime. Technical report, Brandeis University, Department of Economics and International Business School.

Heckman, J. J. and Moktan, S. (2020). Publishing and promotion in economics: The tyranny of the top five. *Journal of Economic Literature*, 58(2):419–470.

Hellmanzik, C. (2010). Location matters: Estimating cluster premiums for prominent modern artists. *European Economic Review*, 54(2):199–218.

Hellmanzik, C. (2016). Historic art exhibitions and modern-day auction results. *Research in Economics*, 70(3):421–430.

Hirsch, F. (1976). *Social Limits to Growth*. Harvard University Press.

Hopkins, E. and Kornienko, T. (2004). Running to keep in the same place: Consumer choice as a game of status. *American Economic Review*, 94(4):1085–1107.

Hsu, D. H. (2004). What do entrepreneurs pay for venture capital affiliation? *The Journal of Finance*, 59(4):1805–1844.

Hwang, S., Ryu, J., and Hong, K. (2025). The paradox of being unsold: hidden signaling

value of bought-in in Korean art auction. *Journal of Cultural Economics*, 49(3):639–658.

Jackson, M. O. (2005). A survey of models of network formation: Stability and efficiency. In Demange, G. and Wooders, M., editors, *Group Formation in Economics: Networks, Clubs and Coalitions*, chapter 1, pages 11–49. Cambridge University Press, Cambridge.

Jackson, M. O. and Wolinsky, A. (1996). A strategic model of social and economic networks. *Journal of Economic Theory*, 71(1):44–74.

König, M. D., Tessone, C. J., and Zenou, Y. (2014). Nestedness in networks: A theoretical model and some applications. *Theoretical Economics*, 9(3):695–752.

Kranton, R. E. and Minehart, D. F. (2001). A theory of buyer-seller networks. *American Economic Review*, 91(3):485–508.

Lippert, S. and Spagnolo, G. (2011). Networks of relations and word-of-mouth communication. *Games and Economic Behavior*, 72(1):202–217.

Lockwood, L. (2013). *Beethoven: The music and the life*. WW Norton & Company.

Manea, M. (2018). Intermediation and resale in networks. *Journal of Political Economy*, 126(3):1250–1301.

Mei, J. and Moses, M. (2002). Art as an investment and the underperformance of masterpieces. *American Economic Review*, 92(5):1656–1668.

Mele, A. (2017). A structural model of dense network formation. *Econometrica*, 85(3):825–850.

Miller, K. S. (1981). On the inverse of the sum of matrices. *Mathematics Magazine*, 54(2):67–72.

Monderer, D. and Shapley, L. S. (1996). Potential games. *Games and Economic Behavior*, 14(1):124–143.

Parise, F. and Ozdaglar, A. (2021). Analysis and interventions in large network games. *Annual Review of Control, Robotics, and Autonomous Systems*, 4(1):455–486.

Pénasse, J., Renneboog, L., and Scheinkman, J. A. (2021). When a master dies: Speculation and asset float. *The Review of Financial Studies*, 34(8):3840–3879.

Pesando, J. E. (1993). Art as an investment: The market for modern prints. *The American Economic Review*, 83(5):1075–1089.

Pesando, J. E. and Shum, P. M. (2007). The law of one price, noise and "irrational exuberance": the auction market for Picasso prints. *Journal of Cultural Economics*, 31(4):263–277.

Pesendorfer, W. (1995). Design innovation and fashion cycles. *The American Economic Review*, 85(4):771–792.

Polanski, A. (2007). Bilateral bargaining in networks. *Journal of Economic Theory*,

134(1):557–565.

Popov, S. V. (2022). Alma mat(t)er(s): Determinants of early career success in economics. *PLOS One*, 17(12):e0278320.

Prendergast, C. (2019). The distribution of contemporary art. *Portable Gray*, 2(2):263–273.

Renneboog, L. and Spaenjers, C. (2013). Buying beauty: On prices and returns in the art market. *Management Science*, 59(1):36–53.

Rider, C. I. (2012). How employees' prior affiliations constrain organizational network change: A study of us venture capital and private equity. *Administrative Science Quarterly*, 57(3):453–483.

