Informal Seniority*

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Abstract

We study how lenders compete for repayment priority in environments with selective default. We document that lenders obtain informal seniority by increasing borrowers' continuation values through credit limits, interest rates, and cross-selling. We show that such competition can lead to either over- or underprovision of relationship benefits relative to the first best. Because of these opposing forces, widely used cross-default provisions may be suboptimal, and reducing competition or raising origination costs can increase welfare. We develop a sufficient-statistic approach to quantify the impact of these policies on welfare. Using quasi-experimental variation in search costs together with the sufficient-statistic approach, we show that lowering search costs decreased welfare in our setup.

Keywords: debt dilution, non-exclusive contracts, selective default, bankruptcy law
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"Everyone in Brazil has four credit cards, and if they don't have money to buy groceries, they use all these cards.[...] But they pay the one from the bank they have the relationship with. Banks don't care if you have a negative score if you keep paying them. So, being the main bank determines the cost of credit. People underestimate the huge relationship between being the main bank and default risk. If you are the principal relationship bank, they might default on the other banks but you will be the last one."

The former CFO of Itaú, the largest private bank in Latin America,
in an interview

1 Introduction

Debt dilution is a central contracting problem in corporate finance.[1] By originating new loans, lenders may impose a default externality on their competitors by affecting the borrower's ability and incentives to repay existing obligations held by other institutions (e.g., Bizer and DeMarzo, 1992; Parlour and Rajan, 2001; Green and Liu, 2021). The vast majority of the literature is theoretical, and analyses debt dilution assuming universal default (e.g., due to cross- default provisions) and the existence of repayment priority rules, exogenously set by the regulator. However, in many environments – such as trade credit, sovereign debt or retail loans – priority rules and cross-default provisions are absent or imperfectly enforceable.[2] We argue that, in such settings, repayment priority is determined endogenously by continuation values: borrowers repay the loan whose continuation value is highest, and lenders compete for repayment priority through contract terms, such as credit limits or cross-selling. We call this phenomenon informal seniority. This paper studies, empirically and theoretically, how informal seniority shapes contract terms, welfare, and optimal regulation.

Unsecured retail credit markets provide a natural setting in which to study informal seniority. Individuals often hold multiple credit cards and loans from different lenders. Unlike corporate credit contracts, household credit contracts typically allow borrowers to become delinquent on one obligation without triggering immediate consequences

[1] See, for instance, Bizer and DeMarzo (1992). It has shaped important parts of creditor/bankruptcy law, including priority rules and antidilution covenants.

[2] Examples include cross-border contracts, such as international trade finance (Antras and Foley, 2015); sovereign debt (Erce and Mallucci, 2018); trade credit (e.g., Cuñat, 2007; Dahis et al., 2024; Brugués, 2026); informal lending; and financial markets in countries where enforcement may be imperfect or prohibitively costly (e.g., Djankov et al., 2007). Even in formal credit markets, bankruptcy procedures are often costly, slow, and uncertain, and financial distress is frequently resolved outside bankruptcy (Dawsey et al., 2013).

from other lenders.[3] Retail borrowers can therefore choose which creditors to repay, making repayment priority endogenous.

In the first part of the paper, we show that selective default is quantitatively relevant in the retail credit card market and analyze its determinants. Our analysis leverages administrative data from the Central Bank of Brazil. Unlike the vast majority of credit registries, we observe all loans held by a given borrower across all financial institutions, including mortgages, car loans, consumer loans, and credit cards. This feature allows us to reconstruct each borrower's complete loan portfolio and document new facts about repayment behaviour in the presence of multiple lending relationships. For instance, we show that borrowers rarely simultaneously default on all their cards: only 10% of default are universal. Second, we show that borrowers prioritize the repayment of more favorable credit contracts: conditional on selective default, they are more likely to continue servicing cards with higher available limits, lower interest rates, those issued by digital banks, and those provided by lenders that also supply other financial products, such as auto loans.

In the second part of the paper, we study the implications of competition for informal seniority theoretically, and develop a sufficient statistic to assess the welfare impact of different policies. To do so, we relax the simultaneous default and equal-recovery assumption made in models of non-exclusive lending (e.g., Bizer and DeMarzo, 1992; Green and Liu, 2021) by allowing borrowers to prioritize repaying contracts with more favorable terms. In the model, lenders impose default externalities on their competitors through two distinct channels. The first is the standard debt-dilution channel: originating a new loan increases overall indebtedness and raises default probabilities across all outstanding loans. The second is novel and operates through repayment prioritisation: issuing a new loan affects which obligations the borrower chooses to repay. We show that distinguishing between these channels is important, as they generate different positive and normative implications.

We begin our empirical analysis by documenting three facts suggesting that debt dilution is a first-order force in the credit card market. First, multi-lender borrowing is pervasive: among 85 million credit card users in June 2022, 39.4 million (46.5%) maintained active cards issued by two or more institutions, collectively accounting for over 70% of the total outstanding credit card balance. Second, borrowing through credit cards is common. Approximately 50% of borrowers revolve their balance at the end of the month.[4] Finally, default rates are high: during the sample period, 9% of credit card borrowers default on at least one card, defined as failing to make the minimum

[3] For evidence on debt prioritization across loan types, such as mortgages versus credit cards, see Federal Reserve Bank of New York, Liberty Street Economics, "When the Household Pie Shrinks, Who Gets Their Slice?" (March 2025).

[4] About 20% of monthly credit card purchases are not repaid at the end of the month due to revolving balances and card-linked buy-now-pay-later arrangements.

payment for 90 days or more.

We then derive new facts about repayment behavior in the credit card market. First, we highlight that borrowers with multiple active cards rarely default on all their active cards. Indeed, among those who default on at least one card, only 16% default on all of their active cards, meaning that 84% continue to make payments on at least one card. This pattern is not driven by lenders preemptively cancelling credit cards. Defaulting on one card has little effect on the credit limits or interest rates of other cards, whereas limits and monthly usage drop to zero on cards that borrowers do not repay.

Second, we test for the presence of the standard debt dilution channel by estimating the causal effect of obtaining an additional card on default on any card. To do so, we exploit a novel, plausibly exogenous variation in the number of cards owned within a difference-in-differences framework. The variation arises when banks form partnerships with a firm. The arrangement requires employees to receive their wages through the partner bank, thereby increasing the likelihood that they obtain a card from that bank. We compare two groups of borrowers employed at the same firm around the time the partner bank changed: those who already had a relationship with the partner bank before the contract (control) and those who did not (treatment). The identification assumption is that, absent the partner bank's change, the two groups of workers would have continued to follow parallel trends. Thus, the observed relative change in default probability between them can be attributed to the increased likelihood of the treatment group obtaining a new card. We find that the relative default probability of the treated borrowers one year after the treatment increases by 1.1 percentage points. Given that treated borrowers acquire, on average, 0.11 new card lenders, obtaining a new card lender increases default rates by 10 percentage points.[5]. This difference-in-differences framework allows variation in the search cost of getting an additional credit card. Later, we exploit those estimates together with our model to assess the impact of lowering search costs on welfare.

We also provide suggestive evidence that lenders can compete to gain loan seniority (i.e., be the bank the borrower prioritizes repaying) through contract terms and cross-selling. We document this pattern by examining which card each financially distressed individual chooses to repay. Estimating regressions of default choices among selective defaulters holding multiple credit cards, we find that, on average, these borrowers default on 45% of their cards and systematically prioritize repayment of cards with higher credit limits (by 4 percentage points), those originated by digital banks (by 14 percentage points), and those issued by lenders that also provide other financial products (e.g., a 14-percentage-point higher repayment probability when the lender also holds the borrower's mortgage and a 5-percentage-point higher probability when the lender

[5]The impact on default probability is of a similar magnitude to the one found in De Giorgi et al. (2023) for retail borrowers in Mexico.

holds the borrower's auto loan).

These estimates should be interpreted with caution, as they are not recovered using quasi-experimental variation. Nevertheless, the richness of our data allows us to absorb a wide range of confounding factors by exploiting within-borrower variation among selective defaulters. For example, we identify the coefficient on credit limit using selective defaulters: we compare the default choices of the same borrower across multiple cards with different limits. This structure allows us to include borrower-time fixed effects and, given multiple observations per bank, bank-time fixed effects, which absorb differences across lenders in factors such as debt collection technology. Remaining bias in the estimated effects of contract terms would have to come from unobserved product-level characteristics correlated with repayment priority, such as rewards programs on premium cards, which also tend to have higher credit limits. Even if the coefficients are biased, the results show that default choices across lenders are not random. Instead, they covary systematically with lender and contract characteristics that lenders can plausibly shape, consistent with competition for repayment priority.

Finally, we use earnings calls and interviews of banks' CFOs and CEOs to show that lenders communicate in ways consistent with informal seniority (see, for instance, the quote at the beginning of the paper). For instance, in Brazil, six out of the largest lenders communicated caring about being perceived as the principal bank and how it affects default probabilities.

We then move to the theoretical analysis. The baseline version of the model features two decision periods. In the first period, lenders simultaneously offer loan contracts, and borrowers may accept any subset of these offers. The key friction is that they cannot commit to exclusive contracts as in Green and Liu (2021). In the second period, borrowers choose which loan to repay, subject to a budget constraint. Lenders can choose the loan interest rate and the credit limit, and can pay a cost to increase the borrower's perceived value of repaying the card by providing relationship benefits. We interpret these relationship benefits as improvements in, for example, the bank's mobile application, the card's rewards program or investment in customer capital. The analysis also applies when the benefits takes the form of cross-selling or offering more favourable terms (credit limits and interest rates) in the future.

We show that there can be over- or underprovision of relationship benefits relative to the first-best due to two opposing forces. First, the standard debt-dilution channel generates underprovision. Because of the default externality, lenders expect borrowers to leverage too much relative to the first-best, and thus to default too frequently.[6] Holding default probabilities fixed, this reduces the expected duration of the lending rela-

[6]As in Bizer and DeMarzo (1992), a lender internalises that originating a new loan increases the borrower's default probability on its own loan, but does not internalise the impact on the default probabilities of loans held by other lenders.

tionship and thus the marginal benefit of relationship benefits for the borrower, thereby weakening incentives to provide them. Second, competition for repayment priority can lead to overprovision or underprovision depending on which contract characteristics the borrower is more sensitive to. For instance, when borrowers care more about (future) credit limits than the debt face value, each lender has an incentive to increase their limits relative to their competitor. This competition leads to an arms race, resulting in overprovision of credit limits. Instead, when borrowers prioritise repaying the debt with the lower face value (and thus default on the other loans), lenders have incentives to lower the credit limit so that the debt face value is lower.

In that setup, we show theoretically that standard bankruptcy regulation features, such as cross-default provisions and increasing competition, can reduce welfare. By imposing cross-default provisions, the regulator cancels competition for informal seniority. When relationship benefits are underprovided, imposing cross-default increases them, which can increase demand for credit cards and worsen the default externality problem. Our empirical results suggest that the underprovision channel dominates. Using a calibrated version of our model, we show that imposing cross-default provision would cut welfare by half (the cost is equivalent to roughly doubling the interest rate).

Using a sufficient-statistic approach in a multi-period model, together with exogenous variation in the cost of getting an additional card, we show that lowering the cost of acquiring a card decreases welfare. The reason is the following. Because of the default externality, borrowers take out too many loans. Holding relationship benefits constant, increasing competition (i.e., lower search costs) can increase demand for cards as lenders decrease the markup. Furthermore, the lower markup decreases incentives to compete on repayment priority, as lenders' marginal profit falls with lower rates. This effect can either decrease or increase the demand for cards depending on whether there is under- or overprovision of benefits. The sufficient statistic captures these forces and suggests that the default-externality effect dominates in our setup. The statistic implies that the cost of lowering search costs so everyone gets one more card is equivalent to having them pay 425 reais (i.e., 10 per cent of average monthly income). Using a mix of the sufficient-statistic approach and parametric assumptions on lending costs, we further show that credit limits after origination are around 8 per cent lower than in the first best.

Related Literature

The paper makes two contributions. The first one is theoretical: we provide the first analysis of informal seniority. The second contribution is empirical. We show selective default in the retail market and develop tools to bring debt dilution models to the data.

The reason we need a model is that debt dilution is difficult to observe directly. One needs to establish not merely that a firm issues new debt, but that the issuance transfers value away from existing creditors relative to a counterfactual in which it did not occur.

We connect to the literature on non-exclusive contracts. Most contributions in this area are theoretical (e.g., Bizer and DeMarzo, 1992; Parlour and Rajan, 2001; DeMarzo and He, 2021; Green and Liu, 2021).[7] A common assumption in this literature is that loans with the same formal seniority receive identical recovery rates in default. We depart from this framework by studying informal seniority. This approach is particularly relevant in settings where formal enforcement is weak or costly. Examples include retail and small-business lending, where bankruptcy proceedings are often too expensive to be used routinely, as well as international trade and many developing-country credit markets, where creditors face substantial limitations in enforcing contractual seniority (e.g., Djankov et al., 2007, 2008). By allowing repayment priority to emerge endogenously, our framework highlights a novel channel through which competition among lenders shapes contract design and repayment outcomes.

Our paper is closely related to Bolton and Jeanne (2007) and Brunnermeier and Oehmke (2013), who study how contractual features such as debt renegotiability and maturity structure affect recovery rates and repayment incentives. Their objective is to explain, respectively, why sovereign debt may be difficult to renegotiate and why firms may choose short-term financing. A key conceptual difference is that, in their models, contractual distortions that increase repayment incentives reduce both default risk and borrower utility. Consequently, these frameworks do not generate a trade-off between overprovision and underprovision, and policies that equalize recovery rates across creditors are generally optimal.

More generally, we contribute to the industrial organisation literature analysing competition in quality. The vast majority of the literature studies the perfect competition case in which quality does not impose an externality on others (see, for instance, Spence, 1975 or more recently Bourreau et al., 2026). We make two contributions. First, we analyse the effect of imperfect competition in a model in which quality provision imposes an externality on other firms via debt dilution. Second, we develop a sufficient-statistic approach to analyse and quantify the welfare impact of policies in this set-up.

Our framework draws on tools from the empirical industrial organisation and public economics literature (Chetty 2009), allowing us to bring the model to the data and quantify the importance of these mechanisms in the credit card market. Our modeling approach is closely related to the industrial organization literature on banking (e.g., Crawford et al., 2018; Clark et al., 2021; Benetton, 2021; Nelson, 2025; Brugués, 2026;

[7] We focus on the debt-dilution consequences of non-exclusivity. Other studies emphasize its benefits, including mitigating hold-up problems (e.g., Ioannidou and Ongena, 2010) and providing insurance against idiosyncratic shocks to lenders' liquidity (e.g., Detragiache et al., 2000).

Egan et al., 2026). Existing models in this literature typically assume exclusive lending relationships and abstract from repayment priority across lenders. We extend this framework to an environment with non-exclusive contracts, debt dilution, and endogenous repayment priority. By combining insights from the contract-theoretic literature on non-exclusive lending with tools from empirical industrial organization, our approach provides a tractable framework for estimating models of informal seniority and quantifying their implications for market outcomes and policy. The paper is related to Brugués (2026), which studies an environment in which repayment is sustained by continuation value. We extend this framework to study debt dilution by allowing the borrower to contract with multiple lenders rather than a single lender.

Although regulation recognizes selective default,[8] the phenomenon has received relatively little empirical attention. Most work in this area focuses on personal bankruptcy (e.g., Indarte and Kanz, 2024; Argyle et al., 2025; Antill and Hunter, 2026; Ben-Ishai et al., 2026), where procedures tend to equalize recovery across creditors within the same class. We instead study defaults outside formal bankruptcy, which account for the majority of credit card charge-offs and are characterized by selective default.[9] A small empirical literature documents selective default and debt dilution in credit markets. Andersson et al. (2013) show that borrowers prioritize mortgage payments over credit card debt, while Degryse et al. (2016) and De Giorgi et al. (2023) document that additional borrowing increases default risk on existing loans. Schlegl et al. (2019) studies repayment priorities across sovereign creditors. We contribute to this literature by documenting repayment priority within a class of loans that share the same formal seniority, namely credit cards. We show that repayment decisions are shaped by the continuation value of lending relationships, implying that seniority cannot be imposed in the contract via the use of, for instance, secured or unsecured debt, but must be sustained through the promise of generous contract terms going forward. This new mechanism is important since it has implications for contract design (e.g., interest rates, credit limits), welfare, and bankruptcy regulation.

Our paper also contributes to the empirical literature on credit card borrowing. Recent work has studied the pricing of credit card contracts through the lenses of competition, consumer behavior, and asymmetric information (e.g., Ru and Schoar, 2016; Stango and Zinman, 2016; Galenianos and Gavazza, 2022; Drechsler et al., 2025; Nelson, 2025). While this literature primarily focuses on contract pricing and credit supply, we study how borrowers allocate repayments across competing lenders and how these repayment decisions affect equilibrium contract design. Our paper is particularly

[8]For example, the CARD Act of 2009 restricts credit card issuers from repricing a consumer solely because the consumer defaulted on another lender's card.

[9]The 1999 Visa Bankruptcy Survey reports that 66.7% of charged-off credit card accounts were lost outside bankruptcy. Similarly, Dawsey and Ausubel (2002) finds that 50.7% of written-off accounts, representing 45.8% of credit losses in dollar terms, did not involve a bankruptcy filing.

related to Ponce et al. (2017) and Gathergood et al. (2019), who show that borrowers often rely on simple heuristics when allocating repayments across debts. We show that repayment behavior differs markedly among delinquent borrowers. When default is imminent and the costs of losing access to credit are substantial, borrowers behave strategically and prioritize the loans that provide the greatest continuation value.

By showing that other loan products affect credit card repayment decisions, we also contribute to the banking literature on cross-selling (e.g., Basten and Juelsrud, 2026). The existing literature has focused on account stickiness (Puri and Rocholl, 2008) or strengthening of the relationship with borrowers (Drucker and Puri, 2005). Our empirical results point to an additional rationale for banks to cross-sell financial products: cross-selling effectively confers implicit seniority in repayment. This mechanism differs from existing explanations for cross-selling, which emphasise account stickiness or the value of lending relationships for future business.

The rest of the paper is organized as follows. Section 2 describes the institutional setting and the data. Section 3 presents the descriptive statistics and stylized facts. Section 4 reports the difference-in-differences results. Section 5 introduces the model, the equilibrium, welfare analysis, and quantification exercise. Finally, Section 8 concludes.

2 Institutional Setting and Data

2.1 Data

We assemble data from multiple sources to construct a monthly panel dataset of retail loans for borrowers in Brazil. We focus our main analysis on 2019-2024, when information on credit card operations is more complete.

We obtain monthly loan-level data from the credit registry of the Central Bank of Brazil (*Sistema de Informações de Créditos*, SCR). The dataset contains the universe of bank loans—mortgages, personal credit, car loans, credit card loans, payroll-deductible loans, and others—above 200 BRL (around 35 USD). For each term loan, we observe outstanding loan amounts, interest rates, maturity, defaults, anonymized bank and borrower identifiers consistent across products, and the date of origination. For credit cards, we observe monthly credit card usage, revolving balances, amounts due each month, and credit limits.[10] If a borrower holds multiple cards with the same bank in a given month (e.g., two cards from different card networks), we observe aggregate amounts across all of their cards. We also observe whether the borrower does not make the minimum payment due on their card at the end of the month.

[10]Credit card interest rate information is available only when the borrower maintains a revolving balance. The accuracy of credit card limit data improved sharply after May 2021. In certain specifications, we will limit the data to the period following this month.

To get information on the borrower characteristics, we supplement credit information with matched employer-employee data from the Ministry of Labor's *Relação Anual de Informações Sociais* (RAIS). These data provide information on the wage, employer, and selected sociodemographic characteristics, including age, sex, and occupation type. For informal workers, we use the lender's assessment of their income. After 2023, we also have data on the borrower's credit score.

To obtain a tractable sample size, we draw a representative sample of the borrower population using a stratified sampling strategy to ensure sufficient coverage of borrowers experiencing financial distress. Within each cohort, defined at the calendar month level, we partition the population of borrowers into two groups and draw random samples. The first group comprises individuals who were current on all cards until failing to make a payment on at least one card in the cohort month. We designate these individuals, who may be transitioning into default (more than 90 days past due), as *borrowers nearing distress*. The second group comprises all remaining borrowers. Our focus on credit cards in the debt dilution analysis, and hence our sample construction, is motivated by the descriptive statistics reported in the next section (Section 2.2). We provide further details on the sampling procedure and sample characteristics in Appendix A1.

2.2 The Credit Card Industry in Brazil

In this subsection, we use aggregate, publicly available data to present descriptive statistics on the Brazilian credit card market and, where possible, compare them with those of the United States. These statistics suggest that debt dilution is a first-order force in the credit card market: individuals hold cards issued by multiple institutions and frequently borrow and default.

Prevalence of multi-lender borrowing. The credit card industry has expanded rapidly in Brazil. After a stagnant period, the number of active credit cards doubled and the total amount of credit card loans grew by 73% between 2018 and 2023 (Appendix Figure A1). The growth in the number of cards was driven by two factors: the addition of new users and the acquisition of cards from different banks by existing users. According to Central Bank of Brazil (2022), the number of credit card users increased by 31% between June 2019 and June 2022, from 65 million to 85 million (52% of the population aged 15 years or older).[11] Over the same period, Panel A of Appendix Figure A2 shows that the share of users with active credit cards from at least two distinct financial institutions increased from 38% to 47%. Panel B shows that the share of outstanding credit card balances held by these users rose from 58% to 70%, consistent with higher average credit card usage among multi-lender users relative to

[11] We use population figures from the 2022 census.

single-lender users. Panel C shows that a substantial portion of the growth in credit card usage is associated with digital banks, which reduced the costs of applying for new cards.[12] In contrast, in the United States, 82% of consumers used a credit card in 2023, and 76.5% of consumers held two or more cards.[13]

Credit card borrowing. Borrowing with a credit card is common. In Panel A of Appendix Figure A3, we decompose the total credit card balance into three parts: non-interest-bearing purchases, revolving debt, and buy now, pay later. Non-interest-bearing purchases refer to transactions made during the billing cycle that are fully due on the next due date and incur no interest charges if paid by that date; revolving debt refers to the accumulation of debt resulting from not paying the full balance of the monthly credit card bill; and buy now, pay later debt refers to card transactions where payment is split into installments, with or without interest charges. The composition of total credit card debt has remained stable, with non-interest-bearing purchases accounting for 80% of total debt, revolving debt for 10%, and buy now, pay later for 10%. Using our micro-level data, we show that 50% of borrowers revolve at the end of the month. This number is similar to the one in the United States (see, for instance, Lee and Maxted, 2025).

Credit card loans are a substantial part of households' balance sheets. In Panel B of Appendix Figure A3, we show that the participation of credit card debt in total household debt increased from 12.7% to 15% between 2018 and 2023.[14] In the United States, credit cards accounted for 6% of total household debt in 2023.[15]

Default. Using our micro-level dataset (see Appendix Table A2), we show that 7% of credit cards (approximately 11% of credit card users) are in default in a given month, where default is defined as failing to make the minimum payment for 90 days. Conditional on borrowing (revolving and BNPL), the number increases to 15%. In the United States, default—defined as a credit card loan being charged off in a given year—occurs at a rate of around 5% (Drechsler et al., 2025) and rises to as high as 10% among borrowers with low FICO scores.

Appendix Figure A4 shows that the share of total balances in default for revolving credit card debt exceeded 50% in 2023, while the corresponding share for *buy now, pay later* balances reached 10%. The figure also shows that, although credit cards account for about 15% of total credit, they represent more than 50% of household debt overdue by more than 15 days, a share that has been increasing since 2018.

[12] Buchak et al. (2018) and Fuster et al. (2019) document faster application processes among fintech lenders in the mortgage market.

[13] Data are drawn from the *2023 Survey and Diary of Consumer Payment Choice* conducted by the Federal Reserve Bank of Atlanta. The information on the number of cards is not fully informative about multi-lender borrowing, since a consumer may hold multiple cards issued by the same financial institution.

[14] Credit cards are the most common loan contract, accounting for more than 50% of total household loan contracts.

[15] New York Fed Consumer Credit Panel.

Household debt collection. The default procedure in Brazil is broadly similar to that in the United States.[16] Following a missed credit card payment, lenders typically send internal reminders via email or phone. During this stage, Brazil's Consumer Protection Code prohibits harassment and other abusive collection practices. Creditors generally wait at least 30 days before reporting the delinquency to private credit bureaus. By contrast, delinquencies must be reported to the Central Bank of Brazil's credit registry, which is updated monthly and accessible to all financial institutions. If the debt remains unpaid, lenders may outsource collections, sell the delinquent receivable to a collection agency, or initiate judicial collection. In practice, few loans are recovered, as legal proceedings are costly and recovery rates are low.

3 Summary Statistics and Facts

In this section, we provide descriptive statistics and stylized facts obtained from the sample described in Section 2.1, covering the period from June 2019 to June 2022.

3.1 Summary Statistics

In Table 1, we report summary statistics for groups of individuals owning different numbers of active cards issued by different institutions. We report these statistics separately for *borrowers nearing distress* (defined in Section 2.1) and for a random sample of the overall borrower population.

While 40% of the overall population holds credit cards from more than one financial institution, the incidence is significantly higher among *borrowers nearing distress*, at 53%. Within this group, 27% hold cards from two institutions, 14% from three, 6% from four, 3% from five, and another 3% from six or more institutions. Figure 3 confirms the trend documented in Section 2.2, namely that the number of active card lenders per borrower has been increasing since 2019.

Individuals with a larger number of credit card lenders are more likely to hold formal employment, have a higher monthly income, and are more financially integrated. For example, 36% of those with a single card lender are formally employed, compared to 46% among those with three card lenders. Wages exhibit an approximately inverted U-shaped relationship with the number of card lenders. Average monthly earnings rise from 3,757 BRL among individuals with a single card to 4,171 BRL among those with three cards, and subsequently decline to 3,812 BRL for individuals with four cards. In-

[16] While Brazil, like many emerging economies, lacks a personal bankruptcy law (Indarte and Kanz, 2024), this institutional difference is less consequential in practice because most household debt is resolved out of court through collections or private renegotiation rather than through formal bankruptcy proceedings (Dawsey et al., 2013). Moreover, even in settings with a formal bankruptcy procedure, lenders can compete for repayment priority before the preferential-payment lookback period begins.

dividuals with more card lenders exhibit higher debt-service-to-income ratios (18%, 38%, and 59% for those with one, three, and five card lenders, respectively), as well as higher average limit utilization across their cards (54%, 58%, and 64%). They also rely more heavily on other forms of credit. For example, 7%, 16%, and 24% of individuals with one, three, or five card lenders, respectively, hold an auto loan, while the corresponding figures for consumer loans are 14%, 27%, and 42%.

Individuals in the *borrowers nearing distress* group tend to have lower incomes (earning roughly 1,000 BRL less per month) and are 5–10 percentage points less likely to hold a formal-sector job. They are also more leveraged, exhibiting higher credit card limit utilization and a greater propensity to hold other types of loans.

In Table 2, we report default rates in the month before the cohort by category of number of card lenders. We define default as failing to make the minimum payment on the card at the end of the month for more than 90 days. By construction, the share of borrowers with at least one card in default in the *borrower nearing distress* subsample is zero across all categories. In the general population, this share increases monotonically with the number of card lenders (from 7% among borrowers with one card lender to 15% among those with four). However, there is no correlation between the number of credit cards and default on other loans. We interpret this as suggestive evidence that debt dilution is more substantial in the credit card market. Yet, in the *borrowers nearing distress* group, borrowers with one card lender have a higher probability of defaulting on other loans compared to borrowers with more than one card lender.

3.2 Four Facts About Default in the Credit Card Market

In this section, we provide four new facts about default behaviour in the credit card market. First, we show that many borrowers default on only a subset of their cards. Second, we show that the default probability of *borrowers nearing distress* six months after missing a payment increases with the number of cards, even after controlling for observable determinants of default. Third, we show that lenders do not cancel the credit limits of borrowers who defaulted on a competitor's card. Finally, we show that borrowers prioritize repayment of cards with more favorable terms and of cards issued by banks with which they have other credit relationships.

Fact 1: Many borrowers default on only a subset of their cards.

To derive the first fact, we focus on the *borrowers nearing distress* group. These individuals were in good standing with all their cards in month $t-1$ and then miss a payment (overdue for more than 15 days) in month t. We investigate whether these individuals default on all or a subset of their loans six months later, defining a loan as in default if it is more than 90 days overdue. We use this measure of default to ensure that our

results are not driven by borrowers who forget to repay on time. Appendix Figure A5 shows the default incidence for alternative horizons. The results of the paper are not driven by the choice to measure default six months after the borrower first misses a payment, as the group of borrowers is relatively stable after 30 days.

In Figure 5, we show that among borrowers with two card lenders who do not make the minimum payment on their loan at time t, 55% recover 6 months later, around 34% become more than 90 days overdue on at least one card lender, and 11% default on both card lenders. The share of selective defaulters increases monotonically with the number of card lenders.

While the term selective default could apply to individuals who repay a subset of their credit card loans, they may default on the card for which they lack sufficient resources to repay, suggesting that the default decision is driven solely by liquidity constraints rather than a deliberate choice. In this case, selective defaulters would default on the cards that constitute the majority of their debt, implying that the distribution of the share of the total balance in default would be concentrated above the 50% threshold. However, as shown in Appendix Figure A8, the distribution does not follow this pattern. Specifically, there is a large group of selective defaulters who default on cards accounting for less than 50% of their balance, suggesting that they had sufficient resources to pay these cards but chose not to.

Motivated by this fact, we create a more conservative measure of selective default. We identify borrowers who default on the cards that account for less than 50% of their total credit card debt. In Table 3, we show that selective defaulters (21%) are comprised of two groups: those that default on less than 50% of their credit card balance (9% of borrowers), and those that default on more than 50% of their credit card balance (12% of borrowers). That is, *at least* 43% = 9/21 of selective defaulters made a deliberate repayment choice. The proportion of these borrowers also increases with the number of cards.

In Table 4, we show that selective defaulters typically hold more cards, have a greater number of bank relationships, and a higher debt-service-to-income ratio than universal defaulters and no-defaulters. While selective defaulters have higher incomes and credit card balances compared to universal defaulters, their income and balances are lower than those of non-defaulters. Regarding other loan types, selective defaulters tend to have more outstanding loans than universal defaulters.

In Appendix Figure A6, we show that all default measures are increasing over time, potentially due to the expansion in the number of credit cards. In Appendix Figure A7, we present the same figures for individuals with at least two card lenders. In this subsample, the incidence of selective default is significantly higher, as we exclude individuals with only one card, who can only default universally. The figures confirm that the incidence of selective default is increasing over time.

Fact 2: Lenders do not cancel the credit limit of borrowers who defaulted on a competitor's card.

We show that selective default is not driven by lenders preemptively canceling credit cards. In Figure 4, we show that selective defaulters retain both the credit limits and card usage (measured as purchases during the billing cycle) on the cards they continue to repay. In other words, as long as banks remain the prioritized lender, they continue to supply credit to selective defaulters. In contrast, lenders cancel the credit limit of borrowers who default on their cards.

In Section 5, we rationalize lenders' behavior in our model as follows. From a lender's perspective, observing a borrower default on another card while continuing to be repaid is not necessarily bad news. Such behavior reveals informal seniority relative to competing lenders. In addition, the loss of access to the defaulted card and the deterioration in the borrower's ability to obtain new credit increase the cost of default. Taken together, these mechanisms may reduce the likelihood of default on the repaid card.[17] This view is also expressed by market participants.[18]

Fact 3: The share of borrowers who default on at least one card increases with the number of card lenders, even after controlling for observable characteristics.

In Tables 2 and 3, we show that default rates (at the borrower level) increase monotonically with the number of card lenders. For instance, six months after missing a payment, the probability that *borrowers nearing distress* default on at least one card is 43% for borrowers with a single card lender, 45% for those with two card lenders, and 47% for those with three card lenders. This relationship may be driven by factors such as income, liquidity, and leverage. In Table 5, we try to control for these variables and additional covariates in the subsample of formal workers, for whom salary information is available. The positive association between borrower-level defaults and the number of card lenders persists even after controlling for wage, the debt-service-to-income ratio (a proxy for liquidity), the total-debt-to-wage ratio (a proxy for leverage), and other characteristics, including age, gender, and tenure at the last job.

[17]The lender that continues to be repaid may also gain market power as the borrower's number of banking relationships declines.

[18]The former CFO of Itaú, the largest private bank in Latin America, mentions in an interview with Euromoney that: "Everyone in Brazil has four credit cards, and if they don't have money to buy groceries, they use all these cards.[...] But they pay the one from the bank they have the relationship with. Banks don't care if you have a negative score if you keep paying them. So, being the main bank determines the cost of credit. People underestimate the huge relationship between being the main bank and default risk. If you are the principal relationship bank, they might default on the other banks but you will be the last one."

Fact 4: Borrowers prioritize repaying cards with better terms.

To identify the characteristics of bank–borrower relationships that predict repayment, we exploit borrower–cohort–bank data for selective defaulters, comparing repaid and defaulted bank relationships within the same borrower. We estimate the following regression:

$$y_{icb} = \beta X_{icb} + \alpha_{ic} + \gamma_b + \epsilon_{icb} \tag{1}$$

where y_{icb} is a binary variable that takes the value one if, six months after becoming overdue for more than 15 days on any card, individual i of cohort c defaults (i.e., is more than 90 days overdue) on the credit card issued by bank b. The vector X_{ibc} contains borrower-bank characteristics measured in the month when the borrower becomes overdue for more than 15 days: limit available, limit utilization, global limit, loan balances, dummies for other active loan types, dummies for non-performance of other loan types, and dummies for bank types (digital or state-owned). Borrower-cohort fixed effects, α_{ic}, absorb all borrower-level determinants of default risk within a delinquency episode. Bank fixed effects, γ_b, absorb persistent differences across issuers in underwriting, risk appetite, servicing, collection, and reporting practices.

We report descriptive statistics of the regression variables in Table 6. 23% of relationships are with digital banks, 11% with state-owned banks, and the remainder (66%) with other banks. 6% of relationships contain an active payroll loan, 1% an active mortgage, 2% an auto loan, 12% a personal (consumer) loan, and 17% other loan types. The coefficient on the interaction between the state-owned bank and mortgage dummies is also 1%, reflecting the fact that state-owned banks originate the majority of mortgages in Brazil.

We report regression results in Table 7. Our preferred specification uses data from May 2021 onward, when the credit registry introduced measures to improve the accuracy of reported credit limits. As shown in column (4), borrowers are 8 percentage points less likely to default on the credit card with the highest contractual credit limit, 21 percentage points more likely to default on the card with the largest outstanding balance, and 3 percentage points less likely to default on the card issued by the bank with which they have the longest relationship. Column (3) excludes bank fixed effects and shows that cards issued by digital banks are 15 percentage points more likely to be repaid, whereas cards issued by state-owned banks are 17 percentage points less likely to be repaid. This pattern suggests that borrowers may value their relationships with digital banks more than with other types of banks, or that digital banks have more effective debt collection infrastructure. Interest rate information is available only for the subsample of borrowers who revolve. Within this subsample, the results in column (8) show that borrowers are 27 percentage points more likely to default on the credit card with the highest interest rate.

More generally, borrowers are less likely to default on credit cards issued by banks with which they also hold other loan products, including salary-backed payroll loans (-13 percentage points), mortgages (-20 percentage points), auto loans (-6 percentage points), and other loans (-5 percentage points). The only exception is (unsecured) personal loans, where borrowers are 3 percentage points more likely to default on the bank's credit card. These findings suggest that cross-selling, particularly of collateralized loan products, strengthens the bank–borrower relationship and increases the bank's position in the repayment hierarchy, thereby increasing the profitability of its credit card business. Consistent with this interpretation, borrowers who have already defaulted on another loan with a given bank are substantially more likely to default on that bank's credit card than on a credit card issued by another bank.

The above results document correlations between relative contract terms and default probabilities. However, the coefficients should not be interpreted causally because of potential omitted-variable bias. Since the analysis focuses on borrowers' decisions about which bank to default on, the primary concern is unobserved characteristics that vary across credit card products offered by the same bank. For example, banks may bundle product features by offering higher credit limits on premium credit cards with more generous rewards programs. Because the data do not contain information on rewards programs, the estimated coefficient on credit limits would capture both the effect of credit limits and the omitted product feature rather than the causal effect of credit limits alone. In that case, although the coefficients would not reflect the credit limit's causal impact, they still indicate that the lender can leverage certain contract characteristics to gain informal seniority. Likewise, the correlation between holding other loan products and default may reflect selection if borrowers who qualify for a mortgage also receive higher-quality credit card products.

Overall, although the results should not be interpreted as identifying the causal effect of individual contract characteristics, they provide suggestive evidence that lenders use contract design to obtain informal seniority. The theoretical analysis then examines the implications of this behavior, including how the choice of contract characteristics affects welfare and the optimal design of bankruptcy policy.

Additional facts about the credit card limits and credit score. In Appendix Figure A9, we also show that defaulting on a credit card decreases the borrower's credit scores significantly. In an event study, we show that the probability of obtaining a new loan (including revolving card loans) also drops significantly after a default.

We also report additional facts about credit limits. The aggregate credit limit, summing all credit cards, is around 3 times the borrower's income. The average credit limit per card is 1.5 times the borrower's income, regardless of the number of cards the borrower owns. In addition, lenders do not reduce the credit limit on existing cards

after the borrower gets a new card and increase the credit limit over time, provided the borrower does not default. This suggests that lenders consider the possibility of the borrower getting additional cards when setting their initial limit.

3.3 Evidence from Earnings Calls and Interviews

Our empirical analysis, in particular fact 4 of Section 3.2, shows that default is not random. Borrowers choose which loan to default on, and this choice is correlated with contract terms.

To provide evidence that lenders internalise this effect, one would need a shock to the borrower's ability to selectively default. Cross-default provisions would have been a good candidate; however, the data contain no such variation.

For a first analysis of the supply side, we report that banks discuss competing for information seniority in their earnings calls or interviews.[19] For instance, the former CFO of Itaú, the largest private bank in Latin America, says in in an interview:

> 'Everyone in Brazil has four credit cards, and if they don't have money to buy groceries, they use all these cards.[...] But they pay the one from the bank they have the relationship with. Banks don't care if you have a negative score if you keep paying them. So, being the main bank determines the cost of credit. People underestimate the huge relationship between being the main bank and default risk. If you are the principal relationship bank, they might default on the other banks but you will be the last one."

Out of the ten largest banks in Brazil,[20] six mentioned a similar narrative in their earnings calls.[21]We report the quotes in Appendix B. We also looked at other Latin American lenders and found similar results.

4 Difference-in-Differences

In Table 5, we show a positive relationship between the number of card lenders and default probabilities at the borrower level, even after controlling for observable default determinants. However, this relationship may arise from both unobserved borrower heterogeneity—for example, higher-risk borrowers may be more likely to obtain additional cards (adverse selection)—and the causal effect of forming new card relation-

[19]Unfortunately, it is not possible to merge the earnings call data with the central bank data as the banks are anonymised.

[20]Itaú Unibanco, Banco do Brasil, Caixa, Bradesco, Santander Brasil, BNDES, BTG Pactual, XP, Nubank and Safra.

[21]Itaú Unibanco, Banco do Brasil, Caixa, Bradesco, Santander Brasil and Nubank.

ships (moral hazard or payment burden). Distinguishing between these channels is important because only the latter directly contributes to debt dilution.[22]

The ideal experiment would randomly assign borrowers additional credit cards, thereby eliminating selection and identifying the causal effect of additional credit card relationships. We approximate this experiment using quasi-experimental variation and find that the debt-dilution channel is quantitatively important: obtaining an additional credit card increases the probability of default on any credit card by 10 percentage points one year after origination.

To determine the causal impact of obtaining a new card, we employ a difference-in-differences approach. We achieve this by leveraging certain institutional features of the Brazilian market. In particular, banks commonly form partnerships with firms to obtain the rights to process employee salary payments. The arrangement effectively mandates that employees receive their wages through the partner bank, increasing the likelihood that employees will obtain a card from that bank. The length of those contracts is typically five years, making the timing of the renewal predetermined. We thus compare two groups of borrowers who worked at the same firm around the time the partner bank changed. The first group already had a relationship with the new partner bank before it won (control), while the second did not (treatment). The identifying assumption is that the relative change in default probability between the two groups is due to the impact of the treatment group getting a new card. Appendix Figure A10 summarizes the research design.

Using the credit registry of the Central Bank of Brazil, we construct a worker-level panel around the month in which a new bank is assigned to manage a firm's payroll.[23] We retain workers who remain employed at the same firm throughout the entire window, are active credit card users, and rely exclusively on this type of loan.[24] We examine events (i.e., changes in banks managing payroll payments) occurring within the twelve months between May 2017 and April 2018.[25] Appendix Table A5 reports the

[22]When the recovery rate conditional on default is zero, debt dilution operates solely through the effect of additional borrowing on default probabilities. When recoveries are positive, debt dilution can also arise through lower recoveries on defaulted loans, even if default probabilities remain unchanged.

[23]The Central Bank uses an algorithm to identify changes in the bank managing a firm's payroll by detecting structural breaks in the volume of wage-backed payroll loans originated by each bank to the firm's employees.

[24]Restricting the sample to borrowers who rely exclusively on credit cards is not ideal, as it conditions on post-treatment borrowing choices. Nonetheless, we make this restriction to reduce concerns that the treatment alters the demand for other loan types, particularly wage-backed payroll loans. Restricting the sample to workers who remain at the same firm throughout the observation window allows us to indirectly control for changes in wages (for instance, when examining the debt-to-wage ratio) and to account for firm-by-time fixed effects.

[25]We base our sample selection on two main considerations. First, in May 2019, a change occurred in the reporting standards for credit card loans, making April 2018 the last month for which we can observe a long enough post-event window (12 months) under a consistent reporting regime. Second, we focus on a period prior to the rise of digital banks as significant market players. Following their entry, the cost of obtaining a new credit card declined substantially, reducing the likelihood that our treatment would

characteristics of treated and control individuals in the month prior to the new bank assuming management of payroll. Control and treated individuals are similar in terms of the number of card lenders and debt-to-wage ratio: on average, control individuals have 1.5 card lenders and a debt-to-wage ratio of 1.5, compared to 1.2 card lenders and a debt-to-wage ratio of 1.3 among treated individuals. However, the groups differ slightly in baseline default rates: 5% of control individuals are delinquent on some card and 4% are in default on at least one card, compared to 2% and 2%, respectively, among treated individuals. Given that differences in baseline default rates may influence future borrowing trajectories, our regression specification allows for heterogeneous trends by baseline default status. The majority of individuals in our sample (80%) belong to the treatment group; only 20% already held a card from the bank that subsequently assumed management of the firm's payroll.

We estimate the following specification at the individual-month level:

$$y_{it} = \beta_1 \times Post_Treat_{it}^{[0-6]} + \beta_2 Post_Treat_{it}^{[7-15]} + \alpha_i + \alpha_{f(i)t} + \alpha_{d(i)t} + \epsilon_{it} \qquad (2)$$

where i indexes the individual, t the calendar month, and $f(i)$ the firm where individual i works. The dummy variables $Post_Treat_{it}^{[0-6]}$ and $Post_Treat_{it}^{[7-15]}$ equal one if individual i belongs to the treatment group and month t falls, respectively, within 0–6 months or 7–15 months after a new bank assumes management of firm $f(i)$'s payroll. We distinguish between these post-treatment periods because default is defined as failing to make the minimum payment for more than 90 days and therefore materializes with a delay. We control for time-invariant worker heterogeneity by including individual fixed effects, α_i, and we control for time-varying shocks to all workers of firm $f(i)$ by including firm-by-time fixed effects, $\alpha_{f(i)t}$. Because baseline differences in default status may shape subsequent borrowing trajectories, we interact time dummies with an indicator for whether borrower i had a default on any card in the month prior to the new bank assuming management of firm $f(i)$'s payroll, $\alpha_{d(i)t}$. We cluster standard errors at the firm level.

We also estimate the following event-study regression to capture the dynamics of the effects:

$$y_{it} = \sum_{\tau=1}^{\tau=6} \beta_{-\tau} \times Treat_{i,-\tau} + \sum_{\tau=0}^{\tau=15} \beta_{\tau} \times Treat_{i,\tau} + \alpha_i + \alpha_{f(i)t} + \alpha_{d(i)t} + \epsilon_{it} \qquad (3)$$

where the variable $Treat_{i,t-\tau}$ is a dummy variable that takes the value one if individual i belongs to the treatment group and the new bank starts managing firm $f(i)$'s payroll τ months after t, and the variable $Treat_{i,t+\tau}$ is a dummy variable that takes the value one if individual i belongs to the treatment group and the new bank starts managing

remain effective.

firm $f(i)$'s payroll τ months before t.

Table 8 reports the estimates from equation 2. The treatment increases the number of card lenders, the debt-to-wage ratio, and the probabilities of delinquency and default. On average, treated individuals acquire 0.11 additional card lenders, while their probability of default increases by 1.1 percentage points. Scaling the reduced-form estimate by the first-stage effect implies that acquiring an additional card lender increases the probability of default by approximately 10 percentage points (average treatment effect on treated). Under the assumption that treated and control borrowers would have followed parallel trends absent the treatment, these estimates can be interpreted as the causal effect of gaining access to an additional credit card lender.

Appendix Figure A11 plots the estimated coefficients from equation 3. Card take-up is gradual and peaks approximately 10 months after a new bank begins managing salary payments. The debt-to-wage ratio also increases gradually. Interestingly, debt utilization begins to rise before the number of cards increases, suggesting that the anticipation of obtaining a new card may affect the use of existing credit lines. Delinquency (more than 15 days past due) and default (more than 90 days past due) also increase, with these effects emerging roughly 12 months after the treatment.

Across all outcomes, treated and control borrowers exhibit broadly parallel trends in the pre-treatment period. Appendix Figure A11 nevertheless reveals a small pre-trend in the number of cards. As a robustness check, we therefore implement a synthetic difference-in-differences estimator (Arkhangelsky et al., 2021). The corresponding estimates, reported in Appendix Figure A12, are quantitatively similar to the baseline estimates, although they are estimated less precisely, as reflected in the wider confidence intervals.

We further examine heterogeneity by separately comparing the control group with compliers (individuals who obtain a new card following the treatment) and non-compliers (individuals who do not). The results, reported in Appendix Figure A13, show that the modest pre-trend is driven by non-compliers. This finding suggests that individuals who receive a card from the new payroll bank are comparable to borrowers who had already received a card from that bank prior to the payroll transfer. One interpretation is that card issuance is primarily driven by the bank's screening decisions rather than by time-varying borrower selection. Since banks' underwriting criteria tend to be stable over time, this mechanism is less likely to generate differential trends in unobserved borrower characteristics. The (scaled) estimated effects when comparing compliers to the control group are similar in magnitude to the baseline and the synthetic difference-in-differences estimates.

We now discuss the identifying assumption and the economic channels that could threaten its validity. The identifying assumption is that the relative change in default rate is due to the causal impact of getting a credit card rather than any other shock

that differentially affects the treated and control groups. One potential concern is that treated and control borrowers differ in unobservable characteristics, such as savings, financial sophistication, or underlying creditworthiness, which could generate differential trends in outcomes. Appendix Figures A11, A12, and A13 show that treated and control borrowers exhibit similar pre-treatment dynamics, making this explanation less likely. A related concern is that treated borrowers may have been on a different borrowing trajectory and would have increased borrowing and defaulted even in the absence of the payroll transition. Again, this concern would manifest itself through differential pre-treatment trends. The absence of such patterns provides evidence against this interpretation. Because treatment does not lead to universal take-up, one potential concern is that borrowers who obtain a new credit card differ systematically from those who do not. Under the identifying assumption that, absent the transition, treated and control borrowers would have followed parallel trends, the intent-to-treat estimate remains causal even when take-up is incomplete.

Finally, our estimates should be interpreted as a local average treatment effect for borrowers whose access to credit changes as a consequence of the payroll-bank transition. As in any quasi-experimental design, external validity depends on the extent to which these marginal borrowers are representative of the broader population. Our estimates are consistent with those reported by De Giorgi et al. (2023), who study a different setting and obtain similar effects of increased credit access on household borrowing and default.

5 Model

Section 3.2 provides suggestive evidence that lenders can use credit card terms (e.g., credit limit) and cross-sell other products to gain informal seniority. We then report, using earnings calls, that lenders are aware of this and compete on repayment prioritisation (see Section 3.3).

To study the impact of competing for informal seniority on contract terms and welfare, we develop a model with non-exclusive loan contracts and endogenous default choices. We first develop a parsimonious two-period model to clarify the underlying economic mechanisms, and then generalise it in our quantification exercise. We discuss the impact of our modelling assumptions in Section 6.

The model features two periods with two decision dates. On the initial date, lenders simultaneously offer a loan contract. Borrowers have utilities over contract terms and choose which loans to accept. The key friction is that they cannot commit to exclusive contracts as in, for instance, Green and Liu (2021). At the second date, borrowers decide which loans to repay, subject to a resource constraint.

We denote by $\mathcal{M}_i$ the set of loans offered in the market to borrower i. $\mathcal{C}_i \subset \mathcal{M}_i$ is the

set of loans held by a borrower i, with individual loans indexed by $c \in \mathcal{C}_i$. Each contract c specifies a loan amount L_c, which is used for consumption during the first period, and a face value R_c due at the beginning of the second period. Conditional on repaying the debt, the borrower obtains the continuation value $V(q_c)$, where q_c denotes the vector of contract characteristics affecting future utility. We refer to q_c as relationship benefits. These benefits can include, for instance, credit card rewards, investment in customer capital, financial tracking via a mobile app, a bank's branch network, or other loan terms offered in the second period.

Lenders set all contract terms (L_c, R_c, q_c) on the initial date. q_c is contingent on the borrower's repayment of the loan. In practice, lenders often use affine pricing, using fees and rates, with a credit limit. Allowing for such a contracting space yields the same equations as in our baseline model (see, for instance, Tirole et al., 1988, pp. 135–136 and §3.3.1). Thus, letting lenders set contract terms is reasonable in our setup. We discuss alternative designs in Section 6.

The key feature of the model is that, by improving contract terms, a lender can change their position in the borrower's repayment hierarchy. Lenders can thus impact the likelihood that their loan is repaid in states where the borrower does not fully honour all obligations.

5.1 Borrowers and Lenders

This section describes borrowers' loan choices and how lenders set contract terms.

5.1.1 Borrowers

There is an arbitrary number of borrowers. Following the public economic literature (see, for instance, Chetty 2009), we employ a parsimonious reduced- form approach when modelling the borrower decision problem. We define the borrower utility as a function of contract terms:

$$U(\mathcal{C}) = \underbrace{\alpha \sum_{c \in \mathcal{C}} L_c - C(\mathcal{C})}_{\text{First period}} + \underbrace{\beta \mathbb{E}_{\mathcal{R}}[\max_{r \in \mathcal{B}(\mathcal{C},\mathcal{R})} \sum_{j \in r} \{\mathbf{1}_{j \in r}[V(q_j) + v_j - \delta R_j] + \mathbf{1}_{j \notin r} \omega_j(\mathcal{C}, \mathcal{R})\}}_{\text{Second period}} \tag{4}$$

$\alpha \sum_{c \in \mathcal{C}} L_c$ is the utility from obtaining loans from the cards in set $\mathcal{C}$, and $C(\mathcal{C})$ is the cost of acquiring the bundle, both incurred in the first period. $\mathbb{E}_{\mathcal{R}}[\max_{r \in \mathcal{B}(\mathcal{C},\mathcal{R})} \sum_{j \in r} \{\mathbf{1}_{j \in r}[V(q_j) + v_j - \delta R_j] + \mathbf{1}_{j \notin r} \omega_j\}$ is the expected utility in the second period. $V(q_c) + v_c - \delta R_c$ is the utility of repaying card c and $\omega_c(\mathcal{C}, \mathcal{R})$ is the cost of defaulting on that card. We allow it to depend on the set of loans $(\mathcal{C})$ and a repayment shock $(\mathcal{R})$. $\mathcal{B}(\mathcal{C}, \mathcal{R})$ is the feasible set of bundles of loans the

borrower can repay conditional on repayment shock $\mathcal{R}$. We normalise $\beta\delta$ to 1. The key assumptions are that default and card choice derive from the same maximisation problem, and that the default, expected default, and demand functions are continuous.

Example default: To illustrate the implication of selective default. Assume that, with probability λ, the borrower can repay at most one card, denoted by $\mathcal{R} = 1$.[26] They can repay all loans absent this shock. To make the expected default function continuous in contract terms, let us assume that the continuation value term v_c is zero in normal times, but is a random variable with an extreme-value distribution with variance σ^{-1} conditional on being in financial trouble. We discuss here the impact of this modelling on default and, in the next paragraph, its impact on borrower utilities and demand. With this modelling, the survival probability on loan c is:

$$\theta_c(\mathcal{C}) = \underbrace{(1-\lambda)}_{survive\ on\ all\ loans} + \underbrace{\lambda \frac{e^{\sigma[V(q_c)-\delta R_c]}}{\sum_{x\in\mathcal{C}} e^{\sigma[V(q_x)-\delta R_x]}}}_{survive\ on\ loans\ c\ only} \tag{6}$$

Equation 6 implies that, if all lenders offer the same contract terms, the default probability is $(1-\lambda)+\lambda\frac{1}{|\mathcal{C}|}$, and the default semi-elasticity with respect to the interest rate and the one with respect to relationship benefits are: $\frac{\partial_R \theta_c(\mathcal{C})}{\theta_c(\mathcal{C})} = -\sigma\delta(1-\frac{1}{|\mathcal{C}|})\cdot\frac{\lambda\frac{1}{|\mathcal{C}|}}{1-\lambda+\lambda\frac{1}{|\mathcal{C}|}} < 0$ and $\frac{\partial_R \theta_c(\mathcal{C})}{\theta_c(\mathcal{C})} = V'(q)\sigma(1-\frac{1}{|\mathcal{C}|})\frac{\lambda\frac{1}{|\mathcal{C}|}}{1-\lambda+\lambda\frac{1}{|\mathcal{C}|}} \geq 0$ as $V'(q) \geq 0$. With this specification, σ parametrises how strategic the borrowers are. When $\sigma \to 0$, borrowers default with probability $\frac{1}{|\mathcal{C}|}$ no matter the contract terms. When $\sigma \to \infty$, borrowers are extremely strategic and repay the lender as soon as they offer a slightly better contract than their competitors.

This model of default thus focuses on which cards borrowers choose to repay, rather than why the borrower chose to default in the first place. Having the shock on the fraction of the face value the borrower can repay instead of on the number of loans does not change the key economic forces — equation 5.1 remains the same, and borrowers still get too many loans due to the default externality (see, for instance, Green and Liu, 2021). Additionally, empirically, the causal effect of interest rates on default (i.e., on λ) is low in the credit card market (see, for instance, Matcham, 2024), so our model is not a bad approximation of reality in that set-up.

Example demand: Similarly, to illustrate the different economic channels driving

[26] To fix ideas, let us assume that the cost of defaulting has the form:

$$\omega^d(\mathcal{R}) = \begin{cases} \omega, & \text{if } |r| > \mathcal{R}, \\ -\infty, & \text{if } |r| \leq \mathcal{R}. \end{cases} \tag{5}$$

demand for multiple loans, suppose the borrower is offered two similar contracts ($L_1 = L_2$, $q_1 = q_2$, $R_1 = R_2$). As in the previous example, the borrower may experience a bad shock allowing them to repay only one card. By accepting two cards instead of one, the borrower receives a higher loan size in the first period but must default on at least one of them in the second period. Formally, the borrower chooses two cards instead of one when:

$$\begin{aligned} & U(\{1,2\}) \geq U(\{1\}) \\ \iff & \underbrace{\alpha L}_{\text{Larger loan size}} + \underbrace{(1-\lambda)\beta[V(q) - \delta R]}_{\text{Larger continuation value conditional on survival}} + \underbrace{\lambda\beta\Delta IV}_{\text{Love for variety}} \\ \geq & \underbrace{\Delta C(2)}_{\text{Additional search cost}} + \underbrace{\beta\lambda\omega}_{\text{Additional default cost}} \end{aligned} \tag{7}$$

Equation 7 illustrates the different costs and benefits of getting an additional card. αL is the utility increase from getting additional credit in the first period. The term $(1-\lambda)V(q) - \delta R$ is the additional utility in the second period conditional on survival. The last term $\Delta IV = \mathbb{E}_v[\sum_{c \in \{1,2\}} \max_c v_c] - \mathbb{E}_v[v_1]$ is the inclusive value. It arises from the random coefficient in the default function. We call this the love-of-variety effect: more cards increase the probability of a higher continuation value because the borrower is more likely to draw a higher v. In practice, one can cancel its effect by normalising the mean of the random shocks (see Appendix F). $\Delta C(2) := C(2) - C(1)$ is the cost of getting an additional card. Finally, $\beta\lambda\omega$ is the utility loss from defaulting on one of the credit cards.

5.1.2 Lenders

There are an arbitrary number of lenders. Each lender b takes their competitors' offers as given and designs a borrower-specific contract. We drop the borrower index for notational simplicity. Each lender chooses the contract terms (L, R, q) to maximise their expected profits. The profit is the probability that the borrower chooses a loan from their bank times the expected profit on that contract.

Lender b's expected profits, conditional on the set of loans $\mathcal{C}$ being chosen, are $NPV_b(\mathcal{C}) := \mathbf{1}_{\{b \in \mathcal{C}\}}[\theta(q, R; \mathcal{C}_{-b})R - \psi(L, q)]$, where $\mathcal{C}_{-b} := \mathcal{C} \setminus \{b\}$ denotes the subset of $\mathcal{C}$ that excludes loan b, and $\theta(q, R, \mathcal{C}_{-b})$ is the repayment probability of loan b. We often use the notation θ_b instead of $\theta(q, R; \mathcal{C}_{-b})$ for notational convenience. The term $\theta(q, R; \mathcal{C}_{-b})R_b$ is the expected amount the lender receives in the second period, while $\psi(L, q)$ is the total cost of issuing a loan.

We consider general demand D and default $1-\theta$ functions consistent with borrowers' utility $U(\mathcal{C})$ defined in Section 5.1.1. We define $\mathbf{M}^b := \{M \subset \mathcal{M} : b \in M\}$ as the collection of subsets of $\mathcal{M}$ that contain card b. Lender b chooses contract terms (R, L, q)

to solve:

$$\max_{R,L,q} \sum_{\mathcal{C}\in\mathbf{M}^b} D(\mathcal{C})\Big[\theta(q,R;\mathcal{C}_{-b})R - \psi(L,q)\Big] \tag{8}$$

As in standard non-linear pricing models (see, for instance, Rochet and Stole 2002), lenders chose contract terms (L,q) to maximize the surplus generated by the contract, and use the price R to split the surplus between borrowers and lenders.[27]

For intuition, let us focus here on the situation in which $U(\mathcal{C})$ is deterministic. In that case, $\mathbf{M}^b$ is a singleton as, given the set of cards offered by their competitors, lenders can infer the number of cards chosen by the borrower (Appendix K reports the general case,[28] and shows the economic mechanisms are the same). Denoting the marginal costs by $mc_L = \frac{\partial\psi}{\partial L}$ and $mc_q = \frac{\partial\psi}{\partial q}$, the first-order conditions imply:

$$R:\ \theta R = \psi(L,q) + \overbrace{\Big[1+\frac{\partial_R\theta_b}{\theta_b}R\Big]}^{\mu}\frac{D(\mathcal{C})}{\partial_U D(\mathcal{C})} \tag{9}$$

$$L:\ \alpha = \frac{mc_L}{1+\frac{\partial_R\theta_b}{\theta_b}R} \tag{10}$$

$$q:\ \frac{mc_q - \partial_q\theta R}{\beta\theta_b V'(q)} = \frac{mc_L}{\partial_L u(\mathbf{L})} \iff \beta V'(q)\Big\{(\partial_V\theta_b + \partial_R\theta_b)R + \theta_b\Big\} = mc_q \tag{11}$$

Proof: See Appendix C.

Equation 9 states that the bank sets the expected face value θR as a markup-up ($[1+\frac{\partial_R\theta}{\theta}]\frac{D(\mathcal{C})}{\partial_U D(\mathcal{C})}$) over the cost of banking ($\psi(L,q)$). The markup is the inverse demand semi-elasticity $\frac{D(\mathcal{C})}{D_U(\mathcal{C})}$ scaled down by $\frac{\partial_R\theta}{\theta}$. The inverse semi-elasticity captures the standard intensive versus intensive margins: an increasing rate lowers the number of customers but increases the revenue per customer (holding default constant). The presence of the default sensitivity $\frac{\partial_R\theta}{\theta}$ is due to the fact that lenders internalise that higher interest rates increase the borrower's default probability, which provides incentives to lower the markup as in Crawford et al. (2018).

Equation 10 captures the cost and benefits of increasing the credit limit in the first period. α is the marginal benefit for the borrower. mc_L is the lender's marginal cost. $1+\frac{\partial_R\theta}{\theta}R$ captures that an increase in loan size is compensated by an increase in the debt face value, which in turn elevates default probabilities and thereby increases the cost of issuing credit. Section 6.1 discusses how loan size changes when lenders offer

[27] There can be jumps in the sensitivity to interest rates at points where increasing the interest rate changes the set of credit cards one can repay; we report here the derivatives outside of those points.

[28] The model presented in the Appendix uses an alternative default function for tractability; this specification does not change the lender first-order conditions.

a linear pricing schedule, and borrowers can choose how much to borrow without a credit limit.

Equation 11 states that the marginal rate of transformation between loan size and relationship benefits should be the same. To understand the economic forces driving relationship benefits, set the default sensitivities $(\partial_V \theta_b + \partial_R \theta_b)$ to zero. In that case, equation 11 states that the marginal benefits of increasing the relationship benefits for the borrower $\beta\theta V'(q)$ should be equal to the marginal cost for the lender (i.e., mc_q). $(\partial_V \theta_b + \partial_R \theta_b)R$ captures an additional effect coming from default. It captures the idea that, taking competitors' behaviour as given, increasing q stirs customers to repay bank b (when $\partial_V \theta_b > 0$), which increases the expected revenue by $\partial_V \theta_b R$. This effect creates incentives to increase the level or relationship benefits. However, the increase in q also raises lending costs and thus the debt face value R. Taking competitors' behaviour as given, it increases default probabilities. $\partial_R \theta_b$ captures this effect. Thus, when $\partial_V \theta_b + \partial_R \theta_b > 0$, the first effect dominates and leads to higher incentives to provide relationship benefits. To the best of our knowledge, the existing literature has not analysed the two forces discussed in this paragraph.

5.2 Welfare: Contract Distortions

We now examine how competition for informal seniority affects welfare. We compare the borrowers' and lenders' behaviour to that of the social planner. The social planner jointly determines the number of loans and contract terms to maximise utilitarian welfare. For simplicity of the exposition, we consider that all lenders offer the same contract. In the next section, we show that a symmetric equilibrium exists and is unique under our preferred parametrisation.

The social planner internalises selective default. To highlight the effect of selective default, we assume borrowers face exogenous repayment shocks with probability λ as in equation 6 from the example in section 5.1. Relaxing this assumption is feasible but not essential to the channel we seek to highlight.[29]

Welfare Definition and Contract Distortions: We define utilitarian welfare as the sum of borrowers' expected utility and lenders' profits. When all lenders offer the same contract (L^*, R^*, q^*), the borrower's expected utility, when they choose N^* cards is:

$$u^*(N^*, L^*, R^*, q^*) := \alpha N^* L^* + \beta N^* \theta^* [V(q^*) - \delta R^*] - C(N^*) \tag{12}$$

[29] As in 5, we normalise the cost of defaulting, ω, to zero for $N-1$ cards when the repayment shock equals one and normalize the inclusive value (i.e., $max_c E[v_r]$). This is withtout loss of generality as we can simply include them in the definition of $C(N)$.

Considering that borrowers and lenders value one dollar equally, the utilitarian welfare function, defined as the sum of borrowers' utility and lenders' profits, is:

$$W^*(N^*, L^*, q^*) := N^*[\alpha L^* + \beta\theta^* V(q^*) - \psi(L^*, q^*)] - C(N^*) \tag{13}$$

Where $[\alpha L^* + \beta\theta^* V(q^*) - \psi(L^*, q^*)]$ is the social surplus generated by each loan.

Contract distortions: Let us now use the welfare function to characterise the contract distortions. We compare the social planner's choices with those of borrowers and lenders. Formally, the social planner chooses the number of cards and contract terms (N, L, R, q) to maximise welfare W, internalising selective default.

Result 1. Contract Distortions

(i) Default externality: *For given contract terms (L, R, q), borrowers choose too many contracts relative to the social planner when the interest rate markups is high. As a result, the repayment probability is too low when competition is high (i.e., μ close to zero) or when default λ is high. Formally:*

$$\partial_N u^*(N, L, R, q) - \partial_N W^*(N, L, R, q) = \psi(L, q) - (1 - \lambda)R \tag{14}$$

$$= \underbrace{(1 - \frac{1-\lambda}{\theta^*})\psi(L, q)}_{\text{Default externality}} - \underbrace{\mu\frac{1-\lambda}{\theta^*}}_{\text{Imperfect competition}} \tag{15}$$

(ii) Selective-default distortion: *For a given number of contracts N, the lenders incentives are distorted relative to the first best. Lending is underprovided, while relationship benefits may be either over- or underprovided. Comparing the first-order conditions of the lenders and the social planner, we get:*

Social planner:	**Lender:**	
	$\overbrace{\quad\quad\quad}^{\text{overprovision when } > 0}$	
$mc_q = \beta V'(q)\theta;$	$mc_q = \beta V'(q)\Big\{\overbrace{(\partial_V\theta_b + \partial_R\theta_b)R}\ +\theta_b\Big\}$	(16)
$mc_L = \alpha;$	$mc_L = \alpha\underbrace{(1 + \frac{\partial_R\theta_b}{\theta_b}R)}_{\text{underprovision}}$	(17)

Proof. Take the first-order condition of the welfare and utility functions. The first-order condition of the borrower with respect to N does not feature repayment in the bad state because the borrower knows they will not repay the additional card (or one of their old ones). □

The term $1 - \frac{1-\lambda}{\theta^*}\psi(L, q)$ in Equation 15 echoes the mechanism in Bizer and DeMarzo

(1992) and Green and Liu (2021). The inefficiency arises from the interaction of a default externality and borrowers' inability to commit to exclusive contracts: obtaining an additional loan reduces the repayment probability of loans held by other lenders, since $\theta \propto 1/N$. Beyond the selective-default channel discussed below, an important departure from the existing debt-dilution literature is that the number of cards is endogenous in our model, rather than being either equal to one or fixed at an exogenous upper bound. As shown in Section 5.3, endogenising contract demand is important for evaluating policy effects on welfare.

Another important difference from Bizer and DeMarzo (1992) and Green and Liu (2021) is the presence of imperfect competition in equation 15 captured by the markup μ. Imperfect competition reduces borrowers' utility from getting an additional loan because they must pay a higher markup. As a result, imperfect competition lowers the default externality. Equation 15 thus highlights the importance of imperfect competition when bringing the model to the data.

The second result is novel and arises from selective default. To isolate this mechanism, consider first the case without selective default, $(\partial_V \theta_b + \partial_R \theta_b) = 0$. For a given number of contracts N, the equilibrium loan size L and relationship benefits q coincides with its first-best level.[30]

Now let us assume that default sensitivity is different than zero. Let us first consider the case in which $(\partial_V \theta_b + \partial_R \theta_b)$ is larger than zero. Economicaly, it means that borrowers are more sensitive to increase in relationship benefits than in crease in debt face value. In that case, lenders have incentives to overprovide relationship benefits. The intuition is that each lender can increase q in order to raise the likelihood of being repaid in states of partial default. Because all lenders engage in this behaviour, competition for repayment priority generates an arms race in q, leading to overprovision. This channel is stronger when default sensitivities $(\partial_V \theta_b + \partial_R \theta_b)$ are high or when profits conditional on repayment (R) are large.

When $(\partial_V \theta_b + \partial_R \theta_b)$ is lower than zero. Borrowers are more sensitive to rate rather than relationship benefits. In that case, there is an incentive to underprovide relationships. The intuition is that it is more effective to cut relationship benefits so that lenders can lower the debt face value. In that case, we have a race to the bottom.

Sufficient Statistic: The previous results can be used to construct a sufficient statistic allowing us to assess the impact of any policy on welfare. The sufficient statistic depends on the elasticities of contract terms with respect to the policy and can

[30] However, we know from equation 15 that the equilibrium number of contracts, N^*, exceeds the social planner's choice, N^{FB}, when the markup μ is low. As a result, the equilibrium survival probability θ^* is too low. The borrower's marginal value of relationship benefits $\theta V(q)$ is thus lower in equilibrium. Therefore lenders have weaker incentives to provide relationship benefits because borrowers are less likely to use them.

be derived without imposing functional-form assumptions on demand or costs. In the quantification exercise of Section 7, we generalise this sufficient statistic to a dynamic model without homogeneous lenders.

Result 2. Sufficient Statistic *Given a set of equilibrium contracts observed in the data* $(R^*, L^*, N^*, \theta^*)$*, and estimates of the causal impact of a policy on contract terms and number of products* $(\frac{dN}{dp}, \frac{dL}{dp}, \frac{dq}{dp})$*, the welfare impact of the policy is given by:*

$$\frac{dW}{dp} = \frac{dN}{dp} w_{N,*} + \frac{dL}{dp} w_{L,*} - \frac{dq}{dp} w_{q,*} \tag{18}$$

$w_{N,*} := (1-\lambda)R^* - \psi(L^*, q^*) \lesseqgtr 0$, $w_{L,*} := N^*[\alpha \varepsilon_d^* \delta R^*] > 0$, $w_{q,*} := N^*[\beta V'(q^*)[\varepsilon_d^*(1-\delta_d)\theta R^*] > 0$ *when* $(1-\delta) > 0$*, with* $\varepsilon_d^* := \partial_V \theta_b$*.*

Proof. Take the derivative of the welfare function and use the borrower and lender first-order conditions to replace the relevant equartions. For instance, taking the derivative with respect to q, we get: $N\beta V'\theta - mc_q = N\beta V'[\partial_V \theta - \partial_R \theta]$. □

The weights are calculated using the envelope theorem as in Chetty (2009). They can be estimated using data on lending costs at equilibrium. On can use bank accounting data to find the average cost spend on cards at equilibrium $\psi(L^*, q^*)$, and credit register date to find default sensitivities $(\varepsilon_d^*, \delta_d)$, as well as willingness to pay for contract terms $(\alpha, V'(q))$.