Rosen, S. (1981). The economics of superstars. *The American Economic Review*, 71(5):845–858.

Sattinger, M. (1993). Assignment models of the distribution of earnings. *Journal of Economic Literature*, 31(2):831–880.

Schich, M., Huemer, C., Adamczyk, P., Manovich, L., and Liu, Y.-Y. (2017). Network dimensions in the Getty provenance index. *arXiv preprint arXiv:1706.02804*.

Schönfeld, S. and Reinstaller, A. (2007). The effects of gallery and artist reputation on prices in the primary market for art: A note. *Journal of Cultural Economics*, 31(2):143–153.

Smithsonian National Postal Museum (2026). British Guiana: World's rarest stamp. Exhibition: *The Sun Never Sets on the Stamps of the British Empire*. Accessed: 2026-05-05.

Sørensen, M. (2007). How smart is smart money? A two-sided matching model of venture capital. *The Journal of Finance*, 62(6):2725–2762.

Stepanova, E. and Sedino, M. (2026). Private, but documented: tracing contemporary art galleries through provenance records. *Journal of Cultural Economics*, pages 1–28.

Sun, Y., Zhao, W., and Zhou, J. (2026). Sequential network design. *Journal of Economic Theory*, 232:106145.

Tirole, J. (1996). A theory of collective reputations (with applications to the persistence of corruption and to firm quality). *The Review of Economic Studies*, 63(1):1–22.

Ursprung, H. W. and Wiermann, C. (2011). Reputation, price, and death: An empirical analysis of art price formation. *Economic Inquiry*, 49(3):697–715.

Velthuis, O. (2003). Symbolic meanings of prices: Constructing the value of contemporary art in Amsterdam and New York galleries. *Theory and Society*, 32(2):181–215.

Velthuis, O. (2011). Art dealers. *A Handbook of Cultural Economics*, 2:28–32.

Wang, P. and Watts, A. (2006). Formation of buyer-seller trade networks in a

quality-differentiated product market. *Canadian Journal of Economics/Revue canadienne d'économique*, 39(3):971–1004.

Appendix A: Proofs

Proof of Proposition 1. The Neumann sum $\sum_{t=0}^{+\infty}(\alpha\beta X^T X)^t$ converges to $[\mathbb{I}_{J\times J} - \alpha\beta X^T X]^{-1}$ if the *spectral norm* of $\alpha\beta X^T X$, $\alpha\beta\rho(X^T X)$, is less than 1.

The spectral norm of $X^T X$, the largest eigenvalue of $X^T X$, is $\rho(X^T X) = \rho(XX^T)$. For two artists i and h, the ihth element of XX^T is $\sum_j x_{ij}x_{hj}$. By Cauchy-Bunyakovsky-Schwarz inequality,

$$\sum_{j=1}^{J} x_{ij}x_{hj} \leq ||(x_{i1}, x_{i2}, .., x_{iJ})|| \cdot ||(x_{h1}, x_{h2}, .., x_{hJ})|| \leq Y_i Y_h,$$

second inequality works by the positivity of x_{ij}. This means that, element-wise, $XX^T \leq YY^T$, where $Y = [Y_1, Y_2, .., Y_I]^T$. By the Perron-Frobenius theorem, this implies

$$\rho(X^T X) = \rho(XX^T) \leq \rho(YY^T) = \rho(Y^T Y) = \rho(\sum_{i=1}^{I} Y_i^2) = \sum_{i=1}^{I} Y_i^2,$$

with equality if for some j_0, $x_{ij_0} = Y_i$, and $x_{ij} = 0 \forall j \neq j_0$. Give all art to the same collector; the only nonzero eigenvalue of $X^T X$ is $\sum_{i=1}^{I} Y_i^2$, the largest possible value. Banach's theorem delivers the unique fixed point.

The identical argument applies to q's analogous fixed point: since the eigenvalues of $X^T X$ and XX^T are identical, no additional assumptions are required. Appendix B explains why these two are not different problems. □

Proof of Proposition 2. A piece by an artist i from collector k to collector j will be sold if

$$\underbrace{(e_j + e_k)^T \overbrace{(R' - R)}^{\Delta R}}_{P_B - P_S} = (e_j + e_k)^T (A'B' - AB) = (e_j + e_k)^T \left(\underbrace{A'(B' - B)}_{\text{direct}} + \underbrace{(A' - A)B}_{\text{network}} \right) > 0, \quad (\star)$$

where the resolvent matrix A and the direct ownership matrix B are based on matrix X before the trade, and A' and B' are analogous except based on after-trade matrix X'.