The weights have an intuitive economic interpretation: they reflect the distortions in the lender and borrower behaviour. Formally, they are the difference between the lender's first-order conditions and the social planner's in Result 1.

The sign of $w_{N,*}$ is ambiguous. It is negative when too many cards are offered in equilibrium. This happens when survival probabilities and competition are low (i.e., λ close to one and μ close to zero). In that case, the default externality is large because borrowers expect not to repay the card often, so borrowers get too many cards. Thus, when $w_{N,0}$ is negative, a small increase in the number of cards $\frac{dN}{dp}$ decreases welfare.

$w_{L,*}$ is positive and reflects the fact that loan size is underprovided conditional on the number of cards. As a result, an increase in loan size $\frac{dL}{dp} > 0$ increases welfare.

$-w_{q,*}$ is negative when $1 - \delta > 0$. In that case, borrowers care more about relationship benefits than debt face value so relationship benefits are overprovided due to the arm race. As a result, an increase in relationship benefits $\frac{dq}{dp} > 0$ decreases welfare.

5.3 Welfare: Equilibrium and Policy Analysis

In this section, we analyze theoretically how different policies affect market inefficiencies in the presence of selective default. We consider policies imposing equal recov-

ery rates and cross-default provisions, as well as changes in competition or the cost of lending. The goal of this section is not to provide a quantitative assessment of these channels, but rather to show that informal seniority introduces new mechanisms that can overturn conventional predictions about how these policies affect equilibrium outcomes and welfare.

We show that imposing cross-default provisions need not be optimal. We also show that reducing competition or increasing lending costs can increase welfare.

Formally, we use the sufficient statistic decomposition and use our model to understand the mechanisms driving the sign of the elasticities. To do so, we need to impose additional structure on the model.

Parametrisation : We consider a continuum of lenders. This approach lets us use the first-order condition in the card demand equation to derive a tractable expression for the equilibrium number of cards and conduct comparative statics. Under this assumption (the technical details are in Appendix F), the equilibrium conditions are unchanged relative to Section 5.1, except that (N) now denotes the mass of cards held by the borrower.

We assume that borrowers' utility is linear in relationship benefits, $(V(q) = \gamma q)$. The cost of lending and providing benefits is additively separable: $\psi(L, q) = mc_L(L) + \kappa(q) = \frac{mc}{\psi}[exp(\psi L) - 1] + \frac{\kappa}{\phi}[exp(\phi q) - 1]$.

We allow for demand and default shocks. They are independent and extreme-value distributed, with (σ_d) and (σ) parameterising the inverse variance of the default shocks and the variance of the demand shocks, respectively. We normalise the random shocks so the love-of-variety effect disappears (i.e., $E[\max_{\mathcal{C}\in\mathcal{B}(N)} e_{\mathcal{C}}] = 0$, see Appendix F for the derivation). We use the same parametrisation of the cost of defaulting as in Section 5, equation 6. We denote the default semi-elasticity and the inverse demand semi-elasticity by $\varepsilon_d := -\frac{\partial_R \theta}{\theta} = \sigma_d \delta(1 - \frac{1}{N}) \cdot \frac{\lambda \frac{1}{N}}{1-\lambda+\lambda\frac{1}{N}} > 0$ and $\varepsilon^{-1} := \frac{D(\mathcal{C})}{\partial_U D(\mathcal{C})} = \frac{\sigma}{1-\frac{N}{B}}$ where B is the mass of lenders (see Appendix F for the derivation). Following the trade literature on firm entry (see, for instance, Melitz, 2003), we assume borrowers first choose the number of cards they will search for $(|\mathcal{C}|)$, then learn about the demand shocks and choose $|\mathcal{C}|$ cards among all cards offered. This assumption allows a continuous demand function and implies that lenders know how many cards a borrower will get. This can be seen as an approximation of the long-run equilibrium, in which lenders learn about the borrower's number of cards. As shown in Appendix K, making the number of cards uncertain does not change the key economic forces at play.

To focus on selective default, we assume that loan demand is sufficiently high that borrowers hold more loans than they can repay following an adverse repayment shock. Formally, we consider that the costs of searching for loans $C(N)$ are low enough so that $\{N : C'(N) = 0\}$ is larger than one.

Lemma 1. Existence and uniqueness of a symmetric equilibrium
The equilibrium is characterised by:

$$Survival\ probability:\ \theta_c^* = \overbrace{1-\lambda}^{repay\ all\ cards} + \overbrace{\lambda\frac{1}{N^*}}^{repay\ one\ card} \tag{19}$$

$$Markup:\ \theta_c^* R_c^* = \psi(L^*, q^*) + \left[1 - \varepsilon_d \delta R^*\right]\varepsilon^{-1} \tag{20}$$

$$Loan\ size: \alpha = \frac{mc'(L_c^*)}{(1-\varepsilon_d \delta R_c^*)} \tag{21}$$

$$Relationship\ benefits: q_c^* := \left\{q : \beta V'(q)\left[\varepsilon_d(1-\delta)\theta_c^* R_c^* + \theta_c^*\right] = \kappa'(q^*)\right\} \tag{22}$$

$$Number\ of\ cards: C'(N^*) = \alpha L^* + (1-\lambda)\beta[V(q^*) - \delta R^*] \tag{23}$$

When the cost C are quadratic (i.e., $C(N) = \frac{\xi}{2}(N-1)^2 + \eta(N-1) + C_0$), the equilibrium exists and is unique when the cost function is concave enough (i.e., $\xi > \sup_N H'(N)$ and $\eta < H(1)$).

Proof. See Appendix M. □

Equation 23 is the continuum-of-contracts version of equation 7. We normalise the cost of default and the love of variety to zero, as these are not channels we focus on in this section. This is without loss of generality, since we can include them in the cost function C.

Comparing this system of equations with that in Section 5 clarifies the role of our parametrization. Importantly, the parametrization does not shut down any of the economic channels of the general model. Instead, our functional-form assumptions deliver closed-form expressions for the key objects governing equilibrium contract terms: the demand semi-elasticity and hence the markup ($(1 + \frac{\partial_R \theta_b}{\theta_b} R)\frac{D(\mathcal{C})}{\partial_U D(\mathcal{C})} = (1 - \varepsilon_d \delta R)\varepsilon^{-1}$), the ratio of the marginal utility of lending to its marginal cost ($\frac{mc_L}{\partial_L u(L)} = \frac{\alpha}{mc} = 1 - \varepsilon_d \delta R$), and the default semi-elasticity ($\partial_{V(q)} \theta_c^* = \varepsilon_d$). Each of these objects plays a central role in the provision of relationship benefits. The parametrization therefore preserves the key economic forces of the general model while making them transparent, allowing us to isolate and characterize the effect of selective default, summarized by (ε_d), on equilibrium contract terms. Notice that the model is highly tractable as, given a number of cards N, the equilibrium contract terms can be solved in closed form.

Bankruptcy policies: We now analyze the effects of imposing, or removing, cross-default provisions across loans. Cross-default clauses are common in corporate lending but are typically absent from consumer credit. This raises the natural question of whether, and under what conditions, such provisions are socially desirable.

To the best of our knowledge, the literature has not studied when cross-default provisions are optimal. One reason may be that much of the existing literature on non-exclusive contracting assumes universal default. Under universal default, borrowers cannot selectively repay some lenders while defaulting on others, eliminating precisely the margin through which cross-default and equal-recovery provisions operate in our model.

Instead, the existing debt-dilution literature has focused on priority rules based on origination date and has shown that granting seniority to earlier lenders can be optimal. In our setting, a rule under which only the first N^{FB} lenders are repaid in full would restore the first-best allocation.[31] In practice, origination-based priority is absent for unsecured loans, credit cards, and sovereign debt, suggesting that such rules may be costly or difficult to implement. Moreover, origination-based priority may be particularly ineffective in credit-card markets, where borrowers often hold several cards simultaneously and borrow on multiple cards over overlapping periods. Our framework therefore complements the existing literature by studying alternative bankruptcy policies.

In our model, cross-default provisions eliminate the selective-default margin, we can thus analyze it by setting $\varepsilon_d = 0$ (i.e., setting $\sigma_d = 0$). We assume equal recovery, as is often the case in practice. Formally, when $\varepsilon_d = 0$, we set the lender's expected recovery rate to $1/N$, as in the selective-default case. Now we study how welfare varies with ε_d (that is, with σ_d). We denote the variable X with an underscore X_0 when they are evaluated at $\varepsilon_d = 0$.

Result 3. Cross-default and Equal recovery policies (changing ε_d).
Moving away from cross default policies increases welfare when:

$$\partial_\sigma N \cdot w_{N,0} > 0 \tag{24}$$

$w_{N,0} := \frac{1-\lambda}{\theta_0}\varepsilon_0^{-1} - (1 - \frac{1-\lambda}{\theta_0})\psi(L_0, q_0)$ *captures whether there are too many cards offered in equilibrium. $\partial_\sigma N$ captures the effect of increasing strategic default on the number of cards.*

The comparative static on $\partial_\sigma N$ reflects three effects:

$$\partial_\sigma N > 0 \iff \underbrace{-\frac{\alpha\delta}{\psi}}_{\text{lower credit limit}} + (1-\lambda)\beta\Big[\underbrace{\frac{\gamma(1-\delta)}{\phi}}_{\text{higher benefits}} + \underbrace{\frac{\delta K_0}{\theta_0}}_{\text{lower repayment}}\Big] > 0 \tag{25}$$

$$\text{With: } K_0 := \underbrace{\frac{\alpha\delta}{\psi}}_{\text{lower loan size}} - \underbrace{\frac{\beta\gamma\theta_0(1-\delta)}{\phi}}_{\text{higher benefits}} + \underbrace{\delta\varepsilon^{-1}}_{\text{lower markup}} \tag{26}$$

[31] Under such a rule, L^* coincides with its first-best level, $N^* = N^{FB}$, and lenders have no incentive to originate additional cards beyond the N^*th lender.

Proof. When $d = 0$, using the enveloppe theorem, we get that the derivative of the welfare function with respect to σ at $d = 0$ is: $\partial_\sigma N[[1-\lambda]\mu - \lambda[mc(L) + \kappa(q)]]$. The rest of the proof to sign $\partial_\sigma N$ is in Appendix N. □

Equation 24 is the sufficient statistic derived in equation 29 when $\sigma_d = 0$. It can be derived without assumptions about the demand and the costs. It tells us when small deviations away from cross-default is optimal. The weights on the loan size and relationship benefits (w_L and w_q) are zero in that case. The intuition comes from Result 1 (ii). When demand semi-elasticity is zero, the social planner and the lender offer the same contracts conditional on the number of cards N.

Equation 25 clarifies the economic forces behind the demand sensitivity $\frac{dN}{d\sigma}$. When we remove equal recovery, borrowers become strategic in their default decisions. As a result, each card's credit limit decreases, as lenders worry about losing priority (captured by $\frac{\alpha\delta}{\psi}$). Lenders also start competing for seniority using relationship benefits and interest rates. When relationship benefits are relatively more important to gain informal seniority (i.e., $(1-\delta) > 0$), lenders prefer to increase relationship benefits at the cost of a higher debt face value. Relationhip benefits thus increase (captured by $\frac{\gamma(1-\delta)}{\phi}$). The effect on competition for informal seniority on the debt face value is ambiguous. The debt face value increases due to the higher relationship benefits, but it also decrease due to the lower loan size and the incentives to lower the markup on their loans (captured by $\beta\frac{\delta K_0}{\theta_0}$).[32]

The sufficient statistic approach helps clarify the economic mechanism at play. However, it becomes less accurate when σ_d deviates substantially from zero. Figures 1 and 2 report the welfare as a function of σ_d in our simulated model (see Appendix R for the calibration). According to our model, the cross-default provisions (i.e., setting σ_d to zero) are optimal when borrowers are extremely strategic. This is because, when borrowers are extremely strategic, the effects on loan size and relationship benefits (see Result 1 (ii)) are no longer negligible. As a result, the distortions in contract terms overcome the effect on card demand, making welfare concave in the demand semi-elasticity. According to our calibration, the Brazilian market is close to the first-best default semi-elasticity. Imposing cross-default provisions would decrease welfare by half (i.e., equivalent to doubling the interest rates).

Figures 1 and 2 can rationalise why the business loans market features cross-default, while the consumer credit market does not. Indeed, firms are likely more strategic than borrowers when it comes to selective default. If so, they are in the region where the over-provision channel dominates, making cross-default (i.e., forcing

[32]Our model also lets us examine the channel driving the debt face value. There are thre terms $\frac{\alpha\delta}{\psi}$, $\frac{\beta\gamma\theta_0(1-\delta)}{\phi}$ and $\delta\varepsilon^{-1}$. The first two capture the effect of the decrease in loan size, increase in relationship benefits. The last one captures that selective default gives incentives to lower the markup, as lenders worry more about default.

$\sigma_d = 0$) better than the current market equilibrium.

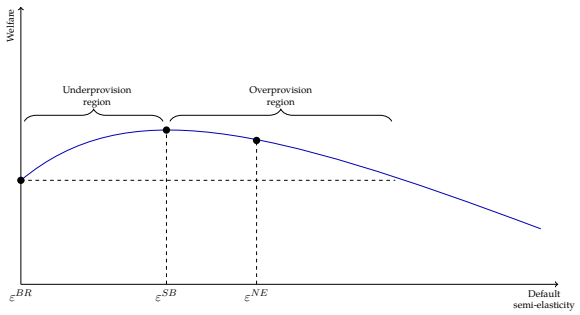


Figure 1: $(\varepsilon^{SB}, \varepsilon^{NE},)$ **are the optimal, current, and under-regulation semi-elasticities.**

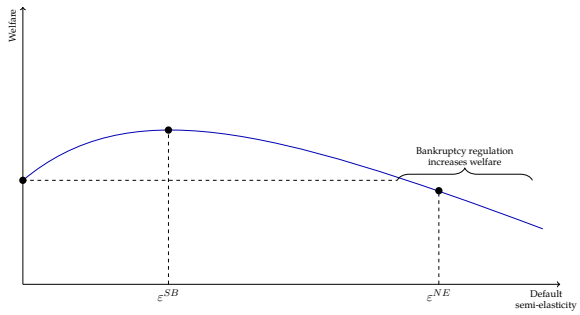


Figure 2: Example where the bankruptcy regulation is welfare increasing.

Competition policies: Antitrust policies are widely used to regulate competition in credit markets. Yet their effects have not been studied in models that combine non-exclusive lending with selective default. One reason is that existing models of non-exclusive lending typically assume perfect competition and therefore abstract from the role of lender market power.

This section studies competition policy in an environment in which lenders compete for informal seniority. This form of competition is particularly relevant in settings in which formal enforcement is weak or repayment priority is not well defined, such as trade credit, consumer lending, and credit markets in developing economies. Our analysis highlights that the degree of lender competition is central to assessing the distortions generated by debt dilution. Competition affects both borrowers' demand for credit relationships and lenders' incentives to provide relationship benefits, and these responses can substantially alter the welfare consequences of non-exclusive lending.

To study the role of competition, we vary the demand semi-elasticity parameter (σ), or, similarly, an increase in the number of lenders B. An increase in (σ or B) lowers the demand semi-elasticity (ε^{-1}), thereby increasing lenders' ability to extract surplus through higher markups. Varying the curvature of demand in this way is standard in the contract-theory literature (see, for instance, Rochet and Stole, 2002 and Azevedo and Gottlieb, 2017) and in empirical industrial organization.

Changes in (σ or B), typically affect the love-for-variety term, which captures the additional utility borrowers obtain from access to a larger set of differentiated products. This channel is absent in our model specification, as we normalised the variance so it disappears (see Appendix F). We thus analyse the effect of competition only through changes in demand semi-elasticity (e.g., due to changes in search technology).

We denote the variable X with an underscore X_* when it is evaluated at the decentralised equilibrium.

Result 4. Effect of competition (changing ε^{-1} using σ or B).

Conditional on the number of cards N, decreasing the demand semi-elasticity increases the debt face value ($\partial_{\varepsilon^{-1}} R(\varepsilon^{-1}, N)) > 0$), lowers the loan size ($\partial_{\varepsilon^{-1}} L(\varepsilon^{-1}, N) < 0$) and increases relationship benefits ($\partial_{\varepsilon^{-1}} q(\varepsilon^{-1}, N) > 0$).

The effect on the number of cards is negative if the reduction in credit limits and the higher repayment burden dominate the increase in quality. Formally:

$$\frac{dN}{d\varepsilon^{-1}} < 0 \tag{27}$$

$$\implies \underbrace{-\frac{\alpha \varepsilon_d^* \delta}{\psi}}_{lower\ loan\ size} + \underbrace{(1-\lambda)\beta(1-\varepsilon_d^* \delta R^*)\left[\frac{\gamma \varepsilon_d^*(1-\delta)}{\phi\left[1+\varepsilon_d^*(1-\delta)R^*\right]} - \delta\right]}_{net\ effect\ of\ relationship\ and\ rate} < 0 \tag{28}$$

The welfare effect of changes in competition is given by the same formula as in Results 2:

$$\frac{dN}{d\varepsilon^{-1}} w_{N,*} + \frac{dL}{d\varepsilon^{-1}} w_{L,*} - \frac{dq}{d\varepsilon^{-1}} w_{q,*} \tag{29}$$

The partial derivatives for q and L have the same sign of the total derivatives when $\frac{dN}{d\varepsilon^{-1}} < 0$ and $\frac{d(\varepsilon_d R)}{d\varepsilon^{-1}} \geq 0$.

Proof. See Appendix O. □

Decreasing competition has an ambiguous effect on the number of cards held. On one hand, it can lower demand because of higher markups and smaller loan sizes on each card. On the other hand, it can also increase demand by affecting relationship benefits. The intuition comes from equation 16. Lenders may overprovide these benefits because of the arms race for informal seniority. The strength of this arms-race channel depends on the value of retaining repayment priority. Lower competition at origination raises lenders' markups and therefore increases the rents associated with being repaid. This, in turn, intensifies competition for informal seniority and strengthens lenders' incentives to provide relationship benefits. The sufficient statistic weights those costs and benefits.

Capital Requirement: Another routinely used policy are capital requierments and monetary policies. Similarly to antitrust policies, their effects have not been studied in models that combine non-exclusive lending with selective default. For this result, the impact of the regulation on R is crucial. With our exponential costs $mc(e^{\psi L} - 1)$ we get that R_{mc} decreases. The reason is that the loan size decreases so that $mc(e^{\psi L}$ stays constant. To generalise, we allow our costs to be $mc(e^{\psi L} + a)$, where $a \geq -1$. This lets us parameterise the effect on R.

Result 4. Effect of capital requierments (changing mc).

Conditional on N, we have $sign(R^*_{mc}) = sign(a),\ sign(q^*_{mc}) = sign(a),\ L^*_{mc} < 0$. The number of cards is ambiguous due to the ambiguous effect on L and q. The welfare formula has the same weights as in equation 29.

Proof. See Appendix P

□

The intuition is the following. After an increase in capital requierments (mc increase), lenders pass through some of that cost. Because of this, holding loan size constant, the debt face value R increases, and so do incentives to provide more relationship benefits. Higher marginal costs also reduce loan size. When marginal costs rise, loan size falls. Depending on the value of a. The loan-size effect can dominate cost increases so that R falls. As in the competition comparative statics, these opposite effects have an ambiguous impact on loan demand and welfare.

6 Discussion about the Modelling Assumptions

In this section, we discuss our key modelling assumptions and how they impact our results.

6.1 Contract terms

Who set the contract terms: We assume that lenders choose contract terms (L, R, q).

We report below the linear pricing case without credit limit. In that case, the loan size equation changes from $mc'(L^*_c) = \ \alpha(1 - \varepsilon_d \delta R^*_c)$ to:

$$u'(L) = \theta[1 - \varepsilon_d \frac{\theta_R}{\theta} \delta R] \tag{30}$$

The rest of the equations have the same form:

$$\theta^*_c = 1 - \lambda + \lambda \frac{1}{N^*} \tag{31}$$

$$q^*_c := \left\{ q : \beta V'(q) \left[\varepsilon_d (1 - \delta) \theta^*_c R^*_c + \theta^*_c \right] = \kappa'(q^*) \right\} \tag{32}$$

$$\theta^*_c R^*_c = \psi(L^*, q^*) + \left[1 - \varepsilon_d \delta R \right] \varepsilon^{-1} \tag{33}$$

$$C'(N^*) = u(L^*) + (1 - \lambda)\beta[V(q^*) - \delta R^*] \tag{34}$$

We can see that now the limited liability on the borrower side leads to overborrowing. This friction can be mitigated by having the lender impose a credit limit. In practice, lenders often use an affine pricing, using fees and rate, with a credit limit.

Allowing for it yields the same equations as in our baseline model model.

What are the relationship benefits: The relationship benefits can have multiple interpretations. Appendix H discusses the micro-foundations for $V_c(q)$ across the three channels that allow lenders to gain seniority: by offering rewards, modifying the card terms conditional on repayments, or cross-selling products. We show that the three channels are equivalent, as they yield a V_c function that is increasing in $q_c \in \mathbb{R}$.

One central result of our paper is the tension between the under- and over-provision channels. For those two channels to move in opposite directions, we assumed that a higher relationship benefits increase both the borrower's utility ($\partial_q V > 0$) and the repayment probability ($\partial_q \theta > 0$).

The sign of $\partial_q V$ and $\partial_q \theta$ depends on the interpretation of the relationship benefits. For the reward, credit limit, interest rate, cross-selling, and app quality interpretation, the sign is likely the same. When interpreting q as the debt-collection process, the sign is ambiguous. Debt collection may not directly affect borrowers' utility (it may be a shrouded attribute or merely a nudge to repay a given bank), affect it positively (when being nice to the borrower is the way to get repaid), or negatively (when the bank harasses the borrower). In interviews with market participants, they pointed out that due to consumer protection, recovery is mainly a matter of being nice to the borrower and reaching out to them once they miss a payment. However, a formal study of debt collection is outside the scope of our paper.

Cancelling relationship benefits upon default: We assume that the borrowers do not get access to relationship benefits conditional on default, so that borrowers choose to fully repay only one credit card. This behaviour is consistent with the data. We show in Appendix L that this behaviour arises in a model in which lenders can set q for the default state as well. The intuition is that lenders optimally set relationship benefits to a minimum in default states to prompt customers to repay them in full.

6.2 Demand

One central element for the underinvestment channel is that demand sensitivities to relationship benefits depend on default probabilities. In our model, the marginal sensitivity to relationship benefits is equal to $\theta_c V'(q)$. $\theta_c V'(q)$ captures the fact that when borrowers are more likely to default, they value a marginal increase in quality q less as they will benefit from it less often. The same formula arises in any model where the survival probability comes from a maximisation problem, so the envelope theorem applies.

Linearity of loan size: This is a standard assumption in the literature (see for instance Green and Liu, 2021). This assumption can be interpreted as a linear approximation of the demand around the equilibrium contract terms. The additive separability can also capture the fact that different types of loans, such as a mortgage and an auto loan, are not used for the same good, so getting one might not affect the marginal utility of getting another.

This assumption removes the insurance benefits associated with the possibility of repaying one loan but not the other and thus not losing all relationship benefits.

The assumption also implies that borrowers are never satiated. Bizer and DeMarzo (1992) use concave utilities. They show that sequential lending nevertheless produces excessive indebtedness and higher default because subsequent borrowing imposes an externality on earlier lenders.

6.3 Supply

Demand function from lenders' perspective: We assume that each lender can observe the set of contracts offered and predict the set the borrower will accept. This assumption is made for tractability and is consistent with the fact that, in practice, lenders do not update their contract terms after the borrower gets a new credit card.[33]

Dynamics: We abstract from the dynamics aspect of lending in our model by having the lender simultaneously offer loan contracts in the first period. This is justified by the fact that selective defaults do not appear to be driven by relationship length in the data. The equilibrium in our model is the same as the one in which lenders sequentially lend and have rational expectations about the other lenders' behaviour. In Appendix L, where we discuss the optimality of canceling the card conditional on default, we allow for lenders to offer a new loan contract after the borrower defaults in the first period. Once default happens, the surviving lender knows he has informal seniority and can extract most of the surplus from lending. The under- and over-provisioning channel is robust to this extension.

7 Quantification

This section brings the model to the data.

We do two quantification exercises. First we use our sifficient statistic together with our difference in difference estimates to assess the impact of lowering search costs on

[33] To microfound the fact that the demand is continuous, one can assume that each card $c \in \mathcal{C}$ represents a card type, that borrowers will get only one card of each type, and that a group of lenders compete on each card type.

welfare. Second, we use the model to provide a sense of how large are the under and overprovision chanels.

7.1 Effect of Changes in Search Costs

We showed in the previous section that, when there is debt dilution, the impact of lowering search costs is ambiguous. On the one hand, this policy may increase competition and thus lead to an increase in loan size. On the other hand, it may increase the demand for credit cards and thus impose a stronger default externality on other lenders.

In this section, we quantify the effect of such policy on welfare in the credit card market. To do so, we extend our model to multiple periods, and remove the assumption about the equilibrium being symmetric. We interpret the relartionship benefits as as credit limit in subsequent periods and the face value R as the amount revolved times the effective interest rate on the loan. The model is presented in Appendix T.

Indexing individuals by i, and defining total welfare $\mathcal{W}$ as the sum of welfare $\mathcal{W}_i$ generated by each market segment (i.e., each individual as there is individual based cerdit limit and pricing), we get the following sufficient statistic:

Result 5. Sufficient Statistic for the credit card market
The impact of a policy on welfare $\frac{d\mathcal{W}}{dp}$ in the dynamic set up is:

$$\frac{d\mathcal{W}}{dp} = \sum_i \frac{d\mathcal{W}_i}{dp} \tag{35}$$

$$\frac{d\mathcal{W}_i}{dp} := \underbrace{\frac{dN_i}{dp} w_{N,i}}_{Number\ of\ cards} + \underbrace{\frac{dL_{0,i}}{dp} w_{L_0,i}}_{Initial\ loan\ size} + \underbrace{\sum_{t=1}^{\infty} \frac{dL_{s,i}}{dp} (\theta_s \beta)^t w_{L_t,i}}_{Relationship\ benefits} \tag{36}$$

The weights are given by:

$$w_N = \sum_{t=1}^{\infty} (\beta(1-\lambda))^t R_t^* - \sum_{t=0}^{\infty} (\beta(1-\lambda))^t \psi_t^* \tag{37}$$

$$w_{L_0} = -\frac{\theta_{R^*}}{\theta} \left(R_0^* + \Pi_1^* \right), \tag{38}$$

$$w_{L_s} = -\frac{1}{\theta_s} \left(\frac{\theta_{L_{s+1}}}{\alpha} + \theta_{R_s} \right) \left(R_s^* + \Pi_{s+1}^* \right), \ s > 0 \tag{39}$$

The formula has the same structure as before, and the weights have the same economic interpretation. One key difference is the presence of the customer lifetime value Π^* in addition to the debt face value for the weights associated with relationship benefits and loan size. For the loan demand, one needs to compare the present values of the

debt face value and the costs of lending instead of just the one-period formulas. The discount rate is the probability of survival on all loans times the time value of money.

Appendix Q reports our calculation of the weights (see in particular section U1). We use accounting data to calculate yearly values for profits and revenues. We use Central Bank of Brazil data to estimate survival probabilities and sensitivities.

The weights imply that too many cards are issued and that borrowers are more sensitive to credit-limit changes than to interest-rate changes. Using the difference-in-differences estimates (10 per cent increase in cards, 100 reais more revolved per card), we get that the change in the cost of getting a card associated with a 10 percentage point increase in the probability of getting a credit card decreases welfare. The reason is that the increase in borrowing capacity does not compensate for the default externality. The statistic implies that the cost if everyone got one more card is equivalent to paying 425 reais per year.

Over versus underprovision: By making a parametric assumption on the costs and using the average costs at equilibrium to calibrate the function (see Appendix U), we can use the ratio of the social planner versus the to quantify the amount of distortion in credit limit relative to the first best:

$$\frac{\psi_L(L_{t+1})}{\psi_L(L^{FB}_{t+1})} = 1 \underbrace{\quad -w_{L_s} \quad}_{\textit{over or under provision}} \tag{40}$$

$$\implies \frac{L^{FB}_{t+1} - L^{*}_{t+1}}{L^{*}} \approx 7.8\% \text{ with a cost elasticity of } 1.5 \tag{41}$$

Because borrowers are more sensitive to the debt face value than future credit limits, the underprovision channel dominates. We find that post-origination credit limits are too low relative to the first-best. Our preferred calculation with a cost elasticity of 1.5 implies an underprovision of 7.8%. This represents 400 reais per month for the average borrower, or 10 per cent of their monthly income. Table A7 reports the results for a wide range of target cost elasticities.

This exercise gives a sense of credit-limit distortions. However, they offer only a partial view of the distortions, as we do not analyse the effect on other dimensions of the relationship benefits, such as cross-selling or debt collection processes. Doing this would require estimates of the causal impact of cross-selling on default sensitivities and average default probabilities, which is outside the scope of our paper.

8 Conclusion

In this paper, we use a rich administrative dataset of household credit in Brazil to document new facts about the credit card market. We show that it is common for borrowers to hold multiple active cards issued by different lenders. We find that default rates rise with the number of lenders, and that, when facing financial distress, borrowers tend to prioritize repaying certain banks over others. Furthermore, we show that prioritized banks keep supplying credit to individuals even after they default on other banks.

While borrowing from multiple lenders may reflect increased competition and greater access to differentiated loan products and services, it also presents challenges, as externalities between lenders can lead to inefficiencies. Banks may respond to it by competing for repayment priority. We study these market inefficiencies through the lens of a model featuring non-exclusivity and repayment decisions.

Although we focus on the credit card market, the idea that lenders endogenously compete for seniority extends more broadly to settings in which contract enforcement is imperfect or regulations—such as consumer protection laws—limit the effective enforcement of legal seniority. Consequently, our framework provides insights into the implications of non-exclusivity in a wide range of credit markets.

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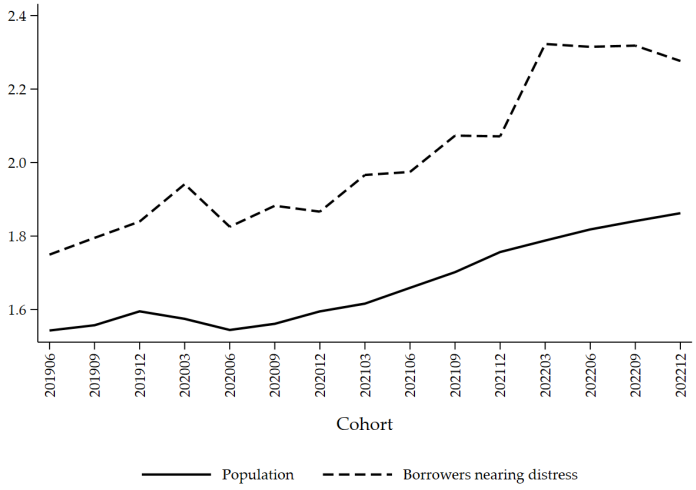
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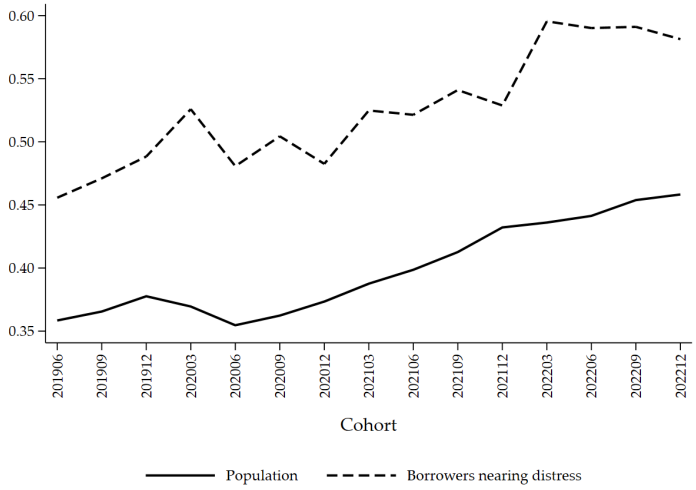
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Figures

Figure 3: Evolution of borrowing from multiple card lenders



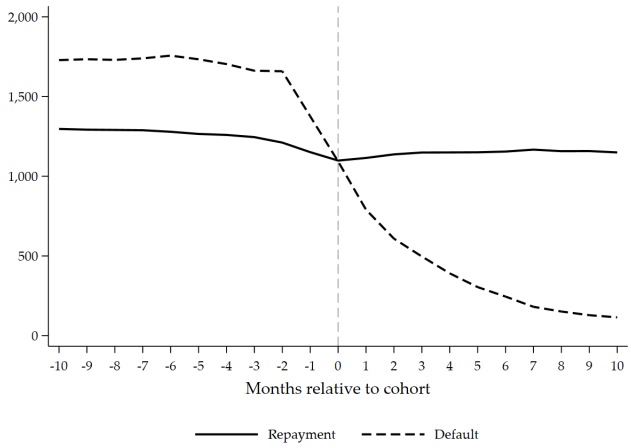
(**a**) Average number of credit card lenders



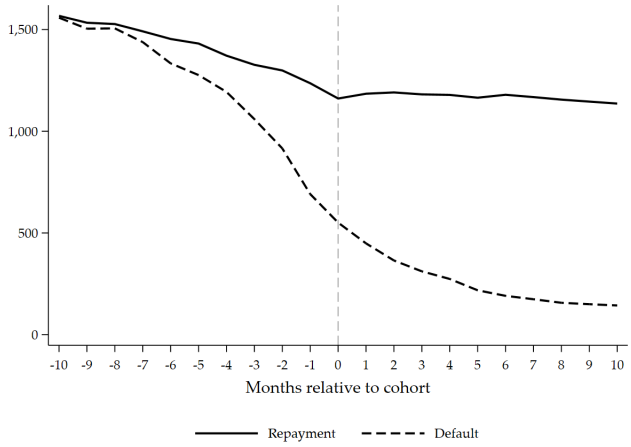
(**b**) Share of card borrowers with two or more credit card lenders

Notes: The unit of observation is the borrower–cohort. Panel (a) shows the evolution of the average number of credit card lenders per borrower. Panel (b) shows the evolution of the share of borrowers with two or more credit card lenders. Population statistics are computed using inverse probability weighting. *Borrowers nearing distress* are borrowers who were current on all credit cards until missing a payment on at least one card in the cohort month.

Figure 4: Evolution of credit card limits and usage among selective defaulters: paid versus unpaid cards



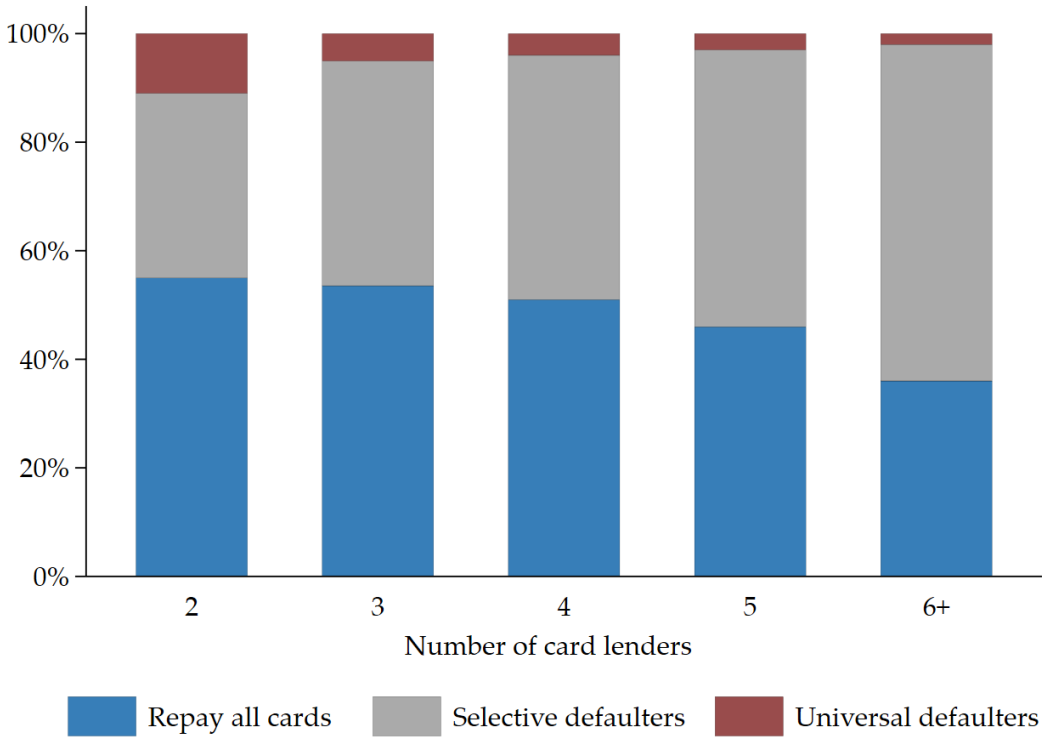
(**a**) Selective defaulters' monthly card usage: paid versus unpaid cards



(**b**) Selective defaulters' limit available: paid versus unpaid cards

Notes: The sample consists of *borrowers nearing distress* who are *selective defaulters*. *Borrowers nearing distress* are borrowers who were current on all credit cards until missing a payment on at least one card in the cohort month. *Selective defaulters* are borrowers who, six months after the cohort month, repay at least one credit card while defaulting on another (payments more than 90 days past due). The figure reports averages separately for repaid and defaulted lenders. Card usage is measured as the total value of purchases made during the billing cycle.

Figure 5: Fraction of selective defaulters 6 months after missing a payment



Notes: The figure reports the shares of *borrowers nearing distress* who, six months after the cohort month, (i) repay all credit cards (*repay all cards*), (ii) default on at least one credit card while repaying at least one other (*selective defaulters*), and (iii) default on all credit cards (*universal defaulters*). *Borrowers nearing distress* are borrowers who were current on all credit cards until missing a payment on at least one card in the cohort month. The shares are reported separately by the number of credit card lenders in the cohort month.

Figure 6: $(\varepsilon^{SB}, \varepsilon^{NE},)$ **are the optimal, current and under regulation semi-elasticities**

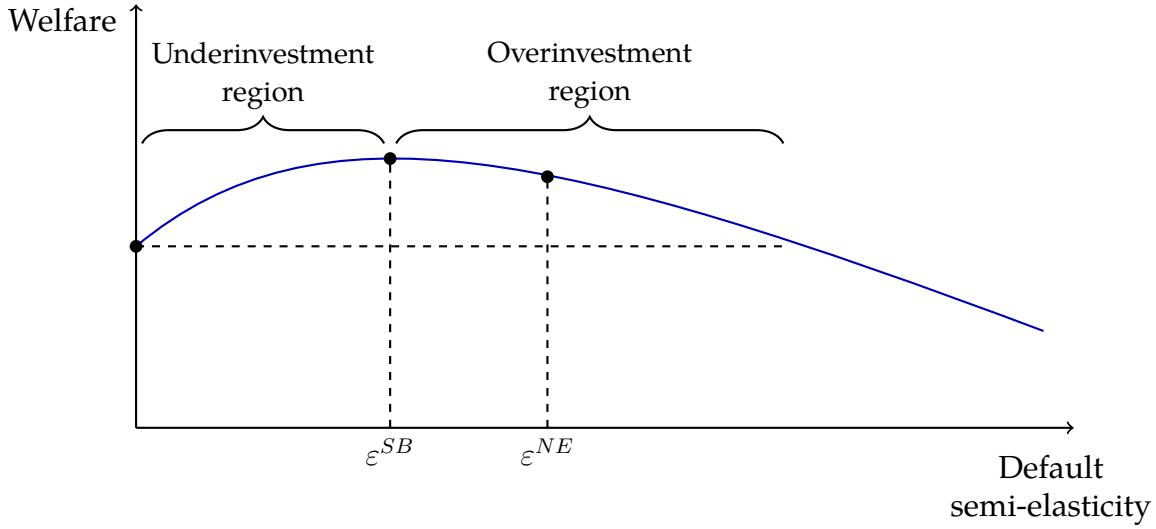
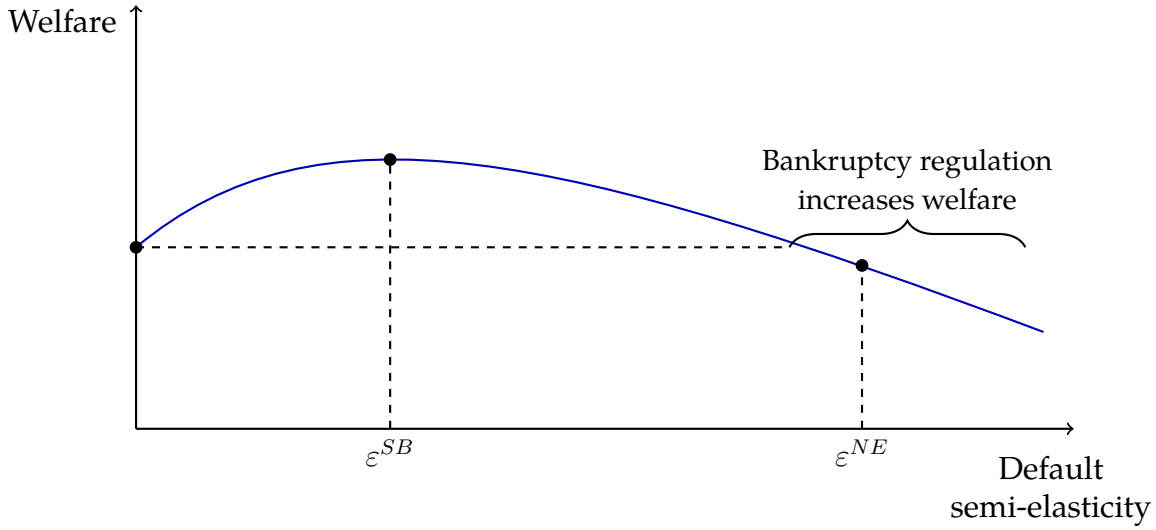


Figure 7: Example where the bankruptcy regulation is welfare increasing



Tables

Table 1: Mean of selected variables by number of credit card lenders

	Number of card lenders					
	1	2	3	4	5	6+
Panel A: Random sample (population)						
Wage (formal)	3,757	4,124	4,171	3,812	3,841	3,920
Wage (reported)	3,214	3,704	3,766	3,607	3,737	3,663
Debt service to income	0.18	0.27	0.38	0.48	0.59	0.85
Formal sector	0.36	0.43	0.46	0.46	0.47	0.47
Age	45	44	44	44	43	41
Total balance card	2,631	5,386	7,985	10,300	13,335	20,069
Average limit utilization (%)	54	55	58	61	64	69
Share cards 100% utilization	0.22	0.22	0.24	0.26	0.29	0.34
Total card limit available	5,129	9,247	11,206	11,793	12,688	13,086
Dummy auto loan	0.07	0.12	0.16	0.20	0.24	0.26
Dummy mortgage	0.06	0.09	0.10	0.11	0.11	0.11
Dummy other loans	0.27	0.35	0.43	0.48	0.53	0.60
Dummy payroll loan	0.17	0.21	0.23	0.26	0.26	0.25
Dummy personal loan	0.14	0.20	0.27	0.34	0.42	0.53
Fraction of borrowers	60%	24%	9%	4%	2%	1%
Panel B: Borrowers nearing distress						
Wage (formal)	2,151	2,943	3,320	3,399	3,548	3,396
Wage (reported)	2,121	2,778	3,054	3,159	3,272	3,374
Debt service to income	0.26	0.41	0.54	0.65	0.77	1.00
Formal sector	0.29	0.38	0.42	0.44	0.44	0.45
Age	39	42	42	42	42	41
Total balance card	1,558	4,234	6,950	9,702	12,621	19,724
Average limit utilization (%)	83	77	75	76	76	78
Share cards 100% utilization	0.74	0.53	0.47	0.45	0.44	0.45
Total card limit available	548	2,431	3,901	4,639	5,233	6,271
Dummy auto loan	0.06	0.11	0.16	0.20	0.22	0.24
Dummy mortgage	0.03	0.06	0.08	0.09	0.10	0.11
Dummy other loans	0.35	0.45	0.52	0.57	0.63	0.69
Dummy payroll loan	0.12	0.22	0.25	0.26	0.28	0.26
Dummy personal loan	0.21	0.30	0.36	0.43	0.48	0.59
Fraction of borrowers	47%	27%	14%	6%	3%	3%

Notes: The unit of observation is the borrower–cohort. In Panel A, we compute population averages using inverse probability weighting. In Panel B, we report sample means for *borrowers nearing distress*, defined as borrowers who were current on all credit cards until missing a payment on at least one card in the cohort month. *Wage* (*formal*) refers to wages recorded in administrative data (RAIS) and are only available for workers employed in formal jobs. *Wage* (*reported*) refers to wages reported by banks. *Formal sector* is a dummy variable that takes the value one if the individual has a formal job. *Debt service to income* is the ratio of debt payments to income. All monetary values are in Brazilian reais (BRL).

Table 2: Mean of default variables by number of credit card lenders

	Number of card lenders					
	1	2	3	4	5	6+
Panel A: Random sample						
Default payroll loan	0.04	0.04	0.03	0.03	0.03	0.02
Default mortagage	0.01	0.01	0.01	0.01	0.01	0.02
Default auto loan	0.04	0.03	0.03	0.02	0.03	0.04
Default personal loan	0.11	0.10	0.10	0.10	0.11	0.13
Default other loans	0.10	0.08	0.09	0.09	0.09	0.11
Default credit card	0.07	0.10	0.13	0.15	0.17	0.21
Panel B: Borrowers nearing distress						
Default payroll loan	0.07	0.04	0.03	0.03	0.02	0.03
Default mortagage	0.05	0.02	0.01	0.01	0.00	0.01
Default auto loan	0.09	0.05	0.03	0.03	0.02	0.02
Default personal loan	0.09	0.08	0.06	0.05	0.05	0.05
Default other loans	0.11	0.07	0.05	0.04	0.04	0.07
Default credit card	0.00	0.00	0.00	0.00	0.00	0.00

Notes: The unit of observation is the borrower–cohort in the month before the cohort. In Panel A, we compute population averages using inverse probability weighting. In Panel B, we compute sample means for the group of *borrowers nearing distress*, defined as borrowers who were current on all credit cards until missing a payment on at least one card in the cohort month.

Table 3: Default rates by number of cards of *borrowers nearing distress*

	(1)	(2)	(3)	(4)	(5)	(6)
Number of	Repay	Default on at	Default on		Selective	
card lenders	all cards	least one card	all cards	Selective	Largest	Smallest
1	0.57	0.43	0.43	-	-	-
2	0.55	0.45	0.11	0.34	0.20	0.13
3	0.53	0.47	0.05	0.41	0.22	0.19
4	0.51	0.49	0.04	0.45	0.23	0.22
5	0.46	0.54	0.03	0.51	0.25	0.26
6+	0.36	0.64	0.02	0.62	0.35	0.27
Total	0.54	0.46	0.24	0.21	0.12	0.09

Notes: The unit of observation is the borrower–cohort. The sample consists of *borrowers nearing distress*, defined as borrowers who were current on all credit cards until missing a payment on at least one card in the cohort month. Columns (1) and (2) partition this group into borrowers who repay all credit cards six months after the cohort month and those who default on at least one card six months after the cohort month, respectively. Columns (3) and (4) further classify borrowers who default on at least one card into those who default on all cards (*universal defaulters*) and those who default on at least one card while continuing to repay at least one other card (*selective defaulters*), respectively. Columns (5) and (6) further split the latter group—*selective defaulters*—into borrowers who default on cards accounting for more than 50 percent and less than 50 percent of their total credit card balance, respectively, measured in the month of the missed payment.

Table 4: Mean of selected variables by default category of *borrowers nearing distress*

	No	Selective		Universal
	default	Largest balance	Smallest balance	default
Number of banks	4.1	4.6	4.8	3.6
Number of card lenders	2.9	3.2	3.3	2.4
Number of digital banks	0.6	1.0	0.9	0.7
Share card balance digital banks	0.15	0.15	0.17	0.16
Number of state-owned banks	0.6	0.7	0.7	0.6
Wage (formal)	3,559	2,735	3,083	2,245
Wage (reported)	3,203	2,719	2,855	2,365
Debt service to income	0.46	0.68	0.56	0.48
Formal sector	0.42	0.40	0.41	0.29
Age	43	39	42	36
Dummy payroll loan	0.25	0.20	0.29	0.12
Dummy mortgage	0.08	0.07	0.08	0.05
Dummy auto loan	0.15	0.15	0.14	0.10
Dummy personal loan	0.31	0.42	0.41	0.36
Dummy other loan types	0.50	0.55	0.53	0.46
Dummy default payroll loan	0.01	0.01	0.01	0.01
Dummy default mortgage	0.00	0.00	0.00	0.00
Dummy default auto loan	0.00	0.01	0.01	0.01
Dummy personal loan default	0.02	0.03	0.04	0.03
Dummy other loan types	0.03	0.05	0.04	0.04
Total card balance	6,432	9,166	7,279	5,295
Total credit balance	38,455	35,735	37,930	20,201
Age bank relationship (years)	8.1	6.2	6.8	4.8

Notes: The unit of observation is the borrower–cohort. The sample consists of *borrowers nearing distress* with two or more credit card lenders in the cohort month. *Borrowers nearing distress* are defined as borrowers who were current on all credit cards until missing a payment on at least one card in the cohort month. *Selective* means that the borrower defaults on at least one card while repaying at least one other card. *Selective largest balance* means that borrowers defaulted on the cards that accounted for more than 50% of their total card balance measured in the month when they missed a payment. *Selective smallest balance* means that borrowers defaulted on the cards that accounted for less than 50% of their total card balance measured in the month when they missed a payment. *Wage* (*formal*) refers to wages recorded in administrative data (RAIS) and only available for workers employed in formal jobs. *Wage* (*reported*) refers to wages reported by banks. *Formal sector* is a dummy that takes the value 1 if the individual has a formal job. *Debt service to income* is the ratio of debt payments to income. All monetary values are in Brazilian reais (BRL).

Table 5: Default rates by number of cards (for *borrowers nearing distress*): controlling for observables

	(1)	(2)	(3)	(4)	(5)	(6)
2 card lenders	0.03***	0.04***	0.04***	0.03***	0.04***	0.03***
	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)
3 card lenders	0.06***	0.07***	0.08***	0.06***	0.07***	0.05***
	(0.011)	(0.011)	(0.011)	(0.009)	(0.011)	(0.009)
4 card lenders	0.09***	0.10***	0.12***	0.09***	0.10***	0.08***
	(0.011)	(0.010)	(0.010)	(0.011)	(0.011)	(0.012)
5 card lenders	0.12***	0.14***	0.15***	0.11***	0.13***	0.10***
	(0.017)	(0.015)	(0.015)	(0.014)	(0.016)	(0.014)
6+ card lenders	0.22***	0.23***	0.24***	0.19***	0.21***	0.17***
	(0.017)	(0.017)	(0.016)	(0.016)	(0.018)	(0.016)
Age		-0.004***	-0.003***	-0.003***	-0.003***	-0.003***
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Female		-0.038***	-0.042***	-0.044***	-0.042***	-0.043***
		(0.003)	(0.004)	(0.004)	(0.003)	(0.004)
Wage			-0.008***	-0.008***	-0.008***	-0.008***
			(0.001)	(0.001)	(0.001)	(0.001)
Tenure			-0.000**	-0.000***	-0.000***	-0.000***
			(0.000)	(0.000)	(0.000)	(0.000)
Debt service to income				0.083***		0.076***
				(0.012)		(0.012)
Total debt to income					0.002***	0.001***
					(0.000)	(0.000)
Constant	0.460***	0.609***	0.604***	0.594***	0.603***	0.595***
	-0.005	(0.006)	(0.006)	(0.007)	(0.006)	(0.007)
Observations	74,468	74,448	74,448	73,792	74,448	73,792
R-squared	0.153	0.161	0.164	0.174	0.167	0.175

Notes: The sample consists of *borrowers nearing distress* with a formal job. *Borrowers nearing distress* are defined as borrowers who were current on all credit cards until missing a payment on at least one card in the cohort month. The dependent variable is an indicator equal to one if the borrower defaults on any card six months after the cohort month. All independent variables are measured at the cohort month. *Age* is measured in years, and *female* is a dummy variable equal to one if the worker is female. We only include borrowers with a formal job in the cohort month. *Tenure* refers to the length of time (in months) the worker has been in the job, and *wage* is the monthly salary (in thousands of BRL). *Debt service to income* is defined as total monthly debt payments divided by wage, while *total debt to income* refers to total outstanding debt divided by wage. The data are at the borrower–cohort level. All regressions include cohort and county fixed effects. Standard errors, reported in parentheses, are clustered at the cohort level. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 6: Descriptive statistics of regression variables

	Mean	SD	p25	p50	p75	N
Default	0.45	0.50	0	0	1	164,613
Limit card	2,931	6,809	500	1,169	2,798	164,613
Digital bank	0.23	0.42	0	0	0	164,613
State-owned bank	0.11	0.31	0	0	0	164,613
Payroll loan	0.06	0.23	0	0	0	164,613
Mortgage	0.01	0.10	0	0	0	164,613
Mortgage $\times$ state-owned	0.01	0.09	0	0	0	164,613
Auto loan	0.02	0.14	0	0	0	164,613
Personal loan	0.12	0.32	0	0	0	164,613
Other loan types	0.17	0.37	0	0	0	164,613
Default payroll loan	0.00	0.03	0	0	0	164,613
Default mortgage	0.00	0.01	0	0	0	164,613
Default auto loan	0.00	0.03	0	0	0	164,613
Default personal loan	0.01	0.08	0	0	0	164,613
Default other loan types	0.01	0.08	0	0	0	164,613

Notes: Data are at the borrower-bank-cohort level for the sample of *borrowers nearing distress* with two or more card lenders who default on a subset of their cards. *Borrowers nearing distress* are individuals who were current on all cards until failing to make a payment on at least one card in the cohort month. All variables are measured in the cohort month, and, except for limit card, which is measured in Brazilian reais, all variables are binary. We report the mean in column (1), the standard deviation in column (2), the 25th percentile in column (3), the median in column (4), the 75th percentile in column (5), and the number of observations in column (6).

Table 7: Predictors of selective default

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Unrestricted sample				Sample with interest rate information			
	All cohorts		Post May 2021		All cohorts		Post May 2021	
Group 1: Card characteristics								
Card with the largest contractual limit	-0.06***	-0.05***	-0.08***	-0.08***	-0.03***	-0.04***	-0.05***	-0.06***
	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Card with the largest balance	0.21***	0.20***	0.23***	0.21***	0.21***	0.20***	0.22***	0.20***
	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Card with the oldest relationship	0.01***	-0.03***	-0.00	-0.03***	0.03***	-0.03***	0.01***	-0.03***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)
Card with the largest interest rate					0.31***	0.26***	0.32***	0.27***
					(0.00)	(0.00)	(0.00)	(0.00)
Group 2: Holdings of other loan types								
Payroll loan	-0.11***	-0.13***	-0.13***	-0.13***	-0.09***	-0.13***	-0.11***	-0.13***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Mortgage	-0.17***	-0.16***	-0.22***	-0.20***	-0.13**	-0.14***	-0.18***	-0.18***
	(0.04)	(0.04)	(0.05)	(0.04)	(0.05)	(0.05)	(0.06)	(0.05)
Mortgage x state-owned bank	0.29***	0.18***	0.29***	0.19***	0.14**	0.13**	0.14**	0.14**
	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)
Auto loan	-0.07***	-0.05***	-0.08***	-0.06***	-0.08***	-0.06***	-0.10***	-0.08***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)
Personal loan	0.03***	0.02***	0.03***	0.03***	0.01*	0.02**	0.01*	0.03***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Other loans	-0.14***	-0.07***	-0.13***	-0.05***	-0.09***	-0.07***	-0.08***	-0.04***
	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Group 3: Default on other products								
Default payroll	0.18***	0.17***	0.29***	0.27***	0.27***	0.24***	0.31***	0.28***
	(0.05)	(0.05)	(0.07)	(0.07)	(0.06)	(0.06)	(0.08)	(0.07)
Default mortgage	-0.04	-0.03	-0.04	-0.00	-0.01	-0.00	0.19	0.20
	(0.17)	(0.18)	(0.24)	(0.25)	(0.20)	(0.20)	(0.27)	(0.27)
Default auto loan	0.21***	0.24***	0.23***	0.28***	0.13*	0.16**	0.20**	0.22**
	(0.06)	(0.06)	(0.08)	(0.07)	(0.08)	(0.08)	(0.09)	(0.09)
Default personal loan	0.20***	0.19***	0.19***	0.19***	0.16***	0.17***	0.16***	0.17***
	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)
Default other loans	0.20***	0.17***	0.16***	0.13***	0.25***	0.20***	0.22***	0.18***
	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)
Group 4: Bank type								
Digital bank	-0.17***		-0.15***		-0.12***		-0.11***	
	(0.00)		(0.00)		(0.01)		(0.01)	
State-owned bank	0.18***		0.17***		0.24***		0.25***	
	(0.01)		(0.01)		(0.01)		(0.01)	
Observations	164,613	164,570	112,634	112,593	101,989	101,950	68,905	68,869
R-squared	0.190	0.272	0.199	0.287	0.276	0.342	0.290	0.358
Borrower FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Bank FE	No	Yes	No	Yes	No	Yes	No	Yes

Notes: The table reports coefficient estimates from Equation 1. The sample is comprised of *borrowers nearing distress* who have two or more credit card lenders, default on at least one card, and repay at least one card. *Borrowers nearing distress* are individuals who were current on all cards until failing to make a payment on at least one card in the cohort month. The data are at the borrower-cohort-bank level. All variables are binary. All regressions include borrower-by-cohort fixed effects. Robust standard errors are reported in parentheses. Columns (3), (4), (7), and (8) restrict the sample to post-May 2021 cohorts, for which the card limit information is more accurate. Columns (5)–(8) restrict the sample to borrower-cohort-bank observations for which interest rate information is available. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 8: Difference-in-differences approach

	(1)	(2)	(3)	(4)
	Number of cards	Card debt to income	Delinquency	Default
$Post_Treat^{[0-6]}$	0.05***	0.092***	0.009**	0.007
	(0.02)	(0.033)	(0.004)	(0.005)
$Post_Treat^{[7-15]}$	0.11***	0.137**	0.019**	0.011*
	(0.03)	(0.053)	(0.008)	(0.006)
Constant	1.3***	1.3***	0.029***	0.025***
	(0.03)	(0.05)	(0.010)	(0.009)
Observations	67,132	67,132	67,132	67,132
R-squared	0.775	0.841	0.648	0.651

Notes: The table reports results from the estimation of Equation 2. Delinquency (default) is a dummy variable equal to one if the individual has any debt payment overdue by more than 15 (90) days. In all regressions, we include individual fixed effects, firm-by-month fixed effects, and baseline default status interacted with time dummies. Standard errors, reported in parentheses, are clustered at the firm level. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively.

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A Empirical part

A1 Sampling

For a given month t (cohort), we take a 1% random sample of individuals who were in good standing with all their credit card loans in month $t-1$ (i.e., no card with payments overdue by more than 15 days) but became more than 15 days overdue on at least one card in month t. We refer to these individuals who are potentially transitioning into default (i.e., more than 90 days overdue) as *borrowers nearing distress*. We apply this procedure every three months, starting in June 2019 and ending in December 2022—a total of 15 cohorts. For each individual in cohort t, we also collect credit information for the months $m \in \{t-12, t-11, ..., t-1, t, t+1, ..., t+11\}$.

For each cohort, we also take a 0.025% random sample of individuals who are not *borrowers nearing distress*. These individuals include those who were not in good standing with all their cards at $t-1$ and those who were in good standing with all their cards at $t-1$ and remained in good standing at month t. We refer to these individuals as *all other borrowers*. The sampling procedure is described in Appendix Table A1. We apply inverse probability weighting to estimate statistics for the population of credit card users. The final data is at the cohort-individual-bank-month level.

In Appendix Table A2, we report the default and delinquency rates for both groups in the month preceding the cohort. For the *all other borrowers* group, 7% of individuals have at least one card with payments overdue by more than 90 days (default), and 10.3% of borrowers have at least one card with payments overdue by more than 15 days (delinquent). By design, both the delinquency and default rates for the *borrowers nearing distress* group are zero. In Appendix Table A3, we report a total of 524,533 observations at the cohort-by-individual level, with 252,844 observations belonging to the *borrowers nearing distress* group. Consistent with the growth of the credit card industry, more recent cohorts are larger than the older ones.

Table A1: Sampling procedure for cohort t

	Repayment Standing at t-1	Repayment Standing at t	Share sampled
Borrowers nearing distress	Good	Bad	1%
All other borrowers	Good	Good	
All other borrowers	Bad	Good	0.025%
All other borrowers	Bad	Bas	

Notes: The table describes our sampling procedure. A borrower is classified as being in good repayment standing if all active cards exhibit payment delays of no more than 15 days. Conversely, an individual is classified as being in bad repayment standing if at least one active card has a payment overdue by more than 15 days. The sample is taken at time t at the individual level, conditional on the individual having at least one active card.

Table A2: Default and delinquency incidence at $t-1$ (month before the cohort)

	Borrowers nearing distress		All other borrowers	
	Card level	Borrower level	Card level	Borrower level
Panel A: All card users				
Delinquency rate	0	0	10.3	13.2
Default rate	0	0	7.0	9.2
Panel B: Card users with 2+ cards				
Delinquency rate	0	0	10.4	17.6
Default rate	0	0	7.0	12.2

Notes: A credit card loan is classified as delinquent if its payments are more than 15 days overdue and as in default if its payments are more than 90 days overdue. A borrower is considered delinquent (in default) if at least one of their credit cards has payments that are more than 15 (90) days overdue.

Table A3: Sample evolution

Cohort	Borrowers nearing distress	All other borrowers	Total
201906	15,186	15,799	30,985
201909	15,435	16,186	31,621
201912	13,348	16,476	29,824
202003	18,293	16,469	34,762
202006	10,128	16,390	26,518
202009	10,323	16,514	26,837
202012	11,281	16,931	28,212
202103	17,696	17,047	34,743
202106	15,804	18,025	33,829
202109	17,780	18,726	36,506
202112	22,750	19,668	42,418
202203	22,817	20,130	42,947
202206	20,407	20,667	41,074
202209	21,499	21,125	42,624
202212	20,097	21,536	41,633
Total	252,844	271,689	524,533

Notes: The table shows the evolution of the number of unique card users per cohort and group classification.

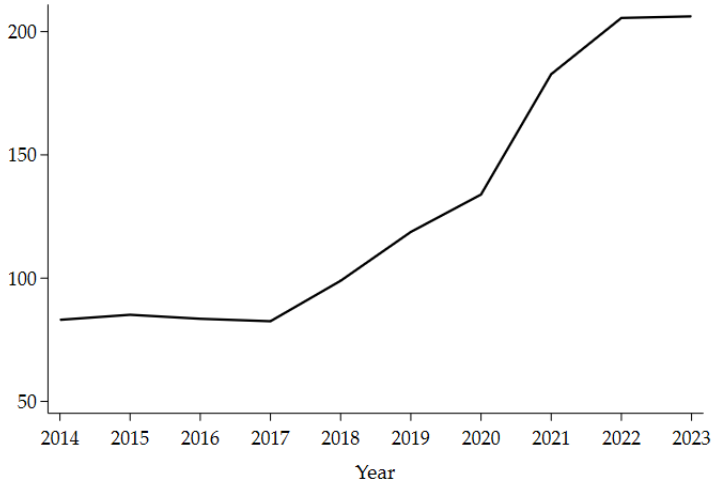
Table A4: Size of groups based on the number of credit card lenders

	Frequency			Shares	
Number of card lenders	Borrowers nearing distress	All other borrowers	Total	Borrowers nearing distress	All other borrowers
1	117,788	163,114	280,902	47%	60%
2	68,462	65,961	134,423	27%	24%
3	34,509	25,363	59,872	14%	9%
4	16,432	9,908	26,340	6%	4%
5	7,793	4,033	11,826	3%	1%
6+	7,860	3,310	11,170	3%	1%
Total	252,844	271,689	524,533	100%	100%

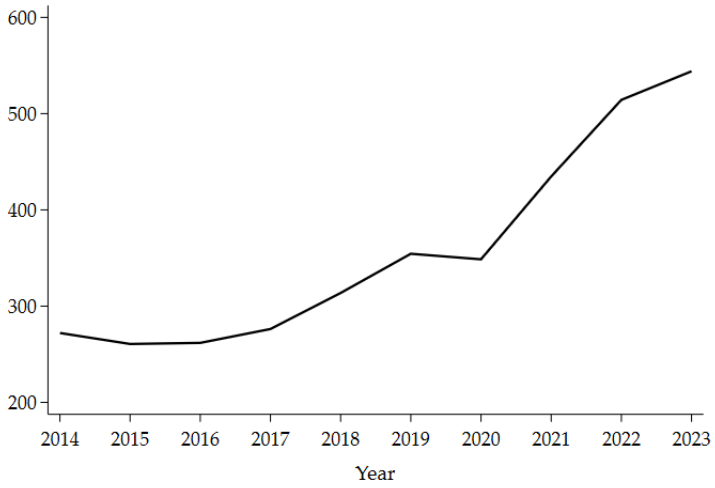
Notes: Borrowers are categorized based on the number of credit card lenders. The data are at the borrower-cohort level.

A2 Institutional setting: additional details from aggregate data

Figure A1: Evolution of the number of credit cards and credit card balance



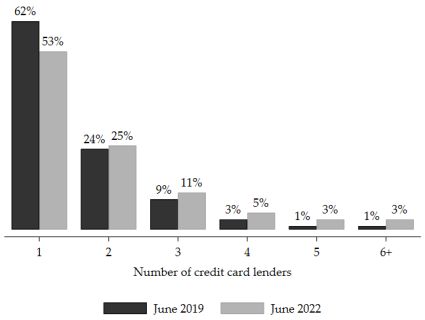
(**a**) Total number of active credit cards (in millions)



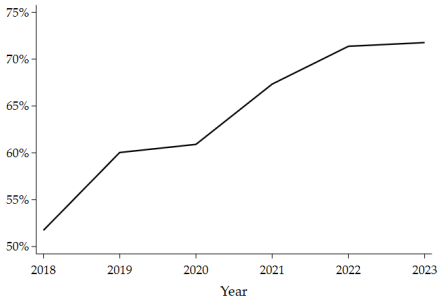
(**b**) Total credit card loans (inflation-adjusted values in billions BRL)

Notes: The figure reports the total number of active credit cards and the aggregate outstanding credit card balance by year. Monetary values are expressed in constant 2023 Brazilian reais and adjusted by the consumer price index (*Índice Nacional de Preços ao Consumidor Amplo*, IPCA). Source: Central Bank of Brazil (*Estatísticas de Meios de Pagamentos* and IF.data).

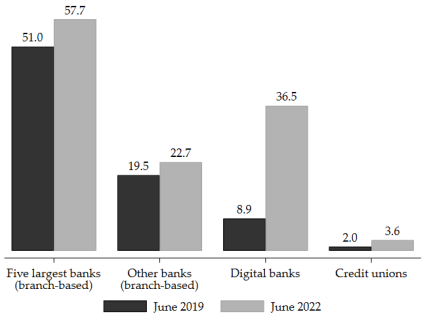
Figure A2: Evolution of borrowers with more than one card lender and growth of digital banks



(**a**) Share of credit card users by number of credit card lenders, 2019 and 2022



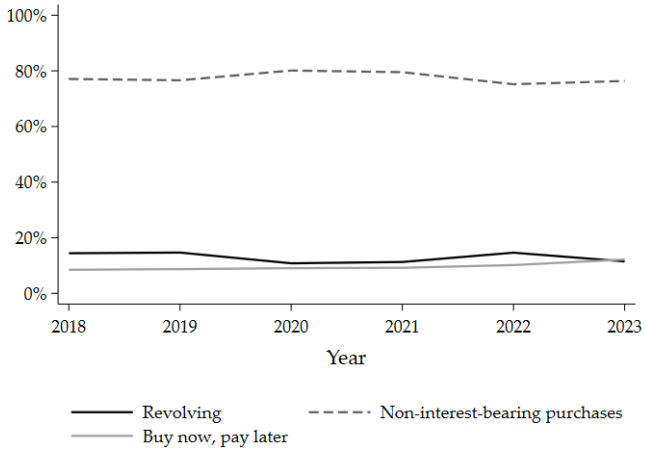
(**b**) Share of total credit card balance held by borrowers with two or more card lenders



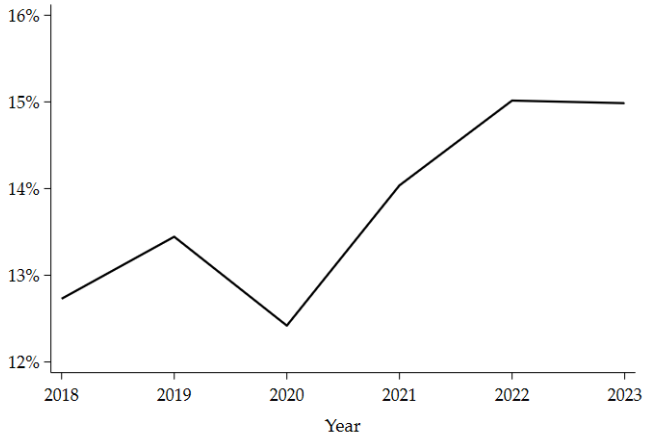
(**c**) Number of credit card users by bank type category, 2019 and 2022

Notes: Panel (a) shows the distribution of credit card users by the number of card lenders in 2019 and 2022. Panel (b) reports the share of total outstanding credit card balances held by users with two or more card issuers. Panel (c) presents the evolution of the number of credit card users per bank type category. All statistics are computed using data from Central Bank of Brazil (2022).