The direct change is $A'(B'-B) = \alpha(1-\beta)\theta_i A'(e_j - e_k)$. Multiplied by $e_j + e_k$ from the left, this yields the direct effect

$$\alpha(1-\beta)\theta_i \left(A'_{jj} - A'_{kk}\right),$$

which is positive if $A'_{jj} > A'_{kk}$. Symmetry of A' cancels out $A'_{kj} = A'_{jk}$.

Now derive the network effect of the transfer:

$$A' - A = A' \left(\overbrace{\mathbb{I}_{J\times J} - \alpha\beta X^T X}^{A^{-1}} - \overbrace{(\mathbb{I}_{J\times J} - \alpha\beta (X')^T X')}^{(A')^{-1}} \right) A$$

so $(A'-A)B = \alpha\beta A'((X')^T X' - X^T X)AB = \alpha\beta A'((X')^T X' - X^T X)R$. So the network effect is $\alpha\beta(e_j + e_k)^T$ times

$$A'((X')^T X' - X^T X)R = A'(\cancel{X^T X} + D^T X + X^T D + D^T D - \cancel{X^T X})R,$$

where $D = E_{ij} - E_{ik} = e_i(e_j - e_k)^T$. Then

$$X^T D + D^T X + D^T D = x_i(e_j - e_k)^T + (e_j - e_k)x_i^T + (e_j - e_k)(e_j - e_k)^T,$$

where x_i is a column of which collectors hold art by artist i.

So the network effect splits into the reputation shift effect due to $X^T D$

$$(e_j + e_k)^T A' x_i (e_j - e_k)^T R = (R_j - R_k) \sum_{l=1}^{J} (A'_{jl} + A'_{kl}) x_{il},$$

the centrality shift effect due to $D^T X$

$$(e_j + e_k)^T A'(e_j - e_k) x_i^T R = (A'_{jj} - A'_{kk}) \sum_{l=1}^{J} x_{il} R_l,$$

and the direct movement effect due to $D^T D$

$$(e_j + e_k)^T A'(e_j - e_k)(e_j - e_k)^T R = (A'_{jj} - A'_{kk})(R_j - R_k). \qquad \square$$

Proof of Corollary 1. Following the proof of Proposition 2, observe that $R' - R =$

$$= \alpha(1-\beta)\theta_i A'(e_j - e_k) + \alpha\beta \left[(R_j - R_k)A'x_i + \Big(\sum_{l=1}^{J} x_{il} R_l \Big) A'(e_j - e_k) + (R_j - R_k)A'(e_j - e_k) \right] =$$

$$= \alpha q_i A'(e_j - e_k) + \alpha\beta(R_j - R_k) \left[A'x_i + A'(e_j - e_k) \right],$$

where the second equality uses $q_i = (1-\beta)\theta_i + \beta \sum_{l=1}^{J} x_{il} R_l$. Multiply by e_j^T and $-e_k^T$:

$$P_B = e_j^T (R' - R) = e_j^T \left(\alpha q_i A'(e_j - e_k) + \alpha\beta(R_j - R_k) \left[A'x_i + A'(e_j - e_k) \right] \right).$$

Use $e_j^T A'(e_j - e_k) = A'_{jj} - A'_{jk}$ and $e_j^T A' x_i = \sum_{l=1}^{J} A'_{jl} x_{il}$ to obtain the statement's equation.

$$P_S = e_k^T (R - R') = -e_k^T \left(\alpha q_i A'(e_j - e_k) + \alpha\beta(R_j - R_k) \left[A'x_i + A'(e_j - e_k) \right] \right).$$

Same matrix manipulations, $A'_{jk} = A'_{kj}$, and sign reverse deliver the statement result. □