Figure A3: Composition of credit card balance and participation of credit card debt in total household debt



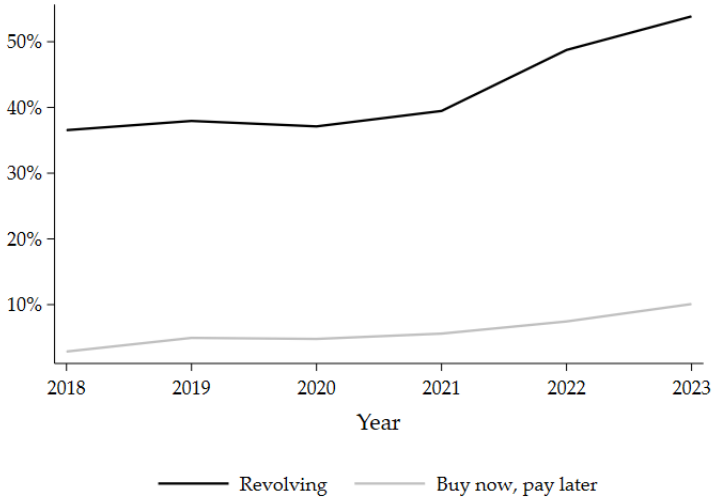
(**a**) Composition of credit card balance



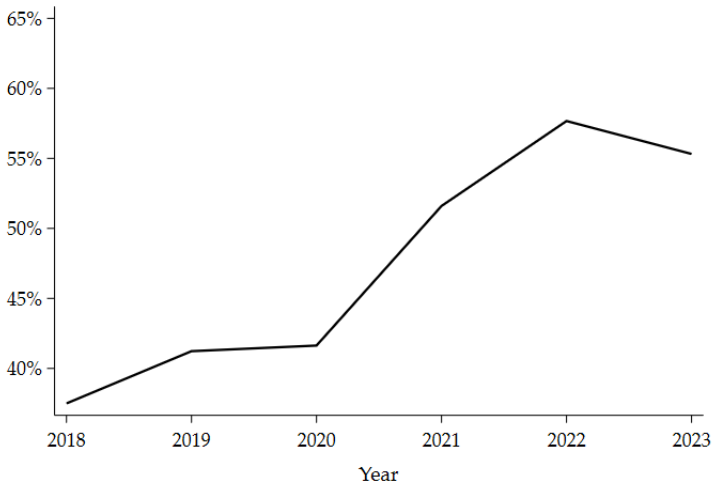
(**b**) Participation in total household debt

Notes: Panel (a) shows the composition of credit card balances. *Revolving* balances arise when cardholders do not pay the full statement balance by the due date and therefore accumulate interest-bearing debt. *Non-interest-bearing purchases* are transactions made during the billing cycle that are fully due on the next payment date and incur no interest if paid on time. *Buy now, pay later* (BNPL) balances correspond to credit card transactions repaid in installments, with or without interest. Panel (b) shows the participation of credit card debt in total household debt. The data are from the Central Bank of Brazil's SCR.data and Time Series Management System.

Figure A4: Participation of credit card debt in total household debt default



(**a**) Share of total balance in default

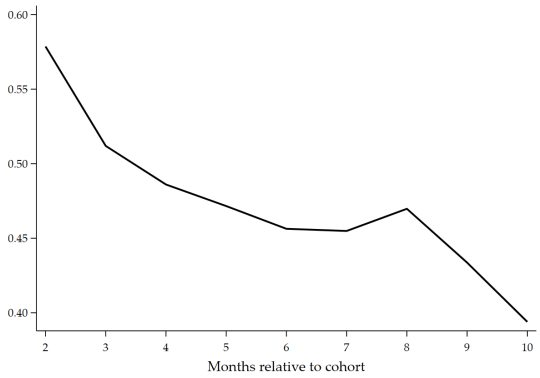


(**b**) Participation in total amount overdue for more than 15 days

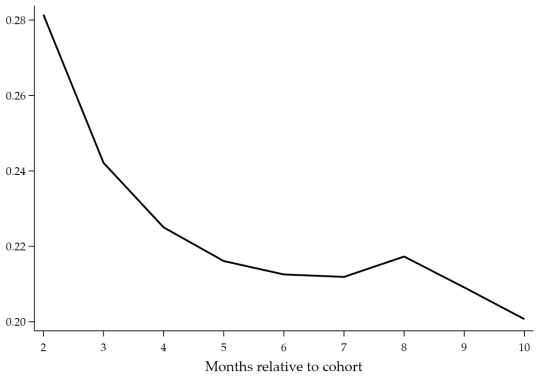
Notes: Panel (a) shows the share of balances in default for the *revolving* and *buy now, pay later* categories. Default is defined as payments more than 90 days past due. Panel (b) shows the share of credit card debt overdue by more than 15 days in total household debt overdue by more than 15 days. We use data from the Central Bank of Brazil's SCR.data and Time Series Management System.

A3 Default and selective default evolution t months after becoming 15+ days overdue

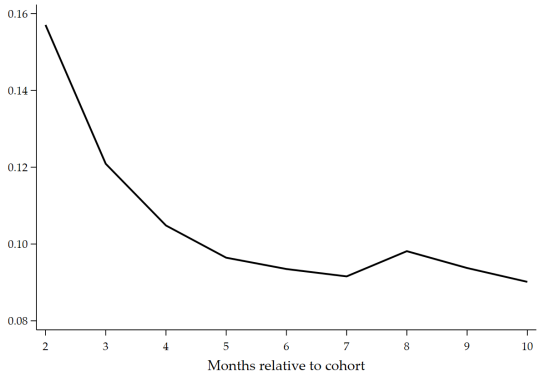
Figure A5: Default and selective default evolution t months after becoming 15+ day overdue



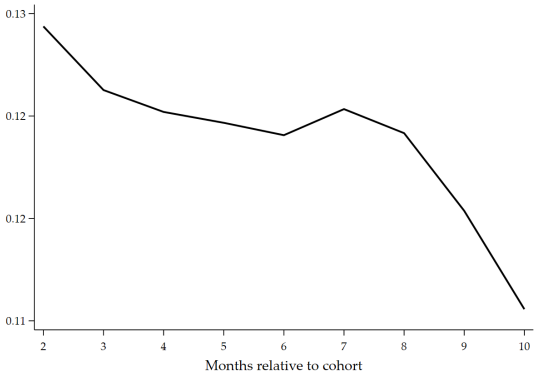
(**a**) Share of *borrowers nearing distress* with at least one card 90+ days overdue



(**b**) Share of *borrowers nearing distress* with at least one card 90+ days overdue and at least one in good standing

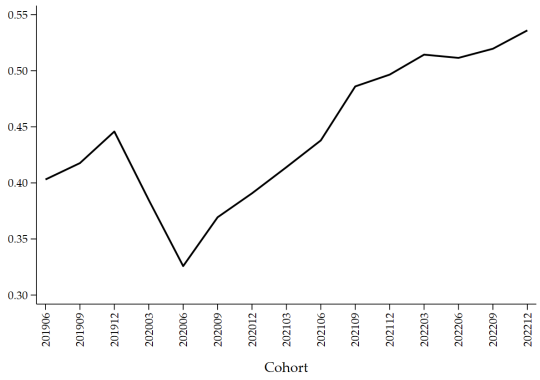


(**c**) Share of *borrowers nearing distress* with at least one card 90+ days overdue, at least one in good standing, and default on less than 50% of the total card balance

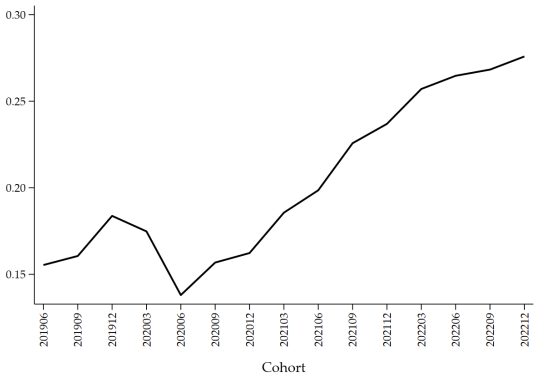


(**d**) Percentage of *borrowers nearing distress* with at least one card more than 90 days overdue and at least one card in good standing, and default on more than 50% of the total card balance

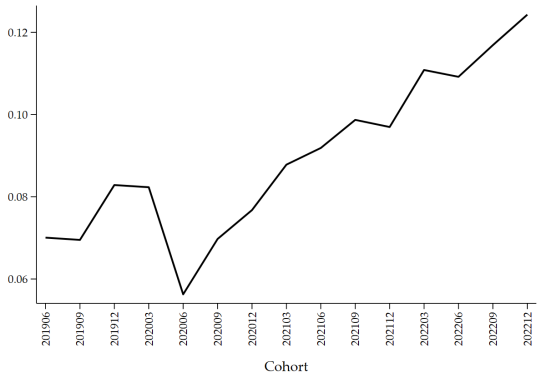
Figure A6: Default and selective default evolution



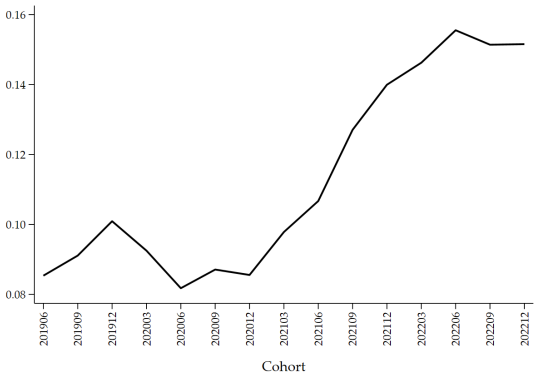
(**a**) Share of *borrowers nearing distress* with at least one card 90+ days overdue



(**b**) Share of *borrowers nearing distress* with at least one card 90+ days overdue and at least one in good standing



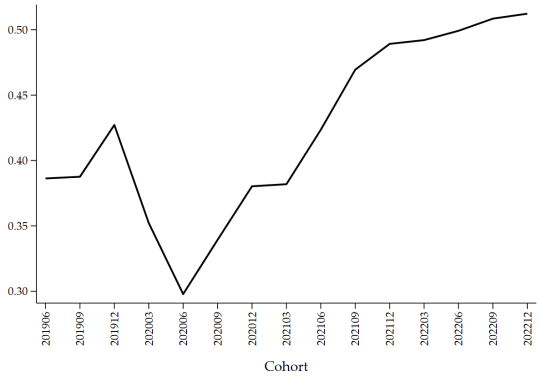
(**c**) Share of *borrowers nearing distress* with at least one card 90+ days overdue, at least one in good standing, and default on less than 50% of the total card balance



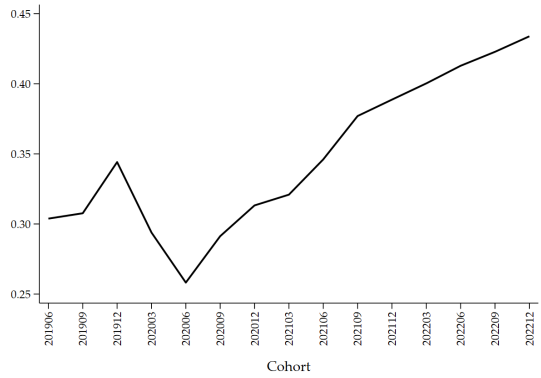
(**d**) Percentage of *borrowers nearing distress* with at least one card more than 90 days overdue and at least one card in good standing, and default on more than 50% of the total card balance

Notes: The sample comprises *borrowers nearing distress* who have one or more cards. We classify a card as in default if there is balance overdue for more than 90 days six months after the borrower becomes more than 15 days overdue.

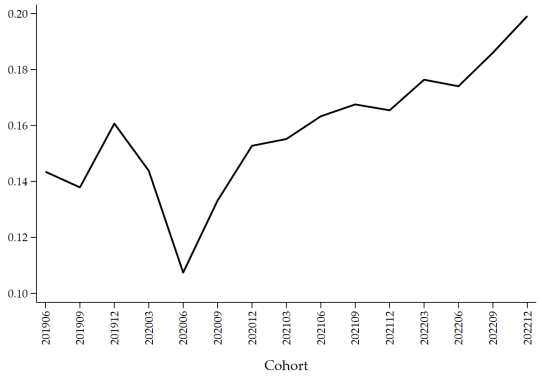
Figure A7: Default and selective default evolution, borrowers with at least two cards



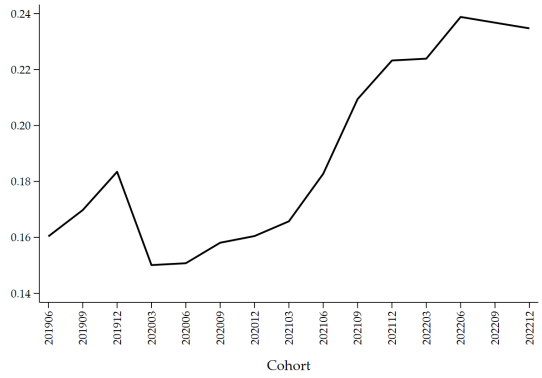
(**a**) Share of *borrowers nearing distress* with at least one card 90+ days overdue



(**b**) Share of *borrowers nearing distress* with at least one card 90+ days overdue and at least one in good standing



(**c**) Share of *borrowers nearing distress* with at least one card 90+ days overdue, at least one in good standing, and default on less than 50% of the total card balance

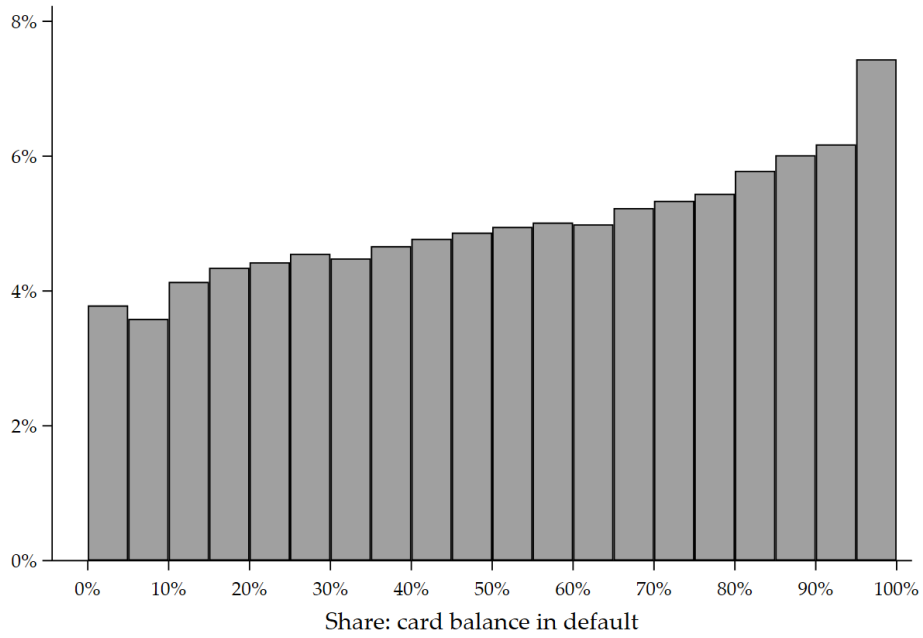


(**d**) Percentage of *borrowers nearing distress* with at least one card more than 90 days overdue and at least one card in good standing, and default on more than 50% of the total card balance

Notes: The sample comprises *borrowers nearing distress* who have two or more cards. We classify a card as in default if there is a balance overdue for more than 90 days six months after the borrower becomes more than 30 days overdue.

A4 Share of balance in default among selective defaulters

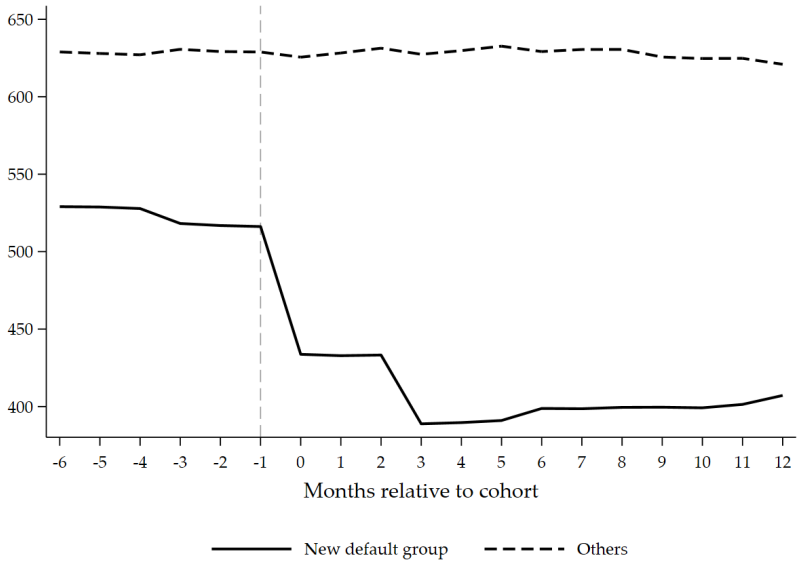
Figure A8: Share of balance in default



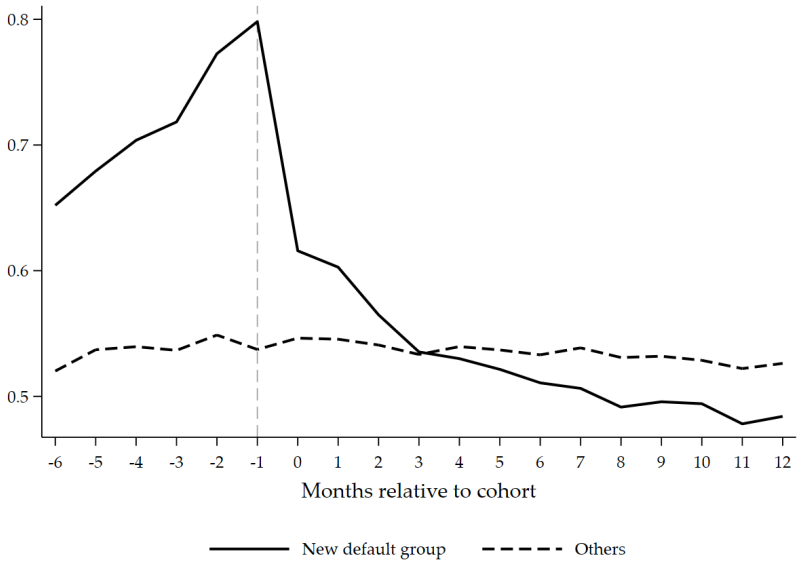
Notes: This figure plots the distribution of the share of total balance in default among selective defaulters. We measure default as payment more than 90 days past due six months after the cohort month. We compute the shares using the balances at the cohort month.

A5 Default and credit scores

Figure A9: Evolution of credit scores around default events



(**a**) Credit score

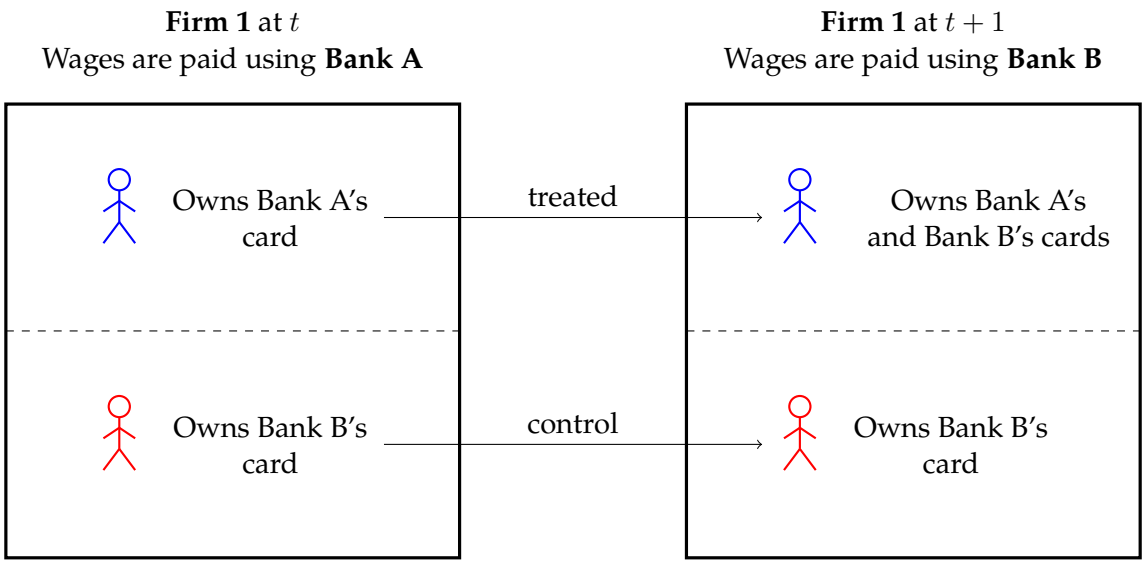


(**b**) Probability of obtaining a new loan

Notes: This figure plots the average credit score and probability of obtaining a new loan for borrowers who experience a new default in a given month (the cohort) relative to borrowers who do not experience a default in the same month.

A6 Difference-in-differences analysis

Figure A10: Identification strategy



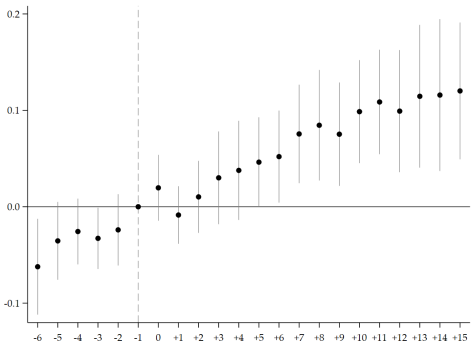
Notes: We compare the relative number of cards and default probability of the blue (treated) and red (control) groups of borrowers before and after the change in the bank that manages the payroll.

Table A5: Descriptive statistics

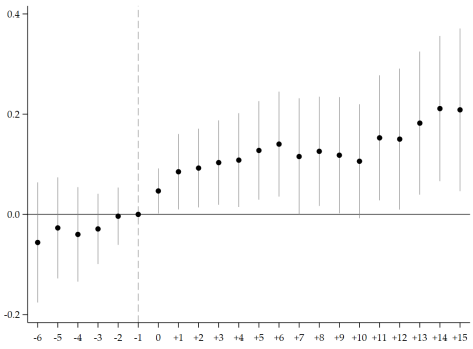
		Treated		
	Control	Complier	Non-complier	All
Number of card lenders	1.5	1.3	1.2	1.2
Debt-to-wage ratio	1.5	1.0	1.3	1.3
Delinquency rate	0.05	0.02	0.02	0.02
Default rate	0.04	0.01	0.02	0.02
Number of individuals	702	163	2,701	2,864

Notes: We report means of selected variables in the month prior to the new bank assuming management of payroll. Means are reported separately for three groups: individuals in the control group, compliers (treated borrowers who use a card from the new bank at some point after treatment), and non-compliers (treated borrowers who never use a card from the new bank in any month of the post-treatment period). Delinquency (default) is defined as a dummy variable equal to one if the individual has any debt payment overdue by more than 15 (90) days

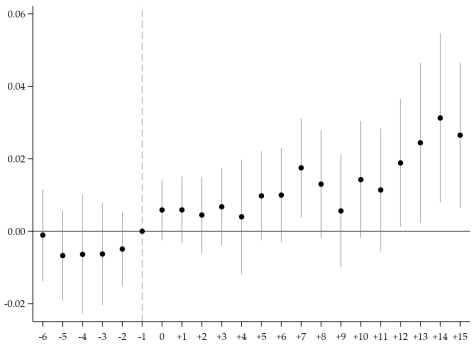
Figure A11: Difference-in-Differences



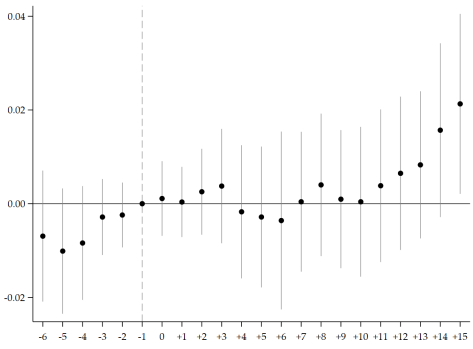
(**a**) Number of cards



(**b**) Debt-to-wage ratio



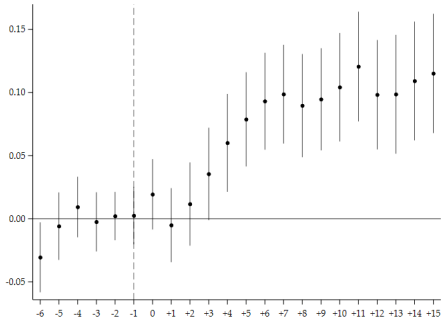
(**c**) Delinquency rate



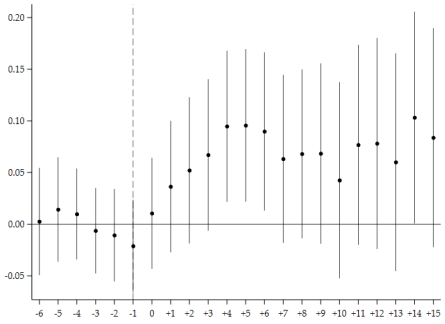
(**d**) Default rate

Notes: The figures present the results from estimating Equation 3. We report 95% confidence intervals calculated using standard errors clustered at the firm level. Delinquency (default) is a dummy variable equal to one if the individual has any debt payment overdue by more than 15 (90) days.

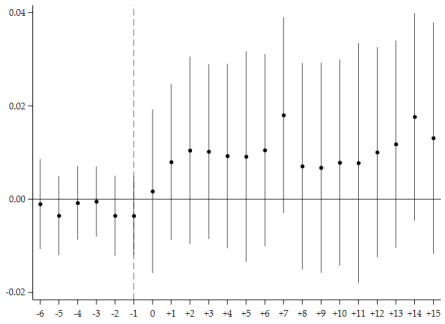
Figure A12: Synthetic Difference-in-Differences



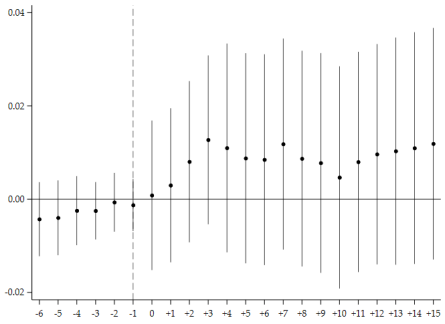
(**a**) Number of cards



(**b**) Debt-to-wage ratio



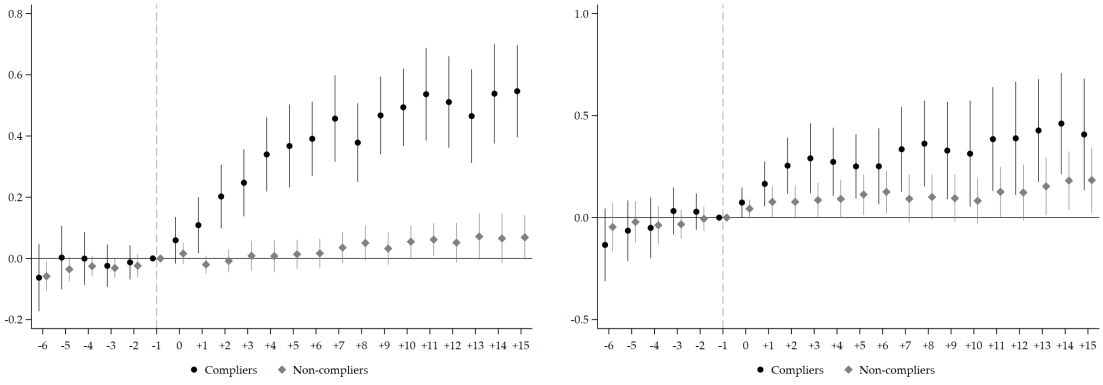
(**c**) Delinquency rate



(**d**) Default rate

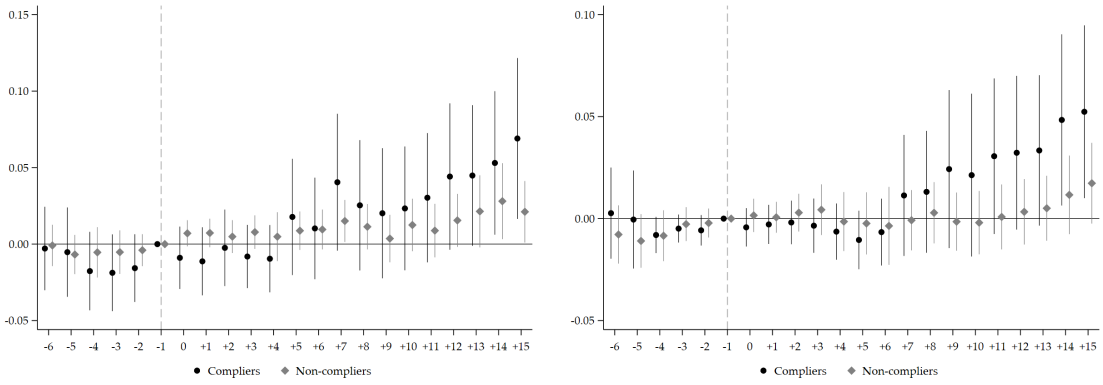
Notes: The figures present estimates from a version of Equation 3 using the synthetic difference-in-differences method proposed by Arkhangelsky et al. (2021). We report 95% confidence intervals. Delinquency (default) is a dummy variable equal to one if the individual has any debt payment overdue by more than 15 (90) days.

Figure A13: Difference-in-Differences: Compliers versus Non-compliers



(**a**) Number of cards

(**b**) Debt-to-wage ratio



(**c**) Delinquency rate

(**d**) Default rate

Notes: The figures show the results from estimating Equation 3 with the following modification: treatment dummies are interacted with post-treatment take-up status—compliers (treated borrowers who use a card from the new bank at some point after treatment) and non-compliers (treated borrowers who never use a card from the new bank in any month of the post-treatment period). We report 95% confidence intervals calculated using standard errors clustered at the firm level. Delinquency (default) is defined as a dummy variable equal to one if the individual has any debt payment overdue by more than 15 (90) days

B Earning Calls and Interviews

Gentera / Banco Compartamos — Enrique Majós Ramírez, CEO — Q2 2026 earnings call. In a discussion explicitly about customers receiving credit from multiple competitors—including Nu, Mercado Pago, Plata, Banco Azteca and BanCoppel—and the resulting risk of over-indebtedness, Majós explains Gentera's response: "our strategy is to remain our customers' first payment priority." He then explains that staying close to customers and maintaining the long-term relationship through loan officers is what allows Gentera to maintain this payment priority.

A second, extremely good one is from Banco Ripley Chile. It is not the CEO, unfortunately, but Julien Labrot, then head of Banco Ripley's Product Division. In a September 28, 2022 interview with Diario Financiero, the interviewer asks about increasing delinquency. Labrot explicitly connects being the customer's principal bank to repayment:

"Si uno es el banco principal, se va a pagar la deuda y la morosidad va a bajar porque los clientes son más fieles."

The surrounding discussion is important: Banco Ripley's objective was specifically to become the principal financial institution among customers who simultaneously had products at more than one bank.

That translates approximately as: "If you are the main bank, the debt will be paid and delinquency will decline because customers are more loyal."

There is also a very useful Nubank statement. After Nubank's Q3 2022 results, CFO Guilherme Lago was asked why its deterioration in delinquency had been smaller than at other Brazilian banks. His answer was explicitly about the banking relationship: he said much of the market's worsening delinquency came from "mar aberto" portfolios—customers for whom the lender did not have "principalidade"—whereas more than half of Nubank's customers had Nubank as their principal banking relationship.

Finally, Santander Brasil CEO Mario Leão made a related statement during the Q2 2024 results. He explained that Santander did not want to use unsecured personal loans simply to acquire customers because that product did not necessarily make Santander the customer's principal bank. Instead, he wanted to concentrate on credit cards to obtain "principalidade."

C Proof Lenders FOCs

We use the shortcut notation D for the demand and $\pi := [\theta(q, R, \mathcal{C}_b)R - \psi(L, q)]$ for the net present value of the loan.

Taking the first-order conditions with respect to an arbitrary characteristic $x \in$

$\{L, q, R\}$, we get:

$$\frac{\partial_x \pi}{\partial_x D} = -\frac{\pi}{D} \tag{42}$$

Let us now calculate $\partial_x \pi$ and $\partial_x D$ for all contract terms. We have:

$$\partial_q \pi = \theta_q R - mc_q \tag{43}$$
$$\partial_R \pi = \theta_R R + \theta \tag{44}$$
$$\partial_L \pi = -mc_L \tag{45}$$

Using the envelope theorem, we have:

$$\partial_q D = D_U \cdot U_q = D_U \cdot \beta\theta V_q \tag{46}$$
$$\partial_R D = D_U \cdot U_R = -D_U \cdot \beta\delta\theta \tag{47}$$
$$\partial_L D = D_U \cdot U_L = D_U \cdot \partial_L u(\mathbf{L}) \tag{48}$$

For equation 47, I used the fact that borrowers chose optimally whether to default or not. To fix ideas, let us assume that the cost of defaulting has the form:

$$\omega^d(|\mathcal{C}|, |\mathcal{R}|) = \begin{cases} 0, & \text{if } |r| > |\mathcal{R}|, \\ -\infty, & \text{if } |r| \leq |\mathcal{R}|. \end{cases} \tag{49}$$

In that case, we have $\partial_R D = D_U \cdot U_R = -D_U \cdot \beta\delta\theta$.

The first equation we report in the main text is $\frac{\partial_R \pi}{\partial_R D} = -\frac{\pi}{D}$, the second is $\frac{\partial_L \pi}{\partial_L D} = \frac{\partial_R \pi}{\partial_R D}$ and the third is $\frac{\partial_L \pi}{\partial_L D} = \frac{\partial_q \pi}{\partial_q D}$.

D Model

D1 Borrower

This section describes borrowers' choice of loans and default. We proceed in two steps. We first characterize the borrower's repayment decision, taking the set of contracts $\mathcal{C}$ as given. We then analyze the borrower's contract choice $\mathcal{C}$ at the initial date. We drop the borrower i index for notational convenience.