Proof of Corollary 2. Rewrite $P_B - P_S$ from Equation $(\star)$ on page 29 as $(e_j + e_k)^T (R' - R) = (e_j + e_k)^T (A'(B' - B) + (A' - A)B) =$

$$= \alpha q_i (e_j + e_k)^T A' \underbrace{(e_j - e_k)}_{=:d} + \alpha\beta(R_j - R_k)(e_j + e_k)^T A'(x + \underbrace{e_j - e_k}_{d}). \qquad (\star\star)$$

It is hard to solve for A' in terms of A, but we don't need A', we need $A'x$ and $A'd$. Observe that $(A')^{-1}Ad = \left(\mathbb{I}_{J\times J} - \alpha\beta(X + E_{ij} - E_{ik})^T (X + E_{ij} - E_{ik}) \right) Ad =$

$$= d - \alpha\beta(xd^T + dx^T + dd^T)Ad = (1 - \alpha\beta(a+b))d - \alpha\beta a x,$$

and $(A')^{-1}Ax = \left(\mathbb{I}_{J\times J} - \alpha\beta(X + E_{ij} - E_{ik})^T (X + E_{ij} - E_{ik}) \right) Ax =$

$$= x - \alpha\beta(xd^T + dx^T + dd^T)Ax = -\alpha\beta(b+c)d + (1 - \alpha\beta b)x.$$

Multiply both sides by A', obtain

$$\begin{bmatrix} Ad \\ Ax \end{bmatrix} = \left[\begin{bmatrix} 1 - \alpha\beta(a+b) & -\alpha\beta a \\ -\alpha\beta(b+c) & 1 - \alpha\beta b \end{bmatrix} \otimes \mathbb{I}_{J\times J} \right] \begin{bmatrix} A'd \\ A'x \end{bmatrix}.$$

Similarly to the case of $J = 1$, this is solved by inverting a 2×2 matrix on the left of $\otimes$:

$$\begin{bmatrix} A'd \\ A'x \end{bmatrix} = \frac{1}{D} \left[\begin{bmatrix} 1 - \alpha\beta b & \alpha\beta a \\ \alpha\beta(b+c) & 1 - \alpha\beta(a+b) \end{bmatrix} \otimes \mathbb{I}_{J\times J} \right] \begin{bmatrix} Ad \\ Ax \end{bmatrix},$$

where D is the determinant of the inverted 2×2 matrix. This yields us

$$A'd = \frac{(1 - \alpha\beta b)Ad + \alpha\beta a Ax}{D}, A'(x+d) = \frac{(1 + \alpha\beta c)Ad + (1 - \alpha\beta b)Ax}{D}.$$

Substitute these into Equation $(\star\star)$ statement, obtain

$$\begin{aligned} P_B - P_S = \alpha q_i (e_j + e_k)^T A'd + \alpha\beta(R_j - R_k)(e_j + e_k)^T A'(x+d) = \\ = \frac{\alpha}{D} \left[q_i \left((1 - \alpha\beta b)(A_{jj} - A_{kk}) + \alpha\beta a \sum_{l=1}^{J} x_{il}(A_{jl} + A_{kl}) \right) + \right. \\ \left. + \beta(R_j - R_k) \left((1 + \alpha\beta c)(A_{jj} - A_{kk}) + (1 - \alpha\beta b) \sum_{l=1}^{J} x_{il}(A_{jl} + A_{kl}) \right) \right]. \end{aligned}$$

D is positive when resolvents exist for both A' and A. Notice that $(A')^{-1} = A^{-1} - \alpha\beta \begin{bmatrix} x & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x & d \end{bmatrix}^T$, and the Sylvester's determinant theorem says

$$\det(A')^{-1} = \det A^{-1} \underbrace{\det \left[\mathbb{I}_{2\times 2} - \alpha\beta \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x & d \end{bmatrix}^T A \begin{bmatrix} x & d \end{bmatrix} \right]}_{=D}.$$

Since both resolvents exist and are positive definite, $D > 0$. □

Proof of Proposition 3. If the space for x_{ij} is discrete: there is a finite number of ownership networks, and one of them has to have a highest value of total reputation. For continuous holdings of x_{ij}, notice the continuity of the total reputation with respect to x_{ij} and boundedness of total supply of art Y_i. $\mathcal{F}$, a product of simplices, is compact, which means Weierstrass theorem applies. □

Proof of Proposition 4. Let X denote the hoarding allocation, with all art held by collector 1; their v_1 is highest among other collectors, and their $R_1 \geq 0$ by assumption.