D1.1 Default

At the second date, we assume the borrower gets a repayment shock so that they can repay at most a given number of cards, denoted by $\mathcal{R}$ and satisfying $|\mathcal{R}| \leq |\mathcal{C}|$. Formally, $|\mathcal{R}|$ is a random variable whose value realize at the second date. The borrower chooses

which cards to repay r to maximise their indirect utility subject to a constraint on the number of cards they can hold:

$$V(\mathcal{C}, \epsilon, |\mathcal{R}|) := \max_{r} \sum_{c \in \mathcal{C}} \left\{ \mathbf{1}_{\{c \in r\}} [\overbrace{V(q_c) + \epsilon_c - \delta R_c}^{benefit\ of\ repaying}] + \mathbf{1}_{\{r_c \neq c\}} \overbrace{v^d(|r|, |\mathcal{C}|, |\mathcal{R}|)}^{cost\ of\ defaulting} \right\} \quad (50)$$

$$s.t., |r| \leq |\mathcal{R}| \quad (51)$$

$V(q_c) + \epsilon_c - R_c$ is the benefit of repaying the card. When repaying card c, the borrowers get the continuation value $V(q_c) + \epsilon_c$ but must pay the full loan amount R_c.

$v^d(|r|, |\mathcal{C}|, |\mathcal{R}|)$ captures the cost of defaulting. We let it depend on the number of cards owned, the number of cards the borrower defaulted on, and the repayment shock $|\mathcal{R}|$. In practice, the impact on the credit score depends on the fraction of cards the borrower repaid. This formulation also captures that defaulting on one card while still having one is less costly than defaulting on your only card. The rationale is that individuals need cards for daily expenses.

Example: To illustrate the implication of our repayment model, let us compute the probability of repaying card c. Let us assume the ϵ shocks are extreme-value distributed with a variance of σ^{-1}, that the borrower can repay only one card ($|\mathcal{R}| = 1$). We also assume the cost of defaulting is high enough to incentivise repayment of one card. Formally:

$$v^d(|r|, |\mathcal{C}|, |\mathcal{R}|) = \begin{cases} 0, & \text{if } |r| > |\mathcal{R}|, \\ -\infty, & \text{if } |r| \leq |\mathcal{R}|. \end{cases} \quad (52)$$

In that case, denoting $r^*(\epsilon)$ as the optimal repayment strategy, we get that the survival probability from the perspective of a lender not knowing the shocks (ϵ) is:

$$E_\epsilon[\mathbf{1}_{r^*=c} \mid |\mathcal{R}| = 1] := \theta_c(\mathcal{C}) = \frac{e^{\sigma[V(q_c) - \delta R_c]}}{\sum_{x \in \mathcal{C}} e^{\sigma[V(q_x) - \delta R_x]}} \quad (53)$$

Equation 53 implies that, if all lenders offer the same contract terms, the default probability is $\frac{1}{|\mathcal{C}|}$, and the default semi-elasticity with respect to the interest rate and the one with respect to relationship benefits are: $\frac{\partial_R \theta_c(\mathcal{C})}{\theta_c(\mathcal{C})} = -\sigma\delta(1 - \frac{1}{|\mathcal{C}|}) < 0$ and $\frac{\partial_R \theta_c(\mathcal{C})}{\theta_c(\mathcal{C})} = V'(q)\sigma(1 - \frac{1}{|\mathcal{C}|}) \geq 0$ as $V'(q) \geq 0$. With this specification, σ parametrises how strategic the borrowers are. When $\sigma \to 0$, borrowers default with probability $\frac{1}{|\mathcal{C}|}$ no matter the contract terms. When $\sigma \to \infty$, borrowers are extremely strategic and repay the lender as soon as they offer a slightly better contract than their competitors.

D1.2 Number of credit cards

At the first date, the borrower chooses the set of cards $\mathcal{C}$ to maximise their expected utility $U(\mathcal{C}) + e_{\mathcal{C}}$ net of any costs associated with getting credit cards $C(N)$. Following the literature on firm entry choice (see, for instance, Melitz, 2003) we allow the borrower to choose first the number of cards they will get N, then the set of cards they choose $\mathcal{C} \in \mathcal{B}(N)$, where $\mathcal{B}(N)$ is the borrower's consideration set. This assumption allows lenders to perfectly predict the number of cards the borrower will get given their competitors' offer. We relax this assumption in the appendix K and show that it does not affect the economic channels at play.[34] Relaxing this assumption makes lenders have to integrate over the possible number of cards the borrowers get. Given that the channels described are the same conditional on the number of cards (see lenders first-order conditions in Appendix K), the resulting forces are similar but are weighted differently.

The expected utility is given by $U(\mathcal{C}) := u(\mathbf{L}) + \beta \mathbb{E}_{\epsilon,|\mathcal{R}|}[V(\mathcal{C}, \epsilon, |\mathcal{R}|)]$, where $\mathbf{L} := (L_c)_{c \in \mathcal{C}}$ is the vector of credit limits available in the first period and $u(\mathbf{L})$ is the utility coming from credit limits. The term $\beta \mathbb{E}_{\epsilon,|\mathcal{R}|}[V(\mathcal{C}, \epsilon, |\mathcal{R}|)]$ is the expected utility in period two, defined in equation 50, discounted by β. Formally, we have:

$$u^*(N^*, \mathcal{C}^*) := \max_N E[\max_{\mathcal{C} \in \mathcal{B}(N)} U(\mathcal{C}) + e_{\mathcal{C}}] - C(N) \tag{54}$$

The cost of getting N credit cards, $C(N)$, captures, for instance, non-monetary costs such as the mental effort of managing multiple cards, as well as application or search costs. We interpret $C(N)$ as a cost, but it can also be interpreted as additional benefits from obtaining multiple cards that $U(\mathcal{C})$ does not capture.

To make the demand function continuous, we add a demand shock for each set of cards, $(e_{\mathcal{C}})$. As shown in the next section, the demand shocks $e_{\mathcal{C}}$ let us introduce an imperfect degree of substitution between products so lenders can apply a markup.

Example: To illustrate the different economic channels driving demand for multiple credit cards, assume the borrower is offered identical contracts ($L_i = L_j$, $q_i = q_j \; \forall j$). In that case, the utility $U(\mathcal{C})$ depends only on the number of contracts. The borrower's number of card choices is thus:

$$u^*(N^*, \mathcal{C}^*) := \max_N \overbrace{U(N)}^{\text{expected utility of } N \text{ cards}} + \underbrace{E[\max_{\mathcal{C} \in \mathcal{B}(N)} e_{\mathcal{C}}]}_{\text{love for variety}} - \overbrace{C(N)}^{\text{cost of getting cards}} \tag{55}$$

[34] The model presented in the Appendix uses an alternative default function for tractability; this specification does not change the lender's first-order conditions.

Equation 55 illustrates the different costs and benefits of getting an additional card in our model. First, getting a new card increases utility through higher credit limits ($\Delta u(L) = u(L_1, L_2) - u(L_1)$). Second, getting a card impacts the utility in period two. To dig further, assume borrowers either repay all cards with probability $1 - \lambda$ or repay only one (i.e., $\mathcal{R} = 1$ with probability λ). In that case, the new card provides an additional utility of $(1 - \lambda)[V(q) - \alpha^d R]$ in the good state, but also increases the cost of defaulting by λv^d in the bad state. Third, getting a new card impacts the term $E[\max_{\mathcal{C} \in \mathcal{B}(N)} e_{\mathcal{C}}]$. We refer to it as the love for variety: A higher number of cards increases the probability of getting a higher continuation value due to getting a higher ϵ draw.[35] Finally, $\Delta C(2) := C(2) - C(1)$ is the cost of getting an additional card.

E Equilibrium Contracts

In this section, we derive the equilibrium contracts. To gain intuition, we focus on a particular parametrisation of the model that yields closed-form formulas for the equilibrium contracts.

E1 Parametrization

The parametrisation is based on the example provided in Section D1. We discuss the impact of our parametrisation on contract terms and the economics channel at play in more depth in Section E2.

Continuum of lenders: We consider a continuum of lenders. This approach allows us to use the first-order condition of equation **??** to derive a tractable expression for the equilibrium number of cards and conduct comparative statics. Under this assumption (the technical details are in Appendix F), the equilibrium conditions are unchanged relative to Section 5.1, except that (N) now denotes the mass of cards held by the borrower.

Parametrisation : We follow the parametrisation used in sections D1 and Section 5.1 examples. We assume that borrowers' utility is linear in loan size and relationship benefits, $(u(\mathbf{L}) = \alpha \sum_c L_c)$ and $(V(q) = \gamma q)$. The cost of lending and providing benefits is additively separable: $\psi(L, q) = mc(L) + \kappa(q) = \frac{mc}{\psi}[exp(\psi L) - 1] + \frac{\kappa}{\phi}[exp(\phi q) - 1]$. The demand and default shocks are independent and extreme-value distributed, with (σ_d) and (σ) parameterising the inverse variance of the default shocks and the variance

[35] Our modeling of default choices with ϵ introduces a love for variety. One can use the cost function C to cancel those effects so that ϵ affects default sensitivities without affecting demand for cards.

of the demand shocks, respectively. We normalise the random shocks so the love-of-variety effect disappears (i.e., $E[\max_{\mathcal{C}\in\mathcal{B}(N)} e_{\mathcal{C}}] = 0$, see Appendix F for the derivation). We denote the default semi-elasticity and the inverse demand semi-elasticity by $d := -\frac{\partial_V \theta}{\theta} = \sigma_d > 0$ and $\varepsilon^{-1} := \frac{D(\mathcal{C})}{\partial_U D(\mathcal{C})} = \frac{\sigma}{1-\frac{N}{B}}$ where B is the mass of lenders (see Appendix F for the derivation).

For the credit market to exist, we assume that the cost of default is sufficiently high that borrowers have incentives to repay a mass of one loans.

To focus on selective default, we assume that loan demand is sufficiently high that borrowers hold more loans than they can repay following an adverse repayment shock. Formally, we consider that the costs of searching for loans $C(N)$ are low enough so that $\{N : C'(N) = 0\}$ is larger than one.

We use the same parametrisation of the cost of defaulting as in Section 5, equation 52. This assumption implies that default choices have a logit form as in equation 53. Our model thus focuses on which cards borrowers choose to repay, rather than why the borrower chose to default in the first place. Having the shock on the fraction of the face value you can repay instead on the number of loans does not change the key economic forces (equation 5.1 remains the same, and borrowers still get too many loans due to the default externality (see, for instance, Green and Liu, 2021). Additionally, empirically, the causal effect of interest rates on default is low in the credit card market (see, for instance, Matcham, 2024), so our model is not a bad approximation of reality in that set-up.

To simplify the notation, we assume that $(|\mathcal{R}|)$ is known and equal to one. These assumptions can be relaxed without affecting the economic forces we seek to highlight. For instance, if $(|\mathcal{R}| = N)$ with probability $(1 - \lambda)$ and $(|\mathcal{R}| = 1)$ with probability (λ), the default semi-elasticity and the equilibrium default formulas change quantitatively, but the qualitative results in this section are unchanged.

E2 Equilibrium Contracts

We look for a Nash equilibrium. At date one, lenders observe the contracts offered by their competitors and choose their own contract to maximize profits, taking competitors' contracts and borrower behavior as given.

We focus on a symmetric equilibrium. When all lenders offer identical contract terms, the default probability, the number of loans, and the equilibrium contract terms admit convenient closed-form expressions. In that case, each lender has the same de-

fault probability. The symmetric equilibrium is characterised by:

$$\theta_c^* = 1 - \lambda + \lambda \frac{1}{N^*} \tag{56}$$

$$mc'(L_c^*) = \alpha(1 - d\delta R_c^*) \tag{57}$$

$$q_c^* := \left\{ q : \beta V'(q) \Big[d(1-\delta)\theta_c^* R_c^* + \theta_c^* \Big] = \kappa'(q^*) \right\} \tag{58}$$

$$\theta_c^* R_c^* = \psi(L^*, q^*) + \Big[1 - d\delta R\Big] \varepsilon^{-1} \tag{59}$$

$$C'(N^*) = \alpha L^* + (1-\lambda)\beta[V(q) - \delta R] + \lambda\omega \tag{60}$$

Proof. Take the first-order condition of the welfare and utility functions. □

Comparing this system of equations with that in Section 5 clarifies the role of our parametrization. Importantly, the parametrization does not shut down any of the economic channels of the general model. Instead, our functional-form assumptions deliver closed-form expressions for the key objects governing equilibrium contract terms: the demand semi-elasticity and hence the markup ($(1 + \frac{\partial_R \theta_b}{\theta_b} R)\frac{D(\mathcal{C})}{\partial_U D(\mathcal{C})} = (1 - d\delta R)\varepsilon^{-1}$), the ratio of the marginal utility of lending to its marginal cost ($\frac{mc_L}{\partial_L u(\mathbf{L})} = \frac{\alpha}{mc} = 1 - d\delta R$), and the default semi-elasticity ($\partial_{V(q)}\theta_c^* = d$). Each of these objects plays a central role in the provision of relationship benefits. The parametrization therefore preserves the key economic forces of the general model while making them transparent, allowing us to isolate and characterize the effect of selective default, summarized by (d), on equilibrium contract terms.

Equation (60) is the continuum-of-cards counterpart of equation (55) from section D1. It follows from the first-order condition of equation (54), using the fact that expected utility conditional on the offered contracts is $U(N) = \alpha L^* N + \gamma q^* - R^*$.

Equation (60) equates the marginal cost of obtaining an additional loan to the marginal utility generated by the additional credit limit. Because borrowers expect to repay a mass of only one loan, obtaining additional loans does not increase their expected relationship benefit q: although the borrower holds multiple contracts, relationship benefits are realised only on the loans that are repaid. As a result, the marginal benefit of an additional loan is driven primarily by its short-term credit provision, L. This feature captures the idea that, under selective default, borrowers value additional loans mainly for the liquidity they provide before making repayment decisions. A more general specification of the repayment shock would allow relationship benefits to enter the condition determining the number of loans. However, their contribution would be attenuated by the probability that the corresponding card is repaid, which is strictly below one under selective default. Our parametrisation therefore isolates this mechanism while preserving a tractable characterisation of the equilibrium.

$$\theta_c^* = \frac{1}{N^*}, \tag{61}$$

$$R_c^* = \frac{\frac{\alpha - mc}{\psi} + \frac{\gamma}{\phi N^*} - \frac{\kappa}{\phi} + \varepsilon^{-1}}{\frac{1}{N^*} + \frac{\alpha d\delta}{\psi} - \frac{\gamma d(1-\delta)}{\phi N^*} + d\delta\varepsilon^{-1}}, \tag{62}$$

$$L_c^* = \frac{1}{\psi}\log\left[\frac{\alpha\left(1 - d\delta R_c^*\right)}{mc}\right], \tag{63}$$

$$q_c^* = \frac{1}{\phi}\log\left[\frac{\gamma\left(1 + d(1-\delta)R_c^*\right)}{\kappa N^*}\right]. \tag{64}$$

$$\Psi N^* = \alpha L_c^* \tag{65}$$

To get intuition about how the parameters affect contract terms, let us approximate the model around δ close to zero. We get that:

$$\theta_c^* = \frac{1}{N^*}, \tag{66}$$

$$\frac{R_c^*}{N^*} = \frac{\alpha - mc}{\psi} + \frac{\gamma}{\phi N^*} - \frac{\kappa}{\phi} + \varepsilon^{-1} \tag{67}$$

$$L_c^* = \frac{1}{\psi}(\log\left[\frac{\alpha}{mc}\right] - d\delta R_c^*) \tag{68}$$

$$q_c^* = \frac{1}{\phi}\log\left[\frac{\gamma\left(1 + d(1-\delta)R_c^*\right)}{\kappa N^*}\right] \tag{69}$$

$$\Psi N^* = \alpha L^* \tag{70}$$

The complete solution is

$$\boxed{\begin{aligned} N^* &= \frac{\alpha(\ell - d\delta G)}{\psi\Psi + \alpha d\delta A}, \\ \theta_c^* &= \frac{\psi\Psi + \alpha d\delta A}{\alpha(\ell - d\delta G)}, \\ L_c^* &= \frac{\Psi(\ell - d\delta G)}{\psi\Psi + \alpha d\delta A}, \\ R_c^* &= \frac{\alpha A\ell + \psi\Psi G}{\psi\Psi + \alpha d\delta A}, \\ q_c^* &= \frac{1}{\phi}\left\{\log\left[\frac{\gamma(\psi\Psi + \alpha d\delta A)}{\kappa\alpha(\ell - d\delta G)}\right] + d(1-\delta)\frac{\alpha A\ell + \psi\Psi G}{\psi\Psi + \alpha d\delta A}\right\}. \end{aligned}} \tag{71}$$

where

$$\boxed{\ell = \log\left(\frac{\alpha}{mc}\right), \qquad A = \frac{\alpha - mc}{\psi} - \frac{\kappa}{\phi} + \varepsilon^{-1}, \qquad G = \frac{\gamma}{\phi}.} \tag{72}$$

F Continuum of lenders

This section first derives the demand and default functions with a discrete number of lenders, then takes the continuum-of-lenders limit.

Default choices: Let us go from the discrete number of cards to a continuum. A convenient way to obtain a well-defined continuum limit is to define $\mu_J = \sum_{j=1}^{J} w_{j,J}\delta_{d_{j,J}}$, where $w_{j,J} > 0$ denotes the mass associated with lender j, and suppose that μ_J tend to a measure μ as $J \to \infty$.

let $\varepsilon_{j,J}. = \widetilde{\varepsilon}_{j,J} + \sigma \log w_{j,J}$ where $\widetilde{\varepsilon}_{j,J} \sim \text{Gumbel}(0, \sigma)$. Using this, we can replace the sum by integrals when calculating the probabilities and demand semi-elasticities.

For instance, the inclusive value is $\mathbb{E}\left[\max_j \left\{u(d_j) + \varepsilon_{j,J}\right\}\right] = \sigma \log\left[\Delta_J \sum_{j=1}^{J} \exp\left(\frac{u(d_j)}{\sigma}\right)\right] + \gamma\sigma$ where γ denotes the Euler–Mascheroni constant. Taking $J \to \infty$ (thus $\Delta_J \to 0$), the Riemann sum converges to an integral $\Delta_J \sum_{j=1}^{J} \exp\left(\frac{u(d_j)}{\sigma}\right) \longrightarrow \int_{\mathcal{D}} \exp\left(\frac{u(x)}{\sigma}\right) d\mu(x)$. Where $d\mu(x)$ is the measure of each borrowers.

To make the equations match the discrete number-of-cards model, we assume borrowers own a set of cards $\mathcal{C}$ of mass N and repay on a set of cards A of mass one.

Denoting $u(x)$ as the utility of repaying the card absent the default shock, the probability of repaying a set of cards of A when owning the set N is thus:

$$Pr(A) = \frac{\displaystyle\int_A \exp\left(\frac{u(x)}{\sigma}\right) d\mu(x)}{\displaystyle\int_{\mathcal{C}} \exp\left(\frac{u(x)}{\sigma}\right) d\mu(x)}. \tag{73}$$

conveniently, ender b default semi-elasticity is:

$$\sigma_d := \frac{1}{\sigma}[1 - \frac{1}{|\mathcal{C}|}] \tag{74}$$

We refer to $Pr(b) = \frac{1}{|\mathcal{C}|} := \frac{d\mu(b)}{\int_{\mathcal{C}} d\mu(b)}$ as the probability of defaulting on borrower b. We refer to $|\mathcal{C}|$ as the mass of cards.

Demand choices: The demand follows the same argument. Suppose the utility

from choosing bundle $\mathcal{C}$ is:

$$U(\mathcal{C}) = V(\mathcal{C}) + \varepsilon_{\mathcal{C}}, \qquad \varepsilon_{\mathcal{C}} = \widetilde{\varepsilon}_{j,J} + \sigma log(\frac{\Delta_J}{\binom{B}{N}}), \text{where } \widetilde{\varepsilon}_{j,J} \sim \text{Gumbel}(0, \sigma) \tag{75}$$

$\binom{B}{N}$ is the number of bundles with N cards one can construct when there are B banks. This normalisation of the random shock is similar to the effect of (Δ_J). It keeps utility from blowing up when there is a continuum of lenders. Its only effect is to normalise the love for variety so that it becomes a constant function of the number of cards, effectively cancelling it. This is great from our modelling perspective as we do not want to focus on this channel.

Denoting M as the set of bundles available, the probability of choosing bundle $\mathcal{C}$ is therefore:

$$P(\mathcal{C}) = \frac{\exp\left(V(\mathcal{C})/\sigma\right)}{\sum_{T \in M} \exp\left(V(T)/\sigma\right)}. \tag{76}$$

Now, denoting C as the set of bundles with card c in it, the probability of choosing a bundle with card c in it is:

$$P(c) = \frac{\sum_{c \in C} \exp\left(V(\mathcal{C})/\sigma\right)}{\sum_{T \in M} \exp\left(V(T)/\sigma\right)}. \tag{77}$$

Similarly, the continuum version is:

$$P(c) = \frac{\int_C \exp\left(V(\mathcal{C})/\sigma\right) d\mu(\mathcal{C})}{\int_M \exp\left(V(\mathcal{C})/\sigma\right) d\mu(\mathcal{C})}. \tag{78}$$

$P(c)$ is the number of bundles with card c over the number of bundles. With discrete number of cards this is equal to the fraction of the number of cards over the number of lenders:

Proof. Assuming a bundle is an unordered set of x distinct balls, drawn from n balls numbered $1, \ldots, n$. The total number of different bundles is:

$$\boxed{\binom{n}{x} = \frac{n!}{x!(n-x)!}}. \tag{79}$$

We denote this number by M.

For bundles that contain ball 1, fix ball 1 in the bundle. You then need to choose the remaining $x-1$ balls from the other $n-1$ balls:

$$\boxed{\binom{n-1}{x-1}}. \tag{80}$$

Therefore, the fraction of all bundles that contain ball 1 is

$$\frac{\binom{n-1}{x-1}}{\binom{n}{x}} = \boxed{\frac{x}{n}}. \tag{81}$$

□

With a continuum of lenders, denoting B as the mass of lenders, we have the same formula as in the standard logit model:

$$\varepsilon^{-1} := \frac{\sigma}{1-\frac{N}{B}} \tag{82}$$

Similar to the default choices section, the inclusive value when all contracts are the same is:

$$V(\mathcal{C}) + E[\max \epsilon_{\mathcal{C}}] = V(\mathcal{C}) + \sigma ln(M(N)) = E[\max \epsilon_{\mathcal{C}}] = V(\mathcal{C}) \tag{83}$$

In its discrete-time version, $M(N)$ is the number of bundles. It decreases with the number of cards: larger bundles mean less variety, so one can create fewer of them. This property does not reflect the economic channel we are interested in. In the continuum case, this number would blow up unless one normalises the random shocks. With our normalisation of the random shock ($\lim_{(B,N)\to\infty} \frac{1}{\binom{B}{N}} \int_T d\mathcal{C} = 1$), the mass of bundles is always one, so the love of variety disappears.

G Proof Linear model

Let us consider that the borrower gets multiple cards and that every competitor of lender b offers a contract that maximizes the debt capacity $R_c = W$ for all c. Let us show that lender b then offers $R_b = W$.

Lender b knows the borrower will default on all but one credit card. If the borrower chooses to repay lender b, given that the marginal rate of transformation $\frac{\alpha}{mc_L} > 1$ on credit card loans is higher than the one on interest rates ($\frac{1}{1+\theta_R} = 1 < \frac{\alpha}{mc_L}$ with θ_R jumping at $W = R$), the lender b then optimally sets $W = R$ (see equation 42 of section C for the derivation of the first order conditions). Setting $W < R$ is not optimal as default is costly (since $V_c - R_c + \epsilon_c > v^d, \forall \epsilon_c$).

Given that $R_c = W$ for all c, borrowers choose multiple cards and default when the condition $\alpha L^* + \beta v^d + \delta(2) > \Delta C(2))$ is satisfied.

Other equilibrium: Due to discontinuities in our model, depending on the parameters, the equilibrium with default might not be the only one. In particular, there might be an equilibrium without default. For instance, let us assume that the borrower is exactly at his budget constaint (i.e., $\sum_c R_c^* = W$). In that case, existing lenders might not have incentive to increase the credit limit and thus the interest rates (as θ_{R_c} jumpings at $R_c = R_c^*$). The reason is that it would generate default, which can lower borrowers' utility (this is the case when $(\alpha - cost)\Delta L + \beta E[\sum \mathbf{1}_{\{r^*(\epsilon)>R\}}(V(q) - R + \epsilon - v^d) + v^d] < \beta E[V(q) - R + \epsilon])$.

Equilibrium Selection: We focus on an equilibrium with default as this is consistent with the data and makes competition for informal seniority relevant. Default would also appear naturally in a more general model when income is a random variable, making the equilibirum we pick relevant to understand selective default forces that would arise in the general model. We focus on the case where income is not random for tractability. While the assumption about default being constants affects the shape of the default function and default elasticity, the first order conditions would still be the same. The economic forces at play would thus remain the same.

Uniqueness: One simple way to make the equilibrium with no default and $R = W$ unique in our model is to assume that the cost of default is increasing in the share of credit card the borrower defaulted (denoted s). One can set those cost so that borrowers have incentives to default on all but one card (i.e., choosing $v^d(s)$ large enough when $s < 1$ so that $V_c - R_c + \epsilon_c > v^d, \forall \epsilon_c$ is satisfied only for $s = \frac{|\mathcal{C}^*|-1}{|\mathcal{C}^*|}$ where $\mathcal{C}^*$ is such that $\alpha L \leq F(|\mathcal{C}^*| + 1))$.

The assumption that defaulting on one card when you have, for instance two, is less costly than when you have one make sense empirically as the credit score impact is lower when you default. It is also arguably less costly for borrowers as they can still use one of the card to make important transactions.

H Examples of micro-foundations for V_c

Let us now discuss different functional forms for V_c that can capture the stylized facts presented in Section 3.2. The empirical section suggests that three main channels can affect repayment behavior. First, the lenders can invest in an app or a reward system that the borrower values, making the threat of cancelling the card upon default more costly. Second, the lender can change the credit limit on the card depending on the

repayment behavior. Third, the lender can offer another loan product in the second period if the borrower does not default on the card.

Those three channels can be captured using the following parametrization of V_c:

$$\text{App or reward system: } V_c(q_c, \mathcal{R}) = q_c(\mathcal{R}) \tag{84}$$

$$\text{Credit limit: } V_c(\tilde{L}_c, \tilde{R}_c, \mathcal{R}) = \tilde{L}_c(\mathcal{R}) - \beta \tilde{R}_c(\mathcal{R}) \tag{85}$$

$$\text{Other Loan product: } \sum_c V_c(\underbrace{\tilde{L}_c, \tilde{R}_c}_{q_c}, \mathcal{R}) = E_{\varepsilon_b}[\max_b \{\tilde{L}_b(\mathcal{R}) - \beta \tilde{R}_b(\mathcal{R}) + \varepsilon_b\}] \tag{86}$$

In Equation 84, q_c can be interpreted as the app quality or the generosity of the reward system. In Equations 85 and 86, $\tilde{L}_c$ and $\tilde{R}_c$ are the loan size and the debt face value for the credit card and the other loan product, respectively. The variables q_c, $\tilde{L}_c$ and $\tilde{R}_c$ are functions of $\mathcal{R}$. That is, lenders can condition their value based on the borrower's repayment behavior, potentially canceling access to the bank app or the rewards if the borrower does not repay the credit card.

The timing assumption behind Equations 85 and 86 is the following, and requires adding one period to the model. After seeing the repayment in period 2, the lender changes the credit card contract to offer a new loan product. The repayment $\tilde{R}$ of the new contract happens at the end of a third period. The benefit of borrowing $\tilde{L}$ happens in the second period. For simplicity, we assume that the borrower cannot get a new credit card contract in period 2 (for Equation 85), or that the other loan contracts are exclusive in the second period (for Equation 86). This assumption is in line with the empirical evidence that other contracts offered are mortgages or car loans, for which borrowers typically only get one. The fact that the borrower does not get a new card (for Equation 85) can be micro-founded by a model in which the repayment shocks in the third period are perfectly correlated with those in the second period. In that case, since the next section shows that the borrower repays only one contract in the second period, the lender learns that if they have seniority in the third period, and are thus the only lender.

The following section I shows that when lending generate a positive net present value, there exist an equilibrium in which it is optimal to exclude borrowers in the second period (i.e., set $q_c = L_c = R_c = 0$ if $c \notin \mathcal{R}$) if they did not repay their debt, and to set $\tilde{R} = \tilde{W}$ if they repays, where $\tilde{W}$ is the borrower wage in the third period. In that case, all three equations are equivalent: They are all increasing functions of one variable (q_c or $\tilde{L}_c$) conditional on repayment. V_c can be normalized to zero conditional on defaulting on card c. This justifies the general approach used in the main text.

I Examples of Micro-foundations for c_c

Using the three micro-foundations discussed in Section H, potential functional forms for the cost function are:

$$\text{Rewards: } c_c(q_c, \theta_c(R_c, q_c)) := c_c(max\{q_c(\mathcal{R})\}) \text{ positive convex and increasing} \quad (87)$$

$$\text{Credit limit: } c_c(R, \tilde{R}, \tilde{L}, \theta_c(R, \tilde{R}, \tilde{L})) = -\theta_c(R, \tilde{R}, \tilde{L})\{\tilde{R} - mc_L\tilde{L}\} \quad (88)$$

$$\text{Other Loan: } c_c(R, \tilde{R}, \tilde{L}, \theta_c(R, \tilde{R}, \tilde{L})) = -\theta_c(R, \tilde{R}, \tilde{L})\tilde{D}(\tilde{R}, \tilde{L})\{\tilde{R} - mc_L\tilde{L}\} \quad (89)$$

In Equation 87, we consider that setting a generous reward system requires the lender to set aside some money in the first period $c(max\{q_c(\mathcal{R})\})$. The operator $max\{q_c(\mathcal{R})\}$ captures the fact that the bank pays the cost in the first period and should ensure to have invested enough to provide the maximum reward it committed to.

In equations 88 and 89, the costs are paid in the second period, after the repayment $\mathcal{R}$ is observed. We drop the $\mathcal{R}$ from the variables $(R, \tilde{R}, \tilde{L})$ to simplify the notation. The cost function can be positive or negative depending on the markup the lender sets for the other product, sold in the second period. In Equation 89, $D(\tilde{R}, \tilde{L})$ is the probability of choosing bank c in the second period. It is derived from Equation 86.

As in section 5.2, when lending is a positive NPV, it is optimal to set $\tilde{R} = \tilde{W}$, where $\tilde{W}$ is the borrower's income in the second period. All the equations are thus functions of one variable (q or $\tilde{L}$).

J Condition Costs Functions

For the lender to know the borrower will always get the same number of cards $|\mathcal{C}|$, we need the cost function to be such that:

$$C(|\mathcal{C}|) \leq u(L_b, L_{-b}(|\mathcal{C}| - 1)) + \min\{E[V(\mathcal{C})] + e_c\} \quad (90)$$

$$\leq u(L_b, L_{-b}(|\mathcal{C}|)) + \max\{E[V(\mathcal{C}')] + e_c\} \leq C(|\mathcal{C}| + 1),\ \forall L_b, q_b \quad (91)$$

$L_{-b}(x)$ is the vector of size x, composed of the x higest loan size contracts other than contract b.

$\mathcal{C}'$ is the set of cards with card b and card $-b$.

L_b, q_b belong to a bounded set. This set is such that the bank has to make positive profits.

$\min\{E[V(\mathcal{C})] + e_c\}$ can be replaced by $|(\mathcal{C}|(v^d + \underline{e}_c)$, where $\underline{e}_c$ is a lower bound on $\underline{e}_c$, and $\max\{E[V(\mathcal{C}')] + e_c\}$ by $|(\mathcal{C}|[V(\bar{q})]$ where $\bar{q}$ is an upper bound on q_c.

One easy way to satisfy this equation is by setting the cost equal to 0 when the number of cards is below a threshold and infinity above.

K Numerical Simulation

Utility : To perform the numerical simulation, we model the default choice as an effort choice problem as in Green and Liu (2021):

$$U(\mathcal{C}) = \max_{e_c} \sum_{c \in \mathcal{C}} \{u(L_c) + \theta(e_c)[u(V(q_c) - R_c) - v^d(e)] + v^d(e)\} \tag{92}$$

$$s.t. \sum_c e_c = W \tag{93}$$

$v^d(e)$ captures the idea that the cost of defaulting is lower when the lender sees that the borrower has put effort into repaying. This modelling assumption is done for tractability and does not change the key economic channels affecting default; they just affect the functional forms.

Default : Using the first order condition and denoting λ the shadow cost, we get:

$$\theta'[u(V(q_c) - R_c) - v^d] + (1-\theta)v^{d'} = \lambda \tag{94}$$

Assuming $\theta(e_c) = \alpha + \beta e$ and $v^d := v + \nu e$, Using the budget constraint, the survival probability, denoted θ_c is:

$$e_c = \frac{\overbrace{\beta[u(V(q_c) - R_c) - v] + (1-\alpha)\nu}^{U_c} - \lambda}{2\beta\nu} \tag{95}$$

$$\implies \sum_{\mathcal{C}} \frac{\beta[u(V(q_c) - R_c) - v] + (1-\alpha)\nu - \lambda}{2\beta\nu} = W \tag{96}$$

$$\iff \lambda = \frac{\sum_{\mathcal{C}}\{\beta[u(V(q_c) - R_c) - v] + (1-\alpha)\nu\} - W2\beta\nu}{|\mathcal{C}|} \tag{97}$$

$$\implies \theta(e_c) = \alpha + \frac{\nu^{-1}}{2}[U_c - E[U_c]] + \beta\nu\frac{W}{|\mathcal{C}|} \tag{98}$$

The survival probability is thus decreasing in the number of credit cards ($|\mathcal{C}|$) and increasing in how good the card is relative to other cards in the portfolio ($U_c - E[U_c]$)

Choice of cards: We consider that the lender observes the cards the borrower chooses $\mathcal{C}$ and offers a contract that depends on that

$$\max_{\mathcal{C}} \overbrace{U(\mathcal{C}) - C(|\mathcal{C}|)}^{\tilde{U}(\mathcal{C})} + e_{U(\mathcal{C})} \tag{99}$$

Where $e_{U(\mathcal{C})}$ is extreme value distributed. We consider that the lender b observes

the set of competitors' cards held by the borrowers ($\mathcal{C}_{-b}$) and then chooses the contract terms conditional on that set. The demand thus has a logit form:

$$D(U(\mathcal{C})) = \frac{exp(\tilde{U}(\mathcal{C}))}{\sum_c exp(\tilde{U}(c))} \tag{100}$$

Contract terms : Lenders set contract terms to maximise profits. Denoting $\mathcal{C}_b$ the set of credit card portfolios containing the lender b card, lender maximization problem is:

$$\sum_{\mathcal{C}\in\mathcal{C}_b} D(U(\mathcal{C}))\underbrace{[\theta(\mathcal{C})R - costL]}_{\pi(\mathcal{C})} \tag{101}$$

Using the envelope theorem, we get that the sensitivity of $D(U(\mathcal{C}))$ to R and q are $-\theta_c((\mathcal{C}))D(U(\mathcal{C}))$ and $\theta_c((\mathcal{C}))V'(q)D(U(\mathcal{C}))$.