We first show that the hoarding allocation is efficient. $X = Ye_1^T$, where Y is an I-long column vector of total existing art counts, and e_1 is a J-long column vector with 1 at the

first coordinate. Therefore $Xv = Yv_1$ and $XX^T = YY^T$. Take any reallocation of art X'.

The artist prestige equation under X is

$$q = (1-\beta)\theta + \beta(1-\alpha)Xv + \alpha\beta XX^T q = (1-\beta)\theta + \beta(1-\alpha)Yv_1 + \alpha\beta YY^T q,$$

and q' is similar but uses X' instead of X, so the second transition does not hold.

$$q' = (1-\beta)\theta + \beta(1-\alpha)X'v + \alpha\beta X'(X')^T q'.$$

Because $v_1 \geq v_j \forall j \in \{2, ..J\}$, $X'v \leq Yv_1$. Proposition 1 proof obtains $X'(X')^T \leq YY^T$. Under X, $q_i = (1-\beta)\theta_i + \beta Y_i R_1 \geq 0$. Subtracting q' from q yields

$$\left[\mathbb{I}_{I\times I} - \alpha\beta X'(X')^T\right](q - q') = \beta(1-\alpha)(Yv_1 - X'v) + \alpha\beta(YY^T - X'(X')^T)q \geq 0.$$

Since the artist side resolvent is component-wise non-negative, this means $q - q' \geq 0$ component-wise for every X'. The total reputation at X' is

$$\mathbf{1}_{1\times J}^T R = (1-\alpha)\mathbf{1}_{1\times J}^T v + \alpha\mathbf{1}_{1\times J}^T (X')^T q' = (1-\alpha)\mathbf{1}_{1\times J}^T v + \alpha Y^T q',$$

so the only thing depending on X' is q', which is maximized pointwise at the hoarding allocation, proving efficiency.

To prove pairwise stability, we need to verify that for every X' that involves only movement from 1 to $k \neq 1$, $R_1 + R_k$ does not increase. Observe that since other collectors have no art, their parameters do not enter q' and do not respond to reallocation of art, and therefore

$$(R_1' + R_k') - (R_1 + R_k) = \mathbf{1}_{1\times J}^T R' - \mathbf{1}_{1\times J}^T R \leq 0$$

by the efficiency argument above. □

Proof of Proposition 5. q is one-dimensional. Let $R_j = (1-\alpha)v_j + \alpha x_j q$, where $x_j := x_{1j}$. Then $R_j' - R_j = \alpha(x_j' q' - x_j q)$ (primes again denote post-trade values) and

$$\Delta(R_j + R_k) = \alpha((x_j' + x_k')q' - (x_j + x_k)q) = \alpha(x_j + x_k)(q' - q),$$

since the trade between j and k keeps $x_j + x_k$ the same. So any trade is only mutually beneficial if it improves q, which means q can serve as an ordinal potential function.

Observe that since $\sum_{j=1}^J x_j = Y$, the total reputation function is $\mathbf{1}_{1\times J}^T R = \mathbf{1}_{1\times J}^T (1 -$

$\alpha)v_j + \alpha Y q$, a linear function of q. □

Online Appendix B: Eigenvalues and Eigenvectors

Both $X^T X$ and XX^T have identical nonzero eigenvalues. This is a consequence of the matrix algebra, and a feature that allows the *singular value decomposition* of X into

$$X = U\Sigma V^T,$$

where V is the eigenvector matrix of $X^T X = V\Lambda V^T$, U is the eigenvector matrix of $XX^T = U\Lambda U^T$, Λ is a diagonal matrix of non-zero eigenvalues (without loss of generality ordered from highest to lowest) supplemented by zeros to maintain dimensionality, and Σ is a *singular value* $I \times J$-sized matrix with square roots of eigenvalues from Λ on the diagonal.[8] The only thing to be careful is the sign consistency: V and U eigenvectors are defined up to a sign change by eigenvalue decomposition of XX^T and $X^T X$, but there is always the right sign for the SVD. To obtain it, pick an appropriate V and calculate