Using the first-order conditions, we get:

$$\begin{aligned} &R: \sum D(U(\mathcal{C}))[\theta_R(\mathcal{C})R + \theta(\mathcal{C})] = \sum D'(U(\mathcal{C}))\theta(\mathcal{C})\pi(\mathcal{C}) \\ \iff &\sum D_R\theta R = \sum D_R mc + D[\theta_R(\mathcal{C})R + \theta(\mathcal{C})] \end{aligned} \tag{102}$$

$$\begin{aligned} &L: mc_L \sum D(U(\mathcal{C})) = \sum D'(U(\mathcal{C}))u'(L)\pi(\mathcal{C}) \\ \iff &u'(L) = mc_L \frac{\sum D(U(\mathcal{C}))}{\sum D'(U(\mathcal{C}))\pi(\mathcal{C})} \end{aligned} \tag{103}$$

$$\begin{aligned} &V: \sum D(U(\mathcal{C}))[\kappa'(q) - \partial_q\theta_c R] = V'(L)\sum D'(U(\mathcal{C}))\theta(\mathcal{C})\pi(\mathcal{C}) \\ \iff &V'(q) = \frac{\sum D(U(\mathcal{C}))[\psi'(q) - \partial_q\theta_c R]}{\sum D'(U(\mathcal{C}))\theta(\mathcal{C})\pi(\mathcal{C})} \end{aligned} \tag{104}$$

In a symmetric equilibrium, we have:

$$\theta_c(\mathcal{C}) = \alpha + \beta\nu\frac{W}{|\mathcal{C}|} \tag{105}$$

$$\theta_X = \frac{\nu^{-1}}{2}[1 - \frac{1}{|\mathcal{C}|}]\partial_X U_c \tag{106}$$

Simplification: If the demand for cards is such that the borrowers get n cards (i.e., $C(|\mathcal{C}|)$ is equal to minus infinity whenever $|\mathcal{C}|$ is different than n), the problem is simpler. In that case, the demand and default functions are the same regardless of the chosen bundle. In that case, the system of equations can be written exactly as in the previous model:

$$R : \theta R = mc(L) + [1 - \tilde{\varepsilon} R]\varepsilon^{-1} \tag{107}$$

$$L : u'(L) = \frac{\psi'(L)}{1 - \tilde{\varepsilon} R} \tag{108}$$

$$V : V'(q)\theta[\frac{mc}{u'(L)} - \tilde{\varepsilon} R] = c'(q) \tag{109}$$

with $\varepsilon^{-1} := \frac{\sum D(U(\mathcal{C}))}{\sum D'(U(\mathcal{C}))}$ which is equal to a constant in a symmetric equilibrium. $\tilde{\varepsilon} := \frac{\partial_R \theta(n)}{\theta(n)}$ is also a contant (equal to $\frac{\frac{\nu^{-1}}{2}[1-\frac{1}{n}]}{\alpha+\beta\nu\frac{W}{n}}$).

Solving for R using equations 107 and 108, we get:

$$R = \frac{mc(L) + \varepsilon^{-1}}{\theta + \tilde{\varepsilon}\varepsilon^{-1}} \tag{110}$$

$$u'(L) = \frac{\psi'(L)}{1 - \tilde{\varepsilon}\frac{mc(L)+\varepsilon^{-1}}{\theta+\tilde{\varepsilon}\varepsilon^{-1}}} \tag{111}$$

Considering that $u'(L) = \gamma log(L)$ and $mc(L) = mc \cdot L$, we get:

$$L^* = \gamma \frac{1 - \tilde{\varepsilon}\frac{\varepsilon^{-1}}{\theta+\tilde{\varepsilon}\varepsilon^{-1}}}{mc(1 + \gamma\frac{\tilde{\varepsilon}}{\theta+\tilde{\varepsilon}\varepsilon^{-1}})} \tag{112}$$

$$R^* = \frac{mcL^* + \varepsilon^{-1}}{\theta + \tilde{\varepsilon}\varepsilon^{-1}} \tag{113}$$

Using it into equation and assuming $V'(q) = V \cdot q$ and $c'(q) = cq$, we get:

$$V'(q)\theta[\frac{mcL^*}{\gamma} - \tilde{\varepsilon} R^*] = c'(q) \tag{114}$$

$$q^* = \frac{V\theta[\frac{mcL^*}{\gamma} - \tilde{\varepsilon}\frac{costL^*+\varepsilon^{-1}}{\theta+\tilde{\varepsilon}\varepsilon^{-1}}}{c} \tag{115}$$

$$q^* = \frac{V\theta[mcL^*[\frac{1}{\gamma} - \tilde{\varepsilon}\frac{1}{\theta+\tilde{\varepsilon}\varepsilon^{-1}}] - \tilde{\varepsilon}\frac{\varepsilon^{-1}}{\theta+\tilde{\varepsilon}\varepsilon^{-1}}}{c} \tag{116}$$

The borrower's welfare is:

$$W = n\{\gamma log(L^*) + \theta[Vq^* - R^* - v^d(e^*)] + v^d(e^*)\} \tag{117}$$

with:

$$\theta = \alpha + \frac{\beta\nu}{n} \tag{118}$$

$$v^d(e^*) = v + \nu e^* \tag{119}$$

$$e^* = \nu\beta\frac{W}{n} \tag{120}$$

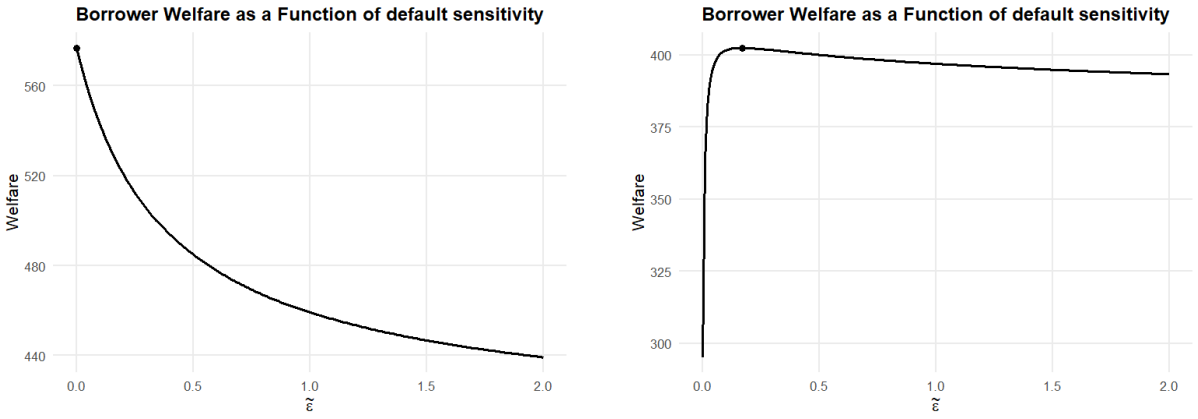
$$L^* = \gamma\frac{1 - \tilde{\varepsilon}\frac{\varepsilon^{-1}}{\theta+\tilde{\varepsilon}\varepsilon^{-1}}}{mc(1 + \gamma\frac{\tilde{\varepsilon}}{\theta+\tilde{\varepsilon}\varepsilon^{-1}})} \tag{121}$$

$$R^* = \frac{mcL^* + \varepsilon^{-1}}{\theta + \tilde{\varepsilon}\varepsilon^{-1}} \tag{122}$$

$$q^* = \frac{V\theta[\frac{mcL^*}{\gamma} - \tilde{\varepsilon}R^*]}{c} \tag{123}$$

The survival probability θ is a parameter (parametrized by α and the number of cards). The markup ε^{-1} is also a free parameter. $\tilde{\varepsilon}$ is parametrized by ν and the effort benefits by β. We can thus focus on changing each parameters $(\varepsilon^{-1}, \tilde{\varepsilon}, \theta)$ holding effort e^* constant. This allows our comparative statics not to depend on our modeling choice of e^*.

The figure below reports the result of the simulation for parameters close to our estimates



(**a**) Welfare when competition is high: equal recovery increases welfare

(**b**) Welfare when competition is low: equal recovery reduces welfare

Figure A14: Borrower Welfare as a function of default sensitivities

L Cancelling card upon not repaying

The section shows that it is optimal for the lender to cancel the card, conditional on the borrower not repaying the full debt face value. To illustrate the case in which q can be interpreted as offering a new credit card, we define relationship benefits as $q = (R, L)$,

where R and L are the face value and credit limit in a third period.

Formally, let us look at the lender problem at date 2 before the borrower makes his repayment choice. Lender b conditions the credit card offer in the second period on repayment r_b made by the borrower. Denoting the credit card offer $(L(r_b), R(r_b))$ in the second period, the lender's objective function is:

$$\max_{L(r_b),R(r_b)} \int r_b + [R_b(r_b)\theta((r_c)_c) - costL(r_b)]dF(r_b) \tag{124}$$

$$\text{s.t. } F(r_b) \text{ defined by problem 126} \tag{125}$$

Let us assume that the borrower repayment shocks ϵ are persistent for the first and second contract, so that lender b expects to lose seniority with probability one with the new loan contract if they lost it with the old one, that is, $\theta((r_c)_c)$ is zero if they repaid the other lender. This simplifies the exposition by making the second contract "exclusive" in the second period, as only one lender is willing to lend.

The borrower in the second period chose which card to repay. Their maximization problem is:

$$\begin{aligned} &\max_{r:=(r_c)_c} W + \sum_c \alpha L_c(r_c) - R_c(r_c) - r_c + \epsilon_c \\ s.t. &\sum r_c \leq W \\ &0 \leq r_c \leq W \end{aligned} \tag{126}$$

Suppose the other lenders offer $\bar{L}_c$ and $R(r_b) = W$ if the borrower repays the debt in full and zero otherwise. We now show that doing this strategy is also the best response for lender b. The maximisation problem implies that the borrower repays only one card.

Using the borrower problem 126, the probability of lender b being repaid is thus: $Pr(\alpha\bar{L}_c + \epsilon_c \leq max_{r_b \leq W} W - \alpha L(r_b) - r_b + \epsilon_b)$. For the lender, the problem is then equivalent to choosing r_b and L instead of the pricing schedule. Lender utility is maximized at $r_b = W$ as $\alpha > cost$. One way to implement the optimal contract is by offering $\bar{L}_c$, $R(r_b) = W$ if the borrower repays the debt in full and zero otherwise. The lender's problem is thus to choose the loan size conditional on repayment L_b. Formally, the lender problem is:

$$\max_{L_b} \overbrace{Pr(\alpha L_b + \epsilon_b \geq \alpha L_c + \epsilon_c \; \forall c)}^{\theta(L_b)}[W_1 + W_2 - costL_b] \tag{127}$$

W_1 is the repayment from the first loan and W_2 from the second. The first-order con-

ditions give the optimal L_b.

M Proof of existence and uniqueness

Equilibrium conditional on N. Consider an arbitrary number of cards N, the following set of equations characterises the unique equilibrium:

$$\theta_c^* = 1 - \lambda + \frac{\lambda}{N^*}, \tag{128}$$

$$\varepsilon_d^* = \sigma\delta\left(1 - \frac{1}{N^*}\right)\frac{\lambda/N^*}{1-\lambda+\lambda/N^*}, \tag{129}$$

$$R_c^* = \frac{\dfrac{\alpha - mc}{\psi} + \dfrac{\beta\gamma\theta_c^* - \kappa}{\phi} + \varepsilon^{-1}}{\theta_c^* + \dfrac{\alpha\varepsilon_d^*\delta}{\psi} - \dfrac{\beta\gamma\theta_c^*\varepsilon_d^*(1-\delta)}{\phi} + \varepsilon_d^*\delta\varepsilon^{-1}}, \tag{130}$$

$$L_c^* = \frac{1}{\psi}\log\left[\frac{\alpha\left(1 - \varepsilon_d^*\delta R_c^*\right)}{mc}\right], \tag{131}$$

$$q_c^* = \frac{1}{\phi}\log\left[\frac{\beta\gamma\theta_c^*\left(1 + \varepsilon_d^*(1-\delta)R_c^*\right)}{\kappa}\right], \tag{132}$$

$$\xi N^* = \alpha L^* + (1-\lambda)\beta[V(q^*) - \delta R^*] \tag{133}$$

Let us denote $C'(N^*) = H(N) := \alpha L^* + (1-\lambda)\beta[V(q^*) - \delta R^*] + \lambda\omega$, to show existence and uniqueness, we need to show that: (i) $C'(1) < H(1)$, (ii) $C'(N) > H(N)$ and (iii) $C'(\infty) > H(\infty)$.

Suppose the cost of holding N cards is quadratic plus affine:

$$C(N) = \frac{\xi}{2}(N-1)^2 + \eta(N-1) + C_0. \tag{134}$$

Let us now show that the conditions for existence and uniqueness are satisfied.

Define $F(N) := C'(N) - H(N)$. At $N = 1$, we have, $F(1) = \eta - H(1)$. Hence, a sufficient condition (i) is:

$$\boxed{\eta < H(1).} \tag{135}$$

WE also have $F'(N) = \xi - H'(N)$. The second condition is thus:

$$\boxed{\xi > \sup_{N \geq 1} H'(N).} \tag{136}$$

These two conditions are independent. The parameter η determines the level of

marginal cost at $N = 1$, while ξ determines the slope of marginal cost.

Finally, if $H(N)$ is bounded above as $N \to \infty$, then condition (iii) is satisfied:

$$C'(N) = \xi(N-1) + \eta \to +\infty, \tag{137}$$

so the upper crossing condition is automatically satisfied.

N Proof Result 2

The derivative of welfare is:

$$\partial_\sigma N[\alpha L - mc(L) - \kappa(q) - C'(N) + \partial_N(N\theta)V(q)] \tag{138}$$
$$+ \partial_\sigma L[\alpha - mc'(L)]N \tag{139}$$
$$+ \partial_\sigma q[\theta V' - \kappa'(q)]N \tag{140}$$

When $\sigma = 0$, we have $\alpha = mc'$ and $[\theta V' = \kappa'(q)]$. Using the borrower first order condition, we carn replace $\alpha L - mc(L) + \theta V - \kappa(q) - C'(N) + \partial_N(N\theta)V(q)$ by $(1-\lambda)R - mc(L) - \kappa(q)$.

When $\sigma = 0$, we have $d^* = 0$. Define

$$\theta(N) = 1 - \lambda + \frac{\lambda}{N}.$$

The equilibrium is then given by

$$\theta_c^* = \theta(N^*), \tag{141}$$
$$L_c^* = \frac{1}{\psi}\log\left(\frac{\alpha}{mc}\right), \tag{142}$$
$$q_c^* = \frac{1}{\phi}\log\left(\frac{\beta\gamma\theta(N^*)}{\kappa}\right), \tag{143}$$
$$R_c^* = \frac{\beta\gamma}{\phi} + \frac{\frac{\alpha - mc}{\psi} - \frac{\kappa}{\phi} + \varepsilon^{-1}}{\theta(N^*)}. \tag{144}$$

Finally, N^* solves

$$\xi N^* = \overbrace{\frac{\alpha}{\psi}\log\left(\frac{\alpha}{mc}\right)}^{lending} + (1-\lambda)\beta\Bigg[\overbrace{\frac{\gamma}{\phi}\log\left(\frac{\beta\gamma\theta(N^*)}{\kappa}\right)}^{Relationship\ benefits} \underbrace{-\frac{\delta\beta\gamma}{\phi} - \frac{\delta\left(\frac{\alpha-mc}{\psi} - \frac{\kappa}{\phi} + \varepsilon^{-1}\right)}{\theta(N^*)}\Bigg]}_{rate} + \lambda\omega. \tag{145}$$

Comparative statics with respect to σ around $\sigma = 0$. Let N_0 denote the equilibrium number of cards when $\sigma = 0$, and define

$$\theta_0 := \theta(N_0) = 1 - \lambda + \frac{\lambda}{N_0}. \tag{146}$$

It is useful to define

$$X := \frac{\alpha - mc}{\psi} - \frac{\kappa}{\phi} + \varepsilon^{-1}, \tag{147}$$

so that, when $\sigma = 0$ and hence $d = 0$,

$$R_0 = \frac{\beta\gamma}{\phi} + \frac{X}{\theta_0}. \tag{148}$$

Define also

$$K_0 := \frac{\alpha\delta}{\psi} - \frac{\beta\gamma\theta_0(1-\delta)}{\phi} + \delta\varepsilon^{-1}. \tag{149}$$

For a given N, the equilibrium interest rate can be written as

$$R = \frac{\theta\frac{\beta\gamma}{\phi} + X}{\theta + dK}, \tag{150}$$

Holding N fixed and differentiating with respect to d, evaluated at $d = 0$, gives

$$\left.\frac{\partial R}{\partial d}\right|_{d=0} = -\frac{R_0 K_0}{\theta_0}. \tag{151}$$

Similarly, since

$$L = \frac{1}{\psi}\log\left[\frac{\alpha(1 - d\delta R)}{mc}\right], \tag{152}$$

we obtain

$$\left.\frac{\partial L}{\partial d}\right|_{d=0} = -\frac{\delta R_0}{\psi}. \tag{153}$$

Moreover,

$$q = \frac{1}{\phi}\log\left[\frac{\beta\gamma\theta\left(1+d(1-\delta)R\right)}{\kappa}\right], \tag{154}$$

which implies

$$\left.\frac{\partial q}{\partial d}\right|_{d=0} = \frac{(1-\delta)R_0}{\phi}. \tag{155}$$

Recall that

$$H(N,\sigma) := \alpha L^*(N,\sigma) + (1-\lambda)\beta\left[\gamma q^*(N,\sigma) - \delta R^*(N,\sigma)\right] + \lambda\omega. \tag{156}$$

Therefore,

$$\begin{aligned}\left.\frac{\partial H}{\partial d}\right|_{d=0} &= \alpha\left(-\frac{\delta R_0}{\psi}\right) \\ &\quad + (1-\lambda)\beta\left[\frac{\gamma(1-\delta)R_0}{\phi} + \frac{\delta R_0 K_0}{\theta_0}\right].\end{aligned} \tag{157}$$

Factoring out R_0,

$$\left.\frac{\partial H}{\partial d}\right|_{d=0} = R_0\left\{-\frac{\alpha\delta}{\psi} + (1-\lambda)\beta\left[\frac{\gamma(1-\delta)}{\phi} + \frac{\delta K_0}{\theta_0}\right]\right\}. \tag{158}$$

Now,

$$d = \sigma\delta\left(1-\frac{1}{N}\right)\frac{\lambda/N}{\theta(N)}. \tag{159}$$

Hence, holding N fixed,

$$\left.\frac{\partial d}{\partial \sigma}\right|_{\sigma=0} = \delta\left(1-\frac{1}{N_0}\right)\frac{\lambda/N_0}{\theta_0}. \tag{160}$$

It follows that

$$\begin{aligned}H_\sigma(N_0,0) &= \delta\left(1-\frac{1}{N_0}\right)\frac{\lambda/N_0}{\theta_0}R_0 \\ &\quad \times\left\{-\frac{\alpha\delta}{\psi} + (1-\lambda)\beta\left[\frac{\gamma(1-\delta)}{\phi} + \frac{\delta K_0}{\theta_0}\right]\right\}.\end{aligned} \tag{161}$$

The equilibrium condition for the number of cards is

$$\xi N^* = H(N^*, \sigma). \tag{162}$$

Implicit differentiation with respect to σ gives

$$\xi \frac{dN^*}{d\sigma} = H_N \frac{dN^*}{d\sigma} + H_\sigma. \tag{163}$$

Therefore,

$$\boxed{\frac{dN^*}{d\sigma} = \frac{H_\sigma}{\xi - H_N}.} \tag{164}$$

Evaluated at $\sigma = 0$,

$$\boxed{\left.\frac{dN^*}{d\sigma}\right|_{\sigma=0} = \frac{\delta\left(1 - \frac{1}{N_0}\right)\frac{\lambda/N_0}{\theta_0} R_0 \left\{-\frac{\alpha\delta}{\psi} + (1-\lambda)\beta\left[\frac{\gamma(1-\delta)}{\phi} + \frac{\delta K_0}{\theta_0}\right]\right\}}{\xi - H_N(N_0, 0)}.} \tag{165}$$

The derivative $H_N(N, 0)$ can also be obtained explicitly. When $\sigma = 0$,

$$H(N, 0) = \text{constant} + (1-\lambda)\beta\left[\frac{\gamma}{\phi}\log\theta(N) - \frac{\delta X}{\theta(N)}\right]. \tag{166}$$

Since

$$\theta'(N) = -\frac{\lambda}{N^2}, \tag{167}$$

we have

$$\begin{aligned} H_N(N, 0) &= (1-\lambda)\beta\theta'(N)\left[\frac{\gamma}{\phi\theta(N)} + \frac{\delta X}{\theta(N)^2}\right] \\ &= -\frac{(1-\lambda)\beta\lambda}{N^2}\left[\frac{\gamma}{\phi\theta(N)} + \frac{\delta X}{\theta(N)^2}\right]. \end{aligned} \tag{168}$$

Under the local uniqueness condition

$$\xi - H_N(N_0, 0) > 0, \tag{169}$$

and assuming $N_0 > 1$ and $R_0 > 0$, the sign of the comparative static is therefore given by

$$\boxed{\text{sign}\left(\left.\frac{dN^*}{d\sigma}\right|_{\sigma=0}\right) = \text{sign}\left\{-\frac{\alpha\delta}{\psi} + (1-\lambda)\beta\left[\frac{\gamma(1-\delta)}{\phi} + \frac{\delta K_0}{\theta_0}\right]\right\}.} \tag{170}$$

Hence,

$$\left.\frac{dN^*}{d\sigma}\right|_{\sigma=0} > 0 \tag{171}$$

if and only if

$$\boxed{(1-\lambda)\beta\left[\frac{\gamma(1-\delta)}{\phi}+\frac{\delta}{\theta_0}\left(\frac{\alpha\delta}{\psi}-\frac{\beta\gamma\theta_0(1-\delta)}{\phi}+\delta\varepsilon^{-1}\right)\right] > \frac{\alpha\delta}{\psi}.} \tag{172}$$

The comparative static reflects three effects:

$$\underbrace{-\frac{\alpha\delta}{\psi}}_{\text{lower credit limit}} + \underbrace{(1-\lambda)\beta\frac{\gamma(1-\delta)}{\phi}}_{\text{higher quality}} + \underbrace{(1-\lambda)\beta\frac{\delta K_0}{\theta_0}}_{\text{lower repayment}} \tag{173}$$

The first effect reduces the incentive to hold additional cards, while the last two increase it when $K_0 > 0$. Thus, the sign of $dN^*/d\sigma$ is in general ambiguous.

N1 Closed form

When $\lambda = 1$, borrowers mainly care about loan size, as they know they will keep only one card. In that case, we have a closed-form solution.

$$\xi N^* = \frac{\alpha}{\psi}\log\left(\frac{\alpha}{mc}\right) + \omega. \tag{174}$$

When $\lambda = 1$, the model has a closed-form solution when δ is small, and each lender is infinitesimal. To get intuition about how the parameters affect contract terms, let us approximate the model around δ close to zero. We get that:

$$\theta_c^* = \frac{1}{N^*}, \tag{175}$$

$$\frac{R_c^*}{N^*} = \frac{\alpha - mc}{\psi} + \frac{\gamma}{\phi N^*} - \frac{\kappa}{\phi} + \varepsilon^{-1} \tag{176}$$

$$L_c^* = \frac{1}{\psi}(\log\left[\frac{\alpha}{mc}\right] - d\delta R_c^*) \tag{177}$$

$$q_c^* = \frac{1}{\phi}\log\left[\frac{\gamma\left(1+d(1-\delta)R_c^*\right)}{\kappa N^*}\right] \tag{178}$$

$$\Psi N^* = \alpha L^* \tag{179}$$

The complete solution is

$$\boxed{\begin{aligned} N^* &= \frac{\alpha(\ell - d\delta G)}{\psi\Psi + \alpha d\delta A}, \\ \theta_c^* &= \frac{\psi\Psi + \alpha d\delta A}{\alpha(\ell - d\delta G)}, \\ L_c^* &= \frac{\Psi(\ell - d\delta G)}{\psi\Psi + \alpha d\delta A}, \\ R_c^* &= \frac{\alpha A\ell + \psi\Psi G}{\psi\Psi + \alpha d\delta A}, \\ q_c^* &= \frac{1}{\phi}\left\{\log\left[\frac{\gamma(\psi\Psi + \alpha d\delta A)}{\kappa\alpha(\ell - d\delta G)}\right] + d(1-\delta)\frac{\alpha A\ell + \psi\Psi G}{\psi\Psi + \alpha d\delta A}\right\}. \end{aligned}} \tag{180}$$

where

$$\boxed{\ell = \log\left(\frac{\alpha}{mc}\right), \qquad A = \frac{\alpha - mc}{\psi} - \frac{\kappa}{\phi} + \varepsilon^{-1}, \qquad G = \frac{\gamma}{\phi}.} \tag{181}$$

O Proof Result 3

Let

$$x \equiv \varepsilon^{-1}, \tag{182}$$

and define

$$\mathcal{U} = \frac{\alpha - mc}{\psi} + \frac{\beta\gamma\theta - \kappa}{\phi} + x, \tag{183}$$

$$\mathcal{D} = \theta + \frac{\alpha d\delta}{\psi} - \frac{\beta\gamma\theta d(1-\delta)}{\phi} + d\delta x. \tag{184}$$

Then,

$$R = \frac{\mathcal{U}}{\mathcal{D}}. \tag{185}$$

We first differentiate conditional on N. Since θ and d only depend on N,

$$\left.\frac{\partial\theta}{\partial x}\right|_N = \left.\frac{\partial d}{\partial x}\right|_N = 0. \tag{186}$$

Moreover,

$$\frac{\partial \mathcal{U}}{\partial x} = 1, \tag{187}$$

$$\frac{\partial \mathcal{D}}{\partial x} = d\delta. \tag{188}$$

Hence,

$$\left.\frac{\partial R}{\partial x}\right|_N = \frac{\mathcal{D} - d\delta\mathcal{U}}{\mathcal{D}^2}. \tag{189}$$

Since $\mathcal{U} = R\mathcal{D}$,

$$\boxed{\left.\frac{\partial R}{\partial \varepsilon^{-1}}\right|_N = \frac{1 - d\delta R}{\mathcal{D}}.} \tag{190}$$

Thus, provided that the equilibrium satisfies $1 - d\delta R > 0$,

$$\left.\frac{\partial R}{\partial \varepsilon^{-1}}\right|_N > 0. \tag{191}$$

For L,

$$L = \frac{1}{\psi} \log\left[\frac{\alpha(1 - d\delta R)}{mc}\right]. \tag{192}$$

Therefore,

$$\left.\frac{\partial L}{\partial x}\right|_N = -\frac{d\delta}{\psi}\frac{R_x}{1 - d\delta R}. \tag{193}$$

Substituting the expression for R_x gives

$$\boxed{\left.\frac{\partial L}{\partial \varepsilon^{-1}}\right|_N = -\frac{d\delta}{\psi\mathcal{D}} < 0.} \tag{194}$$

For q,

$$q = \frac{1}{\phi} \log\left[\frac{\beta\gamma\theta\left(1 + d(1-\delta)R\right)}{\kappa}\right]. \tag{195}$$

Hence,

$$\boxed{\left.\frac{\partial q}{\partial \varepsilon^{-1}}\right|_N = \frac{d(1-\delta)(1 - d\delta R)}{\phi\mathcal{D}\left[1 + d(1-\delta)R\right]} > 0.} \tag{196}$$

Thus, conditional on N,

$$\boxed{R_{\varepsilon^{-1}} > 0, \qquad L_{\varepsilon^{-1}} < 0, \qquad q_{\varepsilon^{-1}} > 0.} \tag{197}$$

Effect on N^*

Define

$$H(N, x) = \alpha L(N, x) + (1 - \lambda)\beta \left[V(q(N, x)) - \delta R(N, x)\right] + \lambda\omega. \tag{198}$$

The equilibrium condition is

$$\xi N = H(N, x). \tag{199}$$

Differentiating with respect to x,

$$\xi \frac{dN}{dx} = H_N \frac{dN}{dx} + H_x. \tag{200}$$

Hence,

$$\boxed{\frac{dN^*}{d\varepsilon^{-1}} = \frac{H_{\varepsilon^{-1}}}{\xi - H_N}.} \tag{201}$$

Under the uniqueness condition

$$\xi > H_N, \tag{202}$$

the denominator is positive. Therefore, the sign of $dN^*/d\varepsilon^{-1}$ is entirely determined by the direct effect $H_{\varepsilon^{-1}}$.

If

$$V(q) = \gamma q, \tag{203}$$

then

$$H_x = \alpha L_x + (1 - \lambda)\beta \left[\gamma q_x - \delta R_x\right]. \tag{204}$$

Substituting the expressions above gives

$$\boxed{H_{\varepsilon^{-1}} = \frac{1}{\mathcal{D}} \left\{ -\frac{\alpha d\delta}{\psi} + (1 - \lambda)\beta(1 - d\delta R) \left[\frac{\gamma d(1 - \delta)}{\phi \left[1 + d(1 - \delta)R\right]} - \delta \right] \right\}.} \tag{205}$$

Therefore,

$$\boxed{\frac{dN^*}{d\varepsilon^{-1}} = \frac{-\dfrac{\alpha d\delta}{\psi} + (1-\lambda)\beta(1-d\delta R)\left[\dfrac{\gamma d(1-\delta)}{\phi\left[1+d(1-\delta)R\right]} - \delta\right]}{\mathcal{D}\left(\xi - H_N\right)}.} \tag{206}$$

A simple sufficient condition for

$$\frac{dN^*}{d\varepsilon^{-1}} < 0 \tag{207}$$

is

$$\boxed{\frac{\gamma d(1-\delta)}{\phi\left[1+d(1-\delta)R\right]} \leq \delta.} \tag{208}$$

Intuitively, increasing ε^{-1} directly raises R, lowers L, and raises q. The effect on the number of cards is negative if the reduction in credit limits and the higher repayment burden dominate the increase in quality.

Finally, once N^* adjusts, the total derivatives are

$$\boxed{\frac{dR^*}{dx} = R_x + R_N\frac{dN^*}{dx}, \qquad \frac{dL^*}{dx} = L_x + L_N\frac{dN^*}{dx}, \qquad \frac{dq^*}{dx} = q_x + q_N\frac{dN^*}{dx}.} \tag{209}$$

Similarly,

$$\boxed{\frac{d\theta^*}{dx} = -\frac{\lambda}{(N^*)^2}\frac{dN^*}{dx}.} \tag{210}$$

Thus, if N^* falls with ε^{-1}, then θ^* necessarily rises.

P Proof 4

Holding N constant, θ and ε_d are fixed. Define

$$\mathcal{D} := \theta + \frac{\alpha\varepsilon_d\delta}{\psi} - \frac{\beta\gamma\theta\varepsilon_d(1-\delta)}{\phi} + \varepsilon_d\delta\varepsilon^{-1}. \tag{211}$$

Under the cost parametrization

$$c(L; mc) = \frac{mc}{\psi}\left(e^{\psi L} + a\right), \tag{212}$$

the equilibrium face value is

$$R^* = \frac{\dfrac{\alpha + a\,mc}{\psi} + \dfrac{\beta\gamma\theta - \kappa}{\phi} + \varepsilon^{-1}}{\mathcal{D}}. \tag{213}$$

Therefore,

$$\boxed{\left.\frac{\partial R^*}{\partial mc}\right|_N = \frac{a}{\psi\mathcal{D}}.} \tag{214}$$

Hence,

$$\operatorname{sign}\left(\left.\frac{\partial R^*}{\partial mc}\right|_N\right) = \operatorname{sign}(a). \tag{215}$$

The equilibrium loan size is

$$L^* = \frac{1}{\psi}\log\left[\frac{\alpha\left(1 - \varepsilon_d\delta R^*\right)}{mc}\right]. \tag{216}$$

Differentiating with respect to mc while holding N fixed gives

$$\begin{aligned}\left.\frac{\partial L^*}{\partial mc}\right|_N &= \frac{1}{\psi}\left[-\frac{1}{mc} - \frac{\varepsilon_d\delta}{1 - \varepsilon_d\delta R^*}\left.\frac{\partial R^*}{\partial mc}\right|_N\right] \\ &= -\frac{1}{\psi mc} - \frac{a\varepsilon_d\delta}{\psi^2\mathcal{D}\left(1 - \varepsilon_d\delta R^*\right)}.\end{aligned} \tag{217}$$

Thus,

$$\boxed{\left.\frac{\partial L^*}{\partial mc}\right|_N = -\frac{1}{\psi mc} - \frac{a\varepsilon_d\delta}{\psi^2\mathcal{D}\left(1 - \varepsilon_d\delta R^*\right)}.} \tag{218}$$

In particular, if $a \geq 0$,

$$\boxed{\left.\frac{\partial L^*}{\partial mc}\right|_N < 0.} \tag{219}$$

Finally, the equilibrium relationship benefit is

$$q^* = \frac{1}{\phi}\log\left[\frac{\beta\gamma\theta\left(1 + \varepsilon_d(1-\delta)R^*\right)}{\kappa}\right]. \tag{220}$$

Therefore,

$$\left.\frac{\partial q^*}{\partial mc}\right|_N = \frac{1}{\phi}\frac{\varepsilon_d(1-\delta)}{1+\varepsilon_d(1-\delta)R^*}\left.\frac{\partial R^*}{\partial mc}\right|_N = \frac{a\varepsilon_d(1-\delta)}{\phi\psi\mathcal{D}\left[1+\varepsilon_d(1-\delta)R^*\right]}. \tag{221}$$

Hence,

$$\boxed{\left.\frac{\partial q^*}{\partial mc}\right|_N = \frac{a\varepsilon_d(1-\delta)}{\phi\psi\mathcal{D}\left[1+\varepsilon_d(1-\delta)R^*\right]}.} \tag{222}$$

Therefore, for $0 \leq \delta < 1$,

$$\boxed{\operatorname{sign}\left(\left.\frac{\partial q^*}{\partial mc}\right|_N\right) = \operatorname{sign}(a).} \tag{223}$$

Thus, conditional on N,

$$a > 0 \implies R^*_{mc} > 0, \quad L^*_{mc} < 0, \quad q^*_{mc} > 0, \tag{224}$$

$$a = 0 \implies R^*_{mc} = 0, \quad L^*_{mc} < 0, \quad q^*_{mc} = 0. \tag{225}$$

Q Calculation of ψ and Π

Our sufficient statistic requires estimates of the costs and profits associated with credit cards at equilibrium.

We use accounting data to recover those costs. The following paragraphs describe the steps we use to calculate the per-period costs ψ and expected profits Π.

Q1 Calibration of Credit-Card Revenues and Costs in Brazil

We calibrate the revenues and costs associated with a credit-card relationship using the financial statements of Banco CSF (Banco Carrefour). Banco CSF is particularly useful for this exercise because its business is concentrated in consumer credit and credit cards. In 2024, Banco Carrefour reported more than six million customers across the Carrefour, Atacadão, and Sam's Club cards (Grupo Carrefour Brasil, 2024). We therefore set the number of card relationships to

$$N^* = 6.0 \text{ million}. \tag{226}$$

Strictly speaking, this is a number of customers rather than physical cards. Hence, the resulting estimates should be interpreted as costs and revenues per card relationship.