$$U_k = XV_k/\sqrt{\lambda_k}.$$

The interpretation of eigenvalue decomposition of the collectors overlap network $X^T X$ is as follows. Vectors V_k—kth column of V—represent which collectors participate in the kth group. V_1 is the weighed group of collectors that corresponds to the highest eigenvalue λ_1; it can be chosen so that all its coordinates are nonnegative by the Perron-Frobenius theorem. A collector whose holdings are aligned with V_1 participates in the strongest feedback loop, which has a total amplification factor of $(1 - \alpha\beta\lambda_1)^{-1}$. V_2, due to orthogonality to V_1 (assuming $X^T X$ is not block-diagonal), has a mixture of positive and negative coordinates but represents a similar thing: a collector whose holdings are most aligned with V_2 has little feedback from the strongest feedback loop, but participates in the second strongest feedback loop, with amplification factor of $(1 - \alpha\beta\lambda_2)^{-1}$, and so on.

To see this, notice that the resolvent matrix A is $[I - \alpha\beta V\Lambda V^T]^{-1} = V[I - \alpha\beta\Lambda]^{-1}V^T$.

[8] Galeotti et al. (2020) use the same spectral decomposition to their resolvent, ordering principal components by eigenvalue to characterize optimal interventions in an exogenous network.

Note that $[I - \alpha\beta\Lambda]$ is diagonal and therefore is easy to invert:

$$A = V \begin{bmatrix} (1-\alpha\beta\lambda_1)^{-1} & 0 & ... & 0 \\ 0 & (1-\alpha\beta\lambda_2)^{-1} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & (1-\alpha\beta\lambda_J)^{-1} \end{bmatrix} V^T.$$

A_{jj} is therefore

$$A_{jj} = \sum_k \frac{V_{jk}^2}{1-\alpha\beta\lambda_k},$$

where V_{jk} is the j's collector weight in kth eigenvector. Since $\sum_k (V_{jk})^2 = 1$ by orthonormality of eigenvectors, if a collector has $V_{j1} = 1$, they also have the highest possible A_{jj}; a collector that owns all the art would be such a collector. Generically

$$A_{jj} = \sum_{k=1}^{J} \frac{V_{jk}^2}{1-\alpha\beta\lambda_k} = \sum_{k:\lambda_k>0} \frac{V_{jk}^2}{1-\alpha\beta\lambda_k} + \sum_{k:\lambda_k=0} V_{jk}^2 = 1 + \sum_{k:\lambda_k>0} \frac{\alpha\beta\lambda_k V_{jk}^2}{1-\alpha\beta\lambda_k},$$

where the last equality is coming from orthonormality of rows of V.

A similar decomposition can be derived for A_{jk}:

$$A_{jk} = \sum_{l=1}^{J} \frac{V_{jl}V_{kl}}{1-\alpha\beta\lambda_l} = \sum_{l:\lambda_l>0} \frac{V_{jl}V_{kl}}{1-\alpha\beta\lambda_l} + \sum_{l:\lambda_l=0} V_{jl}V_{kl} = 0 + \sum_{l:\lambda_l>0} \frac{\alpha\beta\lambda_l V_{jl}V_{kl}}{1-\alpha\beta\lambda_l}.$$

The signs of V_{kl} and V_{jl} don't always align; but since we know that $A_{jk} \geq 0$, we know that groups where they align dominate groups where they don't align.

Similar is the interpretation of U_i, eigenvectors of XX^T: these are the portfolios of artists that yield the strongest feedback loop; the second strongest feedback loop; etc. Collectors similar to V_1 collect art portfolios similar to U_1; collectors similar to V_2 collect art similar to U_2. If there are more collectors than artists, there will be some V_j's that don't yield any feedback.

The duality of reputations of collectors and prestige of artists produces clusterization as a mechanical consequence of linear algebra: because X^TX and XX^T share the same nonzero eigenvalues, the feedback loops are identical whether viewed from the collector side or the artist side. A group of collectors who reinforce each other's reputations necessarily implies a group of artists who magnify each other's prestige, and vice versa.

Online Appendix C: Example 6, Continuous Holdings

Let $\alpha = 0.3$, $\beta = 0.1$, $\theta = [1, -5]^T$, $v = (1, 1, -5)$, $Y = [2, 2]^T$. A "hoarding" approach (all art belonging to one collector) yields $\alpha\beta\rho(X^TX) \leq 0.24$, so there are no issues with existence for any $X \geq 0$. The proof will unravel in the following fashion:

1. Derive a Proposition 2 continuous-holding analogue for arbitrary trade size.
2. Take a derivative with respect to trade size to obtain a marginal reputation change function for a trade in every artist between any two participants for every X. Every pairwise stationary point (with caveats for corners) must exhibit all these derivatives less or equal to zero, this is a necessary condition for stability but not sufficient.
3. Build a numerical contour plot that illustrates that the only solution of the system is nearby $X_{11} = X_{21} = 1$. Establish that the only solution for the system of marginal reputation change functions being equal to zero derived above in feasible domain is $X_{11} = X_{12} = X_{21} = X_{22} = 1$. We use Claude-assisted Sage code to verify this using interval algebra.
4. Establish that a positive-size trade of Artist 1 from Collector 1 to Collector 2, or vice versa, at that point is welfare-improving: the derivative was showing that either direction works locally, not that neither is profitable.

The difference in the total reputation of buyer j and seller k of Δ units of artist i is

$$(R'_j + R'_k) - (R_j + R_k) = (e_j + e_k)^T(A'(\Delta)B'(\Delta) - AB).$$

Take a derivative with respect to Δ. The derivative of B is straightforward:

$$\frac{dB}{d\Delta} = \alpha(1 - \beta)\theta_i(e_j - e_k).$$

The change in the matrix of connections is quadratic: $(X')^TX' = X^TX + \Delta(x_i(e_j - e_k)^T + (e_j - e_k)x_i^T) + \Delta^2(e_j - e_k)(e_j - e_k)^T$, with obvious derivative with respect to Δ. The derivative of A is harder since it's an inverse of a function of X^TX, but take an implicit derivative of $AA^{-1} = \mathbb{I}_{J\times J}$ to obtain the derivative

$$\frac{dA}{d\Delta} = -A\frac{dA^{-1}}{d\Delta}A = \alpha\beta A\frac{dX^TX}{d\Delta}A.$$

Use the product differentiation rule: $\frac{dR}{d\Delta} = \frac{dA}{d\Delta}B + A\frac{dB}{d\Delta}$. Collect together to obtain

$$G^i_{j,k} := \alpha(1-\beta)\theta_i \overbrace{(e_j+e_k)^T A(e_j-e_k)}^{A_{jj}-A_{kk}} + \alpha\beta[(e_j+e_k)^T A x_i \overbrace{(e_j-e_k)^T R}^{R_j-R_k} + \overbrace{(e_j+e_k)^T A(e_j-e_k)}^{A_{jj}-A_{kk}} x_i^T R] =$$

$$= \underbrace{\alpha q_i (A_{jj}-A_{kk})}_{\text{direct}} + \underbrace{\alpha\beta(R_j-R_k)(e_j+e_k)^T A x_i}_{\text{indirect}}.$$

In the interior stable point, each of these should be nonpositive, otherwise we have a local profitable trade. Observing that $x_{11}+x_{12}+x_{13}=2$ and $x_{21}+x_{22}+x_{23}=2$ we can define

$$\Phi(X) = \sum_{i=1}^{2} \sum_{\substack{k=1 \\ X_{ik}>0}}^{3} \sum_{\substack{j=1 \\ j\neq k}}^{3} \max\left(G^i_{j,k}(X), 0\right)^2$$

to be numerically minimized along (x_{12}, x_{22}) space to be plotted on a $[0,2]^2$ space:

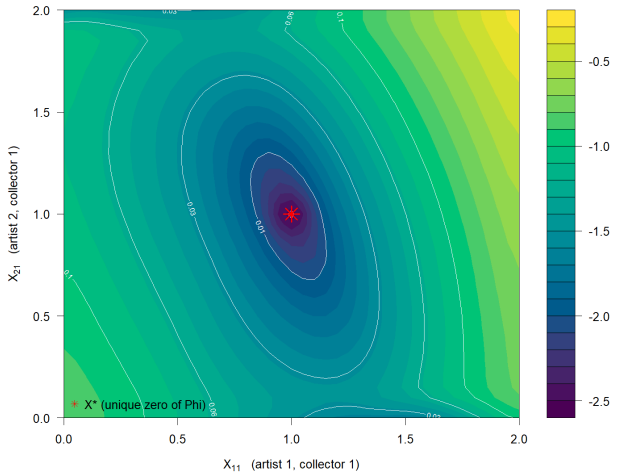


Figure 1: $\min_{x_{12},x_{22}} \Phi(x_{11}, x_{12}, x_{21}, x_{22})$ (numerical, height is log, smoothed). Replication code: `https://sergeyvpopov.github.io/files/Fig1Replicate.R`.

This suggests that $(x_{11}, x_{21}) = (1,1)$ might be part of a pairwise stable allocation. Turns out, $\Phi(1,1,1,1) = 0$. Denote network implied by this point by X^*. At that point, $G^1_{3,1} < 0$ and $G^2_{3,1} < 0$, so collector 3 would sell art away if they had any, so that part is stable; but $G^1_{1,2} = 0$, it could either be a local minimum $R_1 + R_2$ or a maximum. Alas, any positive-size trade away is profitable. Indeed, for every $1 \geq \Delta > 0$, $X' = \begin{bmatrix} 1+\Delta & 1-\Delta & 0 \\ 1 & 1 & 0 \end{bmatrix}$ and

$X = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$, and

$$(R_1' + R_2') - (R_1 + R_2) = \frac{1611\Delta^2}{48400 - 3102\Delta^2} > 0.$$

The same, by symmetry, applies for the reverse trade, and for the trade in artist 2, albeit with different numbers. The only caveat remains to check is that there are no other solutions. Claude helped us design Sage code to verify there are no other solutions, available at `https://sergeyvpopov.github.io/files/Ex6SolutionChecker.sage`.

Computer-assisted proof. We work throughout with exact rational arithmetic and rigorous interval arithmetic (200 bits of precision).

Step 1 (*exact evaluation*). Direct substitution yields $\Phi(X^*) = 0$ and

$$G^1_{3,1}(X^*) < 0, \qquad G^2_{3,1}(X^*) < 0.$$

Step 2 (*global exclusion*). An interval branch-and-bound search on the box $[0, 2]^4$ for the free entries $(x_{11}, x_{12}, x_{21}, x_{22})$ proves that every feasible point outside the neighbourhood $N = [0.98, 1.02]^4$ satisfies $\Phi(X) > 0$.

Step 3 (*local support reduction*). On the feasible part of N (i.e. with third-column entries in $[0, 0.04]$) interval evaluation gives

$$G^1_{3,1}(N) \subset (-\infty, 0), \qquad G^2_{3,1}(N) \subset (-\infty, 0).$$

Consequently any zero of Φ inside N must have $x_{13} = x_{23} = 0$.

Step 4 (*exact algebraic uniqueness on the boundary*). The problem reduces to the two-parameter family

$$X = \begin{bmatrix} a & 2-a & 0 \\ c & 2-c & 0 \end{bmatrix}, \qquad a, c \in [0.98, 1.02].$$

Vanishing of Φ forces the two polynomial equations $G^1_{2,1}(X) = 0$ and $G^2_{2,1}(X) = 0$. After clearing denominators the resultant with respect to c factors as

$$\mathrm{Res}_c(p_1, p_2) = -5992368042253824000000\,(a-1)\,(3a^2-6a-22)\,(68781a^2-137562a-949300).$$

The two quadratic factors have no roots in the interval $[0.98, 1.02]$ (exact algebraic roots

computed in the real algebraic numbers $\mathbb{A}$). Hence $a = 1$. Substituting $a = 1$ yields

$$\gcd(p_1|_{a=1}, p_2|_{a=1}) = c - 1,$$

so $c = 1$. Therefore the only zero inside N is X^*.

Combining Steps 2–4 shows that X^* is the unique zero of Φ on the whole feasible set $\mathcal{F}$. □