Our model separates the cost of supplying credit from the fixed cost of maintaining an additional card relationship:

$$C(L, q, N) = \psi(L, q) + F(N). \tag{227}$$

We map customer acquisition, marketing, and card issuance into $F(N)$, while funding, credit losses, processing, servicing, personnel, network fees, and other operating expenses are included in $\psi(L, q)$.

Q1.1 Revenue per card

Banco CSF reports, under IFRS, total interest and similar income of

$$R\$3.565 \text{ billion} \tag{228}$$

in 2024 and service revenues of

$$R\$2.156 \text{ billion}. \tag{229}$$

Thus, total revenues were

$$\mathcal{R}^* = 3.565 + 2.156 = R\$5.721 \text{ billion}. \tag{230}$$

Dividing by six million customers gives

$$\boxed{\frac{\mathcal{R}^*}{N^*} \simeq R\$954 \text{ per card relationship per year}.} \tag{231}$$

The revenue decomposition is informative. In 2024, Banco CSF reported

$$\text{interest income on loans} = R\$3.427 \text{ billion}, \tag{232}$$

$$\text{interchange income} = R\$0.821 \text{ billion}, \tag{233}$$

$$\text{bank fees} = R\$0.709 \text{ billion}. \tag{234}$$

The bank states that bank fees consist almost entirely of annual card fees: R\$707.7 million out of R\$708.7 million in 2024. Banco CSF also earned R\$189.0 million in commissions on purchases made using its cards in Carrefour-group stores and R\$118.3 million in SMS package fees (Banco CSF, 2026a).

A narrower measure containing only revenues that can be directly attributed to the

card relationship is therefore

$$\begin{aligned}\mathcal{R}^{card} &= 3.427 + 0.821 + 0.709 + 0.189 + 0.118 \\ &\simeq R\$5.264 \text{ billion},\end{aligned} \tag{235}$$

or approximately

$$\boxed{\frac{\mathcal{R}^{card}}{N^*} \simeq R\$877 \text{ per card relationship per year.}} \tag{236}$$

The difference between R$954 and R$877 primarily reflects insurance commissions and income on securities and liquidity holdings. In the baseline accounting-profit calculation below, we use total revenues because this allows the revenue and cost measures to reconcile exactly with Banco CSF's reported profits. The narrower R$877 measure is useful when mapping revenues more directly into the structural model.

Effective interest return on the credit limit. The contractual interest rate on revolving card debt substantially overstates the expected interest revenue generated by one real of credit limit. A large fraction of credit limits is unused, while another substantial fraction finances purchases that do not bear interest, including purchases made through *parcelado sem juros*.

Banco CSF explicitly distinguishes interest-bearing loans from non-interest-bearing payment receivables. The latter include purchases between the transaction and billing dates as well as interest-free installment purchases (Banco CSF, 2026a).

At year-end 2024, Banco CSF had

$$\text{interest-bearing loans} = R\$14.665 \text{ billion}, \tag{237}$$

$$\text{non-interest-bearing payment receivables} = R\$13.710 \text{ billion}, \tag{238}$$

$$\text{unused credit limits} \simeq R\$15.987 \text{ billion}. \tag{239}$$

Hence, total committed credit was approximately

$$L_{2024}^{commit} \simeq R\$44.36 \text{ billion}. \tag{240}$$

The corresponding quantities in 2025 were approximately

$$\text{interest-bearing loans} = R\$18.141 \text{ billion}, \tag{241}$$

$$\text{non-interest-bearing payment receivables} = R\$14.390 \text{ billion}, \tag{242}$$

$$\text{unused credit limits} \simeq R\$17.701 \text{ billion}, \tag{243}$$

giving

$$L_{2025}^{commit} \simeq R\$50.23 \text{ billion}. \tag{244}$$

Using the average of the 2024 and 2025 stocks gives

$$\overline{L}^{commit} \simeq R\$47.30 \text{ billion}. \tag{245}$$

Banco CSF earned R$4.650 billion in loan interest income during 2025. Consequently, the realized interest income per real of committed credit limit is approximately

$$\boxed{r_L = \frac{4.650}{47.30} \simeq 9.8\%.} \tag{246}$$

This measure automatically incorporates the probability that a credit line is unused and the fact that a large fraction of card balances does not bear interest.

Conditional on carrying an interest-bearing balance, the average yield is substantially larger. Average interest-bearing balances between 2024 and 2025 were approximately R$16.40 billion, implying

$$\frac{4.650}{16.40} \simeq 28.3\%. \tag{247}$$

Thus, roughly speaking,

$$9.8\% \simeq 34.7\% \times 28.3\%, \tag{248}$$

where 34.7% is the ratio of interest-bearing balances to total committed limits.

Q1.2 Relationship cost $F(N)$

We interpret $F(N)$ as the cost associated with creating and maintaining additional card relationships rather than the cost generated by the amount of credit actually extended. We therefore include marketing, customer acquisition, and card issuance in $F(N)$.

Banco CSF reported advertising and marketing expenses of

$$R\$75.85 \text{ million} \tag{249}$$

in 2024 (Banco CSF, 2026a).

Banco CSF also reports prepaid expenses associated substantially with the acquisition of new customers and the issuance of new cards. The bank states that these acquisition costs are amortized according to the expected life of the product and the observed activation rate of physical cards. At the end of 2024, the current portion of prepaid expenses was

$$R\$114.92 \text{ million}. \tag{250}$$

We use the current portion as a proxy for the annual flow of customer-acquisition and card-issuance costs. This requires a steady-state assumption: the amount of acquisition costs expected to be amortized over the next twelve months is taken to approximate annual acquisition and issuance expenditure.

The resulting annual relationship cost is

$$\begin{aligned} F(N^*) &\simeq 75.85 + 114.92 \\ &= R\$190.77 \text{ million.} \end{aligned} \tag{251}$$

Therefore,

$$\boxed{\frac{F(N^*)}{N^*} \simeq R\$31.8 \text{ per card relationship per year.}} \tag{252}$$

A convenient linear specification around the observed equilibrium is consequently

$$\boxed{F(N) \simeq 32N,} \tag{253}$$

when N is measured in card relationships and costs are measured in reais per year.

This construction should be regarded as an approximation. In particular, the prepaid-expense balance is a balance-sheet stock rather than a separately disclosed annual expense. However, Banco CSF states that this balance consists substantially of expenses related to acquiring new customers and issuing new cards, making it a useful empirical proxy for $F(N)$.

Q1.3 Cost of supplying credit, $\psi(L, q)$

Rather than attempting to allocate every operating expense individually to card contracts, we construct $\psi(L^*, q^*)$ as the residual cost of operating the credit business after removing the relationship cost $F(N^*)$.

Banco CSF reported 2024 pre-tax income of

$$\Pi_{\text{pre-tax}} = R\$578.43 \text{ million.} \tag{254}$$

Since total revenues were R$5.721 billion, total pre-tax accounting costs were

$$C^* = 5.721 - 0.578 = R\$5.143 \text{ billion.} \tag{255}$$

Therefore,

$$\psi(L^*, q^*) = C^* - F(N^*) \simeq 5.143 - 0.191 = R\$4.952 \text{ billion.} \tag{256}$$

Dividing by the number of relationships gives

$$\boxed{\frac{\psi(L^*, q^*)}{N^*} \simeq R\$825 \text{ per card relationship per year.}} \tag{257}$$

This residual can be partially decomposed using reported accounting quantities.

Funding costs. Banco CSF reported interest expenses of

$$C^{fund} = R\$377.70 \text{ million} \tag{258}$$

in 2024. These expenses include interest on financial bills, time deposits, interbank deposits, and repurchase agreements (Banco CSF, 2026a). Per card relationship,

$$\boxed{\frac{C^{fund}}{N^*} \simeq R\$63.} \tag{259}$$

Credit-loss costs. Banco CSF reported expected credit-loss expenses of

$$C^{loss} = R\$2.464 \text{ billion} \tag{260}$$

in 2024 (Banco CSF, 2026a). These are expenses associated with the expected non-repayment of credit rather than regulatory capital requirements.

Relative to six million customers,

$$\boxed{\frac{C^{loss}}{N^*} \simeq R\$411.} \tag{261}$$

The gross credit portfolio at year-end 2024 was R$28.376 billion, so credit-loss expenses were approximately

$$\frac{2.464}{28.376} \simeq \boxed{8.7\%} \tag{262}$$

of the gross credit portfolio.

We use 2024 rather than 2025 for this calculation because Brazil implemented the expected-credit-loss framework of CMN Resolution No. 4,966 beginning in 2025, generating a break in the accounting treatment of provisions (Conselho Monetário Nacional, 2021).

Other operating costs. The remaining component is

$$\begin{aligned} C^{oper} &= \psi(L^*, q^*) - C^{fund} - C^{loss} \\ &\simeq 4.952 - 0.378 - 2.464 \\ &\simeq R\$2.110 \text{ billion}. \end{aligned} \tag{263}$$

Thus,

$$\boxed{\frac{C^{oper}}{N^*} \simeq R\$352 \text{ per card relationship per year}.} \tag{264}$$

This residual includes card processing, personnel, IT, Visa and Mastercard fees, rewards, collection costs, communications, depreciation and amortization, fraud expenses, operating taxes, and other recurring expenses.

For example, in 2024 Banco CSF reported R$362.3 million in data-processing expenses, which the bank explicitly identifies as expenses associated with processing credit-card transactions. It also reported R$133.9 million in Visa and Mastercard expenses, R$57.0 million in rewards, R$21.2 million in data-transmission expenses, and R$7.5 million in fraud expenses (Banco CSF, 2026a). These expenses are included in ψ rather than F, since they arise from operating the contract rather than simply establishing an additional card relationship.

Combining the components gives

$$\boxed{\frac{\psi(L^*, q^*)}{N^*} \simeq \underbrace{63}_{\text{funding}} + \underbrace{411}_{\text{credit losses}} + \underbrace{352}_{\text{other operating costs}} = R\$825.} \tag{265}$$

Q1.4 Regulatory capital

The accounting cost above does not include an opportunity cost associated with required equity capital. Regulatory capital is a stock rather than an accounting expense, so adding the entire capital requirement to annual costs would be incorrect.

Banco CSF reported credit-risk-weighted assets of

$$RWA^{credit} = R\$16.276 \text{ billion} \tag{266}$$

at the end of 2025. The bank also reports a minimum regulatory capital requirement, including active buffers, of 10.5% (Banco CSF, 2026b). This implies required capital against credit risk of approximately

$$K^{credit} = 0.105 \times 16.276 \simeq R\$1.709 \text{ billion}. \tag{267}$$

If the required return on equity exceeds the cost of debt funding by Δr_K, the annual

economic cost of regulatory capital is

$$C^K = K^{credit} \Delta r_K. \tag{268}$$

For a baseline equity-funding premium of 7.5 percentage points,

$$C^K = 1.709 \times 0.075 \simeq R\$128 \text{ million}. \tag{269}$$

Using approximately seven million customers in 2025 gives

$$\boxed{\frac{C^K}{N} \simeq R\$18 \text{ per relationship per year}.} \tag{270}$$

For an equity-funding premium between 5 and 10 percentage points, the corresponding annual capital cost is approximately R$12–24 per card relationship.

We do not include this imputed capital cost in the accounting-profit calculation below. If the objective is to measure economic rather than accounting profits, it should be added to ψ.

Q1.5 Implied profit per card

Using the accounting definitions above, annual revenue per relationship is

$$\frac{\mathcal{R}^*}{N^*} \simeq R\$954. \tag{271}$$

Total pre-tax costs are

$$\frac{\psi(L^*, q^*) + F(N^*)}{N^*} = 825 + 32 = R\$857. \tag{272}$$

Hence, pre-tax accounting profit is

$$\boxed{\frac{\Pi^*_{\text{pre-tax}}}{N^*} \simeq 954 - 825 - 32 = R\$96 \text{ per card relationship per year}.} \tag{273}$$

Banco CSF reported R$198.1 million in corporate income taxes in 2024 and net income of R$380.3 million. Dividing reported net income by six million customers gives

$$\boxed{\frac{\Pi^*_{\text{net}}}{N^*} = \frac{380.3}{6.0} \simeq R\$63 \text{ per card relationship per year}.} \tag{274}$$

Thus, our preferred accounting calibration can be summarized as

$$\boxed{\begin{aligned} \text{Revenue} &\simeq R\$954, \\ \psi(L^*, q^*) &\simeq R\$825, \\ F(N^*)/N^* &\simeq R\$32, \\ \text{Pre-tax profit} &\simeq R\$96, \\ \text{Net profit} &\simeq R\$63, \end{aligned}} \tag{275}$$

all expressed per card relationship per year.

If the imputed opportunity cost of regulatory capital is included, our baseline economic cost becomes

$$\psi^{econ}(L^*, q^*) \simeq 825 + 18 = R\$843 \tag{276}$$

per relationship, reducing pre-tax economic profit to approximately

$$\boxed{R\$78 \text{ per relationship per year.}} \tag{277}$$

Our baseline calibration, expressed entirely as annual rates per real of committed credit limit, is therefore

$$\boxed{\begin{aligned} \text{Net Revenue} &\simeq 12.90\%\, L, \\ \text{Funding cost} &\simeq 0.85\%\, L, \\ \text{Credit-loss cost} &\simeq 5.55\%\, L, \\ \text{Other operating costs} &\simeq 4.76\%\, L, \\ \psi(L, q) &\simeq L + 11.16\%\, L, \\ F(N^*) &\simeq 0.43\%\, L, \\ \psi(L, q) + F(N^*) &\simeq 11.59\%\, L, \\ \text{Pre-tax profit} &\simeq 1.30\%\, L, \\ \text{Net profit} &\simeq 0.86\%\, L. \end{aligned}} \tag{278}$$

If the opportunity cost of regulatory capital is included, the corresponding economic-cost calibration is

$$\boxed{\psi^{econ}(L, q) + F(N^*) \simeq 11.84\%\, L.} \tag{279}$$

Interpretation and assumptions. The calibration relies on five main assumptions. First, we approximate the number of card relationships by the approximately six million Banco Carrefour card customers reported in 2024. Second, the current portion of prepaid acquisition and card-issuance costs is treated as an annual flow under a

steady-state assumption. Third, marketing and acquisition/issuance costs are allocated to $F(N)$, while processing, servicing, funding, credit losses, and other recurring expenses are allocated to $\psi(L, q)$. Fourth, reported net income is used to validate the aggregate accounting decomposition, while the cost of regulatory equity capital is treated separately as an economic opportunity cost. Finally, the effective return on L is computed using committed limits rather than outstanding balances, thereby accounting for unused limits and interest-free card balances.

The Banco Central's study of the Brazilian payment-card industry provides independent support for this classification. It separates issuer expenses into credit losses, processing, marketing and sales, network fees, risk management, and other expenses. The latter category includes rewards, customer retention, customer service, card production and personalization, delivery, billing, personnel, and administrative expenses (Banco Central do Brasil, 2009). This accounting taxonomy is consistent with separating relationship-creation costs $F(N)$ from the recurring cost of supplying and operating credit contracts $\psi(L, q)$.

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R Calibration Theory

We discipline the quantitative model using moments from the Brazilian credit card market. The objective of the exercise is not to estimate all structural primitives independently. Instead, we combine directly observed empirical moments, externally cal-

ibrated parameters, normalizations, and equilibrium conditions to construct a quantitatively disciplined benchmark economy.

The symmetric equilibrium is characterized by

$$\theta_c^* = 1 - \lambda + \lambda \frac{1}{N^*}, \tag{280}$$

$$\theta_c^* R_c^* = \psi(L^*, q^*) + [1 - \varepsilon_d \delta R^*] \varepsilon^{-1}, \tag{281}$$

$$\alpha = \frac{mc'(L_c^*)}{1 - \varepsilon_d \delta R_c^*}, \tag{282}$$

$$\kappa'(q^*) = \beta V'(q^*) \left[\varepsilon_d (1 - \delta) \theta_c^* R_c^* + \theta_c^*\right], \tag{283}$$

$$C'(N^*) = \alpha L^* + (1 - \lambda)\beta \left[V(q^*) - \delta R^*\right]. \tag{284}$$

We use these conditions to map observable moments into the primitive parameters of the model.

S Baseline targets

Table A6 summarizes the baseline calibration.

Table A6: Baseline Calibration

Parameter / Moment	Symbol	Baseline	Discipline
Credit limit	L^*	5,000	Brazilian credit card data
Number of cards	N^*	2	Multi-card borrowers
Card survival probability	θ^*	0.91	Default frequency
Distress probability	λ	0.18	Implied by (280)
Discount factor	β	0.95	External calibration
Effective contractual return	R^*/L^*	1.129	Brazilian card revenues
Effective repayment	R^*	5,645	Implied
Recovery parameter	δ	0.80	External calibration
Cost elasticity	$\varepsilon_{mc',L}$	1.50	Benchmark curvature
Cost curvature	ϕ	0.874	Implied by elasticity target
Borrower utility weight	α	1	Normalization
Relationship quality	q^*	1	Normalization

Notes: The calibration combines empirical targets, externally calibrated parameters, normalizations, and parameters backed out from the equilibrium conditions. The effective contractual return is not the posted revolving APR. It should be interpreted as the expected annual return generated by a card relationship, incorporating usage, repayment, installment borrowing, revolving credit, fees, and other relevant revenues.

S1 Default and the distress probability

The survival probability of card c is

$$\theta_c^* = 1 - \lambda + \lambda \frac{1}{N^*}. \tag{285}$$

The first term corresponds to states in which the borrower repays all cards. With probability λ, the borrower enters a state in which only one card is repaid. Under symmetry, each card is repaid with probability $1/N^*$ in this state.

We set

$$N^* = 2. \tag{286}$$

This is a natural benchmark for borrowers exposed to non-exclusive lending. In the Brazilian data, multi-card borrowing is quantitatively important: a large fraction of cardholders have relationships with multiple issuers, and these borrowers account for the majority of outstanding credit card balances.

We target a survival probability of

$$\theta^* = 0.91, \tag{287}$$

corresponding approximately to a 9% probability of default.

With $N^* = 2$,

$$0.91 = 1 - \lambda + \frac{\lambda}{2} \tag{288}$$

$$= 1 - \frac{\lambda}{2}. \tag{289}$$

Therefore,

$$\boxed{\lambda = 0.18.} \tag{290}$$

Thus, in the baseline calibration, approximately 18% of borrowers experience the state in which they cannot repay all their cards.

This mapping should be interpreted as an approximation. The empirical default statistic concerns the probability that a borrower defaults on at least one card, whereas θ_c is a card-level survival probability. We therefore examine alternative default definitions and values of λ in robustness exercises.

S2 Credit limit

We normalize the equilibrium credit limit to

$$\boxed{L^* = 5{,}000.} \tag{291}$$

This value provides the scale around which we calibrate both the level and curvature of lending costs.

We parameterize marginal lending costs as

$$mc'(L) = A\left[\exp\left(\phi\frac{L}{5{,}000}\right) - 1\right]. \tag{292}$$

The parameter A determines the level of marginal costs, whereas ϕ determines their curvature.

The elasticity of marginal cost with respect to the credit limit is

$$\varepsilon_{mc',L}(L) = \frac{\partial \log mc'(L)}{\partial \log L}. \tag{293}$$

Using (292),

$$\varepsilon_{mc',L}(L) = \frac{\phi\frac{L}{5{,}000}\exp\left(\phi\frac{L}{5{,}000}\right)}{\exp\left(\phi\frac{L}{5{,}000}\right) - 1}. \tag{294}$$

At the equilibrium limit $L^* = 5{,}000$,

$$\varepsilon_{mc',L}(L^*) = \frac{\phi e^{\phi}}{e^{\phi} - 1}. \tag{295}$$

We choose a baseline elasticity of

$$\boxed{\varepsilon_{mc',L}(L^*) = 1.5.} \tag{296}$$

Solving

$$\frac{\phi e^{\phi}}{e^{\phi} - 1} = 1.5 \tag{297}$$

gives

$$\boxed{\phi \simeq 0.874.} \tag{298}$$

The elasticity of marginal lending costs is difficult to estimate directly. We therefore treat 1.5 as a benchmark and examine a broad range of values between approximately 1 and 3.

S3 Normalization of borrower utility and the level of costs

The first-order condition determining the credit limit is

$$\alpha = \frac{mc'(L^*)}{1 - \varepsilon_d \delta R^*}. \tag{299}$$

Since only the relative scale of borrower utility and lending costs matters, we nor-

malize

$$\boxed{\alpha = 1.} \tag{300}$$

The scale parameter A in the cost function is then backed out from the equilibrium condition:

$$mc'(5{,}000) = 1 - \varepsilon_d \delta R^*. \tag{301}$$

Using (292),

$$A = \frac{1 - \varepsilon_d \delta R^*}{e^{\phi} - 1}. \tag{302}$$

Hence the curvature of costs is calibrated externally through ϕ, while their level is chosen so that $L^* = 5{,}000$ satisfies the equilibrium condition.

S4 Effective contractual repayment

We set the effective annual contractual repayment to

$$R^* = (1 + 0.129) \times 5{,}000 = \boxed{5{,}645}. \tag{303}$$

Equivalently,

$$\frac{R^*}{L^*} = 1.129. \tag{304}$$

This object should not be interpreted as the posted revolving credit card APR. Revolving credit rates in Brazil are much larger. Instead, R^* represents the effective expected annual repayment or return associated with a card relationship.

It aggregates the different ways in which the credit limit generates revenues, including purchases paid in full, installment borrowing, revolving balances, fees, and other contractual payments. The calibration therefore targets an effective return rather than the contractual revolving APR.

S5 Informal seniority and the default semi-elasticity

A central parameter of the model is ε_d, which determines how repayment probabilities respond to differences in contractual terms and relationship quality.

Because the model is expressed in Brazilian reais, the raw value of this semi-elasticity can appear numerically small. A representative calibration is

$$\varepsilon_d = \frac{3.2}{0.9} \times 10^{-5} \simeq 3.56 \times 10^{-5}. \tag{305}$$

The economically relevant statistic is the dimensionless product

$$d^* \equiv \varepsilon_d \delta R^*. \tag{306}$$

For example, setting

$$\delta = 0.8 \tag{307}$$

and using $R^* = 5{,}645$ gives

$$d^* = (3.56 \times 10^{-5}) \times 0.8 \times 5{,}645 \tag{308}$$

$$\simeq 0.161. \tag{309}$$

Thus,

$$\boxed{1 - d^* \simeq 0.839.} \tag{310}$$

Although ε_d is small when expressed per real, it therefore generates an economically substantial wedge in the credit-limit first-order condition.

In the empirical implementation, we discipline ε_d using repayment priority across cards. In particular, differences in credit limits and relationship characteristics across issuers predict which card a borrower chooses to repay when she cannot repay all lenders. These repayment-priority moments provide direct empirical information about the strength of informal seniority.

S6 Relationship benefits

The relationship-quality first-order condition is

$$\beta V'(q^*) \left[\varepsilon_d (1-\delta)\theta^* R^* + \theta^*\right] = \kappa'(q^*). \tag{311}$$

We normalize

$$\boxed{q^* = 1.} \tag{312}$$

For the baseline, suppose

$$V(q) = q \tag{313}$$

and

$$\kappa(q) = \frac{k_q}{2} q^2. \tag{314}$$

Then

$$V'(q) = 1, \qquad \kappa'(q) = k_q q. \tag{315}$$

At $q^* = 1$,

$$k_q = \beta \left[\varepsilon_d (1-\delta)\theta^* R^* + \theta^*\right]. \tag{316}$$

Using

$$\beta = 0.95, \qquad \theta^* = 0.91, \qquad \delta = 0.8, \qquad R^* = 5{,}645, \qquad \varepsilon_d = 3.56 \times 10^{-5},$$

we obtain

$$k_q = 0.95\left[0.91 + (3.56 \times 10^{-5})(1-0.8)(0.91)(5{,}645)\right] \tag{317}$$

$$\simeq 0.899. \tag{318}$$

Thus, a convenient baseline specification is

$$\boxed{V(q) = q, \qquad \kappa(q) = \frac{0.899}{2}q^2.} \tag{319}$$

The curvature of $V(q)$ can be varied in robustness exercises without affecting the normalization $q^* = 1$.

S7 Number of cards and entry costs

We assume that the cost associated with maintaining multiple lending relationships is

$$C(N) = \frac{\xi}{2}(N-1)^2 + \eta(N-1) + C_0. \tag{320}$$

Hence,

$$C'(N) = \xi(N-1) + \eta. \tag{321}$$

The equilibrium entry condition is

$$\xi(N^* - 1) + \eta = \alpha L^* + (1-\lambda)\beta\left[V(q^*) - \delta R^*\right]. \tag{322}$$

We target

$$N^* = 2. \tag{323}$$

Rather than attempting to identify ξ and η separately from a single moment, we choose ξ to satisfy the sufficient condition for uniqueness of equilibrium and back out η from (322):

$$\boxed{\eta = \alpha L^* + (1-\lambda)\beta\left[V(q^*) - \delta R^*\right] - \xi(N^* - 1).} \tag{324}$$

Thus, ξ governs the curvature of relationship costs and is varied in robustness exercises, while η is chosen to ensure that the observed number of card relationships is an equilibrium outcome.

T Sufficient Statistic with Dynamics

Borrower.

Let

$$\theta_t \equiv \theta^s(L_{t+1}, R_t), \tag{325}$$

$$Q_t \equiv (1-\lambda) + \lambda\theta_t, \tag{326}$$

where θ_t is the probability that card c survives conditional on the borrower experiencing financial trouble, and Q_t is the unconditional probability that card c remains active.

The borrower's value in the normal state when N cards are chosen is:

$$V_t = \alpha \sum_{c=1}^{N} L_{c,t} + \beta \left[(1-\lambda) \left(V_{t+1} - \sum_{c=1}^{N} R_{c,t} \right) + \lambda \max_c \left\{ \underline{V}_{c,t+1} - R_{c,t} + v_{c,t} \right\} \right]. \tag{327}$$

Conditional on default, the borrower keeps only one card and never defaults again. The value associated with card c in this absorbing state satisfies

$$\underline{V}_{c,t} = \alpha L_{c,t} + \beta \left(\underline{V}_{c,t+1} - R_{c,t} \right). \tag{328}$$

Conditional on financial trouble, the borrower therefore chooses

$$\max_c \left\{ \underline{V}_{c,t+1} - R_{c,t} + v_{c,t} \right\}. \tag{329}$$

It is useful to define the value generated by one card conditional on remaining in the N-card state:

$$V^{s'} = \alpha L_0 + \sum_{t=1}^{\infty} (\beta(1-\lambda))^t [L_t - R_t] \tag{330}$$

To derive this, we assumed that the inclusive value does not depend on the number of cards:

$$M_N \equiv \mathbb{E} \left[\max_c \left\{ \underline{V} - R + v_c \right\} \right]. \tag{331}$$

This happens naturally when v_c are extreme value distributed and the shock are normalized by the number of cards like in the continuum of lender version of the model.

Lenders.

The profit of lender c satisfies

$$\Pi_t = -\psi(L_t) + \beta \left[(1-\lambda) \left(R_t + \Pi_{t+1} \right) + \lambda\theta_t \left(R_t + \underline{\Pi}_{t+1} \right) \right]. \tag{332}$$

Following borrower default, the lender is the only surviving lender. Its continuation

profit therefore satisfies

$$\underline{\Pi}_t = -\psi(L_t) + \beta\left(R_t + \underline{\Pi}_{t+1}\right). \tag{333}$$

Holding future contract terms fixed, the derivative with respect to the current credit limit, current repayment and future credit limit is:

$$\Pi_{L_t} = -\psi_L(L_t) \tag{334}$$
$$\Pi_{R_t} = \beta\left[Q_t + \lambda\theta_{R_t}\left(R_t + \underline{\Pi}_{t+1}\right)\right]. \tag{335}$$
$$\Pi_{L_{t+1}} = \beta\left[\lambda\theta_{L_{t+1}}\left(R_t + \underline{\Pi}_{t+1}\right) - Q_t\psi_L(L_{t+1})\right]. \tag{336}$$

Denoting by D the demand for the lender's contract, the lender's first-order conditions are

$$D_{L_t}\Pi_t + D\Pi_{L_t} = 0, \tag{337}$$
$$D_{L_{t+1}}\Pi_t + D\Pi_{L_{t+1}} = 0, \tag{338}$$
$$D_{R_t}\Pi_t + D\Pi_{R_t} = 0. \tag{339}$$

Social Planner.

Repayments are transfers between borrowers and lenders and therefore do not enter social welfare directly. The planner's value satisfies

$$W_t = N\left[\alpha L_t - \psi(L_t)\right] + \beta\left[(1-\lambda)W_{t+1} + \lambda\underline{W}_{t+1}\right]. \tag{340}$$

Following default, only one card remains, so that

$$\underline{W}_t = \alpha L_t - \psi(L_t) + \beta\underline{W}_{t+1}. \tag{341}$$

Let total welfare be

$$\mathcal{W} = W - C(N). \tag{342}$$

The planner's first-order condition with respect to the credit limit and with respect to the number of cards is

$$\alpha = \sum_{t=0}^{\infty}(\beta(1-\lambda))^t L_s - \psi_t = C'(N) \tag{343}$$

Sufficient-statistic weights.

Suppose that demand depends on the borrower's utility, $D = D(V)$. The relevant

derivatives of borrower utility are

$$V_{L_t} = \alpha, \tag{344}$$

$$V_{R_t} = -\beta Q_t, \tag{345}$$

$$V_{L_{t+1}} = \beta Q_t \alpha. \tag{346}$$

Define

$$S_t \equiv R_t + \underline{\Pi}_{t+1}. \tag{347}$$

Combining the lender's first-order conditions with respect to L_t and R_t gives

$$\frac{\psi_L(L_t)}{\alpha} = 1 + \frac{\lambda \theta_{R_t}}{Q_t} S_t. \tag{348}$$

Combining the lender's first-order conditions with respect to R_t and L_{t+1} gives

$$1 + \frac{\lambda \theta_{R_t}}{Q_t} S_t = \frac{\psi_L(L_{t+1})}{\alpha} + \frac{\lambda \theta_{L_{t+1}}}{\alpha Q_t} S_t. \tag{349}$$

$$\iff \frac{\psi_L(L_{t+1})}{\alpha} = 1 - [\frac{\lambda \theta_{L_{t+1}}}{\alpha Q_t} - \frac{\lambda \theta_{R_t}}{Q_t}] S_t \tag{350}$$

Thus, defining the sufficient-statistic weights as normalized wedges, we obtain

$$w_{L_t} = -\frac{\lambda \theta_{R_t}}{Q_t} \left(R_t + \underline{\Pi}_{t+1} \right), \tag{351}$$

$$w_{L_{t+s}} = -\frac{\lambda}{Q_{t+s}} \left(\frac{\theta_{L_{t+1}}}{\alpha} + \theta_{R_t} \right) \left(R_t + \underline{\Pi}_{t+1} \right). \tag{352}$$

$$w_N = \sum_{t=1}^{\infty} (\beta(1-\lambda))^t R_t - \sum_{t=0}^{\infty} (\beta(1-\lambda))^t \psi_t \tag{353}$$

$$= \frac{\beta(1-\lambda)R - \psi(L)}{1-\beta(1-\lambda)} \text{When } R_t \text{ and } \psi_t \text{ constant} \tag{354}$$

U Lending costs

We assume that the cost of extending a credit limit L is convex and takes the form

$$\psi(L) = \frac{mc}{\phi} \left(e^{\phi L} - 1 \right), \qquad mc > 0,\ \phi > 0. \tag{355}$$

When the lender's first-order condition is

$$\psi_L(L^*) = \alpha(1-d), \tag{356}$$

then

$$L^* = \frac{1}{\phi} \log \left(\frac{\alpha(1-d)}{mc} \right). \tag{357}$$

We discipline the level and curvature of ψ separately. Our baseline calibration implies annual variable costs of approximately 11.2% of the credit limit at the representative card. Normalizing the benchmark limit to $\bar{L} = \text{R\$}5,000$, we choose the scale parameter so that

$$\psi(\bar{L}) = 1.112\, \bar{L} = \text{R\$}5,560. \tag{358}$$

The curvature parameter is conveniently summarized by the elasticity of lending costs with respect to the credit limit,

$$\eta_\psi(L) \equiv \frac{L\psi_L(L)}{\psi(L)}. \tag{359}$$

Writing $\ell = L/\bar{L}$ and

$$\psi(\ell) = A\left(e^{\phi\ell} - 1\right), \tag{360}$$

the cost elasticity at the benchmark is

$$\eta_\psi(\bar{L}) = \frac{\phi e^{\phi}}{e^{\phi} - 1}. \tag{361}$$

Our baseline sets $\eta_\psi(\bar{L}) = 1.5$, which corresponds approximately to $\phi = 0.87$. Matching the level of costs then implies

$$\boxed{\psi(L) = 4,008.91 \left[\exp\left(0.87 \frac{L}{5,000} \right) - 1 \right].} \tag{362}$$

At the benchmark limit, this yields an annual cost of R\$5,560 and a marginal cost of approximately 16.8% per additional real of credit limit.

U1 Sufficient Statistic calcualtion

We have: $\beta = 0.95$, $\theta = 0.9$, $\lambda = 0.8$, $R^* = (1+12.9\%) \cdot L = 5,645)$, $\psi = (1+11.11\%) \cdot L = 5,555$ and $\Pi = 0.0086 \cdot L \cdot \frac{1}{1-\beta\theta} = 43 \cdot 8.4 = 361$.

Using those values, the weights are: $w_N = \frac{\beta(1-\lambda)R^* - \psi^*}{1-\beta\lambda} = 0.8 * 0.95 * 5,645 - 5,555 = -1,264.8$. $w_{L_0} = \frac{3.2}{0.9} \cdot 10^{-5}(5,645+361) \approx 0.2135$ and $w_{L_s} = -\frac{1.6-3.2}{0.9} \cdot 10^{-5} * (5,645+361) \approx 0.1068$, we thus have $\sum_1^\infty (\beta\theta)^t w_{L_s} = 0.95 * 0.9 * \frac{1}{1-0.95*0.9} * 0.1068 \approx 0.6295$.

Using the difference-in-differences estimates (10 per cent increase in cards, 100 reais

Table A7: First-Best Credit Limit for Different Cost Elasticities

Elasticity of ψ_L	ϕ	L^{FB}	Increase over 5,000
1.05	0.098	5,566.2	11.32%
1.10	0.194	5,537.9	10.76%
1.25	0.464	5,468.0	9.36%
1.50	**0.874**	**5,385.3**	**7.71%**
1.75	1.247	5,327.8	6.56%
2.00	1.594	5,285.5	5.71%
2.50	2.232	5,227.2	4.54%
3.00	2.821	5,188.9	3.78%

more revolved per card), we get that: $dW = 0.10 * (-1264) + 100 * 0.21 + 100 * 0.629 = -42.5$.